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2 Tensorial Permeability Microstructure Model Considering Crystallographic 3 Texture and Grain Size for Evaluation of Magnetic Anisotropy In 4 Polycrystalline Steels

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10 **ABSTRACT**

11 A finite element microstructure model with permeability tensors that considers crystallographic
12 texture and grain size based on magnetic domain theory has been developed for the evaluation
13 of magnetic anisotropy in polycrystalline steels. The model has proved capable of capturing
14 the crystallographic texture, the grain size and the vector induction effects on the effective
15 permeability behaviours for typical textures in steels. The predicted magnetic properties as a
16 function of the magnetic field direction enables a quantitative characterisation of the magnetic
17 anisotropy. The predicted effective permeability maps can serve as a visual indication of the
18 crystallographic texture from magnetic values. These features have been experimentally vali-
19 dated against a commercial grain oriented electrical steel featuring strong texture and magnetic
20 anisotropy.

21 **KEYWORDS**

22 Permeability tensor; Finite Element; Microstructure; Crystallographic texture; Steels

23 **1. Introduction**

24 Iron and steel crystal structures are magnetically anisotropic due to the alignment of mag-
25 netic dipoles in a crystal cell [1]. It has also been experimentally confirmed that the cube
26 edges ($\langle 100 \rangle$) and the cube diagonals ($\langle 111 \rangle$) are the easiest and the hardest directions of mag-

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27 netising respectively in iron [2] and silicon-iron [3] single crystals. This fundamental magnetic
28 anisotropy is inherited by each grain in polycrystalline steels. If grains are randomly orientated,
29 the anisotropy effect averages out and, as a result, the steels exhibit isotropic behaviours. If
30 there are preferred crystallographic orientations present, often referred to as crystallographic
31 texture, the overall average properties have a certain anisotropy associated with the texture.
32 This simple yet useful averaging approach has been applied to predict anisotropic mechanical
33 properties of polycrystalline materials, e.g., elastic modulus, based on the corresponding single-
34 crystal properties, with differences in specific approximations including Reuss [4], Voigt [5],
35 Hill [6] and finite element (FE) [7] models. In a similar manner Daniel et al [8] estimated the
36 scalar effective magnetic permeability of polycrystalline materials based on an empirical single
37 crystal anisotropy and effective medium approximations. L. Kestens [9] took a more basic and
38 simplified approach proposing an ‘A’ parameter, as opposed to a fundamental magnetic prop-
39 erty, that averages the minimum angle between the magnetisation (implicitly assumed to be
40 homogeneous) and the closest easy direction to characterise the so-called magnetic quality of a
41 given texture for non-oriented electrical steels.

42 Both Daniel’s and Kestens’ model overlooked some important aspects of the microstructure,
43 in particular, the morphology of individual grains as well as the microstructure as a whole,
44 which can also influence the magnetic flux behaviours and hence the effective permeability.
45 The models work well for uniform equiaxed single phase material but cannot be extended to
46 more complex microstructures. For example, alignment in the microstructure, especially second
47 phase, often occurs during steel processing, e.g., rolling [10], and sometimes is present in the
48 final product, e.g., in superduplex stainless steel (banded austenite and ferrite structures), dual
49 phase steels (banded ferrite and martensite) or hot rolled C-Mn grades (banded ferrite and
50 pearlite structures) and can also give rise to magnetic anisotropy. Zhou et al [11] predicted the
51 effective permeability for dual-phase steel microstructures represented by digitised and processed
52 (recognising different phases) real micrographs by FE modelling. Whilst the approach enables
53 studying the separate effects of aligned microstructures, phase balance, and more recently grain
54 size [12], their model does not consider the effect of crystallographic texture, which may give
55 misleading prediction and interpretation if textures also play a significant role on magnetic
56 properties in the measurement direction and on anisotropy.

57 There is an important implicit assumption in the scalar permeability models that the mag-
58 netic flux density B always parallels with the applied field H , which is only a valid approximation
59 at low and uniform fields. Some tensor permeability models [13–16] have been reported to be

60 able to address this limitation, which also facilitates finite element modelling [17–19] to solve
61 problems that involve rotational fields and complex geometry. Nevertheless, tensor models re-
62 quire prior knowledge of the permeability for principal directions, along which B parallels with
63 H , to formulate a permeability tensor. Note the principal directions are not readily known or
64 necessarily exist in polycrystalline materials. Some models [13, 14] simply took two orthogonal
65 directions with maximum and minimum permeability as the principal directions and their val-
66 ues as the elements of a diagonal tensor [13]. This basic approach fails when the maximum and
67 the minimum permeability occur in non-orthogonal directions, e.g., in grain oriented electrical
68 steels (GOES). Others went the extra length to formulate a non-diagonal tensor and obtain
69 the principal directions and the corresponding permeability values by finding the eigenvalues
70 and eigenvectors of the tensor [15]. All these empirical permeability tensors can not predict the
71 anisotropy for given crystallographic textures but only deal with rotational fields, which could
72 be experimentally applied by a rotational single sheet tester [20, 21] or be present in electrical
73 steel components in motors, in the presence of known magnetic anisotropy. Some vector hys-
74 teresis models based on the Presiach model, e.g., [22], or the Jiles-Atherton model, e.g., [23, 24],
75 have also been reported to model anisotropy hysteresis behaviours associated with rotational
76 fields. Again, none of these hysteresis models can predict the magnetic anisotropy associated
77 with crystallographic textures.

78 There are no reports of a permeability tensor for a cubic single crystal that can fully de-
79 scribe the observed anisotropy and symmetry. According to the Neumann’s principle, the tensor
80 representing any physical property of a crystal should be invariant with regard to the symmetry
81 operation of the crystal class. In the case of cubic crystals such as electrical steel, the perme-
82 ability tensor that satisfies all the symmetries must reduce to a scalar [25]. It follows that the
83 corresponding magnetic properties should be isotropic, which would be inconsistent with ex-
84 periments [2, 3]. This paradox rendered the tensorial approach inapplicable as far as the cubic
85 crystallographic texture is concerned and thereby make people resort to empirical approaches
86 using scalar permeability e.g., [8]. The fundamental reason is that the magnetic structure of,
87 say, α -iron does not have all the symmetries of the crystal structure. Magnetic domains exist
88 in ferromagnetic materials; there are more than one direction of magnetic domains even in a
89 single crystal. Their magnetic structure has a lower symmetry than the crystal structure itself
90 does as illustrated in Fig. 1 due to the directionality of the magnetic spin. In this paper, we
91 propose a solution to this paradox by formulating the fundamental permeability tensor at the
92 magnetic domain level without violating the general Neumann’s principle and then extend it

93 to single crystals and then polycrystalline grains in turn. Thus, the aforementioned averaging
 94 approach based on single domain properties can be used to predict polycrystalline ones using
 95 the tensor approach.

96 We have developed a new FE model based on the permeability tensors incorporating both
 97 microstructure and crystallographic texture and hence enabling a more accurate and robust pre-
 98 diction of the anisotropic behaviours of effective permeability. Moreover, our model considers
 99 the crystallographic orientation of each individual grain, as opposed to statistics, i.e., orienta-
 100 tion distribution function (ODF) as usually seen in the literature e.g., [8], and hence is capable
 101 of capturing any local anisotropy (the effects of grain boundary misorientation on the electro-
 102 magnetic interactions between adjacent grains and/or spatial distribution of the specific crystal
 103 orientations) as well as global anisotropy (the effects of texture on the effective permeability
 104 anisotropy for the microstructure as a whole).

105 2. Model

106 2.1. Formulation of permeability tensors

107 Assume a cubic crystal is composed of a large number (N) and equal size of elementary magnetic
 108 domains that can only orientate along one of the magnetic easy directions, i.e., cubic edges or the
 109 $\langle 100 \rangle$ directions. The magnetic structure of the elementary domains orientated along direction 1
 110 is illustrated in Fig. 1. When a magnetic field h is applied along the direction 1, the induction of
 111 a consequential elementary domain along the direction 1 will be $\mathbf{B} = \mu_0 \mu_c h \hat{\mathbf{e}}_1$, where μ_0 is the
 112 permeability of free space; μ_c is the scale constant defined as relative elementary permeability,
 113 by analogy to the continuum counterpart, the relative permeability, for the elementary domain
 114 along direction 1; $\hat{\mathbf{e}}_1$ is the unit vector for direction 1. When h is applied along the other
 115 orthogonal directions, i.e., the direction 3 and 5, the induction will be $\mathbf{B} = \mu_0 h \hat{\mathbf{e}}_3$ and $\mathbf{B} =$
 116 $\mu_0 h \hat{\mathbf{e}}_5$ respectively. In other words, the direction 1, 3 and 5 are three principal directions along
 117 which relative elementary permeability values are μ_c , 1 and 1 respectively. Therefore, the relative
 118 elementary permeability tensor for direction 1 can be represented by

$$\boldsymbol{\mu}_1 = \begin{bmatrix} \mu_c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

119 The relative elementary permeability tensors for the other easy directions can be easily obtained
 120 by symmetry and orientation rotation:

$$\boldsymbol{\mu}_2 = \boldsymbol{\mu}_1, \quad \boldsymbol{\mu}_3 = \boldsymbol{\mu}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \mu_c & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{\mu}_5 = \boldsymbol{\mu}_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu_c \end{bmatrix} \quad (2)$$

121 where the subscript denotes the six easy directions as shown in Fig. 1.

122 The following assumptions, after Bozorth [26], are applied:

- 123 (1) When the crystal as a whole is not magnetised, all the domains orientate along the six
 124 easy directions by equal probability.
- 125 (2) When an external magnetic field is applied, the crystal is magnetised by re-distributing
 126 the numbers of the domains across the six directions represented by $\mathbb{N} = \{N_1, N_2, \dots, N_6\}$,
 127 which will be referred to as the domain configuration, favouring those closest to the ex-
 128 ternal field direction.
- 129 (3) The resulting magnetisation must have a component along the given field direction.

130 Heisenberg originally made the first two assumptions in 1930s, which have since become widely
 131 accepted as part of domain theory. Mathematically, the most probable domain configuration for
 132 a single crystal has already been solved by Bozorth [26]:

$$\begin{aligned} N_1 &= e^{\alpha+\beta\gamma_x} & N_3 &= e^{\alpha+\beta\gamma_y} & N_5 &= e^{\alpha+\beta\gamma_z} \\ N_2 &= e^{\alpha-\beta\gamma_x} & N_4 &= e^{\alpha-\beta\gamma_y} & N_6 &= e^{\alpha-\beta\gamma_z} \end{aligned} \quad (3)$$

133 where α and β can be determined from the following equations

$$\frac{\gamma_x \sinh(\gamma_x \beta) + \gamma_y \sinh(\gamma_y \beta) + \gamma_z \sinh(\gamma_z \beta)}{\cosh(\gamma_x \beta) + \cosh(\gamma_y \beta) + \cosh(\gamma_z \beta)} = \frac{B_h}{B_s} \quad (4)$$

$$2e^\alpha (\cosh(\gamma_x \beta) + \cosh(\gamma_y \beta) + \cosh(\gamma_z \beta)) = N \quad (5)$$

$$N = N_1 + N_2 + N_3 + N_4 + N_5 + N_6 \quad (6)$$

134 where B_h is the component of the induction \mathbf{B} along the applied field direction defined by the
 135 direction cosine $(\gamma_x, \gamma_y, \gamma_z)$ with respect to crystal direction 1, 3 and 5; B_s denotes the saturation
 136 induction. The effective permeability tensor for a single crystal with the domain configuration \mathbb{N}
 137 can be obtained by tensor addition as follows

$$\boldsymbol{\mu}_{sc} = \frac{1}{N} \sum_{i=1}^6 N_i \boldsymbol{\mu}_i \quad (7)$$

138 We now have formulated permeability tensors for an ideal single crystal in its own crystal
 139 reference frame. The relative permeability tensor for an arbitrary orientation with respect to
 140 the specimen reference frame, which is conventionally chosen to consist of the rolling direction
 141 (RD), transverse direction (TD) and normal direction (ND) as three axes, can be given as

$$\boldsymbol{\mu}_g = \mathbf{g}^{-1} \boldsymbol{\mu}_{sc} \mathbf{g} \quad (8)$$

142 where \mathbf{g} is the crystal orientation represented by an orientation matrix (refer to [27] for the
 143 definition of \mathbf{g} and more details on relevant crystallography). Now consider a grain in a poly-
 144 crystalline microstructure with orientation \mathbf{g} and grain diameter d . The grain size effect needs
 145 considering. Assume the elementary domains on grain boundaries are not orientated along any
 146 easy directions, N will decrease proportionally with the volume fraction of the grain boundaries
 147 given by $6t/d$, where t denotes the grain boundary thickness. One can correlate t with the mis-
 148 orientation of the grain boundaries to consider the local anisotropy. The domain configuration
 149 in the present model, as a statistical representation of domain directions, does not consider the
 150 locality and morphology of the domains within a grain. The closure domains that are expected
 151 to be present near grain boundaries can be considered as two groups of elementary domains:
 152 one that parallels with any of the easy directions and the other that does not. The effects of
 153 the former are already taken into account as presumably less favoured easy directions; those
 154 of the latter are considered not to contribute to the permeability tensor. For simplicity in the
 155 present paper we consider the overall effects of the loss of the unparallel elementary domains
 156 by modifying each element in the domain configuration per unit volume in the polycrystalline
 157 grain as follows

$$N'_i = N_i \left(1 - \frac{c_g}{d}\right) \quad (9)$$

158 where c_g is a material parameter that can be measured experimentally. Note N'_i reduces to
 159 N_i when d approaches infinity, which is equivalent to a stand-alone single crystal. Combining
 160 Equations (7), (8) and (9) one obtains the permeability tensor for the grains in polycrystalline
 161 microstructures as a function of the crystallographic orientation, grain size and the domain
 162 configuration:

$$\boldsymbol{\mu}'_g = \mathbf{g}^{-1} \left[\frac{1}{N} \sum_{i=1}^6 N_i \left(1 - \frac{c_g}{d}\right) \boldsymbol{\mu}_i \right] \mathbf{g} \quad (10)$$

163 2.2. Finite element microstructure model

164 A FE microstructure model based on the above permeability tensors was developed in MATLAB.
 165 The model considers a magnetostatics problem that involves a uniform static field applied to
 166 the microstructure. Substituting the constitutive equation

$$\mathbf{B} = \mu_0 \boldsymbol{\mu}_r \mathbf{H} \quad (11)$$

167 into the Maxwell's equations for magnetostatics and choosing the Columb gauge condition,
 168 $\nabla \cdot \mathbf{A} = 0$, one obtains the governing partial differential equation

$$-\nabla \cdot \left(\frac{1}{\mu_0 \boldsymbol{\mu}_r} \nabla \mathbf{A} \right) = J \quad (12)$$

169 where \mathbf{A} is the vector potential, J the external current density, $\boldsymbol{\mu}_r$ denotes the relative perme-
 170 ability for the materials, which would be 1 for air and $\boldsymbol{\mu}'_g$ for the microstructure. To simulate
 171 uniform applied fields a Dirichlet boundary condition of uniform magnetic flux density, B_b , is
 172 applied to the model. The vector potential \mathbf{A} can be broken down into two parts as

$$\mathbf{A} = \mathbf{A}_r + \mathbf{A}_b \quad (13)$$

173 where \mathbf{A}_r is the reduced vector potential and \mathbf{A}_b denotes the vector potential that satisfies

$$\mathbf{B}_b = \nabla \times \mathbf{A}_b \quad (14)$$

174 One solution of A_b can be given as

$$\mathbf{A}_b = \begin{bmatrix} -0.5yB_{bz} \\ 0.5xB_{bz} \\ yB_{bx} - xB_{by} \end{bmatrix} \quad (15)$$

175 where x and y are the coordinates; B_{bx} , B_{by} and B_{bz} are the three components of \mathbf{B}_b . No
 176 external current density is applied, i.e., $J = 0$.

177 The geometry of the model consists of the microstructure and a surrounding circular region of
 178 air as shown in Fig. 2. The diameter of the air region is set to five times the maximum dimension
 179 of the micrograph. The microstructure is composed of a number of entities representing the
 180 grains each drawn as a polygon rather than a single-entity micrograph or digital image. The
 181 model accepts either virtual microstructures together with simulated crystallographic texture
 182 data or measured Electron Backscatter Diffraction (EBSD) data. A boundary condition of

$$\mathbf{n} \times \mathbf{A} = \mathbf{n} \times \mathbf{A}_b \quad (16)$$

183 is applied to the outer edge of the air region, Γ , which is considered to be far away from the
 184 microstructure to simulate the external magnetic flux density, where \mathbf{n} denotes the unit normal
 185 vector.

186 It is important to note that $\boldsymbol{\mu}'_g$ is not a constant tensor but dependent on the induction \mathbf{B}
 187 and hence the FE solution, \mathbf{A} . It follows that the FE model is non-linear and hence tends to
 188 be complex and computationally costly to solve. For simplicity and computational efficiency,
 189 we recursively solve the average B across the whole microstructure, as opposed to at all nodes,
 190 at each iteration step, as illustrated in the flow chart, Figure 3. The *Patternsearch* algorithm
 191 in the MATLAB Global Optimization Toolbox, is used and the model usually converges to a
 192 very small residual ($< 0.0001B_s$), typically within 20 iterations. In each *Patternsearch* iteration,
 193 the permeability tensor for each grain, $\boldsymbol{\mu}'_g$, is calculated for the current guess on the B_h value,
 194 referred to as B_{hs} . Now that $\boldsymbol{\mu}'_g$ is known the FE model is linear. The model is solved using
 195 MATLAB's Partial Differential Equation (PDE) Toolbox. From the model solution the B_h
 196 value for the microstructure, B_{hm} , is then calculated. The cost function for the *Patternsearch*
 197 optimisation is $F(B_{hs}) = |B_{hm} - B_{hs}|$. Adjust B_{hs} according to the *Patternsearch* algorithm
 198 and repeat. The solution of the linear FE model at the end of the optimisation process is taken,
 199 at a first approximation, as the solution to the non-linear problem.

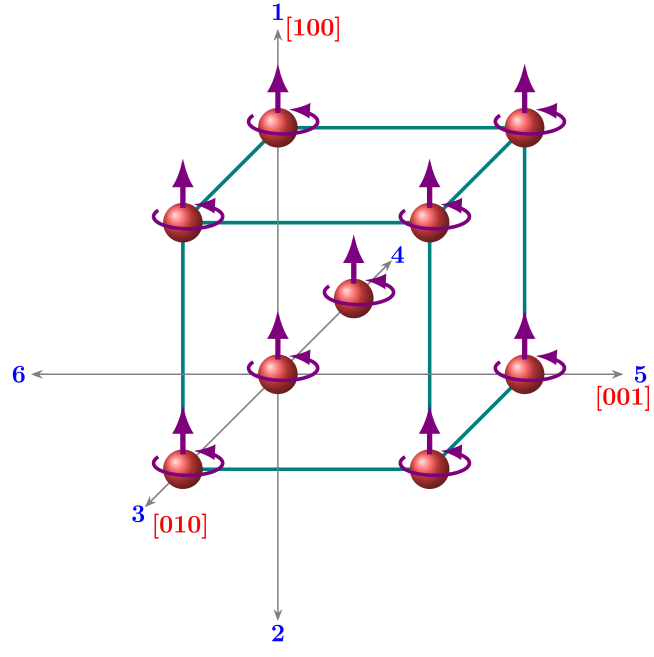


Figure 1. Schematic of magnetic structure of α -iron showing the magnetic moment directions of each atom in a crystal cell in its crystal reference frame represented by [100], [010] and [001] directions. The six magnetic easy directions are marked as numbers.

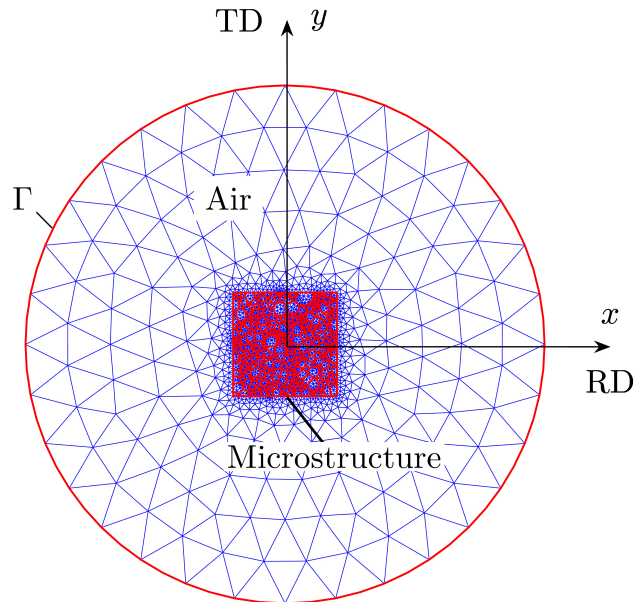


Figure 2. Geometry and meshing of finite element microstructure model for a 1 mm \times 1 mm virtual microstructure consisting of 500 grains. The mesh is generated by MATLAB Partial Differential Equation Toolbox.

200 The weak form of the governing equation for the above linear FE model in a solvable form
 201 by the PDE Toolbox is

$$-\nabla \cdot (\mathbf{c}\nabla\mathbf{A}) + \mathbf{a}\mathbf{A} = \mathbf{f} \quad (17)$$

202 where the coefficients \mathbf{c} , \mathbf{a} and \mathbf{f} are specified as follows as per the rules set out in [28]. For the
 203 air domain, $\mathbf{c} = \frac{1}{\mu_0}$, $\mathbf{f} = \mathbf{A}_b$ and \mathbf{a} is set to 1; for the whole microstructure \mathbf{a} is set to 0 and \mathbf{f}
 204 a 3×1 zero vector; for each individual grain,

$$\mathbf{c} = \begin{bmatrix} \nu_{11} & 0 & \nu_{12} & 0 & \nu_{13} & 0 \\ 0 & \nu_{11} & 0 & \nu_{12} & 0 & \nu_{13} \\ \nu_{21} & 0 & \nu_{22} & 0 & \nu_{23} & 0 \\ 0 & \nu_{21} & 0 & \nu_{22} & 0 & \nu_{23} \\ \nu_{31} & 0 & \nu_{32} & 0 & \nu_{33} & 0 \\ 0 & \nu_{31} & 0 & \nu_{32} & 0 & \nu_{33} \end{bmatrix} \quad (18)$$

205 where ν is the inverse of μ'_g for that grain and the subscripts denote the index of the element
 206 in ν .

207 The effective permeability of the microstructure, $\bar{\mu}$ can be evaluated from the FE solution
 208 by

$$\bar{\mu} \stackrel{\text{def}}{=} \frac{\|\mathbf{B}\|}{\mu_0\|\mathbf{H}\|} \quad (19)$$

209 where $\|\mathbf{B}\|$ and $\|\mathbf{H}\|$ are the magnitude of \mathbf{B} and \mathbf{H} for the microstructure. The angle between
 210 the \mathbf{B} and \mathbf{H} vectors, Θ , is calculated by

$$\Theta = \arccos \frac{\mathbf{B} \cdot \mathbf{H}}{\|\mathbf{B}\|\|\mathbf{H}\|} \quad (20)$$

211 *2.3. Microstructure and texture data*

212 Virtual microstructures of targeted grain size and shape are simulated by the open-source
 213 software Neper [29]. To simulate texture data, ODFs were created given a mode of orientations
 214 and corresponding distribution kernel functions and half width for the spread, in the open-source

215 MTEX toolbox for MATLAB [30]. In this paper, the default values, i.e., the de la Valee Poussin
216 function and 10° half width, were used; cubic crystal symmetry and orthorhombic specimen
217 symmetry are also applied to all texture data. Crystal orientations were then generated from
218 the ODFs and allocated randomly to each grain in the microstructure.

219 A separate MATLAB code was developed to convert the raster EBSD data into polygons
220 with continuous and smoothed grain boundaries, as opposed to discontinuous segments available
221 in commercial EBSD software packages, representing each grain ready for the FE geometry. The
222 average grain orientations for each grain were calculated using the Aztec software package.

223 3. Modelling Results

224 3.1. Effects of crystallographic textures

225 To predict the separate effects of crystallographic texture on magnetic anisotropy, we allocated
226 different textures to a virtual microstructure consisting of 500 grains whilst keeping the material
227 parameters constant. μ_c is set to 1000, which, as a rule of thumb, will give predicted effective
228 values of around 333 for random textures at a very small field (which has been measured
229 experimentally for fully ferritic steel [31]). c_g is set to 0 to exclude the grain size effect. Fig. 4
230 shows the predicted effective permeability for some typical textures in steels including the fibre
231 texture with $\langle 100 \rangle$ in parallel with ND, notated as $\langle 100 \rangle \parallel \text{ND}$ and also known as θ fibre,
232 the γ fibre ($\langle 111 \rangle \parallel \text{ND}$), the η fibre ($\langle 100 \rangle \parallel \text{RD}$), the α fibre ($\langle 110 \rangle \parallel \text{RD}$) and the Goss
233 texture ($\{011\}\langle 100 \rangle$) along a series of directions swept from RD by 10° interval to 180° . All
234 the curves are symmetrical with respect to the 90° axis as expected of the cubic crystal and
235 specimen symmetry. The highest $\bar{\mu}$ values occur at RD for the η fibre and Goss texture which
236 are both $\langle 100 \rangle$ directions, i.e., the magnetic easy direction; and the lowest value occurs at
237 approximately 54.7° from RD for the Goss texture, which parallels with the $\langle 111 \rangle$, i.e., the
238 magnetic hard directions. The γ fibre and the θ fibre exhibit isotropic permeability within the
239 microstructure plane, which would be consistent with their in-plane random orientations, as a
240 ND fibre, averaging out.

241 Fig. 5 shows the effective permeability maps for some selected textures and background field
242 directions. The γ fibre (Fig. 5 (c)) and the Goss texture (Fig. 5 (i)) exhibit distinctive $\bar{\mu}$ maps
243 despite very similar average $\bar{\mu}$ values (see Fig. 4). The former features more or less random $\bar{\mu}$
244 values across the microstructure indicating random in-plane orientations whilst the latter shows
245 much less variation corresponding to the single texture (note the simulated uni-mode ODF for

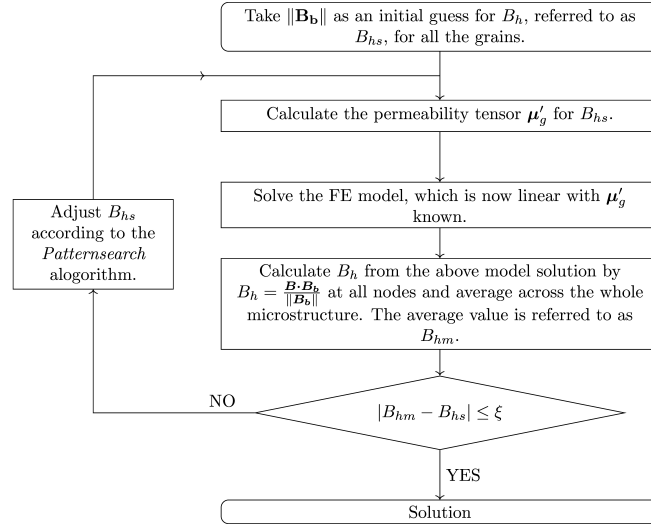


Figure 3. A flow chart illustrating the recursive approach for solving the non-linear FE model. ξ denotes the tolerance for the Patternsearch algorithm.

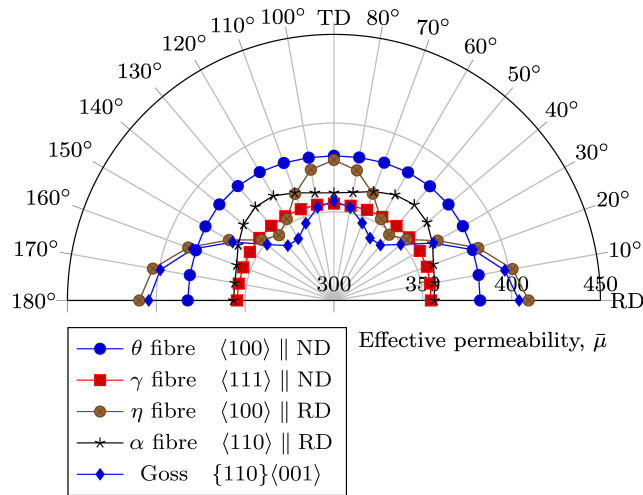


Figure 4. Predicted effective permeability for some important textures using a same virtual microstructure as a function of the directions of the applied field with respect to the RD.

246 the single texture is not an ideal single crystal but has a 10° spread half width). Similarly, the
 247 ND fibres, e.g., the γ fibre (Fig. 5(b)), exhibits more variation than the RD fibre, e.g., the α
 248 fibre (Fig. 5(d)), for the RD field direction. By comparison, the θ fibre map has systematically
 249 higher values than the γ fibre one and similar randomness across the microstructure. The
 250 Goss texture map features predominately uniform colors for each selected directions but are
 251 distinctive between each other.

252 The consistency in all these permeability behaviours demonstrates that the present model
 253 is capable of capturing the crystallographic texture effects on magnetic anisotropy. The pre-
 254 dicted permeability curves serve as a quantitative characterisation of the magnetic anisotropy
 255 associated with texture. In addition, the permeability maps serve as an enhanced visual and
 256 quantitative indication of the textures as a supplement to inverse pole figure (IPF) maps.

257 Thanks to the tensorial permeability, as opposed to scalar ones, the model is also capable
 258 of predicting the angle between the \mathbf{B} and the \mathbf{H} vector, Θ . Fig. 6 shows the predicted average
 259 Θ values for the different textures. Similar to the effective permeability behaviours, the in-
 260 plane isotropy of the ND fibres including the γ and the θ fibres also manifests itself in the Θ
 261 behaviours. It is worth noting that Θ is without regard to the rotation axis direction. Thus, Θ
 262 values do not average out to be zero despite the in-plane isotropy as a whole. The Θ behaviours
 263 of the RD fibres and the Goss texture exhibit more undulated anisotropy than their effective
 264 permeability behaviours. The troughs appear to occur near the $\langle 100 \rangle$ direction, e.g. the RD for
 265 Goss and η fibre, the $\langle 110 \rangle$ direction, e.g., the RD for α fibre, as well as the $\langle 111 \rangle$ direction, e.g.,
 266 55° from RD for the Goss texture and the η fibre. This behaviour would be consistent with the
 267 literature [32, 33] reporting that these directions are the principal directions where $\mathbf{B} \parallel \mathbf{H}$.

268 *3.2. Effects of uniform applied field strength*

269 The uniform applied field magnitude, $\|\mathbf{B}_b\|$, normalised against B_s , is set to 0.1 for all the
 270 above modelling and the average $\|\mathbf{B}\|$ values of the microstructure, \bar{B} , eventually converge at
 271 0.3–0.31. Owing to the non-linearity of the present model the predictions are also dependent
 272 on \bar{B} . Fig. 7 shows the $\bar{\mu}$ and Θ values as a function of \bar{B} for the α fibre and the Goss texture
 273 for the \mathbf{B}_b directions along which the maximum and the minimum $\bar{\mu}$ values occur respectively.
 274 The predicted $\bar{\mu}$ values for all the conditions increase from approximately 333, i.e. one third of
 275 the μ_c value, at different rates, by power laws as illustrated by the fitting lines. Similarly, the
 276 predicted Θ values also increase with \bar{B} by power law. The order of both values for different
 277 conditions remain unaffected throughout the modelled range. The differences between the dif-

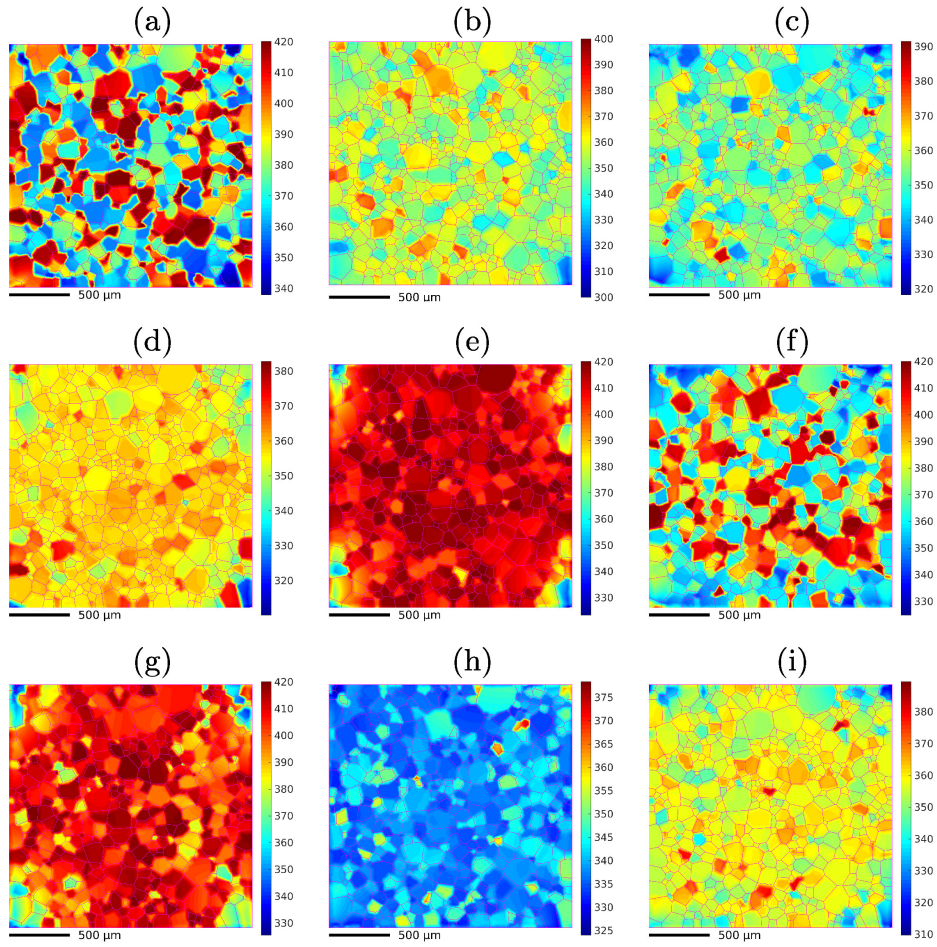


Figure 5. Predicted effective permeability maps for selected textures and field directions. The colors are mapped to the permeability values using a same virtual microstructure. (a) θ fibre along TD, (b) γ fibre along RD and (c) TD, (d) α fibre along RD, (e) η fibre along RD and (f) TD, and Goss texture along (g) RD, (h) 55° from RD and (i) TD.

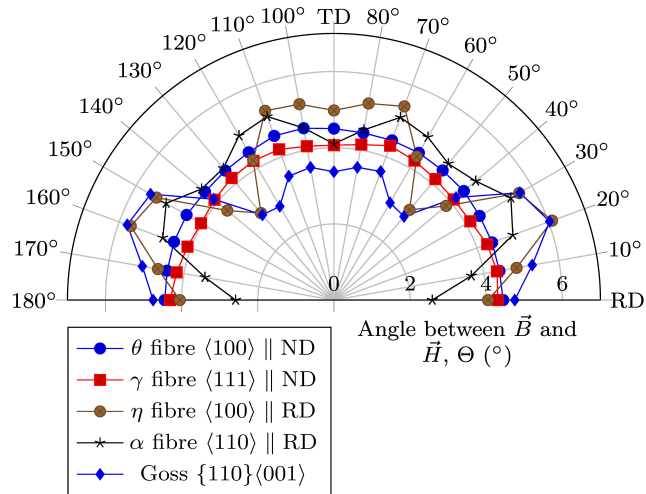


Figure 6. The average angle between the \vec{B} and \vec{H} of the virtual microstructure for some typical textures in steels.

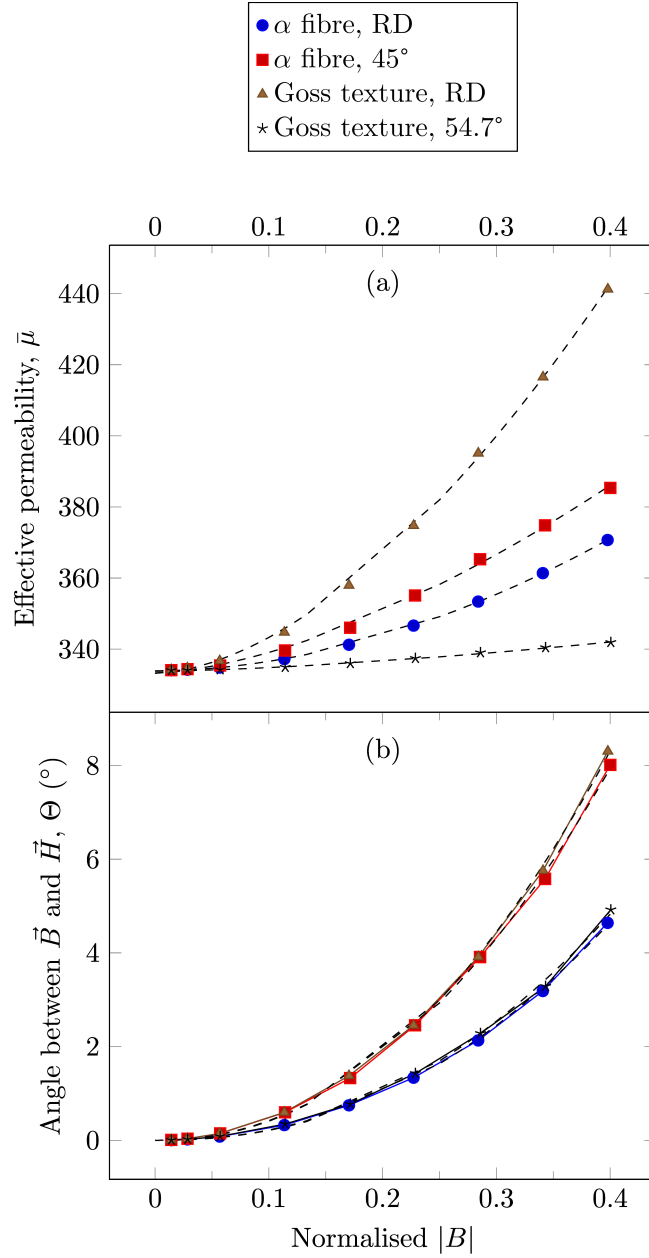


Figure 7. The effective permeability $\bar{\mu}$ (a) and the angle between \vec{B} and \vec{H} (b) as a function of \bar{B} for the α -fibre texture with the B_b along RD and 45° from RD and for the Goss texture with B_b along RD and 54.7° from RD with a virtual microstructure consisting of 500 grains. The dashed lines are power law fitting.

278 ferent \mathbf{B}_b directions increase steadily indicating the anisotropy intensifies with the increase of
 279 the normalised $\|\mathbf{B}\|$ field up to 0.4.

280 **3.3. Effects of grain size**

281 Fig. 8 shows the predicted $\bar{\mu}$ as a function of the average equivalent grain diameter, \bar{d} , for a
 282 series of virtual microstructures with different number of grains in a 1 mm×1 mm square and
 283 hence different \bar{d} , with and without considering the grain size effects. Random grain orientations
 284 were allocated to all the microstructures. The predicted $\bar{\mu}$ values increase with \bar{d} fitting perfectly
 285 well with the power law by

$$\bar{\mu} = 362.2(1 - \frac{6.95}{\bar{d}}) \quad (21)$$

286 for $c_g = 6.95 \mu\text{m}$ whilst remain constant at approximately 362.2 for $c_g = 0$. Note the remarkable
 287 similarity of Eq. (21) to Eq. (9). This behaviour also agrees well with the literature reporting
 288 the magnetic permeability values increase with the ferrite grain size by a similar inverse or
 289 inverse square root relationship in extra-low carbon steels [31, 34] or in non-orientated electrical
 290 steels [35]. The results prove that the present model has captured the grain size effects by consid-
 291 ering the loss of elementary domains to the grain boundaries through introducing the parameter
 292 c_g . It is interesting and perhaps slightly counterintuitive at first view that the interactions of
 293 magnetic flux with the grain boundaries in the FE model, as manifested in the transition region
 294 near grain boundaries in the permeability maps as shown in Fig. 9, does not capture the grain
 295 size effect. Note where there is a decrease in the $\bar{\mu}$ values on one side of the grain boundaries,
 296 as compared to the bulk of the grain, there is increase on the other side cancelling it out. As a
 297 result, the effective $\bar{\mu}$ for the microstructure as a whole remains unchanged.

298 **4. Measurements**

299 **4.1. Experimental details**

300 A commercial grade GOES featuring strong texture was selected for experimental validation of
 301 the present model. EBSD data were collected across a large area of approximately $12 \times 11 \text{ mm}^2$
 302 at a step size of $10 \mu\text{m}$. Fig. 10 shows the inverse pole figure maps exhibiting strong Goss texture
 303 and coarse grains as expected of this steel grade.

304 A small ($32 \times 15 \times 16 \text{ mm}^3$) U-shaped electromagnetic (EM) sensor that can apply a relatively

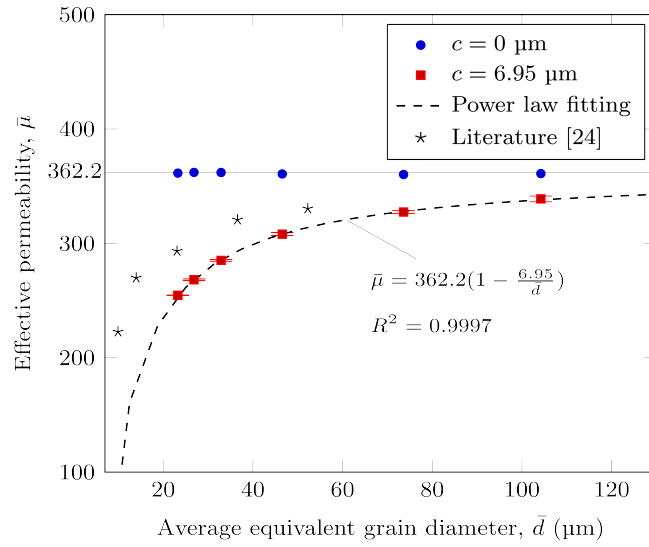


Figure 8. Predicted effective permeability as a function of the average grain size for a series of virtual microstructures.

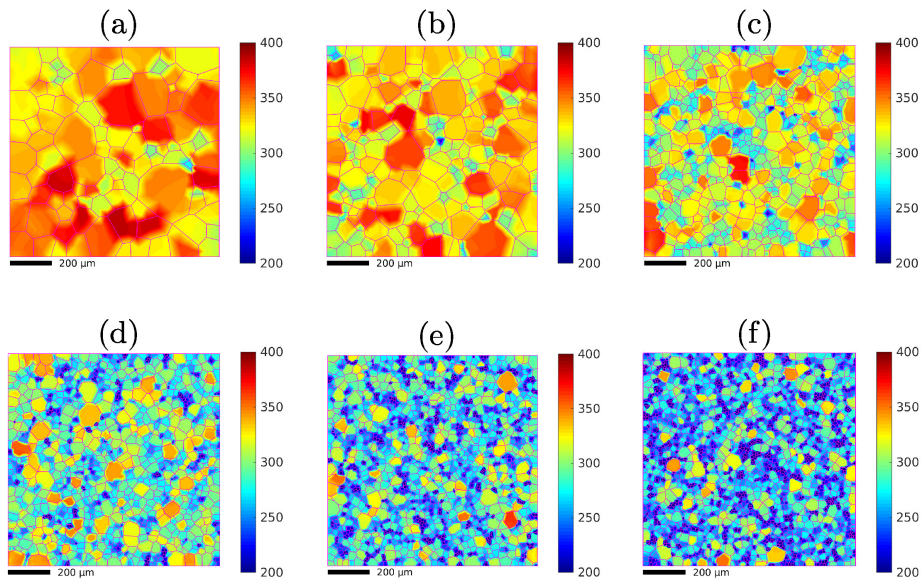


Figure 9. Predicted effective permeability maps for a series of virtual microstructure consisting of different number of grains, $n_{gs} =$ (a) 100, (b) 200, (c) 500, (d) 1000, (e) 1500, (f) 2000.

305 low magnetic field was used to measure an A4-sized thin (0.25 mm) GOES sheet at a series
306 of angles (φ) with respect to the RD. The relative permeability values were then indirectly
307 extracted by non-linear least square regression. More details about the measurement system
308 and the finite element modelling approached can be found elsewhere [36].

309 **4.2. Identification of the model parameters**

310 The grain size parameter c_g was set to 0 for simplicity considering the predominantly coarse
311 grains and hence expectedly insignificant effects of c_g on the permeability values. The unknown
312 material parameter μ_c were identified by *Patternsearch* optimisation algorithm fitting the pre-
313 dicted effective permeability values with the measured ones and at the same time recursively
314 solving \bar{B} . Fig. 11 shows the optimised predictions of the permeability values as a function of
315 φ agreeing reasonably well with the measurements.

316 The permeability behaviours are also generally consistent with the predictions using the
317 generated Goss texture data and virtual microstructure described in Section 3.1. μ_c has been
318 identified to be 1497 with \bar{B} , for example, for $\varphi = 54.7$, having converged at approximately
319 0.6 and the Θ at approximately 24.9° . Fig. 12 shows the predicted $\bar{\mu}$ maps for the identified
320 model parameters and the background field along RD, TD, ND and 54.7° with respect to RD.
321 These maps visualise the following main characteristics of the magnetic anisotropy associated
322 with the Goss texture corresponding to the IPF maps shown in Fig. 10. First, the RD maps
323 shows generally highest permeability values indicating the RD being close to $\langle 100 \rangle$ directions.
324 Second, the TD and ND ones are similar indicating these directions are close to the same crystal
325 direction. The map for 54.7° from RD shows predominantly low permeability value indicating
326 it is close to $\langle 111 \rangle$ directions.

327 It should be noted that the prediction of Θ values and the effects of B on the effective per-
328 meability anisotropy cannot be fully validated using the present measurement technique, which
329 only measure scalar permeability (hence $\mathbf{B} \parallel \mathbf{H}$) and is not capable of changing the applied
330 field strength (which is determined by the sensor geometry). A sensor system that can measure
331 multiple \mathbf{B} and \mathbf{H} components is needed and being developed. The present tensor permeability
332 model is fully capable of modelling 3D microstructures and any direction in 3D space although
333 only 2D microstructures have been modelled in this paper. The 2D microstructure in-plane
334 directions are often of more interest and probably more accurate as far as the microstructural
335 effects are concerned.

336 The present model is anticipated to be used to provide the permeability anisotropy for

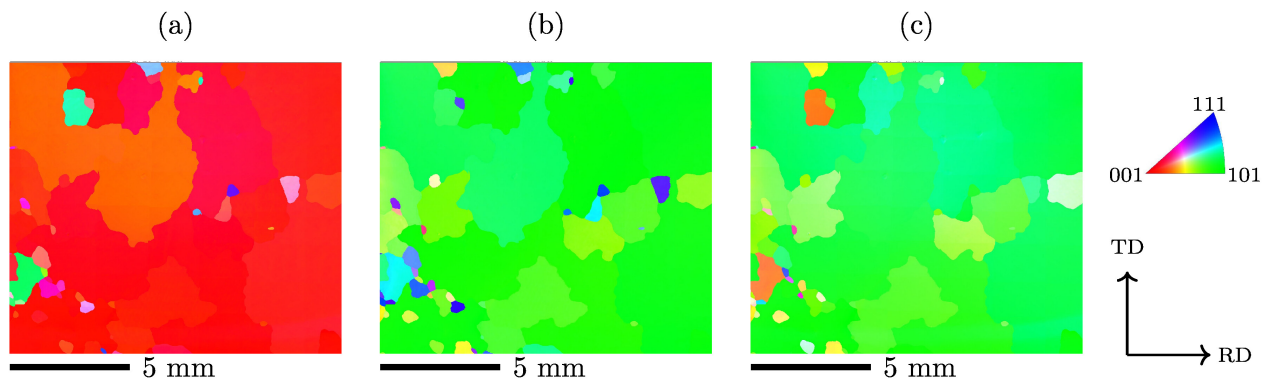


Figure 10. Inverse pole figure maps of the grain oriented electrical steel sample for the (a) rolling direction (RD) (b) transverse direction (TD) and (c) normal direction (ND) pole.

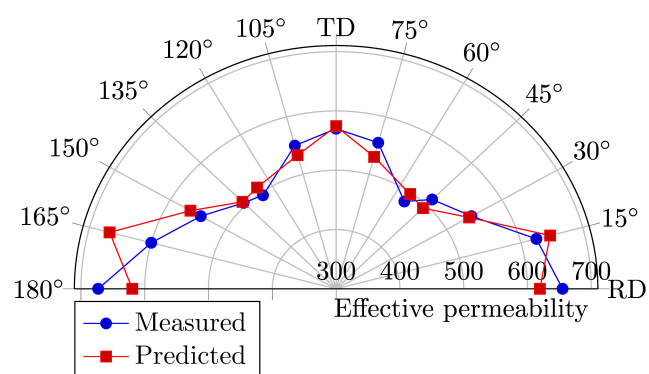


Figure 11. Predicted permeability as a function of the field direction using the identified model parameters compared with the measured ones for the grain oriented electrical steel.

337 predicting sensor measurements for low field EM sensors used for monitoring steel quality during
338 processing (e.g., [37]) and / or for the interpreting the EM sensor signals to infer the texture
339 of the steel. The model could also be used to predict anisotropic effective permeability values
340 that can be input into other macroscopic FE electromagnetic models as the material property,
341 say, the permeability of electrical steel components of an electric motor, to consider the steel's
342 microstructure and texture. As a FE microstructural model the present model may potentially
343 be coupled with other microstructural models, e.g., the microstructure-based crystal plasticity
344 models, for multi-physics modelling.

345 5. Conclusion

346 A tensorial permeability finite element microstructure model that considers crystallographic
347 textures based on magnetic domain theories has been developed for evaluation of magnetic
348 anisotropy of polycrystalline steels. The model can predict consistent and logical effective per-
349 meability behaviours and the angle between B and H for some selected typical textures that
350 are important and common in steel manufacturing. The model has proved capable of capturing
351 the crystallographic texture, the grain size and the background field effects on the magnetic
352 anisotropy of steels based on the magnetic domain theory. The predicted effective permeability
353 curves as a function of the magnetic field directions and the permeability maps can serve as a
354 quantitative characterisation of the magnetic anisotropy as well as an enhanced visual indica-
355 tion of the crystallographic texture from magnetic values. These capabilities have been initially
356 validated against a commercial grain oriented electrical steels featuring strong Goss texture and
357 magnetic anisotropy.

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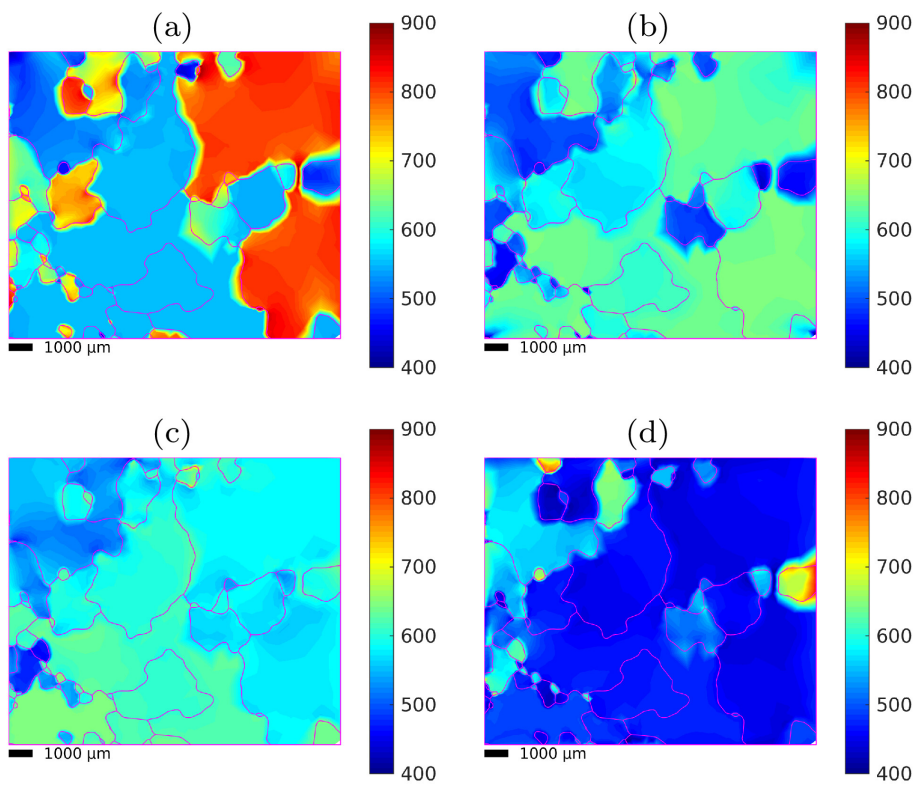


Figure 12. Predicted permeability maps for the grain orientated electrical steel sample for different background field directions. (a) RD, (b) TD, (c) ND, (d) 54.7° from RD.