Unconventional Monetary Policy of UK : The Study through a SOE-DSGE Model

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Abstract

This thesis aims to investigate the impact of unconventional monetary policy through banking lending channel in the UK. The Dynamic Stochastic General Equilibrium (DSGE) model is built following the Smets and Woulters (2007) and adjusted by incorporating the financial accelerator, then extended to an Armington (1969) version small open economy. We evaluate and estimate the model by Indirect Inference method with the un-filtered non-stationary data from 1985Q1 to 2016Q4. The model with estimated parameters significantly passes the Indirect Inference test and describes the UK economy well. The empirical study based on estimated model approves the significance of financial intermediary in the transmission mechanism of quantitative easing, which can substantially enhance the economy's stability by the bank lending channel, particularly during the crisis time of zero lower bound. And we find that the shocks from financial sectors play a significant role in the recession, but they are not sufficient to make a big economic crisis. The alternative monetary policy, including price-level targeting and nominal GDP targeting, are also investigated. By measuring and comparing the frequency of crisis and the welfare cost, the monetary regime with the combination of nominal GDP targeting and monetary reform performs the best.

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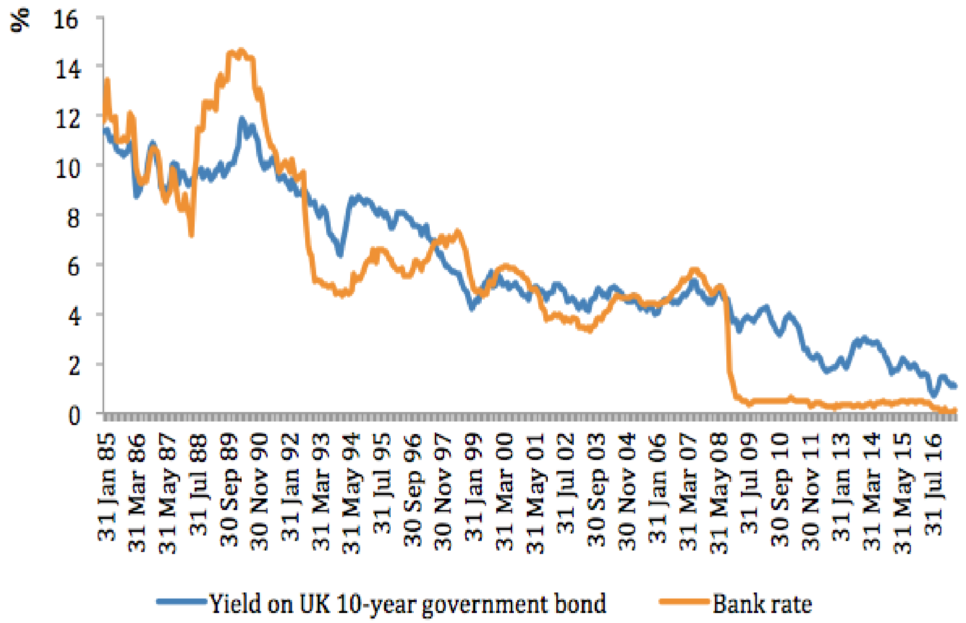
Chapter 1

Introduction

The global financial crisis, which started in August 2007 and then intensified with the collapse of Lehman Brothers in September 2008, brought the severest downturn to the world economy in modern history. In the 1st quarter of 2009, output in the United Kingdom fell with annualised rate approximated 20%, and the unemployment rate rose unprecedentedly to 7.6% (Dale, 2010). In response to the big challenge, central banks internationally took the expansionary monetary policy to support the demand by aggressively cutting the monetary policy rates, in many cases toward a lower bound. For example, Federal Reserve immediately cut down the federal funds rate to the level between 0% and 0.25% in December 2008, and the European central bank adjusted its deposit rate from 3.25% to 0,25% in May 2009. As showing in figure 1.1, the Monetary Policy Committee of the Bank of England (BoE) cut the interest rate from 5% to 0.5% in 2009, where it then remained for a long term till recent years.

Unfortunately, the ultra-low interest rate was not enough to help with the economic depression, and many have argued that the central bank was also "out of ammunition" to save the economy in dealing with the subsequent shocks. The Monetary Policy Committee (MPC) of the BoE announced that without additional measurement, the nominal spending could be challenging to meet the 2% targeted inflation level during a medium term. Reasonably, the central banks started to implement unconventional monetary policy, including direct market intervention with the large-scale asset purchase, which is referred to as " Quantitative Easing (QE)". This policy aimed to increase the liquidity then encourage lending and investment by expanding the balance sheet of the central bank. As showing in figure 1.2, the QE has been widely adopted in the world's central banks and hugely drove up the

Figure 1.1: Bank Rate and 10-year government bond yield in UK (1985 2016)



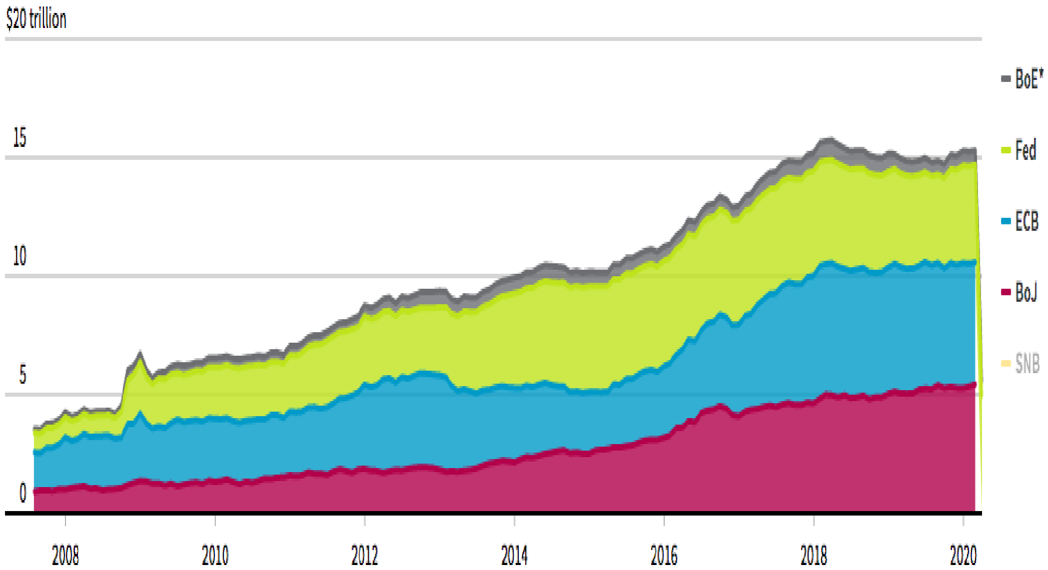
Sources: Data base of Bank of England (IUMAMNZC, IUMAJNB)

bank assets holdings. In the UK, the QE started from March 2009, and the total purchasing increased to £200 billion in 2010, then up to £375 billion in 2012. Until 2013, the UK economy gradually gained strength with the unemployment rate falling to 7%. However, facing the economic uncertainty of Brexit, the BOE announced another round of QE.

Recent experiences have inspired a growing body of work on the unconventional monetary policy. Firstly, they would like to know whether QE could genuinely generate a positive impact on the economy. By reviewing the past study, we found an amount of empirical evidence, particularly by the event study which mainly focuses on the response from a specific economic or financial variable within a term interval between two QE announcements (See Meier (2009), Blinder (2010), Neely (2010), Breedon et al. (2012), Baumeister and Benati (2013) and Chum, Joyce, Kapetanios and Theodoridis (2015)). Then the remaining works mainly study the policy impact by estimating a dynamic structural model with the time-series analysis and depict economic response, for instance, Kapetanios et al. (2012) and Weale and Wieladek (2016).

Besides, another interest is the identification of the transmission mechanism through which the QE can make effects. The most-held

Figure 1.2: Central bank balance sheets



Source: Thomson Reuters Datastream

opinion is portfolio rebalancing channel (See Meier (2009), Joyce et al. (2011), Christensen and Rudebusch (2012), Harrison (2012), Christensen and Krogstrup (2016), Falagiarda (2014) and Koijen et al. (2017)). Based on the theory built up by Tobin (1961,1963, and 1969), that if assets are not perfectly substituted, the change of quantity for a kind of assets will lead to a change of its expected rate of return, they propose that QE can affect the asset prices and yield by rebalancing the assets structure. Regarding other aspects, some economists remarked that, with forward expectations, an announcement of QE would revise people's expectation of future short rate by signalling impacts (See Krishnamurthy and Vissing-Jorgensen (2011) and Christensen and Rudebusch (2012)); Under an open economy, the exchange rate also plays an essential role and allows QE to make effects through the international trade-flow channel.

Notably, being challenged with the recent financial turmoil, more attentions have been paid to the transmission channel with the financial intermediary, especially in a situation of severe financial distress. A number of works have investigated the effect of the QE by the financial systems, and here I would conclude them into two ways: Firstly, an increase on reserve held by financial intermediary allows them to make more lending. Therefore, the investment would be directly boosted with the QE (See Christensen and Krogstrup (2016)). Secondly, a large asset purchasing pushes up the prices of a wide range of financial asset, which can foster the credit of the entrepreneur, then lower the finance premium against the bank lending (See Brunnermeier and Sannikov (2016)).

Meanwhile, macroeconomic modelling has been hugely developed to keep consistent with the changes in the monetary policy, particularly during the devastating aftermath of the big financial crisis. The first critique of modelling is a lack of financial sector to count for an essential part of aggregate fluctuations, such as the financial frictions. Besides, the DSGE model has been employed by an increasing number of central banks, since it can provide more reliable monetary policy analysis than earlier models with explicitly theoretical foundations. Consequently, the DSGE framework has been remarkably developed with the build of the financial system to capture the real-world dynamic status (See, e.g., Brunnermeier and Pedersen (2009), Christiano et al. (2010), Adrian and Shin (2010), Gertler and Karadi (2011)).

Based on the fact, that the evidence which can support the QE effects in the UK through the credit channel is limited, I will build a theoretical model and then empirically evaluate and estimate it to fit the UK data. Based on the facts Great Financial Crisis (GFC) was a crisis started with the collapse of the cash flows, which further triggered the insolvency, we firstly investigate whether the QE would encourage the bank lending

Specifically, with the prevailing framework of financial friction built up by Bernanke et al. (1999) (BGG model), an agency problem will be introduced between financial intermediary and entrepreneur. When there is a deterioration of intermediary asset, the external finance premium required by financial intermediary will be increased, and vice versa. Referring to Le et al. (2016), to catch the response of the model under the ZLB situation, money is not only treated as the loadable liquidity but also the cheapest type of collateral against bank lending. Consequently, with money injected by large asset purchasing, the private sector like the entrepreneur will hold more cash on the account, which can be treated as the collateral to increase their credit and lower the external finance premium required by the financial intermediary.

In the model setup, we follow the framework built by Smets and Wouters(SW, 2007), since it has been widely approved with reasonable imperial properties. (See, Di Cecio and Nelson (2007), Villa and Yang (2011), Kamber and Millard (2012), and Faccini et al. (2013)). To describe the QE effect through the interaction between the financial intermarry and the firms, we augment the financial accelerator into the SW model, referring to BGG (1999). Furthermore, I extend the model with a small open economy sector, referring to the 'two-country model' of Armington (1969). Domestic investors are assumed to be interested in holding foreign assets and pursue a higher rate of return. In particular, during the ZLB crisis, there would be a downward pressure on the sterling exchange rate. Notably, for wage and price setting, we choose to follow the hybrid model (Le et al. 2012), where parts of products and labour are from perfectly competitive goods and labour market, with the remaining parts from imperfect competition market. The closed model to our work can be found in Le et al. (2016), but it is applied to the US data without considering the foreign sector.

How to evaluate the DSGE model properly is one of the unresolved problems in the applied macroeconomic history. Different from other studies with conventional estimation techniques such as the Bayesian method, we evaluate and estimate the model by indirect inference method. This method is firstly proposed by Goureroux et al. (1993), and it judges the model by comparing the behaviour of actual and simulated data through the statistical inferential framework of the auxiliary model. Notably, Le et al. (2011) refined this method in boosting the power and flexibility through the Monte Carlo experiments. In terms of the data, to avoid losing information with filtered data, we investigate the model with un-filtered non-stationary UK data from 1985Q1 to 2016Q4.

We first test the model based on the calibrated parameters, and it is severely rejected. While after performing the indirect inference estimation, the model can significantly pass the test with a better performance. Explicitly, the empirical results based on the estimated model approve the crucial role of financial intermediary in the transmission mechanism of the QE by the bank lending channel, including the crisis time of ZLB. Besides, the forceful impact from the finance premium are captured with its counter-cyclical feature. And the variance decomposition and historical shock decomposition indicate that the finance premium dominates the fluctuations of financial variables, and drives the economic recession.

We further investigate the model by implementing alternative monetary policies in place of the inflation-targeted rule. The idea came from numerous discussions against the practicability and the effectiveness of different monetary regimes. For example, Svensson (1999b) concluded that there is a free lunch of less volatility in both inflation and output by choosing price level targeting instead of Inflation targeting. Eggertsson and Woodford (2003) discussed the price-level targeting as a guide to help the economy with a liquidity trap. Besides, Carney (2012) suggested that nominal GDP targeting is more favourable with the ZLB. Shedding light on the past work, we choose to examine multiple types of alternative monetary regimes, including price-level targeting, nominal GDP targeting and combinations of them with a monetary regime based on our structured model. By comparing the simulation results, the monetary regime with the combinations of nominal GDP targeting and monetary reform behaves best, which can counteract deflation pressures, and stabilize the economy most.

The thesis is structured as follows: In chapter 2, I reviewed the past works on QE by three folders. Firstly, I introduce the background and content of the QE, and then list the QE announcements by BoE during the aftermath of the crisis; Secondly, I collect the evidence in quantifying the impact of QE in the UK. Thirdly, I review the past literature by classifying them into different transmission mechanism, which includes signalling channel, portfolio rebalancing channel, exchange rate and trade-flow channel and the bank lending channel. Lastly, I introduced the prevailing DSGE model developed with large asset purchasing. In chapter 3, I outline a medium-to-large size SOE-DSGE model, including the crisis time of ZLB, when the QE is employed. Then I allow the model to confront the UK data from 1985Q1 to 2016Q4 and then calibrate the model. In chapter 4, I first outline the indirect Inference methodology. Then test the model with the calibration. To search for the best-performed model, I estimate the model and then re-test it. Lastly, I empirically analyse the estimated model with impulse response function, variance decomposition and shock decomposition. In chapter 5, I replace the traditional monetary policy of inflation targeted rule with alternative monetary policies. Then with the simulations under all types of regimes, I measure the frequency of crisis and the welfare cost. In chapter 6, I conclude the research results and bring in possible future extensions of the works.

Chapter 2

Literature Review of Quantitative Easing

2.1 Introduction

The recent experiences inspired a growing body of empirical studying on the QE. Firstly, they contribute amounts of evidence on the impact of the large asset purchasing made to date. Secondly, a large body of works focus on describing the transmission mechanism through which asset purchasing might affect the aggregate demand, and I categorize them into four types: Portfolio re-balancing channel, Signalling channel, Exchange rate and trade-flow Channel, and Bank lending channel.

During the aftermath of the crisis, studying has shown that financial intermediary played a central role in the monetary system in facilitating market activity. Besides, the shocks to the financial market have been approved to make a real effect on bank lending and the firm's investment. For example, Smets Wouters (2007) criticized for the past work in omitting credit channel with financially intermediary and pointed out the importance of banking lending in quantitative transmission. Consistently, Sugo and Ueda (2008) insisted that the disrupted relationships between the economy and financial market should be considered.

Regarding the model framework, we will go through studying based on the Dynamic Stochastic General Equilibrium (DSGE), which can capture the dynamics from the micro perspective. Though the VAR model has been recognized as the most popular method to study the effects of conventional monetary policy, its applicability is limited in identifying the causal relationship of unconventional monetary policy with a short sample (See Bernanke and Blinder (1992)).

The remaining content in this chapter is arranged as followings. First of all, I introduce the content of the QE and list the big QE announcements made by the BoE. Secondly, to support our work, we conclude the empirical evidence on the impact of QE in the UK. Thirdly, we study the channels by which QE transmission mechanism usually work through. Fourthly, we review the developed DSGE models in studying the QE effect. Finally, remark the chapter.

2.2 What is Quantitative Easing

The central bank normally adjusts the bank rate to meet the targeted inflation level. Reasonably, following the collapse of Lehman Brothers, the BoE aggressively cut the interest rate from 5% to 0.5%. But to our knowledge, there is a ceiling to how low the interest rate can go. When the interest rate gets stuck into zero lower bound, the unconventional monetary policy such as the QE will be introduced. QE is proposed to accommodate the shocks by boosting the amount of money in the economy, which can stimulate the nominal demand and thereby ensure the inflation at the targeted level within the medium term. And it is generally implemented through the acquisition of long-term government bonds and financed by an increase in the reserve accounts that commercial banks hold at the central bank. The phrase " QE "was firstly adopted by the central bank of Japan, who dealt with the deflationary pressures, following the burst of the real estate bubble in the 1990s. Then many of the developed countries started to employ it during the recession in 2009 and beyond.

As early as April 2008, BoE started a Special Liquidity Scheme (SLS) to help financial intermediary and building societies to exchange high-quality mortgage-backed securities for liquidity such as the Treasure bills. Then successively there are three rounds of QE in the UK announced by the BoE. In March 2009, facing the risk of zero lower bound, BOE announced to further lose the monetary policy by beginning a large amount of asset purchasing with an initial amount of £75billion, the majority of which were UK government bonds with maturity dates from 5 to 25 years. In the same year, they established the Asset Purchase Facility (APF), which was independent of BOE and targeted to improve market function by open market operation through purchasing private assets. Fearing the inflation would continue to fall below the target value of 2%, the increasing amount of assets purchasing was announced in Aug and Nov of 2009 with total purchasing amount to £200 billion in 2010. Similarly, as before, the assets central bank bought from the private sector were predominantly government securities as well as relatively small purchases of corporate bonds and commercial paper.

GDP of the UK started to recover since late 2009, while with growing concerns of the sovereign debt crisis from some European Countries. In October 2011, BoE announced another round of QE, which made the total amount get to £275 billion. Following by this, in Feb and Jul 2012, another two announcements of QE brought the total amount of asset purchases up to £375 billion. Until 2013, the UK economy gradually gained strength with the unemployment rate falling to 7%, which was faster than expected.

Though the UK economy has been in a stable status afterwards, the BoE developed the third round of QE following the " Brexit" results of leaving European Union (EU) on Aug 2016, with purchasing £60 billion of government bonds and £10 billion of corporate bonds to stabilize the economy. Moreover, the BoE cut the policy rate from 0.5% to 0.25% to encourage market lending. The details of the big announcements of QE in the UK have been listed in Table 2.1.

2.3 Evidence of the QE Effect in the UK

There is a growing amount of literature attempting to quantify the macroeconomic implications of the asset-purchasing program with outcomes in terms of the effects on both the financial and economic variables. Some researchers study the policy impact by estimating a dynamic structural model. Those models rely on time series data and depict the variation of economic response. While some argue it is rather difficult to isolate the effect of BoE's QE policy in the context of broad economic uncertainty, considering the uncertain time lags between the policy actions and effects, and the difficulties in isolating from other effective factors. Consequently, the event study has been frequently used by estimating the changes on the asset prices or the yields over a narrow window surrounding the announcement of the QE. Besides, it has contributed a large amount of evidence, which is very supportive of QE effectiveness. Potter and Smets (2016) concluded that event study could straightforwardly identify the causal relationship and distinguish among different forms of unconventional monetary policy measures.

In the early study, Meier (2009) studied the QE announcement by

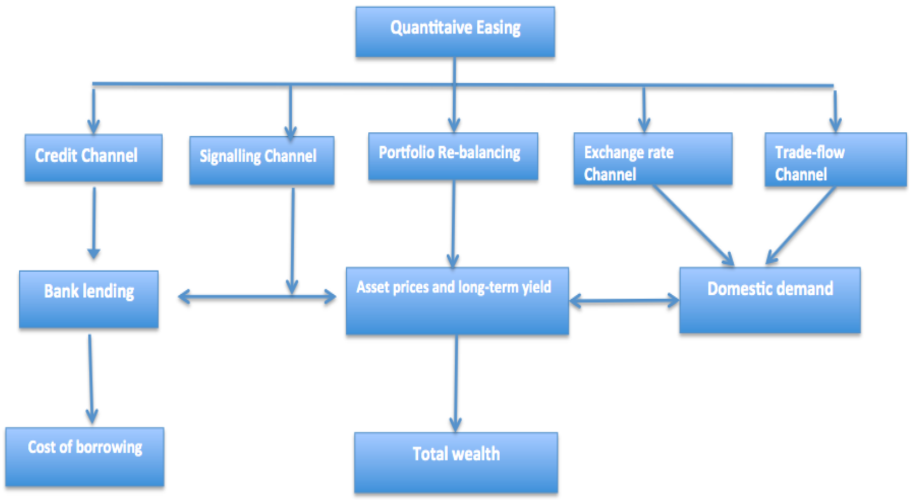
Table 2.1: Key Announcements of Quantitative Easing in UK

Date of QE treatment	Description of QE announcements
19/01/09	Announcement of set up an Asset purchase Facility to facilitate the open market operation
05/03/2009	The MPC cut bank rate to 0.5%. It also announced it would purchase an initial £75bn of assets financed by central bank reserves over the next 3 months. Gilt purchases were to be restricted to bonds with a maturity of between 5 and 25 years.
07/05/09	Announcement of quantitative easing extended to £125 billion
06/08/09	Announcement of QE extended to £175 billion by large amount of asset purchasing, which targeted on all gilts with a maturity of over 3 years
05/11/09	Announcement of quantitative easing extended to £200 billion
06/10/11	Announcement of quantitative easing extended to £275 billion
09/02/12	Announcement of quantitative easing extended to £325 billion
05/07/12	Announcement of quantitative easing extended to £375 billion
04/08/16	Announcement of quantitative easing extended to 435 billion after Brexit vote result comes out

BoE in March 2009; they found that long-term government bond yields declined between 40 and 100 basis points. Then Joyce et al. (2011) set up a comprehensive assessment by event study to two big announcements from BoE. One is between March 2009 and Feb 2010, with £200 billion asset purchases, and the other one is between Oct 2011 and May 2012 with £125 billion. They borrowed the concept of multiplier calculator from work by Markowitz (1952) and found that QE can dramatically lower long-term gilt yields by about 100 basis points. Furthermore, similar empirical evidence can be found in the works by Meaning and Zhu (2011), Bridges and Thomas (2012), Breedon et al. (2012), Baumeister and Benati (2013), Chum, Joyce, Kapetanios and Theodoridis (2015).

Apart from the evidence on the financial market, some of them are related to the effects on the broader economy. For instance, Kapetanios et al. (2012) employed a set of VAR models to measure the impact from the QE and provided new evidence on the potential macroeconomic effect of the QE by BoE program from March 2009 to January 2010. And their estimation results show that QE may have a peak effect on the real GDP with 1.5% and a peak effect on the annual price level with

Figure 2.1: Transmission Mechanism of Quantitative Easing



Sources: Based on models that frequently appear in the QE literature

1.25%. Similarly, Baumeister and Benati (2013) suggested that the first round of QE by BoE have averted significant risk on both deflation and output collapses during the aftermath of the crisis. More recently, Weale and Wieladek(2016) estimated the effects of the QE announced by BoE from 2009 to 2012 and found that an asset purchasing leads to a statistically increase in CPI and GDP of UK with 0.25% and 0.32% respectively. And based on the empirical results, they remarked that the asset purchased was an effective method of supporting the GDP, particularly during the aftermath of the financial crisis.

2.4 Transmission Mechanisms

The monetary transmission mechanism is defined as how the monetary policy decisions are transmitted to the change in the variables like output and price level (See Taylor (1995)). By reviewing the past literature, we can find that there are several potential channels as I described in figure 2.1, and they have been massively discussed and formalized in macroeconomic models (See Gertler and Karadi (2011) and Chen, Curdia, and Ferrero (2012) Radia and Thomas (2014b)). In this part, we unfold them mainly by four ways, and they are signalling channel, portfolio rebalancing channel, exchange rate and trade-flow channel, and bank lending channel.

Signaling channel

Before the financial crisis, the signalling channel has been widely discussed. It was first suggested by Eggertson and Woodford (2003) and Bernanke et al. (2004), who proposed that the QE could affect the economy by changing people's expectations about future monetary policy. Specifically, under the ZLB crisis, with forwarding expectation, the announcement of purchasing large-scale asset may be interpreted as a signal that the policy rates will stay in a lower bound for an even longer period than expected, which then brings a positive effect on the aggregate demand and push up the inflation rate.

The signalling channel has been applied and studied in multiple countries. In Japan, Fujiki, Okina, and Shiratsuka (2004), Oda and Ueda (2005), and Ugai (2007) all remarked that QE could affect the market expectations when central bank confirmed that interest rates would remain low for a certain period. And if the central bank successfully extended the duration of its commitment, the long-term interest rate would be reduced and yield curve could be stabilized as a whole. In the US, for some commentators, this 'signalling' effect represents a vital aspect of the transmission mechanism (See Joyce et al. (2012)). Bauer and Rudebusch (2014) used both model-free analysis and dynamic term structure models to provide statistical evidence for the significant signalling channel of the first long run asset purchasing program of Federal Reserve.

Regarding the evidence of the UK, a minor role of signalling channel was verified. Joyce, Lasaosa, Stevens and Tong (2011) showed that the expectation of long-term rates would not decrease a lot by responding to the QE announcement. The consensus agreement has been achieved by Christen and Rudebusch (2012), through empirical dynamic term structure models, they analyzed and compared the declines of government bond yield on both the US and the UK market. The outcomes by US data shows that more than half of the response of US Treasury yields came from lower expectations for future monetary policy. However, for the UK, portfolio re-balance is an official channel, and it is the term premium that mostly drives the reduction in gilt yield.

Portfolio rebalancing channel

The portfolio rebalancing channel has been broadly discussed before the financial crisis. Different from the signalling channel, the portfolio rebalancing effects have nothing to do with the expectation of the interest rate. According to Tobin (1961,1963, and 1969) and Friedman

(1978), assets are assumed not perfectly substituted, and the change in quantity for a type of asset will lead to a change in its rate of return. If this is the case, large asset purchasing can cause the asset held by the private sector decreased, while with a higher amount of the liquidity on their account. Consequently, higher demand for the asset bid up its price and lowers the yield. Some researchers, such as Joyce et al. (2011), believes that the portfolio rebalancing mechanism is the way by which the QE can affect the economy most. Notably, Harrison (2012) studied the QE effects by a DSGE model in extending the New Keynesian model with imperfect substitution of short-term and long-term bonds. And they confirmed that, through unconventional monetary policy, the change of relative supplies of assets would affect the prices of those assets. Besides, their experiments with the model show that the efficacy of the QE is increasingly substantial when the interest rate gets into the zero lower bound.

Moreover, since the investors who sell the government bond with extra liquidity may also like to purchase alternative assets to rebalance their portfolio, thus the effect should not only reduce the term yields of the purchased asset but also spill over into the yields on others. Gagnon et al. (2011) provided the evidence by studying through the effects of Large-scale asset purchasing of Federal Reserve between Dec 2008 and Mar 2009. And they concluded that large asset purchases could lead to economically meaningful and long-lasting reductions in longer-term interest rates on a range of securities, which even includes securities that were excluded in the purchased programs.

Exchange rate and trade-flow channel

The exchange rate plays a particularly important role for an open economy, and it is also approved to be an essential factor in the transmission mechanism of QE (See Ouyang and Rajan (2013), Bernanke (2015) and Powell (2018)). In particular, through the large amount of asset purchasing, the central banks' balance tended to expand with the asset held by the private sector decreased. Then the yields of asset dominated with the domestic currency would be relatively lower comparing with the one dominated in foreign currency, which then leads to lower demand of the domestic bonds. Presumably, the downward pressure on the domestic currency makes a more competitive export, which can directly boost the demand for domestic products.

Against this background, many researchers are concerned with the spill over effects of the QE to emerging countries. Dahlhaus (2012) found out that when QE is implemented in the presence of global financial market integration, the response of asset in the home country could bring a similar movement in the foreign market. Though the monetary expansion in the home country will increase the net exports and detract from the real GDP of the foreign country, through the income-absorption effects, import of home demand would also rise when the domestic income is driven up by the QE.

The above point has been confirmed by the empirical study on the international transmission channel with the US and Canada. [1] For example, Dahlhaus, Hess and Reza (2014) set up a factor-augmented VAR model and treated balance sheet as the policy instrument to evaluate the spillover effects from the US QE to Canada. They found that QE increases the US and Canada GDP by 2.3 % and 2.2 % separately. Recently, MacDonald and Popiel (2017) generated consistent results. They investigated the unconventional monetary policy effects in a small open economy framework with a focus on the spillover effect from the US to Canada. They found that US unconventional monetary policy increase Canadian output with 0.127 % per month averagely and demonstrated the dramatic spillover effects from a foreign unconventional monetary policy.

Bank lending channel

Assessing the effects of QE through the financial market is a big step in studying the effectiveness of the unconventional monetary policy. The depression following the big financial crisis reminded us that the financial intermediary is a crucial part of the economy in supplying lending. Notably, the role of the bank seems differently from which depicted traditionally. Gambacorta and Marques-Ibanez (2011) remarked that the crisis leads to a change of the whole monetary transmission mechanism caused by the deregulation, financial innovation and the role of the institutional investor. Reasonably, many have addressed the interaction between the financial market and the monetary policy, leading to a new transmission mechanism of the monetary policy.

Under the textbook view, during the period of crisis, the QE by the central bank can directly increase the bank reserves, which are required to hold as a proportion of its deposits. To put the excess reserve into use, the financial intermediary will make more loans, and then stimulates the investment and consumption. This is the so-called Narrow Credit

[1] Canada is a typical case since its economy is strongly depending on the US and the Bank of Canada did not use the QE.

Channel, which focuses on changes in the balance sheet of the financial intermediary (See Gambacorta and Marques-Ibanez (2011)).

Apart from the above view, the bank lending channel can be presented in another way, which considers the impact of QE in augmenting balance sheets of private sectors, such as the entrepreneurs and households, and this is a Broad Credit channel. To be explicit, a large amount of asset purchasing leads to a higher price of the asset, which can increase the value of the net worth on the balance of the firm. With the concept of financial accelerator proposed by Bernanke and Gertler (1995), there will be a lower external finance premium required by the bank, then boost the economy by larger amount of lending. Furthermore, the bank lending will be further boosted while the entrepreneur gets access to a higher credit with a larger balance size.

Some works started to link the narrow and broader credit channels and argued that the external finance premium faced by the bank is determined by its balance sheet strength, which can be affected by the monetary policy. Typically, Gertler and Karadi (2011) developed a DSGE model to evaluate the effects of unconventional monetary policy to fight with a financial shock. In their model, the financial intermediary is incorporated with nominal reignites, and the financial friction is introduced in the closed economy, inducing an equilibrium leverage ratio for the financial intermediary to increase its interest spread. Then the central bank will use unconventional monetary policy to reduce the leverage by providing liquidity or contingent subsidies. Through the study, they found that credit policy could benefit the nominal interest rate even before hitting ZLB. Moreover, with ZLB, the benefit is significantly enhanced. Similarly, Borio and Disyatat (2011) proposed that the monetary policy can affect the economy by bank balance which is determined by the external finance premium. They emphasize the impact of unconventional monetary policy on banks' financial health, by introducing the credit market imperfection in the model setting where firm need to be operated by external funds from the firms.

More recently, Ariccia et al. (2018) presented evidence of the bank lending channel of monetary policy through asset purchasing by the Federal Reserve. Their results indicate that QE is associated with a decline in loan spread, especially for the bank with relatively weak balance sheets. And they also confirmed that through the credit channel, QE could stimulate the economy by reducing inter-mediation cost and promoting bank lending with strengthening the balance sheet of firms and banks.

2.5 Quantitative Easing Study Based on DSGE model

The Dynamic Stochastic General Equilibrium (DSGE) model is now broadly used to explain significant relevance between macro and micro variables. It is built upon the model with nominal frictions both in labour and goods markets, with multiple economic sectors incorporated, and then present the participants in a whole system to reflect the rational decision (See Gambacorta and Marques-Ibanez (2011)).

Regarding the study on the QE, without financial friction, the first attempt has been made by modelling the imperfect substitution among financial assets. For example, Chen et al. (2012) estimated the effects of QE by a standard medium-scale new Keynesian DSGE model, based on the preferred-habitat model proposed by the Vayanos and Vila (2009), where the investors have a heterogeneous preference for the asset with different maturities. Notably, they augmented the exogenous market segmentation into the model by setting up two groups of the household. One is the restricted household who can only buy long-term bonds. And the other is the unrestricted household who can invest both short and long-term bonds with transaction costs, which is defined as a premium over the quantity of long-term issued by the government, in purchasing long-term bonds. And that cost would be reduced when the long-term yield is dropped after the central bank conducts large asset purchasing. Therefore, a central bank can affect the macroeconomy by purchasing the long-term government bonds from the public.

There is a growing number of studies confirmed the crucial role of the financial intermediary in propagating the decision of the monetary policy. For instance, Gerali et al. (2010) reported that the bank deposit in the EU area was more than three-quarters of the short-term financial wealth of the household sectors. And the bank lending accounted for nearly 90% of corporate liabilities. Reasonably, the DSGE model has dramatically developed, particularly after the financial imperfection and bank capital were described by the modellers. And the pioneering works have been done by Curdia and Woodford (2010), Gertler and Kiyotaki (2010), Del Negro et al. (2011), Gertler and Karadi (2011), and Breedon et al. (2012)).

Gertler and Karadi (2010),(2011) studied the effects of unconventional monetary policy based on a DSGE model by introducing financial intermediate and financial friction. The model is built up with the assumption of an agency problem, which constrained the borrowing from the financial intermediary to the household. Hence the net return from lending abstracts monitoring cost must be higher than the payback to the deposit. During the period of the financial crisis, the leveraged financial intermediary is depreciated, and then the central bank can increase the asset price by buying private assets and in turn, improve the balance of the private sector. Besides, they can offer a state-contingent subsidy to directly make effects on bank liability and the quantities of loans. Based on the framework built by Gertler and Karadi (2011), Gertler and Karadi (2012) introduced the long-term government bond in the model, which has a fixed supply and belongs to the bank's balance sheets. They study the effects of two rounds of the QE took place in November 2008 and March 2009 by Federal reserve, and they found that the QE would be less effective if the private asset purchases are replaced by government bond acquisitions.

The closest model to our work is by Le et al. (2016), they set up a DSGE model based on Smets and Wouters (2007, SW) and extended it by augmenting the financial accelerator. More explicitly, the intermediate goods producer is assumed to operate with borrowing from the bank, which requires an external finance premium. By incorporating the idea of financial constraint, the money is treated as the cheapest type of collateral. Then with the unconventional monetary policy like the QE, the net worth will be expanded while holding more liquidity. Consequently, the finance premium would be directly reduced then boost the investment and the whole economy. More importantly, they estimated the model by indirect inference method and demonstrated the monetary policy could affect the bank lending by adjusting the supply of money through the QE, particularly during the period of the ZLB crisis.

2.6 Conclusion

In this chapter, we review the historical studying on unconventional monetary policy of QE. There is a broad consensus that QE can significantly and positively affect the economy by a variety of channels. And I have also collected substantial evidence to support the crucial role of QE in the UK economy during the aftermath of the crisis, including the

ZLB period.

Notably, after being challenged by shocks from the financial market, the crucial role of the financial intermediary is unprecedentedly addressed. And considering the DSGE model has been widely adopted by the central bank during the last two decades, we introduced several pioneering DSGE models with the dynamic changes of the financial system and banking sector.

By review, we found that the modelling on unconventional monetary policy of QE is well established, and more researchers are following the trail. However, few studies are focusing on the credit transmission mechanism in the UK, and very few works are built on DSGE model augmenting the financial system. Thus we will build up a medium-to-large size New Keynesian DSGE model with the transmission mechanism of bank lending. Different from the past, our model will be estimated and tested by the indirect inference method.

Chapter 3

SOE-DSGE Model with Quantitative Easing

3.1 Introduction

In this chapter, I will build up a two-version DSGE model and switch between them covering the situations when the economy gets into the ZLB crisis and out of it. Then calibrate it with quarterly data over 1985Q1 to 2016Q4, which covers the UK circumstance of financial and ZLB crisis. The closest work is from Le et al. (2016). They borrowed the framework set up by SW (2007) but extended it by integrating the concept of the financial accelerator. When the economy gets into the ZLB crisis, beyond the traditional monetary regime, the QE will be employed. Notably, they assumed the role of M0 is not only working in setting the short-term interest rate on the government bond but also in working as collateral against bank lending. Furthermore, they allowed heterogeneity in both price and wage setting with hybrid models following Le et al. (2012). Based on the past works, I would contribute to two aspects. Firstly, I extend the model to an open economy framework following the Armington (1969) and allow the substitution elasticity between the domestic and foreign products by CES preference referring to Meenagh et al. (2010) and Minford (2015). Secondly, I would apply the model to the unfiltered non-stationary UK data, then simulate and estimate with the indirect inference method.

The rest of the chapter is structured as follows: 3.2 introduces the model structure without the QE and ZLB crisis. 3.3 describes the model structure with the QE and ZLB crisis. 3.4 Statistically describes the data and structural shock series. 3.5 Introduces the calibration details

3.6 Concludes the chapter.

3.2 The Model Framework without ZLB Crisis

In this section, I set up a small open economy model with the following sectors: household, entrepreneur (intermediate goods producer), final goods producer, financial intermediary, capital producer, and a central bank. Generally, the household can consume both of domestic and imported goods with a preference bias towards the home products; meanwhile, they work as the labour and receive the income. The intermediate goods producer produce with the input of labour and capital, while the capital needs to be purchased from capital producer by net worth and external finance. Then the basic SW07 framework is modified by augmenting the financial accelerator (See Bernanke, Gertler, and Gilchrist (1999)), which is described by characterizing the contract between financial intermediary and entrepreneur. To be explicit, the entrepreneur is subject to an idiosyncratic shock, which causes a premium over the risk-free rate required by the financial intermediary against lending. In terms of other sectors, perfectly competitive final-goods producer aggregate the intermediary goods as retail goods and then sell them to the household. Notably, the wage and price-setting are following "Hybrid model" proposed by Le et al. (2011), that part of labour and intermediate goods are from the competitive market, and the remaining are from the imperfectly competitive market. Capital producer exists in a perfectly competitive sector, and each period they produce capital with installed capital and new investment. Finally, the aggregate output is converted into consumption, investment, capital utilized goods, and net export.

3.2.1 Representative Household Sector

Each period, a representative household will choose consumption and labour to maximize their non-separable utility function[1], which is de-

[1]Refer to Merola(2014),the non-separable property of the utility function implies that consumption will also depend on expected employment growth. Therefore, when the inverse of elasticity of the intertemporal substitution is smaller than one $(\sigma_c, \sigma_l < 1), consumption and labour supply are complements$

fined as following:

$$U = maxE_0\{\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\sigma_c}(C_t - hC_{t-1})^{1-\sigma_c}\right]exp\left(\frac{\sigma_c - 1}{1+\sigma_l}L_t^{1+\sigma_l}\right)\} \tag{3.1}$$

Where β is the discount factor. C_t is consumption and $h \in (0,1)$ denotes the intensity of habit formation and introduces non-separability of preferences over time. L_t is labour hours supplied. σ_c and σ_l are inverse of the elasticity of inter-temporal substitution for consumption and labour supply respectively. E_t represents the rational expectation operator. Then the inter-temporal budget constraint for household is defined as follows:

$$P_tC_t + B_t + D_t + S_tB_t^f \leq R_{t-1}B_{t-1} + R_{t-1}^fS_tB_{t-1}^f + R_{t-1}D_{t-1} + W_tL_t \tag{3.2}$$

Where W_t is the nominal wage offered by the entrepreneur, and gross nominal interest rate R_t is equal to $(1 + r_t)$.Similarly, Foreign gross nominal interest rate is R_t^f. B_t and B_t^f, represent domestic and foreign bond respectively.The disposable income can be put into the bank as deposit D_t. Q_t is real exchange rate and defined as $Q_t = \frac{P_t^f}{P_t}S_t$, which is treated as import price related to domestic price level. S_t is the nominal exchange rate that is defined as the value of the domestic currency on one unit of foreign currency. P_t^f represents the consumption goods price from a foreign country. According to Minford (2014), the foreign bond price is assumed to be the cost at what foreign consumption basket would cost. P_t is the general price level of the home country.P_t^* is the foreign country general price level, and related to domestic currency will be $P_t^*S_t$. Moreover, here we assume exports goods from the domestic country have little impact on the rest of the world so that $P_t^* \approx P_t^f$.

The household will maximises the utility (equation 3.1) subject to the time and budget constraints (equation3.2), with respect to the $C_t, B_t, B_t^f andL_t$. The Lagrangian function will be set as follows:

$$L_0 = E_0\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{1-\sigma_c}(C_t - hC_{t-1})^{1-\sigma_c}\right]exp\left(\frac{\sigma_c - 1}{1+\sigma_l}L_t^{1+\sigma_l}\right) - \lambda_t$$
$$[C_t + \frac{B_t}{P_t} + \frac{S_tB_t^f}{P_t} + \frac{D_t}{P_t} - \frac{R_{t-1}B_{t-1}}{P_t} - \frac{R_{t-1}^fS_tB_{t-1}^f}{P_t} - \frac{R_{t-1}D_{t-1}}{P_t} - \frac{W_tL_t}{P_t}] \tag{3.3}$$

Then we yield the results by first order condition:

$$\partial C_t : [C_t - hC_{t-1}]^{-\sigma_c} \left(\frac{\sigma_l - 1}{1 + \sigma_l} L_t^{1+\sigma_l} \right) = \lambda_t \tag{3.4}$$

$$\partial L_t : \left[\frac{1}{1 - \sigma_c} (C_{t+s} - hC_{t+s-1})^{1-\sigma_c} \right] exp \left(\frac{\sigma_c - 1}{1 + \sigma_l} L_{t+s}^{1+\sigma_l} \right) L_t^{\sigma_l} = -\lambda_t w_t \tag{3.5}$$

$$\partial B_t : \frac{\lambda_t}{P_t} = E_t \left(\beta R_t \frac{\lambda_{t+1}}{P_{t+1}} \right) \tag{3.6}$$

$$\partial B_t^f : \frac{\lambda_t}{P_t} S_t = E_t \left(\beta R_t^f \frac{\lambda_{t+1}}{P_{t+1}} S_{t+1} \right) \tag{3.7}$$

Combine the optimized condition of consumption (equation 3.4) and domestic bond (equation 3.6) to generate consumption Euler equation as:

$$\frac{[C_t - hC_{t-1}]^{-\sigma_c} \left(\frac{\sigma_l - 1}{1+\sigma_l} L_t^{1+\sigma_l} \right)}{P_t} = E_t \left(\beta R_t \frac{[C_{t+1} - hc_t]^{-\sigma_c} \left(\frac{\sigma_l - 1}{1+\sigma_l} L_{t+1}^{1+\sigma_l} \right)}{P_{t+1}} \right)$$

Re-arrange the above equation:

$$\frac{[C_t - hC_{t-1}]^{-\sigma_c} \left(\frac{\sigma_l - 1}{1+\sigma_l} L_t^{1+\sigma_l} \right)}{[C_{t+1} - hC_t]^{-\sigma_c} \left(\frac{\sigma_l - 1}{1+\sigma_l} L_{t+1}^{1+\sigma_l} \right)} = E_t \left(\frac{P_t}{P_{t+1}} \beta R_t \right) \tag{3.8}$$

The budget constraint can also be represented as following while we use real term to measure the domestic and foreign bonds [2]:

$$C_t + b_t + d_t + S_t \frac{P_t^f}{P_t} b_t^f \leq R_{t-1} b_{t-1} + R_{t-1}^f S_t \frac{P_t^f}{P_t} b_{t-1}^f + R_{t-1} d_{t-1} + W_t l_t$$

The equation can be re-writen as the following with the real exchange rate Q_t.

[2] Since we want to generate a real uncovered interest parity, domestic and foreign bonds here re-presented by real term

$$C_t + b_t + d_t + Q_t b_t^f \leq R_{t-1} b_{t-1} + R_{t-1}^f Q_t b_{t-1}^f + R_{t-1} d_{t-1} + W_t l_t$$

Where b_t and b_t^f are the real amount of domestic and foreign bond respectively. The optimal conditions for domestic and foreign bonds will be :

$$\partial b_t : \lambda_t = E_t \left(\beta R_t \lambda_{t+1} \right)$$

$$\partial b_t^f : \lambda_t Q_t = E_t \left(\beta R_t^f \lambda_{t+1} Q_{t+1} \right)$$

Then we can yield the real uncovered interest parity (RUIP), and the expected expectation or depreciation in real exchange rate can offset by any difference between the domestic and foreign interest rate.

$$E_t \left(\beta R_t \lambda_{t+1} \right) = E_t \left(\beta R_t^f \lambda_{t+1} \frac{Q_{t+1}}{Q_t} \right) \tag{3.9}$$

Since R_t is equals to $(1+r_t)$. Similarly, in the foreign country, $R_t^f = (1 + r_t^f)$.Then the RUIP function will be re-organized as:

$$(1 + r_t) = E_t \frac{Q_{t+1}}{Q_t} (1 + r_t^f) \tag{3.10}$$

3.2.2 Representative Foreign Sector

According to a single-industry version of Armington(1969) model, the total consumption for each household C_t will be differentiated by produced places. Specifically, we distinguish the domestically produced products and imported goods as C_t^d and C_t^f. The utility function for aggregated consumption can be represented via constant elasticity of substitution (CES) index.

$$C_t = [\omega (C_t^d)^{-\rho} + (1-\omega)\zeta_t (C_t^f)^{-\rho}]^{-\frac{1}{\rho}} \tag{3.11}$$

We assume that the domestic consumers have fixed preference bias towards the domestic products, and it is measured by ω; $0 < \omega < 1$. ρ is related to the elasticity of marginal substitution between domestic and foreign goods' variety, which is constant at $\sigma = \frac{1}{1+\rho}$. ζ_t is preference error of demand for imported goods. The total expenditure of consumption is defined as :

$$P_tC_t = P_t^dC_t^d + P_t^fC_t^f \tag{3.12}$$

Where P_t^d is the domestic goods price, and the P_t^f is the foreign goods price in domestic currency. And P_t is the domestic general consumption price index (CPI). The above equation can be re-written as:

$$C_t = p_t^dC_t^d + Q_tC_t^f \tag{3.13}$$

p_t^d and p_t^f are domestic and foreign price relative to the general price level P_t respectively , defined as $p_t^d = \frac{P_t^d}{P_t}$ and $p_t^f = \frac{P_t^f}{P_t}$. As introduced before, $Q_t = \frac{P_t^f}{P_t}S_t$ is real exchange rate. For reiterate, we fix the nominal exchange rate S_t at unity, then the Q_t is assumed as the import price relative to the domestic general price level.

To derive the optimal allocation of domestic and foreign consumption, we set up Lagrangian function as following:

$$L = [\omega(C_t^d)^{-\rho} + (1-\omega)\zeta_t(C_t^f)^{-\rho}]^{-\frac{1}{\rho}} + \lambda(C_t - P_t^dC_t^d - Q_tC_t^f) \tag{3.14}$$

Then through first order condition with respect to C_t^d and C_t^f:

$$C_t^d = (\omega)^\sigma(P_t^d)^{-\sigma}C_t \tag{3.15}$$

$$C_t^f = ((1-\omega)\zeta_t)^\sigma(Q_t)^{-\sigma}C_t \tag{3.16}$$

The symmetric situations for foreign country will be similarly set as :

$$(C_t^d)^* = (\omega^f)^{\sigma f}(P_t^*)^{-\sigma f}C_t^* \tag{3.17}$$

$$(C_t^f)^* = ((1-\omega^f)\zeta_t^*)^{\sigma f}(Q_t^*)^{-\sigma f}C_t^* \tag{3.18}$$

Where $(C^d)^*$ and $(C^f)^*$are foreign demand for their own products and imported goods. Similarly, ω^f is a foreign consumer's home bias. C_t^* is total consumption. And σ^f is a foreign country elasticity of marginal substitution between domestic and imported goods. ζ^* is representing the foreign random preference error to the demand for import. A simple transformation by linearization of equation 3.16 with a first order taylor series expansion around $p^d = \sigma = \zeta = 1$ is:

$$lnC_t^f = lnC_t - \sigma lnQ_t + \widetilde{Constant} + \varepsilon_{im,t} \tag{3.19}$$

Where $\varepsilon_{im,t} = \sigma ln\zeta_t$ denotes the import demand shock.

Similarly, export function with log-linearization will be as:

$$ln(C_t^f)^* = lnC_t^* + \sigma^f ln(1-\omega^f) + \sigma^f ln\zeta_t^* - \sigma^f lnQ_t^* \tag{3.20}$$

Symmetrically, $Q_t^* = \frac{p_t^d}{p_t^*}$, and P_t^* is foreign general price level, so that $lnQ_t^* = lnP_t^d - lnP_t^*$. With the assumption that $P_t^* \simeq P_t^f$, and $Q_t = \frac{P_t^f}{P_t}$. P_t is numerate into one, so $Q_t = P_t^f \simeq P_t^*$, then lnQ_t^* is determined by and P_t^d and Q_t as $lnQ_t^* = lnP_t^d - lnQ_t$. We re-write the equation 3.20 :

$$ln(C_t^f)^* = lnC_t^* + \sigma^f ln(1-\omega^f) + \sigma^f ln\zeta_t^* - \sigma^f lnP_t^d + \sigma^f lnQ_t \tag{3.21}$$

Then we replace the C_t^d, C_t^f with expressions of 3.15 and 3.16 in equation 3.13:

$$\left[\omega\left((\omega)^\sigma (P_t^d)^{-\sigma} C_t)^{-\rho}\right) + (1-\omega)\varsigma_t\left(((1-\omega)\varsigma_t)^\sigma (Q_t)^\sigma C_t\right)^{-\rho}\right]^{-\frac{1}{\rho}} = C_t$$

Continually with:

$$1 = \omega^\sigma (P_t^d)^{\rho\sigma} + [(1-\omega)\zeta_t]^\sigma Q_t^{\rho\sigma}$$

By loglinear approximation for the above equation with first order Taylor expansion,(around the point $P^d \simeq Q \simeq \zeta = 1$) :

$$lnP_t^d = \widetilde{Constant} - \frac{1-\omega}{\omega}\frac{1}{\rho} ln\zeta_t - \frac{1-\omega}{\omega} lnQ_t \tag{3.22}$$

Re-write the export equation by replacing lnP_t^d in equation 3.21 with the expression of 3.22:

$$ln(C_t^f)^* = lnC_t^* + \sigma^f ln(1-\omega^f) + \sigma^f ln\zeta_t^* - \sigma^f(\widetilde{Constant} - \frac{1-\omega}{\omega}\frac{1}{\rho} ln\zeta_t)$$
$$+ \frac{1}{\omega}\sigma^f lnQ_t$$

Therefore , the export demand function will be

$$ln(C_t^f)^* = lnC_t^* + \frac{1}{\omega}\sigma^f lnQ_t + \widetilde{Constant} + \varepsilon_{ex,t} \tag{3.23}$$

$\varepsilon_{ex,t}$ is the export demand shock .And $\varepsilon_{ex,t} = \sigma^f ln\zeta_t^* + \sigma^f \frac{1-\omega}{\omega}\frac{1}{\rho} ln\zeta_t$. $\widetilde{Constant}$ is used to collect the constant term.

In terms of foreign bond market, the evolution of net foreign bond is following the principle that, current account surplus and capital account deficit sum to zero. Current account surplus is the real net exports plus the income flow from foreign bond investment, defined as $(EX_t - Q_t IM_t) + r_t^f b_t^f Q_t$. Capital account deficit captures the decrease in net foreign asset, measured by $(b_{t+1}^f - b_t^f)Q_t$. Thus the evolution of net foreign bond can be expressed as following with the real term:

$$(EX_t - Q_t IM_t) + r_t^f b_t^f Q_t + (b_{t+1}^f - b_t^f)Q_t = 0$$

Re-arragent the above to generate:

$$\Delta b_{t+1}^f = (\frac{EX_t}{Q_t} - IM_t) + r_t^f b_t^f \tag{3.24}$$

3.2.3 Representative Final Goods Producer

Final good producer will package the intermediate goods as final products by Dixit-Stiglitz aggregator $Y_t = \left(\int_0^t Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}$, then sell to households in a perfectly competitive market. To generate the optimal demand of intermediate goods, they maximize the profit function with the constraint of final goods production:

$$Max Y_t P_t - \int_0^1 Y(i)_t P(i)_t di$$

s.t.

$$Y_t = \left(\int_0^t Y_t(i)^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}} \tag{3.25}$$

Where P_t and $P(i)_t$ are the price of final goods and intermediate goods. Y_t and $Y(i)_t$ represent the final goods and intermediate goods respectively. $\lambda_{p,t}$ is an exogenous shocks which cause changes in the elasticity of demand and price mark-up. And it is following the AR(1) process as $ln(\lambda_{p,t}) = \rho_p ln(\lambda_{p,t-1}) + \eta_t^p$.

The optimal demand of intermediate goods can be generated by maximizing their profit:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t \tag{3.26}$$

From zero profit condition, $P_t Y_t = \int_0^1 P_t(i) Y(i)_t di$, the price of final goods will be set as CES aggregate of P(i):

$$P_t = \left(\int_0^t P_t(i)^{\frac{1}{\lambda_{p,t}}} di\right)^{\lambda_{p,t}} \tag{3.27}$$

3.2.4 Representative Intermediate Goods Producer

Following Le et al. (2016), we modify the framework of SW07 model with the concept of financial friction referring to BGG model. Entrepreneurs act as the intermediate goods producer, who hires labour and purchase installed capital with a constant return to scale technology to make intermediate goods $Y_t(i)$. Meanwhile, they purchase the capital from capital producer with the external financed funds and the net worth. They produce the intermediate goods with the following production function:

$$Y_t(i) = K_t^s(i)^\alpha [\gamma^t L_t(i)]^{1-\alpha} \varepsilon_t^\alpha - \gamma^t \Phi \tag{3.28}$$

$K_t^s(i)$ and $L_t(i)$ are two types of input for production : capital services and labour input . α is the parameter to measure the share of capital in production. ε_t^α is productivity shock , which follows ARIMA(1,1,0) process as :

$$ln\varepsilon_t^\alpha = ln\varepsilon_{t-1}^\alpha + \rho_a(ln\varepsilon_{t-1}^\alpha - ln\varepsilon_{t-2}^\alpha) + \eta_t^\alpha \tag{3.29}$$

Each entrepreneur also needs to decide the optimal capital utilization rate by solving the maximizing problem. And the capital services is specified as:

$$K_t^s(i) = Z_t(i) K_{t-1}(i) \tag{3.30}$$

Where $Z_t(i)$ is real capital utilization rate. The income of renting capital services is $R_t^{rental} Z_t(i) K_{t-1}(i)$. And the cost of changing capital utilisation is $a(Z_t(i))K_{t-1}(i)$ [3] . The optimal choice of capital utilisation is

[3]At steady state, $a(1) = 0, andz = 1$.

solved by following equation:

$$maxR_t^{rental}Z_t(i)K_{t-1}(i) - a(Z_t(i))K_{t-1}(i) \tag{3.31}$$

$$\partial z_t : R_t^{rental} = a'(z_t) \tag{3.32}$$

The profit function for entrepreneurs is:

$$P_t(i)Y_t(i) - W_tL_t(i) - R_t^{rental}K_t^s(i) \tag{3.33}$$

To generate the optimal amount of capital and labour, we maximize the profit function with the constraint of production function:

$$\partial L_t(i) : MC_t\gamma^{(1-\alpha)t}(1-\alpha)\varepsilon_t^{\alpha}(\frac{K_t^s(i)}{L_t(i)})^{\alpha} = W_t \tag{3.34}$$

$$\partial K_t^s(i) : MC_t\gamma^{(1-\alpha)t}\alpha\varepsilon_t^{\alpha}(\frac{K_t^s(i)}{L_t(i)})^{\alpha-1} = R_t^{rental} \tag{3.35}$$

MC_t is the marginal cost . Combine equation (3.34) and (3.35) we generate labour demand equation related to the capital:

$$K_t^s = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^{rental}}L_t \tag{3.36}$$

The marginal cost can be derived as:

$$MC_t = \frac{(R_t^{rental})^{\alpha}(W_t)^{1-\alpha}}{\varepsilon_t^{\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{3.37}$$

Following Clavo (1983) contract, each period there will be a fraction ξ_p^s of entrepreneur to re-optimize their setting price. The problem for entrepreneur is how to maximize the profits with the constraint of intermediate goods demand:

$$MaxE_t\sum_{s=0}^{\infty}\beta^s\xi_p^s\frac{\Xi_{t+s}P_t}{\Xi_tP_{t+s}}Y_{t+s}(i)[\widetilde{P_t(i)}(\Pi_{t,t+s}) - MC_{t+s}] \tag{3.38}$$

s.t. intermediate goods demand function

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}Y_t \tag{3.39}$$

Where ξ is used to measure the fraction of intermediate good producer, that will adjust their price level. $\frac{\Xi_{t+s}P_t\beta^s}{\Xi_t P_{t+s}}$ is the firm nominal discount factor. [4] $\Pi_{t,t+s} = \Pi_{k=1}^{s}(\frac{\pi_{t+k-1}}{\pi^*})^{l_p}$. $\widetilde{P_t(i)}$is the chosen optimal price level. MC is marginal cost of intermediate goods production, and has been derived before in equation 3.37. Replace the $Y_{t+s}(i)$ in equation 3.38:

$$MaxE_t \sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}} \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t[\widetilde{P_t(i)}(\Pi_{t,t+s}) - MC_{t+s}] \tag{3.40}$$

Finally, the optimal choice of price can be generated by first order condition with respect to $P_t(i)$:

$$\sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}}[(1-\omega)Y_{t+s}P_t(i)^{-\omega}P_t^{\omega} + \omega Y_{t+s}MC_{t+s}P_t(i)^{-\omega-1}P_t^{\omega}] = 0$$

To be convenient, we set $\omega = -\frac{1+\lambda_{p,t}}{\lambda_{p,t}}$.Then simplify the above equation as:

$$\sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}}(\omega-1)Y_{t+s}P_t(i)^{-\omega}P_t^{\omega}$$
$$= \sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}}\omega Y_{t+s}MC_{t+s}P_t(i)^{-\omega-1}P_t^{\omega}$$

Then the optimal price level chosen by intermediate goods producer is :

$$\widetilde{P_t(i)} = \frac{\sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}} Y_{t+s}MC_{t+s}P_t(i)^{-\omega-1}P_t^{\omega}}{\sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s}P_t}{\Xi_t P_{t+s}} Y_{t+s}P_t(i)^{-\omega}P_t^{\omega}} \frac{\omega}{(\omega-1)}$$

Since each firm would update their price following the same mechanism, the aggregate price index for the intermediate goods from imperfectly competitive market will be :

$$P_t = [\xi_p(P(i)_{t-1}(\frac{\pi_{t-1}}{\pi_t})^{l_p})^{\frac{1}{\lambda_{p,t}}} + (1-\xi_p)(\widetilde{P_t(i)})^{\frac{1}{\lambda_{p,t}}}]^{\lambda_{p,t}} \tag{3.41}$$

Following the Le et al. (2011), we assume the final output is composed of intermediate goods partly from monopoly market (non-perfectly

[4]According to SW2007, the nominal discount factor here is equals the discount factor for the households.

competitive market) measured by ν and partly from perfectly competitive market with $(1 - \nu\ )$. In the competitive market, since the price mark-up is zero, the price of intermediate goods is equal to marginal cost, then the hybrid final goods equation will be defined as follows:

$$P(i)_t^{NC} = MC \tag{3.42}$$

$$P_t^{Hybrid} = \nu_p P_t + (1 - \nu_p) P_t^{NC} \tag{3.43}$$

3.2.5 Labour Union and Labour Packers

According to Smet and Wouters (2003, 2007), the labour markets are consisted of labour unions. Households supplies the homogeneous labour to a labour union, which allocates and differentiates labour services, then sell to labour packers; labour packers pack the labour services from labour union with the aggregator, which is proposed by Kimball (1995), then provide them with intermediate goods producer for the production:

$$L_t = \left(\int_0^t L_t(i)^{\frac{1}{1+\lambda_{w,t}}} di \right)^{1+\lambda_{w,t}} \tag{3.44}$$

L_t and $L_t(i)$ represent the composite labour and differentiated labour services respectively. $\lambda_{w,t}$ measures the shocks to aggregator function, which causes the changes in demand then mark-up,and it is following the AR(1) process as $ln(\lambda_{w,t}) = \rho_w ln(\lambda_{w,t-1}) + \eta_t^w$. The profit function for labour packer is :

$$L_t W_t - \int_0^1 L(i)_t W(i)_t di \tag{3.45}$$

W_t and $W_{t(i)}$are the wage of composite and intermediate labor respectively.Then subject to:

$$L_t = \left(\int_0^t L_t(i)^{\frac{1}{1+\lambda_{w,t}}} di \right)^{1+\lambda_{w,t}} \tag{3.46}$$

By FOC, the optimal demand of labour from labour unions is :

$$L_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t \tag{3.47}$$

The labour unions works as an intermediate between the household and labour packer. Under the Calvo pricing indexation, part of labour unions can adjust their price based on the following optimization problem:

$$MaxE_t \sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} L_{t+s}(i)[\widetilde{W_t(i)}(\Pi^w_{t,t+s}) - W_t^h] \tag{3.48}$$

Where $\Pi^w_{t,t+s} = \Pi^s_{k=1}(\frac{\pi_{t+k-1}}{\pi^*})^{l_w}$. Subject to the labour demand function 3.47, the optimal wage will satisfy the following condition:

$$\sum_{s=0}^{\infty} \beta^s \xi_p^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} [(1-\omega^w) L_{t+s} W_t(i)^{-\omega^w} W_t^{\omega^w}$$
$$+ \omega^w L_{t+s} W_t^h{}_{t+s} W_t(i)^{-\omega^w - 1} W_t^{\omega^w}] = 0$$

Where $\omega^w = -\frac{1+\lambda^w_{p,t}}{\lambda^w_{p,t}}$. Then law of motion of the aggregate wage is:

$$W_t = [\xi_w (W(i)_{t-1} (\frac{\pi_{t-1}}{\pi_t})^{l_w})^{\frac{1}{\lambda_{w,t}}} + (1-\xi_w)(\widetilde{W_t(i)})^{\frac{1}{\lambda_{w,t}}}]^{\lambda_{w,t}} \tag{3.49}$$

Similarly to the price set, here we follow the Le et al. (2012) to build a hybrid wage model. We assume a fixed fraction (ν^w)of labour is from imperfect competitive market and the remaining ((1 − ν^w)) is from competitive market. If the wage is perfectly flexible and mark up equals to zero, then real wage would be equals to the marginal rate of substitution between consumption (equation 3.4) and leisure (equation 3.5). The hybrid wage is then defined as:

$$W_t^{Hybrid} = \nu_w W_t + (1-\nu_w) W_t^{NC} \tag{3.50}$$

3.2.6 Representative Capital Producer

In this subsection, we will discuss the behaviour of capital producer. Refer to SW07, and capital producer takes prices as given in a competitive market. In each period, they purchase the capital left from last period with intermediate goods producer, then combine with the newly invested resources. With every unit of investment, they will produce $\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$ capital. Then the capital evolution equation is:

$$K_t = (1-\delta)K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3.51}$$

According to CEE (2005), capital producers are subject to quadratic investment adjustment costs which is specified as $S\left(\frac{I_t}{I_{t-1}}\right)$, with steady state = 0 , S$' = 0, and S''(\cdot) > 0, I_t$ is investment, and δ is depreciated rate of capital. ε_t^idenotes the random investment shock following AR(1) Progress , specified as : $ln\varepsilon_t^i = \rho_i\varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0, \sigma_i)$. The objective function is profit function of capital producer:

$$MaxE_t \sum_{t=0}^{\infty} \beta^t [P_t^k K_t - P_t^k (1-\delta) K_{t-1} - I_t] \tag{3.52}$$

Then through the first order condition with respect to I_t, we generate the investment Euler equation:

$$\begin{aligned} 1 = \varepsilon_t^i P_t^k (1 - S(\frac{I_t}{I_{t-1}}) - S'\left(\frac{I_t}{I_{t-1}} \frac{I_t}{I_[t-1]}\right) \\ - \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} Pt + 1^k \varepsilon_{t+1}^i S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2\right] \end{aligned} \tag{3.53}$$

3.2.7 External Finance Premium

Due to the asymmetric information friction between borrower and lenders that gives rises to the cost of external finance premium, there is an equation of external finance premium defined for the intermediate goods producer. Each period, intermediate goods producer buys the capital k_t from capital producer with price P_t^k. Then during next period, they can resell capital back to capital producer with the price P_{t+1}^k. Same as assumption set up in BGG (2009), $P(i)_t$ is the relative price of intermediate goods, and $\frac{\alpha Y_{t+1}}{K_{t+1}}$ is the marginal product of capital. Then the expected rate of return of capital for the entrepreneur is:

$$E_t\left[R_{t+1}^k\right] = E_t\left[\frac{P(i)_{t+1}\frac{\alpha Y_{t+1}}{K_{t+1}} + P_{t+1}^k(1-\delta)}{P_t^k}\right] \tag{3.54}$$

Where the δ is the depreciation rate of capital. The $P(i)_{t+1}\frac{\alpha Y_{t+1}}{K_{t+1}}$ is value of marginal product of capital or the rental rate of capital. According to BGG(2009), considering default risk caused by the asymmetry information problem between the entrepreneurs and financial intermediary, the external fiance will be more expensive than internal funds. From the perspective of financial intermediary, if the entrepreneur has defaulted, they will pay the auditing cost and keep what it finds; if the entrepreneur can fully pay back the loan, they will receive the return.

Generally, for the financial intermediary, their return should be no less than the opportunity cost R_tB_t, and B_t is the amount of borrowing of the entrepreneur which can be measured as $(Q_tK_{t+1} - N_t)$.

With the above state-contingent constraints from financial intermediary, the entrepreneur will maximize the profit by choosing the optimal amount of capital. Furthermore, referring to the BGG (2009), the optimal capital purchases should be proportional to the net worth, and determined by the expected discounted rate of return of capital $s_t = E\{\frac{R^k_{t+1}}{R_{t+1}}\}$.

$$P^K_t K_{t+1} = \psi(s_t)N_{t+1}, \psi(\cdot) > 0, \psi(1) = 1 \tag{3.55}$$

Equivalently, we can re-write the above equation as :

$$E[R^k_{t+1}] = \varepsilon^{epr}_t s\left(\frac{N_{t+1}}{P^k_t K_{t+1}}\right) R_{t+1}, s'(\cdot) < 0 \tag{3.56}$$

$s'(\cdot)$ represents the cost of external finance which is related to the leverage ratio. The above equation describes that for each not self-financed entrepreneur, in equilibrium, the discounted rate of return to capital should be equal to the external finance premium. ε^{epr}_t is the finance premium shock, and it is following AR(1) process as $ln\varepsilon^i_t = \rho_i\varepsilon^i_{t-1} + \eta^i_t, \eta^i_t \sim N(0, \sigma_i)$. According to Le et al. (2012), the exogenous premium shock can be treated as a shock to the supply of credit or a shock, which can change the premium. Equation 3.56 expresses the intuition that for the partly self-financed entrepreneur, its return to capital should be equal to the marginal cost of external finance. And the finance premium depends inversely on the net worth to investment ratio. Then in the log-linearized form, the finance premium equation will be as following:

$$E_t r^k_{t+1} - (r_t - E_t\pi_{t+1}) = \chi(qq_t + k_t - n_t) + \xi_t + epr_t$$

The net worth at each period for entrepreneur is given by:

$$N_{t+1} = \varepsilon^{nw}_t \theta V_t \tag{3.57}$$

Where V_t presents the value of entrepreneur's equity. Based on Christen and Dib (2008), we assume that there is a probability θ that entrepreneur survives until the next period. For the entrepreneur who dies out from the market will consume their equity measured by $(1-\theta)V_t$.And ε^{nw}_t represents the shock to equity, and follows the auto-regressive process as :

$$ln\varepsilon_t^{nw} = \rho_{nw} ln\varepsilon_{t-1}^{nw} + \eta_t^{nw} \tag{3.58}$$

The equity for the entrepreneur surviving from last period is measured by gross return on capital minus the external finance cost:

$$R_t^k P_{t-1}^k K_t - E_{t-1}[R_t^k (P_{t-1}^k K_t - NW_{t-1})]$$

Then the net worth evolution can be re-write as following:

$$NW_{t+1} = \varepsilon_t^{nw} \theta V_t = \varepsilon_t^{nw} \theta [R_t^k P_{t-1}^k K_t - E_{t-1}[R_t^k (P_{t-1}^k K_t - NW_{t-1})]] \tag{3.59}$$

We assume that the entrepreneur who is drop out of the market will consume their equity. So here we define entrepreneur consumption as :

$$C_t^e = (1-\theta) V_t \tag{3.60}$$

3.2.8 Monetary and government Policy

In this part,I will firstly introduce the monetary policy without the QE and crisis time of ZLB. In this case, central bank will follow a traditional monetary policy. The nominal interest rate reflects on the deviations of output and inflation from their targeted value.

$$\frac{R_t}{R^*} = \varepsilon_t^r \left(\frac{R_{t-1}}{R^*}\right)^{\rho} \left[\left(\frac{\pi_t}{\pi^*}\right)^{r_p} \left(\frac{Y_t}{Y_t^*}\right)^{r_y}\right]^{1-\rho} \left(\frac{\frac{Y_t}{Y_{t-1}}}{\frac{Y_t^*}{Y_{t-1}^*}}\right)^{r_{\delta y}} \tag{3.61}$$

Where R^*, Y_t^* and π_t^* are the steady state of nominal interest rate, output and inflation respectively. r_p measures the response from inflation . Similarly, r_y and $r_{\delta y}$ determine the response from output and change of output. ρ is the degree of interest rate smoothing. ε_t^r represents the exogenous monetary policy shock, which is following a AR(1) process $ln\varepsilon_t^r = \rho_r ln\varepsilon_{t-1}^r + \eta_t^r$.

Without zero lower bound crisis, we assume M0 is simply determined by the total supply of money M_t via the discount window.

$$M_0 = \psi_0 + \psi_1 M_t + \varepsilon_t^{m0} \tag{3.62}$$

Where $\psi_1 \in (0,1)$. ε_t^{m2} is the money supply shock following AR(1) process $ln\varepsilon_t^{m2} = \rho_{m2} ln\varepsilon_{t-1}^{m2} + \eta_t^{m2}$.

We need another equation to define the money supply. Following Le et al. (2016), the supply of money is assumed to be equal to the (deposits(=credit) + M0)[5] . Then use firms' account balance [6] to express the credit with (K + COLL − NW) . Therefore, total money supply M_t is defined as :

$$M = K + COLL - NW + M0 \tag{3.63}$$

Where the COLL denotes the collateral. Then with the log-linearized form, the equation is :

$$M_t = (1 + v - c - \mu)K_t + \mu m_t - \nu n_t$$

Notably, the collateral is treated as the a fixed proportion of the money. μ ν and c are the ratios of the net worth to money, M0 to money and collateral to money, respectively.

The fiscal authority is set following SW07, and government spending G_t is financed by lump sum taxes. The government budget constraint is defined as follow:

$$P_t G_t + R_{t-1} B_t = T_t + B_{t+1}$$

3.2.9 Market Clearing Condition

The overall resources constraint of whole economy can be integrated by combing household budget constraint and evolution of net foreign assets :

$$Y_t = C_t + I_t + a(Z_t)K_{t-1} + C_t^e + EX_t - IM_t + \varepsilon_t^g \tag{3.64}$$

Where ε_t^g is government spending shock and follow AR(1) process, $ln\varepsilon_t^g = \rho_g ln\varepsilon_{t-1}^g + \eta_t^g$.

[5] Since M2 data captures bank money such as deposit and we assume the deposit move one to one with the bond purchases, the QE's effect would also be detected by the M2

[6] The balance sheet for the firm will be presented in the next section with the balance sheets of other sectors

3.3 The Model Framework with Quantitative Easing and ZLB Crisis

While the interest rate drops into the lower bound, the model will be automatically switched to the version with ZLB crisis, and the unconventional monetary policy of the QE will be employed through the large amount of asset purchasing. As we know, the Great finance crisis(GFC) was more about a liquidity problem rather than an insolvency, which was triggered by the collapse of demand in cash flow. During the GFC period, the financial intermediary were reluctant to make lending to the real economy considering the risk of bankruptcy. Therefore, we assumed the financial intermediary require an amount of collateral against lending as a "skin in the game". Refer to Le et al. (2016), among all types of collateral, money is regarded as the cheapest since it can be recovered directly without loss of value as well as no verification cost. Therefore, under the supplementary tool of the QE, there would be higher liquidity held by private sectors, then finally transferred to the balance of entrepreneur as the net worth, a fraction of which can be held as the collateral to lower the default risk, and then motivates the bank lending with a lower external finance premium.

To be explicit, we assume the central bank makes a deal with the household first. As showing in Table 3.1, the household who holds the extra money will place the disposable income into the bank as a deposit. A higher amount of deposit on the bank balance can lead to more loanable funds, which has been studied by others through the bank lending channel; As explained before, after the liquidity being lent to the entrepreneur who can also use them as the collateral for future lending. In terms of firm' balance, on the asset side, the collateral held in the form of money will rise. With the increasing amount of collateral, the credit of firm will be boosted, so the external finance premium required by the bank will be lower, which can lead to a rise in investment , then further motives the aggregate demand with the counter-cyclical effect of the external finance premium.

Notably, We set ξ as a macro-prudential instrument to regulate bank behaviour. Considering the risk of bankruptcy, we require the financial intermediary to hold a counterpart funds for the assets on their balance. According to Let at al. (2016), Macro-prudential measurement will be built on the Basel Agreements 1 and 2; and it is evolved as an exogenous

I (1) time-series process acting as exogenous shock process rather than modeled with quantities. While processing the empirical study, it will be included in the premium shock term.

When the model is not facing the ZLB crisis, the short-term interest rate is set by traditional Taylor rule. While confronting the ZLB crisis, the traditional monetary policy would be suspended automatically and we fix the short term rate at an exogenous value.

$$r_t = 0.0625\% \tag{3.67}$$

Table 3.1: Balance sheets of each sector in the economy

Firm		Bank		Household		Central bank	
Asset	Liability	Asset	Liability	Asset	Liability	Asset	Liability
$Coll_{NonM}(-)$ $Coll_M(+)$ K(+)	Net worth Credit(+)	Credit(+)	Deposit(+)	Deposit(+) GB (-)	CONS	Borrowing	G B (-) M0 (+)

Note: Resources from Le et al (2014),$Coll_{NonM}$ is the collateral in non-monetary form; $Coll_M$ is the collateral in monetary form; $CONS$ is the consumer saving; $GB$is the government bonds.And + and - are used to describe how the balances change with the quantitative easing.

Based on the analysis above, we will adjust the premium equation with two instruments : ξ and m_t.Then the equation in the log-linearized form will be presented as following:

$$E_t r_{t+1}^k - (r_t - E_t\pi_{t+1}) = \chi(qq_t + k_t - n_t) - \psi m_t + \xi_t + epr_t$$

Where ψ measures how the M0 makes effects to the premium. ξ_t is the macro-prudential instrument. Intuitively, with the given leverage ratio$\frac{N_{t+1}}{K_{t+1}}$,the external finance premium required by financial intermediary would be directly affected by the money supply.

When ZLB is bounded, we allow M0 to target on equilibrium value of credit premium. Explicitly, when credit premium is higher than the steady state, money supply will be adjusted higher to bring it back to normal. ψ_2 is used to measure the response of money supply to the credit premium, and it is expected to be positive.

$$m_t = m_{t-1} + \psi_2(cy_t - cy*) + \varepsilon_{t,zlb}^{m0} \tag{3.66}$$

Where ε_t^m is quantitative easing shock following AR(1) process.

3.4 Log-linearized Model List and Stochastic Shock Process

3.4.1 Log-linearized Model List

In this part, to describe the whole framework, we list all the model equations with the log-linearized form. Each equation is normalised with one endogenous variable. And all the variables are in natural logarithm format, apart from variables are already in the form of percentages and ratios.

Consumption Euler equation

$C_t = C_1 C_{t-1} + C_2 E_t C_{t+1} + C_3(L_t - E_t L_{t+1}) - C_4(r_t - E_t \pi_{t+1}) + eb_t$

$C_1 = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}} C_{t-1} C_2 = \frac{1}{1+\frac{\lambda}{\gamma}} C_3 = \frac{(\sigma_c - 1)\frac{w_*^h L_*}{C_*}}{(1+\frac{\lambda}{\gamma})\sigma_c} C_4 = \frac{1-\frac{\lambda}{\gamma}}{(1+\frac{\lambda}{\gamma})\sigma_c}$

Real Unconverted Interest Rate Parity

$q_t = E_t q_t + r_t^f - r_t$

Labor Demand Equation

$l_t = -w_t + (1 + \frac{1-\psi}{\psi}) rk_t + k_{t-1}$

External Finance Premium Equation without the QE

$E_t cy_{t+1} - (r_t - E_t \pi_{t+1}) = \chi(qq_t + k_t - n_t) + \xi_t + epr_t$

External Finance Premium Equation with the QE

$E_t cy_{t+1} - (r_t - E_t \pi_{t+1}) = \chi(qq_t + k_t - n_t) - \psi m_t + \xi_t + epr_t$

Net Worth Evolution Equation

$n_t = \frac{N}{k}(cy_t - E_{t-1} cy_t) + E_{t-1} cy_t + \theta n_{t-1} + enw_t$

Capital Services Equation

$k_t^s = k_{t-1} + z_t$

Capital Utilisation Equation

$z_t = \frac{1-\psi}{\psi} rk_t$

Hybrid Wage Equation

$w_t^{NK} = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c} l_p} E_t w_{t+1} + \frac{1}{1+\beta\gamma^{(1-\sigma_c)} l_p} w_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t \pi_{t+1} - \frac{1+\beta\gamma^{1-\sigma_c} l_w}{1+\beta\gamma^{1-\sigma_c}} \pi_t - \frac{l_w}{1+\beta\gamma^{1-\sigma_c}} \pi_{t-1} - \frac{1}{1+\beta\gamma^{1-\sigma_c}} (\frac{(1-\beta\gamma^{1-\sigma_c}\xi_w)(1-\xi_w)}{\xi_w(1+(\phi_p - 1)\epsilon_w}) (w_t - \sigma_l l_t - (\frac{1}{1-\frac{h}{\gamma}})(c_t - \frac{h}{\gamma} c_{t-1})) + ew_t$

$w_t^{NC} = \sigma_l l_t - (\frac{1}{1-\frac{h}{\gamma}})(c_t - \frac{h}{\gamma} c_{t-1}) - (\pi_t - E_{t-1}\pi_t) + ew_t^s$

$w_t^{hybrid} = w^w w_t^{NK} + (1 - w^w) w_t^N C$

Hybrid Keynesian Phillips Curve

$\pi_t^{NK} = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c} l_p} E_t \pi_{t+1} + \frac{l_p}{1+\beta\gamma^{(1-\sigma_c)} l_p} \pi_{t-1} - \frac{1}{1+\beta\gamma^{1-\sigma_c} l_p} (\frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{\xi_p(1+(\phi_p - 1)\epsilon_p}) (ar_t^k + (1-\alpha) w_t)) - ep_t$

$\pi_t^{NC} = (1-\alpha)w_t + \alpha r_t^k$
$\pi_t^{hybrid} = w^w \pi_t^{NK} + (1-w^w)\pi_t^{NC}$

Tobin Q Equation

$qq_t = \frac{1-\sigma}{1-\sigma+R_*^k} E_t qq_{t-1} + \frac{R_t^k}{1-\sigma+R_*^k} E_t rk_{t+1} - E_t cy_{t+1}$

Investment Euler Equation

$I_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} I_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t I_{t+1} + \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi} qq_t + ei_t$

Production Function

$y_t = \phi[\alpha k_t^s + (1-\alpha)l_t + ea_t]$

Taylor Rule Equation

$r_t = \rho r_{t-1} + (1-\rho)(r_p \pi_t + r_y y_t) + r_{\delta y}(y_t - y_{t-1}) + er_t$

Quantitative Easing with ZLB crisis

$m_t = m_{t-1} + \psi_2(cy_t - cy*) + errm_{t,zlb}, r_t \leq 0.0625$

Money supply equation without the QE

$m_t = m_{t-1} + \psi_1(M_t - M_{t-1}) + errm_t, r_t > 0.0625$

M2 Equation

$M_t = (1+\nu-\mu)k_t + \mu m_t - \nu n_t$

Foreign Bond Evolution Equation

$b_t^f = (1+r_t^f)d_{t-1}^f + \frac{EX}{Y}\frac{P_*^d}{Q_*} ex_t + \frac{EX}{Y}\frac{P_*^d}{Q_*} q_t - \frac{IM}{Y} m_t$

Export Equation

$x_t = c_t^f + \frac{1}{\omega}\sigma^f q_t + eex_t$

Import Eqution

$m_t = c_t - \sigma q_t + eim_t$

Resource Constraint

$y_t = \frac{c}{y}c_t + \frac{i}{y}i_t + \frac{k}{y}R^k z_t + \frac{c^e}{y}c_t^e + \frac{x}{y}x_t - \frac{m}{y}m_t + eg_t$

3.4.2 Stochastic Shock Process

To determine the dynamics of the model. we set up 15 shocks including two exogenous variable , foreign consumption C_t^f and foreign interest rate r_t^f. The shock process is listed as following:

Government spending shock (market clearing equation)

$eg_t = \rho_1 eg_{t-1} + \rho_2 \eta_t^3 + \eta_t^1$

Preference shock (consumption euler equation)

$eb_t = \rho_2 eb_{t-1} + \eta_t^2$

Productivity shock (production function)

$(ea_t - ea_{t-1}) = \rho_3(ea_{t-1} - ea_{t-2}) + \eta_t^3$

Investment shock (Investment euler equation)

$ei_t = \rho_4 ei_{t-1} + \eta_t^4$

Monetary policy shock (Taylor rule equation)

$er_t = \rho_5 er_{t-1} \eta_t^5$

Price mark-up shock (Hybrid inflation rate equation)

$ep_t = \rho_6 ep_{t-1} + \eta_t^6$

Wage mark-up shock (Hybrid wage equation fro NK)

$ew_t = \rho_7 ew_{t-1} + \eta_t^7$

External finance premium shock (External finance premium equation)

$epr_t = \rho_9 epr_{t-1} + \eta_t^9$

Net worth shock (Net Worth equation)

$enw_t = \rho_{10} enw_{t-1} + \eta_t^{10}$

Money supply shock (M0 equation with crisis)

$errm_t = \rho_{11} errm_{t-1} + \eta_t^{11}$

Money supply shock (M0 equation without crisis)

$errm_t = \rho_{12} errm_{t-1} + \eta_t^{12}$

Export demand shock (Export demand equation)

$eex_t = \rho_{13} eex_{t-1} + \eta_t^{13}$

Import demand shock (Import demand equation)

$eim_t = \rho_{14} eim_{t-1} + \eta_t^{14}$

Exogenous foreign consumption process

$c_t^f = \rho_{15} c_{t-1}^f + \eta_t^{15}$

Exogenous foreign interest rate process

$r_t^f = \rho_{16} r_{t-1}^f + \eta_t^{16}$

3.5 Calibration

In this section, our model will be applied to the UK quarterly data over the period of 1985Q1 to 2016Q4. The whole model contains 128 observations, with all the time series data transferred to the format in per capita, except variables in ratios and percentage. The data resources are mainly collected from the Office of National Statistics (ONS), Bank of England (B0E), Federal Reserve Economic Data (FRED) and Data Stream. The details are introduced in the appendix, Table A.1.

Before evaluate our log-linearized model, I firstly calibrate the structure parameters. I divide them into two groups. The first group determines the dynamics of the model, such as the parameters of the Taylor rule and the elasticity of labour supply. I give the value by the literature for the consistency, and for instance, I make use of the estimated results of the model applied in the Euro area and the US or which has very similar model frameworks to ours. The second group are the steady-state of the models, for example, the investment-output ratio and the capital-output ratio, and the values are obtained from the observable data. I will test the model with the calibrations first, and then if the model cannot be accepted, I will carry out the indirect inference estimation to find out the optimal set of the parameters. The details of the calibration are described as followings.

In the household sector, I set the discount factor β at 0.99 in line with the majority of DSGE model and implies an annually calculated real interest rate equals of 4 per cent [7]. σ_c denotes intertemporal elasticity of consumption, which measures the response of growth rate in consumption to the real interest rate, and the higher the temporal elasticity indicates the consumption growth is more sensitive to the change of real interest rate. According to the SW03 with Euro area data, σ_c is determined at 1.39. Similarly, σ_l is the value of inter-temporal elasticity of labour supply which measures the change of labour with respect to wage, and we define it at 2.83. [8] The external habit formation in consumption λ is equal to 0.7 in line with SW07 and Le et at (2012). The wage stickiness ξ_w is equal to 0.7, indicating there is a relatively high probability of firm sticking to the current wage level, and wage indexation l_w equals to 0.58. The ω_w represents the proportion of labour from

[7] $\bar{R} = \frac{1}{\beta}$, which is equivalent to 4% annually.

[8] In SW03 for Euro data, the inverse of inter-temporal elasticity of consumption $\frac{1}{\sigma_c}$ and inter-temporal elasticity of labour $\frac{1}{\sigma_l}$ are equal to 0.74 and 0.42.

the imperfectly competitive market, and it is defined at 0.7 consistent with Le et al. (2016).

In the firm sector, following SW2007, the degree of price stickiness ξ_p, which measures the probability of the firm can not re-optimize its price, equals to 0.67, implying that the average duration of the price is $\frac{1}{1-\xi_p}$=3. The price indexation l_p referring to the estimated results of the SW model is defined at 0.43. Following Le et al. (2012), we set up price and wage with a hybrid model. The ω_p denotes the proportion of labour and products from the imperfectly competitive market, which is defined at 0.4, then the remaining is from a perfectly competitive market. According to Le et al (2012), the survival rate of capital θ is set equal to 0.99, implying an average duration of the entrepreneur is approximately six years ($\frac{1}{1-\theta}$)), which is also closer to the calibrated value in BGG model. On the production side, the share of capital α is calibrated at 0.3 in line with UK estimates by Gollin (2002). Furthermore, the quarterly depreciation rate of capital is set at 0.0125, indicating an annual rate of 0.05, which is same as the Meenagh et al. (2010) with UK data. The share of fixed cost in production ϕ and elasticity of capital adjustment φ are calibrated with 1.5 and 5.74 respectively which are consistent with Le et al. (2012). Capital utilisation rate ψ is set at 0.05 inconsistent with SW03 and Le et al (2012).

Here we consider the UK as a small open economic entity, and the rest of the world is treated as the foreign country. In the foreign sector, following Meenagh et al. (2010), the preference bias for domestic goods ω is equal to 0.7, implying 70% of the consumption goods is from the own country. Symmetrically, the foreign consumption preference bias ω^f is set at 0.7. The parameter σ represents the elasticity between the domestic goods and imported goods, known as Armington elasticity, and I calibrate its value at unity in line with Meenagh et al. (2010,2012). It indicates that, given a constant amount of domestic goods, one per cent increase in the relative foreign to domestic price leads to one per cent decrease in the number of imported goods. Whereas, σ^f, the equivalent substitution elasticity in a foreign country is defined at 0.7.

For the financial friction, referring to the Bernanke et al. (1999), the elasticity of finance premium with respect to leverage χ is set to equal 0.04, which is the approximately same value as other literature applied in EU or UK. The parameter of the money on the premium equation is φ_1 calibrated at 0.08, implying there will be a 0.08% decrease of credit premium with 1% increase of money supply.

For monetary policy, I choose to keep in line with the Le at al. (2016), where the model is similar to our framework. Regarding traditional monetary policy rule, the response from nominal interest rate to inflationr_p, output r_y and output change $r_{\delta y}$ are valued at 2.3, 0.03 and 0.2 respectively. The interest rate smooth rate ρ is 0.74. And without the ZLB crisis, the money is adjusted by M2 with parameter φ_2 set at 0.05. Moreover, while the economy got stuck in the zero lower constraints, the money supply is targeted on credit premium, and the parameter φ_3 measures the elasticity valued at 0.04.

The log-linearised market clearing condition:

$$y_t = \frac{c}{y}c_t + \frac{i}{y}i_t + \frac{k}{y}R^k z_t + \frac{c^e}{y}C_t^e + \frac{x}{y}X_t - \frac{m}{y}M_t + eg_t$$

The model is calibrated to get certain real and financial ratios by sample average of UK data (1985Q1-2016Q4). Then the consumption over output $\frac{c}{y} = 0.58$; investment to output ratio $\frac{i}{y} = 0.18$;capital to output ratio $\frac{k}{y} = 2.66$;entrepreneur consumption to output ratio $\frac{c^e}{y} = 0.008$;export to output ratio $\frac{x}{y} = 0.24$; and import to output ratio $\frac{m}{y} = 0.25$. The quarterly output growth is equals to 0.55.

Table 3.2: Calibrated Coefficients

Description	Symbols	Value
Household		
Discount factor	β	0.99
Elasticity of consumption	σ_c	1.39
Elasticity of labor supply	σ_l	2.83
External habit formation	h	0.7
Degree of wage stickness	ξ_w	0.7
Degree of Wage indexation	l_w	0.58
Proportion of sticky wage	w^w	0.1
Preference bias in consumption of domestic goods	ω	0.7
Firm		
Degree of price stickness	ξ_p	0.67
Degree of price indexation	l_p	0.43
Proportion of sticky price	w^p	0.4
Entrepreneur Survival rate	θ	0.99
Share of capital in production function	α	0.3
Capital depreciation rate	δ	0.05
Share of fixed cost in production function	ϕ	1.50
Elasticity of capital adjustment	φ	5.74
Elasticity of capital utilisation	ψ	0.05
Monetary policy		
Talyor rule response to inflation	r_p	2.3
Interest rate smoothing	ρ	0.74
Talyor rule response to output	r_y	0.03
Talyor rule response to change of output	$r_{\delta y}$	0.2
M0 response to M2	ψ_1	0.05
Money response to credit growth	ψ_2	0.04
Financial friction		
Elasticity of premium with respect to leverage	χ	0.04
Elasticity of premium with response to money	ψ	0.08

Table 3.3: Steady State values in the Model

Consumption output ratio	$\frac{c}{y}$	0.58
Investment output ratio	$\frac{I}{y}$	0.18
Entreprenuer's consumption output ratio	$\frac{ce}{y}$	0.008
Export output ratio	$\frac{ex}{y}$	0.24
Import output ratio	$\frac{im}{y}$	0.25
Capital output ratio	$\frac{k}{y}$	2.66
Return rate of capital	R_k^*	0.04
Quarterly output growth	$\bar{\gamma}$	0.55

3.6 Conclusion

Based on the aims of the work, to explore how the QE can affect the UK economy with a bank lending channel. In this chapter, I outlined a pair of DSGE models with the unconventional monetary policy of QE to describe the UK economy, including the crisis time of ZLB. Notably, to make the model closer to the UK, we incorporated a small open economy into the model. Then we allowed the model to confront the UK data over the 1985Q1-2016Q4. Before we carry on the indirect inference test in the next chapter, we calibrated the model parameters. And if the calibrated model cannot pass, we will continue with the indirect inference estimation in chapter 4.

Chapter 4

Evaluate and Estimate the Model with Indirect Inference Method

In the previous chapter, we have described the UK economy in reasonable details by a medium-to-large size DSGE model, and then in this chapter, I will evaluate and estimate the model. To my review and knowledge, the DSGE model faced two types of challenges in the past studying. Firstly, though the Bayesian is treated as an advanced technique, the reported confidences are still narrow with fragile evidence of parameters (Schorfheide 2008). Secondly, detrended time-series data may not truly reflect the facts of the economy, since time series data normally exhibit dynamic frequency behaviour and could not be reconciled by the model estimation. Based on the above two points, different from most of the works, I will employ the indirect inference method for model evaluation and estimation. Furthermore, we will employ the un-filtered non-stationary UK data in the period of 1985Q1 to 2016Q4.

The chapter is structured as follows: Section 1 compares the indirect inference (II) method with other popular methods. Section 2 describes the II test procedure and report the results based on calibrated values. In section 3, I point out the advantages of using non-stationary data. In section 4, I study the error properties with ADF and KPSS tests. Section 5, I introduce the II estimation procedures and report the estimation results. Then again test the model based on the estimated parameters. Section 6 studies the empirical performance of the model by impulse response function, variance decomposition and historical shocks decomposition. Section 8 remarks the conclusion.

4.1 Why Indirect Inference ?

During the past decades, numerous econometric methods have been developed for model analysis and estimates[1]. Recent years, Bayesian method is no doubt the most popular approach to analysis and estimates the DSGE model, while its flaws have also been convinced by a growing number of studies. Explicitly, the Bayesian method includes the prior information on the structural parameter, but the justification for the right prior is weak. Thus there is a risk in biasing the results with the incorrect choice of the priors. Additionally, Bayesian judges the model in a non-classical hypothesis testing sense, that treat all modes as false first, then evaluate the probability of being right without a precise line between right and wrong.

Some researchers chose to use maximum likelihood (ML) and generalized method of moments (GMM) to estimate the DSGE model. To identify the parameters of the DSGE models, these estimators mainly rely on the same sample as well as theoretical information on the first moments. so the assumption of the true model can cause the identification problem. Besides, traditional macroeconomic models are usually dynamic specified or data determined so that the models can usually pass the tests. However, the DSGE model is mainly structurally specificity so that the model will be harder to pass the test with classical likelihood test methods for misspecification problem. Evans and Honkapohja(2005) remarked that the likelihood ratio test was rejecting too many good models.

Therefore, how to evaluate a DSGE model more appropriately is one of the critical unresolved-issues in macroeconomic history. In this work, I choose to use a different method called Indirect Inference, which was first proposed by Smith (1990) and then extended by Gregory and Smith (1991,1993), Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2007). The basic idea of indirect inference is to make inferences for the parameters of economic models and find out the best batch of parameters from simulated data and actual data. Different from other simulation-based methods, indirect inference takes use of the auxiliary model, which is entirely independent of the theoretical model. The auxiliary model is featured by a set of parameters, which can be estimated from simulated data or actual data. And the target of the method is to find out a set of parameters, which makes the behaviour

[1] A detailed review can be seen from Canova (2007) and DeJong and Dave (2007)

auxiliary model with simulated data is closest to the one based on the actual data. Durlauf and Blume (2008) concluded: " The auxiliary model serves as a window through which to view both the actual, observed data and the simulated data generated by the economic model: it selects aspects of the data upon which to focus the analysis". Notably, this method can be applied with a non-linear model or the model with any size and complexity.

The indirect inference method applied in this work is firstly proposed by Meenagh et al. (2009) and refined by Le et al. (2011) with Monte Carlo experiments. They compared the power of the indirect inference test with one from the likelihood and found that the power of indirect inference is much higher, especially in the small sample. Details of the indirect inference test and estimation procedures will be introduced in the following sections.

4.2 Indirect Inference Method

The core idea of Indirect Inference is to compare the performances of real data and simulated data by the auxiliary model, which is independent of the theoretical model and represented by a Vector Auto Regression with the exogenous variable model (VARX) following Meenagh et al. (2012b). Generally, it stimulates the data based on the macroeconomic model with given parameters and error distribution. Then use Wald statistic as a criterion to measure the difference between the simulated data and actual data. If the model passes the Wald test, it indicates that the behaviour of simulated data is very similar to the actual data; and the model can explain the economy adequately. If the model fails to pass the test, we will use the indirect estimation to search for the optimal set of coefficients that can minimize the distance between generated data and actual data. The details on how to choose the auxiliary model and the steps of indirect inference test will be introduced in the followings.

4.2.1 Indirect Inference Test Procedures

Step 1 : Calculate the shock process

For a specific variable, we define the residual for a single equation as the difference between the LHS value (actual data) and RHS value. With the values of the structural parameter given, the residuals without rational expectations can be directly backed out by LHS-RHS. While for the equations with rational expectation, firstly, the VAR process will be used to estimate and generated the fitted values one-period ahead for the expectation, then the residuals are calculated by LHS-RHS. We compute the corresponding coefficients (persistence of the shock process) and the innovation of the shock process by OLS regression with the generated residuals series.

Step 2: Generate simulated data by bootstrapping innovations

The innovation we discussed in the previous step is assumed to be the driver of the shock process, so here we bootstrap the shocks' innovation to generate the sample of shock, instead of assuming shocks is following an asymptotic distribution. In more details, we assume that shock sample is at t × n matrix, with t denoted as the number of periods, and n as the number of innovations in the model. While performing the bootstrap, the time vector is set to make sure that all the innovations are randomly chosen due to the interactive volatility of errors. Practically, we randomly draw a time vector from the sample and add it back to the shock process then draw another time vector, to guarantee that each time vector has been drawn with the same possibility. By repeating the procedures for t times, we can generate another sample of innovation, which has the same size as well as distribution with the original shock sample.

Then we firstly simulate the model with the all-zero of innovations (shocks), and then compute the Type II residuals that are defined as the difference between the actual data and Type II iterations (Referring to the definition from Fair and Taylor (1983)). To obtain a sample with the size of 1000 different scenarios, we add back the bootstrapped shocks as well as the Type II residual into the model. Then the difference between the simulated data and original data will be the effects of bootstrapped shocks. In the last step, the effects of deterministic trends (BGP) on the sample will be added back.

Step 3: Compute the Wald statistic

We choose to use the Wald statistic to justify the model performances with the null hypothesis, that the true economic model is our structural model. If the model fits the actual data at 95% confidence level, the wald statistic should be less then the 95the percentile from the simulated data. Notably, because we use bootstrap to generate the small sample distribution instead of using the asymptotic distribution of the Wald statistic, which indicates that the estimated Wald statistic is not following a Chi-squared distribution, and a less than 90 % Wald statistic can not necessarily enough mean to pas the test. Thus we introduce the transformed wald, which is equal to 1.645 when the Wald statistic equals to the 95% from the simulated data. Therefore, when the transformed wald of the actual data is greater than 1.645, the model would be rejected by the actual data; otherwise, we would accept the null hypothesis that the structural model can replicate the performance from actual data.

According to Le et al. (2011), there are two types of Wald statistics: Full Wald and Directed Wald. In the Full Wald, all the endogenous variables form the model will be included in the auxiliary model. However, considering that the more variables and lags incorporated in the model, the higher the possibility that the model will be rejected. Therefore, we choose to focus on specific aspects of the model's performances by considering key endogenous variables in the auxiliary model. Since the target of the work is to evaluate the QE effects on main economic variables, we choose three variables: $y, r, candinflation$.

Before we calculate the wald statistic, the OLS will be employed to compute the parameter vector of auxiliary model for both actual and simulated data. Here we denote the structural parameter vector and the auxiliary parameter vector as θ and β respectively. Hence, given the structural parameter, the estimated auxiliary parameter based on simulated data and given structural parameter will be $\widetilde{\beta}_s(\theta)$, s=1,...S. And the estimated auxiliary parameter from observed data is represented as $\widehat{\beta}$.The Wald statistic is then defined as :

$$WS = (\widehat{\beta} - \overline{\widetilde{\beta}(\theta)})' W(\theta)^{-1} (\widehat{\beta} - \overline{\widetilde{\beta}(\theta)}) \tag{4.1}$$

$\overline{\widetilde{\beta}(\theta)}$ is average value computed from :

$$\overline{\widetilde{\beta}(\theta)} = \frac{1}{1000} \Sigma_{s=1}^{1000} [\widetilde{\beta_S}(\theta)] \tag{4.2}$$

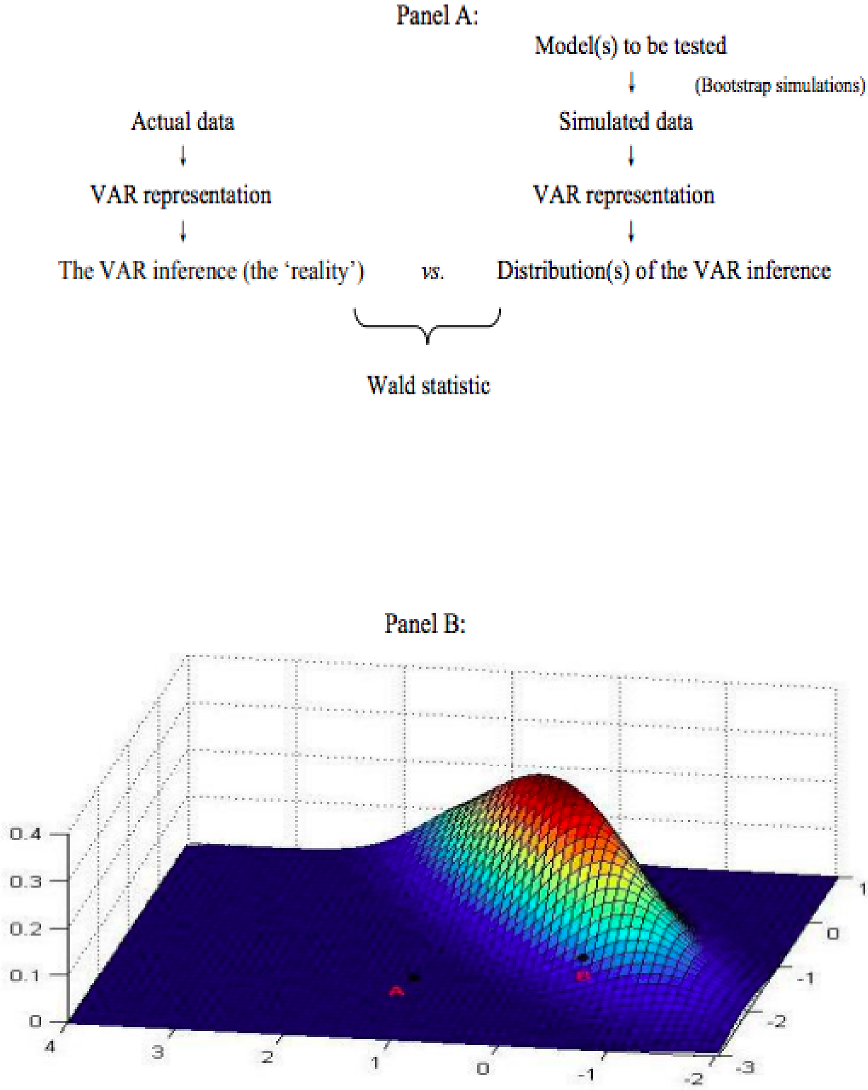
$W(\theta)$ is the variance - co variance matrix of ($\widehat{\beta} - \overline{\widetilde{\beta}(\theta)}$)),which is used to measure the distance between the actual estimated parameter and the average of the simulated ones . Then transformed Mahalanobis Distance defined based on normalised t-statistic as following :

$$T = (\frac{\sqrt{2WS^a} - \sqrt{2k-1}}{\sqrt{2WS_i^{95th}} - \sqrt{2k-1}}) \times 1.645 \tag{4.3}$$

where $w^a$is the Wald statistic on the actual data and w^{95} is the Wald statistic for the 95% of the simulated data. If the T value is less than 1.645, the hypothesis will not be rejected indicating the model fit the observable data.

Overall, the indirect inference testing procedure can be concluded by the diagram and figure 4.1 according to the Minford and Ou (2010). Panel A describes the steps we discussed above; Panel B shows how 'reality' is compared to the prediction by the Wald test while only two parameters are considered. The mountain in the figure represents the corresponding joint distribution from the model simulation. The real data-based estimates are at point A indicating that the theoretical model fails the test since the model predicts is too 'far away' from the suggested ' reality'. If the real data-based estimates are at point b, then the 'reality' is captured by the model's joint distribution. Moreover, the Wald statistic is just used to measure the distance.

Figure 4.1: The Principle of Testing using Indirect Inference



Sources:Minford and Ou (2010)

4.2.2 The Choices of Auxiliary Model

Many literature have discussed the fact that a log-linearized DSGE model can be represented as a vector autoregressive and moving average restricted VARMA model (See Canova (2005), Del Negro et al. (2007) and Killian (2007)), and is rewritten by a finite order reduced VAR model. According to Meenagh et al. (2013) and Le et al. (2015), the approximation of the reduced form of DSGE model can be represented as a co-integrated VAR with exogenous variables (VARX) model even if the shock or exogenous processes are non-stationary. Hence we employ VARX as an auxiliary model to evaluate how closely a DSGE model fits the data. Then we assume that the structural model can be rewritten in the log-linearized form as following:

$$A(L)y_t = B(L)E_t y_{t+1} + C(L)x_t + D(L)e_t \quad (4.4)$$

Where y_t is the vector of endogenous variables, and e_t is the vector of error term with i.i.d. x_t is the vector of exogenous variable which are non-stationary and following a unit root process as :

$$\Delta x_t = \alpha(L)x_{t-1} + d + c(L)\epsilon_t \quad (4.5)$$

Where ϵ_t is same as e_t is following i.i.d with zero means. L is lag operator and A (L), C (L) are polynomial functions with roots outside the unit circle. Then general solution of y_t will be :

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_{tN}(L)\varepsilon_t \quad (4.6)$$

Since y_t and x_t are non-stationary , and there are polynomial functions outside the unit circle. The solution has p co-integrating relationship:

$$y = [I - G(1)]^{-1}[H(1)x_t + f] = \Pi x_t + g \quad (4.7)$$

The long run solution of the model will be as following:

$$\bar{y}_t = \Pi\bar{x}_t + g \quad (4.8)$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \quad (4.9)$$

$$\xi_t = \Sigma_{i=0}^{t-1}\epsilon_{t-s} \quad (4.10)$$

Where $\bar{y}_t$ and $\bar{x}_t$ are long run solution of y_t and x_t respectively. And $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$, where$\bar{x}_t^D$is deterministic trend equals to $[1 - \alpha(1)]^{-1}dt$ and $\bar{x}_t^S$ is the stochastic trend and equals to $[1 - \alpha(1)]^{-1}c(1)\xi_t$ .Then the y_t is re-presented as following VECM form :

$$\Delta y_t = -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\varepsilon_t \quad (4.11)$$

We can combine the last two terms as a mixed Moving Average process. This indicates that the VECM can be represented as VARX form approximately.

$$\Delta y_t = K(y_{t-1} - \Sigma x_{t-1} + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{4.12}$$

Where $\bar{x_t} = [1 - a(1)]^{-1}[d + \xi_t]$ and $\bar{y_t} = \Pi \bar{x_t} + g$, the VECM can be written as :

$$\Delta y_t = K[(y_{t-1} - \overline{y}_{t-1}) - \Pi(x_{t-1} - \overline{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{4.13}$$

Then the equation can be rewritten in the form of a co-integrated VARX (1) that will be used as the auxiliary model:

$$y_t = [1 - K]y_{t-1} + K\Pi x_{t-1} + n + t + q_t \tag{4.14}$$

The time trend and deterministic trend are included in $\overline{x}_t$, which is used to control the impacts from past shocks through a long-run path of endogenous and exogenous variables. Then the estimation of parameters can simply carried out by classical OLS method with equation 4.14. And this procedure has been approved by Meenagh et al. (2012) using Monte Carlo experiments.

4.2.3 Indirect Inference Test Results Based on Calibration

Before we conduct the indirect inference estimation, we perform the indirect inference test with the calibrated parameters set. If the model does not pass the test, the calibrated parameters will be re-defined by Indirect Inference Estimation. Following the procedures discussed above, we generate the results and report in table 4.1. As we mentioned,TMD t statistic is used instead of Wald statistic to justify the results of the Wald test.

Notably, based on the work from Le et al. (2010, 2011, 2014,2105) they made an empirical comparison by test the model with increasing number or order of variables and suggested that researchers should focus on the variables, which can help their model to explain, otherwise the model is likely to be rejected. Additionally, they found that macro models couldn't match the details of consumption and investment, even when they can match the core variables like output, inflation and interest. They claimed the possible reason is that the actual data on the variable like consumption and investment is poor, e.g., the durable consumption goods are regarded as the capital, while are routinely in the consumption. Thus to perform indirect inference Wald test, we choose the output, inflation and interest rate [2] into our auxiliary model. Since these three variables can represent a general inner relationship of the model as well as describe the economy in full. Recall that the VARX (1) in equation 4.14 is employed as the unrestricted auxiliary model for model simulation and estimation. Then a VARX (1) with three endogenous variables will be read as followings.

$$\begin{pmatrix} y_t \\ \pi_t \\ r_t \end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{22} & \beta_{33} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{22} & \beta_{33} & \beta_{34} \end{pmatrix} \begin{pmatrix} T \\ e^{YT} \\ b^f_{t-1} \\ Const \end{pmatrix} + \begin{pmatrix} \zeta 1 \\ \zeta 2 \end{pmatrix}$$

Where e^{YT} is the lagged productivity trend, and measured by the Solow residual, back out from calibrated production function. T is the time trend to capture the deterministic rend. b^f_{t-1} is the lagged level of the net foreign asset to capture the effect of net foreign debt on the fluctuation of real exchange rate. The β^s used in the Wald statistic of equation 4.1 here includes the OLS estimates of ($\beta_{11}\beta_{12}\beta_{13}\beta_{21}\beta_{22}\beta_{23}\beta_{31}\beta_{32}\beta_{33}$) and the variance of the fitted stationary residuals $\zeta 1$ and $\zeta 2$ based on simulated data.

In table 4.1, not surprisingly, the TMD value for auxiliary model is 2.58, which is higher than the critical value of 1.65, and indicates that the model does not fit the data, and cannot explain the data behaviour with calibrated parameters set. Since we cannot know the problem is from the model itself or the calibrated parameters, it is necessary to search for the optimal set of parameters, which can minimise the Wald statistic and re-test the model. Then the Indirect inference estimation will be employed.

Table 4.1: Wald Test Results Based on Indirect Inference Estimation

Variable included	Trans Value	Wald Value
r,inflation,y	2.58	58.89

[2]The target of test is to know whether the simulated data by structural model can mimic the real economy , we still choose to include the policy rate , though the policy rate is almost at ZLB bound since 2009

4.3 Why Non-stationary ?

Macroeconomic data are generally non-stationary data, which is due to the uncertainty around the economy behaviour during the long-term. Generally, we can conclude them into two types: one is trend stationery that the process is around a deterministic trend; the other one is difference stationery where stochastic non-stationary follows a unit root.

Traditionally, the business cycle model focuses on explaining the temporary and short-term impact and examines the policy effects on the economics between the boom and recession. Thus the economist would like to map the data to stationary by detrending methods. However, with the development of empirical works, the flaws of employing stationary data started to be controversially discussed (See Ferroni (2011); Canova and Ferroni (2011); Gorodnichenko and Ng (2010) and Canova (2014)). They proposed that the detrened data eliminate or amplify the dynamics of the data, which may cause potentially non-negligible effects, piratically on the permanent shocks in the stationary detrended data. For instance, Andrle (2008) concluded that detrending data could not explain the movements of data, particularly when the permanent shock has a significant impact on the business cycle.

The mostly applied detrending methods are Hodrick-Prescott (HP) filter or Band Pass (BP), and their flaws have been broadly studied. Firstly, to our knowledge, it has been argued a lot on using a statistically based detrend approach or an economic-based approach in empirical work. Since both BP and HP filters are not based on model theory, but statistical propriety of the data, an amount of evidence shows that extract from the data information can lead model unfit. As Cogley and Nason (1995) remarked, the HP and BP filter are a mathematical tool, which is not based on the model theory and can lead to mistakes in identifying the driving process of trend behaviour.

Secondly, while researchers use HP or BP to filter the data by decomposing the data into the trend and cyclical components, it may generate the cycles, which are not exited. Besides, it can distort key business cycle stylised facts between the cyclical components of the variables. As concluded by Harvey and Jaeger (1993) that the HP filter can lead to the spurious cyclical behaviour problem.

Others may consider using the linear detrending method when the deterministic trend is assumed in the data. Nevertheless, according to Canova (1998), linear detrend is not accurate when the data have a stochastic trend. Since it cannot isolate fluctuations, the data may not be stationary even if we linear detrend it. Apart from that, first-order differencing has also been used to detrend for the data with a stochastic trend. However, it could magnify the component with high-frequency noise, which leads to non-negligible influences of permanent shock.

Overall, detrended data would be no benefit to capture the real dynamics of the business cycle. I choose to use the non-stationary data, which can explain the deviation of time series data from steady time trends in the long term. Additionally, Meenagh and Minford (2012) mention that, Great recession since 2008 clearly shows that OECD economy suffered a massive drop that cannot be forecast and reversed, as well the output may not resume to its former path. Thus considering the UK is a Small open economy experiencing inevitable fluctuations due to size and foreign economy shocks, the universal multiplier should not be suitable for its situation but better for the country like the US which keeps similar developing pace.

Practically, following Le et al. (2011) and Meenagh et al. (2012), the endogenous variables are co-integrated on a set of exogenous variables with a stationary residual, and this correlation can be denoted as a vector error correction model (VECM) model or Vector Auto Regression variable models (VARX) of the auxiliary model. Then the auxiliary model can be used to present the solution of log-linearized models.

4.4 Indirect Inference Estimation

4.4.1 Indirect Inference Estimation Procedures

The result from the Wald test based on calibration shows that the model could not fit the actual data, but we cannot recognize the problem is from the model itself or the calibrated parameters. Therefore, in this section, we use the indirect inference estimation method to find out the set of parameters that can make simulated data and observed data look statistically the same, with the distance between the simulated data and actual data minimal from the auxiliary model. Firstly we assume the model structural is true to the fact, then test by Wald repeatedly with different sets of the parameters.

Notably, a simulated annealing method (SA) [3] is used to search for

[3]Simulated annealing is a method used to find the solution of an optimization problem. https://uk.mathworks.com/discovery/simulated-annealing.html

the optimal parameter set within a pre-defined upper and lower bounds. The SA method takes place over a wide range around the calibrated values (Initial value), and it can automatically loop over the test procedure, avoiding to be trapped in a local minimum until find out a global minimum value of Wald statistics. At each step, it considers neighbouring states of the current status, and decides to move to other states or stay in the current one. Explicitly, the SA process started from an initial choice of the parameter vector to perform the Wald Test. Then try with a new point with a different set of parameter. If the Wald Statistic from the new point is lower than the previous one, then it will move to the new state, which will be regarded as the new current state. It is also possible that the SA algorithm moves to a point with larger Wald, but this probability is decreased by the increasing number of points evaluated. After a number of best points have been tested, the search will be widened by a higher acceptance probability. The indirect inference estimation procedures are as following:

Step 1: we started from an initial parameter set (calibrated parameter set)to conduct an Indirect inference test as we discussed in the previous sector, to obtain first TMD_1.

Step 2:Randomly generated the second set of parameter set , and generate the TMD_2 .

Step 3:We will compare the values of TMD generated , if $TMD_1 < TMD_2$, first set of parameter will still be chosen as starting point to continue search and repeat the step2. If $TMD_1 > TMD_2$, then the parameter set 2 can be chosen as the new starting point. Though there is a chance, we will move to a worse choice, which is the advantage of a mechanism to avoid being trapped in a local optimum, the probability of this happening can be decreased with a higher number of points being tested.

Step 4: The programming will stop until the acceptable value of Wald statistics or TMD value is found, or we repeated the procedures until the up limitation of iterations.

To conclude, it is apparent that the advantage of the indirect inference estimation is to test the model independently against the data. We use Indirect inference estimation to find the optimal structural parameter which can make the model the model fit the data best, while if the minimum value of TMD is still can not be accepted, we can suspect the model problems.

4.4.2 Indirect Inference Estimation Results

By employing the indirect inference estimation method, we have found an optimum set of the parameters for the model. As showing in table 4.2, nearly all the parameters have been adjusted from the calibrated value. [4] . And in the following part, I will tempted to find out how the indirect inference estimation help us to move the calibrated parameter towards the optimal set.

In the household sector, the intertemporal elasticity of consumption σ_c has decreased by 4.83 %, indicating consumption is less sensitive to changes in the real interest rate comparing with the calibrated value. The external habit in consumption is increased to 0.79. The elasticity of labour supply drops by 3.57%, implying the works are less willing to change the working hours compared with the calibrated values when the wage is adjusted.

In terms of firm sector, the elasticity of capital adjustment and capital utilisation are both increased. However, the share of capital in production drop from 0.3 to 0.15, showing a less weight of capital investment in economic growth. Degrees of price indexation are lower than calibrated values with 0.26, while the degrees of wage indexation is increased to 0.68. Comparing both changes, the degree of the price is much lower than the degree of wage, which indicates that wage inflation is more persistent comparing with price inflation. The proportion of sticky price is decreased to 0.38, which means a lower percentage of New Keynesian prices is estimated. Furthermore, the proportion of sticky wage is slightly reduced by 20%.

In the finance friction, the elasticity of premium with the response to the leverage increases to 0.06, implying a more significant impact from leverage to premium. The parameters ψ_1 used to measure the impact of QE on the premium is decreased from 0.08 to 0.06, indicating the premium responses less sensitive than the calibration.

Regarding the monetary policy, the responses from interest rate to inflation is higher, and the parameter is increased to 2.55, implying monetary policy is estimated to be more responsive to the fluctuation of inflation. The estimated parameter of the policy response to output change is nearly the same as calibration, while the response to the output is lower. Besides, interest rate smoothing rate is decreased from

[4] It is necessary to address that the discount factor β, the depreciation rate δ , and entrepreneurs' survival rate θ are all fixed.

Table 4.2: Structural Parameter Estimates

Description	**Symbols**	**Calibration**	**Estimation**
Household sector			
Discount factor	β	0.99	0.99
Elasticity of consumption	σ_c	1.39	1.26
Elasticity of labor supply	σ_l	2.83	2.70
External habit formation	h	0.7	0.79
Degree of wage stickiness	ξ_w	0.7	0.83
Degree of Wage indexation	l_w	0.58	0.68
Proportion of sticky wage	w^w	0.1	0.08
Preference bias in consumption	ω	0.7	0.7
Firm sector			
Degree of price stickness	ξ_p	0.67	0.85
Degree of price indexation	l_p	0.43	0.26
Proportion of sticky price	w^p	0.4	0.38
Entrepreneur Survival rate	θ	0.99	0.99
Capital share in production	α	0.3	0.15
Capital depreciation rate	δ	0.05	0.05
Fixed cost in production	ϕ	1.50	1.54
Elasticity of capital adjustment	φ	5.74	8.02
Elasticity of capital utilisation	ψ	0.05	0.13
Monetary policy			
Talyor rule response to inflation	r_p	2.3	2.55
Interest rate smoothing	ρ	0.74	0.63
Talyor rule response to output	r_y	0.03	0.02
Talyor rule response to output change	$r_{\delta y}$	0.2	0.20
M0 response to M2	ψ_1	0.05	0.01
Money response to credit growth	ψ_2	0.04	0.13
Financial friction			
Elasticity of premium to leverage	χ	0.04	0.06
Elasticity of premium to money	ψ	0.08	0.06

0.74 to 0.63. For unconventional monetary policy, during the normal period, the M0 is less sensitive to the change of M2 by a huge drop from 0.05 to 0.01. When the QE is employed as the unconventional monetary policy to boost the economy during the crisis time of ZLB, the money response to credit change is increased from 0.04 to 0.13, indicating we should have a stronger response from the money supply to boost the economy.

The estimated parameters allow the structured model closer to the actual data, which minimises the TMD t statistic calculated by the auxiliary model.Thus in this section, we will re-test the model with estimated parameters. The results in table 4.3 show that the auxiliary model is significantly not rejected with a Wald percentile of 41.67 and normalised Mahalanobis distance of 0.75, implying that the hybrid model performs well in mimicking the UK economy and generated data is substantially closer to the actual ones. Then we would like to tell whether each estimated parameter of the auxiliary model generated from the actual data can lie between the 95% the upper and lower bound of the parameters from the simulated data. The table 4.4 shows that the individual VARX (1) parameters of the model are all within the 95% bounds based on simulated data, and there is no evidence showing an excessive persistence of any variable. Overall, therefore, the modelling with estimated parameters fits the fact well.

Table 4.3: Wald Test Results Based on Indirect Inference Estimation

Variable included	Trans Value	Wald Value
r,inflation,y	0.75	41.674

Table 4.4: VECM parameters and Bootstrap Bounds for y π r with estimated parameter

y π r	actual VAR coefs	lower bound	upper bound	In/Out
b_{yy}	0.9463	0.1568	1.3762	In
$b_{y\pi}$	-0.0231	-1.3833	2.320	In
b_{yr}	-0.3387	- 1.219	1.087	In
$b_{\pi\pi}$	0.3129	0.1335	0.4382	In
$b_{\pi y}$	0.0712	-0.2248	0.2850	In
$b_{\pi r}$	0.029	-0.008	0.1733	In
b_{rr}	0.8810	0.4195	0.9332	In
b_{ry}	0.0427	-0.011	0.1782	In
$b_{r\pi}$	0.0210	-1.763	0.5482	In

4.5 Error Properties

In this section, we will discuss the error properties on non-stationary data, and we have 13 shocks, which are all extracted based on the structural errors of unfiltered data and estimated parameters. Then for each shock, we use two types of stationary test: Augmented Dickey-Fuller (ADF) test and Kwiatkowski-Phillips-Schmids-Shin (KPSS) test.

The ADF test evaluates the null hypothesis that there is a unit root against the alternative hypothesis that the shock series is stationary. From the table 4.4, all of the shocks can reject the null hypothesis of unit root with 10% 5% or 1% significant levels, except for productivity shock, which appeared to be the integrated process of order one I (1) containing a stochastic trend. And the probability value for the productivity shock is 0.7. Therefore, productivity shock will follow $\delta lnA_t = \mu_A + \rho_A \delta lnA_{t-1} + \eta_{A,t}$ with Solow residual . All others are assumed to exhibit a AR (1) dynamics or AR(1) with deterministic trend as $\varepsilon_{i,t} = \mu_i + \rho_i \varepsilon_{i,t-1} + \eta_{i,t}$. Since ADF unit root test is the lower power against alternative that are close to being I (1)(See Elliott et al. (1996)), next, we perform KPSS stationary test to re-examine the results.

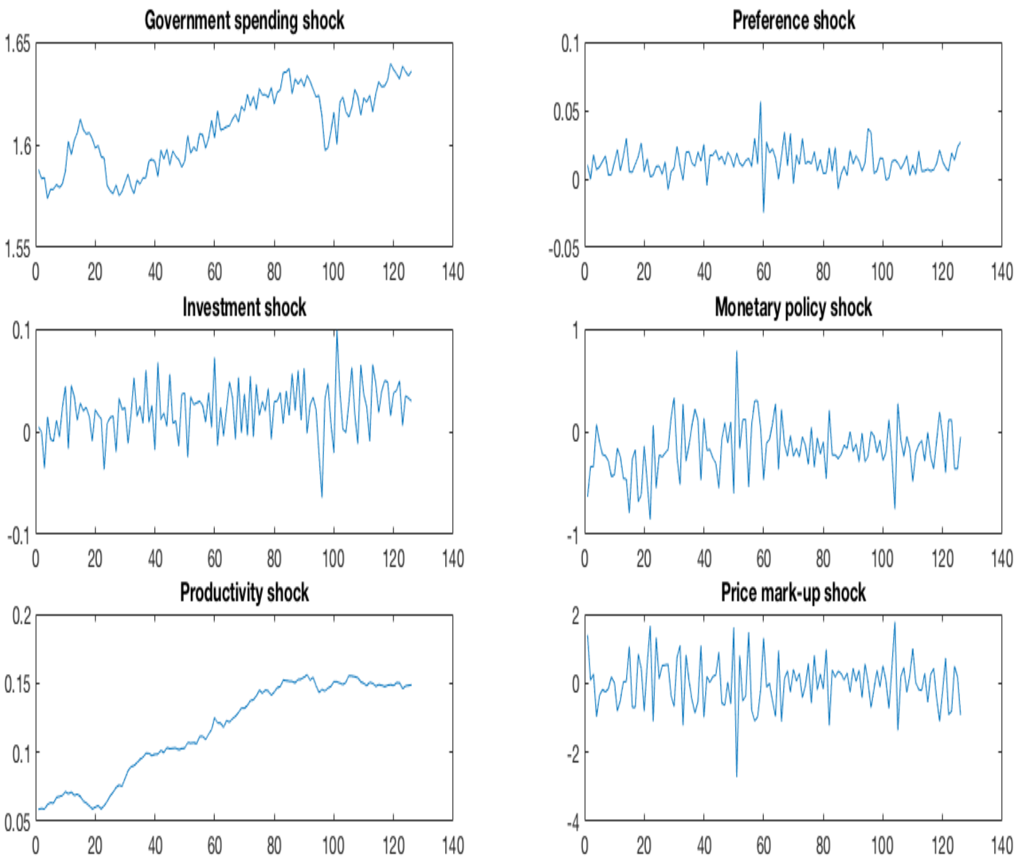
In the KPSS stationary test, the null hypothesis is stationary versus the alternative hypotheses that $\varepsilon_{i,t} \sim I(1)$. From the table, the outcomes are consistent with the ones from ADF tests, that all the shocks fail to reject the null hypothesis with stationary or trend-stationary except for the productivity shock, which rejects the null hypothesis of the stationary at 10% significant level.

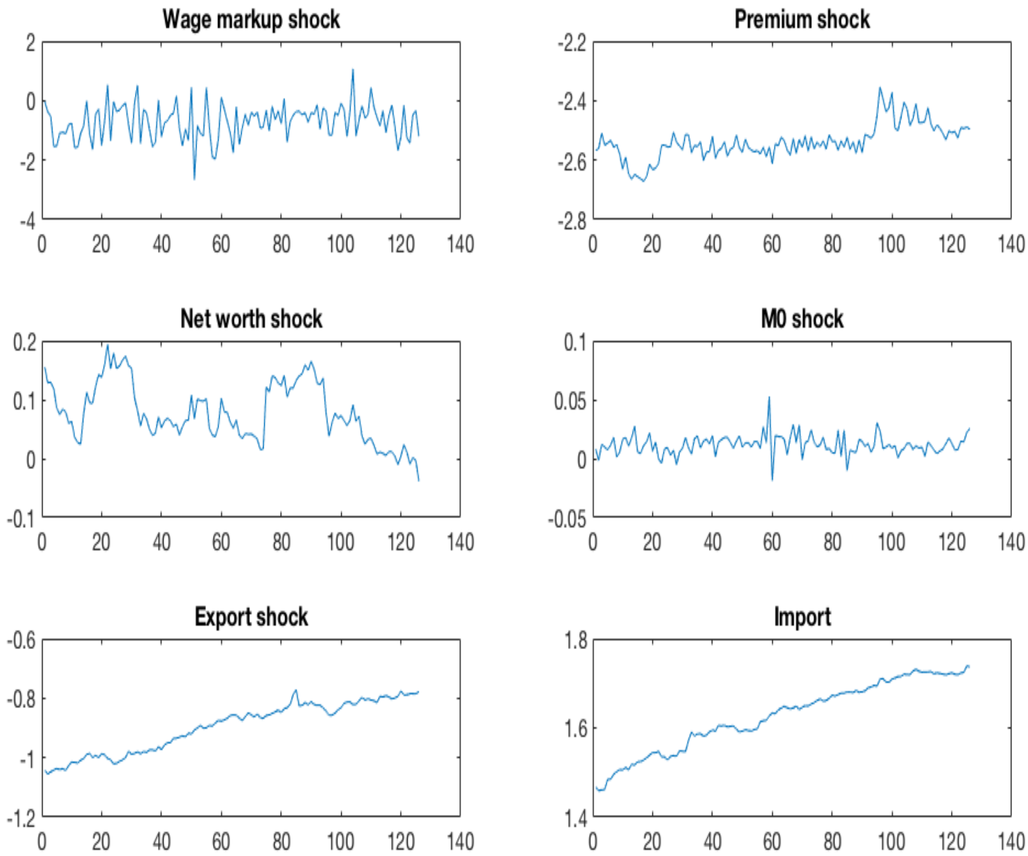
Table 4.5: Testing the Null Hypothesis of Non-stationary

Shocks	ADF p-value[a]	KPSS Statistic[b]	AR(1)Parameters	Process
government spending	0.0324**	0.1285	0.90221850	Trend Stationary
preferences shock	0.0000*	0.3833	0.82106648	Stationary
investment shock	0.0038***	0.1871	-0.11053022	Stationary
monetary policy shock	0.0001**	0.3027	-0.05598822	Stationary
productivity shock	0.7327	1.0547***	-0.09052722	Non-Stationary
price mark-up shock	0.0000***	0.090	-0.27134745	Stationary
wage mark-up shock	0.0000***	0.3136	-0.27134745	Stationary
premium shock	0.070*	0.2275	0.79608210	Trend Stationary
networth shock	0.0541*	0.1510	0.59389231	Stationary
mzero shock (M0 eq)	0.0000***	0.4072	0.02666329	Stationary
mzero shock (crisis)	0.0000***	0.3916	0.02750441	Stationary
export shock	0.0420**	0.3262	0.80832747	Trend Stationary
import shock	0.002***	0.2636	0.95527181	Trend Stationary

Note: a denotes the Augmented Dickey-Fuller (ADF) test, ***, **,* indicate reject the null hypothesis (with unit root) at 10 % 5% and 1% significant level respectively. b denotes the Kwiatkowski-Phillips-Schmidt-Shin (KPSS)test, ***, **,* indicate reject the null hypothesis (stationary) at 10 % 5% and 1% significant level respectively.

Figure 4.2: Residual calculated using estimated parameter





4.6 Indirect Inference Power Test

In previous sections, we use the indirect inference method to evaluate and estimate the model. Now we would like to know how powerful the test is. According to Le et al. (2012)(2016), we will perform a Monte Carlo power statistical test against a falsified model with the misspecified parameter, and the model is assumed to be true with residuals treated as true residuals. As we know, the power of a hypothesis test [5] is the probability of how often we reject a false null hypothesis. Since the number of observations is limited, by the Monte Carlo experiment, we can verify how often the test would reject given the nominal rejection rate, and then generate a degree of accuracy. Thus in this sector, we will follow their procedures using the Monte Carlo experiment to explore how the rejection rate increase when the structural model turns to be more and more false. The test steps are described as followings.

Step 1 Based on the true model generated by the estimated pa-

[5] To out knowledge, there will be Type II error when we fail to reject the null hypothesis while the alternative hypothesis is true. And The power of a test is defined as the one minus the probability of making Type II error

rameters, and actual data, we can generate structural residuals and innovations, then use them to obtain 1000 sets of shocks with moments, including mean, standard deviation, skewness and kurtosis, same as original innovations. Then we will use the sample of shocks to make 1000 sets of artificial data called True data.

Step 2 Following the procedures of the indirect inference estimate introduced, we calculate the Wald statistic for each set of True data based on the following equation.

$$WS^{true} = (\widehat{\beta^{ture}} - \overline{\widetilde{\beta^{true}}(\theta)})'W(\theta)^{-1}(\widehat{\beta^{true}} - \overline{\widetilde{\beta^{true}}(\theta)}) \tag{4.15}$$

Step 3 Falsify structural model by alternately increasing and decreasing the coefficient by x% to get 1000 samples of the false model, which will be used as the misspecified models. Then generate 1000 sets of false data.

Step 4 Similarly to step 2, Estimate each set of False data, and calculate the Wald statistic for each set of false data using the following equation. Then 1000 sets of wald statistics will be obtained and then construct the empirical distribution of Wald statistic to measure the 95 percentile.

$$WS^{false} = (\widehat{\beta^{false}} - \overline{\widetilde{\beta^{true}}(\theta)})'W(\theta)^{-1}(\widehat{\beta^{false}} - \overline{\widetilde{\beta^{true}}(\theta)}) \tag{4.16}$$

Step 5 Calculate how many true data from the true model can reject the false model on calculated distribution with 95% confidence. The rejection rate will be the power of the test for each given level of miss-specified rate.

As we discussed in the previous section, it is not necessary to include all the endogenous variables into the auxiliary model. The policymaker or model users will only focus on specific aspects of the model features, for example, interest rate, output and inflation. We assume that the model can tolerate falseness up to 5%, implying a 95% confidence level.

Table 4.6 presents the rejection rate with parameter falseness at 1%, 5%, 7% and 10%. It is clear that when we increase the falseness rate, there will be a higher rejection rate, which implies that the power is considerably high with a given a significant falseness rate. Specifically, the model is 100 % rejected when we falsify the model with 10%, indicating that for the model users, my estimated coefficients can't be more than 10% away from the true coefficients. Otherwise, the model would get rejected.

Table 4.6: Monte Carlo Power test

Parameter Falseness	True	1%	5%	7%	10%
Rejection rate	5%	12.1%	55.86%	78.4 %	100 %

4.7 Empirical Analysis of Model

In this section, I would analyze the dynamics of the model by the impulse response of macroeconomic and financial variables to the multiple types of shocks. Next, we employ the variance and historical shock decomposition to investigate the drivers of the fluctuations of the variables. To address the importance of financial shocks, the variance decomposition is analyzed based on the time period of 2006Q1 to 2016Q4, covering the crisis and post-crisis periods. Then the historical shock decomposition is carried out with the sample size of 1985Q1 to 2016Q4 based on the estimated parameters.

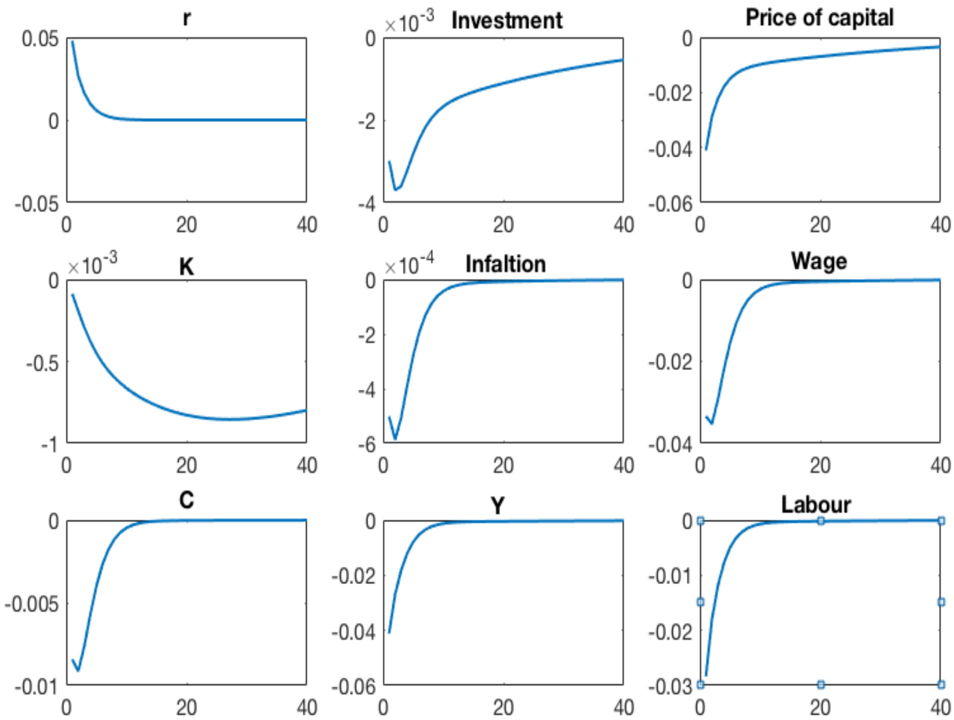
4.7.1 Impulse Response Function

I analyse the impulse responses from the model variables to the shocks from various suspects from different aspects: the monetary policy, fiscal policy, supply side, demand side, as well as the foreign sector. In figure 4.3 to 4.8, the responses from macroeconomic and financial variables to the shocks are represented on y-axis, and the time line on x-axis. In particular, the solid blue line indicates the model without ZLB crisis, and the red dotted line means the model with ZLB crisis when the Taylor rule is switched off, and unconventional monetary policy of the QE is used.

1. **Monetary Policy**
   Figure 3.2 depicts the IRFs to a positive Taylor rule shock, which increases the nominal interest rate of the baseline model. Notably, when the economy gets into the ZLB crisis, the Taylor rule will be temporarily suspended, so the monetary policy shock only applies to the model without a ZLB crisis. To be explicit in the responses, a standard Taylor rule transmission mechanism suggests that a contraction monetary policy usually discourage borrowing, investment and consumption, consequently reduce the output. Then the downward pressures on demand-side are gradually fed through the changes in the output. Meanwhile, the output gap can lead to a lower inflation level. In the labour market, the demand for labour also falls with reduced aggregate demand. In terms of the financial sector, the falls in the capital price lower the new worth of the entrepreneur. Consequently, the external finance premium

is pushed up, then works counter-cyclically with an amplificated impact, that further reduce lending and investment.

In the foreign sector, deflation and higher nominal interest rate appreciate the British pounds with a higher real interest rate, which also reduces the real exchange rate, then makes export less competitive with a higher demand of import. Net foreign bond position decreases overall and gets back after around ten quarters.



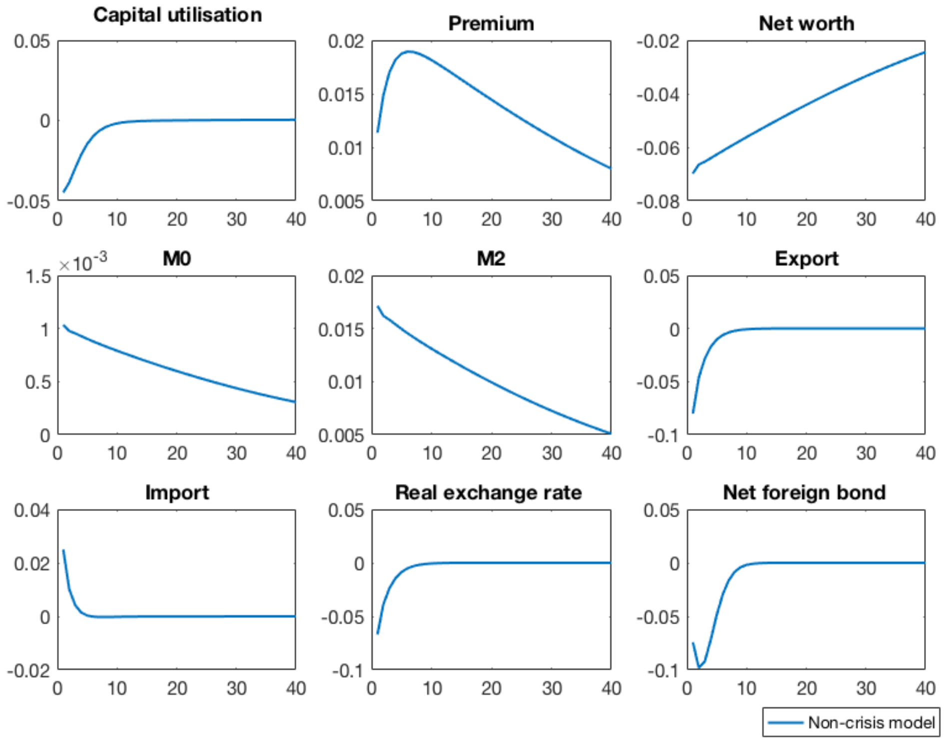
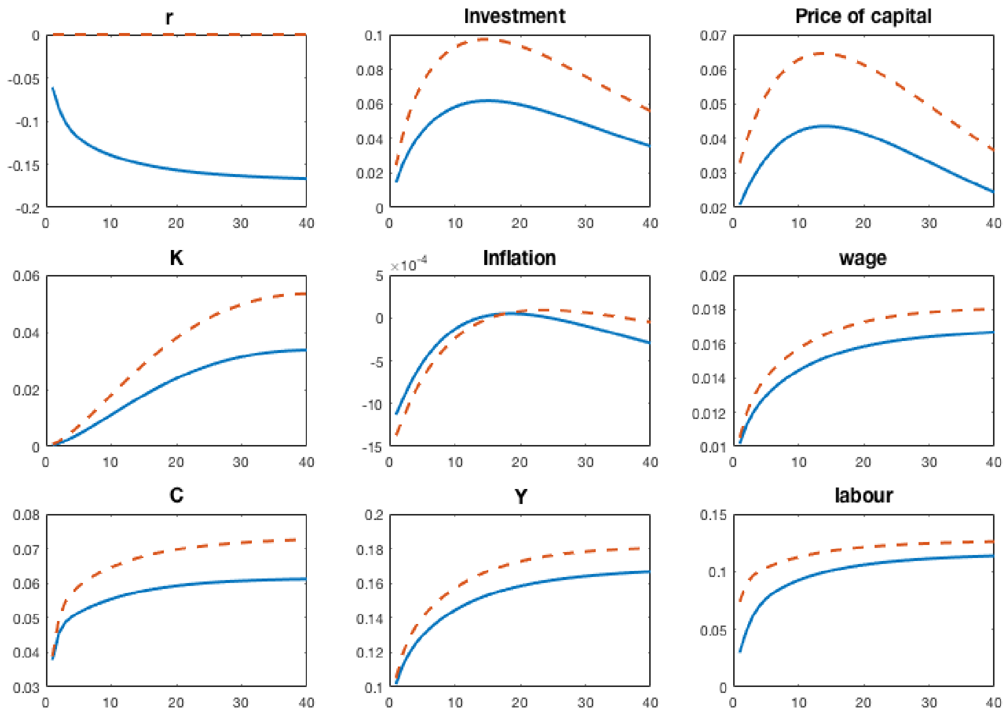


Figure 4.3: Monetary policy shock

2. **Productivity Shock**
   Figure 3.5 captures the model response to a non-stationary positive productivity shock. In both versions of the models, macroeconomic variables including output, consumption, investment, and physical capital react positively, with the permanent effect from productivity shock persistently lasting over 30 quarters. Then the price of capital (Tobin's Q) rises due to a higher demand for capital, which in turn pushes up the value of entrepreneurs' net worth. Thus a lower level of external finance premium will be expected. The counter-cyclical effects from the premium would further increase the investment and capital. Inflation falls with a higher goods supply with superior technology and a decreasing marginal cost of production.

   Regarding the variable in the foreign sector, for the model without ZLB, because the output is boosted, the export must be increased with a lower price level, which would devalue the real exchange rate (a rise in the Q) as shown in figure 3.1. Furthermore, the deprecation of domestic currency decreases the import and makes the export more competitive. Consequently, there is an accumulated net foreign asset.

While the model gets into the ZLB crisis, the QE will be adopted as a supplementary tool to stimulate the economy with traditional Taylor rule suspended. In the foreign sector, since the nominal interest rate is constrained and can not be adjusted any lower, the decrease in inflation will make the real interest rate even higher. Thus the appreciation of currency limits the export. Meanwhile, due to increased domestic production, the demand for import is also decreased, which consequently leads to a negative foreign bond accumulation.



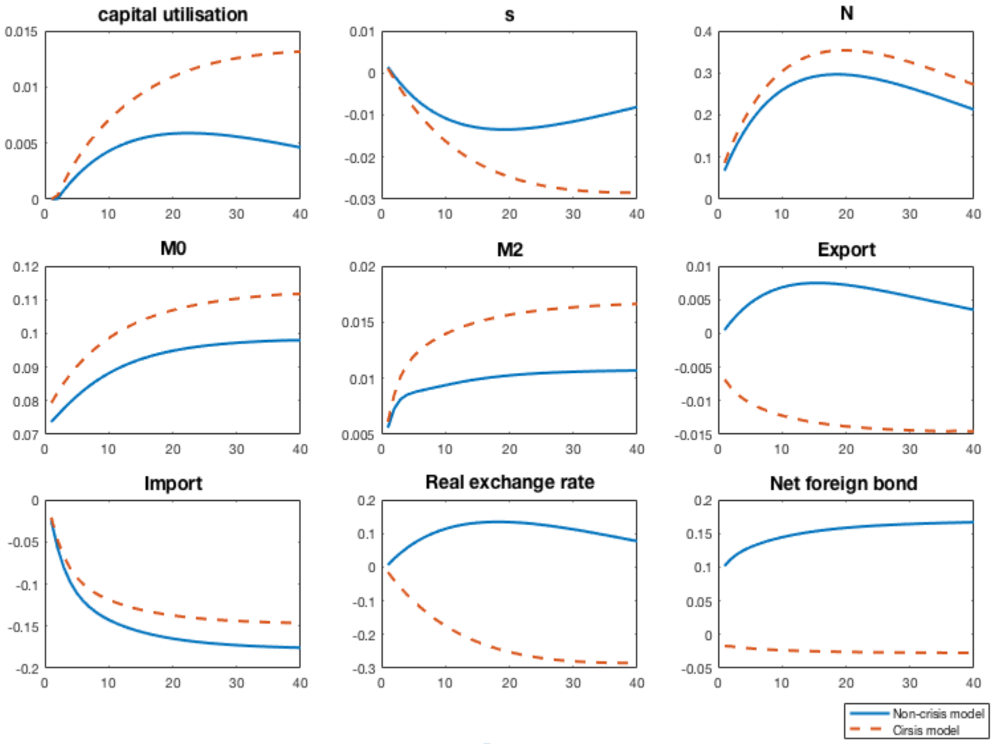


Figure 4.4: Productivity shock

3. **Fiscal Policy**

Figure 3.2 captured the impact from a fiscal multiplier. As we can see, through an expansionary fiscal policy, inflation and the price level are pushed up by higher aggregate demand. And the nominal interest rate rises via the response of Taylor rule. Meanwhile, higher government spending directly increased output. Then the consumer will have a higher expectation on the income, which can induce a higher consumption. In the labour market, the firm will provide a higher wage level to attract labour. Both investment and capital price are increased for the accelerated production process. The capital is now with a higher value, which indicates that the net worth on entrepreneur balance is also increased. Consequently, the external finance premium required by the bank will be lower. Furthermore, from the counter-cyclical effect of premium, the net worth of the entrepreneur would be further increased then leads to more lending and investment.

In the foreign sector, import is increased to satisfy higher domestic demand. This could appreciate the real exchange rate (a decrease in Q) with an appreciation of the British pounds. Meanwhile, weakens the competitiveness of domestic goods in the foreign market with a drop in export. Consequently, the accumulated net foreign asset is decreased.

When the model is constrained by the ZLB, there is a slightly higher response from demand side like consumption, capital and investment, due to a lack of contraction from the monetary policy, which could give a downward pressures on the demand side. In the foreign sector, there is a devaluation of domestic currency due to a lower real interest rate, which makes export more competitive. And the import is also increased slightly to satisfy the excess domestic demands. Finally, the accumulated net foreign asset is increased.

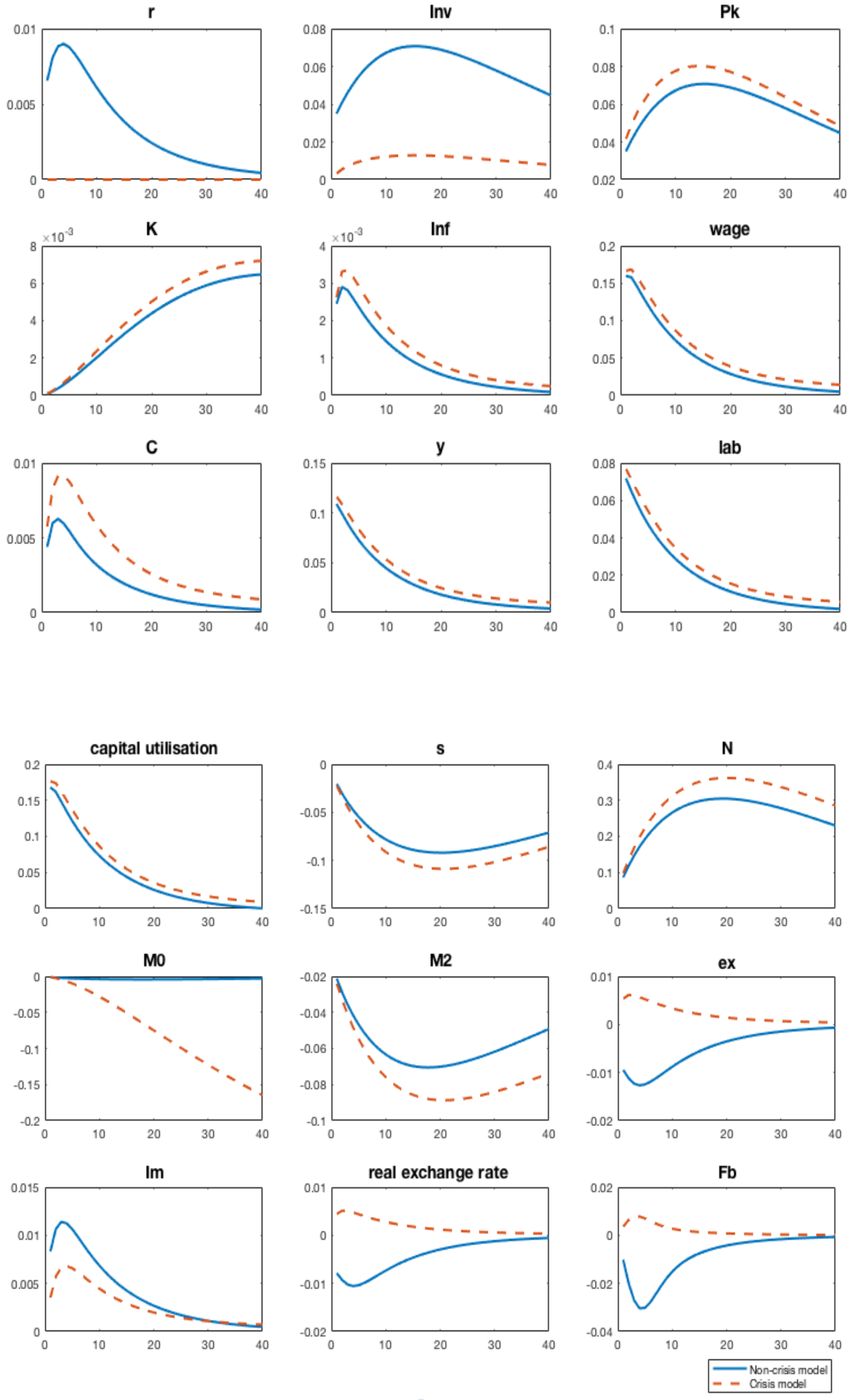


Figure 4.5: Government spending shock

4. **External Finance Premium Shock**

Figure 3.3 captures the responses from an increase in finance premium, and the response from the model with the ZLB clearly shows us how the QE helps. As the figure shows, for both versions of the model, an increase of borrowing cost results in a lower demand of investment, capital, consumption, net worth and the output, also pushes down the price of capital (Tobin's Q). For the model without the ZLB crisis, the Talyor rule is operated, in response from traditional monetary policy, the nominal interest rate is cut to stimulate the economy and it can offset part of the impact of tightening. When the economy is at the ZLB, money supply through the QE will be increased to response for a positive premium shock. Comparing with the model without the QE, money expansion offsets the depression brought from the premium shock, and even more achieved a better economic situation comparing with the baseline model, especially on changes of the variables from demand side.

Regarding on the foreign sector, in the normal time, the interest rate differential relative to abroad is rapidly widened through URIP channel, leads to a rise of the real exchange rate. Then the depreciation of the British pounds would make the export more competitive. Consequently, the net foreign asset will be boosted. While when the ZLB is bounded, a higher real interest rate appreciates the currency and makes the export less competitive.

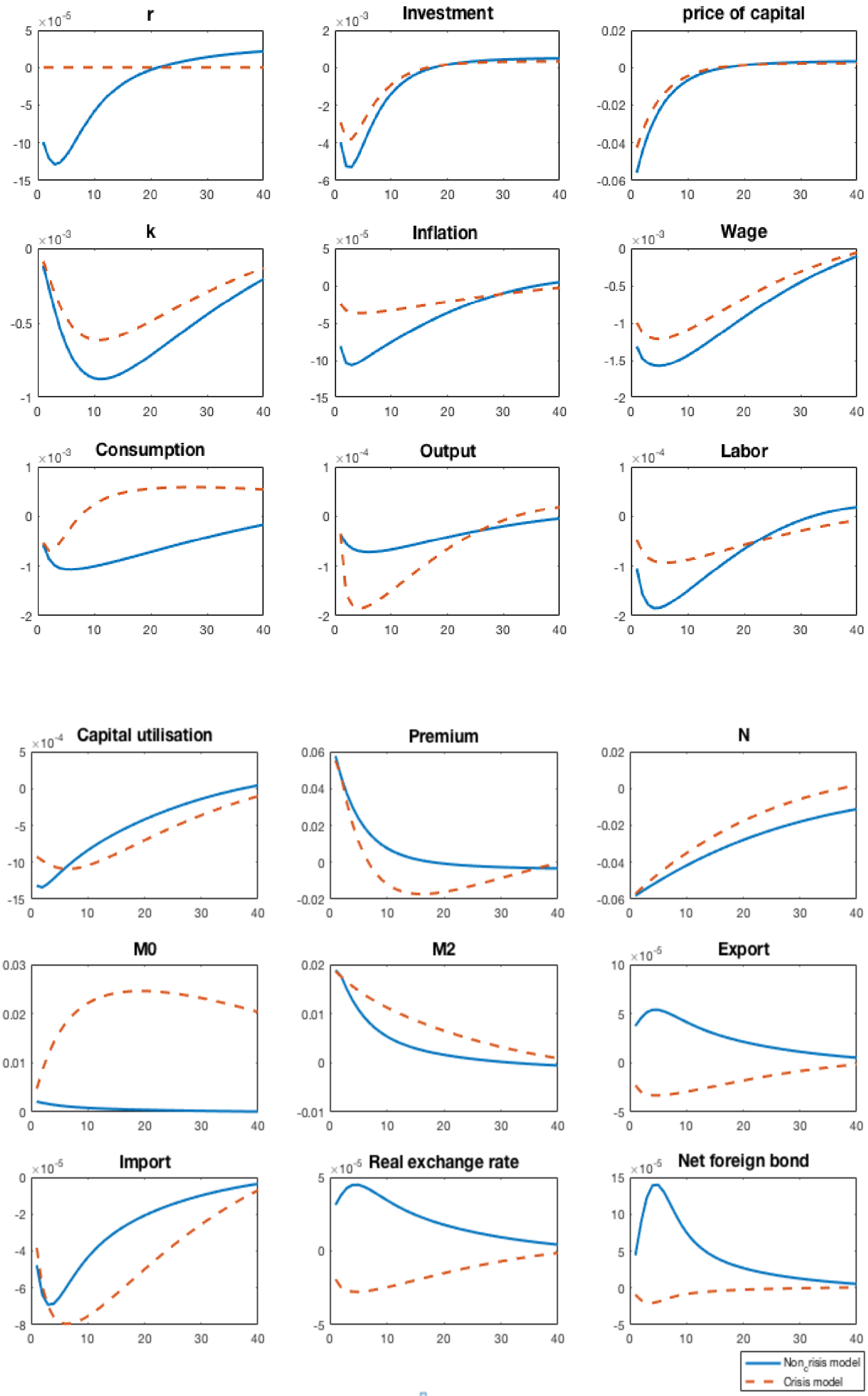


Figure 4.6: Finance premium shock

5. Quantitative Easing

Figure 3.4 shows the effects of a positive money supply shock, which is equivalent to an expansionary unconventional monetary policy. Generally, the responses of variables are similar for both versions of the models. Investment capital, consumption, output and output are all boosted. Then in the labour market, more workers are attracted to a higher wage. Moreover, the external finance premium is reduced, meanwhile, the value of net worth is pushed up with a higher capital price. Without the crisis of the ZLB, traditional monetary policy responses positively. In the foreign sector, higher money supply lowers the value of the currency, then the depreciation of the British pounds makes the export more competitive while the import is decreased. Consequently, there is a positive position in the net foreign bond.

The impulse responses from the model in red dotted line vividly reflected how the QE make effects while the economy is constrained with ZLB. The money injection through the large assets purchasing lowers the default risk and the external finance premium, which then encouraged the variables on the demand side. Comparing with the model without ZLB constraint, the consumption responses stronger, and the boost of inflation quickly brings down the real interest rate. In the foreign sector, a lower real interest rate depreciates the British Pound and then leads to a more competitive export. Different from the baseline environment, the import responses positively, while the money supply is increased without a contractionary effect from traditional monetary policy rule. Therefore, we can conclude that there are more to monetary policy apart from the use of policy rate. Our findings are also consistent with results from other works, as Le et al. (2016) conclude that the QE can help recover the economy during the recession, including the ZLB crisis.

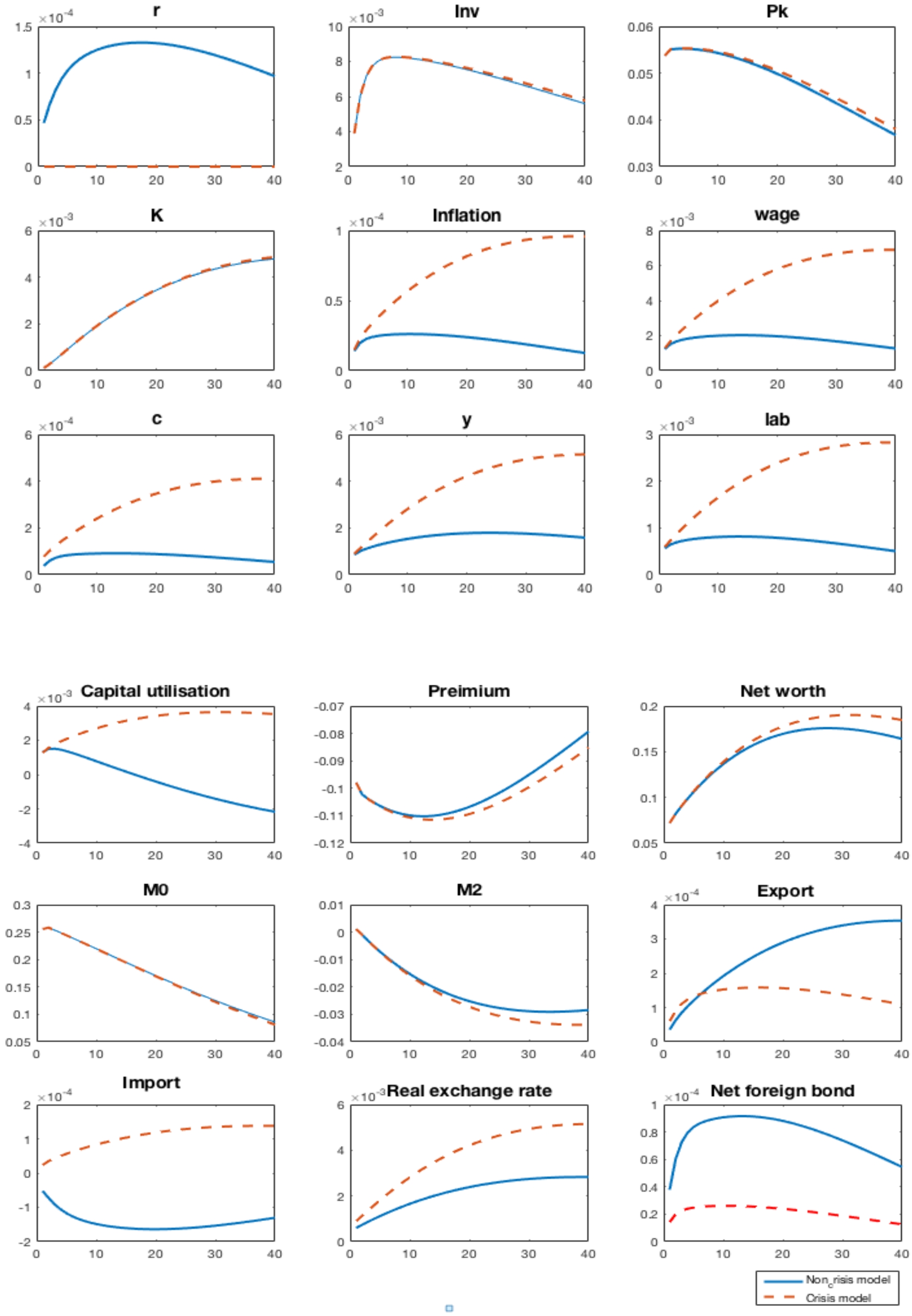
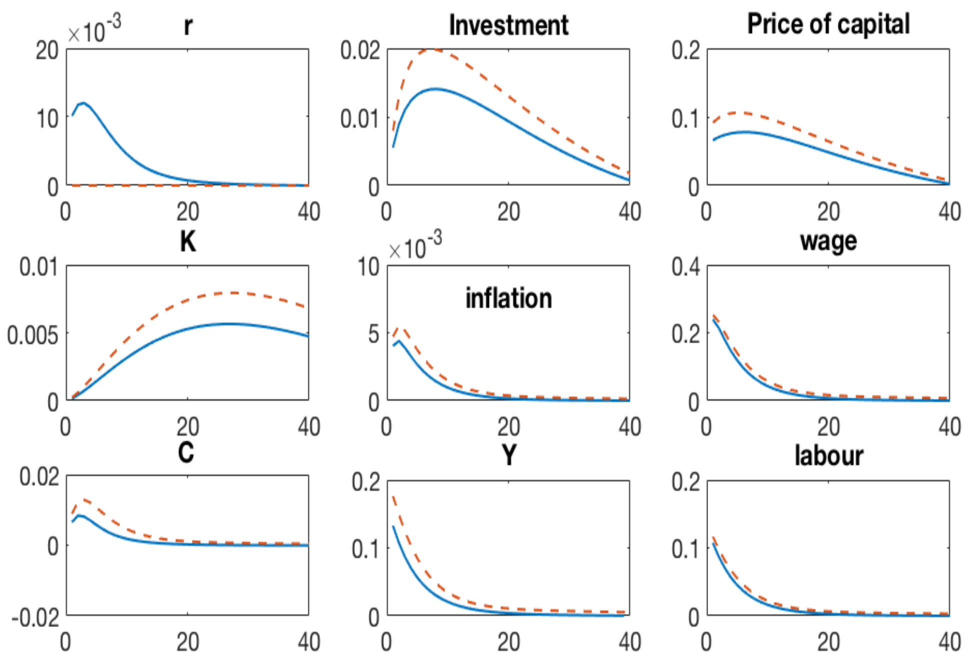


Figure 4.7: Quantitative easing shock

6. **Export Demand Shock**

Figure 3.4 depicted the responses to a positive export demand shock. In this experiment, both versions of the model response similarly. Firstly, it generates a higher aggregate demand through the expenditure switching effects, which stimulates the output, inflation and interest rate. Therefore, the higher demand for capital directly pushes up its price level. Then the net worth of the entrepreneur is increased, which leads to a lower external finance premium required by the bank.

In the foreign sector, when the economy is out of the ZLB crisis, the domestic currency needs to be appreciated (decrease of the real exchange rate) to dampen exports and push the economy back to their steady-state, so the real exchange rate responses negatively. Consequently, there is an increase in the import with appreciated British pounds, however the magnitude of increase is small related to the fluctuations of the export. Eventually, there is an accumulation in the net foreign asset position. With the ZLB crisis, the import response more strongly due to the lack of offset from the traditional monetary policy rule.



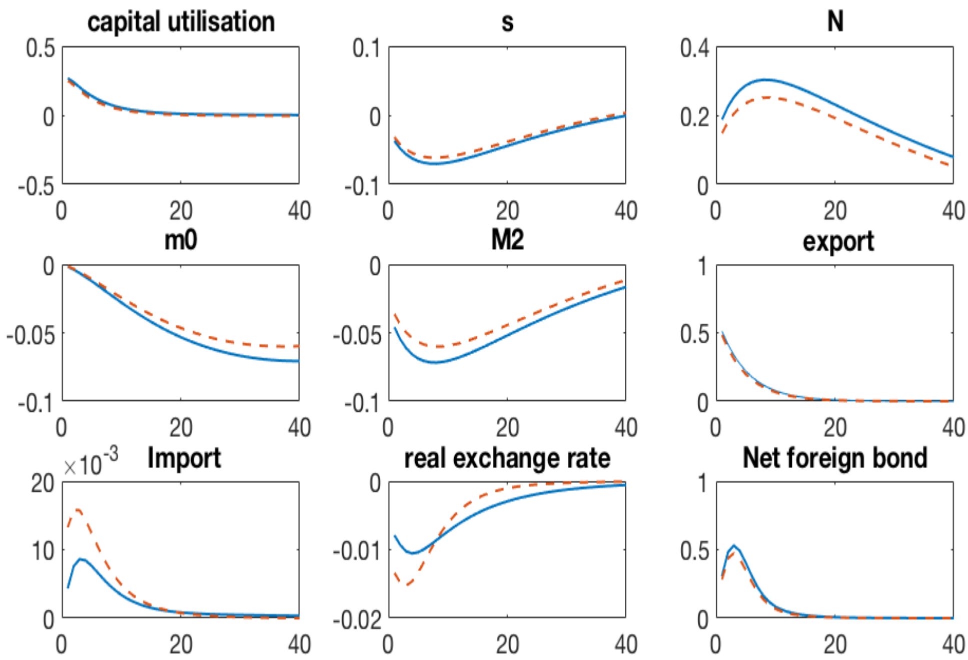


Figure 4.8: Export demand shock

4.7.2 Variance Decomposition

In this section, to answer the question that what drives fluctuation in the UK economy, we will conduct the variance decomposition, which is used to measure the importance of each shock in explaining the variation of the variable. The general equation is defined as the following:

$$\vartheta_{i,j}(t) = \frac{\Theta_{i,t}(t)}{\upsilon_i(t)}$$

Where i and j denote specific variable and shock respectively, and h means the horizon we perform the variance decomposition. Therefore, on the left-hand side of the equation, $\vartheta_{i,j}(t)$ denotes the percentage of the variance for variable i caused by shock j at horizon period h. On the right side, $\upsilon_i(t)$ represents the total variance of variable i at horizon h , and $\Theta_{i,t}(t)$ is the variance of variable i caused by shock j at horizon h.

Table 4.7 shows the variance decomposition of the exchange rate, output, consumption and interest rate. The table presents that a large part of the variance of the exchange rate is contributed by productivity shock with 32.99%. While from the financial disturbance, external finance premium and net worth contribute 6.9% and 3.8% respectively. Besides, international trade is the critical driver for the variance of the exchange rate, with the export shock and import shock playing an essential role on the exchange rate with 14.9% and 18.7 % respectively. Notably, the change of international trade affects net foreign debts position, which then makes the fluctuations on the real exchange rate.

Regarding the variance decomposition of output, we can detect a significant impact from productivity, which contributes 36.37%. As we expected, shocks from the financial market play an important role in describing the fluctuation of the aggregate supply, and external finance premium explains approximately 9.67% of the variance on output. In the foreign sector, both export and import demands provide a significant influences of 6% and 5% respectively. In monetary policy, there is a total impact of 10% with approximately equal contributions from both the traditional monetary policy of Taylor rule and unconventional monetary policy of the QE. The preference shock, wage mark up shock and net worth all have a relatively weak contribution in explaining the output fluctuation.

In terms of variance decomposition for consumption, it is supply-side shock driving the changes. For instance, productivity shock contributes to 22.0%. Same as our expectations, shocks to the financial market make a sizable change, since the costs of borrowing to maintain the standard of consumption are surging while stuck in the financial crisis period. Besides, the inter-temporal preference drives the movement of consumption and plays a significant role in consumption movement with 12.91%.

While turning the analysis to the interest rate, the variance decomposition shows that the productivity shock and monetary policy shock are identified as the two dominated drivers for the fluctuation, with the proportion of 19.16 % and 35.22% respectively. Quantitative easing shock can explain 9.6% of the variance while employed as a supplementary tool of monetary policy, particularly while the economy gets into a ZLB crisis. Besides, the financial shock (mainly indicates the premium shock) plays a vital role with 15%.

4.7.3 Historical Shock Decomposition

In this section, I will focus on how each shock contributes to the volatility of variables over the sample period. Firstly, according to different aspects of shocks, I divide them into five groups including productivity shock (in green), financial premium shock (in orange), M0 (Quantitative

Table 4.7: Variance decomposition of shocks : 2006Q1 to 2016Q4

Shocks	Exchange rate	Output	Consumption	Interest rate
government spending shock	3.432334	5.008639	1.3332652	2.1071515
preferences shock	1.042222	1.108837	12.910514	0.0016258
Investment shock	3.809868	8.071381	2.0001068	4.0051501
Monetary policy shock	4.67901	5.812723	4.9710416	35.217756
Productivity shock	32.98789	36.37000	22.009242	19.161725
price mark-up shock	4.876478	10.862621	9.547019	5.5119655
Wage mark up	1.46E-05	0.000024	2.763E-05	4.439E-07
Labour supply	4.013433	5.033975	2.0003234	0.0061056
Premium shock	6.876755	9.674527	22.0093845	15.003631
Networth shock	3.798575	1.085220	10.000833	3.0006949
Quantitative easing shock	0.876487	5.000011	4.3610951	9.6402881
Export shock	14.89716	6.000595	4.4369885	4.1979336
Import shock	18.70903	5.000301	4.4197725	2.1404486
Total	100	100	100	100

easing) (in grey) shock, Taylor rule shock (in yellow) and other shocks (in blue). The quantitative easing and Taylor rule shocks belong to monetary policy, and financial premium shocks represent the shocks from financial markets. Then figure 4.9 and 4.10 present the shock decomposition for two selected variables, output and interest rate, which are our main concerns.

Figure 4.9 displays the historical decomposition of the shocks to the output. Britain experienced a recession since the second quarter of 2008. Particularly after the collapse of Lehman Brothers, forces from the financial market like the premium shock are main contributors to the economic downturn. And productivity shocks play a large portion of a negative impact on output. Meanwhile, the BoE responded by cutting down the nominal interest rate to stimulate the economy so that the shocks to the interest rates made a positive effect to the volatility of output till the last quarter of 2011. While the nominal interest rate gradually got stuck in the ZLB crisis, money supply used as an unconventional monetary policy to stabilize the economy. Therefore, there is an apparently positive impact from M0 to output after the crisis and till the last quarter of 2016 [6]. Back to the history, during the early 1990s, the UK economy confronted another recession, the productivity shock counted for a significant portion of the drop in the output since the first quarter of 1991, within the recovery period of late 1990s to 2000, the financial premium shock made a primarily positive contribution, while the monetary policy did not make dramatic effects.

[6] last round of quantitative easing announced after the end of Br-exit vote

Figure 4.9: Historical Shock Decomposition of Output

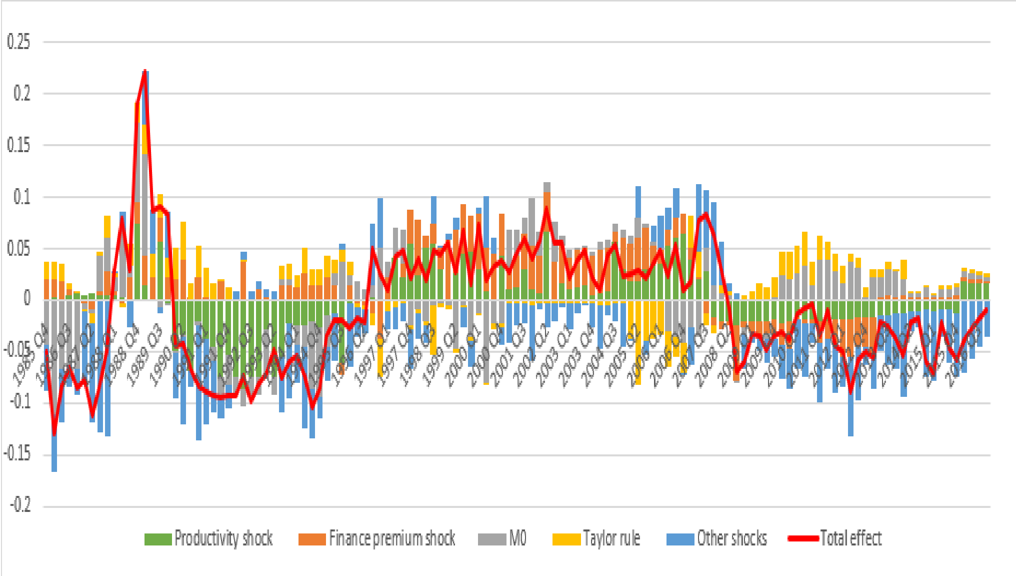
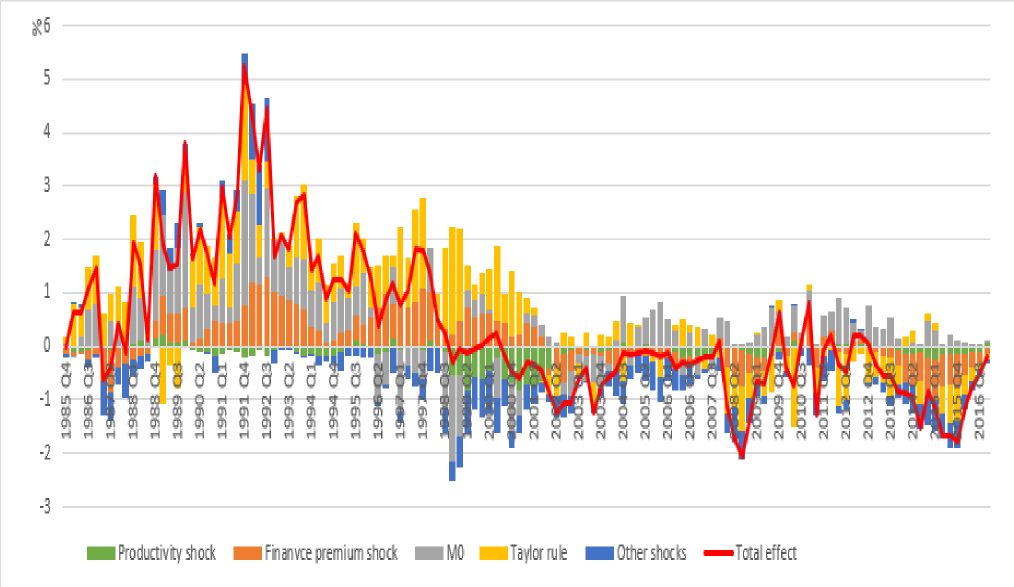


Figure 4.10 explains the contribution of the shocks to the fluctuations of nominal interest. Unsurprisingly, before the burst of the financial crisis in 2008, the monetary policy rule has a major impact on the volatility of the interest rate. More explicitly, between the 1990s to early 2000, together with the financial premium, they lead the interest rate with a positive movement. While after 2008, the nominal interest rate was dramatically cut down by the traditional monetary policy rule (Taylor rule). Then during the period of ZLB crisis, the Taylor rule could not make much effect, and the unconventional monetary policy of the QE was expected to play a significant role. As the figure shows, it helps to bring the nominal interest rate out of the ZLB crisis by contributing a positive effect on the movement.

Figure 4.10: Historical Shock Decomposition of Interest Rate



4.8 Concluding Remarks

In this chapter, I first introduced the procedures of the indirect inference method. Then evaluated and tested the SOE-DSGE model based on the calibrations, and found that without or with ZLB crisis, the model could not pass the Indirect inference test. Consequently, I employed the indirect inference estimation to search for the set of parameters that allows the model to fits the UK data best. With the estimated results, the model can perform well to explain the UK economy with a minimum distance between the simulated data and actual data, and significantly pass the Wald test.

Next, I moved on to the empirical analysis. Regarding the impulse response function, the dynamic response from the variables is mostly similar over both versions of the models. The powerful and positive impact is detected from a money supply shock, which can dramatically boost the economy, particularly while the ZLB is constrained. The shocks from the financial markets bring downward pressures to the aggregate demand. While the model with the presence of the QE would respond more positively, consequently, we can conclude that the monetary policy is not only about the traditional tool of interest, and an unconventional monetary policy like QE can also make profound effects. Furthermore, by exploring the driver of the variability with variance decomposition and historical shock decomposition, we found that the shock from the financial system plays a significant role. In particular, they are the main contributor to the fluctuations in consumption and output.

Chapter 5

Policy Implication in Monetary Regime

The global crisis that started in 2007 challenged the traditional monetary policy. Apart from the call for unconventional monetary policy, another particular suggestion is to conduct an alternative monetary policy rule rather than the inflation targeting. The question about optional choices of targets have excited the economist for decades, but with few experience applied in the real economy[1]. Shedding light on the intensive discussion, Price-level targeting and nominal GDP targeting gained considerable attention. For example, Carney (2012) and Evans (2012) provide a broad discussion about the guidance of using price-level targeting and possibly other variables, including nominal GDP. Besides, some researchers strongly advocated the adoption of nominal GDP targeting (See Sumner (2012) and Sheedy (2014)).

Thus in this chapter, I will argue that alternative monetary policy rule, like the price-level targeting and nominal GDP targeting, can offer big advantages over the inflation targeting without significant drawbacks. And the investigation will be carried out by re-simulating the model under different regimes, then search for the one with better performance by measuring the frequency of crisis and the welfare cost.

The structure of the chapter will be as follows: In this first section, I discuss the limitation of traditional monetary policy with inflation targeting under the circumstance of financial friction and ZLB crisis. Secondly, I carry out an analysis of the cause of the crisis. In the third section, I investigate three types of the monetary policy rule. Lastly,

[1]Sweden experimented with price-level targeting in the 1930s for around two years, but it did not bring much knowledge because of the short duration of adoption. The detailed study can be seen from Jonung (1979) and Berg and Jonung (1999)

remark the conclusion.

5.1 Analysis on the Cause of Crisis

Based on the fact that the great financial crisis in 2008 was triggered by financial friction then led to a long-term constraint of ZLB on the nominal interest rate, we may want to explore the nature of the crisis. And ask whether the financial shock is a sufficient factor to cause the crisis and the ZLB will come with every economic downturn?

I first define the economic and financial crisis, referring to Le et al. (2016). Specifically, the economic crisis is an interruption on output growth for at least eight quarters, and the financial crisis is specified when the ZLB is binding on the nominal interest rate with an economic crisis occurring. Practically, based on UK observations, we bootstrap the shock from 1985Q1 to 2007Q4 to generate a sample named "standard shock scenarios", since during that period massive financial shocks are absent. Meanwhile, the full shock sample from 1985Q1 to 2016Q4 is collected as " crisis-inclusive scenarios". Then two sets of shocks will be used to simulate the model separately. To make the comparison, We graph the two sets of simulated data with randomly drawn examples, as showing in figure 5.3 and 5.4.

They are following findings can be observed from the graphs. Firstly, financial shocks are not sufficient to make a big economic crisis. Under the "standard shock scenarios" without the financial shocks, there are several significant drops in output, indicating the crisis is normally a part of the UK economy. Secondly, The financial shocks are not sufficient to cause the financial crisis. By comparing figure 5.1 and 5.2, we can find that no matter with or without the financial shock, the zero lower bound would bind the nominal interest rate with fractional times of economic crisis. Thirdly, it is quite apparent that the variables fluctuate similarly in both figures. However, several downturns on output go relatively severer when the financial shocks are counted, for example, the decrease of the output around period 100, indicating the financial friction aggravates the crisis and even prolong the recession.

Figure 5.1: Crises without financial crisis

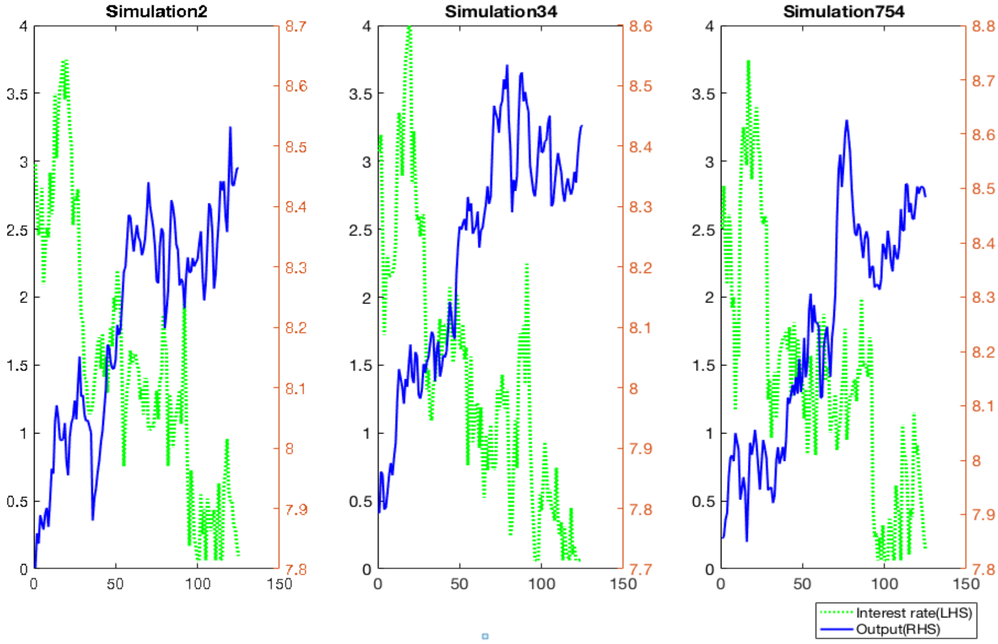
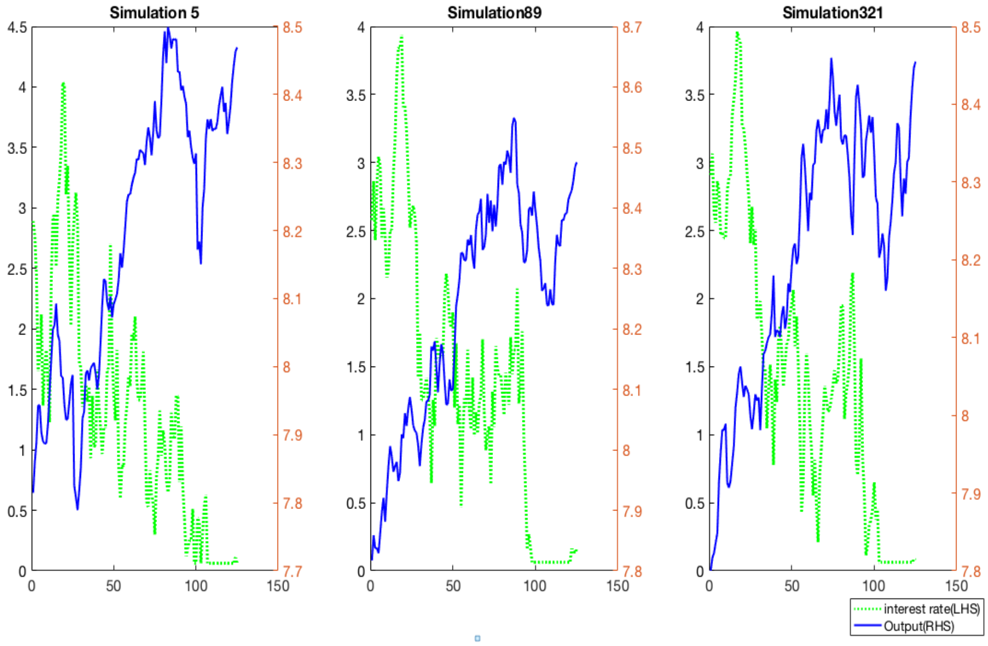


Figure 5.2: Crises with financial crisis



5.2 Investigation on Different Monetary Regimes

5.2.1 Inflation Targeting

Inflation targeting is a traditional type of monetary policy where the central bank sets a specific inflation rate as its target, and stabilise the economy by keeping inflation anchored to its long-run target. It was introduced in the 1970s and 80s to help reduce the inflation expectation and avoid high inflation. Back to history, the inflation targeting has been widely employed by the developed countries, such as the US, UK and the EU countries since the mid-1990s. Based on the definition that the interest rate is adjusted through the deviation of inflation from its targeted value and the output from its potential, the equation is written as follows.

$$r_t = \rho r_{t-1} + (1-\rho)(r_p \pi_t + r_y y_t) + r_{\delta y}(y_t - y_{t-1}) + er_t \tag{5.1}$$

Where ρ measure the interest rate smooth. And $(1-\rho\ )$ captures a short run feedback from inflation and output gap.

Then we perform the bootstrap simulations over the sample period. Then compute the frequency of crisis as well as the welfare cost based on the simulated results. Showing in Table 5.1, both the economic and financial crisis occur with a high frequency of every ten years, approximately 100 times during the estimated 1000 years. In terms of the welfare cost, the variance of output and inflation are 2.8 and 0.03 separately.

Based on the empirical evidence from previous sectors that the expansion of the money can substantially boost the economy, we combine the inflation-targeted rule with monetary reform and implement it into the model with and without the ZLB crisis. As we introduced in the model set up, the money supply rule is adjusted by responding to the credit premium:

$$m_t = m_{t-1} + 0.04 * (s_t - s*) \tag{5.2}$$

Where m_t is quantitative easing, s_t and s^* are finance premium and targeted value of finance premium. The coefficient of $s_t - s^*$ is estimated at 0.04. When finance premium is higher than the targeted value, we will increase the money supply. Back to the finance premium equation $S_{t+1} = \chi(qq_t + k_t - n_t) + (r_t - E_t\pi_{t+1}) - \psi m_t + \xi_t + epr_t$, the premium will be adjusted lower.

Then we simulate the model with the updated monetary regime and generate a set of new results showing on the third column of Table 5.1. Generally, there is a slight drop of frequency on both economics and financial crisis by comparing with the situation where the single inflation-targeted rule adopted. Explicitly, for an expected 1000 years, both the economic crisis and the financial crisis will occur around 90 times, indicating a significant effect from QE in stabilising the economy. Furthermore, same as our expectations, in combing with the monetary reform, there is lower welfare loss with both output and inflation variance reduced to 1.67 and 0.02, respectively.

Though the inflation targeted rule has been controversially discussed during the last several decades since it started to be adopted, the global crisis in 2008 exposed its critical flaws and allowed the argument acquired a new topicality. From the review, I can conclude them into two aspects.

Firstly, facing the ZLB crisis during the aftermath of the financial crisis, some researchers propose to set up a higher inflation target under the inflation targeted rule. As we know, when the nominal interest rate fell into the zero lower bound, the conventional monetary policy outlived its function to stimulate the economy. Under the ZLB circumstance, assuming there is a negative aggregate demand shock to the economy, monetary policy targeted on inflation will cause a higher real interest rate. Then people would have an even lower expectation about the future inflation rate, which could contribute a further downward pressure on aggregate demand, thus a lengthy recession will be ensured. Based on that fact, a growing number of studying came out with potential solutions. For example, by an insightful theoretical work, Krugman (1998), Michael Woodford and Gauti Eggertsson (2003) Bernanke (2017) propose that a higher inflation target is needed. They remarked that with all else being equal, a higher inflation target could increase the steady-state of nominal interest rate, and then reduce the incidence of ZLB. However, it is rather difficult to answer how much should the target be increased in practice, such as Japan, where the economy has been with ZLB since the mid-1990s with a persistently low inflation problem.

Another big concern about the inflation targeting is whether it can sustain a stable financial system. There has been a growing amount of empirical evidence on the linkage between the financial stability and monetary policy (See Brousseau and Detken (2001), Cecchetti et al. (2002), Driffill et al. (2006) and De Gregorio (2010)). They suggest that monetary policymakers should consider how to improve the impact of monetary policy on financial behaviour. While With the experiences on the financial crisis, inflation targeting takes too little account on the financial friction and imbalances. To our knowledge, from the beginning, the inflation targeting has focused on the only one outcome: price stability, but the global financial crisis required the monetary policy to contribute to financial stability.

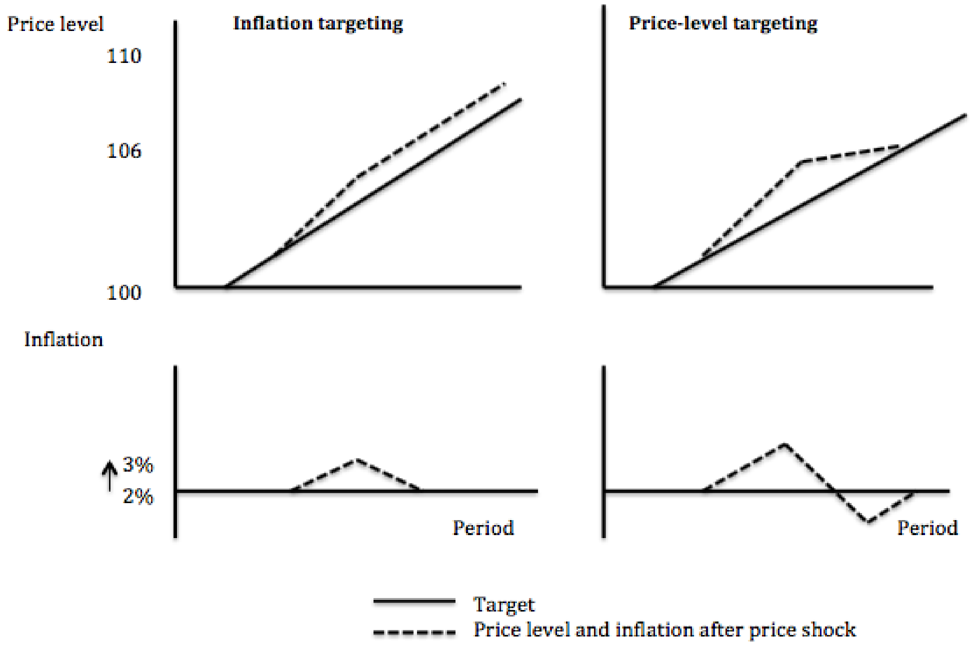
Reasonably, the researchers propose to employ the alternative monetary policy. For example, Hatcher and Minford (2014) concluded that price-level targeting could be a useful mechanism for helping the economy recovers from the deflationary shocks. To compare the ability of different regimes in stabilising the economy, we will investigate the alternative monetary schemes in the same way and make a comparison on the results to identify the winner in the following sectors.

5.2.2 Price-level Targeting

Price-level targeting (PLT) is a strategy, where the central bank defines a targeted path for the price level, then commits itself to correct the deviations along the path within a given period. Different from the inflation targets, which merely targets to stabilise inflation, price-level targeting can provide more guidance to the economy. As Sbensson (1999) addressed, employing a price-level targeting can help solve the time-inconsistency problem, thus under the rational expectation, price-level targeting would deliver a free lunch with lower inflation and output variability. Based on the past works, I will conclude several features, which make the price-level targeting a potential alternative.

Under inflation targeting, bygones are treated as the bygones, that the past deviation from the target will be effectively ignored. Differently, price-level targeting does not treat the past as bygones, and it is history-dependent with the anchor in the past. With the forward-looking public, the corrections of the past deviations will affect their future expectation. To be more explicit, we assume there is an unexpected rise in inflation from 2% to 3%. As showing in figure 5.3, under the inflation targeting regime, a higher inflation rate will be strictly pursed approaching to the target while without the effort to reverse the price deviation. However, the price-level targeting works more effectively with a more precise target. It maintains a price-level path set up before and requires a below-average inflation rate. Though the interest rates respond for longer to an inflation deviation, it maintains the stability of both price and inflation in the long term.

Figure 5.3: Compare between PLT and inflation targeting with a negative demand shock



During the aftermath of the big crisis in 2008, the price targeting becomes a topical rule. Particularly, when the nominal interest rate is constrained with the zero lower bound, an unexpected change in aggregate demand can cause a rise in the real interest rate under the inflation rate targeted scheme. If the private sector realises the monetary policy has temporarily out of ability in moderating the interest rate, their inflation expectation will fall, which leads to real interest rate to rise even further then increase the risk of the recession. In contrast, under the price-level targeting with an inflation target of 2%, people will have an expectation of inflation above the 2%, since the bygones are not bygones and the public holds the credit that the central bank would make up the shortfall. Thus, this would push up the price level with the aggregate demand stimulated. Eggertsson and Woodford (2003) confirmed this intuition that during the financial and ZLB crisis periods, the welfare loss under inflation targeting is higher than the one under price targeting in New Keynesian models. And more recently, Coibion et al. (2012) found that price-level targeting can reduce the frequency and severity of zero bound episodes. Besides, more pieces of evidence can be found in Coletti and Woodford (1999), Dittmar, Gavin, and Kydland (1999) and Dittmar and Gavin (2000), Lalonde (2008) and Nakov (2008).

To perform the empirical study, we specify the price-level targeting as following,

$$r_t = \rho_1 r_{t-1} + (1-\rho_1)\{\rho_p(p_t - p^*) + \rho_y(y_t - y^*) \\ + \rho_{\delta y}[(y_t - y^*) + (y_{t-1} - y^*)]\} + er_t \quad (5.3)$$

Where the steady-state of price level p^* is assumed constant and normalised to 0, practically, we choose the average value of output from actual data as the steady-state value of output as y^*. ρ_1 is interest rate smoothing rate , and ρ_p is the value of Taylor rule response to price level, and $\rho_y$and $\rho_{\delta y}$are Taylor rule response to output and output change respectively. Then I value the parameters in the above equation by minimizing the crisis times [2]:

$$r_t = 0.545 * r_{t-1} + (1-0.545)\{1.745 * (p_t - p^*) + 0.02 * (y_t - y^*) \\ + 0.03 * [(y_t - y_{t-1}]\} + er_t \quad (5.4)$$

From table 5.1, with a single price-level targeting adopted, there has been a significant decrease in the frequency of economic and financial crisis compared with the results generated by inflation targeting. Within the expected 1000 years, the frequency of both crises comes down to 87. The total welfare cost drops from 2.87 to 0.724 with a significant contribution from output variance, which goes down 0.698.

5.2.3 Nominal GDP Targeting

In this section, I will discuss another desirable strategy for monetary policy, nominal GDP targeting or nominal income targeting, which strives to get a certain level or the growth of the nominal GDP. The most attractive feature of the nominal GDP targeting is that it is closely related to both output and prices, which are the variables the central bank cares about most. Frankel concluded that the central bank under nominal GDP makes decisions regarding the importance of inflation and real output, rather than the breakdown between the two (Frankel 2012).

Additionally, superior to the inflation targeting, it can respond effectively to both the demand and supply shock. For example, facing a negative supply shock, there will be a decline in output meanwhile with a rise in inflation. Under inflation targeting, the central bank would choose to carry out contractionary monetary policy to maintain a lower

[2] I search for the values which can allow the model stabilise economy most by simulation

inflation rate, but at the cost of further exacerbating the recession. In contrast, nominal GDP targeting can avoid a worse situation by an expansionary monetary policy, and return the nominal GDP to target. Though the inflation rate will be temporarily above the potential, it can decrease the unemployment rate by letting inflation rise, particularly during the recession.

On the front of financial friction, the NGDP targeting can be more capable of avoiding default and create more financial stability. Koenig (2013) and Sheedy (2014) remarked, if the aggregate income can keep close to the steady growth path by nominal GDP targeting, it would not fall as much during the recession, which allows people to continue to repay their loans, then avoid default and bankruptcy.

Then we bootstrap our model with nominal GDP targeting and the rule is defined as follows.

$$r_t = \rho_1 r_{t-1} + \rho_y(y_t + p_t - \bar{y} - \bar{p}) + er_t$$

Where $y_t + p_t - \bar{y} - \bar{p}$ indicate the deviation of the nominal GDP from targeted value. $\bar{p} = 0$ and $\bar{y}$ follows the real output data. Andρ_y is treated as partial elasticity of interest rate responding to the nominal GDP deviation. Then with the valued parameters, we use the following equation to generate the simulated data.

$$r_t = 0.625 * r_{t-1} + 2.21 * (y_t + p_t - 8.71) + er_t$$

Based on the simulated results, showing in Table 5.1, the frequency of financial crisis is further reduced from 87 to 66 compared with the price-level targeting case, which confirms its higher capacity in lowering the zero bound episodes against other two rules. Regarding the welfare cost, the variance of output gets lowest to 0.690, and the inflation variance is at the same level as the inflation targeting at 0.025. Overall, by comparing the simulated results of three monetary policy rules (without the monetary reform), both of nominal GDP targeting and price-level targeting behave better than the traditional inflation-targeted rule. And the nominal GDP targeting can dramatically stabilize the economic volatility.

5.2.4 Combing the PLT and NGDPT with the Monetary Reform

In this sector, I would also like to investigate the alternative monetary regime where the monetary interest rate rule is combing with the monetary reform. Table 5.1 has put all the outcomes together, and the simulations show that the crisis time is further reduced under either combination related to single adoption. Since the combination with monetary reform for both price targeting and nominal GDP targeting will cause a higher inflation variance, inflation targeting could outperform the other two types only if the central bank uses the measurement of welfare cost only on the inflation variance. Whereas, when accounting for the variance of output, NGDPT+monetary reform is the best with the lowest value around 0.552. In terms of the frequency of crisis, the monetary regime of NGDPT+Monetary reform can generate the least ZLB episodes.

To further present how different monetary regimes behave in stabilising the economy, in figure 5.4 and 5.5, I plot the graphs for simulated data of output with two randomly drawn as the examples. It apparently shows that the alternative monetary regimes can perform better in stabilizing the economy, particularly after combing with the monetary reform. It is easy to detect several big fluctuations with inflation targeting (Solid blue line). However, there is more stability created by other regimes, for instance, the big ups and downs under inflation targeting are squashed with alternative regimes from period 20 to 30. Furthermore, the price-level targeting and nominal GDP targeting can perform better especially when the crisis collapse, around the period 120, there is a big slump under the inflation targeting (solid blue line). While under the PLT+monetary reform (solid red line) and the NGDPT+monetary reform (solid grey line), the big crisis is stabilised into a moderate drop or small swings and lasted for a shorter period.

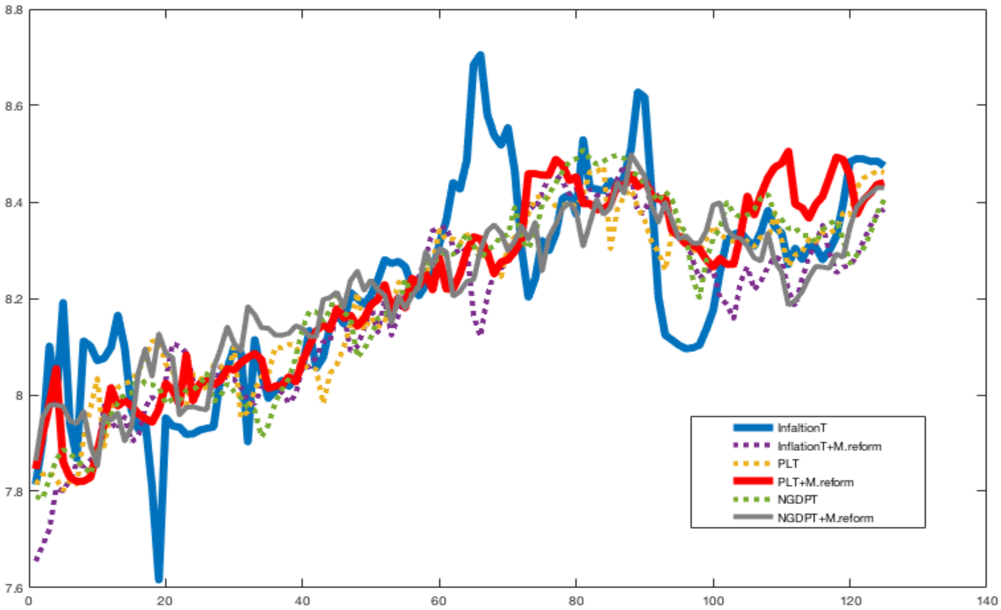
5.3 Conclusion

After the financial crisis, the traditional monetary policy has been challenged as an appropriate role in sustaining the economic stability. Though the inflation targeting has been an unmitigated success in the history, the 2008 financial crisis has called into the question that whether the sole focus on price stability is sufficiently enough to fight with the fi-

Table 5.1: Frequency of Crisis under Different Types of Monetary Regimes

	Inflation targeting	Inflation targeting+Monetary reform	Price level targeting	NGDPT	PLT+Monetary reform	NGDPT+Monetary reform
Economic Crisis						
Duration between two crisis	10.01	10.84	11.41	10.58	18.11	17.24
Frequency of crisis (expected economics crisis per 1000 years)	99.90	91.91	87.64	94.51	55.21	58.00
Financial Crisis						
Duration between two crisis	10.12	10.98	11.42	14.94	19.97	20.04
Frequency of crisis (expected economics crisis per 1000 years)	98.78	91.07	87.62	66.93	50.08	49.90
Welfare cost						
Var(Output)	2.840	1.668	0.698	0.690	0.683	0.552
Var (Inflation)	0.025	0.023	0.026	0.025	0.025	0.028
Total	2.865	1.691	0.724	0.715	0.708	0.580

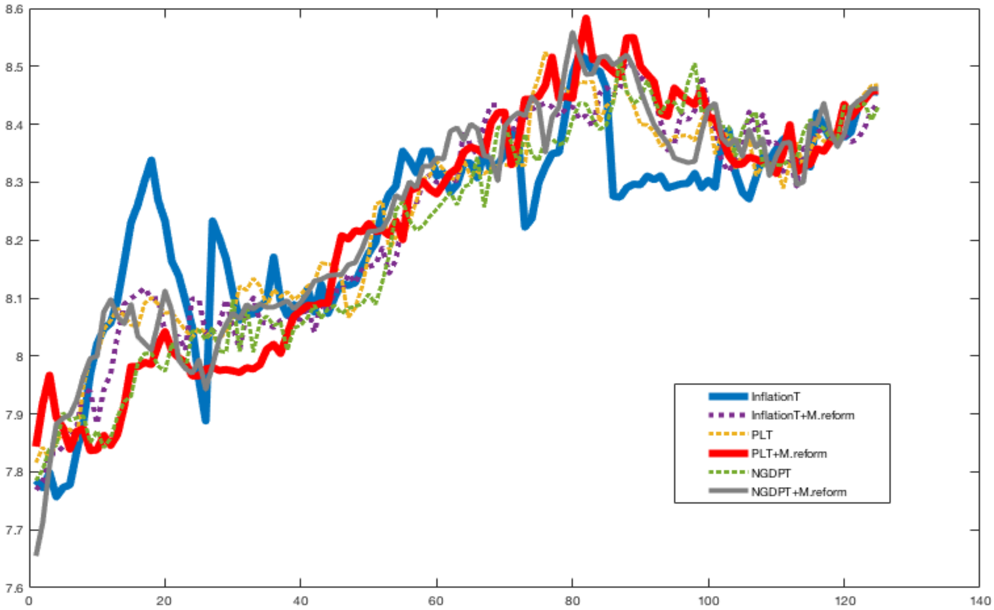
Figure 5.4: Simulated output under different rules (Example1)



nancial and economic crisis. Thus it is crucial to consider what kind of approach can provide a better framework than inflation targeting. Based on that, we replace the traditional monetary rule in the model with two alternatives: price-level targeting and nominal GDP targeting.

By measuring the frequency of the crisis and welfare cost under the monetary regimes with our simulated data, both price-level and nominal GDP targeting can stabilise the economy better than the inflation targeting. After combine with the monetary reform, the frequency of crisis

Figure 5.5: Simulated output under different rules (Example2)



can be reduced further in either way comparing with a single adoption of the monetary policy rule. Among all the types, the winner is no doubt the NGDPT+monetary reform. It not only minimises the crisis frequency but also accounts for fluctuations of both output and price level with the lowest total welfare cost.

Based on the simulation results, it's clear to find out that both NGDP and CPI targeted rule combing with the monetary reform can perform better than the traditional inflation targeting, but BoE has not adopted alternative rules. According to the works have been done by other researchers, there are several possible reasons. To deliver all the technical advantages of price-level targeting, the credibility of central bank must be very high. And the price-level targeting requires the government to commit to a long- run path for inflation. But it might be a doubt for the government to commit to what will happen in ten years' time, when it might be a completely different government power. Besides, since there was not modern example of price level or Nominal GDP targeting, the central bank may not want to try them. Though Sweden experimented with the price-level targeting at the great-depression in 1931, the applicability of the rule to a modern economy is still questionable (Epstein and Yeldan (2009)).

Chapter 6

Conclusion

This work is based on the new challenge for monetary policy during the aftermath of the financial crisis in 2008. Particularly when the interest rate with zero lower bound hampered the ability of the conventional monetary policy, the central banks sought to adopt unconventional monetary policy such as quantitative easing. The magnificent updates in the monetary policy motivate us to have a better understanding of the unconventional monetary tool, for instance, in light of their transmission mechanism through the financial intermediary.

In chapter 2, I review the literature about the unconventional monetary policy. In the existing works, the role of quantitative easing has been unprecedentedly addressed, and the mechanism by which quantitative easing makes effects was massively formalized in macroeconomic models. Whereas, studying mainly through the portfolio re-balance, and the number of researching through the financial sector just started to grow after the big crisis, when challenged by the financial friction. In the UK, the modelling and analysis incorporating financial friction are still limited. Thus my first research question is to remedy the lack of money in modelling the UK economy and capture the dynamic relationship between the quantitative easing and overall economy, through the credit channel.

Because the DSGE model is now broadly used to explain significant relevance between macro and micro variables, and following the question proposed above, I built up a medium-to-large dynamic stochastic general equilibrium model to study the effectiveness of quantitative easing, and incorporate the imperfectness of the financial market with the predominant role of financial intermediary. Notably, the model is adjusted following the framework set up by Smets and Wouters (2003,2007), and three features of the UK economy were added: 1) the model is extended to a small open economy with certain features referring to Meenagh et al. (2007). 2) The model adopts the price and wage-setting with a hybrid model introduced by Le et at. (2011). 3) To capture the dynamic response on the unconventional monetary policy, according to the transmission method proposed in Le et al. (2016), quantitative easing is employed and works through the bank lending channel.

Overall, the model captures the significance of the unconventional monetary policy. Through the outputs of the impulse response function, the model detects the countercyclical features of the external finance premium, which directly causes a higher cost in borrowing and a lower investment; meanwhile, push down the capital price, then weaken the net worth with a further increase of the external finance premium. More importantly, a positive money supply shock can lower the external finance premium and then boost the production, revealing the money supply needs to be controlled to maintain economic stability. Besides, the stimulates to the economy including the zero lower bound period made by the expansionary fiscal policy and traditionally money policy are verified.

Notably, I employed the indirect inference method to investigate whether the model can explain the data behaviour in the UK. Different from most of the studying, I simulated and estimated the model by non-filtered UK data over the period 1985Q1 - 2016Q4. The testing results show that the model could not be accepted using calibrated values. Then the indirect inference estimation is employed, and I found out a set of the parameters that could significantly pass the test and minimize the distance between the simulated data and actual data. The estimation results show that the model can explain the observations well, and mimic the fluctuations of the core endogenous variables: output, inflation and interest rate, which we mostly concern.

Furthermore, the economic recession shows that monetary policy with inflation targeting alone struggled to stabilize the economy or deal with the shocks to the economy. As a result, the economist argued that, apart from the strong impact of quantitative easing under the lower bound, there should be possible changes to a regulative monetary regime that may cope with the economic un-stability better. Hence we discussed some possible alternatives to the inflation-targeted rule. Considering our focus is on the capacity of the policy to stabilize the economy, we measure the frequency of economic and financial crises, as well as the welfare cost under different regimes. The simulated results indicate that the monetary regimes behave much better with the combination of monetary policy and the monetary reform. Specifically, from the view of the simulated output under different schemes, the monetary reform can help to squash the large crisis and stabilize the economy with less big fluctuations. By measuring the welfare lost and crisis times, we found that the monetary regime of nominal GDP targeting plus the monetary reform stands out with the lowest level of welfare cost and crisis frequency. Therefore, we propose that the single Taylor rule is not enough to fight with the financial friction, a better-performed monetary regime like the combination of nominal GDP targeting with monetary reform could be considered.

To conclude, there are three contributions to this work. Firstly, the theoretical SOE-DSGE model is set up incorporating financial friction and unconventional monetary policy of the UK economy. This work provides a framework to describe the UK economy more appropriately and verify the effectiveness of the unconventional monetary policy. Secondly, we evaluate and estimate the model following indirect inference method by unfiltered non-stationary data. We also assess the power of the indirect inference test on the model, which is considerately high with a significant falseness level. Thirdly, based on my empirical study, I provide the policymaker evidence that the alternative monetary policy may work better than the inflation targeting.

Despite the findings from this work, the door is open to future research. Firstly, apart from the progress made, the extension of the model is still necessary to explore. Since the UK is a net debtor country, more attention should be paid on the foreign sector. As we know, when the global financial crisis spikes, the currency will experience dramatic fluctuation. Conventionally, we suppose the changes in the short-term exchange rate should be consistent with the unconverted interest rate parity. Whereas, such volatility of exchange rate cannot support for the UIP any further. Thus the currency risk premium can be introduced in promoting the capacity of our policy and macroeconomic models. The closest work can be seen in Barnett and Engineer (2010), they propose a SOE-DSGE model with nominal and real reignites and the international portfolio balance effects. And derive the deviation for the unconverted interest rate parity by setting short and long term portfolio in both domestic and foreign portfolios. Secondly, based on the recent experience of the UK, the economy gradually out of the ZLB crisis, so how long should the unconventional monetary policy be kept in place optimally?

And whether the unconventional policy should be part of the conventional monetary policy even under the ' Normal' time. Thus it could be an extension for the DSGE model in the next generation and a further study of the unconventional monetary policy in certain aspects.

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Appendix A

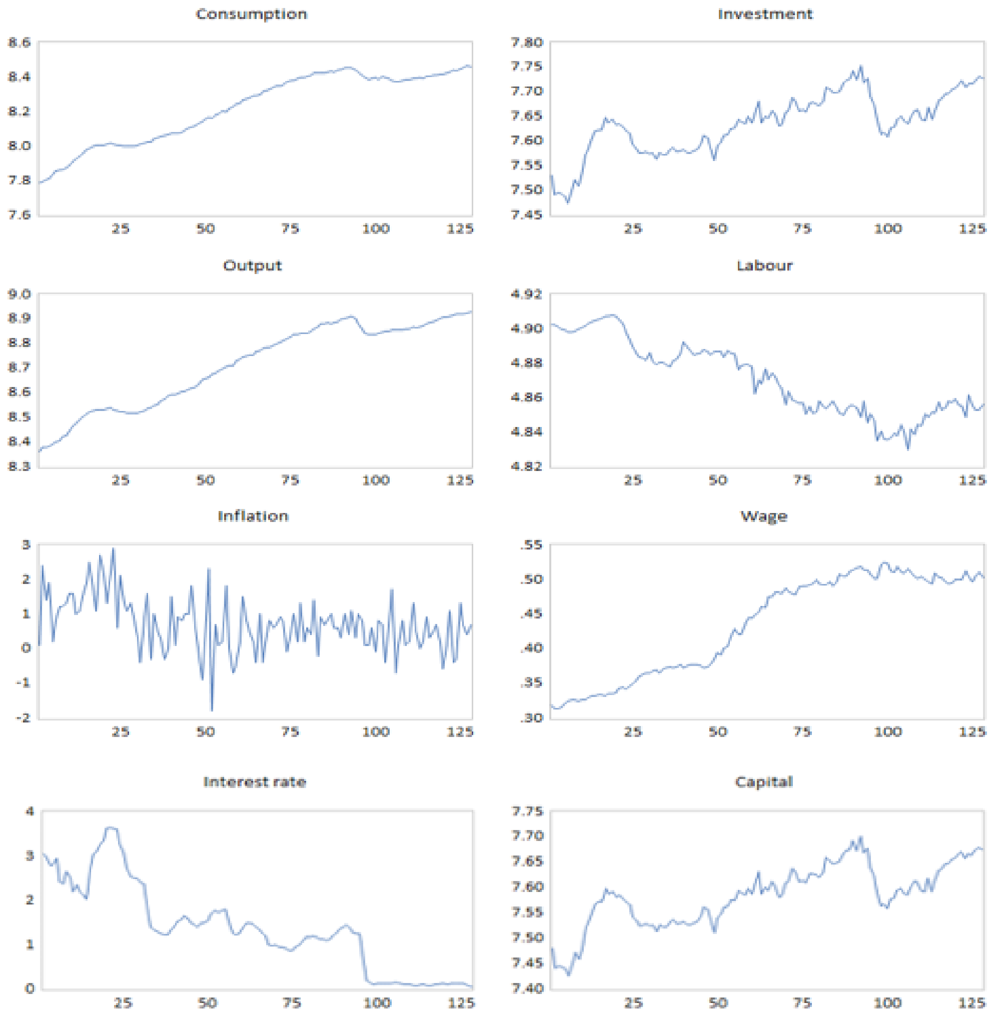
Data

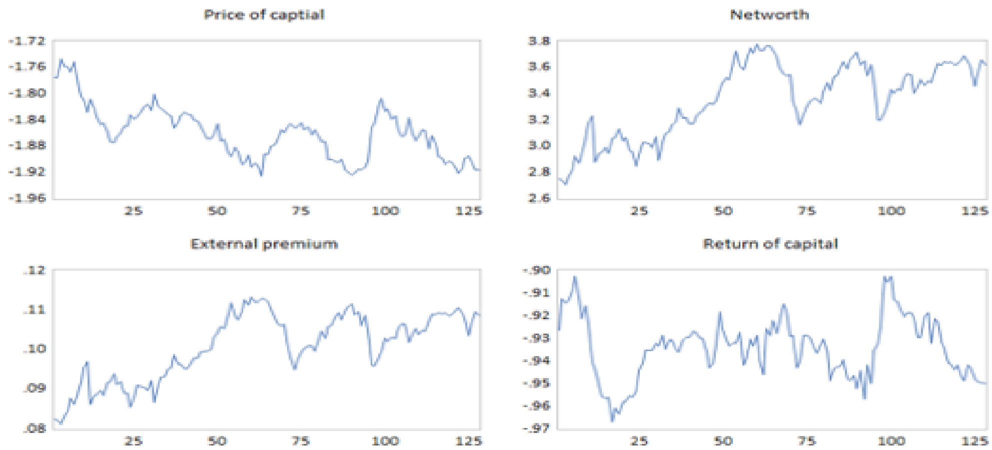
I use the UK data over the period of 1985Q1-2016Q4 on 18 macroeconomic variables, which includes consumption, investment, output, labour, inflation, wage, interest rate, capital,price of capital, net worth, external finance premium, return of capital, M0, M2 , export, import, real exchange rate, and net foreign debt to GDP ratio. Two variables are from the rest of the world: world consumption and foreign interest rate. All the variables are converted into per capita basis by dividing the total working-age population. All the variables are in natural log forms, except the variables expressed in percentages or ratio such as interest rate,inflation and net foreign debt to output. And all the variables are seasonally adjusted.

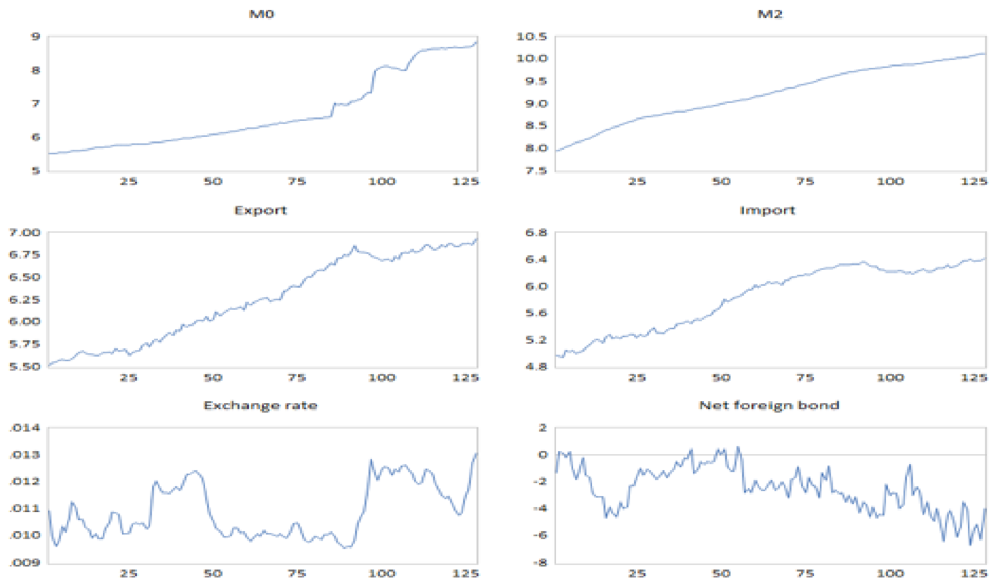
Table A.1: Data and Resources

Symbol	Variable	Definition, Description	Sources
R	Nominal interest rate	3 month average sterling T-bill	BoE
I	Investment	Gross fixed capital formation + Changes in inventions	ONS
P^k	Price of capital	Calculated from model equation	N/A
K	Capital	Calculated from model equation	N/A
π	Inflation	Quarterly percentage change in price GDP deflator	ONS
W	Wage	Average wage and earning / Total actual working hours , diveded by GDP deflator	ONS
C	Consumption	Household final consumption expenditure	ONS
Y	Output	Gross domestic product	ONS
L	Labour	employment / total actual hour worked	ONS
R^k	Rental rate of capital	Calculated from equation	N/A
S	External finance premium	Difference of bank lending rate and risk free rate	BoE
N	Net worth	FTSE all share index , divided by GDP deflator	Data Stream
M0	Quantitative easing	M0 Stock in UK	Federal Reserve Economic Data
M2	Total money supply	M2 money stock in UK	Federal Reserve Economic Data
EX	Export	Total UK export	ONS
IM	Import	Total UK import	ONS
Q	Real exchange rate	Inverse of quarterly average sterling effective exchange rate	ONS
P	General price level	Consumer Price Index of All items in the UK	Feferal Reserve Economic Data

Figure A.1: Actual Data from 1985Q1 to 2016Q4




Price of captial
Networth
External premium
Return of capital


M0
M2
Export
Import
Exchange rate
Net foreign bond