Modelling changes in healthcare demand through geographic data extrapolation.

ARTICLE HISTORY
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ABSTRACT
Stay Well Plans are a new programme of care offered to frail and elderly people in Newport. In 2016 there were plans to roll out the programme so that it would be offered in all five counties of Gwent, a region of South East Wales serviced by Aneurin Bevan University Health Board. This paper presents the data analysis and modelling used to determine the effects of the programme on the demand of the wider system, and the effects of a Gwent-wide roll out. The analysis involves extrapolating information from data from a geographical subset of the model domain to a larger geographical area, adjusting for population sizes, deprivation, and distances to healthcare facilities. These are then used to parametrise a Markov model and a Monte Carlo simulation to predict changes in demand due to different levels of roll out of the Stay Well Plans. Future population projections are also included in the analysis to examine the programme's effect in conjunction with population growth. One of the main conclusions of the case study is that a roll out of the programme may result in a large reduction on demand at residential care services, however at the expense of an increase in demand at community care services.

KEYWORDS
simulation; data extrapolation; evaluation;

1. Introduction

In recent years, partly due to an ageing population, and especially in light of the COVID-19 pandemic, demand for healthcare has been increasing. The most expensive of such care, in-patient services, face the most pressure with their limited resources becoming stretched. Many older people, those 60 years or older, would be considered frail (Clegg et al. (2013)), a state of vulnerability due to ageing that increases the risk of adverse health outcomes. These people contribute considerably to this rise in healthcare demand. This has been seen recently with a higher mortality and hospitalisation rate amongst the elderly during the COVID-19 pandemic (World Health Organisation (2020)). These ageing populations, meaning a larger proportion of frail people needing medical care, is a worldwide phenomenon (United Nations (2020)), including Wales (Older People's Commissioner for Wales (2019)).

One of the most significant system shifts required at Aneurin Bevan University Health Board (ABUHB) is enabling patients, families and carers to become more empowered and informed about the services and support available to them. This comes within a nation-wide context of a move to providing patients with more and more care within their own homes and communities, as stated in the Welsh Government's Healthier Wales plan (Welsh Government (2018)); and similar plans across the UK (NHS England (2016); Scottish Government (2012)). One strategy to achieve this goal in Gwent in South East Wales is a programme called Stay Well Plans (SWPs), which are plans aimed to deliver an integrated healthcare pathway for elderly people living in Newport. A core element of this plan is focused on providing patients, carers and families with the appropriate information, advice and assistance to better manage their needs, enabling continued independence and effective long term conditions management in their own homes. The focus of the pathway is "pro-active patient centred co-ordinated care" (Kenney (2015)); patients at risk of becoming frequent users of healthcare services are offered a single holistic SWP with the intent of using low or no cost services where appropriate.

Patients offered the plan receive an integrated care assessment, carried out by a designated care facilitator, and, together with the individual and a family member or carer, an anticipatory care plan is developed. This plan may include such interventions as (PeopleToo (2016)):

- Bathing aid installations
- Nail cutting
- Domestic services
- Fire safety checks
- Hearing aid
- Grab rail installation
- Wheelchairs
- Benefit checks
- Social support
- Befriending

In this paper, a model of demand at healthcare facilities across large geographic area in South East Wales is built, to investigate the effect of a roll-out of SWPs across the area. It is parametrised using study data from a smaller geographic area only. Data extrapolation techniques are used to scale up parameters and capture inherent differences in behaviour between geographic regions. This is a common problem when using study cohorts to test system wide programmes. A central part of the system being modelled here are specialised frailty services in Gwent, while the intervention being evaluated is a bespoke plan for elderly people in order to alleviate the adverse outcomes of frailty. Therefore this modelling sits within a topical and pressing context for Wales. The main contribution of the paper is to parametrise a model of demand across geographies and scenarios in which data is lacking.

1.1. Background

This work uses mathematical modelling in order to aid with forecasting healthcare demand across a heterogeneous region under various levels of SWP roll out. Forecasting healthcare demand using mathematical techniques is a standard practice, with and overview given in Soyiri and Reidpath (2013). Common techniques include time series analysis, logistic regression models, and probability models. It is argued in Wang (2020) that it is vital to include geographical information in healthcare models, and specifically that treating large geographical areas as homogeneous regions can be inappropriate due to geographic heterogeneity and spacial accessibility. This is evidenced in a number of studies where healthcare demand and attendance probabilities are observed to be variable across geographic regions, due to accessibility and distances to healthcare facilities (Alegana et al. (2012); Yao et al. (2013); Cheng et al. (2020); Lee et al. (2016)).

Mathematical modelling has a strong history of application within healthcare contexts, with comprehensive reviews given in Brailsford et al. (2009); Cooper et al. (2007), and Palmer et al. (2018). Popular techniques include decision trees, Markov models, and discrete event simulation. These are often combined with data analysis and forecasting methods in order to parametrise them.

Models that integrate secondary and community care in a whole systems approach are not common. The authors of Millard et al. (2001) build an analytical model of patient flows in a social care setting, and indicate that this methodology has the potential to be applied for whole systems modelling of secondary and community care. In McClean et al. (2006) and McClean et al. (2007) integrated Markov models of secondary and community care are built, with the latter paper also including rewards in the model for evaluation. Systems dynamics is another methodology that is often used in models integrating secondary and community care, for example in Brailsford et al. (2004); while queueing network models are used in Bidhandi et al. (2019) and discrete event simulation is used in Bae et al. (2017) to model the flow between hospitals and community care. In Cooper et al. (2007) distinction between cohort modelling and population modelling is given, with the former effective in evaluating effects and the latter useful in planning for costs and decision making.

This work combines both cohort and population modelling, extrapolating data from a small cohort to a large population to investigate both the effects of a healthcare intervention, and also to plan and anticipate future demand. Here a Markov model, supported by Monte Carlo simulation, is used to model an integrated system investigating demand at secondary care, community care, and their interactions. A key feature of the model is that is does not consider the geographic region of interest as homogeneous, and accounts for the heterogeneous nature of the populations and differences in distances and access to healthcare facilities.

1.2. Aims and Structure

The aims of this paper are to use cohort and population modelling:

- to build a Markov model of demand across healthcare facilities in Gwent;
- to parametrise that Markov model from data collected from a subset of its geographic domain, adjusting for differences across subregions such as population size, deprivation levels, and distances to healthcare facilities;
- to use that Markov model to investigate demand changes due to various stages of a roll-out of SWPs across the region.

The next section discusses the study cohort from Newport and the data collection period. Section 3 presents the model in its entirety, which can be thought of chronologically as (1) finding empirical external arrival rates to each facility for the study cohort; (2) identifying the effects of accepting or rejecting a plan for the study cohort; (3) scaling up by population sizes for each county in Gwent; (4) adjusting uptake proportions due to the effects of deprivation; (5) adjusting demand at hospitals due to geographical distances; (6) calculating effective arrival rates to each facility from external arrival rates and facility-to-facility transition probabilities; and (7) using these effective arrival rates to calculate yearly demand in terms of bed-days or contact-days at each facility. The geographic adjustments in steps (3), (4) and (5) here are described in Section 4. In addition to this Markov model, Section 5 introduces a Monte Carlo simulation around this demand to account for variability. Finally Section 6 presents

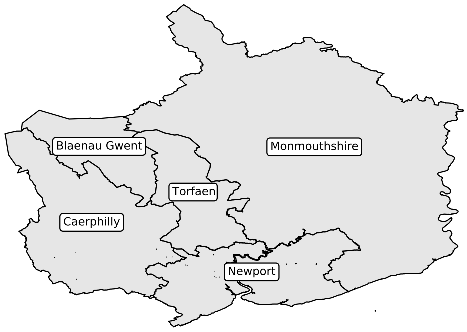

Figure 1. The five counties served by ABUHB (Blaenau Gwent, Caerphilly, Monmouthshire, Newport, and Torfaen), known as Gwent.

the model results.

2. The Study Cohort

SWPs are facilitated by ABUHB, in partnership with Newport City Council and Age Cymru. The health board covers a geographical area of South East Wales encompassing five counties (Aneurin Bevan University Health Board (2018)), shown in Figure 1.

In April 2014, a trial selection of GP practices in Newport county, Wales, began offering a new programme of care for frail and elderly people, to a subset of their patients who were over 60 years of age. SWPs were administered through GP practices. Not all GP practices were offered to participate in the scheme immediately, and some practices opted out of the trial; thus the study cohort included 12605 individuals, corresponding to around 38% of over people over 60 years of age in Newport county. For the purposes of the modelling here we assume that in all characteristics relevant to the model they are representative of the population of Newport county.

Additionally, a proportion of the population are offered a separate type of plan, and referred directly to frailty services to receive specialist bespoke care. As these patients receive fundamentally different care from the rest of the population, and they are not of interest to decision makers, this group of patients are omitted from the model. We do however assume that this proportion of omitted patients would be constant throughout the region.

Data on this cohort's activities was recorded for a period between 01/04/2014 and 30/08/2017, and SWPs were continually offered throughout this period. These activities include whether they were offered an SWP, whether they accepted or rejected an SWP, the date they started an SWP, and the dates and lengths of stay at twelve healthcare facilities of interest, listed in the next section.

Preliminary data analyses (PeopleToo (2015, 2016)) indicate that these interventions may be causing a decrease in demand at a number of healthcare facilities. In this paper Markov modelling and data extrapolation techniques are used to investigate any effect on demand this programme may be having, the effect of offering the plans in all five counties of Gwent, and what this effect looks like in the context of increasing populations.

3. The Model

The population to be modelled is all people over 60 years of age in each of the five counties of Gwent. In addition to the county in which that person resides, a person is classified according to their SWP status: not offered an SWP, offered an SWP but it was rejected, and offered an SWP and it was accepted. These fifteen classes of person (five counties and three SWP statuses) are used to define the population for the model.

Data could only be obtained for a subset of people living in Newport county, discussed in Section 2. This subset of people is called the study cohort. Section 4 outlines some of the steps and techniques used to extrapolate parameters of this data to whole population of Gwent.

The health system is modelled as a Markov model of patients arriving and progressing through a network of healthcare facilities. This methodology does not capture any limits on capacity which may exist, but including them might mask changes in demand, especially with any large population changes, which is the primary focus of this work. Following discussions with healthcare professionals from the ABUHB Workforce Planning department, Public Health Wales, ABUHB Frailty Department, Aneurin Bevan Continuous Improvement (ABCi), and PeopleToo, a network of healthcare facilities was derived. This is shown as a map of patient flows in Figure 2.

Let $\mathcal{F}$ be the set of the twelve healthcare facilities of interest, listed below, indexed by f. These include three hospitals in the region, Royal Gwent, Nevill Hall, and Ysbyty Ystrad Fawr, along with specialised frailty services, community care and residential care services. The set of facilities $\mathcal{F}$ are:

(1) ED Attendance (in any hospital)
(2) general ward admissions at Royal Gwent
(3) general ward admissions at Nevill Hall
(4) general ward admissions at Ysbyty Ystrad Fawr
(5) Assessed Out
(6) Frailty - Rapid Medical
(7) Frailty - Rapid Other
(8) Frailty - Reablement
(9) Frailty - Falls
(10) Community Care - Home/Day Care
(11) Community Care - Other
(12) Community Care - Residential

Frailty services are a multidisciplinary team set up within ABUHB, sitting within both primary and community care, focussing on better care for the elderly. The services they offer are categorised into four areas of expertise, each with different skill mix in its workforce. These are Rapid Medical services for urgent treatment of illnesses, Rapid Other for urgent treatment of other ailments such as physical pain, Falls for response to and prevention of injuries from falling, and Reablement for physical and occupational support for rehabilitation after a health care episode. A number of community care packages are provided, often bespoke, and difficult to describe exactly what is involved in the care. These packages are categorised into three types: Home/Day Care often involve preventative care that is either provided at home or do

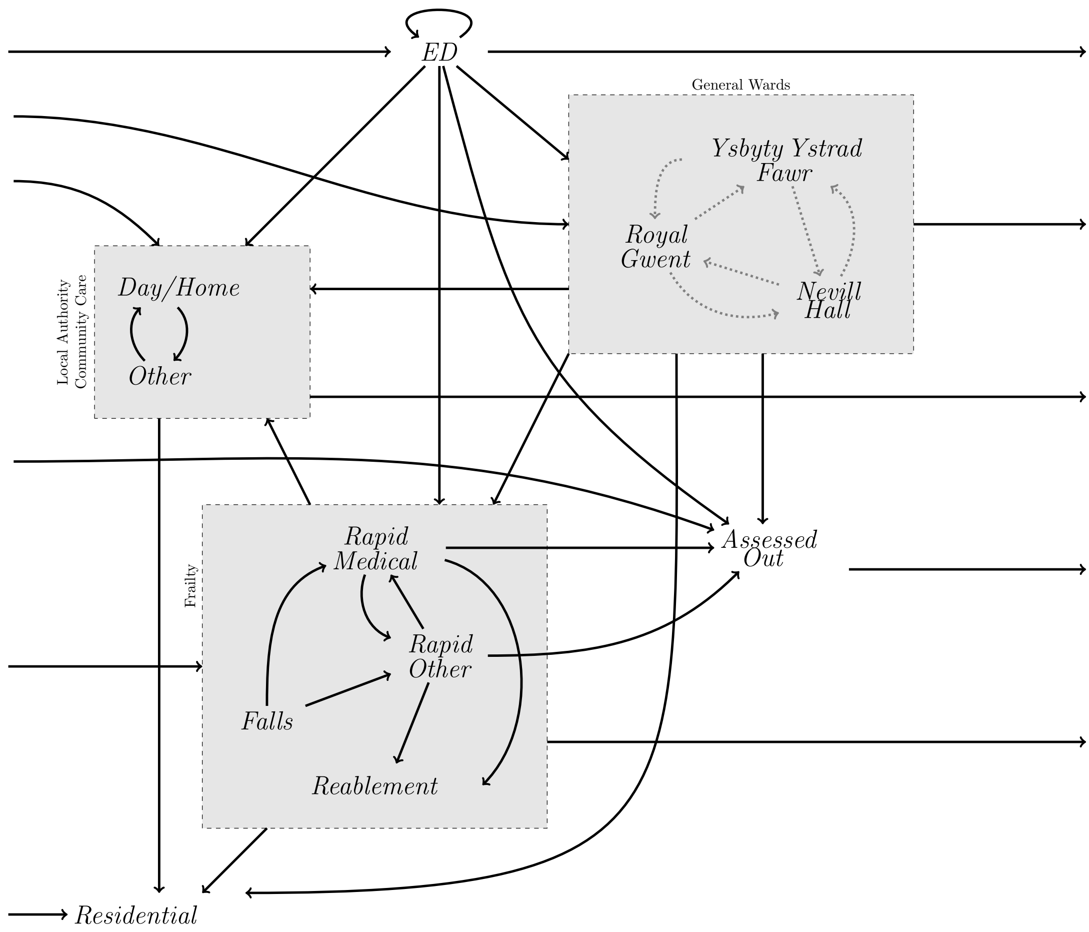

Figure 2. Map of the older person's integrated care network. Greyed out routes within the general wards are not captured in this analysis due to data restrictions.

not involve an overnight hospital say, Other services are generally ongoing care provided via telecommunications, and Residential Care involves moving to a residential, care or nursing home. Note that 'Assessed Out' is not a healthcare facility in the same sense as the others, it is a marker of a certain route out of the healthcare system that is useful during model validation.

Let $\mathcal{C}$ be the set of the five counties that correspond to Gwent, shown in Figure 1, indexed by c. Let $\mathcal{S}$ be the set of SWP statuses a patient can be (not offered a plan, offered and rejected a plan, offered and accepted a plan), indexed by s.

Now the yearly demand at each facility is given by Equation 1:

$$D_f = 365 \sum_{c \in \mathcal{C}} \sum_{s \in \mathcal{S}} \Lambda_{fcs} \frac{1}{\mu_{fs}}, \tag{1}$$

for $f \in \mathcal{F}$, where Λ_{fcs} denotes the effective arrival rate to facility $f \in \mathcal{F}$ of patients of status $s \in \mathcal{S}$ in county $c \in \mathcal{C}$, and μ_{fs} denotes the service rate at facility $f \in \mathcal{F}$ for patients of status $s \in \mathcal{S}$.

The effective arrival rates are found by solving the traffic equations given in Equation 2:

$$\Lambda_{fcs} = \lambda_{fcs} + \sum_{g \in \mathcal{F}} T_{scgf} \Lambda_{gcf} \tag{2}$$

$f \in \mathcal{F}$, $c \in \mathcal{C}$, $s \in \mathcal{S}$, where T_{sgf} denotes the probability of a patient of status $s \in \mathcal{S}$ being transferred to facility $f \in \mathcal{F}$ after completing service at facility $g \in \mathcal{F}$. Here λ_{fcs} is the external arrival rate to facility $f \in \mathcal{F}$ for patients of status $s \in \mathcal{S}$ in county $c \in \mathcal{C}$. These external arrival rates are influences by a number of factors, and so are calculated by Equation 3:

$$\lambda_{fcs} = \begin{cases} \tilde{\lambda}_f t_{fc} \left(\frac{N_c}{N}\right) & \text{if } s \text{ denotes those not offered a plan,} \\ \tilde{\lambda}_f t_{fc} \psi_f \left(\frac{\rho(1-p_c)}{1-\rho}\right) \left(\frac{N_c}{N}\right) & \text{if } s \text{ denotes those rejecting a plan,} \\ \tilde{\lambda}_f t_{fc} \psi_f \gamma_f \left(\frac{\rho p_c}{1-\rho}\right) \left(\frac{N_c}{N}\right) & \text{if } s \text{ denotes those accepting a plan,} \end{cases} \tag{3}$$

$f \in \mathcal{F}$, $c \in \mathcal{C}$, $s \in \mathcal{S}$. Table 1 lists interpretations of all the above components and how each are calculated.

Equation 3 gives all external arrival rates in terms of scaling of a base arrival rate, $\tilde{\lambda}$, that of the study cohort who are not offered an SWP. Though it may seem more intuitive to simply scale up the study cohort's arrivals by county population for each status, this would assume that proportions of the population in each status are identical in each county. This may not be the case, as uptake rates may differ by county, and more importantly we wish to investigate scenarios in which not all counties are offering SWPs. Two key factors then are the effect on arrival rate of being at-risk enough to be offered an SWP, and the effect on arrival rate of receiving an SWP. These two factors, ψ_f and γ_f respectively for facility $f \in \mathcal{F}$, are calculated by Equations 4 and 5.

Component	Interpretation	Derivation
$\mathcal{C}$	The set of counties of Gwent.	Figure 1.
$\mathcal{S}$	The set of SWP statuses of patients.	Not offered a plan, rejected a plan, and accepting a plan.
$\mathcal{F}$	The set of all healthcare facilities.	Listed in Section 3 and Figure 2.
N	The size of the study cohort.	12605, from data.
N_c	The number of people over 60 in county c, excluding the proportion of patients that would be referred directly to frailty.	ONS estimates, given in Table A3.
ρ	The proportion of the population eligible to be offered an SWP.	0.0882, from data.
p_c	The proportion of those offered an SWP from county c that accept the plan.	Derived from a logistic regression model, details in Section 4.1.
T_{scfg}	The probability of a patient of status s from county c being transferred to facility g from facility f.	Values for Newport from data, given in Equations A1, A2 andA3. Transformation for other counties discussed in Section 4.2.
t_{fc}	A transform factor for external arrival rates to facility f for patients from county c.	Discussed in section 4.2.
ψ_f	The effect on the arrival rate to facility f of being at-risk enough to be offered an SWP.	Derived in Equation 4, values given in Table A1.
γ_f	The effect on arrival rate to facility f of receiving an SWP.	Derived in Equation 5, values given in Table A1.
$\tilde{\lambda}_f$	The base arrival rate of the study cohort to facility f, for those not offered an SWP.	From data, values given in Table A1.
λ_{fsc}	The external arrival rate to facility f for patients of status s in county c.	Given by Equation 3.
Λ_{fsc}	The effective arrival rate of facility f for patients of status s in county c.	Given by solving Equations 2.
μ_{fs}	The service rate at facility f for patients of status s.	From data, given in Table A2.
D_f	The expected demand (total yearly contact days) at facility f.	Given by Equation 1.

Table 1. List of interpretations of all components of the model.

$$\psi_f = \frac{\tilde{\lambda}_f^{\star}}{\tilde{\lambda}_f}\left(\frac{1-\rho}{\rho\,(1-p)}\right) \tag{4}$$

$$\gamma_f = \frac{\tilde{\lambda}_f^{\star\star}}{\tilde{\lambda}_f}\psi_f\left(\frac{1-\rho}{\rho p}\right) \tag{5}$$

$f \in \mathcal{F}$, where $\tilde{\lambda}_f^{\star}$ is the observed arrival rate for the study cohort for those rejecting SWPs, and $\tilde{\lambda}_f^{\star\star}$ is the observed arrival rate for the study cohort for those accepting SWPs.

Tables A1 and A2 give observed values of $\tilde{\lambda}_f$, ψ_f, γ_s and μ_{fs} for each $f \in \mathcal{F}$, $s \in \mathcal{S}$. Table A3 gives the population sizes of the five counties of Gwent for people over 60 years of age, obtained from British Office of National Statistics (ONS) Mid-2015 population estimates (ONS (2016b)).

Equations A1, A2 and A3 gives matrices representing T_{sfg}, for Newport county, for each $s \in \mathcal{S}$ representing not offered, rejected, and accepted plans respectively. In these matrices the indices of the rows and columns represent the corresponding numbered healthcare facilities $f \in \mathcal{F}$ numbered above. Section 4.2 describes how these are transformed for the other counties.

4. Geographic Adjustments

The data sets provided by the health board cover a subset of patients residing in Newport county only. Thus to be able to parametrise the whole population of Gwent it is required that the parameters are scaled up from a subset of the population of Newport, any inherent differences across counties.

This is done in three ways: scaling arrival rates to population sizes (described in Equation 3) investigating the effect of deprivation levels on SWP uptake, and adjusting transition rates to consider hospital locations.

4.1. Effect of deprivation

This section will explore whether deprivation levels of an individuals' place of residence affects the likelihood of accepting an SWP once offered. The accepted measure of deprivation in Wales is the Welsh Index of Multiple Deprivation (WIMD) (Welsh Government (2014)), which ranks small geographical areas of Wales called Lower Super Output Areas (LSOAs) by deprivation. Figure 3 shows the WIMD score for each LSOA, and shows that Gwent contains extremes in deprivation, both the most (St Kingsmark, Monmouthshire) and least (St James, Caerphilly) deprived areas in Wales are in the region.

A logistic regression is carried out on the study cohort to determine the effect of WIMD score on the probability of accepting an SWP once offered. This was carried out on the study cohort, a sample from Newport which is most diverse of the five counties in terms of deprivation.

There were some missing WIMD scores, and in general there are two ways of dealing with this (Soley-Bori (2013)), deletion of missing values or average imputation. We

Method	Parameter	Value	p-value
Deletion	Intercept	2.90579	≤ 0.01
	WIMD coefficient	0.00045	0.844
Average Imputation	Intercept	1.71348	≤ 0.01
	WIMD coefficient	0.00079	0.078

Table 2. Logistic Regression Summary

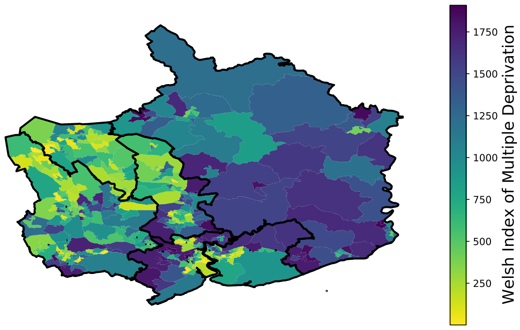

Figure 3. Map showing variation in deprivation scores by the LSOAs served by ABUHB. Low scores indicate high deprivation, and high score indicate low deprivation.

carried out both methods using SciPy (Jones et al. (2001)), summarised in Table 2, and choose to continue with the average imputation model. A plot of the predicted and observed values is given in Figure 4. As shown by the p-values in the table the relationship is weak, however the average imputation logistic regression model is still used in the overall model to capture any effect resulting from deprivation.

This logistic regression model shows that there is a small increase in probability of accepting an SWP as WIMD score increases, that is as deprivation level decreases. Assuming that this effect is constant across counties, an aggregated WIMD score for each county is used to obtain p_c for each county $c \in \mathcal{C}$.

4.2. Adjusting transitions to hospitals

The largest discrepancy between the parameters of the study cohort and the required parameters of the model for each county $c \in \mathcal{C}$ is the arrivals and routing to the three hospitals Royal Gwent, Nevill Hall, and Ysbyty Ystrad Fawr. Royal Gwent, located in Newport, is the most popular hospital for patients in the study cohort to attend, with 98.5% of all general ward admissions going here. Nevill Hall, located in Abergavenny in North Monmouthshire, far away from where the study cohort live, consists of 1.2% of all general ward admissions by the study cohort. Finally Ysbyty Ystrad Fawr is a smaller hospital in Ystrad Mynach in Caerphilly, and only 0.3% of general ward admissions were at this hospital.

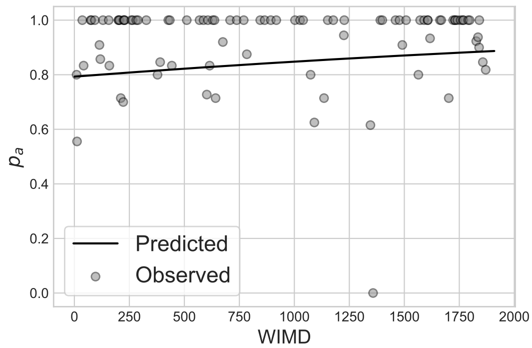

Figure 4. Scatter plot of WIMD score against observed probability of accepting an SWP, along with logistic regression fit.

It is known that GP practices in the same geographic cluster give similar advice on which hospital their patients should attend. In addition to proximity, it is known that different hospitals specialise in different ailments. Therefore a patient may still need to be admitted to a hospital far away from their home.

In order to account for this, the following probabilistic adjustment is proposed. Each county $c \in \mathcal{C}$ has three associated probabilities $a_{(\text{RG},c)}$, $a_{(\text{NH},c)}$, and $a_{(\text{YYF},c)}$; such that $a_{(\text{RG},c)}$ is the probability of a patient from county $c \in \mathcal{C}$ going to Royal Gwent for a general ward admission, $a_{(\text{NH},c)}$ is the probability of a patient from county $c \in \mathcal{C}$ going to Nevill Hall for a general ward admission, and $a_{(\text{YYF},c)}$ is the probability of a patient from county $c \in \mathcal{C}$ going to Ysbyty Ystrad Fawr for a general ward admission. Therefore $a_{(\text{RG},c)} + a_{(\text{NH},c)} + a_{(\text{YYF},c)} = 1$, for each county $c \in \mathcal{C}$. Now the parameters derived from the study cohort (a subset of Newport) can be adjusted by multiplying by the ratio of the probability of entering each hospital from county $c \in \mathcal{C}$, with the corresponding probability for Newport. For example the arrival rate to Nevill Hall for patients from Caerphilly must be multiplied by $\frac{a_{(\text{NH},\text{Caerphilly})}}{a_{(\text{NH},\text{Newport})}}$. That is patients from Caerphilly are $\frac{a_{(\text{NH},\text{Caerphilly})}}{a_{(\text{NH},\text{Newport})}}$ times more likely to be admitted to Nevill Hall than patients from Newport (that is the study cohort).

In order to obtain $a_{(\text{RG},c)}$, $a_{(\text{NH},c)}$, and $a_{(\text{YYF},c)}$, first define the set of LSOAs by $\mathcal{L}$ indexed by l. Then the following steps are taken, making use of the Google Maps API:

(1) Obtain the distances $d_{\text{RG},l}$, $d_{\text{NH},l}$, and $d_{\text{YYF},l}$ from the centre (centroid) of each LSOA $l \in \mathcal{L}$, to each hospital RG, NH and YYF.
(2) Normalise the distances using a weighted inverse power normalisation method, weighted by number of beds at each hospital, to transform the distances to probabilities $a_{\text{RG},l}$, $a_{\text{RG},l}$, and $a_{\text{RG},l}$ for each LSOA $l \in \mathcal{L}$.
(3) Aggregate the probabilities over each county, weighted by the population of people over 60 years of age, to obtain the probabilities $a_{(\text{RG},c)}$, $a_{(\text{NH},c)}$, and $a_{(\text{YYF},c)}$.

The weighted inverse power normalisation used in step 2 is shown in Equation 6, with parameter n. The d's represent the distances, and the w's represent the hospital weights. The hospital weights are taken as the number of beds available at each hospital, thus $w_{\text{RG}} = 774$, $w_{\text{NH}} = 499$, and $w_{\text{YYF}} = 269$.

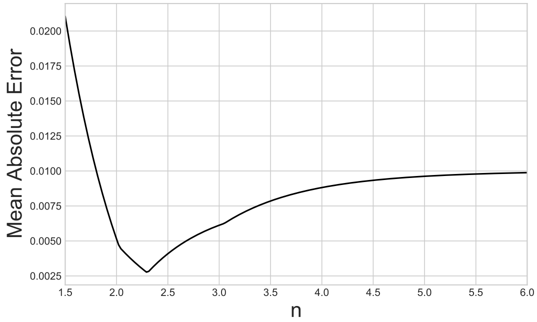

Figure 5. Effect of the choice of the parameter n of the normalisation method on the mean absolute error between the fitted and observed vectors for the Newport parameters.

$$a_{gl} = \frac{w_g d_{gl}^{-n}}{\sum_j w_j d_{jl}^{-n}} \tag{6}$$

for each LSOA $l \in \mathcal{L}$ and each hospital g. This is more appropriate in this case than the usual Softmax normalisation, $a_{gl} = \frac{e^{-d_{gl}/w_g}}{\sum_i e^{-d_{il}/w_i}}$, which is very sensitive to scale. Another advantage with the inverse power normalisation is that it is parametrised by the power n. A parameter n can be chosen that yields a vector for Newport closest to the observed values, using an implementation of the Broyden–Fletcher–Goldfarb–Shanno algorithm (BFGS) (Binnans et al. (2003)) in Scipy (Jones et al. (2001)). This function minimises the mean absolute error between the predicted vector $\left(a_{(\text{RG},c)}, a_{(\text{NH},c)}, a_{(\text{YYF},c)}\right)$ and the corresponding observed values, for Newport.

Figure 5 shows how the mean absolute error between the fitted and observed vectors for the Newport parameters is affected by the choice of n. The optimal value of n using the minimisation algorithm was $n = 2.30056$. Figure 6 shows the obtained distances, probabilities, and aggregated probabilities for each hospital. Table A4 summarise the calculated probabilities $a_{(\text{RG},c)}$, $a_{(\text{NH},c)}$, and $a_{(\text{YYF},c)}$ for each county.

These values transform the arrival rates and transitions in the following ways:

(1) The external arrival rates are multiplied by t_{fc} for each county $c \in \mathcal{C}$ and each facility $f \in \mathcal{F}$, given by Equation 7,

$$t_{fc} = \begin{cases} \frac{a_{(f,c)}}{a_{(f,\text{Newport})}} & \text{if } f \in \{\text{RG, NH, YYF}\} \\ 1 & \text{otherwise.} \end{cases} \tag{7}$$

(2) For each g belonging to the set of three hospitals, T_{cfg} is replaced with $a_{(f,c)} \times q_{fc}$, where q_{fc} is the probability of a patient from county $c \in \mathcal{C}$ being transferred to any hospital, given by Equation 8.

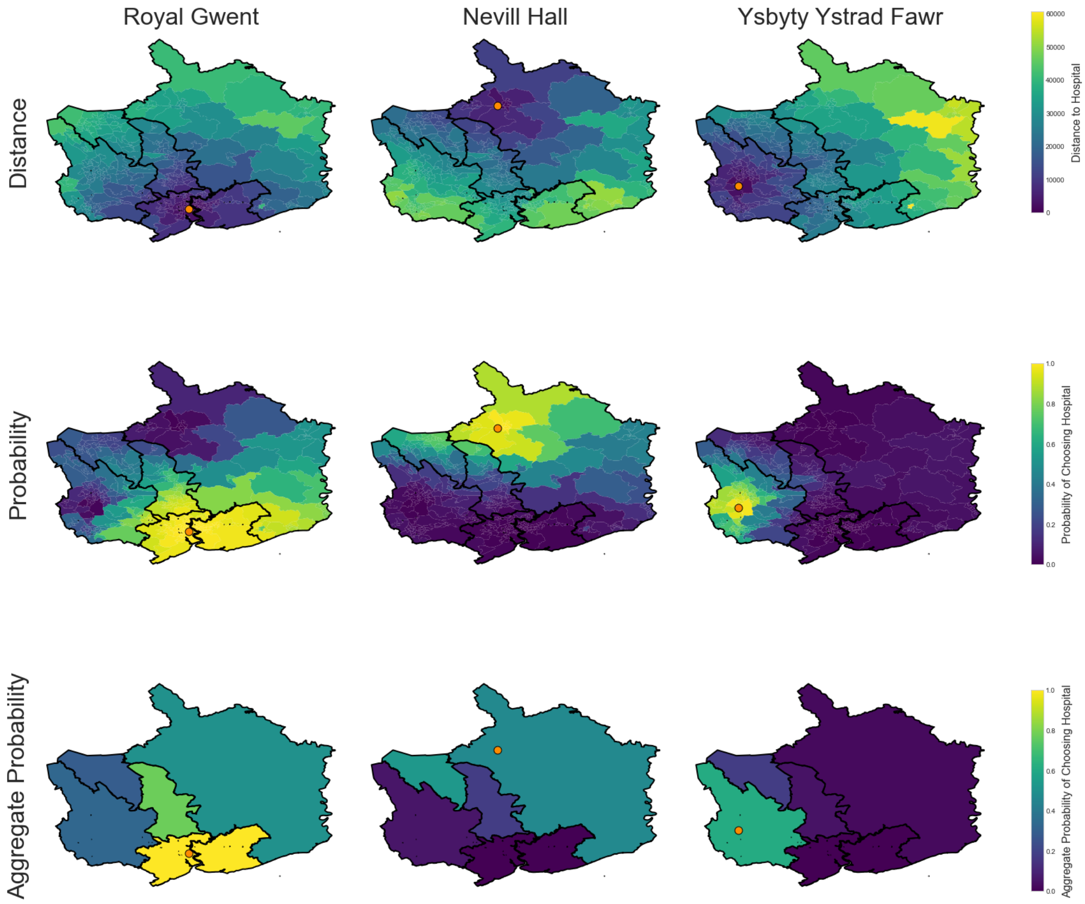

Figure 6. Geographic visualisation of the adjustment method.

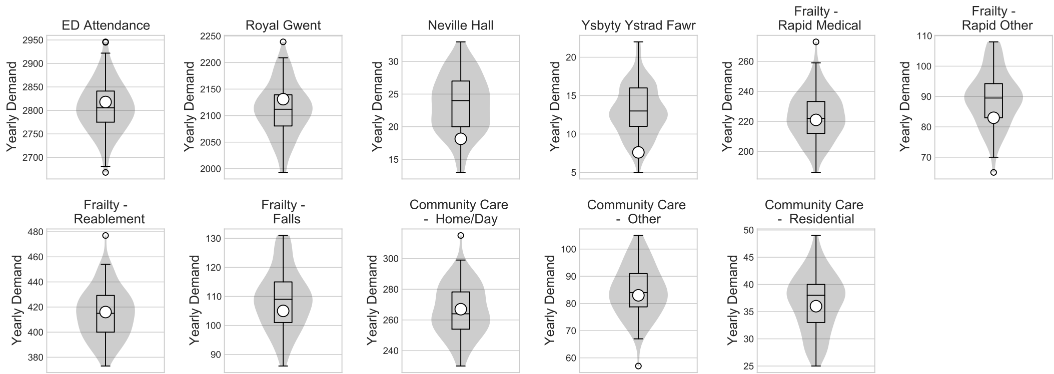

Figure 7. Validation run results for the demand on each facility. Box and violin plots give the variability from the Monte Carlo simulation, white circles gives the observed value form data.

$$q_{fc} = \sum_{g \in \{\text{RG, NH, YYF}\}} T_{cfg} \tag{8}$$

5. Monte Carlo Simulation

The model above estimates the expected demand at each healthcare facility. Along with the assumption that arrivals and lengths of stays are exponentially distributed at each facility, a Monte Carlo simulation is run to gain insight into the variability and around demand and its effect size. First the model is validated by running the simulation using demand for the study cohort only and comparing to observed demand from data. Then three scenarios are run for comparison.

5.1. Validation

In order to validate the model developed in Section 3, the model is run for a simulated population with the size and characteristics of the study cohort. Total demand and total contact days are recorded over the course of a year of simulation time. Contact days are not meaningful for the ED Attendance facility, and further work is given in Section 6.3 to calculate contact days for the Community Care - Residential facility. Figures 7 and 8 show box and violin plots of the results obtained from 100 Monte Carlo observations for attendances and contact days respectively. White circles give the observed values from the last year (May 2016 to May 2017) of data. As the numbers for Nevill Hall hospital and Ysbyty Ystrad Fawr are small, discussed in Section 4.2, these facilities are compared against average yearly demand over the whole data. These visualisations confirm the model reflects reality for the study cohort.

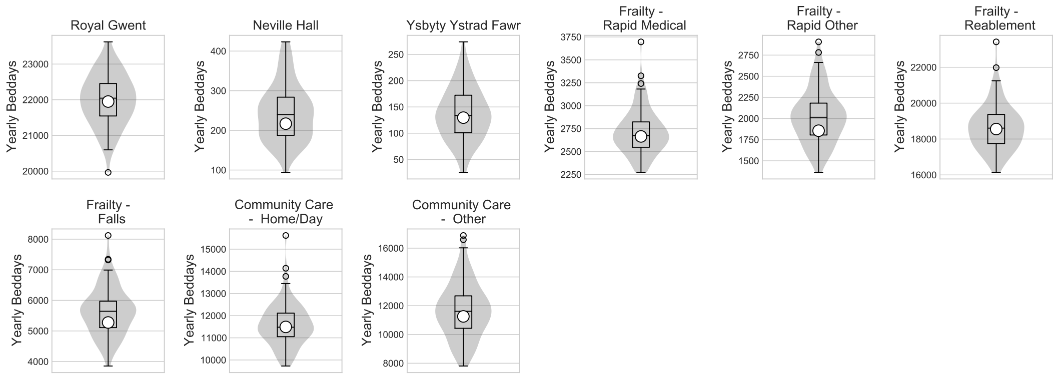

Figure 8. Validation run results for the contact days at each facility. Box and violin plots give the variability from the Monte Carlo simulation, white circles gives the observed value form data.

6. Results

This model is used to investigate the effect of of the SWPs. With a lack of true before and after data, that is patients were not observed before the SWP trail began, this model can be used to observe the effect of the plans by running a scenario in which the plans were not offered in any county in Gwent. Additionally, the model is used to predict the effect of offering plans in all five counties of Gwent. Thus the following three scenarios are run for comparison:

- Scenario 1: Plans not offered anywhere in Gwent (that is $p_c = 0$ for all $c \in \mathcal{C}$)
- Scenario 2: Plans offered in Newport county only (that is $p_c = 0$ for all $c \in \mathcal{C}$ except for Newport county, which takes its value from the logistic regression model)
- Scenario 3: Plans offered everywhere in Gwent (that is p_c taken from the logistic regression model for all $c \in \mathcal{C}$)

All data analysis was conducted with Pandas (McKinney (2010)) and SciPy (Jones et al. (2001)) in Python. The traffic equations are solved numerically using matrix algebra in Numpy (Harris et al. (2020)), while the Monte Carlo simulation was conducted by sampling pseudorandom numbers using Python's standard library.

First results will be presented using populations from the year 2015 in Section 6.1, considering variability using Monte Carlo simulation. Then, considering the ageing population of Gwent and looking at ONS population projections, the expected results are presented using those projected parameters in Section 6.2. Finally, as any changes in demand in residential care are cumulative, a further immigration-death model is given in Section 6.3.

6.1. Results for 2015

Using both the Markov model presented in Section 3, and the Monte Carlo simulation discussed in Section 5, expected demand and variability around this demand can be predicted for each of the three scenarios. For ED Attendance and Community Care - Residential we consider total number of attendances over one year, while for the other facilities we consider total number of contact days used over one year. Box and violin

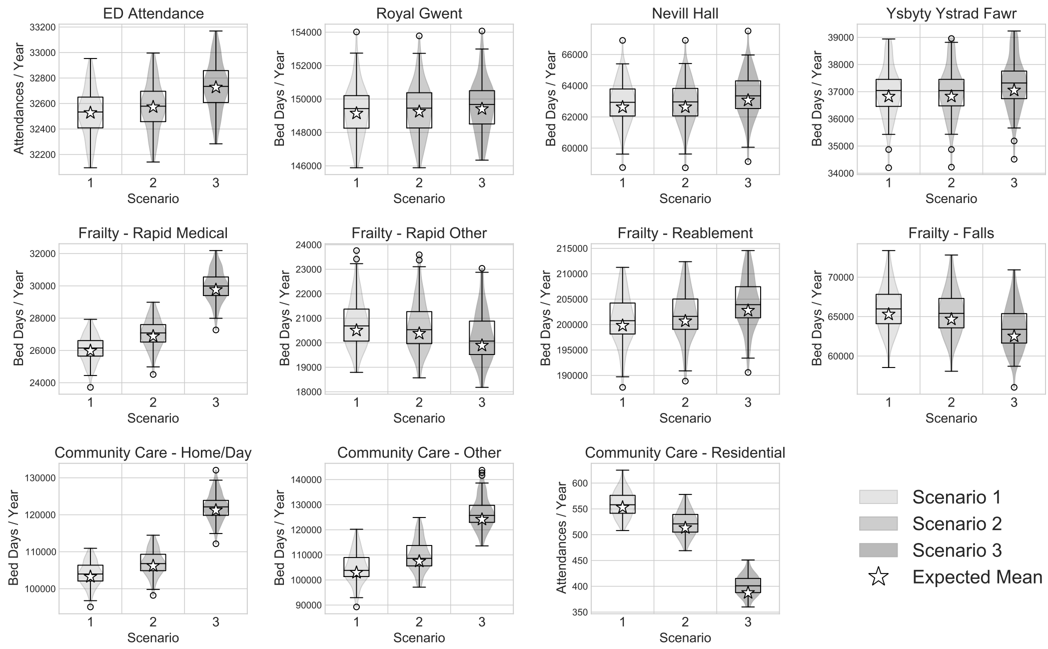

Figure 9. Yearly attendances or contact days for each facility, over the three scenarios.

plots of the results for the three scenarios, for 100 sampled observations, are shown in Figure 9. White stars denote the expected demand given by the Markov model, and their values are given in Table 3.

There are five major conclusions that can be draw from these results:

(1) The adoption of SWPs will cause slight increases in demand at ED.
(2) The adoption of SWPs won't affect demand that the three hospitals.
(3) The adoption of SWPs will cause an increase in demand at Rapid Medical frailty services. This is offset by a fall in demand at Falls frailty services.
(4) The adoption of SWPs will cause large increases in demand at Day, Home and Other Community Care services.
(5) The adoption of SWPs will cause large preventions of admittances to Residential Community Care services.

6.2. Results using population projections

In 2014 the ONS produced 26 year population projections for each county in the UK. The projections for the counties of Gwent, for the population of people over 60 years old, from 2015, are shown in Figure 10. Caerphilly, the county with the largest population of over 60s, is projected to increase the most in terms of absolute numbers. Counting all five counties together as Gwent, the number of over 60s are expected to increase from around 147,000 to nearly 200,000 by 2039. The shape of the projection curves are interesting, the growth in population is expected to ease by around 2030, with a fairly stable population size for the next ten years.

Due to this projected increase in population (nearly 30% increase over the whole of Gwent in 25 years), the model was run for each year between 2015 and 2039, using

Facility	Scenario 1	Scenario 2	Scenario 3
ED Attendance	32527.2	32572.6	32726.7
Royal Gwent	149140.6	149244.0	149407.5
Nevill Hall	62628.8	62632.7	63063.6
Ysbyty Ystrad Fawr	36826.4	36828.1	37051.2
Frailty - Rapid Medical	25982.0	26882.7	29758.8
Frailty - Rapid Other	20505.8	20378.2	19890.1
Frailty - Reablement	199807.1	200688.0	202759.6
Frailty - Falls	65291.5	64657.6	62504.5
Community Care - Home/Day	103256.5	106181.8	121235.2
Community Care - Other	102934.9	107438.8	124120.4
Community Care - Residential	553.0	513.2	387.4

Table 3. Expected number of contact days or attendances for one year, for each facility, for each scenario.

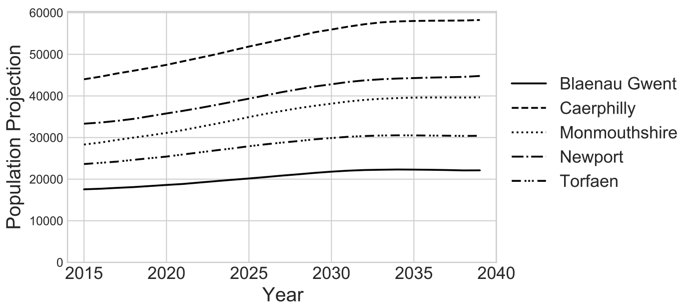

Figure 10. Summary of population projections for the counties of Gwent.

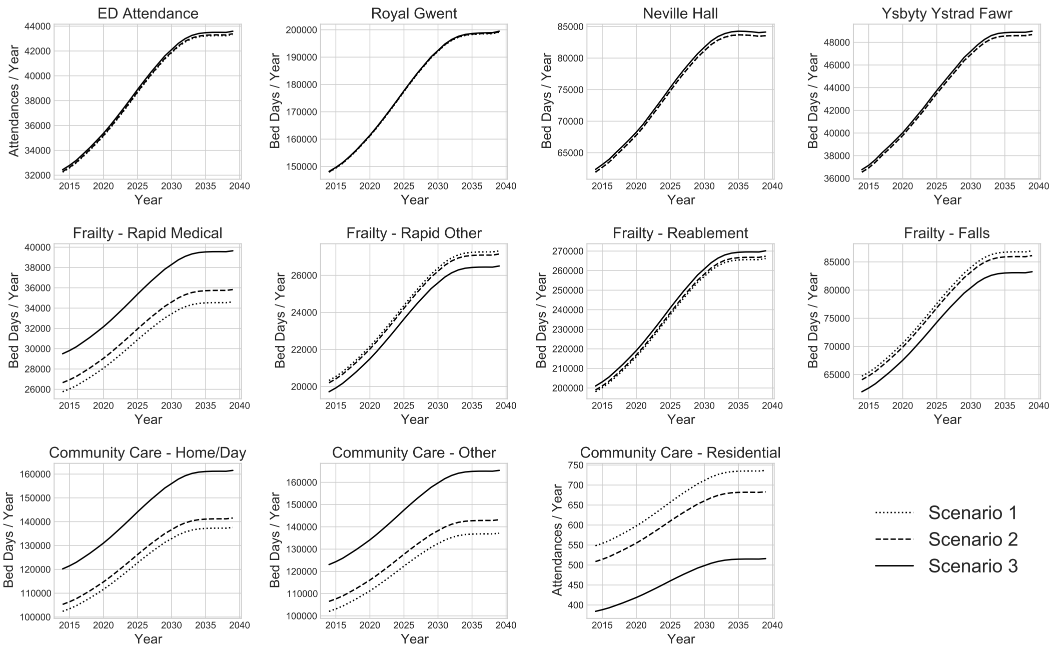

Figure 11. Projected yearly attendance and contact days for each facility, over the three scenarios, for the period 2015 - 2039.

the projected populations in place of N_c for each county $c \in \mathcal{C}$. The results of these simulations show projected future effects of the plans, as well as put any effects on demand into context with any natural increase in demand from population growth.

The resulting predicted demand at each facility for each year between 2015 and 2039 is shown in Figure 11, using the Markov model given in Section 3.

Though these results do not show any scaling of effect with population increase, it does highlight important features on the scale of effects for each facility. ED Attendance and Frailty facilities show that the differences observed in each scenario due to the SWPs are substantially dwarfed by the natural increase in demand by population increase from year to year. Thus any positive or negative effects at these facilities are likely to go unnoticed in comparison with the natural changes in demand caused by population increase. It is in community care services where large scale effects due to SWPs are seen, both increases and decreases in demand. In fact by 2039 the number of yearly admissions to residential care in the scenario where SWPs are offered everywhere is still lower than the number of yearly admissions to residential care in 2015 in the scenario where there are no SWPs. The savings towards residential care admissions are large, even when the cumulative effect of the years is not considered.

6.3. An immigration-death model of residential care

As patients are modelled as remaining in Residential Care services until death, a yearly increase in demand results in a cumulative increase in bed days. Assuming that the lower admissions to Residential care seen in Scenario 3 are due to preventions rather than delays, this may result in larger savings in residential care bed days than indicated in the above model. To investigate this, the cumulative effect is estimated

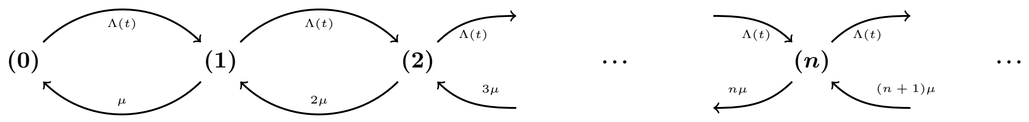

Figure 12. Transition diagram of the time dependant immigration-death process used to model Residential Community Care occupancy.

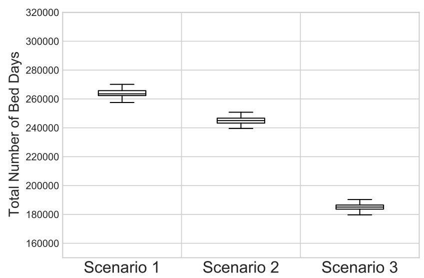

Figure 13. Results of the immigration-death model, with death rate +0%, with expected total number of bed days used in 25 years shown for each scenario.

using a time-dependent immigration-death model. A representation of this Markovian immigration-death model is shown in Figure 12. The states of the continuous time Markov chain correspond to the current number of residents in residential care. Thus summing up the number of residents multiplied by the time since the last event will give the total number of bed days used over the simulation period. The same three scenarios are run.

- Patients are admitted at a time-dependant rate $\Lambda(t)$, the 'immigration rate' of the model. For each scenario this corresponds to the total effective yearly admissions calculated from the Markov model, shown in Figure 11.
- Patients leave residential care when they pass away. Thus the 'death rate' of the model, μ, is the death rate of the patients at Residential care. ONS data (ONS (2016a)) is used to obtain this rate, under the assumption that patients over 60 years of age in Gwent residential care homes have the same death rate as all people over 65 in England and Wales. From mid 2015 estimates the death rate is $\mu = 0.04348$.
- However, this death rate is an estimate only: it is not known whether residential care residents have a higher death rate than the rest of the population as they are more ill than most other people, or if they have a lower death rate as they are being cared for professionally. Therefore two more scenarios are run where the death rate is increased and decreased by 10%.
- The model is run for 25 years (from 2015 to 3039, for which the population projections are estimated). As we are only interested in the gains made from rolling out SWPs, thus the differences in occupancy, then the model can begin from an empty system, that is state (0).

The system is simulated over 100 trials, and Figure 13 show the total number of bed days required over a 25 year period used for each scenario, for the base death rate (+0%).

	Scenario 1	Scenario 2	Scenario 3	p-Value
-10%	237458.43	220464.26	166693.60	< 0.01
+0%	263829.92	244956.75	185048.94	< 0.01
+10%	290135.17	269293.01	203539.72	< 0.01

Table 4. Mean number of residential care bed days for each scenario, for each considered death rate, with reported p-value of the Kruskal-Wallis test.

The model shows a large decrease in bed days as more SWPs are offered. Table 4 summarises the total number of residential care bed days used for each scenario and each death rate (-10%, +0%, +10%). A one-way Kruskal-Wallis test is carried out, yielding a p-values close to zero in each case, and so the improvements seen in the plot are significant. Scenario 3 exhibits a 29.8% decrease in the amount of Residential care bed days used over 25 years compared to the Scenario 1, regardless of the death rate used.

7. Conclusion

The models presented in this paper have given insight into the effects of SWPs on the population of Gwent. The model was used to investigate the effect of the plans across three dimensions: situation, geography, and time.

The three scenarios reflect three different situations that can give insight into the effect of a roll out of the plans across the region. Scenario 1 corresponds to the time before SWPs were offered anywhere, Scenario 2 corresponds to the period 2016 when they were only offered in Newport, and Scenario 3 corresponds to the proposed roll out across the region. Parameters for Scenarios 1 and 3 were inferred by adjusting the observed data that corresponds to Scenario 2. These scenarios we analysed using Markov models and Monte Carlo simulation.

This work contributes to the literature by using both cohort and population modelling to build an integrated model of secondary, community, and residential care. Another significant contribution of this work is the consideration of geography, and the treatment of a large geographical region as a heterogeneous area, a consideration considered key in Wang (2020). This involved the extrapolation of parameters for a subset of the Newport population to the whole population of Gwent, accounting for any inherent geographic effects that might occur. Three geographic adjustments were considered:

- the effect of deprivation levels on willingness to accept a plan;
- the scaling up of parameters due to population sizes;
- adjustments of demand and transitions to account for hospital locations.

This work made a number of assumptions, some dictated by the limited data availability. In scaling arrival rates to health facilities by population size, we assumed that the effect of SWPs on the individual, that is the γ_f values, were equal across all counties. This itself may be a flawed assumption, which may be corrected by comparing with other similar preventative plans in areas with similar diversity in terms of deprivation and urbanicity. Another key assumption was that the proportion of the population unwell enough to be offered an SWP, ρ is consistent across counties. However is is very likely that deprivation may in fact have an affect on this, as health is one of the factors that make up WIMD rankings Welsh Government (2014).

When modelling residential care it was assumed that the lower admissions observed in the results of Section 6.1 were preventions, although the limitation of the data collection period, only three years, means that this observation could mean that patients are experiencing delayed admissions. A longer data collection period would be beneficial in determining this. However as the demand is increasing year upon year, even a delay in admissions would result in a yearly demand less than expected otherwise. Other important considerations have been discussed throughout this paper, for example the assumption that the study cohort is representative of the whole population of Newport, and that the logistic regression model fitted for Newport is representative of the whole of Gwent.

The model was also used to investigate these effects across time. Twenty six year population projections were used to model the situations each year until 2039. The important conclusions of this modelling are listed below. Comparing against no plans offered (Scenario 1), a full roll our of the plans (Scenario 3) give:

- A 0.61% increase in demand for ED Attendance; however this is dwarfed in comparison to the natural increase in demand due to population growth. Therefore negative effects of SWPs are inconsequential here over the span of more than a year.
- Demand at frailty services is expected to be redistributed amongst the four priorities: demand at Rapid Medical is expected to increase by 14.53%, demand at Rapid Other is expected to decrease by 3.00%, demand at Reablement is set to increase 1.48%, and demand at Falls to decrease by 4.27%.
- In comparison to the increase in demand from population growth, the decreases in demand due to SWPs at Rapid Other and Falls frailty services are small. However the increase in demand due to SWPs at Rapid Medical services do seem meaningful.
- Demand at Home/Day and Other community care services, is expected to have large increases in demand, 17.41% and 20.58% respectively.
- The increases in demand due to SWPs at these community care services (Home/Day and Other) are still large in comparison to the increases in demand due to population growth.
- Demand at residential community care services is expected to decrease by a large amount, 29.8%, for all death rates considered.
- The decrease in demand due to SWPs for residential community care is large in comparison to the increases in demand due to population growth. Again here, the demand in 2015 with no SWPs is greater than the demand in 2039 with SWPs everywhere. This effect is cumulative over the 25 years, as residents stay longer than one year. Over the 25 years it is expected that there will be a 29.8% decrease in the amount of residential care bed days used if the plan is rolled out across Gwent.

Due to the current workforce difficulties, ABUHB needs to become more focussed on its approach to workforce planning, ensuring that skills are deployed and maximised to best effect and that new services can demonstrate improved outcomes, benefits and value to the people that access them. Often new systems are established in elements of care pathways that will shift or change the demand for services in other parts of the care pathway and will either increase or decrease the respective needs for services.

This project is used by ABUHB to help inform demand and workforce needs at the facilities modelled here. It has enabled an integrated approach between organisations to assess wider workforce and demand impacts. The results obtained from these models, can allow for improved financial assessment and planning and has the potential to improve workforce planning.

Finally, we have shown that predictive models of demand can be parametrised even when data from only a small geographic region is available. This was achieved by adjusting for factors such as deprivation and distances to healthcare facilities, as well as population sizes.

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Appendix A. Tables of values

$$T_{\text{Not offered}} = \begin{pmatrix} 0.0016 & 0.3174 & 0.0049 & 0.0004 & 0.0029 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0090 & 0.0107 & 0.0271 & 0.0 & 0.0085 & 0.0006 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0217 & 0.0 & 0.0053 & 0.0265 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0256 & 0.0128 & 0.0 & 0.0385 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0211 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0104 & 0.0104 & 0.0 & 0.0 & 0.0 & 0.0104 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0625 & 0.0208 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1667 & 0.0 & 0.0333 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} \tag{A1}$$

Facility	$\tilde{\lambda}_f$	γ_f	ψ_f
ED Attendance	6.6849	1.0543	1.4950
Royal Gwent	2.6904	0.9029	2.3925
Nevill Hall	0.0329	1.0000	1.0000
Ysbyty Ystrad Fawr	0.0082	1.0000	1.0000
Assessed Out	1.2438	0.8169	1.9065
Frailty - Rapid Medical	0.4658	2.7088	0.7274
Frailty - Rapid Other	0.1589	0.8556	1.0000
Frailty - Reablement	0.8795	1.0835	0.9630
Frailty - Falls	0.2630	0.7023	1.2880
Community Care - Home/Day	0.1863	1.6670	11.8197
Community Care - Other	0.0548	2.1069	12.3652
Community Care - Residential	0.0110	0.6354	92.7391

Table A1. Calculated values for the external arrivals $\tilde{\lambda}_f$, the effect of needing an SWP ψ_f, and the effect of receiving an SWP γ_f, for each facility f.

Facility	Not offered	Rejected	Accepted
Royal Gwent	0.0985	0.0941	0.0854
Nevill Hall	0.0985	0.0941	0.0854
Ysbyty Ystrad Fawr	0.0985	0.0941	0.0854
Frailty - Rapid Medical	0.0831	0.2222	0.0785
Frailty - Rapid Other	0.0434	0.0447	0.0877
Frailty - Reablement	0.0226	0.0216	0.0208
Frailty - Falls	0.0192	0.0199	0.0265
Community Care - Home/Day	0.0224	0.0259	0.0234
Community Care - Other	0.0073	0.0056	0.0077

Table A2. Calculated values for μ_{fs} for each facility f and each SWP status s.

County	Over 60
Blaenau Gwent	17557
Caerphilly	43864
Monmouthshire	28293
Newport	33248
Torfaen	23624

Table A3. Mid 2015 estimates of the populations over 60 for each county of Gwent.

	$a_{(\mathrm{RG},c)}$	$a_{(\mathrm{NH},c)}$	$a_{(\mathrm{YYF},c)}$
Blaenau Gwent	0.295	0.535	0.170
Caerphilly	0.346	0.062	0.592
Monmouthshire	0.482	0.483	0.035
Newport	0.985	0.008	0.007
Torfaen	0.775	0.168	0.056

Table A4. Table of the probability vector $(a_{(\mathrm{RG},c)}, a_{(\mathrm{NH},c)}, a_{(\mathrm{YYF},c)})$ for each county.

$$T_{\mathrm{Rejected}} = \begin{pmatrix} 0.0 & 0.4407 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1692 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0385 & 0.0385 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} \tag{A2}$$

$$T_{\mathrm{Accepted}} = \begin{pmatrix} 0.0032 & 0.3859 & 0.0064 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0102 & 0.0034 & 0.0136 & 0.0 & 0.0814 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0333 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0968 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1429 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0276 & 0.0069 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.1458 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix} \tag{A3}$$