CARDIFF UNIVERSITY

BUSINESS SCHOOL

Dynamics of Momentum Effects and Long-run Risk Model

by

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A Thesis Submitted in Partial Fulfilment of the Requirement for the Degree of Doctor of Philosophy of Cardiff University

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March 2021

Acknowledgements

First and foremost, I would like to express my special appreciation and thanks to my supervisory team, especially my first supervisor Dr Woon K. Wong, for their patient guidance, encouragement, and advice during the period of my five-year PhD program. I was extremely lucky to become his student, and to have supervisors who care about my progress.

I would also like to give my gratitude to Prof. Patrick Minford for giving me the precious chance to undertake this postgraduate doctoral research at Cardiff University. I shall never forget his kindness and generosity in providing help. I would like to thank the Julian Hodge bursaries for providing my funding too, for which I am also very grateful.

Thanks go to my friends in Cardiff: Dr. Zheyi Zhu, Dr. Yongdeng Xu, Dr. Congyi Hu, Dr. Xuanyang Zhang, Dr. Jia Cao, Dr. Wenna Lv, Dr. Sisi Ji, Dr. Jiayi Huang, Dr. Wenrui Ma, Dr. Haoling Liu, Jieran Wang, Dr. Xiaoxiao Gong, Dr. Chaowei Wang, and Dr. Ziqing Wang, Xinxin Li for giving me help and support with academia and living.

Finally, a special thanks to my family. I would like to thank my parents for their wise counsel and sympathy, as well as for all of the sacrifices that they have made on my behalf.

Abstract

This thesis examines the dynamics of momentum effects in the two-regime switching model and momentum effects in the long-run risk model. A two-regime switching model was built to analyse the switching that can be controlled by the level of market risk between the momentum effect and return reversal. Further, the study examined the relationship between the three independent variables of domestic market risk, returns in the ranking period, and foreign financial market risk (i.e., US market risk and UK market risk). It found that the momentum return in both stock markets has a significant positive effect on foreign market risk, and a negative effect on return in the ranking period and domestic market risk in the momentum regime. However, the results for the US market were insignificant in the reversal regime when compared with those for the UK market. Further, a cross-sectional long-run risk model was developed at the level of individual security. The assumption of fluctuation economic volatility was extended to allow for the effect of economic uncertainty on the aggregate consumption growth and dividend growth of individual security. Theoretically, the model provides a more significant explanation for momentum returns. In conclusion, the model matches the relative portfolios' returns, dividend growth, and valuation ratio at the portfolio level via its simulation.

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Chapter 1.0 Introduction

In recent years, one of the most controversial debates in finance have been whether the capital market is efficient or not, as well as whether share prices can be forecast into the future. According to traditional finance theory, the market is efficient in terms of financial assets pricing. The Efficient Market Hypothesis (EMH) was established and developed by Malkiel (1962), Fama (1965b) and Samuelson (1965) in the 1960s. Specifically, the EMH proposed that asset prices must fully and instantaneously reflect all the available relevant information. In general, the EMH is related to the concept of random walk, which means that there are random changes in stock prices that the current stock prices failed to forecast from past prices. Thus, the fundamental principle of a random walk means that the continuous changes in stock prices involves independent and identically distributed random variables. This implies that there are no rules about the series of changes in stock prices, and that past trends cannot be used to forecast the future. According to the EMH, changes in stock prices depend on new information regarding the economy, the market, or firms. More importantly, the adjustment in stock prices is immediate, and thus investors fail to trade in a timely way, so they cannot earn returns from the new information. However, the EMH is now facing huge challenges in the current financial economy, and some researchers have found that a large number of investment trading strategies are able to produce huge profits in different markets.

Momentum effect, as a general financial phenomenon, implies a trend of stock returns based on past returns. Stocks that performed well in the past will tend to perform well in the future, and stocks that performed poorly in the past will tend to perform poorly in the future. There has been a large amount of research about momentum in different stock markets, including European stock markets (Rouwenhorst, 1999), Asian stock markets (Chui et al., 2000), African stock markets (Griffin et al., 2003), and Latin American emerging markets (Muga and Santamaria, 2007). Furthermore, the momentum effect exists not only in the stock market but also in the exchange market, bond market, oil market, and in metal stocks. On the other hand, the study of contrarian strategy or return reversal[1] argues that stocks which performed well in the past will tend to perform poorly in the future, and that stocks that performed poorly in the past will tend to perform well in the future. Thus, the return reversal exhibits a tendency for shock returns, or returns to reverse in a different direction. Much literature has proposed that in general, a momentum strategy is more likely to work during periods of calm in the market when risks and volatility are low. When the market suffers a dramatic fluctuation, the reversal effect is more likely. Thus, when the reversal effect occurs in the financial market, investors could suffer enormous losses if they continue to use the momentum strategy. Thus, in this study I build an investment trading strategy based on a regime switching between momentum effect and return reversal in the stock market. More specifically, I use a threshold model that uses information on market risk to predict whether the market is in a momentum state or a return reversal state. This two-regime switching model is built based on market volatility. Further, it can be used to test the relationship between momentum return and the possible determinants of domestic market risk, ranking period return, and foreign market risk.

Further, in the second main chapter, I explain whether momentum profits are produced as a result of exposure to systematic risk factors. I develop the long-run risk model set out by Bansal and Yaron (2004). These authors presented a long-run risk model (LRR) to interpret several critical features of asset market phenomena like equity premium, the risk-free rate, the volatility of the market return, the volatility of the risk-free rate, and the price-dividend ratio using US asset market data. Mehra and Prescott (1985) defined the equity puzzle based on the phenomenon that the fluctuations of observed returns on stock of approximately 6.98% during the studied period were as much due to higher returns on a government bond or the contemporaneous risk-free interest rate of roughly 0.8%. A significant body of literature fails to interpret that the risk premium on stocks is much riskier than that of government bonds, which leads to investors' demand for compensation for the excess risk.

However, traditional financial theories such as the Capital Asset Pricing Model (CAPM),

[1] Return reversal is different from price reversal. In general, price reversal only focuses on the stock price trend, but return reversal focuses on the stock returns trend (including stock prices and dividends).

the International Capital Asset Pricing Model (ICAPM), and Arbitrage pricing theory (APT) all measure the marginal utility of wealth by the behaviours of large portfolios of assets. For example, the CAPM model can estimate returns on market portfolios, while multifactor models can estimate returns on multiple portfolios. According to CAPM and these theories, the uncertainty related to returns on market portfolios is due to a single source of risk in the real economy, but CAPM and related theories have no theoretical and symmetrical structure to identify what it is that makes market portfolios risky. Thus, consumption-based CAPM (CCAPM) was explained by Cochrane (2000), who stated that the asset-pricing model is embedded in stochastic macroeconomics because the function that shows the behaviour of asset prices and return in the CCAPM is transferred from the consumption and asset choice decisions of households. There is no denying that in order to prove a relationship between the real economy and financial markets, the growth in the marginal utility of wealth can be measured by the growth in consumption, because consumption is the payoff on the market portfolio. However, the classic consumption-based CAPM cannot interpret the empirical low risk-free interest rate; also, agents have a negative time preference. Moreover, the long-run risk model related to security returns to macroeconomic fundamentals based on a systematic framework. Thus, the model has a closed relationship with the underlying economic theory. Further, the model can provide the impetus for exploring the underlying factors with which to interpret the momentum effect. To do so, firstly, I set up the core assumptions about preferences, the stochastic discount factor, and the economic environment. Moreover, to solve the equilibrium asset prices in my model, I adopt the standard approximations used by Campbell and Shiller (1998). Further, I match several moments from the actual data with the results of the simulation, basing the calibration and design of my simulation on the previous literature.

The thesis is organised as follows. Chapter 2.1 summarises the effective market hypothesis, uncertainty, the relevant literature regarding the momentum effect and the two-regime switching model. Chapter 2.2 expounds the data processing, portfolio construction, variables description, and Bayesian estimation related to the two-regime switching model. Chapter 2.3 reports the results of momentum effects in the US and UK markets, as well as the presenting the results and discussion of the Bayesian estimation. Chapter 3.1 reviews the development of the long-run risk model and cross-sectional security return in the long-run risk model. Chapter 3.2 explains the assumptions of the long-run risk model, data processing, a cross-section of the model, its calibration, and the simulation design for cross-sectional long-run risk. Chapter 3.3 describes and discusses the findings from the empirical and simulation results. Chapter 4 summarises this thesis and suggests directions for future research.

Chapter 2.0 Dynamics of Momentum Effects

2.1 Literature Review

2.1.1 Efficient Market Hypothesis

There is a considerable controversy as to whether, or to what extent, the capital market is efficient, and further, whether the return on security can be predicted in the future or not. The EMH argues that stock markets are extremely efficient in reflecting new information. When new information arrives its transmission is quite speedy, and thus is promptly reflected in share prices. Therefore, the traditional EMH pointed out that regardless of which form of technological analysis or fundamental analysis was used, they were not able to determine the undervalued stocks or investment portfolio that achieve greater profits than those of randomly selected stocks or portfolios. Fama (1970) explained the EMH theory by pointing out that asset prices immediately embody all the valid, relevant information. In other words, stock prices should follow a random walk. Hence, stocks that will generate abnormal returns in the future cannot be identified, regardless of which technical and fundamental analyses are adopted. Malkiel and Fama (1970) also highlighted that market participants always seek rational profit maximisation behaviour, and that asset prices also always reflect all the available relevant information. Meanwhile, Malkiel (2003, 2005) more recently explained that random walk means that if asset prices can reflect information immediately and fully with unobstructed information flow, then price changes in the future will only reflect the future information but will not impact the current independence of the price changes. Furthermore, Samuelson (1965) also stated that in an efficient market, if changes in price can entirely reflect all market participants' expectations and information, then such price changes must be unpredictable. Due to the random announcements of news, security prices must randomly fluctuate. Therefore, it is unlikely that any relevant information set can be used to forecast future changes in stock prices. The randomisation of price changes does not enable investors to obtain abnormal returns which exceed the market.[2] Allen, Brealey and Myers (2011) redefined the efficient market as one in which it is impossible to gain higher returns than the market return, which implies that the value of shares reflects the fair value of firms and equals their discounted future cash flows. Eakins and Michkin (2012) also supported the EMH view that there are two pillars: incorporation of the available information in share prices, and the inability of investors to earn risk-weighted excess returns in an efficient market.

2.1.2 Uncertainty

Many studies have investigated the reason for stock price drifts based on information uncertainty. Based on the notion of rational belief equilibrium (RBE), Kurz (1997) proposed that agents' beliefs in RBE are generally wrong because they differ from the true probability in the process of equilibrium. In fact, these agents' beliefs are rational. Agents anticipate making mistakes, which are important in the RBE theory. These mistakes can explain the reasons for stock returns, and the aggregation of these mistakes produces endogenous uncertainty that the factor underlying changes in stock prices is endogenous transmission. Dieci and He (2018) explained the Heterogeneous Agent Model (HAM) in finance, stating that agents are heterogeneous and have diverse social interactions (Kirman, 1992 and 2010). The HAM framework sees financial market dynamics as a result of the interaction of heterogeneous investors with various behavioural biases. This view is important to the expectation feedback mechanism in the HAM model. More precisely, agents' decisions are based on forecasts for endogenous variables, and the actual value of endogenous variables are determined by agents' expectations. Earlier HAMs studies built different non-linear models to illustrate different endogenous mechanisms of market fluctuations arising from the interaction of heterogeneous agents, instead of exogenous shocks or news. Thus, these models indicate that asset price fluctuations are generated endogenously. Such models generally involve two main types of beliefs (extrapolative and regressive, or chartists and fundamentalists). Chartists depend on the extrapolative method to predict future prices and to

[2] This followed a review by Kendall (1953), Cootner (1962), Fama (1965a, 1965b), Leroy (1973), Rubinstein (1976), and Lucas (1978).

establish their position in the market. Therefore, they prefer to maintain and reinforce stock price tendencies, and extend the difference from the fundamental price. In contrast, fundamentalists are more interested in the mean reversion of the stock price to its intrinsic price in the long term.

Verardo (2009) revealed an empirical relationship between heterogeneity of beliefs and the momentum effect. Their study used the dispersion of analysts' forecasts of earnings to estimate the diversity in investors' beliefs, and controlled for stock visibility, the speed of information dissemination, the uncertainty of fundamentals, the accuracy of information, and volatility. The findings illustrated that portfolios with high heterogeneity of beliefs have high momentum returns. Jiang, Lee and Zhang (2005) revealed that information uncertainty relative to value ambiguity that knowledgeable and experienced investors estimate the accuracy of firm value. The result showed that companies with high information uncertainty not only earn low expected returns, but also have a strong relationship with price and earnings momentum effect. Makarov and Rytchkov (2009) used a rational expectation equilibrium (REE) with heterogeneously informed agents to find that in a specific condition, asymmetric information can produce positive autocorrelation of returns.

Bouattour and Martinez (2019) indicated that both uncertainty and information asymmetry have a significant effect on the extent of market efficiency and information asymmetry. There is a decline in market efficiency with fluctuations of the fundamental value of stocks. Zhang (2006a) revealed that information uncertainty has an important impact on share price continuation anomalies and cross-sectional variations in stock return. Their hypothesis was based on two prior results from behavioural finance: the first, by Chan, Jegadeesh, and Lakonishok (1996), is that the reason for price continuation is a gradual stock market response to information; the second, by Hirshleifer (2001) and Daniel, Hirshleifer, and Subrahmanyam (1998, 2001), is that investors' psychological biases are prompted by greater information uncertainty. Zhang (2006b) combined these two results and examined the subsequent hypothesis: if the market is slow in reflecting new information because of psychological biases, these biases will be greater, leading to slower price reflection when there is more uncertainty regarding the implications of the information for the company's value. Consequently, Zhang (2006b) found that if price continuation in the short run is because of the behavioural biases of investors (e.g. underreaction to new information), then there is a greater price change with greater information uncertainty. Meanwhile, the author also found that larger information uncertainty results in a decline in expected stock return when bad news arrives, and an increase in the expected stock return when good news arrives, which implies that uncertainty delays the stock price response to the flow of information.

Much empirical evidence has confirmed that stock markets have a delayed reaction to fundamental information, and that information spreads gradually among markets. Hou and Moskowitz (2005) used the delay in stock prices' responses to information to show that the level of market friction influences stocks. They found that the most delayed companies generate a high return premium, which cannot be interpreted by firm size effect, liquidity effect, and micro-structure effect. In HAM, the heterogeneity expectations of agents are shaped based on price trend, as in Chiarella et al. (2006). He and Li (2015) proposed a continuous-time heterogenicity agent model consisting of fundamental, momentum, and contrarian traders in order to examine the effect of various time intervals on the market price and profitability of various strategies. They pointed out that because stock prices are continuous, price trends are formed by the moving average of historical prices, which is important. More specifically, price trends are more sensitive to market shocks when momentum traders continue to be active. This can be described as the instability of momentum trading to market volatility. However, this situation offers a chance to obtain momentum profits in short-run. Westerhoff (2004) established an HAM in multi-asset market that allows fundamentalists and chartists to be active at the same time. Fundamentalists focus on gathering expertise in a certain market, and chartists can switch between markets, relying on short-term profit chances. Chartists are more likely to choose markets that manifest price trends but which do not deviate too much. Interactions between the traders and markets lead to complex dynamics in this model. Schmitt and Westerhoff (2014) proposed a model in which the demand of heterogeneous speculators can suffer various types of exogenous shocks (i.e., global shocks). Investors can switch between different markets and between different strategies according to behavioural effects and market environments. The results displayed that traders' behaviour can expand financial market interconnections and produce stock price co-movement and a cross-correlation of market volatilities.

2.1.3 Momentum effect

In the mid-1980s, a large amount of financial evidence exhibited that, to a large extent, the future stock price depends on the past return. For instance, De Bondt and Thaler (1987) found that the past loser portfolio in the long term tends to outperform the long-run past winner portfolio over 3-5 years, and that the market always tends to mean reversion in the long run. Moreover, Jegadeesh (1990) and Lehmann (1990) also discovered the phenomenon that there appears to be a short-term trend of mean reversion over 1-4 weeks. Summers (1986), Fama and French (1988), Lo and Mackinlay (1988), and Poterba and Summers (1988) presented contradictory evidence regarding the random walk theory, finding that based on psychological and behavioural factors, stock market returns can be forecasted to a large extent.

The momentum strategy is widely-known and accepted in the public and academic arenas. Briefly, the momentum effect or strategy is the theory that investors buy those stocks that have performed well and sell shares that have performed poorly during a specific past period, and then hold this investment portfolio during a specific future period, an approach which can produce a significant positive return. In other words, investors buy past winners and sell past losers. Jegadeesh and Titman (1993) primarily put forward this strategy. More specifically, they selected stocks based on past returns over J months and held them for K months. They defined J as the raking or formation period, and K as the holding period. Further, they ranked all stocks based on past returns in the ranking period in the stock market and divided them into ten equally weighted portfolios. The portfolio with the highest return was called the winner portfolio, and the portfolio with the lowest yield was the loser portfolio. Moreover, they observed the performance of these portfolios in the holding period J. If the winner portfolio continuously has the highest return, and the loser portfolio continuously has the lowest yield, then the momentum effect has occurred. They summarised their research by analysing the American stock exchange and New York stock market exchange from 1965 to 1989, which both produced strong profitability. They also claimed that the results failed to explain the systematic risk and lag stock price reaction.

Grinblatt, Titman, and Wermers (1995) examined the momentum effect in relation to mutual funds. They pointed out that fund managers buy stocks based on past returns, a strategy which can gain positive risk-adjusted excess returns. More than 70% of investors adopted the momentum strategy by preferring to obtain funds that performed well in the past. The authors also expounded that mutual funds that adopted the momentum strategy outperformed others, and there is no evidence to show that fund managers can buy and sell the same stocks simultaneously. Some early studies about momentum strategy, by Lo and Mackinlay (1990) and Lehmann (1990), investigated this matter and found that more than half of trading strategies produce significant profits when they decomposed the stock returns into two dimensions, cross-sectional and time-varying factors, with various ranking and holding periods, and tested 120 different momentum and contrarian strategies. Further, they concluded that the momentum strategy and contrarian strategy had similar profitability between 1927-1947. Also, the profits of momentum and contrarian strategies can each be explained by the changes in mean reversion.

Moskowitz and Grinblatt (1999) illustrated momentum effects in relation to industry components. They found that momentum trading strategies fail to generate significant profits when they controlled for industry momentum. However, the industry momentum in the short run is likely to be stronger than the stock momentum. A persistent momentum effect occurs on the medium horizon, but this effect will be eliminated after one year. They concluded that there is a significant effect of the industry factor on the profitability of the momentum strategy.

Jegadeesh and Titman (2011) presented empirical evidence that stocks that performed well in the past 3-12 months have a higher probability of performing well in the future 3-12 months. This pattern is consistently profitable in the current US and other developed stock markets. At the same time, stocks with high earnings momentum perform continuously better than stocks with poor past earnings momentum. Moreover, there were negative cumulative returns for the momentum portfolio when they tested the returns of winner and loser portfolios from 13 to 60 months, which is similar to the explanation offered by behavioural finance. These authors claimed that some relevant evidence indicated that momentum strength responds to industry and market features, but the scale of profits reflect the level of participation in the company and market activities.

George and Hwang (2004) utilised various patterns to examine the momentum strategy and a “52-week high” investment strategy. More precisely, this investment trading strategy was described as when investors take long winner stocks and short sell loser stocks based on stock prices in the last one month over the highest stock prices in the past 12 months. Further, compared with the investment strategies proposed by Jegadeesh and Titman (1993) and Moskowitz and Grinblatt (1999), they found that the “52-week high” investment strategy can generate higher momentum profitability. In addition, when they controlled for the size effect and the bid-ask bounce, and removed the January return, their investment strategy produced double momentum profits compared with the previous strategy.

Ammann, Moellenbeck and Schmid (2010) examined the profitability of momentum strategies based on the blue-chip stocks listed in Standard and Poor’s (S&P) 100 index. They found that high turnover and the cost of holding a short position leads to high transition costs, particularly in stocks with small capitalisation. Further, they adopted a simple trading strategy that takes long positions on individual stocks and short sells in the stock index, which can produce significant excess returns. They also found that the findings were credible to different risk-adjustments, such as the capital asset pricing model (CAPM) and the Fama-French three factors model.

Li, Brooks and Miffre (2009) explained the influence of the trading cost on the profitability of momentum trading strategies in the London stock exchange. They summarised that the trading costs in the winner portfolio are lower than in the loser portfolio. There is therefore an observed asymmetry regarding the trading cost in the winner portfolio and loser portfolio, which focuses on the high trading cost to sell the loser portfolio that is of a small size and a low trading volume. Thus, they adopted a low-cost relative-strength strategy by choosing the overall lowest transition cost from all the stocks in the winner and loser portfolios. They also stated that the profitability of momentum strategies cannot be interpreted without adjusting for systematic risk, lagging reaction to a specific component, and serial correlation.

Rey and Schmid (2007) used a full sample of large-capitalisation stocks in the Swiss market index, and only invested in single stocks. They adopted a strategy that holds only one winner and one loser stock at the same time, which led to a significant profit of approximately 45%. Griffin, Ji and Martin (2005) found that the momentum strategy makes a profit though investors only holding a long position. Further, if investors ignored the transaction cost, they obtained huge profits due to the large positive price and earnings momentum effects in Europe. Forner and Marhuenda (2003) discovered the momentum effect and its reversal in the Spanish stock market. They claimed that the momentum strategy has profitability on a 12 month basis, and that its reversal produces profit chances, in the long run, over a 60-month period. Naranjo and Porter (2010) researched the sources of cross-country co-movement of momentum profits in developed and developing countries. As a result, country-neutral momentum profits were observed to be closely related across countries, as well as through a time-varying correlation. A failure explanation was also provided, concerning the co-movement of country-neutral momentum profits by using co-movement among industries. Narayan and Phan (2017) investigated the profitability of momentum strategies in Islamic stocks. They reported that the momentum strategy is effective when controlling the stock characteristics, market state, and seasonal mode. Furthermore, they also explained that the market volatility (up or down) could influence the extent of momentum returns, and that risk components can provide a better explanation for momentum profits. Arena, Haggard and Yan (2008) reported that momentum returns are high when investing in stocks with high idiosyncratic volatility, particularly loser stocks with high idiosyncratic volatility. They found that stocks with extreme idiosyncratic volatility suffer a sharp and fast reversal, which is consistent with the reason for momentum returns by underreaction and the limited impact of idiosyncratic volatility on the momentum effect. They also discovered a positive relationship between momentum profits and aggregate idiosyncratic volatility, which assists in interpreting the persistence of the momentum returns reported by Jegadeesh and Titman (1993).

2.1.4 Two regime-switching model

Huang and Tsai (2014) examined whether aggressive growth fund managers adopted a momentum strategy or a contrarian strategy to improve performance in a previous period of financial crisis, as well as testing whether these trading strategies were effective or not in the period of financial crisis. They found that these investment strategies (momentum strategy and contrarian strategy) could both enhance portfolio performance before the financial crisis. Further, during the period of financial crisis, the momentum effect disappeared in the performance of aggressive growth funds. In contrast, the contrarian strategy took up a dominant position, which meant a significant impact on the performance of the aggressive growth funds. The authors therefore suggested that fund managers could switch to the contrarian strategy to enhance their funds' performance when a financial crisis occurs. Dobrynskaya (2017) put forward a simple dynamic investment strategy, because high momentum profits cannot be explained by the risk components. Further, she found that a momentum crash occurs after 1-3 months in a market crash. This dynamic trading strategy means that when a market is calm, investors adopt the standard momentum strategy, but will switch to the contrarian strategy one month after a stock market crash and retain the contrarian strategy for the next three months, before finally reverting to the momentum strategy. They found that this dynamic investment strategy transfers the whole momentum crash into returns, which are roughly 1.5 times greater than returns from the standard momentum strategy.

Wang and Xu (2015) examined the time-series predictability of the momentum strategies, especially the predictability of market volatility. They discovered that there is an economically and statistically significant relationship between market volatility and momentum profits. Moreover, there is a dramatic difference between the time-series predictable power of the momentum strategy and the total predictability of momentum profits in the overall stock market. Further, after a period of low market volatility, the profitability of the momentum strategy has a significant increase. Notably, there are quite low mean monthly returns when a market is down, and aggregate fluctuations are high.

Pastor and Stambaugh (2003) examined the cross-sectional relationship between the expected security returns and sensitivity to volatility in aggregate liquidity from 1966 to 1999. They set up a monthly liquidity indicator. Their hypothesis means that a higher return reversal occurs if there is a lower liquidity signal. They found that the return on stocks with high sensitivities to liquidity is greater than that for stocks with low sensitivities by approximately 7.5% every year. Hwang and Rubesam (2015) built a regime-switching model to analyse the momentum premium based on Pastor and Stambaugh (2003). Their model with multiple structural breaks was able to recognise the correlation between the momentum returns and risk components. Additionally, as the regime-switching model contains multiple structural breaks, it is more flexible and accurate, compared with the limited number of structural breaks included by Hwang and Sarchell (2007).[3] They found an abnormal return by momentum premium during two periods (the 1940s-1960s and 1970s-1990s). Further, Hwang and Rubesam (2015) pointed out that since the early 1990s, the process of the slow disappearance of momentum profits was delayed by the occurrence of the high-tech and telecom stock bubble.

Hamilton and Susmel (1994) found that the persistence of low-frequency variation in volatility can be tested by a discrete Markov-switching model, which relates to discrete variations in business cycle stages during periods of economic expansion and recession. Some other researchers have also investigated the relationship between volatility regimes and economic conditions using the Markov-switching model.[3]

Sinha (1996) used the GARCH model to study the monthly returns in the US stock market. The model adopted probability to measure the switching regime from a high to a low level of volatility, depending on the economic conditions. He found that the macroeconomic components can significantly impact on stock returns, and that stock returns seem to retain a violent volatility in an economic recession.

Wu (2016) examined the asymmetric momentum effect during periods of UP and DOWN market states in the Chinese stock market. They regressed the raw momentum returns through

[3] Schwert (1989), Schaller and Norden (1997), Assoe (1998), Kim et al. (2001, 2004), Hess (2003) and Mayfield (2004).

using dummy variables, which can be followed as,

$$R_{W-L,t} = R_{W-L,UP}UP_t + R_{W-L,DOWN}DOWN_t + e_t$$

where $R_{W-L,t}$ represents the momentum return at time t, that is the difference between returns on the winner and loser portfolios due to the different momentum trading strategies. There are two dummy variables: UP_t and $DOWN_t$. UP_t ($DOWN_t$) is equal to 1 when there is an UP (DOWN) market state, which means a positive (negative) average market return. $R_{W-L,UP}$ ($R_{W-L,DOWN}$) means the average momentum return in a UP (DOWN) market state.

$$R_{W-L,t} = \alpha + R_{W-L,UP-DOWN}UP_t + e_t$$

where $R_{W-L,UP-DOWN}$ denotes the difference in momentum returns under two different market states. They found that momentum returns following UP market states underperform the momentum effect following DOWN market states in the Chinese stock market. Moreover, they also found that the reasons for the subdued asymmetry of market-state-dependent momentum returns are low liquidity, higher market return volatility, and the weak under-reaction of stock prices.

Cao (2014) proposed a threshold regression model to explain the relationship between the momentum effects and the volatility of stock market returns. This threshold model is a two regime-switching model between the momentum regime and the reversal regime, and the switch from one regime to other regime is based on a change in the volatility of stock market returns. Specifically, the two-regime switching model can be expressed as,

$$r_t^H = \left[1 - I_{[\tau,\infty)}(z_{t-1}^R)\right](\alpha_1 + \beta_1 z_{t-1}^R + \gamma_1 r_{t-1}^R)$$

$$+I_{[\tau,\infty)}(z_{t-1}^R)(\alpha_2 + \beta_2 z_{t-1}^R + \gamma_2 r_{t-1}^R) + \varepsilon_t$$

where r_t^H denotes the momentum portfolio's holding-period return. z_{t-1}^R denotes ranking period market volatility. r_{t-1}^R denotes ranking period return. $I_{[\tau,\infty)}(z_{t-1}^R)$ denotes an indicator function with τ as the threshold parameter. When z_{t-1}^{R} is below the threshold parameter, the indication function (I) is equal to 0. Then, the momentum effect is expected. In contrast, when z_{t-1}^{R} is above the threshold parameter, the indication function (I) is equal to 1, which means the reversal occurs. Furthermore, through using Bayesian estimation, the threshold parameter (τ) can be predicted. The author found a negative relationship between the momentum trading return and market volatility in the momentum and reversal regimes, as well as a significant inverse relationship between portfolio performance in the ranking period and the holding return in the momentum regime. More importantly, based on the indications of the threshold regression model, the new trading strategies can be exploited between the momentum effect and the reversal effect, to produce economically significant profitability.

Yang (2016) explored the dynamic momentum strategies in Chinese stock markets from 1991 to 2012. He found no momentum profits in Chinese stock markets, but significant reversal profits across the sample period. In addition, he found that momentum strategies produce highly contrarian returns in UP market states compared to those in DOWN market states. Further, he stated that the reason for this could be that the improved performance of the strategy in the UP market is too extreme, going over a specific threshold value, leading to a price reversal instead of a price continuation (Cooper, Gutierrez Jr and Hameed, 2004).

Abdymomunov and Morley (2011) adopted a two-state Markov-switching model to examine time-variations in betas in the CAPM for the BM ratio and momentum portfolio across market volatility regimes. The model allowed the beta and market risk premium to switch between low and high volatility regimes. They found statistically significant strong evidence of time-variations in betas across market volatility regimes under the unconditional CAPM. Although the regime-switching conditional CAPM could sometimes be rejected, they believed that the beta with time-variations can interpret returns on portfolio well, compared with the unconditional CAPM, particularly in conditions of high stock market volatility.

2.2 Data and Methodology

This section describes the data and core methodology utilised in this research. Firstly, it sets out concise data specifications, including data collection, variable definitions, and the data analysis process. Secondly, the section turns to the momentum trading strategy, which follows the paper by Jegadeesh and Titman (1993). Moreover, dependent and independent variables are described and the two-regime switching model is established. Finally, the Bayesian estimation method is used to examine the switching between the momentum effect and its reversals.

2.2.1 Data specifications

The dataset adopted here is composed of the monthly data of all stocks traded on the US stock market and the UK stock market from January 1980 to December 2018. In total, there are 7704 effective stocks for the US. These comprise all the domestic and primary stocks in the US stock market listed on the three main markets - the New York (NYSE), American (AMEX) and NASDAQ stock exchanges. The data source for the US stock market is the Centre for Research in Security Prices (CRSP). For the UK stock market, the sample contains a total of 5588 companies, obtained from London Share Price Data (LSPD). The primary data includes holding returns, stock prices on the last trading date of the month, dividends, and market capitalisations. These variables are used to process the return in the stock market. For the critical measure of market volatility, the daily market values are used to compute the realised market volatility by the CRSP for the US market and by the Datastream for the UK market, which can measure the omitted market volatility from 1979. The formula for daily realised market volatility can be expressed as below:

$$\sigma_d^2 = \frac{\sum_{t=1}^{T}(r_t - \bar{r})^2}{T-1}$$

where r_t is the daily market return in one month, $\bar{r}$ is the mean of monthly market returns over T trading days in one month, and T is the number of trading days in one month.

Further, to present this volatility in monthly terms, I can multiply the daily market volatility by the trading days, T, which can be followed as,

$$\sigma^2 = \sigma_d^2 * T^2$$

where σ^2 denotes the monthly market volatility.

In order to avoid missing data and survival bias, this study allowed companies in the sample to have incomplete records during the ranking periods and holding periods. However, any firm for which all the values in the holding period or ranking period were missed was removed. Thus, this research avoids survivorship to no small extent, and contains more available samples. To avoid inferior liquidity and illiquidity in the data samples, the US samples excluded all stocks priced below $5 (Jegadeesh and Titman, 2001; Avramov et al., 2007), and removed the stocks of the smallest decile of market capitalisation in the NYSE, in line with Jagadeesh and Titman (2001). In addition, the UK samples excluded shares traded on the Unlisted Securities Market (USM), Third Market companies, over-the-counter (O.T.C.) companies, the Alternative Investment Market (AIM) and the Off-Exchange (OFEX).

2.2.2 Momentum strategies using overlapping portfolios

The momentum effect in stock markets stipulates that the same share price movement will continue in the future. More precisely, stocks that performed well in the past will go on performing well in the future, and stocks that performed poorly in the past will go on performing poorly in the future, which means investors can earn significant positive payoffs. According to Jegadeesh and Titman (1997), momentum strategies can be adjusted as below. Stocks were selected based on past returns. Thus, they ranked the stocks according to the return over the past J months. They also observed the following return in the future K months. The ranking period or formation periods J and holding periods K were 3, 6, 9 and 12 months, and therefore there were a total of 16 momentum strategies. Furthermore, the gap between the portfolios in the ranking period and holding period skipped one month, to avoid some bid-ask spreads, price pressure, and lagged reactions (Jegadeesh, 1990; Lehmann, 1990; Daniel and

Moskowitz, 2016). For example, a 6*9 momentum strategy in July 1980 means that the stocks were sorted over the past 6 months and held in the following 9 months, and hence the portfolio in the first ranking period was formed from January 1979 to June 1979 and was held from August 1979 to Match 1980. Jagadeesh and Titman (1995) and Galariotis, Holmes and Ma (2007) found that the profitability of trading strategies might be magnified due to non-synchronous trading, the bid-ask spread, and infrequent trading if there was a failure to skip one month. The stocks were further divided into ten deciles, representing equally-weighted portfolios with equal weight. The top decile of the "Loser" portfolio has the lowest return and the bottom decile of the "Winner" portfolio has the highest return. In each month, the average return on each decile was observed. According to the description of the CRSP and the LSPD, the return on a delisted security in the past month is computed by its value after delisting and the share price on the last trading date.

To form the momentum trading strategies and evaluate the profitability of trading strategies, all the companies in each of the stock markets can be sorted by past return. Thus, the buy-and-hold return for each company in each month during the past J ranking period was calculated as below,

$$MR_{i,\mathrm{t}} = \left(R_{i,t-1} + 1\right) * \dots * \left(R_{i,t-j} + 1\right) \qquad 2.2.1$$
$$= [\prod_{\mathrm{j}=1}^{J} (R_{i,t-j} + 1)] - 1$$

where,

$$R_{i,t-j} = \frac{P_{i,t-j} - P_{i,t-\mathrm{j}-1} + D_{i,t-j}}{P_{i,t-\mathrm{j}-1}}$$

where $MR_{i,\mathrm{t}}$ denotes the momentum portfolio's return over the past J months. $R_{i,t-j}$ denotes the simple return on security i at time t-j. Ranking period J is only 3, 6, 9 and 12 months in this research. $P_{i,t}$ and $D_{i,t}$ are security i's price and dividend at time t, respectively. $P_{i,t-1}$ is price at time t-1. According to Moskowitz and Grinblatt (1999) and George and Hwang (2004), a self-financing momentum portfolio is also called the momentum portfolio, which means taking a long position in the winner portfolio and a short position in the loser portfolio. In other words, the return on the momentum portfolio or the profitability of momentum strategies can be observed in the stock market, which is also called the momentum return. Further, this study adopted the overlapping trading strategy to observe the profitability of the momentum trading strategy each month, meaning that the profitability of the momentum trading strategies can be obtained each month.

Finally, the return on the momentum portfolio was calculated by working out the difference between the returns on the winner and loser portfolios at the end of the last trading day of the K month. Therefore, the average buy-and-hold return on each momentum strategy or each average momentum return can be computed as per below,

$$MR_t = \{[\frac{1}{n}\sum_{i=1}^{n}\prod_{t=1}^{K}(R_{i,t,W}+1)]-1\} - \{[\frac{1}{n}\sum_{i=1}^{n}\prod_{t=1}^{K}(R_{i,t,L}+1)]-1\} \quad 2.2.2$$

where $R_{i,t,W}$ and $R_{i,t,L}$ are an average return on the winner portfolio and the loser portfolio in the holding period K respectively. n denotes the number of firms, and K is only 3, 6, 9 and 12 months.

An example can be set out to understand the overlapping momentum strategy. First, the momentum trading strategy is set up as 3*6. The first ranking period should be 3 months, starting from January 1980 to March 1980. Then, all the securities' returns are ranked in the ranking period to acquire the momentum portfolio (including the winner portfolio and the loser portfolio), and further skip one month to April 1980. This means that the momentum portfolio has a range of 6 months, from May 1980 to October 1980. Again, the second formation period should be from May 1980. Therefore, all momentum trading strategies should start from January 1980 plus J months to December 2018 minus K months, which contains a total of 468 months. In addition, the firms are removed from the portfolio if they have gone out of business. According to the descriptions in the LSPD database, firms losing all value can do so due to liquidation, quotations being cancelled for unknown reasons, administration/administrative receivership, and cancelled assumed valueless. On the other hand, the money received, or value of stocks is reinvested in the portfolio with equal-weighting when the company is subject to acquisition/takeover/merger, suspension/cancellation, the quotation is cancelled or suspended, it goes into voluntary liquidation, undergoes changes to foreign registration, is converted into an alternative security, or is subject to nationalisation (Arnold and Baker, 2007; LSPD, 2013). In terms of the CRSP dataset, according to the Data Descriptions Guide in 2018, the reasons for delisting may include mergers, exchanges, liquidations, or being delisted by the securities and exchange commission. Notably, the reasons for missing returns can explain valid, current prices, but not valid previous prices, and can include not trading on the current exchange, no data available to calculate returns, and missing returns due to missing prices because of a suspension in trading or trading on an unknown exchange.

2.2.3 Variables description

My model incorporates four variables: the momentum portfolio return in the holding period, the momentum portfolio return in the ranking period, the domestic market volatility in the ranking period, and foreign market volatility in the ranking period. Many studies have examined the relationship between independent variables (i.e. the momentum portfolio return in the ranking period, the domestic market volatility in the ranking period, and foreign market volatility in the ranking period) and the dependent variable (i.e. the momentum portfolio return in the ranking period, the domestic market volatility in the ranking period, and foreign market volatility in the ranking period). More specifically, previous research has found an inverse correlation between the momentum portfolio return in the holding period and the domestic market volatility. Barroso and Santa-Clara (2012) proposed a highly predictable framework between momentum returns and the risk of momentum calculated by the realised variance of daily returns. Wang and Xu (2010) examined whether market volatility has predictability in terms of the time-varying momentum payoff. They found a significant and robust power to predict momentum payoff if using market volatility. Moreover, when controlling the market stated and business cycle variables, the predictability of market volatility remains strong in stock markets. Thus, compared with other variables, only the variable of market volatility retains a significant explanatory power in terms of predictive power regarding momentum profitability. And, the predictable power of momentum payoff comes mainly from the loser stocks, and the performance of winner stocks never deviates from whole market performance in aspects of prediction. Giot (2005) showed that future market returns could be forecast via market volatility, and found that market returns have a negative relationship with the regression parameter of a low volatility level, but a positive relationship with that of a high volatility level. Moreover, many researchers have found a negative relationship between momentum returns in the ranking period and momentum returns in the holding period. There are two possible reasons; one reasonable explanation for this is that when a small return occurs in the ranking period, the market is more likely to under-react, and therefore the momentum portfolio is also more likely to produce strong profitability in the holding period. This is because of the momentum effect that the stock price will continue to move in the same direction. In contrast, when the ranking period return is quite high, the market is more likely to overreact. In the next period, market overreaction is corrected or adjusted in the holding period, leading to weak momentum effects or reversal effects.[4] On the other hand, this financial phenomenon combines with mean reversion.[5] When there is an upward trend in momentum portfolio returns in the ranking period that reaches a high value, according to mean reversion, in the next period, the holding return will be more likely to be low, which leads to a dominant reversal. Finally, many studies have investigated the cross-market equity return and volatility linkages in different markets. They proposed lots of underlying explanations in previous research. With an increase in return correlations among stock markets, stock returns in one market can influence stock returns in other markets, and simultaneously, the market volatility transfers from one stock to another, which leads to a spillover effect. Engle, Ito and Lin (1990) provided two alternative interpretations of volatility spillovers. One is the heat wave hypothesis, that higher volatility, as an internal phenomenon, never spills from one market to another in any serial manner. Conversely, an alternative theory

[4] Daniel, Hirshleifer and Subrahmanyam, 1998; DeLong et al., 1990; Barberis, Shleifer and Vishny, 1998; Hong and Stein, 1999; Grinblatt and Han, 2002

[5] Berk, Green and Naik, 1999; Lee and Swaminathan, 2000; Balvers and Wu, 2006

is the meteor shower hypothesis that higher volatility transfers from one market to another on the next global trading day. The authors found that for the London-New York markets, the evidence persuasively supports the meteor shower hypothesis. In addition, an alternative theory presented by King and Wadhwani (1990) showed that market contagion can transfer across different markets, generating evidence of varying interdependencies among stock markets in the world.

Thus, the previous research has found certain empirical evidence regularities, and has made a reasonable and possible interpretation. The present study examines the relationship between one dependent variable (the momentum portfolio return in the holding period) and three independent variables (the momentum portfolio return in the ranking period, the domestic market volatility in the ranking period, and foreign market volatility in the ranking period) in a two-regime switching model, with regard to whether these regularities can support effective trading strategies.

2.2.4 Two-regime switching model

Many researchers have investigated the regime-switching model based on market volatility. Abdymomunov and Morley (2011) adopted a two-state Markov-switching process based on market returns and portfolio returns, with the transformation based on the beta and the market risk premium between LOW and HIGH volatility regimes. They found strong evidence of time variation in betas across volatility regimes, regardless of the unconditional CAPM or the conditional CAPM. Dobrynskaya (2017) proposed a simple dynamic trading model which follows the standard momentum strategy during the calm period, then switches to the contrarian strategy during the month of a market crash, before reverting again to the momentum position after maintaining the contrarian strategy for three months. This is entirely consistent with Huang and Tsai (2014), who suggested that trading investment can be switched, and focused on contrarian strategies when market risk is high, especially during times of financial crisis.

Based on the previous research, a two-regime switching model was set up between the momentum regime and its reversal regime. More specifically, each regime contains one dependent variable and three independent variables. The dependent variable is the momentum portfolio return in the holding period in both regimes. The three independent variables are the momentum portfolio return in the ranking period, the domestic market volatility in the ranking period, and foreign market volatility in the ranking period between the momentum regime and the reversal regime. On the other hand, the threshold model as a two-regime switching model is used to indicate which regime is at work in a specific period. According to the level of domestic market risk in the ranking period, Bayesian estimation can be used to explore the threshold parameter. This implies that when the threshold parameter is over (below) a specific value, one of the regimes will work.

The two-regime switching model can be shown as below:

$$MR_{H,t} = \left[1 - F_{[0,\tau)}\left(z^D_{R,t-1}\right)\right]\left(\alpha_1 + \beta_1 MR_{R,t-1} + \gamma_1 z^D_{R,t-1} + \theta_1 z^F_{R,t-1}\right) \quad 2.2.3$$

$$+F_{[\tau,\infty)}\left(z^D_{R,t-1}\right)\left(\alpha_2 + \beta_2 MR_{R,t-1} + \gamma_2 z^D_{R,t-1} + \theta_2 z^F_{R,t-1}\right) + \varepsilon_t$$

Where $MR_{H,t}$ denotes the momentum portfolio return in the holding period, and $z^D_{R,t-1}$ denotes the domestic market volatility in the ranking period. Here, the model calculates the average market volatility in the past J period as the measure of market volatility in the ranking period. $MR_{R,t-1}$ is the momentum portfolio return in the ranking period. $z^F_{R,t-1}$ means the level of foreign market volatility in the ranking period, which is the same calculation as for domestic market volatility in the ranking period. $F_{[\tau,\infty)}(z^R_{t-1})$ denotes the Dirac function with τ as the threshold parameter. When $z^D_{R,t-1}$ is below the threshold parameter, the Dirac function (F(.)) is equal to 0. Then, the first regime is at work, which means the momentum effect is expected. In contrast, when $z^D_{R,t-1}$ is above the threshold parameter, the Dirac function (F(.)) is equal to 1, which means the second regime with the reversal effect has occurred.

2.2.5 Bayesian method of estimation

In this section, the Bayesian estimation method is adopted to estimate several values, including the threshold parameter, coefficients of independent variables, and the error term. Bayesian inference is based on posterior distribution, which in turn is based on a sound, reliable and principled pattern of combined prior information with actual data. It allows the incorporation of past information regarding coefficients and the setting up of a prior distribution for the data analysis. Alternatively, the previous posterior distribution can be used as the prior information, following the Bayesian theorem. Moreover, Bayesian estimation also provides inferences that are both conditional on the data and accurate.

Initially, the two-regime switching model 2.2.3 can be rewritten as below:

$$MR_{H,t} = \left(\alpha_1 + \beta_1 MR_{R,t-1} + \gamma_1 z^D_{R,t-1} + \theta_1 z^F_{R,t-1}\right) \quad 2.2.6$$

$$+F_{[\tau,\infty)}\left(z^D_{R,t-1}\right)\left(\alpha_2 - \alpha_1 + (\beta_2 - \beta_1) MR_{R,t-1}\right.$$

$$\left. + (\gamma_2 - \gamma_1) z^D_{R,t-1} + (\theta_2 - \theta_1) z^F_{R,t-1}\right) + \varepsilon_t$$

Then, according to Bauwens, Lubrano and Richard (2000), I can transfer the equation 2.2.6 to the matrix form for practical computations, which can be reparametrized as below:

$$y_t = x'_t \beta + \varepsilon_t \quad 2.2.7$$

where

$$y_t = MR_{H,t} \quad 2.2.8$$

$$x'_t = [1, z^D_{R,t-1}, MR_{R,t-1}, z^F_{R,t-1}, F_{[\tau,\infty)}\left(z^D_{R,t-1}\right), F_{[\tau,\infty)}\left(z^D_{R,t-1}\right) \quad 2.2.9$$

$$* z^D_{R,t-1}, F_{[\tau,\infty)}\left(z^D_{R,t-1}\right)$$

$$* MR_{R,t-1}, F_{[\tau,\infty)}\left(z^D_{R,t-1}\right) * z^F_{R,t-1}]$$

$$\beta' = [\alpha_1, \beta_1, \gamma_1, \theta_1, (\alpha_2 - \alpha_1), (\beta_2 - \beta_1), (\gamma_2 - \gamma_1), (\theta_2 - \theta_1)] \quad 2.2.10$$

Further, my model is a heteroscedastic model, which assumes different variances in the momentum regime and the reversal regime. Thus, it can be shown as,

$$Var(\varepsilon_t) = \sigma_1^2\left[1 - F_{[\tau,\infty)}\left(z_{R,t-1}^D\right) + \delta F_{[\tau,\infty)}\left(z_{R,t-1}^D\right)\right] \quad 2.2.11$$

$$= \sigma^2 h_t(\tau, \delta)$$

where

$$\sigma_1^2 = \sigma^2 \quad 2.2.12$$

$$\delta = {\sigma_2^2}/{\sigma_1^2} \in (0, +\infty)$$

Due to rescale the data, x_t and y_t can be redefined as,

$$y_t(\tau, \delta) = {y_t}/{\sqrt{h_t(\tau, \delta)}} \quad 2.2.13$$

$$x_t(\tau, \delta) = {x_t(\delta)}/{\sqrt{h_t(\tau, \delta)}}$$

The likelihood function of the heteroscedastic model can be shown as,

$$L(\beta, \tau, \delta, \sigma^2; y) \quad 2.2.14$$

$$\propto \sigma^{-T}\left[\prod_{t=1}^{T} h_t(\tau, \delta)\right]^{-\frac{1}{2}} \exp\{-\frac{1}{2\sigma^2}\sum_{t=1}^{T}[y_t(\tau, \delta) - x_t'(\tau, \delta)\beta]^2\}$$

Priors,

$$\varphi(\delta) \propto F_{[\delta_L, \delta_H]}(\delta) \tag{2.2.15}$$

$$\varphi(\tau) \propto F_{[z_L, z_H]}(\tau) \tag{2.2.16}$$

The conditional posterior density of β and σ^2 can be shown as,

$$\varphi(\beta|\tau, \delta, y) = f_t(\beta|\beta_*(\tau, \delta), M_*(\tau, \delta), s_*(\tau, \delta), v) \tag{2.2.17}$$

$$\varphi(\sigma^2|\tau, \delta, y) = f_{IG2}(\sigma^2|(\tau, \delta), v) \tag{2.2.18}$$

where

$$M_*(\tau, \delta) = \sum_{t=1}^{T} x_t\,(\tau, \delta) x_t'(\tau, \delta) \tag{2.2.19}$$

$$\beta_*(\tau, \delta) = M_*^{-1}(\tau, \delta) \sum_{t=1}^{T} x_t\,(\tau, \delta) y_t(\tau, \delta) \tag{2.2.20}$$

$$s_*(\tau, \delta) = \sum_{t=1}^{T} y_t\,(\tau, \delta)^2 - \beta_*'(\tau, \delta) M_*(\tau, \delta) \beta_*(\tau, \delta) \tag{2.2.21}$$

$$v_* = T - K \tag{2.2.22}$$

Therefore, the joint posterior density of τ and δ can be gained from the likelihood function and the prior densities of τ and δ,

$$\varphi(\tau, \delta|y) \propto \left[\prod_{t=1}^{T} h_t(\tau, \delta)\right]^{-\frac{1}{2}} s_*(\tau, \delta)^{-\frac{v_*}{2}} |M_*(\tau, \delta)|^{-\frac{1}{2}} \varphi(\tau) \varphi(\delta) \tag{2.2.23}$$

These can be adopted by a Markov chain Monte Carlo (Metropolis-Hasting algorithm) with uniform distribution.

The marginal posterior densities of τ and δ can be followed as,

$$\varphi(\beta|y) = \iint \varphi(\beta|\tau,\delta,y)\varphi(\tau,\delta|y)d\tau d\delta \quad 2.2.24$$

$$\varphi(\sigma^2|y) = \iint \varphi(\sigma^2|\tau,\delta,y)\varphi(\tau,\delta|y)d\tau d\delta \quad 2.2.25$$

Hence, the Bayesian 90% confidence interval of all parameters' marginal posterior density can be acquired.

2.3 Results and findings for the dynamics of momentum effects

2.3.1 Momentum effect in the UK and the US

Table A1.1 and Table A1.2 in Appendix A show the momentum portfolio returns from the sixteen different momentum strategies from 1980 to 2018. Overall, there is a significant momentum effect in the US market and in the UK market, which is more significant in the UK market than in the US market. There are also many features in common between the US market and the UK market. Initially, it can be found that both markets show a positive momentum effect, only at different magnitudes. There are significant positive momentum effects under the 5% significance level during the 3 months ranking period. Additionally, there are monotonic increases in momentum portfolio returns in the holding period given the constant 3 months ranking period. Further, returns in the remaining momentum trading strategies have dramatic fluctuations given the fixed ranking periods. However, there is the same distribution of the highest returns between the US market and UK market (i.e. the highest return is in 12 months holding period given the constant 3 months ranking period; the highest return is in 9 months holding period given the constant 6 months ranking period; the highest return is in 9 months holding period given the constant 9 months ranking period; the highest return is in 6 months holding period given the constant 12 months ranking period). This implies that in the long run, the probability of the reversal effect is more substantial. In addition, nine of the momentum trading strategies in the full sample in the US market, and four of the 16 momentum trading strategies in the UK market, produce significant profits at the significance level of 5%. Almost all the momentum trading strategies in the 9 months and 12 months ranking periods are non-significant at the significance level of 5%. On the other hand, the most striking difference between the two markets is that all the returns for each of the momentum trading strategies in the UK are always higher than those in the US market. However, there is quite similar overall performance in both markets, which implies that there is a potential relationship between the two markets.

Table A2.1 and Table A2.2 show the performance reliability of different momentum trading strategies in both markets from 1980 to 2018. In total, almost all the momentum trading strategies have high reliability in both markets. I used the ratio of the amount of positive momentum return divided by the whole observation, which can measure the performance reliability of each of the momentum strategies. This means that there is higher performance reliability when the percentage is substantial. The full sample in the UK has stronger dependability across the 16 momentum strategies than in the US market. More specifically, the ratios in both markets score more than 60%, except for momentum strategy 3*3 in the US market. However, although most of the momentum trading strategies in the UK market are over 70% in performance dependability, only one strategy in the US market scores higher than 70%. Further, it was found that the lowest ratios in both markets were for momentum strategy 12*12. Therefore, this underlying evidence, like the lowest momentum payoff and the least dependable momentum trading strategy, is potentially consistent with a reversal effect in the long run.

Table A3.1 and Table A3.2 illustrate the annualised market-adjusted returns on the winner and loser portfolios in the US and UK markets.[6] In the UK market, there are multiple negative returns on the loser portfolios, which are investment strategies during the ranking periods of three, six and nine months, which results in the potential insight that the momentum effect is significant in the UK market. Further, the annualised market-adjusted return on the loser portfolio could be positive when the formation period is nine months or more. Moreover, there are significant positive impacts by the winner portfolios under the 5% significance level, compared with the loser portfolios. In addition, I found that given the fixed formation period in the UK market, annualised market-adjusted returns on the winner portfolios monotonically decrease with the increase of the holding period. On the other hand, all the annualised market-adjusted returns on the winner and loser portfolios are positive in the US market. Similarly, there is also a more significant positive impact by the winner portfolios under the 5% significance level, compared with loser portfolios. Although there is a monotonic increase in

[6] The formula for annualised market-adjusted returns is $MR_p = \frac{\sum_{i=1}^{n} \prod_{t=1}^{K} R_{i,t,p}}{n} - \prod_{t=1}^{k} R_{m,t}$, where $p = w, l$ means the winner portfolio and loser portfolio, K is the holding period ($K = 3,6,9,12$), and $R_{m,t}$ denotes the monthly market return.

returns on the winner portfolios as the holding period lengthens, given the fixed ranking period, the highest returns on the winner portfolios occur in the holding period of 3 months.

According to the profitability of different momentum trading strategies in the UK and US markets shown in figure A4 (A4.1-A4.16), it can be observed that the frequency and duration of the momentum effect will be much greater than that of its reversals in both markets. Given the same momentum trading strategy, the momentum effect in the US market is more powerful than in the UK market, but has a shorter duration than in the UK market before the arrival of its reversal. Simultaneously, the frequency of reversion in the US market is more than in the UK market. Furthermore, the reversals in the UK market are more substantial and stronger in each historical stage, compared with the US market. Moreover, given the constant ranking period (J), it can be seen that a momentum trading strategy with a longer holding period has a longer momentum effect duration, but also a stronger power of reversion in both markets. Also, when the holding period is shorter, momentum volatility is more frequent, leading to the capture of more information. On the other hand, given the fixed holding period (K), one of the findings by observation is that a momentum trading strategy with a longer ranking period has a weaker power of momentum effect, but a stronger power of reversal. As a result, an economic insight is that during the course of nearly 30 years, significant reversion has only happened a few times, especially with momentum trading strategies with a more extended ranking period and holding period.

From figure A5 (A5.1-A5.4), it can be seen that the magnitude of market volatility in the US market is more significant than in the UK market, particularly during periods of increased market volatility. To a large extent, volatility in the US market looks like a leading indicator, compared with UK volatility. Overall, the two markets have a quite similar co-movement in terms of market volatility. From figure A6 (A6.1-A6.4), it can be found that the momentum portfolio returns in the ranking period in the UK market are often more than in the US market. And thus, the volatility of returns in the ranking period is sharper in the UK market, compared with the US market. Given the fixed holding period, a momentum trading strategy with a shorter ranking period has lower returns in the ranking period in both markets, based on examining the results of sixteen momentum trading strategies.

From the empirical evidence set out in figure A7 (A7.1-A7.4), figure A8 (A8.1-A8.4) and figure A9 (A9.1-A9.4), the initial relationship between returns in the holding period and returns in the ranking period, and domestic market volatility and foreign market volatility in the US and UK markets can be observed. Thus, the initial correlation among variables can be judged. It can be found that there is an inverse relationship between momentum portfolio returns in the holding period and domestic market volatility. Indeed, according to the empirical evidence in figure A7 (A7.1-A7.4), returns in the holding period are in an inverse correlation with returns in the ranking period in the 16 momentum strategies in both markets. An interesting finding is that given the constant ranking period, the alphas (constant in linear regression) with the longer holding period is following an upward trend in both markets, and the UK market has a larger scale than the US market under each momentum trading strategy. On the other hand, given the fixed holding period, there is a downward trend of alpha with the decline of the ranking period in the US market. In terms of beta, the UK market has a decreasing value with increasing holding period, given the stable ranking period, which is similar to the US stock market in most momentum strategies. When the holding period is constant, the beta is in a monotone increase in the UK market with increasing ranking period. Figure A8 (A8.1-A8.4) shows a total increase of alpha with rises in the holding period given the constant ranking period in both markets. In respect of beta, the results in both markets fall with the increasing holding period momentum strategy, if maintaining a fixed ranking period. And betas in all momentum strategies are negative, and all alphas are positive. Figure A9 (A9.1-A9.4) illustrates that given the constant ranking period, there is an increase of the alpha with the increasing holding period in both stock markets. Importantly, there is a significant negative link between momentum portfolio returns in the holding period and foreign market volatility. More specifically, there is a monotonic decrease of the beta with the increasing holding period when keeping the constant ranking period.

2.3.2 Results of Bayesian estimation

In this part, the empirical results arising from the estimations of the two-regime switching model are reported and interpreted in relation to 16 momentum trading strategies in the US and UK stock markets, respectively. According to the Bayesian inference, I used the prior distribution of the Tau and the Delta produced by Cao (2014). Cao (2014) used the error and trail method and the Independent Metropolis-Hasting algorithm to determine the prior distribution of the Tau and Delta in the UK market. I used the prior distribution of threshold parameter (τ), which is a uniform distribution with distribution support between 0.035 and 0.045, and the prior distribution of ratio (δ), which is a uniform distribution with distribution support between 0.5 and 6 in the UK market. Similarly, I can use the same method to obtain the prior distribution of the Tau and Delta in the US market. The prior distribution of threshold parameter (τ), a uniform distribution with distribution support, is between 0.025 and 0.05, and the prior distribution of ratio (δ), as a uniform distribution with distribution support, is between 1 and 6.5 in the US market. More specifically, according to the distribution of the domestic market volatility in both markets, I set wide intervals so that the switches were extremely frequent in each of the momentum strategies, mainly focusing on the range [0.025, 0.5][7]. Draws for the posterior distribution of the Tau can be produced via the Independent Metropolis-Hasting algorithm that uses uniform distribution as the candidate density. The prior distribution of the Delta as a uniform distribution with distribution support is between 1 and 6.5 for all 16 momentum strategies.[8] The posterior probability distributions of the Delta are produced using an independent Metropolis–Hastings algorithm with uniform candidate density as well.

[7] To ensure the dependability of regression estimation in both regimes, I selected the support for prior distribution of the Tau so that at least 30 observations were set in each regime.

[8] Because I adopted the uniform distribution, that is a non-informative prior distribution, this distribution is adaptive only if it does not limit the posterior distribution. All the posterior distributions of the Delta were in the interval between 1 and 6.5, thus this distribution was adaptive and dependable.

2.3.2.1 Findings and discussion of the posterior probability distribution of the Tau and the Delta

Regarding the estimation of the posterior probability distribution of the Tau, there was a significant estimation result for most of Tau in both markets. Based on the whole sample, almost all of the posterior probability distribution of Tau were in common intervals between 0.035 and 0.045 in the UK market and between 0.025 and 0.05 in the US market. Obviously, as figure A11 (A11.1-A11.4) shows in the US market and figure A13 (A13.1-A13.4) shows in the UK market, there is a significant switch between the momentum regime and the reversal regime in the majority of momentum trading strategies. In table A14.2 for the US market and table A15.2 for the UK market, this study reported the 90% Bayesian confidence interval for the Tau in the US and the UK markets, which provides exact intervals. For example, under the 12*6 momentum trading strategy, the large number of probabilities have a relatively fixed value for the Tau in the US market, distributed between 0.035 and 0.037. This result implies that in this interval, there is a robust possibility of the switch from the momentum (reversal) regime to the reversal (momentum) regime. In other words, when the market risk in the ranking period is less (more) than the threshold parameter (Tau), the momentum (reversal) effect occurs. Thus, when the momentum regime works, the 12*6 momentum trading strategy is successful or profitable during the coming six months. In contrast, the reversal effect is expected when the market risk in the ranking period exceeds the threshold parameter (Tau), which produces losses via the 12*6 momentum trading strategy in the following six months. To a large extent, the 90% Bayesian confidence intervals for the Tau in the US and the UK markets are in a small range of approximately one fixed value.

In figure A11 (A11.1-A11.4) and figure A13 (A13.1-A13.4), I also report the Bayesian estimation of posterior probability distribution of the Delta in the US and UK markets. Table A14.2 and table A15.2 show the 90% Bayesian confidence intervals of the Delta in the US and UK markets. In sum, all Delta values in the different momentum strategies are more significant than one, which means that the variance in the reversal regime is more than that of the momentum regime in both markets. Moreover, the results of the posterior probability distribution of the Delta also imply that the assumption of heteroskedasticity is reasonable. Specifically, the momentum regime shows a low variance, which means lower risk. On the other hand, the reversal regime has a high variance, which implies a higher risk. For example, in momentum trading strategy 6*6, the 90% Bayesian confidence intervals of the Delta are [3.053, 3.615] in the US market. As a result, the Bayesian estimation of the posterior probability distribution of the Tau and the Delta reveals reasonability and adaptability when applying the two-regime switching model in both markets.

2.3.2.2 Findings and discussion of posterior probability distribution of the Alpha, Beta, Gamma and Theta in the momentum regime

In this section, I report the results of the alpha, beta, gamma, and theta parameters in the momentum regime. Overall, I found more significant results in the UK market than in the US market. Based on the Bayesian estimation of posterior probability distribution of parameters, the interval of the intercept Alpha1 measures the degree of average momentum return. For example, in momentum trading strategy 6*6, the 90% Bayesian confidence intervals of the Alpha1 are [0.083, 0.128] in the US market. To a large extent, the positive Alpha1 means that in a calm market, the momentum investment strategy can produce momentum returns. Further, I find that for the total momentum strategy in the UK market, the values of the Alpha1 are positive, which can potentially explain the significant profitability of a momentum trading strategy in the UK market in the momentum regime. Further, there is a majority of positive Alpha1 values in the US market. In addition, compared with the UK market, the absolute value of the Alpha1 in the US market is smaller overall than it is in the UK market, which potentially means that the level of momentum returns in the US market is lower than in the UK market for each momentum trading strategy, as shown in Table A14.1 and Table A15.1.

The value of Beta1 shows the relationship between domestic market risk and the strength of the momentum effect in the momentum regime. For example, in momentum trading strategy 6*9 in the UK market, the Bayesian estimation of the posterior probability distribution of parameter Beta1 is in the interval [-2.483, -1.285]. This means that there is a negative relationship between domestic market risk and the strength of the momentum effect. In other words, during calm market periods, higher domestic market risk leads to a reduction in the strength of the momentum effect. Further, almost all the momentum trading strategies in the UK market also show that there is an inverse relationship between domestic market risk and the strength of the momentum effect. In the US market, 13 out of the 16 momentum strategies illustrated robust evidence of a negative correlation between domestic market risk and momentum returns.

Parameter Gamma1 shows the relationship between the momentum portfolio returns in the formation period and in the holding period in the momentum regime. For instance, in momentum trading strategy 6*9 in the UK market, the Bayesian estimation of the posterior probability distribution of parameter Gamma1 is in the interval [-0.174, -0.148]. Further, I found that in the 16 momentum trading strategies, all the Bayesian estimations of the posterior probability distribution of parameter Gamma1 fall into the negative interval in the US market. One potential explanation is that there is a dramatic potential strength of momentum effect when returns in the formation period are low in the momentum regime in the UK market.

The value of the Theta1 measures the effect of foreign financial market risk on the strength of the momentum effect in the momentum regime. For example, in the UK market, the Bayesian estimation of the posterior probability distribution of parameter Theta1 for momentum strategy 6*6 is in the interval [1.103, 1.788]. This implies that when the market is calm, there is a positive relationship between US market risk and the momentum return in the UK market in the holding period. Further, across all 16 momentum strategies in the US market, almost all the results are significant positive, except for momentum strategy 3*3 and 3*6.

2.3.2.3 Findings and discussion of the posterior probability distribution of the Alpha, Beta, Gamma, and Theta parameters in the reversal regime

This section reports the results for the alpha, beta, gamma, and theta parameters in the reversal regime, when market volatility is above the threshold parameter Tau. Overall, there was a difference between the Bayesian estimation of the posterior probability distribution of parameter Alpha1, Beta1, Gamma1, and Theta1 in the momentum regime, and Alpha2, Beta2, Gamma2, and Theta2 in the reversal regime. This is because there is a higher risk exposure (volatility) in the reversal regime than in the momentum regime due to the posterior probability distribution Delta. There is a high probability of switching from the momentum regime to the reversal regime when market volatility is above the threshold parameter.

The value of the Alpha2 seems to be small significant under the 90% Bayesian confidence interval. However, I found that under the 90% Bayesian confidence interval, all the Alpha2 values are positive in the UK market, and almost all of the Alpha 2 values are positive in the US market. This means that the initial momentum returns in the reversal regime could be substantial. This could be explained by the information delay that because investors cannot immediately reflect the high market volatility, they cannot make investment decisions in time. The Beta2 measures market risk in the reversal regime. The results for the 16 momentum strategies show a significant negative relationship between market risk and momentum effect in the UK market. However, there is an insignificant relationship between market risk and momentum effect in the US market. The values of the Gamma2 measure the correlation between returns in the ranking period and the strength of the momentum effect. Across the 16 momentum strategies, there is a significant inverse relationship between returns in the ranking period and momentum effect in the UK market but a significant positive relationship in the US market. On the other hand, the value of the Theta2 measures the correlation between foreign market risk and the strength of the momentum effect. Further, there are a significant positive relationship in the UK stock market and a significant negative relationship in the US stock market.

Chapter 3.0 Momentum Effects and Long-run Risk Model

3.1 Literature review for long-run risk model

In this section, I will explain the long-run risk model, and review its development as well as providing a cross-section of security returns in the long-run risk model.

3.1.1 Long-run risk model

Bansal and Yaron (2004) and Hansen, Heaton and Li (2006) summarised the future of the long-run risk model, which is that the model focuses on the low-frequency characteristics of time series of dividends and aggregate consumption. Further, the model has superiority in accounting for time series and cross-sectional asset returns in the financial markets. Bekaert, Engstrom and Xing (2009) utilised an external habit model containing persistent time-varying uncertainty fundamentals, which can match the dynamics of dividends and consumption. Malloy, Moskowitz, and Vissing-Jorgensen (2004) found new empirical evidence regarding long-run risk in asset pricing. They believed that long-run risk shareholders' consumption risk can capture cross-sectional changes better than aggregate consumption risk.

One long-run risk model is the classical asset pricing model by Bansal and Yaron (2004). I will review this model's fundamental assumptions, construction, and results in this part. Regarding the assumptions, Campbell and Cochrane (1999) established an asset pricing model based on the external habit model. Their model assumed that the processes of consumption and dividends are produced by time-varying risk aversion, and thus that investors prefer the late resolution of uncertainty that is backwards-looking. Further, historical consumption significantly impacts on asset pricing movement. However, Bansal and Yaron (2004) make entirely different assumptions. They put forward a long-run risk model which can adequately explain asset market phenomena such as the risk-free rate, equity premium (Mehra and Prescott, 1985), and volatility puzzles (Shiller, 1981). More specifically, the model includes two components in explaining asset pricing puzzles, such as a small, persistent, predictable component showing long-run fluctuations in two expected growth processes and fluctuating economic uncertainty with time-variance in consumption volatility. The model is built on the basis of the standard Epstein and Zin (1989) preferences. This life-time utility separates out the intertemporal elasticity of substitution (IES) and risk aversion. Theoretically, the measure of risk aversion correlative to the reciprocal of the IES can be determined by whether the agent prefers an early resolution of uncertainty or a late one regarding the consumption process in the future. In the long-run risk model, it is assumed that investors prefer to resolve uncertainty early, and hence the parameter of risk aversion exceeds the reciprocal of the IES. It implies that shocks in the model to the expected growth and consumption volatility are part of the long-run process, and therefore, that they change investors' expectations of future growth and volatility. Theoretically, the price depends on the expected return and consumption, as they contribute to forecast future growth and uncertainty. Thus, the long-run risk model has forward-looking predictability, and the core driving force of the financial market in this model is the volatility of the long-run economic growth expectation and the content of the economic uncertainty. There are also three different sources of risks in the original model: short-run risk, long-run risk, and consumption volatility risk. As a result, there is a closed match regarding risk correlated to growth expectation and fluctuating economic uncertainty with time-variance between the observed market data and the simulation of the model.

3.1.2 Development of the long-run risk model

Bansal, Khatchatrian and Yaron (2005) illustrated the significant empirical result that the expected magnitudes of economic uncertainty can be forecast by the valuation ratio. They also found that with increased economic uncertainty, asset valuation declines, which means that the financial market is averse to economic uncertainty. Additionally, to a large extent, changes in asset prices are more likely to contribute to volatility and economic uncertainty, and the expected growth of cash flow. Thus, they concluded that fluctuations in economic uncertainty and the predicted growth of cash flow are significant in explaining the asset market.

Bansal, Kiku and Yaron (2007a) presented the long-run risk co-integrated extension. This paper deployed a long-run risk model to interpret asset returns using asset pricing Euler equation-GMM based estimation. They found that investors care more about long-run risk, which along with economic uncertainty, is critical to understanding asset returns. More specifically, the risk component in the long-run risk model is strongly persistent and examines fluctuations longer than those relative to the business cycle. Importantly, it is also economically and statistically predictable by theoretically motivated variables. Moreover, it is more sensitive for assets with substantial mean returns from innovation in the long-run risk variable and from news about economic uncertainty. They also found that the long-run risk model can interpret most risk premia well, and cannot be rejected by the over-identifying limitation. More importantly, there was evidence that the estimation of risk aversion and the IES are more vulnerable to time-variance and finite sample biases. The market price of the long-run risk model at risk aversion and the IES value for preference parameters was more closed relative to that of short-run and volatility risks. Therefore, the authors summarised that the long-run risk model, as opposed to short-run risks, was more important for understanding asset prices. Similarly, Bansal, Gallant and Tauchen (2007) examined and compared two models; the first, with three main channels, including the long-run risk model, low-frequency movements, and time-varying uncertainty in aggregate consumption growth, constructed the empirical reasonability of the asset pricing model. The other model was by Campbell and Cochrane (1999), in which the significant channel was habit formation. They found that the two models were fitted with annual observations data from 1929 to 2001, using simulation estimators. Both models could trail a measurement of real annual volatility quite tightly. The results show that the long-run risk model could be better. Bansal, Kiku and Yaron (2009) estimated that the forward-looking long-run risk model was different to the backwards-looking habit, and produced three key results. The first was that there is material evidence in time-varying expected growth and consumption volatility data. The second was that the long-run risk model matched the primary asset market data characteristics. The third was that past consumption growth cannot forecast future asset prices, but lagged consumption could predict future price-dividend ratios. The new equation for dividend growth was therefore as follows:

$$g_{d,t+1} = \mu_d + \varphi x_t + \pi \sigma_t \eta_{t+1} + \varphi \sigma_t \mu_{d,t+1}$$

They found that although there was persistent volatility in the calibration of the model, the volatility of the price-dividend ratio was not large enough for the price-dividend rate to forecast excess returns. Bansal, Kiku and Yaron (2010) developed a generalised long-run risk model to test the influence of cyclical volatility and macroeconomic risk on asset prices and expected returns. Particularly, they sought to assess the cyclical risks, so the model contained a cyclical element that is stationary in the consumption growth path. The model also examined surges in consumption and volatility of consumption. They discovered that the measurement of risk compensation for cyclical risks in consumption relies to a large extent on the scale of the IES. Thus, they found that when the IES is above 1, there is a small risk premium for cyclical risk, and that when the IES is approximately 0 or more than 1, the risk compensation for cyclical risk is quite large, a finding consistent with Lucas (1987) and Bansal, Dittmar and Kiku (2009). Bansal and Shaliastovich (2013) developed and examined a long-run risk model with time-varying fluctuations in expected growth and inflation in the future. The model showed that an increase in bond risk premia leads to an increase in the uncertainty of expected inflation and a reduction in the uncertainty of expected growth. They also identified several significant factors for asset markets, such as preference for the early resolution of uncertainty, time-varying volatilities, and non-neutral impacts of inflation on economic growth. Bansal, Kiku and Shaliastovich (2014) proposed a framework based on the long-run risk model with a stochastic discount factor, and thus that the risk premium can be decided by cash flow risk, discount rate risk, and volatility risk. As a result, they presented three major findings. The first was that increased volatility is linked to an increasing discount rate and falling expected consumption. The second is that it is important for the role of volatility risk to explain the joint dynamics of return with human capital and equity. The last is that volatility risk with a significantly positive risk premium can contribute to interpreting the magnitude of, and cross-sectional dispersion in, expected returns in the future. Ferson, Nallareddy and Xie (2012) developed an out-of-sample asset return sample from 1931 to 2009. They discovered that the long-run risk model implemented the momentum effect reasonably well, and also that a cointegrated version of the model was better than the classical stationary version. They evidenced that generally, the models' average pricing errors were relatively small from the 1950s to the 1990s. If the risk premium was restricted to identifying structural parameters, it would lead to a larger average pricing error but smaller error variances. In addition, the average squared errors were not able to dramatically improve on those of the traditional CAPM, in addition to momentum.

Piazzesi and Schneider (2006) introduced the role of inflation into the long-run risk model by using treasury inflation-protected securities data in the US. They found that inflation was bad news for consumption growth, that the nominal yield curve was an upwards trend, and that the level of nominal interest rate and term extension were high in time when inflation news was difficult to explain. Ai (2010) used a production-based long-run risk model to enhance understanding of asset pricing implications in three respects. These were the increase in the equilibrium equity premium, the declining volatility of consumption growth, and the risk-free interest rate. He found that due to the different assumptions in the production economy, there is a low relationship between information and high equity premiums, low volatility consumption growth, and the risk-free interest rate. Additionally, he also optimised the long-run risk model to gain a strong improvement regarding volatility of returns on aggregate wealth and the wealth-consumption ratio. Sasaki (2016) extended the long-run risk model by bringing in stochastic jump intensity and the variance of the consumption growth rate, to offer an explicit representation of the skewness risk premium and the volatility risk premium in equilibrium. They claimed a superior predictive ability for future expected aggregate stock market index returns between the skewness risk premium and the variance risk premium. At the same time, the skewness risk premium played an independent and essential role in forecasting market index returns.

Kaltenbrunner and Lochstoer (2010) examined the endogenous formation of long-run consumption risk with Epstein–Zin preferences. They found that optimal consumption smoothing generates long-run risk with highly persistent fluctuations in expected growth, even with the assumption of i.i.d. technology growth. Therefore, their model has the ability to explain the high price of risk even though both the volatility of consumption growth and relative risk aversion parameters are low. They argued that the endogenous long-run risk impact on asset pricing relies on the persistence of technology shocks, as well as agents' performance in the timing of resolution of uncertainty to a large extent. Hartzmark (2016) found that the long-run risk model was helpful in supporting and describing the basic association between economic uncertainty and the interest rate. They evidenced that this relation is large, which lends strong support to the basic finance theory. Colacito and Croce (2011) induced the exchange rate in the long-run risk model, which successfully reconciled international prices and quantities, and thus solved the international equity premium puzzle using US and UK data. As a result, a closed relationship between common long-run growth perspectives and exchange rate movement was identified. Hasseltoft (2012) accounted for the primary characteristics of equity and bond markets, and then the interactions between asset pricing and the macroeconomy in a long-run risk model. They discovered that shocks to future expected consumption growth and time-varying macroeconomic volatility could accurately reflect the measure of risk premia and its changes over time in both markets, using a simulation estimator and a wide set of moment conditions. Croce (2014) studied the probability of long-run productivity risk, using the original long-run risk model by Bansal and Yaron (2004). They found a predictable component in US productivity growth, which impacts not only on aggregate market prices but also on primary macroeconomic variables. Further, they adopted a production-based dynamic stochastic general equilibrium (DSGE) model with long-run productivity shocks, leading to an improved asset price explanation. Kaltenbrunner and Lochstoer (2010) illustrated that an endogenous production-based model can predict the long-run expected fluctuations in consumption growth.

However, several potential problems in the development of the long-run risk model also need to be analysed and explained. First of all, there has been debate about the existence of a long-run risk component in future expected consumption growth. More precisely, this question is challenging to explore statistically using univariate methods as consumption is closed to the random walk. Besides, the impact on asset prices relies on investors exploring it, and the model counterfactually makes consumption growth predictable via the price-dividend ratio. Furthermore, a large number of adjustments and calibrations of the long-run risk model are needed. For example, one study’s calibration (Bansal, Kiku and Yaron, 2007b) changed the weighting towards the second source of long-run risk, that is, persistent volatility reducing the predictability of consumption growth. Finally, another debate focusses on the value of the IES greater than one. Beeler and Campbell (2009) summarized, revaluated, and discussed the long-run risk model, which they argued contained several crucial difficulties as a quantitative depiction of historical financial data in the US. Firstly, there was scarcely any evidence either for persistent fluctuations in consumption and dividend growth ratios, or for the capability of the stock market predicably reflecting these growth ratios. This implied that the long-run risk model could not utilise persistent variations in consumption growth to interpret changes in the stock market. Bansal, Kiku and Yaron (2007b) also recognised this issue and recalibrated the long-run risk model to highlight persistent changes in consumption. However, doing so produced another problem, which was that even though stock prices could forcefully forecast expected consumption volatility, they failed to contribute a predictable power to the expected volatility of returns on the stock. In addition, the authors also argued that aggregate consumption growth does not react to changes in the short-term real interest rate, which was required by the model’s assumption of an IES of larger than one.

Bonomo et al. (2010) presented an asset-pricing model with generalised disappointment, aversion preferences, and long-run volatility risk. Compared with Bansal and Yaron (2004), the model had two features containing more predictability of excess returns by price-dividend ratio and less predictability of the consumption growth ratio by price-dividend ratios. More importantly, their results do not rely on a value of the IES of larger than one. The last argument was that the model results in extremely low yields and negative term premia on long-run inflation-indexed bonds. Bansal (2007) stated that the relative asset price can be driven by long-run expected growth and economic uncertainty in the future. Both channels of economic risk were able to explain the risk premium and the volatility of asset price. A significant result is that the long-run risk model is able to provide a comprehensive and systematic model for analysing the financial markets. Bansal, Kiku and Yaron (2009) investigated the significant discrepancy between the habit model and the long-run risk model. They provided empirical evidence for the long-run risk model for time-varying expected consumption growth and consumption fluctuation, as well as matching the primary asset market data. However, the weak explanatory aspect of the long-run risk model is that lagged consumption growth fails to forecast the expected price-dividend ratio.

3.1.3 Cross-section of security returns in the long-run risk model

Constantinides and Ghosh (2011) proposed a new approach to testing the long-run risk model proposed by Bansal and Yaron (2004). This method is fundamentally based on the observation that the potential state variables can be found as the given function. The model can interpret the cross-sectional return by a higher persistence of consumption and dividend growth, compared with the actual data. Furthermore, the model can effectively match the real data for the unconditional moments of consumption and dividend growth. However, there are differences in the higher risk-free rate, and lower fluctuations in the price-dividend ratio, risk-free rate, and market return, compared with the observed data. One problem with this model is that the conditional variance of the long-run risk variable cannot capture the considerable time variation in the equity premium. Kiku (2006) claimed that the long-run risk model has the ability to interpret cross-sectional variations in average returns. Their model is successful in capturing the overall transition density of stock value returns and growth returns, which emphasises the significance of long-run risk in explaining the behaviour of stock markets.

Zurek (2007) estimated the cross-section of equity securities by using the long-run risk model in the US stock market. Their core contribution is to explain the momentum selection mechanism in the long-run risk model. More specifically, the mechanism is the difference between expected returns in different portfolios, as investors generally believe that securities in the winner portfolio have a higher risk than those in the loser portfolio. Thus, due to this systematic risk, these stocks are more likely to enter the winner portfolio. Once these securities are chosen for the winner portfolios, they can outperform the loser portfolios, and even the market portfolios, since the expected returns in the winner portfolios are still higher. Bansal, Dittmar and Lundblad (2005) also found that cross-sectional diversity can be interpreted well by long-run risk exposure in the expected returns of portfolios that are formed by the historical return, firms' scale, and book-market ratio. As a consequence, Zurek (2007) pointed out that according to their observations, the consumption beta at the cross-sectional portfolio level can effectively interpret most of the returns in the momentum portfolio. Their model can be simulated to generate a strong momentum effect and a comparatively large equity premium, and thus match relative moments like returns in the holding return, dividend growth, and price-dividend ratio in the momentum portfolio. However, Zurek's model (2007) has a potential problem, which is that it is short of relative macroeconomic risks. This is because the original long-risk model by Basal and Yoran (2004) was based on fluctuations with time-variance in periods of economic uncertainty, which is a potential merit in explaining the asset market. As they discovered, risks relative to varying growth expectations and fluctuations in economic uncertainty can be used to estimate a large number of observed characteristics of asset market data.

Further, Bansal, Kiku and Yaron (2007b) also explained that fluctuations in economic uncertainty can directly influence the price-dividend ratio, as well as increasing the expected economic volatility, producing a decline in asset prices. As a result, they found that the consumption volatility path can effectively capture the volatility feedback impacts in a way that implies a negative relationship between returns and volatility. This is mainly because the consumption growth path includes the overall economic volatility with time-variance. On the other hand, the model by Zurek (2007) allowed three different risk sources: long-run risk, short-run risk, and consumption risk, based on the different volatility and shocks in the model. Zhou and Zhu (2014) extended the long-run risk model based on multiple macroeconomic volatilities, which allowed the coexistence of long-run and short-run volatility components. The assumptions of the model may be consistent with the actual volatility, and their model can match different modes, including the scale of the market risk premium and the predictability of dividend yields. Moreover, Alizadeh, Brandt and Diebold (2002), Adrian and Rosenberg (2008) and Chacko and Viceira (2005) have all also studied multiple volatilities in financial markets. Nakamura, Sergeyev and Steinsson (2012) identified many relative factors of consumption volatility using developed countries' panel data. They found that the long-run risk model which includes multiple volatility variables can closely match predictability moments. Moreover, Boguth and Kuehn (2013) tested a dual-volatility process by separating the two factors of the aggregate consumption path. They concluded that volatility risks are significant for the cross-section of securities returns.

Bansal et al. (2009) built a dynamic asset-pricing model to evaluate the volatility risk relative to the dynamics of asset prices and macroeconomic fluctuations. They revealed three main results. The initial finding was that increased macroeconomic volatility leads to increased discount rates and falling expected consumption. The following finding was that the volatility risks can effectively explain the joint dynamics of returns to human capital and equity. The last outcome was that a considerable positive risk premium was observed for volatility risks, which contributes to interpreting the extent and cross-sectional dispersion of predictable returns. Further, the model with three different sources of risk (cash flow, discount rate, and volatility risk) can make estimations using the observed macro and financial data. Simultaneously, Drechsler and Yaron (2011) also established a long-run risk model with a volatility path. The first process can capture persistent long-run movements. The other shows a quickly mean-reverting and shorter-run component of consumption volatility. These extensions of the long-run risk model are able to enhance the predictability of consumption growth and abnormal returns. Chordia and Shivakumar (2002) accounted for returns in the momentum portfolio using lagged macroeconomic variables and the disappearance of returns in the momentum portfolio. Their results contained time-varying expected returns, which can be used to interpret momentum payoffs.

3.2 Data and methodology

In this section, I will explain and discuss the methods I used to conduct the present research. Initially, I will set out the data collection and selection methods. Then, I will demonstrate the assumptions of the model, including investors' preferences, the construction of stochastic discount factors, and the economic environment. Additionally, I will develop the long-run risk model and show the exact results of the model's deductions. Finally, I will expound the process of calibration and simulation on the basis of the long-run risk model.

3.2.1 Data collection

There were two components to my data collection. On the one hand, I obtained monthly market data on individual securities from the Center for Research in Security Prices (CRSP) through the Wharton Research Data Services (WRDS). This monthly market data contained the holding return with and without dividends, and used the New York Stock Exchange and American Stock Exchange data from January 1970 and December 2018. Further, the per capita real consumption expenditure quarterly data for nondurable goods and services could be collected from the US Bureau of Economic Analysis. According to Hansen and Singleton (1983) and Bansal, Dittmar and Lundblad (2005), aggregate consumption can be computed as seasonally adjusted real personal consumption expenditures per capita of nondurable goods plus services. The present sample ranged from the first quarter in 1970 to the fourth quarter in 2018. The monthly Consumer Price Index (CPI) was collected as well, in order to calculate the inflation-adjusted return.

3.2.1.1 Momentum portfolio using non-overlapping portfolio

In this part, I calculate the return on each portfolio, following Jagadeesh and Titman (1993), similar to the above discussion. Firstly, I adjust the monthly holding period return (with and without dividends) to the real return:

$$r_{real,t} = \frac{1 + r_{nominal,t}}{1 + \pi_t} \qquad 3.2.1$$

$$\pi_t = \frac{CPI_t}{CPI_{t-1}} - 1$$

Where $r_{nominal,t}$ denotes the monthly nominal holding period return, and π_t indicates the inflation rate calculated by the CPI.

Further, the momentum portfolio can be established based on the past adjusted inflation real returns with dividends over the ranking period (J), then observing the future gains in the holding period (K). Here, the ranking period is 6 or 12 months, and the holding period is 3, 6 or 12 months, which is convenient in transferring the quarterly return data and in terms of the availability of quarterly consumption data. Moreover, due to the microstructure effects, for example non-synchronous trading, the bid-ask bounce, and infrequent trading, the one-month interval is skipped in the ranking period. Therefore, due to the construction of the non-overlapping portfolio, there is no overlapping calculated return in the strategies holding period, but there could be overlapping in ranking periods longer than three months in duration. The whole sample is divided into the five portfolios on the basis of the past cumulative returns in the ranking period. Similarly, the winner portfolio has the highest payoff, and the loser portfolio the lowest payoff in the formation period. Future returns in the holding period are observed in each portfolio, but the skipped month is excluded at the start of the holding period. For example, for non-overlapping momentum strategy 6*3, the first formation period is from January 1970 to May 1970. Then, one month, June 1970, is skipped, followed by a holding period from July 1970 to September 1970, which is the first construction of the non-overlapping momentum strategy 6*3.

Following the example of Jagadeesh and Titman, I established a momentum portfolio with non-overlapping generation. Then, as per Bansal, Dittmar and Lundblad (2005), the dividend can be computed by the adjusted inflation returns with dividends (total returns, $r_{d,t}$) and adjusted inflation returns without dividends (capital gains, $r_{x,t}$). Thus, there are two-time series returns, including the total return and capital gains in five momentum portfolios. The dividend formula can be shown as below,

$$DIV_{t,p} = V_{t-1,p}(r_{d,t} - r_{x,t}) \quad 3.2.2$$

Where $V_{t-1,p}$ denotes the overall value of each momentum portfolio p. p represents the five momentum portfolios, particularly the loser portfolio for $p = 1$ and the winner portfolio for $p = 5$. And $R_{d,t} - R_{x,t}$ means the dividend yield. Further, the total value of each portfolio $V_{t,p}$ is assumed by a recursive process,

$$V_{t,p} = V_{t-1,p} * (1 + r_{x,t}) \quad 3.2.3$$

Obviously, the equation implies that the value of each portfolio depends on the previous value of the momentum portfolio and the growth in capital gains. There is no effect of dividend on the growth in the value of each portfolio since the dividends are extracted each month. Further, the monthly dividend series can be transferred to a quarterly frequency so that each monthly dividend can be summed in the holding period. Thus, each dividend in a non-overlapping series represents the sum of the holding months' dividends. Because of the strong seasonality in dividend yields, I followed a method used by Hodrick (1992), Heaton (1993), Bollerslev and Hodrick (1995), and Bansal, Dittmar and Lundblad (2005), by adopting a preceding four-quarter moving average between the current and past dividends, for the seasonally adjusted dividend. Therefore, the return with dividends ($R_{T,p}$), the growth of dividends ($\Delta DIV_{T,p}$) and valuation ratio ($VR_{T,p}$) in each quarter, half-year or year can be represented as follows:

$$R_{T,p} = \frac{V_{T,p} + D_{T,p}}{V_{T-1,p}} \quad 3.2.4$$

$$\Delta DIV_{T,p} = log\ (DIV_{T,p}) - log(DIV_{T-1,p}) \quad 3.2.5$$

$$VR_{T,p} = log(V_{T,p}) - log\ (D_{T,p}) \quad 3.2.6$$

Where T denotes the interval of the holding period, and T is 3, 6, or 12 months. Hence, $T-1$ shows the last quarter, half-year, or year.

3.2.2 Preferences, stochastic discount factor, and the environment

In this part, I propose a new model based on Bansal and Yaron (2004) and Zurek (2007). The main reasons for using the long-run risk model are its two ingredients. Initially, the model uses the Epstein and Zin recursive preferences. These permit a separation between the intertemporal elasticity of substitution (IES) and risk aversion, and both parameters (IES and risk aversion) can be greater than 1 at the same time. When the IES is more than one, this means that agents require a higher equity risk premium due to their worries about expectations for the reduction of economic growth. Furthermore, the model contains two core components, a small, persistent, and predicable component in the growth rate component, and fluctuating volatility with time-varying economic uncertainty. More specifically, a representative agent relies on the Epstein and Zin recursive preferences, which satisfies below,

$$E_t\left[\delta^{\theta} G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{x,t+1}\right] = 1 \quad 3.2.7$$

Where G_{t+1} denotes the aggregate gross growth rate of consumption. $R_{a,t+1}$ and $R_{x,t+1}$ denote the return on an asset with aggregate consumption as its dividends every period, and the asset pricing restriction on any continuous asset return, respectively. $\delta \in (0,1)$ is the time discount factor (time-preference). $\theta = \frac{1-\gamma}{1-\psi^{-1}}$, where γ is the parameter of risk aversion (sensitivity) and ψ is the parameter of the intertemporal elasticity of substitution, which are non-negative. So, when γ and ψ are simultaneously greater than 1, θ will be negative. When the risk aversion is equal to the reciprocal of the IES ($\gamma = \frac{1}{\psi}$), this means that $\theta = 1$, which reflects a standard case of expected utility. In other words, both the indifference between agents and the timing of the resolution of uncertainty are reflected. Moreover, when the risk aversion is more than the reciprocal of the IES ($\gamma > \frac{1}{\psi}$), agents prefer an early solution to the uncertainty of the consumption path. In contrast, when the risk aversion is less than the reciprocal of the IES ($\gamma < \frac{1}{\psi}$), agents prefer a late solution to the uncertainty of the consumption path. It is vital to build the agents' preferences into the dynamic asset-pricing model, which is based on an underlying assumption. As illustrated by Bansal and Yaron (2004) in the long-run risk model, the assumption that γ and ψ are above 1 means that the agent prefers an early solution to the uncertainty of the consumption path. According to Lucas (1978), in an exchange economy without labour income, aggregate dividends are equal to the aggregate consumption of the representative agent. This means that the return on the aggregate portfolio corresponds to the return on a claimed aggregate dividend. In fact, consumption should not be equal to dividends, since the discrepancy can be generated by labour income. Thus, the original long-run risk model separates the aggregate consumption process and the aggregate dividend process. However, my model follows Zurek (2007) by focusing on the level of the individual security. This is because the assumption of infinitely lived securities can reveal the momentum selection mechanism, which is intrinsically consistent with aggregate consumption and the equity market.

Further, the model is solved using approximate analytical solutions, following Campbell and Shiller (1988):

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \qquad 3.2.8$$

Where $r_{a,t+1} = \log R_{a,t+1}$ denotes the return on the claim to aggregate wealth. According to Campbell and Shiller (1988) and Bansal and Yaron (2004), κ_0 and κ_1, as two constant parameters, rely on the log price-consumption ratio, $log\ (P_t/C_t) = z_t$.[9] One crucial assumption made by Campbell and Shiller (1988) and Zurek (2007) was that there is a stationary price-dividend ratio for individual securities. Further, under this assumption, there is an approximate expression for the continuously compounded returns that could be seen as a linear function of dividend-price ratios and log dividend growth rates. They identified a potential problem in that an individual firm could be influenced by the firm life cycle effect,

[9] Bansal, Kiku and Yaron (2009) and Zurek (2007) recognised two approximate constants, $\kappa_1 = exp\ (\bar{z})/(1 + exp\ (\bar{z}))$ and $\kappa_0 = log(1 + exp(\bar{z})) - \kappa_1 \bar{z}$. They believed that these can ensure that any variation in parameters that alters the average price-consumption ration $\bar{z}$, can be contained in the approximation constants. Therefore, the endogenous solution for $\bar{z}$ could be shown as: $\bar{z} = A_0(\bar{z}) + A_2(\bar{z})\sigma^2$.

which implies a permanent change in their valuation ratios. Hence, Zurek (2007) pointed out that security i can be seen as a representative security of a company in phase i of the life cycle, as well as being transferred to distinct entities as time goes on from one period of the life cycle into others. Thus, it is similar to using the standard approximations for the solution of the corresponding total return on security i, which can be followed below,

$$r_{i,t+1} = \kappa_{i,0} + \kappa_{i,1} z_{i,t+1} - z_{i,t} + g_{i,t+1} \tag{3.2.9}$$

Where $r_{i,t+1} = log\, R_{i,t+1}$ denotes the return on equity asset i. $\kappa_{i,0}$ and $\kappa_{i,1}$ are also constants, but different for different securities depending on the log price-dividend ratio for the claim on dividends, $log\,(P_t/D_t) = z_{i,t}$.

The logarithm of the intertemporal marginal rate of substitution (IMRS) based on the Epstein and Zin (1989) preferences, can be rewritten to reflect the state variables which define the pricing kernel,

$$m_t = \theta\, ln\delta - \frac{\theta}{\psi} g_t + (\theta - 1) r_{a,t} \tag{3.2.10}$$

Further, as shown in Appendix B, innovation in the pricing kernel depends on the innovation in the aggregate gross growth rate of consumption g_t and innovation in return on an asset that delivers aggregate consumption as its dividends each period $r_{a,t}$.

3.2.3 Long-run risk model in cross-section

Based on the previous work by Bansal and Yaron (2004) and Zurek (2007), I expand a rational equilibrium model that is a long-run risk model. Some supporters of the rational market have argued that returns on the momentum portfolio result from exposure to systematic risk factors. Further, I built a cross-section of equity securities in the long-run risk model. More specifically, the momentum portfolio has time-varying economic uncertainty contained in the consumption path and expected returns.

The long-run risk model can be expressed as below:

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} \quad 3.2.11$$

Equation 3.2.11 shows the process of long-run risk transfer, which is an autoregressive process (AR(1)). x_{t+1} denotes a small but persistent predictable component, which refers to long-run risk. $\rho \in (-1,1)$ controls for the persistence of long-run risk. $\varphi_e > 0$ is a scaling parameter which identifies the relationship between the volatility of e_{t+1} and the aggregate volatility process σ_t. e_{t+1} is a normal distribution $N(0,1)$ and the i.i.d. shock, which is a factor that interprets the behaviour of the momentum portfolio on the basis of an expected return mechanism. σ_t is aggregate volatility with time-varying economic uncertainty. It implies risk transfer in the future that can acquire risk compensation in the long-term process.

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \quad 3.2.12$$

Equation 3.2.12 describes the consumption growth path. g_{t+1} denotes the growth rate of log aggregate consumption, which is a logarithm form. μ is a constant, and $\mu + x_t$ is the conditional expectation of consumption growth. Here, x_t, as a persistently varying component, captures the long-run risk in the expected consumption growth. η_{t+1} represents purely transitory shocks, with a normal distribution $N(0,1)$ and i.i.d. shock.

$$g_{i,t+1} = \mu_i + \phi_i x_t + \varphi_i \sigma_t u_{i,t+1} + \varphi_{i,m} \sigma_t v_{t+1} \quad 3.2.13$$

Equation 3.2.13 illustrates the dividend growth path in the equity market, which is linked to the aggregate consumption decisions of economic agents. This is because markets can provide risk compensation to owners of assets that can be utilised to finance consumption. $g_{i,t+1}$ denotes the dividend growth rate of the infinitely lived security i. μ_i, as a constant, estimates a security's average tendency to dividend growth over time. ϕ_i is the leverage parameter of security i, and the absolute value of ϕ_i is generally more than one. This implies that dividend and consumption growth rates can share the common component x_t that is persistently predictable, adjusted by the leverage parameter ϕ_i, although their correlation is imperfect. Abel (1999) also explained that ϕ_i, as a leverage ratio on the expected consumption path implies that a firm's profits are more sensitive to fluctuating economic uncertainty. Furthermore, the two scaling parameters φ_i and $\varphi_{i,m}$ are non-negative (and typically greater than one), which can calibrate the volatility of dividend growth relative to aggregate economic volatility. In addition, two shocks $u_{i,t+1}$ and v_{t+1} are normally distributed $N(0,1)$ and i.i.d. shocks. More precisely, the independent shock $u_{i,t+1}$ can capture purely transitory and idiosyncratic shocks to the dividend growth process of the security i. v_t denotes market uncertainty, which can be idiosyncratic at the market level. This means that this shock is independent and uncorrelated with e_{t+1} and η_{t+1}, but incorporates a cross-sectional correlation across securities.

$$\sigma_{t+1}^2 = \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \qquad 3.2.14$$

Equation 3.2.14 shows the process of aggregate volatility with fluctuating economic uncertainty. σ_{t+1} denotes the conditional variance of the time-varying economic uncertainty, which is the AR(1) process with parameter v_1 that can control the permanence of volatility shocks since the fluctuation of economic uncertainty is a long-term process. σ^2 denotes the unconditional mean. It implies that the current economic uncertainty has an internal relationship between the constant value and past economic volatility. w_{t+1} indicates the shock to consumption volatility, which is a normally distributed $N(0,1)$ and i.i.d. shock.

3.2.4 Solution for the model

According to the solution for my model in Appendix B, the return of security i in excess of its prior period expectation can be expressed as,

$$\begin{aligned} r_{i,t} - E_{t-1}[r_{i,t}] & \\ &\approx \beta_{i,e}\sigma_{t-1}e_t + \varphi_i\sigma_{t-1}u_{i,t} + \varphi_{i,m}\sigma_{t-1}v_t \\ &+ \beta_{i,w}\sigma_w w_t \end{aligned} \qquad 3.2.15$$

$A_{i,1}$, $\beta_{i,e}$ and $\beta_{i,w}$ can be solved by the model solution in Appendix B. Four different sources of shocks can be found - long-run consumption expectations, firm-specific transitory volatility, idiosyncratic risk at the market level, and aggregate consumption volatility.

Further, the risk premia on asset i can be determined by the conditional covariance between the return on asset i and the innovation in the pricing kernel, as below,

$$E_t(r_{i,t+1} - r_{f,t}) + \frac{1}{2} var_t(r_{i,t+1}) = -cov_t[m_{t+1} - E_t(m_{t+1}), r_{i,t+1} - E_t(r_{i,t+1})] \tag{3.2.16}$$

$$= \beta_{i,e}\lambda_{m,e}\sigma_t^2 + \beta_{i,w}\lambda_{m,w}\sigma_w^2$$

Where $r_{f,t}$ denotes the risk-free rate in time t. $\beta_{i,e}$, $\lambda_{m,e}$, $\beta_{i,w}$ and $\lambda_{m,w}$ are solved by the model solution in Appendix B. Obviously, the equity premium can be determined by the two sources of systematic risk - risk in expected consumption growth, and risk in consumption volatility. Further, compensation for stochastic volatility risk exposure depends on $\lambda_{m,e}$. The risk premium on each security is time-varying in the economy because of the fluctuation of σ_t. Moreover, $\beta_{i,e} = \kappa_{i,1}A_{i,1}\varphi_e$, denotes the consumption beta for each security i. And $\kappa_{i,1}$ is approximately constant, following Bansal and Yaron (2004). And there is a fixed correlation between consumption beta and leverage parameter ϕ_i. Further, due to the assumption of time-variance, the risk premium on risk-free assets is time-varied as well.

3.2.5 Measurement of long-run risk component

Importantly, x_T, as a state variable, represents the time-series data of the conditional mean of real aggregate per capita consumption growth (only the nondurable data and service consumption data). Due to the assumption that it is known by the representative agent, it is difficult to observe by economists. One main reason for this is that economists fail to obtain all the information available to representative agents when they make consumption decisions. However, e_t, as the innovation to x_T, can be obtained by the autoregressive model. Thus, I estimated the x_T as $\hat{x}_T$ under the homogenous assumption, which implies that the aggregate consumption rate volatility is constant ($\sigma_t = \sigma$). According to Bansal, Dittmar and Lundblad

(2005), $\hat{x}_T$ can be estimated as below:

$$\hat{x}_{T-1} = \frac{1}{8}\sum_{k=1}^{8}(g_{T-k} - \frac{1}{T}\sum_{T=1}^{T} g_T) \tag{3.2.17}$$

Where T denotes the length of the whole sample. Obviously, the estimated value, $\hat{x}_T$ used the trailing eight quarters moving average of the aggregate consumption growth. Thus, following Zurek (2007), I suppose that the estimated value is consistent with the actual value ($\hat{x}_T = x_T$). This implies that the true value of long-run risk components with constant aggregate consumption growth volatility can be measured by the estimation, $\hat{x}_T$, which leads to any measurement errors being ignored. Thus, the innovation to $\hat{x}_T$, $\hat{e}_T$ can be estimated as below if the time-varying economic fluctuation is constant:

$$\hat{\varepsilon}_T = \hat{x}_T - \hat{\rho}\hat{x}_{T-1} \tag{3.2.18}$$

Where x_T can be estimated as the AR(1) process with the autocorrelation coefficient ρ.

$$\hat{e}_T = \frac{(\hat{x}_T - \hat{\rho}\hat{x}_{T-1})}{\sigma_{T-1}} \tag{3.2.19}$$

Where $\hat{e}_T$ can be estimated through using the stochastic volatility model[10]. I assume the aggregate economic fluctuation, followed by the stochastic volatility (Bansal and Yaran, 2004). This is because the expected realised level of consumption volatility can be forecast by the current price/dividend ratio[11]. Consumption volatility without time-variation means a zero-slope coefficient on the price-dividend ratio. However, in fact, Bansal, Khatchatrian and Yaron (2005) and Bansal and Yaran (2004) have evidenced that information about the persistent volatility of economic uncertainty is incorporated into asset prices.

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[10] I used the "stochvol" package in R that adopted the Bayesian estimation of stochastic volatility using the Markov chain Monte Carlo (MCMC) approach, which can directly obtain the $\hat{e}_T$.

[11] The approximation solution for the price-dividend ratio is $z_{i,t} = A_{i,0} + A_{i,1}x_t + A_{i,2}\sigma_t^2$.

3.2.6 Calibration

The main aim of the calibration of the present model is to develop its powers of theoretical prediction, and to examine and interpret the primary empirical forecasts. In this section I will explain and describe the reasons for my calibration, based on the previous research. Moreover, it also contributes to providing a reasonable explanation for the long-run risk cross-sectional economy. More specifically, I built several fundamental parameters to match the annualised aggregate consumption growth, risk-free rate, and market return. Further, the constraint of the mean values of the cross-sectional parameters $(\mu_i, \phi_i, \varphi_i, \varphi_{i,m})$ must be considered when I match equity market moments. Initially, I calibrated the investment preference and economic environment parameters, including risk aversion (γ), I.E.S. (ψ) and the time discount factor (δ), based on the previous literature. As per Bansal, Kiku and Yaron (2007), γ and ψ were set at 10 and 2 respectively. The discount factor δ and ρ were set to 0.9988 and 0.9827, following Zurek (2007).

The fluctuating economic uncertainty process was calibrated by Bansal and Yaron (2004), including the unconditional mean of volatility (σ), and parameter σ_w, which were set to 0.0078 and 0.23e-5. Further, based on Zurek (2007), I calibrated the rest of the parameters of the volatility of consumption growth like μ=0.001541, $\varphi_e = 0.034404$ and $v = 0.987$.

3.2.7 Simulation design

The simulation design depends on the consumption growth path of equation 3.2.12 and the dividend growth path of equation 3.2.13. Initially, based on the calibration and model solution in the Appendix B, the parameters of consumption growth path can be obtained such as A_0, A_1, A_2, κ_0 and κ_1.[12] According to the equation 3.2.13, each individual security can be determined by the value of the parameters $(\mu_i, \phi_i, \varphi_i, \varphi_{i,m})$. Further, following the assumption made by Zurek (2007), due to infinitely lived securities with time-varying long-run risk exposure φ_i, they can be seen as representative securities in various phases of the

[12] The value of κ_0 and κ_1 are required by the function of $\bar{z} = A_0(\bar{z})$, following Zurek (2007).

life cycle. Further, there are changes in identification when the securities move from one phase into others. I assumed the existence of five basic types of securities in the market. The characteristics of each fundamental type were based on the calibrated cross-sectional coefficients (i.e. $\mu_i, \phi_i, \varphi_i, \varphi_{i,m}$). Initially, all securities were different even within the same type via specific realisations of $u_{i,t}$ shocks. Calibrated average dividend growth (μ_i) followed a monotonic increase trend from type1 to type5. Thus, type1 had a small average dividend growth or even a negative growth, and type5 had a large average dividend growth. The value of the long-run risk consumption beta (ϕ_i) followed by the monotonic increase from type1 to type5, matched the same pattern of unconditional dividend growth. Further, φ_i followed a U-shape pattern from type1 to type5. $\varphi_{i,m}$ also followed a monotonic increase from type1 to type5. These attempts to match the empirical evidence of a U-shaped pattern in regression residuals followed the method used by Zurek (2007).

On the other hand, the non-linearised equation of the model means the price/dividend ratio $z_{i,t}$ depends on the intercept $A_{i,0}$, the coefficient of x_t, $A_{i,1}$, and the coefficient of σ_t^2, $A_{i,2}$, which is solved in Appendix B. The value of $\kappa_{i,1}$ is approximately one.[13] Following the dividend growth path in equation 3.13, I simulated the dividend growth in each security with five securities types, given the shock $u_{i,t}$ and v_t. Further, the price/dividend ratio were simulated through the process of x_t and σ_t^2. Finally, the monthly simulated series of total returns with dividend and capital gains were generated for each security, which can be expressed as follows:

$$sr_{d,i,t} = \exp\big(\log\big(1 + \exp(z_{i,t})\big) - z_{i,-1} + g_{i,t+1}\big) - 1 \qquad 3.2.20$$

$$sr_{x,i,t} = \exp\big(z_{i,t} - z_{i,-1} + g_{i,t+1}\big) - 1$$

Where $sr_{d,i,t}$ denotes the total return with dividend, $sr_{x,i,t}$ denotes capital gains, which means returns without dividend. Moreover, total return was used to produce a momentum portfolio that ranks all the securities from the lowest to the highest return, and allocates them

[13] The value of $A_{i,0}$, $A_{i,1}$, $A_{i,2}$, $\kappa_{i,0}$ and $\kappa_{i,1}$ are required by the function of $\bar{z}_i = A_{i,0}(\bar{z}_i)$.

into the five different momentum portfolios.

3.3 Results and findings

3.3.1 Findings and discussion of the empirical evidence

3.3.1.1 Results for the momentum portfolios

In this part, I report the empirical evidence arising from the different momentum trading strategies. Further, I examine the arithmetic mean returns, logarithmic dividend growth, and logarithmic valuation ratios based on the three dimensions of mean, standard deviation, and autocorrelation. According to Tables C1.1, C1.2, C1.3, and C1.4, there are many similar findings under the different momentum trading strategies. For example, under momentum trading strategy 6*3 (Table C1.1), there is a monotonic increase in returns and dividends from the loser portfolio to the winner portfolio. More specifically, the arithmetic average return in the loser portfolio (P1) is 1.16%, and in the winner portfolio (P5) it is 2.84% in the full sample. This finding implies a significant momentum effect due to positive returns on the momentum portfolio (1.68%) from 1970 to 2018, which makes it substantially profitable. Moreover, the standard deviation of the return from P1 to P5 follows a U-shape pattern, as well as lower autocorrelations in each portfolio. This means there is high volatility in both the loser portfolio and the winner portfolio. In addition, there is a monotonic rise in dividend growth from close to zero in P1 to 0.018 in P5. This can be explained with reference to Zurek (2007) in that although the constant dividend growth rate in the winner portfolio is claimed over time, the extent of the dividend rises due to the increasing amount of investment by capital gains. Thus, there is a higher average dividend growth rate in the winner portfolio than in the loser portfolio. On the other hand, the level of dividend may be lower in the loser portfolio due to a decreasing amount of investment by capital gains even given the fixed growth rate. This is a potential explanation for why there is a lower dividend growth rate in the loser portfolio. Furthermore, there is a similar U-shape pattern in the standard deviation of dividend growth from P1 to P5. The autocorrelation of dividend growth is high since the seasonally adjusted dividend uses the trailing 12-month moving average. As discussed in Chapter 4.3, this approach was taken to eliminate the seasonal effect. However, there is a potential problem that leads to the extremely smooth transition of a variable. Additionally, the mean and standard deviation of log valuation ratio also illustrated a U-shaped mode, which implies the highest value in the loser portfolio and the winner portfolio. A reasonable interpretation of this is the low dividend growth in the loser portfolio and the high capital gains in the winner portfolio. Further, there is quite a high autocorrelation in the valuation ratio.

Tables C1.2, C1.3, and C1.4 show quite similar results compared with Table C1.1. Initially, the arithmetic mean returns maintain the monotonic increases in these four investment strategies from P1 to P5. Additionally, compared with table C1.1 and C1.2, I found that given the constant ranking period, portfolios with longer holding periods have higher returns, which is consistent with tables C1.3 and C1.4. However, in the long run, comparing the 2.48%[14] of momentum return for the three months holding period with the 3.83%[15] of momentum for the twelve months holding period, it seems that the extent of the momentum effect is reduced with the longer formation period. In other words, the strength of the momentum strategy has a downward trend, which is consistent with the long reversals. Simultaneously, there is the same U-shaped mode in the standard deviation of returns in each of the momentum investment strategies. On the other hand, the log dividend growth in each strategy has a significant monotonic increase as well. Likewise, there is a higher dividend growth by the trading strategy with a longer holding period. For example, averagely dividend growth of each portfolio in momentum strategy 6*6 is quite higher than that of momentum strategy 6*3. Likewise, the U-shaped pattern is illustrated by the standard deviation of dividend growth. At the same time, the valuation ratio decreases when a momentum investment strategy has a longer holding period. The main reason for this is the decreasing

[14] In momentum strategy 12*3, the momentum return equals the return on winner portfolio (P5) minus the return on loser portfolio (P1), which is 3.23%-0.75%=2.48%.

[15] In momentum strategy 12*6, the momentum return equals the return on winner portfolio (P5) minus the return on loser portfolio (P1), which is 6.31%-2.48%=3.83%.

strength of the momentum effect.

3.3.1.2 Results for time-varying risk exposure

In equation 3.13, I estimate the unconditional dividend growth processes for different portfolios p. This is in order to examine my hypothesis that the winner (loser) portfolio obtaining a higher profit depends on higher (lower) risk exposure. Simply, it shows a higher return at a higher risk. This is mainly because, with a longer ranking period, stocks with low expected returns (holding a low beta) are unlikely to perform as well as stocks with high expected returns (holding a high beta). As a result, the winner portfolio will hold fewer stocks with a low beta due to lower expectations. Thus, an unconditional portfolio beta can be estimated to use the average security beta in each portfolio. Therefore, the unconditional dividend growth processes can be shown as,

$$g_{T,p} = \mu_p + \phi_p x_{T-1} + error_{T,p} \quad 3.3.1$$

Where ϕ_p is the unconditional average beta for each portfolio p ($p = 1,2,3,4,5$). T denotes the quarterly data with non-overlapping generation. I reported the results of the regression, which are described in Table 2.1, Table 2.2, Table 2.3, and Table 2.4. Here, I will explain the results of Table 2.1 as an example. Initially, ϕ_p, as the slope of this linear regression, demonstrates a monotonic rise from -6.103 in P1 to 8.744 in P5. The main reason for the monotonic increase of the beta is the momentum selection mechanism, which means that the winner (loser) portfolio has a high (low) average beta due to the high (low) expected return. Further, there is an increase in the beta coefficient when the security moves from the loser portfolio to the winner portfolio. Moreover, the intercept also shows a monotonic increase from P1 to P5. Moreover, there is a U-shaped pattern of average beta's standard deviation, which means significant fluctuations in the loser and winner portfolios. A reasonable explanation for this is the comparatively small observation in my full sample because of adopting quarterly data. The adjusted R-squared is quite small, which implies that the unexplained variance (non-model risk) plays an essential role. Eventually, there is a similar U-shaped mode regarding the regression residual from P1 to P5. These findings are consistent with Zurek (2007).

Further, tables C2.2, C2.3, and C2.4 expand the results of my regression. Similarly, there is a monotonic increase in intercept and average beta (consumption beta) from P1 to P5 in each momentum strategy, and a U-shaped mode of standard deviation of intercept and average beta and residual error. Furthermore, the sensitivity of the average beta is higher, and the volatility of the consumption beta and intercept is intense when the holding period is longer. A reasonable explanation for this is that there is an increase in risk or uncertainty when the holding period is longer. Moreover, the winner portfolio with the longer ranking period has a low beta, which means that the momentum selection mechanism is more effective in the short term and has a weaker performance in the long run. Additionally, there is a much larger residual error in each portfolio when the holding period is longer.

I regressed the time-varying equation with the homoscedasticity of shocks to test the impact of the long-run risk consumption beta, which can be shown as below:

$$g_{T,p} = \mu_{L,p} + \phi_{1L,p} x_{T-1} + \phi_{2L,p}(x_{T-1} * \hat{\varepsilon}_{T-1}) + error_{L,T,p} \qquad 3.3.2$$

Where $\mu_{L,p}$ is the intercept, $\phi_{1L,}$ is the average long-run risk consumption beta, and $\phi_{2L,p}$ is the conditional impact between consumption growth x_{T-1} and the realisation of long-run risk component $\hat{\varepsilon}_{T-1}$. According to Zurek (2007), risk exposures for momentum portfolios rely on the cumulative realisation of the long-run risk component $\hat{\varepsilon}_{T-1}$ in the ranking period. The estimated value, $\hat{\varepsilon}_{T-1}$, depends on the duration of the ranking period. An example of a momentum strategy with a 6 month (2 quarter) formation period can be followed as,

$$\phi_p = \phi_{1L,p} + \phi_{2L,p} * (\varepsilon_{T-1} + \varepsilon_{T-2}) \qquad 3.3.3$$

Thus, $\phi_{2L,p}$ can measure the realisation of long-run risk components in each portfolio. Theoretically, the winner portfolio with the positive $\phi_{2L,p}$, holds the high portfolio beta and the negative $\phi_{2L,p}$, shows the low portfolio beta. In contrast, the loser portfolio with the negative $\phi_{2L,p}$, holds the high portfolio beta and the positive $\phi_{2L,p}$, shows the low portfolio beta. The parameter $\phi_{1L,p}$ denotes the average beta, which is similar to the ϕ_p in equation 3.3.3.

Table C3 reports the consumption beta which is conditional on the long-run risk component in different momentum strategies. According to equation 3.3.2, the regression shows the dividend growth rate in each portfolio's effect on the aggregate consumption growth rate with time variation, and its interaction with the long-run risk component. In table C3.1, there is a monotonic increase in average beta from P1 to P5. To a large extent, this maintains the consistency of the average beta (see equation 3.3.2) due to the approximate value. Besides, the standard deviation of the average beta is similar to the previous results of the consumption beta in Table C2.1. More importantly, the long-run risk beta also exhibited reliable explanatory power for a different portfolio. Likewise, a monotonic relation was observed in relation to the long-run risk beta from the loser portfolio to the winner portfolio, as well as a U-sharped pattern in the standard deviation. Further, the adjusted R-squared dramatically increased compared with equation 3.3.3, which implies that the new equation with conditional information has a significant explanatory impact. To a large extent, the predictability of the model is also supported by the consumption beta with time-variance and constant aggregate volatility. Further, when combined with table C3.2 with the same ranking period, it can be seen that the levels of average beta and long-run risk beta are higher than in table C3.1. This means that the winner with the longer holding period expects higher momentum returns due to the higher uncertainty in the future. Similarly, portfolios with longer formation periods have a low long-run risk beta. Furthermore, table C3.3 demonstrates that the level of consumption beta and long-run risk beta are lower in the longer formation period than in table C3.1. This means that the momentum portfolio is expected to have a high value in the short-run since the short formation period captures more volatility, which is consistent when comparing table C3.2 for momentum trading strategy 6*6 with table C3.4 for momentum trading strategy 12*6.

Table C4.1 which follows equation 3.2.18 shows a consumption beta conditional on the long-run risk component with time-varying aggregate economic fluctuations in various momentum strategies. More specifically, I added the aggregate fluctuations in economic volatility with time-variance. Thus, the regression with time-varying aggregate economic fluctuation can be expressed as:

$$g_{T,p} = \mu_{SW,p} + \phi_{SW,p}x_{T-1} + \phi_{SW,p}(x_{T-1} * \hat{e}_{T-1}) + error_{SW,T,p} \quad 3.3.4$$

$$\hat{e}_{T-1} = \frac{(\hat{x}_{T-1} - \hat{\rho}\hat{x}_{T-2})}{\sigma_{T-2}} = \frac{\hat{\varepsilon}_{T-1}}{\sigma_{T-2}}$$

$$\sigma_{T-2}^2 = \sigma^2 + v_1(\sigma_{T-3}^2 - \sigma^2) + \sigma_w w_{t-2}$$

Where, I assume the aggregate economic fluctuation followed by the stochastic volatility (Bansal and Yaran, 2004). As a result, the new equation with conditional impact has strong explanatory power in relation to the portfolio level long-run risk consumption beta, with fluctuating economic uncertainty. More specifically, there is a similar monotonic increase in average beta and consumption beta from P1 to P5, as well as the same U-shaped standard deviation pattern. Further, the value of average beta is quite similar between regressions with constant aggregate economic volatility and with fluctuating economic uncertainty. However, in terms of the long-run rusk beta, table C4.1 shows a higher level than table C3.1. This means that the long-run risk beta with fluctuating economic uncertainty has a higher requirement for expected momentum returns when the momentum selection mechanism discriminates between stocks with high or low expected returns. Moreover, the adjusted R-squared is basically consistent with constant consumption volatility regression, and is sometimes better than that. Although the residual standard errors are slightly higher than the regression with constant consumption volatility, these results appear in the forecast of the model regarding consumption betas with time-varying consumption volatility at the portfolio level. Further, tables C4.2, C4.3, and C4.4 expand the findings reported in Table C4.1. There are some common findings regarding the different durations of the formation and holding periods. More precisely, there is a higher level of average beta and long-run risk beta with longer holding periods. Again, the expected return on the winner portfolio is high with the longer holding period, because of the increased uncertainty with a longer duration. Similarly, the portfolio level with the longer formation period has a lower long-run risk beta. In conclusion, the results of the model with time-varying aggregate economic fluctuations are typically consistent with the model with constant consumption volatility.

In Tables C5.1, C5.2, C5.3, and C5.4, I report the predictability of momentum investment strategies based on the positive and negative realisation of the long-run risk component with time-varying economic fluctuations in the ranking period. For example, in table C5.1, the average momentum return following a positive realisation of the long-run risk component is 2.6%[16], and the negative realisation of the long-run risk component is 0.5%. Further, following the positive realisation of the long-run risk component, the standard deviation is lower. This is because the momentum investment strategy is determined to take a long position in the winner portfolio and a short position in the loser portfolio, which means a fairly high level of idiosyncratic (unsystematic) risk. Thus, following the negative realisation of the long-run risk factor, a momentum strategy that is a long winner portfolio and short loser portfolio, holds a portfolio with a lower average long-run risk beta, leading to lower systematic risk. Thus, the source of the high volatility following the negative realisation of long-run risk is idiosyncratic risk (unsystematic risk). Therefore, when the realisation is negative, there is a rising beta in the winner portfolio and a falling beta in the loser portfolio, which narrows the beta gap, and further lowers long-run consumption risk. Furthermore, the t-value shows statistical significance based on the null hypothesis of mean zero return at the 1% significance level following the positive realisation of the long-run risk component, but fails following the positive realisation of the long-run risk component. These findings are consistent with tables C5.2, C5.3 and C5.4 for the different momentum trading strategies.

3.3.2 Findings and discussion of the model solution and simulation results

Table C6.1 concludes the calibration in each type of security. Following table C6.1, the results of the model solution and expected return implications in tables C6.2 and C6.3 are

[16] The momentum return equals the return on the winner portfolio (P5) minus the return on the loser portfolio (P1).

shown. $A_{i,0}$ has a U-shaped pattern and there is a monotonic increase in $A_{i,1}$ and a monotonic decrease in $A_{i,2}$ from Type1 to Type5. Moreover, expected returns also have a monotonic increase from Type1 to Type5, as does the risk premium, which is consistent with the empirical evidence. In addition, the valuation ratio follows a U-shaped pattern.

Table C7.1 shows simulated return, dividend, and valuation results for momentum trading strategy 6*3. Given its different calibration for the different type of securities, the model can explain the comparative observed momentum profits. This is because there is a reasonable and acceptable difference in terms of expected returns for different individual securities. Due to the universality and size of the momentum effects, there is no single source of risk that accounts for this effect. Hence, in my model, I placed the overall five risks to explain the momentum effects, which is a reasonable and feasible approach. As a result, there is a monotonic increase in average return, which generates a significant momentum effect. In addition, the model can match the U-shaped standard deviation and negative autocorrelation in the loser portfolio and the positive autocorrelation in the winner portfolio, and the autocorrelation is insignificant. Further, there is a rising dividend growth rate from P1 to P5, and the U-shaped standard deviation is similar to the empirical evidence. In addition, the approximate magnitude of the valuation ratio is a little more than the empirical evidence on the same momentum strategy, as is the U-shaped standard deviation for the valuation ratio. On the other hand, from the results of the simulation, based on the different momentum strategies in Tables 7.2, 7.3 and 7.4 it can be seen that given the constant ranking period, portfolios with longer holding periods have higher returns and a higher standard deviation. At the same time, the standard deviation of the return is higher when the holding period is longer. Then, the log dividend growth in each strategy also has a significant monotonic increase in the different simulated momentum strategies. Likewise, there is higher dividend growth in trading strategies with longer holding periods. On the other hand, given the same holding period, winner and loser portfolio returns are higher when the formation period is longer.

Moreover, the magnitudes of simulated momentum effects reduced with longer formation periods, and the same U-shaped mode of standard deviation of returns is present for each of the simulated momentum investment strategies, which is consistent with the empirical evidence.

Chapter 4.0 Conclusion

Momentum effects, as a common phenomenon in the financial market, have been widely and systematically explored. The present thesis can be divided into two dimensions: it has explored the dynamics of momentum effects in the two-regime switching model, and the momentum effect in the long-run risk model. As such, the thesis makes many contributions based on the work presented in both main chapters. In the first, I examined the momentum effects in the UK and US markets from 1980 to 2018. Further, I compared the UK and US markets, and found that the results of momentum returns in the UK market are more significant than in the US market in the total of 16 momentum trading strategies. Further, performance dependability in the UK market is higher than in the US market in all momentum trading strategies. In addition, compared to the 16 different momentum trading strategies, I found potential evidence of the reversal effect in the long run. When the holding period is 12 months, there is a reduction in momentum returns, compared with the shorter holding period. Furthermore, the results of the Bayesian estimation verified the relationship between momentum return and four variables (i.e. domestic market volatility, ranking period return and foreign market volatility). Importantly, I found that a transition threshold parameter does exist to switch between the momentum regime and the reversal regime. When market volatility is above or below the threshold parameter, changes occur in the regime. Particularly, the reasonable cause that the study did not use the VIX to measure the market volatility is that the VIX index is asymmetrical for market volatility, meaning that it responds much more when market price decrease than when they increase. Thus, the study used the realised market volatility. In addition, I also developed Cao's threshold model. Through adding foreign market volatility, I found that momentum returns in the UK and US markets have a significant positive effect on the average momentum return and foreign market risk, as well as a negative effect on the return in the ranking period and domestic market risk in the momentum regime. However, the results for the US market are insignificant when compared with the UK market in the reversal regime. This indirectly indicates that the US market has a significant impact on the UK market.

In the second section, based on the long-run risk model by Basal and Yaron (2004), I explained the reason for the momentum portfolios' returns with fluctuating economic uncertainty. Further, I developed the cross-sectional model at the level of individual security by Zurek (2007). I added the assumption of fluctuating economic volatility and hence allowed for economic uncertainty to have an effect on the aggregate consumption and dividend growth of individual securities. Theoretically, my model identifies a more significant explanation for momentum returns. Specifically, I found that with fluctuating economic uncertainty, there is a monotonic increase between conditional consumption risk and momentum returns from the loser portfolio to the winner portfolio, a finding which is consistent with Zurek (2007). However, consumption beta is higher, which means there are more sensitivities between the growth dividend of individual securities and the consumption growth path with fluctuating economic uncertainty. Further, I reported that all five momentum portfolios have time variations in long-run risk and expected return, which is consistent with the model-implied component structure of momentum returns. The time-variance may be driven by the realisation of long-run risk factors and the aggregate consumption growth volatility in the ranking period. Furthermore, I operated the cross-sectional model at the level of individual security and then endogenously produced the return, dividend growth and valuation ratio at the portfolio level. Thus, my model matches portfolio return, dividend growth, and valuation ratio at the portfolio level in the simulation. Overall, my model is practical and realistic, and the results are close to the actual ones, thus explaining momentum returns effectively in each portfolio and matching aggregate consumption growth dynamics and equity premium well.

I suggest future research following on from the first chapter here. Firstly, the model could be tested using different financial data in different countries or financial markets. Thus, the robust power of my model could be verified in the future. Moreover, a weak significant effect in the reversal regime is observed here among several variables. Thus, future research could be based on identifying the correlation of factors in the reversal regime. Based on the long-run risk model of cross-sectional dynamics at the security level, there are several potential research directions. Currently, the long-run risk model only has empirical evidence for the US market. Thus, the model could also be applied to other stock markets, to test its robustness and power.

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Appendix

Appendix A

	3M	6M	9M	12M
3M	0.021	0.047	0.066	0.075
t-value	2.931	3.269	3.725	3.354
6M	0.025	0.055	0.072	0.067
t-value	2.609	2.244	2.394	1.804
9M	0.026	0.053	0.056	0.048
t-value	2.652	2.168	1.589	1.022
12M	0.019	0.032	0.028	0.013
t-value	1.814	1.200	0.756	0.268

Table A1.1- Different momentum portfolio returns in terms of holding returns in the UK market: full sample 1980-2018

	3M	6M	9M	12M
3M	0.009	0.019	0.039	0.040
t-value	2.041	1.978	2.855	2.164
6M	0.011	0.029	0.041	0.029
t-value	1.779	1.822	1.599	0.932
9M	0.014	0.025	0.027	0.013
t-value	1.882	1.258	0.884	0.366
12M	0.011	0.015	0.011	0.000
t-value	1.281	0.695	0.350	0.006

Table A1.2- Different momentum portfolio returns in terms of holding returns in the US market: full sample 1980-2018

These table report the returns of the different momentum strategies (winner minus loser portfolios' returns) in the US stock market (based on NYSE, AMEX and NASDQ) and the UK stock market (based on LSPD). Return is defined here as the geometric average monthly return as a percentage. The monthly sample was collected from 1980 to 2018. The momentum trading strategy J*K was implemented by ranking all stocks in descending order based on the past return from t-J to t-1. The first column shows the formation period, and the first row shows the holding period. The momentum portfolio (buy past winner and sell past loser) is held from t+1 to t+K, skipping month t. Thus, the formation and holding periods used four different horizons, such as 3, 6, 9 and 12. A total of 16 momentum trading strategies were adopted to compute the average monthly holding return. A two-tailed test was required for heteroscedasticity and autocorrelation of consistent standard error based on Newey-West with one lag. The t values that test the significance of momentum return are reported below for each momentum trading strategy, which corresponds to the significance level of critical value at 1% (2.576), 5% (1.96) and 10% (1.645).

	3M	6M	9M	12M
3M	0.675	0.693	0.701	0.674
6M	0.736	0.745	0.714	0.693
9M	0.755	0.759	0.711	0.660
12M	0.745	0.723	0.684	0.626

Table A2.1- Performance dependability of different momentum trading strategies in the UK market

	3M	6M	9M	12M
3M	0.593	0.604	0.622	0.632
6M	0.644	0.672	0.651	0.633
9M	0.705	0.691	0.651	0.638
12M	0.668	0.634	0.620	0.608

Table A2.2- Performance dependability of different momentum trading strategies in the US market

	3M	6M	9M	12M
3M_Loser	-0.016	-0.029	-0.028	-0.015
t-value	-0.358	-0.432	-0.712	-0.379
3M_Winner	0.100	0.096	0.089	0.086
t-value	3.729	2.376	3.443	3.902
6M_Loser	-0.011	-0.028	-0.024	-0.005
t-value	-0.215	-0.381	-0.495	-0.107
6M_Winner	0.130	0.121	0.108	0.094
t-value	5.031	2.527	4.004	3.613
9M_Loser	-0.010	-0.020	-0.011	0.007
t-value	-0.188	-0.270	-0.205	0.134
9M_Winner	0.134	0.124	0.100	0.084
t-value	5.538	2.599	3.737	3.210
12M_Loser	0.013	0.006	0.013	0.024
t-value	0.240	0.072	0.237	0.460
12M_Winner	0.128	0.106	0.083	0.070
t-value	5.827	2.111	3.099	2.798

Table A3.1- Annualised market-adjusted returns of winner and loser portfolios based on the different momentum trading strategies in the holding period in the UK market: full sample 1980-2018

This table reports the returns from different momentum strategies for the winner and loser portfolios in the US stock market (based on NYSE, AMEX and NASDQ) and the UK stock market (based on LSPD). Return is defined here as the geometric average monthly return as a percentage. The monthly sample was collected from 1979 to 2018. The momentum trading strategy J*K was implemented by ranking all stocks in descending order based on the past return from t-J to t-1. The first column shows the formation period, and the first row shows the holding period. The winner portfolio and loser portfolio (buy past winner and sell past loser) are held from t+1 to t+K, skipping month t. Thus, the formation and holding periods used four different horizons, 3, 6, 9 and 12. A total of 16 momentum trading strategies were adopted to compute the average monthly holding return. A two-tailed test was required for heteroscedasticity and autocorrelation of consistent standard error based on Newey-West with one lag. The t values that test the significance of momentum return are reported below for each momentum trading strategy, which corresponds to the significance level of critical value at 1% (2.576), 5% (1.96) and 10% (1.645).

	3M	6M	9M	12M

3M_Loser	0.045	0.035	0.019	0.025
t-value	2.103	1.274	0.978	1.341
3M_Winner	0.077	0.073	0.069	0.061
t-value	4.072	4.398	4.531	4.151
6M_Loser	0.053	0.037	0.028	0.039
t-value	1.874	1.008	1.144	1.746
6M_Winner	0.098	0.094	0.080	0.066
t-value	5.175	4.917	5.127	4.632
9M_Loser	0.049	0.046	0.044	0.054
t-value	1.472	1.109	1.692	2.247
9M_Winner	0.103	0.091	0.075	0.066
t-value	5.399	4.320	5.042	4.624
12M_Loser	0.062	0.060	0.057	0.062
t-value	1.818	1.412	1.986	2.341
12M_Winner	0.093	0.083	0.068	0.062
t-value	4.867	3.887	4.411	4.071

Table A3.2- Annualised market-adjusted returns of winner and loser portfolios based on the different momentum trading strategies in the holding period in the US market: full sample 1980-2018

This table reports the returns from different momentum strategies for winner and loser portfolios in the US stock market (based on NYSE, AMEX and NASDQ) and the UK stock market (based on LSPD). Return is defined here as the geometric average monthly return as a percentage. A monthly sample was collected from 1979 to 2018. The momentum trading strategy J*K was implemented by ranking all stocks in descending order based on the past return from t-J to t-1. The first column shows the formation period, and the first row shows the holding period. The winner portfolio and loser portfolio (buy past winner and sell past loser) are held from t+1 to t+K, skipping month t. Thus, the formation and holding periods used four different four horizons, 3, 6, 9 and 12. A total of 16 momentum trading strategies were adopted to compute the average monthly holding return. A two-tailed test was required for heteroscedasticity and autocorrelation of consistent standard error based on Newey-West with one lag. The t values that tested the significance of momentum return are reported below for each momentum trading strategy, which corresponds to the significance level of critical value at 1% (2.576), 5% (1.96) and 10% (1.645).

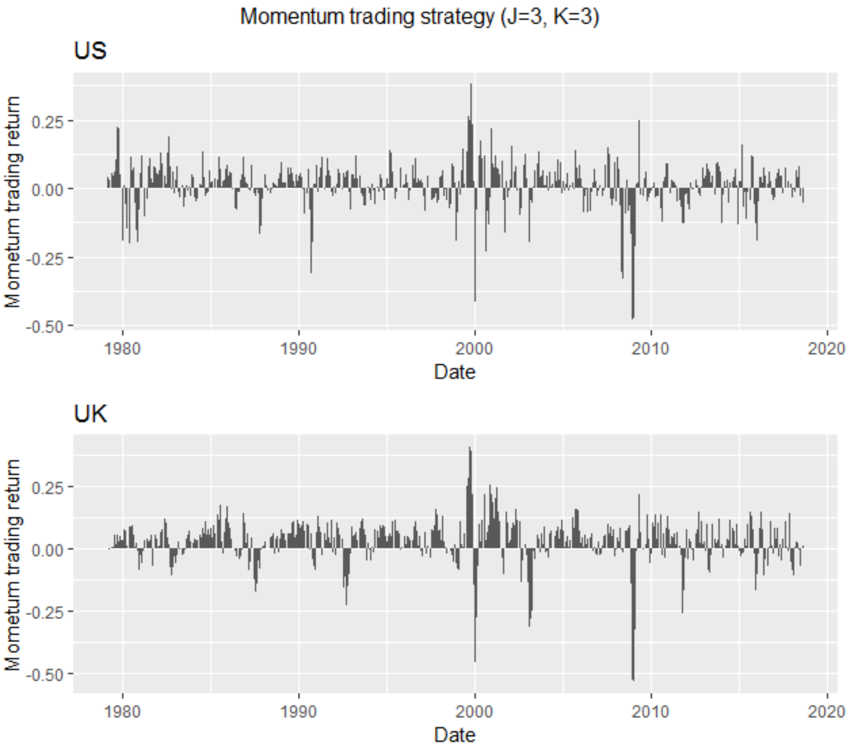

Figure A4.1- Profitability of the momentum investment strategies (3*3) in the US and UK market

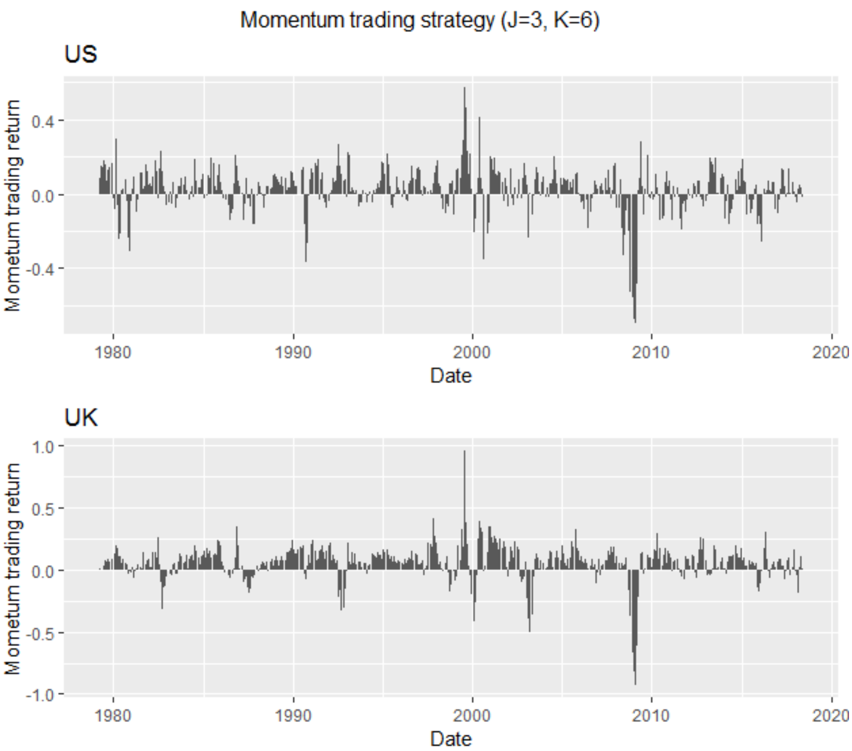

Figure A4.2- Profitability of the momentum investment strategies (3*6) in the US and UK market

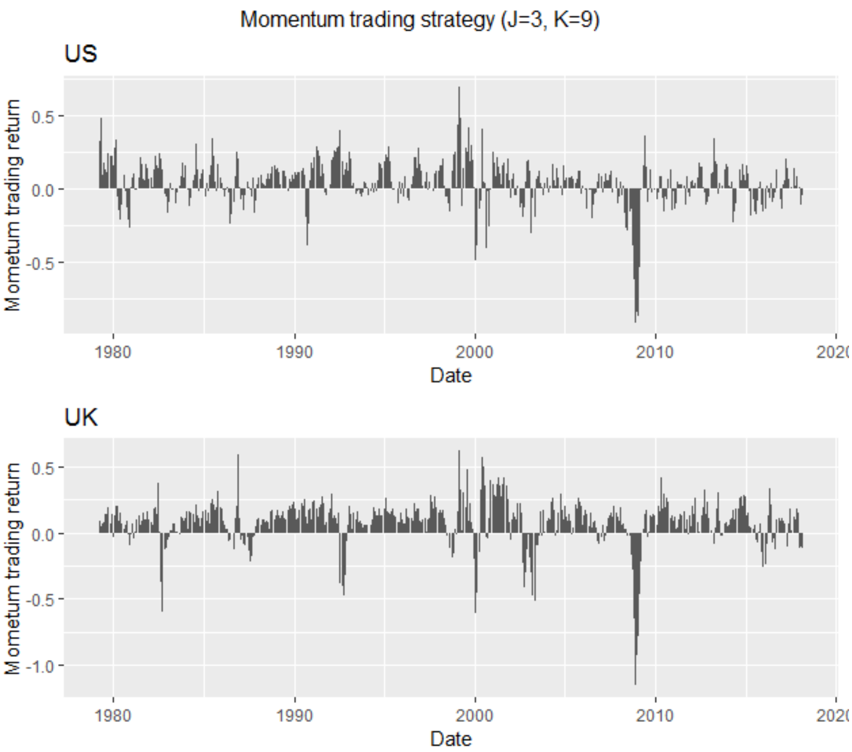

Figure A4.3- Profitability of the momentum investment strategies (3*9) in the US and UK market

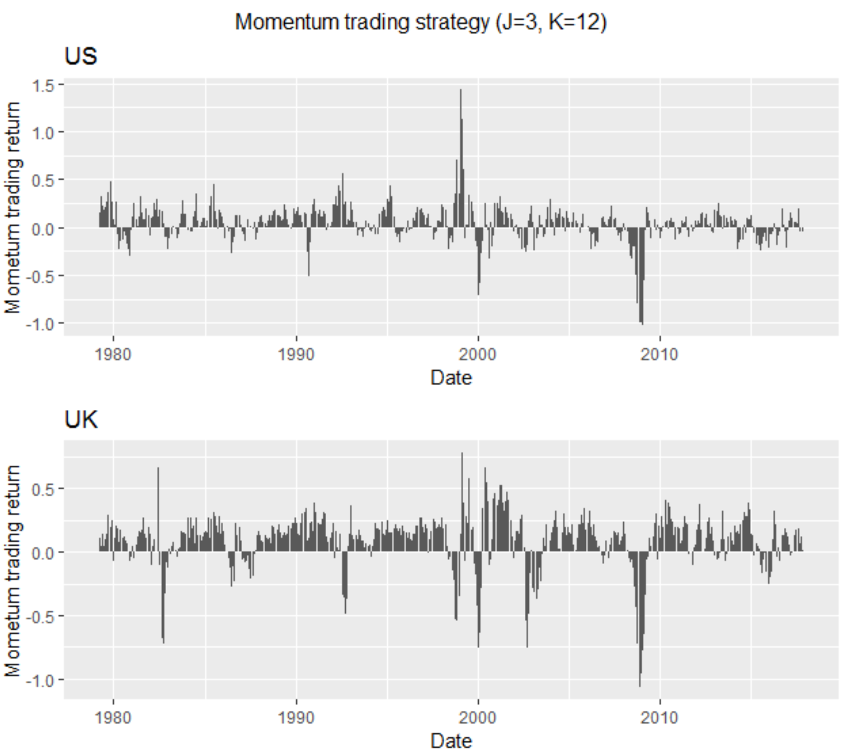

Figure A4.4- Profitability of the momentum investment strategies (3*12) in the US and UK market

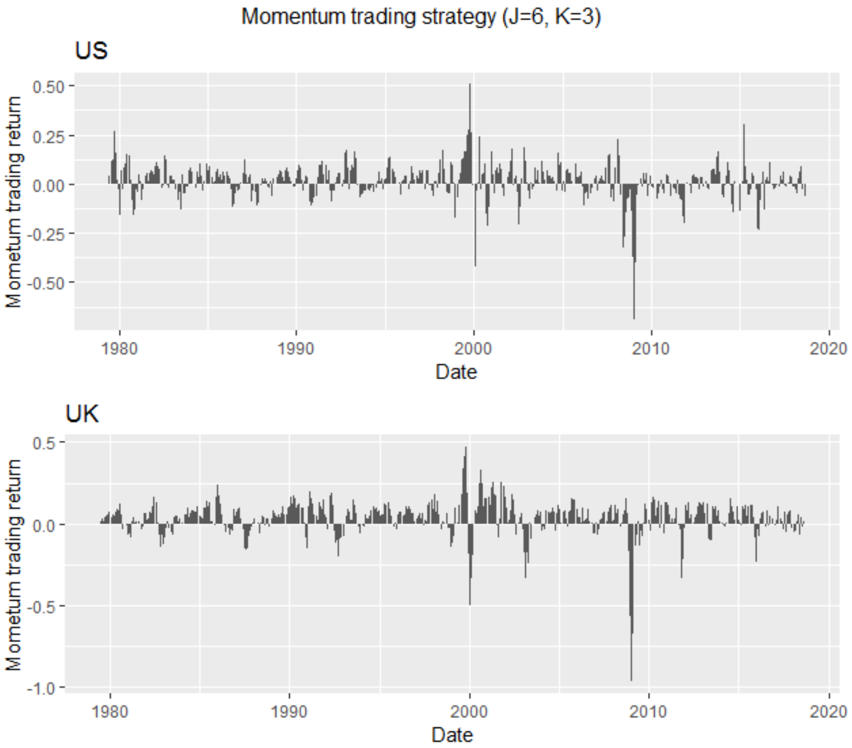

Figure A4.5- Profitability of the momentum investment strategies (6*3) in the US and UK market

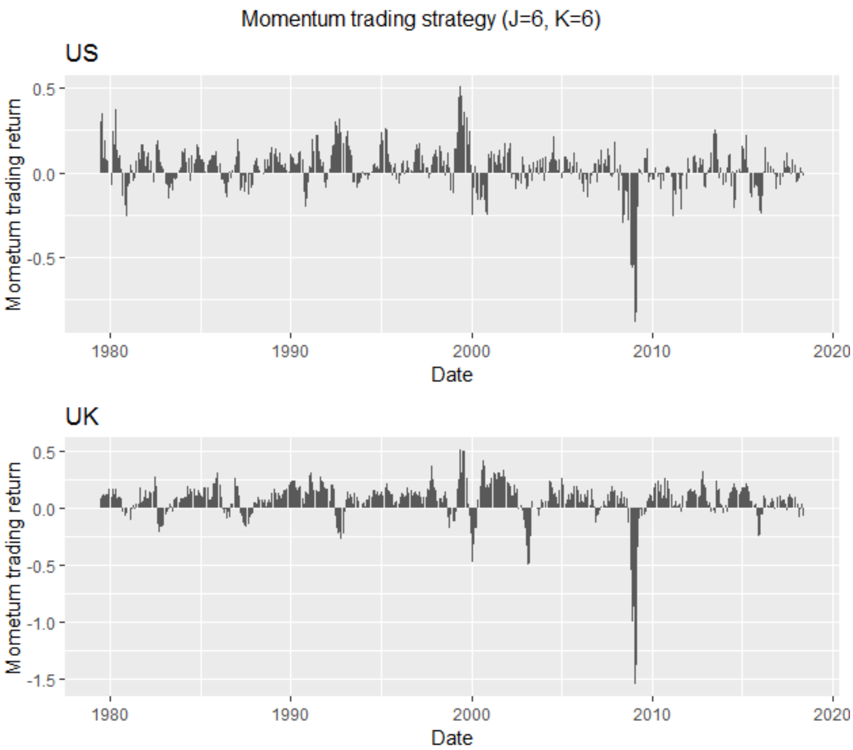

Figure A4.6- Profitability of the momentum investment strategies (6*6) in the US and UK market

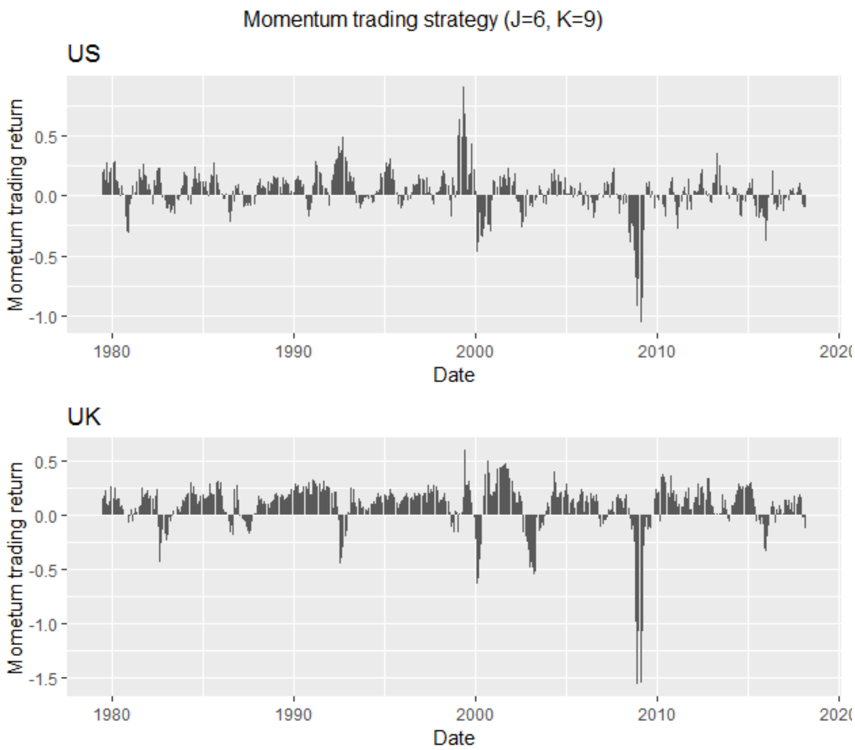

Figure A4.7- Profitability of the momentum investment strategies (6*9) in the US and UK market

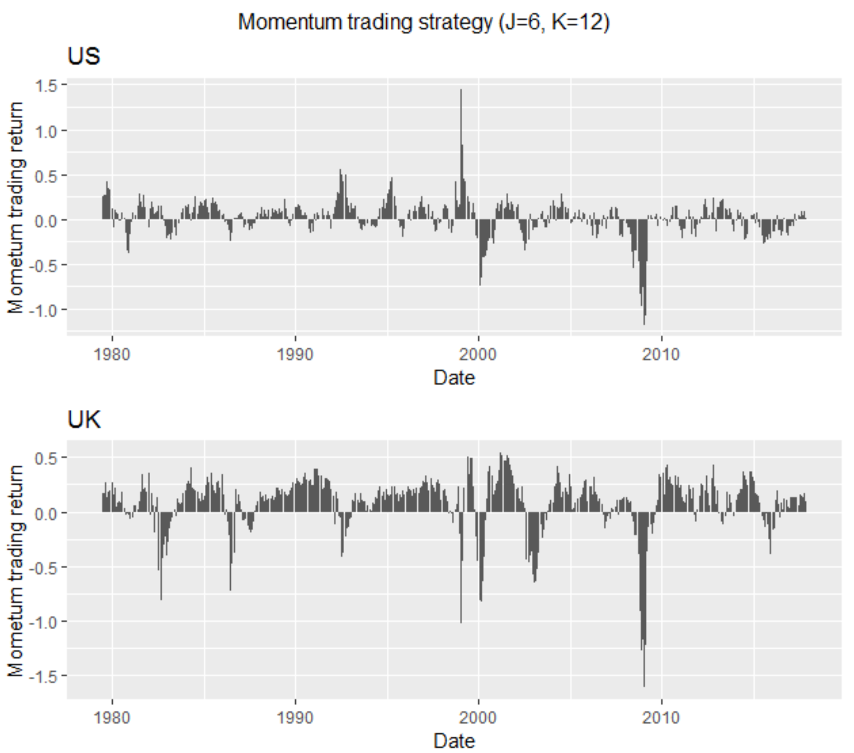

Figure A4.8- Profitability of the momentum investment strategies (6*12) in the US and UK market

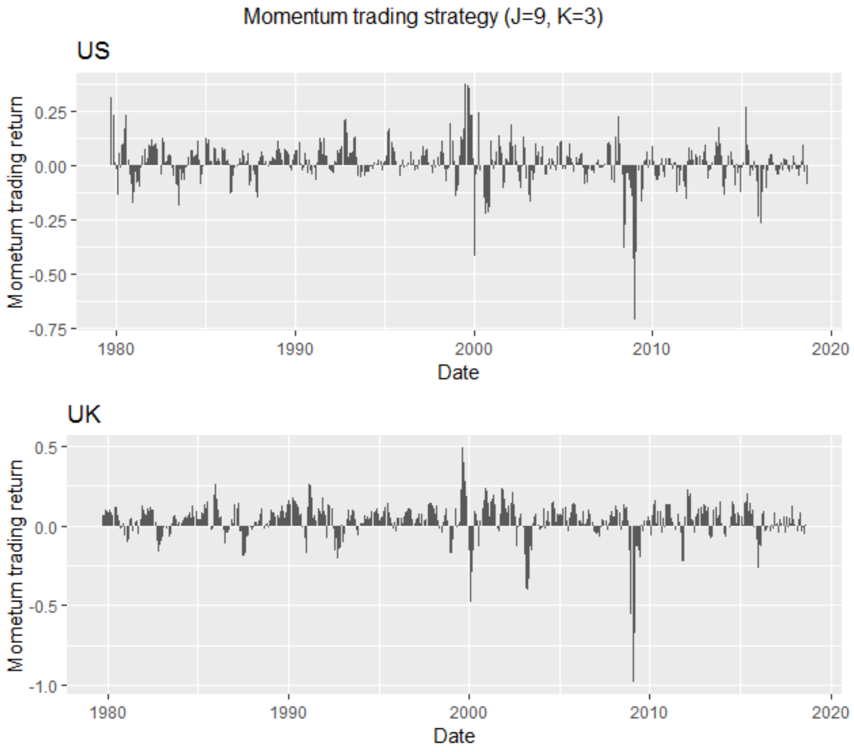

Figure A4.9- Profitability of the momentum investment strategies (9*3) in the US and UK market

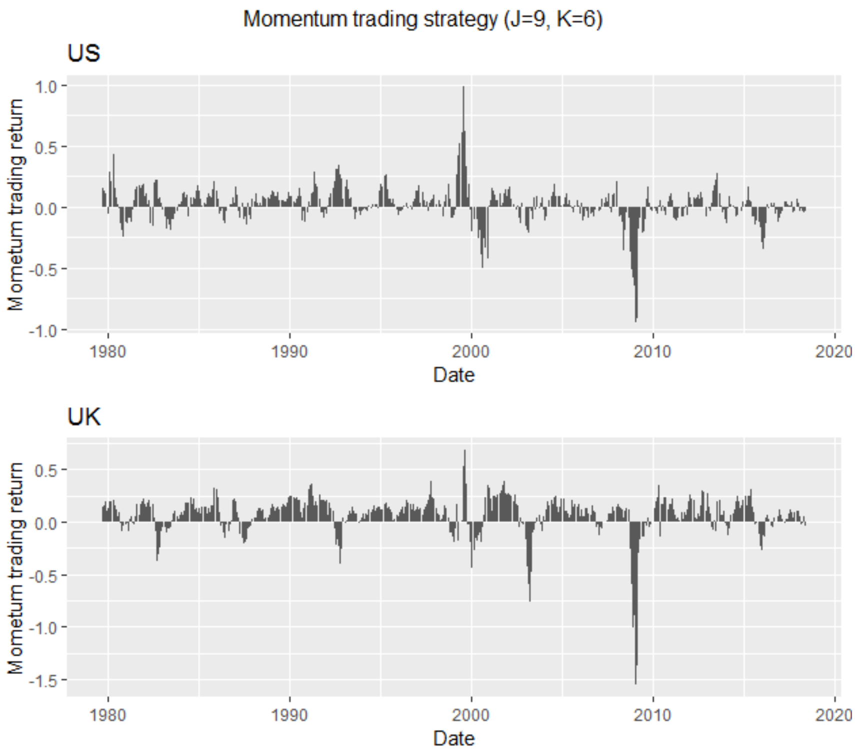

Figure A4.10- Profitability of the momentum investment strategies (9*6) in the US and UK market

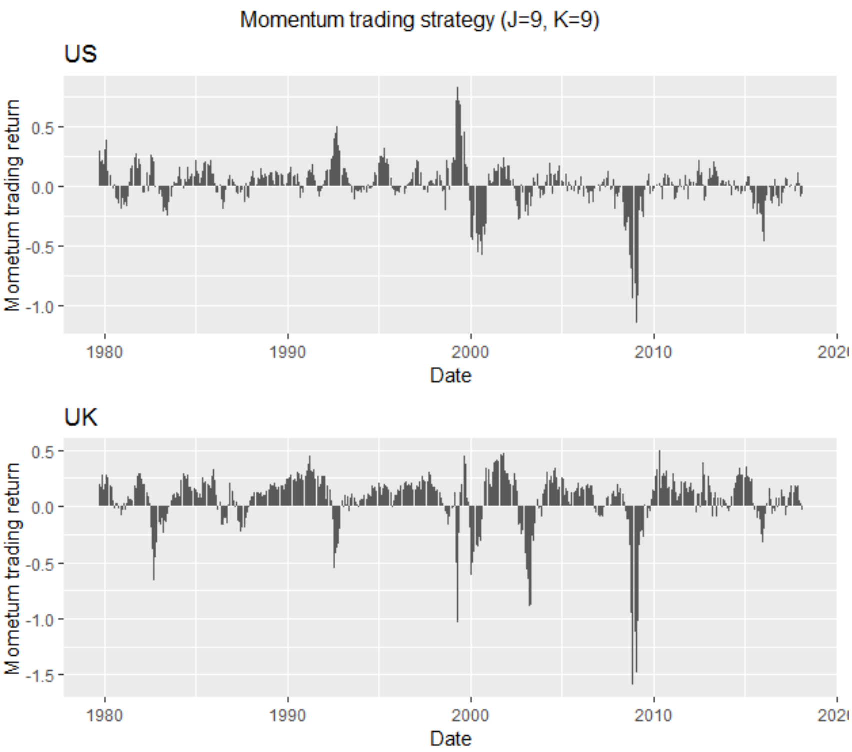

Figure 4.11- Profitability of the momentum investment strategies (9*9) in the US and UK market

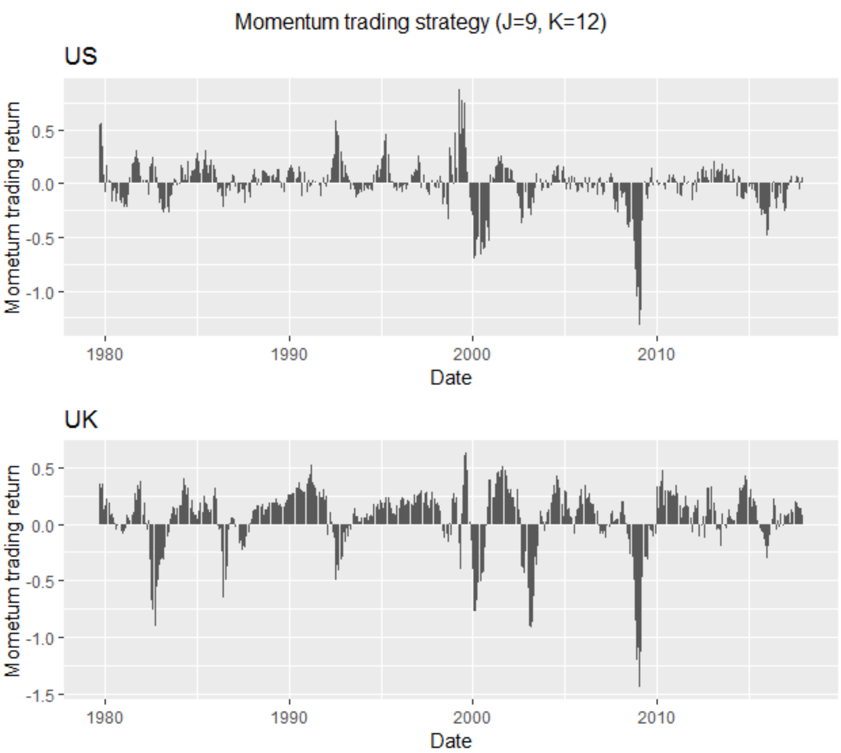

Figure A4.12- Profitability of the momentum investment strategies (9*12) in the US and UK market

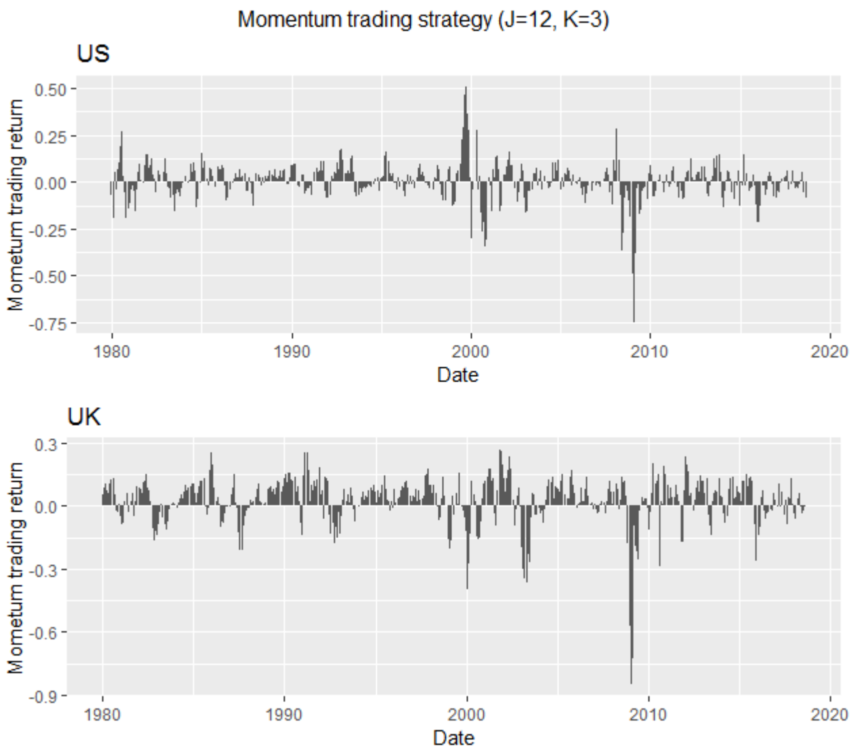

Figure A4.13- Profitability of the momentum investment strategies (12*3) in the US and UK market

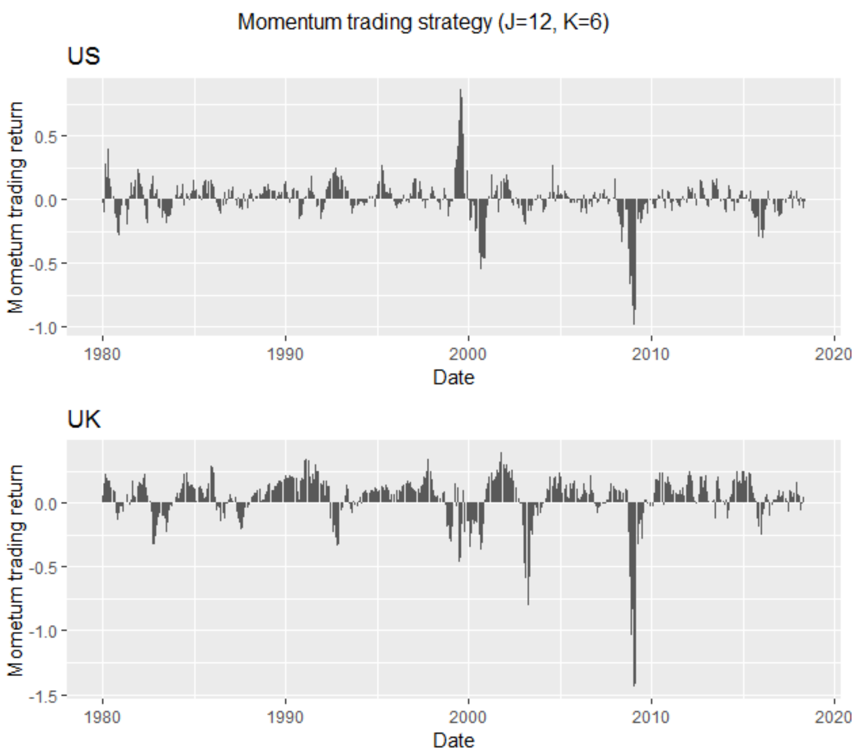

Figure A4.14- Profitability of the momentum investment strategies (12*6) in the US and UK market

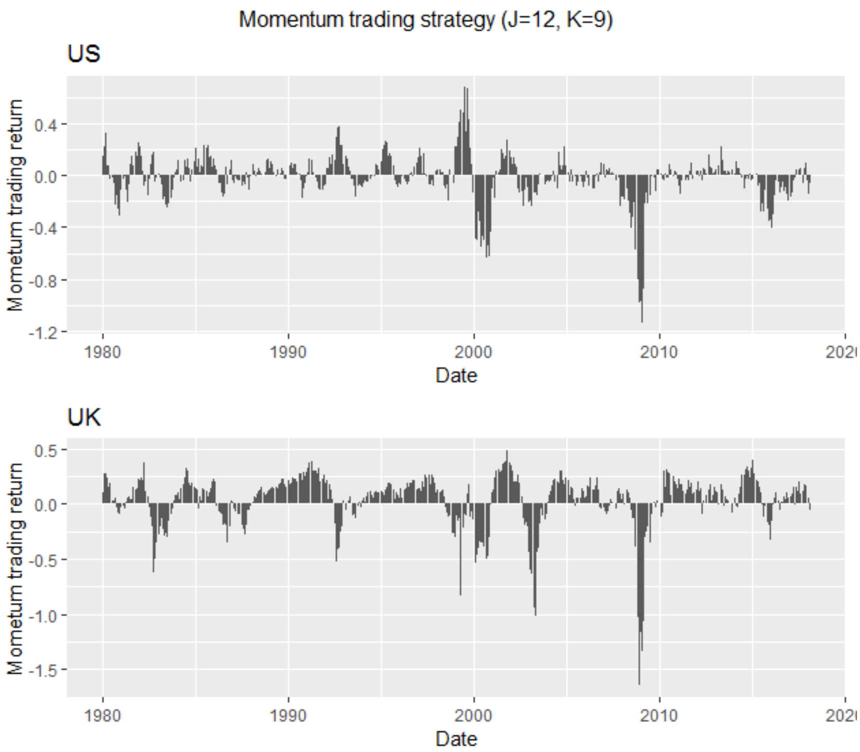

Figure A4.15- Profitability of the momentum investment strategies (12*9) in the US and UK market

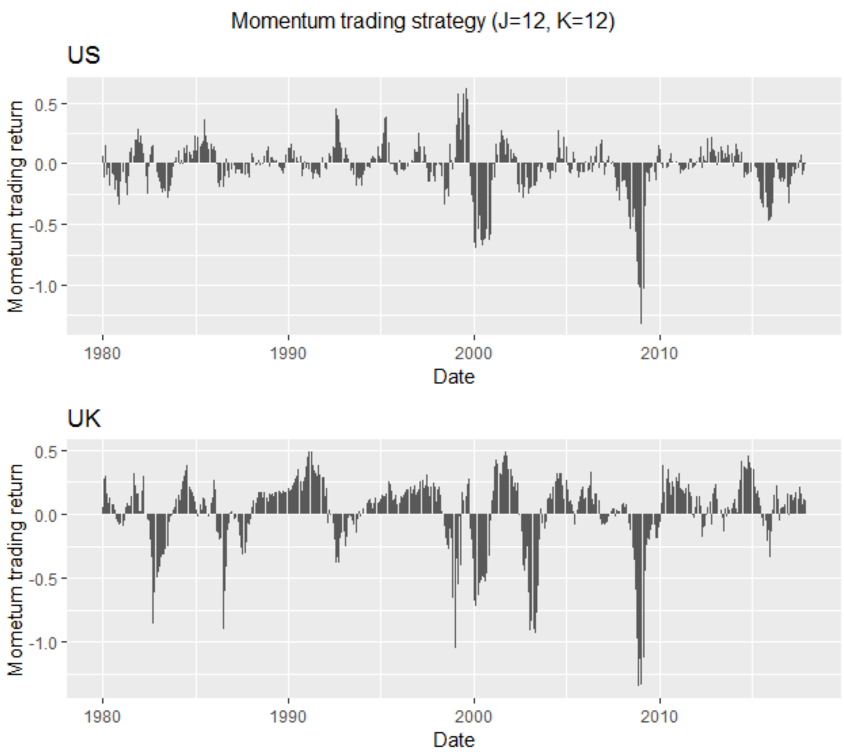

Figure A4.16- Profitability of the momentum investment strategies (12*12) in the US and UK market

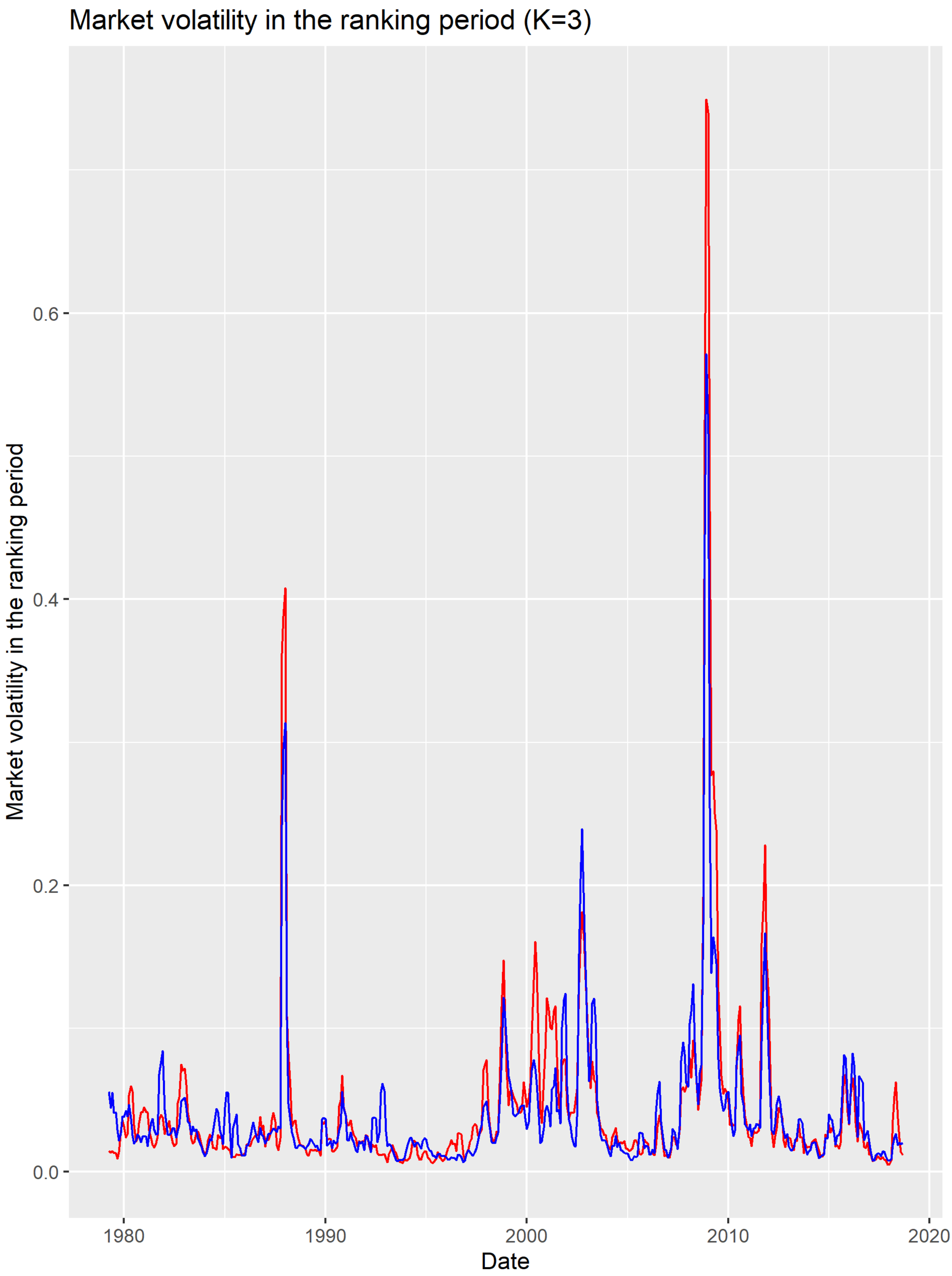

Figure A5.1- Market Volatility during the ranking period of 3 months

Figure A5.1 shows market volatility in the US stock market and the UK stock market during the ranking period of 3 months. The red line shows the US stock market, and the blue line shows the UK stock market from 1979 to 2018.

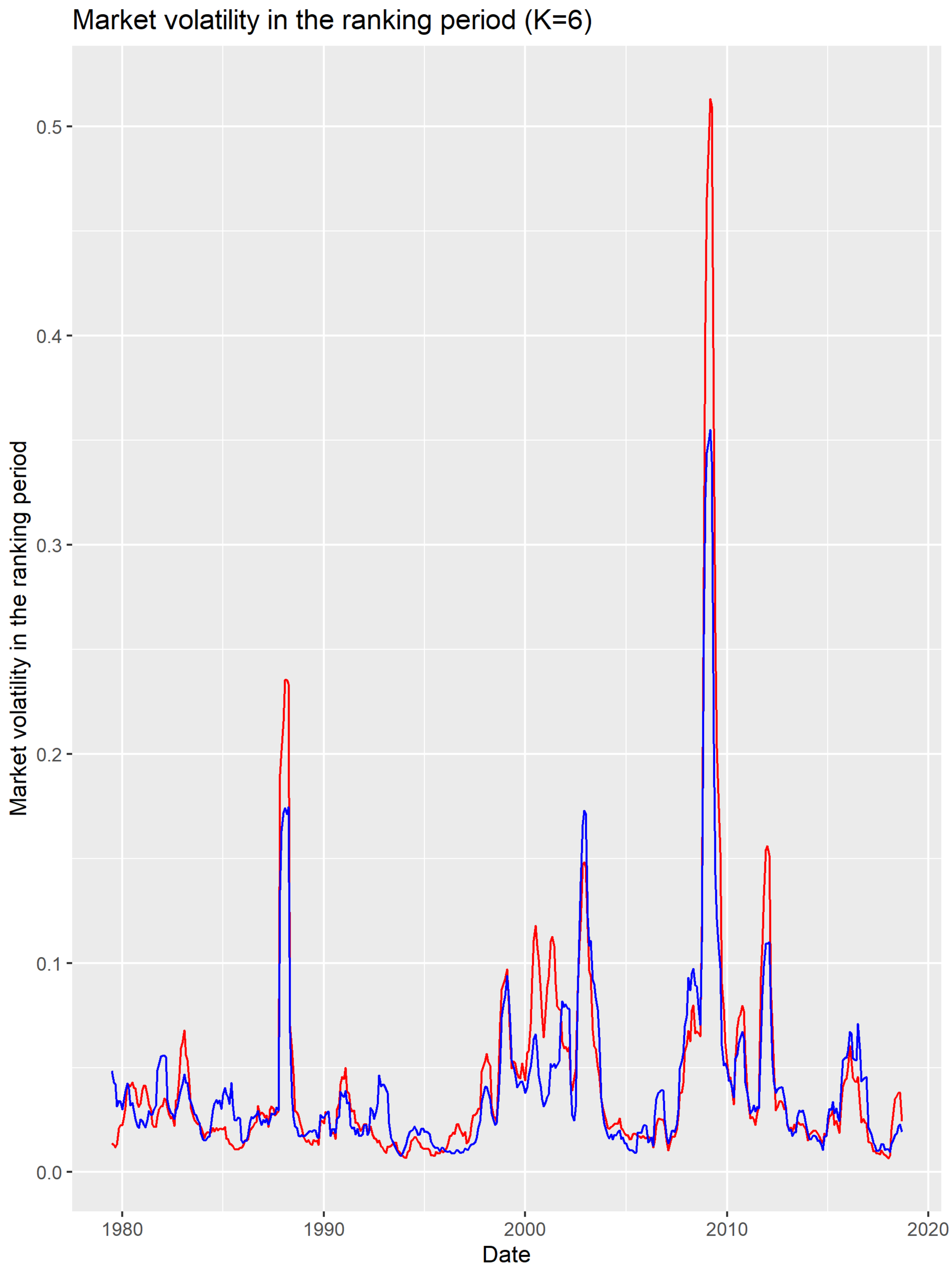

Figure A5.2- Market Volatility during the ranking period of 6 months

Figure A5.2 shows market volatility in the US stock market and the UK stock market during the ranking period of 6 months. The red line shows the US stock market, and the blue line shows the UK stock market from 1979 to 2018.

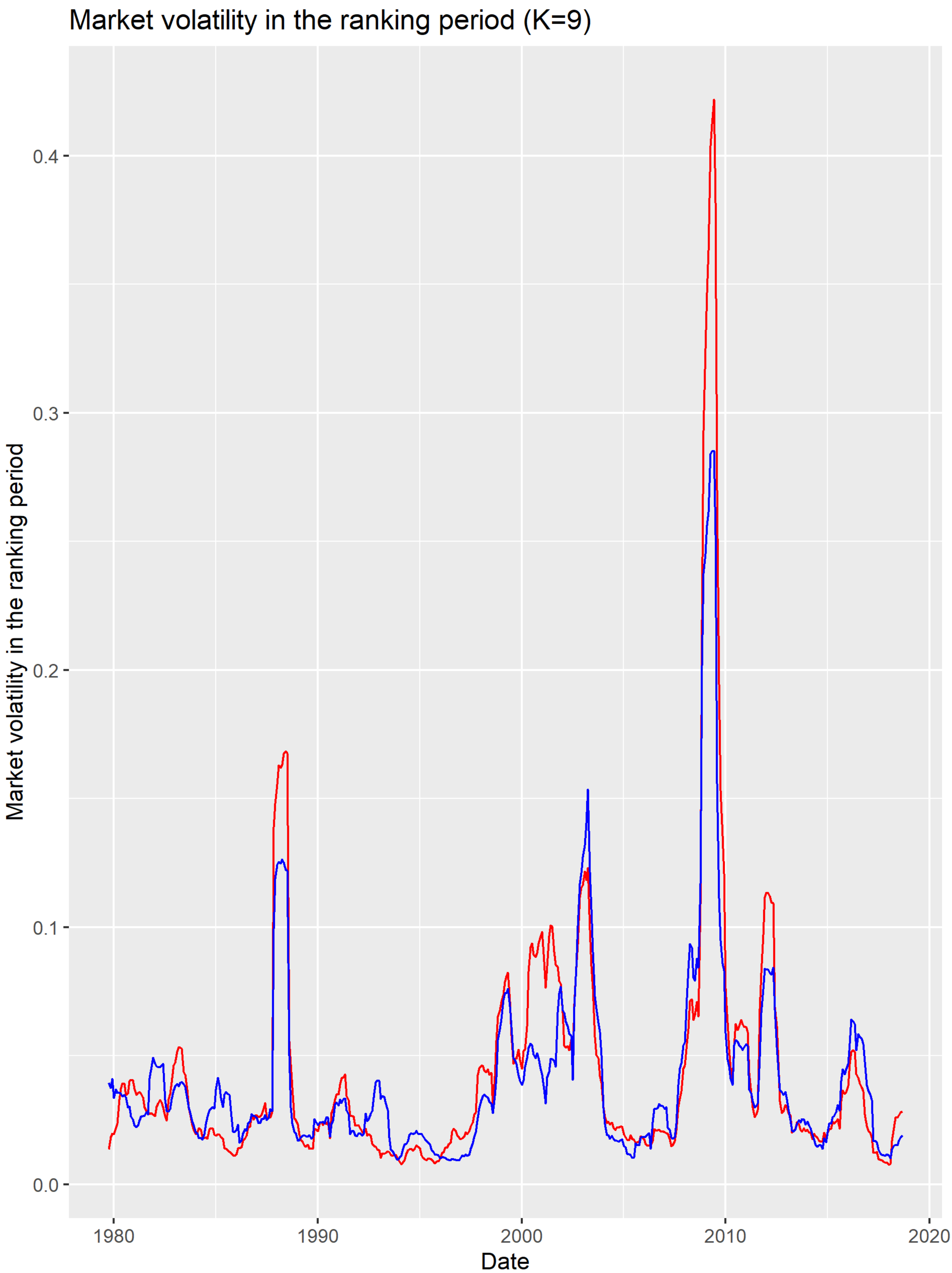

Figure A5.3- Market Volatility during the ranking period of 9 months

Figure A5.3 shows market volatility in the US stock market and the UK stock market during the ranking period of 9 months. The red line shows the US stock market, and the blue line shows the UK stock market from 1979 to 2018.

Figure A5.4- Market Volatility during the ranking period of 12 months

Figure A5.4 shows market volatility in the US stock market and the UK stock market during the ranking period of 12 months. The red line shows the US stock market, and the blue line shows the UK stock market from 1979 to 2018.

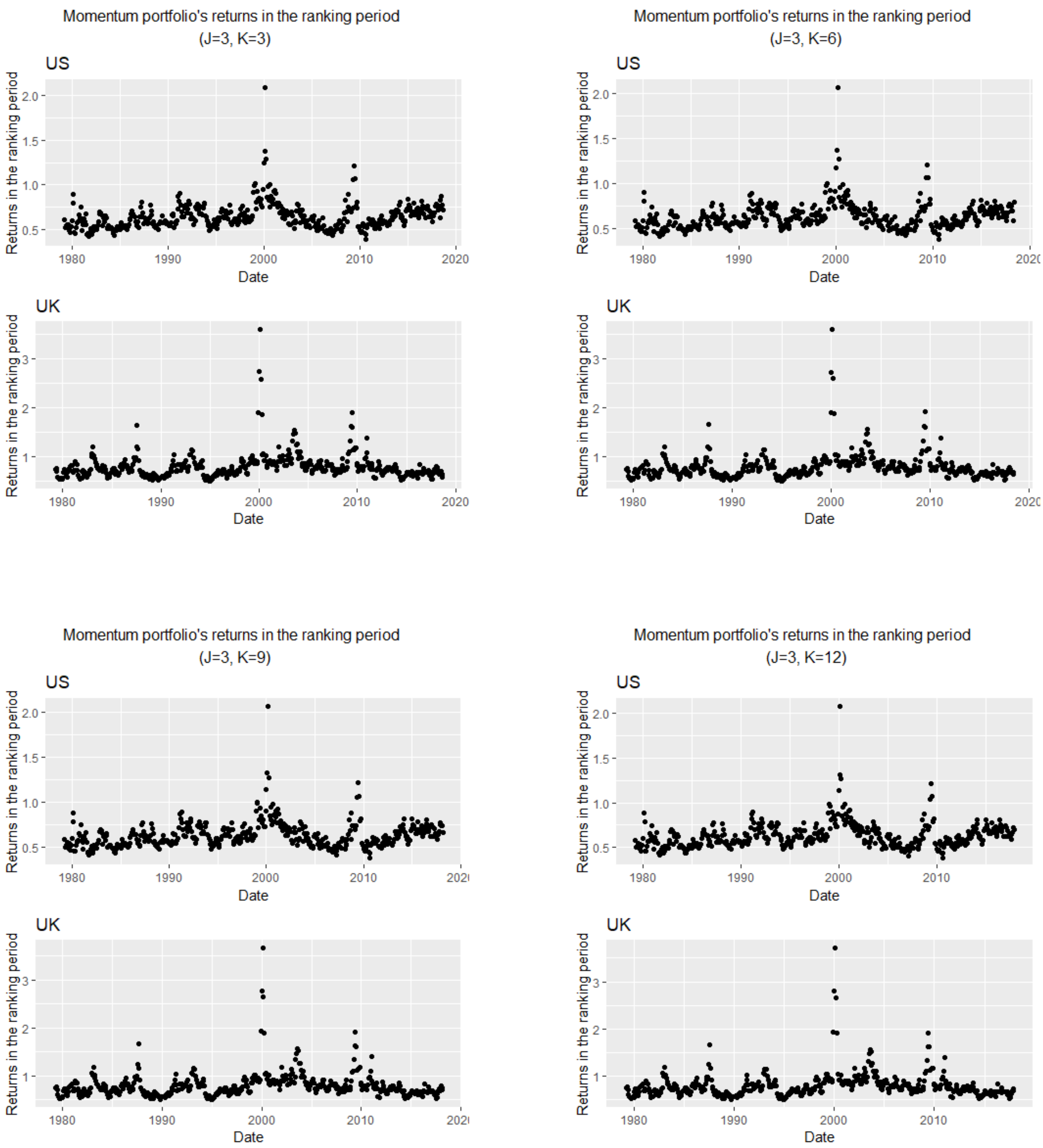

Figure A6.1 – Ranking returns during the ranking period of 3 months

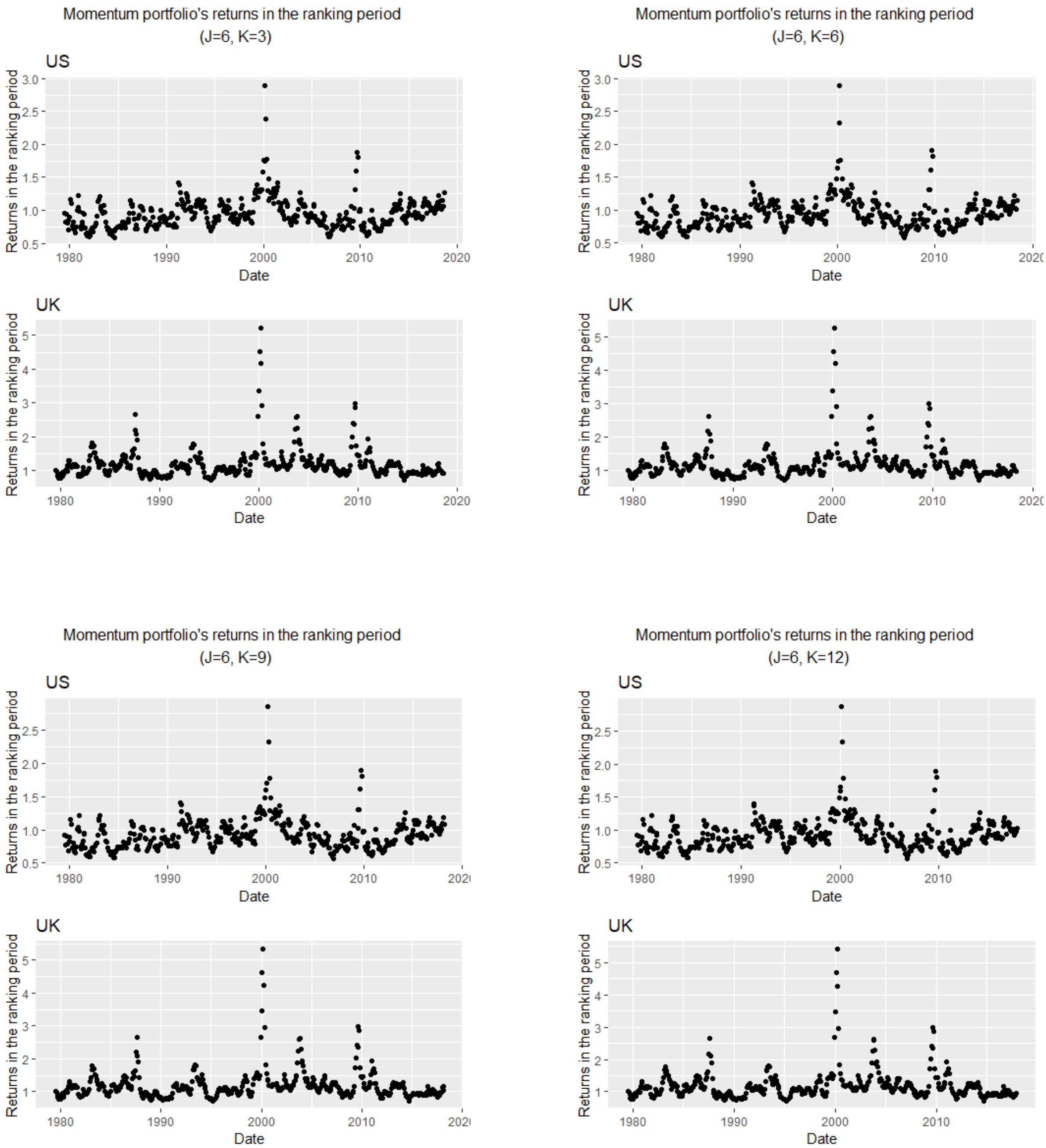

Figure A6.2 – Ranking returns during the ranking period of 6 months

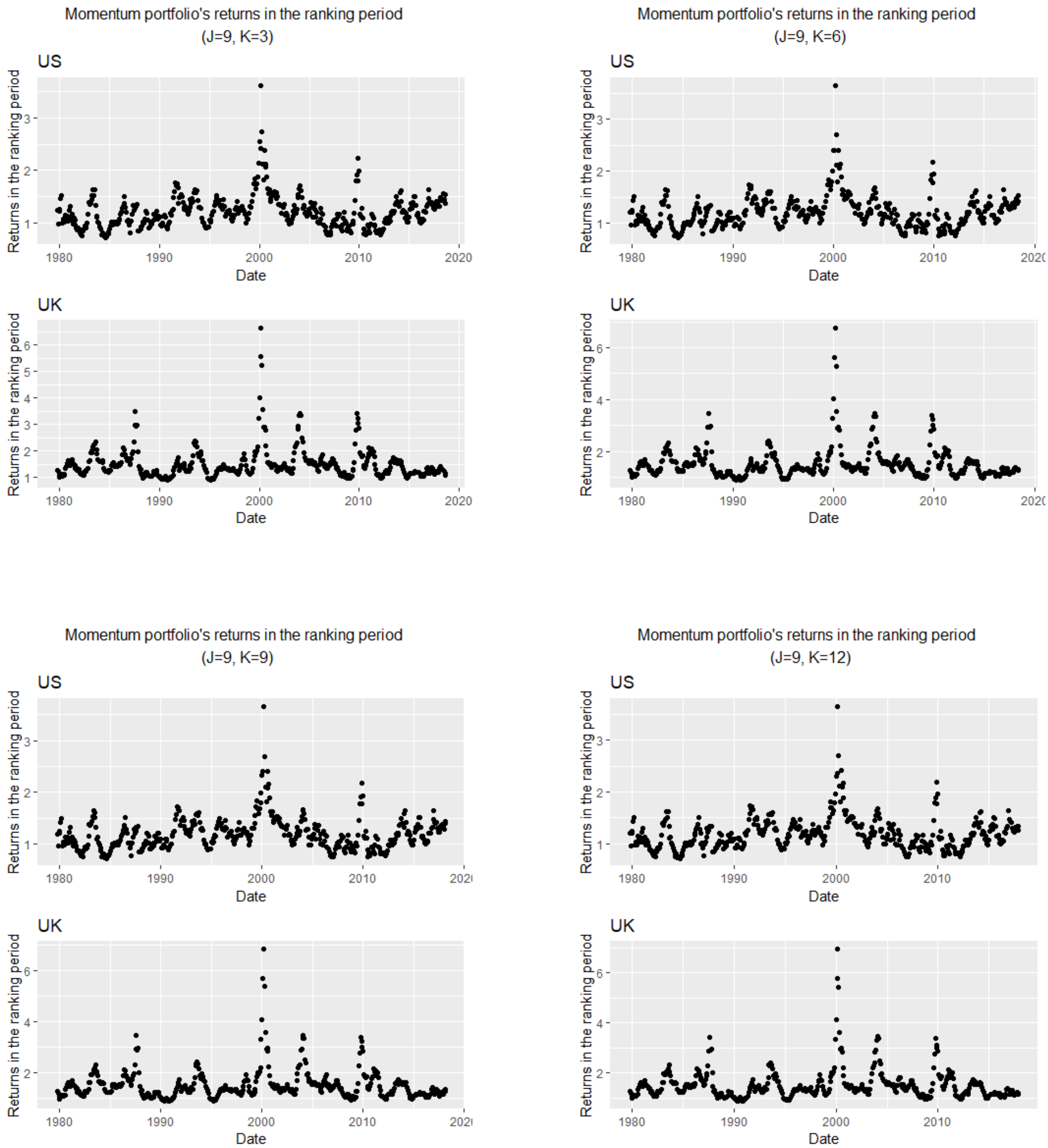

Figure A6.3 – Ranking returns during the ranking period of 9 months

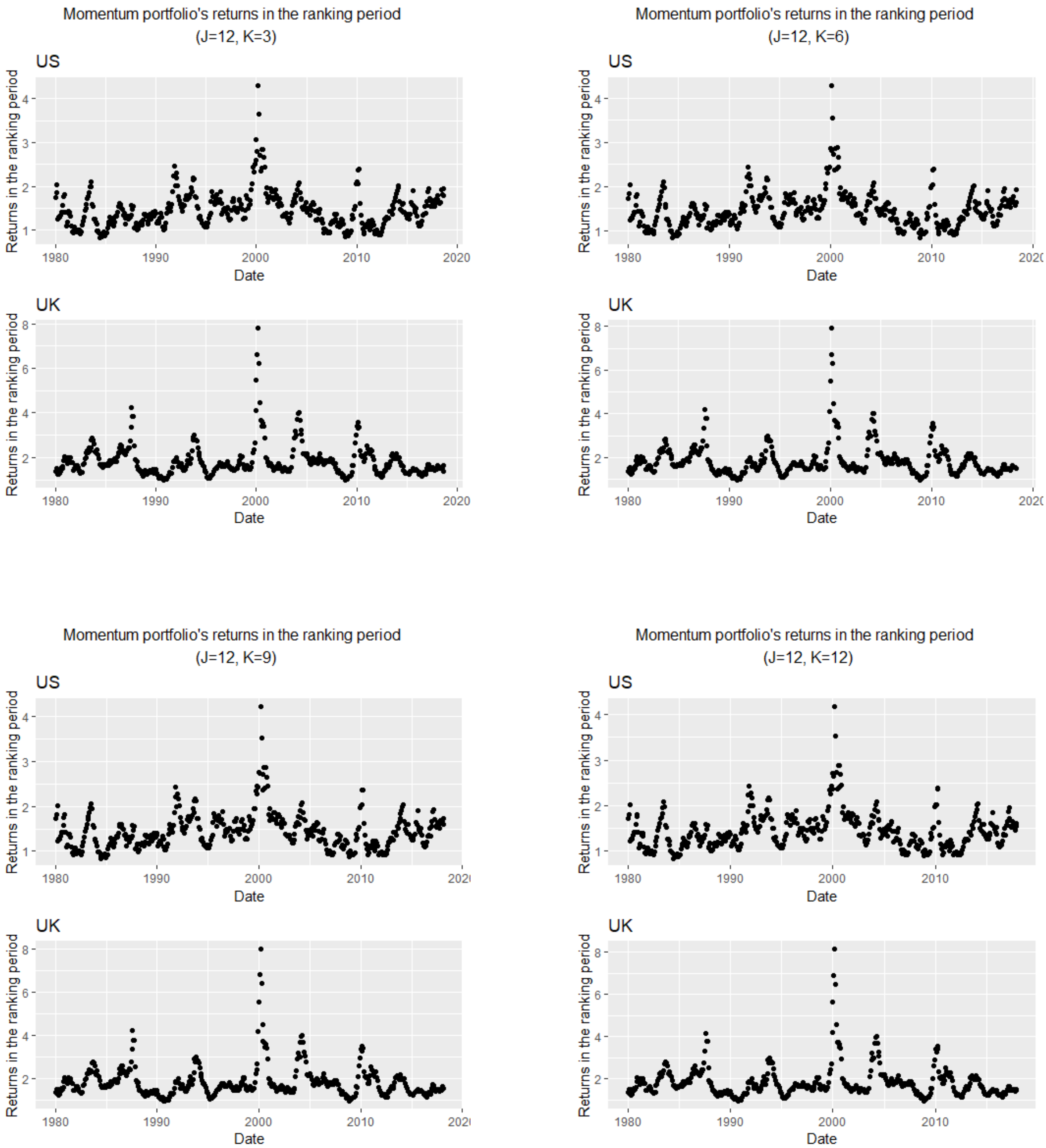

Figure A6.4 – Ranking returns during the ranking period of 12 months

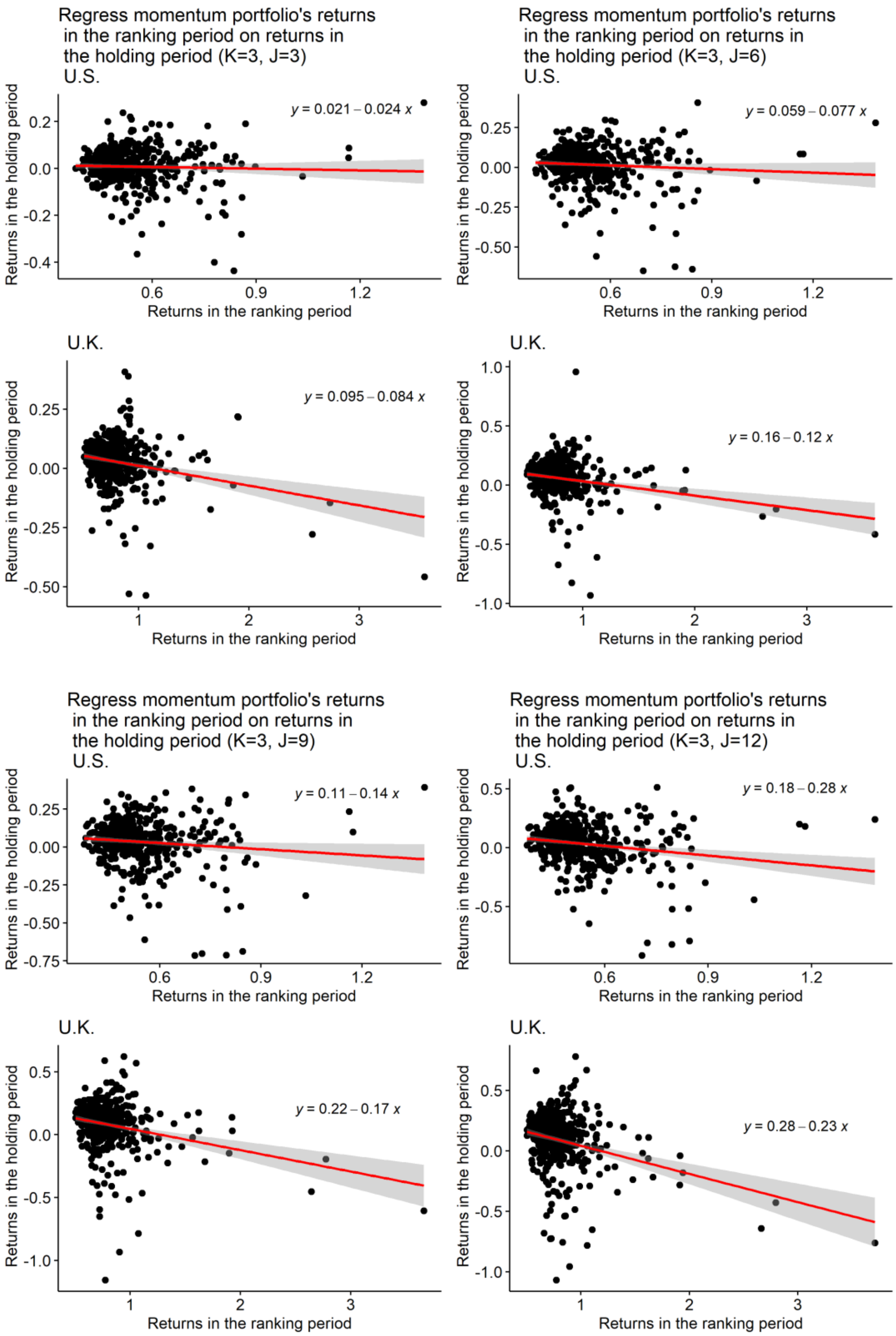

Figure A7.1 – Relationship between returns in the ranking period and the holding period when the ranking period is 3 months

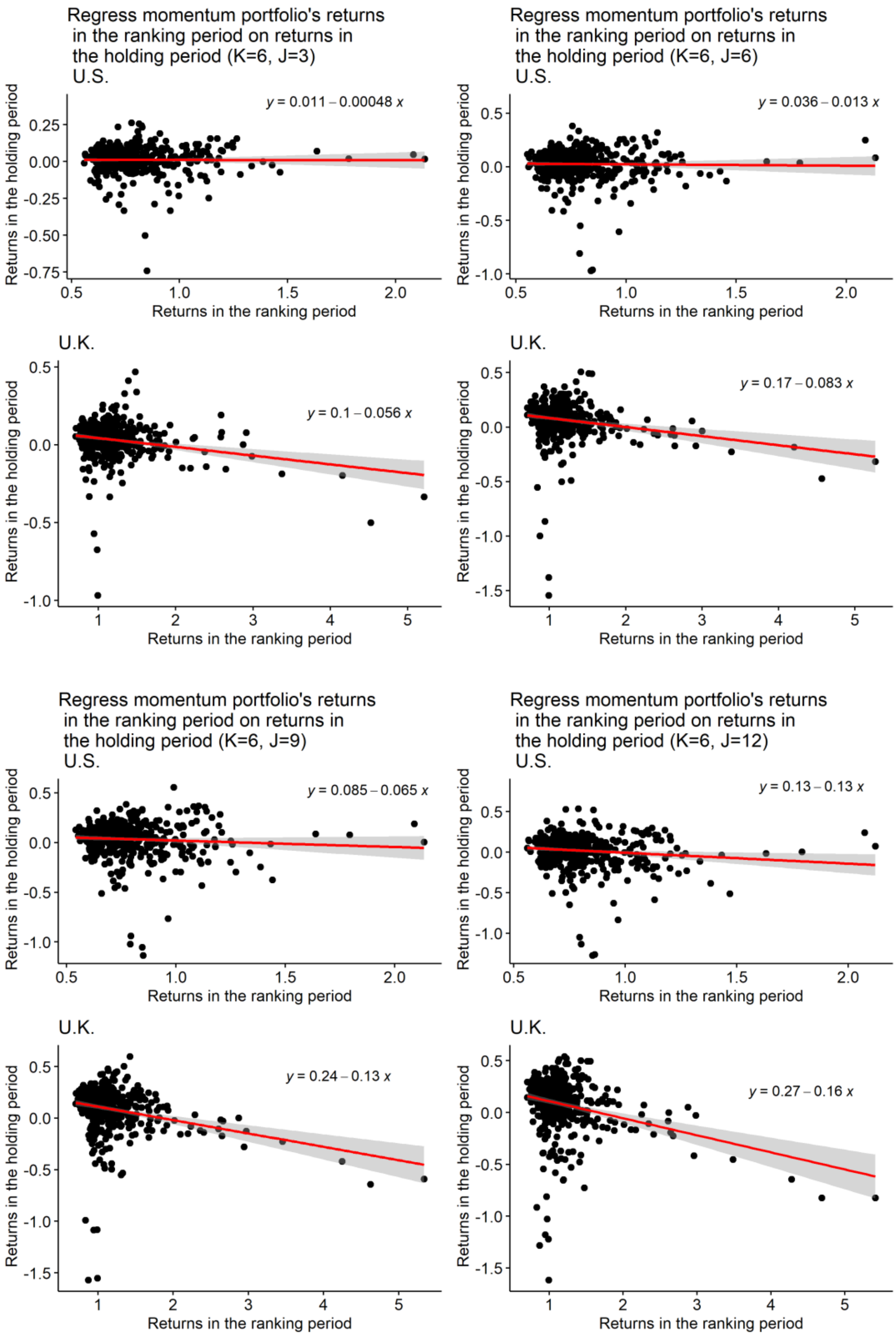

Figure A7.2 – Relationship between returns in the ranking period and the holding period when the ranking period is 6 months

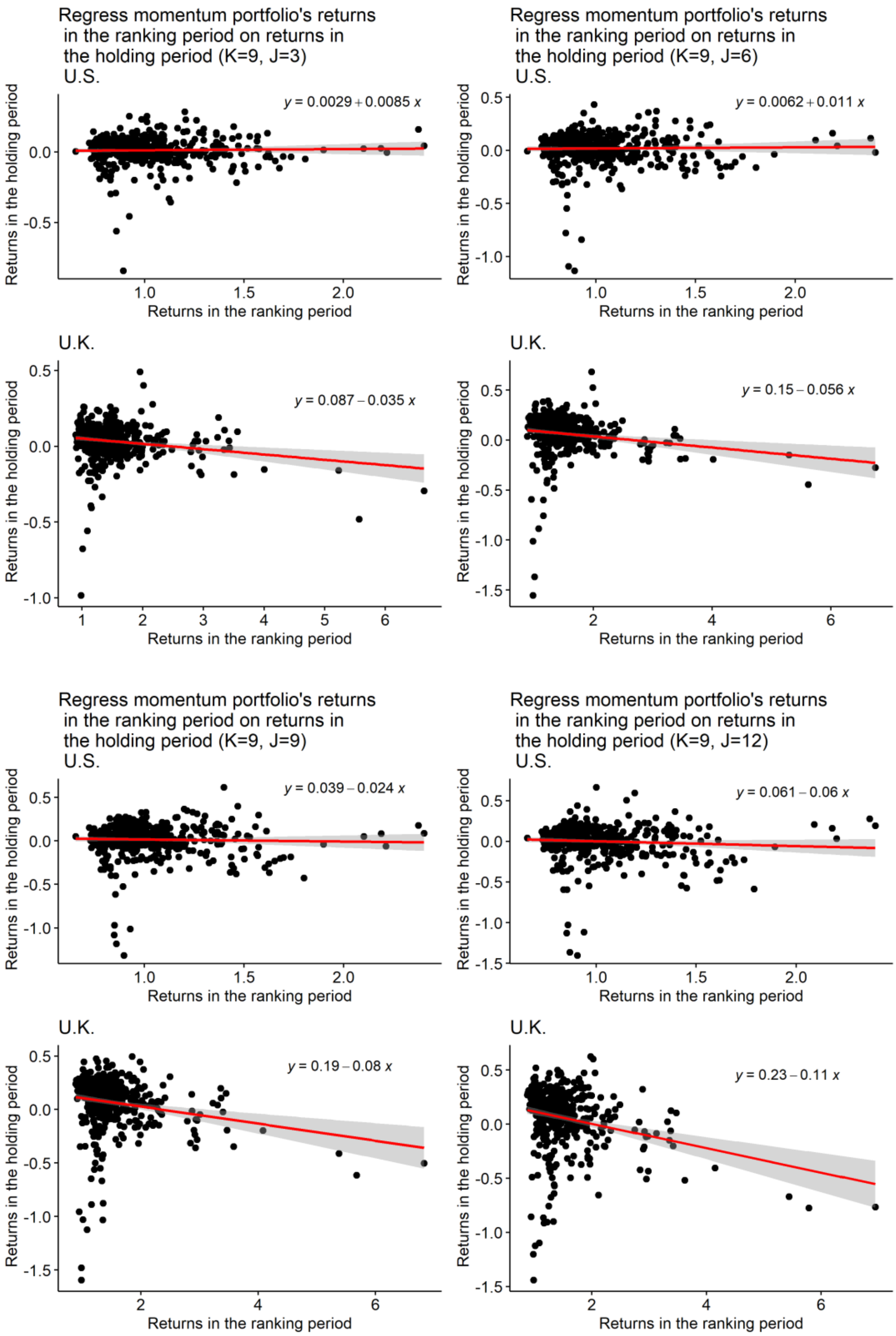

Figure A7.3 – Relationship between returns in the ranking period and the holding period when the ranking period is 9 months

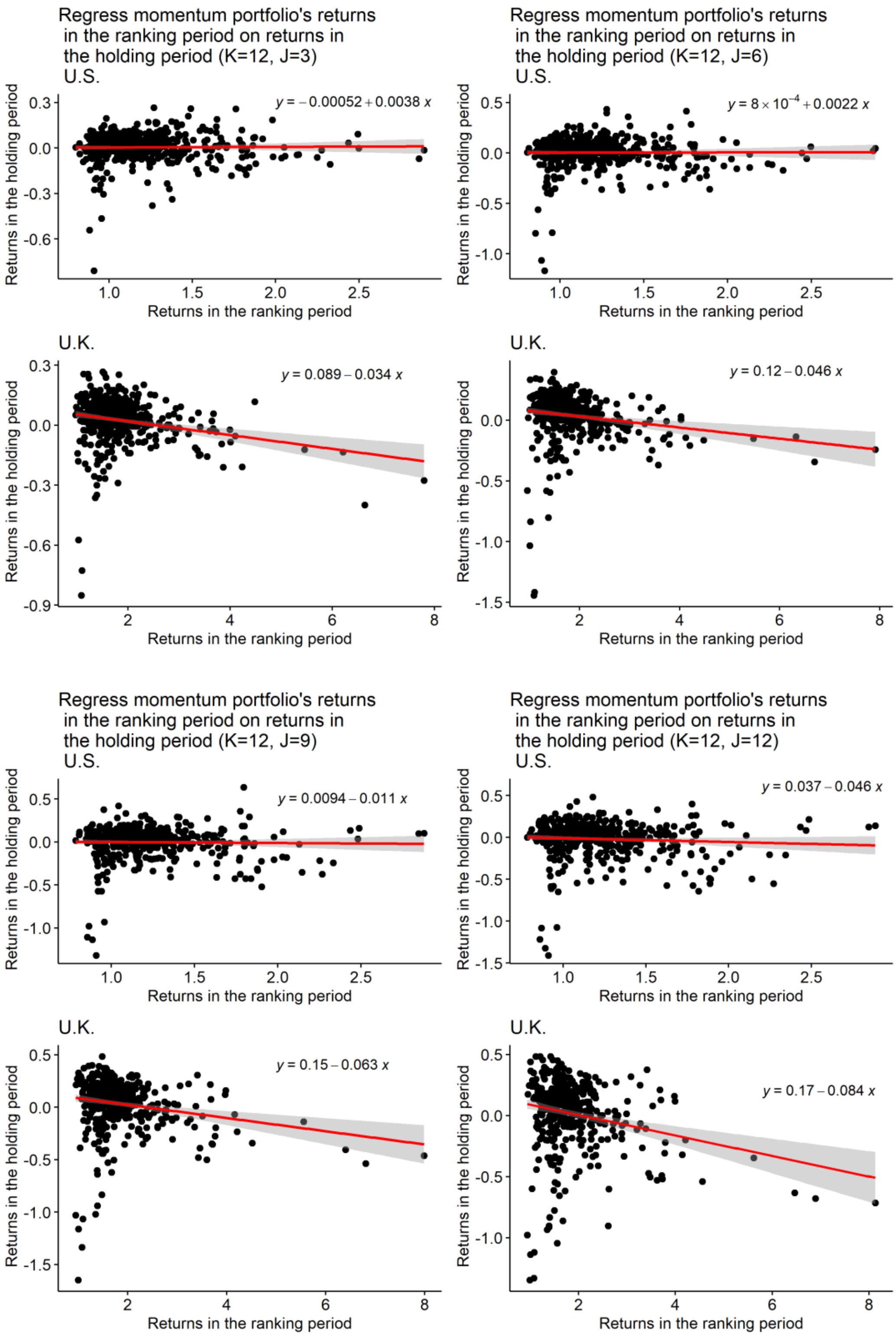

Figure A7.4 – Relationship between returns in the ranking period and the holding period when the ranking period is 12 months

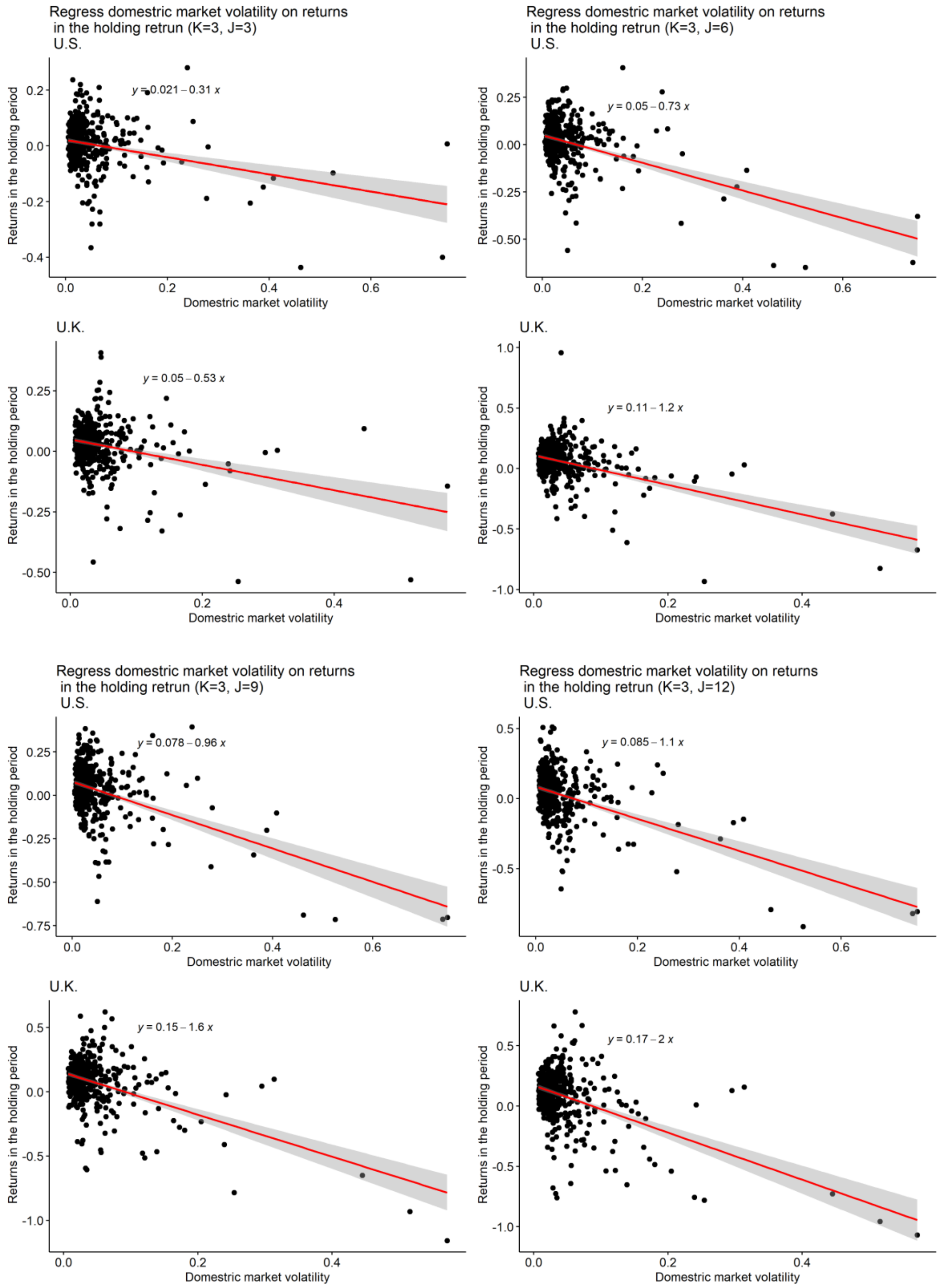

Figure A8.1 – Relationship between domestic market volatility and momentum returns when the ranking period is 3 months

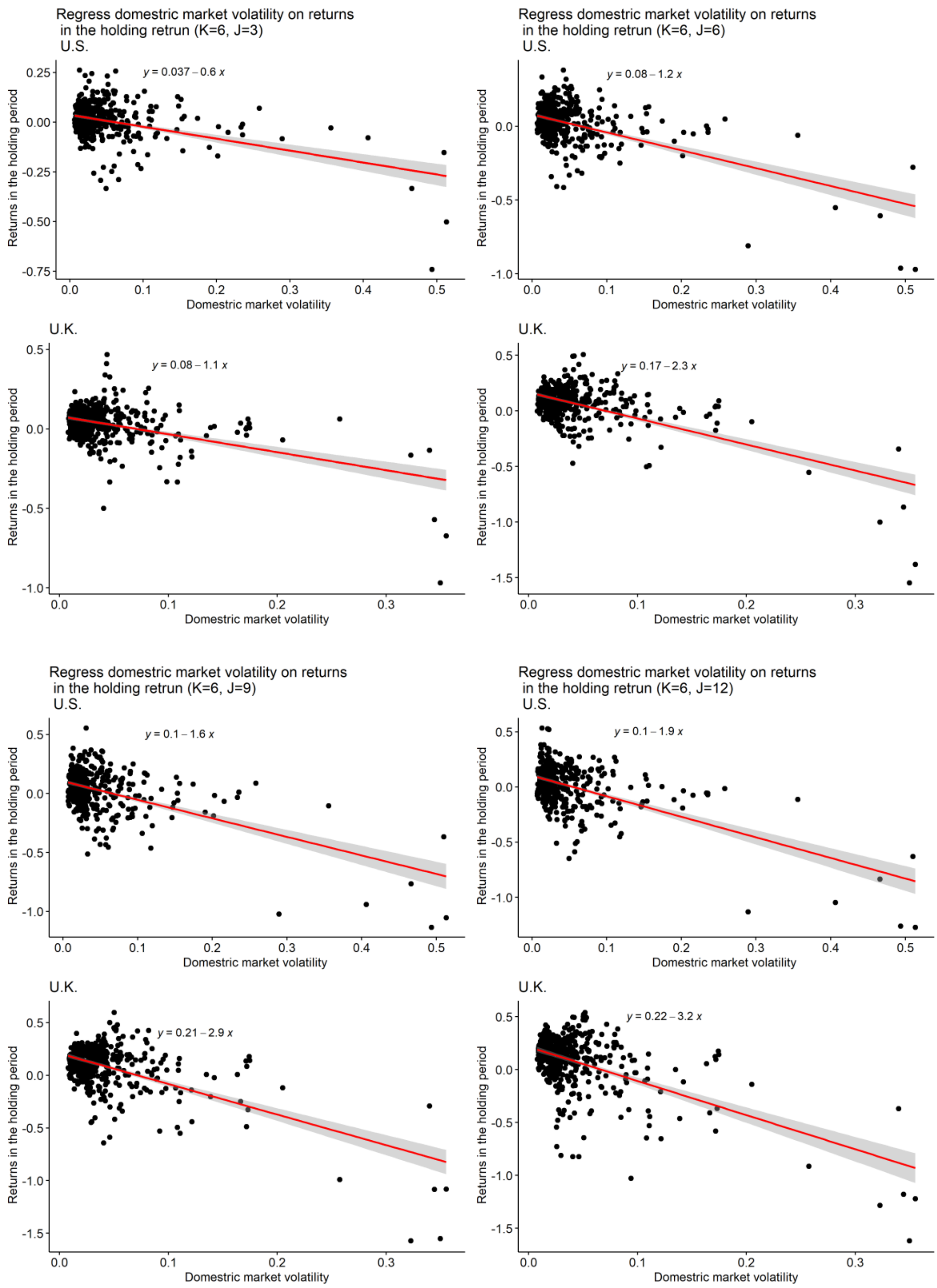

Figure A8.2 – Relationship between domestic market volatility and momentum returns when the ranking period is 6 months

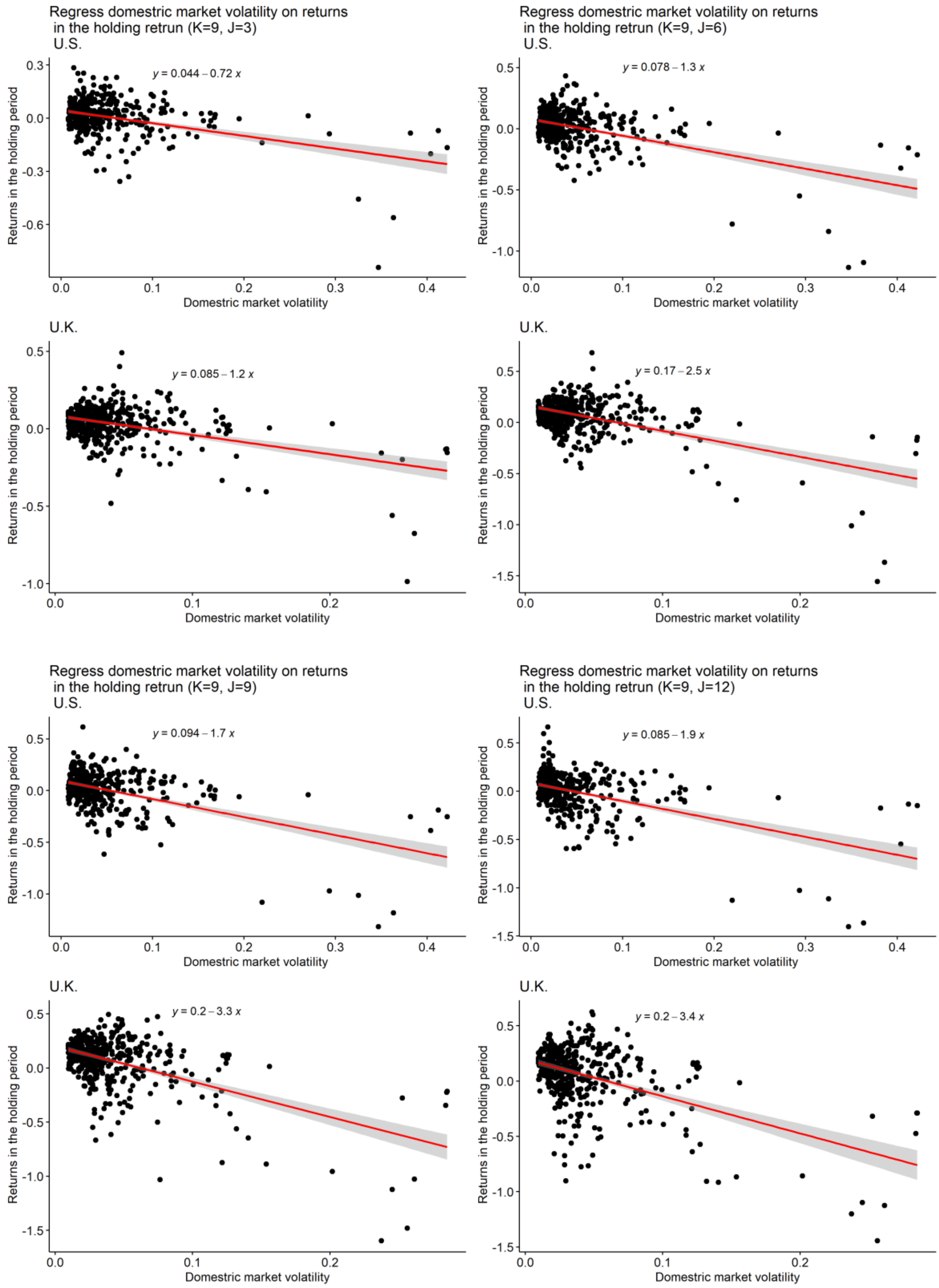

Figure A8.3 – Relationship between domestic market volatility and momentum returns when the ranking period is 9 months

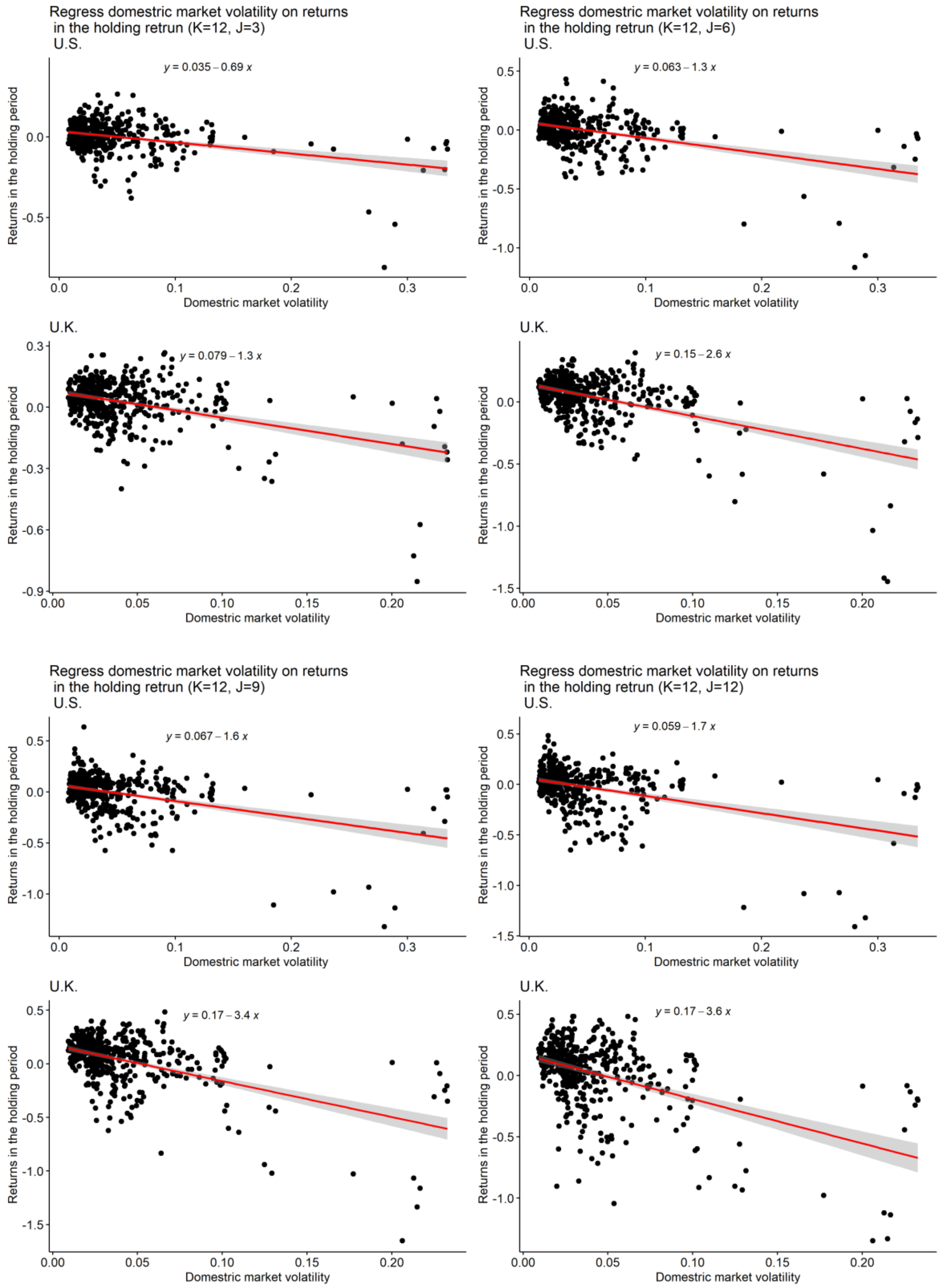

Figure A8.4 – Relationship between domestic market volatility and momentum returns when the ranking period is 12 months

Alpha	**U.K.**				**U.S.**			
	3	6	9	12	3	6	9	12
3	0.05	0.11	0.15	0.17	0.021	0.05	0.078	0.085
6	0.08	0.17	0.21	0.22	0.037	0.08	0.1	0.1
9	0.085	0.17	0.2	0.2	0.044	0.078	0.094	0.085
12	0.079	0.15	0.17	0.17	0.035	0.063	0.067	0.059

Beta	**U.K.**				**U.S.**			
	3	6	9	12	3	6	9	12
3	-0.53	-1.20	-1.60	-2.00	-0.31	-0.73	-0.96	-1.10
6	-1.1	-2.3	-2.9	-3.2	-0.6	-1.2	-1.6	-1.9
9	-1.2	-2.5	-3.3	-3.4	-0.72	-1.3	-1.7	-1.9
12	-1.3	-2.6	-3.4	-3.6	-0.69	-1.3	-1.6	-1.7

Figure A8.5- Summary of value of alpha and beta for relationship between domestic market volatility and momentum returns

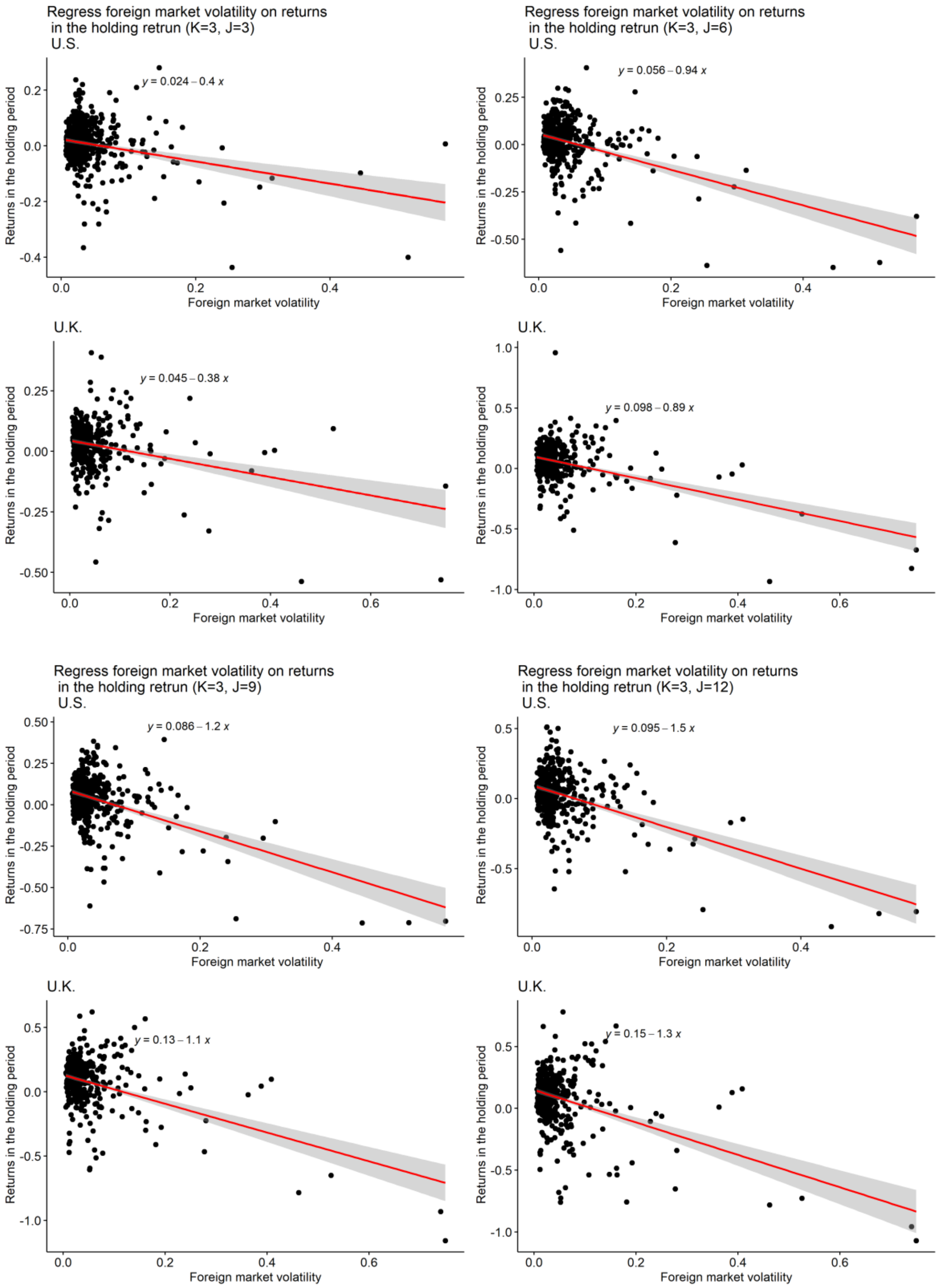

Figure A9.1 – Relationship between foreign market volatility and momentum returns when the ranking period is 3 months

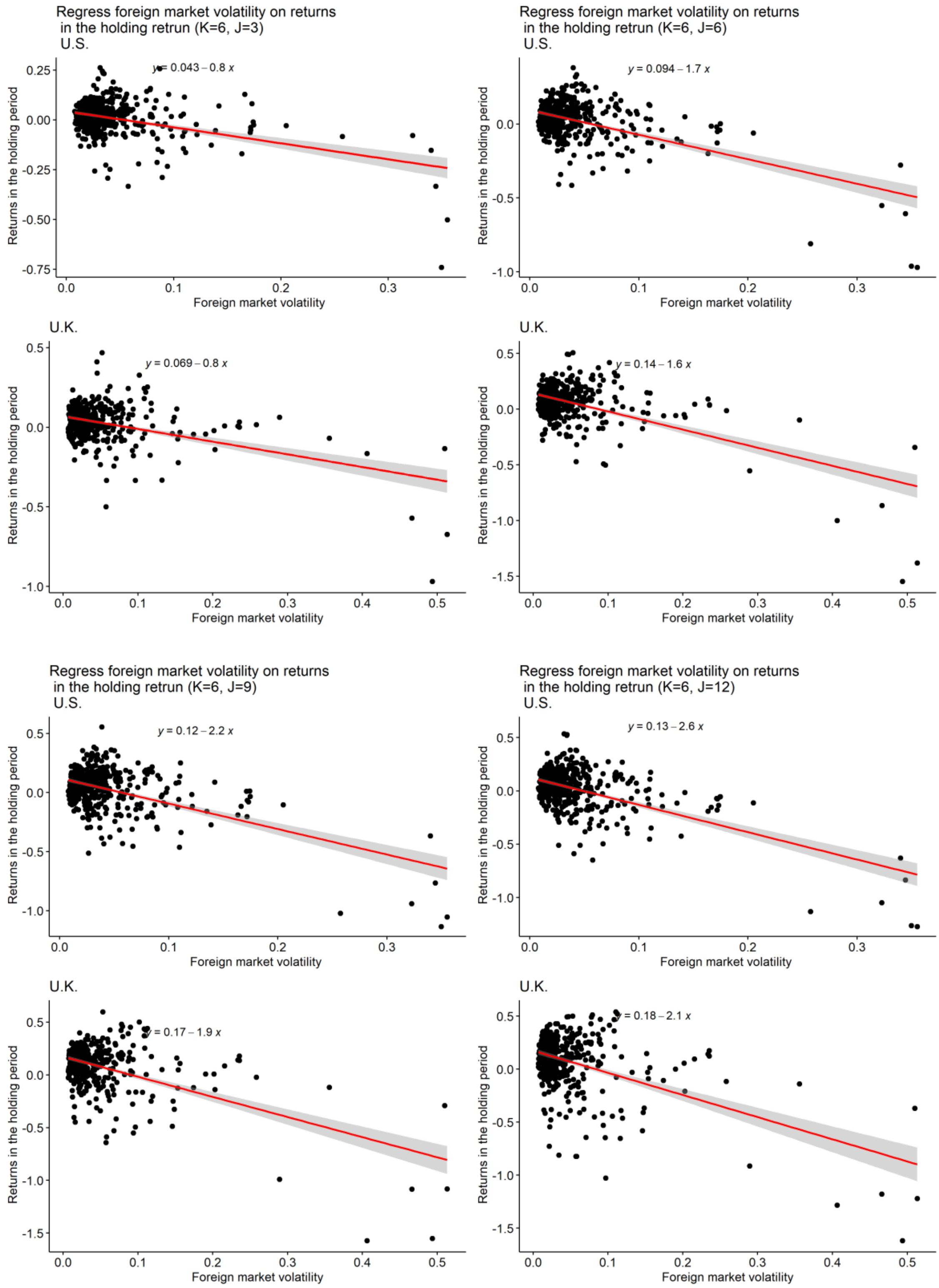

Figure A9.2 – Relationship between foreign market volatility and momentum returns when the ranking period is 6 months

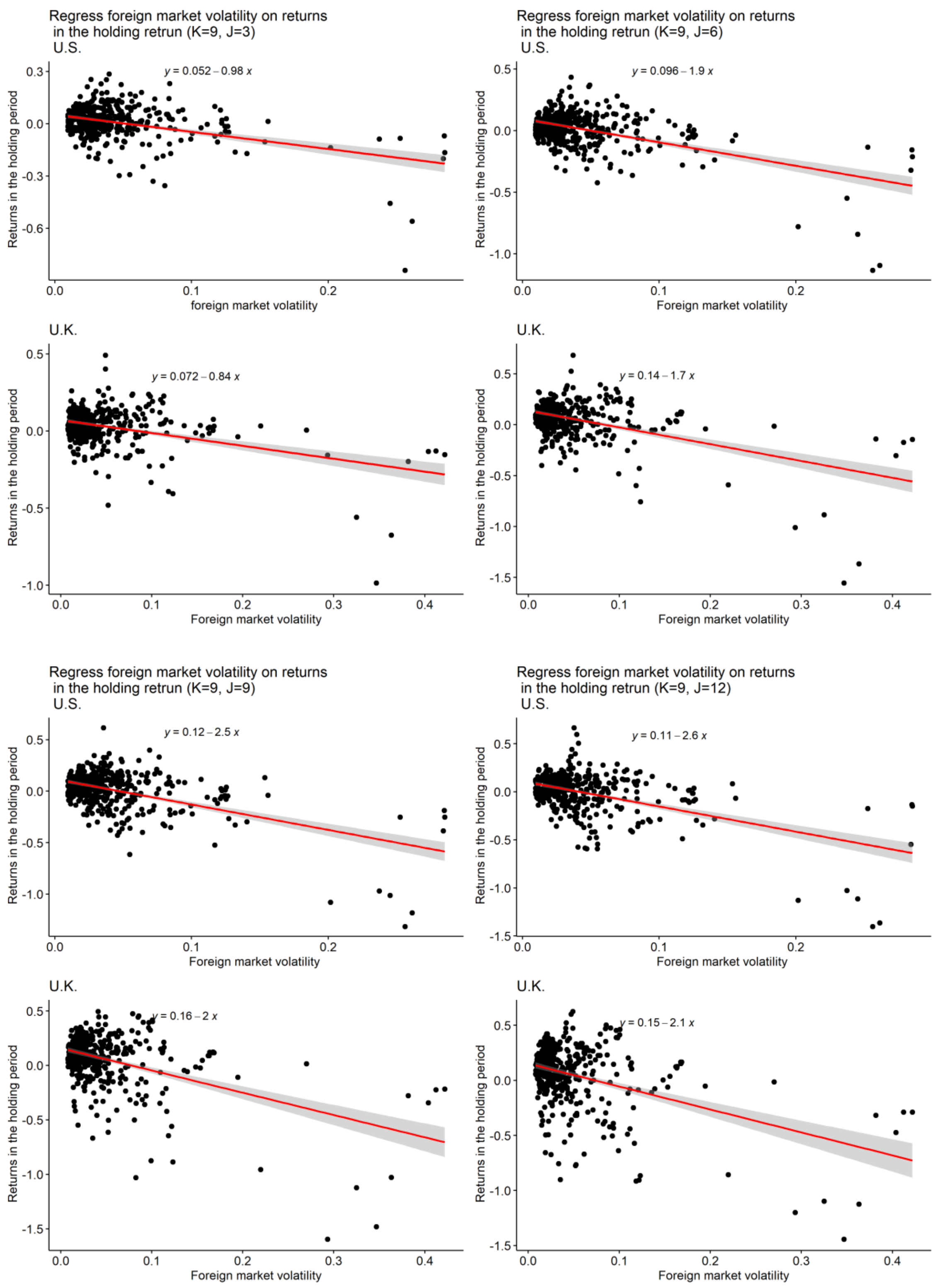

Figure A9.3 – Relationship between foreign market volatility and momentum returns when the ranking period is 9 months

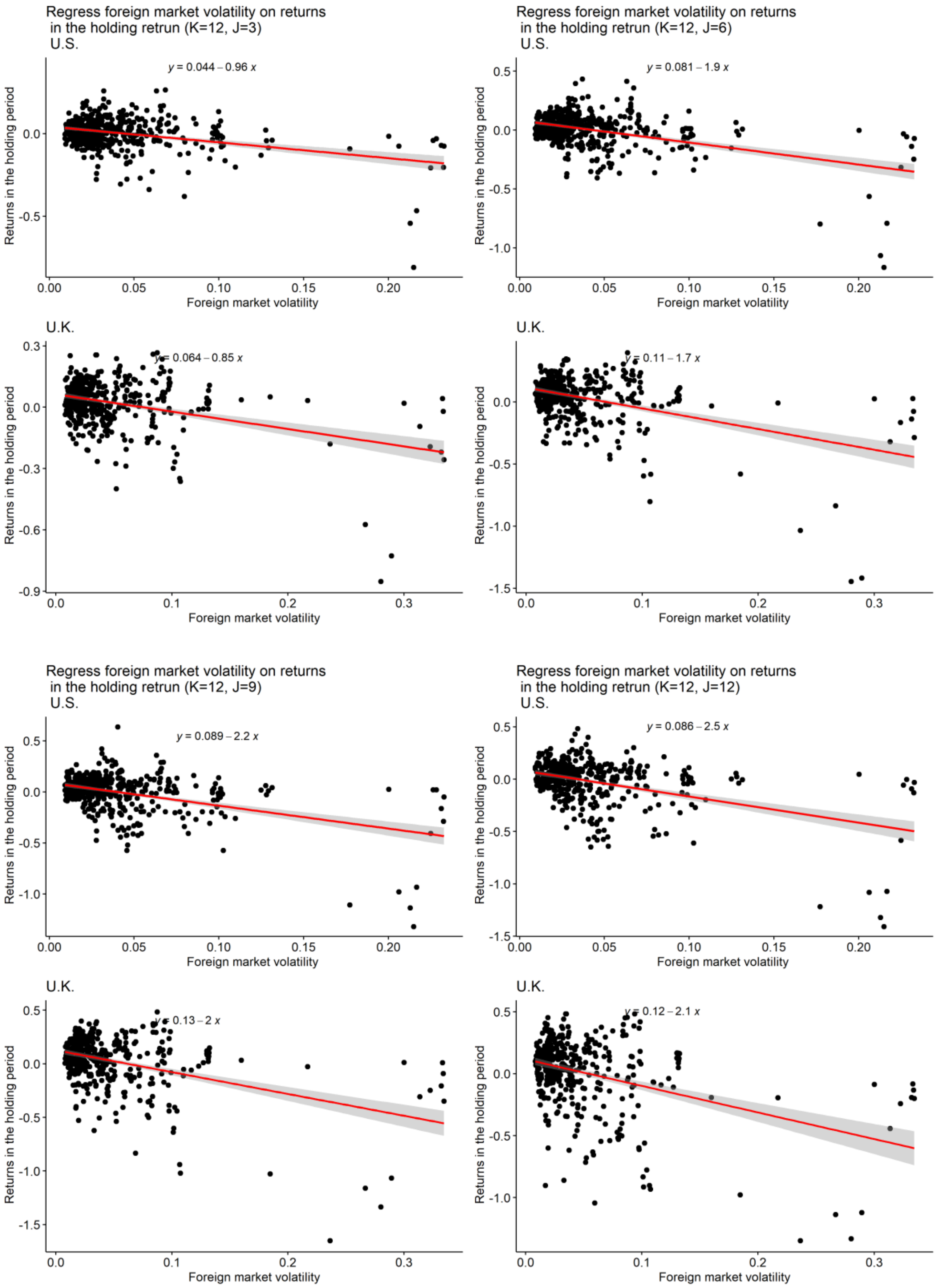

Figure A9.4 – Relationship between foreign market volatility and momentum returns when the ranking period is 12 months

Alpha	U.K.				U.S.			
	3	6	9	12	3	6	9	12
3	0.045	0.098	0.13	0.15	0.024	0.056	0.086	0.095
6	0.069	0.14	0.17	0.18	0.043	0.094	0.12	0.13
9	0.072	0.14	0.16	0.15	0.052	0.096	0.12	0.11
12	0.064	0.11	0.13	0.12	0.044	0.081	0.089	0.086

Beta	U.K.				U.S.			
	3	6	9	12	3	6	9	12
3	-0.38	-0.89	-1.1	-1.3	-0.4	-0.94	-1.2	-1.5
6	-0.8	-1.6	-1.9	-2.1	-0.8	-1.7	-2.2	-2.6
9	-0.84	-1.7	-2.0	-2.1	-0.98	-1.9	-2.5	-2.6
12	-0.85	-1.7	-2.0	-2.1	-0.96	-1.9	-2.2	-2.5

Figure A9.5- Summary of value of alpha and beta for relationship between foreign market volatility and momentum returns

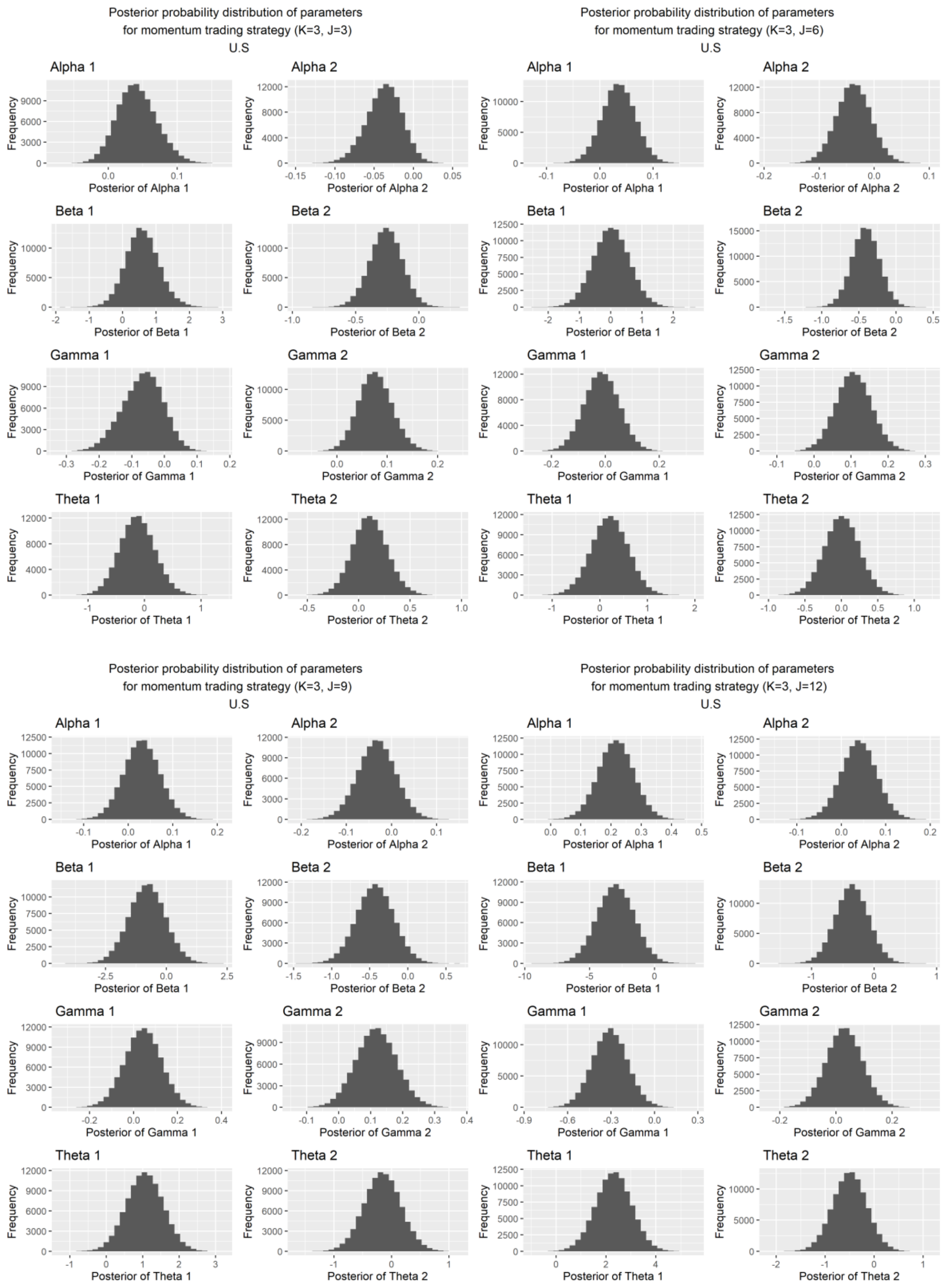

Figure A10.1-Posterior probability distribution of parameters Alpha, Beta, Gamma and Theta in the two-regime switching model during the 3 months ranking period in the US market.

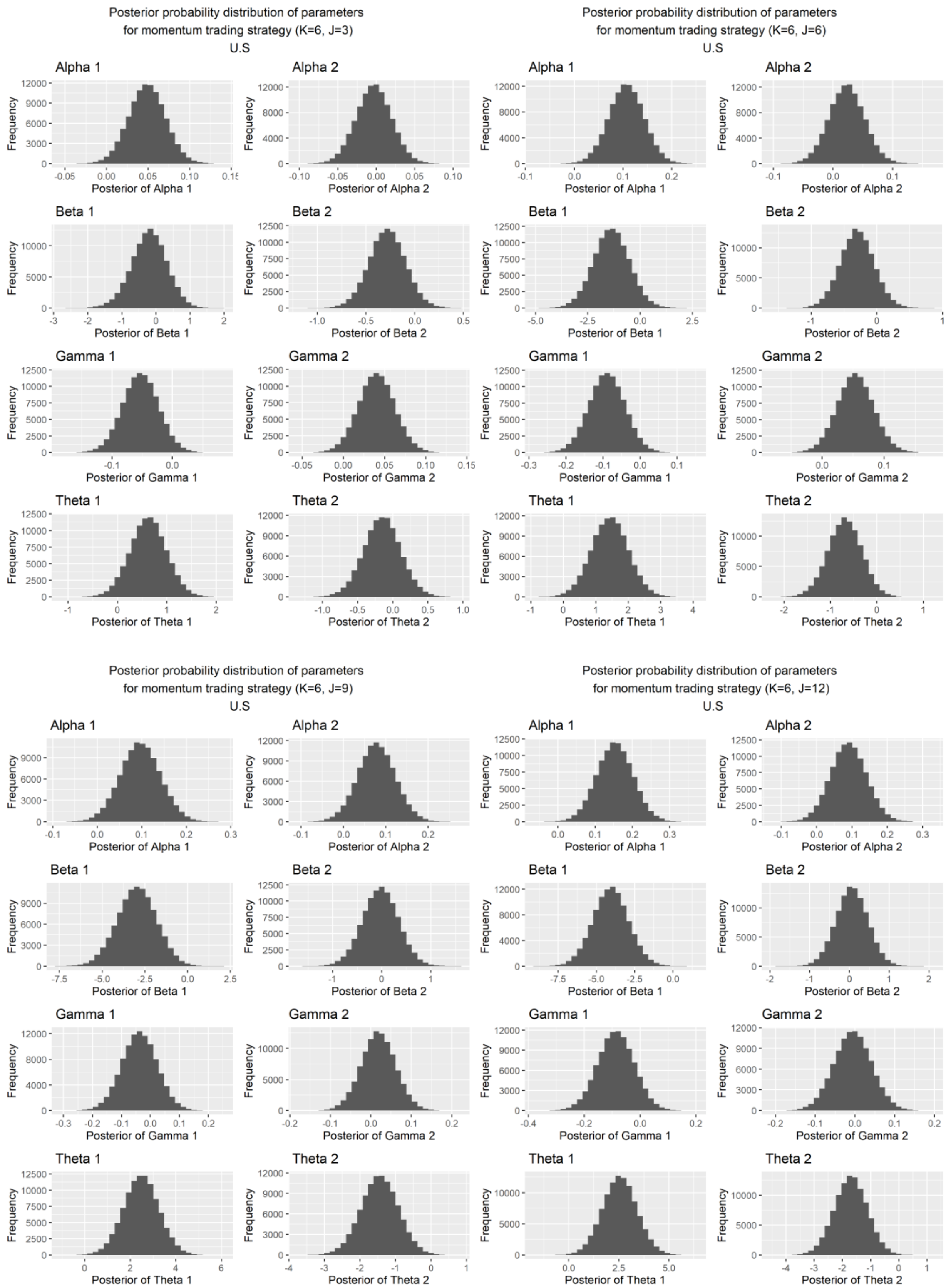

Figure A10.2-Posterior probability distribution of parameters Alpha, Beta, Gamma and Theta in the two-regime switching model during the 6 months ranking period in the US market.

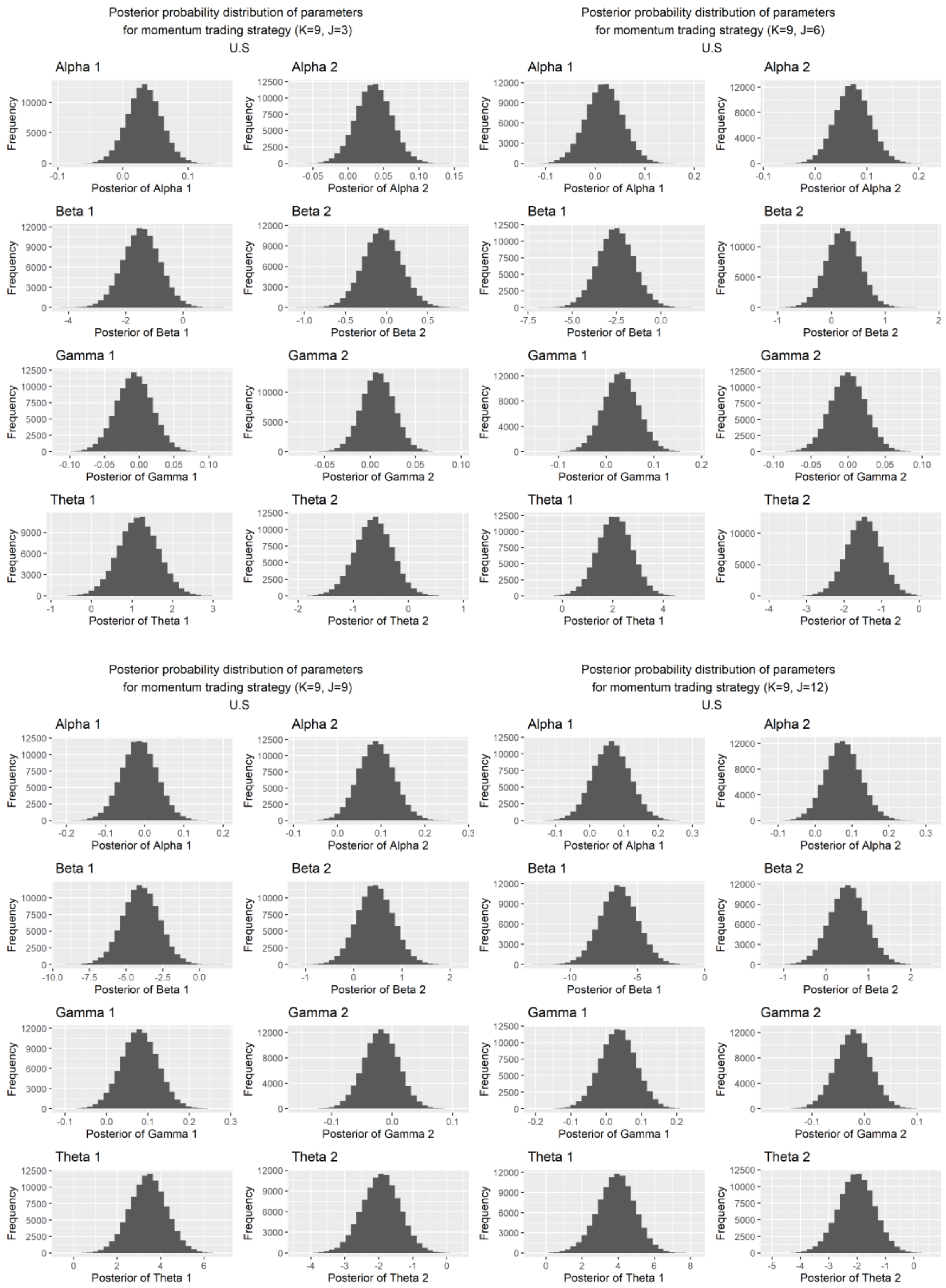

Figure A10.3-Posterior probability distribution of parameters Alpha, Beta, Gamma and Theta in the two-regime switching model during the 9 months ranking period in the US market.

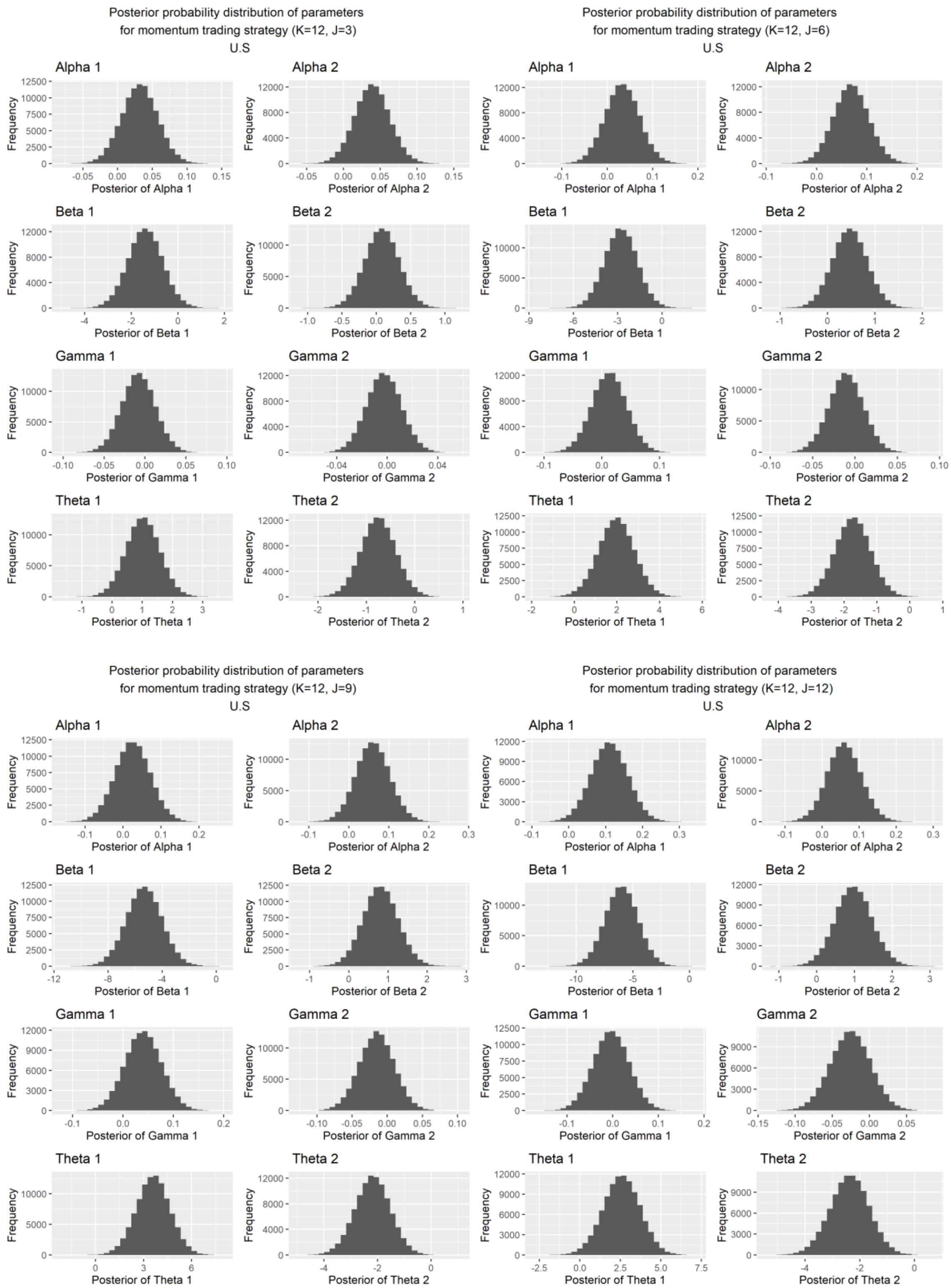

Figure A10.4-Posterior probability distribution of parameters Alpha, Beta, Gamma and Theta in the two-regime switching model during the 12 months ranking period in the US market.

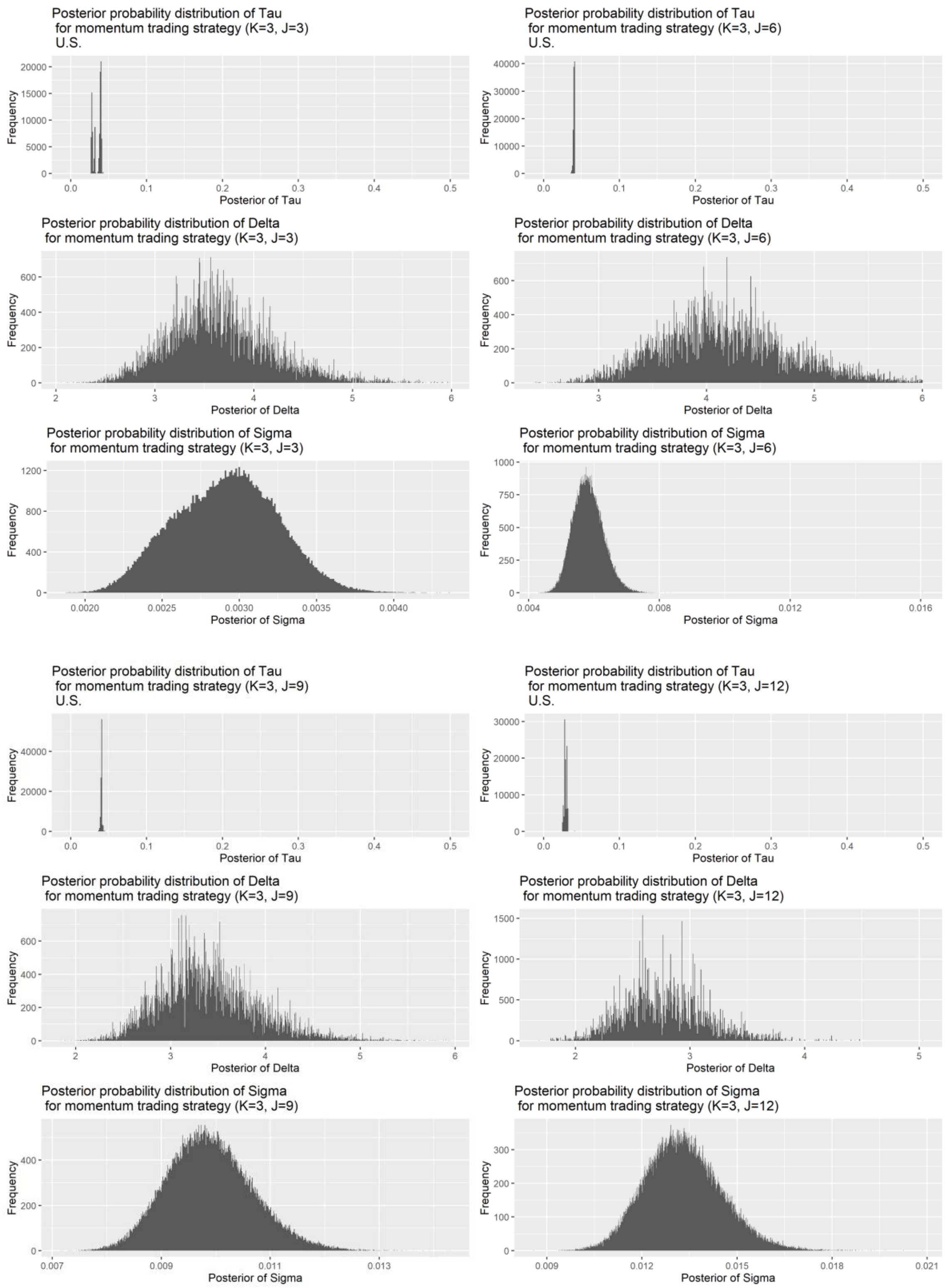

Figure A11.1-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 3 months ranking period in the US market.

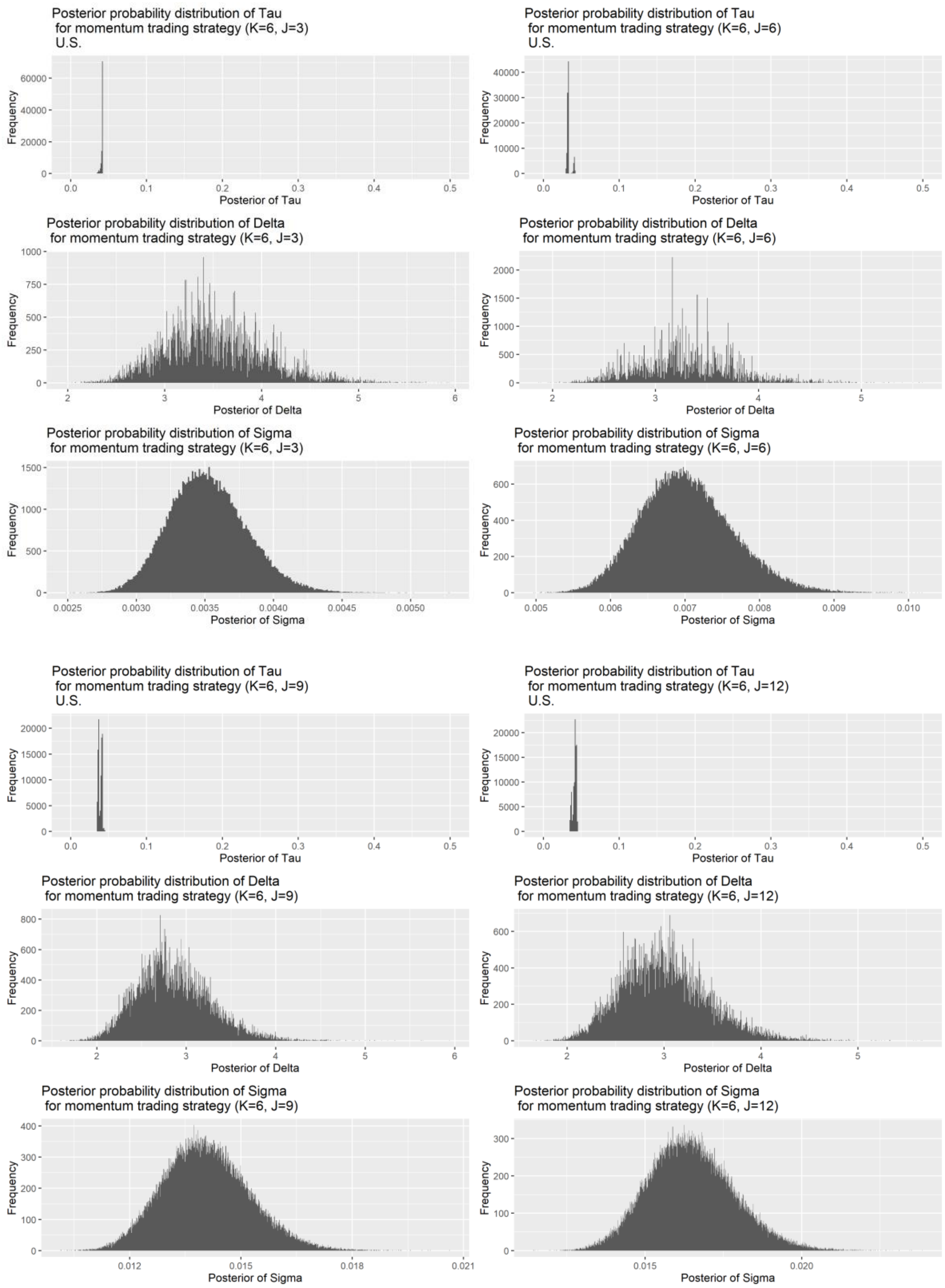

Figure A11.2-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 6 months ranking period in the US market.

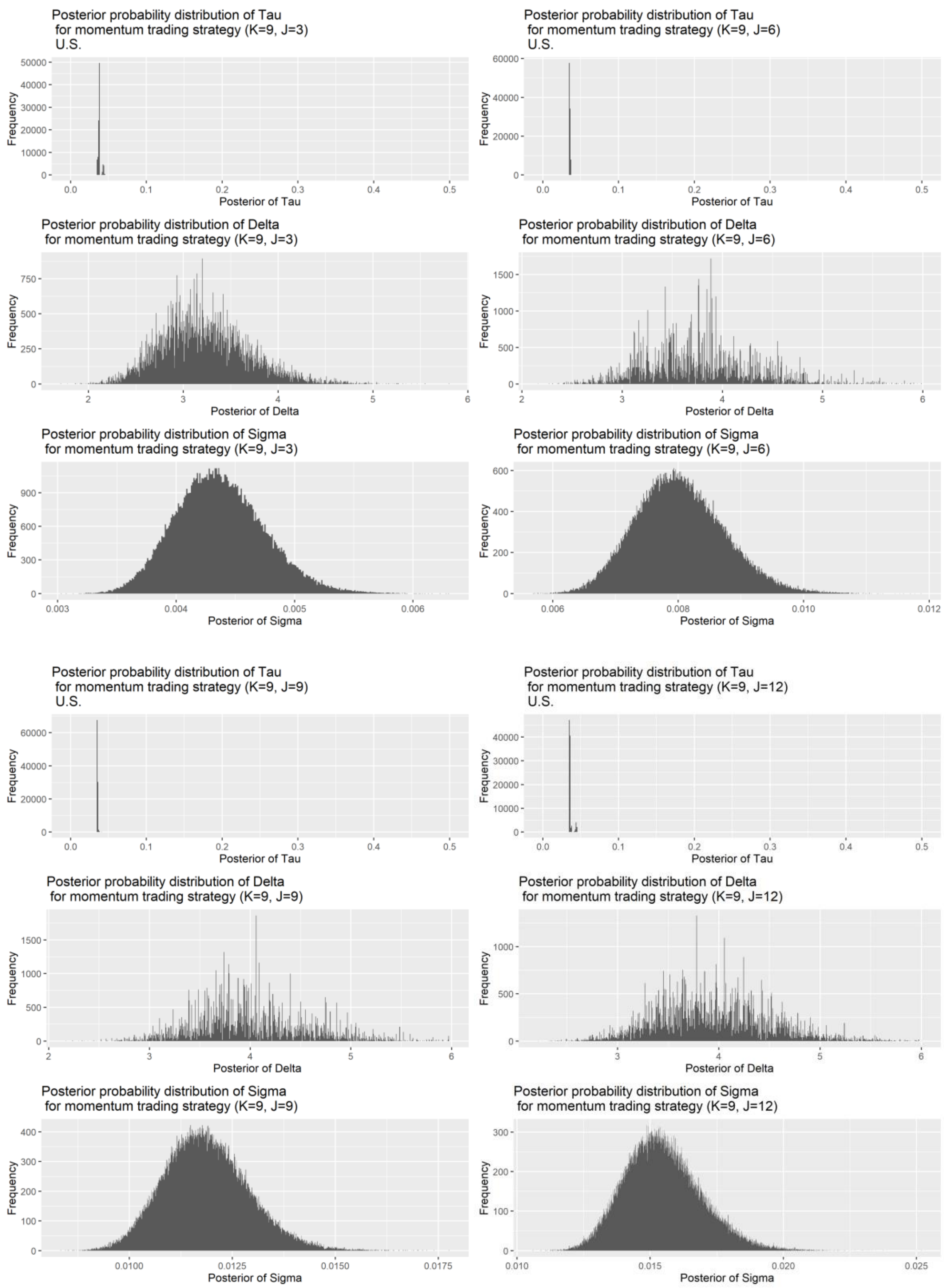

Figure A11.3-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 9 months ranking period in the US market.

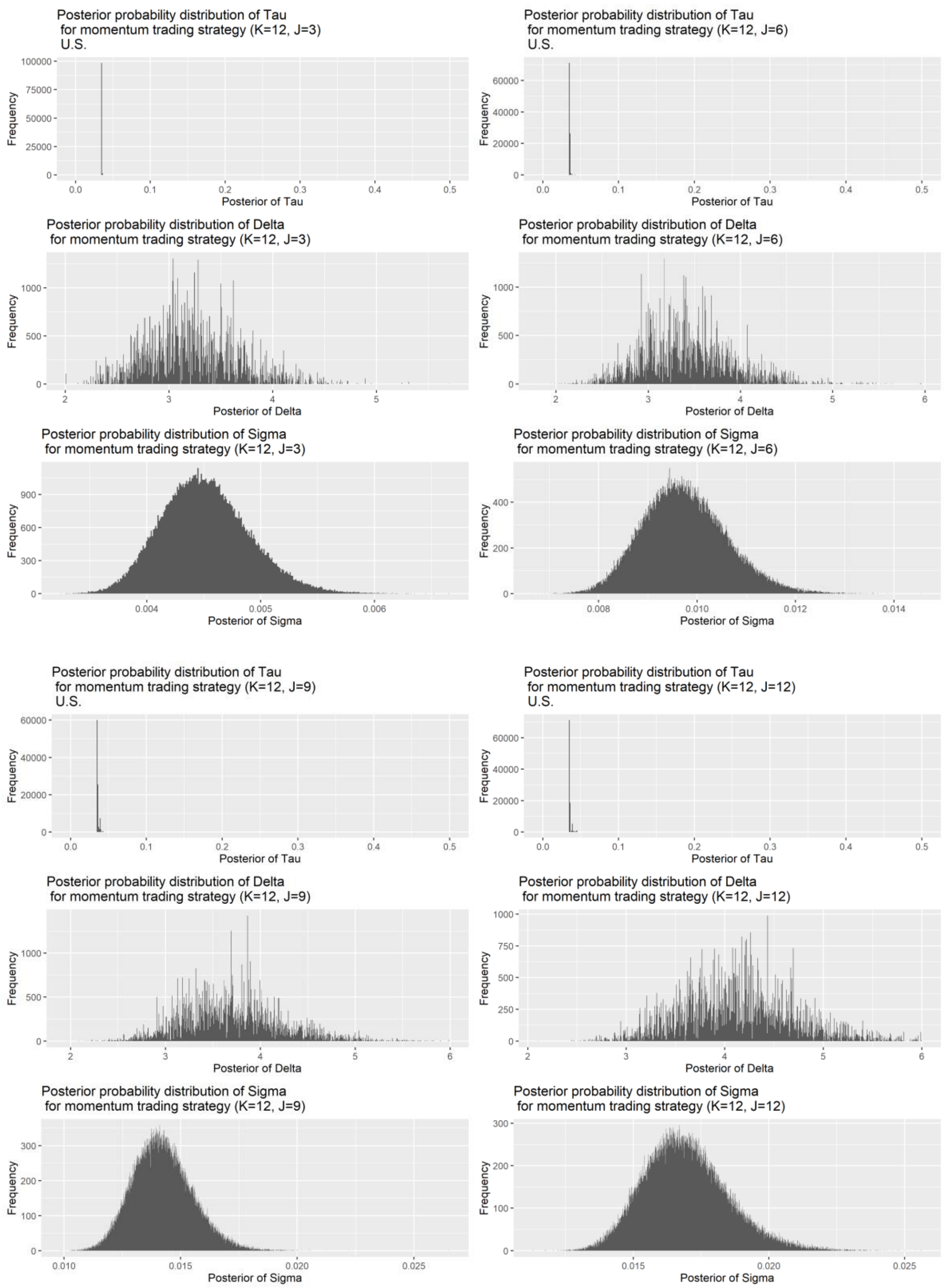

Figure A11.4-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 12 months ranking period in the US market.

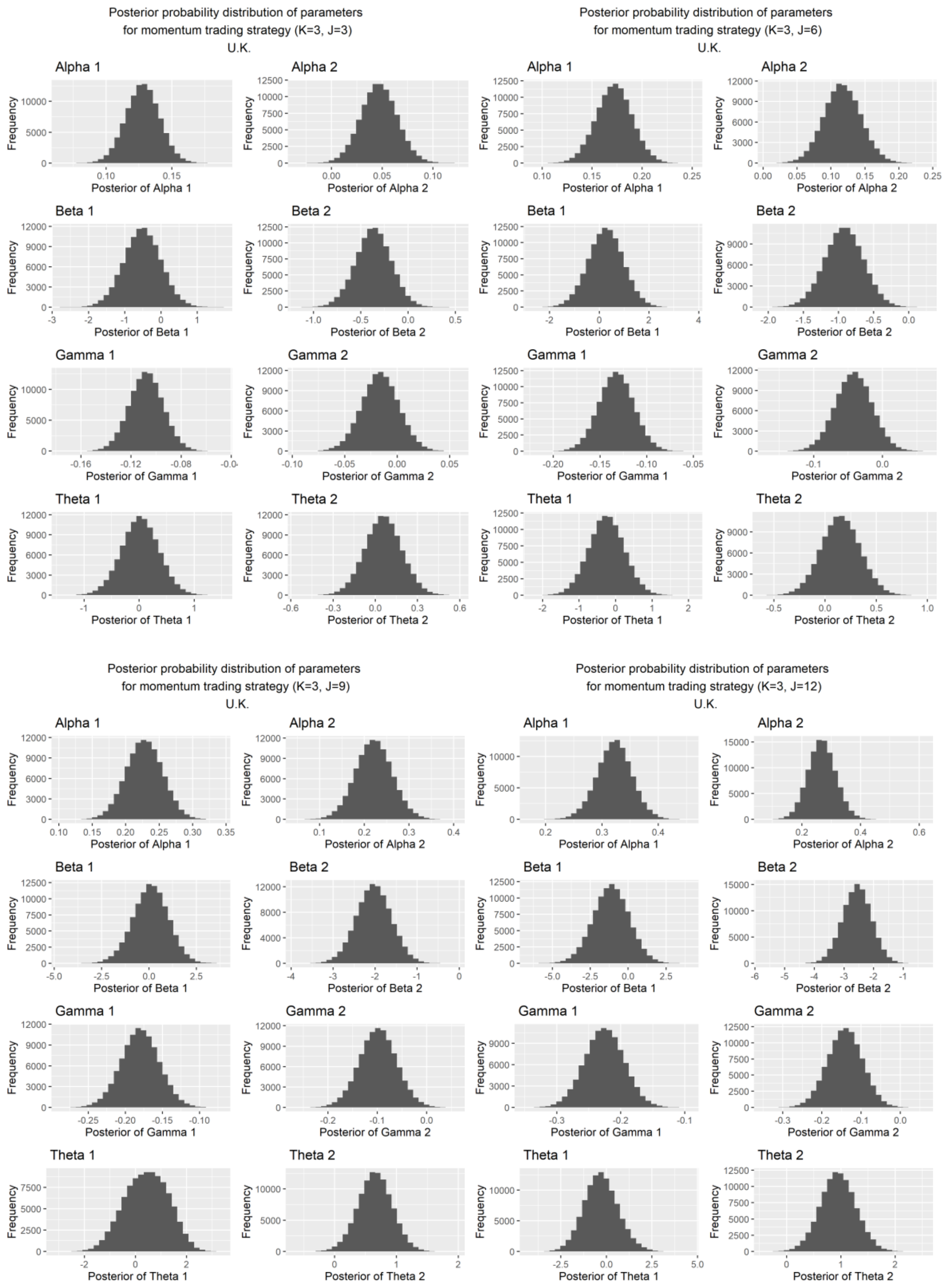

Figure A12.1-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 3 months ranking period in the UK market.

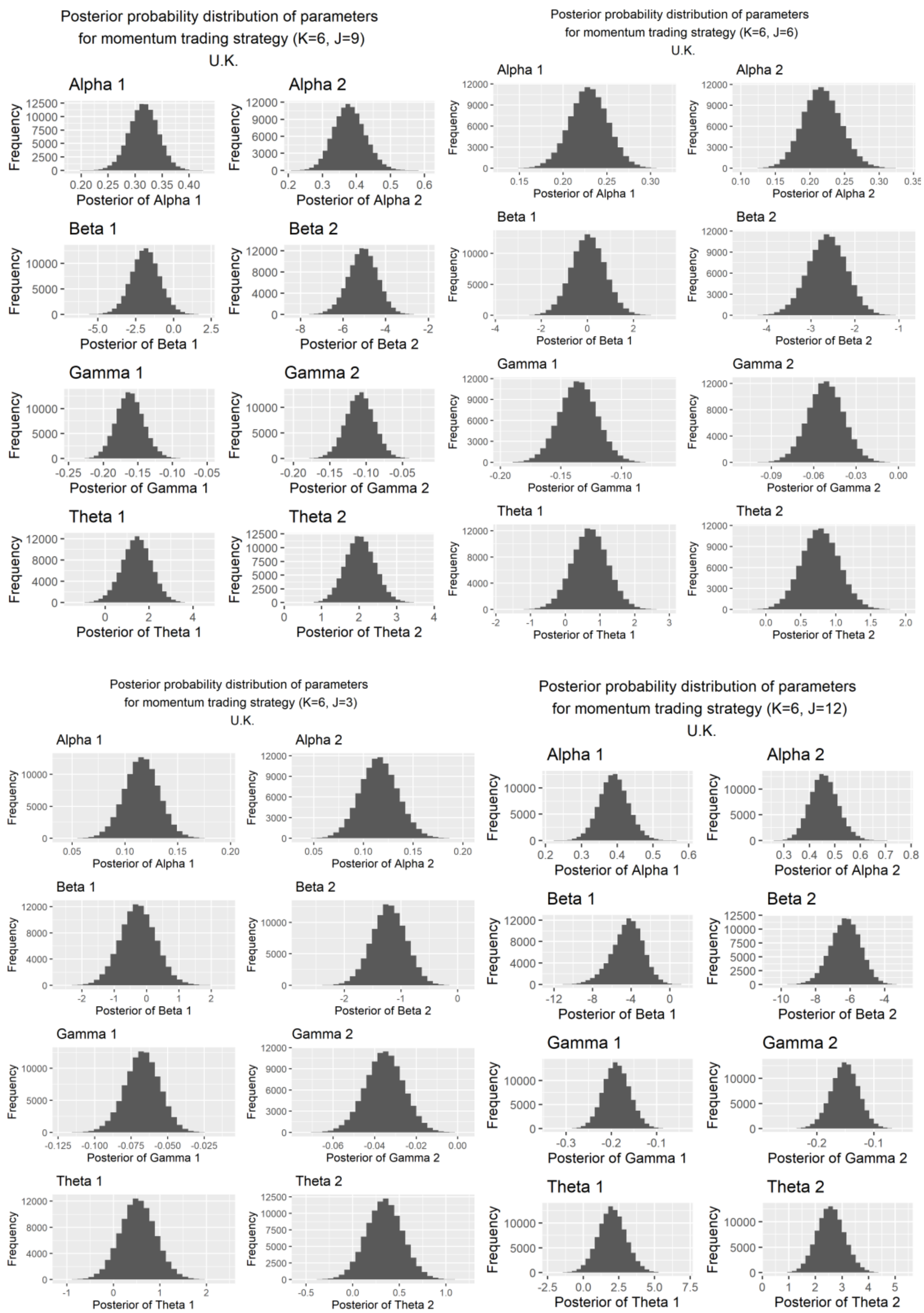

Figure A12.2-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 6 months ranking period in the UK market.

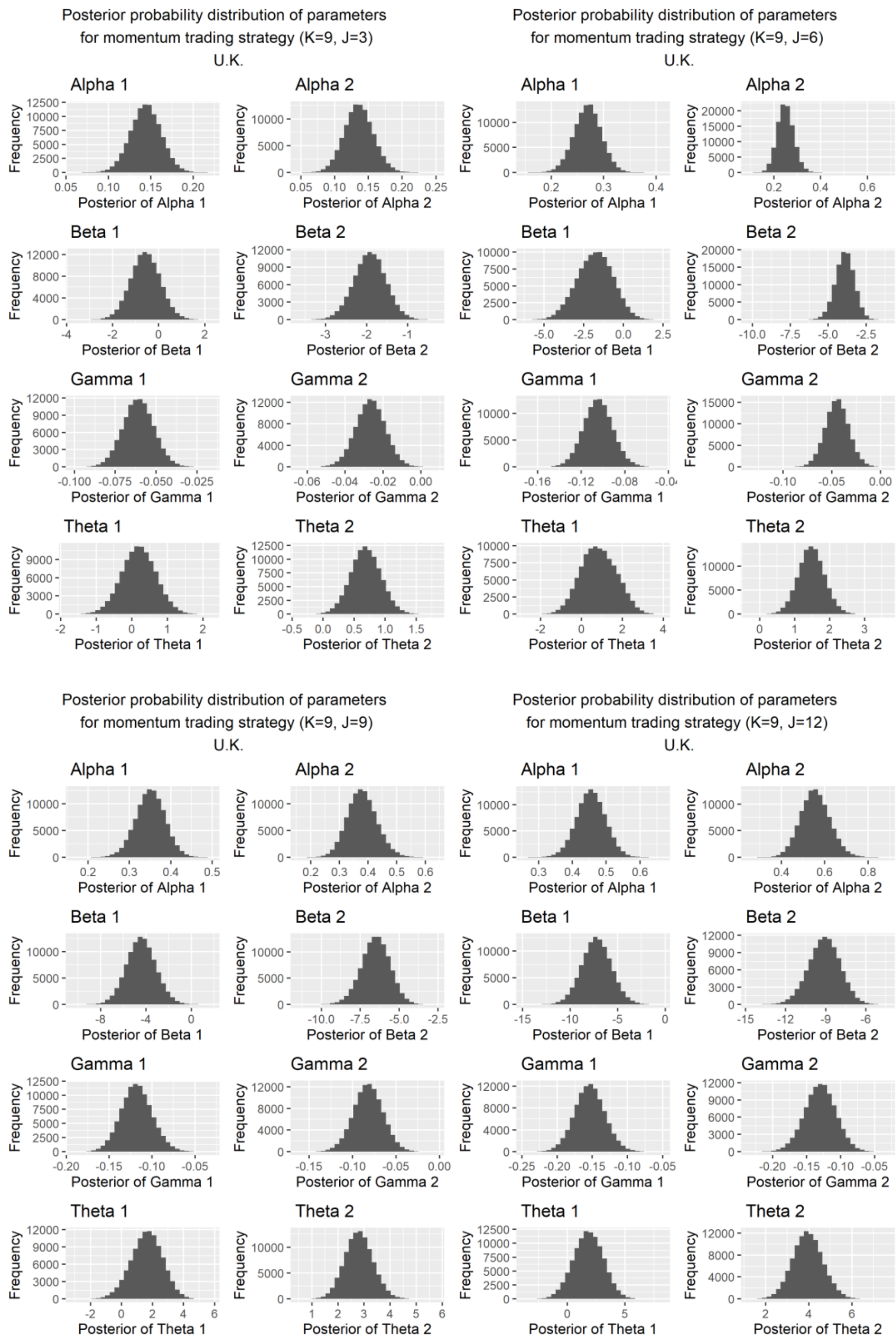

Figure A12.3-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 9 months ranking period in the UK market.

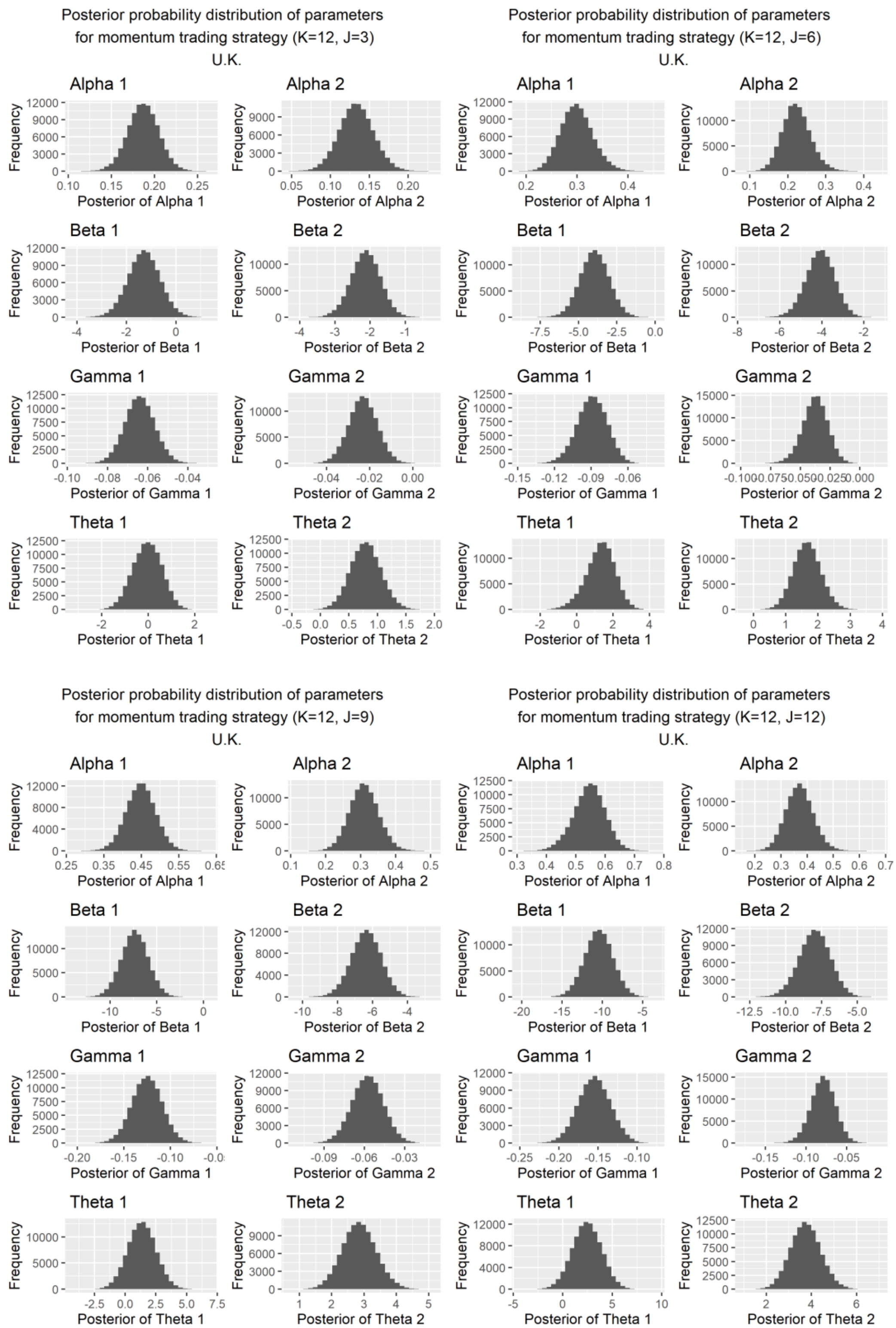

Figure A12.4-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 12 months ranking period in the UK market.

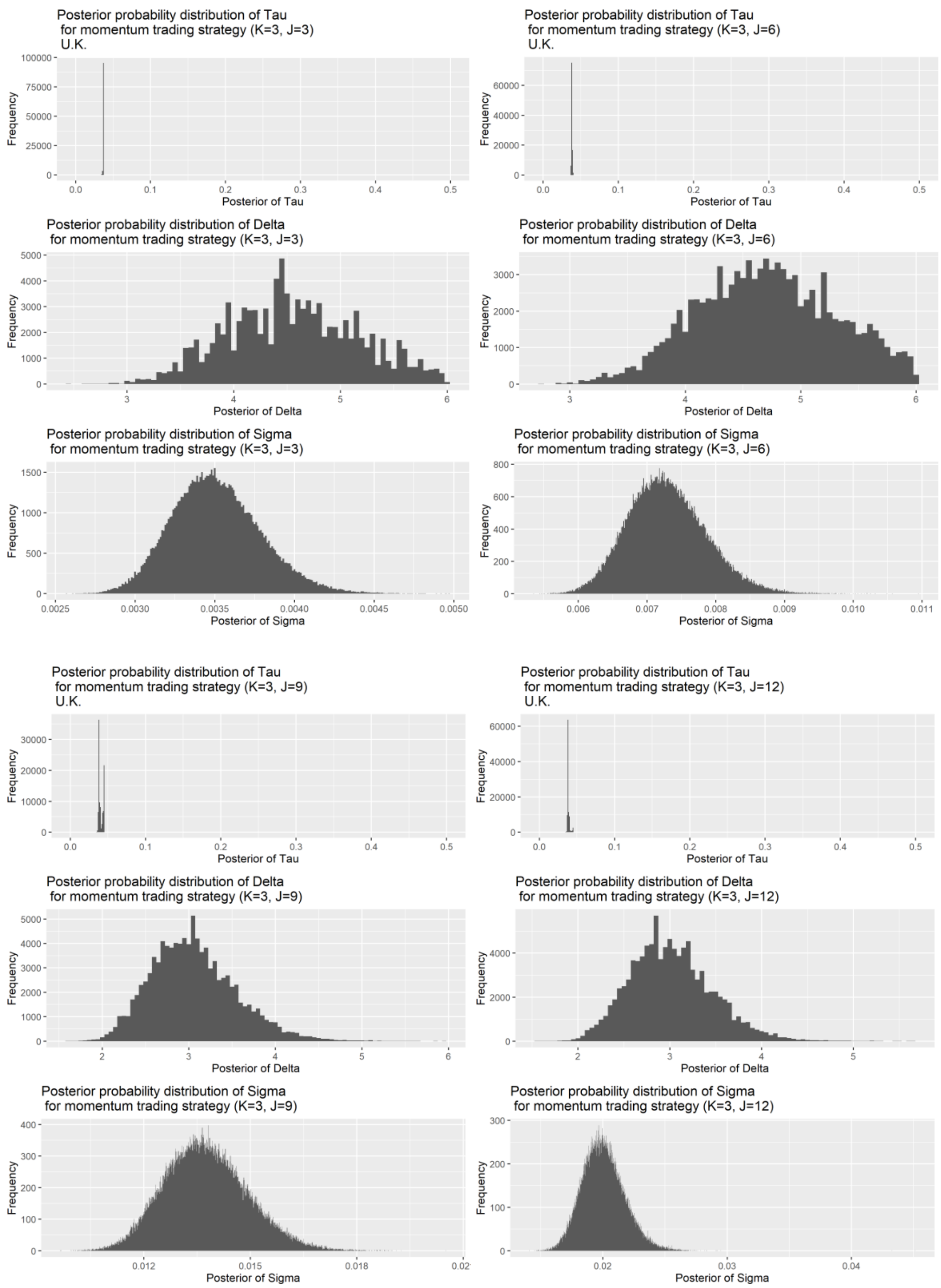

Figure A13.1-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 3 months ranking period in the UK market.

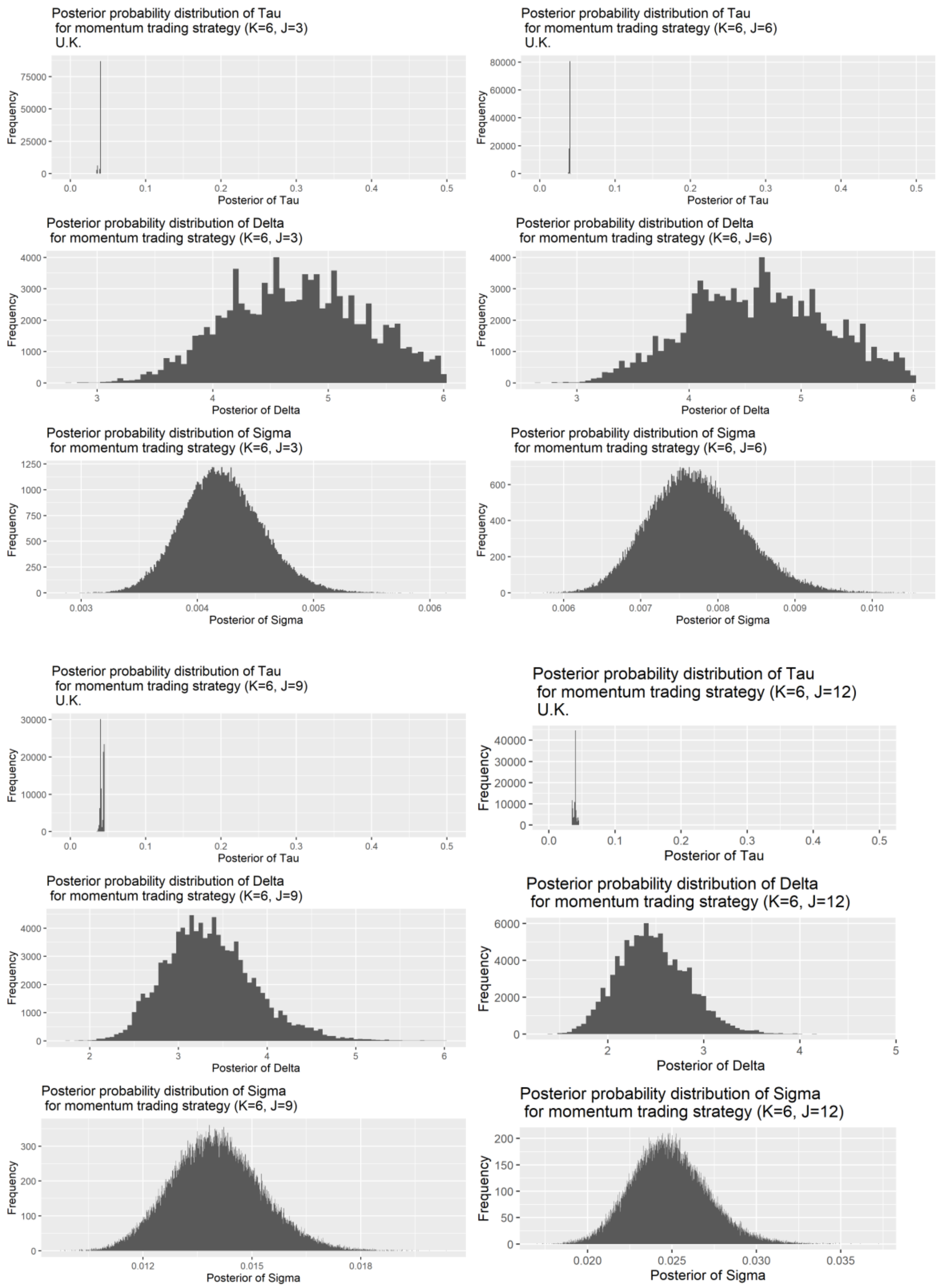

Figure A13.2-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 6 months ranking period in the UK market.

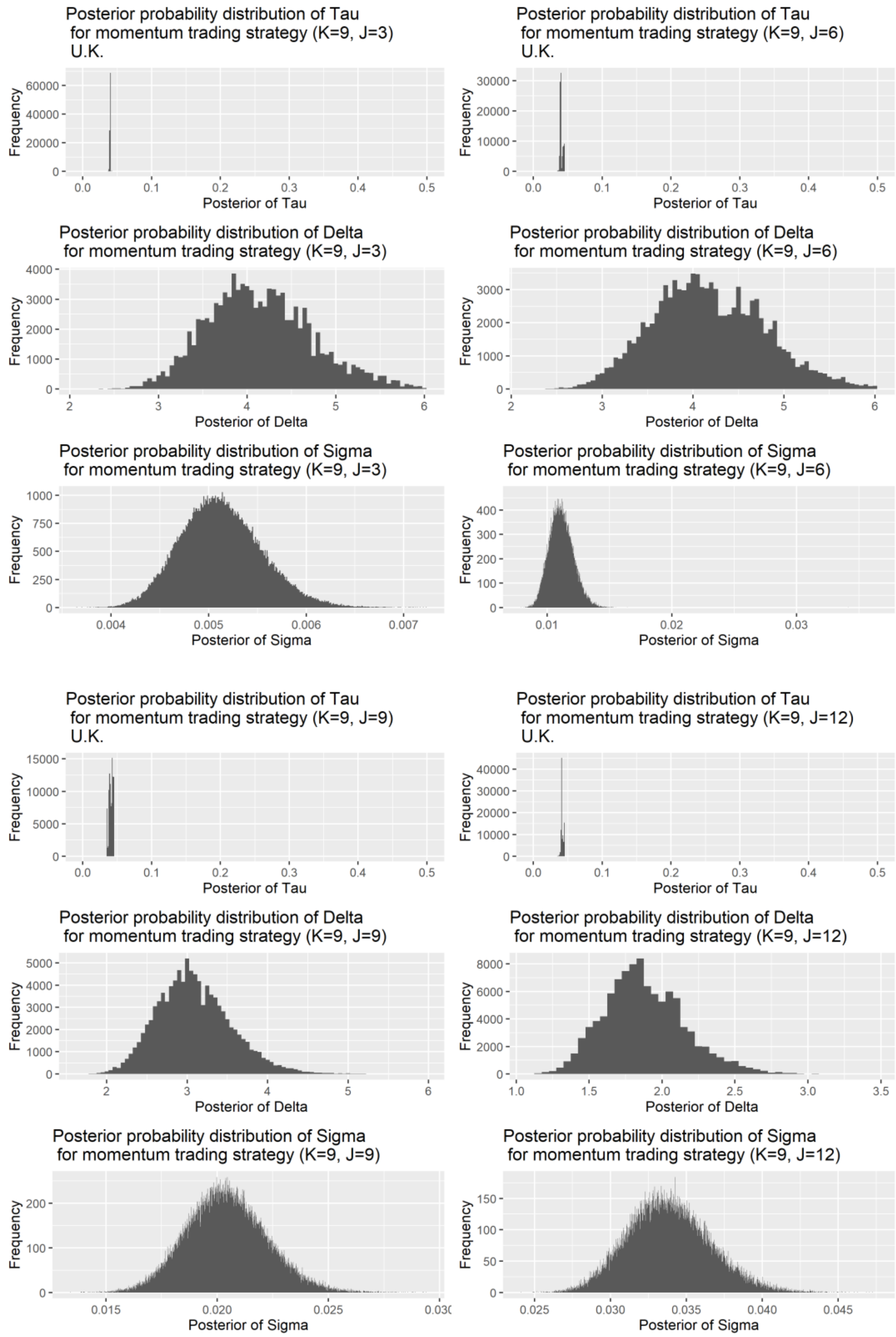

Figure A13.3-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 9 months ranking period in the UK market.

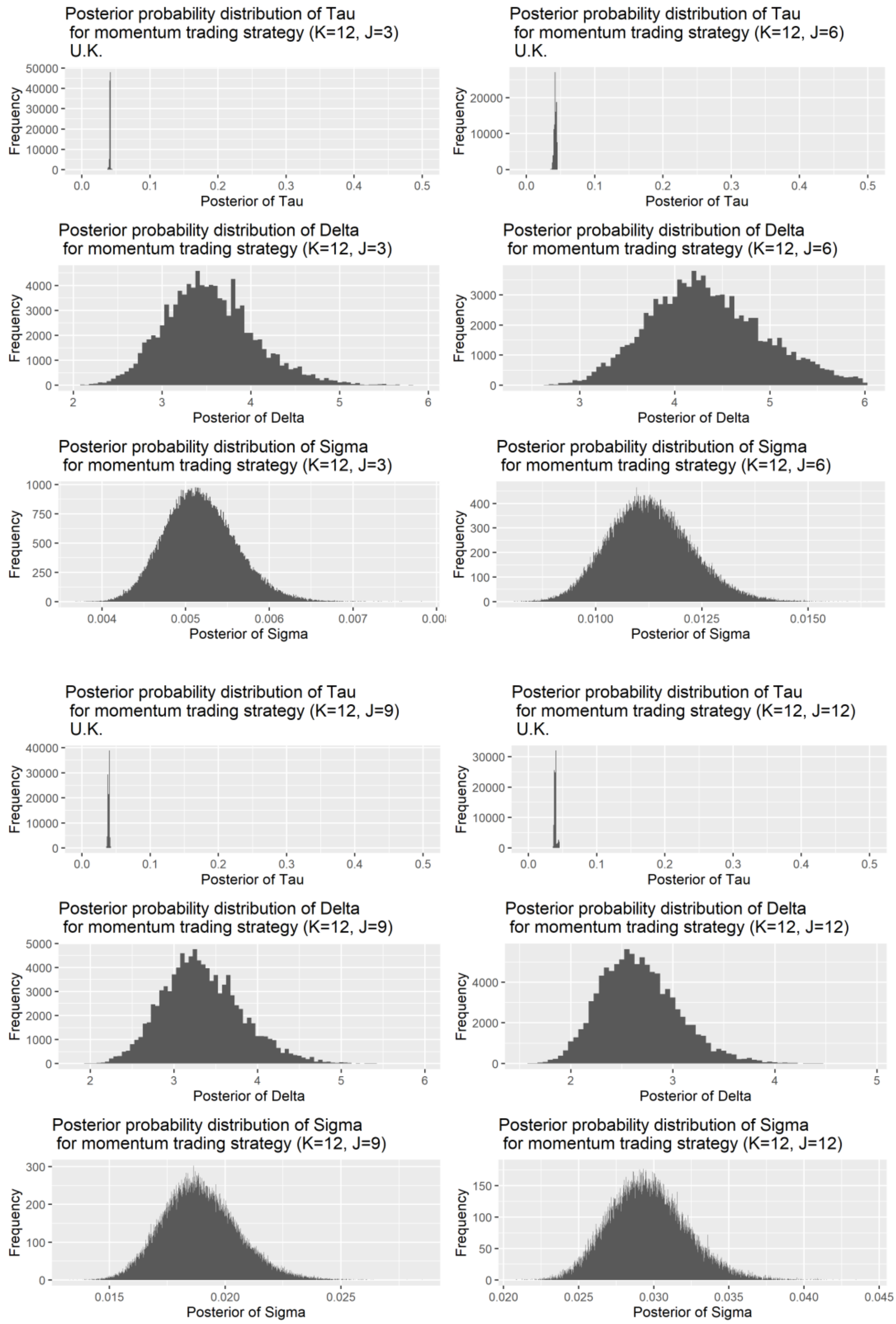

Figure A13.4-Posterior probability distribution of parameters Delta, Tau and Sigma in the two-regime switching model during the 12 months ranking period in the UK market.

	3		6		9		12	
Alpha1								
3	0.025	0.059	0.014	0.054	0.006	0.056	0.179	0.252
6	0.035	0.062	0.083	0.128	0.071	0.123	0.123	0.183
9	0.016	0.049	-0.003	0.042	-0.041	0.015	0.031	0.096
12	0.016	0.048	0.007	0.056	-0.002	0.056	0.081	0.142
Beta1								
3	0.265	0.911	-0.330	0.377	-1.276	-0.364	-3.852	-2.138
6	-0.534	0.111	-1.885	-0.830	-3.637	-2.286	-4.759	-3.319
9	-1.832	-1.056	-3.187	-1.938	-4.723	-3.215	-7.225	-5.568
12	-1.868	-0.929	-3.541	-2.080	-6.151	-4.520	-7.020	-5.105
Gamma1								
3	-0.103	-0.032	-0.051	0.031	-0.008	0.093	-0.380	-0.229
6	-0.071	-0.034	-0.118	-0.058	-0.075	0.001	-0.127	-0.047
9	-0.022	0.010	0.007	0.053	0.054	0.107	0.001	0.064
12	-0.021	0.006	-0.005	0.033	0.018	0.062	-0.027	0.022
Theta1								
3	-0.312	0.077	-0.012	0.455	0.751	1.360	1.855	2.738
6	0.390	0.840	1.103	1.788	2.011	2.975	1.987	3.078
9	0.874	1.447	1.646	2.510	2.897	3.918	3.269	4.503
12	0.662	1.383	1.452	2.489	2.836	4.208	1.866	3.241
Alpha2								
3	-0.052	-0.024	-0.057	-0.018	-0.057	-0.007	0.013	0.062
6	-0.017	0.012	0.006	0.043	0.052	0.105	0.060	0.122
9	0.021	0.052	0.051	0.094	0.065	0.115	0.049	0.109
12	0.025	0.055	0.047	0.091	0.034	0.088	0.032	0.091
Beta2								
3	-0.351	-0.180	-0.575	-0.289	-0.574	-0.282	-0.527	-0.182
6	-0.402	-0.174	-0.474	-0.143	-0.256	0.193	-0.213	0.327
9	-0.193	0.079	0.030	0.434	0.223	0.682	0.268	0.793
12	-0.081	0.237	0.244	0.708	0.522	1.083	0.687	1.277
Gamma2								
3	0.056	0.099	0.077	0.136	0.081	0.154	-0.007	0.072
6	0.027	0.054	0.035	0.071	-0.008	0.046	-0.034	0.023
9	-0.003	0.020	-0.015	0.015	-0.037	0.000	-0.042	0.000
12	-0.012	0.006	-0.023	0.003	-0.031	0.001	-0.041	-0.010
Theta2								
3	0.011	0.224	-0.132	0.171	-0.355	0.032	-0.725	-0.281
6	-0.310	0.006	-0.908	-0.438	-1.770	-1.142	-2.082	-1.317
9	-0.829	-0.425	-1.800	-1.214	-2.268	-1.610	-2.435	-1.653
12	-0.978	-0.524	-2.077	-1.406	-2.569	-1.748	-2.803	-1.959

Table A14.1- Parameters for Alpha, Beta, Gamma and Theta in the two-regime switching model under 90% Bayesian confidence intervals in the US market

	3		6		9		12	
Delta								
3	3.365	3.968	3.694	4.472	3.143	3.738	2.573	3.170
6	3.262	3.843	3.053	3.615	2.633	3.248	2.778	3.352
9	2.980	3.560	3.499	4.070	3.723	4.295	3.667	4.237
12	3.011	3.555	3.162	3.751	3.359	4.011	3.830	4.418
Tau								
3	0.033	0.038	0.040	0.040	0.040	0.041	0.028	0.032
6	0.041	0.042	0.032	0.035	0.038	0.040	0.040	0.042
9	0.037	0.039	0.035	0.036	0.035	0.036	0.035	0.037
12	0.035	0.036	0.035	0.037	0.035	0.037	0.035	0.037
Sigma								
3	0.003	0.003	0.005	0.007	0.009	0.010	0.013	0.014
6	0.003	0.004	0.007	0.007	0.013	0.015	0.016	0.017
9	0.004	0.005	0.008	0.009	0.011	0.013	0.015	0.017
12	0.004	0.005	0.009	0.010	0.013	0.016	0.016	0.018

Table A14.2- Parameters for Delta, Tau and Sigma in the two-regime switching model under 90% Bayesian confidence intervals in the US market

	3		6		9		12	
Alpha1	3		6		9		12	
3	0.119	0.136	0.161	0.183	0.211	0.244	0.302	0.341
6	0.106	0.126	0.216	0.242	0.299	0.332	0.367	0.418
9	0.133	0.155	0.253	0.288	0.327	0.372	0.429	0.483
12	0.176	0.198	0.282	0.320	0.425	0.474	0.514	0.578
Beta1								
3	-0.829	-0.215	-0.154	0.722	-0.551	0.609	-1.854	-0.409
6	-0.595	0.091	-0.447	0.503	-2.483	-1.285	-5.439	-3.507
9	-0.985	-0.177	-2.409	-1.167	-5.238	-3.602	-8.217	-6.260
12	-1.683	-0.920	-4.583	-3.382	-8.273	-6.337	-11.692	-9.407
Gamma1								
3	-0.117	-0.100	-0.144	-0.121	-0.195	-0.165	-0.244	-0.208
6	-0.075	-0.060	-0.144	-0.126	-0.174	-0.148	-0.207	-0.168
9	-0.066	-0.055	-0.113	-0.096	-0.128	-0.106	-0.166	-0.140
12	-0.069	-0.059	-0.097	-0.082	-0.136	-0.116	-0.167	-0.143
Theta1								
3	-0.180	0.225	-0.528	0.071	-0.057	0.877	-0.853	0.332
6	0.297	0.774	0.370	1.011	1.024	1.858	1.430	2.699
9	-0.080	0.481	0.344	1.308	1.008	2.225	1.034	2.655
12	-0.458	0.333	0.755	1.773	0.606	2.095	1.525	3.392
Alpha2								
3	0.036	0.058	0.101	0.134	0.197	0.246	0.239	0.311
6	0.105	0.127	0.201	0.234	0.355	0.410	0.427	0.499
9	0.123	0.150	0.224	0.302	0.349	0.414	0.515	0.600
12	0.121	0.146	0.202	0.251	0.283	0.336	0.339	0.409
Beta2								
3	-0.476	-0.244	-1.084	-0.759	-2.285	-1.767	-2.958	-2.248
6	-1.413	-1.025	-2.899	-2.384	-5.520	-4.664	-6.840	-5.771
9	-2.161	-1.698	-4.667	-3.470	-7.151	-5.960	-9.864	-8.442
12	-2.393	-1.868	-4.561	-3.681	-6.914	-5.852	-8.662	-7.352
Gamma2								
3	-0.027	-0.006	-0.059	-0.026	-0.121	-0.077	-0.169	-0.114
6	-0.041	-0.030	-0.059	-0.044	-0.121	-0.096	-0.167	-0.136
9	-0.032	-0.022	-0.056	-0.037	-0.093	-0.071	-0.144	-0.117
12	-0.027	-0.019	-0.045	-0.030	-0.065	-0.051	-0.091	-0.069
Theta2								
3	-0.025	0.133	0.041	0.265	0.477	0.825	0.757	1.159
6	0.221	0.460	0.623	0.945	1.783	2.272	2.264	2.944
9	0.542	0.844	1.245	1.766	2.479	3.188	3.563	4.436
12	0.632	0.959	1.399	1.963	2.552	3.165	3.349	4.175

Table A15.1- Parameters for Alpha, Beta, Gamma and Theta in the two-regime switching model under 90% Bayesian confidence intervals in the UK market

	3		6		9		12	
Delta								
3	4.225	4.812	4.356	4.956	2.802	3.426	2.693	3.351
6	4.417	4.971	4.313	4.883	3.090	3.699	2.275	2.768
9	3.826	4.426	3.686	4.474	2.846	3.454	1.739	2.082
12	3.288	3.855	4.023	4.603	3.092	3.674	2.427	3.048
Tau								
3	0.037	0.038	0.038	0.038	0.039	0.042	0.038	0.039
6	0.039	0.040	0.039	0.040	0.041	0.043	0.038	0.040
9	0.039	0.040	0.040	0.042	0.040	0.042	0.041	0.042
12	0.041	0.042	0.041	0.043	0.038	0.040	0.039	0.040
Sigma								
3	0.003	0.004	0.007	0.008	0.013	0.015	0.019	0.023
6	0.039	0.040	0.007	0.008	0.013	0.015	0.024	0.026
9	0.005	0.005	0.010	0.014	0.019	0.022	0.032	0.036
12	0.005	0.006	0.011	0.012	0.018	0.020	0.028	0.032

Table A15.2- Parameters for Delta, Tau and Sigma in the two-regime switching model under 90% Bayesian confidence intervals in the UK market

Appendix B

A candidate assets pricing model for log-normal approximation,

$$E_t\left[M_{t+1}R_{x,t+1}\right] = 1 \qquad \text{B1.1}$$

Where M_{t+1} denotes the stochastic discount factor or pricing kernel in this model, and $R_{x,t+1}$ denotes the gross return on any assets x.

Taking logs of asset pricing equation for equity returns,

$$0 = log\ [E_t\left(M_{t+1}R_{x,t+1}\right)] \qquad \text{B1.2}$$

According to the log-normal distribution that if Y is a normal distribution, then the exponential function of Y, $y = ln\ (Y)$, is a normal distribution. Thus, if Y follows a normal distribution with mean $E(y) = \pi$ and variance $var(y) = \omega^2$, $Y \sim N(\pi, \omega^2)$, the expectation of the exponential function can be shown as,

$$E[exp(Y)] = exp\ (E(y) + \frac{var(y)}{2})$$

Or,

$$logE[Y] = E(y) + \frac{var(y)}{2} \qquad \text{B1.3}$$

Then, if $Y = XZ$, and $y = logY$, $x = logX$ and $z = logZ$

$$logE[Y] = E(\log(Y)) + \frac{var(\log(Y))}{2} \qquad \text{B1.4}$$

$$E(\log(Y)) = E(\log(XZ)) + \frac{var(\log(XZ))}{2}$$

$$= E(\log(X) + \log(Z)) + \frac{var(\log(X) + \log(Z))}{2}$$

$$= E(\log(X)) + E(\log(Z)) + \frac{var(\log(X) + \log(Z))}{2}$$

$$= E(x) + E(z) + \frac{var(x+z)}{2}$$

Thus, taking logs of B1.2 for the risk-free rate equation, $R_{x,t+1} = R_{f,t+1}$,

$$0 = log\left(R_{f,t+1}\right) + logE_t(M_{t+1})$$

Or,

$$r_{f,t+1} = -logE_t(M_{t+1}) \qquad \text{B1.5}$$

If the pricing kernel is conditionally log-normal due to the B1.3,

$$logE_t(M_{t+1}) = E_t(m_{t+1}) + \frac{1}{2}var_t(m_{t+1}) \qquad \text{B1.6}$$

Where $m_{t+1} = \log(M_{t+1})$. And therefore,

$$r_{f,t+1} = -E_t(m_{t+1}) - \frac{1}{2}var_t(m_{t+1}) \qquad \text{B1.7}$$

Moreover, according to the B1.2 and B1.3, if equity returns and pricing kernel are jointly log-normal,

$$\begin{aligned} &logE_t\left(M_{t+1}R_{x,t+1}\right) \qquad \text{B1.8} \\ &= E_t(m_{t+1}) + E_t\left(r_{x,t+1}\right) + \frac{1}{2}var_t(m_{t+1}) \\ &+ cov_t\left(m_{t+1}, r_{x,t+1}\right) + \frac{1}{2}var_t\left(r_{x,t+1}\right) = 0 \end{aligned}$$

Rearrange B1.8, and substitute B1.7,

$$E_t(r_{x,t+1}) = -E_t(m_{t+1}) - \frac{1}{2}var_t(m_{t+1}) \qquad \text{B1.9}$$

$$- cov_t(m_{t+1}, r_{x,t+1}) - \frac{1}{2}var_t(r_{x,t+1})$$

$$= r_{f,t+1} - cov_t(m_{t+1}, r_{x,t+1}) - \frac{1}{2}var_t(r_{x,t+1})$$

Thus, the risk premium for $r_{x,t+1}$ can be written as below,

$$E_t(r_{x,t+1}) - r_{f,t+1} = -cov_t(m_{t+1}, r_{x,t+1}) - \frac{1}{2}var_t(r_{x,t+1}) \qquad \text{B1.10}$$

$$= E_t(r_{x,t+1} - r_{f,t+1})$$

This section describes the long-run risk model with time-varying volatility of consumption growth and dividend growth,

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1} \qquad \text{B2.1}$$

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1} \qquad \text{B2.2}$$

$$g_{i,t+1} = \mu_i + \phi_i x_t + \varphi_i \sigma_t u_{i,t+1} + \varphi_{i,m} \sigma_t v_{t+1} \qquad \text{B2.3}$$

$$\sigma_{t+1}^2 = \sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \qquad \text{B2.4}$$

$$e_{t+1}, \eta_{t+1}, u_{i,t+1}, v_{t+1}, w_{t+1} \sim N.i.i.d.(0,1) \qquad \text{B2.5}$$

The intertemporal marginal rate of substitution (IMRS) or pricing kernel for this economy can be shown as,

$$ln\, M_{t+1} = m_{t+1} = \theta\, ln\, \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1)\, r_{a,t+1} \qquad \text{B2.6}$$

Again, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. According to the standard asset pricing condition $E_t[M_{t+1}R_{x,t+1}] = 1$,

$$E_t\left[exp\left(\theta\, ln\,\delta - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{a,t+1} + r_{x,t+1}\right)\right] = 1 \qquad \text{B2.7}$$

Where $r_{a,t+1}$ denotes the return on the aggregate consumption claim, $R_{x,t+1}$ means asset pricing restriction on any continuous return, and $r_{x,t+1} = \log R_{x,t+1} = r_{a,t+1}$ for solving the special case.

Initially, a particular method is used to solve the return on the consumption claim asset ($r_{a,t+1}$) with a Campbell-Shiller approximation, which can be followed as,

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \qquad \text{B2.8}$$

Here, z_t is the log price-consumption ratio, which is the endogenous variable. As a conjecture, an expression form of z_t,

$$z_t = A_0 + A_1 x_t + A_2\sigma_t^2 \qquad \text{B2.9}$$

Then, substitute equation B2.9 to B2.8,

$$r_{a,t+1} = \kappa_0 + \kappa_1(A_0 + A_1 x_{t+1} + A_2\sigma_{t+1}^2) - (A_0 + A_1 x_t + A_2\sigma_t^2) + g_{t+1} \qquad \text{B2.10}$$

$$= \kappa_0 + \kappa_1 A_0 + \kappa_1 A_1 x_{t+1} + \kappa_1 A_2 \sigma_{t+1}^2 - A_0 - A_1 x_t - A_2\sigma_t^2 + g_{t+1}$$

Further, substitute B2.1, B2.2 and B2.4 into B2.10,

$$r_{a,t+1} = \kappa_0 + \kappa_1 A_0 + \kappa_1 A_1(\rho x_t + \varphi_e \sigma_t e_{t+1}) + \kappa_1 A_2[\sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}] - A_0 - A_1 x_t - A_2\sigma_t^2 + \mu + x_t + \sigma_t \eta_{t+1} \qquad \text{B2.11}$$

Again, due to $r_{x,t+1} = r_{a,t+1}$ and B2.11, B2.7 can be rewritten as follows,

$$E\left\{exp\left[\theta ln\delta-\frac{\theta}{\psi}(\mu+x_t+\sigma_t\eta_{t+1})+\theta r_{a,t+1}\right]\right\} \qquad \text{B2.12}$$

$$= E\{exp[\theta\, ln\,\delta-\frac{\theta}{\psi}(\mu+x_t+\sigma_t\eta_{t+1}) + \theta[\kappa_0+\kappa_1A_0+\kappa_1A_1(\rho x_t+\varphi_e\sigma_te_{t+1}) + \kappa_1A_2[\sigma^2+v_1(\sigma_t^2-\sigma^2)+\sigma_ww_{t+1}]-A_0 - A_1x_t-A_2\sigma_t^2+\mu+x_t+\sigma_t\eta_{t+1}]]\}$$

To obtain the A_1 and A_2, due to the B1.3, Y, π and ω^2 can be obtained from equation B2.12,

$$Y=\theta\, ln\,\delta-\frac{\theta}{\psi}(\mu+x_t+\sigma_t\eta_{t+1}) + \theta[\kappa_0+\kappa_1A_0+\kappa_1A_1(\rho x_t+\varphi_e\sigma_te_{t+1}) + \kappa_1A_2[\sigma^2+v_1(\sigma_t^2-\sigma^2)+\sigma_ww_{t+1}]-A_0-A_1x_t-A_2\sigma_t^2+\mu+x_t + \sigma_t\eta_{t+1}]$$

$$\pi=\theta\, ln\,\delta-\frac{\theta}{\psi}\mu-\frac{\theta}{\psi}x_t+\theta\kappa_0+\theta\kappa_1A_0+\theta\kappa_1A_1\rho x_t+\theta\kappa_1A_2(\sigma^2+v_1\sigma_t^2-v_1\sigma^2)-\theta A_0 - \theta A_1x_t-\theta A_2\sigma_t^2+\theta\mu+\theta x_t$$

$$\omega^2=\frac{\theta^2}{\psi^2}\sigma_t^2+\theta^2\kappa_1^2A_1^2\varphi_e^2\sigma_t^2+\theta^2\kappa_1^2A_2^2\sigma_w^2+\theta^2\sigma_t^2$$

Since the $e_{t+1}, \eta_{t+1}, u_{i,t+1}, w_{t+1} \sim N.i.i.d.(0,1)$, $w_{t+1}^2=e_{t+1}^2=\mu_{i,t+1}^2=\eta_{t+1}^2=1$. Thus, due to B1.3,

$$\pi+\frac{\omega^2}{2}=\theta\, ln\,\delta-\frac{\theta}{\psi}\mu-\frac{\theta}{\psi}x_t+\theta\kappa_0+\theta\kappa_1A_0+\theta\kappa_1A_1\rho x_t+\theta\kappa_1A_2(\sigma^2+v_1\sigma_t^2-v_1\sigma^2) - \theta A_0-\theta A_1x_t-\theta A_2\sigma_t^2+\theta\mu+\theta x_t+\frac{1}{2}\frac{\theta^2}{\psi^2}\sigma_t^2+\frac{1}{2}\theta^2\kappa_1^2A_1^2\varphi_e^2\sigma_t^2 + \frac{1}{2}\theta^2\kappa_1^2A_2^2\sigma_w^2+\frac{1}{2}\theta^2\sigma_t^2-\frac{\theta^2}{\psi}\sigma_t^2$$

$$= \theta\, ln\, \delta - \frac{\theta}{\psi}\mu + \theta\kappa_0 + \theta\kappa_1 A_0 + \theta\kappa_1 A_2\sigma^2 - \theta\kappa_1 A_2 v_1\sigma^2 - \theta A_0 + \theta\mu + \frac{1}{2}\theta^2\kappa_1^2 A_2^2\sigma_w^2$$
$$+ (-\frac{\theta}{\psi} + \theta\kappa_1 A_1\rho - \theta A_1 + \theta)x_t + (\theta\kappa_1 A_2 v_1 - \theta A_2 + \frac{1}{2}\frac{\theta^2}{\psi^2} + \frac{1}{2}\theta^2\kappa_1^2 A_1^2\varphi_e^2$$
$$+ \frac{1}{2}\theta^2 - \frac{\theta^2}{\psi})\sigma_t^2$$

$\pi + \frac{\omega^2}{2} = 0$, all terms involving x_t and σ_t^2 must be satisfied as,

$$\left(-\frac{\theta}{\psi} + \theta\kappa_1 A_1\rho - \theta A_1 + \theta\right)x_t = 0 \qquad \text{B2.13}$$

$$\left(\theta\kappa_1 A_2 v_1 - \theta A_2 + \frac{1}{2}\frac{\theta^2}{\psi^2} + \frac{1}{2}\theta^2\kappa_1^2 A_1^2\varphi_e^2 + \frac{1}{2}\theta^2 - \frac{\theta^2}{\psi}\right)\sigma_t^2 = 0 \qquad \text{B2.14}$$

Then, equation 2.13 and 2.14 divided by θx_t and $\theta\sigma_t^2$, respectively, and rearrange both and A_0,

$$A_0 = \frac{ln\,\delta + (1 - \frac{1}{\psi})\mu + \kappa_0 + \kappa_1 A_2(1 - v_1)\sigma^2 + \frac{1}{2}\theta\kappa_1^2 A_2^2\sigma_w^2}{1 - \kappa_1} \qquad \text{B2.15}$$

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1\rho} \qquad \text{B2.16}$$

$$A_2 = \frac{0.5[(\theta - \frac{\theta}{\psi})^2 + (\theta A_1\kappa_1\varphi_e)^2]}{\theta(1 - \kappa_1 v_1)} \qquad \text{B2.17}$$

Therefore, given the solution of z_t, the innovation to the return $r_{a,t+1}$ can be derived by B2.11,

$$r_{a,t+1} - E_t(r_{a,t+1}) = \kappa_1 A_1\varphi_e\sigma_t e_{t+1} + \kappa_1 A_2\sigma_w w_{t+1} + \sigma_t\eta_{t+1} \qquad \text{B2.18}$$

$$= C\sigma_t e_{t+1} + \kappa_1 A_2\sigma_w w_{t+1} + \sigma_t\eta_{t+1}$$

Where $C = \kappa_1 A_1\varphi_e$. Hence, the conditional variance of $r_{a,t}$ can be shown as,

$$var_t(r_{a,t+1}) = (1 + C^2)\sigma_t^2 + \kappa_1^2 A_2^2 \sigma_w^2 \qquad \text{B2.20}$$

The intertemporal marginal rate of substitute

According to the pricing kernel B2.6, B2.2 and B2.11 are substituted,

$$\begin{aligned} m_{t+1} = \theta\, ln\, \delta - \frac{\theta}{\psi}(\mu + x_t + \sigma_t \eta_{t+1}) + (\theta - 1)\{\kappa_0 + \kappa_1 A_0 \\ + \kappa_1 A_1(\rho x_t + \varphi_e \sigma_t e_{t+1}) \\ + \kappa_1 A_2[\sigma^2 + v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}] - A_0 \\ - A_1 x_t - A_2 \sigma_t^2 + \mu + x_t + \sigma_t \eta_{t+1}\} \end{aligned} \qquad \text{B3.1}$$

Hence, the conditional mean for m_t by substituting the A_1 can be obtained as,

$$\begin{aligned} E_t(m_{t+1}) = \theta\, ln\, \delta - \frac{\theta}{\psi}(\mu + x_t) + (\theta - 1)\{\kappa_0 + \kappa_1 A_0 \\ + \kappa_1 A_1 \rho x_t + \kappa_1 A_2[\sigma^2 + v_1(\sigma_t^2 - \sigma^2)] - A_0 \\ - A_1 x_t - A_2 \sigma_t^2 + \mu + x_t\} \end{aligned} \qquad \text{B3.2}$$

$$\begin{aligned} = \Big\{\theta\, ln\, \delta - \frac{\theta}{\psi}\mu \\ + (\theta - 1)[\kappa_0 + \kappa_1 A_0 + \kappa_1 A_2(\sigma^2 - v_1 \sigma^2) - A_0 \\ + \mu]\Big\} + \left[-\frac{\theta}{\psi} + (\theta - 1)(\kappa_1 A_1 \rho - A_1 + 1)\right] x_t \\ + (\theta - 1)(\kappa_1 A_2 v_1 - A_2)\sigma_t^2 \end{aligned}$$

$$\begin{aligned} = C_m + \left[-\frac{\theta}{\psi} + (\theta - 1)(\kappa_1 \rho \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} - \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} + 1)\right] x_t \\ + (\theta - 1)(\kappa_1 A_2 v_1 - A_2)\sigma_t^2 \end{aligned}$$

$$= C_m - \frac{1}{\psi} x_t + A_2(\theta - 1)(\kappa_1 v_1 - 1)\sigma_t^2$$

Where C_m denotes the constant in this equation, which equals $\theta\, ln\, \delta - \frac{\theta}{\psi}\mu + (\theta - 1)[\kappa_0 +$

$\kappa_1 A_0 + \kappa_1 A_2(\sigma^2 - v_1\sigma^2) - A_0 + \mu]$. Then, the innovation to pricing kernel can be written as,

$$m_{t+1} - E_t(m_{t+1}) = -\frac{\theta}{\psi}\sigma_t\eta_{t+1} + (\theta-1)(\kappa_1 A_1\varphi_e\sigma_t e_{t+1} + \kappa_1 A_2\sigma_w w_{t+1} + \sigma_t\eta_{t+1}) \quad \text{B3.3}$$

$$= \left(-\frac{\theta}{\psi} + \theta - 1\right)\sigma_t\eta_{t+1} + (\theta-1)\kappa_1 A_1\varphi_e\sigma_t e_{t+1} + (\theta-1)\kappa_1 A_2\sigma_w w_{t+1}$$

$$= \lambda_{m,\eta}\sigma_t\eta_{t+1} - \lambda_{m,e}\sigma_t e_{t+1} - \lambda_{m,w}\sigma_w w_{t+1}$$

Where $\lambda_{m,\eta}$, $\lambda_{m,e}$ and $\lambda_{m,w}$ are the market prices of short-run, long-run and volatility risks,

$$\lambda_{m,\eta} = -\frac{\theta}{\psi} + \theta - 1 = -\gamma$$

$$\lambda_{m,e} = (1-\theta)C$$

$$\lambda_{m,w} = (1-\theta)\kappa_1 A_2$$

Thus, the conditional variance of m_{t+1}, can be shown as,

$$Var_t(m_{t+1}) = \left(\lambda_{m,\eta}^2 + \lambda_{m,e}^2\right)\sigma_t^2 + \lambda_{m,w}^2\sigma_w^2 \quad \text{B3.4}$$

Risk premium for $r_{a,t+1}$

The risk premium for any asset can be determined by the conditional covariance between the return on asset, $r_{a,t+1}$ and pricing kernel, m_{t+1}. So, due to B1.9, the risk premium for $r_{a,t+1}$ can be shown as,

$$E_t(r_{a,t+1} - r_{f,t}) = -cov_t(m_{t+1}, r_{a,t+1}) - \frac{1}{2}var_t(r_{a,t+1}) \quad \text{B4.1}$$
$$= -cov_t[m_{t+1} - E_t(m_{t+1}), r_{a,t+1}$$
$$- E_t(r_{a,t+1})] - \frac{1}{2}var_t(r_{a,t+1})$$

Therefore, exploiting the innovations in B2.18 and B3.3,

$$E_t(r_{a,t+1} - r_{f,t}) \quad \text{B4.2}$$
$$= -\lambda_{m,\eta}\sigma_t^2 + \lambda_{m,e}C\sigma_t^2 + \lambda_{m,w}\kappa_1 A_2\sigma_w^2$$
$$-\frac{1}{2}var_t(r_{a,t+1})$$

Equity premium and volatility for $r_{i,t+1}$

Further, the risk premium for individual security i, $r_{i,t+1}$ can be shown as,

$$E_t(r_{i,t+1} - r_{f,t}) \quad \text{B5.1}$$
$$= -cov_t[m_{t+1} - E_t(m_{t+1}), r_{i,t+1} - E_t(r_{i,t+1})]$$
$$-\frac{1}{2}var_t(r_{i,t+1})$$

Here, the innovation in m_{t+1} has been obtained. Innovation to individual security needs to be derived. Thus, the price-dividend ratio for individual security i is,

$$z_{i,t} = A_{i,0} + A_{i,1}x_t + A_{i,2}\sigma_t^2 \quad \text{B5.2}$$

And, substituting into the return on individual security i by a Campbell-Shiller approximation,

$$r_{i,t+1} = \kappa_{i,0} + \kappa_{i,1}z_{i,t+1} - z_{i,t} + g_{i,t} \quad \text{B5.3}$$

$$= \kappa_{i,0} + \kappa_{i,1}\left(A_{i,0} + A_{i,1}x_{t+1} + A_{i,2}\sigma_{t+1}^2\right) - A_{i,0} - A_{i,1}x_t$$
$$- A_{i,2}\sigma_t^2 + \mu_i + \phi_i x_t + \varphi_i \sigma_t u_{i,t+1}$$
$$+ \varphi_{i,m}\sigma_t v_{t+1}$$

$$= \kappa_{i,0} + \kappa_{i,1}A_{i,0} + \kappa_{i,1}A_{i,1}(\rho x_t + \varphi_e \sigma_t e_{t+1}) + \kappa_{i,1}A_{i,2}(\sigma^2 +$$
$$v_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}) - A_{i,0} - A_{i,1}x_t - A_{i,2}\sigma_t^2 + \mu_i +$$
$$\phi_i x_t + \varphi_i \sigma_t u_{i,t+1} + \varphi_{i,m}\sigma_t v_{t+1}$$

$$= \kappa_{i,0} + \kappa_{i,1}A_{i,0} + \kappa_{i,1}A_{i,2}(1 - v_1)\sigma^2 - A_{i,0} + \mu_i$$
$$+ \left(\kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t + (\kappa_{i,1}A_{i,2}v_1$$
$$- A_{i,2})\sigma_t^2 + \kappa_{i,1}A_{i,1}\varphi_e\sigma_t e_{t+1} + \kappa_{i,1}A_{i,2}\sigma_w w_{t+1}$$
$$+ \varphi_i\sigma_t u_{i,t+1} + \varphi_{i,m}\sigma_t v_{t+1}$$

$$= C_{r_i} + \left(\kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t + (\kappa_{i,1}A_{i,2}v_1 - A_{i,2})\sigma_t^2$$
$$+ \kappa_{i,1}A_{i,1}\varphi_e\sigma_t e_{t+1} + \kappa_{i,1}A_{i,2}\sigma_w w_{t+1}$$
$$+ \varphi_i\sigma_t u_{i,t+1} + \varphi_{i,m}\sigma_t v_{t+1}$$

Where $C_{r_i} = \kappa_{i,0} + \kappa_{i,1}A_{i,0} + \kappa_{i,1}A_{i,2}(1 - v_1)\sigma^2 - A_{i,0} + \mu_i$. And, conditional mean of $r_{i,t+1}$, can be expressed as,

$$E_t\left(r_{i,t+1}\right) = C_{r_i} + \left(\kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t + (\kappa_{i,1}A_{i,2}v_1 - A_{i,2})\sigma_t^2 \quad \text{B5.4}$$

Thus, innovation to individual return $r_{i,t+1}$ can be shown as,

$$r_{i,t+1} - E_t\left(r_{i,t+1}\right) = \varphi_i\sigma_t u_{i,t+1} + \kappa_{i,1}A_{i,1}\varphi_e\sigma_t e_{t+1} + \kappa_{i,1}A_{i,2}\sigma_w w_{t+1} + \varphi_{i,m}\sigma_t v_{t+1} \quad \text{B5.5}$$

$$= \varphi_i \sigma_t u_{i,t+1} + \beta_{i,e} \sigma_t e_{t+1} + \beta_{i,w} \sigma_w w_{t+1} + \varphi_{i,m} \sigma_t v_{t+1}$$

Where $\beta_{i,e}$ and $\beta_{i,w}$ can be expressed as,

$$\beta_{i,w} \equiv \kappa_{i,1} A_{i,2} \quad \text{B5.6}$$

$$\beta_{i,e} = \kappa_{i,1} A_{i,1} \varphi_e$$

Then, the variance of $r_{i,t+1}$ can be obtained,

$$var_t(r_{i,t+1}) = (\beta_{i,e}^2 + \varphi_i^2 + \varphi_{i,m}^2)\sigma_t^2 + \beta_{i,w}^2 \sigma_w^2 \quad \text{B5.7}$$

The equity premium for individual security i (B5.1) can be expressed to adopt the innovation in individual security's return $r_{i,t+1}$ and pricing kernel,

$$E_t(r_{i,t+1} - r_{f,t}) \quad \text{B5.8}$$

$$= -cov_t[m_{t+1} - E_t(m_{t+1}), r_{i,t+1} - E_t(r_{i,t+1})] - \frac{1}{2} var_t(r_{i,t+1})$$

$$= \beta_{i,e} \lambda_{m,e} \sigma_t^2 + \beta_{i,w} \lambda_{m,w} \sigma_w^2 - \frac{1}{2} var_t(r_{i,t+1})$$

To solve the $A_{i,0}$ $A_{i,1}$ and $A_{i,2}$, according to the B1.1, the Euler condition cn be obtained,

$$E_t[M_{t+1} R_{i,t+1}] = 1 \quad \text{B5.9}$$

$$E_t[\exp(m_{t+1} + r_{i,t+1})] = 1$$

Where $m_{t+1} = \log(M_{t+1})$ and $r_{i,t+1} = \log(R_{i,t+1})$, and due to the B1.4,

$$E_t[\exp(m_{t+1} + r_{i,t+1})] \qquad \text{B5.10}$$

$$= E_t(m_{t+1}) + E_t(r_{i,t+1}) + 0.5var_t(m_{t+1} + r_{i,t+1})$$

$$= C_m - \frac{1}{\psi}x_t + A_2(\theta - 1)(\kappa_1 v_1 - 1)\sigma_t^2 + C_{r_i} + (\kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i)x_t + (\kappa_{i,1}A_{i,2}v_1 - A_{i,2})\sigma_t^2 + 0.5var_t(\lambda_{m,\eta}\sigma_t\eta_{t+1} - \lambda_{m,e}\sigma_t e_{t+1} - \lambda_{m,w}\sigma_w w_{t+1} + \varphi_i\sigma_t u_{i,t+1} + \beta_{i,e}\sigma_t e_{t+1} + \beta_{i,w}\sigma_w w_{t+1} + \varphi_{i,m}\sigma_t v_{t+1})$$

$$= C_m + C_{r_i} + \left(-\frac{1}{\psi} + \kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t + [A_2(\theta - 1)(\kappa_1 v_1 - 1) + \kappa_{i,1}A_{i,2}v_1 - A_{i,2}]\sigma_t^2 + 0.5[(\lambda_{m,\eta}^2 + (\beta_{i,e} - \lambda_{m,e})^2 + \varphi_i^2 + \varphi_{i,m}^2)\sigma_t^2 + (\beta_{i,w} - \lambda_{m,w})^2\sigma_w^2]$$

$$= H_c + \left(-\frac{1}{\psi} + \kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t + [A_2(\theta - 1)(\kappa_1 v_1 - 1) + \kappa_{i,1}A_{i,2}v_1 - A_{i,2} + 0.5H_{var}]\sigma_t^2 + 0.5(\beta_{i,w} - \lambda_{m,w})^2\sigma_w^2$$

Where $H_c = C_m + \kappa_{i,0} + \kappa_{i,1}A_{i,0} + \kappa_{i,1}A_{i,2}(1 - v_1)\sigma^2 - A_{i,0} + \mu_i$ and $H_{var} = \lambda_{m,\eta}^2 + (\beta_{i,e} - \lambda_{m,e})^2 + \varphi_i^2 + \varphi_{i,m}^2$, since there is a correlation between $-\lambda_{m,e}\sigma_t e_{t+1}$ and $\beta_{i,e}\sigma_t e_{t+1}$. Thus, all the x_t and σ_t^2 in the equation B5.10 can be followed as,

$$-\frac{1}{\psi} + \kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i = 0 \qquad \text{B5.11}$$

$$A_2(\theta - 1)(\kappa_1 v_1 - 1) + \kappa_{i,1}A_{i,2}v_1 - A_{i,2} + 0.5H_{var} = 0 \qquad \text{B5.12}$$

Therefore, the $A_{i,1}$ and $A_{i,2}$, can be solved and shown as,

$$A_{i,0} = \frac{C_m + \kappa_{i,0} + \kappa_{i,1}A_{i,2}(1-v_1)\sigma^2 + \mu_i + 0.5(\beta_{i,w} - \lambda_{m,w})^2\sigma_w^2}{1-\kappa_{i,1}}$$

$$A_{i,1} = \frac{\phi_i - \frac{1}{\psi}}{1-\kappa_{i,1}\rho} \quad \text{B5.13}$$

$$A_{i,2} = \frac{A_2(\theta-1)(\kappa_1 v_1 - 1) + 0.5H_{var}}{1-\kappa_{i,1}v_1} \quad \text{B5.14}$$

Therefore, the conditional variance of return on individual security can be expressed i, $var_t(m_{t+1} + r_{i,t+1})$,

$$var_t(m_{t+1} + r_{i,t+1}) = \left(\lambda_{m,\eta}^2 + (\beta_{i,e} - \lambda_{m,e})^2 + \varphi_i^2 + \varphi_{i,m}^2\right)\sigma_t^2 + (-\lambda_{m,w} + \beta_{i,w})^2\sigma_w^2 \quad \text{B5.15}$$

And, to derive the unconditional variance of return on individual security i, the unconditional mean of $r_{i,t+1}$ must be obtained by equation B5.3,

$$\begin{aligned} E(r_{i,t+1}) = {} & \kappa_{i,0} + \kappa_{i,1}\left(A_{i,0} + A_{i,1}E(x_{t+1}) + A_{i,2}E(\sigma_{t+1}^2)\right) \\ & - A_{i,0} - A_{i,1}E(x_t) - A_{i,2}E(\sigma_t^2) + \mu_i \\ & + \phi_i E(x_t) + \varphi_i E(\sigma_t)E(u_{i,t+1}) \\ & + \varphi_{i,m}E(\sigma_t)E(v_{t+1}) \end{aligned} \quad \text{B5.16}$$

$$\begin{aligned} = {} & \kappa_{i,0} + \kappa_{i,1}A_{i,0} + \kappa_{i,1}A_{i,1}E(x_t) + \kappa_{i,1}A_{i,2}v_1E(\sigma_t^2) - A_{i,0} \\ & - A_{i,1}E(x_t) - A_{i,2}E(\sigma_t^2) + \mu_i + \phi_i E(x_t) \end{aligned}$$

$$= C_{ui} + \left(\kappa_{i,1}A_{i,1} - A_{i,1} + \phi_i\right)E(x_t)$$
$$+ \left(\kappa_{i,1}A_{i,2} - A_{i,2}v_1\right)E(\sigma_t^2)$$

$$= C_{ui} + \left(\kappa_{i,1}A_{i,2} - A_{i,2}v_1\right)E(\sigma_t^2)$$

Where $E(e_{t+1}) = E(w_{t+1}) = E\left(u_{i,t+1}\right) = E(v_{t+1}) = 0$ and $C_{ui} = \kappa_{i,0} + \kappa_{i,1}A_{i,0} - A_{i,0} + \mu_i$. And, for $E(x_t)$, according to the B2.1,

$$E(x_{t+1}) = \rho E(x_t) + \varphi_e E(\sigma_t)E(e_{t+1})$$

$$E(x_t) = \rho E(x_t) \Rightarrow E(x_t) = 0$$

And therefore, to obtain the unconditional variance of return on individual security i,

$$r_{i,t+1} - E\left(r_{i,t+1}\right) \qquad \text{B5.17}$$
$$= \left(\kappa_{i,1}A_{i,1}\rho - A_{i,1} + \phi_i\right)x_t$$
$$+ \left(\kappa_{i,1}A_{i,2}v_1 - A_{i,2}\right)\sigma_t^2 + \kappa_{i,1}A_{i,1}\varphi_e\sigma_t e_{t+1}$$
$$+ \kappa_{i,1}A_{i,2}\sigma_w w_{t+1} + \varphi_i\sigma_t u_{i,t+1} + \varphi_{i,m}\sigma_t v_{t+1}$$
$$- (\kappa_{i,1}A_{i,2}v_1 - A_{i,2})E(\sigma_t^2)$$

$$= -\frac{1}{\psi}x_t + \varphi_i\sigma_t u_{i,t+1} + \beta_{i,e}\sigma_t e_{t+1} + \beta_{i,w}\sigma_w w_{t+1}$$
$$+ \varphi_{i,m}\sigma_t v_{t+1} + A_{i,2}\left(\kappa_{i,1}v_1 - 1\right)(\sigma_t^2 - E(\sigma_t^2)$$

So, the unconditional variance is,

$$Var(r_i) = \frac{Var(x)}{\psi^2} + \left(\beta_{i,e}^2 + \varphi_i^2 + \varphi_{i,m}^2\right)\sigma_t^2 + \beta_{i,w}^2\sigma_w^2 \qquad \text{B5.18}$$
$$+ \left[A_{i,2}\left(\kappa_{i,1}v_1 - 1\right)\right]^2 var(\sigma_t^2)$$

Further, the unconditional variance of $z_{i,t}$ can be acquired via equation B5.2,

$$Var(z_{i,t}) = A_{i,1}^2 var(x_t) + A_{i,2}^2 var(\sigma_t^2) \qquad \text{B5.19}$$

Moreover, the innovation to an individual security's return volatility needs to be described. Hence, the innovation to individual security's return in time t+1 can be rewritten via equation B5.5,

$$r_{i,t+2} - E_{t+1}(r_{i,t+2}) \qquad \text{B5.20}$$

$$= \varphi_i \sigma_{t+1} u_{i,t+2} + \beta_{i,e} \sigma_{t+1} e_{t+2} + \beta_{i,w} \sigma_w w_{t+2}$$

$$+ \varphi_{i,m} \sigma_{t+1} v_{t+2}$$

Thus, the conditional variance of an individual security's return can be shown as,

$$Var_{t+1}(r_{i,t+2}) = \varphi_i^2 \sigma_{t+1}^2 + \beta_{i,e}^2 \sigma_{t+1}^2 + \beta_{i,w}^2 \sigma_w^2 + \varphi_{i,m}^2 \sigma_{t+1}^2 \qquad \text{B5.21}$$

$$= (\varphi_i^2 + \beta_{i,e}^2 + \varphi_{i,m}^2)\sigma_{t+1}^2 + \beta_{i,w}^2 \sigma_w^2$$

$$= (\varphi_i^2 + \beta_{i,e}^2 + \varphi_{i,m}^2)(v_1 \sigma_t^2 + \sigma_w w_{t+1}) + \beta_{i,w}^2 \sigma_w^2$$

And, the expectation of variance of an individual security's return,

$$E_t[Var_{t+1}(r_{i,t+2})] = (\varphi_i^2 + \beta_{i,e}^2 + \varphi_{i,m}^2) v_1 E_t(\sigma_t^2) + \beta_{i,w}^2 \sigma_w^2 \qquad \text{B5.22}$$

$$= (\varphi_i^2 + \beta_{i,e}^2 + \varphi_{i,m}^2) v_1 \sigma_t^2 + \beta_{i,w}^2 \sigma_w^2$$

Therefore, the innovation to an individual security's return volatility can be shown as,

$$Var_{t+1}(r_{i,t+2}) - E_t[Var_{t+1}(r_{i,t+2})] \qquad \text{B5.23}$$

$$= (\varphi_i^2 + \beta_{i,e}^2 + \varphi_{i,m}^2) \sigma_w w_{t+1}$$

The risk-free rate and volatility

According to equation B1.7, the risk-free rate, $E_t\left[M_{t+1}R_{f,t+1}\right]=1$ can be derived,

$$E_t\left[exp\left(m_{t+1}+r_{f,t}\right)\right]=1 \qquad \text{B6.1}$$

$$\exp\left[E_t(m_{t+1})+E_t\left(r_{f,t}\right)+0.5Var_t\left(m_{t+1}+r_{f,t}\right)\right]=1$$

$$E_t(m_{t+1})+r_{f,t}+0.5Var_t(m_{t+1})=0$$

$$r_{f,t}=-E_t(m_{t+1})-0.5Var_t(m_{t+1})$$

$$=-\theta\ ln\,\delta+\frac{\theta}{\psi}E_t(g_{t+1})+(1-\theta)\,E_t(r_{a,t+1}) - 0.5Var_t\left(-\frac{\theta}{\psi}g_{t+1}+(1-\theta)\,r_{a,t+1}\right)$$

Then, both sides subtract $(1-\theta)r_{f,t}$ and divide by θ, as well as substitute equation B3.4, giving

$$r_{f,t}=-\,ln\,\delta+\frac{1}{\psi}E_t(g_{t+1})+\frac{1-\theta}{\theta}E_t\left(r_{a,t+1}-r_{f,t+1}\right) -\frac{1}{2\theta}Var_t\left(-\frac{\theta}{\psi}g_{t+1}+(\theta-1)\,r_{a,t+1}\right) \qquad \text{B6.2}$$

$$=-\,ln\,\delta+\frac{1}{\psi}E_t(g_{t+1})+\frac{1-\theta}{\theta}E_t\left(r_{a,t+1}-r_{f,t+1}\right) -\frac{1}{2\theta}[\left(\lambda_{m,\eta}^2+\lambda_{m,e}^2\right)\sigma_t^2+\lambda_{m,w}^2\sigma_w^2]$$

And, substituting the equation unconditional mean of $r_{f,t}$

$$E_t\left(r_{f,t}\right)=-\,ln\,\delta+\frac{1}{\psi}E(g)+\frac{1-\theta}{\theta}E\left(r_{a,t+1}-r_{f,t+1}\right) -\frac{1}{2\theta}[\left(\lambda_{m,\eta}^2+\lambda_{m,e}^2\right)\sigma_t^2+\lambda_{m,w}^2\sigma_w^2] \qquad \text{B6.3}$$

According to equation B2.2 and equation B4.2 and B2.20,

$$E(g)=\mu+x_t \qquad \text{B6.4}$$

$$E\left(r_{a,t+1} - r_{f,t+1}\right) = -\lambda_{m,\eta} E(\sigma_t^2) + \lambda_{m,e} C E(\sigma_t^2) + \lambda_{m,w} \kappa_1 A_2 \sigma_w^2 - \frac{1}{2} E[var_t\left(r_{a,t+1}\right)]$$

$$= -\lambda_{m,\eta} E(\sigma_t^2) + \lambda_{m,e} C E(\sigma_t^2) + \lambda_{m,w} \kappa_1 A_2 \sigma_w^2 - 0.5[(1 + C^2) E(\sigma_t^2) + \kappa_1^2 A_2^2 \sigma_w^2]$$

Then, substituting the equation B6.4 into B6.3,

$$E_t\left(r_{f,t}\right) = -\ln\delta + \frac{1}{\psi}(\mu + E(x_t)) + \frac{1-\theta}{\theta}\{-\lambda_{m,\eta} E(\sigma_t^2) + \lambda_{m,e} C E(\sigma_t^2) + \lambda_{m,w} \kappa_1 A_2 \sigma_w^2 - 0.5[(1 + C^2) E(\sigma_t^2) + \kappa_1^2 A_2^2 \sigma_w^2]\} - \frac{1}{2\theta}[\left(\lambda_{m,\eta}^2 + \lambda_{m,e}^2\right) E(\sigma_t^2) + \lambda_{m,w}^2 \sigma_w^2] \qquad \text{B6.5}$$

Hence, unconditional variance of $r_{f,t}$, can be

$$var\left(r_{f,t}\right) = \frac{1}{\psi^2} var(x_t) + \{\frac{1-\theta}{\theta}\left[-\lambda_{m,\eta} + \lambda_{m,e} C - 0.5(1 + C^2)\right] - \frac{1}{2\theta}\left(\lambda_{m,\eta}^2 + \lambda_{m,e}^2\right)\}^2 var(\sigma_t^2) \qquad \text{B6.6}$$

$$= \frac{1}{\psi^2} var(x_t) + (\frac{1-\theta}{\theta} L_1 - \frac{1}{2\theta} L_2) var(\sigma_t^2)$$

Where $L_1 = -\lambda_{m,\eta} + (1-\theta) C^2 - 0.5(1 + C^2)$ and $L_2 = \lambda_{m,\eta}^2 + \lambda_{m,e}^2$.

Appendix C

		P1	P2	P3	P4	P5
Return	Mean	1.16%	2.26%	2.33%	2.36%	2.84%
	Stdev.	0.120	0.087	0.077	0.080	0.098
	AC(1)	0.077	0.122	0.044	-0.012	0.010
Dividend	Mean	0.000	0.011	0.011	0.012	0.018
	Stdev.	0.155	0.095	0.051	0.080	0.155
	AC(1)	0.358	0.202	0.144	0.256	0.149
Valuation	Mean	5.226	4.681	4.572	4.685	5.093
	Stdev.	0.369	0.220	0.160	0.209	0.366
	AC(1)	0.853	0.819	0.832	0.873	0.883

Table C1.1- Return, dividend, and valuation in momentum trading strategy 6*3

		P1	P2	P3	P4	P5
Return	Mean	2.30%	4.02%	4.55%	4.91%	5.85%
	Stdev.	0.181	0.129	0.111	0.112	0.138
	AC(1)	0.055	0.004	0.039	0.078	0.057
Dividend	Mean	0.002	0.017	0.021	0.026	0.039
	Stdev.	0.289	0.159	0.085	0.133	0.244
	AC(1)	0.083	0.051	0.093	-0.008	-0.020
Valuation	Mean	4.575	3.977	3.862	3.970	4.412
	Stdev.	0.377	0.197	0.150	0.211	0.379
	AC(1)	0.640	0.552	0.615	0.702	0.761

Table C1.2- Return, dividend, and valuation in momentum trading strategy 6*6

		P1	P2	P3	P4	P5
Return	Mean	0.75%	1.90%	2.20%	2.67%	3.23%
	Stdev.	0.126	0.087	0.077	0.078	0.097
	AC(1)	0.118	0.121	0.056	-0.039	-0.013
Dividend	Mean	-0.002	0.008	0.011	0.016	0.023
	Stdev.	0.155	0.094	0.049	0.077	0.142
	AC(1)	0.555	0.495	0.226	0.421	0.406
Valuation	Mean	5.294	4.698	4.543	4.668	5.094
	Stdev.	0.424	0.252	0.163	0.214	0.386
	AC(1)	0.890	0.861	0.849	0.883	0.904

Table C1.3- Return, dividend, and valuation in momentum trading strategy 12*3

		P1	P2	P3	P4	P5
Return	Mean	2.48%	4.23%	4.76%	5.59%	6.31%
	Stdev.	0.193	0.137	0.119	0.116	0.142
	AC(1)	-0.046	-0.074	-0.021	-0.049	-0.075
Dividend	Mean	0.001	0.017	0.022	0.031	0.040
	Stdev.	0.304	0.200	0.094	0.158	0.274
	AC(1)	0.218	0.114	0.173	0.148	0.089
Valuation	Mean	4.598	4.006	3.848	3.958	4.404
	Stdev.	0.451	0.266	0.170	0.216	0.347
	AC(1)	0.686	0.578	0.615	0.666	0.709

Table C1.4- Return, dividend, and valuation in momentum trading strategy 12*6

	P1	P2	P3	P4	P5
Intercept	-0.29%	0.97%	1.19%	1.37%	2.23%
Std.Error	0.011	0.007	0.004	0.006	0.011
Consumption Beta	-6.103	-1.735	1.157	3.772	8.744
Std.Error	4.150	2.571	1.386	2.132	4.125
Adj. R-squared	0.006	-0.003	-0.002	0.011	0.018
Residual Error	0.154	0.095	0.051	0.079	0.153

Table C2.1- Unconditional long-run risk beta in momentum trading strategy 6*3

	P1	P2	P3	P4	P5
Intercept	-0.37%	1.38%	2.12%	2.98%	4.61%
Std.Error	0.030	0.017	0.009	0.014	0.026
Consumption Beta	-12.577	-5.730	0.538	7.040	14.159
Std.Error	11.013	6.078	3.255	5.063	9.255
Adj. R-squared	0.003	-0.001	-0.011	0.010	0.014
Residual Error	0.289	0.159	0.085	0.133	0.242

Table C2.2- Unconditional long-run risk beta in momentum trading strategy 6*6

	P1	P2	P3	P4	P5
Intercept	-0.48%	0.65%	1.12%	1.91%	2.62%
Std.Error	0.012	0.007	0.004	0.006	0.011
Consumption Beta	-4.991	-2.983	0.964	5.891	7.186
Std.Error	4.169	2.540	1.311	2.029	3.801
Adj. R-squared	0.002	0.002	-0.002	0.039	0.014
Residual Error	0.155	0.094	0.049	0.075	0.141

Table C2.3- Unconditional long-run risk beta in momentum trading strategy 12*3

	P1	P2	P3	P4	P5
Intercept	-0.49%	1.28%	2.27%	3.59%	4.49%
Std.Error	0.032	0.021	0.010	0.017	0.029
Consumption Beta	-11.542	-9.067	0.693	9.743	9.820
Std.Error	11.546	7.569	3.595	5.957	10.438
Adj. R-squared	0.000	0.005	-0.011	0.018	-0.001
Residual Error	0.304	0.199	0.095	0.157	0.275

Table C2.4- Unconditional long-run risk beta in momentum trading strategy 12*6

	P1	P2	P3	P4	P5
Intercept	0.010	0.019	0.015	0.010	0.015
Std.Error	0.012	0.008	0.004	0.006	0.012
Average beta	-9.411	-4.158	0.400	4.747	10.601
Std.Error	4.299	2.645	1.449	2.233	4.322
Long run risk beta	-9.379	-6.871	-2.147	2.764	5.266
Std.Error	3.734	2.298	1.259	1.940	3.755
Adj. R-squared	0.034	0.038	0.009	0.017	0.023
Residual standard error	0.152	0.048	0.019	0.027	0.153
Observation	185.000	185.000	185.000	185.000	185.000

Table C3.1- Consumption beta conditional on the long-run risk component in momentum strategy 6*3

	P1	P2	P3	P4	P5
Intercept	0.020	0.035	0.030	0.026	0.026
Std.Error	0.032	0.017	0.009	0.015	0.027
Average beta	-18.984	-11.317	-1.898	8.183	19.439
Std.Error	11.375	6.087	3.318	5.317	9.566
Long run risk beta	-17.718	-15.453	-6.737	3.162	14.602
Std.Error	9.359	5.008	2.729	4.375	7.871
Adj. R-squared	0.031	0.085	0.043	0.005	0.040
Residual standard error	0.285	0.152	0.083	0.133	0.239
Observation	90.000	90.000	90.000	90.000	90.000

Table C3.2- Consumption beta conditional on the long-run risk component in momentum strategy 6*6

	P1	P2	P3	P4	P5
Intercept	0.007	0.018	0.016	0.016	0.018
Std.Error	0.013	0.008	0.004	0.007	0.012
Average beta	-8.167	-5.901	-0.234	6.658	9.247
Std.Error	4.537	2.736	1.422	2.222	4.153
Long run risk beta	-3.942	-3.622	-1.487	0.951	2.559
Std.Error	2.284	1.377	0.716	1.119	2.091
Adj. R-squared	0.013	0.033	0.015	0.037	0.016
Residual standard error	0.154	0.093	0.048	0.075	0.141
Observation	183.000	183.000	183.000	183.000	183.000

Table C3.3- Consumption beta conditional on the long-run risk component in momentum strategy 12*3

	P1	P2	P3	P4	P5
Intercept	0.004	0.037	0.030	0.025	0.025
Std.Error	0.038	0.025	0.012	0.020	0.034
Average beta	-13.690	-15.136	-1.239	12.520	14.857
Std.Error	12.687	8.172	3.922	6.512	11.406
Long run risk beta	-2.759	-7.794	-2.480	3.566	6.469
Std.Error	6.602	4.253	2.041	3.389	5.935
Adj. R-squared	-0.009	0.030	-0.005	0.019	0.001
Residual standard error	0.305	0.197	0.094	0.157	0.274
Observation	89.000	89.000	89.000	89.000	89.000

Table C3.4- Consumption beta conditional on the long-run risk component in momentum strategy 12*6

	P1	P2	P3	P4	P5
Intercept	0.009	0.020	0.015	0.009	0.014
Std.Error	0.013	0.008	0.004	0.007	0.013
Average beta	-8.493	-3.692	0.512	4.624	10.368
Std.Error	4.256	2.612	1.428	2.200	4.259
Long run risk beta	-9.730	-7.964	-2.622	3.470	6.611
Std.Error	4.498	2.761	1.509	2.326	4.501
Adj. R-squared	0.025	0.035	0.009	0.018	0.024
Residual standard error	0.153	0.094	0.051	0.079	0.153
Observation	185.000	185.000	185.000	185.000	185.000

Table C4.1- Consumption beta conditional on the long-run risk component with time-varying economic fluctuation in momentum strategy 6*3

	P1	P2	P3	P4	P5
Intercept	0.018	0.034	0.029	0.026	0.029
Std.Error	0.033	0.018	0.010	0.015	0.028
Average beta	-18.766	-11.221	-1.750	8.230	18.989
Std.Error	11.538	6.213	3.381	5.370	9.716
Long run risk beta	-17.867	-15.851	-6.604	3.433	13.942
Std.Error	10.835	5.834	3.175	5.043	9.124
Adj. R-squared	0.022	0.064	0.025	0.004	0.029
Residual standard error	0.286	0.154	0.084	0.133	0.241
Observation	90.000	90.000	90.000	90.000	90.000

Table C4.2- Consumption beta conditional on the long-run risk component with time-varying economic fluctuation in momentum strategy 6*6

	P1	P2	P3	P4	P5
Intercept	0.008	0.018	0.015	0.018	0.019
Std.Error	0.014	0.009	0.004	0.007	0.013
Average beta	-7.231	-5.064	0.330	6.047	8.433
Std.Error	4.414	2.667	1.390	2.161	4.040
Long run risk beta	-7.580	-7.043	-2.145	0.526	4.220
Std.Error	5.042	3.047	1.588	2.469	4.615
Adj. R-squared	0.009	0.025	0.002	0.034	0.013
Residual standard error	0.154	0.093	0.049	0.076	0.141
Observation	183.000	183.000	183.000	183.000	183.000

Table C4.3- Consumption beta conditional on the long-run risk component with time-varying economic fluctuation in momentum strategy 12*3

	P1	P2	P3	P4	P5
Intercept	0.003	0.038	0.028	0.025	0.024
Std.Error	0.039	0.025	0.012	0.020	0.035
Average beta	-13.383	-14.927	-0.488	12.200	14.547
Std.Error	12.721	8.206	3.952	6.537	11.444
Long run risk beta	-4.490	-14.297	-2.880	5.995	11.533
Std.Error	12.727	8.210	3.954	6.540	11.449
Adj. R-squared	-0.010	0.027	-0.016	0.016	-0.001
Residual standard error	0.305	0.197	0.095	0.157	0.275
Observation	89.000	89.000	89.000	89.000	89.000

Table C4.4- Consumption beta conditional on the long-run risk component with time-varying economic fluctuation in momentum strategy 12*6

	Positive	Negative
Return_Mean	0.026	0.005
Return_Stdev.	0.065	0.071
Risk/Return	2.444	13.837
t_value	4.153	0.666
Observation	103.000	85.000

Table C5.1- Momentum returns conditional on the long-run risk component in momentum strategy 6*3

	Positive	Negative
Return_Mean	0.046	0.021
Return_Stdev.	0.084	0.139
Risk/Return	1.811	6.540
t_value	4.019	0.967
Observation	53.000	40.000

Table C5.2- Momentum returns conditional on the long-run risk component in momentum strategy 6*6

	Positive	Negative
Return_Mean	0.033	0.016
Return_Stdev.	0.071	0.088
Risk/Return	2.154	5.420
t_value	4.525	1.760
Observation	95.000	91.000

Table C5.3- Momentum returns conditional on the long-run risk component in momentum strategy 12*3

	Positive	Negative
Return_Mean	0.060	0.016
Return_Stdev.	0.074	0.143
Risk/Return	1.223	8.763
t_value	5.547	0.774
Observation	46.000	46.000

Table C5.4- Momentum returns conditional on the long-run risk component in momentum strategy 12*6

	Type1	Type 2	Type 3	Type 4	Type 5
μ_i	-0.0175	-0.002	0.001	0.002	0.0045
ϕ_i	-4.2	3	5.2	7.2	11
φ_i	9	9	5	6	7
$\varphi_{i,m}$	6	7	8	10	12
Weight	0.12	0.24	0.26	0.24	0.12

Table C6.1- Cross-sectional calibration for five basic types of securities

	Type 1	Type 2	Type 3	Type 4	Type5
$\kappa_{i,0}$	0.0039	0.0214	0.023	0.0198	0.0021
$\kappa_{i,1}$	0.9996	0.9968	0.9965	0.9971	0.9998
$A_{i,0}$	7.7163	5.7535	5.6644	5.845	8.3997
$A_{i,1}$	-264.97	122.51	227.1028	332.7365	599.2811
$A_{i,2}$	2.0694e+04	901.1206	-2.9499e+03	-3.4560e+03	-3.2515e+03

Table C6.2- Model solution for five basic types of securities

	Type 1	Type 2	Type 3	Type 4	Type5
$E[R_t]$	-0.009	0.0058	0.0103	0.0146	0.0259
$E[R_t - R_{f,t-1}]$	-0.01	0.0047	0.0085	0.0124	0.0234
$E[log\ (P/D)]$	7.851	5.8097	5.4876	5.7386	8.2088

Table C6.3- Expected return implication for five basic types of securities

		P1	P2	P3	P4	P5
Return	Mean	1.84%	2.29%	2.31%	2.57%	3.21%
	Stdev.	0.142	0.118	0.115	0.118	0.169
	AC(1)	-0.12	-0.04	0.042	0.075	0.14
Dividend	Mean	0.002	0.009	0.010	0.014	0.024
	Stdev.	0.172	0.105	0.091	0.152	0.232
	AC(1)	-0.19	-0.14	-0.29	0.002	-0.17
Valuation	Mean	7.342	5.741	5.5124	5.687	7.924
	Stdev.	0.430	0.351	0.234	0.245	0.378
	AC(1)	0.863	0.786	0.734	0.876	0.891

Table C7.1- Simulated results of return, dividend, and valuation in momentum trading strategy 6*3

		P1	P2	P3	P4	P5
Return	Mean	3.27%	4.65%	4.84%	5.17%	6.52%
	Stdev.	0.269	0.204	0.192	0.212	0.272
	AC(1)	-0.062	0.045	0.041	-0.024	0.062
Dividend	Mean	0.007	0.020	0.023	0.031	0.053
	Stdev.	0.234	0.170	0.103	0.224	0.312
	AC(1)	0.021	-0.061	0.054	0.001	-0.014
Valuation	Mean	6.821	5.734	5.64	5.965	6.917
	Stdev.	0.412	0.203	0.163	0.254	0.367
	AC(1)	0.744	0.542	0.614	0.721	0.768

Table C7.2- Simulated results of return, dividend, and valuation in momentum trading strategy 6*6

		P1	P2	P3	P4	P5
Return	Mean	2.01%	2.10%	2.31%	2.72%	3.61%
	Stdev.	0.142	0.104	0.134	0.098	0.144
	AC(1)	-0.124	-0.073	0.074	0.062	-0.016
Dividend	Mean	0.018	0.008	0.011	0.017	0.027
	Stdev.	0.164	0.101	0.087	0.129	0.183
	AC(1)	-0.23	0.078	-0.245	0.014	-0.097
Valuation	Mean	7.412	5.841	5.510	6.024	7.894
	Stdev.	0.481	0.374	0.254	0.234	0.401
	AC(1)	0.870	0.812	0.722	0.897	0.903

Table C7.3- Simulated results of return, dividend, and valuation in momentum trading strategy 12*3

		P1	P2	P3	P4	P5
Return	Mean	3.43%	4.97%	5.12%	5.75%	7.28%
	Stdev.	0.213	0.147	0.124	0.134	0.151
	AC(1)	-0.024	-0.042	-0.031	-0.051	-0.068
Dividend	Mean	0.007	0.021	0.024	0.034	0.048
	Stdev.	0.351	0.212	0.134	0.197	0.285
	AC(1)	0.013	0.094	-0.152	0.124	-0.101
Valuation	Mean	6.845	5.710	5.531	5.823	6.931
	Stdev.	0.502	0.447	0.241	0.312	0.421
	AC(1)	0.720	0.687	0.654	0.682	0.712

Table C7.4- Simulated results of return, dividend, and valuation in momentum trading strategy 12*6