A personal history of Hawkes processes

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1 Introduction

I was very honoured to be asked to contribute something on the history of Hawkes processes to a special issue of a journal produced by such an illustrious Institute. At first it was suggested that I might answer a few questions, but I thought that I would like to say a bit more: the result is this article which covers more topics as well as including answers to those questions. My work on this topic falls distinctly into two parts: the early work and a late return starting in 2012. The latter would never have happened without my colleague, Professor Jing Chen, who has contributed so much. I therefore asked her to be co-author. The first part of this article has been written by me (Alan) and Jing is the main author of the second part, but we both contributed to each section.

1.1 Background

I graduated in Mathematics from King's College London in 1960 and moved to the Statistics Department in University College London (UCL) to study a one-year postgraduate diploma in Statistics. At the end of that year I started as a research student under the direction of Professor Maurice Bartlett, working on queueing theory applied to problems in road traffic. One year later I was appointed to a lectureship in the department and continued my research alongside teaching and other duties. My thesis was completed in 1965: the external examiner was Professor David Cox who, amongst many of his important original works, introduced a well-known point process model called the Cox process.

On completion of my thesis, I published two more queuing theory papers then decided that it was time to broaden my research area. I published another four papers on a variety of topics. UCL provided excellent facilities for research. London University had seminars that enabled one to meet scholars from other colleges in London (such as Cox and David Brillinger) and distinguished visitors from abroad, including Jerzy Neyman, to name just a few who made contributions to point process theory. Cox and Lewis [1] published an interesting monograph with some theoretical models, but mostly concentrating on data analysis. Most of all, I was stimulated by a paper by my PhD supervisor, Bartlett [2], in which he introduced what soon became known as the Bartlett spectrum for a point process. I decided that point processes might provide an interesting area to concentrate on for a while.

2 Some exciting point processes

In this section I discuss the time, 1971 – 1974, when those five early papers on Hawkes processes, [3] – [7], were written. Of course, they were not called Hawkes processes at that time but self-exciting and mutually-exciting point processes. What I have to say includes answers to three of the five questions that I mentioned were posed to me.

Question 1: At the time when the Hawkes process was proposed, what point process models were popular? What were the main difficulties of applying Hawkes models and other models?

When I moved to Durham University 1n 1969 I had already begun to think seriously about point processes. There was some interest in point processes at that time but, it seemed to me then, not many models apart from the obvious Poisson, renewal, semi-Markov processes (Hawkes, 1970, [8] was on the latter topic) together with Neyman-Scott, Bartlett-Lewis and Cox processes. I was influenced particularly by Bartlett so that I thought maybe I could introduce a new model and find its Bartlett spectrum.

Nowadays I have a much broader definition of point processes to include anything that models streams of events, such as queueing theory, continuous-time Markov chains (i.e. Markov processes with discrete states in continuous time), neuron firing, epidemic processes, systems reliability etc.

The main difficulties in those days were computing and availability of ample accurate data. Computers were physically large and remote, though small in capacity and slow in speed. Software was limited in scope. Today you can have an incredibly powerful machine on your desk and a huge range of software, much of it freely available as the result of work of scholars from around the World. The developments in computer hardware and software have also made it much easier today to get large amounts of reliable data, for example in physiology or recording transactions on financial markets every few milliseconds.

Question 2: How did the process come out in your idea? This process appeared before I was born. I wonder many researchers who are younger than I might be interested in the answer to this question.

I do not have a clear memory of exactly how or why things turned out the way they did. The motivation was purely mathematical, although some possible applications were mentioned, particularly epidemic processes. Many people begin articles on Hawkes processes saying that they were originally invented to describe earthquake sequences. This is clearly not true, although the interesting paper by Vere-Jones [9] was cited in Hawkes [3]. At this time I acquired a PhD student and I thought that he should do something practical, so I set him to work on analysing earthquake data. Hawkes [5] introduced marked exciting processes and included the remark "A marked process model is being fitted to earthquake data by L. Adamopoulos at Durham".

This came to fruition in Hawkes and Adamopoulos [6], which was indeed the first application of Hawkes processes to any practical subject: but earthquakes were certainly not on my mind when those processes were first introduced. It was not a particularly good paper, because we knew little about earthquakes, but Seismologists felt that it had interesting possibilities. Since they do know about earthquakes they, particularly the famous Japanese experts, have been able to do much better with sensible exciting kernels and developing the well-known ETAS models, including important spatio-temporal versions (Ogata, 1998, [10]). These have also been taken up by people predicting criminal activity, see for example Mohler et al, 2011, [11].

Question 3: On statistical inference, in my impression, spectrum, generating functional and moment-based methods were the main techniques used in point process data analysis. Likelihood based method (MLE) was just introduced. Is this true?

I think that is true. The methods most in use were described in Cox and Lewis [1] while Lewis et al (1969, [12]) provided useful computer programs (SASE IV) to apply those methods. Computer-intensive MCMC Bayes methods, such as Mertropolis-Hastings, were in their infancy but (as far as I can recall) had not yet become common tools for analysing point processes. Hawkes and Adamopoulos [6] introduced "a spectral likelihood" method that relied on the idea that the ratio of the sample spectrum (sometimes called periodogram) and the theoretical Bartlett spectrum computed at various well-chosen frequencies were approximately distributed as i.i.d exponential random variables.

I moved from Durham to Swansea university in 1974 and about 1978 David Vere-Jones visited Swansea to give a talk in which he described the now well-known likelihood formula for Hawkes processes published by Osaki (1979, [13]) following a result of Rubin (1972,[14]) for general point processes. I believe Vere-Jones had some part in encouraging Osaki to work on that problem. The EM algorithm for maximum likelihood was not generally known before the notable work of Dempster et al (1977, [15]): now it is often used on the branching process representation of the Hawkes process [7].

3 The middle years

Question 4: As many people have noticed, you gave up to continue your work in self-exciting process for many years, what are the main reasons? During these years what were your main research focuses?

Throughout my academic career I have always thought of myself primarily as an applied probabilist. I have not been particularly interested in developing mathematical theory for its own sake. I like to work on a concrete problem that has some practical importance. Those self-exciting processes did not seem to have generated all that much excitement: according to Google Scholar, over the next 30 years the first Hawkes paper [3] received on average about 1.5 citations per year: nowadays it gets about 200 citations per year. I did not feel that I had sufficient knowledge to make a practical contribution to, say, earthquakes or epidemics. But I found something much more interesting to do.

I had known pharmacologist, David Colquhoun, from my time at UCL. I had made a small contribution to a book he was writing, on biostatistics [16] by helping him with the chapter on stochastic processes. Also Hawkes [8] was written in response to a question he asked me about how often you might expect to see extra-large spikes when observing a series of spikes in a nerve system. The paper generalised it and abstracted it out of all recognition, but that was the question he wanted answered.

Then he started asking me questions about minute electric currents passing through ion-channels, an essential part of the complex communication system that carries electrical signals around the body, without which we would be unable to feel, think, move or do anything at all. They are also important sites of drug action. For example, local anaesthetics close channels so that no signal can pass and therefore the patient feels no pain. I did not know much about ion channels, but David did and was able to communicate what he needed to know and to understand the mathematics that I produced.

So began a wonderful collaboration between us that lasted for about 40 years. David is immensely talented: he became professor of Pharmacology at UCL, a Fellow of the Royal Society (FRS) who worked with three Nobel prize-winners. Such a nice man and good friend, it was a great pleasure to work with him.

Together we pioneered the stochastic modelling of single-channel dynamics which came to be indispensable to the worldwide community of pharmacologists, physiologists and biophysicists who were thereby enabled to extract detailed understanding of the experimental results that they routinely measure. I regard this as the most important work that I ever did, having practical importance for our understanding of the bodies of people, animals and even plants. And the mathematics was nice too! We published 21 papers, of which 10 of the most important are included in the bibliography [17] –[26]. I should also mention that my colleague, Dr Assad Jalali, also made a significant contribution to solving the important problem of time interval omission ([20], [21], [27]), when some open or shut channel periods may not be observed because they last for such a short time; less than about 25 μs. Under this topic I also note my only previous publication in Japanese [28].

I also published 14 papers on systems reliability. I was not so motivated by this topic, although some of the mathematics was similar to that used for describing ion-channels. I was mainly drawn into this by students or former students of mine who were interested in that subject and wanted my help from time to time. The bibliography includes 8 of these papers, [29] – [36], mostly in collaboration with Professor Lirong Cui, from Beijing Institute of Technology, and some of his research students.

There were also 15 papers of little lasting importance on a miscellaneous collection of topics.

4 Return to Hawkes processes

In 2012 I got to know Jing Chen, who persuaded me to return to the subject of Hawkes processes, which she assured me was a "hot topic in finance" and invited me to work with her on that subject. I knew nothing about finance but once again I found myself working with a talented specialist in their field who understood the mathematics sufficiently well to guide my efforts. At the same time I thought I might try to catch up with all the new mathematics and the very broad range of applications of Hawkes processes. This proved to be an impossible task: the original paper [3] now gets over 200 citations per year.

As Jing motivated and organised almost everything we did on Hawkes processes in Finance (workshops, journal special issues…), she will continue the story.

4.1 Recent history of Hawkes processes and Finance (Jing Chen)

In 2010 I joined Swansea School of Management as a lecturer in Finance. In 2011 I realized that Hawkes processes were becoming important tools in Finance research, particularly in high frequency trading (HFT) and market microstructure. I also realized that Emeritus Professor Alan Hawkes, in my department, was actually the author of Hawkes processes. One of the important features of HFT is that the events (e.g. trades and quotes) occur at extremely high speed (in milliseconds) and, often, in clusters. This provides a natural environment to apply Hawkes processes. This, indeed, has proved to be the case and the early Hawkes process applications in finance are to understand how limit order book activities related to bids and asks that would move trading prices (See Bowher, 2007 [37] and Large, 2007 [38]). In insurance, a few researches also appeared using Hawkes processes to model the clusters in arrivals of insurance claims (see Stabile & Torrisi, 2010 [39] and Dassios & Zhao, 2011, [40]). The idea is to consider the claim process as a “dynamic contagion process” that exhibits the self- or mutual-exciting characteristics. Such applications are often found highly useful in credit risk modelling as they ultimately relate to the classic problem of the probability of ruin

These early applications of Hawkes processes are fascinating, and I hesitated no more to convince Alan that Hawkes processes will be hugely important and helpful in many more areas in Finance. In 2012, we started to work together, Alan teaching me Stochastic processes — especially Hawkes processes - and Statistics, while I taught him Finance. Part of his initial training in Finance was to check the Mathematics and Statistics in several papers that I was submitting to Finance journals.

Our first research idea that we worked on was to incorporate a Hawkes jump process into a GARCH financial model. Having developed a theoretical model, we needed a method of identifying jumps in a series of financial returns. We started using one of the most popular methods: we found that it was sometimes very poor at identifying jumps, especially when the jumps showed a contagious property. Therefore, we devised a new method that would behave well in those circumstances. Unfortunately, we have so far found it difficult to persuade quality journals to publish what we think are quite important results — but we keep trying. Our method of jump detection was, however, well received when presented in a talk to the Commodities Futures Trading Commission (CFTC), a US government financial regulatory body in Washington DC. Eventually, we returned to our planned jump-GARCH model, using our new methods of detection, and are close to completion. We hope, through our model, to understand how intra-day behaviour, such as occurrence of jumps, is related to volatility. Subsequently, traders may be able to get more accurate forecasts of volatility at the end of each day, which are practically achievable and feasible for their routine decision making.

Meanwhile, I persuaded the journal *Quantitative Finance* to publish a special issue on Hawkes processes, and they asked me to be its chief editor. The issue was published in early 2018. It included three papers written by Chen and Hawkes with various other co-authors, [41], [42] md [43], and a review paper by Hawkes [44].

First, we model order flows in a financial market through a birth-death-immigration Hawkes model in Khashanah, Chen and Hawkes [41]. This is opposite to most literature using positively exciting processes: we model trades and quotes in a mutual-exciting bi-variate setting so that one stream of events occurs at a decreasing rate due to interactions with the other type of events.

Second, Chen, Hawkes, Scalas and Trinh [42] compare three typical information criteria for choosing from a collection of possible exciting kernels: Akaike's information criterion (AIC), Bayesian information criterion (BIC) and the Hannan-Quinn criterion (HQ).

In the meanwhile, behaviour finance research has been prevailing. Especially, it is clear that news sentiment exists and drives market prices to move. Yang, Liu, Chen and Hawkes [43] applied a mutually-exciting Hawkes process to understand how positive/negative price and sentiment movements interact. We find that, in particular post the 2008 financial crisis, the news sentiment that reflects traders' belief about the market has become more dominant in leading the market to fluctuate.

Another half dozen excellent papers are also published in this special issue, addressing various theoretical and practical issues such as high-dimensional, non-linear Hawkes model for limit order book (Achab et al., [45]; Lu & Abergel, [46]), dark pool trading (Gao et al., [47]), financial jumps and co-jumps (Calcagnile et al., [48]), liquidity/illiquidity spillover (Schneider et al., [49]) and constant proportion portfolio insurance (Buccioli & Kokholm, [50]).

To further advance the applications of Hawkes processes in Finance, we are currently editing another special issue of *The European Journal of Finance*. Many more important issues in finance such as volatility clustering, financial jumps, financial networks, portfolio optimisation etc. will be discussed. This will also include a review by Hawkes (2021, [51]) on Hawkes jump-diffusions in Finance.

We have also promoted Hawkes processes by a series of three Workshops in Cardiff (2017), Swansea (2018) and Stevens Institute in Hoboken, New Jersey (2019). The Swansea meeting was a 2-day affair to celebrate Alan's 80th birthday. David Colquhoun talked about Alan's work on ion channels. Most of the other talks were about Hawkes processes in Finance. Main speakers were Mathieu Rosenbaum, Frédéric Abergel. Steve Hardiman and Judith Rousseau (Paris); Valérie Chavez-Demoulin and Matthias Kirchner (Switzerland); Khaldoun Khashanah and Steve Yang (New Jersey); Fabrizio Lillo (Italy); Mark Tippett (Australia), Lirong Cui (China), Anton Merlushkin (Credit Suisse, London), Enrico Scalas (Sussex, UK) and, of course, Alan Hawkes. It was a great birthday!

Another strand of development in Hawkes processes and their applications is to consider using more complex kernels.: Chen, Hawkes and Scalas (2020, [52]) introduce a Mittag-Leffler type kernel to replace the classic ETAS models' powerlaw kernel. This can take advantage of the well-studied Laplace transform of the Mittag-Leffler function, thus providing a simpler calculation of some properties of such a Hawkes model. Cui. Hawkes and Yi (2020, [53]) develop a method for finding moments of properties of some Hawkes processes.

We also continue the research in news sentiment and price behaviour. Two papers by Liu, Yang, Chen and Hawkes (2020) do not involve Hawkes processes at all, but are based on entropy. [54] introduces entropy-based measures to quantify information flows driven by prices or news, and thus identify different types of trading behaviours and market regimes. Based on 11 years of news and market data, we find that the financial market has been dominated by different information flows before and after the double crises period (the 2008 liquidity and euro-zone debt crises). The responses to information flows are well studied in finance literature of Granger causality studies. [55] again uses entropy to further find traders' responses to news have become much more pre-dominant during the crisis and, therefore, the more effective price discovery process will need to adopt both price- and news-driven information.

5 The present and the future of Hawkes processes

And now the final question.

> ***Question 5**:* How do you think about recent developments and applications of Hawkes models? How do you expect future development of Hawkes models will be?

It has been amazing to see the volume and scope of applications of Hawkes processes over the last five years, considering that they were virtually ignored for about 30 years. The traditional users. the Seismologists, are still active. Among the more recent users we find plenty of activity in Finance, Social media and Mathematical theory, with some regular work also in Neuroscience; crime and acts of violence. We suppose that the frequency of publications using Hawkes processes will die down eventually but, like the Poisson process, it will never entirely fade away: it is just too useful. The processes are flexible in choice of base process, and exciting kernels, with marks and spatial effects if needed, and interaction with exogenous processes. They can also be modified in various ways to suit particular circumstances. We expect an increasing number of researches in various fields of study will continue to make ingenious use of these properties.

It is interesting to note that, despite the suggestion in [3], that these processes might be useful for modelling disease, and the obvious implications of the title ETAS, Epidemiologists have not made a great deal of use of Hawkes processes. The arrival of COVID-19 has brought a sudden spurt of interest. We do not refer to any particular publication of this type in the bibliography as it is too early to decide which are the important ones. It is interesting to note that the branching ratio, often called R, is a regular part of the daily news reporting of the pandemic.

For the general use of Hawkes processes we would like to see development of a coherent set of model-fitting techniques with well-tested, freely available, software. Bayesian methods show particular promise, combining time-varying properties and model-learning features.

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