Wage Inequality In The United States

CARDIFF BUSINESS SCHOOL

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PhD Thesis

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Funded By The Economic and Social Research Council

May 26, 2021

Abstract

This research seeks to assess the factors affecting unskilled wages. This is a means to analyse the development of low paid wages, and whether it is sustainable.

Chapter 1 seeks to measure relative income effects coming from outside of the individuals control as opposed to the the increase in wage inequality coming from an increase in relative level of skills. I find that skill levels attributed to high earners grew at a steady rate in the US from the 1940s. The income these skills earned stagnated after the 1970s, while unskilled wages actually decreased, after growing rapidly before the 1970s. As skilled workers can increase their earnings by growing their skill levels, not just by relying on the income growth for the pre-existing abilities, their wages continued to grow, resulting in rising wage inequality. The stagnation of income going to already obtained skills and unskilled wages is indicative of a structural shift in the economy and rising relative demand for skilled workers even as skill levels increased. This shift ensured the level of pay for Individuals' skills grew faster than the unskilled wage.

The second Chapter looks at the role of factor inputs and relative scarcity to explain an increase in demand for skilled labour. I conclude that the transition from unskilled to skilled biassed growth is a result of bottlenecks in certain inputs to production. Using US manufacturing data, I find that capital equipment predominantly complements better paid non-production workers, while energy predominantly complements worse paid production workers. As a result of the elasticity values and relative rise of inputs, there has been a large net increase in demand for non-production workers.

The third chapter analyses the implications of factor inputs on technology growth and how this further impacts income distribution in a way that helps explain what type of shock or structural change has initiated the increasing inequality that has been observed. This finds that observed changes in prices and skill premium are consistent with being induced by a permanent shock in energy prices. The finding indicates potential benefits of technologies that reduce energy prices that in Chapter 2 were shown to complement lower paid workers.

Contents

CHAPTER 1

ADVANCED HUMAN CAPITAL

There has been much recent concern about rising income and wealth inequality and its effect on welfare (Ribeiro et al. (2017), Halfon et al. (2017), Findling et al. (2020), Anderson et al. (2019)). But there has been no notable research into wage inequality attributable either to increased skills of high earners or to stronger demand for the skills that high earners generally possess.

The Mincer equation (Mincer (1974)) estimates the total human capital based on schooling and experience and their returns. I find it more appropriate to define human capital as the productive skills that workers obtain. The important distinction is whether growth in earnings comes from increased skills or their rents. On the one hand increased inequality is because of economic growth induced by high earners that can benefit everyone, including low earners. On the other hand, high earners are the predominant beneficiaries of structural changes that increase inequality.

To distinguish between the two possible sources of rising wage inequality I will separate skilled human capital from 'basic' labour. This allows the relative income of a unit of basic labour to be compared to income per unit of human capital to change income distribution, as lower income individuals will have their income from the former, whereas high income earners will receive their income from their human capital. This can be used to explain the change in wage inequality over the last century, which can be a result of either relative skill accumulation or relative income for skills already possessed.

Using data going back to the 1940s I find that human capital, derived from time spent learning, grew at a steady rate in the US from the 1940s. The income these skills earned stagnated after the 1970s, while unskilled wages actually decreased, after growing rapidly before the 1970s. As skilled workers can increase their earnings by growing their skill levels, not just by relying on the income growth for the pre-existing abilities, their wages continued to grow, resulting in rising wage inequality. The stagnation of income going to already obtained skills and unskilled wages is indicative of a structural shift in the economy and rising relative demand for skilled workers even as skill levels increased. This shift ensured the level of pay for Individuals' skills grew faster than the unskilled wage.

1.1 Introduction

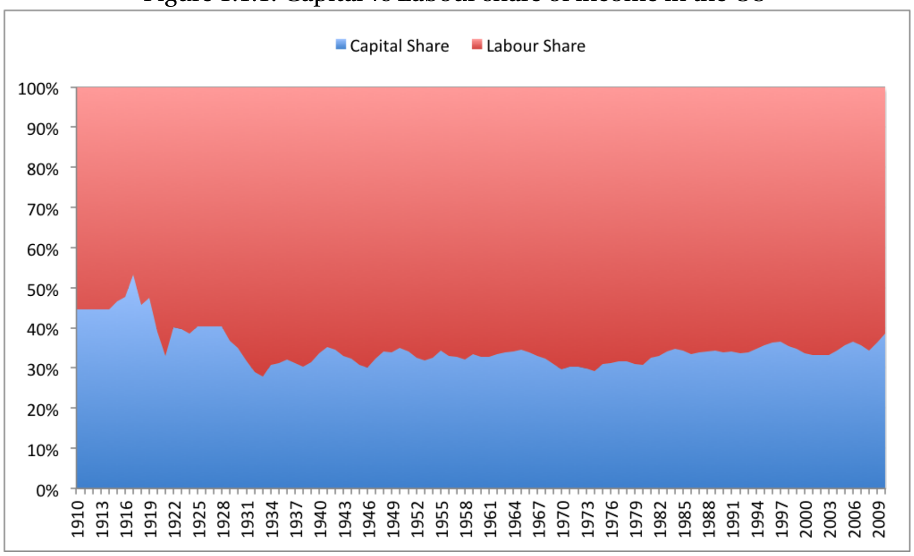
1.1.1 Motivation

The great recession that started in 2008, like all periods of economic turmoil, led to polarisation of political and economic viewpoints. Due to concern for low income workers, income inequality has once again become a controversial topic. This is because in most countries, including developed and developing economies, income has become more unequal since the 1970s. As marginal utility is generally higher for lower wage individuals, who are generally less wealthy, increasing income inequality could mean growth in output overstates change in welfare. There are voices of concern surrounding the sustainability of the current trajectory, mainly related to the income inequality (Berg and Ostry (2017)), and many of the voting public are looking to more interventionist policies as a solution. In recent literature globalisation and technology change have dominated equilibrium analysis when explaining income inequality (Moore and Ranjan (2005) and Svizzero and Tisdell (2002)). However, what has induced growth to benefit higher income earners, and whether this itself has been influenced by globalisation and trade, is still unclear. It is the aim of this research to better understanding the root causes of this trend.

While inequality wouldn't be such a pressing issue if it were purely from wealth creation at the top end, for many developed countries there seems to be a stagnation of living standards at the poorer end of society. In the US, for example, the real minimum wage has not had a sustained rise for over 60 years according to the US department of labour. As can be interpreted from the rising Gini coefficients, growth in lower paid worker's pay has lagged general output growth. The increased disparity in income within countries is creating growing social discontent and frictions, as it is argued by Robert Frank (Frank (2005)) that welfare is dependant on relative as well as total consumption. The fact income inequality has sparked public debate and is widely perceived as a social issue in itself creates relevance to the topic. This in turn makes its way through to taxes and interventionist policies as politicians come under pressure to redistribute wealth as predicted by median voter theory. These policies generally only create a one off redistributive benefit, and so do not deal with the secular trend of rising inequality. Many of the policies (e.g. income taxes) are also distortive. Because of this, it has been argued, gross income inequality is bad for overall growth (Alesina and Rodrik (1994), Persson and Tabellini (1994)), as the taxes, etc that are used to redistribute wealth reduce economic growth. It would be preferable if a more sustainable and less distortive solution can be found. Galor and Moav (2004) further argues that inequality has a negative effect on growth in richer countries due to disincentivizing aggregate human capital accumulation, mainly from lower income workers.

Whether the standards of living and treatment of poorer workers are at levels that should be deemed objectionable by modern standards is more of a matter of opinion and is not the objective of this thesis to assess. However, the explanation, sustainability and development of this fundamental issue is of great relevance and worth studying. The question of sustainability is reflected in the secular decline in interest rates, from decreasing marginal returns, that has occurred since the 1980s. This is possibly a result of increased saving rates worldwide from increased inequality, as wealthier individuals have a higher marginal propensity to save. When growth shows an unbalanced trend in this way, it is relevant to understand whether it is a temporary deviation from a longer term equilibrium, a movement to a new equilibrium or a violation of balanced growth. According to the Kuznets curve inequality temporarily increases during economic development, though the hypothesis itself doesn't give a strict explanation as to why. This would imply increased inequality is simply a temporary by product of economic growth and the trend should reverse eventually, supporting supply side economics, which justifies a much more laissez faire approach by government. However, as shown in some of the charts in this chapter, such a redistribution does not seem to be materialising, calling into doubt this idea. To better predict whether this trend will continue a deeper understanding is required as to why inequality initially grows, and why it hasn't yet declined again on its own accord, like it has in the past.

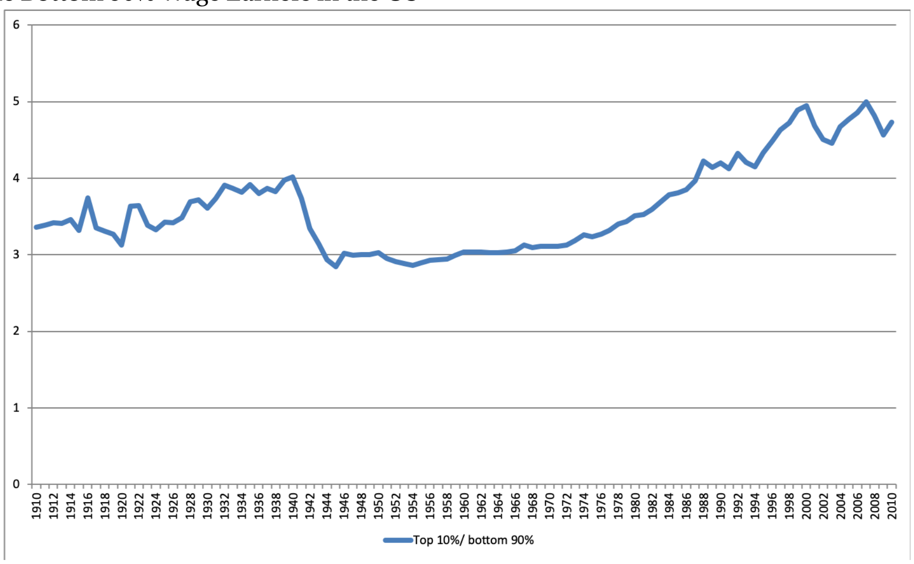
Figure 1.1.1: Capital vs Labour share of income in the US



Data from Bengtsson and Waldenstrom (2015) shows the long term trend in labour and capital share of income for the US

The first element of analysing these phenomena would be looking at the functional distribution of income, i.e. the amount of total income that is going to wages compared to paying capital, as capital owners are generally wealthier individuals. In the context of this type of research the relevance of the labour share would be down to how it exposes income inequality, through dividing agents into entrepreneurs, or capitalists, and workers. However, analysing factor shares as a gauge for wealth inequality has potentially become less reliable. The increased concentration of income within the labour share has shown to be more relevant than the change in labour share itself. This is evident as figure 1.1.1 shows that long term labour share hasn't changed much, using US data from Bengtsson and Waldenstrom (2015). Data from the national accounts confirm this, as when including supplements to wages, like pension, labour share has actually increased. However, Acemoglu (2002) has shown a large rise in college premium, and IRS returns shows the wage premium of the top 10% of earners has increased substantially more in the US over the last century, as shown in figure 1.1.2. Consequently it is appropriate to look at the share of income attributed the basic (unskilled) labour in particular when studying the development of low paid workers incomes. As it is the welfare of low paid workers that is of interest, there is an important distinction to be made between outcomes that decrease unskilled Labour's share of income but still increase basic wages, and factors that decrease basic wages.

Figure 1.1.2: Ratio of the Average Wage of the Top 10% Wage Earners vs Average Wage of the Bottom 90% Wage Earners in the US



This shows measure of wage distribution based on IRS returns as reported by https://wid.world/ in 2018

Until the late 20th century the measured labour share of income remained extraordinarily consistent. John Maynard Keynes noted in 1939 that this consistency was a bit of a miracle (Keynes (1939)) referring to the previous half century. Since the 1980s there has been, as reported by many measures, a secular decline in the labour share reported for most countries (Bengtsson and Waldenstrom (2015)). Over the course of the whole of the last century however, most countries have have not seen a significant increase in capital share of income. This can be observed clearly with data from the US (figure 1.1.1). As a result explaining the trend in wage distribution is the primary aim of this research in order to understand the main drivers of equilibrium basic wages which in turn gives a guide to the drivers of overall inequality.

The approach of this thesis will be to better understand the role of fundamental changes in the economy, specifically production, in the changing wage distribution. Chapter 1 seeks to measure income effects coming from outside of the individuals control as opposed to the the increase in wage inequality coming from an increase in relative level of skills. The second Chapter looks at the role of factor inputs in production to explain an increase in demand for skilled labour. Outsourcing, globalisation and price shocks can create a change in factor inputs that will incorporate structural changes. The third chapter analyses the implications of factor inputs on technology growth and how this further impacts income distribution in a way that helps explain what type of shock or structural change has initiated the increasing inequality that has been observed.

1.1.2 Aim

As I will discuss, the literature on wage inequality mainly uses proxies to measure the levels of skilled vs unskilled work and pay. The problem with these measures is not only that these proxies are non-extensive, but also their relevance may change through time and between individuals, making the parameters that link this measures to marginal product not constant, both on a cross-sectional and time series basis. In the modern economy, skills come in many, mostly unobservable, forms. A comprehensive method of comparing skill levels in a single time period is to assume the varying marginal productivity (and so wage) is determined by the skills the workers possess. However, as an economy evolves the demand for these skills may increase or decrease. This means wage premium can be influenced by both the level of skills and the demand for them.

Within this chapter I will develop a theoretical basis for showing the extent of wage inequality and how this has occurred in terms of distinguishing between the accumulation of skills and the change in the wage or income going to skills already possessed. Through this it can be understood how much of the change in distribution has been to relatively more skills being learned at the top end of the curve (i.e. an increase in human capital) and how much from the increase in income for those skills at the top end. For example, if group A increases their wage compared to group B, it may be because group A have developed relatively more skills, which gives them increased economic value. Alternatively it could be because their existing skills have become relatively more valuable because of a structural change in the economy. Distinguishing between the two will help understand how wage inequality has progressed and whether it is merited. As in the second principle of the theory of justice (Rawls (1971)), I judge greater inequality to be justified if it makes the worst off better off, but not if the better off simply experience windfall gains to existing skills. It could be part of the cause of economic growth if it is through skill accumulation, or a consequence of economic growth as more skilled labour become relatively more valuable.

The main contribution of this chapter will be to distinguish between an increase in higher end skills, relative to basic skills, and an increase in relative value of skills when explaining the increasing wage inequality. This can be seen as relevant on multiple levels, economically, socially and econometrically. Firstly, if the extra relative wage is due to human capital quantity, it can attributed to their contribution to economic growth, as the individuals involved will be getting paid more due to increased skills (i.e. human capital) which is causing economic growth. This means the inequality created can only be undone by curbing this growth in human capital, and so restricting economic growth, or creating the same accelerated skill growth among low wage earners. This leads to the social issue, that if the relative wage growth is from human capital quantity, it can be seen as warranted or deserving, due to being earned by increasing skills through extra hard work and learning. As mentioned earlier, if it is a result of the marginal product of skill changing, it is a more circumstantial consequence of other changes in the economy. In this case high wage earners have been the lucky beneficiaries of structural changes in the economy. This is something that can potentially be addressed by looking further into what structural changes have taken place to cause such an effect, as is done in later chapters. The final benefit from distinguishing the two sources of rising wage inequality is it enables further econometric research, to assess and test the implications.

In summary the main aim for chapter 1 in the context of this thesis is to test the hypothesis that the increase in wage inequality has come largely from relative increase in income for skilled human capital as opposed to an accelerated increase in skills.

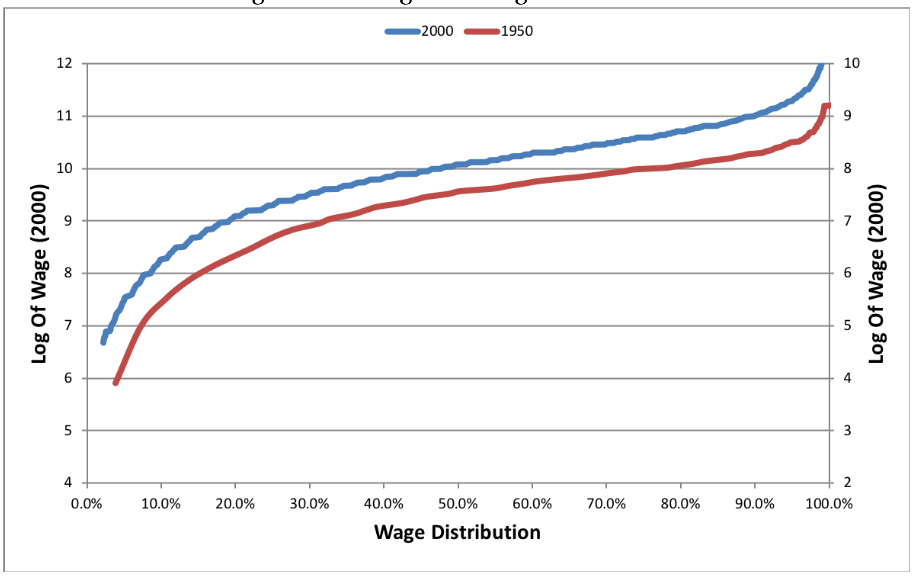
1.1.3 LITERATURE

The clearest measure of wage inequality can be seen from the wage distribution curve (figure 1.1.3) which shows the log of wages for employed worker in the US in 1950 and 2000. However the distribution curve's econometric use in inequality literature has been limited as it supplies no natural way of showing heterogenous agents through splitting workers into skilled and unskilled. Other measures, e.g. educational attainment or job role, do naturally divide up the population, but don't attempt to quantify skill levels. A general issue with all methods is that while the amount paid to individuals is observable, there is no natural way of determining whether changes in income comes from a change in the quantity of human capital or a change in the return to human capital. In other words whether skilled individuals are seeing increased relative pay through a further accumulation of skills. This is because, in human capital literature, high and low paid workers possess the same human capital, just different levels of it. As a result any rise in return to human capital would effect skilled and unskilled workers equally, leaving only the quantity of human capital to explain changing wage inequality. To make the distinction between whether highly skilled workers are paid progressively more due to more accumulated skills, or the a higher value being put on skills already obtained, units of human capital between skilled and unskilled workers must be shown as heterogenous.

In macroeconomics, the topic of human capital is generally used to explain growth, with accumulation of human capital being the driver of endogenous growth. There is not usually a distinction made between basic labour input and more advanced human capital, they are both measured in homogeneous units, with a consistent relative distribution among individuals. This means the set up is limited in the way that it can only explain wage inequality through an accumulation of skills, and not a change in value of already obtained skills. In micro econometrics however, human capital has generally been measured by the following methods:

- The educational attainment approach - this is commonly used in literature that divides labour into skilled and unskilled individuals (Acemoglu (2002)). Here a measure like number of years schooling, enrolment rates or literacy rates are used as a proxy for human capital.
- The cost based approach - using the value of the inputs that enter the production of human capital. This values the human capital stock as being the depreciated value of the amount spent on investment in human capital (Kendrick (1976)).

Figure 1.1.3: Log of US wage distribution



US census data from IPUMs based on individuals who are currently employed and worked over 40 weeks in the previous year

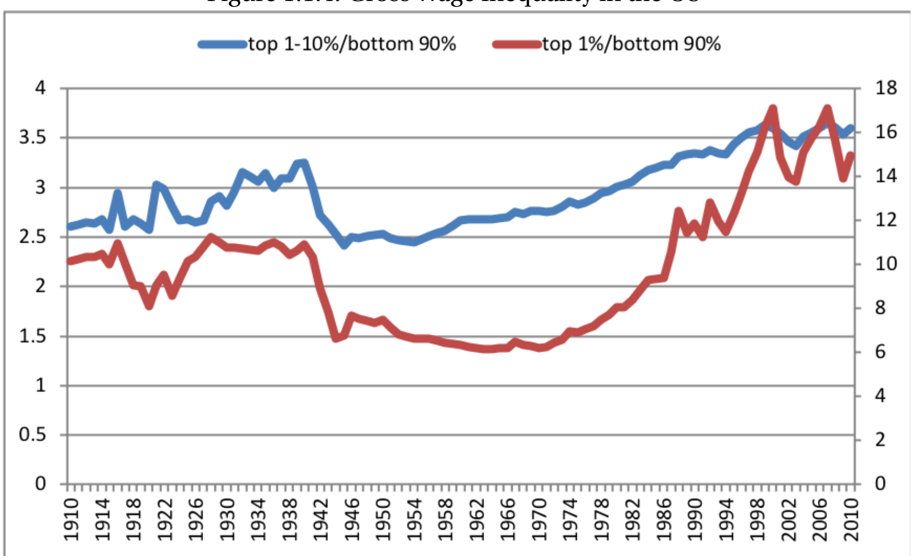
This approach gives an estimate of the resources used in human capital. Schultz (1961) used this approach to estimate that between 36 and 70 percent of the unexplained rise in earnings comes from the increase in human capital as measured by the return to education.

- The income based approach - human capital that is typically measured by labour market income. Expected human capital income is measured for each demographic by taking future incomes with the probability of a change in demographic and a discount factor. This income is used to estimate the value of human capital.

The main limitation with any approach that uses education is that they miss most of human capital attained beyond that elementary level, such as soft skills, self obtained skills and experience. This makes them flawed proxies for human capital, especially in developed countries where alternative ways of obtaining skills are more available according to Judson (2002). It is also dependant on the relevance or quality of the college, which has debatably been reduced during the aggressive roll out of college education, being consistent. For this and other reasons the relevance of education in accumulating human capital may change through time due to factors such as institutional change. With a cost based approach the amount spent is also technically quite hard to measure. Overall the two input measures (the education attainment approach and the cost based approach) are not fully inclusive of all skills that increases an individuals marginal productivity, as most are not easily observable. According to Ashton and Green (1996), it is necessary that the link between measured accumulation of human capital and economic performance should be considered within a social and political context to precisely measure the human capital. This would include linking an input based approach with final wage to compensate for quality and actual added value.

Further investigating the change in inequality that has occurred, as can be seen in figure 1.1.4 which compares the top 1% to the rest of the top 10% to the bottom 90%, most of the rise in wage inequality occurred between 1950 and 2000 (though it was much more pronounced after 1970). Between these two dates the wage distribution didn't actually change much among the bottom 63% of earners in the US. It was only the top 37% that saw increased inequality as their wages increased more than the rest, as shown in figure 1.1.5 which shows the wage distribution in 1950, 1980 and 2000 as percentage of the 37th percentile of each corresponding year. This indicates changes for individuals at the higher end of the distribution curve is driving inequality.

Figure 1.1.4: Gross Wage Inequality in the US



Data: Picketty, Based on IRS income tax returns, downloaded 2018 from https://wid.world/

Some methods that disaggregate the population require a level of homogeneity between different demographics, and so does not account for heterogeneity between agents within certain categories where there is vast differences in wage and so implied human capital. As seen in figure 1.1.6, there is increasing levels of wage inequality among college graduates and among non-college graduates. Another issue with the idea of college education as a proxy for skilled labour in terms of explaining the wage premium is that the increase in wage premium was more general when there were very few college graduates before 1980, and then the increase was much more concentrated afterward when there was actually many more college graduates. From 1950 to 1980 the increase in wage premium was predominantly seen among the top third, which is far broader than the increase in college graduates from around 7% to 17%. From 1980 to 2000 the increase in inequality was mostly among the top 15% of individuals, which is a less broad than

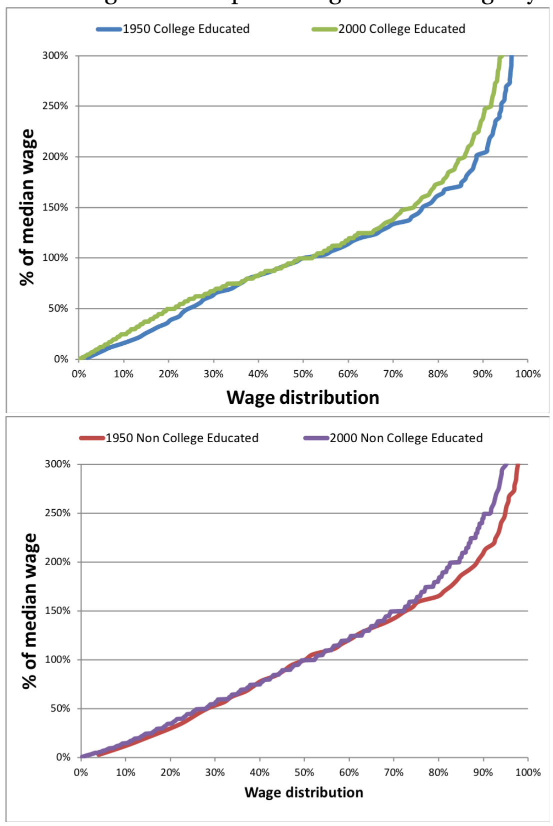
Figure 1.1.5: US wage curve as percentage of 37th percentile wage

US census data from IPUMs based on individuals who are currently employed and worked over 40 weeks in the previous year. All wages are shown as a percentage of the 37th percentile wage of each corresponding year

the number of college graduates which rose to around 25%. These observations are not consistent with what one would expect to see if college education were the main driver or proxy for wage inequality. As such there is reason to believe that using college education is biased in terms of its application as a proxy to skilled labour over time, with the resulting growth in skilled labour being overestimated and their relative earnings being underestimated.

Recently wage inequality has commonly been represented by this method of splitting workers into two heterogeneous groups. Literature on Capital Skill complementarity (Krusell et al. (2000)) and Skill biased technical progress (Acemoglu (2002)) have used educational attainment to represent skill. Typically those with a college education are considered skilled whereas everyone else is unskilled. As seen in figure 1.1.7 the number of college educated individuals started at a very low base, only accounting for around 5% of the workforce in 1940, rising to over a third more recently. With college premium rising dramatically as well (also shown in figure 1.1.7), clearly the relevance of college has increased. However, the question remains whether this is a good measure for explaining the wage distribution curve. I have already argued it isn't for a time series or panel data approach. Supporting this it can be seen with the comparison of college to non college educated wage distribution curves (figure 1.1.6) there is still much wage inequality within both of these groups, which is comparable to the level of inequality

Figure 1.1.6: US wage curve as percentage median wage by education

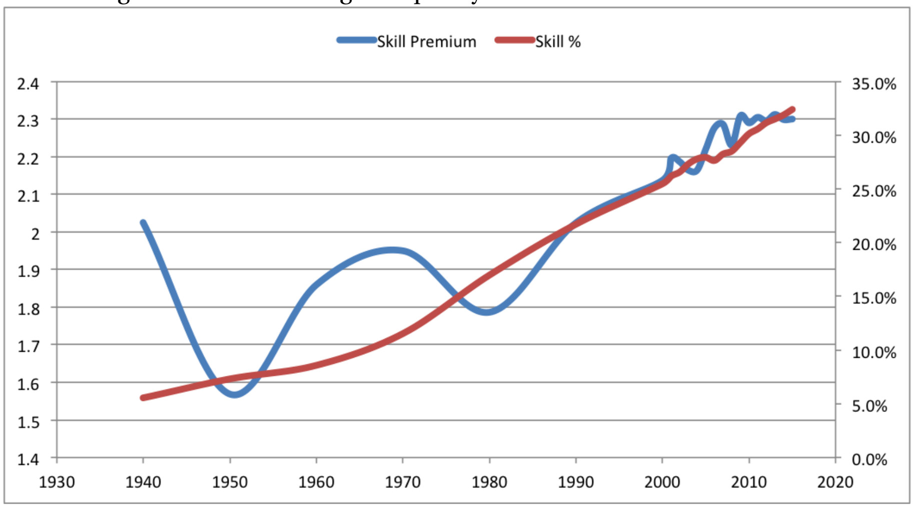


Charts show wage distribution for college graduates and non college graduates between 1950 and 2000. Data from IPUMS, College educated is defined as 4+ years of college

in the two groups combined. As there is a college premium that has been rising, an increasing premium on college education does go some way to signalling wage inequality, but it looks like it might not be a prominent driver.

Research done on historical inequality going back the last few centuries in the UK has used job role to determine skilled labour (Clark (2005)). In Clark's paper carpenter wages, or craftsmen, are compared to that of basic manual labourers. One issue with this is that this measure of skill premium is affected by any structural change away from skilled craftsmanship and towards automation. In the US for example, due to a structural change away from manufacturing and also due to increased automation (making craftsmen less needed), wages for job roles like carpentry are approximately equal to with average wage, whereas there used to be a noticeable premium back in the mid 20th century, according to census data. This is in contrast to the direction of overall wage inequality, which is increasing over the same time period. An updated method of defining skilled job roles shows the wage premium in certain broad category jobs over time like managerial roles and professionals which, on average, are the highest paid job

Figure 1.1.7: Gross Wage Inequality in the US based on education



Using IPUMs data, skill premium, on the left axis, shows the average ratio of the wage of college educated individuals vs non college educated with skill defined by 4+ years of college. The red line shows the percentage of workers who are college educated using the right axis.

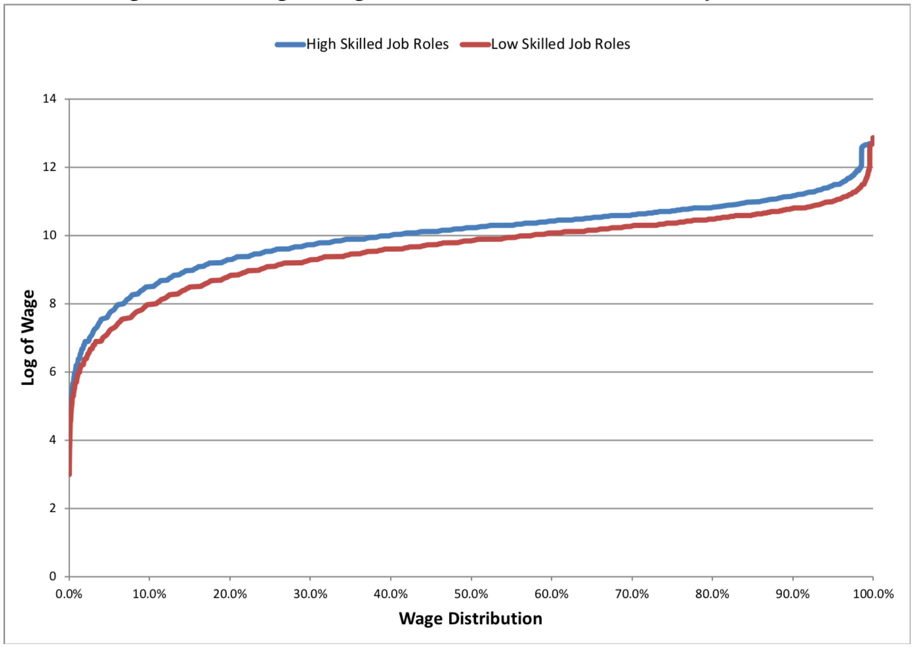
roles as figure 1.1.9 shows by comparing the wage ratio of skilled vs unskilled job roles. This shows a similar pattern to college premium. Ideally it would be more revealing to use more specific job titles, however it is difficult to get consistency in the data due to changes in methodology. With the broad definitions once again the highly skilled job roles don't explain the majority of the wage inequality, as inequality within those roles is larger than between them as shown in figure 1.1.8 which compares the wage distribution between skilled job roles and other job roles. And so the conclusion for this method is the same as with college education in terms of its flaws when using time series data.

1.1.4 Free Market Wages

One complication with the overall approach to wage inequality being a proxy for welfare is, with the introduction of pension schemes and social security, the wealth of the average household is more exposed to capital than it was before. (Burtless (2007)) argues that between 2000 and 2005 over two thirds of the wage increase was not from cash wages. In terms of measuring the effective pay of workers this is a limited issue, as my figures include pension pay. This welfare is paid for through increased taxes, which is why I have included taxes in the model.

Another consideration is the idea that wage levels are a result, in part, due to political decisions as apposed to economic forces. Non free market factors like unionisation and minimum wages are thought by some to be the main influences involved. Part of the problem with many of observed political effects is that these factors often change to-

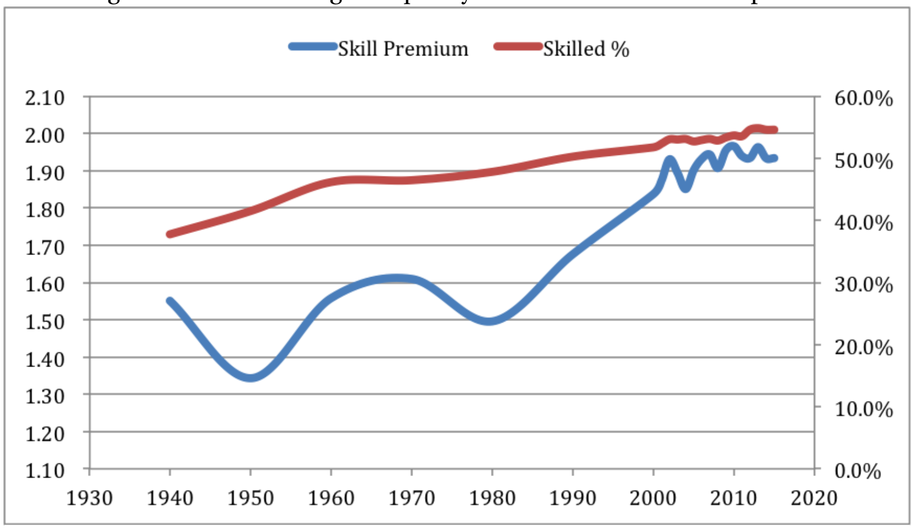
Figure 1.1.8: Log of wage curve of skilled vs less skilled job roles



Data: IPUMS (2010), skill defined as being professionals, managerial roles or professors/instructors

gether as part of a larger political change, which can also create structural changes in the economy. As result isolating the effect of one policy becomes difficult. For example, since the 1990s minimum wage in Brazil has been rapidly rising, and at the same time there has been decreasing inequality. However during the same time there has been a significant regime change, as well as a change in terms of trade. In fact, cross-sectional data has indicated that some of the evidence points to minimum wage having had adverse effects on lower-income families in Brazil according to Neumark et al. (2006). Similarly in the US, while a strong decline in real minimum wage in the 1980s coincides with increasing inequality, there are many other potential causes during that decade, as energy prices, trade barriers and taxes all changed during the same period. Outside of that decade minimum wage doesn't seem to explain the persistent movements in inequality. However, various papers (Acemoglu et al. (2001), Pontusson (2013)) claim that deunionisation has had an effect on inequality, or at least has provided less protection against the effects of globalisation and technology changes. These papers don't necessarily consider a potential problem with the endogeneity of union power. It is likely that union power is dictated by the equilibrium wage (i.e. marginal product) of member workers. If the equilibrium wage stagnates, then unions will likely be trying to push wage further from free market equilibrium, which would increase unemployment, eroding union power. Alternatively if workers become more productive, unions may find it relatively easy to negotiate progressively favourable contracts. This shows how unions probably aren't exogenous when deciding wages, as has been found in Duncan

Figure 1.1.9: Gross Wage Inequality in The US based on occupation



Using IPUMs data, skill premium, on the left axis, shows the average wage of ratio of skilled job roles (defined as professionals, managerial roles or professors/instructors) vs other job roles. The red line shows the percentage of workers who are in skilled job roles using the right axis.

and Leigh (1985). Ultimately, through this process, wages will still mainly be driven by equilibrium market forces. Even if it can be argued that union power provides some protection against free market forces, this and redistribution effects are not only distortionary, but also cannot permanently protect poorer workers from the secular effects of wage inequality bought about by relative changes in marginal product. As a result, I would argue that free market forces are still the main drivers of basic wages.

It can also be argued that other non free market factors in the form of rent seeking can impact inequality. In this case individuals obtain higher wages, not through productivity, but by negotiating or otherwise, directing income that formerly went elsewhere to themselves. An increase in rent seeking is distinguishable from increase in skills or technology as rent seeking discourages growth as apposed to encouraging it (McCombie and Spreafico (2015)) leading to the claim that much of the recent inequality and economic slowdown has been due to rent seeking. There is little evidence, however, that increased rent seeking has accounted for the change in wage equality, as most of the increase in wage inequality happened before the year 2000, when there was decent economic growth. It also runs against the idea of functioning capital markets for owners to allow CEO's (who McCombie's paper argues are receiving large increases in economic rents) to be paid beyond what they are worth. However, if inequality is driven mainly by equilibrium forces, then it may then be possible to address the issue in a way that doesn't discourage growth and creates a sustainable secular trend.

1.1.5 Defining Human Capital

As the aim of this thesis is to explain how wage inequality in the United States has increased over the last century, there is an important difference between wages being driven by individuals obtaining more skills and the income they get from those skills increasing compared to the skills others have. If every worker earns their income through the homogenous human capital it is not possible for the change in human capital income to affect wage distribution, as it would affect every individual at the same rate. Those with higher human capital would gain more however, as a ratio nothing would change. So if the rent per unit of human capital increases 10% everyone's wage increases 10%, and there is no change in the wage premium. This would leave only the quantity of human capital to explain a change in wage premium. However, I distinguish a higher end human capital that more skilled individuals have, allowing for the wage of this to be compared to the wage of more basic labour and impact the wage distribution.

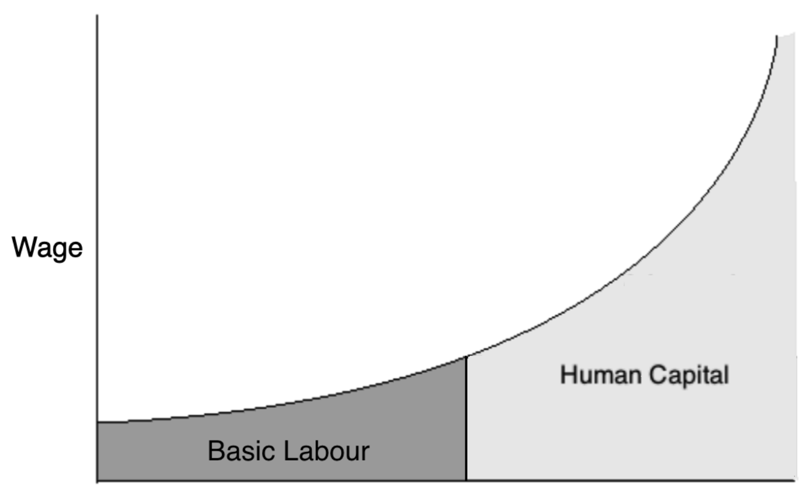
The Mincer equation (Mincer (1974)) is widely used for measuring human capital, including with Penn World Tables (Feenstra et al. (2015)). This treats schooling and experience as synonymous with human capital. With this definition the quantity of human capital can not sustainably rise above a fixed limit as there is only so much time the individuals can spend in education or experience. As generations can't sustainably have more experience and education than the previous generation, years of schooling becomes increasingly inadequate as human capital has little room to grow. This has resulted in the measured level of human capital growth decreasing in the US according to Davis (2020). Despite this, the value or efficiency of skills learned can continue to increase at a constant rate as each year of schooling or experience can give the individual more productive capabilities from generation to generation. For example, the know how gained from one year of learning computing skills that individuals learn today arguably gives them greater productive potential than was possible with one year of learning before computers were invented. In the classic definition of human capital this would have the same value, and would not account for the increased level of skill the individual has. In this chapter, human capital is defined by the actual skill level the individual obtains.

Distinguishing between changes in skilled human capital income and quantity to explain increasing wage inequality requires having separate human capital and basic labour inputs, as well as having idiosyncratic characteristics creating varying levels of human capital. This variable human capital allows for wage inequality to exist in the model. Keeping basic labour and human capital as separate inputs allows the relative income of a unit of basic labour compared to income per human capital to change income distribution, as lower income individuals will have their income from the former, whereas high income earners will receive their income from their human capital.

In this theoretical setup wage inequality is bought about by different levels of skill resulting in different marginal productivity levels. I allow for the concept of what affects human capital to remain less specified, simply assuming it is a mixtures of skills and abilities that lead to a higher marginal product of labour. As an example this can be knowledge of how to use a machine, piece of software, manage processes, etc. As a result any unobservable or unmeasurable characteristic of the individual is included in human capital, and it also allows for a changing composition of what would effect human capital.

Past models of human capital, in the context of inequality, focus on changing quantities of human capital being the driver (e.g. Galor and Moav (2004)). These don't factor in the previously mentioned impact of a change in the return to different labour inputs, as all labour has homogenous units of human capital. This ignores more skilled human capital units experiencing higher relative demand compared to a more basic labour (or unskilled labour) input, as mentioned above. This is important as in this research I focus on macro effects that have increased the income of higher skilled individuals as opposed to those skilled individuals increasing the quantity of skills they posses. And so I introduce the distinction between basic labour (that exists just from what is considered to be basic skills/education) and more advanced human capital from more advanced education, experience and know-how that is expandable on the individual level. In this case I separate the two in order to distinguish between the work done on a basic level and what requires more advanced skills and knowledge. Conceptually the resulting pay for the two can be shown by the distribution curve in figure 1.1.10.

Figure 1.1.10:



The theoretical wage distribution of unskilled (basic) labour and skilled workers who utilise their human capital (or skill).

In this setup, lesser paid job roles utilise the most basic of skills that are in abundance. For example, a labourer in a warehouse may only use basic skills that are possessed by most, e.g the ability to lift, carry, stack and maybe basic technical skills that are easy to learn. Taxi drivers need to be able to drive, but this is a common skill and so would not earn a premium to the basic labour wage. Unique skills like computer programming, operating advanced machinery in a factory, being specifically good at strategy or people managing are all skills that individuals take time to develop and are more scarce and sought-after enough to justify earning a wage beyond what is considered basic. These types of skills are defined as human capital within this model. These skills can come in other forms and be developed differently, e.g. a professional footballer develops unique skills through practice, or an entrepreneur might develop a range of skills though their experience with business, but the theoretical framework remains that these more advanced skills are ones that aren't generic and can't be adopted by others easily. As a result of their scarcity these skills lead to higher marginal productivity, and so they lead to higher levels of pay.

The implication of such a model is that basic skills are limited in their use beyond a certain point whereas the more unique and advanced skills can continue to add to marginal productivity. Empirical work has estimated that for lower education there are increasing returns to scale of further education, yet for higher education these returns to scale are significantly decreasing (Trostel (2004)). If it was possible to continuously add to lower education type skills then one would do so due to the higher returns to scale, implying there is a cap to these type of skills. Although the human capital measured in this model is supposed to be much more inclusive than just formal education, education should serve as a signal to the returns to scale in human capital. It would also help explain why the start of the wage distribution curve is fairly flat.

In practice there is still some room for interpretation on what counts as skill and so what wage level to consider the basic wage level. For example, today it is considered as standard to be able to read and write but for most of the 19th century less than a fifth of the world was literate according to OECD data. More recently it is normal to be able to use a computer however only decades ago that would have been a rare skill. Consequently is is not suitable to define human capital or basic labour by specific skills. It can be noted from figure 1.1.5 that for half a century the shape of the lower end of the wage curve remained more or less unchanged. It is only the top part of the wage curve that has seen a notable steepening from 1950 to 2000. This is consistent with the idea that it is pay for more advanced and unique skills that have driven wage inequality, as opposed to a change in relative levels of more general or basic skills among middle and lower paid workers.

So wage inequality has been driven by pay and accumulation of fairly advanced skills and, even though human capital does exist in the form of more basic skills, the latter doesn't seem to change significantly in terms of its distribution. What level is the cut-off between basic labour pay and human capital pay is not clear cut. It is best judged by taking a job role that is generically basic and looking at the pay within that role. Pay for basic labourers in the more modern economy would unlikely include much added value beyond basic pay, and so can be used to benchmark the cut off between skilled and unskilled workers.

As mentioned, the main econometric problem with using human capital in this way is, while the total income paid to skilled human capital can be measured once the basic wage is defined, this number is the product of quantity of human capital and wage per unit of human capital. Separating the two is not an exact science but is a key aim of the following model as it aids further analysis and also gives a perspective of how much of inequality is driven by changes in the marginal product of different skills as opposed to the accumulation of additional skills. The definition of a unit of human capital is vague due to its variable and abstract nature. Conceptually, any rise in wage that comes from obtaining skills is from human capital, whereas any rise beyond that would be due to increase in income per unit of human capital, that results from change in productivity or factor inputs. It follows that quantifying the human capital obtained from learning different skills is a case of consistency of how much that skill can earn in a given time period, similar to how physical capital is accounted for. For example, if learning a computer program and operating a certain piece of machinery both increase a worker's wage by the same amount, they both represent the same amount of human capital for that time period. If these skills become more valuable the increase in wages caused by its increase in value represents an increase in the return to human capital, not an increase in human capital. However if the computer program or machinery is expanded on in a way that requires further training, human capital is increased through the further training.

1.2 MODEL

In this model wage inequality occurs due to some individuals having more human capital than others. This assumes there is no inequality through rent seeking activity or through inherited advantageous contacts, etc. This assumption has many exceptions on the micro level but, as asserted earlier, changes in comparative skill sets should still be broadly valid in explaining changes in wage differentials. Human capital is obtained through investing time in accumulating it (i.e. time spent learning). However, the workers are heterogeneous in the way that they have a different value on free time, representing their different natural propensity to learn.

1.2.1 ENVIRONMENT

The production function is generalised to be a function of capital (K), human capital (H), total hours worked by unskilled workers and skilled workers (L_u and L_s) and a matrix of productivity values (A). Here human capital is the added value from more advanced skills and is only used by skilled workers. Total human capital can be defined as skilled labour multiplied the average human capital per skilled worker (h). A worker's 'human capital' simply means their general skill level, so that a worker with human capital h_t is the productive equivalent of two workers with $\frac{1}{2}h_t$ each, or a half-time worker with $2h_t$. The theory of human capital focuses on the fact that an individuals natural ability and the way they allocate their time over various activities in the current period affects their productivity, or his h_t level, in future periods. Whether a worker decides to do skilled or unskilled work depends on whether their wage will be higher if they use their human capital compared to a basic labourer's wage.

$$Y_t = F(A_t, K_t, L_{u,t}, H_t) \tag{1.2.1}$$

$$H_t = L_{s,t} h_t \tag{1.2.2}$$

$$w_{j,t} = max(w_{u,t}, I_t h_{j,t}) \tag{1.2.3}$$

If the worker is skilled, wage per hour (w) is human capital income (human capital times income per unit of human capital (I)) or basic wages (w_u) if the worker is unskilled. w_u is a theoretical level, below which the individual has not learned any discernible, unique skills that gives them added productivity compared to the any other unskilled worker. As such they are not paid any premium to the most basic wage level. Output (Y) is used for consumption and investment in capital, which has a deprecation rate of δ. Output is made up of capital rents and total wages. Consumption and investment have both public (government spending from taxes) and private elements.

$$Y_t = C_t + i_t \tag{1.2.4}$$

$$C_t = C_{g,t} + C_{p,t} \tag{1.2.5}$$

$$i_t = i_{g,t} + i_{p,t} \tag{1.2.6}$$

$$i_t = K_{t+1} - K_t(1-\delta) \tag{1.2.7}$$

$$Y_t = w_t L_t + r_t K_t \tag{1.2.8}$$

Here i is investment, C is consumption (which can be either private or government funded), r is return on capital and subsets p and g is for private and government sectors.

Various dynamic features of the economy, exogenous to individuals who are price takers, impact on income per unit of human capital, or rents (It). While individuals will have expectations of these features being stable, they will vary with shocks to the economy and policy. All that needs to be known about these rents is that they deviate from expectations randomly with an i.i.d ($\epsilon_{I,t}$) that takes the form:

$$I_t = E(I_t)e^{\epsilon_{I,t}} \tag{1.2.9}$$

1.2.2 Agents

Agents have a utility function which is dependant on consumption and leisure time (L_j). Time can be spent on leisure, working (l_j) or learning advanced skills (τ_j):

$$U_{j,t} = \Gamma(c_{j,t}) + \rho_j L_{j,t} \tag{1.2.10}$$

The individual has a total time (T) that can be spent on leisure, working (l_j) or learning advanced skills (τ_j). As a result $L_{j,t} = T - l_{j,t} - \tau_{j,t}$ resulting in the utility function being:

$$U_{j,t} = \Gamma(c_{j,t}) + \rho_j(T - l_{j,t} - \tau_{j,t}) \tag{1.2.11}$$

$c_{j,t}$ and $k_{j,t}$ are the individual's consumption and capital respectively. Unlike the firm, the consumer cannot separate hours worked from total wage obtained through human capital, as at any specific time period they have a fixed level of human capital utilised per unit of time worked if skilled (h_j). As a result the level of human capital used in a time period is linked to working hours for that individual (l_j).

Agent j maximises the sum of their future time discounted utilities subject to the resource constraint.

$$\sum_0^T \beta^t E_0(U_{j,t}) \tag{1.2.12}$$

Subject to their budget constraint:

$$((1 - T_{w,t})w_{j,t}l_{j,t} + (1 - T_{r,t})r_t k_{j,t} = c_{j,t} + k_{j,t+1} - k_{j,t}(1-\delta)) \tag{1.2.13}$$

Both the worker and capital are taxed at a marginal rate ($T_{w,t}, T_{k,t}$).

1.2.3 FIRMS

Competitive firms maximise profits. They can choose the amount of human capital they use not just by hiring more or less workers but by employing workers with more or less human capital. For this reason they can choose to use amount of human capital independently of how many labour hours they use:

$$\pi_t = Y_t - w_{u,t}L_{u,t} - I_t H_t - r_t K_t \tag{1.2.14}$$

$$I_t = \frac{dY_t}{dH_t} \tag{1.2.15}$$

$$w_{u,t} = \frac{dY_t}{dL_{u,t}} \tag{1.2.16}$$

$$r_t = \frac{dY_t}{dK_t} \tag{1.2.17}$$

1.2.4 FIRST ORDER CONDITIONS

First order conditions with regard to consumption (c), capital (k), labour hours (l) and time spent learning (τ) using 1.2.12 and 1.2.13 gives the following:

$$\frac{\Gamma'(c_{j,t})}{\Gamma'(c_{j,t-1})} = \frac{1}{\beta(1-\delta+(1-T_{k,t})r_t)} \tag{1.2.18}$$

$$\Gamma'(c_{j,t}) = \frac{\rho_j}{(1-T_{w,t})w_{j,t}} \tag{1.2.19}$$

$$E(\Gamma'(c_{j,t})) \geq \frac{\rho_j}{\beta E[(1-T_{w,t})l_{j,t}]\frac{dE(w_{j,t})}{d\tau_{j,t-1}}} \tag{1.2.20}$$

1.2.18 is the standard first order results from the agent's optimisation of consumption and capital. 1.2.19 is from optimising labour hours. 1.2.20 is from optimising time developing skilled human capital. Both 1.2.19 and 1.2.20 come from the cost, in terms of utility, of time spent either learning or working (ρ_j) being equal to the present value of the extra income from learning or working multiplied by the extra marginal utility from income ($\Gamma'(c_{j,t})$). Since not every agent chooses to obtain skilled human capital, the marginal benefit of time learning may be less than the marginal benefit of future consumption. For them to be equal, I will assume that those included obtain skilled human capital in the future, so:

$$\rho_j = \beta E[(1-T_{w,t})l_{j,t}]\frac{dE(w_{j,t})}{d\tau_{j,t-1}}E(\Gamma'(c_{j,t})) \tag{1.2.21}$$

In 1.2.21 the right side shows the marginal benefit of time spent learning at t-1, while the left side shows the marginal cost of less leisure time (ρ_j). The marginal benefit includes the marginal wage gain, at time t, of time learning at t-1 is $\frac{dE(w_{j,t})}{d\tau_{j,t-1}}$. The total marginal net wage, factoring tax and hours worked, is $E[(1-T_{w,t})l_{j,t}]\frac{dE(w_{j,t})}{d\tau_{j,t-1}}$. Then the marginal gain to utility of this future income, discounted for time, is shown by multiplying this by the time discount (β) and the gain in utility of extra consumption ($E(\Gamma'(c_{j,t}))$).

From this it can be seen how different individual wages, reflective of different human capital levels, is influenced by the idiosyncratic cost of time (ρ_j) in the optimisation of individual outcomes. Without the need for different abilities, potentially impacting returns from time learning, this explains different levels of time learning or education amongst individuals.

1.3 Linear Model Specification

A linear relationship between education and learning variables occurs if certain assumptions are made surrounding the law of motion of human capital. The main assumption is that the idiosyncratic capabilities of the individual does not impact the level of human capital. In this model different levels of human capital still exist due to different personal cost of time between individuals (ρ_j) which is used to accumulate human capital. Here, an individual's time spent learning becomes exogenous in the the law of motion of capital, and so is not impacted by natural capability. This assumption does not hold up, as there is an estimated bias in the basic OLS regression of education and wages that results from these assumptions (Carneiro et al. (2003)). This I discuss later as well as estimate how this bias affects the result.

The level of human capital can now be defined by the accumulated time spent learning by an individual, plus a standard starting level of human capital (η).

$$ln(h_{j,t}) = \eta + \frac{\beta_{x,t}}{b} \sum_{i=1}^{t} \tau_{j,i} \tag{1.3.1}$$

$\tau_{j,i}$ is a vector of measures for time spent learning different types of skills. Time spent learning translates into human capital through vector $\frac{\beta_{x,t}}{b}$. Time spent learning in terms of educational signals can be expressed as:

$$\sum_{i=1}^{t} \tau_{j,i} = bX_{j,t} \tag{1.3.2}$$

$X_{j,t}$ is a vector of variables that measure the skill or human capital accumulated by that individual. These variables signal the time spent learning with a weighting using a vector of parameters (b). It includes dummy variables for completing school and each year of college. It also includes the experience of the individual and whether they live in an urban/metropolitan area. This will include a log variable measure for experience as well as the normal measure to capture the likely concave relationship it has with the log of wages, as it is well known that the marginal increase in earnings declines as workers get older.

This gives the following expression for human capital:

$$ln(h_{j,t}) = \eta + \beta_{x,t} X_{j,t} \tag{1.3.3}$$

Variables translate into human capital through vector $\beta_{x,t}$. As $\beta_{x,t}$ changes with time it doesn't define the relationship between change in variables and change in human capital, yet can be used as a guide. Human capital levels are not observable so I use the term for wages, which multiplies human capital by human capital rents (I_t). So I use $ln(w_{j,t}) = ln(h_{j,t} I_t) = ln(h_{j,t}) + ln(I_t)$ to produce an expression that can be used econometrically:

$$ln(w_{j,t}) = \eta + ln(I_t) + \beta_{x,t} X_{j,t} \tag{1.3.4}$$

This expression is fundamentally similar to The Mincer Equation. One difference is that the variables of interest ($X_{j,t}$) aren't just schooling, but any variable that acts as an indicator for the level of human capital or skill (e.g. experience, etc). For the regression I also control for the inclusion of other factors, e.g. effects of discrimination, rent seeking or other circumstantial factors, which may vary in significance over time. All parameters have a time subscript to show the varying relationship between each ten year period that these might have with wages, as the economy and environment structurally change between generations. The resulting regressions will be 1.3.5 as a series of cross sectional regressions.

$$ln(w_{j,t}) = \eta + ln(I_t) + \beta_{x,t} X_{j,t} + \beta_{y,t} Y_{j,t} + \epsilon_{j,t} \tag{1.3.5}$$

$Y_{j,t}$ is the vector of personal characteristics, and other control variables that control for labour market discrimination and non free market forces. The error term ($\epsilon_{j,t}$) also controls for this. These include dummy variables for whether the individual is white, Asian, Male, lives with at least one family member and whether they live with at least two family members. It also includes measures for weeks worked over the last year and hours worked per week. Both these measure will include a log variable measure as well as the normal measure to capture the potential concave or convex relationship these have with the log of wages.

To use the regression results from 1.3.5 to establish the impact of a growth in time learning on wages I introduce a term for the impact on wages brought about by the rise in time learning. 1.3.6 is the resulting term showing the change in the vector of educational variables multiplied by a vector of parameters.

$$\beta_{X,t} \Delta X_{j,t} \tag{1.3.6}$$

$\beta_{X,t}$ is not defined above, as it shows the relationship between variables and wages between t-1 and t, where as the regressions from 1.3.5 give parameter estimates for each time period but not between them. From this we it is highly likely that the true value for $\beta_{X,t}$ lie between $\beta_{x,t-1}$ and $\beta_{x,t}$. Because of this I will take the estimate for $\beta_{X,t}$ ($\hat{\beta}_{X,t}$) to be:

$$\hat{\beta}_{X,t} = \frac{\hat{\beta}_{x,t-1} + \hat{\beta}_{x,t}}{2} \tag{1.3.7}$$

Where $\hat{\beta}_{x,t}$ is the estimate for β_x in 1.3.5 at time t. This assumption might seem slightly arbitrary but it is worth noting, from experimenting with the results, that the resulting estimate for the growth in skills induced from increased time spent varies a trivial amount compared to the assumptions that $\hat{\beta}_{X,t}$ is taken from the lagging parameters in the first time period ($\hat{\beta}_{X,t} = \hat{\beta}_{x,t-1}$) or $\hat{\beta}_{X,t}$ is taken from the second time period ($\hat{\beta}_{X,t} = \hat{\beta}_{x,t}$).

The resulting value for $\hat{\beta}_{X,t}\Delta X_{j,t}$ shows the part of wage growth that can be explained by the growth in skills, as opposed to changes in the value of those skills, for skilled individuals. The remaining growth in wages for these individuals is a result of changes in the value of skills, which is from a change in human capital income including changes in returns to skills obtained. These can be compared to the wage level for the basic worker for a general narrative of the causes behind the growth in wage inequality.

1.3.1 OLS Bias

To find an appropriate measure for change in skills using (in part) educational variables, I will first discuss what is actually being measured in this model. 1.3.5 above more or less resembles the Mincer Equation using standard OLS methods. There are a couple of factors that does make this method slightly biassed. One is the effect of the individuals ability on optimal learning decisions. The choice of education for an individual cannot necessarily be regarded as independent from expected earnings of that person. This reflects the fact that more capable individuals, on average, will get higher returns from education, and so are more likely to go to college, etc. This creates a selection bias in 1.3.5 as part of the explanation for the correlation between education and wages is that these are both endogenous to ability. This would create an upward bias in the OLS results.

Methods for controlling for ability have included household dummies (Abbas and Foreman-Peck (2007)), proxies for parents backgrounds and the Armed Forces Qualifying Test (AFQT) Carneiro et al. (2003). These all support that controlling for ability does create a lower estimation for returns to schooling. However, when IVs have been used in estimations of the Mincer equation to control for endogeneity, the parameter estimation is actually higher than when using the standard OLS methods, as discussed in Heckman et al. (2006).

To show the likely explanation for why OLS gives estimations which are downward biassed, it is important to first discuss what the OLS actually measures. If the wage of the individual without a level of schooling/college ($W_{0,j}$) and the wage with that level of schooling ($W_{1,j}$) is given by:

$$W_{0,j} = \alpha + U_{0,j} \tag{1.3.8}$$

$$W_{1,j} = \alpha + \beta + U_{1,j} \tag{1.3.9}$$

Where β is the return to schooling and $U_{0,j}$ and $U_{1,j}$ are random variables that result from the individuals natural ability. For it to be worth the individual going through that level of education, the benefit of schooling $W_{1,j} - W_{0,j} = \beta + U_{1,j} - U_{0,j}$ has to be larger than the total cost (In this model cost is in the form of time learning, but in reality it includes cost of college, missed wage, etc). For this explanation I will assume that this cost is the same for each individuals and there are no liquidity constraints.

The basic argument, outlined above, for why OLS should be upward biassed assumes $U_{0,j} = U_{1,j}$, i.e. if a person is generally more able they are more able whether they choose to work as an educated or less educated individual. However this may not be the case. If all the educated individuals became scientists, and everyone else became plumbers, it could be the case that many of the scientists would have made relatively poor plumbers. In their case $U_{1,j}$ is high and $U_{0,j}$ is low ,where as the plumbers may have a low $U_{1,j}$ but a high $U_{0,j}$. The value of education for the scientist may be higher than comparing their wage to plumbers, as if they were plumbers they would not earn as much as current plumbers. Put more formally, OLS measures the benefit of education as:

$$E(W_{1,j}|S_j = 1) - E(W_{0,j}|S_j = 0) \tag{1.3.10}$$

Where S_j=1 if the individual if the individual partook in that level of education and 0 if they did not. The actual benefit of education for educated individuals is:

$$E(W_{1,j} - W_{0,j}|S_j = 1) \tag{1.3.11}$$

Which would be above the cost of education. This measure would be less than the cost of education for uneducated individuals:

$$E(W_{1,j} - W_{0,j}|S_j = 0) \tag{1.3.12}$$

As a result OLS estimates are downwardly biased if $COV(U_1, U_0) < 0$.

This exposes that there are different measures for education returns, meaning it should be established what measure is suitable for this purpose, and how it compares to the OLS estimation. The measured returns to education shows the average returns among those who have completed that education. Those who have already decided to complete a certain level of education will experience a higher net benefit from it ($W_{1,j} - W_{0,j}$ minus cost of education) than those who don't, and so have higher returns to education. As such the average educated individual should have higher returns to education than individuals on the margin, for whom it is marginal whether there is a net benefit of further education. If we then say some of the individuals that wouldn't have previously decided to complete a certain level of education, then decide to do so, the increase in returns may not be the same as the previous average returns to education.

As the increased human capital or skills from education is measured by an increase educational variables, the benefit of this increase is best represented by the returns to education of the individuals who would previously have not gone through that level of education due to it not quite being worth while. However, every ten years, a higher number of individuals do educate themselves, either because the cost of education goes down or the benefit increases. These individuals would have previously been on the margin of whether they benefit from the education. Written formally the measurement of interest is:

$$E(W_{1,j} - W_{0,j}|S_j = S) \tag{1.3.13}$$

Where S is the individual who is indifferent between taking on further education, as there is the same future net benefit in both cases. This is the measure that is of interest in this research.

The IV method estimates the return to schooling for individuals induced to go to school by changes in the values of the instruments. This is dependant on certain assumptions that the instruments do not affect the returns to education (Heckman et al. (2006)). While there is less bias in IV estimations, they are not necessarily much better than OLS. The estimates produced for many of the commonly used instruments have large standard errors in producing any particular parameter of interest except for parameters defined by instruments. Many of the instruments used in this literature are controversial. Parental education and number of siblings produce smaller standard errors. However, some of these measures are seen to be invalid. Heckman et al. (2006) discusses how most of the candidates for instrumental variables in the literature are also correlated with cognitive ability. Therefore, in data sets where cognitive ability is not available, most of these variables are not valid instruments since they violate the crucial IV assumption of independence.

It is worth noting that with the Census data I use, IVs and control variables for ability are not possible. Much of the CPS-Census literature on the returns to schooling ignores the choice of schooling and also ignores ability bias for this reason. Methods for controlling for general ability such as household dummies, parents backgrounds and the Armed Forces Qualifying Test (AFQT) are not available using census data. Common IVs are based on the individual's background, such as parental education, number of siblings and other characteristics from an individual's childhood like distance from college of their home at the age of 14. None of these are available for census data, which doesn't go into this level of history of the individuals background. To control for ability Abbas and Foreman-Peck (2007) uses household dummies to find returns to schooling in Pakistan. While US census data does identify households, the average household for the Pakistan data contained six male paid employees. Most households in the US have one or two working adults in total, making this method impractical. As I use census data, which takes a snapshot of each individuals situation, there is little data on the backgrounds of parents for each individual.

There has been a comprehensive study on the different measures of returns to education (Carneiro et al. (2003)) which includes estimates for the returns of individuals on the margin, as well as comparing this result to the result for OLS for US data from 1990 to 1994. The instruments used are the number of siblings, parental education, distance to college at 14, tuition at 17, local wage and local unemployment variables at the age of 17. They also include controls for general ability using AFQT scores, which is higher for those who attend college. The results give a guide to the difference in estimation between a standard OLS estimate, and an estimate for the average marginal effect of college, factoring in ability.

1.3.2 DATA

The data source used is IPUMS who report census data. I only use those in full time employment (excluding the self employed) in the private sector. I also only include those between the ages of 18 and 65. From the perspective of collecting historical data, I don't just rely on income data but also link income with certain demographics. This presents complications in terms of linking these demographics with income. In the US there is data on wage distribution going back to 1910 based on US tax returns, however it is not known how much of this income was earned by each demographic, meaning that IRS data used in figure 1.1.2 can't be used in the regression. The data from IPUMS use census data for historical records. Because of this, prior to 2000 there is only data for every 10 years going back to 1940, before which income data wasn't collected. This 10 year gap for each unit of time is reasonable considering the long term nature of what is being measured. It is reasonable to think that when an individual increases their skills, they are not planning one year ahead but are looking to improve their income many years into the future.

For $w_{j,t}$ I use the real annual wage of individuals above the basic wage, defined as the average wage of a labourer in that year[1] in 2000 dollars. I didn't use hourly wage per hour worked as this would have to be calculated manually using average wage and an estimate for hours worked. This creates a potential mismatch of measures between time worked and annual wage. The annual wage is measured as a person's wage over the last year. However, weeks worked over the last year is measured in intervals (as explained below) and is not taken accurately. Similarly responses for hours worked is given in intervals up until 1980, and are responses to hours worked over the last week which, despite being an indicator, may not resemble hours worked per week during the whole year. From 1980 and after, the measure for hours worked per week can be give as usual hours. However even this may misrepresent how many hours the individuals actually worked over the year, as what they give as hours "usually" worked may not actually be the hours they worked if they switched work or had a period of unemployment during the year. These measures would probably even out if hourly wage was estimated as an average over a large group, however any attempt to give hourly wages on the individual level will likely give many erroneous measurements.

The dummy variable for Gender (1 for male and 0 for female) is based on labour market discrimination (women withdrawal from the labour market for children before becoming more senior is likely to be captured in the experience variable). The same goes for race dummies for white and Asian individuals. The dummy for Asian workers was not significant in 1940, however grew in significance and had roughly the same coefficient value as for white workers in 2010. This indicates that though there was a historical disadvantage of being Asian compared to white, that disadvantage has been decreasing and disappeared in 2010.

[1] The basic annual wages of unskilled workers for each year (in 2000 dollars), based on the average labourers wage, are 16,328.91 for 2010, 19,627.84 for 2000, 18,255.70 for 1990, 20,394.22 for 1980, 19,769.97 for 1970, 16,665.36 for 1960, 13,227.69 for 1950 and 8,458.28 for 1940

To capture family dynamics, there is no consistent data for whether individuals are married or have children. However, there is data for how many family members live in the same household. For most individuals we can assume that if they are living with one family member then it is with a partner. If there is more than one family member then either they have children or still live with their parents. I use dummies for when there is one or more other family member in the same household, and a dummy for when there is two or more other family members. The latter, though having significance in the regression, has very little impact on wages. Generally, as long as the individual isn't living alone, they earn more. However, there is no clear explanation for this as explained below.

The method used in census data that can be used to measure hours worked per week and weeks worked per year changes. For hours worked there are three measures that have historically been used, usual hours per week (UHRSWORK), hours worked in the previous week (HRSWORK1) and hours worked in the previous week in intervals[2] (HRSWORK2). HRSWORK1 is not available for 1960, 1970 and from 2000. HRSWORK2 is available up until 1990 and UHRSWORK is available from 1980. To create as much consistency in data used as possible I use HRSWORK2 up until 1980, and then UHRSWORK (which is the most preferred measure) from 1980. For 1980 I do two regressions, one using HRSWORK2 to compare to 1970 and one using UHRSWORK to compare with 1990.

Data for how many weeks worked last year is not available for 1960, 1970 or 2010. However weeks worked last year given in intervals is available for every year[3]. To create consistency between time periods I use the interval data.

The measure for urbanisation is based on whether the individual is in a metropolitan area ("Metro") are for all time periods apart from 1970 and 1990, when answers for that measure do not exist. For 1990 the measure "Urban" can be used, which is a similar to the "Metro" measure. For 1970 this measure is omitted due to lack of data. This makes comparison between time periods that include 1970 and 1990 slightly inconsistent. In terms of impact on the regression results, omitting this dummy variable has little impact on the parameter estimates, as discovered by trialing this with other time periods. As this variable is used as one of the signals of skill levels, I show estimates in the results of growth impact of the growth in this variable. To do this I take estimates for "Metro" in 1970 and 1990 for parameter and variable values based on the average values in the time period before and after (by taking these as half way between the values in 1960 and 1980, for 1970, as an example). I am able to do this as the parameter and average values for "Metro" do not vary much, and create a small impact on estimated results in any case.

Experience is defined as age minus 16, minus the individuals years of schooling or college after 16. This won't always be accurate in determining years experience, as the individual may have past periods of unemployment or alternative education. However generally this calculation will be reflective of past experience.

[2]These interval values are are 1 for 1-14 hours, 2 for 15-29 hours, 3 for 30-34 hours, 4 for 35-39 hours, 5 for 40 hours, 6 for 41-48 hours, 7 for 49-59 hours and 8 for 60+ hours

[3]These interval values are are 1 if they worked 1-13 weeks, 2 for 14-26 weeks, 3 for 27-39 weeks, 4 for 40-47 weeks, 5 for 48-49 weeks and 6 for 50-52 weeks

For education, I use dummy variable for completing grade 12 of school, as well as dummy variables for 1 year of college, 2 years of college, 3 years of college (up to 1980), 4 years of college and 5 plus years of college. A key advantage of using dummy variables for each stage of education is to account for the various returns of each stage. As well as there being diminishing returns to schooling, different years have different levels of significance in terms of qualifications, e.g. in generally takes 4 years to complete a degree in the US. As data for completing year 3 is only available until 1980, the 1980 regression for comparing 1980 to 1990 will not include a dummy variable for completing 3 years of college.

In justifying the treatment of each variable, as the aim of this regression is to separate variables that are reflective of skill levels, and not just schooling like the Mincer Equation. Considerations have to be made with each variable whether each variable is best treated as a control variable or an indicator of skill. For experience and school/college education it is obvious that these are measurements of time spent learning new skills. It is also estimated by Yankow (2006) that that the higher urban wage can be mostly explained by cities attracting workers of higher unmeasured skills. As such any measure of urbanisation is also reflective of an increasing level of skill among the working population, though this increase is rather small.

The explanation on why being married positively affects wages, which is the main reason for the significance of the number of family members in the household, is uncertain, as discussed in Chiodo et al. (2002). So whether this is a variable that is reflective of variable human capital or should be left as just a control variable is not obvious. Luckily, growth in this variable has an insignificant impact on wage growth in the data. As such the assumption that a worker's family situation is not reflective of unobservable skill is a trivial assumption.

Any claim that race or gender is somehow reflective of a higher level of unmeasured skill would mean that this change in demographic has caused a downward effect on skill, mainly due to less men and white workers. However, there is little evidence that this is the case. The main variable that could explain the gender pay gap, that is not controlled for in these regressions, is occupation and industry, according to Chamberlain (2016). I assume that workers are free to move between occupations and industries, but gender may simply be a reflection of preference between these industries and occupations.

The only other variables to mention are measures for hours (per week) and weeks (per year) worked. One would expect that these variables show a concave relationship with wages, as the marginal benefit of time working for each individual declines. This is the case for hours worked per week but the relationship with weeks worked and wage is convex. Whether some of this pattern is due to weeks worked causing a higher growth in skills through experience is uncertain and it is not easily distinguishable from other possible explanations. However, it is worth noting, based on the parameter values, that the implied wage per week declines as number of weeks worked increases. This makes the explanation that weeks worked is a reflection of experience growth less likely, as one would expect weeks worked to have a positive affect on hourly wages in that case. As such, keeping weeks worked as a control variable, as opposed to a variable that causes higher skill growth, seems appropriate.

Table 1.1: Summary of Averages IPUMS 1940-2010

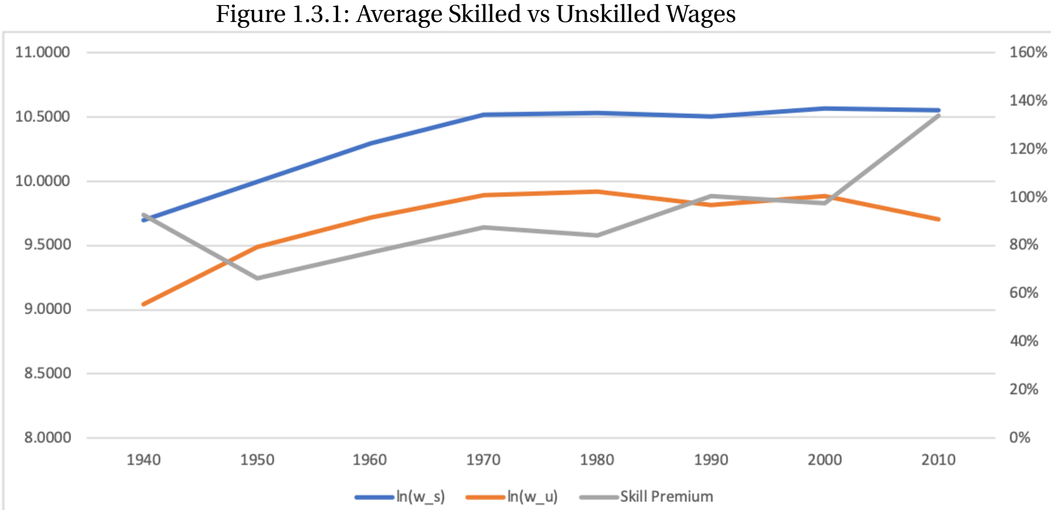
	1940	1950	1960	1970	1980	1990	2000	2010
white	**0.963**	**0.946**	**0.936**	**0.914**	**0.888**	**0.859**	**0.811**	**0.808**
asian	**0.002**	**0.002**	**0.005**	**0.008**	**0.017**	**0.028**	**0.037**	**0.053**
male	**0.806**	**0.810**	**0.795**	**0.756**	**0.704**	**0.622**	**0.597**	**0.550**
1+ family members	**0.878**	**0.897**	**0.913**	**0.900**	**0.835**	**0.853**	**0.836**	**0.840**
2+ family members	**0.670**	**0.654**	**0.679**	**0.650**	**0.579**	**0.577**	**0.541**	**0.522**
uhrswork					**42.127**	**43.242**	**43.914**	**43.025**
uhrswork (log)					**3.718**	**3.747**	**3.762**	**3.741**
hrswork2 (8 categories)	**5.085**	**5.121**	**5.022**	**4.769**	**4.965**			
hrswork2 (log)	**1.653**	**1.650**	**1.646**	**1.618**	**1.641**			
weeks work (6 categories)	**5.365**	**5.472**	**5.550**	**5.594**	**5.589**	**5.614**	**5.634**	**5.746**
weeks work (log)	**1.643**	**1.681**	**1.692**	**1.701**	**1.700**	**1.703**	**1.706**	**1.732**
metro	**0.729**	**0.736**	**0.698**		**0.753**		**0.754**	**0.764**
urban						**0.717**		
experience	**20.570**	**21.644**	**22.580**	**22.134**	**19.834**	**19.973**	**21.492**	**23.758**
experience (log)	**2.802**	**2.862**	**2.911**	**2.848**	**2.716**	**2.790**	**2.889**	**2.988**
school	**0.395**	**0.463**	**0.548**	**0.679**	**0.824**	**0.915**	**0.939**	**0.959**
college 1	**0.168**	**0.184**	**0.240**	**0.307**	**0.442**	**0.597**	**0.578**	**0.660**
college 2	**0.140**	**0.149**	**0.196**	**0.254**	**0.370**	**0.375**	**0.420**	**0.510**
college 3	**0.102**	**0.108**	**0.148**	**0.194**	**0.281**			
college 4	**0.086**	**0.091**	**0.125**	**0.169**	**0.242**	**0.290**	**0.334**	**0.410**
college 5+	**0.023**	**0.032**	**0.051**	**0.075**	**0.116**	**0.103**	**0.120**	**0.159**
Skilled Wage (log)	**9.700**	**9.999**	**10.293**	**10.520**	**10.533**	**10.507**	**10.566**	**10.551**
Unskilled Wage (log)	**9.043**	**9.490**	**9.721**	**9.892**	**9.923**	**9.812**	**9.885**	**9.701**

Based on Census data. All individuals are full time workers in the private sector, between the ages of 16 and 65 and earn above the threshold amounts described in the data section. White, Asian, male and family members, metro, urban school and colleges results are all dummy variables. uhrswork is the response to how many hours usually worked in a week. hrswork2 is how many hours worked in the previous week as categorical data. Weeks work shows how many weeks worked over the last year as categorical data. Experience is Age minus 16 minus the number of years schooling after the age of 16. Unskilled wage (w_u) is the average annual wage of a labourer. Skilled wage ($h1$) is the average wage of every worker who earns above w_u.

In summary, education dummies, experience and urbanisation are treated as variables causing skill growth ($X_{j,t}$) in 1.3.5, where as the other variables are all control variables ($Y_{j,t}$). A summary of the averages of each variable is shown in table 1.1. For the dummy variables the average will show the portion of the workers in the data that the variable describes. It is notable that as well as the portion of workers who complete all levels of education increasing, the average level of experience has also increased, as higher college uptakes has resulted in a higher average age in the workforce. Skilled wages have outperformed unskilled wages since 1950, largely because of a stagnation of unskilled wages from 1970. While they were both rising before 1950, unskilled wages actually grew faster.

Figure 1.3.1 shows the log of skilled vs unskilled labour, and the difference between them (skill premium), as used in the data. This is a different measure to that used in 1.1.2 earlier, where the annual wage of the top 10% of earners was compared to the bottom 90%. The comparison used in 1.3.1 compares those with relatively low incomes with everyone else, so middle earners would be considered as skilled. In the comparison in 1.1.2 middle earners would be grouped with the unskilled category. As middle earners have underperformed high earners, the comparison in 1.1.2 shows a slightly more extreme trend, but apart from that they are very similar. Both reach new heights by 1990 when comparing with the previous high in 1940. Both also show a consistent rise in skill premium from 1950. What is notable when looking at the actual wage levels in 1.3.1 is that both stagnate from 1970. However, unskilled wages decline slightly in real terms while skilled wages rise a trivial amount.

Figure 1.3.1: Average Skilled vs Unskilled Wages



This shows the log of the average annual wage of a labourer (unskilled wage - w_u) in orange. The log of the average wage for every individual being paid more than the unskilled wage (skilled wage- hI) is in blue. The higher pay for skilled vs unskilled wages as a percentage, or skill premium, is in black and uses the right axis.

1.3.3 Results

I regress 1.3.5 estimating the parameters for variables discussed in the previous section. I then use these results and 1.3.7 to derive values for skill growth in 1.3.6, and subsequently growth in income these skills earn, for each decade.

Table 1.2 shows the results using a linear OLS estimation. These parameters show how the variables that are signals of skill gauge time spent learning and wage earning skills, and so can be used to derive skill growth.

It can be seen that all education dummies, urbanisation (urban and metro) and experience (i.e. all the variables that indicate skill) are significant in each time period. While the experience parameter itself is sometimes negative, along with the log of experience parameter, the measured effect of experience is positive for each year for any possible value for experience. Generally, returns to college education have risen, mainly since 1980. This may be part of the explanation for the increase in college graduates. Interestingly, the impact of experiences dropped dramatically from 1940 to 1950, then slowly recovered until 2010. One could speculate that the drop after 1940 was related to the World War, as structural changes in production may have made previous experience less relevant. It could also be the case that the increasing returns to experience since 1950 is due to increased specialisation in industry, making experience more relevant to wages.

All control variables show high levels of significance for most years. The exception are in 1940 there was no benefit to being Asian as opposed to any other non-white race. After this there is a benefit to being Asian, which increased until it is similar to being white in 2010. Living with more than one other family member also had no significance in 1940 and 1950, though for these years those living with at least one family member had higher wages.

From these estimates, the average return to one year of college starts off between 6% and 9% up until 1980. After this it increases to just over 10% in 1990 and around 13% in 2010. The basic OLS regression used by Carneiro et al. (2003) from 1990 to 1994 puts this number at around 10%, showing consistency with these results. This consistency is important as it is this paper that I use to adjust my OLS results for endogeneity and bias.

The same paper estimates that experience would have grown earnings around 7.5% for the average years of experience of 9.4. My results, for the same average level of experience, give an average gain of 4.5% in 1990. This is slightly lower, however Carneiro et al. (2003) does use a younger sample which may partly explain the difference.

Table 1.2: US Cross-Section Wage Regressions

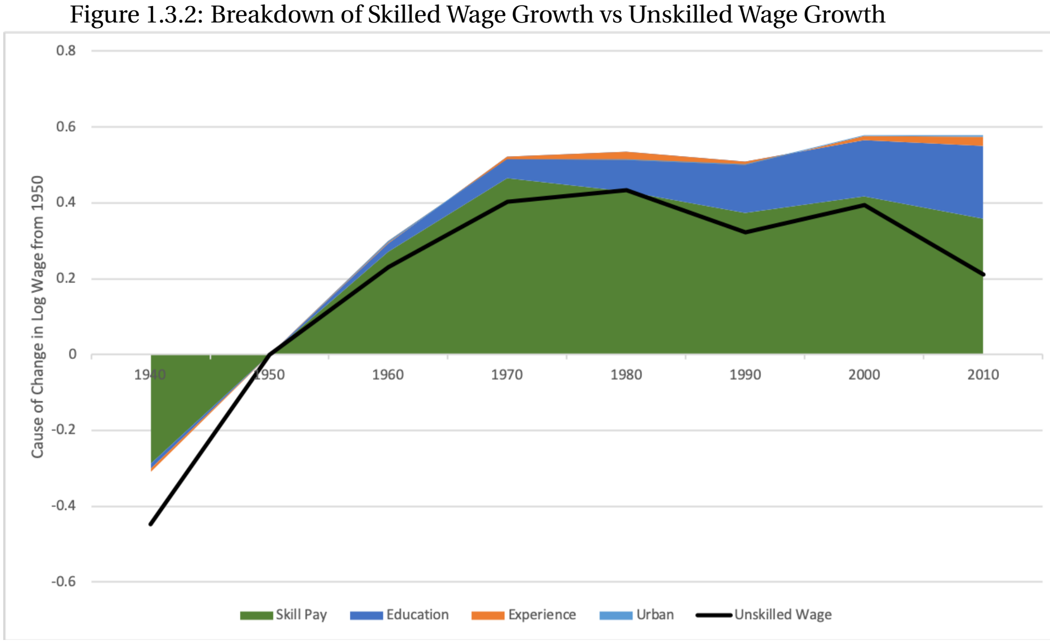
	1940	1950	1960	1970	1980	1980(2)	1990	2000	2010
white	0.305***	0.193***	0.205***	0.123***	0.0824***	0.0805***	0.0873***	0.0744***	0.0965***
	(74.90)	(47.57)	(228.38)	(98.99)	(112.91)	(114.34)	(138.11)	(124.40)	(71.23)
asian	-0.00250	0.0581**	0.0731***	0.0736***	0.0149***	0.0128***	0.0365***	0.0526***	0.0990***
	(-0.12)	(2.74)	(22.10)	(17.26)	(7.95)	(7.01)	(24.53)	(38.45)	(36.96)
male	0.288***	0.264***	0.317***	0.313***	0.303***	0.299***	0.252***	0.206***	0.201***
	(145.40)	(106.18)	(568.72)	(384.05)	(655.72)	(672.80)	(577.61)	(450.43)	(198.81)
1+ family members	0.0608***	0.0371***	0.0403***	0.0149***	0.0206***	0.0213***	0.0354***	0.0333***	0.0322***
	(19.95)	(9.43)	(41.37)	(10.84)	(29.91)	(32.03)	(51.05)	(49.06)	(21.39)
2+ family members	-0.00368	0.00112	0.0230***	0.0289***	0.0240***	0.0248***	0.00669***	0.0171***	0.0261***
	(-1.77)	(0.44)	(37.26)	(31.78)	(44.77)	(47.83)	(13.12)	(32.61)	(22.64)
uhrswork						0.00788***	0.00319***	0.00918***	0.0105***
						(105.81)	(28.06)	(81.43)	(40.58)
uhrswork (log)						-0.127***	0.143***	0.00868	0.0969***
						(-52.42)	(27.47)	(1.77)	(8.83)
hrswork2	-0.0454***	0.0256***	0.0200***	0.0446***	0.0659***				
(8 categories)	(-20.69)	(7.17)	(27.69)	(41.37)	(107.14)				
hrswork2 (log)	0.176***	-0.0711***	-0.0498***	-0.141***	-0.173***				
	(19.05)	(-4.49)	(-16.18)	(-31.00)	(-64.96)				
weeks work	0.156***	0.106***	0.0964***	0.0914***	0.0949***	0.0967***	0.122***	0.136***	0.110***
(6 categories)	(37.09)	(15.99)	(53.19)	(32.75)	(62.31)	(73.24)	(98.31)	(109.14)	(33.69)
weeks work (log)	-0.300***	-0.243***	-0.150***	-0.182***	-0.203***	-0.202***	-0.272***	-0.384***	-0.145***
	(-17.59)	(-8.61)	(-18.98)	(-14.94)	(-31.15)	(-36.41)	(-53.89)	(-76.86)	(-11.01)
metro	0.132***	0.0853***	0.116***		0.0955***	0.0934***		0.141***	0.144***
	(71.03)	(36.09)	(222.09)		(189.79)	(192.07)		(290.24)	(130.63)
urban							0.0891***		
							(187.66)		
experience	-0.00361***	-0.00338***	-0.00427***	-0.00512***	-0.00299***	-0.00282***	-0.00165***	-0.000986**	-0.00413***
	(-19.79)	(-14.95)	(-80.51)	(-65.82)	(-63.44)	(-62.38)	(-33.35)	(-18.02)	(-35.96)
experience (log)	0.253***	0.159***	0.190***	0.189***	0.190***	0.184***	0.194***	0.173***	0.240***
	(93.09)	(45.40)	(238.58)	(167.33)	(283.12)	(283.28)	(269.52)	(212.19)	(133.01)
school	0.171***	0.106***	0.119***	0.120***	0.115***	0.113***	0.125***	0.112***	0.175***
	(77.97)	(41.19)	(203.85)	(135.02)	(178.82)	(182.54)	(161.66)	(119.75)	(78.77)
college 1	0.0939***	0.0680***	0.0674***	0.0730***	0.0474***	0.0471***	0.105***	0.0991***	0.116***
	(17.14)	(11.02)	(52.55)	(43.87)	(55.96)	(57.36)	(186.01)	(161.25)	(81.79)
college 2	0.0645***	0.0398***	0.0490***	0.0553***	0.0396***	0.0528***	0.0556***	0.0445***	0.0760***
	(9.29)	(4.83)	(28.28)	(24.90)	(37.41)	(55.21)	(68.10)	(52.50)	(41.45)
college 3	0.0631***	0.0429***	0.0534***	0.0541***	0.0429***				
	(7.49)	(3.98)	(24.00)	(17.95)	(32.15)				
college 4	0.134***	0.103***	0.130***	0.161***	0.121***	0.149***	0.207***	0.226***	0.245***
	(16.71)	(9.85)	(61.37)	(55.17)	(92.94)	(168.75)	(234.05)	(262.16)	(137.27)
college 5+	0.0896***	0.0485***	0.0526***	0.0803***	0.0804***	0.0808***	0.149***	0.163***	0.207***
	(11.72)	(5.63)	(31.76)	(37.99)	(83.51)	(85.48)	(151.72)	(169.66)	(113.21)
_cons	7.902***	8.845***	8.836***	9.348***	9.252***	9.470***	8.503***	8.933***	7.974***
	(725.63)	(492.77)	(1954.38)	(1372.63)	(2469.98)	(1380.80)	(570.90)	(636.25)	(255.71)
N	192138	76073	1627881	787453	2444014	2626089	3286791	3767291	834948
r2	0.332	0.278	0.333	0.345	0.330	0.322	0.321	0.287	0.349

t statistics in parentheses
="* p<0.05 ** p<0.01 *** p<0.001"

Taking the average parameter values for the beginning and end of each decade, and multiplying it by the change in the average for that variable, gives the impact of the growth in that variable on skilled wages. The resulting impact of each skill indicator on the change in the log of skilled wages can be shown in figure 1.3.2, using 1950 as a starting point, due to this being the point when skill premium started to increase (though slowly at first). The remaining level of the growth in the log of skilled wages not accounted by skill growth, i.e. the growth in pay for a set level of skills, is shown in green. These are compared to the wage growth (logged) for unskilled workers. It can be seen that unskilled wage growth has been lower than the income for skills. This is despite those skills accumulating, so increasing in supply, for skilled workers.

These results, however, do suffer from the bias mentioned earlier when estimating the returns to education. To adapt the estimates for skill bias (which decreases the parameter estimates) and general endogeneity (which increases the parameter estimates) I use results generated by Carneiro et al. (2003) that compare different estimation methods and measurements. For this I am interested in how education might affect the marginal individual when it comes to wages. Their unbiased parameter estimates for the returns to education on marginal individuals were just under 52% higher than with the standard OLS method. Using this as a guide, 1.3.3 shows the same as 1.3.2 but including the adjusted estimated impact of education on the marginal individual.

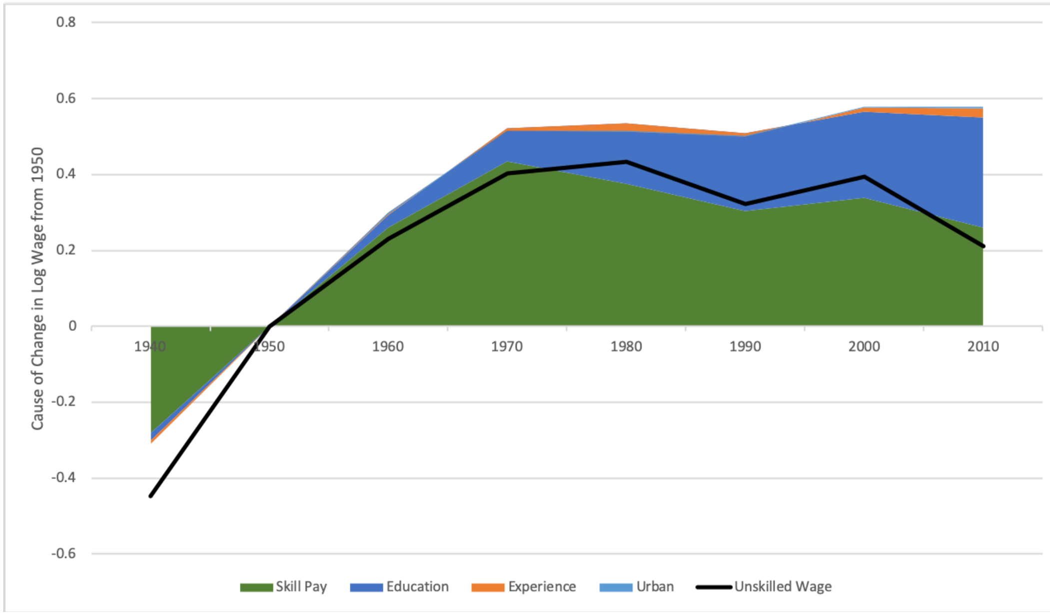
Figure 1.3.2: Breakdown of Skilled Wage Growth vs Unskilled Wage Growth



From tables 1.2 and 1.3 this shows the implied impact of growth in education variables, experience and urbanisation on the log of skilled wages. The amount in green accounts for the rise in income for skills. The is compared to the growth in the log of unskilled wages (black line)

Figure 1.3.3 shows that, even after adjusting for endogeneity, the rise in pay for wage earning skills has still been marginally higher than the growth in unskilled wages since

Figure 1.3.3: Breakdown of Skilled Wage Growth vs Unskilled Wage Growth (adjusted)



This shows the same as 1.3.2 but adjusted for OLS bias and to show the education returns for marginal workers using Chiodo et al. (2002) results. The returns to education increases 51.87% as a result.

1950, though these two measures have tracked each other quite closely. This is despite skills increasing in supply per worker through extra education, etc. Skill growth has been fairly consistent, accelerating slightly after 1970, possibly due to the higher returns to education making college education more popular. As skilled workers can earn more money from the rise in skills, as well as the rise in pay for those skills, they have increased their wage premium over unskilled workers, who don't expand their skills to earn more income. This rise in skill has been signalled mainly through a rise in the uptake of college education. Both unskilled wages and income paid for a set levels of skills grew rapidly before 1970, and both have decline since. This indicates a structural change in the economy from 1970, which will be discussed in later chapters.

This trend indicates that demand for skilled work has risen more than for unskilled work. This is because the supply of skill that exists has increased, yet the pay for those skills has slightly increased compared to unskilled pay, which has not seen a rise in supply. This is a similar conclusion to previous research observing that, taking college educated individuals as skilled, the total relative demand for skill has increased due to the the quantity of college graduates and skill premium for college workers both increasing (Acemoglu (2002)). The results of this chapter is derived from similar observations, but uses returns to education, as well as other indicators of skill, to quantify skill growth and income beyond simply observing the quantity and pay of graduates.

1.4 CONCLUSION

This exercise uniquely distinguishes between a change in skill and a change in income from skills already possessed for high wage earners in a way that the standard use of the Mincer equation does not. Wage growth includes both the rise in wages and the rise in skills and the income those skills earn. Without disaggregating workers creating heterogeneous human capital, where all workers have the same percentage rise in human capital rents, such a model would be unable to explain rising inequality. This distinction influences the interpretation of rising relative wages of high earners. If the period of increasing wage inequality was bought about by high earners learning skills at an accelerated rate, that add to output, and so improve welfare for everyone, it would be a necessary and acceptable aspect of rising living standards for low paid workers. On the other hand, if increasing wage inequality is a result of a structural change in the economy, that predominantly benefits high wage earners, the welfare of low paid workers may not be benefiting from the changes taking place. This model does not indicate what structural change may trigger a rise in human capital rent, as it could be from external factors increasing demand for more skilled workers, or a decline in the supply of human capital. The parameters of this exercise are simply to determine whether the rise in inequality is from the quantity of human capital (or skills) or from the income going to pre-existing skills.

The large increase in wage inequality occurred between 1950 and 2010. During this period this model estimates the level of skill per skilled worker grew fairly consistently. The growth in income for these skills has followed a trend very similar to the growth in unskilled wages, measured by the average wage of a labourer, which grew until 1970 and then stagnated. While these two forms of income stagnated at the same rate, skilled workers continued to grow their skills, meaning their wage didn't stagnate like they did for unskilled workers, who in this model have a fixed (basic) level of skill. Between 1940 and 1950 unskilled wages did grow rapidly, outpacing the income for skilled workers by a large margin. As the 1940s did include the end of the World War, it was a highly unusual decade economically, as the redeployment of workers after the war could be responsible for a resetting of skill.

While skill premium was increasing from the 1950s, in the first few decades it was still rebounding after a steep fall in the 1940s. Since the 1980s it reached new heights based on the measure used in this model. Part of the reason for this was the stagnation in unskilled pay from 1970. While skill based pay did also stagnate, this is partly to be expected when the level of skill is constantly rising. What caused the sudden stagnation of unskilled wages, and skill income, from the 1970s needs further study, but there is a clear indication of a structural shift from this period. This coincides with many political and supply side changes in the economy, the latter coming partly from the oil shocks in the 1970s and a revolution in computing technology. Investigating this is worth further study, and is the aim of the following chapters in this research.

On top of the shift in basic wages and human capital quantity and rent growth, another sign of a structural shift during this period is a change in what forms of education are connected to wage growth. For example, since the 1980s the premium for college education grew rapidly in the US. The impact of each year of experience also grew steadily since 1950, after declining before that. The changing relevance of these parameters emphasises the relevance of using time sensitive parameters when linking education to earnings, as these parameters are shown not to be structural in that their value and relevance changes with time.

The growth in college education, as well as experience (reflecting higher average age of workforce), have been indications of skilled workers spending more time learning and increasing their future wages. This does not sustainably increase, as they have done this at the expense of labour hours, mainly when they are young and decide to go to college instead of paid work. We can Interpret from this that skilled workers have experienced higher wages partly due to skill growth, but the increased level of skill should result in a decline in pay for these skills relative to unskilled labour (which hasn't happened), unless there has also been an increase in demand for skilled labour, implying there has been external factors benefiting skilled labour.

In a balanced growth economy, while skilled capital may be able to increase their skills from generation to generation, it would be expected that the returns for each skill (or rent per unit of human capital) would decline from this increase in skill relative to the supply of unskilled labour, which remains fixed per worker in this model. In terms of the notation used earlier, change in unskilled wage ($w_{u,t}$) and average skilled wage should be the same for balanced growth. The change in skilled wage includes the change in skills per worker (or human capital - h_t) and the income for these skills (I_t), giving the following condition in balanced growth.

$$\dot{I}_t \dot{h}_t = \dot{w}_{u,t} \tag{1.4.1}$$

Where $\dot{x}_t = \frac{x_t}{x_{t-1}}$. From this, accumulation in h_t should result in a relative decline in I_t relative to unskilled wages. This is similar to how, in a standard economic model, growth in capital per worker would cause a decrease capital rent compared to wages, as capital is the faster growing input. In this human capital model, skill growth among skilled workers should decrease income per unit of skill compared to unskilled wages, keeping incomes shares constant. As income per unit of skill has seen a similar trend to unskilled wages, despite skill growth, there is an indication of increased demand for skilled labour relative to unskilled workers. This has benefited high earners and, though admittedly their gain has skill has contributed to the income gain, the relative wage has increased beyond what one would expect in a balanced growth model. And so the answer whether skilled workers have increased their premium due to skill growth or external factors, the answer is both. This is an important conclusion when evaluating how we perceive the inequality growth experienced and the welfare consequences mentioned earlier in this chapter. Some have already claimed this increase in skill demand from simply observing an increase in college graduates and their premium (Acemoglu (2002)). However, this model gives a more thorough investigation of this conclusion, as well as investigates whether other skill indicators explain rising premiums, going back to 1940 in the US.

Chapter 2

Explaining Income Shares Through Heterogenous Capital

Chapter 1 identified the role of marginal product growth of skill, as opposed to accumulation of skills, in increasing wage inequality. In chapter 2 I investigate what may cause this growth in the marginal product of skilled labour relative to unskilled. In this chapter I take an econometric approach to understanding the skill biased growth experienced by using different physical inputs that complement different types of labour. Measuring skill biased growth through observable tangible inputs enables econometric measurements to take place. Chapter 3 then takes a theoretical approach to examine whether technical progress is likely a cause of wage inequality growth.

To better understand how income shares are impacted by exogenous shocks to available inputs, I introduce a process for identifying elasticities of substitution in multi-stage production process. This allows for substitution effects to be identified and measured between two inputs while taking into account indirect effects from other factors of production. I demonstrate the consequences of disaggregating inputs into components that are complementary respectively with skilled and with unskilled labour. This potentially gives insight into the cause of transitions in the type of growth experienced as well as into inflection points of fluctuations in income shares.

I conclude that the transition from unskilled to skilled biassed growth is a result of bottlenecks in certain inputs to production. Using data from US manufacturing plants, I find that capital equipment predominantly complements better paid non-production workers, while energy predominantly complements worse paid production workers. A similar percentage rise in both energy and capital would have a net effect of increasing demand for non-production workers over production workers. On top of this, capital has increased substantially more than energy, with a 64% increase in capital equipment among manufacturing industries between 1987 and 2011, compared to a 17% increase for energy. As a result capital equipment's contribution to increasing relative skill demand is greater than energy's contribution to decreasing it.

This research can be revealing as to the potential benefits of a certain types of capital, beyond overall economic growth. As it is theorised that more resource or energy dependant capital generally complements unskilled labour more a technological revolution in energy, as seen in the industrial revolution, would create a decline in inequality. This is in contrast to a more computer based technical revolution which may have the opposite effect.

2.1 INTRODUCTION

2.1.1 AIM

The aim of this research is to create a framework for predicting the development of functional income shares in the future, in a way that understands the drivers and what type of exogenous shocks will affect these trends. The main contribution of this paper is to demonstrate the consequences of disaggregating inputs into components that are complementary respectively with skilled and with unskilled labour. This potentially gives insight into the cause of transitions in the type of growth experienced as well as into inflection points of fluctuations in income shares. These insights can then be combined with the existing theory on endogenous growth to help complete the narrative in the next chapter. This can also explain the change in capital's income share and where relative augmented technology has affected income shares. The result is a theory that better explains structural changes in growth through multiple economic cycles.

The conclusion of this theory is that the transition from unskilled to skilled biassed growth is a result of bottlenecks in certain inputs to production. Skilled labour has a more elastic supply of capital complementing it, being less dependent on finite natural resources than unskilled biased growth. The more elastic capital that complements skilled labour is free to grow faster and experiences less supply side shocks than natural resources. This is best demonstrated by the fluctuations in the supply of natural resources, which are more prone to supply shocks and inelastic supply than capital in general. These natural resources are predominately used in the end production part of the supply chain, which are more commonly done by poorer "blue collar" workers, where as "white collar" workers would be more involved in areas like sales and R&D. This research can be revealing as to the potential benefits, beyond overall economic growth, of a certain types of technical progress. As it is theorised that more resource or energy dependant capital generally complements unskilled labour more, a technological revolution in energy, as seen in the industrial revolution, would create a decline in inequality. This is in contrast to a more computer based technical revolution which may have the opposite effect. As will be explored in the next chapter, this can make (endogenous) technical progress switch to being skill biased. This is a relevant exercise in the task of trying to understand the future progression of income inequality, as technology behind alternative progresses rapidly, and there is a chance of another period of energy based productivity gains like in the industrial revolution.

2.1.2 MOTIVATING EVIDENCE

Previous research (Krusell et al. (2000)) has noted that in the last century capital has complemented skilled labour, rather than unskilled labour, resulting in an increase in the skill premium through capital deepening. This result has been backed up by more modern estimates (Karabarbounis and Neiman (2013)). However, this contrasts with what happened in previous centuries. There have been claims that technical progress was unskilled labour biased during the Industrial Revolution (James and Skinner (1985)), (Pleijt and Weisdorf (2014)). Consequently capital was complementing unskilled labour in this period. Hence, capital-skill complementarity seems only relevant in the last hundred years, or even more recently.

It would seem there has been a structural change to the economy that has led to skill complementary capital growth in more recent years. To understand this it is worth discussing how the composition of capital may have changed. One can casually observe that as technology has advanced, the nature of capital has changed somewhat. It has been estimated that capital that is more technologically advanced has a higher level of complementarity with skilled labour than less technologically advanced capital (Correa et al. (2014)). This would potentially cause a gradual increase in the skill complementarity of labour. For example, in the modern era a large part of capital formation is Information and Communications Technology (ICT) [Colecchia and Schreyer (2002)]. This has some cross over with Intellectual Property Products (IPP), which have also been growing (Koh et al. (2015)).

Breaking down the more technology based or intangible forms of capital, ICT includes computers and related hardware, communications equipment and software. IPP includes artistic originals, R&D and software. Software is included in both. However, the rest of IPP has only recently been included in capital measurements. The growth of both these types of investment represents new challenges when accounting for capital and its changing composition. Another example, that is of interest, is when the capital used in production goes from being resource intensive to technology-intensive and certain resources become more scarce. As an example, using a translog function Correa et al. (2014) finds that capital-skill complementarity is more prominent with highly technological capital than in non-technological capital. Another paper (Autor et al. (1998)) claims that increases in demand for skilled labour have been greater in more computer intensive industries. Advanced technology complementing skilled labour is also significant with the rise of IPP and computer equipment. It is clear that these changes can have a significant impact on complementarity or substitution effects within the economy.

To better understand what structural change may have happened to cause growth to go from unskilled complementing to skill complementing, it is worth looking at the historical trends. A popular consideration for the recent rise in inequality is non free market factors, e.g. minimum wages, taxes and union power. I discussed this in chapter 1, concluding there is little to support that this is a long term explanation for diverging wages. As I now discuss observations going back hundreds of years, it is worth also noting that during the time of the industrial revolution there was little in terms of minimum wages, union power or income taxes to redistribute income. However it is during this time that wage inequality is thought to have decreased. In the UK, there exists estimates of skill premium going back until the 13th century (Brown and Hopkins (1955) and Clark (2005)). This is based on the wage of craftsmen or carpenters vs labourers, the idea being the former is classified as skilled and latter is unskilled. These estimate that the skill premium declined substantially from the late 1800's (during the second industrial revolution) until the mid 1900's, during which there was huge advances in energy and the ability to extract natural resources. On the other hand, the largest increase in skill premium occurred after the two oil crisis in the 1970's, when commodity prices rose substantially from a restriction in supply. Before these periods the British skill premium was remarkably consistent from the 1400's. Stern and Kander (2012) has also found the during the first half of the 20th century the expansion of energy services explains a large amount of the economic growth in Sweden, where as during the second half of the century growth was driven more by labour augmenting technological progress. From this one could speculate that energy and resource-intense capital is more complementary to unskilled labour, where as, as has been estimated in Correa et al. (2014), more technologically advanced capital complements skilled labour more. This supports the theoretical framework I will use. These observations have to be taken lightly though, as the skill premium demanded by craftsmen are subject to change through structural changes in production as well as overall inequality variations, as what constitutes as skilled should change in the long term. As production becomes more automated the need for hands on skills becomes less, as jobs that go towards the application and administration of automated processes become more valuable, creating less of a need for craftsmen as other job roles become skilled. This outlines the issue of using specific job roles to define skill.

An important part of the application of this theory is distinguishing between inputs used predominantly by skilled and unskilled workers. If non labour inputs are treated homogeneously, the general change in composition of capital would create a structural change to the parameters of production. To obtain a more general theory of capitals relationship with skilled and unskilled labour, that is more robustly applicable throughout these changes, the elements of non labour factors of production should be separated. This would give insight into the potential impact of declines in energy prices or a new energy revolution, and whether it would likely create a decline in wage inequality, as observed during the industrial revolution.

2.1.3 Intuition Of Model

When discussing wage inequality, a question arises of why are modern professional footballers paid so much more than previous footballers. While they might be better players, it is unlikely to the extent of the many multiples extra they get paid compared to just a few decades ago. In the context of structural changes in capital and technology, it can be answered that it is due to an increase in media. Taking the top professional footballers in the world as skilled, their talents can be displayed and generate revenue from all corners of the world due to improved media resulting in more exposure. Previously this income would have likely been going to more local entertainment. And so this is an example of the rising level of technological capital increasing the relative marginal product of highly skilled individuals.

This example can be extended to the stylised fact that in the modern world, economies which are wealthier specialise in more service based industries, like finance, as apposed to manufacturing. Some richer economies, like Germany or Japan, that still specialise in manufacturing, do so with more technology based manufacturing, where higher portion of added value and jobs will be involved in the development of technology that goes into the product as opposed to directly in production and increasing scale. The overall narrative is that the part of the supply chain that requires manual input, e.g. assembly and production, mainly simply requires a physical input and less leverages on intellectual ability or advanced know how than other jobs. Though they might require more technology with modern production lines, the technological knowledge required for these production jobs are assumed to be relatively basic compared to work elsewhere in the supply chain.

The setup in this model follows this reasoning with a nested multi-stage production process. Skilled factors of production and skilled labour are used in one stage of production, for example R&D researches using intellectual property and ICT. Similarly unskilled factors of production and unskilled labour utilise each other in the other stage of production, for example assembly line workers using raw materials that create the good. This setup is similar to Acemoglu's endogenous growth theory (Acemoglu (2002)), though using capital accumulation instead of technical progress, as both skilled and unskilled labour have separately complementary inputs. Using capital instead of technical change does not explain the persistence in skilled or unskilled biased growth (this is partly why I introduce endogenous growth in the next chapter), however it does have the potential to explain the reason behind shocks causing growth to complement a different factor input. It also explains growth through an observable input (i.e. quantities of types of capital as apposed to technology), which means it has more empirical use.

The resulting production function for the economy is based on skilled and unskilled intermediate processes (Y_s and Y_u). The skilled intermediate process utilises effective skilled labour (s_t) skilled capital ($K_{s,t}$), and the unskilled intermediate process mirrors this:

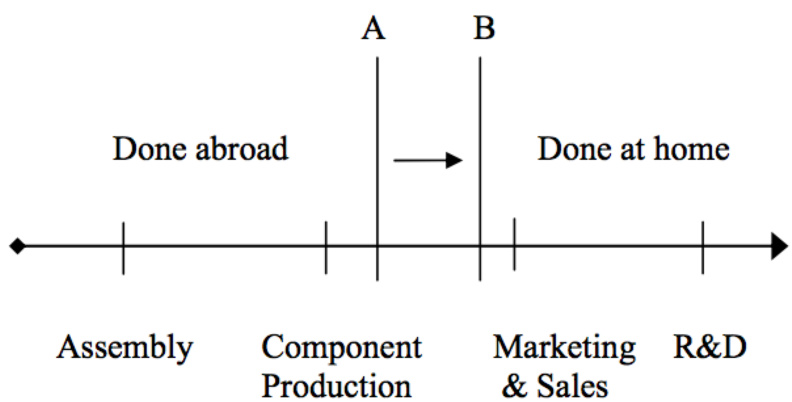
$$Y_t = Y(Y_u(u_t, K_{u,t}), Y_s(s_t, K_{s,t})) \tag{2.1.1}$$

The key parameters that deicide how these inputs interact are the elasticity of substitution between intermediate processes Y_s and Y_u, as well as elasticity of substitutions within these processes (between s_t and $K_{s,t}$ and also between u_t and $K_{u,t}$). It is worth noting that factor inputs that are utilised by unskilled labour, so are used in the less skilled stage of production, does not necessarily complement unskilled labour more than skilled. The same is true for the skilled stage of production. As will be shown in the model, this depends on the elasticity of substitution between the two processes, as well as between labour and non labour inputs within those processes. For example, if the skilled and unskilled processes has a very low elasticity of substitution with each other, and labour and non labour inputs turn out to be easily substituted within those processes, then inputs used by unskilled labour could complement skilled labour more than unskilled.

In distinguishing between factors of productions that utilise skilled labour and capital that complements unskilled labour, I apply a concept applied by Robert Feenstra

(Feenstra (2007)) when explaining the effect of outsourcing on demand for skill from the point of view of a developed (skill abundant) economy. He uses the value chain of a firm in order to show how much skilled labour is required compared to unskilled (figure 2.1.1). Roles like assembly line work uses the least amount of skilled labour relative to unskilled labour, followed by component production, then marketing and sales, and finally R&D. In his model, for a skilled abundant economy, the unskilled part of the value chain is outsourced while the skilled part of production expands in the home economy. In Feenstra's setup this outsourcing causes a structural change from A to B for the already skilled abundant economy, resulting in an increase in relative demand for skilled workers within industries, but not necessarily between industries. Using the same logic, it follows from this that certain types of capital are more necessary for the skilled part of production, whereas other capital is used more in the unskilled part. This is the basis for conceiving a multi-stage production process between skilled and unskilled intensive work, represented by two intermediate goods, which in turn use skilled and unskilled factors of production.

Figure 2.1.1: Freensta's Value Chain



Feenstra (2007)

Feenstra's model explains why changes in the relative size of industries won't necessarily be responsible for changing incomes. It shows how changes can happen within industries through structural changes in production. This explains why previous papers have pointed out that structural transformations doesn't seem to explain a significant amount of the change in income shares (Karabarbounis and Neiman (2013)).

The use of change in job role as opposed to change in industry to proxy skilled labour is supported by observed changes in skilled labour between industries. Using college education as a proxy for skill, figure 2.1.2 shows how changes across broad industries and within industries account for the composition of labour between 2003 and 2016. This shows the change in share of these industries. It also shows the college education (skilled labour) for each industry. From this we can calculate how much the increase in skilled labour was due to skilled industries growing and how much was because of a change in skill within industries. In total the number of college graduates increased from 32.9% of workers to 39.7%, or by 6.8%. If the portion of workers that have college education remained the same within industries, and only changed in total because of the relative growth of skilled industries, then the total number of college graduates would have grown by less than 0.9%. The rest of the growth in college graduates (around 6%) came from an increase in the number of college graduate within these industries. This shows that the growth in college graduates happened within industries, and not as a result of structural transformation.

Figure 2.1.3 shows the same data but breaks down workers by occupation instead of industry. Here occupation fares slightly better in explaining the increase in skilled labour than industry, as 2.1% of the 6.8% change is explained by a growth in certain occupations. This still leaves nearly 5% as resulting from an increase in the number of college graduate within these job roles. This method of disaggregating labour would likely explain more if the occupations and industries were defined less broadly. This generally outlines how the change in demand for different types of work explains shifts in income share better than the change in demand for each industry, supporting measures that reflect job roles as opposed industries to explain changing inequality.

Figure 2.1.2: Employment By Industry

	Employment Share		% College Graduates	
	2003	2016	2003	2016
Industry (Employed Civilians Only)	**100%**	**100%**	**32.7%**	**39.7%**
Educational and health services	22%	24%	49.1%	55.6%
Information	3%	2%	45.2%	55.5%
Professional and business services	11%	13%	47.4%	53.7%
Financial activities	8%	7%	42.3%	51.2%
Public administration	5%	5%	40.6%	49.2%
Mining	0%	1%	23.6%	33.7%
Manufacturing	13%	11%	23.1%	29.8%
Wholesale and retail trade	13%	12%	22.8%	27.8%
Other services	5%	5%	20.2%	27.0%
Leisure and hospitality	6%	7%	21.0%	23.9%
Transportation and utilities	6%	6%	17.3%	21.8%
Agricultural, forestry, fishing, and hunting	1%	2%	14.4%	21.1%
Construction	7%	7%	12.7%	14.9%

Comparison employment shares of each industry on the left and the portion of workers who are graduates in each industry on the right using March CPS data

In figure 2.1.3 it can be observed, from comparing the more skilled based occupations to less skill based occupations, that the unskilled parts of the supply chain generally include tangible production, while skilled parts of the supply chain are processes not associated with handling of tangible goods or materials. Another way to characterise this is that unskilled are generally blue collar workers, while white collar workers are more skilled. Figures 2.1.2 and 2.1.3 supports this, as both by industry and occupation, lower paid labour is more prominent where tangible goods are being handled. Figure 2.1.2 shows that the top 5 industries in terms of pay are all industries where there is little to no reliance on tangible goods. The bottom two (Construction and Agriculture)

Figure 2.1.3: Employment By Occupation

	Employment Share		% College Graduates	
	2003	2016	2003	2016
Occupation (Employed Civilians Only)	**100%**	**100%**	**32.7%**	**39.7%**
Professional and related occupations	22%	25%	68.1%	73.5%
Management, business, and financial occupations	16%	18%	53.4%	60.1%
Sales and related occupations	11%	9%	31.6%	35.8%
Office and administrative occupations	14%	11%	17.7%	25.1%
Service occupations	14%	15%	11.6%	15.0%
Production occupations	7%	6%	7.3%	10.7%
Farming, forestry, and fishing occupations	1%	1%	6.9%	10.5%
Installation, maintenance, and repair occupations	4%	3%	6.8%	10.4%
Transportation and material moving occupations	6%	6%	7.1%	10.3%
Construction and extraction occupations	6%	5%	8.2%	8.2%

Comparison employment shares of each occupation on the left and the portion of workers who are graduates working in each occupation on the right using March CPS data

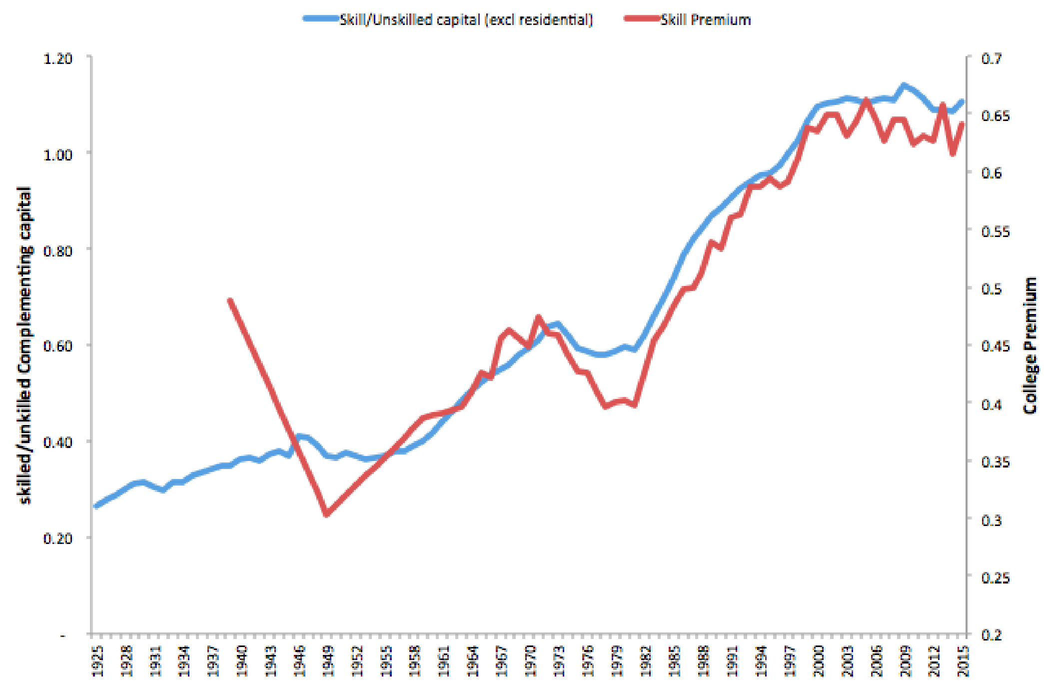
are especially heavily reliant on manual labour. The two above this (Transportation and hospitality) also would rely on tangible goods. Occupations show a similar trend, where the top 4 paid are, once again, professional and office jobs. Construction, Installation and Farming are all reliant on manual labour, with Transportation also requiring large levels of physical capital.

The narrative of production goods complementing less skilled labour, while more technological capital capital complements skilled labour, fits in with nineteenth century history which saw mostly unskilled biased growth. During this time period the famous inventions mostly encouraged the tangible side of producing goods (e.g. the steam engine, railway and the sewing machine). More recently however, innovations happen more in the form of technology, the internet and communications. Keeping all this in mind I have split capital into skilled and unskilled complementing based on which part of the value chain it is most associated with in Feenstra's model above (figure 2.1.1), using BEA statistics for US data[1]. This excludes residential capital equipment and structures. Figure 2.1.4 shows the ratio of the value of both types of capital compared to the college premium in the US.

Consistent with the narrative of more technological capital complementing skilled labour, it has been found that the internet roll out in Brazil has benefited wages of workers in non-routine cognitive tasks relative to employees who did routine cognitive tasks (Poliquin (2018)), and so increasing in firm inequality. A similar trend has been found in Norway (Akerman et al. (2015)). Not only did these results show inequality increase, but the unskilled workers outcomes actually worsened after this technology change.

[1] Skill complementing capital includes the following types of capital: Information processing equipment, Commercial and health care, Communication, Religious, Educational and vocational, Lodging, Amusement and recreation and Intellectual property products.
Unskilled complementing capital includes the following capital: Industrial equipment, Transportation equipment, Other equipment, Manufacturing, Power, Mining exploration, shafts, wells, Transportation and Farming

Figure 2.1.4: College Premium

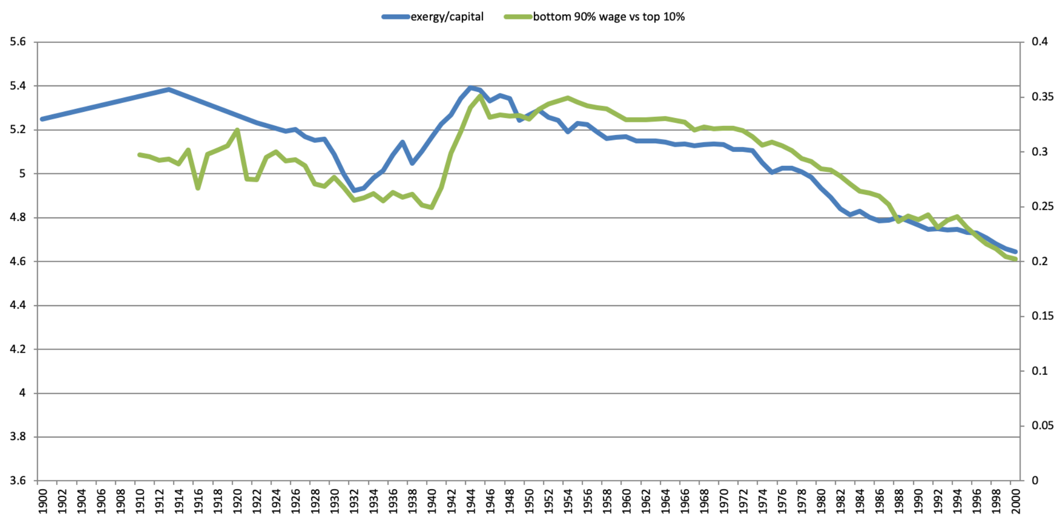


Skill premium shows college premium figures using Census and Current Population Survey data as of 2018. Capital data is from the Bureau of Economic Analysis as of 2018. Skill complementing capital includes the following types of capital: Information processing equipment, Commercial and health care, Communication, Religious, Educational and vocational, Lodging, Amusement and recreation and Intellectual property products.
Unskilled complementing capital includes the following capital: Industrial equipment, Transportation equipment, Other equipment, Manufacturing, Power, Mining exploration, shafts, wells, Transportation and Farming

One part of figure 2.1.4 that doesn't show a correlation between type of capital and skill premium is when there was a large drop in skill premium from 1939 to 1949, and there are estimates that it was also dropping before this [Goldin and Katz (2007)]. Here, skill premium is the ratio of the average wages of college graduates compared to non college graduate. The trend doesn't fit as well at end of the second world war when there was a large drop in wage inequality but little change in the composition of capital. This break in trend during World War Two could well be related to a redeployment of the work force, or use of previously idle capital.

An alternative way of capturing a structural change towards a more production-based workforce is to look at the energy intensity of capital. Figure 2.1.5 shows a measure of the energy intensity of capital vs wage inequality (Exergy is actually used which is a measure of energy used in a system, wage inequality is measured by comparing the wages of the top 10% of earners to everyone else). Energy intensity increased dramatically during the war, which is likely indicative of more production intensity among utilised capital

Figure 2.1.5: Exergy per unit of Capital vs Wage Inequality



Capital data is from the Bureau of Economic Analysis as of 2018. Wage inequality is from IRS returns as reported by https://wid.world/ in 2018. Exergy is from Ayres et al. (2003)

in a way that isn't captured by the composition of the type of capital.

2.2 THE MODEL

In this model wage inequality occurs due to some workers being skilled and others unskilled. The premium skilled workers are paid can change as augmented productivity and factor inputs change. There is a lot of literature on equilibrium levels of wage inequality that tries to weigh in on whether it is driven by outsourcing or skilled biased growth. This model does not make that distinction, as the changes in factor inputs and productivity in the model can be due to a structural change in production from outsourcing or it can be from structural change brought about from modern technology improvements. The purpose of this study is to further understand the mechanism in which the change in wage inequality has occurred. For example I distinguish between whether it is the growth in energy used in production that is stagnating the growth in marginal productivity for lower skilled production workers. This not only looks at how new technology and capital can complement skilled workers but how bottlenecks in other factor inputs may affect unskilled wages. This creates a more thorough foundation for further understanding and predicting how these wage trends might evolve in the future.

This is an equilibrium model and so assumes there is no inequality through rent seeking activity or through inherited advantageous contacts, etc. Human capital is obtained through investing in learning and becoming skilled. This assumption has many exceptions on the micro level if these non equilibrium factors are considered, however comparative skill sets should still be valid in explaining growing wage differentials, as there is no reason to believe non equilibrium factors have grown in relevance.

The set up to this model follows methods by Fallon and Layard (1975) and Krusell et al. (2000). The key difference is that in these papers there are three key inputs of interest, as the relationship of capital is examined with both skilled and unskilled labour using an embedded CES production function, finding that capital in general seems to complement skilled labour more than unskilled. In this paper there are two types of non labour factors of production, allowing for a more precise look at what what inputs complement the different labour types. As such the functional form of the production function follows the general form proposed earlier in 2.1.1, but with a CES form as used by McAdam et al. (2011). This seeks to find an explanation that shows how the composition of capital, as apposed to the absolute levels of capital, is creating growing inequality among workers.

2.2.1 FIRMS

In this section relative wages can be obtained through the equilibrium supply and demand for labour in a free market economy. The dynamics of factor complementarity can be seen in the following production function, which shows the output for competitive final goods producers. This final good is made only with intermediate goods, with the following production function for competitive firm j:

$$Y_{j,t} = A_{j,t}[\alpha Y^{\gamma}_{u,j,t} + (1-\alpha) Y^{\gamma}_{s,j,t}]^{\frac{1}{\gamma}} \tag{2.2.1}$$

Where $Y_{u,j}$ is the intermediate good intensive in unskilled labour (U_j) and the factor input that unskilled labour mainly uses ($K_{u,j}$) while $Y_{s,j}$ is the intermediate good intensive in skilled labour (S_j) and capital that skilled labour mainly uses ($K_{s,j}$). These intermediate goods actually represent the skilled and unskilled processes in the value chain, as in Feenstra's model in figure 2.1.1. For simplicity I will take the case where these forms of production use only unskilled or skilled factor inputs in a CES production function:

$$Y_{u,j,t} = [\alpha_u u^{\gamma_u}_{j,t} + (1-\alpha_u) K^{\gamma_u}_{u,j,t}]^{\frac{1}{\gamma_u}} \tag{2.2.2}$$

$$Y_{s,j,t} = [\alpha_s s^{\gamma_s}_{j,t} + (1-\alpha_s) K^{\gamma_s}_{s,j,t}]^{\frac{1}{\gamma_s}} \tag{2.2.3}$$

Where u and s are effective quantities of unskilled labour and skilled labour respectively. Each γ represents the substitution parameter, which is connected with the elasticity of substitution (σ) such that $\gamma = \frac{\sigma-1}{\sigma}$. In this setup, if the elasticity of substitution between $K_{u,j}$ and U_j ($\sigma_u = \frac{1}{1-\gamma_u}$) is less than the elasticity of substitution between intermediate goods (σ) then $K_{u,j}$ complements unskilled labour more than skilled. However if the opposite was true then it would actually be the case that $K_{u,j}$ complements skilled labour more. This is because the complementarity effects between intermediate goods would be more signifiant than within intermediate goods. In such an outcome, though unskilled labour may use raw materials in the component production or assembly, this part of the value chain may still strongly require more skilled areas like R&D. If the assembly part of production is heavily dependant on R&D to produce the finished product, raw materials could theoretically complement skilled labour more than unskilled labour, despite it being unskilled labour that directly utilises the raw materials. As such, the key component of this model is finding whether processes of production have greater or smaller elasticities with each other than between factors within those processes. If the processes have lower elasticities with each other, then factors would experience greatest complementary effects with factors predominant in other processes (or intermediate goods in the model).

Combining intermediate and final goods production functions gives the production for final goods based on the raw inputs.

$$Y_{j,t} = A_{j,t}[\alpha[\alpha_u u^{\gamma_u}_{j,t} + (1-\alpha_u) K^{\gamma_u}_{u,j,t}]^{\frac{\gamma}{\gamma_u}} + (1-\alpha)[\alpha_s s^{\gamma_s}_{j,t} + (1-\alpha_s) K^{\gamma_s}_{s,j,t}]^{\frac{\gamma}{\gamma_s}}]^{\frac{1}{\gamma}} \tag{2.2.4}$$

Included in the effective labour terms ($s_{j,t}$ and $u_{j,t}$) are specifically skilled and unskilled augmenting technical progress variables ($A_{u,j,t}$ and $A_{s,j,t}$). This gives:

$$u_{j,t} = A_{u,j,t} U_{j,t} \tag{2.2.5}$$

$$s_{j,t} = A_{s,j,t} S_{j,t} \tag{2.2.6}$$

Where S and U are actual levels of skilled and unskilled labour. As a result, each competitive firm (j) that makes an intermediate good (with price normalised at 1) and experiences the following production function:

$$Y_{u,j,t} = [\alpha_u (A_{u,j,t} U_{j,t})^{\gamma_u} + (1-\alpha_u) K_{u,j,t}^{\gamma_u}]^{\frac{1}{\gamma_u}} \tag{2.2.7}$$

$$Y_{s,j,t} = [\alpha_s (A_{s,j,t} S_{j,t})^{\gamma_s} + (1-\alpha_s) K_{s,j,t}^{\gamma_s}]^{\frac{1}{\gamma_s}} \tag{2.2.8}$$

The cost of inputs, resulting in factor incomes, comes from the aggregation of this demand for inputs coming from firms. However, no one firm is large enough to have an impact on factor prices, as a result firms are price takers. Maximising profits in competitive markets results in the aggregate outcome of zero profits for the marginal firm producing final goods from the intermediate group. This gives the following outcome where output is equal to the total cost of inputs for each firm:

$$Y_{j,t} = Y_{s,j,t} P_{s,j,t} + Y_{u,j,t} P_{u,j,t} \tag{2.2.9}$$

With P denoting the price for each intermediate good, measured in terms of the consumption good. Similarly the intermediate good producers experience the following conditions as a result of zero profits.

$$Y_{u,j,t} P_{u,j,t} = U_{j,t} w_{u,t} + K_{u,j,t} r_{u,t} \tag{2.2.10}$$

$$Y_{s,j,t} P_{s,j,t} = S_{j,t} w_{s,t} + K_{s,j,t} r_{s,t} \tag{2.2.11}$$

Where $w_{u,t}, w_{s,t}, r_{u,t}$ and $r_{s,t}$ are the costs of the corresponding inputs. In this set up firms can make intermediate and final goods. As there is a free market for goods, firm j experiences local prices for intermediate goods. If the firm has an idiosyncratic exogenous shock in wages or rents, it doesn't affect the price they pay or receive for the intermediate good. For example, rents and wages impact firm j's production of intermediate goods, however the firm's price of intermediate goods would not change. As a result, the quantity of the final good would not change according to 2.2.12 and 2.2.13 below. So firm j would change the output of intermediate goods without changing the final good output or affecting prices.

This is different to if there was no free market in intermediate goods, meaning all firms would have to be vertically integrated to be able to supply the intermediate goods that they will use for final good production. In that case a decrease w_u, for example, would decrease P_u and affect final good output. In effect, when there is no market for intermediate it forces vertical integration, so the prices of intermediate goods (P_u and P_s) become endogenous to the firms production of intermediate goods. But that is not the case in this model.

From the above conditions derived from zero profits, first order conditions for the final goods producers (2.2.9), give:

$$\alpha(\frac{Y_{j,t}}{Y_{u,j,t}})^{1-\gamma} = P_{u,j,t} \quad (2.2.12)$$

$$(1-\alpha)(\frac{Y_{j,t}}{Y_{s,j,t}})^{1-\gamma} = P_{s,j,t} \quad (2.2.13)$$

Similarly the first order conditions for the intermediate goods producers (2.2.10 and 2.2.11) are:

$$MPK_{u,j,t} = \alpha(1-\alpha_u)(\frac{Y_{u,j,t}}{K_{u,j,t}})^{1-\gamma_u} P_{u,j,t} = r_{u,t} \quad (2.2.14)$$

$$MPK_{s,j,t} = (1-\alpha)(1-\alpha_s)(\frac{Y_{s,j,t}}{K_{s,j,t}})^{1-\gamma_s} P_{s,j,t} = r_{s,t} \quad (2.2.15)$$

$$MPL_{u,j,t} = \alpha\alpha_u A_{u,j,t}^{\gamma_u}(\frac{Y_{u,j,t}}{U_{j,t}})^{1-\gamma_u} P_{u,j,t} = w_{u,t} \quad (2.2.16)$$

$$MPL_{s,j,t} = (1-\alpha)\alpha_s A_{s,j,t}^{\gamma_s}(\frac{Y_{s,j,t}}{S_{j,t}})^{1-\gamma_s} P_{s,j,t} = w_{s,t} \quad (2.2.17)$$

The key component to estimating the impact of disaggregated factor inputs on wages is through the elasticities of substitutions. The first stage is to derive the substitution parameters within the intermediate goods (γ_u and γ_s), which is the same technique used in Fallon and Layard (1975). Comparing the wages and rents for the unskilled intensive intermediate good (2.2.14 and 2.2.16) gives:

$$\frac{w_{u,t}}{r_{u,t}} = \frac{\alpha_u}{1-\alpha_u} A_{u,j,t}^{\gamma_u}(\frac{K_{u,j,t}}{U_{j,t}})^{1-\gamma_u} \quad (2.2.18)$$

$$(\frac{K_{u,j,t}}{U_{j,t}})^{1-\gamma_u} = \frac{1-\alpha_u}{\alpha_u A_{u,j,t}^{\gamma_u}} \frac{w_{u,t}}{r_{u,t}} \quad (2.2.19)$$

$$\frac{K_{u,j,t}}{U_{j,t}} = (\frac{1-\alpha_u}{\alpha_u})^{\frac{1}{1-\gamma_u}} A_{u,j,t}^{\frac{\gamma_u}{\gamma_u-1}}(\frac{w_{u,t}}{r_{u,t}})^{\frac{1}{1-\gamma_u}} \quad (2.2.20)$$

$$ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u ln(\frac{1-\alpha_u}{\alpha_u}) + (1-\sigma_u)ln(A_{u,j,t}) + \sigma_u ln(\frac{w_{u,t}}{r_{u,t}}) \quad (2.2.21)$$

Where it should be noted the $\frac{1}{1-\gamma_u}$ is equal to the elasticity of substitution, σ_u (or σ_s for the skilled intermediate good), between U and K_u. Mirroring this for skilled capital and labour:

$$ln(\frac{K_{s,j,t}}{S_{j,t}}) = \sigma_s ln(\frac{1-\alpha_s}{\alpha_s}) + (1-\sigma_s)ln(A_{s,j,t}) + \sigma_s ln(\frac{w_{s,t}}{r_{s,t}}) \quad (2.2.22)$$

In these partial equilibrium equations, factor inputs are shown as endogenous, while factor prices, or incomes (wages and rents), and productivity growth are the exogenous variables. As mentioned, the incomes are a result of aggregated demand from firms, but are not decided on by any individual firm and so is exogenous to the firm. Productivity is considered exogenous, as is defined below, as firms only choose inputs with no control over productivity growth.

Since the assumed Hicks neutral element of productivity growth ($A_{j,t}$) increases the marginal productivity of all inputs by the same percentage, the ratio of marginal products is unaffected by this type of growth. The two types of labour augmenting productivity do affect the ratio of incomes, though their effect depends upon the substitution parameters which are defined by the elasticity of substitution.

2.2.2 Specification and Endogeneity Concerns

In 2.2.21 it can be seen, by a lack of a j subscript, that each firm will face the same factor income ratio ($\frac{w_{u,t}}{r_{u,t}}$), leaving the only variable creating a difference in factor inputs is the unobservable augmented productivity ($A_{u,j,t}$). Without any variation in the $\frac{w_{u,t}}{r_{u,t}}$ term the parameters would not be identifiable. However the data implies variable wage and rental levels. For this reason my identification strategy relies upon differences in regional and industry wages and rents across locations in the US, thus introducing varying factor price ratios between manufacturing firms as $w_{u,t}$ and $r_{u,t}$ becomes $w_{u,j,t}$ and $r_{u,j,t}$. This is similar to the assumption in Raval (2011) that there are differences in wages across local areas in the US. 2.2.21 Then becomes:

$$ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u ln(\frac{1-\alpha_u}{\alpha_u}) + (1-\sigma_u) ln(A_{u,j,t}) + \sigma_u ln(\frac{w_{u,j,t}}{r_{u,j,t}}) \qquad (2.2.23)$$

It should be mentioned at this point that the stochastic element of this equation ($e_{u,j,t}$) is included in the specification of the exogenous augmented productivity ($A_{u,j,t}$), and will be defined in the next section.

The varying prices in 2.2.23 are treated as exogenous, as in the perfectly competitive model, individual firms do not have control over these factor prices. One potential concern is that industry wage levels are endogenous to factors that affect firm factor ratios and productivity, for example fluctuations in multiple $e_{u,j,t}$ values from the same area, due to a local policy change, may affect local prices. To explore this, I will discuss what causes differences in wages across locations in the US. If there were no frictions preventing people from moving around the country, people move seeking higher wages until the real wage that the worker faces is the same across locations. The wage relevant to this study is the wage that the factory pays its workers. In areas with a higher cost of living (due to varying rents etc), nominal wages faced by manufacturers will be high even if real wages for workers are consistent across locations. This cost of living differences is one reason for area wage differences, though in a way that is not obviously endogenous.

Migration costs are another reason for variable local area wages. Increases in labour demand will increase wages in the short run when labour supply is relatively fixed. labour demand can be affected by many factors, from shocks to local industries, policy, structural capital, etc. As evident from the absence of the hicks neutral productivity variable (A_t) from 2.2.23, local demand shocks from Hicks neutral productivity improvements, or any shock that uniformly affect all inputs, may increase labour demand but will not affect the plant's optimal ratio of factor costs, as it affects all inputs equally. However improvements in manufacturers labour augmenting productivity, which is another exogenous input in the model, could both change the wage, by increasing labour demand, and change the plant's factor cost ratios. Even in this specific case, manufacturing may not affect local area wages very much. Manufacturing is a small percentage of total employment, from around 20% in the 1980's to less than 10% over the last decade according to the Bureau of Labour Statistics. Though it is conceivable that broadly defined manufacturing bases would endogenise prices if the units used in a data sample were cumulated into broadly defined industries, as the technology fluctuations in large industries are more likely to impact prices. In response, each unit can not be defined as a broad industry.

Another source of variable wages between firms is the slight heterogeneity of skills between industries. The skills required by labour, which are more basic for unskilled workers but still exist in some form, will vary between industries and firms. As such some firms will have to pay slight more or less for their workers depending on the precise level of skills required, as well as any temporary shortages or oversupply in the skills required.

For capital, a national market for equipment should mean that the rental price is the same across local areas. However according to estimates on rents from the Bureau of Labour Statistics from 1987 to 2017, different types of capital have different external and internal rent, causing wildly different rental rates between industries. This is largely because different industries have different capital compositions, creating heterogeneity of capital between industries, with different types of capital demanding different rental rates. Some types of capital demand higher rents than others, partly due to differences in depreciation or different risk levels. On top of this, due to many types of capital being less mobile, local area wages and prices of materials can affect the price of capital and so the rental market. Similar to labour, as capital and other inputs are not perfectly mobile, it is possible the price of this input, and so the rental market or prices can slightly vary between locations.

Another source of concern comes from the augmented productivity variable. While technical progress is assumed to be exogenous in this model, it can be argued that the plant's choice of its labor augmenting technical progress depends on the local and industry wage. If firms adjust their level of labour augmenting technology because of the wage they experience, A_u and A_s will be related to wages. When unskilled wages are high, for example, augmenting technology that saves on unskilled labour is more valuable. Such wage based technology adoption would cause high wage firms to have high levels of labour augmenting technology and high capital shares. If it is assumed technical progress is a strict time trend, this would bias the estimate of the elasticity of substitution. However technical progress is measured as an exogenous stochastic trend variable, it could create some multicollinearity between technical progress and prices. I will compare the two methods to investigate whether the is significant difference in measured outcomes.

In this model I introduce variable factor prices, caused by imperfectly mobile capital and labour. This is still consistent with perfect competition product markets but not factor markets. Firms are still producing an homogenous good with a consistent selling price. In this model, while there is perfect mobility within labour markets, there is not perfect mobility between them. Having different costs for inputs doesn't make firms uncompetitive as there are diseconomies of scale, meaning firms with higher costs will simply reach a point where marginal revenue and marginal costs equate at an earlier stage. As such having variable local wages and a perfectly competitive set up is not uncommon (e.g. Helm (2019)).

2.2.3 Technical Progress and Stochastic Element Specification

The model's stochastic components are the pair of efficiency factors of the two types of labour. These efficiency factor variables are assumed to be unobservable. To make this interpretation I specify the stochastic process governing labour quality of the two types as the following process, which is similar to the method used in Krusell et al. (2000). This once again shows the case for unskilled labour but with skilled labour mirroring this.

$$ln(A_{u,j,t}) = ln(A_{u,j,0}) + \phi_{u,t} + e_{u,j,t} \tag{2.2.24}$$

$$\Delta ln(A_{u,j,t}) = \Delta\phi_{u,t} + \Delta e_{u,j,t} \tag{2.2.25}$$

Where 2.2.24 shows either a unit root or trend stationary process and 2.2.25 is stationary. $A_{u,j,0}$ represents the starting productivity level for the firm, which may represent different productivity levels than other firms, but this element is fixed in time. $\phi_{u,t}$ is the total (since time zero) growth in productivity across the economy, and is an unit root or trend stationary exogenous variable . However, as mentioned above, it may be desirable to restrict $\phi_{u,t}$ so that it follows a time trend:

$$\phi_{u,t} = \phi_u t \tag{2.2.26}$$

ϕ is assumed to be equal for all firms, but some firms will have higher or lower productivity growth depending on the random variable $e_{u,j,t}$ as well as the fixed effect $A_{u,j,0}$.

Whether the deviation of a firm's productivity is unit root or trend stationary will have an effect on the characteristic of this error term. Either $e_{u,j,t}$ or $\Delta e_{u,j,t}$ is assumed to be serially uncorrelated and can be treated as an i.i.d, while the other would be a stochastic component but not an i.i.d. If $\Delta e_{u,j,t}$ is an i.i.d then labour augmented productivity levels will be the result of accumulated productivity shocks, that is captured in the $e_{u,j,t}$ term. In this case the deviation of a firm's productivity is unit root. If $e_{u,j,t}$ is an i.i.d then any productivity shock is strictly temporary and the firms productivity is trend stationary. 2.2.23 now can be written as:

$$ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u ln(\frac{1-\alpha_u}{\alpha_u}) + \sigma_u ln(\frac{w_{u,j,t}}{r_{u,j,t}}) + (1-\sigma_u) ln(A_{u,j,0}) + (1-\sigma_u)\phi_{u,t} + (1-\sigma_u) e_{u,j,t} \tag{2.2.27}$$

This is in the familiar form of a fixed effects estimation of:

$$ln(\frac{K_{u,j,t}}{U_{j,t}}) = a_u + \sigma_u ln(\frac{w_{u,j,t}}{r_{u,j,t}}) + \rho_{u,j} + \delta_{u,t} + \omega_{u,j,t} \tag{2.2.28}$$

With $a_u = \frac{1}{1-\gamma_u} ln(\frac{1-\alpha_u}{\alpha_u}), \rho_{u,j} = \frac{\gamma_u}{\gamma_u - 1} ln(A_{u,j,0}), \delta_{u,t} = \frac{\gamma_u}{\gamma_u - 1}\phi_{u,t}$ and $\omega_{u,j,t} = \frac{\gamma_u}{\gamma_u - 1} e_{u,j,t}$. $\rho_{u,j}$ is the individual fixed effect and $\delta_{u,t}$ is the time fixed effect. The unobserved individual effects are coefficients on dummies for each individual firm while the year effects are coefficients on time dummies.

At this point one of two methods can be used for estimation (Angrist and Pischke (2009)) in order to eliminate the unmeasurable variables included in $\rho_{u,j}$. Either individual fixed effects can be used by subtracting a within estimator from 2.2.28, or a first differences estimation can be used. As will be seen, which one is most suitable will depend on whether the error term in 2.2.28 suffers from serial correlation. If productivity shocks are unit root we would likely see that the stochastic element $e_{u,j,t}$ would create serial correlation. In this case $\Delta e_{u,j,t}$ would be a more suitable stochastic element. If productivity is mean reverting however, $e_{u,j,t}$ would be more suitable.

2.2.4 Test of Model Using Morishima Elasticities

In the process above the setup between energy and capital is asymmetric, as it is assumed that unskilled labour directly utilise energy and skilled labour directly utilise capital. To test the findings I will use a method that treats both inputs the same to see if the same result occur. As such a fair test is required to tests the hypothesis that complementarity effects are the same as observed in the above exercise.

When analysing multi-stage production functions, it is crucial to distinguish two distinct types of substitution between inputs from different stages of production: that among factors within a given process (say, within the production process) and that among factors from different processes (say, marketing sales and R&D). As claimed in Anderson et al. (2019), when nested production functions are used, it can be preferable to use Morishima elasticity measures as they capture both. While the Hicksian two-variable elasticity of substitution, as standardly used, connects relative demand with relative prices, the Morishima elasticity of substitution (Morishima (1967)) allows for the price of inputs to have different effects on demand. While the Morishima Elasticity of Substitution set up is very generalised and doesn't offer as much in terms of economic intuition, it can be used to test if the complementary outcomes are consistent with the complementary outcomes of the model.

Here I demonstrate the issues of taking direct Hicksian elasticity of substitutions in this model. From taking the wage conditions for the production of intermediate goods

(2.2.16 and 2.2.17), how prices affect intermediate good and labour demand can be shown with marginal productivity equations.

$$\frac{U_{j,t}}{Y_{u,j,t}} = \alpha_u A_{u,j,t}^{\sigma_u - 1} (\frac{P_{u,j,t}}{w_{u,j,t}})^{\sigma_u} \tag{2.2.29}$$

$$\frac{S_{j,t}}{Y_{s,j,t}} = \alpha_s A_{s,j,t}^{\sigma_s - 1} (\frac{P_{s,j,t}}{w_{s,j,t}})^{\sigma_s} \tag{2.2.30}$$

Comparing these gives how prices exogenously impact relative labour and intermediate good outputs for firm j:

$$\frac{S_{j,t} Y_{u,j,t}}{U_{j,t} Y_{s,j,t}} = \frac{\alpha_s}{\alpha_u} \frac{A_{s,j,t}^{\sigma_s - 1}}{A_{u,j,t}^{\sigma_u - 1}} (\frac{P_{s,j,t}}{w_{s,j,t}})^{\sigma_s} (\frac{w_{u,j,t}}{P_{u,j,t}})^{\sigma_u} \tag{2.2.31}$$

Here the relationship between the two wage prices does not have a simple relationship with labour demand. In this case a change in $w_{u,j,t}$ or $w_{s,j,t}$ have asymmetric effects on relative labour demand. They are affected by factors within intermediate goods or process, shown by wages ($w_{u,j,t}$ and $w_{s,j,t}$) having a direct impact on relative labour inputs $\frac{S_{j,t}}{U_{j,t}}$. They are also affected by factors from different process, as intermediate goods prices ($P_{u,j,t}$ and $P_{s,j,t}$) are endogenous to wages. Intermediate good prices in turn impact intermediate good quantities ($\frac{Y_{u,j,t}}{Y_{s,j,t}}$), meaning wages have a direct and indirect effect on quantities.

This shows how the Hicksian elasticity of substitution is sometimes not suitable when there are more than two inputs. It has also been demonstrated in Blackorby and Russell (1989) that the Morishima Elasticity of Substitution may be more suitable when there are more than two factor inputs. The main insight of Morishima (1967) is that elasticities of substitution for input ratios are generally asymmetric given more than two inputs, where it becomes important which input price changes for all ordered, pairwise combinations of inputs. Importantly, Blackorby and Russell (1989) demonstrate that the Morishima Elasticity of Substitution is a natural generalisation of the Hicks two-input elasticity of substitution in settings of three or more inputs.

As a result of multiple price variables directly and indirectly affecting relative demand for wages, I will measure Morishima elasticities as presented in Anderson et al. (2019) to verify the complementary effects between K_u and U as well as K_s and S.

$$\frac{S_{j,t}}{U_{j,t}} = a + b_{w_s} w_{s,j,t} + b_{w_u} w_{u,j,t} + b_{r_s} r_{s,j,t} + b_{r_u} r_{u,j,t} \tag{2.2.32}$$

In such a regression, each wage has a different impact of relative demand. Also due to the nested production function, the relative demand for two inputs is exogenously driven by changes in the prices of all inputs, not just wages. From the values of b_{r_u} and b_{r_s}, it is evident whether inputs K_u and K_r respectively complement skill or unskilled labour more. A rise in an inputs cost decreases the demand of that input, which decreases the demand for any complementing labour. As such, if either of these input costs parameters are more than zero, it is evidence that the corresponding input complements unskilled labour. Likewise if either of these input costs parameters are less than zero, it is evidence that the corresponding input is skill complementing.

2.2.5 International Trade

It is worth mentioning the part trade has in this setup. In this model prices are exogenous to firms. As this is a partial equilibrium model, international markets impacting prices does not invalidate model but is simply part of the explanation behind exogenous movements in these prices. This includes the increase or decrease in global demand for different goods. For example, during the relevant time period, there has been further integration of China and other poorer countries that are abundant in unskilled labour to global trade. This would likely have increased the global supply of the unskilled intermediate good (Y_u) thus decreasing it's price domestically (P_u). The result would be an exogenous impact on $P_{u,j,t}$ in 2.2.9 above.

The literature that studies equilibrium wage inequality tends to distinguish between trade and technology effects on wage inequality. In this case both are accounted for in the model, but are not necessarily distinguishable from each other. As already mentioned, trade has an exogenous impact on the prices used, which may include capital rents. Technology growth that impact relative wages is obviously clearly shown in the augmented productivity variables. However changes in investment prices can also be a key part of growth (Karabarbounis and Neiman (2013)), in how it drives capital accumulation. In this model this capital deepening has an important impact on relative wages. It is for this reason it is hard distinguish between the effects of trade vs technical progress in this model, as both can drive the relative price of inputs. Supporting this Parro (2013) finds that the impact of trade on wage inequality is a result of the negative impact trade has on capital goods, which increases wage inequality due to capital skill complementarity.

2.3 Substitution Parameters Estimations

While a typical way to measure the elasticity of substitution is through regressing the output on the production function, as the production function here includes unobservable labour augmenting productivities with four inputs, such a method is more problematic in this model. As mentioned earlier, either productivity shocks are unit root, and $\Delta e_{u,j,t}$ is an i.i.d, or are trend stationary, and $e_{u,j,t}$ is an i.i.d. It is important to make this distinction because using the wrong method will result in residual serial correlation and make the results invalid. In the unit root case first differences can be used to estimate the parameters while canceling out unobservables, in the trend stationary case a fixed effects estimation is used. To find which one is more suitable I will perform both and assess them based on analysis of the residual serial correlation.

In the regression I will use panel data that has unobservable fixed effects ($\rho_{u,j}$ in 2.2.28) bought about by firms being heterogeneous due to different starting levels of technical progress ($A_{u,j,0}$) in 2.2.24. As well as firm specific fixed effects, there are also time specific fixed effects ($\delta_{u,t}$ in 2.2.28) , due to growing systematic technical progress, which is represented by $\phi_{u,t}$ in 2.2.24. The methods used below follow fixed effect methods where, given panel data, the causal effect of initial augmented technology on factor inputs can be derived by treating $\rho_{u,j}$ as a fixed parameter for each firm.

2.3.1 Fixed Effects Estimation

The unobserved individual effects ($\rho_{u,j}$) are coefficients for each individual firm, while the year effects are coefficients on time dummies. First, the individual unit averages over time gives:

$$ln(\frac{\bar{K}_{u,j}}{\bar{U}_j}) = a_u + \sigma_u ln(\frac{\bar{w}_{u,j}}{\bar{r}_{u,j}}) + \rho_{u,j} + \bar{\delta}_u + \bar{\omega}_{u,j} \tag{2.3.1}$$

Where $\bar{x_j}$ is variable $x_{j,t}$'s average value over all time periods. Subtracting this within estimator from 2.2.28 gives:

$$ln(\frac{\dot{K}_{u,j,t}}{\dot{U}_{j,t}}) = \sigma_u ln(\frac{\dot{w}_{u,j,t}}{\dot{r}_{u,j,t}}) + (\delta_{u,t} - \bar{\delta}_u) + (\omega_{u,j,t} - \bar{\omega}_{u,j}) \tag{2.3.2}$$

Where $\dot{x}_t = \frac{x_t}{\bar{x}}$ shows the proportionate change over time for that variable. The relationship between prices and inputs can be quite "sticky" as firms can take time to react to prices for various reasons. For this reason it is not necessarily desirable to simply regress current periods. As such I will be lagging the exogenous variable (prices) to allow for these delayed reactions. $\delta_{u,t} - \bar{\delta}_u$, which represents the effect of the systematic technical progress (ϕ). The stochastic changes in technical progress can be both idiosyncratic and systematic. The idiosyncratic element is part of the error term, however changes in systematic technical progress impacts the $\delta_{u,t}$ value. It can be assumed that this technical progress is constant, giving less endogeneity concerns surrounding technical progress mentioned earlier. Alternatively, allowing technical progress growth to vary makes the model less restricted. So $\delta_{u,t}$ can be treated as constant growth or as exogenously stochastic. In the former case $\delta_{u,t}$ can be written as a linear function of time ($\delta_u t$):

$$ln(\frac{\dot{K}_{u,j,t}}{\dot{U}_{j,t}}) = \sigma_u ln(\frac{\dot{w}_{u,j,t}}{\dot{r}_{u,j,t}}) + \delta_u t - \bar{\delta}_u + (\omega_{u,j,t} - \bar{\omega}_{u,j}) \tag{2.3.3}$$

In the latter case the $\delta_{u,t} - \bar{\delta}_u$ can be controlled using time dummy variables:

$$ln(\frac{\dot{K}_{u,j,t}}{\dot{U}_{j,t}}) = \sigma_u ln(\frac{\dot{w}_{u,j,t}}{\dot{r}_{u,j,t}}) + (\delta_{u,t} - \bar{\delta}_u) D_t + (\omega_{u,j,t} - \bar{\omega}_{u,j}) \tag{2.3.4}$$

With D being a dummy variable for each time period and $\delta_{u,t} - \bar{\delta}_u$ being the parameter for each time dummy variable. In the form of the models variables this can be shown as:

$$ln(\frac{\dot{K}_{u,j,t}}{\dot{U}_{j,t}}) = \sigma_u ln(\frac{\dot{w}_{u,j,t}}{\dot{r}_{u,j,t}}) + (1-\sigma_u)(\phi_{u,t} - \bar{\phi}_u) + (1-\sigma_u)(e_{u,j,t} - \bar{e}_{u,j}) \tag{2.3.5}$$

Although ϕ is unobserved, dummy variables can be taken to control for each different time period. From this the effect of changes in technical progress can be absorbed. These dummy variables may also contain the impact of other, more systematic, shocks e.g. the effect of taxes. For this method to be valid $e_{u,j,t}$, as defined in 2.2.24, has to be a valid i.i.d random variable. However this might well not be the case if $e_{u,j,t} - \bar{e}_{u,j}$ shows signs of being serially correlated. This might be the case if augmented technical change ($ln(A_{u,j,t})$) displays unit root characteristics rather than being mean reverting. In other words is the varying augmented productivity consistently inherent to each sector, or does this change with each time period. If augmented productivity is mean reverting $\delta e_{u,j,t}$ would be a more suitable i.i.d random variable, for which case first differences would be a more efficient estimation.

2.3.2 First Differences Estimation

If first differences is used, instead of subtracting average averages from 2.2.28 and taking deviations from the average, the previous time period is subtracted taking the first differences. With Δ as a backward first difference operator, the above equations then become:

$$\Delta ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u \Delta ln(\frac{w_{u,j,t}}{r_{u,j,t}}) + \Delta\delta_{u,t} + \Delta\omega_{u,j,t} \tag{2.3.6}$$

Similar to before I will look at lags in the independent variable for delayed effects. Also similar to before, technological gain can be restricted to being constant, and so $\Delta\delta_{u,t}$ becomes constant, or it can be stochastic with time dummies used to show the variable growth (in that each year has a different growth in technical progress) in technical change, creating:

$$\Delta ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u \Delta ln(\frac{w_{u,j,t}}{r_{u,j,t}}) + \Delta\delta_{u,t} D_t + \Delta\omega_{u,j,t} \tag{2.3.7}$$

Where $\Delta\delta_{u,t}$ is the parameter for each time dummy variable. In the form of the models variables this can be shown as:

$$\Delta ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u \Delta ln(\frac{w_{u,j,t}}{r_{u,j,t}}) + (1-\sigma_u)\Delta\phi_{u,t} + (1-\sigma_u)\Delta e_{u,j,t} \tag{2.3.8}$$

It can be seen here that $\Delta e_{u,j,t}$ becomes the i.i.d random variable of concern. Once both of these methods have been used, a Durbin Watson test can be performed on the error terms to see for which estimation the i.i.d assumption is valid. From this a suitable estimation for the elasticity of substitution (σ_u) can be derived.

2.3.3 Substitution Between Intermediary Goods

A key step to evaluating the relationship between inputs and prices, and validating this model, is estimating the elasticity of substitution between intermediate goods from the substitution parameter (γ). My previous estimations of the elasticity of substitution within the two intermediary goods (γ_u and γ_s) were estimated by comparing relative prices and inputs within each intermediary good, canceling out γ and making it possible to measure γ_u and γ_s. Now, to measure γ, it is necessary to compare input prices using the first order conditions between skilled and unskilled inputs. I will show this by comparing skilled and unskilled prices from the first order equations. By comparing first order conditions for the production of the final good (2.2.12 and 2.2.13), equation 2.3.10 shows relative expenditure on intermediate goods depends on their prices and the elasticity of substitution.

$$\frac{\alpha Y_{s,j,t}}{(1-\alpha)Y_{u,j,t}}^{1-\gamma} = \frac{P_{u,j,t}}{P_{s,j,t}} \tag{2.3.9}$$

$$\frac{Y_{s,j,t}P_{s,j,t}}{Y_{u,j,t}P_{u,j,t}} = \frac{1-\alpha}{\alpha}^{\frac{1}{1-\gamma}} \frac{P_{u,j,t}}{P_{s,j,t}}^{\frac{\gamma}{1-\gamma}} \tag{2.3.10}$$

In this the firm subscript j has been introduced for intermediate prices. As mentioned earlier, firms will actually experience different input prices, in terms of wages and rents, which would create varying intermediate goods prices. In the earlier stage, productivity shocks as well as stochastic variations in wages and rents, caused changing input levels. In this stage, all these create exogenous shocks are passed on to intermediate goods prices. Taking logs gives:

$$ln(\frac{Y_{s,j,t}P_{s,j,t}}{Y_{u,j,t}P_{u,j,t}}) = \sigma ln(\frac{1-\alpha}{\alpha}) + (1-\sigma)(\frac{P_{s,j,t}}{P_{u,j,t}}) \tag{2.3.11}$$

This is the regression that is used to find the elasticity of substitution between intermediate goods ($\sigma = \frac{\gamma}{1-\gamma} + 1$). To achieve this, estimates for unobservable intermediate goods prices, $P_{u,j,t}$ and $P_{s,j,t}$, need to be derived. The relative price can be derived from the supply of intermediate goods, comparing 2.2.14 and 2.2.15:

$$\frac{P_{u,j,t}}{P_{s,j,t}} = \frac{\alpha_s}{\alpha_u} \frac{A_{s,j,t}}{A_{u,j,t}} \frac{Y_{s,j,t}}{A_{s,j,t}S_{j,t}}^{1-\gamma_s} \frac{A_{u,j,t}U_{j,t}}{Y_{u,j,t}}^{1-\gamma_u} \frac{w_{u,j,t}}{w_{s,j,t}} \tag{2.3.12}$$

One problem with this estimation is that it defines the exogenous variable (intermediate prices) by Y_u and Y_s, which are not observable. As a result a two step estimation process can be used if the intermediate goods output are divided by labour, giving intermediate output per unit of labour. This provides terms which are dependant on $\frac{K_{u,j,t}}{U_{j,t}}$ and $\frac{K_{s,j,t}}{S_{j,t}}$ respectively, which can be derived from these exogenous calculations to create estimates of variables. Taking $y_{u,j,t} = \frac{Y_{u,j,t}}{A_{u,j,t}U_{j,t}}$ and $y_{s,j,t} = \frac{Y_{s,j,t}}{A_{s,j,t}S_{j,t}}$, for the unskilled intermediate good, this leads to:

$$y_{u,j,t} = ((1-\alpha_u)(\frac{K_{u,j,t}}{A_{u,j,t}U_{j,t}})^{\gamma_u} + \alpha_u)^{\frac{1}{\gamma_u}} \tag{2.3.13}$$

$$\hat{y}_{u,j,t} = ((1-\alpha_u)(\frac{\hat{K}_{u,j,t}}{\hat{A}_{u,j,t}\hat{U}_{j,t}})^{\gamma_u} + \alpha_u)^{\frac{1}{\gamma_u}} \tag{2.3.14}$$

Where ˆ denotes an estimated value. From the last section there are estimates for $\frac{K_{u,j,t}}{U_{j,t}}$ and $\frac{K_{s,j,t}}{S_{j,t}}$ which are formed from exogenous prices. From this it is possible to derive an exogenous estimation of $y_{s,j,t}$ and $y_{s,j,t}$, and so relative intermediate good prices can be used as estimates of variables in 2.3.12.

$$\frac{\hat{P}_{u,t}}{\hat{P}_{s,t}} = \frac{\alpha_s}{\alpha_u} \frac{A_{s,j,t}}{A_{u,j,t}} \frac{\hat{y}_{s,j,t}^{1-\gamma_s}}{\hat{y}_{u,j,t}^{1-\gamma_u}} \frac{w_{u,j,t}}{w_{s,j,t}} \tag{2.3.15}$$

For this an assumptions on α_u and productivity growth rates still needs to be estimated. An estimation for the unobservable productivity variables ($A_u$and A_s) needs to be made. Productive growth estimates from 2.3.5 and 2.3.8 can be used. These can be checked against a range of assumed growth values to check the sensitivity of the estimations to various assumptions. Arbitrary values of α_u and α_s also need to be assumed. Fortunately this turns out to have little effect on the results.

The endogenous side of equation 2.3.10, which shows the total incomes (or costs) for intermediate goods, is much simpler. As shown in equations 2.2.10 and 2.2.11 with a competitive industry, the total revenue for intermediate goods is the same as the total cost of inputs, which is known. As such the left hand side of 2.3.10 can be derived form the data as:

$$ln(\frac{Y_{s,j,t}P_{s,j,t}}{Y_{u,j,t}P_{u,j,t}}) = ln(\frac{S_{j,t}w_{s,j,t} + K_{s,j,t}r_{s,t}}{U_{j,t}w_{u,j,t} + K_{u,j,t}r_{u,t}}) \tag{2.3.16}$$

2.4 DATA

The data used will be low level manufacturing industry panel data, using 6 digit level industry codes (of which there are 469) from 1958 to 2011. I can use these industries as "de facto" firms, as they are small enough to be price takers, so the assumption of exogenous prices still holds.

To keep the comparisons of different industries structurally similar, so that the assumption of consistency among industries can be seen to hold, only manufacturing industries will be used. Non labour inputs can be split into skilled and unskilled based on whether it is considered to be directly complementary to the production process. The theory above is clear that actual production processes, as opposed to design, management etc, is considered predominantly unskilled. As such change in materials, energy and machinery used are all potential complements for production workers.

One way of defining K_u, for which there is the most data, would be to take a perishable input used in production. The most appropriate being energy as, unlike materials, which would be an alternative, it remains homogenous. Energy also serves as an appropriate input as part of the inspiration for this chapter was some of the historic correlation of energy supply and relative wages. This data, along with production and non-production worker levels and wages, are available from the NBER-CES Manufacturing Industry Database[1] for 6 digit level industry codes from 1958 to 2011. Capital equipment is not broken down however, for this case, it can be assumed to be skilled complementing as, on aggregate, it is generally found to be in Krusell et al. (2000) and in more recent papers (Correa et al. (2014)). Though investment prices are included in this dataset, rental rates are not available, however can be taken from BLS data[2] for 3 digit industry (of which there are 18) data from 1987. For rental rates I will assume all 6 digit industries within each 3 digit codes have the same rental rates, adjusting the rents according to prices, similar to the assumption made in Raval (2011). Obviously this data can only be taken from 1987, though this is potentially desirable. If a longer time frame was to be used it is more likely that structural breaks caused by structural changes in the economy would make the results less reliably. As such, if a longer time frame was used the parameters that are being measured may be unstable or prone to change.

From this I obtain panel data from 1987 to 2011 for 469 industries. Even though the input data goes back to 1958, as mentioned earlier, capital rent prices can only be estimated from 1987. As I will use four lags, the data points start in 1991. This gives 21 years of data with nearly 10k data points (first differences has slightly less due to having one less time period). A summary of the relative variables used is shown in table 2.1.

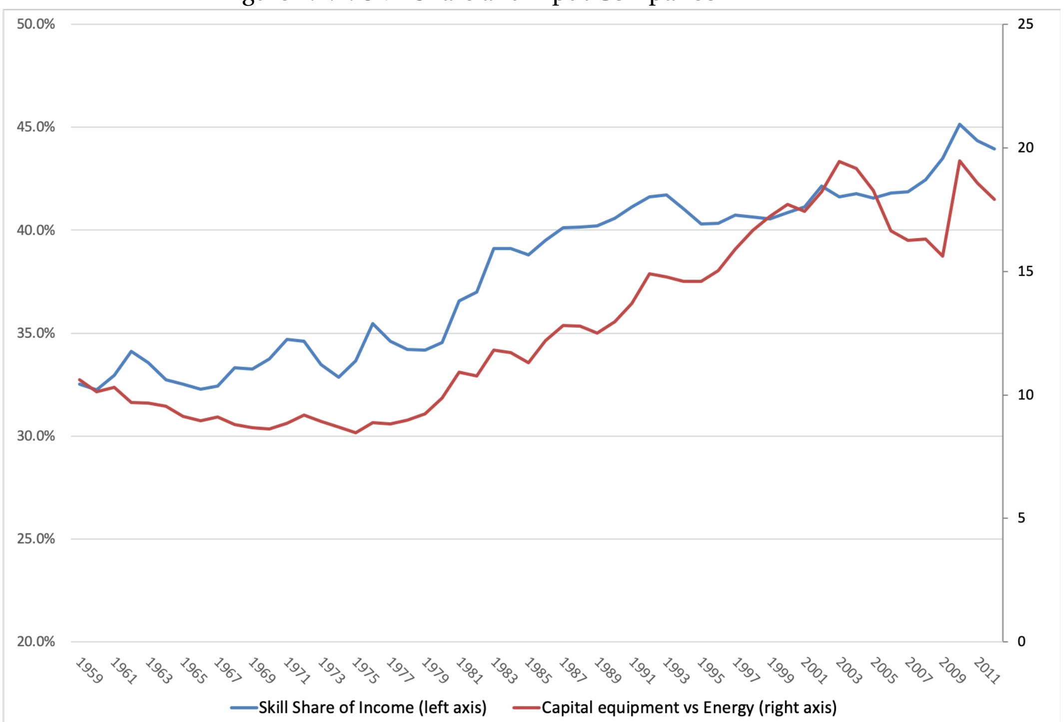
Through the aggregation of the industries in this data, figure 2.4.1 shows how the skill share of income in US manufacturing (measured by comparing production and non-production workers) compares to quantities of capital equipment vs energy inputs since 1958. The former can be seen as a measure of relative demand for skilled labour. The latter is the aggregation of $\frac{K_{s,t}}{K_{u,t}}$ in the model, i.e. the relative level of skill complementary inputs. It can be seen that both have been growing steadily since the late 1970's,

[1] This can be found at http://www.nber.org/nberces. Downloaded April 2020

[2] This can be found at https://www.bls.gov/mfp/mprdload.htm

when there was a rapid decrease in the supply of energy from two oil shocks. Before this both measures were relatively stationary. While the complementarity between capital equipment and skilled labour has been fairly well established, the possible unskilled complementarity of energy is less recognised.

Figure 2.4.1: Skill Share and Input Comparison



Summary of the total share of income going to non production workers (skilled workers) vs capital equipment/ energy usage ratio

Here I use low level industry data as apposed to firm level data. This potentially creates issues concerning the aggregation of a constant elasticity of substitution production function. The CES production functions in the model above are at firm level, however the data used is effectively an aggregation of groups of firms. This theoretically may create bias in some estimates, as the firms aggregated CES functions don't create a precise CES function itself. van der Loeff and Harkema (1975) find that, in the case of aggregation of CES functions, there can be upward bias of productive measures and bias in share parameters, but not the substitution estimation. Since I am mainly interested in the substitution parameter this does not cause an issue. However it does emphasise that the exact readings of augmented technical progress are biased.

Table 2.1 shows a summary of the data used, with figure 2.4.1 showing how the aggregated capital and labour shares develop over time. The log of the premium non-production workers are paid ($ln(W_s) - ln(W_u)$) is consistently positive with an average that is over 2.5 standard deviations above zero. The skill premium is similar to the wage premium shown for college graduates in figure 2.1.4. It can also be seen that there is a higher level of variation in relative inputs than in relative prices. This is likely reflective of some level of mobility between firms and industries meaning the relative cost of inputs, which are exogenous to the firm, doesn't vary as much as it otherwise would.

Table 2.1: Summary Of Manufacturing Industries from 1987 to 2011

Variable	Observations	Mean	Std Deviation	Min	Max
$ln(K_u) - ln(U)$	9,930	1.44	1.06	-1.85	6.30
$ln(K_s) - ln(S)$	9,930	6.00	1.00	2.33	10.3
$ln(W_u) - ln(R_u)$	9,930	3.88	0.351	2.47	5.10
$ln(W_s) - ln(R_s)$	9,930	6.37	0.571	4.44	8.15
$ln(S) - ln(U)$	9,930	-0.972	0.560	-3.01	1.50
$ln(W_s) - ln(W_u)$	9,930	0.523	0.203	-0.434	1.94
$ln(R_s) - ln(R_u)$	9,930	-1.962	0.426	-3.59	-0.987
$ln(K_s) - ln(K_u)$	9,930	3.58	0.731	-0.677	7.38

U is the number of production workers, S the number of non-production workers, K_u units of energy used in production, K_s capital used in production, R_u is the price of energy used, R_s is capital rents, W_u is the real wage of production and W_s is the real wage of non production workers

2.4.1 Structure of Regressions

The relationship between prices and inputs can be quite "sticky" as firms can take time to react to prices for various reasons. For this reason it is not necessarily desirable to simply regress current periods. As such it is necessary to include lags the exogenous variable (prices) to allow for these delayed reactions. The latter would also help avoid any endogeneity that exists in prices.

Equations with lags for both first differences and fixed effects estimations still seem to suffer from a high level of autocorrelation in the residuals, though much less for first differences. A natural solution to this would be to introduce a lagged dependant variable into the regression creating an AR(1) process. An issue with this in a fixed effects is that, as exposed by Nickell (1981), the lagged dependant variable is endogenous to the fixed effect. This is especially the case as there are lagged independent variables included, which may also make the lagged dependant variable endogenous. Because of this Nickell recommends an alternative structural form for AR(1) which includes an AR(1) process in the residual (effectively an MA(inf) process). For example the first difference residual for 2.3.8 is defined as:

$$\Delta\omega_{u,j,t} = \rho\Delta\omega_{u,j,t-1} + \eta_{u,j,t} \tag{2.4.1}$$

Where η is an iid.

2.5 RESULTS

The results of the first regressions (2.3.5 and 2.3.8) comparing energy and production workers elasticities are shown in table 2.2. This is shown for both constant productive growth and variable productive growth, which includes time dummy variables. As mentioned, to take into account stickiness of prices and inputs, I have included up to four time lags for the exogenous prices. I have dropped lags that were not shown to be relevant using the 5% significance level. I have used both Fixed Effect and First Differences estimations, as well as used two different definitions of capitals. Table 2.3 shows the same relationship within the other intermediate good, between non-production workers and capital equipment. It can be generally noted that, from these results and across all the regressions, first differences proved much better at avoiding autocorrelation in the error term. This can be seen from the AR disturbance coefficient, which shows the estimate for ρ from 2.4.1. As a result first differences is a more reliable method, implying the error term derived from productivity growth behaves more like a unit root process than a mean reverting one. From the estimated values for σ_u and σ_s, it can be noted that the complementary effects within the skilled intermediate good, which includes non-production workers and capital equipment, is stronger than within the unskilled intermediate good, between production workers and energy. σ_u estimates range from around 0.44 to 0.62, while σ_s estimates are all between 0.33 and 0.36.

From these estimations it is possible to derive estimates for intermediate prices, which is then used to get values for the elasticity of substitution between intermediate goods from 2.3.12. This result for this is shown in table 2.4. It can be seen that σ is generally higher than both σ_u and σ_s, with estimate between 0.67 and 0.94. This confirms complementary effects are stronger within intermediate goods than between them, meaning that energy complements production workers more than non-production. Likewise capital equipment complements non-production workers more. Also the complementary effects are stronger within the skilled intermediate good, as evident by $\sigma - \sigma_s$ being larger than $\sigma - \sigma_u$. This means capital has a stronger complementing effect on non-production workers than energy does on production workers.

Table 2.5 shows the Morishima elasticities for relative labour demand (2.2.32). As the error terms acts more like a unit root process than a mean reverting one, I have only used first differences for this validation. The coefficient values for capital rents is negative. This is consistent with capital equipment being skill complementing, as increasing rents will restrict skill completing capital and so decrease skill demand. The coefficient values for energy price is positive when assuming constant productive growth, implying unskilled complementarity. If productive growth isn't assumed to be constant however, energy prices don't show any significance. As seen in table 2.4, the difference between σ and σ_u is much smaller with variable productive growth. This implies, by this method, energy has a relatively small complementing effect on unskilled labour compared to skilled. As such it doesn't seem to have been captured in the Morishima regression when allowing for variable productive growth.

All regressions were tested for stationarity. Along with the AR disturbance measure, this could give more of a sign as to whether technical progress is best treated as unit root, where a first difference estimation is more suitable, or trend stationary, where a fixed effects estimation is more suitable. in this case all the regressions used tested positive for cointegration using Dickey Fuller and Phillips-Perron tests, rejecting the null hypothesis that residuals are unit root. The one exception to this was the fixed effects estimation between intermediate goods and with time dummies. This didn't quite reject the null at the 5% level using the Modified Phillips-Perron test, however it is the first differences estimations that are regarded as more reliable in this exercise, due to less residual serial correlation shown by the AR disturbance.

These results conclude that capital equipment does complement skill and energy does seem to complement unskilled labour more. Generally, labours elasticity of substitution with energy is higher than between labour and capital.

Table 2.2: Measuring the Elasticity of Substitution Between Energy and Production Workers (1991-2011)

$$ln(\frac{K_{u,j,t}}{U_{j,t}}) = \sigma_u ln(\frac{1-\alpha_u}{\alpha_u}) + (1-\sigma_u) ln(A_{u,j,t}) + \sum_{i=0}^{4} \sigma_{u,i} ln(\frac{w_{u,j,t-i}}{r_{u,j,t-i}})$$

	Constant Productive Growth		Variable Productive Growth	
	First Differences	Fixed Effect	First Differences	Fixed Effect
σ_(u,0)	0.374***	0.406***	0.512***	0.505***
	(14.10)	(16.13)	(15.48)	(16.67)
σ_(u,2)	0.0694*	0.0938***	0.109**	0.105***
	(2.55)	(3.61)	(3.22)	(3.34)
R-sq (within)	0.0217	0.115	0.0671	0.164
R-sq (between)	0.128	0.385	0.125	0.385
AR Disturbance	-0.210	0.662	-0.229	0.639
σ_u (ES)	**0.443**	**0.500**	**0.621**	**0.610**

t statistics in parentheses
="* p<0.05 ** p<0.01 *** p<0.001"

Note: Lags that were not shown to be relevant using the 5% significance level have been dropped. $ln(A_{u,j,t})$ includes a productive growth trend, which is the same for all firms, and an idiosyncratic error term. First differences assumes that productivity growth follows a unit root trend where as fixed effect assumes a trend stationary trend. This being incorrectly identified would likely create a significant Autoregressive (AR) disturbance in the residuals.
Variable productive growth uses a time dummy variable for each time period for the different productive growth rates. Constant productive growth uses a constant, assuming the same growth for each time period. The constant and parameters to the dummy variables have been estimated but are not reported.

Table 2.3: Measuring the Elasticity of Substitution Between Capital Equipment and Non-Production Workers (1991-2011)

$$ln(\frac{K_{s,j,t}}{S_{j,t}}) = \sigma_s ln(\frac{1-\alpha_s}{\alpha_s}) + (1-\sigma_s) ln(A_{s,j,t}) + \sum_{i=0}^{4} \sigma_{s,i} ln(\frac{w_{s,j,t-i}}{r_{s,j,t-i}})$$

	Constant Productive Growth		Variable Productive Growth	
	First Differences	Fixed Effect	First Differences	Fixed Effect
σ_(s,0)	0.212***	0.213***	0.207***	0.214***
	(24.68)	(24.95)	(23.01)	(24.48)
σ_(s,1)	0.0599***	0.0633***	0.0350***	0.0362***
	(6.94)	(7.32)	(3.85)	(4.06)
σ_(s,2)	0.0324***	0.0408***	0.0228*	0.0196*
	(3.66)	(4.66)	(2.45)	(2.17)
σ_(s,3)	0.0321***	0.0373***	0.0390***	0.0342***
	(3.39)	(4.06)	(3.93)	(3.62)
σ_(s,4)			0.0296**	0.0329***
			(2.94)	(3.47)
R-sq (within)	0.0651	0.262	0.121	0.301
R-sq (between)	0.180	0.0372	0.177	0.0387
AR Disturbance	-0.110	0.804	-0.106	0.806
σ_s	**0.3364**	**0.3544**	**0.3334**	**0.3369**

t statistics in parentheses
="* p<0.05 ** p<0.01 *** p<0.001"

Note: Lags that were not shown to be relevant using the 5% significance level have been dropped. $ln(A_{s,j,t})$ includes a productive growth trend, which is the same for all firms, and an idiosyncratic error term. First differences assumes that productivity growth follows a unit root trend where as fixed effect assumes a trend stationary trend. This being incorrectly identified would likely create a significant Autoregressive (AR) disturbance in the residuals.
Variable productive growth uses a time dummy variable for each time period for the different productive growth rates. Constant productive growth uses a constant, assuming the same growth for each time period. The constant and parameters to the dummy variables have been estimated but are not reported.

Table 2.4: Measuring the Elasticity of Substitution Between Intermediate Goods (1991-2011)

$$ln(\frac{Y_{s,j,t}P_{s,j,t}}{Y_{u,j,t}P_{u,j,t}}) = \sigma ln(\frac{1-\alpha}{\alpha}) + \sum_{i=0}^{4}(\sigma_i - 1)(\frac{P_{u,j,t-i}}{P_{s,j,t-i}})$$

	Constant Productive Growth		Variable Productive Growth	
	First Differences	Fixed Effect	First Differences	Fixed Effect
σ_0	-0.0614***	-0.420***	-0.0356***	-0.472***
	(-13.45)	(-31.83)	(-10.61)	(-36.47)
σ_1	-0.00926*		-0.0125***	
	(-2.00)		(-3.64)	
σ_2		0.0598***		0.0318*
		(4.42)		(2.40)
σ_3	-0.00927	-0.0360*		
	(-1.88)	(-2.55)		
σ_4	-0.0303***	0.0691***	-0.0187***	0.0863***
	(-5.77)	(4.81)	(-4.52)	(6.08)
R-sq (within)	0.0301	0.140	0.101	0.235
R-sq (between)	0.0342	0.000647	0.00851	0.000232
AR Disturbance	-0.215	0.732	-0.177	0.735
σ	**0.891**	**0.6729**	**0.935**	**0.6461**
σ-σ_u	**0.4476**	**0.1731**	**0.314**	**0.0361**
σ-σ_s	**0.5546**	**0.3185**	**0.6016**	**0.3092**

t statistics in parentheses
="* p<0.05 ** p<0.01 *** p<0.001"

Note: Lags that were not shown to be relevant using the 5% significance level have been dropped.
First differences assumes that productivity growth follows a unit root trend where as fixed effect assumes a trend stationary trend. This being incorrectly identified would likely create a significant Autoregressive (AR) disturbance in the residuals.
Variable productive growth uses a time dummy variable for each time period for the different productive growth rates. Constant productive growth uses a constant, assuming the same growth for each time period. The constant and parameters to the dummy variables have been estimated but are not reported.

Table 2.5: Morishima Elasticities (1991-2011)

$$\frac{S_{j,t}}{U_{j,t}} = a_{j,t} + \sum_{i=0}^{4} [b_{w_s,i} w_{s,j,t-i} + b_{w_u,i} w_{u,j,t-i} + b_{r_s,i} r_{s,j,t-i} + b_{r_u,i} r_{u,j,t-i}]$$

	Constant Productive Growth First Differences	Variable Productive Growth First Differences
b_(Wu,0)	0.258***	0.240***
	(10.21)	(9.07)
b_(Wu,2)	0.0741**	0.0998***
	(2.86)	(3.64)
b_(Wu,4)	0.0655*	0.0685*
	(2.41)	(2.42)
b_(Ws,0)	-0.777***	-0.768***
	(-54.84)	(-53.36)
b_(Ws,1)		0.0348*
		(2.42)
b_(Ru,0)	0.0964***	
	(3.56)	
b_(Ru,4)	0.0795**	
	(2.68)	
b_(Rs,0)	-0.0278***	-0.0220**
	(-3.53)	(-2.63)
b_(Rs,1)	-0.0351***	
	(-4.46)	
b_(Rs,3)	0.0175*	
	(2.00)	

	Constant Productive Growth First Difference:	Variable Productive Growth First Difference
b_Wu	0.3976	0.4083
b_Ws	-0.777	-0.7332
b_Ru	0.1759	0
b_Rs	-0.0445	-0.022
R-sq (within)	0.274	0.285
R-sq (between)	0.229	0.241
AR Disturbance	-0.195	-0.201

t statistics in parentheses
="* p<0.05 ** p<0.01 *** p<0.001"

Note: Lags that were not shown to be relevant using the 5% significance level have been dropped. $a_{j,t}$ includes a productive growth trend, which is the same for all firms, and an idiosyncratic error term. First differences assumes that productivity growth follows a unit root trend where as fixed effect assumes a trend stationary trend. This being incorrectly identified would likely create a significant Autoregressive (AR) disturbance in the residuals.
Variable productive growth uses a time dummy variable for each time period for the different productive growth rates. Constant productive growth uses a constant, assuming the same growth for each time period. The constant and parameters to the dummy variables have been estimated but are not reported.

For the economy as a whole, the aggregated first order conditions (2.2.14, 2.2.15, 2.2.16, 2.2.17, 2.2.12 and 2.2.13) gives terms for wages and prices:

$$w_{u,t} = \alpha_u A_{u,t}^{\gamma_u} (\frac{Y_{u,t}}{U_t})^{1-\gamma_u} P_{u,t} \tag{2.5.1}$$

$$w_{s,t} = \alpha_s A_{s,t}^{\gamma_s} (\frac{Y_{s,t}}{S_t})^{1-\gamma_s} P_{s,t} \tag{2.5.2}$$

$$P_{u,t} = \alpha (\frac{Y_t}{Y_{u,t}})^{1-\gamma} \tag{2.5.3}$$

$$P_{s,t} = (1-\alpha)(\frac{Y_t}{Y_{s,t}})^{1-\gamma} \tag{2.5.4}$$

As a result, relative wages for the whole economy can be given as:

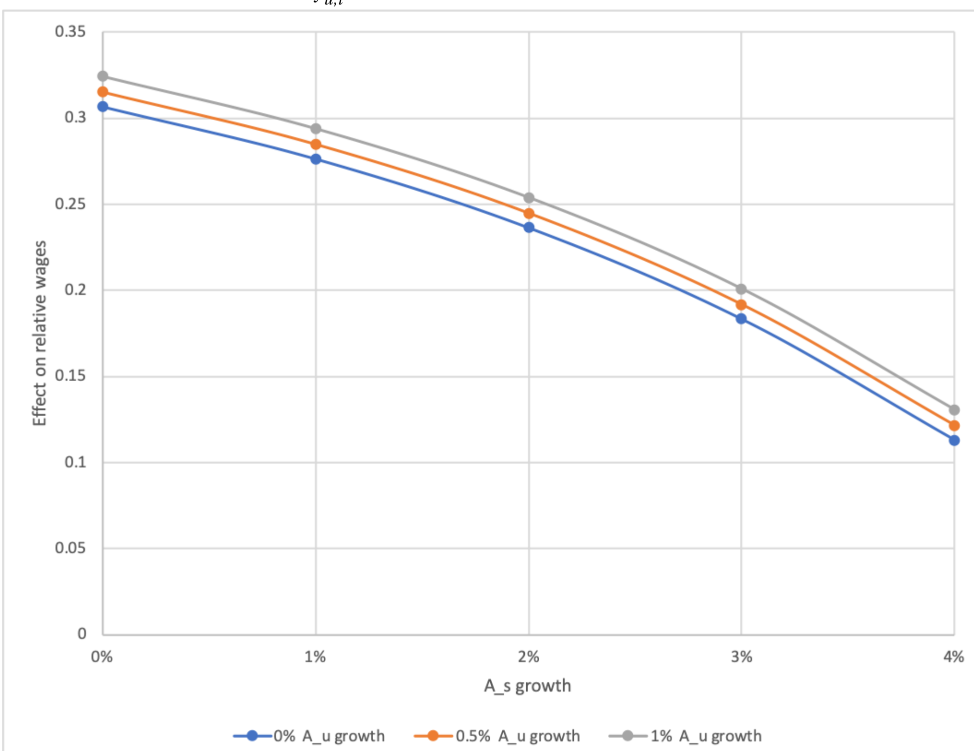
$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\alpha)\alpha_s}{\alpha\alpha_u} (\frac{A_{s,t}}{A_{u,t}})^{\gamma} (\frac{U_t}{S_t})^{1-\gamma} \frac{y_{s,t}^{\gamma-\gamma_s}}{y_{u,t}^{\gamma-\gamma_u}} \tag{2.5.5}$$

With $y_{u,t} = \frac{Y_{u,t}}{A_{u,t}U_t}$ and $y_{s,t} = \frac{Y_{s,t}}{A_{s,t}S_t}$. Unlike elasticity estimates, the models implication for relative labour demand is highly sensitive to small changes in augmented productivity. In the model estimates can be made for the time dummy parameters or constants. However these values will also control for other systematic changes in relative input quantities outside of the model, e.g. policy change. This makes these measures unreliable for estimating changes in technical progress. As such the below sensitivity analysis is the best way to asses the models impact on relative labour demand. One thing that is clear and consistent from the results of the model is that skill complementary technical progress is far larger than unskilled complementing technical progress. This is a result of skilled capital per labourer being far in excess of what can be explained by prices. Consistent this these findings, I will compare scenarios of $A_{u,t}$ growth between 0 and 1 percent, and $A_{s,t}$ between 0 and 4 percent, which give aggregate productive growth estimates consistent with other findings (Jorgenson et al. (2008)). I will use 2.5.5 to assess the impact of intermediate output per effective labour ($\hat{y}_{u,t}$ and $\hat{y}_{s,t}$, which are driven by capital gains) and the total implied change in relative wages.The total impact of intermediate output per effective labour, shown in figures 2.5.1 and 2.5.3 with different productive growth assumptions, shows the impact of capital growing at a rate that isn't balanced compared to growth in productivity and labour within the same sector. This excludes the impact of $(\frac{A_{s,t}}{A_{u,t}})^{\gamma}(\frac{U_t}{S_t})^{1-\gamma}$ in 2.5.5, which shows labour or productivity growth of skilled an unskilled sectors growing at a different rate, so exclude the impact of capital.

As mentioned earlier, van der Loeff and Harkema (1975) finds that technical progress estimations are likely biased using this method. To assess what technical progress levels are consistent with observed outcomes, I will compare outcomes using different assumptions. In the regression, time dummies allow for different productivity growth rates each year, where as the exclusion of time dummies enforces constant productive growth. Figures 2.5.1 and 2.5.2 show the first differences estimation with the assumption of constant technical progress (no time dummy variables) where as figures 2.5.3 and 2.5.4 show this the first differences estimation with time dummy variables. Figures 2.5.1 and 2.5.3 show the effects of intermediate output per effective labour where as figures 2.5.2 and 2.5.4 show the the total predicted change in relative demand according to 2.5.5. The graphs compares different levels of $A_{s,t}$ annual growth and resulting effects for $A_{u,t}$ growth of 0, 0.5 and 1 percent. In the data, despite a relative increase in skilled wages of 12 percent over the relevant time period, skilled labour relative to unskilled increased 4 percent over the same period. A comparable outcome would be observed if $A_{s,t}$ growth is assumed to be around 3 percent per annum with negligible $A_{u,t}$ growth. This would imply that growth in intermediate output per effective labour, driven by capital gains, caused an increase in relative wages of around 20 percent over the 20 years. It is also possible the unskilled augmenting technical progress experienced higher growth of 0.5 or 1 percent per annum. This would also mean skill augmenting technical progress grew at nearer 4 percent per annum, and impact of capital gains is around 15 percent in total. However this would assume higher aggregate productive growth beyond what is generally observed in the long run, making the former case more likely.

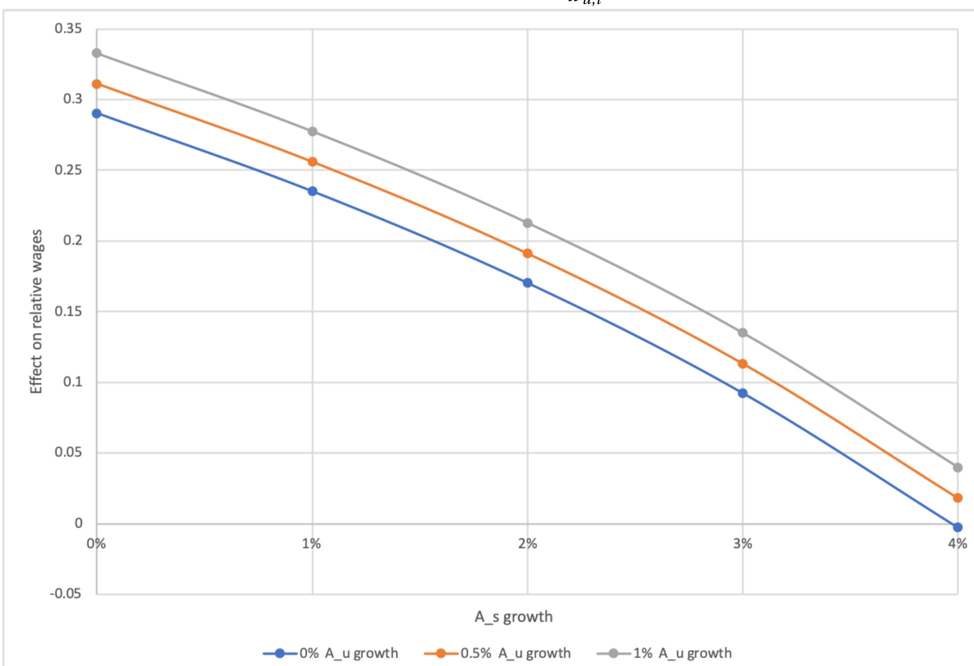
Comparing these graphs an unbalanced growth in inputs relative to labour and productivity growth accounts for the full impact wage premium. Unless skilled labour augmenting growth is zero, which would seem unrealistic, other factors (i.e. relative labour and productivity levels) actually had a negative impact on skill premium.

Figure 2.5.1: Effect of $\frac{\hat{y}_{s,t}^{1-\gamma_s}}{\hat{y}_{u,t}^{1-\gamma_u}}$ on relative wages($\frac{w_{s,t}}{w_{u,t}}$) without time dummies



Growth in $\frac{\hat{y}_{s,t}^{1-\gamma_s}}{\hat{y}_{u,t}^{1-\gamma_u}}$ *shows the impact of capital growing at a rate that isn't balanced compared to growth in productivity and labour within the same sector*
This is based on various assumptions of A_s *growth (x axis) and* A_u *growth (shown by different lines)*

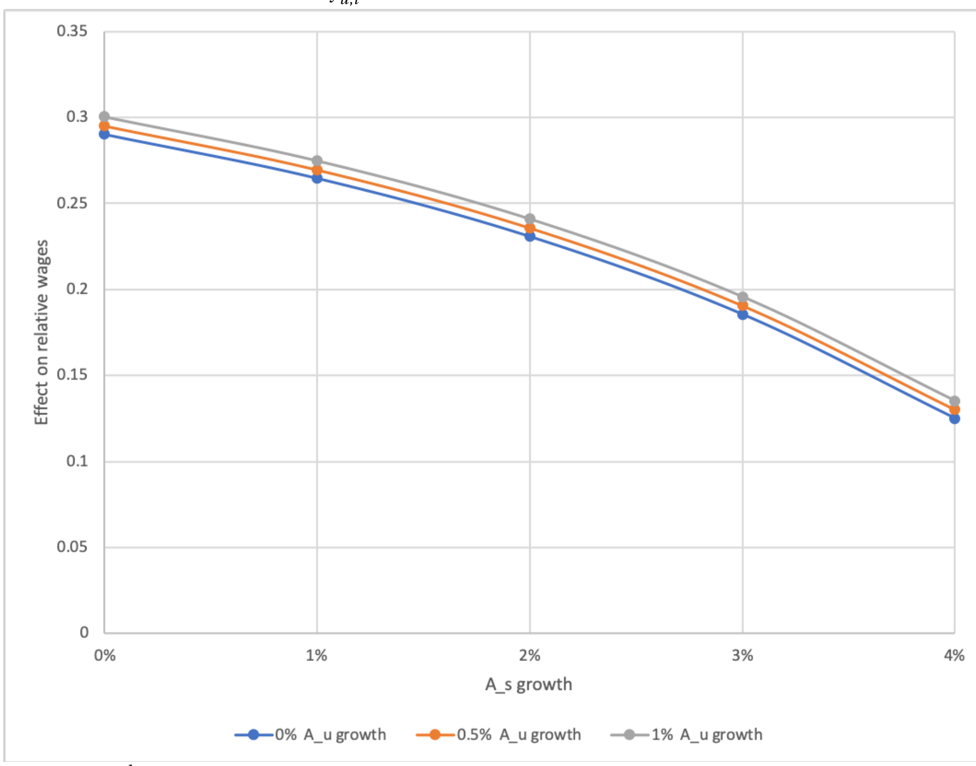
Figure 2.5.2: Total change in relative wages ($\frac{w_{s,t}}{w_{u,t}}$) without time dummies



The figure shows the total expected change in wage premium according to the model and exogenous variables

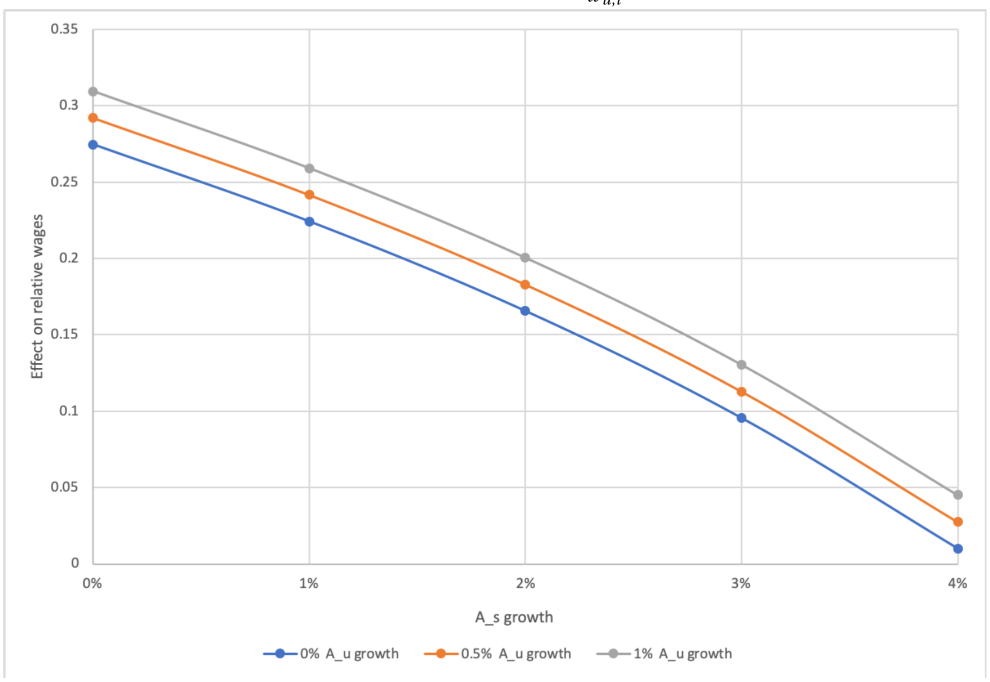
This is based on various assumptions of A_s growth (x axis) and A_u growth (shown by different lines)

Figure 2.5.3: Effect of $\frac{\hat{y}_{s,t}^{1-\gamma_s}}{\hat{y}_{u,t}^{1-\gamma_u}}$ on relative wages ($\frac{w_{s,t}}{w_{u,t}}$) with time dummies



Growth in $\frac{\hat{y}_{s,t}^{1-\gamma_s}}{\hat{y}_{u,t}^{1-\gamma_u}}$ *shows the impact of capital growing at a rate that isn't balanced compared to growth in productivity and labour within the same sector*
This is based on various assumptions of A_s *growth (x axis) and* A_u *growth (shown by different lines)*

Figure 2.5.4: Total change in relative wages ($\frac{w_{s,t}}{w_{u,t}}$) with time dummies



The figure shows the total expected change in wage premium according to the model and exogenous variables

This is based on various assumptions of A_s growth (x axis) and A_u growth (shown by different lines)

2.6 Conclusion

In this paper I introduce a process for identifying elasticities of substitution in multi-stage production process. This allows for substitution effects to be identified and measured between two inputs while taking into account indirect effects from other factors of production. Morishima elasticities similarly can be used to account for these indirect effects as in Anderson et al. (2019). This paper differs in that it has a specified setup provides economic intuition and reveals the dynamic relationships between labour and non labour factors of production that impact relative labour demand, as opposed to simply assuming a linear relationship between factor input demand and prices.

Using data from US manufacturing plants, I find that capital equipment predominantly complements better paid non-production workers, while energy predominantly complements worse paid production workers. This is because complementary effects within the processes of production are stronger than between processes of production. Additionally, I find the elasticity of substitution between energy and production workers is larger than between capital equipment and non-production workers. As a result capital equipments contribution to increasing relative skill demand is greater than energy's contribution to decreasing it. This means a similar percentage rises in both energy and capital would have a net effect of increasing demand for non-production workers over production workers. On top of this, capital has increased substantially more than energy, with a 64% increase in capital equipment among manufacturing industries between 1987 and 2011, compared to a 17% increase for energy. This has occurred as capital rents have decreased around 36% due to the decreasing price of capital, substantially more over the relevant period relative to energy prices, which have declined 22% in real terms over the same time period.

Most modern papers comparing skilled and unskilled labour have them as substitutes. For example McAdam et al. (2011) find this while using a very similar model to the one used in this chapter. In my model I have skilled and unskilled workers as very slight complements, but this could be because they are defined differently. Modern papers generally use education levels, as opposed to job roles, as a proxy for skill level. Even though workers may have different education levels, in many scenarios this won't necessarily mean they can't hold the same skill relevant to a specific job role, causing a high elasticity of substitutions. However using job roles as a proxy for skill makes it less likely the different types of workers have the skills to easily replace each other.

There is some history of capital, or a breakdown of capital, accounting for wage inequality. In Krusell et al. (2000) it was estimated that capital skill complementarity alone (without the impact of energy supply) accounted for 60% of the skill premium growth between 1963 and 1991 in the US. Feenstra and Hanson (1999) found a specification in which capital complementarity accounted for 75% of the relative growth of non-production workers relative to production workers from 1979 to 1990 in the US. More recently, Parro (2013) estimates that, without capital skill complementarity, skill premium would not have grown between 1990 and 2007 in the US. There is little literature on the impact of energy on the skill premium and I do not estimate this to be as significant as capital skill complementarity. Added to the impact of capital skill complementarity, the results shown in this chapter seem plausible and consistent with other literature.

This chapter provides further evidence that modern capital is skill complementing, yet at the same time provides some insight into the apparent unskilled biased growth that occurred during the industrial revolution. As energy complements lower paid production workers, it follows that a growth in energy services and technologies that occurred during the industrial revolution would increase the productivity of these lower paid workers. As concluded by Pearson and Foxon (2012), a new energy revolution would be a slow process and require large scale supply side changes. This paper supports that, on top of the overall productive gain such an event would produce, there would likely also be social gains in the form of decreased inequality.

An interesting extension of these findings would be to investigate the impact of endogenous technical progress within a multi-stage production model, as is done in chapter 3. The outcomes of this model is highly sensitive to the type of technical progress experienced and the observed factor input changes discussed above would naturally impact endogenous technology, something that would be a key element of the balanced growth path. This has been discussed in a simple two factor model (Acemoglu (2002)). However a multi stage process, with exogenous shocks in non labour inputs, has the potential to create further insights.

CHAPTER 3

ENDOGENOUS GROWTH IN A TWO STAGE PRODUCTION PROCESS

In the immediate post Second World War period the wage skill premium declined rapidly, and then increased after the two oil crises of the 1970s. Chapter 3 attempts to explain this pattern with a similar theoretical set up as chapter 2.

In this chapter I introduce endogenous technical progress with the same theoretical setup as Acemoglu (2002). However, in this exercise I look at the outcomes when production is split into more inputs, with two types of labour and two type of capital, as in chapter 2. This introduces an interaction between growth and capital, which act as the main endogenous variables in this chapter. From this a better understanding of what pattern of technology shocks or capital price shocks may best explain skill premium growth in a way that is constant with observed outcomes.

I find that a technology shock boosting the demand for skills paradoxically generally reduces the demand for skill-complementing inputs. By contrast, a shock reducing investment prices increases both skill demand and skill-complementing inputs. Observed changes in prices and skill premium are consistent with being induced by a permanent shock in energy prices. The finding identifies potential benefits of technologies that reduce energy prices that in Chapter 2 were shown to complement lower paid workers. In general this setup helps forecast wage inequality after factor input prices changes take place.

3.1 INTRODUCTION

In the model used in chapter 2 the skill premium and capital share is dictated not only by input quantities but also the relative augmented productivity or technical progress. Endogenous factor biased technical progress can be used to model the progression of augmented growth, relaxing the assumption of neutral productivity growth. As such the model in this chapter differs from chapter 2 in that each factor input has endogenous factor augmenting technical progress. Incorporating endogenous productivity growth in the model helps explain some of the movements in skill premium and capital share of income since 1945 that aren't easily explained by relative inputs. It also helps explain the persistence of biassed technical progress as well as some of the fluctuations in capital share of income.

The model I will use for endogeneous technical change is based on Acemoglu's theoretical paper (Acemoglu (2002)) where the type of technical change (in terms of which factor it augments) is dependent on a profit incentive for inducing technology growth. Acemoglu explains how augmented technical change is effected through factor inputs. If the elasticity of substitution is greater than one, i.e. the factor inputs are gross substitutes, then an increase in the factor augmented technology complements that input, and vice versa. This means if one input increases, the technical progress augmenting that input increases (Acemoglu uses the variety of machines in his abstraction for technical progress). If the elasticity of substitution is less than one, i.e. the factor inputs are gross complements, it means augmenting one input complements the other. The two inputs reliance on each other means if one rises the other inputs augmented technical progress becomes more valuable.

A generalised example of this can be seen, with Y as output from using two intermediate goods (Y_u and Y_s). The two intermediate goods are produced using skilled and unskilled labour (U and S) and their corresponding augmented productivity (A_u and A_s).

$$Y = f(Y_u, Y_s) \tag{3.1.1}$$

$$Y_u = g(U, A_u) \tag{3.1.2}$$

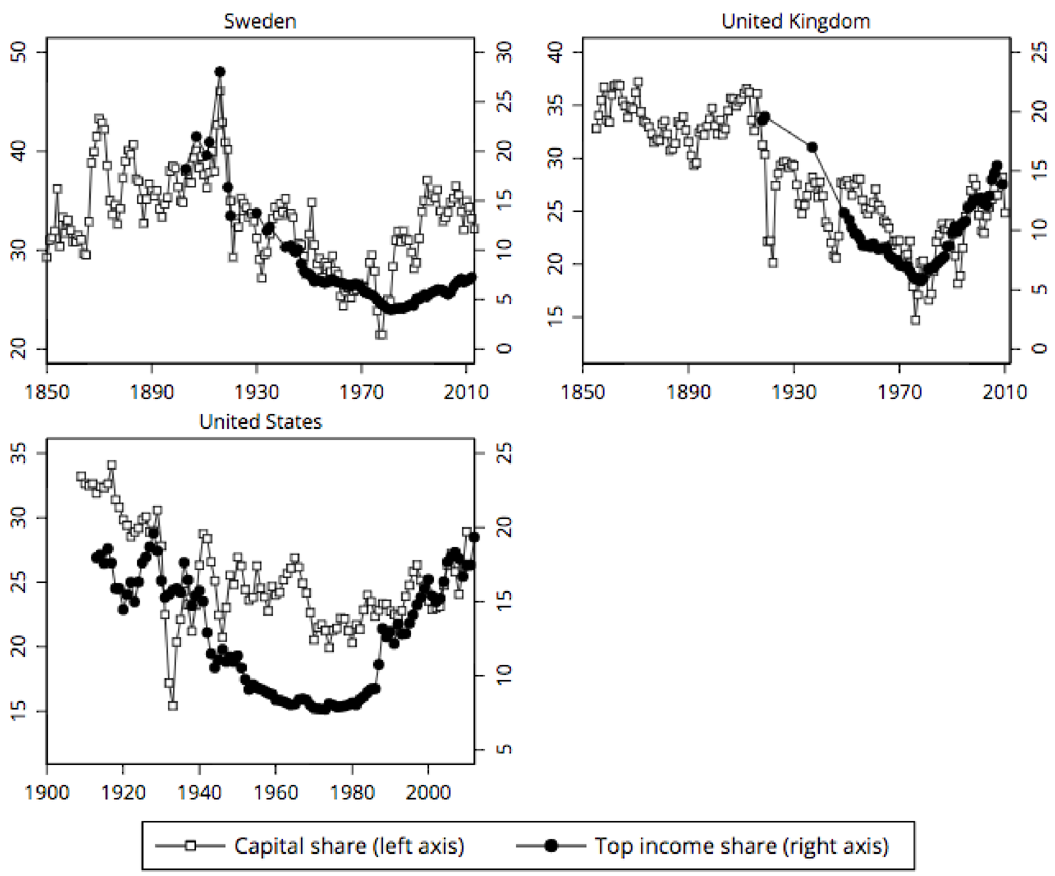
$$Y_s = g(S, A_s) \tag{3.1.3}$$

Though Acemoglu's model explains the reaction of technical progress to shocks in inputs or technology itself, it doesn't go far in fundamentally explaining what caused the transition from growth being unskilled biased to skilled bias. In the model, wages can be represented as simply a function of skilled and unskilled labour (as factor inputs affect incomes both directly and through technology incentives).

As a result, Acemoglu examines the idea of a large exogenous shock in relative supply of skilled labour causing a transition from unskilled biased to skilled biased growth in the 1950s. This could debatably be a result of a change in education policy following WWII. However, there was only a small increase in the relative supply of skilled labour leading up to the 1950s when the skill premium started to rise. Acemoglu's theory tries to explain the persistence in skilled-biased growth, without identifiying the root cause of the transition from one type of growth to the other. As a result there is no ability to predict whether there will be a similar transition back again in the future. The implication of Acemoglu's paper is that the cause of a reverse in growth trend (from unskilled complementing to skilled complementing) would be a large deskilling, possibly caused by the disruption of the Second World War, which would promote unskilled biased technical change. Such a deskilling in the future would be self-defeating in terms of promoting growth, as it would include discouraging building human capital, which is a key source of economic growth. This conclusion can be challenged with a more comprehensive model. Allowing complementarity in observable and tangible inputs that experience exogenous shocks themselves allows a clearer narrative as to what causes these transitions in growth.

Any theory on income shares should be consistent with the observed pattern of labour and capital income shares. In a later paper (Acemoglu (2003)) Acemoglu attempts to explain the pattern of labour share that has been experienced in the long run. Here Acemoglu points to more capital deepening in labour intensive industries. As capital rises capital intensive industries grow faster. However they are gross complements with labour intensive industries, meaning labour intensive industries attract more capital. The outcome of this is the capital share converging downwards (as labour intensive industries grow relative to capital intensive industries) to a terminal level showing consistency with Kaldor's facts (in this case the long run constancy of capital share). This does not explain the more recent persistent rise in capital share, and it does not address the changing structure of capital or what is causing it. Since the 1970s capital share has been constantly increasing in most countries (Bengtsson and Waldenstrom (2015)) as seen in figure 3.1.1. Because capital and labour are gross complements, this could be explained by a decline or slow down of growth in capital per labourer, yet there is no explanation for why capital per labourer would decline, or any evidence of it. Alternatively, Acemoglou's endogenous growth theory says an increase in the productivity of labour increases the demand for the other factor, capital, by more than the demand for labour, effectively creating "excess demand" for capital. As a result, the marginal product of capital increases by more than the marginal product of labour, pushing up capital's share relative to labour's.

Figure 3.1.1: Net Capital Share and Top Percentile Income Share



Top income share is the share of total income of top earners. Copied from Bengtsson and Waldenstrom (2015)

3.2 Observations and Assumptions

There is evidence from manufacturing industries in Chile that the skill complementarity of capital is stronger the more technological the capital is (Correa et al. (2014)). In America, according to data from the Bureau of Economic Analysis, intellectual property has steadily grown from accounting for less than 1 percent of total fixed private assets in the 1920s to around 7 percent today. Similarly, information processing equipment has grown from around 0.5 percent to around 4 percent. Combining these observations implies it is more skill complementing capital that rises faster. While most capital needs some level of a specific skill to operate or utilise it, more technologically advanced capital is likely to require higher levels of skill.

Chapter 2 results imply that skill augmenting technical change grows faster than unskilled augmenting technical change. This is a result of the particular set up in chapter 2, where skill is complemented by the more elastic (and faster growing) input, something that is supported by faster growing technological capital being more skill complementary. This results in an outcome where skilled augmented productivity has to grow faster than unskilled augmented productivity for a steady state between skilled and unskilled income shares, resulting in unbalanced growth. Even outside this set up, there is evidence that productivity growth is increasingly concentrated in high-skill industries in US manufacturing (Kahn and Lim (1998)), which also implies productive growth has been predominantly skilled augmenting.

Whether skill and unskilled workers are complements or substitutes may depend on their exact definition. Generally, previous research has found more and less educated workers are considered substitutes, e.g. Mollick (2011) and Ciccone and Peri (2005). Using production (unskilled) and non-production (skilled) workers for distinguishing between skilled and unskilled workers has historically produced mixed results regarding the substitutability of these inputs (Bachtiar et al. (2015)). This was the definition used in chapter 2 which, using more recent data than other papers that use this definition, found them to be slight complements.

3.3 The Model

3.3.1 The Environment

In this I extend Acemoglu's model on endogenous technical progress Acemoglu (2002) using a production function that uses a similar functional form as in chapter 2 and McAdam et al. (2011) in a neo-classical model. I will use the same theory of factor biased endogenous technical change as Acemoglu, but expand the number of inputs to include complementary capital, which allows complementing capital and technical progress to interact. I do this by splitting capital into capital which complements skilled and unskilled labour. These, including skilled and unskilled labour themselves, mean there is a total of four types of input being used. While capital levels are dictated by returns, I will keep labour levels as exogenous. Not only are changes in types of labour "sticky", in the case where labour augmenting productivity is endogenous and elasticity of substitution is less than one, the labour augmented technical progress acts similarly to a rise in labour itself. This is the case as it turns out that, when the elasticity of substitution is below one, both labour and labour augmented productivity decrease the relative marginal product for that type of labour through making effective labour more abundant. Also 3.3.22 shows that the demand for both labour and the corresponding technical change is dictated by the same variables, the intermediate good demand and the intermediate good price. As such both labour and its augmented technical change have the same directional change to other variables, and have the same direction of impact on relative wages.

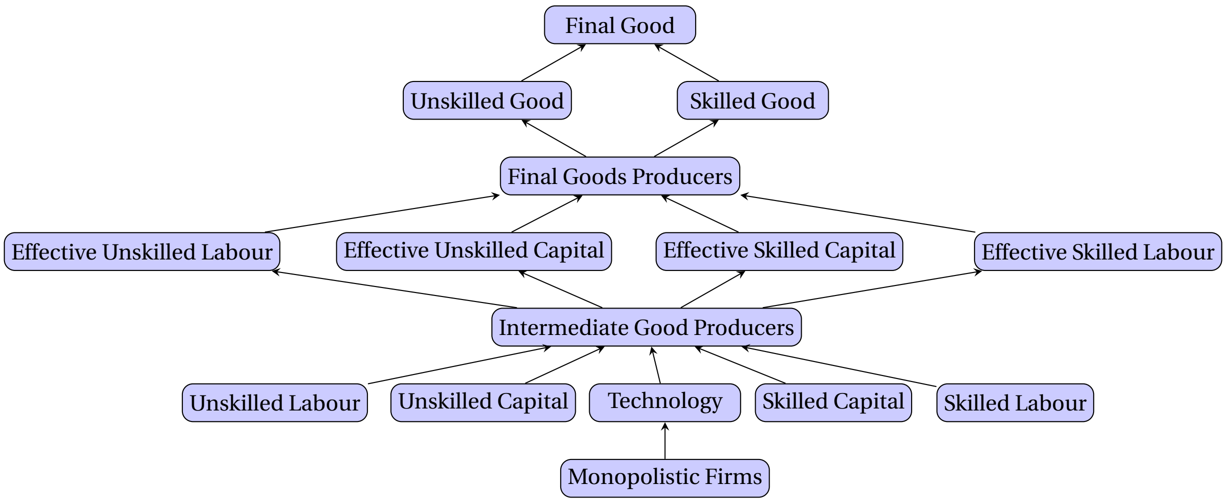
Like Acemoglu, to break down the stages and help with the intuition, I will introduce intermediate producers. Technology is made by monopolistic technology companies, who are free to make technology which augments any of the four inputs. As shown in the diagram below, intermediate producers use these technologies with the four inputs to create the intermediate goods. It is worth noting that these four intermediate goods are the same as the effective values of the inputs, i.e. the combined effect of the input and its augmented productivity, and so would have to grow at the same rate for balanced growth. The first stage for final good producers uses these intermediate goods, combining either effective skilled or unskilled labour, along with the corresponding effective capital that complements that type of labour, to create either a skilled or unskilled good. As a second stage, these final goods producers use these two goods to make the consumption good. A summary of this structure can be seen in 3.3.1.

Consider an economy that admits a representative consumer with the usual constant relative risk aversion preferences:

$$max[\sum_{0}^{\infty}\beta^{t}\frac{c_t^{\lambda-1}-1}{\lambda-1}] \tag{3.3.1}$$

Where β is the time preference and λ is the relative risk aversion. Unlike chapter 2 I do not have to treat prices as exogenous to identify parameters. As a result I can use a full

Figure 3.3.1: Structure Of Supply Of Goods



equilibrium model as opposed to a partial equilibrium one. For this I have introduced consumer preferences. Later this will not be relevant for the equilibrium relative incomes or relative income shares that are of interest. This is because the outcomes that are of interest are all relative levels, and so these consumer demand influences cancel each other out. For example, if the demand has an equal proportionate impact on skilled and unskilled wages, the wage premium itself won't be affected by it, even though each wage separately is. The one place where consumer preferences will have an impact is when there are shocks, as these consumer preferences will decide the speed in which investment and consumption reacts to any shock.

Capital can be split into unskilled complementing capital (K_u, t) and skilled complementing capital (K_s, t). The resource constraint shows output consisting of consumption and the investment in both types of capital.

$$Y_t = C_t + I_{K_u,t} + I_{K_s,t} \tag{3.3.2}$$

$$I_{K_u,t} = P_{K_u,t}(K_{u,t+1} - K_{u,t}(1-d)) \tag{3.3.3}$$

$$I_{K_s,t} = P_{K_s,t}(K_{s,t+1} - K_{s,t}(1-d)) \tag{3.3.4}$$

Where d is capital depreciation, $P_{K_u,t}$ and $P_{K_s,t}$ are the prices of the respective capital compared to the consumption good, which is normalised to one. This gives relatively standard Euler equations which includes investment prices:

$$(\frac{C_{t+1}}{C_t})^{\lambda} = \beta E(\frac{(1-d)P_{K_u,t+1} + r_{u,t+1}}{P_{K_u,t}}) = \beta E(\frac{(1-d)P_{K_s,t+1} + r_{s,t+1}}{P_{K_s,t}}) \tag{3.3.5}$$

Firms are broken down into final goods producers, intermediate good producers and technology producers.

3.3.2 Final Goods Production

Competitive firms making the consumption good from the the unskilled good ($Y_{u,t}$) and skilled good ($Y_{s,t}$) produce output based on a Cobb Douglas production function:

$$Y_t = A_t Y_{u,t}^{\alpha} Y_{s,t}^{1-\alpha} \tag{3.3.6}$$

Final goods producers maximise their profit, which equals zero in the competitive market:

$$Y_t - P_{y_u,t} Y_{u,t} - P_{y_s,t} Y_{s,t} \tag{3.3.7}$$

First order conditions give:

$$P_{y_u,t} = \alpha \frac{Y_t}{Y_{u,t}} \tag{3.3.8}$$

$$P_{y_s,t} = (1-\alpha) \frac{Y_t}{Y_{s,t}} \tag{3.3.9}$$

Integrated in this process, the final goods producers also make the skilled good and unskilled good out of the goods made from intermediate producers (u, s, k_u and k_s). Note it is not necessary for the final goods producers to have two steps to the production process as it may be integrated. This is done purely for simplifying the exposition.

$$Y_{u,t} = [(1-\alpha_u) k_{u,t}^{\gamma} + \alpha_u u_t^{\gamma}]^{\frac{1}{\gamma}} \tag{3.3.10}$$

$$Y_{s,t} = [(1-\alpha_s) k_{s,t}^{\gamma} + \alpha_s s_t^{\gamma}]^{\frac{1}{\gamma}} \tag{3.3.11}$$

Where u, s, k_u and k_s are intermediate goods, which are produced with technology as well as unskilled labour, skilled labour and capital that complements unskilled and skilled labour respectively. The elasticity of substitution between each type of labour and its complementary capital is $\epsilon = \frac{1}{1-\gamma}$ with γ being the substitution parameter. As each capital is complementary to the corresponding labour type, it is assumed that $\epsilon < 1$ meaning $\gamma < 0$. I assume the elasticity of substitution between u and K_u is the same as between s and K_s for simplicity.

Once again firms maximise their profit functions for both goods which equal zero for the representative firm:

$$Y_{u,t}P_{y_u,t} - P_{u,t}u_t - P_{k_u,t}k_{u,t} \tag{3.3.12}$$
$$Y_{s,t}P_{y_s,t} - P_{s,t}s_t - P_{k_s,t}k_{s,t} \tag{3.3.13}$$

First order conditions give:

$$P_{u,t} = \alpha_u(\frac{Y_{u,t}}{u_t})^{\frac{1}{\epsilon}}P_{y_u,t} \tag{3.3.14}$$
$$P_{k_u,t} = (1-\alpha_u)(\frac{Y_{u,t}}{k_{u,t}})^{\frac{1}{\epsilon}}P_{y_u,t} \tag{3.3.15}$$
$$P_{s,t} = \alpha_s(\frac{Y_{s,t}}{s_t})^{\frac{1}{\epsilon}}P_{y_s,t} \tag{3.3.16}$$
$$P_{k_s,t} = (1-\alpha_s)(\frac{Y_{s,t}}{k_{s,t}})^{\frac{1}{\epsilon}}P_{y_s,t} \tag{3.3.17}$$

3.3.3 Intermediate Goods

Unskilled capital, skilled capital and their complementary capitals (U, S, K_u and K_s), are the raw inputs used by intermediate producers. u, s, k_u and k_s are intermediate goods, which can be interpreted as the effective values for each input, meaning they are the raw variables augmented by technology. Technology is determined endogenously from the type and quality of machines supplied by technology monopolists. Firms producing the intermediate good x (u, s, k_u and k_s) which uses the raw input X (U, S, K_u or K_s) experiences the following production function:

$$x_t = \int_0^{A_{x,t}} Z_{x,t}(j)^{1-\theta} dj X_t^{\theta} \tag{3.3.18}$$

Z denotes the machines and A_x is the variety of machines in industry x. The term $\int_0^{A_{x,t}} Z_{x,t}(j)^{1-\theta} dj$ acts as the augmented productivity variable. As this increases when there are more varieties ($A_{x,t}$), the more variety of machines there are the higher the productivity. Machine varieties represent heterogeneous units of innovation that, despite being different, have the same input value. Intermediate good firms have the same production function as in Acemgolu's paper [Acemoglu (2002)]. Firms will maximise the following profit function for each good:

$$x_t P_{x,t} - X_t w_{x,t} - \int_0^{A_{x,t}} R_{x,t}(j) Z_{x,t}(j) dj \tag{3.3.19}$$

Here w is the cost of input X, R_x is the rent on machines in the industry that uses factor input X and A_x is the range of these machines. In the version, which is used in this exercise, where scientists create machines, the machines rents would be paid to the scientists as a wage. Even though this creates another wage, the role of the scientists is simply to create a theoretical framework where technical progress is created by a finite resource. As a result the income of such scientist are not of interest when investigating wage inequality. The first order conditions with respect to machines and input X respectively give:

$$Z_{x,t}(j) = (\frac{(1-\theta)P_{x,t}}{R_{x,t}(j)})^{\frac{1}{\theta}} X_t \tag{3.3.20}$$

$$w_{x,t} = \theta P_{x,t} \frac{\int_0^{A_{x,t}} Z_{x,t}(j)^{1-\theta} dj}{X_t^{1-\theta}} \tag{3.3.21}$$

Substituting 3.3.20 into the production function (3.3.18) for intermediate good producers, and using the constant for rent of technology (shown later- 3.3.27), gives:

$$x_t = (\frac{(1-\theta)^2 P_{x,t}}{\psi})^{\frac{1-\theta}{\theta}} A_{x,t} X_t \tag{3.3.22}$$

Where ψ is the technology producers cost of creating a new machine. In terms of prices:

$$P_{x,t} = \frac{\psi}{(1-\theta)^2} (\frac{x_t}{A_{x,t} X_t})^{\frac{\theta}{1-\theta}} \tag{3.3.23}$$

This shows the price of intermediate goods in terms of the supply of the inputs (technology and input X_t) and the demand for the intermediate good (x_t).

3.3.4 Supply From Technology Producers

Up until now I have outlined what determines the demand for innovation, by calculating how producers in the economy determine the return to different types of innovation. To complete this model I now determine the supply side, or cost, of innovations. This is done through the "innovation possibilities frontier" which was introduced by Kennedy (1964). This captures the trade-off between different types of innovation, which then drives different types of technical progress. In this case, the innovation possibilities frontier specifies how augmenting technical change can be traded off due to resources that are distributed based on varying relative factor inputs. This happens because factor inputs change the profit incentive for technology producers. In my model the innovation possibilities frontier has a level of "state dependence", which is when current R&D directed to a factor inputs technical progress reduces the relative cost of that inputs R&D in the future.

Assuming full depreciation, technology monopolists maximise their profit function for each industry using factor input X, with ψ as their cost for creating a new machine:

$$\pi_{x,t}(j) = max[Z_{x,t}(j)(R_{x,t}(j) - \psi)] \tag{3.3.24}$$

As before, Z_x denotes machines and R_x is the rent on machines going towards the technical progress for input X. As in Acemoglu's 2002 paper, it is assumed that profits and interest rates are expected to be constant in the future in steady state. As a result the total value created is the proportional to the current periods profit, and so the quantity that maximises profit also maximises present value created. Substituting 3.3.20 into the profit function to substitute out $R_{x,t}$ gives:

$$max[(1-\theta)P_{x,t}U_t^{\theta}Z_{x,t}(j)^{1-\theta} - \psi Z_{x,t}(j)] \tag{3.3.25}$$

First order conditions give:

$$P_{x,t} = \frac{\psi}{(1-\theta)^2 X_t^{\theta} Z_{x,t}(j)^{-\theta}} \tag{3.3.26}$$

Combining this with 3.3.20 gives the markups of machine rents:

$$R_{x,t}(j) = \frac{\psi(1-\theta)X_t^{\theta}Z_{x,t}(j)^{-\theta}}{(1-\theta)^2 X_t^{\theta} Z_{x,t}(j)^{-\theta}} = \frac{\psi}{1-\theta} \tag{3.3.27}$$

This shows there are constant mark ups, and so from 3.3.20 a simpler term can be derived for the demand for machines from intermediary goods producers:

$$Z_{x,t}(j) = (\frac{(1-\theta)^2}{\psi})^{\frac{1}{\theta}} P_{x,t}^{\frac{1}{\theta}} X_t \tag{3.3.28}$$

Putting this back into the profit equation (3.3.24) gives the profits of technology producers in terms of prices and factor inputs:

$$\pi_{x,t} = (\frac{(1-\theta)^2}{\psi})^{\frac{1}{\theta}} P_{x,t}^{\frac{1}{\theta}} X_t(\frac{\psi}{1-\theta} - \psi) \tag{3.3.29}$$

$$\pi_{x,t} = \theta(1-\theta)^{\frac{1-\theta}{\theta}} P_{x,t}^{\frac{1}{\theta}} X_t \tag{3.3.30}$$

As technical change will go where there are the most profits, this relationship shows how the direction of technical change responds to market size and price effects. On one hand there will be a greater incentive to invent technologies producing more expensive goods, as shown in the $P_{x,t}^{\frac{1}{\theta}}$ term. At the same time a larger market for the technology leads to more innovation, as shown in the X_t term. Since the market for a technology consists of the input that uses it, the market size effect encourages innovation for the more abundant factor.

In this model I use the more general method for knowledge-based R&D specification, first introduced by Rebelo (1991), where spillovers from past research to grow productivity are used for further growth. Technology itself is grown by scientists, of which there are a fixed amount ($S_t = S_{u,t} + S_{s,t} + S_{k_u,t} + S_{k_s,t}$). As there are scarce factors (scientists) they need to become more and more productive over time. This is the essence of the knowledge-based R&D specification, whereby spillovers imply that current researchers ensure that the marginal productivity of research does not decline by utilising and building on previous research.

Here I introduce state dependence (δ). This can be seen as the opposite of spillover effects. It means that expanding one type of augmented technology is relatively cheaper the higher it is (i.e. technology builds on similar previous technology). There is a variety of specifications with different degrees of state dependence, because productivity in each sector can depend on the state of knowledge in all sectors depending on the level of state dependence. Full state dependence ($\delta = 1$) means the efficiency of growth in technical progress is purely dependent on the level of that specific type of technical progress. Zero sate dependence ($\delta = 0$) means the efficiency of growth in technical progress is dependent on current levels of all types of technical progress:

$$A_{x,t+1} = A_{x,t} + \eta_x A_{x,t}^{\delta} \hat{A}_t^{1-\delta} S_{x,t} \tag{3.3.31}$$

Where $\hat{A}_t$ is the geometric mean of $A_{x,t}$ values:

$$\hat{A}_t = (A_{u,t} A_{s,t} A_{k_u,t} A_{k_s,t})^{\frac{1}{4}} \tag{3.3.32}$$

η_x is a parameter that shows the productivity of scientist growing the technical progress that augments factor input X. With complete state dependence, the growth of one type of augmented technology does not benefit the other. Complete state dependence is an unrealistic assumption as one would assume some spill over effects or cross benefits of technology growth in terms of augmenting inputs. Acemoglu himself admits the evidence suggests there is limited state dependence.

For the market clearing condition all scientist have to be be paid the same, and so their marginal product must equate, so comparing skilled and unskilled labour augmenting technical progress:

$$\eta_u A_{u,t}^{\delta} \hat{A}_t^{1-\delta} \pi_{u,t} = \eta_s A_{s,t}^{\delta} \hat{A}_t^{1-\delta} \pi_{s,t} \tag{3.3.33}$$

$$\eta_u A_{u,t}^{\delta} \pi_{u,t} = \eta_s A_{s,t}^{\delta} \pi_{s,t} \tag{3.3.34}$$

Inserting the formula for profits in terms of price and quantities (3.3.30) gives the necessary condition for technology producers to have equal profits:

$$\eta_u A_{u,t}^{\delta} P_{u,t}^{\frac{1}{\theta}} U_t = \eta_s A_{s,t}^{\delta} P_{s,t}^{\frac{1}{\theta}} S_t \tag{3.3.35}$$

$$\frac{P_{s,t}}{P_{u,t}} = (\frac{\eta_u}{\eta_s} \frac{U_t}{S_t} (\frac{A_{u,t}}{A_{s,t}})^{\delta})^{\theta} \tag{3.3.36}$$

By symmetry the following conditions apply between labour and capital:

$$\frac{P_{k_u,t}}{P_{u,t}} = (\frac{\eta_u}{\eta_{k_u}} \frac{U_t}{K_{u,t}} (\frac{A_{u,t}}{A_{k_u,t}})^{\delta})^{\theta} \tag{3.3.37}$$

$$\frac{P_{k_s,t}}{P_{s,t}} = (\frac{\eta_s}{\eta_{k_s}} \frac{S_t}{K_{s,t}} (\frac{A_{s,t}}{A_{k_s,t}})^{\delta})^{\theta} \tag{3.3.38}$$

These show how the supply of each input and corresponding technology affect relative intermediate good prices. These are later used for determining incomes and income shares, as it dictates technical progress based on factor inputs (and prices which are in turn based on factor inputs).

It can be seen without any state dependence ($\delta = 0$) technology has no impact on these prices. In Acemoglu's 2002 paper he experiments with full state dependence, which means technology in one sector builds on technology only in that sector. If there is full state dependence ($\delta = 1$) then income shares are constant along the innovation possibility frontier (i.e. the steady state possibilities that result from exogenous changes in inputs). Such an outcome can be seen in 3.6.1 as if $\delta = 1$, factor inputs have no impact on relative incomes shares. This result occurs because, in the case of full state dependence, the marginal product of scientists goes up linearly with the increase in machine varieties (as can be seen if $\delta = 1$ in 3.3.31). This results in relative technical progress going up exactly inversely to relative factor input levels (this can be seen if $\delta = 1$ in .A.11 of Appendix .A), creating constant effective input levels and so constant incomes shares. However, as mentioned before, evidence suggests there is limited state dependence based on research on patent data (Trajtenberg et al. (1992)). This does go to show the importance of how technology is created when discussing incomes shares, and how changes in δ can lead to different observed outcomes over different time periods. Acemoglu (2002) even shows that when state dependence is high, the model becomes less stable in response to changes in factor inputs when elasticities are greater than one. Though I find the opposite is true when the elasticity of substitution is less than one. This is because a higher state dependence leads to a greater response to change in factor inputs, meaning countervailing productivity responses to factor input changes are more prominent.

3.4 Steady State Outcomes

3.4.1 The Innovation Possibility Frontier

As with the initial model, I will first show the relationships between the inputs and the resulting relative augmented technology growth. This shows how exogenous changes or shocks to factor inputs may impact the direction of technical progress, which is important to understanding secondary (outside of their direct effects) effects of factor input levels on incomes shares, something that isn't captured in a model that doesn't include endogenous technical change. The relative technology growth between unskilled labour and its complementary capital is (.A.11 of appendix .A):

$$(\frac{A_{u,t}}{A_{k_u,t}})^{1-\delta\sigma} = (\frac{\alpha_u}{1-\alpha_u})^{\epsilon}(\frac{\eta_u}{\eta_{k_u}})^{\sigma}(\frac{K_{u,t}}{U_t})^{1-\sigma} \tag{3.4.1}$$

Here I introduce a new variable σ where $\sigma = 1+\theta(\epsilon-1)$. As defined earlier, ϵ is the elasticity of substitution and θ is the elasticity parameter for each factor input compared to its corresponding technical progress. It is worth noting that if ϵ is less than one then so is σ and if ϵ is more than one then so is σ. As θ is less than one, σ is closer to one than ϵ.

And by symmetry:

$$(\frac{A_{s,t}}{A_{k_s,t}})^{1-\delta\sigma} = (\frac{\alpha_s}{1-\alpha_s})^{\epsilon}(\frac{\eta_s}{\eta_{k_s}})^{\sigma}(\frac{K_{s,t}}{S_t})^{1-\sigma} \tag{3.4.2}$$

As can be seen, as $\sigma < 1$ (as a result of $\epsilon < 1$, i.e. capital and labour are gross complements) an increase in one factor would induce an increase in the other factor's augmented technical progress. Finding the relative skilled versus unskilled technology growth is more complicated as it is also effected by the relative prices of secondary intermediate goods. The relative technologies are as follows (proof in appendix .C):

$$\frac{A_{u,t}}{A_{s,t}}^{1-\delta} = \frac{\alpha}{(1-\alpha)}\frac{\eta_u}{\eta_s}\frac{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1} \tag{3.4.3}$$

From these outcomes it can be seen that relative unskilled labour augmenting technology increases if relative relative unskilled capital increases, as this increases the marginal benefit of effective unskilled labour relative to effective skilled labour. Relative unskilled labour augmenting technology also increases if unskilled labour decreases relative to skilled. This is because declining labour levels decreases effective labour levels, once again increasing the marginal benefit of effective unskilled labour relative to effective skilled labour. These effects are such that if capital to labour ratios remain constant then relative machine varieties levels (and so relative productivity levels) remain constant.

In this theory I assume $\epsilon < 1$. This is because both skilled and unskilled labour are reliant on their specific type of capital. Here I show how technology will change based on inputs in this model depending on specific input intensity.

From this it is clear the higher the level of state dependence the more reactive technical change is to a change in factor inputs. In the event of full state dependence there is no natural equilibrium for technology producers from .A.11, as technology producers maximisation outcome does not involve machine variety levels. However full state dependence does result in constant factor input income shares. This can be shown between labour and capital from .A.18 (if δ is set to 1) as a result of the condition in .A.11. .A.11 results in the share of income with the skilled (or unskilled) industries being the same, and these industries themselves have constant income share due to being Cobb Douglas, income shares between labour types are always the same. This constant relative incomes of labour is shown when δ is equal to 1 in the in .B.17, which shows the derived relative income going to skilled and unskilled workers as a function of factor inputs. The constant constant income shares with full state dependence comes about as the elasticity of substitution is less than one in this model. This means relative technical change, which is higher with full state dependence, counters the income effects of changes in factor inputs.

The outcome of zero state dependence ($\delta = 0$) does not change the mechanics of the model (compared to some state dependence) and, as mentioned earlier, is closer to reality. This results in the following innovation possibility frontier:

$$\frac{A_{u,t}}{A_{s,t}} = \frac{\alpha}{(1-\alpha)}\frac{\eta_u}{\eta_s}\frac{(\frac{1-\alpha_s}{\alpha_s})^{\epsilon}(\frac{\eta_s}{\eta_{k_s}})^{1-\sigma}(\frac{S_t}{K_{s,t}})^{1-\sigma}+1}{(\frac{1-\alpha_u}{\alpha_u})^{\epsilon}(\frac{\eta_u}{\eta_{k_u}})^{1-\sigma}(\frac{U_t}{K_{u,t}})^{1-\sigma}+1} \tag{3.4.4}$$

In this case, similar .A.11, relative technical change is less reactive to factor inputs with lower state dependence, as the cost of growing technologies all change at the same rate. As the elasticity of substitution is less than one in this model, relative technical change counters the income effects of changes in factor inputs. This means a lower state dependence allows more variation in income levels.

3.4.2 Wages

In Acemoglu's model (Acemoglu (2002)), factor inputs are exogenous. From this came a measure of different steady state incomes from different input levels. In my model, capital is endogenous, yet labour supply is still exogenous. This once again allows for different steady states depending on the ratio of exogenous inputs. While this observation is more of an area of focus Acemoglu's paper than in this exercise, it is worth mentioning how the outcomes compare.

In Acemoglu's paper, the steady state relationship between skilled and unskilled labour behaves as one would expect without exogenous technical progress. If the elasticity of substitution (ϵ) is less than one, then an exogenous increase in skill would decrease the skilled share of wages. However, if the elasticity of substitution is more than one then an increase in skilled labour increases the skilled share of wages. What does come uniquely out of endogenous technical change is that it is possible for an increase in skilled labour to increase the skill premium. This outcome would require a very high elasticity of substitution (one that is well over two). Obviously elasticity of substitution estimates between skill and unskilled workers vary, and this scenario is the case according to some papers but not others, e.g. Mollick (2011) estimate the elasticity of substitution of between 2 and 3.21 whereas Ciccone and Peri (2005) have an estimate of around 1.5 in the US. As a result, to explain the rising skill premium, Acemoglu's paper may still require a shock as well as the skilled vs unskilled labour input ratio.

Due to the added complexity of this model, caused by having more inputs, it isn't as easy to achieve a clear linear relationship between labour inputs and wages alone. However in appendix .D I have shown a relationship that dictates the behaviour. If it is assumed that there is a constant required rate of return for both skilled and unskilled capital, then $r_{s,t}$ and $r_{u,t}$ can be treated as constant in steady state. As a result skilled wages and skill share of wages show a positive relationship, if the elasticity of substitution between capital and labour (ϵ) is less than one, and a negative relationship if it is more than one. This shows that U and S behave like complements if $\epsilon < 1$ and like substitutes if $\epsilon > 1$.

3.4.3 Capital Rents

Apart from technical progress the main endogenous variable is capital. To understand how this model responds to shocks I will first establish how capital accumulates. From 3.3.5, which is the result of the agents optimisation of welfare where net returns are the same for both types of capital, and assuming full depreciation for simplicity:

$$\frac{E(r_{u,t+1})}{P_{K_u,t}} = \frac{E(r_{s,t+1})}{P_{K_s,t}} \tag{3.4.5}$$

In equilibrium, expectations of returns and actual returns equate and remain at $\bar{r}_u$ and $\bar{r}_s$. Prices remain constant at $\bar{P}_{K_u}$ and $\bar{P}_{K_s}$ giving:

$$\frac{\bar{r}_u}{\bar{r}_s} = \frac{\bar{P}_{K_u}}{\bar{P}_{K_s}} \tag{3.4.6}$$

Capital is completely elastic (however I will discuss what would be observed if the opposite is true and capital is inelastic). That means that price of capital is not affected by capital levels, and so relative capital prices are exogenously defined as:

$$ln(\frac{P_{K_u,t}}{P_{K_s,t}}) = ln(\frac{P_{K_u,t-1}}{P_{K_s,t-1}}) + e_{p,t} \tag{3.4.7}$$

Where $e_{p,t}$ is an i.i.d. This means equilibrium relative rents are exogenous.

3.5 EXOGENOUS SHOCK OUTCOMES

In this section I will look at the relative observable outcomes of each type of permanent shock. As the shocks are permanent, each finds a new steady state, using a new steady state for relative technical progress in accordance with the previous section. In doing this I can establish what type of exogenous shock is consistent with the observed real world outcomes. The outcomes I will be looking for consistency with are the relative levels of capital and relative wages in response to each type of shock. As has been discussed, the has been a relative rise in both skilled labour and skilled capital.

3.5.1 CAPITAL RESPONSE TO TECHNOLOGY SHOCKS

In this set up there are 4 different shocks to technical progress, each augmenting one of the 4 different inputs. What type of shock is experienced when there is an advancement in technology or productivity is convoluted as it depends on how the different inputs utilise the technology being developed. A typical example shock in technical progress would be an advancement in the use of the internet or computing. But whether this augments skilled labour, unskilled labour or a subset of capital depends on the micro interactions between these inputs and the new technology. As such, the best guide for the type of technology shock that has occured may be the observed outcomes on a macro level. In this section, identifying which type of shock may have been experienced is not necessary. This is because the more pressing question is whether wage inequality growth is likely the result of any type of technology shock and, if so, the mechanism by which it has caused a large rise in skill premium.

To investigate what happens when there is a shock to technical progress, I will first treat the relative η values, which represent the efficiency of different types of technical growth, as being prone to one off exogenous shocks. As such:

$$\frac{\eta_{k_u}}{\eta_u} = \frac{\eta_{k_{u,t}}}{\eta_{u,t}} = \frac{\eta_{k_{u,t-1}}}{\eta_{u,t-1}} + e_{\eta_u,t} \tag{3.5.1}$$

$$\frac{\eta_{k_s}}{\eta_s} = \frac{\eta_{k_{s,t}}}{\eta_{s,t}} = \frac{\eta_{k_{s,t-1}}}{\eta_{s,t-1}} + e_{\eta_s,t} \tag{3.5.2}$$

Where $e_{\eta_u,t}$ and $e_{\eta_s,t}$ are i.i.d's. Mirroring the relative income as a function of factor inputs (derived in Appendix .B), relative rent based on factor inputs is given as:

$$\frac{r_{s,t}}{r_{u,t}} = \frac{K_{u,t}}{K_{s,t}} \frac{(1-\alpha)}{\alpha} \frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_{u,t}}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_{s,t}}}{\eta_{s,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{3.5.3}$$

In this equation the $\frac{K_{u,t}}{K_{s,t}}$ term on the right shows the direct effect that a relative abundance in each type of capital has on relative rents. The longer (non-linear) fraction to the right of this equation shows how each type of capital is additionally impacted by its dependancy on complementary labour. The latter magnifies the impact of scarcity or abundance of each type of capital.

Differentiating 3.5.3 alone to find the change in relative capital during a technology growth shock doesn't give a definitive response to shocks (in terms whether the differential is always negative or positive). This is due to the capital over labour terms on the right side making it not just the relative values of capital that matter, but also the nominal values, meaning the solution is still unspecified from the relationship shown in 3.5.3. To comprehensively show what happens when there is a change in relative growth parameters such as $\frac{\eta_{k_u}}{\eta_u}$, I will assume adjustments in capital happen in two stages. In the first stage rents (and so returns) change as capital is initially inelastic. However in the long run, without an exogenous change in prices, 3.4.6 shows that relative returns go back to the same ratio. This happen as capital adjusts so that the net returns for the two types of capital equate. For the first stage, with capital being inelastic and so fixed and not affected by changes in rent or any η value, then it is clear from 3.5.3 that the relative rents follows the following rule:

$$\frac{d(\frac{r_{s,t}}{r_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})}|\bar{K}_{u,t},\bar{K}_{s,t} > 0 \tag{3.5.4}$$

In the second stage increased relative rents, causing higher returns, in one industry causes relative capital to increase to that industry, which logically follows the agent's optimisation decisions. As a result of this a relative rise in capital, corresponding rents causes a relative decline in skilled capital until returns equate again.

So for returns to equate again, $\frac{K_{s,t}}{K_{u,t}}$ must increase when $\frac{r_{s,t}}{r_{u,t}}$ increases, and vice versa. As a result the following ends up always being true:

$$\frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{r_{s,t}}{r_{u,t}})} > 0 \tag{3.5.5}$$

From 3.5.4 and 3.5.5 it follows that:

$$\frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} > 0 \tag{3.5.6}$$

Mirroring 3.5.6 for a shock in skilled industries:

$$\frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{k_{s,t}}}{\eta_{s,t}})} < 0 \tag{3.5.7}$$

These results are quite intuitive and show that if there is a positive shock in the technical progress of a certain type of capital, then the relative level of that capital won't increase as much as the other type of capital, due to it being less needed. It also shows that if there is a positive shock for a specific type of labour, then the relative level of capital in the same industry increases due to it complementing a more abundant effective resource.

From these outcomes, the types of shocks that could create an increase in the relative skilled capital levels, as has been experienced over the last half century, are rises in $\eta_{k_{u,t}}$ or $\eta_{s,t}$ or declines in $\eta_{k_{s,t}}$ or $\eta_{u,t}$.

3.5.2 Wage Response To Technology Shocks

In this section I will investigate the response of relative wages to a shock in technical progress. From .B.17 in appendix .B, relative wage based on factor inputs is given as:

$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\alpha)}{\alpha}\frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{u,t}}{\eta_{k_{u,t}}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{s,t}}{\eta_{k_{s,t}}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}\frac{U_t}{S_t} \tag{3.5.8}$$

In order to know what happens to relative wages following a shock to technical progress, it is important to first understand the different parts that move following the shock, in terms of the direct effects of technical change, and then the reaction of capital. It is clear from 3.5.8 that if there was a shock in relative technical progress within unskilled industries, such that $\frac{\eta_{u,t}}{\eta_{k_{u,t}}}$ increases, then this causes a direct rise in the wage premium $(\frac{w_{s,t}}{w_{u,t}})$, if we exclude any change in capital (if capital was inelastic for example). This can also be seen by combining .A.17 for skilled and unskilled industries gives the following term for skill premium, keeping in mind relative renta end up being constant assuming no shock in capital prices:

$$\frac{w_{s,t}}{w_{u,t}} = \frac{r_{s,t}}{r_{u,t}}(\frac{\alpha_s(1-\alpha_u)}{(1-\alpha_s)\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_{s,t}}\eta_{u,t}}{\eta_{s,t}\eta_{k_{u,t}}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_tK_{s,t}}{S_tK_{u,t}})^{\frac{2-\sigma-\delta}{1-\delta\sigma}} \tag{3.5.9}$$

The positive relationship between wage premium and unskilled labour augmenting technical progress is due to this type of technical progress creating a higher relative abundance of the unskilled labour product (u_t) decreasing its value relative to the skilled skilled product (s_t). The positive relationship between wage premium and unskilled capital augmenting technical progress is due to this type of technical progress augmenting a type of labour that specifically complements unskilled labour.

This shows for inelastic capital $\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} < 0$, and mirroring this $\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{k_{s,t}}}{\eta_{s,t}})} > 0$. The opposit happens to $\frac{K_{s,t}}{K_{u,t}}$ in response to shocks, as shown in the previous section. This means one would expect, if wage premium have increased due to one of these shocks that the relative level of skilled capital would decrease. However, as will be discussed later, wage premiums and relative levels of skill complementing capital have both increased over the last half century. One explanation for this might be that the wage premium has actually risen due to a shock in the relative price of capital, increasing $\frac{K_{s,t}}{K_{u,t}}$. This will be discussed later. Alternatively, if technology shocks are responsible, the direct cause of the rise in wage premium would have to be due to capitals response to the technology shock, which is only possible in certain circumstances as I will discuss now.

On top of these direct effects, these shocks also cause a countervailing relative change in skilled capital ($\frac{K_{s,t}}{K_{u,t}}$) which will move the wage premium in the other direction to the initial direct effect of the shock. This response in capital counters the effect of the initial shock in technical progress. However, the resulting change in $\frac{w_{s,t}}{w_{u,t}}$ will depend whether capital changes substantially enough to cause an increase in wage premium, or if the wage premium will still decline. Working out the net effect on wages of changes in relative efficiency of technical progress ($\frac{\eta_{u,t}}{\eta_{k_{u},t}}$) is more complicated.

I first note that, from the differential of wage premium, $\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})}$ is larger than zero if the following is true (proof: appendix .F):

$$\frac{\eta_u}{\eta_{k_u}} + (1-\delta)\frac{U_t}{K_{u,t}}\frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u,t}}{\eta_{u,t}})} - (1-\delta)\frac{S_t}{K_{s,t}}\frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u,t}}{\eta_{u,t}})}\left[\frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_u,t}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_s,t}}{\eta_{s,t}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}\right] > 0 \tag{3.5.10}$$

The first term ($\frac{\eta_u}{\eta_{k_u}}$) comes from the direct effect of a technical progress shock on relative wages. The other two terms come from the changes in the two types of capital, the first of these other terms showing the impact of unskilled capital ($K_{u,t}$) and the last term the effect of skilled capital ($K_{s,t}$). It is worth pointing out that both types of capital generally grow in response to a positive capital shock and decline in response to a negative shock. As such $\frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u}}{\eta_u})}$ and $\frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u}}{\eta_u})}$ will either both be minus or both be positive.

Similarly, from the differential of capital, appendix .E shows it is also known that, if $\frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{s,t}}{\eta_{k_s,t}})} > 0$ (which I have previously shown is always the case), the following term must be larger than zero for every possible parameter and exogenous variable combination:

$$\frac{\eta_{u,t}}{\eta_{k_u,t}} + (1-\delta)\frac{U_t}{K_{u,t}}\frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u,t}}{\eta_{u,t}})} - (1-\delta)\frac{S_t}{K_{s,t}}\frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u,t}}{\eta_{u,t}})}\left[\frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{u,t}}{\eta_{k_u,t}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}\right] > 0 \tag{3.5.11}$$

Here it is noted that 3.5.11 and 3.5.10 are similar as only the term in the square brackets differ. While we know 3.5.11 is always true, if 3.5.10 is also true then the condition $\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} > 0$ holds.

It is clear to see that the terms in brackets in 3.5.11 and 3.5.10 are equal if:

$$(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_u,t}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{(\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} = (\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_s,t}}{\eta_{s,t}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{s,t}}{S_t})^{(\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} \quad (3.5.12)$$

Or, using 3.5.9, this is when:

$$\frac{S_t w_{s,t}}{K_{s,t} r_{s,t}} = \frac{U_t w_{u,t}}{K_{u,t} r_{u,t}} \quad (3.5.13)$$

This reflects that when labour and capital shares of income are the same within both skilled and unskilled sectors, capital will not completely counteract the effect of a shock in technical progress. However, when this balance significantly changes, from either skilled or unskilled capital becoming particularly scarce or abundant, it is theoretically possible that a capital response reverses the effect of a technology change.

The environment where capital can reverse the effect of a technical change on relative wages is not only based on a relative scarcity or abundance of one type of capital, but other characteristics make this more or less likely the case. Firstly, with full state dependance ($\delta = 1$), as can be seen in 3.5.8, wage premium is dictated by η values, i.e. by technical progress, but not capital, as the capital terms are to the power of 0 when $\delta = 1$. This is also seen in the differentials 3.5.11 and 3.5.10. In this case capital still changes in response to a shock, however the effect of capital movements are counteracted by changes in technical progress. The lower state dependance is the less technology counteracts a change in capital and the more likely it is the capital will have a significant effect on outcomes. As discussed elsewhere, full state dependance is unlikely, however at the same time there is likely a small level of state dependance according to Acemoglu et al. (2001).

Another factor that impacts the likeliness of capital completely counteracting the impact of technical change on relative wages is the elasticity of capital. I have assumed here that capital is completely elastic, however if capital has less freedom to react, it would obviously make it less likely to counteract and reverse the effect of technology change.

Overall there is very specific circumstances which cause an environment where capital will respond to a shock in technical progress to an extent that reverses its effect. What makes it less likely is that $\frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u}}{\eta_u})}$ and $\frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u}}{\eta_u})}$, in 3.5.10, will either both be minus or both be positive, as either both capitals go up or both go down depending on whether a positive or negative shock takes place. As they have opposing effects on relative wages, the capital the capital that counteract the direct effect of a shock to technical change, will have to also fully counter the effect of the change in the other type of capital. And so, though this is theoretically possible, it would seem an unlikely explanation for a large change in skill premium as has been experienced. If it is the case, then wage premium growth could be initiated from a shock in technical progress, but will directly be the result of capital changes and an initial scarcity or abundance in one type of capital. If this was the case, the role capital plays in wage inequality is pivotal, as it is capital that would directly drive wage inequality growth.

3.5.3 RESPONSE TO PRICE CHANGES

The alternative to a shock in technical progress is a shock to relative capital prices as in 3.4.7. An example of such a shock may be seen with the oil shocks in the 1970s, which had a dramatic effect on prices of energy, and which arguably had knock on effects to other prices . Demand from other nations can also influence prices. Since the 1990s China has gone through a boom in infrastructure spending. This has pushed up the relative price of natural resources like Iron ore, Copper and (once again) energy. This demand shift in a major, and growing, economy like China impacts the supply experienced by other nations for these inputs.

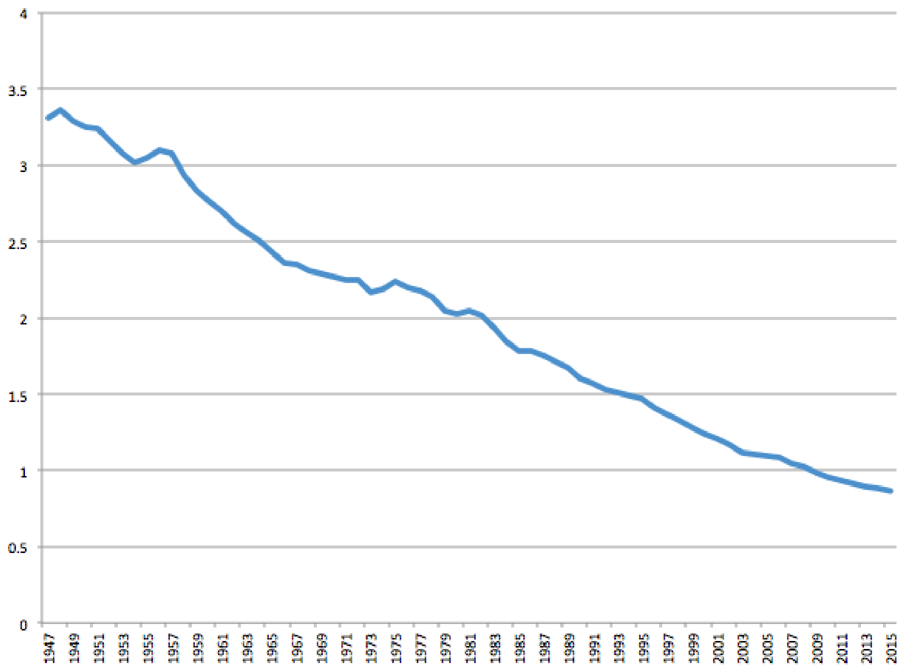
The outcome to this is much simpler and intuitive compared to shocks in technical progress. 3.4.6 shows how this then changes relative rents. If the relative price of a certain type of capital increases, the relative returns to that type of capital decreases, so relative capital decreases as agents can get better returns in the other type of capital. This happens until the relative rent increases to the point that returns equate again. The change in capital then has consequences regarding wage premium as show in 3.5.8.

This gives a very simple outcome that is consistent with observed outcomes. The relative price of unskilled capital increasing would cause an increase in relative skilled capital, which would then increase the skill premium. This is not only a very plausible explanation, but is consistent with the increase in the price of energy in the 1970's, which in chapter 2 I show complements unskilled labour. It is also consistent with the decreasing price of mainly technology based capital, which complements skilled labour, as is discussed below.

3.6 INVESTMENT PRICES AND TERMINAL OUTCOME

It is clear from the continuing decline in relative investment prices shown in figure 3.6.1, that part of the productive growth experienced is in the form of decreasing investment prices, creating capital deepening. There are similarities between a decreasing price of investment and capital augmented technical progress, in that both impact wage premium through an increase in effective capital. The key difference is that a decline in investment prices causes capital levels to rise, whereas a rise in capital augmenting technical progress would cause relative capital to fall.

Figure 3.6.1: Figure 5: log of US Real Investment Prices



DiCecio, Riccardo, Relative Price of Investment Goods [PIRIC], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PIRIC, August 10, 2020.

On top of this Greenwood et al. (1997) estimate that 60 percent of postwar productivity growth can be attributed to investment-specific technology change, which creates capital deepening. The significance of this is that shocks in investments prices (which reflect general capital prices) are a key driver in structural changes in the economy.

From .B.17 in appendix .B, the relative labour shares can be expressed in terms of factor inputs:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{3.6.1}$$

In a simpler form this can be written as:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = b_0 \frac{b_1(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{b_2(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{3.6.2}$$

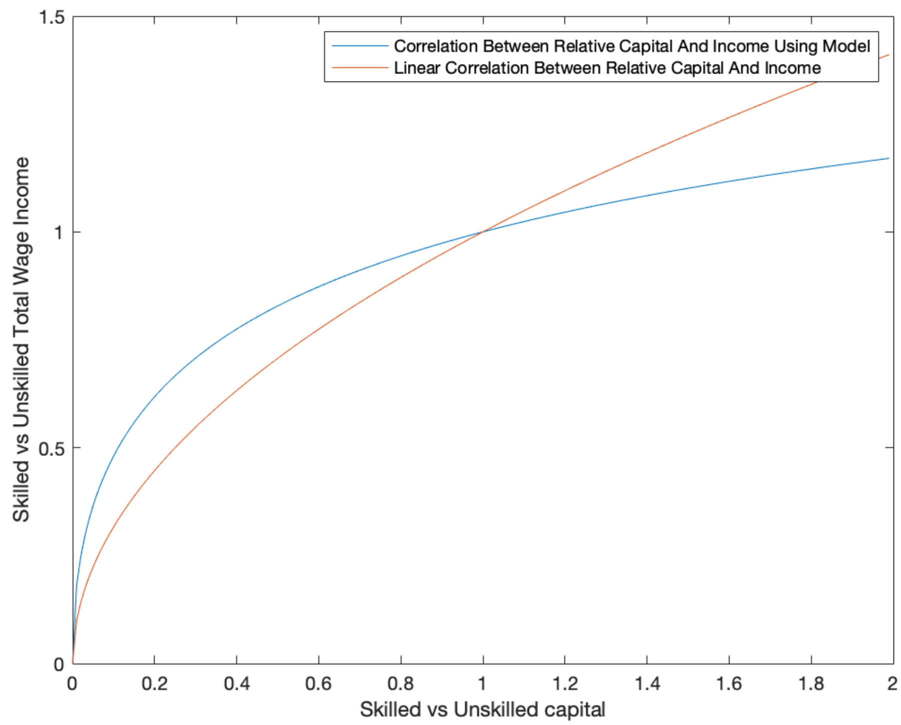
It is a stylised fact that a decrease in investment prices encourages capital deepening, increasing the quantity of the type of capital that experiences the price decline. As expected, with an elasticity of substitution between capital and labour less than one, increasing $K_{s,t}$ would increase the income share of skilled labour.

3.6.2 shows the relationship between relative capital and relative labour shares is non linear. Compared to a linear relationship between capital and income shares, this results in capital deepening in K_s, with other variables not changing much, having a very steep impact when levels are low, though much less as it grows. 3.6.2 shows that as K_s grows, and the other variables remain fairly constant, the denominator goes towards one. The resulting secular outcome of the relative incomes is given by the remunerator:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} \longrightarrow b_0[b_1(\frac{U_t}{K_{u,t}})^{1-\sigma} + 1] \tag{3.6.3}$$

Treating labour and K_u as inelastic, figure 3.6.2 shows how income shares change as the relative capital levels change. I compare this to a linear relationship given by $b_0(\frac{U_t K_{s,t}}{S_t K_{u,t}})^{1-\sigma}$. The accelerated levelling off shows how the terminal outcome that results in stable income shares despite K_s increasing faster than K_u, as the relative income share moves towards this ceiling. The share of income going to skilled labour (as well as other inputs) has a ceiling as skilled labour is paid from the income going to $Y_{s,t}$, which itself is limited due to the demand for $Y_{s,t}$ being Cobb Douglas. This reflects skilled labour being limited in total income as there are certain job types or sectors skilled labour is used for, where as other job types will used lower paid workers.

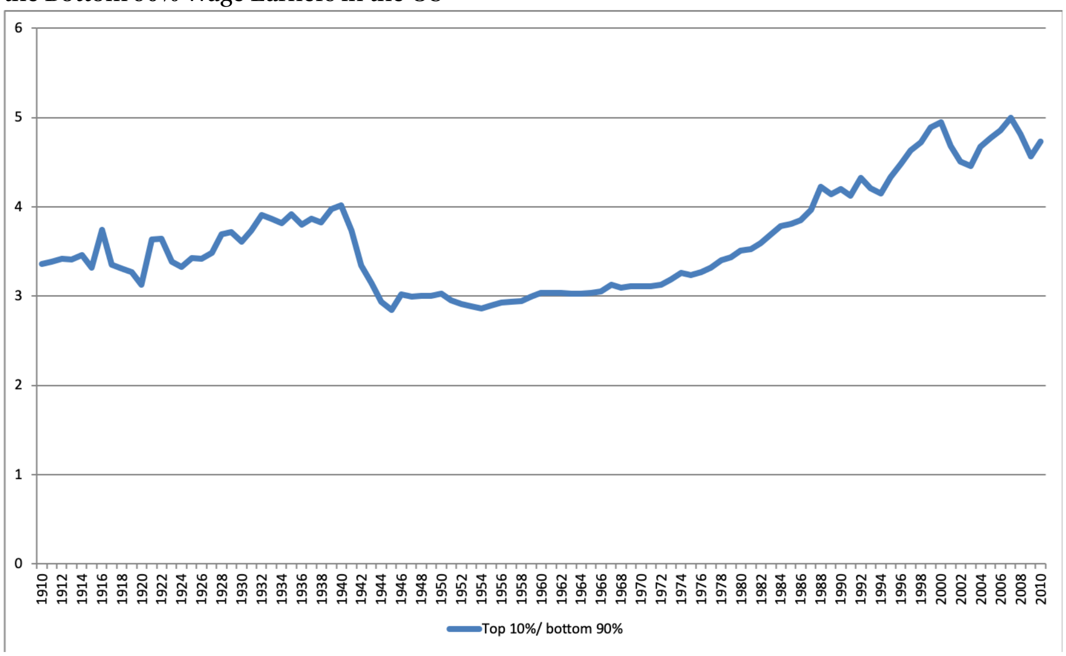
Figure 3.6.2: Growing Relative Skilled Capital Effect On Wage Shares



3.7 Conclusion

In the US the wage inequality was lowest during the first 25 years of the post-war era. In this period the wage of the top ten percent of earners was about three times that of the bottom ninety percent. This was lower than before the World War, where this ratio was between three and four, representing slightly higher wage inequality. As seen in figure 3.7.1, more recently the top ten percent have earned around five times more than other workers. This represents the highest wage inequality for the last 100 years, when the IRS data began.

Figure 3.7.1: Ratio of the Average Wage of the Top 10% Wage Earners vs Average Wage of the Bottom 90% Wage Earners in the US



IRS returns as reported by https://wid.world/ in 2018

As I argue in chapter one, the increased disparity in income within countries is creating growing social discontent and frictions. Frank (2005) shows that welfare is dependant on relative as well as total consumption. This in turn makes its way through to taxes and interventionist policies as politicians come under pressure to redistribute wealth as predicted by median voter theory. These policies are not only temporary expedients, and so do not deal with the secular trend, but they are also distortive. These taxes, etc, which are used to redistribute wealth, reduce economic growth (Alesina and Rodrik (1994), Persson and Tabellini (1994)).

The outcomes in this chapter indicate that this recent rise in wage inequality has likely been driven by exogenous shocks in the supply of factor inputs rather than factor productivity shocks. The only possible exception to this being the case is if a technical shock that directly decreases wage inequality induces a large countervailing factor input response that then increases wage inequality. In either case the relative level of factor inputs is the driver of the inequality growth experienced.

Levels of factor inputs are endogenous. However, exogenous shocks can be caused by supply side distribution, which includes factor input supply shocks as well as productive shocks. In this paper I have investigated both possibilities, and found that shocks in the supply of factor inputs is more consistent with observed outcomes. As a result, the key driver of wage inequality is based on measurable variables, as opposed to being dependant on unobservable efficiency units, with the energy intensity of capital being a strong exogenous indicator of skill premium. This further allows for a clear understanding of what shocks or structural changes may increase or decrease wage inequality (e.g. energy shocks or efficiency of productive growth). This in turn helps forecast wage inequality after price changes take place.

The econometric results in chapter 2 supports the theory that energy supply can form part of this narrative, as it shows signs of complementing lower skilled workers more than skilled workers in the US. This is consistent with the timing of the rise in wage inequality, which accelerated shortly after the two oil shocks in the 1970s. As shown in chapter 2, the trends in energy intensity of capital support this theory going back to the start of the 1900s. Up until World War 2 energy intensity of capital was falling and wage inequality was slowly rising. After World War 2, energy intensity sharply rose and the wage inequality sharply fell. This all implies that energy usage in production is linked with lower wage inequality. At the moment it can be argued that we are at the early stages of another energy revolution (Pearson and Foxon (2012)) as the threat of climate change has created a renewed push for clean energy, driving a technology push that may see lower energy prices. And so my thesis outlines another potentially very desirable outcome of such a revolution, in that the politically sensitive rises in inequality over the last few decades is a trend that may reverse.

The steady increase in skill premium outside of World War 2 (even outside of the sharp rises since the 1970s it has slowly increased) is consistent with skill complementing capital coupled with capital deepening in the model. This is consistent with the trend of structural transformations moving towards service industries and away from manufacturing and agriculture, which has been persistent over the last century. This research has concentrated more on the rise of "white collar" workers over "blue collar" workers to capture the effect that is largely seen within industries. The Second World War caused a one-off structural transformation towards industry, as manufacturing, construction and mining went from 28 percent of output in 1935 to 38 percent in 1950, according to Census Bureau data[1]. This was against the long term trend that had started in the early 1900s, as the industry portion of output was declining before this period and started to decline again from the 1960s onwards. This one off shift during the World War increased the energy intensity of capital and decreased the wage inequality.

Reassuringly the model of this chapter implies stable income shares with any terminal outcome, and so doesn't support increasing secular inequality. As can be concluded from 3.6.3 and figure 3.6.2, even if the trend of growth per worker of relative factor inputs complementing skilled more than unskilled wages continues, the wage premium

[1] this data was obtained from the Statistical Abstract of the United States series

should still plateau after a certain period of time. Without positive energy supply shocks, this may become important as growth seems to come increasingly from capital with a high technological component, which is the type of capital that most strongly complements skill labour relative to unskilled as is supported by Correa et al. (2014). Levels of state dependence also decide how much incomes vary, with higher state dependence leading to smaller changes in incomes from capital, due to stronger countervailing changes in technical progress. Conversely, if elasticities were more than one, larger state dependence would be destabilising. It is possible that the way modern economies grow technical progress varies between economics and form the past, to the extent that levels of state dependence may vary. However, completely stable income share would require full state dependence, which is an unlikely reality.

In summary, I have shown how interactions between factor inputs can impact wage inequality. I show how different types of shocks impact inequality, showing that historical observations are consistent with price shocks as opposed to productive shocks. I also show, in this chapter, the impact of diverging relative input levels on wage inequality, and how it plateaus, resulting in stable income shares. Chapter 2 identified energy and general capital as two inputs complementary to unskilled skilled labour respectively. This is supported by studying US manufacturing data and is also consistent with historical observations. Useful further study would involve further investigating disaggregated capital to understand how different types of capital impacts relative wages. This may reveal what other observable indicators may be driving wage inequality.

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APPENDIX .A CALCULATING CAPITAL VS LABOUR INCOME SHARES

$$u_t = (\frac{(1-\theta)^2 P_{u,t}}{\psi})^{\frac{1-\theta}{\theta}} A_{u,t} U_t \tag{.A.1}$$

$$\frac{P_{k_u,t}}{P_{u,t}} = \frac{1-\alpha_u}{\alpha_u}(\frac{u_t}{k_{u,t}})^{\frac{1}{\epsilon}} \tag{.A.2}$$

$$u_t = (\frac{\alpha_u}{1-\alpha_u}\frac{P_{k_u,t}}{P_{u,t}})^{\epsilon} k_{u,t} \tag{.A.3}$$

From 3.3.22 the comparative effective capital to labour ratios can be show. I will show this in the unskilled industry but the same applies for skilled.

$$\frac{k_{u,t}}{u_t} = (\frac{P_{k_u,t}}{P_{u,t}})^{\frac{1-\theta}{\theta}} \frac{A_{k_u,t} K_{u,t}}{A_{u,t} U_t} \tag{.A.4}$$

And combining the prices of the two intermediate goods (3.3.14 and 3.3.15):

$$\frac{P_{u,t}}{P_{k_u,t}} = \frac{\alpha_u}{1-\alpha_u}(\frac{k_{u,t}}{u_t})^{\frac{1}{\epsilon}} \tag{.A.5}$$

Combining this with .A.4:

$$\frac{P_{u,t}}{P_{k_u,t}} = \frac{\alpha_u}{1-\alpha_u}(\frac{P_{k_u,t}}{P_{u,t}})^{\frac{(1-\theta)}{\theta\epsilon}}(\frac{A_{k_u,t} K_{u,t}}{A_{u,t} U_t})^{\frac{1}{\epsilon}} \tag{.A.6}$$

$$(\frac{P_{u,t}}{P_{k_u,t}})^{\frac{1+\theta(\epsilon-1)}{\theta\epsilon}} = \frac{\alpha_u}{1-\alpha_u}(\frac{A_{k_u,t} K_{u,t}}{A_{u,t} U_t})^{\frac{1}{\epsilon}} \tag{.A.7}$$

$$\frac{P_{u,t}}{P_{k_u,t}} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\theta\epsilon}{1+\theta(\epsilon-1)}}(\frac{A_{k_u,t} K_{u,t}}{A_{u,t} U_t})^{\frac{\theta}{(1+\theta(\epsilon-1))}} \tag{.A.8}$$

With 3.3.37:

$$(\frac{1-\alpha_u}{\alpha_u})^{\frac{\theta\epsilon}{1+\theta(\epsilon-1)}}(\frac{A_{u,t} U_t}{A_{k_u,t} K_{u,t}})^{\frac{\theta}{1+\theta(\epsilon-1)}} = (\frac{\eta_u}{\eta_{k_u}}\frac{U_t}{K_{u,t}}(\frac{A_{u,t}}{A_{k_u,t}})^{\delta})^{\theta} \tag{.A.9}$$

$$(\frac{A_{u,t}}{A_{k_u,t}})^{1-\delta(1+\theta(\epsilon-1))} = (\frac{\alpha_u}{1-\alpha_u})^{\epsilon}(\frac{\eta_u}{\eta_{k_u}})^{1+\theta(\epsilon-1)}(\frac{K_{u,t}}{U_t})^{1-\sigma} \tag{.A.10}$$

$$(\frac{A_{u,t}}{A_{k_u,t}})^{1-\delta\sigma} = (\frac{\alpha_u}{1-\alpha_u})^{\epsilon}(\frac{\eta_u}{\eta_{k_u}})^{\sigma}(\frac{K_{u,t}}{U_t})^{1-\sigma} \tag{.A.11}$$

Here I introduce a new variable σ where $\sigma = 1+\theta(\epsilon-1)$. It is worth noting that if ϵ is less than one then so is σ and if ϵ is more than one then so is σ. As θ is less than one, σ is closer to one than ϵ.

3.3.20 and 3.3.21 result in:

$$\frac{w_{u,t}}{r_{u,t}} = (\frac{P_{u,t}}{P_{k_u,t}})^{\frac{1}{\theta}} \frac{A_{u,t}}{A_{k_u,t}} \tag{.A.12}$$

Combining this with .A.8:

$$\frac{w_{u,t}}{r_{u,t}} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon}{1+\theta(\epsilon-1)}} (\frac{A_{k_u,t}}{A_{u,t}})^{\frac{1-\sigma}{1+\theta(\epsilon-1)}} (\frac{K_{u,t}}{U_t})^{\frac{1}{1+\theta(\epsilon-1)}} \tag{.A.13}$$

$$(\frac{w_{u,t}}{r_{u,t}})^{\sigma} = (\frac{\alpha_u}{1-\alpha_u})^{\epsilon} (\frac{A_{k_u,t}}{A_{u,t}})^{1-\sigma} \frac{K_{u,t}}{U_t} \tag{.A.14}$$

This with .A.11:

$$\frac{A_{u,t}}{A_{k_u,t}} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon}{1-\delta\sigma}} (\frac{\eta_u}{\eta_{k_u}})^{\frac{\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{1-\sigma}{1-\delta\sigma}} \tag{.A.15}$$

$$(\frac{w_{u,t}}{r_{u,t}})^{\sigma} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon\sigma(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_u}}{\eta_u})^{\frac{\sigma(1-\sigma)}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\delta\sigma)-(1-\sigma)^2}{1-\delta\sigma}} \tag{.A.16}$$

$$\frac{w_{u,t}}{r_{u,t}} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_u}}{\eta_u})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{2-\sigma-\delta}{1-\delta\sigma}} \tag{.A.17}$$

$$\frac{U_t w_{u,t}}{K_{u,t} r_{u,t}} = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_u}}{\eta_u})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} \tag{.A.18}$$

Mirroring this:

$$\frac{S_t w_{s,t}}{K_{s,t} r_{s,t}} = (\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_s}}{\eta_s})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} \tag{.A.19}$$

APPENDIX .B CALCULATING RELATIVE LABOUR SHARES

First it is necessary to use what has been done to find terms for the quantity of first stage intermediate goods in terms of the inputs. Then we can substitute this into the relative quantities of secondary intermediate goods and repeat the process. From .A.4 and .A.11:

$$\frac{k_{u,t}}{u_t} = (\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon}{1-\delta\sigma}}(\frac{\eta_{k_u}}{\eta_u})^{\frac{\sigma}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{\frac{\sigma(1-\delta)}{1-\delta\sigma}}(\frac{P_{k_{u},t}}{P_{u,t}})^{\frac{1-\theta}{\theta}} \tag{.B.1}$$

.B.1 with .A.5 gives:

$$\frac{k_{u,t}}{u_t} = (\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon}{1-\delta\sigma}}(\frac{\eta_{k_u}}{\eta_u})^{\frac{\sigma}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{\frac{\sigma(1-\delta)}{1-\delta\sigma}}(\frac{1-\alpha_u}{\alpha_u}(\frac{u_t}{k_{u,t}})^{\frac{1}{\epsilon}})^{\frac{1-\theta}{\theta}} \tag{.B.2}$$

$$\frac{k_{u,t}}{u_t} = (\frac{1-\alpha_u}{\alpha_u})^{\epsilon+\frac{\delta\theta\epsilon^2}{1-\delta\sigma}}(\frac{\eta_{k_u}}{\eta_u})^{\frac{\theta\epsilon}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{\frac{\theta\epsilon(1-\delta)}{1-\delta\sigma}} \tag{.B.3}$$

By symmetry the following applies:

$$\frac{k_{s,t}}{s_t} = (\frac{1-\alpha_s}{\alpha_s})^{\epsilon+\frac{\delta\theta\epsilon^2}{1-\delta\sigma}}(\frac{\eta_{k_s}}{\eta_s})^{\frac{\theta\epsilon}{1-\delta\sigma}}(\frac{K_{s,t}}{S_t})^{\frac{\theta\epsilon(1-\delta)}{1-\delta\sigma}} \tag{.B.4}$$

Now the relative price of secondary intermediate goods can be found in terms first stage intermediate goods and inputs. From 3.3.8 and 3.3.9:

$$\frac{P_{y_s,t}}{P_{y_u,t}} = \frac{(1-\alpha)}{\alpha}\frac{Y_{u,t}}{Y_{s,t}} \tag{.B.5}$$

From 3.3.14, 3.3.16 and .B.5:

$$\frac{P_{s,t}}{P_{u,t}} = \frac{\alpha_s(1-\alpha)}{\alpha_u\alpha}(\frac{u_t}{s_t})^{\frac{1}{\epsilon}}(\frac{Y_{s,t}}{Y_{u,t}})^{\frac{1-\epsilon}{\epsilon}} \tag{.B.6}$$

Using $\hat{Y}_{u,t} = \frac{Y_{u,t}}{u_t}$ and $\hat{Y}_{s,t} = \frac{Y_{s,t}}{s_t}$:

$$\frac{P_{s,t}}{P_{u,t}} = \frac{\alpha_s(1-\alpha)}{\alpha_u\alpha}(\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}})^{\frac{1}{\epsilon}}\frac{Y_{u,t}}{Y_{s,t}} \tag{.B.7}$$

From 3.3.22:

$$\frac{s_t}{u_t} = (\frac{P_{s,t}}{P_{u,t}})^{\frac{1-\theta}{\theta}}\frac{A_{s,t}S_t}{A_{u,t}U_t} \tag{.B.8}$$

Combining these two:

$$\frac{P_{s,t}}{P_{u,t}}^{\frac{1}{\theta}} = \frac{\alpha_s(1-\alpha)}{\alpha_u \alpha}(\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}})^{\frac{1-\epsilon}{\epsilon}} \frac{A_{u,t}U_t}{A_{s,t}S_t} \tag{.B.9}$$

From 3.3.20 and 3.3.21:

$$\frac{w_{s,t}}{w_{u,t}} = (\frac{P_{s,t}}{P_{u,t}})^{\frac{1}{\theta}} \frac{A_{s,t}}{A_{u,t}} \tag{.B.10}$$

Combining this with .B.9

$$\frac{w_{s,t}}{w_{u,t}} = \frac{\alpha_s(1-\alpha)}{\alpha_u \alpha}(\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}})^{\frac{1-\epsilon}{\epsilon}} \frac{U_t}{S_t} \tag{.B.11}$$

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = \frac{\alpha_s(1-\alpha)}{\alpha_u \alpha}(\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}})^{\frac{1-\epsilon}{\epsilon}} \tag{.B.12}$$

From the production functions 3.3.10 and 3.3.11:

$$\frac{Y_{s,t}}{Y_{u,t}} = [\frac{(1-\alpha_s)k_{s,t}^{\frac{\epsilon-1}{\epsilon}} + \alpha_s s_t^{\frac{\epsilon-1}{\epsilon}}}{(1-\alpha_u)k_{u,t}^{\frac{\epsilon-1}{\epsilon}} + \alpha_u u_t^{\frac{\epsilon-1}{\epsilon}}}]^{\frac{\epsilon}{\epsilon-1}} \tag{.B.13}$$

This with .B.3 and .B.4:

$$\frac{Y_{s,t}}{Y_{u,t}} = [\frac{(1-\alpha_u)(\frac{\alpha_u}{1-\alpha_u})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_u}{(1-\alpha_s)(\frac{\alpha_s}{1-\alpha_s})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_s}]^{\frac{\epsilon}{1-\epsilon}} \frac{s_t}{u_t} \tag{.B.14}$$

$$\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}} = [\frac{(1-\alpha_u)(\frac{\alpha_u}{1-\alpha_u})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_u}{(1-\alpha_s)(\frac{\alpha_s}{1-\alpha_s})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_s}]^{\frac{\epsilon}{1-\epsilon}} \tag{.B.15}$$

$$\frac{\hat{Y}_{s,t}}{\hat{Y}_{u,t}}^{\frac{1-\epsilon}{\epsilon}} = \frac{(1-\alpha_u)(\frac{\alpha_u}{1-\alpha_u})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_u}{(1-\alpha_s)(\frac{\alpha_s}{1-\alpha_s})^{1-\epsilon+\frac{\delta\epsilon(1-\sigma)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + \alpha_s} \tag{.B.16}$$

The full term for relative labour shares is given from inserting .B.16 into .B.12:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{.B.17}$$

APPENDIX .C MACHINE VARIETIES IN TERMS OF FACTOR INPUTS

From .B.9 and 3.3.36

$$\frac{A_{u,t}}{A_{s,t}}^{1-\delta} = \frac{\alpha_u \alpha}{\alpha_s(1-\alpha)} \frac{\eta_u}{\eta_s} (\frac{\hat{Y}_{u,t}}{\hat{Y}_{s,t}})^{\frac{1-\epsilon}{\epsilon}} \tag{.C.1}$$

And inserting .B.16 gives the relative variety of machines for labour industries in terms of factor inputs:

$$\frac{A_{u,t}}{A_{s,t}}^{1-\delta} = \frac{\alpha}{(1-\alpha)} \frac{\eta_u}{\eta_s} \frac{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_u}{\eta_{ku}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{.C.2}$$

If zero sate dependance is assumed ($\delta = 0$) this becomes:

$$\frac{A_{u,t}}{A_{s,t}} = \frac{\alpha}{(1-\alpha)} \frac{\eta_u}{\eta_s} \frac{(\frac{1-\alpha_s}{\alpha_s})^{\epsilon} (\frac{\eta_s S_t}{\eta_{k_s} K_{s,t}})^{1-\sigma} + 1}{(\frac{1-\alpha_u}{\alpha_u})^{\epsilon} (\frac{\eta_u U_t}{\eta_{k_u} K_{u,t}})^{1-\sigma} + 1} \tag{.C.3}$$

Appendix .D Relative Labour Shares In Terms of Incomes With No State Dependance

Assuming $\delta = 0$, .B.17 becomes:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = \frac{1-\alpha}{\alpha} \frac{(1-\alpha_u)^{\epsilon} \alpha_u^{-\epsilon} (\frac{\eta_u U_t}{\eta_{k_u} K_{u,t}})^{1-\sigma} + 1}{(1-\alpha_s)^{\epsilon} \alpha_s^{-\epsilon} (\frac{\eta_s S_t}{\eta_{k_s} K_{s,t}})^{1-\sigma} + 1} \tag{.D.1}$$

Assuming $\delta = 0$, .A.17 becomes:

$$(\frac{U_t}{K_{u,t}}) = (\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon}{2-\sigma}} (\frac{\eta_{k_u}}{\eta_u})^{\frac{1-\sigma}{2-\sigma}} (\frac{r_{u,t}}{w_{u,t}})^{\frac{1}{2-\sigma}} \tag{.D.2}$$

This can be mirrored with the skilled good. These combined gives:

$$\frac{S_t w_{s,t}}{U_t w_{u,t}} = \frac{1-\alpha}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})(\frac{\alpha_u \eta_u r_{u,t}}{(1-\alpha_u)\eta_{k_u} w_{u,t}})^{\frac{1-\sigma}{2-\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})(\frac{\alpha_s \eta_s r_{s,t}}{(1-\alpha_s)\eta_{k_s} w_{s,t}})^{\frac{1-\sigma}{2-\sigma}} + 1} \tag{.D.3}$$

APPENDIX .E TECHNOLOGY SHOCK AND CAPITAL

Rearranging 3.5.3 for capital:

$$\frac{K_{s,t}}{K_{u,t}} = \frac{r_{u,t}}{r_{s,t}} \frac{(1-\alpha)}{\alpha} \frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_{u,t}}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_{s,t}}}{\eta_{s,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \tag{.E.1}$$

Taking differentiation by parts splits the differential to give the form:

$$\frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} = \frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})}|\bar{K}_{u,t}, \bar{K}_{s,t} + \frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{K_{u,t}}{U_t})} \frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})}|\bar{K}_{s,t} + \frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{K_{s,t}}{S_t})} \frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})}|\bar{K}_{u,t} \tag{.E.2}$$

Relative rents are constant in the long run, assuming there is no shock in prices, so they can be treated as exogenous. Labour levels are also exogenous. This can be written as:

$$\begin{aligned} \frac{d(\frac{K_{s,t}}{K_{u,t}})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_{u,t}}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_s}}{\eta_s})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \frac{r_{u,t}}{r_{s,t}} \frac{1-\sigma}{1-\delta\sigma} \\ [\frac{\eta_{u,t}}{\eta_{k_{u,t}}} + (1-\delta)\frac{U_t}{K_{u,t}} \frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} - \\ (1-\delta)\frac{S_t}{K_{s,t}} \frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{u,t}}{\eta_{k_{u,t}}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}] \end{aligned} \tag{.E.3}$$

As we know this is larger than zero from 3.5.6, the term in square brackets must be larger than zero. And so, for every possible parameter combination, as well as variables (S_t and U_t), $\frac{K_{s,t}}{S_t}$ and $\frac{K_{u,t}}{U_t}$ are such that:

$$\begin{aligned} \frac{\eta_{u,t}}{\eta_{k_{u,t}}} + (1-\delta)\frac{U_t}{K_{u,t}} \frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} - \\ (1-\delta)\frac{S_t}{K_{s,t}} \frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_{u,t}}}{\eta_{u,t}})} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{u,t}}{\eta_{k_{u,t}}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} > 0 \end{aligned} \tag{.E.4}$$

APPENDIX .F TECHNOLOGY SHOCK AND WAGES

Rearranging .B.17 gives:

$$\frac{w_{s,t}}{w_{u,t}} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{u,t}}{\eta_{k_u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{s,t}}{\eta_{k_s,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \frac{U_t}{S_t} \tag{.F.1}$$

Following the same process as .E.2:

$$\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_u}{\eta_{k_u,t}})} = \frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} |\bar{K}_{u,t}, \bar{K}_{s,t} + \frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{U_t}{K_{u,t}})} \frac{d(\frac{U_t}{K_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} |\bar{K}_{s,t} + \frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{S_t}{K_{s,t}})} \frac{d(\frac{S_t}{K_{s,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} |\bar{K}_{u,t} \tag{.F.2}$$

The first term on the right shows the direct effect of a technical progress shock on relative wages. The following two terms show how the countervailing changes in capital affect wages.

$$\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{u,t}}{\eta_{k_u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{s,t}}{\eta_{k_s,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \frac{U_t}{S_t} \frac{1-\sigma}{1-\delta\sigma}$$
$$[\frac{\eta_{k_u,t}}{\eta_{u,t}} + (1-\delta)\frac{K_{u,t}}{U_t} \frac{d(\frac{U_t}{K_{u,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} - \tag{.F.3}$$
$$(1-\delta)\frac{K_{s,t}}{S_t} \frac{d(\frac{S_t}{K_{s,t}})}{d(\frac{\eta_{u,t}}{\eta_{k_u,t}})} \frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_u,t}}{\eta_{u,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_s,t}}{\eta_{s,t}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}]$$

$$\frac{d(\frac{U_t}{K_{u,t}})}{d(\frac{\eta_u}{\eta_{k_u}})} = (\frac{\eta_{k_u}}{\eta_u} \frac{U_t}{K_{u,t}})^2 \frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u}}{\eta_u})} \tag{.F.4}$$

$$\frac{d(\frac{w_{s,t}}{w_{u,t}})}{d(\frac{\eta_u}{\eta_{k_u}})} = \frac{(1-\alpha)}{\alpha} \frac{(\frac{1-\alpha_u}{\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_u}{\eta_{k_u}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{U_t}{K_{u,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}}{(\frac{1-\alpha_s}{\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_s}{\eta_{k_s}})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{S_t}{K_{s,t}})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1} \frac{U_t}{S_t} \frac{1-\sigma}{1-\delta\sigma} (\frac{\eta_{k_u}}{\eta_u})^2$$
$$[\frac{\eta_u}{\eta_{k_u}} + (1-\delta)\frac{U_t}{K_{u,t}} \frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u}}{\eta_u})} - \tag{.F.5}$$
$$(1-\delta)\frac{S_t}{K_{s,t}} \frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u}}{\eta_u})} \frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_u}}{\eta_u})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}} (\frac{\eta_{k_s}}{\eta_s})^{\frac{1-\sigma}{1-\delta\sigma}} (\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}} + 1}]$$

Similar to .E.4, this is larger than zero if:

$$\frac{\eta_u}{\eta_{k_u}} + (1-\delta)\frac{U_t}{K_{u,t}}\frac{d(\frac{K_{u,t}}{U_t})}{d(\frac{\eta_{k_u}}{\eta_u})} - \\ (1-\delta)\frac{S_t}{K_{s,t}}\frac{d(\frac{K_{s,t}}{S_t})}{d(\frac{\eta_{k_u}}{\eta_u})}\frac{(\frac{\alpha_u}{1-\alpha_u})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_u}}{\eta_u})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{u,t}}{U_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1}{(\frac{\alpha_s}{1-\alpha_s})^{\frac{\epsilon(1-\delta)}{1-\delta\sigma}}(\frac{\eta_{k_s}}{\eta_s})^{\frac{1-\sigma}{1-\delta\sigma}}(\frac{K_{s,t}}{S_t})^{\frac{(1-\sigma)(1-\delta)}{1-\delta\sigma}}+1} > 0 \tag{.F.6}$$