State-dependent pricing turns money into a two-edged sword: A new role for monetary policy

Vo Phuong Mai Le*
(Cardiff Business School, Cardiff University)

David Meenagh†
(Cardiff Business School, Cardiff University)

Patrick Minford‡
(Cardiff Business School, Cardiff University, and CEPR)

July 2021

Abstract

Strong evidence exists that price/wage durations are dependent on the state of the economy, especially inflation. We embed this dependence in a macro model of the US that otherwise does well in matching the economy’s behaviour in the last three decades; it now also matches it over the whole post-war period. This finding implies a major new role for monetary policy: besides controlling inflation it now determines the economy’s price stickiness. We find that, when backed by fiscal policy in preventing a ZLB, by targeting nominal GDP monetary policy can achieve high price stability and avoid large cyclical output fluctuations.

Keywords: State-dependence; New Keynesian; Rational Expectations; Crises; Price Stability; Nominal GDP

JEL Classification: E2; E3

1 Introduction

Modern applied macroeconomic modelling is dominated by the New Keynesian model, in which wages and prices are fixed for a set duration, either in an explicit or implicit contract. This contract duration enables monetary policy to have effects on output. However, a long line of classical thought has emphasised wage-price flexibility, and its contract equivalent, state-contingent contracts, as the way in which agents could reach optimal outcomes. According to this view, the apparent fixed duration of wage-price contracts conceals a latent variability in response to the state of the economy. This Classical view has underlain the dominant divide, even schism, in macroeconomics, between those willing to accept the idea of contract-based price/wage rigidity and those who reject this as necessarily a violation of optimising behaviour by free agents. One way of bridging this divide would be to acknowledge that it could be optimal for agents to hold off changing prices in response to small shocks for some duration because of what we might call marketing costs such as changing price lists — upsetting consumers’ expectations — and are generally termed ‘menu costs’; and yet that the duration for which they would be willing to do this and the size of shocks they would ignore in this way would be state-contingent. This is still not the same as the classical assumption of fully-state-contingent and flexible prices and wages; however it gets fairly close once one concedes the existence of menu costs and the strong evidence that prices and wages are not in general fully flexible, whether straightforwardly

*Email: LeVP@cardiff.ac.uk
†Corresponding author: Cardiff Business School, Cardiff University, Colum Drive, Cardiff, UK, CF10 3EU. Tel.: +44 (0)29 2087 5198. Email: MeenaghD@cardiff.ac.uk
‡Email: MinfordP@cardiff.ac.uk
or indirectly within fully state-contingent contracts. It is this hypothesis of state-dependent variation in price/wage rigidity that we will examine in this paper.

There are many studies using microeconomic level data across different countries and finding evidence of state-dependent price and wage changes (Alvarez et al., 2019; Gagnon, 2009; Grigsby, 2021; Konieczny and Skrzypacz, 2005; Nakamura et al., 2018; Sigurdsson and Sigurdardottir, 2016; Wulfsberg, 2016). At the macroeconomic level prominent macroeconomic models such as Smets and Wouters (2007) and Christiano et al. (2005) assume time-dependent price and wage adjustments. To our knowledge there are only two papers incorporating these state-dependent assumptions into DSGE models (Costain et al., 2019 and Takahashi, 2017). In this paper we wish to check whether there is evidence at the macro level that corroborates the state-dependence found at the micro level. Unlike previous studies who use calibrated models, our aim is to use Indirect Inference to estimate and test a model with this state dependence against the data over a long post-war period from 1959, much like that used by Smets and Wouters (2007) — we will call it, for brevity, the ‘full post-war period’. We include the state-dependent variation by modelling an economy in which a fraction of firms face nominal rigidities, whilst the rest of the firms set prices flexibly. Similarly, a fraction of labour unions face nominal rigidities, with the remainder setting wages flexibly. The fraction of flexible price firms and flexible wage unions is state dependent and is an increasing function of past inflation. This framework will allow us to see how this state dependence affects macroeconomic behaviour, and how monetary policy could be used to stabilise the economy when state-dependent variation is present.

To anticipate our findings and our contributions in this paper, it turns out that the model which includes state-dependence can indeed fit the facts of the post-war period. Furthermore, we find that this state-dependence opens up a key new role for monetary policy in influencing the degree of price/wage stickiness, and we find that there are new optimal monetary policy rules in this new context. Because state-dependence interacts with the ZLB to create high price and output volatility in ZLB episodes that cannot be controlled by unconventional monetary measures such as QE, we also find that these rules need supplementing by a fiscal commitment to stop ZLB episodes in their tracks.

In what follows we review the literature in Section 2, then set out in Section 3 a simple micro-founded model of price and wage setting in which the recent behaviour of inflation affects the variances of all sectors’ cost-shock distributions, so changing their Calvo probability of price/wage change. In Section 4 we apply this model to the full sample of US postwar data, test it by indirect inference, and describe the properties of the adjusted SW macro model. In Section 5 we consider its implications for monetary policy rules. Section 6 concludes.

2 Literature Review

There has been a long list of studies trying to establish the facts about the relationship between the state of the economy, usually just inflation, and the frequency and size of price changes across different countries and across different data episodes. These studies utilise different sets of micro data on retail prices to obtain these estimates.

For the US, Bils and Klenow (2004) used the BLS micro data set from 1995–1997 and found that the median frequency of price change including price changes that occur because of sales and product substitution is 20.9%, that is, the median duration is 4.3 months. They also adjust this for sales, and report the sales-adjusted median duration as 5.5 months. They then use the price setting equation in time-dependent Calvo and Taylor models to check their ability to mimic the persistence and volatility of inflation across goods categories. They find for the goods with more infrequent price changes the models predict too much inflation persistence and too little inflation volatility, compared with the micro data; so the time-dependent models of price stickiness cannot account for the microdata evidence. Nakamura and Steinsson (2008) use more detailed data over a longer period (the data series on prices underlying the CPI index from 1988–2005): on this microdata sample they find higher median durations of 8–11 months for regular prices. With a longer sample they observe that the frequency of price changes and inflation have a relationship, i.e. the frequency of price increases covaries strongly with inflation, whereas the frequency of price decreases do not. Klenow and Kryvtsov (2008) include sale prices in their analysis and find that price changes are frequent (4–7 months depending on the treatment of sale prices) and usually large in absolute size. For a given item, price durations and absolute price changes vary over time. Like Nakamura and Steinsson (2008) they show
that the fraction of items increasing prices correlates most with inflation, but unlike Nakamura and Steinsson (2008), the fraction of price decreasing items also varies with inflation. These movements of fraction of price changes offset each other, and as a result, the inflation movement is driven by the size of price changes rather than the fraction of prices changing. Using partial equilibrium versions of macro models to reproduce this micro evidence, they find that none of the time-dependent and state-dependent models they considered can explain all of micro evidence about the price setting behaviour.\(^1\)

One disadvantage of these earlier studies is that they use data from the Great Moderation period where inflation was low and stable, which is a unique episode; hence they do not provide strong and conclusive evidence on the role of variation in inflation on the economy and the behaviour of prices. Nakamura et al. (2018) extend this data set by also including data from 1977 to capture the US Great Inflation period during the late 1970s and early 1980s. They find that firms raise the frequency of price change during the period of high inflation. Similar results are also found in other studies using micro data sets for other countries. Gagnon (2009) found that in Mexico at low inflation levels, the aggregate frequency of price changes responds little to movements in inflation because movements in the frequency of price decreases partly offset movements in the frequency of price increase. But during a period of high inflation in the mid 1990s while the absolute size of price changes varies little with inflation, the frequency of price changes becomes more responsive to inflation. He found that this behaviour can be replicated well by a simple menu-cost model with idiosyncratic technology shocks.

Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (2019) use product-level-data underlying Argentina’s CPI during the period of 1988–1997 with a mixed experience of deflation and very high inflation. They find that high inflation leads to more frequent price changes across all products, whereas idiosyncratic firm-level shocks would drive this frequency when inflation is low. In a similar fashion, Wulfsberg (2016) looks at another micro data set for both high inflation periods in the 1970s and 1980s and the low inflation period since the early 1990s — in this case for Norway. He finds that when inflation is high and volatile, prices change more frequently and in smaller absolute size; and when inflation is low, the frequency of price changes is low and the size of changes is high. There are some more studies in countries with high inflation. Konieczny and Skrzypacz (2005) look at a large disaggregated data set for Poland in the period 1990–1996 and find that the size and frequency of price changes are both positively correlated with the inflation rate. For the UK, Zhou and Dixon (2018) also find that prices are indeed fixed for average durations but these are state-contingent. They interpret this to mean that price-setters responded to larger macro shocks with larger and quicker than usual price changes, because the costs of not responding are unusually high, the disequilibrium being unusually large. The key implications for contract duration of varying inflation are shown in Table 1. It can be seen that duration varies very substantially with inflation, with median duration potentially moving from nearly a year to as low as one week.

<table>
<thead>
<tr>
<th>Country</th>
<th>Duration in high inflation</th>
<th>Duration in low inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nakamura et al. (2018)</td>
<td>USA</td>
<td>6.6 months</td>
</tr>
<tr>
<td>Gagnon (2009)</td>
<td>Mexico</td>
<td>3.1 months</td>
</tr>
<tr>
<td>Alvarez et al. (2019)</td>
<td>Argentina</td>
<td>1 week</td>
</tr>
<tr>
<td>Wulfsberg (2016)</td>
<td>Norway</td>
<td>6.7 months</td>
</tr>
<tr>
<td>Konieczny+Skrzypacz (2005)</td>
<td>Poland</td>
<td>1.7 months</td>
</tr>
</tbody>
</table>

Table 1: Summary of Findings in the Literature

This literature shows that to establish and understand the relationship between inflation and price stickiness in macroeconomic models, we might want to use state-dependent models.

\(^1\)The other branch of literature argues that the inconsistency between microdata evidence and macro models might be corrected by introducing heterogeneity across sectors in price stickiness. That is, macro models allow for Multiple Calvo (MC) contracts for different sectors. Kara (2015) uses the SW model with MC features, where the share of each product sector is based on micro evidence, and found that the model fits the low degree of persistence in actual inflation and the low variability of reset price inflation relative to actual inflation. Nevertheless this approach does not account for state-dependence.
There is also evidence of state-dependent wage-setting at the microeconomic level in many countries. Sigurdsson and Sigurdardottir (2016) used an administrative monthly wage dataset during 1998-2010 to provide new insight into nominal wage rigidity in Iceland. They find that timing of wage changes is both time-dependent and state-dependent. The study shows that wage cuts are rare but their frequency increases with large macroeconomic shocks and the timing depends on cumulative inflation and unemployment over the current wage spell and on the state when wages were adjusted in the past. Other studies also find the presence of state dependency in the nominal wage changes for other European economies. Using European survey data, Drueg et al. (2009) find that wage-setting in 15 European countries has both a time component and a state component where more than half of firms report that they change wages in a specific month, and one-third of firms have an internal policy of adapting wages to inflation. Montornés and Sauner-Leroy (2009), using survey data for French companies, find evidence of state-dependent and backward-looking wage-setting behaviour. For the US, Grigsby et al. (2021) used administrative payroll data to measure the extent of nominal wage rigidity during the period 2008 to 2016. They find a similar results to that in Sigurdsson ad Sigurdardottir (2016), that is nominal wage adjustment exhibits both time and state dependence properties. The time dependence shows through the facts that wage adjustment is found to be periodic with the majority of adjustment occurs every 12 months and synchronised within a firm. The state-dependent adjustment is shown through the fact that although nominal wage cuts are very rare over the whole sample, 6% workers received nominal base wage cuts during the Great Recession and more than 10% of workers in industries hit hardest during the Great Recession experienced nominal base wage declines. Also, the share of workers who see any base wage changes falls during the Great Recession. Further evidence for the state dependent wage adjustment in the US is reported in Cajner et al. (2020). Using weekly administrative payroll data, they investigate the condition of the US labour market during the first four months of the global COVID-19 pandemic and reported that nearly 7% of workers received nominal base wage cuts during the first few months of the Pandemic Recession.

At the macro level, it is nevertheless usual in the dominant New Keynesian modelling approach to assume fixed contract duration. Probably the most widely used model of the US, that of Smets and Wouters (2007), hereafter SW, which in turn was derived from Christiano, Eichenbaum and Evans (2005), is a New Keynesian model of substantial size, with structural equations for consumption and investment as well as for price- and wage-setting under imperfect competition. It follows Calvo’s (1983) framework, in which the probability of adjustment is constant. SW estimated the model by Bayesian means and fitted it to a long post-war period from 1966 to 2004. However, Le et al. (2011) found that when tested by the powerful method of indirect inference, the model was rejected by the sample data behaviour for the complete postwar sample period from 1947 to 2004. They also found evidence of two structural breaks at 1964 and 1984 which they interpreted as being due to the beginning of serious inflation and the move to inflation targeting respectively. For the period from 1984 until 2004, ‘the Great Moderation’, they found that if a second flexprice sector was introduced side by side with the sticky-price one, the SW model was not rejected. They found that the weight on the flexprice sector was close to zero for both wages and prices in this period; the Calvo parameters for the other sector were both around 0.7. However, for the previous two sub-samples they could find no model that could pass the test. Le et al. (2016a) reestimated the model for the later sample but extended it until 2011, and so included the financial crisis. They also extended the model to allow for a banking sector, for the Zero Lower Bound (ZLB) and for Quantitative Easing (QE). They found that this extended model again could pass the test over the longer period from mid-1980s to the present. However, in this subperiod the weight on the flexprice sector, for both wages and prices, rose considerably from near zero to 0.56 and 0.91 respectively; the Calvo parameters in the other sector fell to 0.63 for wages and rose to 0.97 for prices (with only the most sticky sub-sectors left in the sticky-price sector). This weight can be thought of as measuring the proportion of sectors that have price/wage rigidity of less than three months; thus they approximate to changing prices at once within the quarterly model context and so act as if flexprice. What all these findings seem to imply is that when the stochastic environment changes so does the duration of price- and wage-setting, i.e. macroeconomic models should allow for price and wage adjustment to be state dependent rather than merely time dependent, thus letting price/wage-stickiness be state-contingent.

One possible reason for the New Keynesian model’s failure to fit the whole time period with fixed duration pricing could be the basic assumption of rational expectations under full information. There is a large literature considering the empirical claims of models without this assumption — e.g. see Mankiw and Reis (2007), Dräger (2016), Knotek (2010), Del Negro and Eusepi (2001). We agree that these models
are of interest but they lie beyond the scope of our work here. We have not included these features in our specification, as they belong to a quite different class of models to those we are considering, which are models with full rational expectations, full information and fixed policy regimes, in the manner of Christiano et al. (2005) and Smets and Wouters (2007). We focus on these models because this aspect of them, state-dependence, has not been explored in terms of ability to fit the data; as we will see, if such a model turns out to fit the data and so proves close to the true model, then it has important policy implications of which we should be aware in the framing of policy. Plainly abandoning full rationality or fixed policy regimes introduces important changes in the model behaviour and may match the data better, though we may note that if the model we have identified here is the true one, then other models will be mis-specified and so are very likely to be rejected by our tests; but if accepted, these different models would no doubt entail yet other policy implications. We simply leave these issues for future work. It is true that the model here contains endogenous switches between fixed policy regimes — with and without the ZLB. But the policy regimes in the model are fixed: the switching is forced by shocks to the model, not by changing regime choices.

In this empirical paper we explore the implications at the macro level of allowing both wage and price contracts to be state-dependent. These pricing features have been explored at the macro level in Costain, Nakov and Petit (2017). They incorporated both state-contingent wage stickiness and price stickiness, and the state-dependent adjustment mechanism is based on the control cost model, where the price/wage decision is a random variable defined over a set of feasible alternatives and the decision-maker faces a cost function that increases with the precision of that random variable. The authors calibrated the micro data evidence of frequency of price and wage adjustments into a DSGE model for the US in which duration depends on inflation. They find that sticky wages play a big role in creating monetary non-neutrality and that the model with both forms of stickiness has larger real effects of monetary shocks than does the model with just price stickiness. Takahashi (2017) also studies a DSGE model with state-dependent price and wage setting, where the state-dependent pricing mechanism is based on a stochastic menu costs model, i.e. households face different fixed wage-setting costs that evolve independently over time. He calibrated the distribution of wage setting costs to match the US data of the fraction of unchanged wages for a year. It turns out that this distribution is very similar to the Calvo-type distribution and thus the responses to monetary shocks in this state-dependent model are very longlasting just as in the time-dependent model. However, both these papers focus on micro-data relationships from a sample period, the Great Moderation, where inflation did not vary much; this may well account for their macro models turning out quite similar to the Smets-Wouters model.

Our contribution here, to recapitulate on our relationship to this literature, is that we bring to bear a full model that is estimated to match closely the data behaviour over the bulk of the post-war sample, and hence approximately the full range of inflation dependence. Our model thus contains substantial state-dependence side by side with the many real rigidities in Smets and Wouters (2007), financial frictions as in Bernanke et al. (1999), and the ability to deal with the ZLB as in Le et al. (2016a); we do not calibrate but rather estimate and test the model as a whole by indirect inference on unfiltered, and therefore nonstationary, macro data. In our model price/wage duration depends on the variance of lagged inflation according to a linear parameter that we estimate, and inflation in turn depends on duration. We had in mind that such state-dependency could account for the failure of the SW model to pass our test for the earlier data subsamples (Le et al, 2011). It could be that the problem lay with shifting behaviour in wage/price-setting within these subsamples in response to a fluctuating macro environment: notoriously, inflation rose steadily during the 1960s, and extremely sharply during the 1970s before collapsing in the early 1980s. Possibly too the structural breaks found by Le et al. (2011) could be accounted for by this shifting wage/price behaviour. If we could find a single model that would match the data behaviour in the whole sample sufficiently well to pass our test, then this would constitute strong evidence in favour of these hypotheses. We think that the link from the macro state distributions to price-setting will be reinforced and possibly modified at the macro level because of the strong feedback in both directions, from price-setting to macro distributions and from the latter to price-setting. Thus our aim is to check whether there is evidence at the macro level that corroborates the evidence of state-dependence at the micro level, and if so just what the final macro relationships turn out to be, as well as their implications for monetary policy.

To anticipate our findings, it turns out that the model which includes state-dependence can indeed fit the facts of the full post-war period most closely\(^2\). We also find that this state-dependence model opens

\(^2\) Notice that in testing this model indirect inference differs from direct inference Maximum Likelihood
up a key new role for monetary policy in influencing the degree of price/wage stickiness, and we make a
search for optimal monetary policy rules in this new context. Because state-dependence interacts with the
ZLB to create high price and output volatility in ZLB episodes that cannot be controlled by unconventional
monetary measures such as QE, we find that these rules need supplementing by a fiscal commitment to stop
ZLB episodes in their tracks.
In what follows we set out in Section 2 a simple micro-founded model of price and wage setting in which
the recent behaviour of inflation affects the variances of idiosyncratic cost-shock distributions, so changing
their Calvo probability of price/wage change. In Section 3 we apply this model to the full sample of US
postwar macro data and test it by indirect inference. In Section 4 we describe the properties of the adjusted SW
macro model. In Section 5 we consider its implications for monetary policy rules. Section 6 concludes.

3 Model

The model we use here is a development of the well-known model of Smets and Wouters (2007) which in
turn was derived from Christiano, Eichenbaum and Evans (2005). In a series of papers starting with Le et
al. (2011), we have sought to test the ability of this model to match the empirical behaviour of US data from
the beginning of the post-war period to the present day. In the process of doing this, we have introduced
various new features to get the model closer to the data. In the first place, the model assumes that a fraction
of goods markets are flexprice while the rest set prices for longer durations; similarly with labour markets.
This ‘hybrid’ element in the model turned out to enable the model to fit the Great Moderation period. In
order to fit the period of run-up to the Financial Crisis and the post-crisis period we incorporated financial
frictions as proposed by Bernanke et al. (1999) and also allowed for collateral as in Le et al. (2016a) to make
monetary policy effective at the zero lower interest rate bound via the provision of cheap money collateral
through Quantitative Easing. With these additions we were able to fit the data behaviour from the Great
Moderation through to today, as detailed in Le et al. (2016a). However, as mentioned above, we still could
not fit the behaviour of earlier periods, which brings us to the modifications of the model we implement in
this paper; these are to introduce state-dependent price and wage setting, in a way we now set out in detail.

In Le et al. (2011, 2016a) it was assumed that imperfectly competitive ﬁrms and labour unions decide on
changing their prices/wages based on Calvo ﬁxed probabilities, but there were ﬁxed weights on the fractions
of goods and labour markets where there is ‘long’ duration of more than one quarter, and those in a ‘short
duration’/flexprice sector where prices and wages change continuously each quarter. That is, we assumed the
structure of price/wage durations is ﬁxed. In this paper we relax this assumption and assume this structure
changes with the state of the economy, i.e. these durations vary as more ﬁrms/labour unions decide, in the
face of aggregate shocks, to change their prices and wages more frequently; and in particular continuously,
so shifting from the long to the short duration sector. The short duration sector we describe as ‘flexprice’
(FP) since it is continuously in a quarterly context keeping prices equal to marginal costs plus the same
constant mark-up as in the long-duration sector. The long duration sector we call ‘New Keynesian’ (NK)
since it conforms to the Calvo sticky-price model.

We assume these agents will only change prices/wages if the shock is larger than some particular value,
representing the menu cost of changing prices: below this point at which, as Calvo(1983) puts it, the signal
to change prices ‘lights up’, they would rather stabilise the price in order to insure their customers against
uncertainty, which is how we may interpret the menu cost. However, above it the cost of providing this
insurance is too great compared with the benefit it gives. We also assume that this idiosyncratic distribution’s
variance is related to the size of recent inflation shocks to the economy, denoted by $H$ and measured by a
moving average of inflation discussed below. These shocks to other prices set oﬀ price shocks to particular
markets because they are shocks to the product’s relative price. Thus if prices in general, i.e. other prices,
have moved substantially then demand and supply for the particular product must also be affected; hence as recent inflation rises, so does the variance of the idiosyncratic distributions being used by price setters. A rise in the variance implies that the critical shock size now comes at a lower percentile of this more volatile distribution, as illustrated in Figure 1. This percentile is then the Calvo percentage of firms not changing their price. This Calvo parameter is therefore a function of the sector distribution, which in turn depends on $\Pi$.

![Figure 1: Distribution of Idiosyncratic Shocks](image)

Hence the probability of not changing price is reduced by $\Pi$ and so too the Calvo parameter. As a result more sectors will become flexprice (i.e. have an overall duration of 1 quarter) and in the remaining sectors the Calvo parameter may fall. However, we should note that the Calvo parameter for the sticky-price sector may rise, fall, or remain unchanged in response to $\Pi$. The sectors closest to the short duration sector tend to migrate to it, leaving behind the sectors that have higher Calvo parameters. This 'abandonment effect' is opposite to the reduction effect on these remaining sectors' Calvo parameters, which we estimate in the usual way with the other model parameters, but allowing for this net response to $\Pi$. The model wage/price parameters are changing so that the model is now nonlinear — its behaviour is changing in response to the history of shocks. This nonlinearity will feed back into macro variables' volatility which in turn will react on the wage/price parameters.

We now turn to our assumptions on the parameters driving these shifts. We are looking for a function relating wage/price parameters to the past variance of inflation. A natural candidate is the square of a moving average of inflation over the recent past, say four years; this is our $\Pi$. It allows for offsetting effects where inflation rises have been later reversed by inflation falls; but it will strongly register a sustained rise in inflation or a sustained fall into deflation. The idea behind using past inflation variance to proxy the rational expectation of future variance is that the variance of the exogenous shocks (driving the excess demand shocks causing the raised inflation shocks) is highly persistent — for example the Great US inflation of the 1960s and early 1970s was driven by the Vietnam War and the Great Society Programme’s joint effect on excess demand; both of these were long-lasting sources of high-variance shocks. The response to this higher inflation variance of the short-duration sector weights we allow to be determined empirically, by indirect inference estimation. The weights on the NK sectors are calculated according to the function $\omega^i = \exp(-\theta, \Pi)$, where $i = \pi, w^i$ (for the Calvo weights we looked for a quadratic function of $\Pi$ but our estimates turned up a zero net response). We add this price/wage setting state-dependence to the model of Le et al. (2016a), a model that includes a variant monetary policy based on QE when the ZLB is triggered. The resulting nonlinear, shifting-weights, model is then estimated and evaluated using the method of Indirect Inference on unfiltered

\footnote{From this function we can see that the weights are bounded by 0 and 1, $\omega^i \in [0,1]$.}
US quarterly data from 1959–2017. The current model, including the previous and these latest modifications is listed in Appendix A.

### 3.1 Indirect Inference Estimation

We evaluate the model’s capacity in fitting the data using the method of Indirect Inference as set out in Le et al. (2011). A detailed description of the method can be found in Le et al. (2016b). The approach employs an auxiliary model that is completely independent of the theoretical one to produce a description of the data against which the performance of the theory is evaluated indirectly. Such a description can be summarised either by the estimated parameters of the auxiliary model or by functions of these; we will call these the descriptors of the data. While these are treated as the ‘reality’, the theoretical model being evaluated is simulated to find its implied values for them. In estimation the parameters of the structural model are chosen such that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the actual data. The optimal choices of parameters for the structural model are those that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model.

When testing the model’s fit to the data the structural model is simulated and the auxiliary model is fitted to each set of simulated data, from which we obtain a sampling distribution of the coefficients of the auxiliary model. A Wald statistic is computed to determine whether functions of the parameters of the auxiliary model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

The auxiliary model should be a process that would describe the evolution of the data under any relevant model. It is known that for non-stationary data the reduced form of a macro model is a VARMA where non-stationary forcing variables enter as conditioning variables to achieve cointegration (i.e. ensuring that the stochastic trends in the endogenous vector are picked up so that the errors in the VAR are stationary). This in turn can be approximated as a VECM. So we use as the auxiliary model a VECM which we reexpress as a VAR(1) for the three macro variables of interest (interest rate, output and inflation), with a time trend and the productivity residual entered as an exogenous non-stationary process (these two elements having the effect of achieving cointegration). We treat as the descriptors of the data the VAR coefficients on the lagged dependent variables and the VAR error variances, so that the Wald statistic is computed from these. Thus effectively we are testing whether the observed dynamics and volatility of the chosen variables are explained by the simulated joint distribution of these at a given confidence level. The Wald statistic is given by:

\[
(\Phi - \Phi') \sum_{(\Phi\Phi)}^{-1} (\Phi - \Phi)
\]

where \(\Phi\) is the vector of VAR estimates of the chosen descriptors yielded in each simulation, with \(\Phi\) and \(\sum_{(\Phi\Phi)}\) representing the corresponding sample means and variance-covariance matrix of these calculated across simulations, respectively.

The joint distribution of the \(\Phi\) is obtained by bootstrapping the innovations implied by the data and the theoretical model; it is therefore an estimate of the small sample distribution\(^4\). Such a distribution is generally more accurate for small samples than the asymptotic distribution.

This testing procedure is applied to a set of (structural) parameters put forward as the true ones (\(H_0\), the null hypothesis). The test then asks: could these coefficients within this model structure be the true (numerical) model generating the data? We extend our procedure by a further search algorithm, in which we seek other coefficient sets that minimise the Wald test statistic — in doing this we are carrying out indirect estimation.

Thus we calculate the minimum-value Wald statistic using a powerful algorithm based on Simulated Annealing (SA) in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the space. The merit of this extended procedure is that we are using the best possible version of the model when finally doing our comparison of model compatibility with the data.

---

\(^4\)The bootstraps in our tests are all drawn as time vectors so contemporaneous correlations between the innovations are preserved.
4 Results

4.1 Parameter Estimation

The model is estimated on unfiltered US data for the period 1959–2017. Table 2 reports the model’s parameter estimates. This model matches the data behaviour well, with a p-value of 0.21, comfortably above the usual 0.05 level of model rejection. The parameters of the non-pricing functions are much the same as estimated in Le et al. (2016a) without state-dependent pricing. However, whereas the model in Le et al. (2016a) was only able to fit the Great Moderation period, having introduced state-dependent pricing to the model it can now explain the dynamic behaviour of major macroeconomic variables for the much longer sample of 1959–2017.

<table>
<thead>
<tr>
<th>Models’ Coefficients</th>
<th>Estimate Model (state dependent)</th>
<th>Le et al. (2016)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of capital adjustment</td>
<td>( \varphi )</td>
<td>6.881</td>
</tr>
<tr>
<td>Elasticity of consumption</td>
<td>( \sigma_c )</td>
<td>1.283</td>
</tr>
<tr>
<td>External habit formation</td>
<td>( \lambda )</td>
<td>0.767</td>
</tr>
<tr>
<td>Probability of not changing wages</td>
<td>( \xi_w )</td>
<td>0.635</td>
</tr>
<tr>
<td>Elasticity of labour supply</td>
<td>( \sigma_L )</td>
<td>2.865</td>
</tr>
<tr>
<td>Probability of not changing prices</td>
<td>( \xi_p )</td>
<td>0.746</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>( r_w )</td>
<td>0.376</td>
</tr>
<tr>
<td>Price indexation</td>
<td>( i_p )</td>
<td>0.107</td>
</tr>
<tr>
<td>Elasticity of capital utilisation</td>
<td>( \psi )</td>
<td>0.128</td>
</tr>
<tr>
<td>Share of fixed costs in production ((+1))</td>
<td>( \Phi )</td>
<td>1.083</td>
</tr>
<tr>
<td>Taylor Rule response to inflation</td>
<td>( r_p )</td>
<td>2.913</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>( \rho )</td>
<td>0.732</td>
</tr>
<tr>
<td>Taylor Rule response to output</td>
<td>( r_y )</td>
<td>0.019</td>
</tr>
<tr>
<td>Taylor Rule response to change in output</td>
<td>( r_{\Delta y} )</td>
<td>0.019</td>
</tr>
<tr>
<td>Share of capital in production</td>
<td>( \alpha )</td>
<td>0.222</td>
</tr>
<tr>
<td>Elasticity of the premium with respect to leverage</td>
<td>( \chi )</td>
<td>0.032</td>
</tr>
<tr>
<td>Money response to premium ( \psi_2 )</td>
<td>0.059</td>
<td>0.065</td>
</tr>
<tr>
<td>Elasticity of the premium to M0 ( \psi_1 )</td>
<td>0.058</td>
<td>0.055</td>
</tr>
<tr>
<td>Money response to credit growth</td>
<td>( \psi_1 )</td>
<td>0.052</td>
</tr>
<tr>
<td>Parameter response of NK weight — prices</td>
<td>( \theta_\pi )</td>
<td>0.052</td>
</tr>
<tr>
<td>Parameter response of NK weight — wages</td>
<td>( \theta_w )</td>
<td>0.071</td>
</tr>
<tr>
<td>Wald ((Y, \pi, R))</td>
<td>15.525</td>
<td>21.904</td>
</tr>
<tr>
<td>p-value</td>
<td>0.21</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 2: Coefficient Estimates

This shows that, as the parameters have not changed a lot, the state-dependent mechanism is vital in fitting long samples with possible regime changes. The p-value of the current model is also much higher than that of the model without state dependence. Since the main model parameters are largely independent of the state-dependent mechanism they are much the same as those in Le et al. (2016a) shown in column 3. The residuals and shocks extracted from the estimated model and data can be found in Appendix B.

Figure 2 shows how the weights on NK prices and wages change over time due to fluctuations in inflation. As inflation increases in the 1970s the NK weights decrease, then rise back close to 1 for the Great Moderation period. These weights produce durations that are in line with Nakamura et al. (2018).

4.2 Impulse Response Function Analysis

We now discuss the model’s behaviour in response to shocks and examine its behaviour according to how New Keynesian (NK) it is. We consider IRFs under two extremes: at the one extreme is an entirely NK version (NK weights are maximum with corresponding Calvo parameters) and at the other is a flexprice
Figure 2: Time Varying NK Weights

(FP) version (NK weights are at their minimum). As the weight on the NK sectors increases/decreases the model behaviour will move towards/away from the NK IRFs.

In the IRFs that follow — and are shown fully in the Appendix C — there are two pure demand shocks, a government spending shock and a shock to the Taylor Rule in Figures 3 and 4. In both, one sees a much larger output fluctuation in the NK versus the FP case, where inflation responds sharply to stabilise output.

Figure 3: IRFs to a Government Spending Shock

Notice that the shocks we identify may include both supply and demand effects or elements which we distinguish from the originating shocks themselves. For example, a productivity shock, shown in Figure 5, (which has a permanent effect here) raises supply (directly and via the capital it induces); it also raises demand (consumption reacts to its implied permanent income; investment via the need for more capital). The IRF shows higher output fluctuation under NK than FP around the same long-run change.
The other shocks are all mainly demand shocks: the consumer preference shock plainly is, while the net worth, premium and investment shocks all disturb investment demand, leaving long run supply the same. Accordingly, all show more output fluctuations under NK than NC. We show these output IRFs to all the shocks in Figure 6 (as noted the full set of IRFs is shown in the appendix). We omit the labour supply shock (to the utility cost of labour) from the output IRFs here because it has no demand element: under NK it has no effect on employment or output, as it has virtually no effect on wages; it simply has a temporary effect on employment and so output under FP.

The IRF analysis shows that an NK model acts like an old Keynesian model, producing high multipliers on output for demand effects; with fixed prices demand directly affects output. Hence demand elements create output turbulence. Inflation does not react much in the short run but in the medium run reacts substantially to the resulting persistent output gaps. By contrast under an FP model demand elements affect prices, with little effect on output; prices move with marginal costs and so the output gap and, with interest rates, clear the goods market. On the other hand, supply elements affect output directly in the FP model through the production function generating output supply; prices and interest rates adjust to bring demand into balance with this supply. In the NK model supply elements affect prices, with an effect on
output indirectly via the Taylor Rule; these effects are weak because pass through of supply elements to prices is very limited, prices being fixed for long periods.

Hence an NK model, relative to an FP model, stabilises output against supply disturbances but destabilises it against demand ones. For prices, the NK model stabilises inflation via the Calvo mechanism, while the FP model keeps it related to marginal costs; on balance the NK model stabilises inflation the most, maximising the duration of fixed prices and wages.

4.3 Variance Decomposition

We show in Tables 3 and 4 how the model responds to shocks on average, via its variance decomposition for the long- and short-run respectively. As we would expect, demand shocks, notably government spending, dominate output in the short run while in the long run the non-stationary productivity, and government spending jointly dominate. This result is in line with the fact that the weights on the NK sectors are high. Monetary policy shocks account for about a tenth of output variance in both the short and long run.

Table 3: Long Run Variance Decomposition

<table>
<thead>
<tr>
<th>Shock</th>
<th>Int. Rate</th>
<th>Inv.</th>
<th>Infl.</th>
<th>Wage</th>
<th>Cons.</th>
<th>Output</th>
<th>Hours</th>
<th>Premium</th>
<th>Net Worth</th>
<th>M0</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Govt Spending</td>
<td>2.47</td>
<td>3.60</td>
<td>3.34</td>
<td>4.02</td>
<td>7.71</td>
<td>23.10</td>
<td>13.09</td>
<td>2.87</td>
<td>4.16</td>
<td>42.47</td>
<td>2.99</td>
</tr>
<tr>
<td>Consumer Pref.</td>
<td>1.27</td>
<td>1.56</td>
<td>1.81</td>
<td>2.18</td>
<td>18.31</td>
<td>2.83</td>
<td>1.96</td>
<td>1.12</td>
<td>1.52</td>
<td>1.33</td>
<td>61.58</td>
</tr>
<tr>
<td>Investment</td>
<td>3.14</td>
<td>55.31</td>
<td>3.92</td>
<td>4.83</td>
<td>5.01</td>
<td>9.87</td>
<td>6.46</td>
<td>3.77</td>
<td>8.44</td>
<td>2.79</td>
<td>8.92</td>
</tr>
<tr>
<td>Interest Rate Rule</td>
<td>15.82</td>
<td>10.16</td>
<td>9.11</td>
<td>7.86</td>
<td>9.81</td>
<td>9.22</td>
<td>10.47</td>
<td>8.10</td>
<td>10.19</td>
<td>8.92</td>
<td>8.40</td>
</tr>
<tr>
<td>Price Mark-up</td>
<td>5.28</td>
<td>7.76</td>
<td>9.46</td>
<td>6.15</td>
<td>6.68</td>
<td>6.49</td>
<td>7.13</td>
<td>5.67</td>
<td>7.61</td>
<td>6.46</td>
<td>6.27</td>
</tr>
<tr>
<td>Wage Mark-up</td>
<td>2.09</td>
<td>3.46</td>
<td>3.12</td>
<td>3.93</td>
<td>3.13</td>
<td>3.34</td>
<td>3.33</td>
<td>2.48</td>
<td>3.31</td>
<td>2.91</td>
<td>2.88</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>4.48</td>
<td>3.07</td>
<td>3.90</td>
<td>6.65</td>
<td>3.43</td>
<td>3.98</td>
<td>2.93</td>
<td>2.49</td>
<td>3.17</td>
<td>2.70</td>
<td>2.69</td>
</tr>
<tr>
<td>Credit Premium</td>
<td>3.63</td>
<td>0.82</td>
<td>4.34</td>
<td>6.50</td>
<td>1.67</td>
<td>2.81</td>
<td>0.76</td>
<td>29.91</td>
<td>1.45</td>
<td>0.49</td>
<td>0.60</td>
</tr>
<tr>
<td>Net Worth</td>
<td>3.70</td>
<td>1.97</td>
<td>4.57</td>
<td>5.92</td>
<td>2.99</td>
<td>4.20</td>
<td>2.68</td>
<td>7.46</td>
<td>35.67</td>
<td>1.59</td>
<td>1.69</td>
</tr>
<tr>
<td>Monetary Base</td>
<td>29.66</td>
<td>2.77</td>
<td>26.05</td>
<td>30.38</td>
<td>11.81</td>
<td>11.13</td>
<td>2.40</td>
<td>23.48</td>
<td>10.54</td>
<td>1.45</td>
<td>1.45</td>
</tr>
</tbody>
</table>

5 What is the potential role of monetary policy in a model with state-dependent pricing?

We now turn to a discussion of how monetary policy can best respond to shocks within this model of the US economy. The estimated model reveals that the changing duration of pricing has major effects on how shocks impact on the economy; and that in turn monetary policy, through its influence on inflation, has major effects on price-duration.
Table 4: Short Run Variance Decomposition (5 year)

To compare different monetary policy rules we analyse various welfare measures. The standard way in New Keynesian models is to evaluate welfare through the variance of inflation since this is related to the extent that relative prices are disturbed from their zero margin optimum over marginal cost; to generate this optimum it is usually assumed that a government subsidy offsets the steady state margin. However, since the Financial Crisis and the Great Recession attention has also been focused on the ability of policy to avoid crises, viewed as long recessions — Le et al. (2016a) showed via simulation analysis how many crises were likely under various rules. An alternative way of measuring this output tendency is to measure output volatility directly, around a measure of trend output\footnote{We construct this trend output measure as the balanced growth path we find in the data plus simulated productivity shocks; these two elements together constitute the deterministic plus the stochastic trend in output, that we estimate constitute the optimum equilibrium output path.}. Since there is little agreement in the policy debate on any one of these measures of stabilisation success, we use our simulation analysis to generate all of them, in the hope that we can find a monetary rule that would produce a broadly attractive result from most viewpoints — and so is generally robust.

In our discussion that follows, we aim to review first of all the economy’s behaviour under the default option of the monetary behaviour we have estimated for the model: namely a Taylor Rule in normal times, accompanied by a QE rule under the ZLB whenever this hits. We then compare the results we obtain when we substitute new policy rules, notably those targeting Nominal GDP.

![Figure 7: Two examples of simulated output under different rules](image-url)
NK-flexprice weights which in turn feed back into inflation variance. Notice that output remains moderately smooth.

To summarise the difference between the new endogenous-duration model and the previous fixed-duration one, the Taylor Rule default monetary policy loses the ability to stabilise inflation whereas previously it lacked the ability to stabilise output.

Figure 8: Taylor Rule with ZLB Simulation Example

To restore the powers of monetary policy we again need to turn towards Nominal GDP targeting. To do this the central bank specifies an intermediate target for the official interest rate as follows:

$$r_t = \rho_1 r_{t-1} + \rho_o (\hat{y}_t + p_t - \hat{y}_t - \hat{p}) + \varepsilon_t$$

where $\hat{y}_t + \hat{p}$ is the target for nominal GDP, $\hat{p} = 0$ and $\hat{y}_t$ follows the trend path of real output.

However, we also need a policy that switches off the ZLB, with its destabilising effect on inflation. It is obvious that monetary policy is powerless to push interest rates up off the zero bound, except at the cost of creating a severe output loss, which would much worsen output stability. Hence we need to look for another way that does not interfere with monetary policy’s stabilising function. We find this in fiscal policy, which we can use solely to prevent the zero bound, in effect kicking in with whatever demand expansion at the zero bound is sufficient to push interest rates away from zero; we do this by specifying a government spending shock sufficient to achieve a positive rate. Illustrative simulation results for this policy can be seen in the RED lines of simulation 15 in Figure 9. Here the existence of this fiscal guarantee, or ‘fiscal backstop’, switches off the ZLB, while nominal GDP targeting stabilises inflation, output remaining broadly smooth.

This fiscal backstop is needed for the optimisation of policy, which we believe to be an important new normative finding. It might well be argued that the fiscal authorities would be unwilling to deploy the backstop, having turned a deaf ear throughout the Great Recession aftermath of the financial crisis to pleas from central banks for active fiscal policy to support demand — see for example the speech by the Fed chairman on October 6th (Powell, 2020). If that were the case, then the fiscal backstop would have no practical relevance inspite of the model’s normative implications. However, during the Covid crisis there has
been a fundamental shift of fiscal authorities’ attitudes towards fiscal activism. Even as a strong post-Covid recovery is occurring, US fiscal policy remains highly stimulative, suggesting that the US government is not averse to rising interest rates to curb rising inflation; indeed long term rates are already rising in response. This is precisely how the fiscal backstop would be deployed at the ZLB.

When we employ this nominal GDP targeting interest rate rule with the fiscal backstop preventing the ZLB, we broadly recover the stability we found in our previous work from Nominal GDP targeting with the ZLB but without state-dependence (Le et al., 2016). Table 5 summarises our average simulated results for each Targeting Rule. If we compare the stability results for this rule with those for our estimated baseline Taylor Rule we find that it greatly reduces the variance of inflation and the variation of output around our target trend, so also our chosen welfare cost measure which combines the two, with weights determined by the relative variance. It also keeps the number of long crises (4–6 years long, Great Recessions) down to one per century,[6] roughly matching the Taylor Rule.

This is achieved while also largely keeping the world close to totally NK, with NK weights between 0.8 and 1.0 and so high price stability.

<table>
<thead>
<tr>
<th></th>
<th>Crises/1000 years</th>
<th>var(σ)</th>
<th>var(y)*</th>
<th>Welfare*</th>
<th>Av. NK weight wage</th>
<th>Av. NK weight price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor Rule</td>
<td>8.10</td>
<td>0.127</td>
<td>25.2419</td>
<td>0.1755</td>
<td>0.9377</td>
<td>0.9516</td>
</tr>
<tr>
<td>NOMGDPT (noZLB)</td>
<td>9.72</td>
<td>0.0176</td>
<td>16.8902</td>
<td>0.0598</td>
<td>0.9534</td>
<td>0.9658</td>
</tr>
</tbody>
</table>

* Deviation from target trend
* Weighted welfare=0.9975*var(σ)+0.0025*var(y)

Table 5: Crises and Welfare Comparison

According to our model, the trend path of real output may not be the true estimate of the FP model solved path which corresponds to the welfare-maximising path; this would rather be the balanced growth path plus the simulated effect on output of all shocks under the flexprice model. Therefore we will check

[6] We treat a long crisis as a drop in GDP that takes between 4 and 6 years for GDP to recover to the previous peak.
the robustness of our welfare measures for our chosen rule to using a model-estimated optimum equilibrium output path. For this alternative measure we used the BGP trend plus the flexprice model solution for the effects on output of all the model shocks. It can be seen in the Table 6 that the welfare results for the chosen Nominal GDP target rule still show a marked improvement on the Taylor Rule, though a smaller one on the output element.

<table>
<thead>
<tr>
<th></th>
<th>var((\pi))</th>
<th>var((y))^*</th>
<th>Welfare^+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor Rule</td>
<td>0.1127</td>
<td>20.8553</td>
<td>0.16453</td>
</tr>
<tr>
<td>NOMGDPT (noZLB)</td>
<td>0.0176</td>
<td>20.1508</td>
<td>0.06791</td>
</tr>
</tbody>
</table>

^* Deviation from Optimum output under FP model

^† Weighted welfare=0.9975*var(\(\pi\)) + 0.0025*var(\(y\))

Table 6: Welfare Comparison for Mistaken Equilibrium Output Path

To illustrate what is going on in Table 5 we show in Figure 10 a number of illustrative simulations with results for the Taylor Rule and Nominal GDP target and show the target trend for output. It can also be seen that this rule (in Red) keeps output on a rather stable course, relative to the status quo Taylor Rule case (in Blue), while eliminating unstable behaviour in inflation and price/wage duration.

6 Conclusions

In this paper we have investigated how US macroeconomic behaviour is affected by state-dependence in price/wage duration. Current major macro models assume constant duration but there is considerable evidence now both in macro and micro data that duration varies with the state of the economy, especially with inflation. We have reestimated a fairly successful DSGE model to include state-dependence and found that with this extension it can match US behaviour over the bulk of the postwar period, whereas with constant duration it failed to match it before the mid-1980s. We found that duration fluctuated over the whole period quite substantially, between strongly New Keynesian periods such as during the Great Moderation and much closer to flexprice periods such as during the Great Inflation and the Great Recession.
The role of monetary policy becomes two-fold in such a world, since any monetary rule does not merely respond to shocks but also affects the extent to which the economy is New Keynesian and hence its fundamental responses to shocks. We investigate how such a powerful twin role might be best discharged; and we find that an interest rate rule targeting Nominal GDP, with a differential response to prices and output, the first relative to a simple loglinear trend, the second relative to a flexprice equilibrium trend, performs well according to a number of welfare criteria, provided it is buttressed by a fiscal backstop that prevents the Zero Lower Bound taking hold by pushing interest rates away from it. Notably this rule achieves a world in which prices are heavily stabilised much as they would have been under the gold standard, leading to long price/wage durations; but also one where the demand shocks to which such a New Keynesian world is highly vulnerable are strongly stabilised.

References


7 APPENDIX A: Model

7.1 Smets-Wouters Model Extensions

Our starting point is the Smets and Wouters (2007) model of the US. The model is then extended as in Le et al. (2011) to a hybrid version in which the price and wage setting equations are assumed to be a weighted average of a New Keynesian and New Classical (Flexprice) equation. This is done by assuming that wage and price setters supply labour and intermediate output in two markets; a competitive market with price/wage flexibility and a market with imperfect competition so that inflation and wage are set as follows:

\[ \pi_t = \theta_\pi \pi_{tNK}^N + (1 - \theta_\pi)\pi_{tFP}^F \]  

(2)

\[ w_t = \theta_w w_{tNK}^N + (1 - \theta_w)w_{tFP}^F \]  

(3)

where \( \pi_{tNK}^N \) and \( \pi_{tFP}^F \) are inflation in a New Keynesian and Flex Price respectively. Similarly for wages. \( \theta_\pi \) and \( \theta_w \) are the fixed weights on the New Keynesian sector.

The model is then extended further to incorporate a banking sector, following Bernanke et al. (BGG, 1999). The BGG model introduces credit, extended by banks to entrepreneurs. The difference of BGG from SW lies in the nature of entrepreneurs. Whilst still producing intermediate goods, they now do not rent capital from households (who do not buy capital but only buy bonds or deposits) but must buy it from capital producers and in order to buy this capital they have to borrow from a bank which converts household savings into lending. On their production side, entrepreneurs face the same situation as in Le et al. (2011). They hire labour from households for wages that are partly set in monopolistic, partly in competitive labour markets; and they buy capital from capital producers at prices of goods similarly set in a mixture of monopolistic and competitive goods markets. Thus the production function, the labour demand and real marginal cost equations are unchanged. It is on their financing side that there are major changes. Entrepreneurs buy capital using their own net worth \( (n_t) \), pledged against loans from the bank, which thus intermediates household savings deposited with it at the risk-free rate of return. The net worth of entrepreneurs is kept below the demand for capital by a fixed death rate of these firms \( (1 - \theta) \); the stock of firms is kept constant by an equal birth rate of new firms. Entrepreneurial net worth therefore is given by the past net worth of surviving firms plus their total return on capital \( (cy_t) \) minus the expected return (which is paid out in borrowing costs to the bank) on the externally financed part of their capital stock — equivalent to

\[ n_t = \theta n_{t-1} + \frac{K}{N} (cy_t - E_{t-1}cy_t) + E_{t-1}cy_t + enw_t \]  

(4)

where \( \frac{K}{N} \) is the steady state ratio of capital expenditures to entrepreneurial net worth, \( \theta \) is the survival rate of entrepreneurs and \( enw_t \) is a net worth shock. Those who die will consume their net worth, so that entrepreneurial consumption \( (c^e_t) \) is equal to \( (1 - \theta) \) times net worth. In logs this implies that this consumption varies in proportion to net worth so that:

\[ c^e_t = n_t \]  

(5)

In order to borrow, entrepreneurs have to sign a debt contract prior to the realisation of idiosyncratic shocks on the return to capital: they choose their total capital and the associated borrowing before the shock realisation. The optimal debt contract takes a state-contingent form to ensure that the expected gross return on the bank’s lending is equal to the bank opportunity cost of lending. When the idiosyncratic shock hits, there is a critical threshold for it such that for shock values above the threshold, the entrepreneur repays the loan and keeps the surplus, while for values below it, he would default, with the bank keeping whatever is available. From the first order conditions of the optimal contract, the external finance premium is equated with the expected marginal product of capital which under constant returns to scale is exogenous to the individual firm (and given by the exogenous technology parameter); hence the capital stock of each entrepreneur is proportional to his net worth, with this proportion increasing as the expected marginal product rises, driving up the external finance premium. Thus the external finance premium increases with the amount of the firm’s capital investment that is financed by borrowing:

\[ E_tcy_{t+1} - (r_t - E_t\pi_{t+1}) = \chi (qq_t + k_t - n_t) + epr_t \]  

(6)
where the coefficient $\chi > 0$ measures the elasticity of the premium with respect to leverage. Entrepreneurs leverage up to the point where the expected return on capital equals the cost of borrowing from financial intermediaries. The external finance premium also depends on an exogenous premium shock, $e_{pr_t}$. This can be thought of as a shock to the supply of credit: that is, a change in the efficiency of the financial intermediation process, or a shock to the financial sector that alters the premium beyond what is dictated by the current economic and policy conditions.

Entrepreneurs buy capital at price $qq_t$, in period $t$ and use it in $(t + 1)$ production. At $(t + 1)$ entrepreneurs receive the marginal product of capital $rk_{t+1}$ and the ex-post aggregate return to capital is $cy_{t+1}$. The capital arbitrage equation (Tobin’s Q equation) becomes:

$$ qq_t = \frac{1-\delta}{1-\delta+R^K}E_tqq_{t+1} + \frac{R^K}{1-\delta+R^K}E_trk_{t+1} - E_tcy_{t+1} $$

The resulting investment by entrepreneurs is therefore reacting to a Q-ratio that includes the effect of the risk-premium. There are as before investment adjustment costs. Thus, the investment Euler equation and capital accumulation equations are unchanged from Le et al. (2011). The output market-clearing condition becomes:

$$ y_t = C\gamma c_t + I\gamma inn_t + R^K_k \frac{1-\psi}{\psi} r_k t + c^e c_t + e_g $$

Further, the model is extended to allow the effects of Quantitative Easing. We note that in BGG firms put up no collateral. Net worth by construction is all invested in plant, machinery and other capital. However, once so invested, this amount cannot be recovered at original value plainly: it will have less value as second hand sales when the firm goes bankrupt because it has become specialised to the firm’s activities. The cost of bankruptcy recovery (costly state verification) applies to the valuation of the activity this capital still allows.

It is in fact normal for banks to request an amount of collateral from the firms to which they lend. This gives firms more incentive to avoid bankruptcy. (Some models underpin bank contracting entirely on the basis that banks will only lend against collateral — Kiyotaki and Moore (1997) — however we do not adopt this extreme position here.) We therefore supplement the BGG model by the assumption that banks require firms to put up the amount of collateral, $c$, as a fraction of their net worth. We also assume that recovery of this typical collateral costs a proportion, $\delta$, of its original value when posted — we can think of the example of a house being put up and it costing this proportion in fees and forced-sale losses to sell the house and recover its value in cash.

It is at this point we introduce the idea of cash as collateral. If a firm holds some cash on its balance sheet, this can be recovered directly with no loss of value and no verification cost; thus it eliminates the cost $\delta$ and lowers the credit premium for given leverage; it therefore permits firms to increase leverage and so raise their expected returns. We therefore assume that banks and firms have an interest in firms holding as much cash as can be acquired for collateral. Thus as M0 is issued we assume that it is acquired by firms from banks to be held as collateral. This effect of the monetary base on collateral echoes Williamson (2013) in a search model.

The government/central bank issues this cash through open market operations (QE) to households in exchange for government bonds they hold. They deposit this cash with the banks. Firms wish to acquire as much of this cash as possible for their collateral needs. We can think of them as investing their net worth in cash (to the maximum available), with the rest going into other collateral and capital. In practice of course their profits (which create their net worth) are continuously paid out as dividends to the banks which provide them with credit, so they have nothing with which to acquire these assets if they do not collaborate with banks. So they achieve this balance sheet outcome by agreeing with the banks that, as a minimum counterpart to the credit advanced they will hold the maximum cash collateral available, which is M0. Thus all of M0 at once finds its way into firms’ balance sheets, where it is securely pledged to the banks in the event of bankruptcy (for example by being actually lodged with them); in practice as we explain below in the balance sheets it would be held as a counterpart deposit by firms and the M0 held by the banks.

Finally, the short-term interest rate is set by the central bank according to some rule, such as the Taylor Rule. In our model here only firms hold M0; households have no use for it and deposit it at once in banks where as we have seen it is lent to firms to hold as collateral, in effect M0’s only use. In New Keynesian models it is implicitly assumed that the Taylor Rule is enforced by open market operations of some sort, presumably
in money and Treasury Bills. Here we make the assumption that it is enforced by open market operations in public debt; households hold part of their savings in government bonds, the rest in bank deposits, which pay the short term interest rate also obtainable on Treasury Bills (treated here as an equivalent asset). The Taylor Rule represents the short term interest rate at which the government debt office will borrow; hence it sets the Treasury Bill rate and so the bank deposit rate.

Therefore, monetary authorities have two instruments, \( M_0 \) and \( r \), and thus they need, apart from their interest rate setting rule, an operating rule for \( M_0 \).

First, we set out the balance sheets of the agents in the economy and discuss how they are altered by acts of policy (see Table 7).

<table>
<thead>
<tr>
<th>Firms</th>
<th>A</th>
<th>L</th>
<th>A</th>
<th>L</th>
<th>A</th>
<th>L</th>
<th>A</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLL</td>
<td>=</td>
<td>( exM_0 )</td>
<td>( (\text{COLL} - C\text{DEP}(M_0)+) )</td>
<td>( (\text{CDEP}) )</td>
<td>( (\text{DEP}) )</td>
<td>( (\text{CUMSAV}) )</td>
<td>( (\text{CUMDEF}) )</td>
<td>( (\text{GB}) )</td>
</tr>
</tbody>
</table>

Table 7: Balance Sheets of the Agents of the Economy

where \( \text{COLL} \) = collateral (exM0=held as non-monetary; \( M_0 \)=held as money); \( K \)=capital investment; \( NW \)=net worth; \( CR \)=credit; \( DEP \)=deposits; \( GB \) = government bonds; \( \text{CUMSAV} \)=stock of private savings; \( \text{CUMDEF} \)=accumulated government borrowing; \( M_0 \)=monetary base. For simplicity we have written as if the firms hold \( M_0 \) directly; in practice of course they would hold it indirectly as a marked deposit with the bank, and the bank would hold the \( M_0 \) on its behalf — ready to seize it as collateral in the event of bankruptcy. This is shown in the above balance sheets as \( C\text{DEP}(M_0) \) which is an asset of firms corresponding to their \( M_0 \) deposit; in turn it is a liability of banks, which hold the corresponding \( M_0 \) as an asset. Thus injections of \( M_0 \) by the central bank wind up being held as liquid collateral by banks to back up their credit operations.

Consider now how an open market operation (QE) by buying GB for \( M_0 \) would change these balance sheets — as indicated by + and − in this table. Households place the extra cash on deposit; the banks then lend it to firms who are able to use it as collateral in a future lending deal with the banks, so that a larger part of collateral is held as \( M_0 \). With collateral cheaper (\( \delta \) falls) the bank credit premium falls (which will induce a future rise in investment and leverage); the other collateral is converted into capital stock. These are the partial equilibrium or direct effects, which then lead to further general equilibrium changes in response to the fall in the credit premium.

To adjust the model for these additional features, we need to introduce the effect of \( M_0 \) on the credit premium via its effect on the cost of liquidating collateral, \( \delta \); and we need to add \( \xi \), the macro-prudential instrument directly raising the credit friction, into the credit premium equation. We can think of \( \xi \) as being like a buffer of \( M_0 \) that the banks need to hold for reasons of liquidity, and that is hence unavailable for use as collateral; hence it is equivalent to negative \( M_0 \). The credit premium equation now has additional terms in \( m = \ln(M_0) \) and \( \xi \), as follows:

\[
E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = s_t = \chi (qq_t + k_t - n_t) - \psi m_t + \xi_t + epr_t
\]

where \( \psi \) is the elasticity of the premium to \( M_0 \) via its collateral role. This effect comes about, conditional on leverage (\( k - n \)), through the willingness of banks (under their zero profit condition) to reduce the credit premium for given leverage. Now that they will recover more in the event of bankruptcy, the equilibrium contract, for given leverage, now has a lower bankruptcy threshold and a lower required rate of return on firm assets. Both produce a lower credit premium for given leverage.

We now need equations for the supply of \( M_0 \) and for the setting of \( \xi \). QE programmes have sought to raise money supply growth (and implicitly therefore credit growth); before these programmes \( M_0 \) seems to have been set to accommodate the supply of credit/broad money generated at the interest rates set by the Taylor Rule.

Macro-prudential measures have been built on the Basel Agreements nos 1 and 2; clearly they have been made more harsh over this period in response to the crisis, which was unpredicted by officials. Before that there was a gradual tightening of regulation at least in the Agreements, if not always in practical application by individual countries.
What these considerations suggest, as argued above, is that the supply of M0 was supplied via the discount window, before the crisis when interest rates were above the zero bound, as required to support the supply of money (M in logs); after the crisis, when interest rates were at the zero bound (which we take to be 0.25% p.a.), M0 (i.e. QE) seems to have been targeting the credit premium around its steady state, \( s^* \), aiming to bring credit conditions back to normal. For macro-prudential measures the above suggests that they have evolved as an exogenous \( I(1) \) time-series process, with the crisis acting as an exogenous shock to the process.

So we write the equation for M0 in two parts:

\[
\begin{align*}
m_t &= \psi_0 + \psi_1 M_t + errm_{2t} \quad \text{for } r_t > 0.0625% \\
\Delta m_t &= \psi_2 (s_t - s^*) + errm_{2t} \quad \text{for } r_t \leq 0.0625%
\end{align*}
\]

where \( \psi_1, \psi_2 \) are both positive. The credit premium tends to be correlated inversely with the broad money supply, so that one may think of this approximately as a policy of money targeting; however the money element in the banks’ balance sheet fluctuates with other things and so from a welfare viewpoint it is the credit premium that should be targeted with as much information on it as can be amassed, including that from M itself.

Finally, we need an equation now additionally for the supply of money, which we define as equal to deposits (= credit) + M0. Here we simply use the firms’ balance sheet \( (M = CR + M0 = K + COLL - NW + M0) \) which can be written in loglinearised form as:

\[
M_t = (1 + \nu - c - \mu)K_t + \mu m_t - \nu n_t
\]

where \( M, K, m, n \) are respectively the logs of Money, capital, M0 and net worth, we have omitted the constant (which includes collateral, assumed fixed as a proportion of money); \( \nu, \mu, c \) are respectively the ratios of net worth, M0 and collateral to money.

### 7.2 Variable Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>Consumption</td>
</tr>
<tr>
<td>( l )</td>
<td>Hours worked</td>
</tr>
<tr>
<td>( r )</td>
<td>Interest Rate</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Inflation</td>
</tr>
<tr>
<td>( inn )</td>
<td>Investment</td>
</tr>
<tr>
<td>( qq )</td>
<td>Tobin’s Q</td>
</tr>
<tr>
<td>( k )</td>
<td>Capital</td>
</tr>
<tr>
<td>( cy )</td>
<td>External finance rate</td>
</tr>
<tr>
<td>( rk )</td>
<td>Rental rate of capital</td>
</tr>
<tr>
<td>( w )</td>
<td>Wages</td>
</tr>
<tr>
<td>( y )</td>
<td>Output</td>
</tr>
<tr>
<td>( n )</td>
<td>Net worth</td>
</tr>
<tr>
<td>( m )</td>
<td>M0</td>
</tr>
<tr>
<td>( M )</td>
<td>M2</td>
</tr>
<tr>
<td>( s )</td>
<td>Premium</td>
</tr>
</tbody>
</table>

### 7.3 Model Listing

**Consumption Euler equation**

\[
c_t = \frac{\lambda}{1 + \frac{\lambda}{\gamma}} c_{t-1} + \frac{1}{1 + \frac{\lambda}{\gamma}} E_t c_{t+1} + \frac{(\sigma_c - 1) W_t L_t}{\gamma} (l_t - E_t l_{t+1}) - \left( \frac{1 - \frac{\lambda}{\gamma}}{\frac{\lambda}{\gamma}} \right) (r_t - E_t r_{t+1}) + eb_t \quad (10)
\]

**Investment Euler equation**
\[
\begin{align*}
\text{Tobin Q equation} \\
qq_t &= \frac{1 - \delta}{1 - \delta + R^K_t E_t q_{t+1}} E_t q_{t+1} + \frac{R^K_t}{1 - \delta + R^K_t E_t q_{t+1}} E_t k_{t+1} + E_t c_{t+1}
\end{align*}
\]

\[
\begin{align*}
\text{Capital Accumulation equation} \\
k_t &= \left(1 - \frac{1}{\gamma} \right) k_{t-1} + \left(1 - \frac{1}{\gamma} \right) in_{t-1} + \left(1 - \frac{1}{\gamma} \right) \left(1 + \beta \gamma (1-\sigma_e) \right) \gamma^2 \varphi (c in_{t-1})
\end{align*}
\]

\[
\begin{align*}
\text{Labour demand} \\
l_t &= -w_t + \left(1 + \frac{1 - \psi}{\psi} \right) r k_t + k_{t-1}
\end{align*}
\]

\[
\begin{align*}
\text{NK Price Setting equation } (\pi_t^{NK}) \\
\pi_t &= \frac{\beta \gamma (1-\sigma_e) \pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} E_t \pi_{t+1} + \frac{\pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} \pi_{t-1} - \frac{1 + \beta \gamma (1-\sigma_e) \pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} \left(\frac{1 - \beta \gamma (1-\sigma_e) \xi_P (1 - \xi_P)}{\xi_P (1 + \psi - \psi) \xi_P} \right) (\alpha r k_t + (1 - \alpha) w_t - e a_t) + e P_t
\end{align*}
\]

\[
\begin{align*}
\text{NK Wage Setting equation } (w_t^{NK}) \\
w_t &= \frac{\beta \gamma (1-\sigma_e) \pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} E_t w_{t+1} + \frac{\pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} w_{t-1} + \frac{\beta \gamma (1-\sigma_e) \pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} E_t \pi_{t+1} + \frac{1 + \beta \gamma (1-\sigma_e) \pi_P}{1 + \beta \gamma (1-\sigma_e) \pi_P} \pi_{t-1} - \frac{1}{1 + \beta \gamma (1-\sigma_e) \pi_P} \left(\frac{1 - \beta \gamma (1-\sigma_e) \xi_P (1 - \xi_P)}{\xi_P (1 + \psi - \psi) \xi_P} \right) (\alpha r k_t + (1 - \alpha) w_t - e a_t) + e P_t
\end{align*}
\]

\[
\begin{align*}
\text{FP Marginal Product of Labour } (w_t^{FP}) \\
r k_t &= \frac{1}{\alpha} \left[ (1 - \alpha) w_t + e a_t \right]
\end{align*}
\]

\[
\begin{align*}
\text{FP Labour Supply } (\pi_t^{FP}) \\
w_t &= \sigma l_t + \left(1 - \frac{1}{\psi} \right) \left( c_t - \frac{\lambda}{\gamma} c_{t-1} \right) - (\pi_t - E_t \pi_t) + e w_t^S
\end{align*}
\]

\[
\begin{align*}
\text{Weighted Inflation} \\
\pi_t &= \theta_{\pi} \pi_t^{NK} + (1 - \theta_{\pi}) \pi_t^{FP}
\end{align*}
\]

\[
\begin{align*}
\text{Weighted Wage} \\
w_t &= \theta_w w_t^{NK} + (1 - \theta_w) w_t^{FP}
\end{align*}
\]

\[
\begin{align*}
\text{Market Clearing condition in goods market} \\
y_t &= \frac{C}{V} c_t + \frac{I}{V} in_{t-1} + R^K_t k_{t-1} \frac{1 - \psi}{\psi} - r k_t + c^p c_t + e g_t
\end{align*}
\]

\[
\begin{align*}
\text{Aggregate Production equation}
\end{align*}
\]
\[ y_t = \phi \left( \alpha \frac{1 - \psi}{\psi} r_k t + \alpha k_{t-1} + (1 - \alpha) l_t + e a_t \right) \]  

(21)

Taylor Rule

\[ r_t = \rho r_{t-1} + (1 - \rho) (r_p \pi_t + r_y y_t) + r \Delta y_t (y_t - y_{t-1}) + er_t \text{ for } r_t > 0.0625 \]  

(22)

Premium

\[ E_t c_{yt+1} - (r_t - E_t \pi_{t+1}) = pm_t = \chi (q q_t + k_t - n_t) - \psi m_t + \xi_t + e p r_t \]  

(23)

Net worth

\[ n_t = \frac{K}{N} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + \theta n_{t-1} + e n w_t \]  

(24)

Entrepreneurial consumption

\[ c_t^e = n_t \]  

(25)

M0

\[ \Delta m_t = \psi_1 \Delta M_t + err m_{2t} \text{ for } r_t > 0.0625 \text{ and } \Delta m_t = \psi_2 (s_t - c^*) + err m_{2t} \text{ for } r_t \leq 0.0625 \]  

(26)

M2

\[ M_t = (1 + \nu - \mu) k_t + \mu m_t - \nu m_t \]  

(27)
8  APPENDIX B: Residuals and Shocks

Figure 11: Residuals

Figure 12: Shocks
9 APPENDIX C: all model IRFs

Figure 13: IRFs to a Consumer Preference Shock

Figure 14: IRFs to a Government Spending Shock
Figure 15: IRFs to an Investment Shock

Figure 16: IRFs to a Labour Supply Shock
Figure 17: IRFs to a Money Supply Shock

Figure 18: IRFs to a Taylor Rule Shock
Figure 19: IRFs to a Net Worth Shock

Figure 20: IRFs to a Premium Shock
Figure 21: IRFs to a Price Mark-up Shock

Figure 22: IRFs to a Wage Mark-up Shock
Figure 23: IRFs to a Nonstationary Productivity Shock