Optimal Second-Harmonic Current Injection for Capacitor Voltage Fluctuation Reduction in Hybrid MMCs Under Grid-side SLG Faults

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Abstract — The second-harmonic injection strategy for modular multilevel converters (MMCs) can effectively suppress the capacitor voltage fluctuation in steady states. However, the capacitor voltage fluctuation under grid-side single-line-to-ground (SLG) faults will be more severe. Moreover, there is no research on the mitigation of capacitor voltage fluctuation under grid-side SLG faults for the hybrid MMC with mixed half-bridge and full-bridge sub-modules. To address this issue for hybrid MMCs, an optimal second-harmonic current injection scheme for mitigating the capacitor voltage fluctuation in the hybrid MMC under grid-side SLG faults is proposed in this paper. The mathematical models of arm power fluctuations of the hybrid MMC are firstly developed under both steady states and grid-side SLG faults. Then, the principle and optimal strategy of the proposed capacitor voltage fluctuation strategy are presented. The capacitor voltage fluctuation under grid-side SLG faults can be reduced by suppressing the three-phase arm power fluctuations. A hybrid MMC model with the proposed scheme has been established in PSCAD/EMTDC to verify its effectiveness and feasibility. Simulation results show that the average voltage fluctuation reduction rate can reach 55.77% with the proposed strategy.

Index Terms — Second-harmonic current injection; hybrid MMC; HB and FB submodule; grid-side SLG faults; voltage fluctuation.

I. INTRODUCTION

Modular multilevel converter (MMC) has attracted more attention compared to line commutated converter (LCC) for high-voltage direct-current (HVDC) transmission, due to their advantages of low harmonics, flexible control, modularity and no commutation failure [1], [2]. At present, MMCs have been widely used in high voltage and large capacity scenarios [3], [4]. However, the high volume/weight and cost of MMC-HVDC are still the concerns of manufacturers and market players. Submodule (SM) capacitors of an MMC account for about 1/2 of its volume [5], [6]. Therefore, it is necessary to reduce the capacitance of SM capacitors so as to reduce the overall volume/weight and cost of the MMC.

The capacitance can be reduced if the fluctuations of SM capacitor voltages can be regulated within a minimum range. SM voltage fluctuation suppression schemes of half-bridge (HB) MMC have been widely investigated in the open literature. In [7], a circulating current elimination method has been proposed to reduce the HB-SM voltage fluctuation. It has been found in [8] that the capacitor voltage ripple can be further reduced by injecting a second-harmonic circulating current properly. Based on [8], an optimal design of the injected second-harmonic circulating current is carried out in [9]. In [10] and [11], mathematical models of the capacitor voltage fluctuation and energy storage requirements for HB-MMC have been developed. Reference [12] investigates how to reduce the capacitor voltage fluctuation using a third-harmonic voltage injection. The above methods all contribute to reducing the SM voltage fluctuation of HB-MMC.

It is known that the HB-MMC does not have DC fault blocking capability like the full-bridge (FB) MMC [13]-[14]. However, the FB-MMC exhibits the challenges of their high capital cost and power losses. The HB-SM and FB-SM based hybrid MMC combines the merits of the two types of MMCs [15]-[16]: 1) The hybrid MMC has DC fault-ride-through capability; 2) The hybrid MMC has a wider modulation index range compared to HB-MMC, which provides more space to improve the suppression of capacitor voltage fluctuation. Under steady states, the modulation index of 1.414 can significantly reduce the fundamental frequency component of the SM capacitor voltage, and at the same time, suppress the fluctuation of the SM capacitor voltage [19], [20]. In [19], an optimal second-harmonic current injection of amplitude and phase has been developed based on the defined fluctuation function of SM capacitor voltage. An optimal circulating current injection method considering the limitation of the current rating of semiconductor (switch) has been studied in [20]. In addition, the hybrid MMC may experience unipolar arm currents under reduced DC voltage conditions, which will affect the charge/discharge balance of HB-SM capacitors in each fundamental cycle [21]-[23]. A second-harmonic circulating current injection to achieve arm currents in hybrid MMC bipolar during reduced DC voltage conditions has been proposed in [23]. However, the objective of [23] is not to
investigate the control of MMC under SLG faults but to achieve maximum power transfer under a reduced DC voltage, in which case the AC side voltages are not affected by any faults.

However, the abovementioned work only analyzes the harmonic injection method under steady states. The impact of hybrid MMC’s grid-side unbalanced faults on the capacitor voltage fluctuation suppression schemes has been underresearched. It is well known that the single-line-to-ground (SLG) fault is one of the most common faults in power systems. The modeling and control of HB-MMC subject to grid-side SLG faults have been investigated in [24]. Following [19], [25] further investigates the impact of FB-MMC’s modulation index on the SM capacitor voltage fluctuation under SLG faults. However, the above studies have not considered the impact of SLG faults on the harmonic current injection strategy which is one of the most important methods to suppress capacitor voltage fluctuation. Moreover, these methods do not apply to the hybrid MMC because of its special converter configuration of mixed HB- and FB-SMs. A dual harmonic current injection strategy for reducing the SM voltage fluctuation in hybrid MMCs has been proposed in [26]. However, the work mainly focuses on SM voltage fluctuation mitigation under steady-state operation. Although tests under grid-side SLG faults have been conducted, there is no further theoretical analysis and optimal design of the harmonic current injection for the SM voltage fluctuation under SLG faults.

This paper aims to address the abovementioned issues. The main contributions of this work include: 1) Theoretically analyze the impact of converter grid-side SLG faults on the second-harmonic injection strategy in the hybrid MMC; 2) Develop mathematical models of the arm power fluctuation under both steady states and grid-side SLG faults; 3) Propose an optimal second-harmonic current injection scheme to minimize arm power ripples and therefore, reduce the SM voltage fluctuation under grid-side SLG fault conditions. The studies reveal that, under grid-side SLG faults, the SM voltage fluctuation with the second-harmonic current injection strategy will be larger than that without the injection strategy. This means that the second-harmonic current injection strategy brings negative effects on the hybrid MMC under grid-side SLG faults. The analysis is verified in PSCAD/EMTDC. Simulation results indicate that the SM capacitor voltage fluctuation under an SLG fault can be effectively mitigated by the proposed second-harmonic current injection scheme. In addition, the accuracy of the proposed mathematical models of the arm power model is high.

II. MODELING OF ARM POWER FLUCTUATION IN HYBRID MMCs

A. Structure and Arm Power Fluctuation of Hybrid MMCs

Fig. 1 gives the structure of hybrid MMCs, which consists of six arms. Each arm consists of N SMs, including n HB-SMs and m FB-SMs. $U_{dc}$ is the DC voltage, $I_{dc}$ is the DC current, $u_a$, $u_b$ and $u_c$ are the AC voltages and $L_{arm}$ is the arm inductor.

As the three-phase arm power fluctuations are symmetrical under steady states, phase $A$ is taken as an example. The positive directions of the voltage and current are defined in Fig. 1. Then, the voltages $u_{a0}$ and $u_{a1}$ across the upper and lower arms can be expressed as:

$$
\begin{align*}
    u_{a0} &= \frac{1}{2} U_{dc} \left(1 - m \sin \omega t \right), \\
    u_{a1} &= \frac{1}{2} U_{dc} \left(1 + m \sin \omega t \right),
\end{align*}
$$

where $\omega$ is the fundamental angular frequency. $m$ is the modulation index of the hybrid MMC, which can be expressed as:

$$
    m = \frac{2 U_{ac}}{U_{dc}},
$$

where $U_{ac}$ is the amplitude of AC voltage. Considering the second-harmonic circulating current, the upper and lower arm currents $i_{a0}$ and $i_{a1}$ can be expressed as:

$$
\begin{align*}
    i_{a0} &= \frac{1}{2} I_m \left( -m \cos \phi_1 + \sin \left( \omega t - \phi_1 \right) + k_2 \sin(2 \omega t + \phi_2) \right), \\
    i_{a1} &= \frac{1}{2} I_m \left( -m \cos \phi_1 - \sin \left( \omega t - \phi_1 \right) + k_2 \sin(2 \omega t + \phi_2) \right),
\end{align*}
$$

where $\phi_1$ is the power factor angle, $I_m$ is the amplitude of AC current and $\phi_2$ is the initial phase angle of the second-harmonic circulating current. $k_2$ is the second-harmonic injection coefficient which is calculated by $2 I_2/I_m$ and $I_2$ is the amplitude of the second-harmonic circulating current.

The instantaneous arm power can be obtained by substituting (1) and (3) into the following expression:

$$
\begin{align*}
    P_{a0} &= u_{a0} \times i_{a0}, \\
    P_{a1} &= u_{a1} \times i_{a1},
\end{align*}
$$

i.e.,

$$
\begin{align*}
    P_{a0} &= \frac{U_{a0} U_m}{4} \left( -\frac{1}{2} m^2 \cos \phi_1 \sin \omega t + \sin(\omega t - \phi_1) + \frac{1}{2} m \cos(2 \omega t - \phi_1) \right. \\
    &+ k_2 \sin(2 \omega t + \phi_2) + \frac{1}{2} m k_2 \cos(3 \omega t + \phi_2) - \frac{1}{2} m k_2 \cos(\omega t + \phi_2) \right), \\
    P_{a1} &= \frac{U_{a1} U_m}{4} \left( -\frac{1}{2} m^2 \cos \phi_1 \sin \omega t - \sin(\omega t - \phi_1) + \frac{1}{2} m \cos(2 \omega t - \phi_1) \right. \\
    &+ k_2 \sin(2 \omega t + \phi_2) - \frac{1}{2} m k_2 \cos(3 \omega t + \phi_2) + \frac{1}{2} m k_2 \cos(\omega t + \phi_2) \\
    &+ k_2 \sin(2 \omega t + \phi_2) - \frac{1}{2} m k_2 \cos(3 \omega t + \phi_2) + \frac{1}{2} m k_2 \cos(\omega t + \phi_2) \right).
\end{align*}
$$
B. Mathematical Model of Arm Power Fluctuation under Grid-side SLG Faults

To block the zero-sequence current, the Y/Δ winding configuration is usually used for the converter interface transformer. Therefore, the AC voltages are only comprised of positive- and negative-sequence components. To suppress the negative-sequence current under grid-side faults, the negative-sequence double-closed loop controller is applied. Fig. 2 presents the control diagram of hybrid MMCs with the second-harmonic current injection.

Fig. 2. Control diagram of hybrid MMCs with second-harmonic current injection.

The control system consists of two parts: the AC reference voltage controller and the second-harmonic current injection controller. The AC reference voltage controller includes positive- and negative-sequence closed loop controllers which are typical control in MMC[27]. $i_{d_{refP}}$ and $i_{q_{refP}}$ are the references of positive-sequence $dq$-axis currents, which generate the positive-sequence AC reference voltage $e_{d_{refP}} (j=α, β, γ)$ through $dq$ inverse transformation. $i_{d_{refN}}$ and $i_{q_{refN}}$ are the references of negative-sequence $dq$-axis currents. $e_{d_{refN}}$ is negative-sequence AC reference voltage. The second-harmonic current injection controller uses the references of the second-harmonic currents of $dq$-axis $i_{2d_{ref}}$ and $i_{2q_{ref}}$ to generate the reference of the common-mode voltage $U_{cm_{ref}}$. It is noted that $i_{2d_{ref}}$ and $i_{2q_{ref}}$ are determined by the amplitude and phase of the injected second-harmonic circulating current. This part takes a solid SLG fault in phase $A$ as an example to analyze. After the fault, the positive- and negative-sequence AC voltages are:

\[
\begin{align*}
    u_{as,p} &= \frac{2}{3} U_m \sin\omega t \\
    u_{bc,p} &= \frac{2}{3} U_m \sin(\omega t - \frac{2}{3} \pi) \\
    u_{cs,p} &= \frac{2}{3} U_m \sin(\omega t + \frac{2}{3} \pi) \\
    u_{as,n} &= \frac{1}{3} U_m \sin(\omega t - \frac{2}{3} \pi) \\
    u_{bc,n} &= \frac{1}{3} U_m \sin(\omega t + \frac{2}{3} \pi) \\
    u_{cs,n} &= \frac{1}{3} U_m \sin(\omega t + \frac{2}{3} \pi)
\end{align*}
\]

According to (6) and (7), the three-phase upper arm voltages can be obtained:

\[
\begin{align*}
    u_{as,f} &= \frac{1}{2} U_{dc} - \frac{2}{3} U_m \sin\omega t - \frac{1}{3} U_m \sin(\omega t - \frac{2}{3} \pi) \\
    u_{bc,f} &= \frac{1}{2} U_{dc} - \frac{2}{3} U_m \sin(\omega t - \frac{2}{3} \pi) - \frac{1}{3} U_m \sin(\omega t + \frac{2}{3} \pi) \\
    u_{cs,f} &= \frac{1}{2} U_{dc} - U_m \sin(\omega t + \frac{2}{3} \pi)
\end{align*}
\]

To avoid overcurrent during the fault, the transmission power will be reduced to two-thirds of the rated power. Then, the upper arm currents can be expressed as:

\[
\begin{align*}
    i_{as,f} &= \frac{1}{2} \sqrt{3} \tan \phi_l \sin(\omega t + \phi_l) + I_m \cos(\omega t - \frac{2}{3} \pi) \\
    + I_2 \sin(2\omega t + \phi_l) \\
    i_{bc,f} &= \frac{1}{2} \sqrt{3} \tan \phi_l \sin(\omega t + \phi_l) + I_m \cos(\omega t - \frac{2}{3} \pi) \\
    + I_2 \sin(2\omega t + \phi_l + \frac{2}{3} \pi) \\
    i_{cs,f} &= \frac{1}{2} I_m \cos(\omega t - \phi_l + \frac{2}{3} \pi) \\
    + I_2 \sin(2\omega t + \phi_l - \frac{2}{3} \pi)
\end{align*}
\]

Substituting (8) and (9) into (5), the upper arm power can be obtained:
fluctuation of phase states will increase the capacitor voltage fluctuation under C.

Before \( t=2.0 \text{ s} \), the system is in the steady state. The SLG fault occurs at \( t=2.0 \text{ s} \). Before \( t=3.0 \text{ s} \), the circulating current elimination method in [7] is applied. Then, after \( t=3.0 \text{ s} \), the second-harmonic current injection scheme proposed in [19] is activated. It can be seen that, after \( t=3.0 \text{ s} \), the capacitor voltage fluctuation becomes severer, which may damage the semiconductor devices and capacitors.

III. PROPOSED OPTIMAL SECOND-HARMONIC CURRENT INJECTION SCHEME

The arm power fluctuation changes along with the capacitor voltage ripples. A large voltage ripple would lead to a large arm power fluctuation. The fundamental-frequency and second-harmonic components are dominating the three-phase arm power fluctuations, which will be derived in this section. Based on this, an optimal second-harmonic current injection strategy in hybrid MMCs under SLG Faults is proposed to suppress the three-phase arm power fluctuation.

A. Extraction of the Amplitude of Arm Power Fluctuation

From equation (10), the fundamental-frequency and second-harmonic components \( P_{a_f1} \) and \( P_{a_f2} \) of the upper arm power fluctuation in phase \( A \) can be obtained:

\[
P_{a_f1} = \frac{U_u I_m}{4} \left[ -\frac{1}{3} \left( \frac{1}{2} + \sqrt{3} \tan \phi \right) m^2 \cos \phi \sin \omega t + \frac{1}{6} \tan \phi \right] \cos \phi \sin \omega t
\]

\[
- \frac{1}{3} \frac{1}{2} \left( \frac{1}{2} + \sqrt{3} \tan \phi \right) m^2 \cos \phi \sin \omega t + \cos \phi + \frac{2}{3} \pi \right] + \frac{1}{3} \frac{1}{2} \left( \frac{1}{2} + \sqrt{3} \tan \phi \right) m^2 \cos \phi \sin \omega t + \cos \phi + \frac{2}{3} \pi \right] + \frac{1}{3} \frac{1}{2} \left( \frac{1}{2} + \sqrt{3} \tan \phi \right) m^2 \cos \phi \sin \omega t + \cos \phi + \frac{2}{3} \pi \right] + \frac{1}{3} \frac{1}{2} \left( \frac{1}{2} + \sqrt{3} \tan \phi \right) m^2 \cos \phi \sin \omega t + \cos \phi + \frac{2}{3} \pi \right]
\]

It can be observed from (5) and (10) that the power fluctuation of phase \( C \) remains constant as pre-fault and the power fluctuations of phases \( A \) and \( B \) are quite different from the pre-fault state.

C. SM Voltage Behavior under SLG Faults

In this section, it is proved through simulation that the second-harmonic current injection strategy used for steady states will increase the capacitor voltage fluctuation under grid-side SLG faults. This part takes the steady state with \( m=1 \) and \( \cos \phi =0.8 \) as an example to analyze. Fig. 3 presents the capacitor voltage fluctuation of hybrid MMC with the second-harmonic current injection strategy used for steady states.

Before \( t=2.0 \text{ s} \), the system is in the steady state. The SLG fault occurs at \( t=2.0 \text{ s} \). Before \( t=3.0 \text{ s} \), the circulating current
B. Parameter Determination Method of the Proposed Injection Strategy

According to the previous analysis, the behaviors of arm power fluctuation of the three phases are different from each other. For the SLG fault in phase \( C \), the power fluctuation of phase \( A \) remains constant as pre-fault. However, the power fluctuations of phases \( A \) and \( B \) are quite different from the pre-fault state. It can be seen from (11)-(14) that the factors that affect the amplitudes of power fluctuation include the power factor angle \( \varphi_1 \), modulation ratio \( m \), second-harmonic injection coefficient \( k_2 \) and initial phase angle of second-harmonic circulating current \( \varphi_2 \). Therefore, the objective function to minimize the three-phase power fluctuation can be defined as:

\[
\begin{align*}
\text{min } P_f (\varphi_1, m, k_2, \varphi_2) &= P_{f1}(\varphi_1, m, k_2, \varphi_2) + P_{f2}(\varphi_1, m, k_2, \varphi_2) \\
\text{s.t. } & \begin{aligned}
P_{f1}(\varphi_1, m, k_2, \varphi_2) &= |P_{su_{f1}}| + |P_{bu_{f1}}| + |P_{su_{f2}}| \\
P_{f2}(\varphi_1, m, k_2, \varphi_2) &= |P_{su_{f1}}| + |P_{bu_{f1}}| + |P_{su_{f2}}| 
\end{aligned}
\end{align*}
\]  
(15)

In order to ensure that the power fluctuation with the proposed strategy is less than that without the proposed strategy, the following inequality constraint should be satisfied:

\[
\begin{align*}
P_{f1}(\varphi_1, m, k_2, \varphi_2) &\leq P_{f1}(\varphi_1, m, 0, 0) \\
P_{f2}(\varphi_1, m, k_2, \varphi_2) &\leq P_{f2}(\varphi_1, m, 0, 0) 
\end{align*}
\]  
(16)

In order to avoid the significant increase of the maximum arm current after activating the second harmonic injection, the second harmonic injection coefficient should meet:

\[|k_2| \leq 1.\]  
(17)

The optimal process of the proposed method is shown in Fig. 4. First, the system parameters will be uploaded, and the initial value and traversal step of each variable will be determined. Considering that a dynamic adjustment of \( m \) may occur during operation, the optimal process can be divided into two modes. One mode can optimize \( k_2 \) and \( \varphi_2 \) to minimize the objective function when \( \varphi_1 \) and \( m \) are fixed, which can obtain a local optimum. The other one can optimize \( m, k_2 \) and \( \varphi_2 \) to minimize the objective function. The selection of the mode depends on the user demands. Dynamic adjustment of \( m \) can be realized by adjusting the DC voltage, which will affect the amplitude of power fluctuation. According to (11)-(14), increasing the modulation ratio dynamically, which means reducing the DC voltage, is helpful to reduce the amplitude of power fluctuation. Therefore, the optimal modulation ratio needs to be greater than the modulation ratio \( m_{ori} \) in steady states.

Supposing \( m \in [m_{ori}, 3] \), \( \varphi_2 \in [-\pi, \pi] \), \( k_2 \in [-3, 3] \), \( m_{ori}, \varphi_{20} \) and \( k_{20} \) are the initial values of \( m, \varphi_2 \) and \( k_2 \) (\( m_{ori}=m_{ori}, \varphi_{20}=-\pi, k_{20}=-3 \)). \( d_1, d_2 \) and \( d_3 \) are the corresponding traversal steps. Then, the variables are inserted to (14) to calculate the amplitude of the fundamental frequency power fluctuation and second-harmonic power fluctuation of each phase, i.e., \( |P_{su_{f1}}| \) and \( |P_{su_{f2}}| \) (\( x=a, b, c \)). \( |P_{su_{f1}}| \) and \( |P_{su_{f2}}| \) are used to calculate the value of objective function \( P_f \) which is recorded with corresponding parameters as a data group \( (\varphi_1, m, k_2, \varphi_2, P_f) \). Then the values of \( \varphi_2, m \) and \( k_2 \) will be updated and the producers will keep repeating until each parameter reaches their maximum. Finally, the optimal parameters can be obtained by finding the data group with the minimum objective function value.

It should be mentioned that the proposed optimal process is an offline process. The proposed strategy can obtain the appropriate optimal harmonic injection parameters for all operating conditions. Generally, the steady-state operation condition of the converter station is known. According to the steady-state modulation ratio and power factor, the amplitude and phase of the second harmonic needed to be injected after the fault can be calculated in advance and embedded in the control system. Upon an SLG fault, the proposed strategy can be put into operation immediately after fault detection. [28]
proposed an SLG fault detection method which takes 10 ms to detect SLG faults.

To clearly show the performance of the proposed strategy, the arm power fluctuation calculated based on (10) with \( m=1 \) and \( \cos \theta_1=1 \) is illustrated in Fig. 5. The power fluctuation is in per unit under a base of \( U_{dc}/4 \). It can be seen that the three-phase arm power fluctuations are effectively reduced with the proposed strategy.

**C. Performance Evaluation**

Under steady states, the capacitor voltage fluctuation of the three phases is symmetrical in hybrid MMCs, so the performance of the second-harmonic current injection strategy under steady states can be evaluated by the reduction of the capacitor voltage fluctuation of one phase. The single-phase SM voltage fluctuation mitigation rate \( \eta_j \) is defined as:

\[
\eta_j = \frac{\Delta U_{jx} - \Delta U_{jy}}{\Delta U_{jx}} ,
\]

where \( \Delta U_{jx} \) and \( \Delta U_{jy} \) are the amplitudes of capacitor voltage fluctuation of phase \( j \) without and with the second-harmonic current injection strategy. However, under SLG faults, the three-phase capacitor voltage fluctuations are different from each other. \( \eta_j \) is not enough to evaluate the performance of the second-harmonic current injection strategy under SLG faults. Then, the average voltage fluctuation mitigation rate \( \eta_{ave} \) is defined:

\[
\eta_{ave} = \frac{\eta_a + \eta_b + \eta_c}{3} .
\]

**IV. Simulation Results**

In order to verify the effectiveness of the proposed optimal second-harmonic current injection strategy, a hybrid MMC simulation model has been developed in PSCAD/EMTDC. System parameters of the hybrid MMC are provided in Table I.

### Table I Parameters of the Simulation Model

<table>
<thead>
<tr>
<th>Items</th>
<th>Values</th>
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<tbody>
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<td>DC voltage</td>
<td>460 kV</td>
</tr>
<tr>
<td>Grid-side voltage (rms)</td>
<td>380 kV</td>
</tr>
<tr>
<td>Arm reactor</td>
<td>0.056 H</td>
</tr>
<tr>
<td>Leakage inductance</td>
<td>0.1 p.u.</td>
</tr>
<tr>
<td>Converter capacity</td>
<td>800 MW</td>
</tr>
<tr>
<td>Number of FB-SMs per arm</td>
<td>188</td>
</tr>
<tr>
<td>Number of HB-SMs per arm</td>
<td>80</td>
</tr>
<tr>
<td>Capacitance of SM</td>
<td>5100 μF</td>
</tr>
<tr>
<td>Capacitor voltage reference</td>
<td>2 kV</td>
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</tbody>
</table>

**A. Operation Under Unit Power Factor Conditions**

Fig. 6 presents the simulation results of hybrid MMC with the proposed strategy under the unit power factor condition.
Phase A

Fig. 6. Simulation results with different modulation ratio in steady states. (a) m = 0.8 and cosφ1 = 0.8. Simulation results of hybrid MMC with different modulation ratio in steady states are shown in Fig. 8. It can be concluded that the proposed strategy can effectively reduce the capacitor voltage fluctuation with different mori. The smaller the modulation ratio in steady states is, the greater the average voltage fluctuation reduction rate is. The average voltage fluctuation reduction rate is 55.77% when mori is 0.8, which is close to 3.5 times of that when mori is 1.3.

B. Impact of Modulation Ratio

To study the impact of the modulation ratio m ori in steady states on the proposed strategy, m ori = 0.8 and m ori = 1.3 are selected for simulations with cosφ1 = 1. The parameters optimized by the proposed strategy are m = 1.2, k2 = 0.47 and φ2 = -90° with m ori = 0.8 and that are m = 1.3, k2 = 0.44 and φ2 = -90° with m ori = 1.3. Simulation results of hybrid MMC with different m ori are shown in Fig. 7.

C. Impact of Power Factor

To investigate the impact of the power factor on the proposed strategy, two operating conditions are considered with m ori = 1: cosφ1 = 0.9 and cosφ1 = 0.8. The parameters optimized by the proposed strategy are m = 1.3, k2 = -0.53 and φ2 = 57.78° with cosφ1 = 0.9 and m = 1.4, k2 = 0.57 and φ2 = 134.74° with cosφ1 = 0.8. Simulation results of hybrid MMC with different cosφ1 are shown in Fig. 8.

Figs. 6 and 8 show that the amplitude of voltage fluctuation is inversely proportional to cosφ1. It can be seen that when the
power factor decreases, the reactive power will increase, which will lead to the increase of voltage fluctuation. To quantify the impact of $\cos \phi_1$, the voltage fluctuation mitigation results under different $\cos \phi_1$ are investigated. The obtained results are listed in Table III.

According to Table III, the power factor does not affect the effectiveness of the proposed strategy. Among the three operating conditions, the proposed strategy with $\cos \phi_1=0.8$ has the best performance in restraining voltage fluctuation, as $\eta_{\text{ave}}$ reaches 52.07%.

<table>
<thead>
<tr>
<th>$\cos \phi_1$</th>
<th>$\eta_a$</th>
<th>$\eta_b$</th>
<th>$\eta_c$</th>
<th>$\eta_{\text{ave}}$</th>
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<tbody>
<tr>
<td>1</td>
<td>33.14%</td>
<td>29.9%</td>
<td>66%</td>
<td>43%</td>
</tr>
<tr>
<td>0.9</td>
<td>42.56%</td>
<td>28.88%</td>
<td>73.7%</td>
<td>48.38%</td>
</tr>
<tr>
<td>0.8</td>
<td>49.53%</td>
<td>30.69%</td>
<td>76%</td>
<td>52.07%</td>
</tr>
</tbody>
</table>

**D. Verification of Arm Power Fluctuation Model**

In order to verify the effectiveness of the proposed arm power fluctuation model under grid-side SLG faults, results of simulation and calculation under a grid-side SLG fault are shown in Fig. 9.

It is observed from Fig. 9 that whether the proposed strategy is put into operation or not, the proposed arm power model shows a good agreement with the simulation results. The results show that the proposed model can well characterize the arm power fluctuation characteristics during the fault.

**E. Comparison of Steady-State Strategy and the Proposed Strategy**

A dual harmonic current injection strategy for reducing the SM voltage fluctuation in hybrid MMCs has been proposed in [26]. However, it aims to mitigate the SM voltage fluctuation under the steady-state operation. In order to compare strategy in [26] and the proposed strategy after an SLG fault occurs in phase $A$ are given in Fig. 10.

Fig. 10. Capacitor voltage fluctuation under the control strategies in [26] and our paper.

The simulation conditions in Fig. 10 are the same as those in Fig. 6. It can be calculated that the voltage fluctuation mitigation rates of phases $A$, $B$, and $C$ with the strategy in [26] are 4.09%, 6.8% and 32.34%, while the proposed strategy achieves 33.14%, 29.9% and 66%. Simulation results show that the strategy in [26] performs less effective in suppressing the capacitor voltage fluctuation under grid-side SLG faults compared to the proposed strategy in this paper.

**V. Conclusion**

In this paper, the mathematical models of arm power fluctuations of MMC have been developed under steady states and grid-side SLG faults. It has been proved that the second-harmonic current injection strategy used for steady states will increase capacitor voltage fluctuation under grid-side SLG faults. Based on the theoretical analysis, an optimal second-harmonic current injection strategy is proposed for capacitor voltage fluctuation reduction in hybrid MMCs under grid-side SLG faults. With the proposed strategy, the three-phase power fluctuation under grid-side SLG faults can be reduced by optimizing the modulation ratio, second-harmonic injection coefficient and initial phase angle of the second-harmonic circulating current. In this case, the capacitor voltage fluctuation can be reduced at the same time. Simulation results show that the average voltage fluctuation mitigation rate can be 55.77% if the proposed strategy is applied. Moreover, it is concluded that the modulation ratio in steady states and the power factor do not affect the effectiveness of the proposed strategy.

**REFERENCES**


From equation (10), the fundamental-frequency and second-harmonic components of the upper arm power fluctuation in phase B can be obtained:

\[
P_{bu,1} = \frac{U_L I_m}{4} \left\{ -\frac{1}{3} \left( \frac{1}{2} \sqrt{3} \sin \phi_1 \right) \right. \cos \phi_1 \sin (\omega t + \frac{2}{3} \pi) \\
- \frac{1}{2} \frac{1}{3} \sqrt{3} \sin \phi_1 \right. \cos \phi_1 \sin (\omega t - \frac{2}{3} \pi) \\
- \frac{1}{2} \frac{1}{3} \sqrt{3} \sin \phi_1 \left. \cos \phi_1 \sin (\omega t + \frac{2}{3} \pi) \right] \\
- \frac{1}{2} \frac{1}{3} \sqrt{3} \sin \phi_1 \left. \cos \phi_1 \sin (\omega t - \frac{2}{3} \pi) \right] \\
+ k_2 \sin (2\omega t + \phi_2 + \frac{2}{3} \pi) \\
+ k_2 \sin (2\omega t - \phi_2 - \frac{2}{3} \pi)
\]

Using the trigonometric transformation, \( P_{bu,1} \) and \( P_{bu,2} \) can be rewritten as:

\[
P_{bu,1} = A_{bu,1} \sin \omega t + B_{bu,1} \cos \omega t
\]

\[
A_{bu,1} = \frac{U_L I_m}{4} \left[ \cos (\phi_1 + \frac{2}{3} \pi) + \frac{1}{3} m \sin (\phi_1 - \frac{2}{3} \pi) \right]
\]

\[
B_{bu,1} = \frac{U_L I_m}{4} \left[ \frac{1}{3} \sqrt{3} \sin \phi_1 \sin (\omega t + \frac{2}{3} \pi) \right]
\]

\[
P_{bu,2} = A_{bu,2} \sin 2\omega t + B_{bu,2} \cos 2\omega t
\]

\[
A_{bu,2} = \frac{U_L I_m}{4} \left[ k_2 \cos (\phi_2 + \frac{2}{3} \pi) - \frac{1}{3} m \sin \phi_2 - \frac{\sqrt{3}}{12} \sin \phi_1 \right]
\]

\[
B_{bu,2} = \frac{U_L I_m}{4} \left[ k_2 \sin (\phi_2 + \frac{2}{3} \pi) + \frac{\sqrt{3}}{12} \cos \phi_1 \right]
\]

From equation (10), the fundamental-frequency and second-harmonic components of the upper arm power fluctuation in phase C can be obtained:

\[
P_{cu,1} = \frac{U_L I_m}{4} \left[ -\frac{1}{2} m \cos \phi_1 \sin (\omega t + \frac{2}{3} \pi) + \sin (\omega t - \phi_1 + \frac{2}{3} \pi) \right]
\]

\[
- \frac{1}{2} m \cos (\omega t + \phi_1 + \frac{2}{3} \pi)
\]

\[
P_{cu,2} = \frac{U_L I_m}{4} \left[ -\frac{1}{2} m \cos (2\omega t - \phi_1 + \frac{4}{3} \pi) + k_2 \sin (2\omega t + \phi_2 + \frac{4}{3} \pi) \right]
\]
Using the trigonometric transformation, $P_{cu_f1}$ and $P_{cu_f2}$ can be rewritten as:

$$
P_{cu_f1} = A_{cu_f1} \sin(\omega t + \frac{2}{3} \pi) + B_{cu_f1} (\omega t + \frac{2}{3} \pi)$$

$$
A_{cu_f1} = \frac{U_m L_m}{4} \left[ - \frac{1}{2} m k_1 \cos \phi_1 + \cos \phi_1 + \frac{1}{2} m k_2 \sin \phi_2 \right],
$$

$$
B_{cu_f1} = \frac{U_m L_m}{4} \left[ - \sin \phi_1 + \frac{1}{2} m k_2 \cos \phi_2 \right]
$$

$$
P_{cu_f2} = A_{cu_f2} \sin(2\omega t + \frac{4}{3} \pi) + B_{cu_f2} \cos(2\omega t + \frac{4}{3} \pi)$$

$$
A_{cu_f2} = \frac{U_m L_m}{4} \left[ \frac{1}{2} m \sin \phi_1 + k_2 \cos \phi_2 \right],
$$

$$
B_{cu_f2} = \frac{U_m L_m}{4} \left[ \frac{1}{2} m \cos \phi_1 + k_2 \sin \phi_2 \right].
$$

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