

Cardiff Economics Working Papers



Working Paper No. E2021/31

A General Model of International Tax Competition with Applications

Helmuts Azacis and David R Collie

December 2021

ISSN 1749-6010

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

This working paper is produced for discussion purpose only. These working papers are expected to be published in due course, in revised form, and should not be quoted or cited without the author's written permission. Cardiff Economics Working Papers are available online from:
<http://econpapers.repec.org/paper/cdfwpaper/> and
business.cardiff.ac.uk/research/academic-sections/economics/working-papers
Enquiries: EconWP@cardiff.ac.uk

A General Model of International Tax Competition with Applications

Helmut Azacis and David R Collie

Cardiff Business School, Cardiff University

December 2021

Abstract

A general version of the ZMW model of international tax competition is presented that confirms and extends the results of the existing literature about the choice of tax policy instruments in the symmetric case when the tax externality is positive for both countries. In the asymmetric case when the tax externality is positive for one country and negative for the other country, it is shown that the results are reversed. This demonstrates the importance of the sign of the tax externality in models of international tax competition. This general model is then used to analyse a couple of policy-relevant applications: depreciation allowances and interest payment deductibility.

Keywords: Tax Competition; Proportional Taxes; Per-Unit Taxes; Capital Taxes.

JEL Classification: H21; H25; H77; F21; F23, F53, C72.

1. Introduction

Corporate income tax is currently a controversial issue with many commentators concerned about the small amounts of tax paid by some firms, especially multinational firms, that is often believed to be the result of increased international tax competition that has led governments to lower corporate income tax rates over recent decades.¹ Although corporate tax revenue is significant in many countries, it is not one of the main sources of tax revenue. According to the OECD (2021), on average in 2018, corporate tax revenue accounted for 15.3% of total tax revenue or 3.2% of GDP for the 105 jurisdictions for which data was available.² Faced with growing budget deficits due to the Covid-19 pandemic, governments seem to be inclined to increase corporate income taxes but are concerned about the limitations on their ability to raise these taxes when capital is internationally mobile. On the 8th October 2021, 136 countries and jurisdictions representing more than 90% of global GDP agreed to a minimum rate of corporate income tax of 15% on multinational firms.³ For firms, what matters is the effective tax rate, which depends upon the various allowances available to firms such as R&D and depreciation allowances. Therefore, governments that agree to a minimum statutory rate of corporate income tax could still engage in international tax competition by aggressively using these allowances. According to OECD (2021), on average in 2020, accelerated depreciation reduced the effective rate of tax by 1.5%, while the largest reductions were 3.5% in the United States and 3.4% in Italy. Governments are also concerned about multinational firms engaging in profit-shifting by using debt interest payments to move profits from high-tax jurisdictions to low-tax jurisdictions. Action 4 of the OECD/G20 Inclusive Framework on Base Erosion and Profit Shifting (BEPS) has recommended limits on the deductibility of debt

¹ These important policy issues are discussed in a couple of recently published books, see de Mooij, Klemm, and Perry (2021) and Devereux et al. (2021).

² For further details, see the [OECD Corporate Tax Statistics Database](#).

³ See [Statement on a Two-Pillar Solution to Address the Tax Challenges Arising from the Digitalisation of the Economy – 8 October 2021](#).

interest, and limits have been introduced by a number of countries including the UK and all the member states of the EU.⁴

The substantial literature that models tax competition as a game between countries (or regions) setting taxes began with the articles by Zodrow and Mieszkowski (1986) and Wilson (1986), and the resulting ZMW model soon became the standard or workhorse model of international tax competition.⁵ In a world with two or more countries, capital is internationally mobile and countries tax capital at source to fund a public good. When a country increases its tax rate, capital will move to the other countries thereby increasing the tax base and welfare of the other countries. This positive externality in tax setting leads countries to set lower taxes on capital and provide less of the public good than if the countries were maximising their joint welfare.⁶ Hence, co-operation as recently suggested by the US can make all countries better off if an agreement can be reached and enforced. An important issue in this literature is how the choice of tax policy instrument affects the outcome of the tax competition game. Using the ZMW model, Wildasin (1988) shows that the Nash equilibrium when governments set per-unit taxes on capital is not the same as the Nash equilibrium when governments set the level of expenditure on the public good.⁷ Extending this analysis, but with only two countries, Wildasin (1991) considers a two-stage game where governments first commit to the type of tax policy instrument and then set the level of the tax policy instrument. Assuming symmetry and that the production technology is quadratic, he shows that using per-unit taxes dominates setting the level of expenditure on the public good for both countries. Per-unit taxes are compared with proportional (*ad valorem*) taxes on capital by Lockwood (2004) who shows that tax revenue,

⁴ See [OECD BEPS Action 4](#). The EU (and the UK) now limit debt interest deductibility to 30% of earnings before interest, tax, depreciation, and amortisation (EBITDA).

⁵ For surveys of the theoretical literature, see Wilson (1999), Wilson and Wildasin (2004), and Keen and Konrad (2013).

⁶ In their survey, Wilson and Wildasin (2004) consider models where tax competition may have a positive effect on welfare such as the Leviathan model analysed by Edwards and Keen (1996).

⁷ In the same way that under oligopoly the Nash equilibrium in quantities is not the same as the Nash equilibrium in prices.

expenditure on the public good, and welfare will be higher when countries use per-unit taxes. This analysis has been extended by Akai, Ogawa, and Ogawa (2011) in a symmetric model with two countries and a quadratic production function to show that per-unit taxes dominate proportional taxes on capital in a two-stage game. Tax competition between asymmetric countries is considered by Ogawa (2016) who assumes a quadratic production function and that the private consumption and the ‘public good’ are perfect substitutes. In a two-stage game, he shows that the country that imports capital will set a proportional tax on capital and the country that exports capital will set a per-unit tax on capital.⁸

This paper will address a number of these issues using a general model of international tax competition. In particular, it will consider how the type of tax policy instrument (for instance, a proportional or a per-unit tax) affects international tax competition in a one-shot, simultaneous-move game; in a Stackelberg game where one country acts as a leader in the setting of tax; and how it affects the sustainability of international tax agreements in an infinitely-repeated game. This general model will also be used to analyse how depreciation allowances and interest rate deductibility affect international tax competition.

The novel contribution in this paper is to use a general approach in the analysis of the international tax competition game. Rather than assuming symmetry and/or making restrictive assumptions about the production function and/or the utility function, the analysis will start from assumptions about how the taxes of the countries affect the world net return on capital, total tax revenue, and the welfare of the countries. This analysis will bring to the fore the importance for the results of the sign of the tax externality, which is usually assumed to be positive. It will also allow a comparison between three types of tax policy instrument rather than just the binary comparisons in the existing literature. The technique used to compare the

⁸ Since the private consumption good and the ‘public good’ are perfect substitutes, the motivation for taxes on capital is to improve the terms of trade and hence the capital exporter uses a subsidy to capital and the capital importer uses a tax on capital.

tax policy instruments will be the same as that used by Vives (1985) to compare prices and quantities under Cournot and Bertrand oligopoly. He analysed Cournot oligopoly by assuming that each firm was setting prices to maximise profits subject to a constraint that its output was a given quantity, where the constraint was the demand function. By using this technique, the best-reply functions under Cournot and Bertrand oligopoly can all be shown in the price-space, which allows easy comparisons between Cournot and Bertrand oligopoly. This approach was used by Wildasin (1991), but not using the general approach that will be employed in this paper.

2. A Two-Country Tax Competition Game

A general model of international tax competition with two countries will be presented using only the minimal assumptions required to compare tax policy instruments. In order to justify and motivate the assumptions of the general model, a version of the standard ZMW model of international tax competition will first be presented. There are two countries in the world and two factors production, where capital is internationally mobile, but labour is internationally immobile. It is assumed that there is perfect competition, so firms are price-takers in the goods market and in the factor markets, and hence factors of production are paid their marginal products. The factor endowment of the j th country is \bar{L}_j of labour and \bar{K}_j of capital, where $j = 1, 2$. A single consumption good that acts as the numeraire is produced using a constant returns to scale production function $y = f_j(K_j, \bar{L}_j)$, where K_j is the amount of capital employed in the j th country. Since the labour employed in the country is equal to the labour endowment, which is exogenous, the production function can be written without L_j as $f_j(K_j)$, where as usual it is assumed that $f_j' > 0$ and $f_j'' < 0$. The marginal product of capital $f_j'(K_j)$ is the gross return to capital, and the net return to capital is: $r_j = f_j'(K_j) - \delta$, where δ is the depreciation rate that is assumed to be the same in both countries. In the j th country,

there is a proportional tax τ_j on the gross return to capital employed in the country so the after-tax net return to capital is $(1-\tau_j)r_j - \tau_j\delta$, which will be the same in both countries due to international capital mobility, and will be equal to the world net return to capital r .⁹ The total tax revenue of each country is used to pay for a public good g_j , and it is assumed that each unit of the public good is produced using one unit of the numeraire good so $g_j = \tau_j(r_j + \delta)K_j = \tau_j f_j'(K_j)K_j$. Countries are assumed to tax capital at source and to have no other sources of tax revenue available to them to fund the public good. Consumption of the numeraire good is c_j and the utility of the representative consumer in the j th country is $u_j(c_j, g_j)$, which is assumed to be strictly increasing and strictly quasi-concave in both goods. Consumption of the numeraire good is given by the after tax income of the representative consumer, who supplies all the labour and owns the endowment of capital, so $c_j = f_j(K_j) - f_j'(K_j)K_j + r\bar{K}_j$, where $f_j(K_j) - f_j'(K_j)K_j$ is labour income since there are constant returns to scale, and $r\bar{K}_j$ is the net capital income. Since the model is static and represents the steady-state of the economy, it is assumed that the representative consumer invests $\delta\bar{K}_j$ to maintain the capital endowment.¹⁰

International capital mobility will equalise the after-tax net return to capital in the two countries, which will be equal to the world net return to capital:

$$(1-\tau_1)f_1'(K_1) - \delta = r = (1-\tau_2)f_2'(K_2) - \delta \quad (1)$$

⁹ For a detailed review of the effects of taxation on the user cost of capital, see Creedy and Gemmell (2017).

¹⁰ However, it does not matter if the representative consumer allows the capital endowment to depreciate as the depreciation rate and the capital endowment are exogenous so it will not affect the qualitative results.

Totally differentiating this equation, and noting that $dK_2 = -dK_1$ since the world supply of capital is fixed, $K_1 + K_2 = \bar{K}_1 + \bar{K}_2$, yields the derivatives of capital and the world net return to capital with respect to the two taxes:

$$\begin{aligned} \frac{\partial K_1}{\partial \tau_1} = -\frac{\partial K_2}{\partial \tau_1} = \frac{f_1'}{\Delta} < 0, & \quad \frac{\partial K_1}{\partial \tau_2} = -\frac{\partial K_2}{\partial \tau_2} = \frac{-f_2'}{\Delta} > 0 \\ \frac{\partial r}{\partial \tau_1} = \frac{-(1-\tau_2)f_1'f_2''}{\Delta} < 0, & \quad \frac{\partial r}{\partial \tau_2} = \frac{-(1-\tau_1)f_2'f_1''}{\Delta} < 0 \end{aligned} \quad (2)$$

Since $\Delta \equiv (1-\tau_1)f_1'' + (1-\tau_2)f_2'' < 0$, these results are unambiguous given the assumptions about the production function. An increase in a country's tax rate decreases the amount of capital employed in that country and increases the amount employed in the other country, and decreases the world net return to capital.

The welfare of each country is given by the utility of the representative consumer in that country so the welfare of the j th country is:

$$W_j = u_j(c_j, g_j) = u_j\left(f_j(K_j) - f_j'(K_j)K_j + r\bar{K}_j, \tau_j f_j'(K_j)K_j\right) \quad (3)$$

It will turn out that the results on the comparison of tax policy instruments depend crucially upon the sign of the tax externality. The tax externality is positive (negative) if an increase in the other country's tax rate increases (decreases) a country's welfare, $\partial W_j / \partial \tau_i > (<) 0$, and in models of tax competition it is generally assumed to be positive. To see whether there is a positive tax externality, differentiate (3) with respect to the tax rate of i th country, which yields:

$$\frac{\partial W_j}{\partial \tau_i} = \frac{\partial u_j}{\partial c_j} \left[\left(\bar{K}_j - \frac{K_j}{1-\tau_j} \right) \frac{\partial r}{\partial \tau_i} \right] + \frac{\partial u_j}{\partial g_j} \left[-\tau_j f_j'' K_j (\varepsilon_j^K - 1) \frac{\partial K_j}{\partial \tau_i} \right] \quad i \neq j \quad (4)$$

where $\varepsilon_j^K = -f_j' / f_j'' K_j > 0$ is the elasticity of the derived demand for capital in the j th country. The effect of τ_i on the welfare of the j th country through consumption of the numeraire good is the terms of trade effect as the world net return to capital decreases, $\partial r / \partial \tau_i < 0$.¹¹ When $\tau_j = 0$, this effect is negative (positive) if the j th country is a net exporter (importer) of capital, $\bar{K}_j - K_j > (<) 0$, and equal to zero in the symmetric case when both countries are identical so that $K_j = \bar{K}_j$. When $\tau_j > 0$, this terms of trade effect would be positive in the symmetric case and if the two countries are sufficiently similar, $(\bar{K}_j - K_j) / \bar{K}_j < \tau_j$ if the j th country is a net exporter. This condition says that the proportion of the capital endowment exported by a country is less than its tax rate. The effect of τ_i on the welfare of the j th country through consumption of the public good is the tax base effect, $\partial g_j / \partial \tau_i$, which will be positive if there is an increase in the tax revenue of the j th country. Total tax revenue (and hence total expenditure on the public good) will increase, $\partial g_j / \partial \tau_i > 0$, if $\tau_j > 0$ and the derived demand for capital is elastic, $\varepsilon_j^K > 1$. Hence, if the two countries are sufficiently similar and the derived demand for capital is elastic then both the terms of trade effect and the tax base effect will be positive so the tax externality will be positive, $\partial W_j / \partial \tau_i > 0$ if $\tau_j > 0$ for both countries.

2.1 A General Two-Country Model of International Tax Competition

Now consider a general two-country model of international tax competition that makes minimal assumptions about the world net return to capital, the total tax revenue (expenditure on the public good), and the welfare of each country as functions of the capital tax rates of the two countries.¹² This allows general results to be obtained about the comparison of tax policy

¹¹ With a proportional tax, it is not exactly the terms of trade effect as it does not include the tax revenue from imported or exported capital, which is included in the second term in (4).

¹² A similar general approach is used by Azacis and Collie (2020) to compare *ad valorem* and specific trade taxes, and to show that The Lerner Symmetry Theorem does not hold for specific trade taxes in a trade war.

instruments, and highlights the importance of the assumption about the tax externality. Residents in each country own endowments of labour, which is internationally immobile, and capital, which is internationally mobile. Perfectly competitive firms in both countries employ capital and labour to produce a consumption good using a constant returns to scale technology.¹³ A public good is financed by a tax on capital, and one unit of the public good is produced using one unit of the consumption good. Capital is taxed at source, and it is assumed that there are no other sources of government revenue. The world net return to capital $r(\tau_1, \tau_2)$ is a function of the proportional taxes on the gross return to capital of the two countries, τ_1 and τ_2 . The world gross return to capital is $r(\tau_1, \tau_2) + \delta$, where δ is the depreciation rate, which is assumed to be exogenous.¹⁴ In line with the standard ZMW model, see (2), it is assumed that an increase in the tax rate of either country will decrease the world net return to capital. It is assumed that if both countries increase their tax rate by the same amount, then the user cost of capital will not decrease in either country, which implies that $r + \delta + (1 - \tau_j)(\partial r / \partial \tau_j + \partial r / \partial \tau_i) \geq 0$.¹⁵ The key assumptions about the world net return to capital are:

Assumption A1: *The world net return to capital $r(\tau_1, \tau_2)$ is decreasing in the tax rates, $\partial r / \partial \tau_j < 0$ for $j = 1, 2$. The user cost of capital in a country is non-decreasing if both countries increase their tax rate by the same amounts so $r + \delta + (1 - \tau_j)(\partial r / \partial \tau_j + \partial r / \partial \tau_i) \geq 0$.*

¹³ In fact, there could be any number of goods and factors of production as long as the assumptions below are satisfied.

¹⁴ Most models of international tax competition do not consider depreciation and it does not affect the results until the analysis is applied to depreciation allowances.

¹⁵ In the ZMW model, since the world endowment is fixed, both countries imposing taxes at the same rate will not increase the user cost of capital. The assumption implies that one country increasing its tax rate will increase the user cost of capital in that country.

To allow for the possibility that the governments maximise tax revenue rather than welfare or that their strategic variable is expenditure on the public good rather than tax rates as in Wildasin (1988, 1991), some assumptions will be made about total tax revenue (or government expenditure on the public good). The total tax revenue of a country $g_j(\tau_1, \tau_2) = \tau_j(r + \delta)K_j / (1 - \tau_j)$ is a function of the proportional tax rates on the gross return to capital of the two countries. Total tax revenue is assumed to be strictly quasi-concave in its own tax rate and, to ensure that expenditure on the public good (and the tax rate) is always positive, it is assumed that $\partial g_j(0, \tau_i) / \partial \tau_j > 0$ for all $\tau_i \geq 0$. In line with the standard ZMW model when the derived demand for capital is elastic as in (4), it is assumed that the total tax revenue of a country is increasing in the other country's tax rate. The key assumptions about the total tax revenue of each country are:

Assumption A2: *The total tax revenue (expenditure on the public good) of the j th country $g_j(\tau_1, \tau_2)$ is strictly quasi-concave in its own tax rate and increasing in the tax rate of the other country, $\partial g_j / \partial \tau_i > 0$ for $\tau_j > 0$ and $i \neq j$. Also, $\partial g_j(0, \tau_i) / \partial \tau_j > 0$ for all $\tau_i \geq 0$.*

Note that a welfare-maximising government will never operate on the downward-sloping part of the Laffer curve so $\partial g_j / \partial \tau_j > 0$.

The welfare of each country $W_j(\tau_1, \tau_2)$, given by the utility from consuming the consumption good and the public good, is a function of the proportional tax rates of the two countries, and the welfare of each country is assumed to be strictly quasi-concave in its own tax rate. Also, to ensure that expenditure on the public good (and the tax rate) is always positive, it is assumed that $\partial W_j(0, \tau_i) / \partial \tau_j > 0$ for all $\tau_i \geq 0$. In line with the ZMW model when the tax base effect and the terms of trade effect are both positive, as in (4), the tax externality is assumed to be positive. The key assumptions about the welfare of each country are:

Assumption A3: The welfare of the j th country $W_j(\tau_1, \tau_2)$ is strictly quasi-concave in its own tax rate and increasing in the tax rate of the other country, $\partial W_j / \partial \tau_i > 0$ for $\tau_j > 0$ and $i \neq j$. Also, $\partial W_j(0, \tau_i) / \partial \tau_j > 0$ for all $\tau_i \geq 0$.

Note that no assumption has been made about whether tax rates are strategic complements, $\partial^2 W_j / \partial \tau_i \partial \tau_j > 0$, or strategic substitutes, $\partial^2 W_j / \partial \tau_i \partial \tau_j < 0$. The usual assumption in models of tax competition is that tax rates are strategic complements, and diagrams will be drawn for this case, but the analysis holds for both strategic substitutes and strategic complements. Also, note that the assumptions about the welfare function and the total tax revenue function are the same so a similar analysis can be used for the case of a welfare maximising government or a tax revenue maximising government.

Each country has two decisions to make about tax policy in this model. First, each country has to decide whether to use a proportional tax on the gross return to capital and this choice is denoted by T_j^r ; a per-unit tax on capital and this choice is denoted by T_j^t ; or to use total tax revenue (or expenditure on the public good) as its strategic variable and this choice is denoted by T_j^g , where $j = 1, 2$.¹⁶ Then, each country has to decide the rate for the chosen tax policy instrument with the proportional tax rate denoted by τ_j , the per-unit tax rate denoted by t_j , and total tax revenue (or expenditure on the public good) denoted by \bar{g}_j for $j = 1, 2$.

Now consider international tax competition when both countries choose to use proportional taxes on the gross return to capital. As usual, this can be modelled as the Nash equilibrium (NE) in proportional taxes. In the NE, each country independently and simultaneously sets its proportional tax on capital to maximise its welfare given the

¹⁶ Lockwood (2004) describes the proportional tax as an *ad valorem* tax and the per-unit tax as a specific tax, but that terminology seems more relevant to consumption taxes. Wildasin (1988) suggests expenditure on the public good as a strategic variable in tax competition models.

proportional tax set by the other country. Hence, when both countries use proportional taxes, the first-order conditions for the NE in taxes are:

$$\frac{\partial W_1(\tau_1, \tau_2)}{\partial \tau_1} = 0, \quad \frac{\partial W_2(\tau_1, \tau_2)}{\partial \tau_2} = 0 \quad (5)$$

The equation on the left implicitly defines the best-reply function of country one, $\tau_1 = \tau_1(\tau_2, T_2^r)$, and the equation on the right implicitly defines the best-reply function of country two, $\tau_2 = \tau_2(\tau_1, T_1^r)$, where T_1^r and T_2^r denote that both countries have chosen to use proportional taxes. The best-reply functions are shown in figure 1 for the usual case when taxes are strategic complements, and under the assumption that the two countries are symmetric. If one country sets a zero tax then the other country will set a proportional tax $\tau_j^* > 0$, $j = 1, 2$. The intersection of the two best-reply functions is the NE in proportional taxes, which is assumed to be unique. The welfare of each country is represented in figure 1 by the iso-welfare loci where $W_1^N(T_1^r, T_2^r)$ and $W_2^N(T_1^r, T_2^r)$ are the welfare of country one and country two, respectively, in the NE in proportional taxes. The shape of the iso-welfare loci follows from assumption A3 that the tax externality is positive for both countries, $\partial W_j / \partial \tau_i > 0$. If the two countries cooperate in the setting of taxes on capital, then they could reach the (symmetric) cooperative outcome at C where the iso-welfare loci of the two countries are tangential. Clearly, the cooperative outcome yields higher tax rates and higher welfare than the non-cooperative NE, which shows that tax competition leads to lower taxes and lower welfare for both countries.

In this NE in proportional taxes, since firms are perfectly competitive, each country is indifferent between using a proportional tax on the gross return to capital or an equivalent per-unit tax on capital given the capital tax set by the other country. However, as Lockwood (2004)

has shown the type of capital tax chosen by the other country affects the best-reply function of a country. Now consider the best-reply function of country 1 when country 2 uses a per-unit tax so now country 1 sets its proportional capital tax given that country 2 sets a per-unit tax t_2 . This can be analysed in the (τ_1, τ_2) -space in figure 1 by considering a constrained optimisation problem as in Vives (1985), who compared the Cournot oligopoly and Bertrand oligopoly outcomes in the price-space.¹⁷ This requires that τ_2 is interpreted as the proportional tax that is equivalent to the per-unit tax set by country 2. For country 2, a per-unit tax t_2 is equivalent to a proportional tax on the gross return to capital τ_2 if the tax burden on each unit of capital is the same so the equivalence condition (or constraint) is:

$$t_2 = \tau_2 \frac{r(\tau_1, \tau_2) + \delta}{1 - \tau_2} \quad (6)$$

This constraint implicitly defines an equivalent proportional tax rate for country 2 that is equivalent to its per-unit tax rate t_2 , and this equivalent proportional tax rate is a function of the proportional tax rate set by country 1. Totally differentiating the equivalence constraint while keeping the per-unit tax rate, t_2 , constant, yields that:

$$\left. \frac{d\tau_2}{d\tau_1} \right|_{t_2=\bar{t}_2} = - \frac{\partial t_2 / \partial \tau_1}{\partial t_2 / \partial \tau_2} = \frac{-\tau_2(1-\tau_2)(\partial r / \partial \tau_1)}{r + \delta + \tau_2(1-\tau_2)(\partial r / \partial \tau_2)} > 0 \quad (7)$$

The numerator is clearly positive, and the denominator is positive given assumption A1 about the user cost of capital. Furthermore, assumption A1 implies that the derivative is less than one, which will ensure that the NE taxes when all countries use per-unit taxes are lower than the cooperative taxes. Hence, an increase in the proportional tax rate of country 1 increases

¹⁷ Under oligopoly, firms are indifferent between setting prices or quantities given their conjecture about the prices or quantities set by their competitors, but their choice affects the best-reply functions of their competitors. Wildasin (1991) discussed what he called *constrained* non-cooperative games in his conclusion, but did not use this approach to obtain general results.

the equivalent proportional tax rate of country 2 so the equivalence constraint is upward sloping in (τ_1, τ_2) -space as shown in figure 2. When country 1 sets its proportional tax in response to country 2 setting a per-unit tax, it maximises its welfare subject to the equivalence constraint (6), which yields the first-order condition:

$$\frac{dW_1}{d\tau_1} = \frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2} \frac{d\tau_2}{d\tau_1} \Big|_{t_2=\bar{t}_2} = 0 \quad (8)$$

Since the slope of the equivalence constraint is positive (7) and the tax externality is positive, $\partial W_1/\partial \tau_2 > 0$, $\partial W_1/\partial \tau_1 < 0$ and quasi-concavity implies that the proportional capital tax set by country 1 is higher when country 2 sets a per-unit tax than when country 2 sets a proportional tax. The situation is shown in figure 2 where the best-reply function of country 1 when country 2 uses a proportional tax $\tau_1(\tau_2, T_2^r)$ is obtained by maximising the welfare of country 1 given the proportional tax of country 2, so when $\tau_2 = \tau_2^\alpha$ then the optimum is at α where $\tau_1 = \tau_1^\alpha = \tau_1(\tau_2^\alpha, T_2^r)$.¹⁸ When country 2 uses a per-unit tax and the per-unit tax set by country 2 is equivalent to the proportional tax τ_2^α then the optimum is at β where $\tau_1 = \tau_1^\beta = \tau_1(\tau_2^\alpha, T_2^t) > \tau_1^\alpha = \tau_1(\tau_2^\alpha, T_2^r)$. Hence, when country 2 switches from using a proportional tax to using a per-unit tax, the best-reply function of country 1 swivels clockwise around $(\tau_1^*, 0)$. A similar analysis can be used to derive the best-reply function for country 2 when country 1 uses a per-unit tax $\tau_2(\tau_1, T_1^t)$.

Wildasin (1988, 1991) analysed tax competition when the strategic variable chosen by the governments was the level of expenditure on the public good and compared it to a per-unit tax. Now consider the best-reply function of country 1 when country 2 sets its expenditure on

¹⁸ The diagram is similar to that used by Cheng (1985) to illustrate the analysis in Vives (1985).

the public good so country 1 sets its proportional tax optimally given that country 2 sets expenditure on the public good equal to \bar{g}_2 . This requires that the government of country 2 raises sufficient total tax revenue to finance expenditure on the public good so $\bar{g}_2 = g_2(\tau_1, \tau_2)$, which is the equivalence constraint that defines the proportional tax τ_2 that is equivalent to \bar{g}_2 and is a function of τ_1 . Totally differentiating the equivalence constraint while holding expenditure on the public good \bar{g}_2 constant yields that:

$$\left. \frac{d\tau_2}{d\tau_1} \right|_{g_2=\bar{g}_2} = -\frac{\partial g_2/\partial \tau_1}{\partial g_2/\partial \tau_2} < 0 \quad (9)$$

The numerator is positive if the tax base effect is positive as assumed in A2, $\partial g_2/\partial \tau_1 > 0$, and the denominator is positive if country 2 is maximising welfare and hence on the upward-sloping section of the Laffer curve, $\partial g_2/\partial \tau_2 > 0$. Hence, an increase in the proportional tax rate of country 1 decreases the equivalent proportional tax rate of country 2 so the equivalence constraint is downward sloping in (τ_1, τ_2) -space as shown in figure 3. Country 1 maximises its welfare subject to the equivalence constraint, hence the first-order condition is:

$$\left. \frac{dW_1}{d\tau_1} = \frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2} \frac{d\tau_2}{d\tau_1} \right|_{g_2=\bar{g}_2} = 0 \quad (10)$$

Since the slope of the equivalence constraint is negative (9) and the tax externality is positive, $\partial W_1/\partial \tau_2 > 0$, $\partial W_1/\partial \tau_1 > 0$ and quasi-concavity implies that the proportional capital tax set by country 1 is lower when country 2 sets the level of expenditure on the public good than when country 2 sets a proportional tax. The situation is shown in figure 3. When country 2 sets the level of expenditure on the public good and the level of expenditure on the public good set by country 2 is equivalent to the proportional tax τ_2^α then the optimum is at γ where

$\tau_1 = \tau_1^g = \tau_1(\tau_2^\alpha, T_2^g) < \tau_1^\alpha = \tau_1(\tau_2^\alpha, T_2^r)$. Hence, when country 2 switches from using a proportional tax to setting the level of expenditure on the public good, the best-reply function of country 1 swivels anti-clockwise around $(\tau_1^*, 0)$. A similar analysis can be used to derive the best-reply function for country 2 when country 1 sets the level of expenditure on the public good $\tau_2(\tau_1, T_1^g)$.

The best-reply functions for both countries for all three types of tax policy instrument are shown in figure 4. The three best reply functions of country 1 intersect the three best-reply functions of country 2 nine times and are labelled from i to ix . In a static one-stage game where both countries independently and simultaneously choose the type of tax policy instrument and the level of the instrument then all nine intersections are NE. Assuming that the countries are symmetric, which is the assumption in figure 4, and that they use the same type of tax policy instrument, $T^z = \{T_1^z, T_2^z\}$ where $z = \tau, t, g$, then the NE will be symmetric in terms of equivalent proportional taxes so $\tau_1^N(T^z) = \tau_2^N(T^z) = \tau^N(T^z)$, and welfare so $W_1^N(T^z) = W_2^N(T^z) = W^N(T^z)$. Comparing the symmetric NE (ii , iv and ix) the NE equivalent proportional taxes are lowest when both countries set the level of expenditure on the public good and highest when both countries use per-unit taxes, $\tau^N(T^g) < \tau^N(T^r) < \tau^N(T^t)$. The welfare of both countries is increasing along the diagonal, where $\tau_1 = \tau_2 = \tau$ so $dW_j(\tau, \tau)/d\tau > 0$ for $j = 1, 2$, up to the symmetric cooperative outcome, C . Hence, the NE welfare is lowest when both countries use the level of expenditure on the public good and highest when both countries set per-unit taxes, $W^N(T^g) < W^N(T^r) < W^N(T^t)$. This leads to the following proposition:

Proposition 1: Under assumptions A1 to A3 and assuming symmetry, the equivalent proportional taxes in the NE are such that $\tau^N(T^g) < \tau^N(T^\tau) < \tau^N(T^t)$ and welfare in the NE is such that $W^N(T^g) < W^N(T^\tau) < W^N(T^t)$.

This proposition generalises the results of Wildasin (1988, 1991), who showed that $\tau^N(T^g) < \tau^N(T^t)$ and $W^N(T^g) < W^N(T^t)$, and Lockwood (2004), who showed that $\tau^N(T^\tau) < \tau^N(T^t)$ and $W^N(T^\tau) < W^N(T^t)$, and provides a seemingly novel result that $\tau^N(T^g) < \tau^N(T^\tau)$ and $W^N(T^g) < W^N(T^\tau)$. The key insight from this proposition is that the results are quite general as the only significant assumption is that the tax externality is positive, and notably it does not matter if taxes are strategic substitutes or strategic complements.¹⁹ Also, since assumptions about total tax revenue A2 and welfare A3 are the same, the results would hold if both governments maximised total tax revenue or they both maximised a convex sum of welfare and government revenue, $V_j = (1-\lambda)W_j + \lambda g_j$ where $\lambda \in [0,1]$.

In the symmetric case, although there are multiple NE, it seems reasonable to suggest that both countries will choose to use per-unit taxes since this NE Pareto dominates the other symmetric NE where both countries use the same tax policy instrument. A unique outcome can be ensured if the structure of the game is changed.

2.2 Two-Stage International Tax Competition Game

Consider the two-stage game where each country independently and simultaneously chooses the type of tax policy instrument (expenditure on the public good, T_j^g ; a proportional tax, T_j^τ ; or a per-unit tax, T_j^t ; where $j = 1, 2$) to use at stage one, and then sets the level of the

¹⁹ If the tax externality was negative then the best-reply functions would swivel in the opposite directions and the ranking of the equivalent proportional taxes in the NE would be reversed, but the ranking of welfare in the NE would remain the same as both countries would be better off with lower taxes in the symmetric NE.

tax policy instrument (\bar{g}_j , τ_j , or t_j) at stage two. The nine possible NE of the second stage of the game are shown in figure 4 in the (τ_1, τ_2) -space in terms of the equivalent proportional taxes, and these NE depend upon the type of tax policy instrument chosen by the countries in the first stage. At the first stage, if country 2 chooses to use a proportional tax then the best-reply function of country 1 will be $\tau_1(\tau_2, T_2^r)$ so the NE in the second stage will be *i* if country 1 uses a per-unit tax, *ii* if it uses a proportional tax, and *iii* if it sets the level of expenditure on the public good. Country 1 will choose to use a per-unit tax as the NE *i* gives it a higher level of welfare than *ii* or *iii*, as the proportional tax of country 2 is higher in the NE *i* than in *ii* or *iii* and there is a positive tax externality. A similar argument can be used to show that country 1 will choose to use a per-unit tax if country 2 uses a per-unit tax or sets the level of expenditure on the public good. Hence, choosing to use a per-unit tax is a dominant strategy for country 1, and by the same argument it is a dominant strategy for country 2. Therefore, the subgame-perfect NE of this two-stage international tax competition game is for both countries to choose to use per-unit taxes, and for the outcome to be given by *iv* in figure 4. In the symmetric case, as shown in Proposition 1, both countries are better off choosing to use per-unit taxes than when they both use proportional taxes or set the level of expenditure on the public good. This leads to the following proposition:

Proposition 2: *In the subgame-perfect NE of the two-stage game, under assumptions A1 to A3, both countries choose to use per-unit taxes. In the symmetric case, both countries are better off than when they both choose to use proportional taxes or set the level of expenditure on the public good.*

This proposition generalises the results of Wildasin (1991), who showed that countries choosing to use per-unit taxes dominates choosing to set the level of expenditure on the public good, and Akai, Ogawa, and Ogawa (2011) who showed that countries choosing to use per-

unit taxes dominates proportional taxes in an extension of Lockwood (2004) using a quadratic production function and where countries maximise total tax revenue. They both assumed symmetry whereas Proposition 2 allows for asymmetries as long as the tax externality is positive. It also provides a novel result that countries choosing to use proportional taxes dominates choosing to set the level of expenditure on the public good.

However, it is possible that the tax externality could be negative for one of the countries if it was a sufficiently large exporter of capital while the tax externality would be positive for the other country since it would be an importer of capital.²⁰ Figure 5 shows the situation when the tax externality is positive for country 1, $\partial W_1/\partial \tau_2 > 0$, and negative for country 2, $\partial W_2/\partial \tau_1 < 0$. The best-reply functions of country 1 are unaltered compared to figure 4, but the best-reply functions of country 2 swivel in the opposite direction, see (8) and (10), so their relative positions are reversed compared to figure 4. In the first stage, if country 2 chooses to use proportional taxes then the best-reply function of country 1 is $\tau_1(\tau_2, T_2^r)$ so the NE will be *i* if country 1 chooses to set the level of expenditure on the public good, *ii* if it chooses to use a proportional tax, and *iii* if it chooses to use a per-unit tax. Clearly, country 1 will choose to set the level of expenditure on the public good as NE *i* gives it a higher level of welfare than *ii* or *iii* since for it the tax externality is positive so it prefers country 2 to set a higher proportional tax. A similar argument can be used to show that country 1 will choose to set the level of expenditure on the public good if country 2 chooses to set the level of expenditure on the public good or to use a per-unit tax. Hence, choosing to set the level of expenditure on the public good is a dominant strategy for country 1. If country 1 chooses to set a proportional tax at stage one then the best-reply function of country 2 is $\tau_2(\tau_1, T_1^r)$ so the NE will be *viii* if country 2 chooses

²⁰ In the ZMW model, the tax externality would be negative for a country that was a sufficiently large exporter of capital as then the negative terms of trade effect would outweigh the positive tax base effect, see (4).

to set the level of expenditure on the public good, *ii* if it chooses to use a proportional tax, and *v* if it chooses to use a per-unit tax. Country 2 will choose to set the level of public expenditure on the public good as NE *viii* gives it a higher level of welfare than *ii* or *v* since for it the tax externality is negative and it prefers country 1 to set a lower proportional tax. A similar argument can be used to show that country 2 will choose to set the level of expenditure on the public good if country 1 chooses to set the level of expenditure on the public good or to use a per-unit tax. Therefore, for both countries, choosing to set the level of expenditure on the public good is a dominant strategy, and the outcome is the NE *vii* in figure 5. These results lead to the following proposition:

Proposition 3: *In the subgame-perfect NE of the two-stage game, when the tax externality is positive for one country and negative for the other country, both countries choose to set the level of expenditure on the public good.*

Surprisingly, introducing an asymmetry in terms of the tax externality between the countries, leads both countries to change from using per-unit taxes to setting the level of expenditure on the public good. Note that in this case proportional taxes dominate per-unit taxes. It may seem that this Proposition 3 is contradicted by the result of Ogawa (2016) who finds that the capital exporter uses a per-unit tax and the capital importer uses a proportional (*ad valorem*) tax. In his model, which assumes a quadratic production function, the public good and the consumption good are perfect substitutes in the utility of the representative consumer, but consumption of the public good may be negative so it is really a government lump-sum transfer to (or tax on) the representative consumer. This implies that the sole motivation for the government to use a capital tax is the terms of trade effect, which is positive for the capital importer and negative for the capital exporter, and hence the capital importer uses a capital tax and the capital exporter uses a capital subsidy.

This situation can be handled quite easily by the current model with some adaption and is shown in figure 6. Suppose that country 1 is the capital importer so $\tau_1^* > 0$ and the tax externality is positive, $\partial W_1 / \partial \tau_2 > 0$, due to the positive terms of trade effect, while country 2 is the capital exporter so $\tau_2^* < 0$ and the tax externality is negative, $\partial W_2 / \partial \tau_1 < 0$, due to the negative terms of trade effect. Since $\tau_2 < 0$, the slope of the equivalence constraint (7) is now negative and, when country 2 switches from using a proportional tax to using a per-unit tax, the best-reply function of country 1 shifts to the left, see (8), but still swivels clockwise around $(\tau_1^*, 0)$. Since $\tau_2 < 0$ and hence $g_2 < 0$, the slope of the equivalence constraint (9) becomes positive since $\partial g_2 / \partial \tau_1 < 0$ and, when country 2 shifts from using a proportional tax to setting the level of expenditure on the public good, the best-reply function of country 1 shifts to the right, see (10), but still swivels anti-clockwise around $(\tau_1^*, 0)$. The shifts in the best-reply functions of country 2 are the same as in figure 5 since $\tau_1 > 0$, but $\tau_2^* < 0$. Hence, from figure 6, it can be seen that for country 1 (the capital importer) setting the level of expenditure on the public good dominates using a proportional tax, which dominates using a per-unit tax. For country 2 (the capital exporter), using a per-unit tax dominates a proportional tax, which dominates setting the level of expenditure on the public good. These results are consistent with Ogawa (2016), and also show that setting the level of expenditure on the public good dominates using a proportional tax for the capital importer. Clearly, the results of Ogawa (2016) are dependent on the capital exporter using a capital subsidy whereas if it uses a capital tax (the usual assumption in the literature on tax competition) then Proposition 3 will be the relevant result.

2.3 Stackelberg International Tax Competition Game

Rather than countries setting taxes simultaneously, it has been suggested that one country may be a leader and the other country a follower so that the appropriate game is a Stackelberg leader-follower game. Altshuler and Goodspeed (2015) find evidence for the US having a leadership role in international tax competition, but they do not find evidence that the UK or Germany had a leadership role. For example, in the 2021 Budget the UK Chancellor of the Exchequer announced a future increase in corporation tax in anticipation of the US government increasing business taxation.²¹ Therefore, it seems reasonable to consider a Stackelberg game where both countries choose the type of the tax policy instrument that they will use at stage one of the game then the leader (assumed to be country 1) sets the level of its tax policy instrument in the second stage and the follower (country 2) sets the level of its tax policy instrument in the third stage.

The situation when the tax externality is positive for both countries is shown in figure 7. As is well known, the Stackelberg equilibrium occurs where the iso-welfare loci of the leader is tangential to the best-reply function of the follower. The Stackelberg equilibrium is at *i* when the leader (country 1) chooses to use a per-unit tax in stage one, at *ii* when it chooses to use a proportional tax, and at *iii* when it chooses to set the level of expenditure on the public good. Hence, at stage one, the leader (country 1) will choose to use a per-unit tax as welfare is higher at *i* than at *ii* or *iii* since there is a positive tax externality, and the follower is indifferent about its choice of a tax policy instrument since it does not affect the outcome in stage three of the game. This leads to the following proposition:

Proposition 4: *In the Stackelberg game, under assumptions A1 to A3, the leader will choose to use a per-unit tax and the follower is indifferent about the type of tax policy instrument used.*

²¹ See [‘Sunak plans corporate tax rise to bolster public finances’](#), *Financial Times*, 24th February 2021.

Since some asymmetry between countries is presumably the reason that one country is a leader and the other is a follower, it seems worthwhile to consider the Stackelberg game when there is an asymmetry in terms of the tax externality. The situation when the tax externality is negative for the leader (country 1) and positive for the follower (country 2) is shown in figure 8. The Stackelberg equilibrium is at *i* when the leader (country 1) chooses to use a per-unit tax in stage one, at *ii* when it chooses to use a proportional tax, and at *iii* when it chooses to set the level of expenditure on the public good. Hence, at stage one, the leader (country 1) will choose to set the level of expenditure on the public good as welfare is higher at *iii* than at *i* or *ii* since it has a negative tax externality, and the follower is indifferent about its choice of a tax policy instrument since it does not affect the outcome in stage three of the game.

The situation when the tax externality is positive for the leader (country 1) and negative for the follower (country 2) is shown in figure 9. With a negative tax externality, the best-reply functions of country 2 swivel in the opposite direction, see (8) and (10), so their relative positions are reversed compared to figure 7 and figure 8. The Stackelberg equilibrium is at *i* when the leader (country 1) chooses to set the level of expenditure on the public good in stage one, at *ii* when it chooses to use a proportional tax, and at *iii* when it chooses to use a per-unit tax. Hence, at stage one, the leader (country 1) will choose to set the level of expenditure on the public good as welfare is higher at *i* than at *ii* or *iii* since it has a positive tax externality. These results on the asymmetric cases lead to the following proposition:

Proposition 5: *In the Stackelberg game, when the tax externality is positive for one country and negative for the other country, the leader will choose to set the level of expenditure on the public good and the follower is indifferent about the type of tax policy instrument used.*

Surprisingly, with an asymmetry in terms of the tax externalities, the leader will choose to set the level of expenditure on the public good at stage one whether its tax externality is positive or negative.

2.4 Infinitely-Repeated International Tax Competition Game

International tax competition leads to lower levels of capital taxation than would be globally optimal and this creates the possibility that countries could all be better off if they were to cooperate in the setting of taxes. The recent global agreement on a minimum rate of corporate income tax on multinational firms is an example of cooperation. In game theory, cooperation can be sustained in an infinitely-repeated game by the use of Nash-reversion trigger strategies. Suppose that the international tax competition game is repeated infinitely with the countries choosing the type of tax policy instrument at the beginning of the game. With Nash-reversion trigger strategies, each country could set the cooperative tax rate (at point C in figure 1) as long as the other country does the same, but if either country deviates then the two countries revert to the NE tax rates forever thereafter. Then, if the discount factor is $\delta \in [0, 1]$, cooperative tax rates can be sustained if the present discounted value of welfare from cooperation exceeds the present discounted value from deviation followed by the NE welfare forever thereafter. Assuming symmetry so that the welfare function is the same for both countries, and assuming that both countries use the same type of tax policy instrument so $T_1^z = T_2^z$ for $z = \tau, t, g$. Welfare from cooperation is the same for both countries, $W_1^C = W_2^C = W^C$, and obviously does not depend upon the type of tax policy instrument chosen by the two countries.

The situation when country 1 deviates from the cooperative outcome is shown in figure 10. If the countries chose to use a proportional tax rate then when country 1 deviates from the cooperative outcome, C , it takes the proportional tax set by country 2, τ_2^C , as given and will maximise its welfare at ii , where its welfare is $W_1^D(T^\tau)$. If the countries chose to set the level of expenditure on the public good then when country 1 deviates from the cooperative outcome, C , it takes the level of expenditure on the public good set by country 2, \bar{g}_2^C , as given

and will maximise its welfare (subject to the equivalence constraint) at i , where its welfare is $W_1^D(T^g)$. If the countries chose to use a per-unit tax then when country 1 deviates from the cooperative outcome, C , it takes the per-unit tax set by country 2, t_2^C , as given and will maximise its welfare (subject to the equivalence constraint) at iii , where its welfare is $W_1^D(T^t)$. Clearly, the welfare of country 1 from deviation is highest when the countries set the level of expenditure on the public good and lowest when the countries set per-unit taxes, $W_1^D(T^g) > W_1^D(T^\tau) > W_1^D(T^t)$, and since the countries are symmetric, $W_1^D(T^z) = W_2^D(T^z) = W^D(T^z)$ for $z = \tau, t, \bar{g}$. Following any deviation, the countries will revert to the NE in taxes where welfare depends upon the type of tax policy instrument chosen by the countries. According to Proposition 1, welfare in the NE is highest when the countries use per-unit taxes and lowest when they set the level of expenditure on the public good, $W^N(T^g) < W^N(T^\tau) < W^N(T^t)$.

Hence, when the two countries are symmetric, cooperation can be sustained in the infinitely-repeated game using Nash-reversion trigger strategies if:

$$\frac{1}{1-\delta}W^C > W^D(T^z) + \frac{\delta}{1-\delta}W^N(T^z) \quad (11)$$

Cooperation is sustainable if the discount factor is greater than the critical value obtained by making the above expression into an equality and solving for the discount factor, which depends upon the type of tax policy instrument chosen by the countries:

$$\delta > \delta_N(T^z) \equiv \frac{W^D(T^z) - W^C}{W^D(T^z) - W^N(T^z)} \quad z = \tau, t, g \quad (12)$$

An increase in the welfare from deviation increases the critical discount factor as does an increase in welfare in the NE. Hence, since $W^D(T^g) > W^D(T^\tau) > W^D(T^t)$ and $W^N(T^g) < W^N(T^\tau) < W^N(T^t)$, the effect of the choice of tax policy instrument on the critical discount factor is ambiguous. This leads to the following proposition:

Proposition 6: *In the infinitely-repeated game, the welfare from deviation is highest when countries set the level of expenditure on the public good and lowest when the countries use per-unit taxes, $W^D(T^g) > W^D(T^\tau) > W^D(T^t)$, while welfare in the NE is highest when countries use per-unit taxes and lowest when they set the level of expenditure on the public good, $W^N(T^t) > W^N(T^\tau) > W^N(T^g)$.*

Alternatively, the countries could use per-unit taxes in the cooperative phase, which minimises the incentive to deviate, and then following any deviation set the level of expenditure on the public good, which maximises the punishment in the NE for deviating from the cooperative outcome.

3. Applications

The modelling techniques employed in Section 2 can be used to analyse how the tax treatment of depreciation allowances and debt interest deductibility affect the outcome of international tax competition.

3.1 Depreciation Allowances

The first example is the case of depreciation allowances where firms are allowed to deduct depreciation in the calculation of capital taxes so that they are taxed on the net return to capital rather than the gross return to capital. Since proportional taxes are the most relevant form of taxation from a policy perspective, a proportional tax on the net return to capital (i.e. with a depreciation allowance) will be compared to a proportional tax on the gross return to

capital. Assume that the tax externality is positive for both countries. The best reply functions of the two countries when both countries use a proportional tax on the gross return to capital, $\tau_1(\tau_2, T_2^r)$ and $\tau_2(\tau_1, T_1^r)$ are shown in figure 11, where i is the NE. Suppose that the proportional tax on the net return to capital is σ_j in the j th country and let T_j^σ denote that it has chosen to use a proportional tax on the net return to capital. Then, a proportional tax on the net return to capital is equivalent to a proportional tax on the gross return to capital if $\sigma_j r_j = \tau_j (r_j + \delta)$. Since the gross return to capital in the j th country less capital taxes is equal to the world gross return, $(1 - \tau_j)(r_j + \delta) = r + \delta$, the gross return in the j th country is $r_j + \delta = (r + \delta) / (1 - \tau_j)$. Hence, a proportional tax on the net return to capital is equivalent to a proportional tax on the gross return if $\sigma_j = \tau_j (r + \delta) / (r + \tau_j \delta)$ so a proportional tax on the net return has to be higher than one on the gross return, $\sigma_j > \tau_j$, if they are both to raise the same total tax revenue. Now consider the best-reply function of country 1 when country 2 uses a proportional tax on the net return to capital so that country 1 sets its proportional tax on the gross return given that country 2 sets a proportional tax on the net return, σ_2 . For country 2, the equivalence constraint is:

$$\sigma_2 = \tau_2 \frac{r(\tau_1, \tau_2) + \delta}{r(\tau_1, \tau_2) + \tau_2 \delta} \quad (13)$$

Totally differentiating the equivalence constraint while holding the proportional tax on the net return constant, yields that:

$$\left. \frac{d\tau_2}{d\tau_1} \right|_{\sigma_2 = \bar{\sigma}_2} = - \frac{\partial \sigma_2 / \partial \tau_1}{\partial \sigma_2 / \partial \tau_2} = \frac{\delta \tau_2 (1 - \tau_2) (\partial r / \partial \tau_1)}{r(r + \delta) - \delta \tau_2 (1 - \tau_2) (\partial r / \partial \tau_2)} < 0 \quad (14)$$

The numerator is clearly negative and the denominator is clearly positive. Hence, an increase in the proportional tax on the gross return by country 1 will decrease the equivalent proportional tax on the gross return of country 2 so the equivalence constraint is downward sloping. When country 1 sets its proportional tax on the gross return, it maximises its welfare subject to the equivalence constraint (13), which yields the first-order condition:

$$\left. \frac{dW_1}{d\tau_1} = \frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2} \frac{d\tau_2}{d\tau_1} \right|_{\sigma_2 = \bar{\sigma}_2} = 0 \quad (15)$$

Since the constraint is downward sloping according to (14) and $\partial W_1 / \partial \tau_2 > 0$, $\partial W_1 / \partial \tau_1 > 0$ so the best-reply function shifts inwards, $\tau_1(\tau_2, T_2^\sigma) < \tau_1(\tau_2, T_2^\tau)$, so it swivels anti-clockwise as shown in figure 11. A similar analysis can be used to show that the best-reply function of country 2 also shifts inwards so it swivels clockwise in figure 11 when country 1 switches from setting a proportional tax on the gross return to capital to setting one on the net return.

The best-reply functions for both countries for both types of tax policy instrument are shown in figure 11. The two best-reply functions of country 1 intersect the two best-reply functions of country 2 four times and are labelled from i to iv . In a static one-stage game where both countries independently and simultaneously choose the type of tax policy instrument and the level of the instrument then all four intersections are NE. Assuming that the countries are symmetric, which is the assumption in figure 11, and that they both use the same type of tax policy instrument, $T^z = \{T_1^z, T_2^z\}$ where $z = \tau, \sigma$, then the NE will be symmetric in terms of equivalent proportional tax rates (on the gross return to capital) so $\tau_1^N(T^z) = \tau_2^N(T^z) = \tau^N(T^z)$ and in terms of welfare so $W_1^N(T^z) = W_2^N(T^z) = W^N(T^z)$. Similarly to Proposition 1, comparing the two symmetric NE (i and iv) the NE equivalent proportional taxes and NE

welfare are lower when both countries have depreciation allowances, $\tau^N(T^\sigma) < \tau^N(T^\tau)$ and $W^N(T^\sigma) < W^N(T^\tau)$. Taxes and welfare would be higher in the NE if countries did not give depreciation allowances.

In the two-stage game, where countries choose the type of tax to use at stage one and then set the level of the tax at stage two, it should be clear from figure 11 that whatever type of tax country 2 chooses at stage one then country 1 is better off if it chooses to use a tax on the gross return to capital. Hence, similarly to Proposition 2, choosing to use a proportional tax on the gross return to capital is a dominant strategy for both countries so the outcome will be the NE at i in figure 11. Countries giving depreciation allowances is dominated by them not giving depreciation allowances and using a proportional tax on the gross return to capital. In a Stackelberg international tax competition game, similarly to Proposition 4, it can be shown that the leader will choose not to give depreciation allowances and will use a proportional tax on the gross return to capital.²²

3.2 Interest Rate Deductibility

The second example is the case of interest rate deductibility where firms subtract interest payments on debt in the calculation of capital taxes so that firms are taxed on the net return to capital (net of interest payments on debt, but with no depreciation allowance) rather than the gross return.²³ Suppose that the proportional tax on the net return to capital is μ_j in the j th country and let T_j^μ denote that it has chosen to use a proportional tax on the net return. Then, a proportional tax on the net return to capital is equivalent to a proportional tax on the

²² Both of these results will be reversed if the tax externality is positive for one country and negative for the other country, see Proposition 3 and Proposition 5.

²³ The OECD Base Erosion and Profit Shifting (BEPS) Project recommended in Action Item 4 that countries limit the deductibility of interest against corporate profits, <https://www.oecd.org/tax/beps/beps-actions/action4/>, and the UK has followed this recommendation in the 2017 Finance Bill.

gross return to capital if $\tau_j(r_j + \delta) = \mu_j(r_j + \delta - d_j r)$, where the interest rate on debt is assumed to be the world net rate of return on capital, $r(\tau_1, \tau_2)$, and d_j is debt per unit of capital in the j th country, which is assumed to be exogenous and $d_j \in [0, 1]$. Since $(1 - \tau_j)(r_j + \delta) = r + \delta$ or $r_j + \delta = (r + \delta)/(1 - \tau_j)$, a proportional tax on the net return is equivalent to a proportional tax on the gross return if $\mu_j = \tau_j(r + \delta)/(r + \delta - (1 - \tau_j)d_j r)$, so a proportional tax on the net return has to be higher than one on the gross return, $\mu_j > \tau_j$, if they are both to raise the same total tax revenue. Now consider the best-reply function of country 1 when country 2 uses a proportional tax on the net return to capital so country 1 sets its proportional tax on the gross return given that country 2 sets a proportional tax on the net return, μ_2 . For country 2, the equivalence constraint is:

$$\mu_2 = \tau_2 \frac{r(\tau_1, \tau_2) + \delta}{r(\tau_1, \tau_2) + \delta - (1 - \tau_2)d_2 r(\tau_1, \tau_2)} \quad (16)$$

Totally differentiating the equivalence constraint while holding the proportional tax on the net return constant, yields that:

$$\left. \frac{d\tau_2}{d\tau_1} \right|_{\mu_2 = \bar{\mu}_2} = - \frac{\partial \mu_2 / \partial \tau_1}{\partial \mu_2 / \partial \tau_2} = \frac{-\delta d_2 \tau_2 (1 - \tau_2) (\partial r / \partial \tau_1)}{(r + \delta) ((1 - d_2) r + \delta) + \delta d_2 \tau_2 (1 - \tau_2) (\partial r / \partial \tau_2)} > 0 \quad (17)$$

The numerator is positive if the depreciation rate is positive, $\delta > 0$, and the denominator is positive given assumption A1 about the user cost of capital and the assumption that debt per unit of capital is less than one, $d_2 \in [0, 1]$. Hence, an increase in the proportional tax on the gross return by country 1 will increase the equivalent proportional tax on the gross return of country 2 so the equivalence constraint is upward sloping. When country 1 sets its

proportional tax on the gross return, it maximises its welfare subject to the equivalence constraint (17), which yields the first-order condition:

$$\frac{dW_1}{d\tau_1} = \frac{\partial W_1}{\partial \tau_1} + \frac{\partial W_1}{\partial \tau_2} \frac{d\tau_2}{d\tau_1} \Big|_{\mu_2 = \bar{\mu}_2} = 0 \quad (18)$$

Since the constraint is upward sloping according to (17) and $\partial W_1 / \partial \tau_2 > 0$, $\partial W_1 / \partial \tau_1 < 0$ so the best-reply function shifts outwards, $\tau_1(\tau_2, T_2^\mu) > \tau_1(\tau_2, T_2^\tau)$, swivelling clockwise as shown in figure 12. A similar analysis can be used to show that the best-reply function of country 2 also shifts outwards (swivelling anti-clockwise in figure 12) when country 1 switches from setting a proportional tax on the gross return to capital to setting one on the net return.

The best-reply functions for both countries for both types of tax policy instrument are shown in figure 12. In a static one-stage game where both countries independently and simultaneously choose the type of tax policy instrument and the level of the instrument then all four intersections are NE. Assuming that the countries are symmetric, which is the assumption in figure 12, and that they both use the same type of tax policy instrument, $T^z = \{T_1^z, T_2^z\}$ where $z = \tau, \mu$, then the NE will be symmetric in terms of equivalent proportional tax rates (on the gross return to capital) so $\tau_1^N(T^z) = \tau_2^N(T^z) = \tau^N(T^z)$ and in terms of welfare so $W_1^N(T^z) = W_2^N(T^z) = W^N(T^z)$. Similarly to Proposition 1, comparing the two symmetric NE (*i* and *iv*) the NE equivalent proportional taxes and NE welfare are higher when both countries allow interest rate deductibility, $\tau^N(T^\mu) > \tau^N(T^\tau)$ and $W^N(T^\mu) > W^N(T^\tau)$. Taxes and welfare would be higher in the NE if both countries allowed interest rate deductibility, which suggests that the OECD BEPS recommendation on interest rate deductibility may have an undesirable effect on international tax competition.

In the two-stage game, where countries choose the type of tax to use at stage one and then set the level of the tax at stage two, it should be clear from figure 12 that whatever type of tax country 2 chooses at stage one then country 1 is better off if it choose to use a tax on the net return to capital. Hence, similarly to Proposition 2, choosing to use a proportional tax on the net return to capital is a dominant strategy for both countries so the outcome will be the NE at i in figure 12. Countries allowing interest rate deductibility dominates them not allowing interest rate deductibility and using a proportional tax on the gross return to capital. In a Stackelberg international tax competition game, similarly to Proposition 4, it can be shown that the leader will choose to allow interest rate deductibility and will use a proportional tax on the net return to capital.²⁴

4. A Multi-Country Tax Competition Game

The model can readily be extended to consider multi-country tax competition. First, assuming symmetry, the NE when all the countries use a proportional tax on capital will be compared with the NE when all the countries use a per-unit tax on capital. Second, the two-stage game where the countries choose the type of tax instrument at the first stage will be analysed.

4.1 NE of Symmetric International Tax Competition Game

At the first stage of the game, the J countries have to choose whether to use a proportional tax on capital, a choice that is denoted by T_j^r , or a per-unit tax on capital, a choice that is denoted by T_j^t , where $j = 1, \dots, J$ denotes the country. Then, each country has to decide the rate for the chosen capital tax with the proportional tax rate denoted by τ_j and the per-unit tax denoted by t_j . As before, it will be assumed that the world net rate of return to capital $r(\boldsymbol{\tau})$

²⁴ Both of these results will be reversed if the tax externality is positive for one country and negative for the other country, see Proposition 3 and Proposition 5.

is decreasing in the tax rates of all the countries, $\partial r/\partial \tau_j < 0$ for $j=1, \dots, J$, where $\boldsymbol{\tau} = (\tau_1, \dots, \tau_J)'$. Also, it will be assumed that if all countries increase their tax rates by the same amount then their user cost of capital, $(r + \delta)/(1 - \tau_j)$, will not decrease, which implies that $r + \delta + (1 - \tau_j) \sum_{i=1}^J \partial r/\partial \tau_i \geq 0$. Assumptions A1 and A3 can be replaced by the following two assumptions:

Assumption A1*: *The world net return to capital $r(\boldsymbol{\tau})$ is decreasing in the tax rates, $\partial r/\partial \tau_j < 0$ for $j=1, \dots, J$, and when all countries increase their tax rates by the same amount their user cost of capital is increasing so $r + \delta + (1 - \tau_j) \sum_{i=1}^J \partial r/\partial \tau_i \geq 0$.*

Assumption A3*: *The welfare of the j th country $W_j(\boldsymbol{\tau})$ is strictly quasi-concave in its own tax rate and increasing in the tax rate of the other countries, $\partial W_j/\partial \tau_i > 0$ for $\tau_j > 0$ and $i \neq j$. Also, $\partial W_j(0, \boldsymbol{\tau}_{-j})/\partial \tau_j > 0$ for all $\boldsymbol{\tau}_{-j} \geq 0$.*

When all countries use proportional taxes, $T^\tau = \{T_1^\tau, \dots, T_J^\tau\}$, the first-order condition for the symmetric (interior) NE, which is assumed to be unique, are:

$$\frac{\partial W_j(\boldsymbol{\tau}^N(T^\tau))}{\partial \tau_j} = 0 \quad j = 1, \dots, J \quad (19)$$

where $\boldsymbol{\tau}^N(T^\tau) = (\tau_1^N(T^\tau), \dots, \tau_J^N(T^\tau))'$ is the vector of NE tax rates when all countries use proportional taxes and, since the countries are assumed to be symmetric, $\tau_j^N(T^\tau) = \tau_h^N(T^\tau) = \tau^N(T^\tau)$ for all $j, h = 1, \dots, J$.

When all countries use per-unit taxes, $T^t = \{T_1^t, \dots, T_J^t\}$, each country maximises its welfare given the per-unit taxes set by the other $J-1$ countries. Hence, each country sets τ_j

to maximise its welfare $W_j(\boldsymbol{\tau})$ subject to the $J-1$ equivalence constraints that $t_h = \tau_h(r(\boldsymbol{\tau}) + \delta)/(1 - \tau_h)$ for all $h \neq j$. The derivative of the equivalence constraint can be obtained by totally differentiating the $J-1$ constraints then setting $\tau_i = \tau_h$ and noting that $\partial r/\partial \tau_h = \partial r/\partial \tau_i$ for all $i, h \neq j$:

$$\frac{d\tau_h}{d\tau_j} = \frac{-\tau_h(1-\tau_h)\partial r/\partial \tau_j}{r + \delta + (J-1)\tau_h(1-\tau_h)\partial r/\partial \tau_h} > 0 \quad (20)$$

The numerator is clearly positive, and the denominator is positive given assumption A1* about the user cost of capital. Furthermore, assumption A1* implies that the derivative is less than one, which ensures that the NE taxes when all countries use per-unit taxes are lower than the cooperative taxes. Hence, the first-order conditions for the symmetric NE when all countries use a per-unit tax are:

$$\frac{\partial W_j(\boldsymbol{\tau}^N(T^t))}{\partial \tau_j} + \sum_{h \neq j} \frac{\partial W_j(\boldsymbol{\tau}^N(T^t))}{\partial \tau_h} \frac{d\tau_h}{d\tau_j} = 0 \quad (21)$$

where $\boldsymbol{\tau}^N(T^t) = (\tau_1^N(T^t), \dots, \tau_J^N(T^t))'$ is the vector of NE tax rates when all countries use per-unit taxes and, since the countries are assumed to be symmetric, $\tau_j^N(T^t) = \tau_h^N(T^t) = \tau^N(T^t)$ for all $j, h = 1, \dots, J$. Since the slope of the constraint is positive, (20), and the tax externality is positive, $\partial W_j/\partial \tau_h > 0$, the derivative $\partial W_j(\boldsymbol{\tau}^N(T^t))/\partial \tau_j$ is negative in the symmetric NE where all countries use a per-unit tax. Since there is assumed to be a unique symmetric NE in proportional tax rates and $\partial W_j(\mathbf{0})/\partial \tau_j > 0$, if $\tilde{\boldsymbol{\tau}} = (\tilde{\tau}, \dots, \tilde{\tau})'$ then $\partial W_j(\tilde{\boldsymbol{\tau}})/\partial \tau_j > (<) 0$ when $\tilde{\tau} < (>) \tau^N(T^t)$. Hence, $\partial W_j(\boldsymbol{\tau}^N(T^t))/\partial \tau_j$ being negative implies that the NE taxes are higher when all countries use per-unit taxes than when all countries use proportional taxes,

$\tau^N(T^t) > \tau^N(T^\tau)$. Since the welfare of all countries is increasing in the common tax rate for tax rates below the cooperative level, the welfare of all countries is higher when all countries use per-unit taxes than when all countries use proportional taxes, $W^N(T^t) > W^N(T^\tau)$. This leads to the following proposition:

Proposition 7: *Under assumptions A1* and A3* and assuming symmetry, the equivalent proportional taxes in the NE are such that $\tau^N(T^\tau) < \tau^N(T^t)$ and welfare in the NE are such that $W^N(T^\tau) < W^N(T^t)$.*

This extends the results of Proposition 1 to the case of many countries, and provides an alternative proof of Lockwood (2004) in a more general setting. The analysis can be extended easily to compare other tax policy instruments.

5. Conclusions

This paper has analysed a general version of the ZMW model of tax competition that started from assumptions about how the equilibrium world net return to capital and welfare depend upon the capital taxes of the countries rather than starting from assumptions about the production function and utility function. This general approach confirmed and extended the results in the existing literature while demonstrating the key role of the assumption that the tax externality is positive. In the two-stage, simultaneous-move game, it was shown that governments setting a per-unit tax dominates setting a proportional tax that dominates setting total government expenditure when the tax externality is positive for both countries. However, when the tax externality was positive for one country and negative for the other country, setting total government expenditure dominated a proportional tax that dominated setting a per-unit tax for both countries. The same was true in the Stackelberg game for the leader while the follower was indifferent about the type of tax policy instrument it employed. In the infinitely-

repeated game, the type of tax policy instrument employed by the countries affected the incentives to deviate from any co-operative agreement and the punishment for any deviation, but in such a way that the effect on the critical discount factor was ambiguous.

The general model was applied to a couple of policy-relevant applications: depreciation allowances and interest rate deductibility. For depreciation allowances, it was shown in the two-stage, simultaneous move game that not having depreciation allowances was a dominant strategy for both countries in the symmetric case when the tax externality is positive for both countries. This suggests that countries should refrain from the aggressive use of depreciation allowances. For interest rate deductibility, it was shown that allowing interest rate deductibility was a dominant strategy for both countries in the symmetric case when the tax externality is positive for both countries. This suggests that the OECD BEPS recommendation that countries limit interest rate deductibility may decrease welfare, but this recommendation was mainly intended to limit profit-shifting rather than lessening the effects of international tax competition.

References

- Akai, Nobuo, Hikaru Ogawa, and Yoshitomo Ogawa. 2011. "Endogenous choice on tax instruments in a tax competition model: unit tax versus ad valorem tax." *International Tax and Public Finance* 18 (5):495-506. doi: <https://doi.org/10.1007/s10797-011-9170-3>.
- Altshuler, Rosanne, and Timothy J. Goodspeed. 2015. "Follow the Leader? Evidence on European and US Tax Competition." *Public Finance Review* 43 (4):485-504. doi: <https://doi.org/10.1177/1091142114527781>.
- Azacis, Helmut, and David R. Collie. 2020. The Non-Equivalence of Import Tariffs and Export Taxes in Trade Wars: Ad Valorem vs Specific Trade Taxes. In *Cardiff Economics Working Papers*: Cardiff University, Cardiff Business School, Economics Section.
- Cheng, Leonard. 1985. "Comparing Bertrand and Cournot Equilibria: A Geometric Approach." *The RAND Journal of Economics* 16 (1):146-152. doi: <https://doi.org/10.2307/2555596>.
- Creedy, John, and Norman Gemmill. 2017. "Taxation and the User Cost of Capital." *Journal of Economic Surveys* 31 (1):201-225. doi: <https://doi.org/10.1111/joes.12137>.
- de Mooij, Ruud A., Alexander D. Klemm, and Victoria J. Perry, eds. 2021. *Corporate Income Taxes under Pressure: Why Reform Is Needed and How It Could Be Designed*.

- Washington, DC: International Monetary Fund. doi: <https://doi.org/10.5089/9781513511771.071>.
- Devereux, Michael P., Alan J. Auerbach, Michael Keen, Paul Oosterhuis, Wolfgang Schön, and John Vella. 2021. *Taxing Profit in a Global Economy*. Oxford: Oxford University Press. doi: <https://doi.org/10.1093/oso/9780198808060.001.0001>.
- Edwards, Jeremy, and Michael Keen. 1996. "Tax competition and Leviathan." *European Economic Review* 40 (1):113-134. doi: [https://doi.org/10.1016/0014-2921\(95\)00057-7](https://doi.org/10.1016/0014-2921(95)00057-7).
- Keen, Michael, and Kai A. Konrad. 2013. "Chapter 5 - The Theory of International Tax Competition and Coordination." In *Handbook of Public Economics*, edited by Alan J. Auerbach, Raj Chetty, Martin Feldstein and Emmanuel Saez, 257-328. Elsevier. doi: <https://doi.org/10.1016/B978-0-444-53759-1.00005-4>.
- Lockwood, Ben. 2004. "Competition in Unit vs. Ad Valorem Taxes." *International Tax and Public Finance* 11 (6):763-772. doi: <https://doi.org/10.1023/B:ITAX.0000045330.53417.1f>.
- OECD. 2021. *Corporate Tax Statistics*. Third ed. Paris: Organisation for Economic Cooperation and Development.
- Ogawa, Hikaru. 2016. "When ad valorem tax prevails in international tax competition." *International Review of Economics & Finance* 46:1-9. doi: <https://doi.org/10.1016/j.iref.2016.08.002>.
- Vives, Xavier. 1985. "On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation." *Journal of Economic Theory* 36 (1):166-175. doi: [http://dx.doi.org/10.1016/0022-0531\(85\)90086-9](http://dx.doi.org/10.1016/0022-0531(85)90086-9).
- Wildasin, David E. 1988. "Nash equilibria in models of fiscal competition." *Journal of Public Economics* 35 (2):229-240. doi: [https://doi.org/10.1016/0047-2727\(88\)90055-2](https://doi.org/10.1016/0047-2727(88)90055-2).
- Wildasin, David E. 1991. "Some rudimentary 'duopoly' theory." *Regional Science and Urban Economics* 21 (3):393-421. doi: [https://doi.org/10.1016/0166-0462\(91\)90065-U](https://doi.org/10.1016/0166-0462(91)90065-U).
- Wilson, John D. 1986. "A theory of interregional tax competition." *Journal of Urban Economics* 19 (3):296-315. doi: [https://doi.org/10.1016/0094-1190\(86\)90045-8](https://doi.org/10.1016/0094-1190(86)90045-8).
- Wilson, John Douglas. 1999. "Theories of Tax Competition." *National Tax Journal* 52 (2):269-304.
- Wilson, John Douglas, and David E. Wildasin. 2004. "Capital tax competition: bane or boon." *Journal of Public Economics* 88 (6):1065-1091. doi: [https://doi.org/10.1016/S0047-2727\(03\)00057-4](https://doi.org/10.1016/S0047-2727(03)00057-4).
- Zodrow, George R., and Peter Mieszkowski. 1986. "Pigou, Tiebout, property taxation, and the underprovision of local public goods." *Journal of Urban Economics* 19 (3):356-370. doi: [https://doi.org/10.1016/0094-1190\(86\)90048-3](https://doi.org/10.1016/0094-1190(86)90048-3).

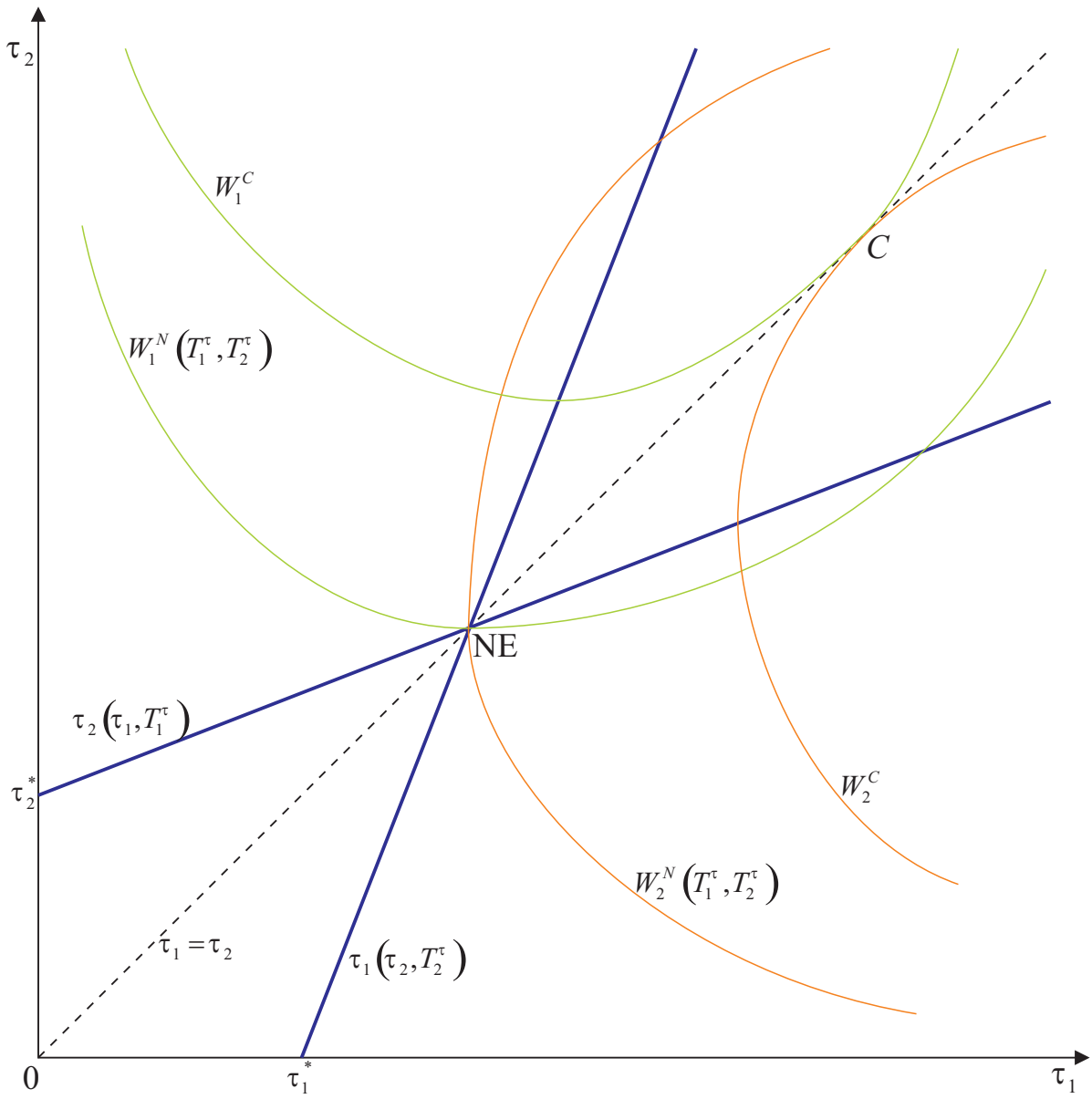


Figure 1: Nash Equilibrium in Proportional Taxes on Capital

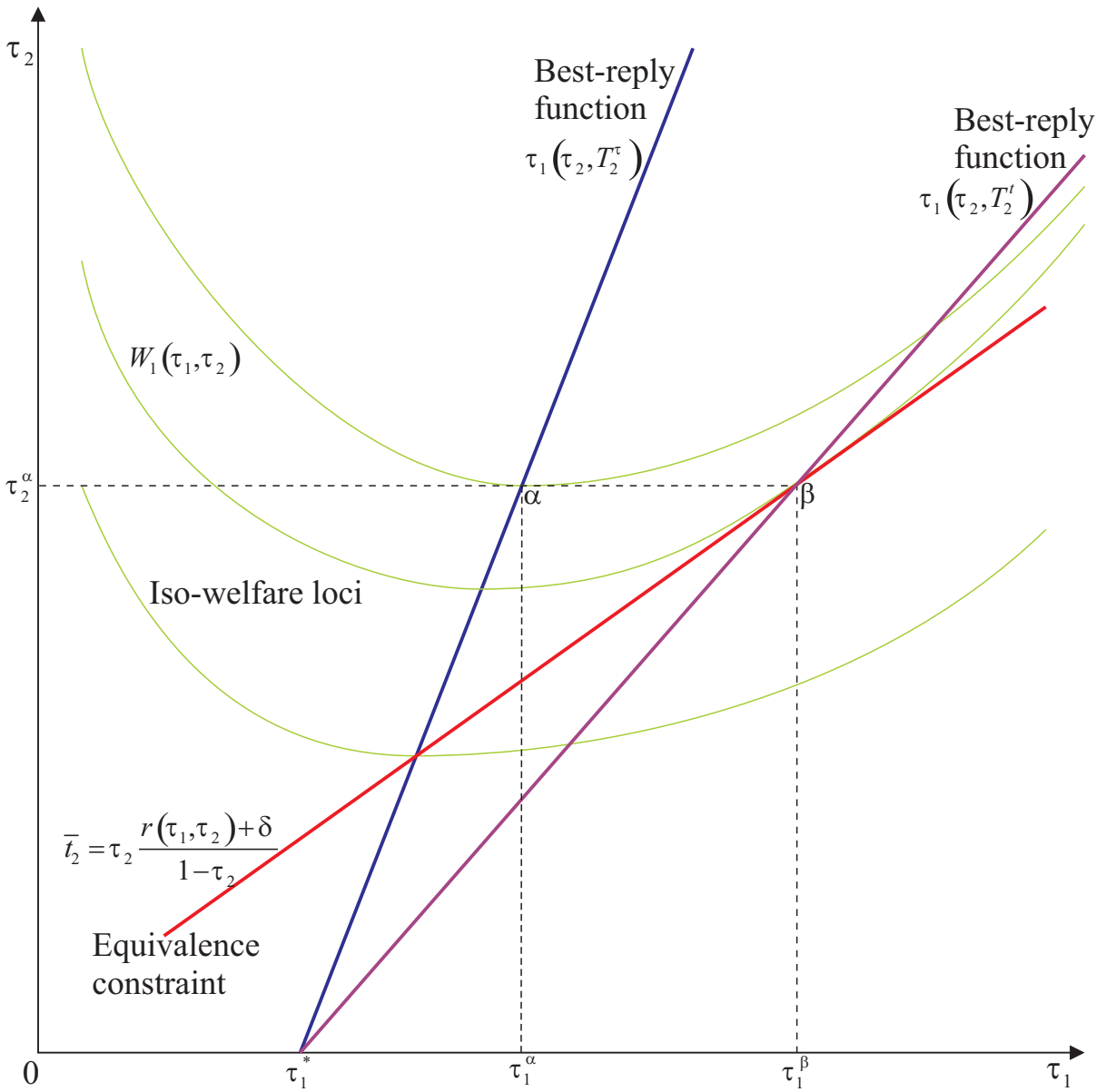


Figure 2: Best Reply Function for Country 1 to a Per-Unit Tax on Capital

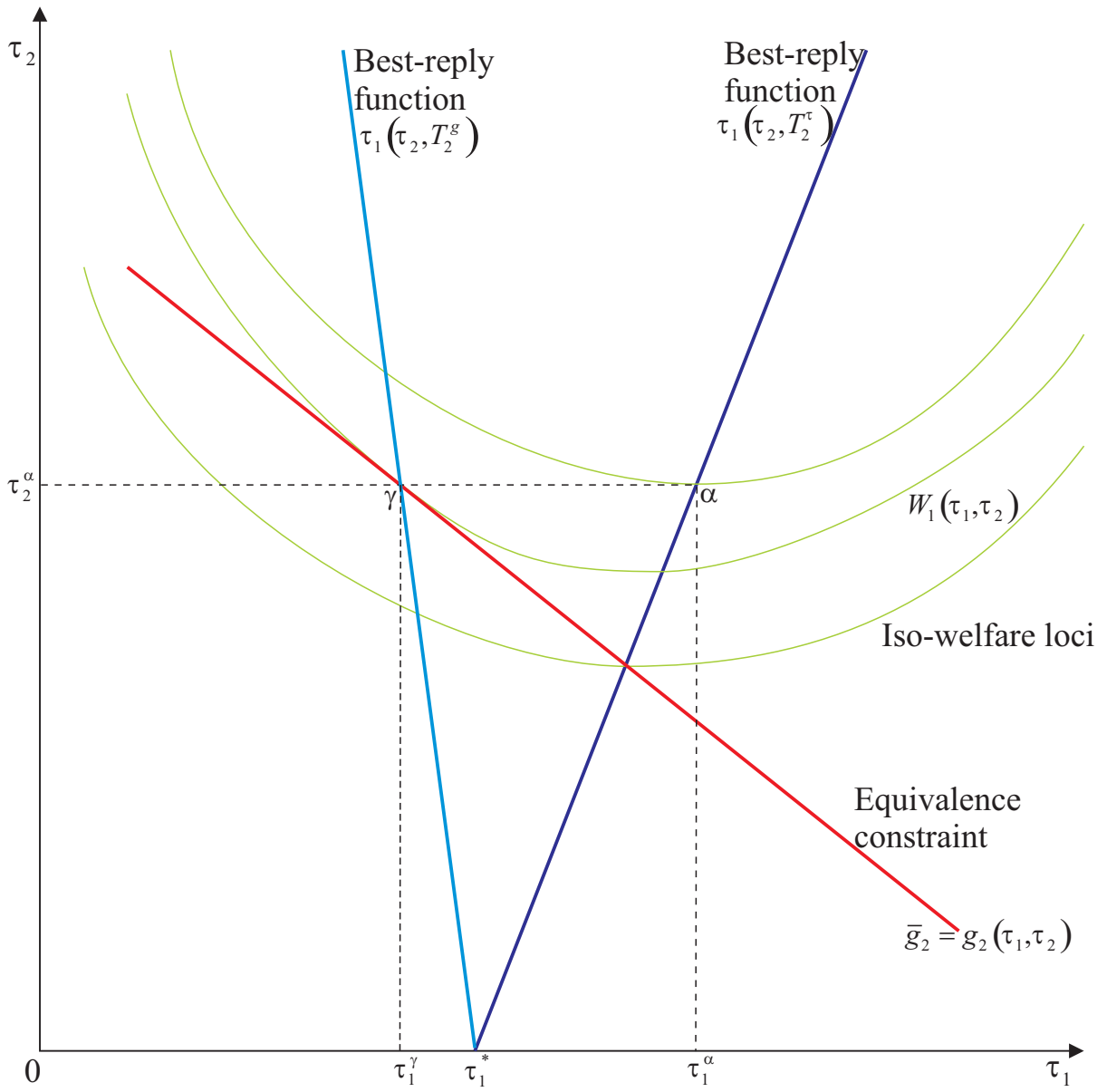


Figure 3: Best Reply Function for Country 1 to Expenditure on the Public Good

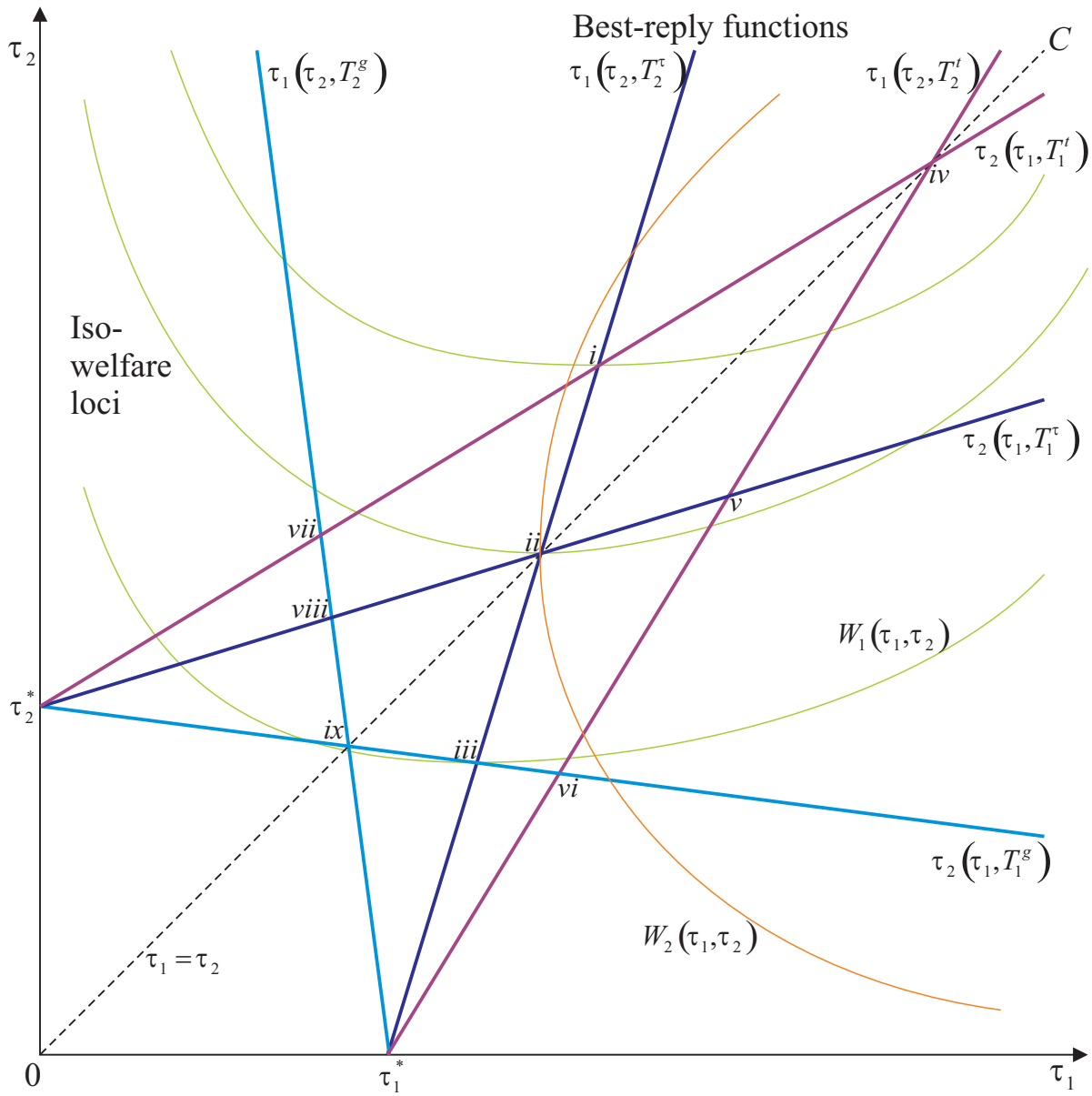


Figure 4: International Tax Competition Game with Positive Tax Externalities

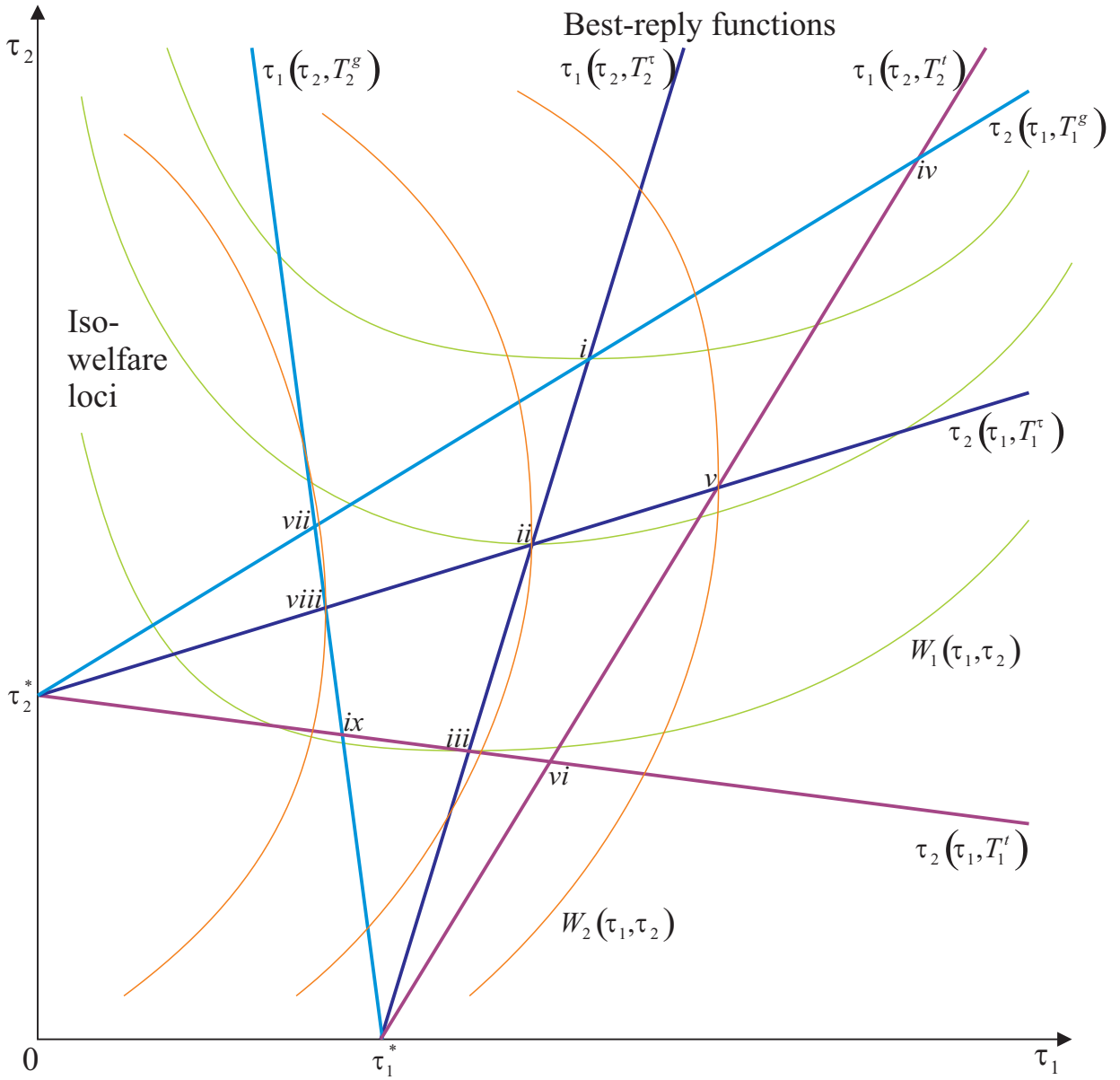


Figure 5: International Tax Competition Game with Positive and Negative Tax Externalities

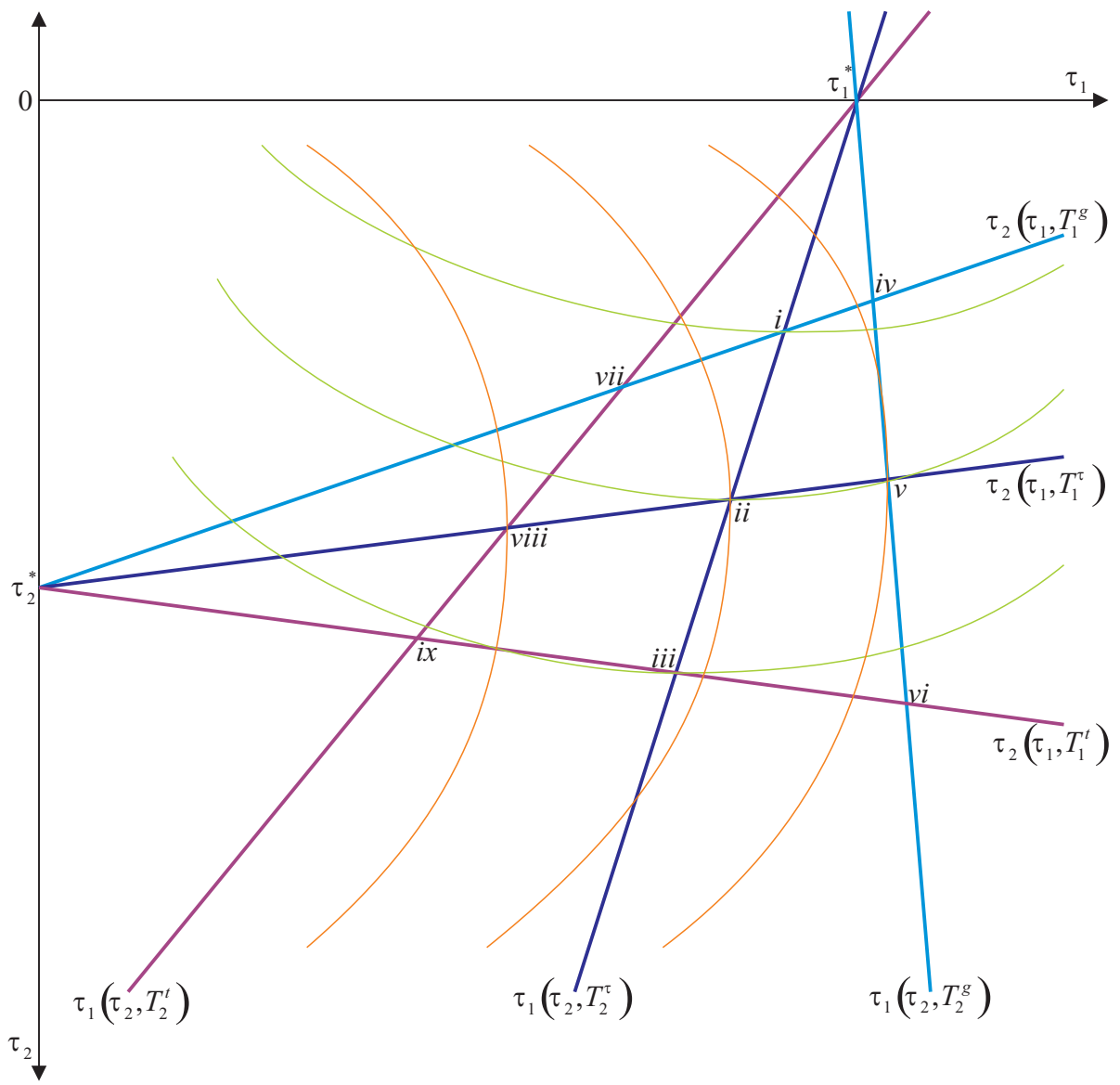


Figure 6: International Tax Competition Game with only the Terms of Trade Effects

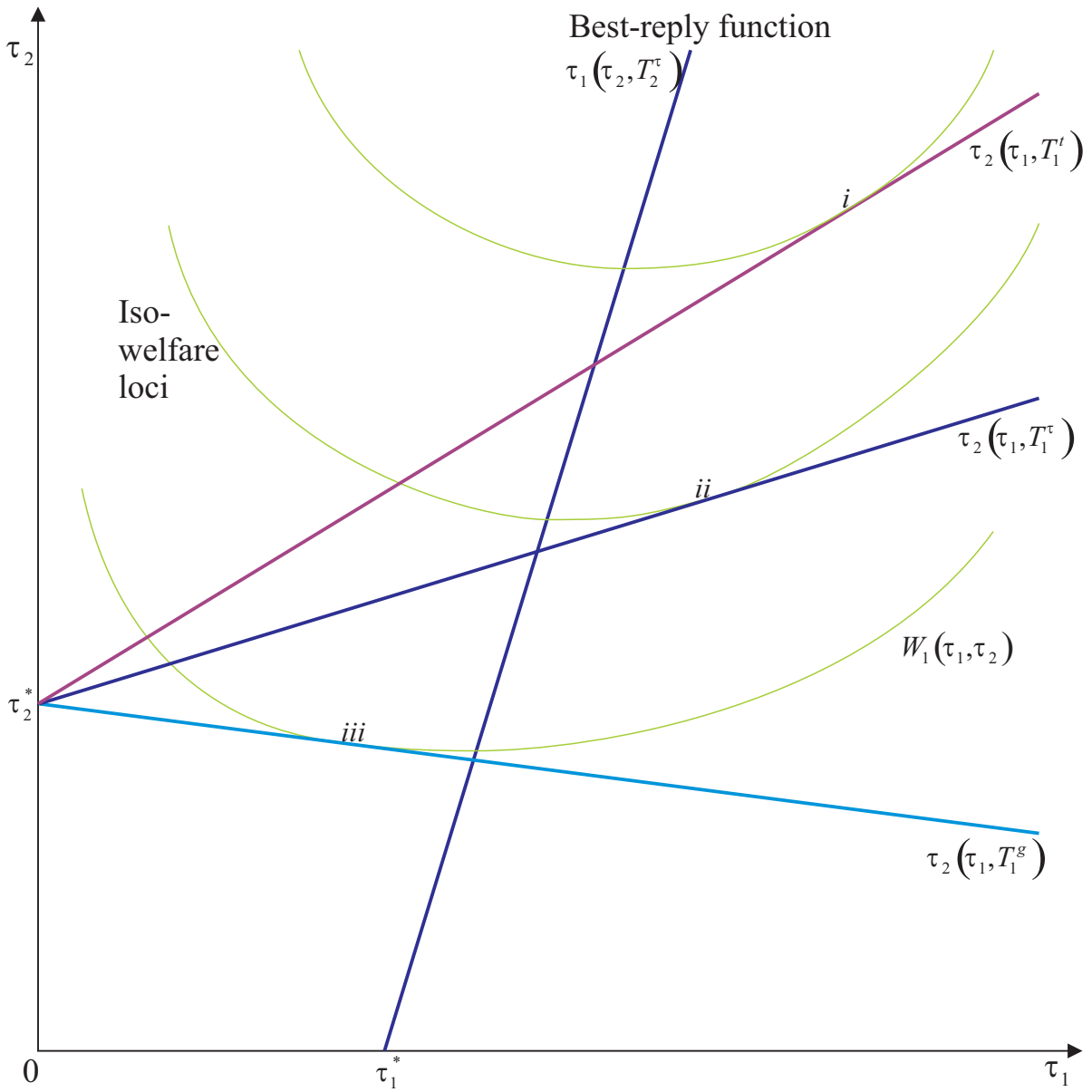


Figure 7: Stackelberg International Tax Competition Game:
 Positive Tax Externalities

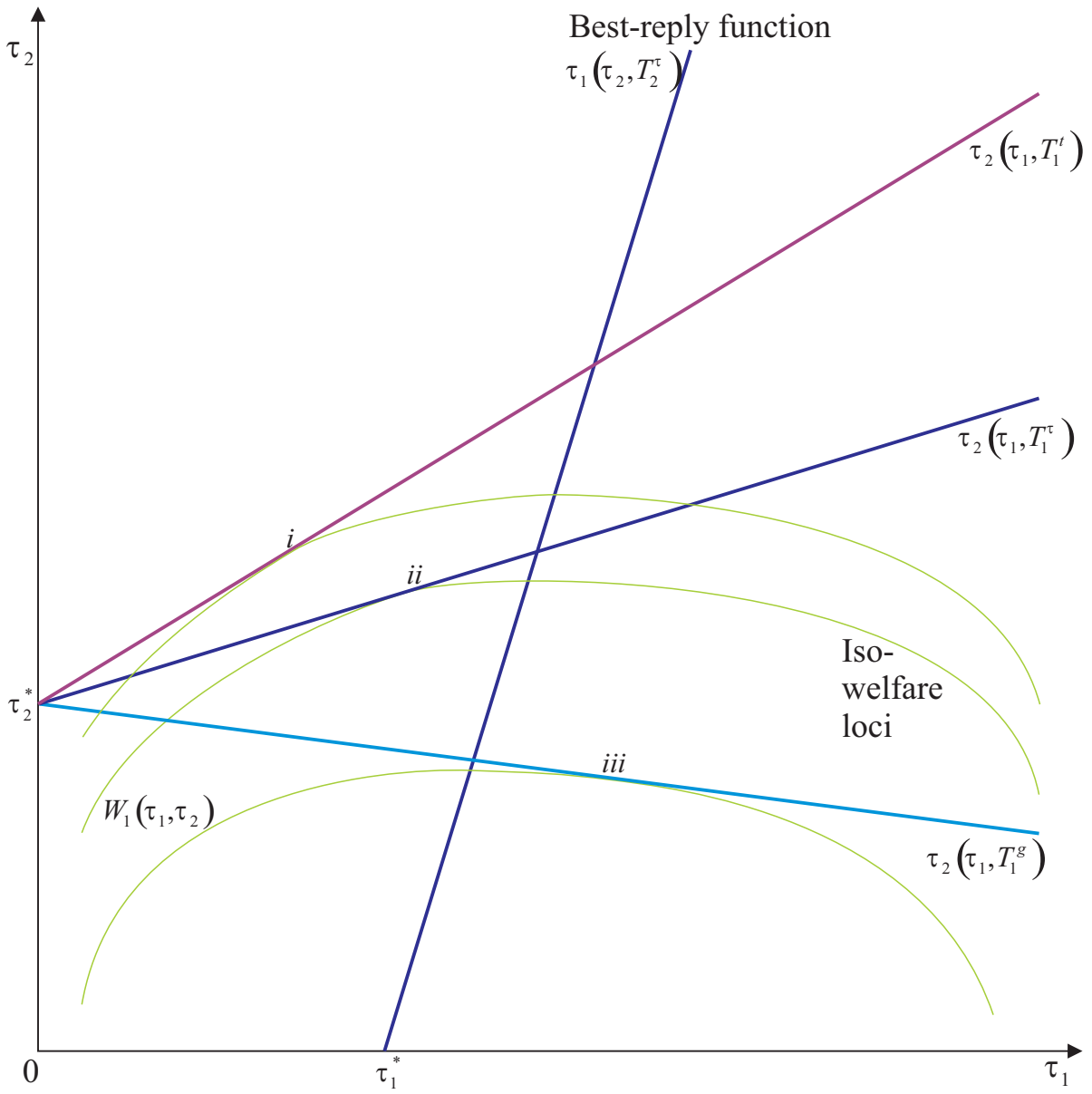


Figure 8: Stackelberg International Tax Competition Game:
Negative Tax Externality for Country 1

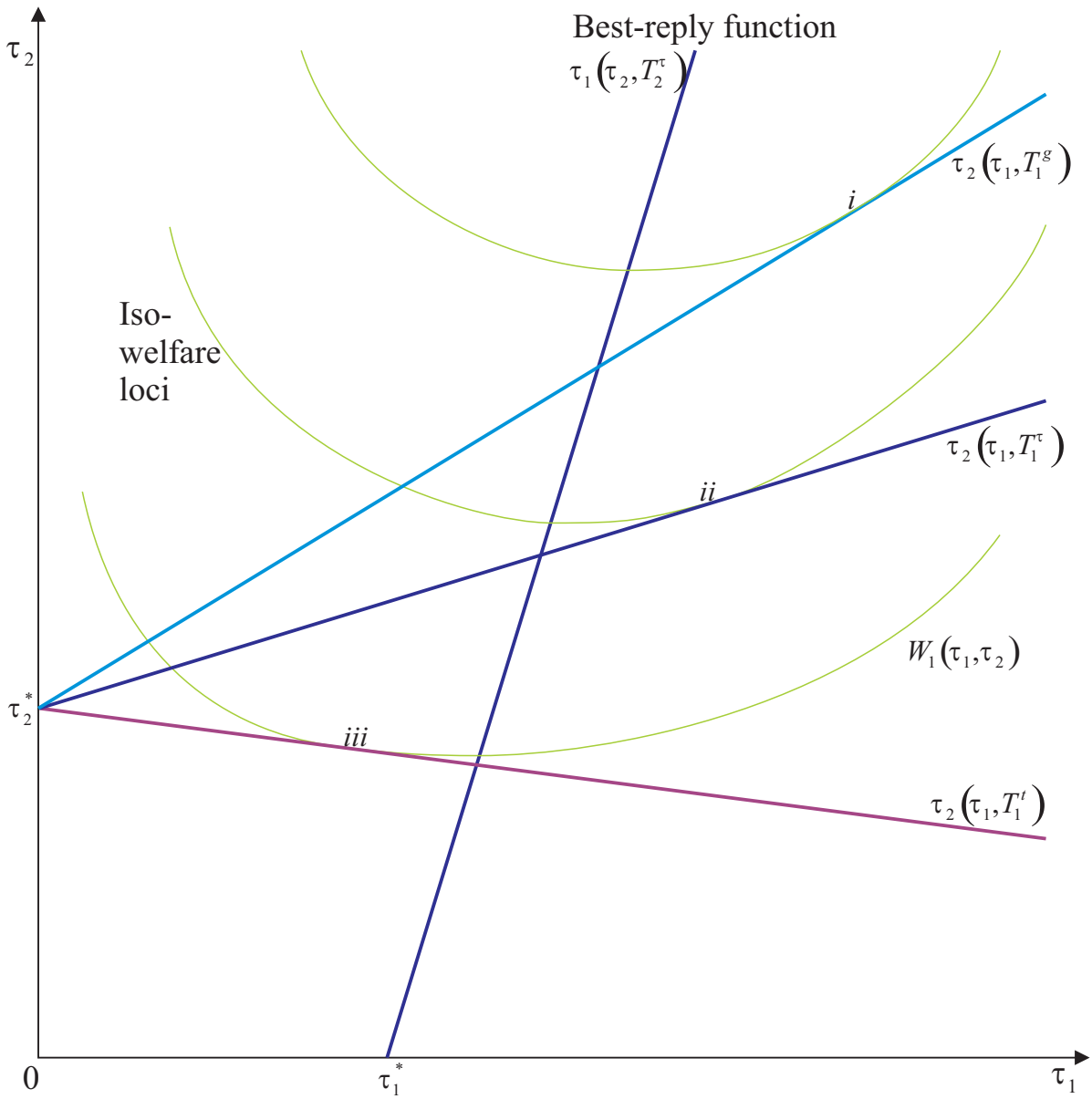


Figure 9: Stackelberg International Tax Competition Game:
Negative Tax Externality for Country 2

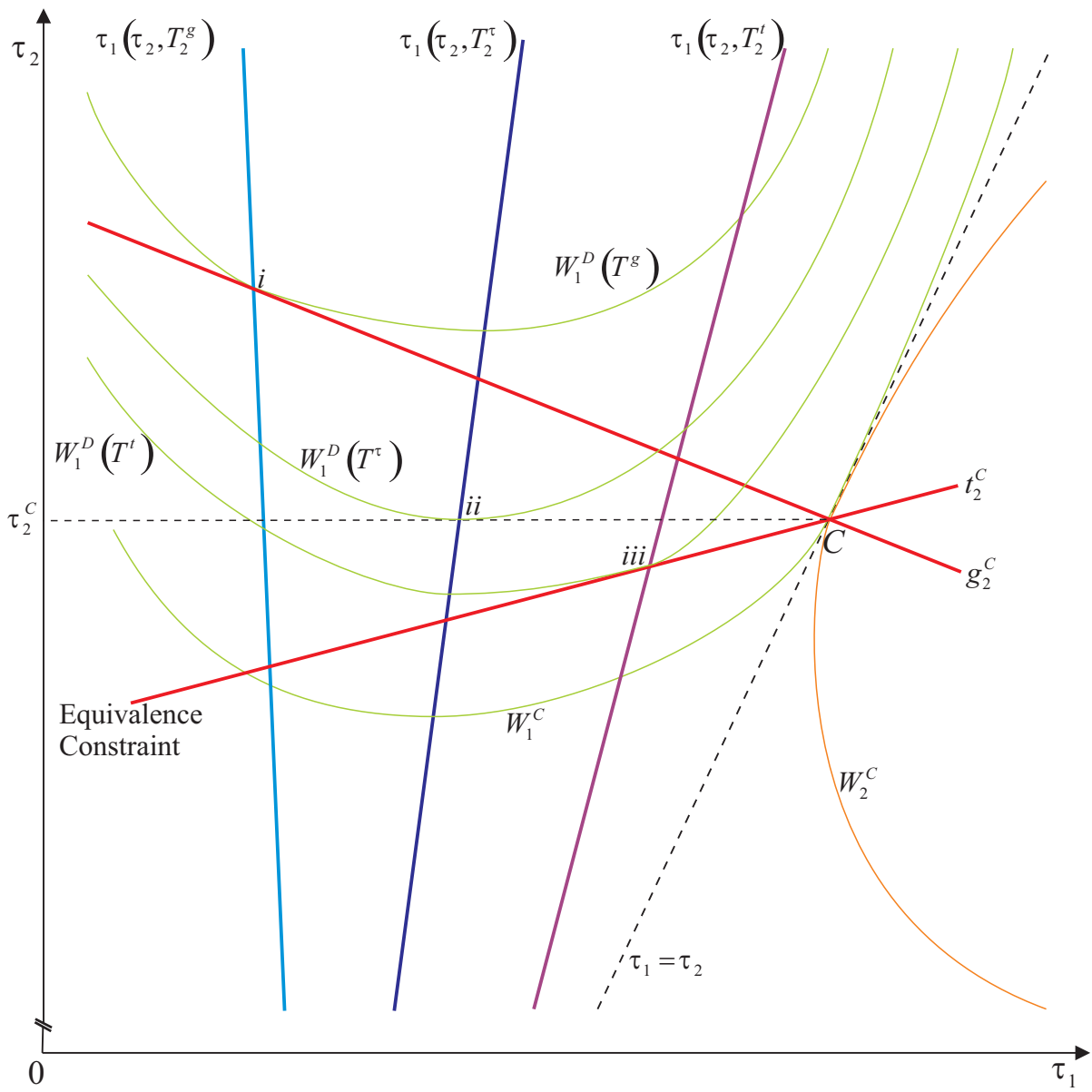


Figure 10: Deviation in Infinitely-Repeated Game

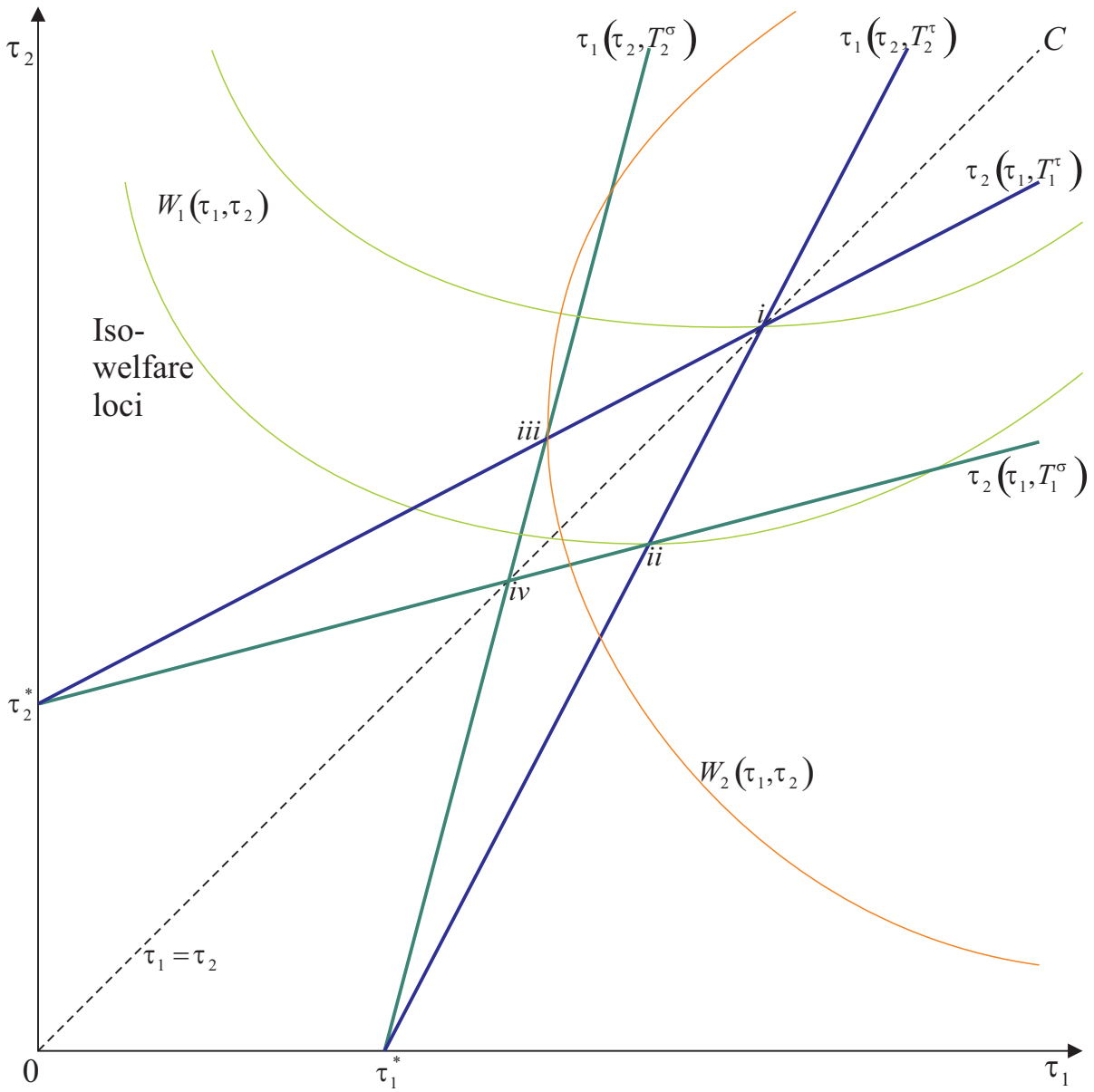


Figure 11: Depreciation Allowances with Positive Tax Externalities

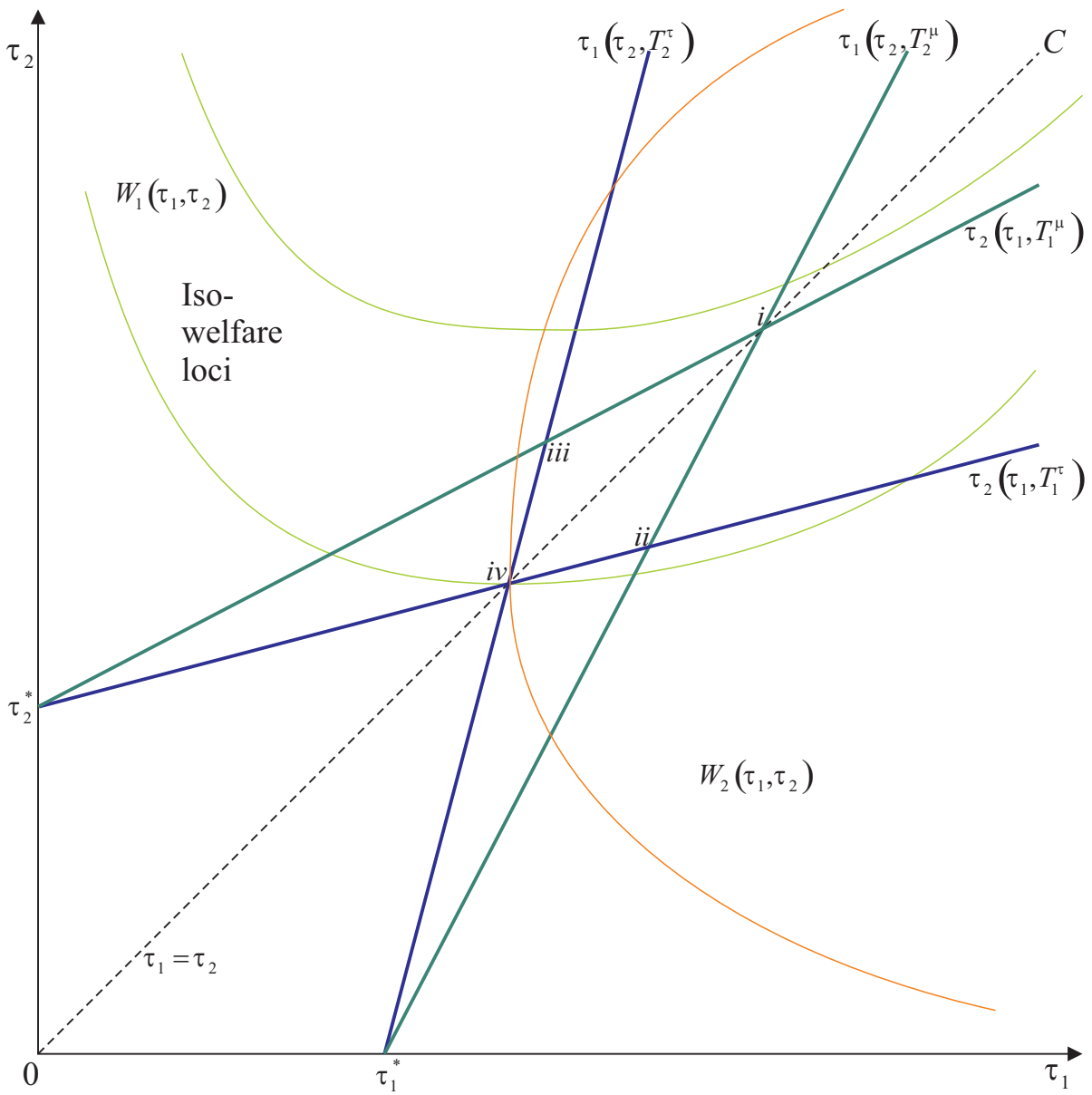


Figure 12: Interest Rate Deductibility with Positive Tax Externalities