Hawkes Processes in Finance: Market Structure and Impact

by

Jing Chen[a], Nick Taylor[b], Steve Yang[c], Qian Han[d]

[a]*School of Mathematics, Abacws, Cardiff University, Senghennydd Road, Cathays, Cardiff, CF24 4AG. UK*

[b]*School of Accounting and Finance, University of Bristol, 15-19 Thyndall Park Road, Bristol, BS8 1PQ. UK*

[c]*School of Business, Stevens Institute of Technology, 1 Castle Point Terrace, Hoboken NJ07030, USA*

[a]*Wang Yanan Institute for Studies in Economics, Xiamen University, Xiamen, 361000, China*

1. Introduction

In response to recent financial crises, financial markets have experienced rapid and profound changes. For example, the belief that global banks are "too big to fail" has awoken the Federal Reserve to lean toward passing the Dodd-Frank Wall Street Reform and Consumer Protection Act [1] (https://www.congress.gov/111/plaws/publ203/PLAW-111publ203.pdf). The deep reasons that led to the 2008 financial crisis following overzealous lending to an overheated housing market remain an unresolved long-term problem. The creation of new insurance instruments against risky mortgage products and mortgage backed

[1]The Dodd–Frank Wall Street Reform and Consumer Protection Act is commonly referred to as Dodd–Frank. It is a United States federal law that was enacted on July 21, 2010. Apart from the US Congress, it is also available at the Commodity Futures Trading Commission (CFTC, https://www.cftc.gov/sites/default/files/idc/groups/public/@swaps/documents/file/hr4173_enrolledbill.pdf).

securities further increased the complexity of the market. The Troubled Asset Relief Program (https://home.treasury.gov/data/troubled-assets-relief-program) was proposed to deal with the aftermath of the crises. Classic issues in finance, such as systemic risk modelling, inevitably become more complex.

Further, the advance of financial technology has enabled trading to take place at the microscopic level (see Huang and Li, 2017). High frequency trading means that market structure is presented in much more profound ways: the price formation processes and price discovery of financial assets are driven by information flows that are highly interactive and very fast moving. Other market features such as order book dynamics, liquidity provisions and resilience also affect price movements. Thus, trading has become both extremely complicated and dynamic. Markets that are interconnected through the new trading dynamic have become more complex and are exposed to a greater number of and more variable types of extreme events. For example, the rampant risk inherent in proprietary trading across Wall Street firms has prompted the regulators' proposal of the Volcker Rule (https://www.sec.gov/divisions/marketreg/faq-volcker-rule-section13.htm)[2] to potentially legislate to prevent banks from taking on too much risk and incurring the consequent risk of default.

---

[2] The Volcker Rule is named after the former Federal Reserve Chair Paul Volcker and particularly refers to section 619 of the Dodd–Frank Wall Street Reform and Consumer Protection Act. It establishes rules for implementing section 13 of the Bank Holding Company Act of 1956 (BHC Act).

A major lesson learnt from these market events is that more rigorous scientific approaches, including modelling techniques, are urgently needed to understand and interpret complex market and financial phenomena. Despite their simplicity and past efficiency, classical asset pricing theory and risk modelling have limitations in their ability to capture the new features of contemporary finance in today's markets. For instance, stock prices almost certainly do not evolve as a random walk and return distributions often deviate far from the normal.

Features observed in return time series such as fat tails, high peaks and extreme skewness are accepted stylised facts of financial markets in the post-crisis era. The occurrence of mini-flash crashes or so-called black swan events are no strangers to traders. They result in many practitioners seeking to neutralize their positions before the closing bell of a trading day. Price movements are also deeply influenced by news that often dominates investment sentiment and leads to either excessive risk taking or contagion.

Recognising the need for substantial further development of new techniques to model price movements in post-crisis financial markets, we set out to apply Hawkes processes to a variety of problems in finance. These processes, introduced by Alan Hawkes in his original works (Hawkes, 1971a, b), are a family of stochastic point processes that model the arrival of events. The essence of Hawkes processes is that they describe the natural occurrence of events and trace their path. Hawkes processes posit that the historical record of a given event would increase the intensity of its recurrence. We can describe this as contagion, in the sense that the occurrence of cases of a contagious disease tends to lead to more cases. For example, the arrival of quotes for a specific stock tends to drive more quotes subsequently, with the increased probability of active quotes more likely result in final trades (Bowsher, 2007). As an example of the usefulness of Hawkes processes, Yang et al. (2018) use a bi-variate model to demonstrate interesting and informative features of investors' behaviour. On the one hand, investors tend to chase positive returns, positive market sentiment also prolongs upward pressure on prices. On the other hand, when investors start to see negative returns, the market falls fast. Negative sentiment, once formed, tends to have contagious effects. More recently, inspired by Yang et al. (2018) using a bi-variate Hawkes process to model stock returns and investment sentiment, Chen et al. (2020) further introduce an entropy approach to identify whether the market trading is driven by return movements or news impacts. They find strong evidence that the financial market trading after the 2008 crisis is dominated by sentiment trading and often suffers from the contagion. This further confirms that, in comparison to classic Brownian Motion, the ability of Hawkes processes to model contagion would seem to be closer to the organic evolution of many financial problems in real markets. Hawkes processes are very flexible as they can accommodate a wide range of probability distributions, especially those suitable to describe extreme events or market conditions.

2. Background to the Special Issue

To further promote the use of Hawkes processes in financial economics, we organised a conference entitled Hawkes Processes in Finance. An important aim was to introduce this family of stochastic processes to finance researchers and practitioners in the belief that they may become part of the econometric toolbox for the analysis of contemporary finance problems. Knowledge transfer on a large scale, however, takes time and substantial efforts from finance researchers. The "Hawkes Process in Finance Conference" was held at The Stevens Institute of Technology in Hoboken, New Jersey (https://hawkes.stevens.edu/home) on 7th and 8th June 2019. This special issue of The European Journal of Finance entitled "Hawkes processes in finance, market structure and impact" contains a selection of nine papers which were presented at the conference and subsequently passed successfully through the journal's review process.

3. Articles in this special issue

Brownian motion, introduced by Bachelier (1900), has formed the backbone of many works in financial mathematics, especially the diffusion models that still prevail in finance. These fundamental works include, as Hawkes (2020) rightly points out, Samuelson (1972) for a Gaussian random walk, Black and Scholes (1973) for the Nobel Prize winning option pricing model and Merton's generalisation in 1973. With consideration of financial jumps, Merton (1976) proposed Poisson jumps while Cont and Tankov (2004) comprehensively discussed Lévy jumps. Hawkes (2020), however, noticed that both processes do not explain the contagious nature of jump occurrence and thus suggests that a Hawkes jump process model that combine the diffusion and Hawkes processes can competently deal with a wide range of risk modeling and pricing issues even no matter of the market complexity. He gave a few examples in problems including trading across multiple markets, pricing and hedging, portfolio optimisation and many more. The general conclusion is that Hawkes jump models are generally better than traditional diffusion or jump-diffusion models in terms of their fitness, forecasting performance or robustness. In addition, these prove that Hawkes processes are highly flexible and can be adapted to suit a good variety of finance problems.

Kirchner and Vetter (2020) model limit order book (LOB) dynamics using a multivariate marked Hawkes process. Unlike the existing limit order book models such as Cont and de Larrard (2013) and Bacry et al. (2016), the proposed model defines events by selecting a set of non-zero excitement activities, the shape of the decay kernels, and the impact function nonparametrically. The event estimation methodology based on 'Hawkes skeleton' introduced by Embrechts and Kirchner (2017), offers a scalable approach to estimate a large number of financial events. Through defining high dimensional events using time stamps, order types, and order sizes, the authors are able to identify rich features of the limit order book using Hawkes skeleton and Hawkes graph edges: i) the market orders are the driving force among all the order book activities; ii) the price impact function of market orders follows a linear function; iii) bid ask activities exhibit symmetric properties under the nonparametric approach; iv) LOB imbalance provides a good future order arrival prediction. Overall, the authors introduce an alternative approach to model LOB dynamics using Hawkes processes, and new insights have been generated for better understanding the market price formation process and predicting future events.

Benzaquen et al. (2021) suggest the non-parametric calibration procedure for general Quadratic Hawkes models that are used to model the order book activities. The nature of the Hawkes processes enables to capture the whole history of event occurrence like the order arrivals. But Benzaquen et al. (2021) apply the calibration in such a way that the full path of the price movements under the impact of the order book event arrivals can also be examined. Essentially, their model calibration shows that the effective quadratic kernel would be rank-one Zumbach kernel. It follows a power law and could indicate the true rate of the exogenous events like flash crashes or Covid 19 in proportion to the total order book events. This further enhances the model and makes it practically useful in finding the points where the system capacity to deal with extreme conditions such as liquidity squeeze is challenged. They interpret this as the microstructural origin of the Zumbach effect and shows that the past trends tend to reduce the liquidity in the order book and subsequently cause the future realized volatility to increase.

Cai (2020) develops a Hawkes process model with a hidden marked process that represents extra random errors (ERE) caused by the data collection mechanisms and certain data cleaning procedures. Related to Kirchner and Vetter (2020) in estimating financial events, the author proposes a quantile approach to augment the basic Hawkes process in addressing noise data issues faced by financial modellers. The author introduces the hidden marked process to take account of the effect of ERE using the generalized Lambda distribution (GLD), and a MCMC-based method for parameter estimation is also provided for model calibration. Through thorough analyses on both simulation and empirical market data, Cai (2020) demonstrates a clear benefit of introducing a hidden marked process in the presence of ERE and shows that when ERE causes information of the underlying process to be lost, the intensity function may be underestimated. The proposed method can be generalized to address more complex Hawkes processes when ERE is a major concern for intensity underestimation.

Zhang et al. (2020) present yet another extension of the basic Hawkes process where the market prices and investor sentiment are combined into a single Hawkes-Contact model. The Contact process is a stochastic process introduced by Zhang and Wang (2010) in modelling a disease spreading mechanism. Zhang et al. (2020) introduce the Contact process into the Hawkes process to better fit the empirical data. The authors argue that ~~while~~ the news sentiment formation can be represented as a dynamic social Contact process but, when it is combined with the Hawkes-style event branching process, the model becomes more powerful in capturing richer statistical properties in returns such as, probability density function (PDF); complementary cumulative distribution function (CCDF); and Lempel-Ziv Complex (LZC). The authors show that the real return distribution is often far from normal but the simulated returns through the proposed Hawkes-Contact model can achieve better fit to the real returns. More interestingly, the proposed model offers two weighting parameters which is able to show the strength of both the Hawkes contagion process and the finite range contact process, and altogether the full model can fulfil the goal of capturing multiple information flows that jointly drive the underlying price process.

Asset prices are characterised by occasional abrupt changes (jumps). Understanding how these jumps are distributed over time has recently gained the attention of researchers. The vast of majority of the studies either adopt a univariate approach or consider the joint dynamics of jumps for assets within the same market. Few studies investigate these dynamics across different markets. It is this gap in the literature that the following papers contribute. Ferriani and Zoi (2020) examine, inter alia, whether jumps transfer between US and German stock markets, and thus whether contagion-type behaviour presents itself as abrupt price adjustments. Buccioli and Kokholm (2021) take the 'flight-to-safety' behaviour as their central hypothesis and consider whether gold and stock markets are causally related in terms of their contribution to each other's jump propensity.

Ferriani and Zoi (2020) consider a generalized Hawkes process in which the evolution of intraday jump intensity in the US and German stock markets is allowed to be determined by jump size and volatility (in addition to allowing for self and cross-excitation). Using five-minute returns to the S&P500 and Euro Stoxx 50 indices observed between 13/09/2007 and 30/04/2014, their results show that that contagion effects do not appear to be present in jumps – a finding consistent with that found by Corradi, Distaso, and Fernandes (2012). Their analysis also finds, contrary to intuition, that jump intensity and volatility are inversely related (implying that large price changes during stress periods are driven by continuous volatility and not jumps). Thus, the importance of jump risk (either via jump occurrence or jump contagion across markets) may have been previously overstated.

Buccioli and Kokholm (2021) make use of a Hawkes process driven model in their study of the joint dynamics of stock and gold prices. Their model allows for jumps in each market such that both self and cross-excitation is permitted. The maintained assumption of 'flight-to-quality' is imposed on the model by focusing on the dynamics of negative jumps in the stock market and positive jumps in the gold market. The direction of causality in this jump-structure is left unrestricted. Using a long span of daily returns to the S&P 500 index and gold (observed in the London Bullion Market) observed between 02/09/1976 to 13/09/2018, their results confirm the presence of a significant flight-to-quality effect with negative stock market price jumps leading to positive gold price jumps. Moreover, they find that this effect is present for approximately 20 days after the stock market price fall. Additional analysis even shows that their jump intensity model can be used to predict future intensity levels, with forecast accuracy superior to those based on GARCH and VIX-based model predictions.

Their findings underline the importance of incorporating flight-to-safety effects within an asset pricing and risk management context.

Empirically, Yang, Liu, Yu and Mo (2021) take advantage of a multivariate Hawkes process model to examine the jump contagion behaviour across energy ETFs and other representative assets. Using five-minute high frequency trading data, authors first detect return jumps by applying the bi-power variation jump detection method as in Barndorff-Nielsen and Shephard (2004, 2006), then model these jumps under a multi-variate Hawkes process framework. Their main findings include: negative jumps tend to self-contagion more than positive ones across all assets. Within the energy ETF group, the ETF with Master Limited Partnership (MLP) segment exhibits less negative self-contagion and stronger positive self-contagion than other ETFs. In addition, the influence of negative jumps on ETFs from both the equity index and the energy future index is stronger than that of the positive jumps.

How much price variation is produced within the market, i.e. endogeneity, is an interesting and important research question in the literature of market microstructure (Shiller, 1980; LeRoy and Porter, 1981; Bouchaud, 2009). Mark, Sila and Weber (2020) fit a Hawkes process model with two parametric kernel specifications to the high frequency mid-prices of Bitcoin traded in one of the largest cryptocurrency exchanges. Authors then construct a reflexivity index to quantify the extent of endogeneity in the Bitmex market. By comparison to traditional assets, authors find that the Bitcoin market performs similar to fiat-currency markets with the mean reflexivity value being around 80%, and that the Bitcoin mid-prices exhibit long-memory properties. Another contribution of the article is that authors pinpoint a way of identifying the optimal estimation horizon to deal with the trade-off problem between estimation accuracy and robustness to regime changes.

4. Directions for Future Research

With the studies reported here, we express the hope that finance researchers and practitioners will appreciate the significance of a modelling approach that more closely describes how the real markets and the associated financial time series behave. Hawkes processes can be effectively and efficiently applied to a variety of finance problems. As Hawkes (2019) points out, the classic diffusion models in finance need to be updated. New phenomena such as financial jumps, contagion, endogeneity, non-Gaussian return and other phenomena can be effectively dealt with by Hawkes processes. Going forward, the booming of the fintech sector and technological advancement in the finance sector surely means that questions in finance about risk will become more prominent and more complex. We believe that Hawkes processes will play a substantial role in developments in the years ahead.

To conclude, we thank all those researchers who have contributed to this special issue: presenters at the conference; authors whose papers appear; the referees; those who participated in the conference; and those particularly those at the Stevens Institute, who organised it. Finally, we express our thanks and gratitude to Alan Hawkes not only for his pioneering work, but for being an active participant in and supporter of our efforts.

References

Bachelier, L. (1900). Théorie de la Speculation. *Annales scientifiques de l'É.N.S. 3e série*, tome 17: 21-86

Bacry, E., T. Jaisson and J. Muzy. 2016. Estimation of Slowly Decreasing Hawkes Kernels: Application to High-frequency Order Book Dynamics. *Quantitative Finance* 16: 1179–1201. DoI: 10.1080/14697688.2015.1123287.

Barndorff-Nielsen, O. E. and Shephard, N. (2004). Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of financial econometrics* 2(1): 1-37.

Barndorff-Nielsen, O. E. and Shephard, N. (2006). Econometrics of Testing for Jumps in Financial Economics using Bipower Variation. *Journal of financial Econometrics*, 4(1): 1-30.

Benzaquen, M., Fosset, A. and Bouchaud, J.P. (2021). Non-parametric Estimation of Quadratic Hawkes Processes for Order Book Events. *The European Journal of Finance*. Forthcoming.

Black, F., and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* ,81 (3): 637–654.

Bouchaud, J. P. (2009). The (unfortunate) Complexity of the Economy. *Physics World,* 22(04): 28.

Bowsher, C.G. (2007). Modelling Security Market Events in Continuous Time: Intensity Based, Multivariate Point Process Models. *Journal of Econometrics*, 141: 876–912.

Buccioli, A. and Kokholm, T. (2020). Shock Waves and Golden Shores: The Asymmetric Interaction between Gold Prices and the Stock Market. *The European Journal of Finance*. DoI: 10.1080/1351847X.2021.1897026.

Cai, Y. (2020). Hawkes processes with hidden marks. *The European Journal of Finance*. DoI: 10.1080/1351847X.2020.1820356.

Chen, J., Hawkes, A., Liu, A. and Yang, S. Y. (2020). The Flow of Information in Trading: An Entropy Approach to Market Regimes. *Entropy* 22(9): 1064. DoI: 10.3390/e22091064.

Cont, R., and de Larrard, A. (2013). Price Dynamics in a Markovian Limit Order Market. *Journal of Financial Mathematics* 4: 1–25. DoI: 10.1137/110856605.

Cont, R., and P. Tankov. 2004. *Financial Modelling with Jump Processes*. Boca Raton, Florida: Chapman & Hall/CRC.

Corradi, V., Distaso, W. and Fernandes, M. (2012). International Market Links and Volatility Transmission. *Journal of Econometrics*, 170 (1): 117 – 141. http://www.sciencedirect.com/science/article/pii/S0304407612000759.

Embrechts, P. and Kirchner, M. (2017). Hawkes Graphs. *Theory of Probability and Its Applications* 62: 163–193.

Ferriani, F. and Zoi, P. (2020). The dynamics of price jumps in the stock market: an empirical study on Europe and U.S. *The European Journal of Finance*. DoI: 10.1080/1351847X.2020.1740288.

Hawkes, A. G. (1971a). Point Spectra of Some Mutually Exciting Point Processes. *Journal of Royal Statistical Society Series*, B 33: 438–443.

Hawkes, A. G. (1971b). Spectra of Some Self-exciting and Mutually Exciting Point Processes. *Biometrika* 68 (1): 83–90.

Hawkes, A. G. (2020). Hawkes Jump-Diffusions and Finance: A Brief History and Review. *The European Journal of Finance.* DoI: 10.1080/1351847X.2020.1755712.

Huang, C. and Li, H. (2017). An Evolutionary Method for Financial Forecasting in Microscopic High-Speed Trading Environment. *Computational Intelligence and Neuroscience* 1: 1-18. DOI: 10.1155/2017/9580815.

Khashanah, K., Chen, J. and Hawkes, A. (2018). A Slightly Depressing Jump Model: Intraday Volatility Pattern Simulation. *Quantitative Finance* 18(2): 213-224. DoI: 10.1080/14697688.2017.1403139.

Kirchner, M. and Vetter, S. (2020). Hawkes model specification for limit order books. *The European Journal of Finance*. DoI: 10.1080/1351847X.2020.1784974.

LeRoy, S. F. and Porter, R. D. (1981). The Present-value Relation: Tests Based on Implied Variance Bounds. Econometrica: Journal of the Econometric Society, 555-574.

Mark, M., Sila, J. and Weber, T. A. (2020). Quantifying endogeneity of cryptocurrency markets. *The European Journal of Finance*. DoI: 10.1080/1351847X.2020.1791925.

Merton, R. C. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science,* 4: 141–183.

Merton, R. C. (1976). Option Pricing When Underlying Stock Returns are Discontinuous. *Journal of Financial Economics,* 3: 125–144.

Samuelson, P. A. (1972). Mathematics of Speculative Price. In *Mathematical Topics in Economic Theory and Computation*, edited by R. H. Day, and S. M. Robinson, 1–42. Philadelphia: SIAM. Reprinted 1973 in *SIAM Review* 15 (1): 1–41.

Shiller, R. J. (1980). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *The American Economic Review*. 71(3): 421-436.

U.S. Congress. “The Dodd–Frank Wall Street Reform and Consumer Protection Act (2010). The United States Federal Law”. https://www.congress.gov/111/plaws/publ203/PLAW-111publ203.pdf. Accessed Nov. 22, 2021.

U.S. Department of the Treasury. “Troubled Assets Relief Program (TARP)” https://home.treasury.gov/data/troubled-assets-relief-program. Accessed Nov. 22, 2021.

U.S. Securities and Exchange Commission. "Responses to Frequently Asked Questions Regarding the Commission's Rule under Section 13 of the Bank Holding Company Act (the 'Volcker Rule')." https://www.sec.gov/divisions/marketreg/faq-volcker-rule-section13.htm. Accessed Nov. 22, 2021.

Yang, S. Y., Chen, J., Hawkes, A. and Liu, A. (2018). Applications of Multi-variate Hawkes Process to Joint Modelling of Sentiment and Market Return Events. *Quantitative Finance* 18(2): 295-310. DoI: 10.1080/14697688.2017.1403156.

Zhang, J. and Wang, J. (2010). Modeling and Simulation of the Market Fluctuations by the Finite Range Contact Systems. *Simulation Modelling Practice and Theory,* 18 (6): 910–925.

Zhang, J., Wen, J. and Chen, J., (2021). Modelling market fluctuations under investor sentiment with a Hawkes-contact process. *The European Journal of Finance*. DoI: 10.1080/1351847X.2021.1957699.