Prescriptive Healthcare Analytics: A Tutorial on Discrete Optimization and Simulation

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***Abstract*—Mathematical Modelling, as a paradigm, has been used in many different industries and healthcare is no exception. In this tutorial, which is split into three parts, we will firstly provide an introduction to mathematical modelling techniques. This includes methods such as queuing theory, discrete event simulation (DES), and mathematical programming. The second part of the tutorial will focus on Integer and Linear Programming as part of Mathematical Programming. This part includes case studies which help participants learn to develop spreadsheet-based models with Open Source solvers. The third part of the tutorial is focused on DES modelling. In healthcare operations, especially in urgent and emergency care, there is a significant variation in demand. This requires careful consideration of statistical distributions in the inter-arrival time of and service duration to treat patients. The tutorial will close with a discussion of different pros and cons of techniques and highlight an analytics and modelling academy that Cardiff University runs in collaboration with the National Health Service in the U.K.**

***Index Terms*—Computer and Information Science Education, Queuing Theory, Simulation, Modeling Methodologies, Healthcare, Optimization of Service Systems**

I. Description

The overall scope of the tutorial is to improve the capability of healthcare analysts in using operational research and mathematical modelling tools and techniques with a particular focus on Mathematical Programming and Discrete-Event Simulation. Improving analytical and modelling capability will enable individuals to gain better insight and understanding of complex decision problems in terms of the immediate and latent impact of when change, or no change, is introduced to their system. It also enables participants to gain awareness of potential, otherwise unforeseen, unintended consequences for their service, or for services in another part of the system. This means managers can make more informed choices when planning, managing and improving the operation of healthcare services and thus deliver more effective and efficient care to patients. Furthermore, the audience of the tutorial will develop a joint language and shared agenda for analytics-driven process innovation.

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II. Audience

Participants of the tutorial are expected to have at least novice proficiency in Microsoft Excel and functions including basic arithmetic, conditional logic functions, and manipulating data. Since the tutorial is intended for both analysts and decision makers in the healthcare sector, the use of case studies will help participants develop a better understanding of complex parts of the stochastic nature of health system such as Emergency Departments and mental health services.

Participants will be expected to use their laptops. Prior to the tutorial, Excel files will be distributed and worked through during the session. Also, a Simul8 Online login will be provided to the participants prior to the session.

III. Aims and Objectives

A. Part I: Introduction to Mathematical Modelling in Healthcare

At the end of this part, participants should be able to:

- Understand the concept of mathematical modelling, the broad range of application of models, and their limitations.
- Explain the potential role of models for decision making in health care.
- Develop and test a simple decision model.
- Understand how to use a model for 'what-if' analyses.
- Understand the importance of considering variation.

B. Part II: Mathematical Programming

At the end of this part, participants should be able to:

- Appreciate differences between location planning, staffing, routing and scheduling.
- Develop an understanding of utilization and coverage factor for core level staffing problems in healthcare facilities.
- Categorize patient scheduling problems using the framework Gartner and Padman (2017)'s framework [1].

C. Part III: Discrete Event Simulation

At the end of this part, participants should have an understanding of the following performance metrics in healthcare queuing systems:

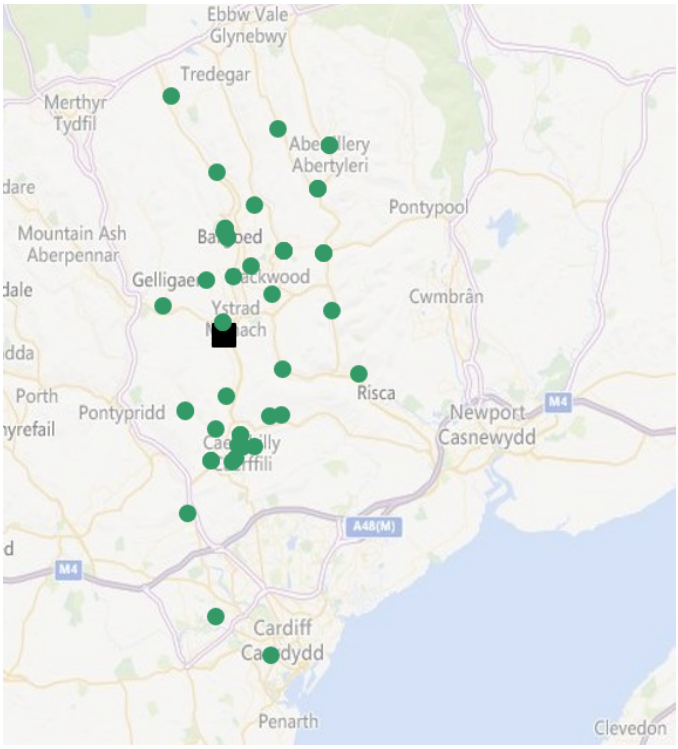

Fig. 1: Pathology Routing Problem

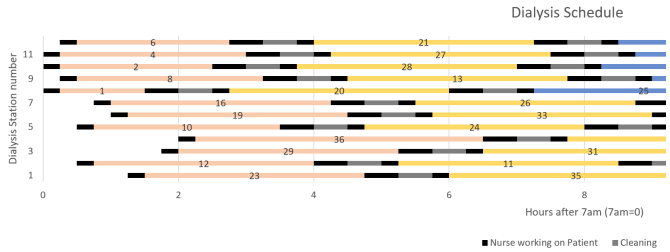

Fig. 2: Dialysis Scheduling Problem

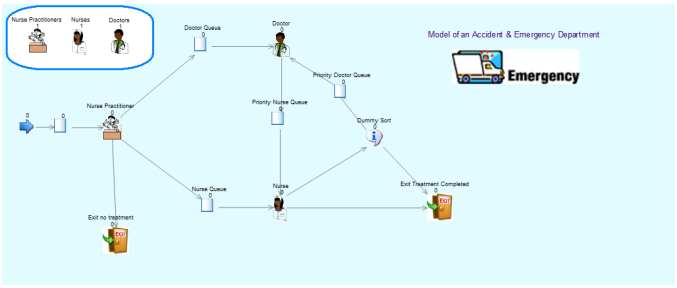

Fig. 3: Discrete-Event Simulation Model of an Accident and Emergency Clinic

- Average number of patients waiting (in queue or in the system).
- Average time the patients wait (in queue or in the system).
- Capacity utilization.
- Costs of a given level of capacity.
- Probability that an arriving patient will have to wait for service.

As a consequence, the quantitative methods covered in this part of the tutorial include skills particularly useful for evaluating service systems and in particular queues in healthcare. The practical aspect of this course uses the functionality of DES software – Simul8 to aid the course participants' ability to analyse and visually display real-life queuing situations. The aims and learning objectives at the end of this tutorial are:

- Describe queuing systems and their use in healthcare services.
- Recognize queuing concepts and their relationship to capacity planning.
- Design a pathway model in Simul8, validate the model and collect results.
- Identify potential bottlenecks in the system.
- Test various 'what-if' scenarios.

IV. Case Studies

We will use four case studies, two within the mathematical programming part and two within the discrete event simulation part.

The first real-world discrete optimization model will be from a case study in the domain of specimen pickup and routing (pathology services). Figure 1 provides a geographical solution of a problem in South Wales, U.K. using a VRP spreadsheet solver from [2]. The problem can be described as follows: Given a set of vans, pickup points (GP practice), time windows and service constraints, what are the optimal routes to minimize travel time.

Furthermore, we will use a Dialysis patient scheduling decision support tool that relies on mathematical programming and is based on Fleming et al. (2019) [3] and shown in Figure 2. Given a set of patients and dialysis machines, the problem is to find an optimal sequence of patients on these machines such that waiting times and overtime of nurses is minimized. The Excel-based decision support tool has been developed in collaboration with NHS Wales.

For the simulation part, we will use a generic model in Accident and Emergency. The model includes variation in the arrival, service time and patient flows. Figure 3 provides a sketch of the model and it can be used to study what-if analyses, for example: Would the A&E department's capacity be able to cope with an increase in demand? Performance metrics such as waiting time distributions and time spent in the system can be reported by the model.

Finally, we will guide the participants of our tutorial through the development of a mental health planning model. Mental health is a significant global concern, not only for public health but also for economic development and societal welfare [4]. We focus on a DES model developed by Howells et al. (2022) [5]. They carried out a what-if analysis i.e. the possible introduction of care coordinators to support mental health practitioners. In their analyses, they were able to find out optimal staffing patterns. The model is shown in Figure 4.

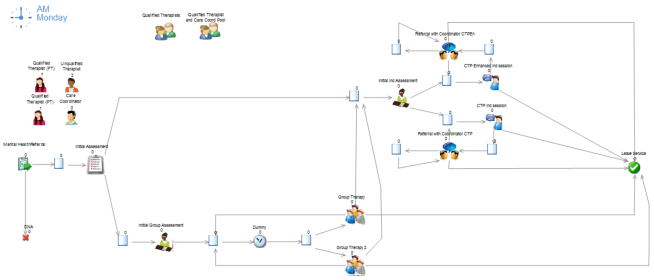

Fig. 4: Mental Health Capacity Planning Model

V. CVs

Professor Daniel Gartner, PhD is a Senior Fellow of the Higher Education Academy and a Personal Chair in Operational Research at Cardiff University, School of Mathematics. He is a Researcher-in-Residence at the National Health Service (NHS) in Wales where he is involved in an analytics and modelling academy to increase the use of quantitative approaches in healthcare. Prior to joining Cardiff University, he worked as a Postdoc at Carnegie Mellon University's H. John Heinz III College and earned his PhD at TU Munich, Germany, linking Machine Learning and Patient Scheduling [6].

Elizabeth Williams is a final year PhD student at Cardiff University, School of Mathematics working in collaboration with NHS Wales on the planning of frail and elderly pathways [7]. Prior to joining the PhD program, she earned her MMath degree from the University of South Wales working on a project around cancer waiting times, A&E admissions and patient pathway predictions. She is a student demonstrator and tutor in the School of Mathematics and president of the SIAM-IMA student chapter at Cardiff University.

Professor Paul Harper, PhD is Director of Cardiff University's Data Innovation Research Institute and a sub-panel member for the UK's Research Excellence Framework REF2021 (Mathematical Sciences). He is founding editor-in-chief of an international leading journal Health Systems (Taylor & Francis) and an Elected Fellow of the Learned Society of Wales. Together with Professor Daniel Gartner and other colleagues in the UK, Paul´s work was awarded the U.K.'s OR Society's Impact Medal in 2021 [8].

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