Microstructurally related model for predicting behaviour of unsaturated soils with double porosity in triaxial space

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ABSTRACT

Microstructure can have an important impact on the hydraulic and mechanical behaviour of unsaturated soil and so it is necessary for it to be considered in constitutive models to enable accurate predictions of soil behaviour. This paper focuses on constitutive modelling of soils exhibiting a dual-porosity structure. Based on the assumption that macro and micropores contained in the double porosity structure have different influences on the mechanical and hydraulic behaviour, the effective degree of saturation was selected as a microstructural index. This microstructural index was implemented within a Bishop's effective stress based approach and the Glasgow Coupled Model and the Modified Camclay Model were adopted as the basic framework for the development of a constitutive model. Typical samples of low-expansive, non-expansive and collapsible soils with dual porosity were selected to validate the model's

performance, with the model found to perform well when compared with experimental data in terms of isotropic compression, triaxial shear and wetting tests.

Key words: unsaturated soils; coupled constitutive model; effective degree of saturation; microstructure

INTRODUCTION

Recently, there has been growing interest in how the microstructure affects the hydraulic and mechanical behaviour of unsaturated soils (Cai et al. 2014, 2020b; a; Cuisinier et al. 2011; Jia et al. 2020; Low et al. 2008; Manahiloh et al. 2016; Ranaivomanana et al. 2017; Romero and Simms 2008; Sánchez et al. 2016; Sergeyev et al. 1980; Tian-er and Lin 2010; Trzciński and Wójcik 2019). The microstructure can be classified as having single or double porosity (Alazaiza et al. 2017; Bagherieh et al. 2009; Russell 2010) according to features of pore size distribution (PSD). The PSD for double porosity manifests significant bimodality, which is related to the presence of both macro and micropores (Casini et al. 2012; Lewandowska et al. 2004; Musso et al. 2014; Ngien et al. 2012). According to Li et al (2019), the macropores are mainly deformed during loading while the micropores are less affected; however, during drying and wetting process, the micropores will exhibit significant changes and be accompanied by reversible volumetric deformation.

A review of the literature (Cai et al. 2018; Li et al. 2013, 2019; Mašín 2013; Mašín et al. 2005) suggests that there are two ways to consider the effects of double porosity on the hydraulic and mechanical behaviour of unsaturated soils. The first way is adopting a specific microstructural index to represent the main features of double porosity. For example, the effective degree of saturation can be adopted as this index considers the different impacts of water within macropores and micropores, which provides an indirect way to take features of double porosity into consideration. Adopting this approach, Cai et al. (2018) established a hydro-mechanical coupled constitutive model based on the Glasgow Coupled Model that was validated against isotropic compression tests of double-porosity soils. Li et al. (2019) also used the effective degree of saturation to establish a model for unsaturated soils with double porosity. Their model used average effective stress, deviator stress and modified suction as the main stress variables. However, the model established by Cai et al (2018) is limited to isotropic loading and so cannot consider shear response. Also whilst the model established by Li et al. (2019) can consider shear response it adopts average effective stress as one of the main stress variables and so cannot consider how the three principal stresses will change respectively in a triaxial space, such as during shear at a constant net mean stress. A second way is to establish mutually independent models for the loading behaviour of macropores and micropores. For example, Mašin (2013) used different constitutive models for the behaviour of macropores and micropores and satisfactorily reproduced macro-mechanical properties by establishing a corresponding coupling relationship. However, this model has a complex form and requires determination of numerous material parameters, which limits its practical application.

This paper introduces a microstructurally based constitutive model for unsaturated soils with double porosity. The key contribution is that the model adopts effective degree of saturation as the microstructural index and uses the Glasgow Coupled Model as the basic framework to acquire a concise constitutive relationship, which needs only nine physically meaningful parameters. The generalized tensor and Modified Camclay Model (Roscoe and Burland 1968) were adopted to extend the model to a three-dimensional space where both isotropic compression and shear loading can be simulated. The model is validated through experimental results of low-expansive Speswhite kaolin (Sivakumar 1993), low-plastic clay (Almahbobi 2018) and non-expansive Jossigny silt (Cui and Delage 1996) in terms of wetting, saturated and unsaturated, isotropic compression and shear tests.

CONSTITUTIVE MODEL

The effective degree of saturation was adopted to consider the effects of double porosity on the mechanical and hydraulic behaviour of unsaturated soils. The model was established based on the framework of the Glasgow Coupled Model (GCM) (Wheeler et al. 2003). The GCM is an important constitutive model which considers the impact of degree of saturation on effective stress and behaviour of unsaturated soils. Unlike other models which adopt a single loading-collapse yield curve in the net mean stress-suction plane to study the elasto-plastic mechanism during suction decrease or mechanical loading, the GCM utilises yield curves in the effective stress-modified suction plane (see **Fig. 1**). These yield curves are: the loading-collapse line (LC), which serves as the boundary between elastic volumetric change and plastic volumetric change when the effective stress increases or decrease; the suction-decrease line (SD), which acts as the boundary between elastic and plastic changes of degree of saturation when suction decreases; and the suction-increase line (SI) which is defined to represent the position where plastic change of degree of saturation will occur during suction increase. According to Lloret-Cabot et al. (2013), the generalized stress tensor was adopted to study both mechanical and hydraulic in a unified framework and Modified Camclay Model (Roscoe and Burland 1968) was adopted to extend the model to a three-dimensional space to study the shear behaviour of soils.

Effective Degree of Saturation

According to Alonso et al. (2010), pores within double-porosity soils can be grouped into two categories: macropores partially filled with freely available water (the main source of suction) and micropores filled with bound water. Correspondingly, degree of saturation S_r can also be separated into two components: macroscopic degree of saturation S_r^M, which contributes to the mechanical behaviour of soils, and microscopic degree of saturation S_r^m, which is considered constant and independent of applied suction and load ($S_r = S_r^m + S_r^M$). Subsequently the effective degree of saturation can be defined as:

$$S_e = \frac{S_r - S_r^m}{1 - S_r^m} \tag{1}$$

with this definition, effective degree of saturation is a microstructurally based variable, which considers the feature of double porosity and establishes a link between this structure and the hydro-mechanical behaviour of soils. The effective degree of saturation can be represented by the residual degree of saturation because it is related to water stored within small occluded pores or intercrystalline pore space, where physico-chemical bonds hold the water to the solid and are not strongly related to applied suction or mechanical load, resulting in a very limited impact on effective stress (Alonso et al. 2010).

Stress Variables

According to Houlsby's theory (Houlsby 1997), the increment of work input dW per unit volume of unsaturated soil can be written as:

$$dW = [\sigma_{ij} - (S_{\mathrm{r}} u_{\mathrm{w}} + (1 - S_{\mathrm{r}}) u_{\mathrm{a}} \delta_{ij})] d\varepsilon_{ij} - (u_{\mathrm{a}} - u_{\mathrm{w}}) n dS_{\mathrm{r}} \tag{2}$$

where σ_{ij} is the stress tensor, $d\varepsilon_{ij}$ is the strain increment tensor, δ_{ij} is the Kronecker delta, u_{w} is the water pressure, u_{a} is the air pressure, S_{r} is the degree of saturation and n is the porosity.

Incorporating the Bishop's effective stress (Bishop 1959) and the effective degree of saturation S_{e} into (2), the expression of the Bishop's stress tensor σ_{ij}^{*} and the modified suction s^{*} can be derived as:

$$\sigma_{ij}^{*} = \sigma_{ij} - (S_{\mathrm{e}} u_{\mathrm{w}} + (1 - S_{\mathrm{e}}) u_{\mathrm{a}}) \delta_{ij} \tag{3}$$

$$s^{*} = (u_{\mathrm{a}} - u_{\mathrm{w}}) n \tag{4}$$

Correspondingly, the stress and strain increment vectors have been adopted as follows:

$$\mathbf{d\sigma}^{*} = (d\sigma^{*}_{xx}, d\sigma^{*}_{yy}, d\sigma^{*}_{zz}, d\sigma^{*}_{xy}, d\sigma^{*}_{yz}, d\sigma^{*}_{xz})^{T} \tag{5}$$

$$\mathbf{d\varepsilon} = (d\varepsilon_{xx}, d\varepsilon_{yy}, d\varepsilon_{zz}, d\varepsilon_{xy}, d\varepsilon_{yz}, d\varepsilon_{xz})^{T} \tag{6}$$

Following the approach of Lloret-Cabot et al. (2013) and taking the modified suction s^{*} and effective degree of saturation S_{e} as generalized stress and strain invariants, then the generalized stress and strain increment vectors can be written as:

$$\mathbf{d\sigma}^{*} = (d\sigma^{*}_{xx}, d\sigma^{*}_{yy}, d\sigma^{*}_{zz}, d\sigma^{*}_{xy}, d\sigma^{*}_{yz}, d\sigma^{*}_{xz}, ds^{*})^{T} \tag{7}$$

$$\mathbf{d}\tilde{\boldsymbol{\varepsilon}} = (d\varepsilon_{xx}, d\varepsilon_{yy}, d\varepsilon_{zz}, d\varepsilon_{xy}, d\varepsilon_{yz}, d\varepsilon_{xz}, -dS_{\mathrm{e}})^{T} \tag{8}$$

For a hydro-mechanical coupled model, the increment of stress is the collective output of the force, suction and effective degree of saturation. According to (3) and (4), the formulas for the increments of generalized stress can be defined as:

$$d\sigma^{*}_{ij} = d(\sigma_{ij} - u_{\mathrm{a}}\delta_{ij}) + (s dS_{\mathrm{e}} + S_{\mathrm{e}} ds)\delta_{ij} \tag{9}$$

$$ds^{*} = n ds - \frac{s d\varepsilon_{\mathrm{v}}}{v} \tag{10}$$

where $s = u_{\mathrm{a}} - u_{\mathrm{w}}$ is the suction, v is the specific volume and $d\varepsilon_{\mathrm{v}}$ is the increment of volumetric strain.

Yield Surfaces

Three yield surfaces, including the loading-collapse surface (LC), suction increase surface (SI) and suction decrease surface (SD) were adopted based on Lloret-Cabot et al., (2013), as seen in **Fig. 1**, in the q: p^{*}: s^{*} stress space (where q is the deviatoric stress, p^{*} is the mean Bishop's stress). The Modified CamClay Model (MCC), with a unique M (assuming that a unique Critical State Line in the q: p^{*} plane exists), was adopted as the reference model for the saturated condition. Then the algebraic expressions of the three yield surfaces are given as follows:

$$F_{\mathrm{LC}} = q^2 - M^2 p^* (p_0^* - p^*) = 0 \tag{11}$$

$$F_{\mathrm{SI}} = s^* - s_{\mathrm{I}}^* = 0 \tag{12}$$

$$F_{\mathrm{SD}} = s_{\mathrm{D}}^* - s^* = 0 \tag{13}$$

where p_0^* is the Bishop's pre-consolidation pressure which defines the position of F_{LC}, s_{I}^* and s_{D}^* are the modified suctions that locate F_{SI} and F_{SD}, respectively.

Coupling between Yield Surfaces

Yielding on either of the three surfaces will lead to the movement of the other two surfaces. The plastic mechanisms and couplings of yield surfaces are defined as follows:

1) yielding on the LC yield surface (F_{SI}) will bring about plastic volumetric strain with no irreversible change of the effective degree of saturation S_e, which in turn triggers upward movements of the SI and SD surfaces. This coupling is established, with a coupling parameter k_2, by:

$$\frac{ds_{\mathrm{I}}^*}{s_{\mathrm{I}}^*} = \frac{ds_{\mathrm{D}}^*}{s_{\mathrm{D}}^*} = k_2 \frac{dp_0^*}{p_0^*} \tag{14}$$

2) yielding on the SI/SD yield surfaces ($F_{\mathrm{SI}} / F_{\mathrm{SD}}$) will lead to a plastic reduction of S_e but no change of plastic volumetric strain, which in turn induces upward/downward movement of the SD/SI surfaces and outward/inward movement of the LC surface. This coupling is established, with a coupling parameter k_1, by:

$$\frac{dp_0^*}{p_0^*} = k_1 \frac{ds_{\mathrm{I}}^*}{s_{\mathrm{I}}^*} = k_1 \frac{ds_{\mathrm{D}}^*}{s_{\mathrm{D}}^*} \tag{15}$$

Flow Rules

Flow rules define the orientation of the generalized plastic strain increments during yielding. This paper assumes associated flow rules and adopts the generalized expression presented by Lloret Cabot et al., (2013) :

$$d\tilde{\boldsymbol{\varepsilon}}_j^{\mathrm{p}} = d\lambda_l^j \frac{\partial F_l}{\partial \boldsymbol{\sigma}^*} \quad \text{with} \quad l = \mathrm{LC}, \beta; j = \mathrm{LC}, \beta, \mathrm{LC}+\beta; \beta = \mathrm{SI} \quad \text{or} \quad \mathrm{SD} \tag{16}$$

where $d\lambda_l^j$ is the plastic multiplier with j related to the plastic mechanism which is active (e.g. when yield on LC yield surface is activated j is LC and for yield on SI or SD j is LC is β) and l is associated with plastic changes of effective degrees of saturation (when l is β) or volumetric strains (when l is LC).

Hardening Laws

The generalized hardening laws determining the relationships between increments of plastic volumetric strain $d\varepsilon_{\mathrm{v}}^{\mathrm{p}}$, increments of the plastic effective degree of saturation $dS_{\mathrm{e}}^{\mathrm{p}}$, and increments of the hardening variables dp_o^*, ds_{I}^*, ds_{D}^* are as follows:

$$dp_o^* = p_0^* \left[\frac{v d\varepsilon_{\mathrm{v}}^{\mathrm{p}}}{\lambda - \kappa} - \frac{k_1 dS_{\mathrm{e}}^{\mathrm{p}}}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}} \right] \tag{17}$$

$$ds_{\mathrm{I/D}}^* = s_{\mathrm{I/D}}^* \left[-\frac{dS_{\mathrm{e}}^{\mathrm{p}}}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}} + k_2 \frac{v d\varepsilon_{\mathrm{v}}^{\mathrm{p}}}{\lambda - \kappa} \right] \tag{18}$$

Generalized Stress-Strain Relationship

Current knowledge on the elasto-plastic mechanisms of unsaturated soils indicates that the generalized strains can be separated into elastic and plastic components (Lloret-

Cabot et al. 2013), so the increment of total generalized strains can be expressed as:

$$d\tilde{\boldsymbol{\varepsilon}} = d\tilde{\boldsymbol{\varepsilon}}^{\mathrm{e}} + d\tilde{\boldsymbol{\varepsilon}}_j^{\mathrm{p}} \quad \text{with} \quad j = \mathrm{LC}, \beta, \mathrm{LC}+\beta; \beta = \mathrm{SI} \quad \text{or} \quad \mathrm{SD} \tag{19}$$

where j is as defined in equation (16).

When only one elastic mechanism is activated, the generalized stress-strain relationship can be expressed as:

$$d\boldsymbol{\sigma}^{*} = \mathbf{D}_{\mathrm{e}}^{*} d\tilde{\boldsymbol{\varepsilon}}^{\mathrm{e}} \tag{20}$$

where $\mathbf{D}_{\mathrm{e}}^{*}$ is the generalized elastic matrix written in terms of the elastic bulk modulus K, the slope of the scanning curve in the water retention plane κ_{s} and the Poisson's ratio μ. Full details of $\mathbf{D}_{\mathrm{e}}^{*}$ and $\mathbf{D}_{\mathbf{ep}}^{*}$, the generalized elasto-plastic matrix, are presented in Appendix A.

There are nine parameters required in this microstructurally related constitutive model, namely λ, κ, k_1, k_2, M, μ, λ_{s}, κ_{s} and S_{res}. λ and κ are the slopes of normal consolidation line for saturated soil and slope of rebound curve, and they are related to the volumetric deformation during loading. k_1 and k_2 reflect the coupled movement between yield surfaces. To determine k_1 and k_2, two sets of data where both plastic volumetric strain and plastic change of degree of saturation are witnessed during loading (the parameters can be determined by using this data to solve equation (17) and equation (18)). M is a parameter that represents the relationship between deviator stress and effective stress under critical state and can be acquired by considering standard triaxial test data. μ is Poisson's ratio. λ_{s} 、 κ_{s} and S_{res} are the slopes of the main drying/wetting curve, the slope of scanning curve and residual degree of saturation, respectively. The residual degree of saturation can be gained by drawing two tangents at the start point and end point of the residual phase in a soil-water characteristic curve and taking the degree of saturation at the intersection as the residual degree of saturation (Eyo et al. 2022).

MODEL VALIDATION

To test the applicability of the proposed model, reported behaviour of a series of statically compacted samples of soils with double porosity (Speswhite kaolin, low-plastic clay and Jossigny silt) are considered. Speswhite kaolin is a low-expansive clay with dominant mineral as kaolinite and 75% clay fraction, which has a higher rate of consolidation compared with other clay soils (Sivakumar 1993). The low-plastic soil is a mixture of 40% Leighton Buzzard sand, 40% M400 silt and 20% Speswhite kaolin (Almahbobi, 2018). The soil showed significant collapsibility, in that it had a 16.2% vertical strain in a single oedometer test where the specimens were compacted at water content of 10% and dry unit weight of 14kN/m^3, and applied vertical pressure was increased to 50kPa. The plasticity index is 8%. According to the Unified Soil Classification System (USCS), the soil can be classified as low-plastic clay. Jossigny silt originates from the eastern region of Paris and typically shows little swelling on wetting. The soil is composed of illite, kaolinite and interstratified illite-smectite. The liquid limit and plastic limit are 37% and 19%, respectively (Cui and Delage 1996).

Experimental data in terms of wetting, isotropic compression and shear response of these soils are used to validate the model's performance.

Speswhite Kaolin

Isotropic compression

Suction-controlled triaxial tests reported by Sivakumar (1993) on compacted samples of Speswhite kaolin and conducted under both isotropic compression and shear have been considered. For isotropic loading, Path 1 was carried out by compressing the initial sample (mean net stress $\bar{p}$ =50kPa) to $\bar{p}$ =250kPa at a constant suction of 300kPa; Path 2 was carried out by wetting the initial sample (mean net stress $\bar{p}$ =50kPa) to s=100kPa and then compressing it to $\bar{p}$ =200kPa at a constant suction of 100kPa (see **Fig. 2**). Model parameters and initial states given by Lloret-Cabot et al. (2013) are shown in **Table 1**.

For both isotropic loading paths, the increasing net mean stress $\bar{p}$ resulted in overall compression and a decrease in void ratio. As noted by Lloret-Cabot et al. (2013), the initial stress state lay on the SD yield surface for both paths. This meant that as the modified suction reduced, yielding invariably occurred on the SD yield surface, which brought about a coupled inward movement of the LC yield surface and downward movement of SI yield surface. Before yielding on the SD surface there was no significant change in the void ratio or degree of saturation mainly because the modified suction did not decrease rapidly. In path 1, when the mean effective stress reached about 275kPa, the sample yielded on both the LC and SD yield surface. This yielding induced considerable amounts of plastic volumetric strain resulting in a significant decrease in void ratio (see **Fig. 3** (b)). Since the yielding on the LC yield surface results in a coupled upward movement of the SD yield surface, this also generated a significant increase in degree of saturation (see **Fig. 3** (c)). **Fig. 3** also presents a similar pattern of behaviour for Path 2. Overall, it can also be seen that the proposed model is able to reproduce well the isotropic loading behaviour of the Speswhite kaolin sample.

Shear loading

The loading of Speswhite kaolin samples included shear tests at constant confining stress and constant mean net stress. Both tests were carried out while suction remained unchanged. Before conducting the shear test at constant confining stress the sample was compressed from initial conditions of $\overline{p}$=50kPa and s=300kPa to $\overline{p}$=150kPa. The axial stress σ_1 was then increased until reaching failure (σ_2=σ_3=150kPa , $s = 300\text{kPa}$). For shear tests at constant net mean stress, the loading path included wetting the initial sample ($\overline{p}$=50kPa and s=300kPa) to s=100kPa and then compressing it to $\overline{p}$=200kPa before shearing it until failure. The model parameters and initial states are again as presented in **Table 1**.

Fig. 4 shows the simulated and experimental results of shear tests at constant confining stress and net mean stress. The constant confining stress path yielded on both the LC and SD yield surface due to the previous isotropic compression where the net mean stress was increased to 150kPa. Yielding on the LC yield surface brought about significant plastic volumetric deformation and a decrease of void ratio (see **Fig. 4** (b)). Meanwhile, the coupled upward movement of the SD yield surface resulted in a significant increase in degree of saturation (see **Fig. 4** (c)). As shown in **Fig. 4**, the simulated stress path is consistent with the experimental path, however the developed axial strain ε_a is larger compared with the observed behaviour. This over prediction of axial strain whilst a soil tends towards critical state is a previously reported phenomenon which is attributed to the deficiency of the Modified Camclay Model (Lloret-Cabot et al. 2013). Generally, it can be observed that the proposed model can satisfactorily reproduce the development of void ratio and degree of saturation during shear.

For the shear test at constant net mean stress, the yielding also occurred on the LC and SD yield surfaces and generated a large amount of plastic change of void ratio and degree of saturation (see **Fig. 4** (e) and (f)). Although the proposed model overestimates the axial strain at critical state (see **Fig. 4** (d)), it can generally represent the mechanical and hydraulic behaviour of the Speswhite kaolin sample.

Low-Plastic Clay

The soil samples were prepared by adding quantities of distilled water to dry mixtures with a target initial water content of 10%. Then the soil-water mixtures were compacted in a mould until an axial pressure equivalent to 998kPa was reached. The final dry unit weight of the soil samples was 15kN/m^3. The measured initial void ratio, degree of saturation and suction were 0.732, 36.2% and 563kPa, respectively, and for the tests considered here the sample was first isotropically loaded under a confining stress of 20kPa. Model parameters and initial states are estimated from calibrating experimental data of wetting, isotropic compression and shear tests provided by

Almahbobi (2018). λ , κ , k_1 and k_2 are from isotropic compression tests. M is attained from shear tests and μ is assumed to be 0.3. λ_s , κ_s and S_{res} are obtained from the reported soil-water characteristic curve. Model parameters and initial states are shown in **Table 2**.

Wetting

The sample was initially wetted to a suction equivalent to 500kPa (at constant confining stress of 20kPa) and compressed to net mean stress equivalent to 100kPa (at constant suction of 500kPa), the suction of the sample was then decreased in a stepwise manner to 5kPa. Experimentally observed behaviour showed the void ratio monotonically decreasing during wetting, along with a decrease in effective stress. This behaviour is consistent with the highly collapsible nature of the soil (Almahbobi 2018). Degree of saturation increased significantly with suction decrease and was close to saturation at the final suction of 5kPa. The proposed model simulated yielding on both the LC and SD yield surfaces throughout the wetting stress path (the initial stress state was located at the corner intersection of both) leading to a prediction of a significant decrease in void ratio and increase of degree of saturation. The modelled behaviour is consistent with the experimental behaviour, with the model successfully reproducing the collapsibility and saturation increase observed during wetting (see **Fig. 5**).

Isotropic compression

For saturated compression, the initial samples were wetted to saturation at a confining stress of 20kPa and then compressed to target net mean stresses of 100, 250 and 400kPa, respectively.

Table 3 compares experimentally reported and proposed model calculated void ratio and water content at the end of compression. Both the void ratio and water content decreased during compression due to the volumetric compression and water discharge induced by the increasing net mean stress. The differences between model results and experimental results are small with relative differences always less than 10% and below 5% for some data. It can be seen that the model performs well when predicting the void ratio and water content in a saturated compression test (see **Table 3**).

For unsaturated compression, the initial samples were wetted to a suction equivalent to 300kPa at the confining stress of 20kPa and then compressed to target net mean stresses of 100, 250 and 400kPa. Experimental and model results are again shown in Table 3 and it can be seen that the model matches the experimental behaviour well (relative differences are around 1% for void ratio and less than 10% for degree of saturation), demonstrating the effectiveness of the model in reproducing hydro-mechanical behaviour in unsaturated compression (see **Table 3**).

Shear loading

After the compression stage described above each of the saturated and unsaturated samples were sheared at a constant confining stress equivalent to the net mean stress at the end of compression. The experimentally measured void ratio and water content at the start of the shear loading stage were used in the proposed model.

In the saturated shear tests the deviatoric stress was observed to increase with increase in axial strain and reached a peak at an axial strain of 25%. The samples with higher confining stress had a higher peak of deviatoric stress. The peaks of deviatoric stress were 172, 436 and 660kPa, respectively, for the samples with confining stress equivalent to 100, 250 and 400kPa. The volumetric strain also increased and the sample with larger confining stress generated more significant volumetric deformation. In the model, the stress path remained yielding on both LC and SD yield surfaces and so volumetric strain developed further during shear. The modelled deviatoric stresses were consistent with the experimental results (see **Fig. 6**(a)). The model also performs well in terms of void ratios (see **Fig. 6** (b)). Similar conclusions can also be drawn from the unsaturated shear tests with the model demonstrating satisfactory performance (see **Fig. 6** (c), (d)). For **Fig. 6** (d), the model satisfactorily reproduced the change of void ratio when the confining stress is 400kPa. However, whilst the model correctly captures the overall trends of decreasing volume during shear, it overpredicts the magnitude of changes in void ratio when the confining stresses are 100kPa and 250kPa.

Jossigny Silt

Cui and Delage (Cui and Delage 1996) studied the yielding and plastic behaviour of unsaturated compacted samples of the Jossigny silt through isotropic compression and shear tests. According to the experimental results reported by Cui and Delage (1996), λ, κ, k_1 and k_2 are based on the isotropic compression tests. M is attained from shear tests and μ is assumed to be 0.3. λ_s and κ_s are obtained from the reported soil-water characteristic curve. The model parameters and initial states are presented in

Table 4. The microscopic degree of saturation S_{res} was estimated by Alonso et al (2010).

The samples were firstly wetted to target suctions (200, 400, 800 and 1500kPa) and then compressed to target net mean stresses (50, 100, 200, 400 and 600kPa) before they were sheared at constant cell pressures (50, 100, 200, 400 and 600kPa). The shear stage terminated when the critical state was reached.

Isotropic compression

Table 5 presents the comparison between model results and experimental results for isotropic compression at both a constant suction equivalent to 200kPa and at a constant net mean stress equivalent to 200kPa. It can be concluded that the model performs well in isotropic compression with the simulated development of void ratio and degree of saturation satisfactorily consistent with the experimental results.

Shear

Shear tests were conducted after the target suctions and net mean stresses were reached in the compression tests mentioned above. Shear tests continued until the critical state was reached. **Fig. 7** presents the deviatoric stress/void ratio-axial strain relationship in shear tests at constant suction equivalent to 200kPa and **Table 6** shows the comparison on degree of saturation between model results and experimental results when critical states are reached.

In general, the model can reproduce the changing trend of deviatoric stress with respect to the axial strain despite the model tending to overestimate the axial strain at a specific deviatoric stress (see **Fig. 7** (a)). The model also performs well in predicting the trends in degree of saturation (see **Table 6**) during shear but does overpredict the magnitude of the change of void ratio, as shown in **Fig. 7** (b). It is noted that as the wetting and compression steps, undertaken before shearing commenced, are also included in the simulation there is different in the predicted and measured void ratios at zero axial strain. It can also be noticed in **Fig. 7** (a) that there are discontinuities of the gradient for shear tests at cell pressure equivalent to 50 and 200kPa because the stress paths for these tests reached the LC yield surface during shearing. The stress path for the shear test at cell pressure equivalent to 600kPa had reached the LC yield surface during the isotropic compression stage so there is no discontinuity observed on the curve.

For shear tests at constant cell pressure equivalent to 200kPa, **Fig. 7** (c), (d) present the deviatoric stress/void ratio-axial strain relationship and **Table 6** shows the comparison on degree of saturation between model results and experimental results at the end of shear tests. It is clear that the model performs well under different suctions by satisfactorily reproducing the trends of behaviour but does exhibit a tendency to overpredict the magnitude of changes in void ratio.

CONCLUSIONS

A hydro-mechanical coupled constitutive model for unsaturated soils with double porosity is proposed in this paper, based on the framework of Glasgow Coupled Model. Since the Glasgow Coupled Model does not consider microstructure, the model established in this paper adopts effective degree of saturation as a microstructural index and introduces this index into Bishop's effective stress to establish the Bishop's effective stress formula that can consider the characteristics of the double porosity. The model also adopted the Modified Camclay Model in the average effective stress-deviatoric stress plane. The expressions of the model were derived by combining the generalized stress-strain tensor, associated flow law, hardening law and consistency conditions. The novelty of the established model lies in the combination of effective degree of saturation, Glasgow Coupled Model and Modified Camclay Model to provide a convenient and straightforward way to model the behaviour of double-porosity soils utilising only nine physically meaningful parameters.

The model has been validated against experimentally observed behaviour of double-porosity samples, such as the compacted samples of Speswhite kaolin (low-expansive), low-plastic clay (highly collapsible) and Jossigny silt (non-expansive). The validation results show that the model is able to predict with reasonable accuracy the hydro-mechanical coupled characteristics of unsaturated soils with double porosity, whether the soil is collapsible or non-expansive. In terms of isotropic compression and wetting behaviour, the model satisfactorily reproduces both the mechanical behaviour (the variation of the modified suction and the void ratio) and hydraulic behaviour (the variation of degree of saturation). The model also performs well in terms of shear response. The model results are generally consistent with the experimental results in terms of void ratio and degree of saturation, but the model, to some extent, overpredicts the axial strain at specific deviatoric stress. This feature is not unexpected as the Modified Camclay Model usually overestimates the axial strain developed during shear on the path to reach critical state. Overall, it is demonstrated that the inclusion of effective degree of saturation in such model is beneficial to consider the effect of microstructure, especially double porosity, on the behaviour of unsaturated soils and the proposed model is reliable in predicting the hydro-mechanical coupled behaviour of collapsible and non-expansive soils with double porosity.

APPENDIX A

The full expression for $\mathbf{D}_{\mathrm{e}}^{*}$ is given as follows:

$$\mathbf{D}_{\mathrm{e}}^{*} = \begin{pmatrix} E_{11} & E_{12} & E_{13} & 0 & 0 & 0 & 0 \\ E_{21} & E_{22} & E_{23} & 0 & 0 & 0 & 0 \\ E_{31} & E_{32} & E_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & E_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & E_{77} \end{pmatrix} = \begin{pmatrix} \mathbf{D}_{\mathrm{e}\ 6\times6} & 0 \\ 0 & \gamma_{\mathrm{e}\ 1\times1} \end{pmatrix} \tag{21}$$

$$E_{11} = E_{22} = E_{33} = K + \frac{4}{3G} = \left(\frac{v}{\kappa}\right) p^{*} + \frac{4}{3G} \tag{22}$$

$$E_{44} = E_{55} = E_{66} = G = \frac{3K\left(1-2\mu\right)}{2\left(1+\mu\right)} \tag{23}$$

$$E_{12} = E_{23} = E_{13} = K - \frac{2}{3G} \tag{24}$$

$$E_{77} = \gamma_{\mathrm{e}} = \left(\frac{1}{\kappa_{\mathrm{s}}}\right) s^{*} \tag{25}$$

where K is the bulk modulus and G is the shear modulus.

When a plastic mechanism is active, the generalized stress-strain relationship can be also assumed as follows:

$$d\boldsymbol{\sigma}^{*} = \mathbf{D}_{\mathrm{e}}^{*} d\tilde{\boldsymbol{\varepsilon}}^{\mathrm{e}} = \mathbf{D}_{\mathbf{ep}}^{*} d\tilde{\boldsymbol{\varepsilon}} \tag{26}$$

$\mathbf{D}_{\mathbf{ep}}^{*}$ is the generalized elasto-plastic matrix, depending on the specific plastic mechanism(s) active during loading. $\mathbf{D}_{\mathbf{ep}}^{*}$ is unknown hitherto, but the expression of it can be obtained by the following approach.

Considering the general case that two plastic mechanisms are active, the expression (26) can be written as

$$d\boldsymbol{\sigma}^{*} = \mathbf{D}_{\mathbf{e}}^{*} d\tilde{\boldsymbol{\varepsilon}}^{\mathbf{e}} = \mathbf{D}_{\mathbf{e}}^{*}\left(d\tilde{\boldsymbol{\varepsilon}} - d\tilde{\boldsymbol{\varepsilon}}_{\mathrm{LC}+\beta}^{\mathrm{p}}\right) \tag{27}$$

Then considering the flow rule (16),

$$d\tilde{\boldsymbol{\varepsilon}}_{\mathrm{LC}+\beta}^{\mathrm{p}} = d\lambda_{\mathrm{LC}}^{\mathrm{LC}+\beta}\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^{*}} + d\lambda_{\beta}^{\mathrm{LC}+\beta}\frac{\partial F_{\beta}}{\partial \boldsymbol{\sigma}^{*}} \tag{28}$$

To keep the end of the stress path invariably attached on yield surfaces after yielding, the consistency condition on F_{LC} and F_{β} should be considered:

$$dF_{\mathrm{LC}} = \left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^{*}}\right)^{T} d\boldsymbol{\sigma}^{*} + \frac{\partial F_{\mathrm{LC}}}{\partial p_{0}^{*}} dp_{0}^{*} = 0 \tag{29}$$

$$dF_{\beta} = \left(\frac{\partial F_{\beta}}{\partial s^{*}}\right)^{T} ds^{*} + \frac{\partial F_{\beta}}{\partial s_{\beta}^{*}} ds_{\beta}^{*} = 0 \tag{30}$$

Incorporating (29), (30) and hardening laws (17) and (18), the following expressions can be obtained:

$$\left(\frac{\partial F_{\beta}}{\partial s^{*}}\right)^{T}\left(-\gamma_{\mathrm{e}}\right)\left(dS_{\mathrm{e}} - d\lambda_{\beta}^{\mathrm{LC}+\beta}\frac{\partial F_{\beta}}{\partial s^{*}}\right) + \frac{\partial F_{\beta}}{\partial s_{\beta}^{*}} s_{\beta}^{*}\left[-\frac{d\lambda_{\beta}^{\mathrm{LC}+\beta}\dfrac{\partial F_{\beta}}{\partial s^{*}}}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}} + k_{2}\frac{vm^{T} d\lambda_{\mathrm{LC}}^{\mathrm{LC}+\beta}\dfrac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^{*}}}{\lambda - \kappa}\right] = 0 \tag{31}$$

$$\left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right)^T \mathbf{D}_{\mathrm{e}}\left(d\boldsymbol{\varepsilon} - d\chi_{\mathrm{LC}}^{\mathrm{LC}+\beta}\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right) + \frac{\partial F_{\mathrm{LC}}}{\partial p_o^*} p_o^* \left[v \frac{m^T d\lambda_{\mathrm{LC}}^{\mathrm{LC}+\beta}\dfrac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}}{\lambda - \kappa} - \frac{k_1 d\lambda_{\beta}^{\mathrm{LC}+\beta}\dfrac{\partial F_{\beta}}{\partial s^*}}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}} \right] = 0 \quad (32)$$

Then the plastic multiplies can be derived as:

$$d\lambda_{\mathrm{LC}}^{\mathrm{LC}+\beta} = \frac{\mathbf{a}_{\mathrm{LC}+\beta}^T d\boldsymbol{\varepsilon} + b_{\mathrm{LC}+\beta} dS_{\mathrm{e}}}{h_{\mathrm{LC}+\beta}} \quad (33)$$

$$d\lambda_{\beta}^{\mathrm{LC}+\beta} = \frac{\mathbf{c}_{\mathrm{LC}+\beta}^T d\boldsymbol{\varepsilon} + d_{\mathrm{LC}+\beta} dS_{\mathrm{e}}}{h_{\mathrm{LC}+\beta}} \quad (34)$$

where

$$\mathbf{a}_{\mathrm{LC}+\beta}^T = \left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right)^T \mathbf{D}_{\mathbf{e}} \left(\frac{\partial F_{\beta}}{\partial s^*}\gamma_{\mathrm{e}}\frac{\partial F_{\beta}}{\partial s^*} - \frac{\partial F_{\beta}}{\partial s_{\beta}^*} s_{\beta}^* \frac{1}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}}\frac{\partial F_{\beta}}{\partial s^*} \right) \quad (35)$$

$$b_{\mathrm{LC}+\beta} = -k_1 \frac{\partial F_{\beta}}{\partial s^*}\gamma_e \frac{\partial F_{\mathrm{LC}}}{\partial p_0^*}\frac{p_0^*}{\lambda_s - \kappa_s}\frac{\partial F_{\beta}}{\partial s^*} \quad (36)$$

$$\mathbf{c}_{\mathrm{LC}+\beta}^T = k_2 \left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right)^T \mathbf{D}_{\mathbf{e}} \frac{\partial F_{\beta}}{\partial s_{\beta}^*} v \frac{s_{\beta}^*}{\lambda - \kappa} m^T \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*} \quad (37)$$

$$d_{\mathrm{LC}+\beta} = \frac{\partial F_{\beta}}{\partial s^*}\gamma_{\mathrm{e}} \left(-\left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right)^T \mathbf{D}_{\mathbf{e}} \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*} + \frac{\partial F_{\mathrm{LC}}}{\partial p_0^*}\frac{v}{\lambda - \kappa} m^T \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*} \right) \quad (38)$$

$$\begin{aligned} h_{\mathrm{LC}+\beta} = & \left[-\left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\right)^T \mathbf{D}_{\mathbf{e}} \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*} + \frac{\partial F_{\mathrm{LC}}}{\partial p_0^*}\frac{v}{\lambda - \kappa} p_0^* m^T \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*} \right] \\ & * \left[-\frac{\partial F_{\beta}}{\partial s^*}\gamma_{\mathrm{e}}\frac{\partial F_{\beta}}{\partial s^*} + \frac{\partial F_{\beta}}{\partial s_{\beta}^*} s_{\beta}^* \frac{1}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}}\frac{\partial F_{\beta}}{\partial s^*} \right] \\ & -k_1 k_2 \frac{\partial F_{\beta}}{\partial s_{\beta}^*} p_0^* \frac{v}{\lambda - \kappa} s_{\beta}^* \frac{1}{\lambda_{\mathrm{s}} - \kappa_{\mathrm{s}}} \frac{\partial F_{\mathrm{LC}}}{\partial p_0^*} m^T \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^*}\frac{\partial F_{\beta}}{\partial s^*} \end{aligned} \quad (39)$$

Substituting these two plastic multipliers (33) and (34) into (26) and after some algebra, the generalized elasto-plastic matrix can be obtained as follows:

$$\mathbf{D}_{\mathbf{ep}}^* = \begin{pmatrix} \mathbf{A}_{\mathbf{LC}+\beta} & \mathbf{B}_{\mathbf{LC}+\beta} \\ \left(\mathbf{C}_{\mathbf{LC}+\beta}\right)^T & D_{\mathrm{LC}+\beta} \end{pmatrix} \quad (40)$$

where

$$\mathbf{A}_{\mathbf{LC}+\beta} = \mathbf{D}_{\mathbf{e}} \left(\mathbf{I}_{6\times6} - \frac{\mathbf{a}_{\mathrm{LC}+\beta}^{T}}{h_{\mathrm{LC}+\beta}} \frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^{*}} \right) \tag{41}$$

$$\mathbf{B}_{\mathbf{LC}+\beta} = \mathbf{D}_{\mathbf{e}} \left(\frac{\partial F_{\mathrm{LC}}}{\partial \boldsymbol{\sigma}^{*}} \frac{b_{\mathrm{LC}+\beta}}{h_{\mathrm{LC}+\beta}} \right) \tag{42}$$

$$\left(\mathbf{C}_{\mathbf{LC}+\beta}\right)^{T} = -\gamma_{\mathrm{e}} \left(\frac{\partial F_{\beta}}{\partial s^{*}} \frac{\mathbf{c}^{T}{}_{\mathrm{LC}+\beta}}{h_{\mathrm{LC}+\beta}} \right) \tag{43}$$

$$D_{\mathrm{LC}+\beta} = \gamma_{\mathrm{e}} \left(1 + \frac{\partial F_{\beta}}{\partial s^{*}} \frac{d_{\mathrm{LC}+\beta}}{h_{\mathrm{LC}+\beta}} \right) \tag{44}$$

DATA AVAILABILITY STATEMENT

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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Table 1. Model parameters and initial states for Speswhite kaolin (Lloret-Cabot et al. 2013)

Parameters		Initial states	
λ	0.124	$\overline{p}$ / kPa	50.00
κ	0.006	s / kPa	300.00
λ_s	0.098	e	1.208
κ_s	0.0076	S_r	60.1%
k_1	0.662	S_r^m	5.0%
k_2	0.803	p_0^* / kPa	273.00
M	0.71	S_D^* / kPa	164.00
μ	0.3	S_I^* / kPa	/

Table 2. Model parameters and initial states for the mixture

Parameters		Initial states	
λ	0.07	$\overline{p}$ / kPa	20.00
κ	0.008	s / kPa	563.00
λ_s	0.12	e	0.732
κ_s	0.02	S_r	36.2%
k_1	0.6	S_r^m	10.0%
k_2	0.3	p_0^* / kPa	250.00
M	1.076	S_D^* / kPa	237.94
μ	0.3	S_I^* / kPa	/

Table 3. Void ratios and water contents/degrees of saturation at the end of saturated and unsaturated compression

	P_{net} (kPa)	e (model)	e (experiment)	w (model)	w (experiment)
Saturated	100	0.579	0.529	21.87%	19.90%
	250	0.490	0.476	18.49%	17.90%
	400	0.439	0.438	16.58%	16.50%
				S_r (model)	S_r (experiment)
Unsaturated	100	0.713	0.702	43.8%	40.7%
	250	0.665	0.661	46.2%	42.2%
	400	0.632	0.626	47.8%	43.6%

Table 4. Model parameters and initial states for the Jossigny silt samples

Parameters		Initial states	
λ	0.091	$\overline{p}$ / kPa	25.00
κ	0.013	s / kPa	200.00
λ_s	0.131	e	0.629
κ_s	0.008	S_r	76.4%
k_1	0.65	S_r^m	56.0%
k_2	0.66	p_0^* / kPa	374.60
M	1.02	S_D^* / kPa	77.23
μ	0.3	S_I^* / kPa	103.84

Table 5. Results for the compression at constant suction/net mean stress

s (kPa)	p_{net} (kPa)	e (model)	e (experiment)	S_r (model)	S_r (experiment)
200	600	0.502	0.569	82.0%	84.0%
200	200	0.617	0.599	76.5%	77.0%
200	50	0.626	0.621	76.4%	77.0%
400	200	0.545	0.577	75.1%	74.0%
800	200	0.570	0.599	70.1%	70.0%

Table 6. Results of degree of saturation in the critical state

s /kPa	σ_3 /kPa	S_r (model)	S_r (experiment)
200	600	94.3%	98.0%
200	200	85.7%	79.0%
200	50	79.2%	79.0%
400	200	76.3%	77.0%
800	200	70.1%	70.0%

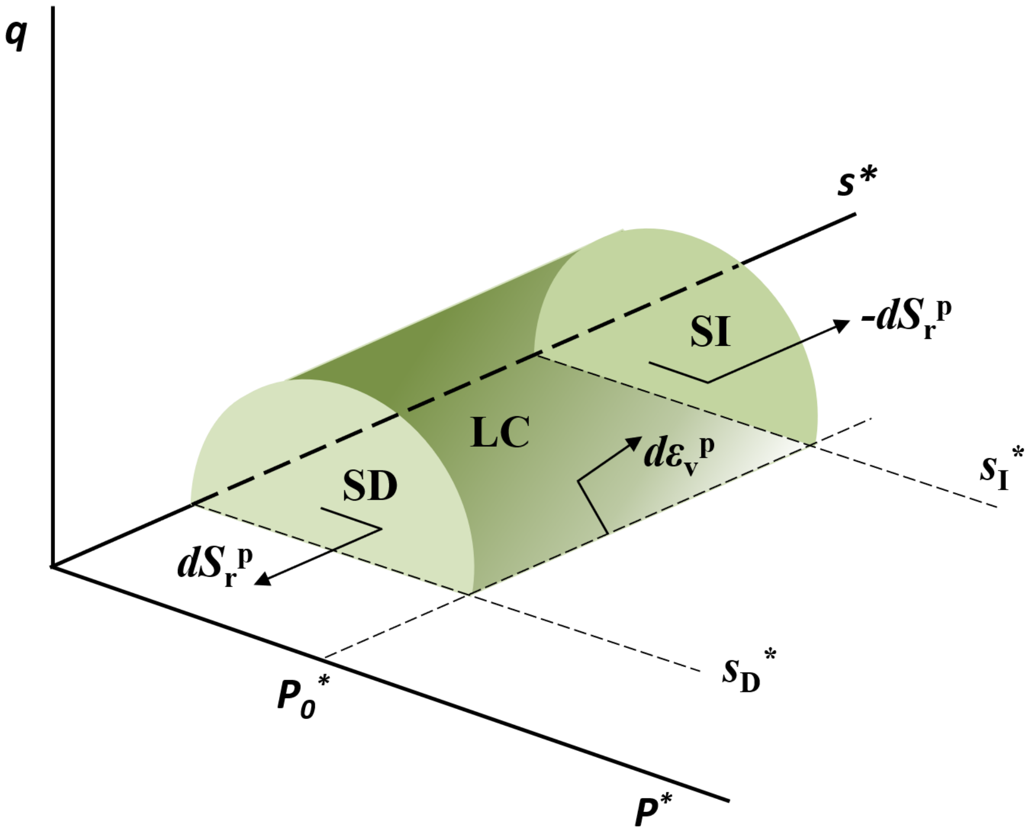


Fig. 1. Yield surfaces of the 3D generalized model (after Lloret-Cabot et al., 2013)

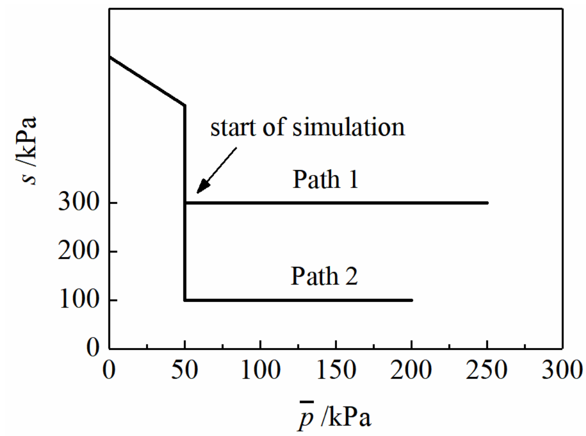
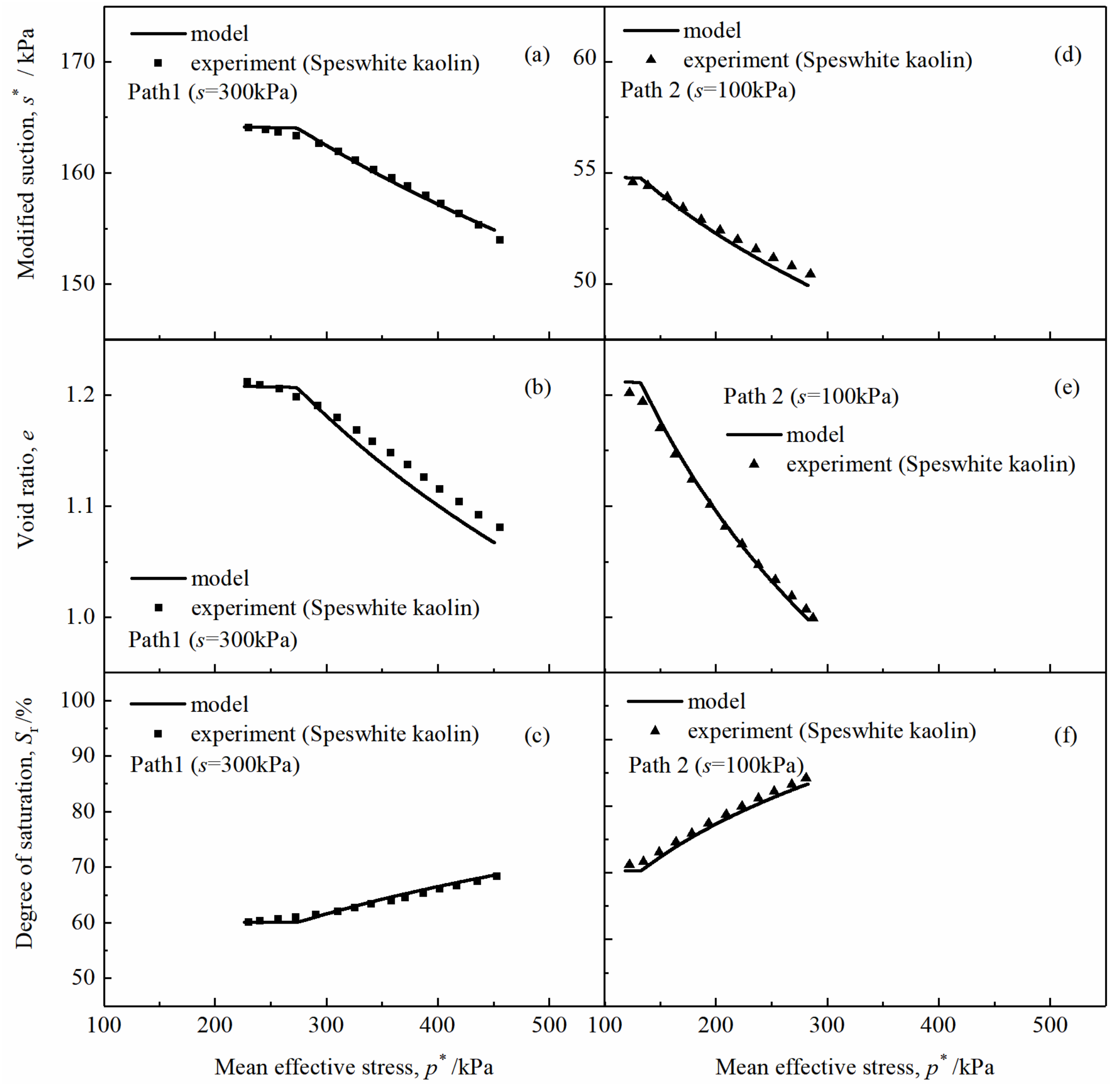


Fig. 2. Isotropic loading paths conducted by Sivakumar (Sivakumar, 1993)

611 

612 **Fig. 3.** Comparison between model results and experimental results on Speswhite

613 kaolin (Sivakumar, 1993) for Path 1and Path 2: (a/d) $p^* - s^*$; (b/e) $p^* - e$; (c/f)

614 $p^* - S_r$

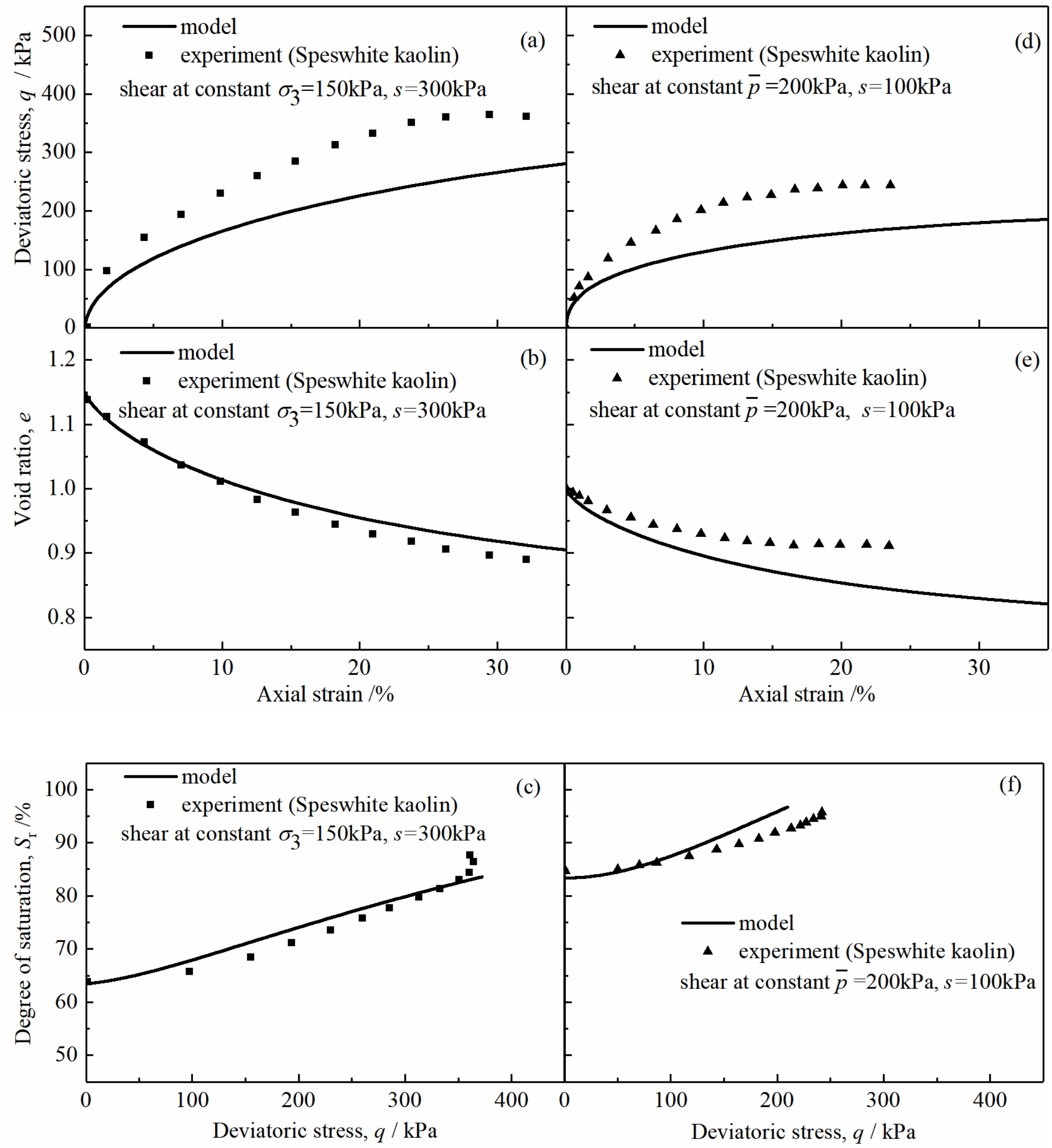


Fig. 4. Comparison between model results and experimental results (Sivakumar 1993) for shear at constant confining stress and net mean stress: (a/d) $\varepsilon_a - q$; (b/e) $\varepsilon_a - e$; (c/f) $q - S_r$

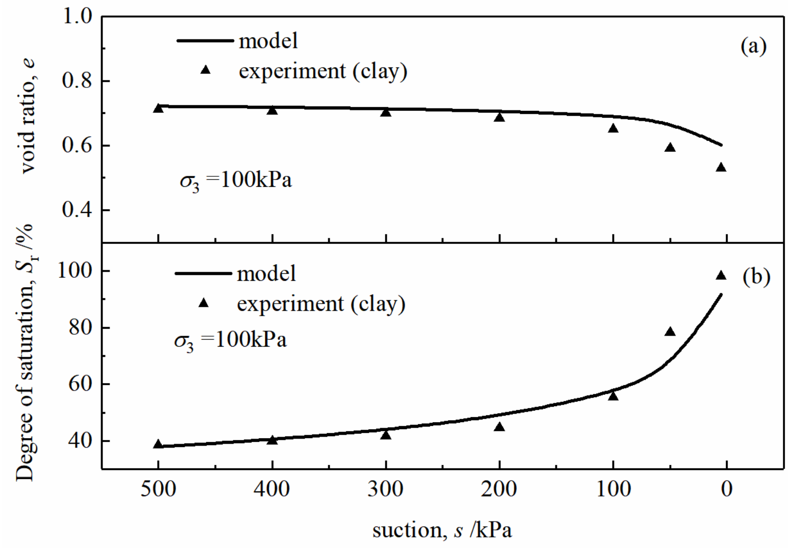


Fig. 5. Comparison between model results and experimental results on low-plastic clay: (a) $s-e$; (b) $s-S_r$

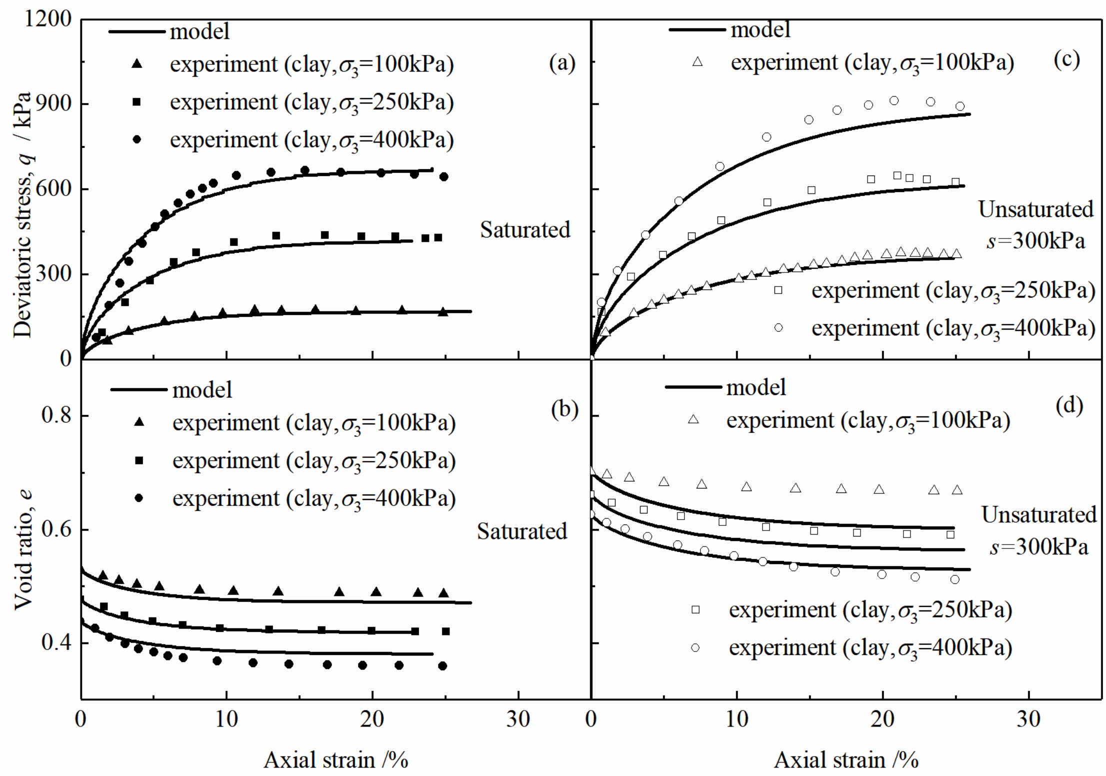


Fig. 6. Comparison between model results and experimental results (Almahbobi 2018) for saturated and unsaturated shear: (a/c) q; (b/d) e

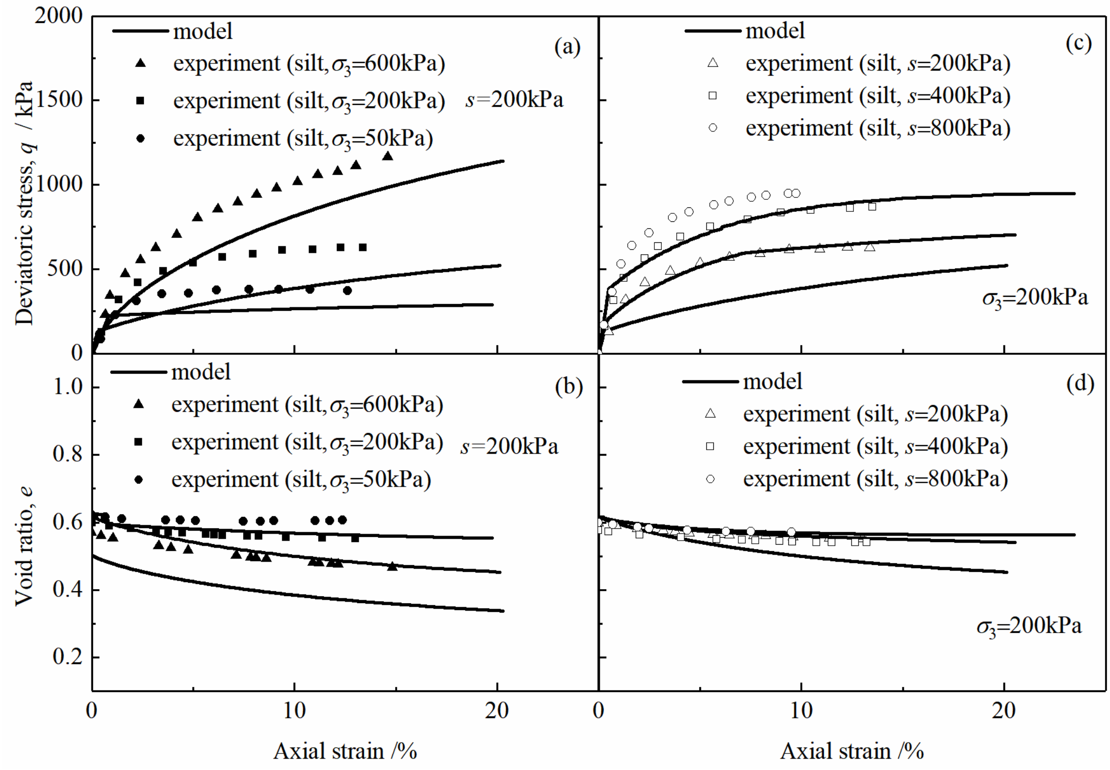


Fig. 7. Comparison between model results and experimental results on Jossigny silt (Cui and Delage 1996) for shear at constant suction or cell pressure: (a/c) q; (b/d) e