CARDIFF UNIVERSITY
DOCTORAL THESIS

POLICIES FOR REVIVING THE JAPANESE ECONOMY

SMALL OPEN ECONOMY DSGE WITH INDIRECT INFERENCE METHOD

by

YUJI FANG

Supervisors:
Dr. David Meenagh
Prof. Vo Phuong Mai Le
Prof. Patrick Minford
Dr. Zheyi Zhu

Economics Section
Cardiff Business School
Cardiff University

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Table of Contents

List of Figures

List of Tables

Abstract

The Japanese economy has remained stagnant under various policy stimuli in the past three decades. This study is planned to explore monetary and fiscal policy effectiveness in the Dynamic Stochastic General Equilibrium (DSGE) model. The model is built following Le et al. (2016) and extended to a small open economy based on Minford and Meenagh (2020). We use the indirect inference method to test and estimate the model using un-filtered nonstationary data in 1981Q3 to 2019Q2. The findings are concluded into three aspects: 1) The model with estimated parameters describes Japan's economy well; 2) the shocks from monetary and fiscal policy play a significant role in the recession, but fiscal policy performs better than monetary one; 3) The optimal regime is the current monetary policy regime- Taylor rule in standard situation and Quantitative Easing (QE) in Zero Lower Bound (ZLB) constraints- together with a strong fiscal response. A robust fiscal policy has been the key missing ingredient in Japanese policy.

1. Introduction

Since the economic bubble burst at the beginning of the 1990s, Japan has experienced the most prolonged economic stagnation. The average annual growth rate of real GDP from 1990 to 2019 was only 1.1%. Under the impact of the global financial crisis in 2008, the actual GDP even showed negative growth in the next few years. The inflation rate has been hovering near zero for the past 30 years. With the economic recession, Japan's unemployment rate has continued to rise. The unemployment rate in 2010 was more than twice the unemployment rate in 1990, increasing from 2.2% to 5.1%. Fortunately, in the past ten years, the unemployment rate has been well controlled, and the unemployment rate in 2019 has returned to the same level as in early 1990. However, it is undeniable that the return of the unemployment rate is closely related to the decline in Japan's population. We conclude that Japan's current economic level has hardly developed compared to the 1990s, but its debt-GDP ratio has reached a very high level at the end of 2019, 234.86%.

Policymakers and practitioners are committed to finding ways to restore Japan’s economy. The first method is to increase profitable investment opportunities through deregulation and structural reforms. Japan’s former central bank governor, Toshihiko Fukui, believed that macroeconomic textbooks could not solve Japan’s low output and low inflation problems. Japanese banks are burdened with massive amounts of non-performing loans, making it difficult to expand the scale of lending. Some economists stated that cleaning up the banking system was the key to the effective operation of the money transmission mechanism and that unprofitable companies should be closed to reduce excess capacity. Facing excessive investment, capital spending by the private sector would fall to a certain level to compensate for the much lower rate of potential growth and to meet the low level of workforce caused by the shrinking population.

However, restructuring the economy will not solve all problems. In a depressed economic environment, not only are banks unwilling to lend, but companies with heavy debts are unwilling to borrow more. Even if the banking system is fixed, new borrowing and subsequent economic growth may remain weak. Moreover, although the write-off of non-performing loans is crucial in the long run, accelerated write-offs in the short term will lead to severe bankruptcies and unemployment, further aggravating deflationary pressures. Thus, a second approach has been proposed: to provide aggregate demand stimulus through demand-side policy.

During Japan's economic recession, the inflation rate dropped to zero or even turned negative. Although policymakers thought of many ways to counter this, the inflation rate still did not rebound, so that people's expectations for inflation became negative. Therefore, even if nominal interest rates fall to zero, real interest rates cannot fall to stimulate the economy; instead, they remain positive, prolonging Japan's economic slump. According to this view, the necessary policy is to raise prices and form positive inflation expectations, so that real interest rates will fall, and the economy will get rid of stagnation. The quantitative easing policy so far has been controversial. It has helped many countries get out of trouble, but for Japan, it has not passed the test on a prolonged basis, although it has performed well in a short period. Some economists think that Japan's economy is stuck in a liquidity trap. Those who question the effectiveness of the quantitative easing policy pointed out that as long as the public lacks confidence in all policies taken by the Bank of Japan to restore inflation, it will be challenging to reverse entrenched inflation expectations.

Other economists believe that substantial and appropriate fiscal stimulus may reverse Japan's declining economic environment. But opponents point out that the substantial increase in government borrowing so far has failed to revive the economy. If the fiscal expansion continues, it will only increase the uncertainty of the results, which may cause households and businesses to take preventive measures and increase the pressure on the government for fiscal expansion.

This study aims to identify which policies can revive the Japan’s economy, monetary policy or fiscal policy, through testing and estimating a DSGE model. The model setting mainly refers to Le et al. (2016a). Specifically, the basic framework of the model follows Smets and Wouters (2007), which is largely based on Christiano et al. (2005). SW07 is now a widely accepted and recognized model that conforms to actual data. (See DiCecio and Nelson (2007), Villa and Yang (2011), Kamber and Millard (2018), and Ferrario et al. (2013)) In order to better describe the relationship between financial institutions, banks, and enterprises, and to find the impact of borrowing rates on the business cycle, the financial accelerator from Bernanke et al. (1999) is added to the model. Le et al. (2016a) consider the collateral used in bank lending so as to make the model more realistic. Therefore, the role of quantitative easing is not only limited to setting bank interest rates but also plays a vital role in investment effects. In addition, the model is set to switch arbitrarily above and below zero interest rates. When the bank interest rate is close to or below zero, the Taylor rule is considered invalid, and the interest rate is exogenously set as the lower limit of the zero interest rate; when the bank interest rate is far from the zero interest rate, the exchange rate is determined by the Taylor rule. The setting of price and wage in the model is consistent with those in the combined model of Le et al. (2016a). Some product and labour types come from a perfectly competitive market with price flexibility, and the rest come from imperfectly competitive markets with price rigidity. Le et al. (2016a) model gets somewhat closer to the US data's behaviour than other models that do not have those new elements. We extend the model to a small open economy, referring to the two-country model of Armington (1969), and apply the extended model to the Japanese economy.

Instead of the widely used Bayesian method, the model is tested and estimated using Indirect Inference, a simulation-based method for estimating the parameters of economic models (Gourieroux et al. (1993); Gourieroux and Monfort (1995)). This approach suggests a simple but fundamental insight: if one finds Nature's data generation process, then data generated (by simulation) from this process should leave a trail that in all aspects resembles the actual data; otherwise, evidence of discrepancies should appear. This match can be made in terms of data generation process's score, or moments associated with this process. However, the match can also be made in terms of the score or other moments of a misspecified data generation process chosen for its simplicity or tractability; this is the idea of indirect inference (Wang and McFadden 1996). More specifically, this method first chooses an auxiliary model as a window to describe a set of simulated data and then compares it with the actual data description. The choice of auxiliary model involving key variables determines the aspects we want to emphasize. If the model is well constructed, the simulated data description generated by the auxiliary model will not significantly differ from that generated by historical data. In this study, we first use calibrated parameters to find out the simulated data and use Wald statistic to reflect the difference between two sets of data; when Wald statistic is less than the critical value, the model can pass the test. The model is severely rejected during the initial testing phase. Then we apply the Simulated Annealing algorithm to find the optimal set of coefficients that minimise the Wald statistic. The model can significantly pass the test with a better performance after doing model estimation.

We further investigate the optimal policy combination. The economy in ZLB episodes generates high price and output volatility because of the interaction between state-dependence and the ZLB. This cannot be controlled by implementing unconventional monetary policies, like QE. We find that these rules need supplementing by a fiscal commitment to stop ZLB episodes in their tracks. We first compare three fiscal policy regimes and then use alternative monetary policy- nominal GDP targeting monetary policy to determine the optimal policy combination. The simulation results show that the strong active fiscal policy regime with normal Taylor rule has the best performance in stabilizing the economy. Such fiscal policy expansion would not cause a solvency problem in Japan because of its high national savings propensity.

The thesis is structured as follows. Chapter 1 starts by reviewing and analysing historical episodes of Japan's development after World War II and then discussing various suggestions in the economic literature on how to revitalize the Japanese economy through non-traditional monetary policy and expansionary fiscal policy. Chapter 2 builds a DSGE model that can be switched arbitrarily under the crisis (with zero lower bound) and standard environment (without zero lower bound). Chapter 3 introduces the indirect inference method used for model test and estimation. The results of model testing and estimation are shown in Chapter 4. This chapter also analyses the estimated model with impulse response function, variance decomposition, and shock decomposition. Chapter 5 is dedicated to finding the optimal policy combination. Chapter 6 concludes the research results and provides possible future extensions of the works.

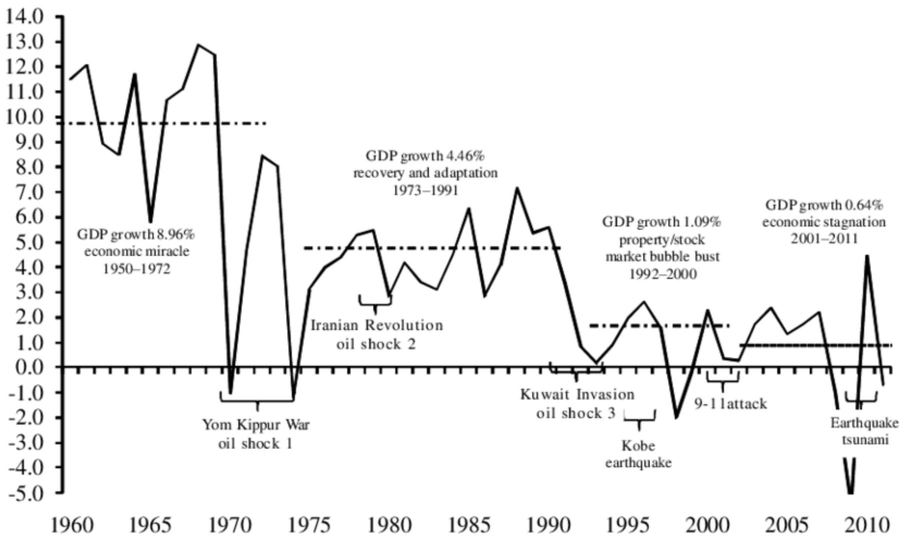
2. Post-war economic review and literature review

2.1 Japan's post-war economic development and policy implementation

2.1.1 Japan's economic development after World War II: from ''miracle recovery'' to ''bubble burst''

Japan was severely damaged during World War II (1941-45). The human loss amounted to about 4% of the entire population and material losses accounted for about 25% of national wealth (Economic Stabilization Board report of 1949). After the War, industrial production dropped to one-tenth of its pre-war level; Consumer goods and industrial inputs were 24% and 8% of pre-war levels, respectively. The Japanese economy suffered a devastating blow. The shortages of goods and increased budgetary expenditure had caused Japan to issue a large number of currencies; coupled with the relaxation of material control after the war and the impact of subsequent policies, the currency in circulation on the market had continued to increase and prices had skyrocketed. The people lived in dire straits. It was hardly imaginable that Japan, which had collapsed in economy and society, could quickly usher in its golden age and become the world's second-largest economy. Some people call it the Japan Economic Miracle.

Figure 2.1.1 Real GDP Growth Japan, 1960-2011



Note: Dotted lines represent average growth rate over the period.

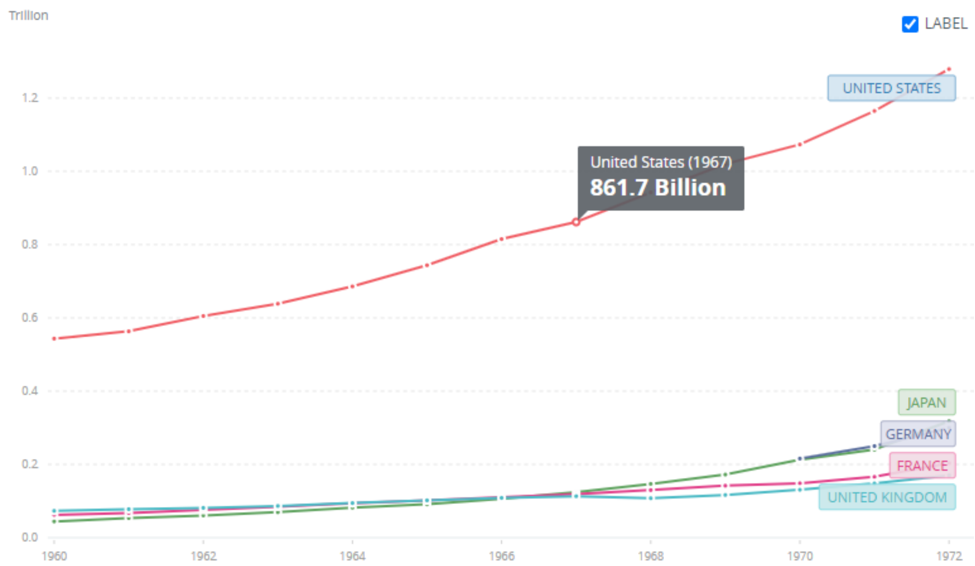
Source: World Bank (2012).

The first stage of economic development (1945-1960s)

The first stage of economic development after World War II was from 1945 to the 1960s. The economic goal at this stage was to catch up with the industrial economies of North America and Europe. The government adopted the "inclined production mode" to solve the most severe problem of savings shortage and focused its funds on basic industries centred on coal and steel. The government adopted the "inclined production mode" to solve the most severe problem of savings shortage and focused its funds on basic industries centred on coal and steel. Relevant industries can buy coal and steel at low prices with high government-backed subsidies. In 1947, the subsidy amounted to a quarter of the total budget. This policy enabled the factory to introduce the most advanced production equipment rapidly and improve production efficiency. Fiscal policy in terms of fiscal support also played an important role. The government allocated funds to provide loans for basic industries. The interest rate on the loan is aimed at the central bank's benchmark interest rate, which is well below inflation. This government-led financing, which essentially required the central bank to issue more money to fund businesses, has yielded remarkable results in various fields. The government also contributed to the rescue of financial institutions. Twenty percent of the government budget in 1946 was used as a special expense to compensate the bank to prevent bankruptcy. Further protections were implemented in the 1960s, neither allowing any financial institution to go bankrupt nor allowing new financial institutions to participate.

In addition, Japan established a so-called 'Japanese-style market' to achieve long-term and stable relations between economic agents. First, foreign exchange management and the main-bank system are carried out simultaneously. The former stipulates that foreign exchange can only be exchanged in designated banks, and the entry of imported goods is restricted through "import licenses", which protects domestic industries from external shocks to a certain extent. The latter refers to the Bank of Japan having absolute control over bank loans, with control of loans to enterprises, so that the government had the right to decide which industries the domestic capital should flow to. Second, the shareholder status was weakened by the bank's funding, corporate governance built on cross-shareholdings among businesses and with other financial institutions. Unlike Western-style trade unions and enterprises, Japanese trade unions were based on enterprises, living and dying with enterprises. Finally, active public policies and guidance also played a role in Japan's economic development. From 1955 to 1970, Japan's GDP doubled almost every 5 years, with an average annual growth rate of 9.6%. It only took 7 years to double the national income. At this stage, Japanese society had realized the development process from agriculturalization to industrialization. In 1950, agriculture and forestry accounted for 49% of GDP. In 1965, it fell sharply to 22%. In the second half of the 1960s, it was even lower than 12%. In 1968, Japan emerged from the shadow of a defeated country, and its GDP ranked second in the world, second only to the United States.

Figure 2.1.2 Real GDP in 1960s: United States, Japan, Germany, France, United Kingdom



Source: World Bank national accounts data, and OECD National Accounts data files.

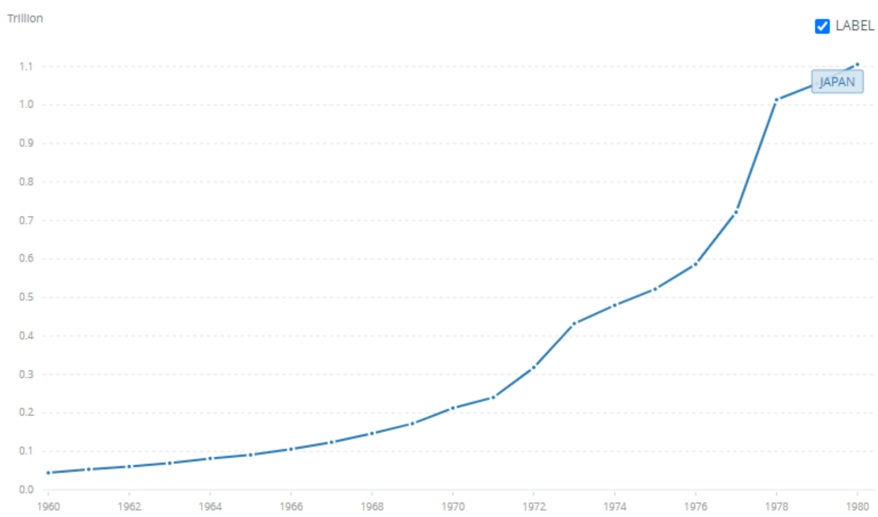
The second stage of economic development (1970s – 1980s)

The second stage is full of twists and turns. Japan experienced two oil crises at this stage and finally managed to get out of the crisis and its economy stabilized. In 1971, the complete collapse of the Bretton Woods system brought the world into an era of floating exchange rates, and Japan was no exception. In the 1970s, the Vietnam War and the Fourth Middle East War caused oil prices to soar from US$3.01 per barrel by 70% to US$5.11 per barrel in a short period. The first oil crisis began. Oil is the most basic energy source, and its price increased directly led to high inflation. The Bank of

Japan increased the benchmark interest rate to reduce currency circulation and further lower inflation. During 1973, the benchmark interest rate was raised five times. However, Japan's fiscal policy had not responded to the increase in oil prices. Until the end of 1973, the Japanese government maintained an expansionary fiscal policy and the public was still guided to invest in accordance with the principle of "peace and welfare" promised by the government. The "Japanese archipelago reconstruction theory", which raised land prices across the country, was a product of this period. That is, the government was committed to building highways and Shinkansen to form a developed transportation network centred on Tokyo. The reality soon caused them to put aside the so-called theory of the transformation of the Japanese archipelago, and shift from an expansionary policy to a policy that suppressed aggregate demand. In the next few years, the impact of this crisis slowed down Japan's economic growth, and it did not improve until 1978.

The good times did not last long, and the second oil crisis came in 1979. The oil price of around US$15 was directly pushed up to US$39. Japan's economic growth has slowed again. However, from the perspective of social influence, after having the first experience, Japan's ability to fight the crisis had also improved. Compared with the previous panic, this crisis had even brought opportunities to Japan. For example, Japanese cars, known for their fuel economy, entered the American market during this period.

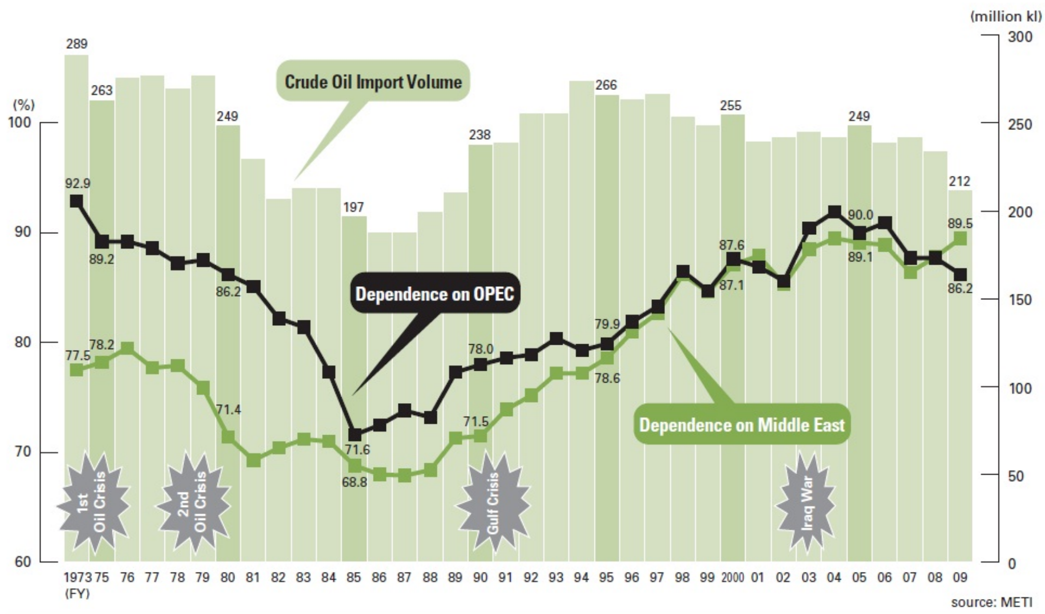
Figure 2.1.3 Japan GDP from 1960s-1970s



Source: World Bank national accounts data, and OECD National Accounts data files.

After the 1980s, Japan quickly walked out of the shadows and continued to achieve rapid development, apparently having overcome the oil crisis. There are three main reasons for its victory over the crisis. The first was that it reduced dependence on a single external source of energy. Japan began to implement a series of energy policies in 1980. Through the strategy of "reducing expenditure and increasing income ", Japan reduced its dependence on oil to the lowest level. In terms of cost reduction, Japan adopted a policy of reducing operations for most industries to save energy; in terms of increasing revenue, Japan actively developed alternative energy sources such as atomic energy, coal, and liquefied natural gas. At the same time, Japan established good diplomatic relations with oil-producing countries. The second was the floating exchange rate. After the collapse of the Bretton Woods system, the yen began a long-term appreciation path like a runaway horse. The appreciation meant that the purchasing power of the yen became stronger, which indirectly offset part of the impact of rising crude oil prices. The third was the unique trade union mechanism. In Japan, the corporate-interest-oriented trade unions overturned the ‘‘wages drive inflation’’ theory, which avoided the vicious circle of rising wages leading to a rise in prices and then causing an increase in wages again. In 1974, the rate of increase in wages in Japan reached 32.9%; by 1980, the growth in wage rate was only 6.9%.

Figure 2.1.4 Crude oil import trend in Japan



Source: Crude Oil Peak

In general, the two oil crises brought shocks to the Japanese economy. Some people think that the excessive dependence on external resources is the weakness of the Japanese economy. However, in the next ten years, Japan entered the golden era in Japan's economic history. When recalling this history in the 1980s, Former US Federal Reserve Chairman Alan Greenspan said that "Since the Sputnik crisis, this period (the United States) has felt more intensely foreign threats than at any other time." Some people would use "golden glitter" to describe this period of Japan. During this period, GDP nearly tripled.

The third stage of economic development (From 1980s)

In 1985, the Japanese government signed the Plaza Agreement, which can be considered a huge turning point in Japan's historical economic development. Before the Plaza Agreement, one U.S. dollar could be exchanged for 240 yen. After that, the central banks of various countries dumped the U.S. dollar, and it took only a few days to convert 1 U.S. dollar to 225 yen, and this number was still declining. The gradual appreciation of the yen put tremendous pressure on the Japanese manufacturing industry. Japanese companies began to find different ways to reduce production costs, in order to maintain the price advantage that was reduced by the appreciation of the yen. One year after the signing of the Plaza Agreement, the yen had appreciated by 20%, but because the cost was compressed to the greatest extent, Japanese goods still had an advantage. However, the yen had been rising, and the means to reduce costs will always be used up. Japanese companies can only move their production lines to places where had cheaper human and material resources, such as Southeast Asia and China. If factories were opened in foreign countries, Japanese workers would face unemployment. Since 1985, overseas production of Japanese cars had gradually increased. Japan's large-scale manufacturing industry can still maintain operation in this environment, but more and more small and low-end manufacturing industries had to declare bankruptcy. Nevertheless, Japanese society was still a prosperous scene. The yen's appreciation was also controlled between 10% and 20%, that is, between 222 yen and 202 yen per U.S. dollar. By the end of 1985, this number was still 202.786. The Japanese felt that the market would not get out of control. But at the beginning of 1986, the yen exchange rate achieved 190. Japan's foreign exchange market lost confidence in government control, and the U.S. dollar was sold wildly, which further promoted the unstoppable rise of the yen. The uncontrolled appreciation of the yen caused more and more small businesses in Japan to fail, causing a crisis. Two years after the signing of the Plaza Agreement, the yen had appreciated by 100%, with 1 U.S. dollar exchanging for 120 yen.

Figure 2.1.5 Japan / U.S. Foreign Exchange Rate

Date	Japan/US
Oct-85	214.6805
Nov-85	204.0737
Dec-85	202.7881
Jan-86	199.8905
Feb-86	184.8516
Mar-86	178.6938
Apr-86	175.0918
May-86	167.0314
Jun-86	167.5419

Source: FRED

Faced with an uncontrolled exchange rate, the Japanese government cut interest rates. The government regarded interest rate cuts as the best weapon to stimulate corporate loans, thereby reducing the pressure of appreciation. At the same time, the cut in interest rate would reduce the amount of yen held in the market, with people more inclined to buy currencies with high interest rates, slowing down the appreciation of the yen. At the end of January 1986, the Bank of Japan gradually reduced the bank interest rate from 5% to 2.5% within a year. However, the interest rate cut did not achieve the desired results. Investors bought into the stock market because low bank interest rates boosted profits of the real industry. From 1983 to 1987, the Nikkei index had more than doubled, spawning many high-value companies. Seven of the top ten companies by market capitalization were Japanese companies in the 1989 world rankings.

The place where prices were blown up was not only the stock market, but also the housing market. The real estate market soared, with low-cost mortgages used to buy land. Under the real estate frenzy that ensued, Japan's total land assets reached 2000 trillion yen at the end of 1989, four times the total land assets of the United States.

"You can buy the United States by only selling Tokyo." This seemingly exaggerated sentence now vividly shows the situation in Japan at that time. Here is another crazy example to show how Japan was rich in 1980s. In 1988, many people on the streets of Japan stopped a taxi to a place just one kilometre away with large banknotes of 10,000 yen (about 480 dollars). This was a very common phenomenon in Japan in the mid to late 1980s. Without the support of the real economy (i.e., manufacturing industry), the inflated stock prices and housing prices created a big bubble in the Japanese economy. On the last day of 1989, Japan’s Nikkei Index hit a high point of 34050.

In order to prevent the bubble from bursting, in 1989, the Japanese government decided to adjust interest rates again to cool down the booming stock and house market. From May 1989 to August 1990, the Bank of Japan substantially increased the central bank's interest rate five times, from 2.5% to 6%.

Figure 2.1.6 Loan interest rate and deposit interest rate in the middle of 1980s

Note: The upper line is loan interest rate and the lower line is deposit interest rate.

Figure 2.1.7 Top 8 by market capitalization (1989 V.S. 2019)

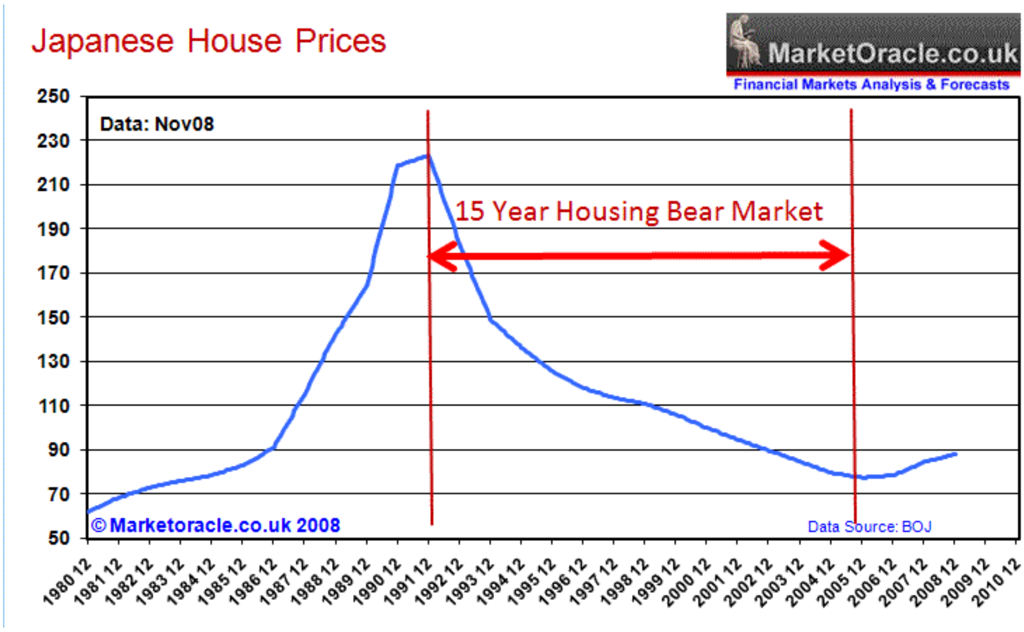
Rank	First year of Heisei (1989)	Last year of Heisei (2019)
1	NTT US$163.8 billion	Microsoft US$940.8 billion
2	Industrial Bank of Japan US$71.5 billion	Apple Inc. US$895.6 billion
3	The Sumitomo Bank US$69.5 billion	Amazon.com US$874.7 billion
4	Fuji Bank US$67.0 billion	Alphabet Inc. US$818.1 billion
5	Dai-Ichi Kangyo Bank US$66.0 billion	Berkshire Hathaway US$493.7 billion
6	IBM US$64.6 billion	Facebook US$475.7 billion
7	Mitsubishi Bank US$59.2 billion	Alibaba Group US$472.9 billion
8	Ericsson US$54.9 billion	Tencent US$440.9 billion

Note: Market Capitalization = Number of Shares Outstanding x Price

Source: Financial Time (1989)–based list is up to date as of 30 September 1989 and YCharts (2019) for market capitalization of each industry.

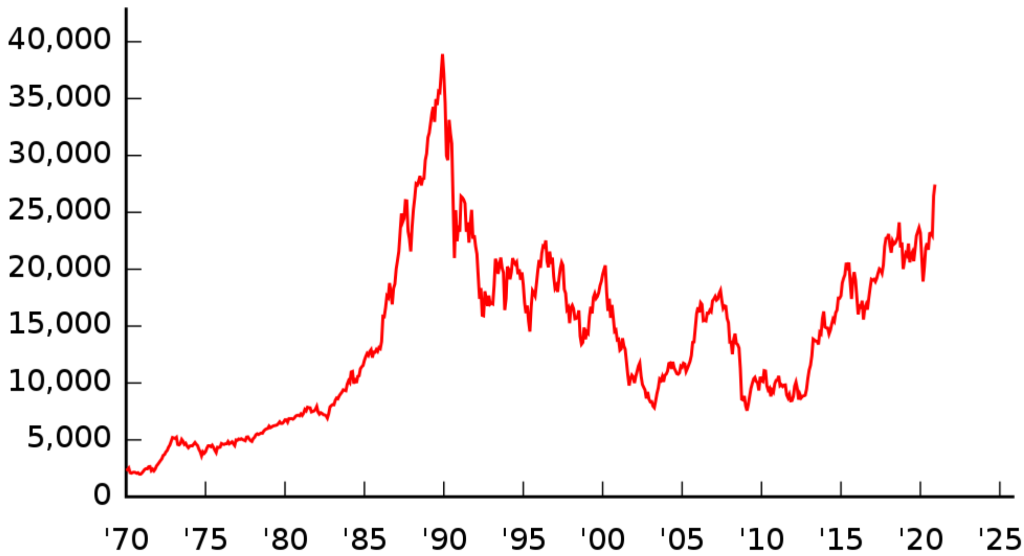
However, the response of the financial market exceeded the expectations of the Japanese government again. After the Bank of Japan raised interest rates in 1989, the stock market fell to 202 yen on the first trading day in 1990. Investors began to sell stocks, but the more people sold, the faster the price fell. The stock market collapsed, behaving like a set of dominoes. One year later, the housing market plummeted. Housing prices fell by 65% from 1992 to 2005. The Japanese economic bubble burst and the economy continued to shrink. Japan entered the era of ‘ the lost decades’’.

Figure 2.1.8 Japanese housing price from 1980 to 2010



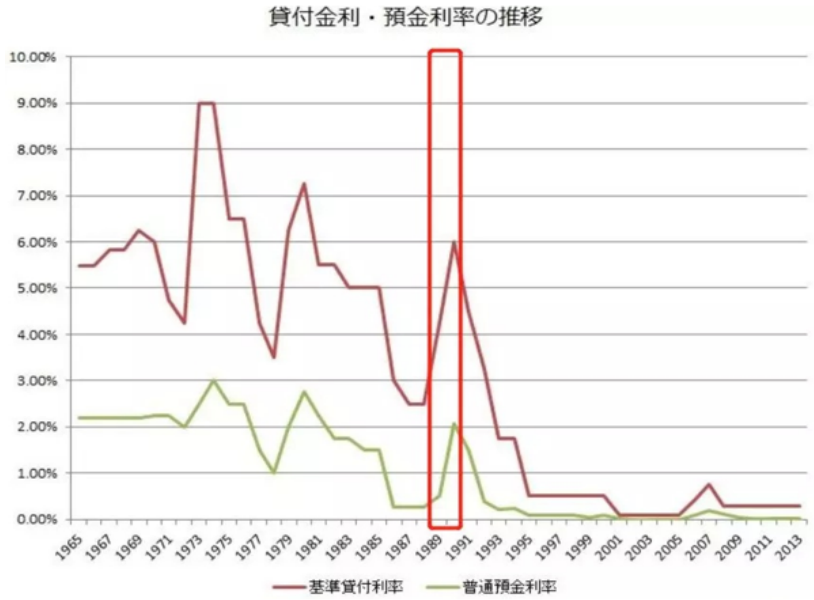
Source: Goldonomic - Real Estate \| Goldonomic, 2021

Figure 2.1.9 Nikki 225 stock average, index



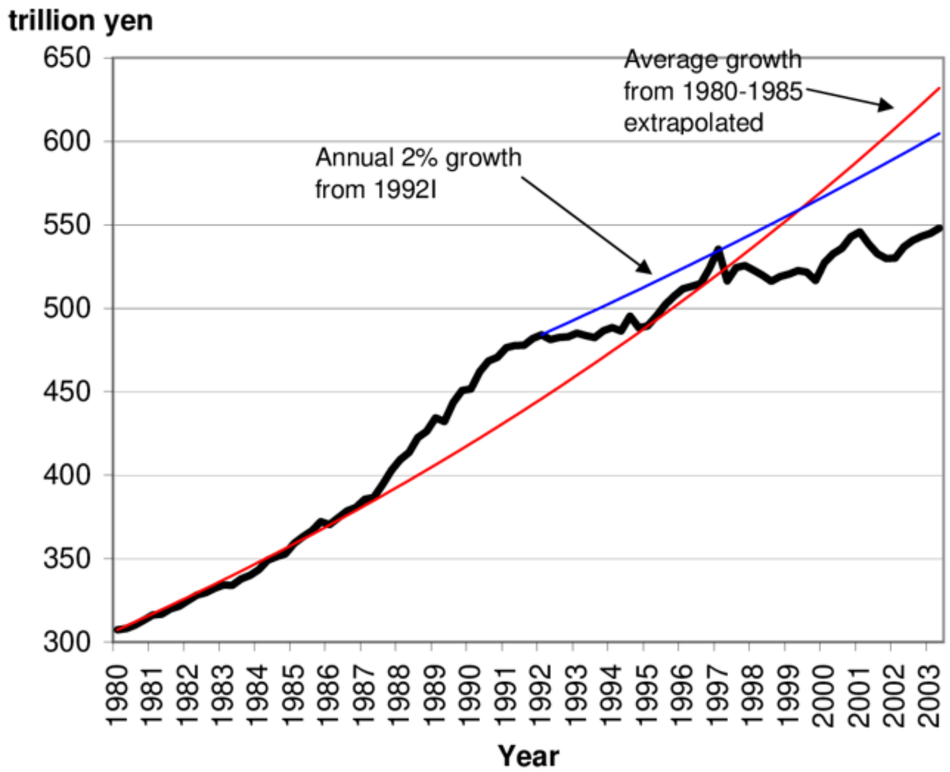
Source: Historical Data-Nikkei 225 Profile, 2021

Figure 2.1.10 Loan interest rate and deposit interest rate in the end of 1980s



Note: The upper line is loan interest rate and the lower line is deposit interest rate.

Figure 2.1.11 Real GDP of Japan: 1980-2003



Source: Hoshi and Kashyap, 2004

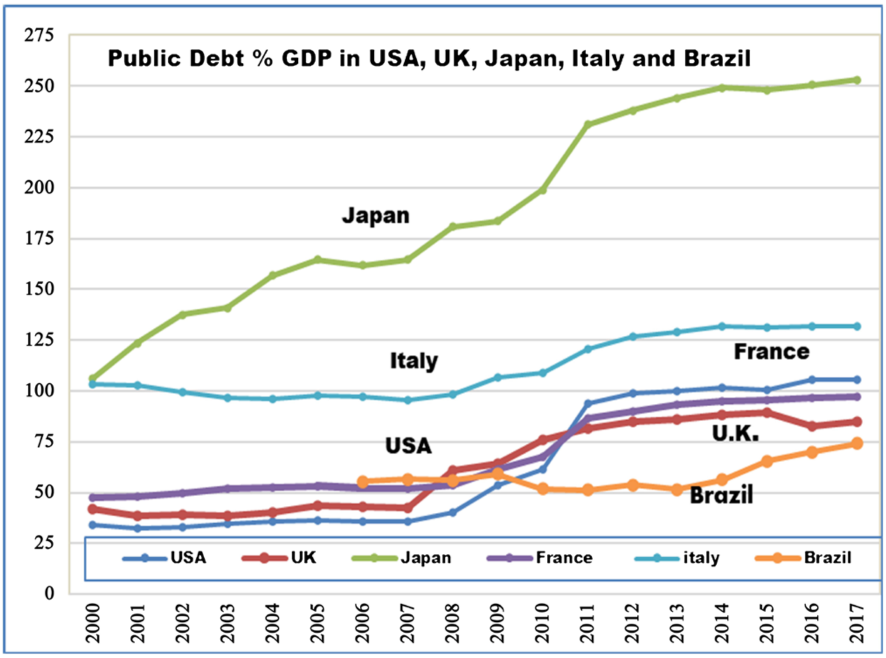
The fourth stage of economic development (From 1990s- Today)

The "lost decade" originally referred to the decade after the bubble burst (from 1991 to 2001). During this period, the Japanese economy grew by only 1.14% of GDP, much lower than other countries. From 1989 to 1992, stock values fell by 60%, and by 2001, land values fell by 70%, as we discussed in last section. However, due to the continued state of sluggish growth, the description of the "lost decade" of Japan's economy may be extended to the present day.

In the recent thirty years since the bubble burst, BOJ is still under intense pressure to do more to stimulate the economy, with interest rates stuck around zero for so long. When implementing monetary policy, the BOJ affects the formation of interest rates through operational tools such as money market operations to achieve monetary regulation. Moreover, the BOJ should announce a positive inflation target rate for the next 2-3 years and be held accountable for achieving the target. Policymakers by any means increase the money supply more substantially, buying non-traditional assets in open market operations. Previously, the Bank of Japan only bought and sold government bonds. Additionally, they also consider the yield curve control policy to release the pressure on bank balance sheet.

Fiscal policy has been expansionary over the past few decades, although some say it is not enough relative to what they think is sufficient. According to official estimates for 2021, Japan has far more public debt than any other developed country, at 266 percent of GDP. 45 percent of the debt is held by the Bank of Japan. In fact, governments often have to choose between the need for fiscal consolidation and the need for fiscal stimulus. In recent years, Japan's fiscal policy has tended to be expansionist due to the deteriorating economic situation. Many believe that "reform is impossible unless the economy improves first". But the effectiveness of fiscal policy in this situation is an open question.

Figure 2.1.12 Public debt in percentage of GDP in USA, UK, Japan, Italy, Brazil.



In general, the Japanese economy has remained stagnant since the 1990s, and the performance of various macro data is pessimistic. The Japanese economy is caught in a whirlpool of “three low” (i.e., low inflation, low interest rates, low growth) and “three high” (i.e., high welfare, high currency, high debt). In addition, from the labour market, although Japan's unemployment rate has improved in recent years, the expected wage growth is still low. The slowdown in productivity growth has ultimately led to disappointingly low wage growth. According to the Japan Productivity Centre’s International Comparison of Labour Productivity Report, Japan's labour productivity ranks 21st out of 36 countries in the Organization for Economic Cooperation and Development. Japan has been the last in the G7 since survey records were first provided in 1970. picture. . . It shows that Japan's hourly and per capita labor productivity is only more than 60% of that of the United States. The gap between the two countries has been steadily widening, compared with about 70% in 2000 and 65% in 2010.

Figure 2.1.13 Inflation rate from 1986 to 2026(compared to the previous year)

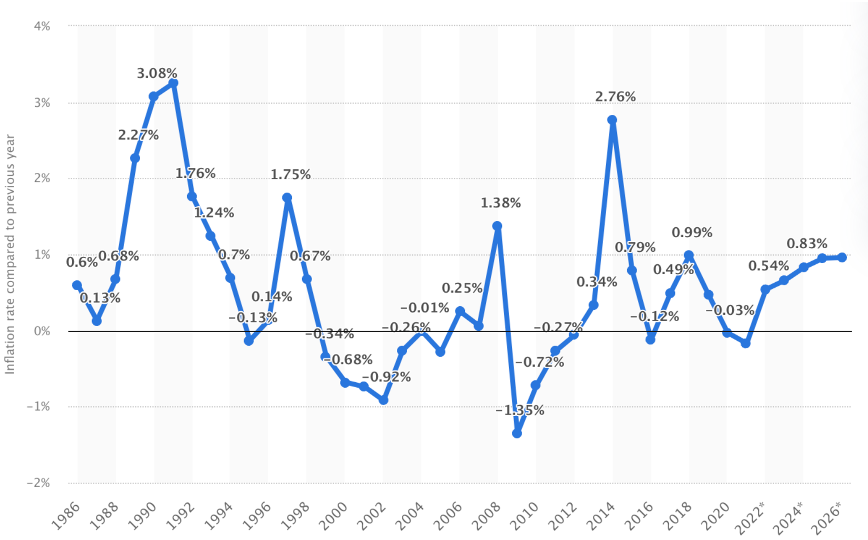
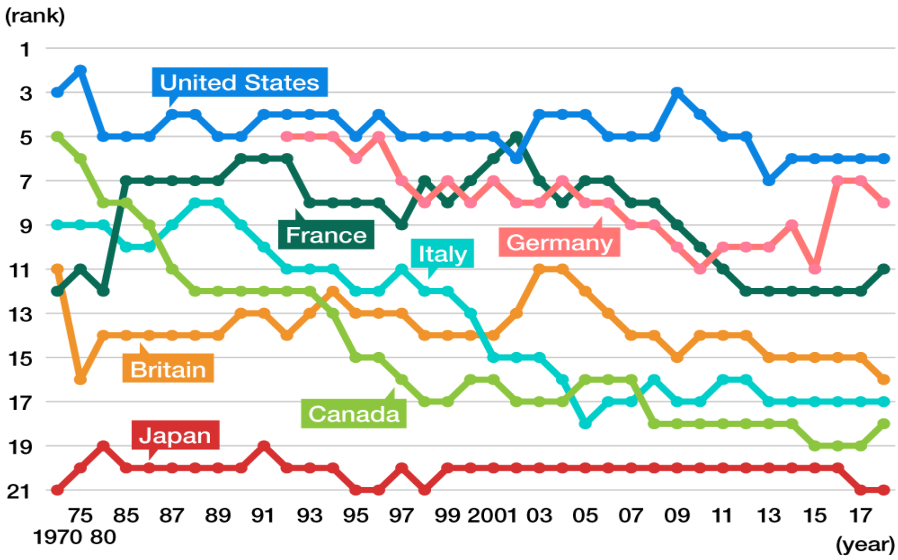


Figure 2.1.14 Ranking of G7 Nations for Per-Hour Labour Productivity



2.1.2 What caused the Japanese economy slowdown?

The Japanese economy has undergone a series of changes in 30 years, from the rapid recovery after World War II to the bubble generation and bubble burst. These last changes weakened aggregate demand through adverse wealth effects and hindered the function of financial intermediation. Because companies used the land as collateral for corporate loans, banks held a large number of stocks, which led to a further decline in macroeconomic activity in Japan. (Bayoumi 2001) This section discusses why the bubble burst and explores why the Japanese economy after the bubble could not recover as quickly as it did after World War II.

There have been two explanations of the cause of the asset bubble in Japan. The first view is that the bubble was structurally caused by bank deregulation. The second view is that the loose money in the late 1980s caused the bubbles. The former explains why banks started financing reckless projects, while the latter explains why the bubble lasted so long.

Bank Deregulation- Before the 1980s, Japanese banks were strictly regulated by the Ministry of Finance. Although the banks have almost no incentive to innovate beyond this regime, they could obtain a sufficient profit margin to prevent bankruptcy. With the disintegration of this system and the increasing competitiveness of financial markets, the banks lost the advantage of guaranteeing assured profits. Large corporate customers moved away from bank borrowing toward other financings, including retained profits, corporate bond issuance, and access to international financial markets. Japanese banks lost large corporate clients and were eager to find new borrowers and projects in small and medium-sized enterprises (SMEs) or in the investment of land and property, which had a high level of risk. However, banks in Japan lacked the ability to properly evaluate these new borrowers and projects. Also, the Bank of Japan relaxed the loan review and scale control of commercial banks. Enterprises, individuals, and non-bank financial institutions could quickly obtain loans from banks for speculative activities. In 1985, the loans from banks to non-financial institutions were less than 27 trillion yen, but they soared to 62 trillion yen in 1990, an increase of about 2.3 times. Approximately 40.2% of these funds are invested in real estate and construction. During the bubble period, about three-quarters of the 81.5 trillion yen required for real estate investment in the real estate industry in Japan came from bank loans. Masaru Yoshitomi pointed out that excessive bank lending in the late 1980s turned these loans into mountains of bad debts after the bubble burst.

Easing money too much and too long - The policy reaction function from The Bank of Japan is such that traditionally, monetary policy responds positively to yen appreciation and domestic recession. In 1985, the yen appreciated sharply, and the Bank of Japan responded by lowering short-term interest rates and relaxing the currency. Statistics show that from 1981 to 1988, Japanese M2+CD (cash currency + reserve currency + transferable deposits) increased rapidly at a rate of 7%-10% pa.

From 1987 to 1989, the growth rate of broad money (M2+CD) accelerated to more than 10%, this is too high for an economy with a growth rate of about 4%. With the increase in the money supply, parts of the money went into equipment investment. Other parts became a source of speculation in financial and land assets, leading to rapid stock and land price rises. Many economists believe that the easing policy promulgated by Satoshi Sumita, the governor of the Bank of Japan at the time, was too loose and took too long. But the Bank of Japan could not find a good way to tighten the currency and end the increase in asset price when inflation was close to zero. At the end of 1999, the new governor of the Bank of Japan, Yasushi Mino, tightened the currency and raised interest rates, which caused the economic bubble to burst.

Figure 2.1.12 Japanese money supply M2 – annualized growth rate

Source: Tenebrarum (2013)

2.1.3 the reasons for the failure of economic recovery

Economists discussed the reasons for the failure of Japan's recovery after the burst of the Japanese economic bubble. It can be classified into the following aspects.

Excessive savings- Japan's national savings rate has always been high. During the bubble economy, housing and land prices soared, which overdraft the country's investment in advance. At that time, Japanese companies and residents believed that investing in other industries was far inferior to real estate with high returns. After that, real estate prices fell sharply, and the fair value of the house was only one-fifth of its previous value. That is to say, the property that used to be loaned for twenty years of salary investment can now be bought with only three to five years of salary, which is equivalent to fifteen-eighteen years of salary income being overdrawn in advance. With investment opportunities contracting, the savings excess increased.

The banking system and the liquidity trap - Japanese commercial banks are controlled by chaebols, who also own industrial enterprises, and the chaebols hold mutual shares. The government invests in or substantively influences the chaebols, thus forming a highly organized and policy-oriented banking system. In addition, banks tend to provide long-term loans for up to 10 years, 15 years, or even 20 years, and the way of repayment is usually an annuity. At the same time, Japan's financial system has also derived annuity-based wealth management products or pension insurance funds based on these annuities as the basis of cash flow. The financial system has many derivatives that are paid in annuities, and those long-term loans are the basic assets. During rapid industrialization before the 1980s, such a banking system played a significant role in boosting Japan's economic growth. It provided long-term loans to many companies, especially emerging industry companies, and reduced the burden of corporate interest expenses. Thus, the banking system was conducive to the long-term and healthy development of manufacturing enterprises. However, after the bubble economy burst, the vitality of banks became limited under the old system. Banks could not find suitable investment projects that could provide long-term stable returns. At the same time, many real estate properties were mortgaged to banks. These dead assets could not generate cash flow. In addition, Japan's traditional manufacturing industry stagnated, and investment risks in emerging industries such as the Internet were high. The financial system under the Japanese banking system was in a dilemma: it was challenging to find projects that could maintain a stable cash flow for a long time, and it was unwilling to invest in high-risk projects. Therefore, Japanese commercial banks tended to invest heavily in Japanese national debt, U.S. national debt, and other fixed-income securities secured by government credit. Commercial banks did not actively expand the scale of loans. Instead, they chose a safe investment option, holding treasury bonds.

Cyclical -The massive overcapacity caused by the bubble period made Japan take time to reduce capital stocks and inventories.

Rise in consumption tax- Faced with a heavy debt burden, Hashimoto's cabinet implemented a tight fiscal policy in terms of fiscal policy. In 1997, the consumption tax was raised from 3% to 5%. This adjustment was reasonable in theory, but the timing of Japan to implement this policy was obviously wrong. Under the environment of the Asian financial crisis and the necrosis of Japan's financial system, the sharp increase in consumption tax greatly shrunk the domestic demand market. Banks were reluctant to lend, resulting in poor capital turnover of enterprises, and a large number of firms going bankrupt.

Other reasons - Japan's economic structure is still highly dependent on traditional manufacturing industries, such as automobiles, electronic components, steel, and machinery manufacturing. As mentioned earlier, when the Internet wave came in the 1990s, it is failed to provide sufficient investment for the start of the new Internet era. Japan's super-aged society is also one of the reasons, which makes the government debt rise at an alarming rate. The Japanese people are full of uncertainty about their future, especially regarding the sustainability of increasing tax burdens, employment opportunities, medical care, and pension plans. This pessimism slowed consumer spending and business investment. It seems that the failure of political leadership to solve these problems is a fatal factor in Japan's social uncertainty and declining confidence. People do not believe that the current government can solve these problems.

2.2 literature review

2.2.1 monetary policy

The Bank of Japan was the first country to implement unconventional monetary policies on a large scale, and its economic development is considered a good case to study the effectiveness of such policies.

Hosono and Isobe (2014) find that non-conventional monetary policy effectively reduces long-term Treasury bond yields and local currency exchange rates. The effect on corporate bond spreads, interbank loan spreads, and stock prices are also favourable. Furthermore, policy announcements with forward-looking guidance often significantly impact a wider range of assets than those without forward-looking guidance. Ugai (2007) confirmed that the unconventional monetary policy has an apparent effect. The commitment to maintain the quantitative easing policy has contributed to the expectation that the zero interest rate will continue. This reduces the yield curve centred on the short- and medium-term range. He also expressed concern that the Bank of Japan's balance sheet composition changes caused by the expansion of the monetary base, create uncertainties in its impact on portfolio rebalancing. However, in general, this potential adverse effect is much smaller than the effect of commitment. Michaelis and Watzka (2017) affirmed the effectiveness of quantitative easing policy in their report and stated that both core CPI and real GDP hsve responded. Changes in core CPI during the "Abenomics" period are more pronounced. The response of the price level is stronger and more significant than the response of real GDP. Wang (2019) constructs a shadow interest rate DSGE model. He finds that Japan's economy would have had a worse performance without implementing unconventional monetary policies. His findings in 2021 are more detailed, pointing out that quantitative easing implemented through portfolio rebalancing channels has a real impact on Japan's economy. It pushes up output and inflation and depresses the long-term interest rates to stimulate investment. As the period of quantitative easing becomes longer, its effect will become greater and greater. Otsubo (2019) found that except during full monetary easing, the impulse response of positive shocks to output under quantitative easing is positive; the impulse response to inflation is also positive after 2003. Abe et al. (2019) estimated the Markov switching DSGE model (MS-DSGE) to measure the interactions between Japan's monetary policy and fiscal policies. The finding shows that quantitative and qualitative easing policies can stabilize the economy and push up inflation in the face of adverse shocks in Japan. Hohberger et al. (2020) use a DSGE model to find the distributional effects of Japanese conventional and unconventional monetary policies; they suggest that expansionary conventional monetary policies and quantitative easing policies alleviate income and wealth inequality in Japan's economy.

Ueda (2012) believes that implementing the quantitative easing policy does not play a role in moving asset prices in the expected direction and devaluing the yen. The trend of deflation in Japan's economy cannot be significantly suppressed. Oda and Ueda (2007) find that the purchase of Japanese government bonds by the Bank of Japan has no significant impact on the expected future short-term interest rate or risk premium of JGB. This may be due to the relatively short remaining maturity of the JGBs purchased by the Bank of Japan. Saiki and Frost (2014) focused on exploring the relationship between Japanese monetary policy and income inequality, and substantial empirical evidence shows that unconventional policy led to increased income inequality in Japan. Taghizadeh-Hesary et al. (2021) reached a similar conclusion that the gap between the top and bottom of income distribution continues to widen. However, Inui et al. (2017) did not find a significant impact of Japanese monetary policy on household income and expenditure inequality. Chuffart and Dell'Eva (2020) deny Japan's monetary policy's effectiveness and point out that the reason for its failure is speculative positions in the foreign exchange market. Arbitrage trading modifies the channel for portfolio rebalancing. Large purchases of long-term securities led investors to invest in other currencies in the foreign exchange market. Therefore, the policy does not increase the incentive for investors to purchase corporate assets to increase credit and growth. Kimura and Nakajima (2016) report that the changes in output and inflation are not significant after the shock of quantitative easing. The estimation results of Koeda (2017) show that excess bond returns are not very sensitive to bond supply under the constraint of the zero lower bound. Huber and Punzi (2020) apply a time-varying parameter vector autoregression (TVP-VAR) model to explore the effect of unconventional monetary policy on the housing market. Their empirical evidence shows that for most advanced economies, like in the US, the EU, and Japan, the transmission mechanism of monetary policy to the real estate market has not changed after implementing quantitative easing policies or forward-looking guidelines.

In addition to using two commonly used tools, forward guidance and quantitative easing, the Bank of Japan launched Yield Curve Control (YCC) in 2006. The Bank of Japan pledged to fix the yield on 10-year Japanese government bonds (JGB) at around zero to push up persistently low inflation and further implement ultra-easy monetary policy. Ideally, yields on all other maturities are expected to be aligned with short and long run policy rates, allowing banks to determine the shape and position of the yield curve (Kuroda 2016). Kortelainen (2020) discusses whether Japanese yield curve control can help the economy and reduce deflationary pressures. Empirical results show that the Japanese economy is still unable to escape low inflation under the new policy. And he further points out that Japan's yield curve control may be less effective because it was only implemented after deflationary pressures had become ingrained in the economy. However, market participants increasingly believe that the policy of expanding the monetary base through large-scale purchases is unsustainable. Haruhiko Kuroda, Governor of the Bank of Japan, noted that quantitative easing without yield curve control targets the amount of government bond purchases, and its impact on real long-term interest rates may be too high or too low relative to what the central bank sees fit. He maintained a positive stance on quantitative easing with yield curve control in his speech, and clarified the importance of long-term interest rates as a policy objective. (Kuroda, 2006) Higgins and Klitgaard (2020) also affirmed the positive influence of YCC in financial markets. Ten- year JGB converged to the target soon in next two years. Yield volatility declined even further, with the standard deviation of monthly changes falling by about half relative to the QQE period.

2.2.2 fiscal policy

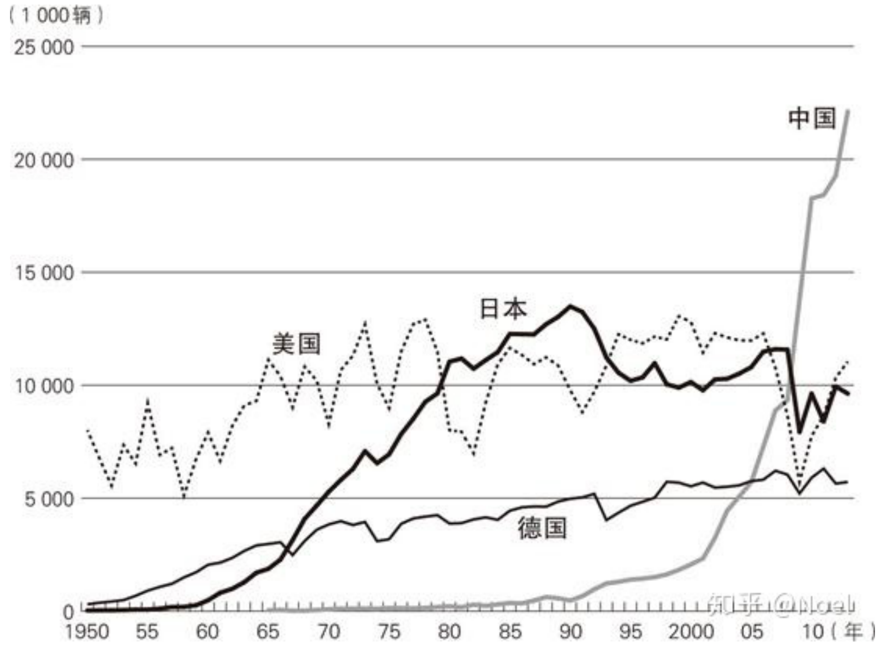
In the presence of enormous debt, expansionary fiscal policy is considered to damage the economy, and the continued expansion of Japan's national debt might be the fuse for the future fiscal crisis. Hoshi and Ito (2014) used simulation methods to assess whether Japan's budget deficit is sustainable and concluded that Japan's fiscal situation is in a dangerous zone. Kameda (2014a) indicates that the actual budget deficit led to increased government bond yields, further decreasing real GDP in 2008. Arai and Nakazawa (2014) conducted an empirical analysis to show that the government needs to prepare a vast primary surplus, 13.8-18.7% of total GDP, to avoid the debt-GDP ratio from expanding. Ko and Morita (2015) found that around 2% growth, or Ricardian fiscal stance, can keep the debt-GDP ratio sustainable. Miyazaki et al. (2004) and Velinov (2015) show that Japan does not have sustainable fiscal policies and the government target of fiscal consolidation is hard to achieve by the year 2020. McNelis and Yoshino (2012) used Bayesian estimation in an open-economy DSGE model of Japan, and they find that even a small risk premium on government debt will cause severe instability of interest rate, real exchange rate, and real sector performance. An exchange-rate rule for monetary policy could significantly ease the instability caused by the rising risk premium. Yoshino and Mizoguchi (2013) investigate the features of the flow of funds in Japan over time and the stabilizing conditions for government bond markets. They believe that those who demand JGBs play an essential role in the stability of the market. An increased volume of corporate savings was deposited as liquid savings and used to purchase Japanese government bonds through financial institutions via the flow of funds. They suggested that the government should restrain the issuance of debt-covering government bonds and guide the flow of Japan's funds to private capital accumulation, to promote the recovery of Japanese economic growth. Yoshino and Vollmer (2014) and also get similar findings.

However, more and more researchers have held different attitudes on the effect of expansionary fiscal policy recently; they believe that expansionary fiscal policy can increase GDP faster and less persistently than expansionary monetary policy. Faced with increasing government debt, Hansen and İmrohoroğlu (2016) developed a Neoclassical growth model to explore alternative ways of financing the increasing government expenditures. They find that the extremely high taxes, like consumption tax and labour income tax, highlight the importance of considering alternatives that attenuate the projected increases in public expenditures and expand the tax base. The results in the study of Afonso and Jalles (2014) also cannot reject the long-run fiscal sustainability. İmrohoroğlu et al. (2019) point out that a combination of fiscal policies can achieve long-run fiscal sustainability in Japan: cut pensions, increase consumption tax, raise the retirement age, and so on. Compared to the collapse of the Greek fiscal system, the Japanese bond market remains stable despite holding the most significant debt-to-GDP ratio. It is because the demand for Japanese government bonds has been maintained at a high level. Its muscular economic strength, high creditworthiness, and good stability provide investors with a comfortable environment with a lower level of credit risk, which results in the domestic and foreign investors having great confidence in this economy. The demand for long-term and short-term bonds is not affected by the debt-to-GDP ratio. (Yoshino and Vollmer (2014); (Yoshino and Taghizadeh-Hesary 2018)

Blanchard and Tashiro (2019) believe that the government should focus on keeping economic growth in the current Japanese environment rather than decreasing government deficits and debt. Since 2003, the GDP nominal growth rate has exceeded the interest rate, breaking down the traditional debt dynamics analysis. The shallow current and expected interest rates provide advantages for expansionary fiscal policies, which lower the fiscal and economic costs of debt but raise the benefit of public deficits. Firstly, the government can run a debt deficit while maintaining a stable and unchanged debt-to-GDP ratio, indicating a decline in fiscal costs. The crowding out of capital as the leading economic cost of public debt is also limited when the interest rate is less than the growth rate. If expansionary deficits increase demand and output successfully, capital investment may also increase rather than decrease. Moreover, they point out that the risk-adjusted social rate of return for many government-invested projects can be higher than its borrowing rate. Public investment in human capital is particularly effective for Japan. For instance, the investment in childcare and family can stimulate the fertility rate and future population growth, further relieving future fiscal pressure and improving future production and the whole economic environment. In conclusion, the benefits of government deficits are substantial, both in sustaining demand in the short run and improving supply in the long run.

Additionally, expansionary fiscal policy has a positive impact on the stock price. In general, the continued rise in government deficit results in government bonds being required to provide a higher premium, which increases the interest rate paid on government bonds. The public investment with a higher interest rate crowds out private investment, which hits the vitality of the economy and negatively affects the stock market. However, the positive externality of future public capital increases stock returns, which could offset the negative crowding-out effect. Thus, the expansionary fiscal policy driven by the increase in government debt raises stock prices in Japan. Miyazaki et al. (2004) compared the stock returns of manufacturing and non-manufacturing and found that the stock returns in both industry groups were positive during the ZLB period. A similar finding in Miyamoto et al. (2018) and Zhang (2016) indicates that the expansionary fiscal policy stimulates private consumption and investment. Morita (2015) shows that expansionary fiscal policy effectively boosts consumption when there are many rule-of-thumb consumers. Miyazaki et al. (2004) examines the effects of fiscal stimulus packages with environmental benefits using a mixed vector autoregression approach. He finds that tax relief and subsidy for the environmentally friendly automobile industry can help stimulate automobile production, while similar programs in industries producing energy-saving appliances have no effect.

Figure 2.2.1 Sales of Japanese Automobiles in US, Japan, China and Germany



Note: The dotted line represents United States, thick solid line represents Japan, solid grey line represents China and thin solid line represents Germany.

To avoid a fiscal crisis, further fiscal tightening, that is, the consumption tax was raised to ten percent, sparked discussions among researchers and politicians. Theoretically, an increase in consumption tax could have a negative macroeconomic impact, leading to higher commodity prices and thus lower aggregate demand. Hayashida et al. (2022) reach consistent conclusions by examining a DSGE model that only include consumption tax. However, in the study, they also show that if the increase in consumption tax can be paired with productive government spending policies (such as increased public investment by the government), then the increase in consumption tax can stimulate the GDP growth. This is because the increase in the production level under the policy increases the marginal labour productivity and thus the wage rate, which will offset the negative impact of the consumption tax on total consumption, and final consumption can also be increased.

Lecznar and Lubik (2018) examine the dynamic changes in consumption, real interest rates, and labour input in Japan before and after consumption rate increases between 1985 and 2014. The results show that Japanese households have formed stronger consumption habits and preferences after the policy change, and real interest rates are more sensitive. Specifically, households were less risk averse and exhibited a higher degree of habit formation during periods of tax hikes and low nominal interest rates. Some scholars have also compared the impact of other types of taxes with consumption taxes. Yoshikawa (2018) emphasize the importance of fiscal reconstruction and social security reform, arguing that the government should raise the consumption tax as soon as possible to increase public revenue. Most Japanese citizens want a social security scheme like that of the EU, however, as it stands, maintaining the existing social security system is still difficult. The result shown in the study states that politicians must lead the way in forging a national consensus on raising the consumption tax rate. Akram (2018) agrees that fiscal and structural policies can stimulate effective demand and real disposable income and overcome low inflation and deflationary trends. However, he believes the planned consumption tax hike, albeit a smaller increase than before, could be counterproductive in an environment of gradual economic recovery. He suggested delaying rate hikes until observed inflation exceeds his target for an extended period.

In addition, some scholars have also compared the impact of other types of taxes with consumption taxes. Watanabe et al. (2015) consider financing through consumption tax and income tax, respectively. Compared with income tax, the consumption tax case can achieve higher income growth. Doi (2010) studies the effect of corporate taxes on factor prices: wage rates and interest rates. Its impact on macro variables is also slightly less than the consumption tax. Imrohoroglu et al. (2016) construct an overlapping generational model incorporating existing pension laws and current fiscal policy. They set the future path for government spending and tax revenue, with implications for government debt and public pension funds. The results suggest that additional pension reforms, higher consumption taxes, and higher female working participation help achieve fiscal stability. More detail, if the consumption tax is raised to ten percent, the non-pension deficit improves significantly as the ratio of non-pension transfers and government spending to GDP starts to rise, and then this deficit gradually rises over time. If the consumption tax is further increased to 20%, this would result in a surplus in non-pension balances.

3. A Small Open Economy DSGE Model

This chapter will discuss the micro-based DSGE model in a small open economy. Our purpose is to capture Japan's policy performance under the prolonged economic recession that followed the collapse of the fabled economic bubble of the 1980s. The model is constructed mainly based on (Le et al. 2016a); Le et al. (2016b). They started by borrowing the framework set up by the well-known reference model of Smets and Wouters (2007). However, a perfect financial market assumption is easily overturned under a financial crisis. The interest rate set by the central bank might not uniquely determine the cost of credit for borrowers. (Le et al. 2016a); Le et al. (2016b) incorporate the financial accelerator mechanism from Bernanke et al. (1999), which considers a financial market imperfection and integrates the effects of financial shocks into business cycle behaviour. They also proposed a hybrid price/wage setting which is assumed to be a weighted average of the corresponding New classical and New Keynesian equations. To be more specific, the aggregate price/wage setting is composed of the price/wage setting in the perfectly competitive market and those in the imperfectly competitive market in a certain proportion. This setting method make the model fit the data much better when the rigidity was quite limited. Japan is a pioneer in the use of non-traditional monetary policy, i.e., Zero Lower Bound and Quantitative Easing. (Le et al. 2016a); Le et al. (2016b) allow for an effective monetary policy via QE under the ZLB of nominal interest rate. We extend (Le et al. 2016a); Le et al. (2016b) by adding a CES preference structure to allow substitution elasticity between domestic and foreign goods to accommodate the small open economy model. (Meenagh et al. 2007; Meenagh et al. 2010)

3.1 Households

The economy is assumed to be populated by a continuum of infinitely lived households. Each household has a monopoly power in supplying its differentiated labour type. Each household's preference depends on its consumption $C_{t+s}(j)$ and labour force $N_{t+s}(j)$ and it maximises an intertemporal utility function

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\sigma_c} \left[C_{t+s}(j) - hC_{t+s-1}(j)\right]^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} N_t(j)^{1+\sigma_l}\right)$$

where $\beta \in (0, 1)$ is the discount factor and $h \in (0, 1)$ represents the extent of internal habit formation in consumption. $\sigma_c$and σ_l are the coefficient of relative risk aversion of households and the inverse of elasticity of labour with respect to the real wage.

The household's expenditures are consumption $C_{t+s}(j)$, domestic bonds $B_{t+s}(j)$ with nominal interest rate R_{t+s}, foreign bonds hold $B^f_{t+s}(j)$ with nominal interest rate R^f_{t+s}, and real tax payment T_{t+s}. These expenditures are financed by wage earnings $W_{t+s}(j)N_{t+s}(j)$, the interest income on last period deposits and foreign bonds and dividends from firms Π_{t+s}. The budget constraint is shown as

$$C_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon^b_t R_{t+s} P_{t+s}} + \frac{S_{t+s} B^f_{t+s}(j)}{R^f_{t+s} P_{t+s}} \leq \frac{W_{t+s}(j) N_{t+s}(j)}{P_{t+s}} + \frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{S_{t+s} B^f_{t+s-1}(j)}{P_{t+s}} + \frac{\Pi_{t+s}}{P_{t+s}} - T_{t+s}$$

where S_t is the nominal exchange rate and $P_t$is the domestic price level. ε^b_t is an exogenous premium in the return to deposit, it follows the stochastic process

$$\varepsilon^b_t = \rho_b \varepsilon^b_{t-1} + \eta^b_t, \quad \eta^b_t \sim (0, \sigma_b)$$

The first order condition after dropping j is given by:

Consumption Euler equation

$$\beta U'(C_{t+1}) \frac{1}{P_{t+1}} = U'(C_t) \frac{1}{\varepsilon^b_t R_t P_t}$$

Labour supply equation

$$\frac{U'(N_t)}{U'(C_t)} = -\frac{W_t}{P_t}$$

where

$$U'(C_t) = \ [C_t - hC_{t-1}]^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} {N_t}^{1+\sigma_l}\right)$$

$$U'(N_t) = \ \frac{1}{1 - \sigma_c} (C_t - hC_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} {N_t}^{1+\sigma_l}\right) (\sigma_c - 1) {N_t}^{\sigma_l}$$

so that

$$\frac{U'(N_t)}{U'(C_t)} = -{N_t}^{\sigma_l} (C_t - hC_{t-1})$$

Uncovered interest parity

$$\frac{1}{\varepsilon_t^b R_t} = \frac{S_t}{S_{t+1} R_t^f}$$

Define Q_t is real interest rate and $Q_t = S_t \frac{P_t^f}{P_t}$ with S_t denoting the nominal exchange rate and P_t^f being the general foreign price level. The last equation can be rewritten in real terms as

$$\frac{R_t^f P_t^f}{\varepsilon_t^b R_t Q_t P_t} = \frac{P_{t+1}^f}{Q_{t+1} P_{t+1}}$$

Households can choose to consume between domestic goods and imported goods. Thus, the level of C_t should satisfy the following expenditure constraint:

$$C_t = P_t^d C_t^d + Q_t C_t^f$$

where C_t^f is the consumption of imported goods and C_t^d is the consumption of domestic goods. P_t^d is the relative price of domestic goods to the domestic general price and Q_t is the real exchange rate.

Household chooses C_t^d and C_t^f to maximise aggregate consumption, in which the aggregate consumption is given by a constant elasticity of substitution (CES) function. The consumption maximisation problem is that:

$$\widetilde{C_t} = [\omega\left(C_t^d\right)^{\frac{\vartheta-1}{\vartheta}} - (1-\omega)\varepsilon_t^m\left(C_t^f\right)^{\frac{\vartheta-1}{\vartheta}}]^{\frac{\vartheta}{\vartheta-1}} \qquad 0 < \omega < 1$$

subject to expenditure constraint:

$$\widetilde{C_t} \leq C_t$$

where ω is the share of domestic goods in aggregate consumption. ϑ is related to the elasticity of substitution across consumption goods. The constraint binds at a maximum. ε_t^m is the shock of demand in imported goods, which follows the stochastic process

$$\varepsilon_t^m = \rho_m \varepsilon_{t-1}^m + \eta_t^m, \quad \eta_t^m \sim (0, \sigma_m)$$

The FOCs determine the demand for imported and domestic goods as follows:

$$IM_t = C_t^f = \left(\frac{(1-\omega)\varepsilon_t^m}{Q_t}\right)^{\vartheta} C_t$$

$$C_t^d = \left(\frac{\omega}{P_t^d}\right)^{\vartheta} C_t$$

Likewise, the demand for exports from the domestic country is given by

$$EX_t = \left(\frac{(1-\omega^F)\varepsilon_t^{ex}}{\frac{1}{Q_t}} \right)^{\vartheta^F} C_t^*$$

where ω^F, ϑ^F and ε_t^{ex} is foreign equivalent to ω, ϑ and ε_t^m. C_t^* is the aggregate consumption of the foreign country. The shock of demand in exported goods ε_t^{ex} follows the stochastic process:

$$\varepsilon_t^{ex} = \rho_{ex}\varepsilon_{t-1}^{ex} + \eta_t^{ex}, \quad \eta_t^{ex} \sim (0, \sigma_{ex})$$

Export and imports together with interest receipts/payments determine the net foreign assets as

$$\frac{Q_t B_{t+1}^f}{R_{t+1}^f} = Q_t B_t^f + P_t^d EX_t - Q_t IM_t$$

3.2 Labour Union and Labour Packer

Smets and Wouters (2007) assume that households supply their labour to the labour union who differentiates the labour services and sells on to labour packers. To match the US data, Le et al. (2016a) assume that intermediate goods producers have a production function that combines in a fixed proportion labour from the labour union in imperfect competitive market and labour from perfectly competitive market. The model follows the latter paper's assumption and assumes that there is a labour bundler who supplies labour to intermediate firms and thus the total labour supply used by the intermediate firms, N_t, is:

$$N_t = N_{1t} + N_{2t} = \left\{ \left(\int_0^1 N_{1t}(l)^{\frac{1}{1+\lambda_{w,t}}} dl \right)^{1+\lambda_{w,t}} + \left(\int_0^1 N_{2t}(l) dl \right) \right\}$$

where $N_{1t}(l)$ is labour supply in competitive markets supplied by the labour union and $N_{2t}(l)$ is labour inputs from perfectly competitive market. $\lambda_{w,t}$ measures the shocks to aggregator function, which causes the changes in demand then mark-up. It follows AR (1) process:

$$\lambda_{w,t} = \rho_w \lambda_{w,t-1} + \eta_t^w, \quad \eta_t^w \sim (0, \sigma_w)$$

It is assumed that the share of unionised labour in the total labour supply is ω_{NK}^w, so that $N_{1t} = \omega_{NK}^w N_t$ and $N_{2t} = (1 - \omega_{NK}^w) N_t$. The weighted average wage is

$$W_t = \omega_{NK}^{w} W_{1t} + (1 - \omega_{NK}^{w}) W_{2t}$$

W_{1t} is set according to Calvo wage setting and W_{2t} is set equal to the current expected marginal monetary disutility of work. These wages are passed to labour packers who offer the weighted wage for each unit of labour to the intermediate goods producers.

The construction of labour services in imperfectly competitive market follows Smets and Wouters (2007). Households supply homogenous labour to labour unions and to the perfectly competitive market directly to the labour packer at the wage equal to the marginal rate of substitution between consumption and leisure. The labour unions then differentiate those labour services and then offer them to labour packers at price $W_{1t}(l)$. Labour packer bundles and sells them to intermediate good producers as labour input used in production at price W_{1t}.

The profit maximization problem of labour packer is given by:

$$\max_{N_{1t}(l)} W_{1t}\, N_{1t} - \int_0^1 W_{1t}(l) N_{1t}(l) dl$$

subject to $N_{1t} = \left(\int_0^1 N_{1t}\,(l)^{\frac{1}{1+\lambda_{w,t}}} dl \right)^{1+\lambda_{w,t}}$. The FOC condition determines the demand for labour as $N_{1t}(l) = \left(\frac{W_{1t}(l)}{W_{1t}} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_{1t}$.

Labour unions bridge between households and labour packer. They have a monopolistic power to set their wages for differentiated labour services. However, under Calvo pricing idea they only can do this when an opportunity arises, so that in every period only $(1 - \xi_w)$ fraction of unions can change their wages optimally and ξ_w fraction would index their wages partially to last period's inflation $(\pi_{t-1})^{\iota_w} W_{t-1}\,(l)$, where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and ι_w is the degree of wage indexation, The wage setting decision of these unions is

$$\max_{\widetilde{W_{1t}}(i)} E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Theta_{t+s} P_t}{\Theta_t P_{t+s}} \left[\widetilde{W_t}(l) X_{t+s} - MRS_{t+s} \right] N_{1,t+s}(l)$$

$$s.t. N_{1t+s}(l) = \left(\frac{\widetilde{W_t}(l)X_{t+s}}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} N_{1t+s}$$

where $\widetilde{W_t}(i)$ is reoptimized wage level and MRS_t is marginal rate of substitution between consumption and labour. and $X_{t+s} = \begin{Bmatrix} \pi_t \times \pi_{t+1} \times \dots \times \pi_{t+s-1}\, for\ s \geq 1 \\ 1\, for\ s = 0 \end{Bmatrix}$.

The FOC is:

$$E_t \sum_{s=0}^{\infty} \xi_w^s \frac{\beta^s \Theta_{t+s} P_t}{\Theta_t P_{t+s}} N_{1,t+s}(l) \frac{1}{\lambda_{w,t+s}} \left[\left(1 + \lambda_{w,t+s}\right) W_{t+s} - X_{t+s} \widetilde{W_t}(l) \right] = 0$$

The wage is determined as:

$$W_t = \left[(1 - \xi_w) \widetilde{W_t}^{-\frac{1}{\lambda_{w,t}}} + \xi_w \left(\pi_{t-1}^{\iota_w} W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

In the perfectly competitive market, labour input N_{2t} is simply expressed by:

$$N_{2t} = \int_0^1 N_{2t}(l) dl$$

and wages are perfectly flexible and equals to the nominal marginal rate of substitution between consumption and leisure.

3.3 Final Goods Producer

It is assumed that final goods producer's output, Y_t,made up in a fixed proportion of intermediate goods purchased from an imperfectly competitive market, Y_{1t}, and perfectly competitive markets, Y_{2t}. as

$$Y_t = Y_{1t} + Y_{2t} = \left[\int_0^1 Y_{1t}(i)^{\frac{1}{1+\lambda_{p,t}}} di \right]^{1+\lambda_{p,t}} + \left[\int_0^1 Y_{2t}(i) di \right]$$

$\lambda_{p,t}$ is an exogenous shock which causes changes in the elasticity of demand and price mark-up. It follows the AR (1) process:

$$\lambda_{p,t} = \rho_p \lambda_{p,t-1} + \eta_t^p, \quad \eta_t^p \sim (0, \sigma_p)$$

Suppose w_{NK}^{p} is the share of good provided from imperfectly competitive market, we have $Y_{1t} = w_{NK}^{p} Y_t$ and $Y_{1t} = (1 - w_{NK}^{p}) Y_t$ so that

$$P_t = w_{NK}^{p} P_{1t} + (1 - w_{NK}^{p}) P_{2t}$$

where P_{1t} is the price in the imperfectly competitive market which follows the Calvo pricing and P_{2t} is the price in competitive market which equals to marginal costs. The final good producer combines the intermediate goods as above in a bundle which it sells to households, investors and government at the weighted average price P_t.

In an imperfectly competitive market, the final good Y_{1t} is a composite of a continuum of intermediate goods $Y_{1t}(i)$ and its profit maximisation is:

$$max_{Y_{1t}(i)}\, P_{1t} Y_{1t} - \int_0^1 P_{1t}(i) Y_{1t}(i)\, di$$

$s.t. Y_{1t} = \left[\int_0^1 Y_{1t}(i)^{\frac{1}{1+\lambda_{p,t}}} di\right]^{1+\lambda_{p,t}}$ final goods Y_{1t} and $P_{1t}(j)$ is the price of intermediate goods $Y_{1t}(i)$. The demand for intermediate goods is decreasing in the relative price:

$$Y_{1t}(i) = \left(\frac{P_{1t}(i)}{P_{1t}}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_{1t}$$

3.4 Intermediate Goods Producer

Intermediate goods $Y_t(j)$ is produced by using the following production function:

$$Y_t(j) = A_t {K_t}^s(j)^{\alpha} [\gamma^t N_t(j)]^{1-\alpha} - \gamma^t \Phi$$

where ${K_t}^s(j)$ and $N_t(j)$ are the capital services and labour used in production respectively. α is capital share and γ^t is labour augmenting deterministic growth rate. Φ is fixed cost of producing products. A_t is *total factor productivity* and follows the ARIMA (1,1,0) process:

$$A_t - A_{t-1} = \rho_a (A_{t-1} - A_{t-2}) + \eta_t^a, \quad \eta_t^a \backsim (0, \sigma_a)$$

The cost minimization problem is:

$min_{{K_t}^s(j), N_t(j)} R_t^{rental} K_t^s(j) + W_t N_t(j)$

s.t.

$$A_t {K_t}^s(j)^{\alpha} [\gamma^t N_t(j)]^{1-\alpha} - \gamma^t \Phi \geq Y_t(j)$$

where R_t^{rental} and W_t are the rental rate on capital and wages, respectively.

The FOCs determined demand for capital services and labour:

$$MC_t(j)\gamma^{(1-\alpha)t}\alpha A_t\left(\frac{N_t(j)}{K_t{}^s(j)}\right)^{1-\alpha} = R_t^{rental}$$

$$MC_t(j)\gamma^{(1-\alpha)t}(1-\alpha)A_t\left(\frac{N_t(j)}{K_t(j)}\right)^{-\alpha} = W_t$$

where MC_t is the marginal cost and determined by

$$MC_t = \frac{\left(R_t^{rental}\right)^\alpha (W_t)^{1-\alpha}}{A_t\alpha^\alpha(1-\alpha)^{1-\alpha}}$$

All intermediate goods firms have the same marginal cost because they pay the same rental rate and labour wages. The capital-labour ratio is therefore equal to:

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t^{rental}}$$

Following the Calvo price setting, it is assumed that in every period a fraction $1-\xi_p$ of intermediate goods firms reoptimize their prices and the remainder ξ_p adjust their prices partially to the previous period's inflation, $\pi_{t-1}^{\iota_p}$ where ι_p is the degree of price indexation. The price setting problem is given as below:

$$\max_{\widetilde{P_1t}(i)} E_t\sum_{s=0}^{\infty}\xi_p^s\frac{\beta^s\Xi_{t+s}P_t}{\Xi_tP_{t+s}}\left[\widetilde{P_t}(i)X_{t+s} - MC_{t+s}\right]Y_{t+s}(i)$$

$$s.t.\, Y_{t+s}(i) = Y_{t+s}\left(\frac{P_t(i)X_{t+s}}{P_{t+s}}\right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}}$$

where $\widetilde{P_t}(i)$ is the newly optimal new price level and the indexation is

$$X_{t,s} = \begin{Bmatrix} 1\, for\ s=0 \\ \Pi_{l=1}^{s}\pi_{t+l-1}^{\iota_p}\, for\ s=1,\dots,\infty \end{Bmatrix}$$

The newly set price is determined by the FOC

$$E_t\sum_{s=0}^{\infty}\xi_p^s\frac{\beta^s\Xi_{t+s}P_t}{\Xi_tP_{t+s}}Y_{t+s}(i)\left[X_{t,s}\widetilde{P_t}(i) - MC_{t+s}(1+\lambda_{p,t+s})\right] = 0$$

The aggregate price of all these intermediate producers in the imperfectly competitive market is:

$$P_t = \left[(1-\xi_p)\left(P_t(i)\right)^{\frac{1}{\lambda_{p,t}}} + \xi_p\left(\pi_{t-1}^{\iota_p} P_{t-1}\right)^{\frac{1}{\lambda_{p,t}}} \right]^{\lambda_{p,t}}$$

In the competitive market, the price of intermediate goods is equal to marginal cost.

3.5 Capital Producer

Capital producers are competitive. At the end of each period t, they they buy undepreciated capital goods from entrepreneurs at price Q_t^K and invest I_t to produce new capital. These capital goods would be sold to entrepreneurs, and they would rent out as one of inputs production in period $t+1$. The capital accumulation is given by:

$$K_t = (1-\delta)K_{t-1} + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

where $S\left(\frac{I_t}{I_{t-1}}\right)$ is a quadratic investment adjustment cost with $S(\cdot) = 0, S'(\cdot) = 0$, $S''(\cdot) > 0$, and δ is the depreciation rate. ε_t^i is the random investment shock following AR (1) process.

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0, \sigma_t^i)$$

The profit maximization problem is shown

$$\max_{I_{t+s}} \sum_{s=0}^{\infty} \frac{\beta^s \Xi_{t+s}}{P_{t+s}} \left[P_{t+s}^K K_{t+s} - I_{t+s} - P_{t+s}^K (1-\delta) K_{t+s-1}\right]$$

where P_{t+s}^K is the capital price. The optimal investment is determined by the FOC:

$$\varepsilon_t^i P_t^K \left(1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right) - \beta E_t\left[\varepsilon_{t+1}^i \frac{\Xi_{t+1} P_t}{P_{t+1} \Xi_t} P_{t+1}^K S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] = 1$$

3.6 Entrepreneurs, Bank and Optimal Contract

Standard Optimal contract

There is a continuum of entrepreneurs who at the end of period t resell all undepreciated capital back to the capital producer and use their own wealth and the loans from the bank to acquire capital for future production. The loan rate is Z_{t+1}. The amount of borrowing at the end of period t is

$$B_{t+1} = P_t^K K_{t+1} - NW_t \quad (1)$$

The entrepreneurs choose the utilisation level of capital U_t by paying a cost in terms of output equal to $\Upsilon(U_t)$ per each unit of capital to form the capital services. They rent these capital services to the intermediate goods firms in the next period and earns a rental rate R_{t+1}^{rental} per unit of capital services. Entrepreneurs are subject to an i.i.d. random idiosyncratic shock ω[1] that changes their capital values, i.e. their revenue may not be enough to repay their loans. The default threshold, $\bar{\omega}$, is determined by

$$\bar{\omega}R_{t+1}^{k}P_t^K K_{t+1} = Z_{t+1}B_{t+1} = Z_{t+1}(P_t^K K_{t+1} - NW_t) \quad (2)$$

where R_{t+1}^{k} is the gross nominal return to capital for entrepreneurs and is expressed as

$$R_{t+1}^{k} = \frac{R_{t+1}^{rental}U_{t+1} + (1-\delta)P_{t+1}^{K} - P_{t+1}\Upsilon(U_{t+1})}{P_t^K}$$

It states that the return on capital (external financing cost) depends on marginal profit from the production of the intermediate goods and the capital gain. and the optimal utilisation choice is determined by $R_{t+1}^{rental} = P_{t+1}\,\Upsilon'(U_{t+1})$ independently from the capital purchased and idiosyncratic shock.

Define $L_t = \frac{P_t^K K_{t+1}}{NW_t}$ as leverage, the loan rate Z_{t+1} can be written as:

$$Z_{t+1} = \frac{\bar{\omega}R_{t+1}^{k}\,L_t}{L_t - 1}$$

The higher threshold value of $\bar{\omega}$, the higher leverage and higher loan rates. When $\omega > \bar{\omega}$, the entrepreneur repays the promised repayment to the bank and receives the net revenue $\omega R_{t+1}^{k}P_t^K K_{t+1} - Z_{t+1}B_{t+1}$. When $\omega < \bar{\omega}$, the entrepreneur chooses to default and gets nothing.

The optimal debt contract maximizes entrepreneurial payoff subject to the bank's zero profit condition. Thus, banks' zero profit condition is given by:

$$[1 - F(\bar{\omega})]Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\bar{\omega}} \omega R_{t+1}^{k}P_t^K K_{t+1}\,dF(\omega) = R_{t+1}B_{t+1} \quad (3)$$

It states that the bank's payoff must be equal to its repayment to the depositors. $F(\bar{\omega})$ is the probability of default. In left hand side, the first term is the repayment to the bank when the entrepreneur operates the firm well; the second term is the expected capital return when entrepreneur chooses to default; the third term is the collateral

[1] ω is an identically independent distribution (i.i.d.) random variable and it follows the cumulative distribution function (c.d.f.) $F(\omega)$. $\omega \in (0,\infty)$ and $E(\omega) = 1$. $f(\omega)$ is the pdf of ω .

after liquidation under bankruptcy. We define $\Gamma(\overline{\omega})$ as the share of entrepreneurial expected capital return accrued to banks

$$\Gamma(\overline{\omega}) = \overline{\omega}[1 - F(\overline{\omega})] + G(\overline{\omega}) \quad (4)$$

where:

$$G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega \, dF(\omega), \qquad \Gamma'(\overline{\omega}) = 1 - F(\overline{\omega}), \qquad \Gamma''(\overline{\omega}) = -f(\overline{\omega})$$

Thus, substituting equations (1)(2)(4) into equation (3), the banks' zero profit condition can be rewritten as:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R^k_{t+1} P^K_t K_{t+1} = R_{t+1}(P^K_t K_{t+1} - NW_t)$$

Dividing both side by NW_t and rearranging the equation above:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R^k_{t+1} L_t = R_{t+1}(L_t - 1)$$

The banks' leverage offer curve can be expressed by:

$$L_t = \frac{R_{t+1}}{R_{t+1} - (\Gamma(\overline{\omega}) - \mu G(\overline{\omega})) R^k_{t+1}}$$

which is an increasing and convex curve with respect to $\overline{\omega}$.

The entrepreneur makes a profit if it does not breach the contract, i.e. drawing $\omega > \overline{\omega}$. The expected entrepreneurial earning from getting a loan is:

$$\int_{\overline{\omega}}^{\infty} \omega R^k_{t+1} P^K_t K_{t+1} \, dF(\omega) - [1 - F(\overline{\omega})] Z_{t+1} B_{t+1} \quad (5)$$

The first term is the expected return on capital, the second term is the expected repayment to banks. By substituting equation (1) and using the definition of leverage above, equation (5) can be reduced to:

$$\left(1 - \Gamma(\overline{\omega})\right) R^k_{t+1} L_t$$

where

$$1 - \Gamma(\overline{\omega}) = \left[\int_{\overline{\omega}}^{\infty} \omega \, dF(\omega) - [1 - F(\overline{\omega})]\overline{\omega}\right]$$

Hence the formal contracting problem for the entrepreneur is shown as:

$$\max_{\overline{\omega}, L_t} E_t R^k_{t+1} \left(1 - \Gamma(\overline{\omega})\right) L_t$$

s.t.

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R^k_{t+1} L_t = R_{t+1}(L_t - 1)$$

The Lagrangian is:

$$L = E_t \left\{ R^k_{t+1} \left(1 - \Gamma(\overline{\omega})\right) L_t + \Lambda_t \left[[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R^k_{t+1} L_t - R_{t+1}(L_t - 1) \right] \right\}$$

The FOCs are:

$$\frac{\partial L}{\partial \overline{\omega}} = -\Gamma'(\overline{\omega}) + \Lambda_t[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})] = 0$$

$$\frac{\partial L}{\partial L_t} = R_{t+1}^k\big(1-\Gamma(\overline{\omega})\big) + \Lambda_t\{[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]R_{t+1}^k - R_{t+1}\} = 0$$

$$\frac{\partial L}{\partial \Lambda_t} = [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]R_{t+1}^k L_t - R_{t+1}(L_t - 1) = 0$$

Given ex post realisation of R_{t+1}^k and predetermined value of R_{t+1}, the optimal choice of $\overline{\omega}$ can be found out. Let $s_t = \frac{R_{t+1}^k}{R_{t+1}}$ be external financial premium. The bank leverage curve provides the optimal choice of L_t, so that we can have the total $(\overline{\omega}, L_t)$ solution. Note that, the optimal choice of $\overline{\omega}$ is a function of s_t, and L_t is a function of $\overline{\omega}$ and s_t. Thus, capital expenditure is proportional to net worth, which can be simply written as:

$$P_t^K K_{t+1} = \psi(s_t) NW_{t+1} \qquad with\, \psi(1) = 1, \psi'(\cdot) > 0$$

The equivalent expression is shown as:

$$E_t R_{t+1}^k = s\left(\frac{NW_{t+1}}{P_t^K K_{t+1}}\right) R_{t+1} \quad s'(\cdot) > 0$$

The equilibrium condition state that the external financial premium, $\frac{E_t R_{t+1}^k}{R_{t+1}}$, depends inversely on the share of the capital investment.

Optimal contract with collateral

In this section, we assume that banks require c proportion of net worth as collateral and δ proportion of collateral used for liquidating collateral. The optimal contact problem is outlined with new formation due to collateral intervention. We start from the bankruptcy threshold. The original one is given by

$$\overline{\omega} R_{t+1}^k P_t^K K_{t+1} = Z_{t+1} B_{t+1} = Z_{t+1}(P_t^K K_{t+1} - NW_t) \quad (2)$$

With collateral demanded by banks, the new bankruptcy threshold becomes:

$$\overline{\omega} R_{t+1}^k P_t^K K_{t+1} + cNW_t = Z_{t+1} B_{t+1} = Z_{t+1}[P_t^K K_{t+1} - (1-c)NW_t] \quad (2a)$$

where the amount of borrowing B_{t+1} becomes to $P_t^K K_{t+1} - (1-c)NW_t$. The new equation states that the sum of the firm's gross return and collateral cost from borrowing (LHS) equal to the sum of firm's loan repayment to bank and collateral received from bank in the non-default case. The leverage is defined same as before, i.e. $L_t = \frac{P_t^K K_{t+1}}{NW_t}$, the loan rate Z_{t+1} is given by:

$$Z_{t+1} = \frac{\overline{\omega} R_{t+1}^k L_t + c}{L_t - (1-c)}$$

The bank's zero profit condition is originally given by:

$$[1 - F(\overline{\omega})] Z_{t+1} B_{t+1} + (1-\mu) \int_0^{\overline{\omega}} \omega R_{t+1}^k P_t^K K_{t+1} \, dF(\omega) = R_{t+1} B_{t+1} \quad (3)$$

With collateral present, it can be written as:

$$[1 - F(\overline{\omega})] Z_{t+1} B_{t+1} + (1-\mu) \int_0^{\overline{\omega}} \omega R_{t+1}^k P_t^K K_{t+1} \, dF(\omega) + F(\overline{\omega})(1-\delta) c NW_t = R_{t+1} B_{t+1} \quad (3a)$$

where $F(\overline{\omega})(1-\delta)cNW_t$ in LHS represents the recovery of collateral after deducting the liquidation cost in the event of default. By substituting equations (2a)(4) into equation (3a), the banks' zero profit condition is shown as:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R_{t+1}^k P_t^K K_{t+1} + \left(1 - \delta F(\overline{\omega})\right) c NW_t = R_{t+1}(P_t^K K_{t+1} - (1-c) NW_t)$$

Dividing both side by NW_t and rearranging the equation above:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R_{t+1}^k L_t - R_{t+1}(L_t - 1) = c[R_{t+1} - 1 + \delta F(\overline{\omega})]$$

$$\{[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})] R_{t+1}^k - R_{t+1}\} L_t = c[R_{t+1} - 1 + \delta F(\overline{\omega})] - R_{t+1}$$

We can solve the banks' leverage offer curve:

$$L_t = \frac{R_{t+1} - c[R_{t+1} - 1 + \delta F(\overline{\omega})]}{R_{t+1} - (\Gamma(\overline{\omega}) - \mu G(\overline{\omega})) R_{t+1}^k} \quad (6)$$

which is an increasing and convex curve with respect to $\overline{\omega}$ (see figure 3.7).

Consider the entrepreneur's utility maximization problem, his original expected total return from getting a loan is:

$$\int_{\overline{\omega}}^{\infty} \omega R_{t+1}^k P_t^K K_{t+1} \, dF(\omega) - [1 - F(\overline{\omega})] \overline{\omega} R_{t+1}^k P_t^K K_{t+1}$$

With collateral given to the bank, in the case of non-default, it is written as:

$$\int_{\overline{\omega}}^{\infty} \omega R_{t+1}^k P_t^K K_{t+1} \, dF(\omega) - [1 - F(\overline{\omega})][\overline{\omega} R_{t+1}^k P_t^K K_{t+1} + cNW_t] + [1 - F(\overline{\omega})] cNW_t$$

The terms of collateral have been eliminated, which mean that the firm's expected return is unaffected by the amount of collateral. The firm's expected return is reduced to:

$$\big(1-\Gamma(\overline{\omega})\big)R_{t+1}^{k}P_{t}^{K}K_{t+1}$$

where $1-\Gamma(\overline{\omega}) = 1-\Gamma(\overline{\omega}) = \left[\int_{\overline{\omega}}^{\infty}\omega\, dF(\omega) - [1-F(\overline{\omega})]\overline{\omega}\right]$. Hence, the formal contracting problem for the entrepreneur is shown as:

$$\max_{\overline{\omega},L_t} E_t R_{t+1}^{k}\big(1-\Gamma(\overline{\omega})\big)L_t$$

s.t.

$$\{[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]R_{t+1}^{k}-R_{t+1}\}L_t = c[R_{t+1}-1+\delta F(\overline{\omega})]-R_{t+1}$$

The Lagrangian is:

$$L = E_t\Big\{R_{t+1}^{k}\big(1-\Gamma(\overline{\omega})\big)L_t + \Lambda_t\{\big[\big(\Gamma(\overline{\omega})-\mu G(\overline{\omega})\big)R_{t+1}^{k}-R_{t+1}\big]L_t - c[R_{t+1}-1+\delta F(\overline{\omega})] + R_{t+1}\}\Big\}$$

The FOCs are:

$$\frac{\partial L}{\partial \overline{\omega}} = -\Gamma'(\overline{\omega})R_{t+1}^{k}L_t + \Lambda_t\left[[\Gamma'(\overline{\omega})-\mu G'(\overline{\omega})]R_{t+1}^{k}L_t - c\delta F'(\overline{\omega})\right] = 0$$

$$\frac{\partial L}{\partial L_t} = R_{t+1}^{k}\big(1-\Gamma(\overline{\omega})\big) + \Lambda_t\{[\Gamma(\overline{\omega})-\mu G(\overline{\omega})]R_{t+1}^{k}-R_{t+1}\} = 0$$

$$\frac{\partial L}{\partial \Lambda_t} = [\Gamma(\overline{\omega})-\mu G(\overline{\omega})]R_{t+1}^{k}L_t + \big(1-\delta F(\overline{\omega})\big)c - R_{t+1}\big(L_t-(1-c)\big) = 0$$

Given ex post realisation of R_{t+1}^{k} and predetermined value of R_{t+1}, the optimal choice of $\overline{\omega}$ is the solution of:

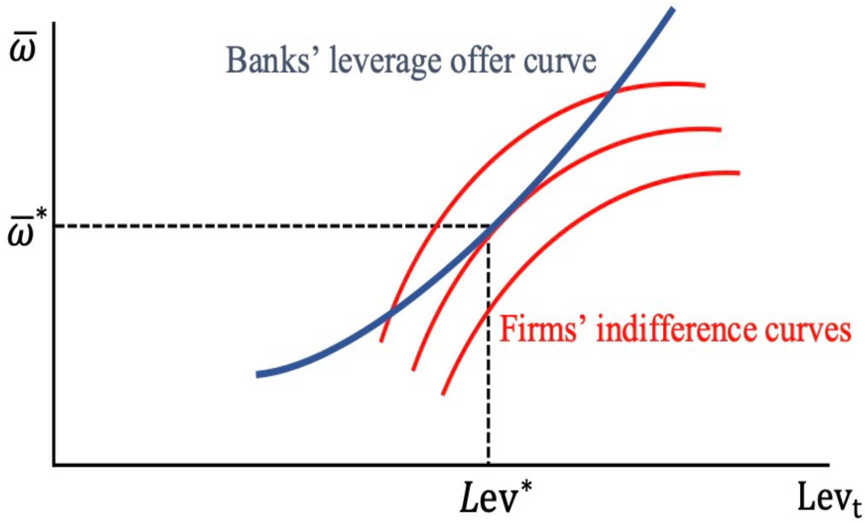
$$\{R_{t+1}-\Omega'(\overline{\omega})R_{t+1}^{k}\}\big[R_{t+1}-c[R_{t+1}-1+\delta F(\overline{\omega})]\big] = -c\delta\frac{F'(\overline{\omega})\big(1-\Gamma(\overline{\omega})\big)}{\Gamma'(\overline{\omega})}\big(R_{t+1}-\psi(\overline{\omega})R_{t+1}^{k}\big)$$

Where $\psi(\overline{\omega}) = \Gamma(\overline{\omega})-\mu G(\overline{\omega})$ and $\Omega'(\overline{\omega}) = \frac{\psi'(\overline{\omega})}{\Gamma'(\overline{\omega})} + \left[1-\frac{\psi'(\overline{\omega})}{\Gamma'(\overline{\omega})}\psi(\overline{\omega})\right] \approx 1$.

We have the leverage curve (see equation (6)) which defines L_t in terms of $\overline{\omega}$. The firm's optimal choice using banks' leverage can be expressed as:

$$L_t[R_{t+1}-\Omega'(\overline{\omega})R_{t+1}^{k}] = -c\delta\frac{F'(\overline{\omega})\big(1-\Gamma(\overline{\omega})\big)}{\Gamma'(\overline{\omega})} \quad (7)$$

Figure 3.3.6.1 The optimal contract for $(\overline{\omega}^{*}, L^{*})$ for given R^{k}, R and δ



We now explore the comparative static properties of changes around the equilibrium by taking the total differential of this two-equation system (i.e. (6) and (7)) in dL, $d\overline{\omega}$, $d\delta$ and dR^k. For convenience, we will evaluate the derivatives at an equilibrium where $\delta = 0$. Note that in the rest of this model, leverage L_t is determined by other variables while δ is determined by the provision of M0 as an alternative to illiquid collateral. Thus, L_t and δ are treated as exogenous variables, which then solve for $\overline{\omega}$ and R^k_{t+1}. These two elements are internal to the bank contract decision and unobservable in the public domain. However, loan rate Z_t can be observed from bankruptcy threshold.

The total differential for equation (7) is written as:

$$[R_{t+1} - \Omega'(\overline{\omega})R^k_{t+1}]dL_t + L_t\big(-\Omega'(\overline{\omega})\big)dR^k_{t+1}$$
$$= (deriviative = 0)\overline{\omega} - c\frac{F'(\overline{\omega})\big(1-\Gamma(\overline{\omega})\big)}{\Gamma'(\overline{\omega})}d\delta \quad (8)$$

The total differential for equation (6) is written as:

$$dL_t = L_t\left[\frac{\psi'(\overline{\omega})R^k_{t+1}}{R_{t+1} - \psi(\overline{\omega})R^k_{t+1}}\right]d\overline{\omega} + L_t\left[\frac{\psi(\overline{\omega})}{R_{t+1} - \psi(\overline{\omega})R^k_{t+1}}\right]dR^k_{t+1}$$
$$+ L_t\left[\frac{-cF(\overline{\omega})}{R_{t+1} - \psi(\overline{\omega})R^k_{t+1}}\right]d\delta \quad (9)$$

We are interested in investigating the effect of δ on the equilibrium values of R^k_{t+1}, ω and Z_{t+1}. Note that $\Gamma'(\overline{\omega}) = 1 - F(\overline{\omega})$. From equation (8) we can find that

$$\frac{dR^k_{t+1}}{d\delta} = \frac{cF'(\overline{\omega})\big(1-\Gamma(\overline{\omega})\big)}{L_t\Omega'(\overline{\omega})\Gamma'(\overline{\omega})} = \frac{cF'(\overline{\omega})\big(1-\Gamma(\overline{\omega})\big)}{L_t\Omega'(\overline{\omega})\big(1-F(\overline{\omega})\big)} > 0$$

From equation (9), we get:

$$\frac{d\overline{\omega}}{d\delta}=\frac{d\overline{\omega}}{dR_{t+1}^{k}}\frac{dR_{t+1}^{k}}{d\delta}+\frac{d\overline{\omega}}{d\delta}=\left[\frac{-\psi(\overline{\omega})}{\psi(\overline{\omega})R_{t+1}^{k}}\right]\left[\frac{cF'(\overline{\omega})\left(1-\Gamma(\overline{\omega})\right)}{L_t\Omega'(\overline{\omega})\Gamma'(\overline{\omega})}\right]+\frac{cF(\overline{\omega})}{L_t\Omega'(\overline{\omega})R_{t+1}^{k}}$$

$$=\frac{cF(\overline{\omega})}{L_t\Omega'(\overline{\omega})R_{t+1}^{k}}\left[1-\frac{F'(\overline{\omega})}{F(\overline{\omega})}\frac{\psi(\overline{\omega})\left(1-\Gamma(\overline{\omega})\right)}{\Omega'(\overline{\omega})\Gamma'(\overline{\omega})}\right]$$

$$=\frac{cF(\overline{\omega})}{L_t\Omega'(\overline{\omega})R_{t+1}^{k}}\left[1-\frac{F'(\overline{\omega})}{F(\overline{\omega})}\frac{\psi(\overline{\omega})\left(1-\Gamma(\overline{\omega})\right)}{\Omega'(\overline{\omega})\left(1-F(\overline{\omega})\right)}\right]>0$$

Finally, we have:

$$\frac{dZ_{t+1}}{d\delta}=\frac{L_t}{L_t-1+c}\left[R_{t+1}^{k}\frac{d\overline{\omega}}{d\delta}+\frac{dR_{t+1}^{k}}{d\delta}\right]$$

$$=\frac{c}{L_t-1+c}\left[\frac{F(\overline{\omega})}{\psi'(\overline{\omega})}\left(1-\frac{F'(\overline{\omega})}{F(\overline{\omega})}\frac{\psi(\overline{\omega})\left(1-\Gamma(\overline{\omega})\right)}{\Omega'(\overline{\omega})\left(1-F(\overline{\omega})\right)}\right)\right.$$

$$\left.+\frac{\overline{\omega}F'(\overline{\omega})\left(1-\Gamma(\overline{\omega})\right)}{\Omega'(\overline{\omega})\left(1-F(\overline{\omega})\right)}\right]>0$$

Optimal contract with money collateral – introduction of Quantitative Easing (QE)

QE is the most important and noticeable unconventional policy tool implemented by central banks, is first introduced by the Bank of Japan in the early of 2000s. After the global financial crisis broke out in 2008, many countries, like United States, United Kingdom and other Euro area, have also used this policy tool and economic began to recovery quickly. By the end of 2018, the Bank of Japan had expanded its balance sheet to 552 trillion in Yen, which was about 100 percent of Japanese GDP.

As in Le et al. (2016a), quantitative easing injected into economy through the idea of cash (M_0) as collateral. Compared with other traditional collaterals (such as mortgages), the advantage of cash collateral lies in its liquidity. Banks can seize the collateral directly without any cost of liquidation (i.e., no recovery cost δ). The elimination of this cost decreases credit spread for given leverage (i.e. $\frac{\partial R^k}{\partial \delta}>0$).

Now, we mainly state how the central bank issues currency (M_0) as cash collateral to operate in the market through quantitative easing policy. When financial crisis hits the economy, central bank issues M_0 in exchange for the government bonds held by households. Recall the assumption that money is not considered in household utility function, households deposit it at once into banks, and then banks lend M0 to intermediate good producer for future collateral. M_0 enters the balance sheet.

Note that intermediate good producer wants to acquire as much as possible for future collateral needs. The more cash he holds, the lower external premium he faces. He invests part of his net worth in cash until the amount cash reach the maximum that can be obtained, and the remaining part is used for capital or other collateral. Intermediate good producer maintains a long-term cooperative relationship with banks. He will return profits as dividends to banks which provide them with credit. Hence, he can hold the maximum cash collateral (M_0) available, as the minimum counterpart to credit advanced. Once the firm goes bankrupt, the M_0 injected by the central bank through quantitative easing was eventually hold as liquid collateral for pledged to banks.

With M0 as the cheapest collateral, the spread between risky rate and the risk-free rate is decreased for given leverage in the log-linearized form:

$$E_t R_{t+1}^k - R_{t+1} = s\left(P_t^k + K_{t+1} - NW_{t+1}\right) - \psi m_t + \varepsilon_t^{pr}, \qquad \psi > 0$$

where m_t is the instrument of M_0. ε_t^{pr} is risk premium shock which follows AR(1) process:

$$\varepsilon_t^{pr} = \rho_{pr}\varepsilon_{t-1}^{pr} + \eta_t^{pr}$$

This equation captures the loosening effect of QE operations on the credit condition. We have verified that $\frac{dR_{t+1}^k}{d\delta} > 0, \frac{d\bar{\omega}}{d\delta} > 0$, and $\frac{dZ_{t+1}}{d\delta} > 0$, it can be conclude that since δ is reduced by M0 rejections, the increase in M0 will reduce the required return on capital and the credit premium.

Optimal contract- introduction of Bank regulation

Macroprudential policy, as another policy tool introduced here, has been implemented worldwide in a growing tendency since the global financial crisis. The Boston Fed President Eric Rosengren pointed out that banking regulations aimed at reducing risks in the financial system, if the Bank of Japan could use macroprudential tools effectively and timely, the financial crisis caused by bursting of the real estate bubble in 1990s could have lessened to a large extent. (Rosengren,2014) Unfortunately, macroprudential policies were taken seriously by central bank until after the financial crisis of 2008. The idea of modelling macroprudential tool is provided by Le et al. (2016a). What the financial regulation does is to increase the cost of lending of intermediate financial producers and to further control the lending and borrowing. Specifically, banks hold as counterpart funds for the credit assets they hold. The regulation believes that deposits are not only type of assets banks hold. They also hold some expensive assets, such as equity. These assets require higher compensation for the risk of bank losses. The macroprudential policies is complicate to measure for now. For simplicity, we set ξ_t as macro-prudential instrument. It is directly added to external premium equation as a part of risk premium shock. The external premium equation in the log-linearized form is presented as given:

$$E_t R_{t+1}^k - R_{t+1} = s\left(P_t^k + K_{t+1} - NW_{t+1}\right) - \psi m_t + \xi_t + \varepsilon_{t,pr}$$

The entrepreneur's equity and net worth

The net worth for entrepreneur in period t depends on the current value of the entrepreneur's equity V_t and the firms surviving rate θ, which is shown as:

$$NW_t = \varepsilon_t^{nw} \theta V_t$$

and

$$V_t = R_t^k P_{t-1}^k K_t - E_{t-1}\left[R_t^k\left(P_{t-1}^k K_t - NW_{t-1}\right)\right]$$

where V_t is calculated by the gross return on capital minus the external finance costs. ε_t^{nw} is the shocks to equity, which follows the AR (1) process:

$$\varepsilon_t^{nw} = \rho_{nw} \varepsilon_{t-1}^{nw} + \eta_t^{nw}$$

Thus, the net worth equation is rearranged and given by:

$$NW_t = \varepsilon_t^{nw}\left[R_t^k P_{t-1}^k K_t - E_{t-1}\left[R_t^k\left(P_{t-1}^k K_t - NW_{t-1}\right)\right]\right]$$

For those who die in current period, the entrepreneurial consumption C_t^e is represented by:

$$C_t^e = (1-\theta) NW_t$$

3.7 Monetary Policy

Taylor rule - When the economic environment is healthy, the interest rate, as an instrument in response to the developments of output and inflation, is adjusted according to Taylor rule. The rule is shown as its log-linearized form:

$$R_t = \rho R_{t-1} + (1-\rho)\big(r_p \pi_t + r_y y_t\big) + r_{\Delta y}(y_t - y_{t-1}) + \varepsilon_t^r \quad r > 0$$

Where ρ measures the degree of interest rate smoothing. r_p, r_y and $r_{\Delta y}$ represents Taylor's rule responses to inflation, output and change in output, respectively. Here the monetary policy shock is determined as:

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$$

Supply in M_0 in different regime- Before the financial crisis, the interest rate controlled by Taylor rules were above zero bound. The supply of M_0 in this period is the supply of credit/broad money. After the financial crisis, the supply of M_0 is set in case that the target of the credit premium is around its steady state. This goal is formulated by the quantitative easing policy, with the purpose of pushing the credit condition back to its original state. The equations of m_t is therefore shown as:

$$m_t = \begin{cases} \psi_0 + \psi_1 M_1 + \varepsilon_t^{m0} & for\ r_t > 0 \\ m_{t-1} + \psi_2\big(R_t^k - R^k\big) + \varepsilon_t^{m0} & for\ r_t < 0 \end{cases}$$

Where ψ_1 and ψ_2 are positive. The money supply shock ε_t^{m0} follows AR(1) process:

$$\varepsilon_t^{m0} = \rho_{m0} \varepsilon_{t-1}^{m0} + \eta_t^{m0}$$

The measurement of macro-prudential instrument is provided by Basel Agreement nos 1 and 2. As we discussed, it is represented as a shock in the equation under financial crisis. As in Le, Meenagh and Minford (2016), it follows an exogenous $I(1)$ time-series process, which is written as:

$$\Delta \xi_t = errxi_t$$

Where $errxi_t$ is simply included into the error term of external premium equation.

In the end of this model, we also consider the additional supply of money M_2, which is determined by bank' balance sheets quantities. The board money supply is that $M_2 = M_0 + household\ deposits$, where the amount of deposit is same as the amount of lending to entrepreneurs. Thus, that $M_2 = M_0 + (capital\ expenditure + collateral\ to\ money - net\ worth)$. We also set the supply of M0 is proportional to the supply of M2. The equation is written as:

$$M_t = (1 + v - c - \mu)K_t + \mu m_t - v n w_t$$

Where $v = \frac{nw}{m2}$, $c = \frac{collateral}{m2}$ and $\mu = \frac{m0}{m2}$ are the ratios of net worth, M_0 and collateral to money, respectively.

3.8 Government Policy

As set in Smert and Wouters(2007), government spending relate to the steady state output path $\varepsilon_g = \frac{G_t}{Y\gamma^t}$, it follows the process that

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga} \varepsilon_t^a + \eta_t^g, \ \ \eta_t^g \sim \left(0, \sigma_g\right)$$

Where $0 < \rho_g < 1$ and the government spending is affected by the productivity process. The government spending comes from collecting lump sum taxes T_t and issues B_t. The budget constraint is shown as:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$$

3.9 Close the Model

By integrating the behaviours among households, firms, entrepreneurs, central bank and government, the good market clearing condition in log-linearized form is:

$$Y_t = \frac{C}{Y} C_t + \frac{I}{Y} I_t + CKY * MPK^* \frac{1-\psi}{\psi} MPK_t + \frac{CE}{Y} CE_t + \frac{EX}{Y} EX_t - \frac{IM}{Y} IM_t + \varepsilon_t^g$$

4. Evaluate and estimate the model: Indirect inference method

This chapter mainly evaluates the ability of the model to simulate actual economic characteristics. The quantitative evaluation of DSGE models was carried out without formal statistical methods for a long time. As simplifications of reality, while DSGE models represent a complete multivariate stochastic process for the data, the models are highly abstract and may have empirical flaws in some aspects. The models impose extreme restrictions on the actual time series, and they are often rejected against less restrictive specifications (e.g., vector autoregressions-VAR) (An and Schorfheide 2007). ''They are necessarily false, and statistical hypothesis testing will reject them,'' said Prescott (1986), who holds a similar view. He also states that "This does not imply, however, that nothing can be learned from such quantitative theoretical exercises." Since then, many researchers have developed econometric frameworks. They formalize various aspects of calibration methods by explicitly considering and including model misspecification (Smith and Roberts (1993), Geweke (1999), Schorfheide (2000), Bierens (2007), DeJong and Dave (2011)). The improvement of the structural models by macroeconomists relaxes many of the mis specified restrictions of old DSGE models. Thus, the new DSGE models are attractive from a theoretical point of view and are also valuable tools for forecasting and quantitative policy analysis in macroeconomics.

The Bayesian method has been developed in recent years and is popular to estimate DSGE models. The Bayesian method is system-based which fits the solved DSGE model to time-series data. The estimation relies on the likelihood function generated by the DSGE model. It incorporates the prior distribution to capture additional information, further used in the parameter estimation. However, the Bayesian method has also been criticized by a growing number of studies. Blanchard (2016) illustrates two problems. The first problem is about the standard in any system estimation. Specifically, the misspecification of one part in the model causes incorrect estimates of the parameters in other parts of the model. For instance, the misspecification of aggregate demand may produce wrong estimation of prices and wage adjustment. The second one is about ''the complexity of mapping from parameters to data''. Indeed, Bayesian estimation is well-performed if the prior for the coefficients are reasonable and tight. However, Blanchard (2016) continually states that ‘‘in some cases, the justification for the tight prior is weak at best, and what is estimated reflects more the prior of the researcher than the likelihood function.’’

This study chooses the indirect inference test to evaluate the empirical performance of the DSGE model, which was first proposed by Guvenen and Smith (2010) and further developed by Gourieroux et al. (1993). It attempts to test how close the structural model gets to the behaviour of a set of variables. The basic idea of indirect inference is to use an auxiliary model that is entirely independent of the theoretical model to produce a description of data, as a way to evaluate the performance of the theoretical model indirectly. When the parameters of the auxiliary model from the simulations are close to the parameters of the auxiliary model from the actual data, the theoretical model is considered to fit the data well. Indirect inference can be applied to linear model, non-linear models, or models of any sizes and complexity. More details of the indirect inference procedure are discussed in section 4.1.

4.1 model evaluation: indirect inference test

4.1.1 Indirect inference test procedure

The indirect inference used in this study was initially applied in Meenagh et al. (2009) and subsequently refined by Le et al. (2010); (2011,2012,2015). The test procedure is explained as below.

Step 1: Calculate the shock process

The number of structural shocks is less than the number of endogenous variables. To get the shock processes, we first calculate residuals using the actual data and model's equations conditional on the calibrated parameters. Note that, for those equations with expected variables, the values of expectations are estimated in advance by constructing a VAR process. The shocks are then backed out through performing OLS regressions with generated residual series and their past values. The corresponding coefficients are the persistence of shocks and error terms are the disturbances of shocks.

Step 2: Generate simulated data by bootstrapping

Holding the persistence of shocks, the disturbances produced in step 1 are bootstrapped by time vector to preserve their simultaneity. These re-ordered disturbances are added back to shock processes. Each sample is drawn independently, the process above is repeated, obtaining the S bootstrapped simulations we need. In this study, S is set as 1000.

Step 3: Calculate the Wald Statistic

A Wald statistic is used to evaluate the performance of the structural model. We first need to find the auxiliary model, which is constructed as a VAR. The Wald statistic is then computed to capture the distance between the vector of VAR coefficients from the simulated data and actual data, which is set as:

$$WS = \left(\beta^{\alpha} - \overline{\beta^{s}(\widehat{\theta_0})}\right)' \Omega^{-1} \left(\beta^{\alpha} - \overline{\beta^{s}(\widehat{\theta_0})}\right)$$

Where β^{α} is the true vector of the descriptor derived from actual data and $\overline{\beta^{s}(\widehat{\theta_0})} = E\left(\beta^{s}(\widehat{\theta_0})\right) = \frac{1}{S}\sum_{i=0}^{s}\beta^{i}(\widehat{\theta_0})$ represents the sample average of S sets of simulated VAR parameters from the simulations. $\Omega = cov\left(\beta^{i}(\widehat{\theta_0}) - \overline{\beta^{s}(\widehat{\theta_0})}\right) = \frac{1}{S}\sum_{i=0}^{s}\left(\left(\beta^{i}(\widehat{\theta_0}) - \overline{\beta^{s}(\widehat{\theta_0})}\right)\left(\beta^{i}(\widehat{\theta_0}) - \overline{\beta^{s}(\widehat{\theta_0})}\right)'\right)$ is the variance-covariance matrix of the distribution of simulated coefficients β^{i}. If the structural model is correct, the simulated data produced by the model should be similar to the actual data, so that the VAR estimates from actual data and simulated data do not have significant differences. Statistically, if the Wald statistic is smaller than its critical value, the null hypothesis is not rejected, which states that the model fits the data well and has the ability to reflect real economic phenomena. For the model to fit the data at the 95% confidence level, the Wald statistics for the actual data should be less than the 95th percentile of the Wald statistic from simulated data. To make the results more intuitively observed, the same information can be presented in the transformed Mahalanobis distance, which is computed as:

$$T = 1.645\left(\frac{\sqrt{2WS} - \sqrt{2k-1}}{\sqrt{2WS_i^{95th}} - \sqrt{2k-1}}\right)$$

with $p\ value = \frac{(100 - wald\ percentile)}{100}$, where Wald percentile is 95. In the expression above, k is the length of β^{α}; WS is the Wald statistic from the actual data and WS_i^{95th}

is the Wald statistic for 95th percentile of the simulated data. The critical value of T-statistic is set as 1.645 at 95% point of distribution, which means that $WS = WS_i^{95th}$. If the null hypothesis is accepted, T-statistic should be less than 1.645.

4.1.2 The Choices of Auxiliary Model

Durlauf and Blume (2016) state the importance of the auxiliary model: ''The auxiliary model serves as a window through which to view both the actual, observed data and the simulated data generated by the economic model: it selects aspects of the data upon which to focus the analysis'' Generally, the log-linearised DSGE model can be expressed in the form of restricted vector autoregressive and moving average (VARMA), or approximately in the form of a finite order reduced vector autoregressive (VAR). Based in Le et al. (2016a), a DSGE model can take the form of a co-integrated VAR with exogenous variables (VARX), whether the shocks and exogenous variables are stationary or not. The auxiliary model is chosen to evaluate how the model fits the data; thus it is suggested to have the same form as the DSGE model, which is VARX. The formation of the DSGE model is given by:

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t$$

Where y_t is the vector of endogenous variable and x_t is the vector of exogenous variables. The exogenous variables are assumed to be the ARIMA processes:

$$\Delta x_t = a(L)x_{t-1} + d + c(L)\varepsilon_t$$

The error terms e_t and ε_t are both following i.i.d. with zero means. L represents lag operator, $A(L)$ etc. are polynomial functions with roots outside the unit cycle. The general solution of DSGE can be further written as:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\varepsilon_t$$

x_t and y_t are non-stationary and they supposed to be cointegrated, the long-run relationship is

$$\overline{y_t} = \Pi\overline{x_t} + g$$

$$\overline{x_t} = [1 - a(1)]^{-1}[dt + c(1)\xi_t]$$

$$\xi_t = \sum_{i=0}^{t-1} \varepsilon_{t-s}$$

where long run solution of x_t consists of a deterministic trend $[1 - a(1)]^{-1}dt$ and a stochastic trend $[1 - a(1)]^{-1}c(1)\xi_t$. Therefore, y_t is eventually written as following VECM form:

$$\Delta y_t = -[1 - G(1)](y_{t-1} - \Pi x_t) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\varepsilon_t$$

Combining the last two errors into one term ($w_t = M(L)e_t + N(L)\varepsilon_t$), new disturbance follows a mixed moving average process, the VECM can be approximated as the form of VARX.

$$\Delta y_t = K(y_{t-1} - \Pi x_t) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + w_t$$

With holding the long-run condition that $\overline{y_t} - \Pi\overline{x_t} - g = 0$, the VECM can be also written as:

$$\Delta y_t = K[(y_{t-1} - \overline{y_{t-1}}) - \Pi(x_{t-1} - \overline{x_{t-1}})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + w_t$$

It can be further re-expressed as a co-integrated VARX (1), which will be used as the auxiliary model:

$$y_t = (1 - K)y_{t-1} + K\Pi x_{t-1} + n + t + \varsigma_t$$

Where t denotes deterministic trend, which affects both endogenous variables and endogenous variables; the stochastic trend is included in x_{t-1}, which is used to control the effects of past shocks on the long-run path of endogenous variables and endogenous variables. Two different effects should be distinguished in the data. n is the intercept and ς_t is the error terms which contains the suppressed lagged difference regressors. The parameter in equation can be estimated using the OLS method. This method is proved in the study of Meenagh et al. (2012) with Monto Carlo experiments, the results are extremely accurate. Le et al. (2012) proposed two types of Wald test: the full Wald test and direct Wald test. The former refers to the complete set of variables, while the latter only focus on the subset of variables. Due to the strength of the test increasing as the number of variables in the auxiliary VAR increases this study selects the direct Wald test to ensure the test has enough power, but not too much to reject all models. It is delivered from one or several aspects of model performance. Practically, I choose output y_t, inflation π_t and interest rate r_t as endogenous variables, the auxiliary model is expressed as VARX process in matrix form.

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = B \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + C \begin{bmatrix} T \\ \varepsilon_{a,t} \\ b^f_{t-1} \\ cons \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \\ \varepsilon_{r,t} \end{bmatrix}$$

where

$$B = \begin{bmatrix} \beta_{yy} & \beta_{y\pi} & \beta_{yr} \\ \beta_{\pi y} & \beta_{\pi\pi} & \beta_{\pi r} \\ \beta_{ry} & \beta_{r\pi} & \beta_{rr} \end{bmatrix} \text{ and } C = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24} \\ \theta_{31} & \theta_{32} & \theta_{33} & \theta_{34} \\ \theta_{41} & \theta_{42} & \theta_{43} & \theta_{44} \end{bmatrix}$$

In the equation above, the second variable matrix in RHS includes the deterministic time trend T, the lagged productivity trend $\varepsilon_{a,t}$ which is measured by the Solow residual, the lagged net foreign debt b_{t-1}^{f} and the intercept $cons$. $\varepsilon_{a,t}$ is calculated from production function. Because of the influence of its stochastic motion on the long-term solution path, this non-stationary exogenous variable is included to provide cointegration and it is said to be indispensable. The final matrix in RHS contains the error terms of endogenous variables. Matrix B and C are coefficient matrices of related variables. We can currently show that β^{s}, as the parameter vector in Wald test, consists of 9 parameters in matrix B (to describe the dynamic of the data) and the variance of the 3 disturbances (to observe the size of variations), it is shown as:

$$\beta^{s} = [\beta_{yy}, \beta_{y\pi}, \beta_{yr}, \beta_{\pi y}, \beta_{\pi\pi}, \beta_{\pi r}, \beta_{ry}, \beta_{r\pi}, \beta_{rr}, \sigma^{\varepsilon_{y,t}}, \sigma^{\varepsilon_{\pi,t}}, \sigma^{\varepsilon_{r,t}}]$$

Only if the model jointly matches 12 coefficients, it can pass the Wald test.

4.2 indirect inference estimation

In section 4.1.2, we concluded that the model with the calibrated parameters could not fit the actual data. However, the failure of the Wald test is because the model itself or the accuracy of the calibration parameters cannot be determined. This section introduces the method of indirect inference estimation. It is used to find out the optimal choice of parameters to minimize their distance from the auxiliary model's parameters to make the model perform well to fit the data.

We use different sets of parameters to calculate the Wald statistic. The choices of parameter sets depend on the algorithm based on Simulated Annealing (SA), which can search in a large range around the calibrated parameters with random jumps within the predefined upper and lower bounds and automatically loop over the test procedure until it finds the minimum value of Wald statistic. Specifically, the SA process starts using the initial choice of parameters and generating the first Wald statistic. It then figures out the next set of parameters to obtain the new statistic. If the new statistic is greater than the previous one, the system will move to the new point; otherwise, it will keep in the current one. Although it is possible to move to a point where the Wald statistic is significant, this possibility decreases as the number of evaluation points increases. After finding a certain number of optimal points, the search range is expanded by increasing the acceptable probability. There are many different stopping rules available for this algorithm. In this paper, the bounds are set within 30% of the initial calibrated parameters and the maximum number of iterations is set to 1000.

In conclusion, indirect inference estimation has the advantage of testing model independently based on data. We use this estimation to determine the optimal set of parameters to make the model fit the data well. If the minimum statistic value still cannot be accepted, we can suspect that the model setting does not hold.

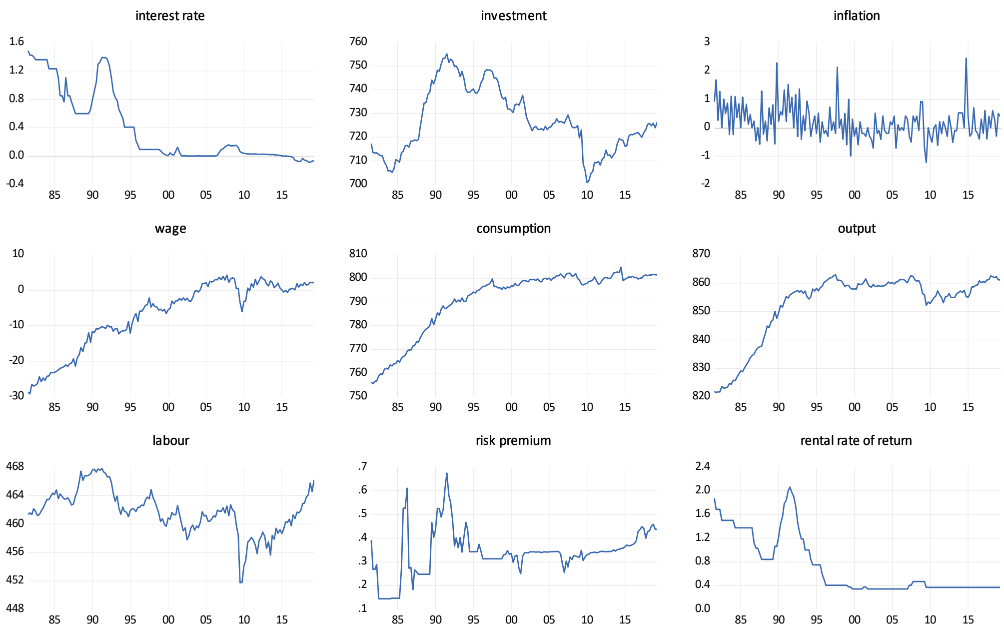
5. Empirical results

In this chapter, the model presented in the previous section will be evaluated and estimated with Indirect Inference using Japan's data. The model will be tested at first by using calibrated parameters. If the model is rejected, I will use indirect inference estimation to find out the optimal set of parameters to ensure the model can fit the data well.

5.1 Data analysis

This study collects Japan's quarterly data from the first quarter of 1981 to the fourth quarter of 2019. A full description of the data sources is shown in the Appendix-3. Except for variables in the ratio and percentage, all other time-series variables are converted to the format in per capita. Figure 5.1.1 plots all series employed in the model.

Figure 5.1.1 Data Series (1981Q3 - 2019Q2)



5.2 Calibration and test results

The structure parameters are calibrated before evaluating the model. For those parameters that determine the dynamics of the model, their values are directly taken from some empirical characteristics of the economy in the DSGE literature. I fix most parameters following Le et al. (2016a). For steady-state parameters, their values come from the observable data. Table 5.2 lists the initial set of parameters, and the details of the calibration are shown below.

The quarterly discount factor β is set at 0.99, which implies the steady-state interest rate of around 1%; the quarterly depreciation rate δ is set at 0.025, which implies the annualized depreciation rate is 10%. The degree of external habit formation h is defined at 0.62. The elasticity of substitution σ_c is set at 1.48, which follows the average coefficient among developed countries provided by Grandelman and Hernandez-Murillo (2015). The elasticity of labour supply σ_l is assumed to be 3.07 within the range of 2 to 4, in line with most macro- econometric estimates. The setting in this rage is proved to match the observed fluctuations in the total number of hours worked throughout the business cycle. The elasticity of capital adjustment cost φ is 6.99 and the elasticity of capital utilisation cost ψ is set at 0.085. The Calvo-type price rigidity ξ_p is defined at 0.89. The Calvo-type wage rigidity ξ_w is set at 0.59. The degree of price indexation ι_p and the degree of wage indexation ι_w are 0.16 and 0.39, respectively. In the hybrid model, the proportion of sticky price ω_p is set at 0.11, which represents the 11% of price setting is determined in imperfectly competitive market and the remainder comes from perfectly competitive market. The proportion of sticky wage ω_w is equal to 0.51. The share of capital α and the share of fixed cost Φ in production function are set equal to 0.2 and 0.73, respectively. In the sector of financial friction, the elasticity of financial premium with respect to leverage χ is set at 0.03. The survival rate of firms θ equals to 0.99, in line with Bernanke et al. (1999), implying that the average duration of entrepreneurs is more than 6 years.

The parameters in the foreign sector are set relying on the empirical study of Meenagh et al. (2010). Preference bias for domestic goods ω is set at 0.7, implying 70% of the consumption goods is from the domestic country. The parameters in the monetary policy rule equation are based on the values taken in Le et al. (2016a). The Taylor rule response from nominal interest rate to inflation r_p, output r_y and output gap $r_{\Delta y}$ are set at 2.86, 0.027 and 0.025, respectively. Interest rate smoothing rate ρ is 0.62. Without ZLB, money responds to the change in M2 ψ_1 with parameter set at 0.05. To reflect a situation similar to ZLB, the money supply is adjusted on credit premium and the money response to credit premium ψ_2 is set to 0.07. Steady state ratios in the log-linearised market-clearing condition is chosen to be consistent with the sample average of Japan data.

Figure 5.2.1 Value of Calibrated Parameters

Symbol	**Description**	**Value**
Households		
β	Discount factor	0.99
h	External habit formation	0.62
σ_c	Elasticity of consumption	1.48
σ_l	Elasticity of labour supply	3.07
ξ_w	Degree of wage rigidity	0.59
ι_w	Wage indexation	0.39
ω_w	Proportion of sticky wages	0.51
ω	Preference bias towards in consumption of domestic goods	0.7

Firms		
α	Share of capital in production function	0.20
Φ	Share of fixed cost in production function	1.73
δ	Capital depreciation rate	0.025
φ	Elasticity of capital adjustment	6.99
ψ	Elasticity of capital utilisation	0.085
ξ_p	Degree of price rigidity	0.89
ι_p	Price indexation	0.16
ω_p	Proportion of sticky prices	0.11
Financial Friction		
θ	Entrepreneur survival rate	0.99
χ	Elasticity of premium with respect to leverage	0.03
ψ	Elasticity of premium to M0	0.044
Monetary Policy		
r_p	Taylor rules response to inflation	2.86
ρ	Interest rate smoothing	0.62
r_y	Taylor rules response to output	0.027
$r_{\Delta y}$	Taylor rules response to change in output	0.025
ψ_1	M0 response to M2	0.05
ψ_2	Money response to credit growth	0.07

Table 5.2.2 shows that the T-statistic for the auxiliary model is 4.209, which is greater than the critical value of 1.65. Therefore, we can conclude that the structure model with calibrated parameters does not fit the data well. This could be for two reasons. One is that the selection of the calibrated parameters may be inappropriate, and the second is that the structural model itself has defects. Therefore, indirect inference estimation is used to find out whether the structural model can be rejected. If the structural model passes the test, the model's most suitable set of parameter estimates can also be found through indirect inference estimation.

Figure 5.2.2 Test Results from Calibration

Subsets	Trans value	Wald value
y, π, r	4.209	48.169

5.3 Estimation and test results

The application of indirect inference estimation provides II estimates of the parameters model as shown in table 5.3.1, which improve the fitness of the model to the observed data. It should be noted that almost all parameters move some away from their calibrated values except for the discount factor, depreciation rate and firm's survival rate. We will compare the estimated and calibrated set of parameters below.

In terms of household sector, the intertemporal elasticity of consumption σ_c has increased by 10.8%, indicating that the consumption growth in estimation is less sensitive to the real interest rate than the one in calibration. The external habit in consumption decreases from 0.62 to 0.52. The elasticity of labour supply σ_l has increased by 33.6%, implying that workers are more reluctant to smooth working hours as the wage rate changes, compared to the calibrated moment. Given the constant marginal utility of wealth constant, a 1% rise in wage rates results in a 0.75% increase in working hours. The Calvo wage parameter has been estimated to 0.57, and the degree of wage indexation has decreased to 0.31. We compare the estimated price setting parameters here. The Calvo price rigidity has decreases from 0.89 to 0.70, the price indexation is 0.15 which like the calibrated one. The degree of price indexation is lower than wage indexation. It implies that wage inflation shows more persistent than price inflation. The proportion of sticky wages that encodes the weight of New Keynesian price is estimated to 0.3764 while the proportion of sticky price is equal to 0.1082. The proportion of sticky wage and price as the part from in NK model change to 0.58 and 0.11 respectively. The former has increased, while the latter remains unchanged. On the firm side, share of capital and share of fixed cost in production function are both increase. It shows that capital investment has a greater weight in economic growth. The capital adjustment cost has increased by 3.6% and the capital utilisation has dropped by 2.4%. For those parameters in financial friction sector, the 33% increases in elasticity of premium with respect to leverage represents a more significant performance of leverage to premium; the 9.1% increase in the elasticity of premium to M0 indicates a more sensitive premium response to M0. In terms of monetary policy, the interest rate's response to inflation has fallen from 2.86 to 2.71, indicating that monetary policy is estimated to be more responsive to the fluctuation of inflation. Meanwhile, relative to the calibrated value, the estimated value shows a higher policy response to output, but a lower policy response to output changes. The two estimates are 0.034 and 0.019, respectively. The interest rate smoothing rate has also been slightly reduced, by 6%. The sensitivity of M0 to changes in M2 has been lower both in the normal period and in the ZLB period, which implies that the influence of the money supply on boosting the economy may be limited.

Testing the model with the estimated parameters allows the model to perform well to fit the observed data through minimising the T statistic calculated by the auxiliary model. We re-test the model by holding the estimated parameters and the result is shown in Table 5.3.2. Wald percentile is 25.67 and normalised Mahalanobis distance is 1.38, which is smaller than the critical value. The auxiliary model is significantly not rejected with estimated parameters, and the data generated in the auxiliary model is close to the actual data. Therefore, we conclude that our hybrid model has good performance to describe Japan's economy.

Figure 5.3.1 Value of Calibrated Parameters

Symbol	**Description**	**Calibration**	**Estimation**
Households			
β	Discount factor	0.99	0.99
h	External habit formation	0.62	0.52
σ_c	Elasticity of consumption	1.48	1.64
σ_l	Elasticity of labour supply	3.07	4.10
ξ_w	Degree of wage rigidity	0.59	0.57
ι_w	Wage indexation	0.39	0.31
ω_w	Proportion of sticky wages	0.51	0.58
ω	Preference bias towards in consumption of domestic goods	0.7	0.7
Firms			

α	Share of capital in production function	0.20	0.27
Φ	Share of fixed cost in production function	1.73	2.06
δ	Capital depreciation rate	0.025	0.025
φ	Capital adjustment	6.99	7.24
ψ	Capital utilisation	0.085	0.083
ξ_p	Degree of price rigidity	0.89	0.70
ι_p	Price indexation	0.16	0.15
ω_p	Proportion of sticky prices	0.11	0.11
Financial Friction			
θ	Entrepreneur survival rate	0.99	0.99
χ	Elasticity of premium with respect to leverage	0.03	0.04
ψ	Elasticity of premium to M0	0.044	0.048
Monetary Policy			
r_p	Taylor rules response to inflation	2.86	2.71
ρ	Interest rate smoothing	0.62	0.58
r_y	Taylor rules response to output	0.027	0.034
$r_{\Delta y}$	Taylor rules response to change in output	0.025	0.019
ψ_1	M0 response to M2	0.05	0.04
ψ_2	Money response to credit growth	0.07	0.06

Figure 5.3.2 Test Results from Estimation

Subsets	Trans value	Wald value
y, π, r	1.383	25.668

5.4 Error properties

This model contains 13 shocks to observe how the model reacts to different volatility. The process of the fundamental impact comes from solving the model with estimated parameters and historical data. In this section, we first apply stationarity tests to analyse the properties of residuals and then describe the basic characteristics of shocks and residuals. Model implied residual and shock histories are plotted in Table 5.4.1

and 5.4.2, respectively. Note that the shocks fluctuate around zero and the residuals are the accumulation of shocks over sample periods.

We submit the stationarity tests on the residuals, using the Augmented Dickey-Fuller (Dickey and Fuller 1979), Phillips–Perron (Phillips and Perron 1988) and Kwiatkowski–Phillips–Schmidt–Shin (Kwiatkowski et al. 1992) tests, respectively. The KPSS test is used to avoid some potential shortcomings of the first two tests (e.g. the regression includes redundant trend items, which makes the ability to reject zeros low). Contrary to the null hypothesis of ADF and PP that series has unit roots, the null hypothesis of KPSS test is that the sequence is stationary. Moreover, due to factors such as lag length selection and differences in limited sample performance, the results from the three tests may differ. (Maddala and Kim 1998). Table 5.4.1 reports the results of the stationary test. For the stationarity test of fourteen series, ten tend to be stationary or the trend stationary. Four exceptions, which are total factor productivity, export, labour supply and risk premium shock, have the possibility of following I (1) processes. They cannot reject the null hypothesis of ADF and PP tests. These two tests reach the same conclusions in most cases, although there are some subtle differences in testing the stationary of labour supply shock. The KPSS test confirms the non-stationarity of the TFP shock but holds the opposite results to the ADF test on the stationarity of the other three. We can conclude that only the TFP shock follows the I (1) process which can be confirmed by three tests simultaneously, while the others all show evidence of stability in at least one test. The assumption of shock processes thus cannot be rejected by the data. That is, the TFP shock is assumed to be ARIMA (1,1,0), while the rest are assumed to be AR (1). The degree of persistence and standard deviations of their innovations is described in Table 5.4.2.

Figure 5.4.1 Model implied shock histories

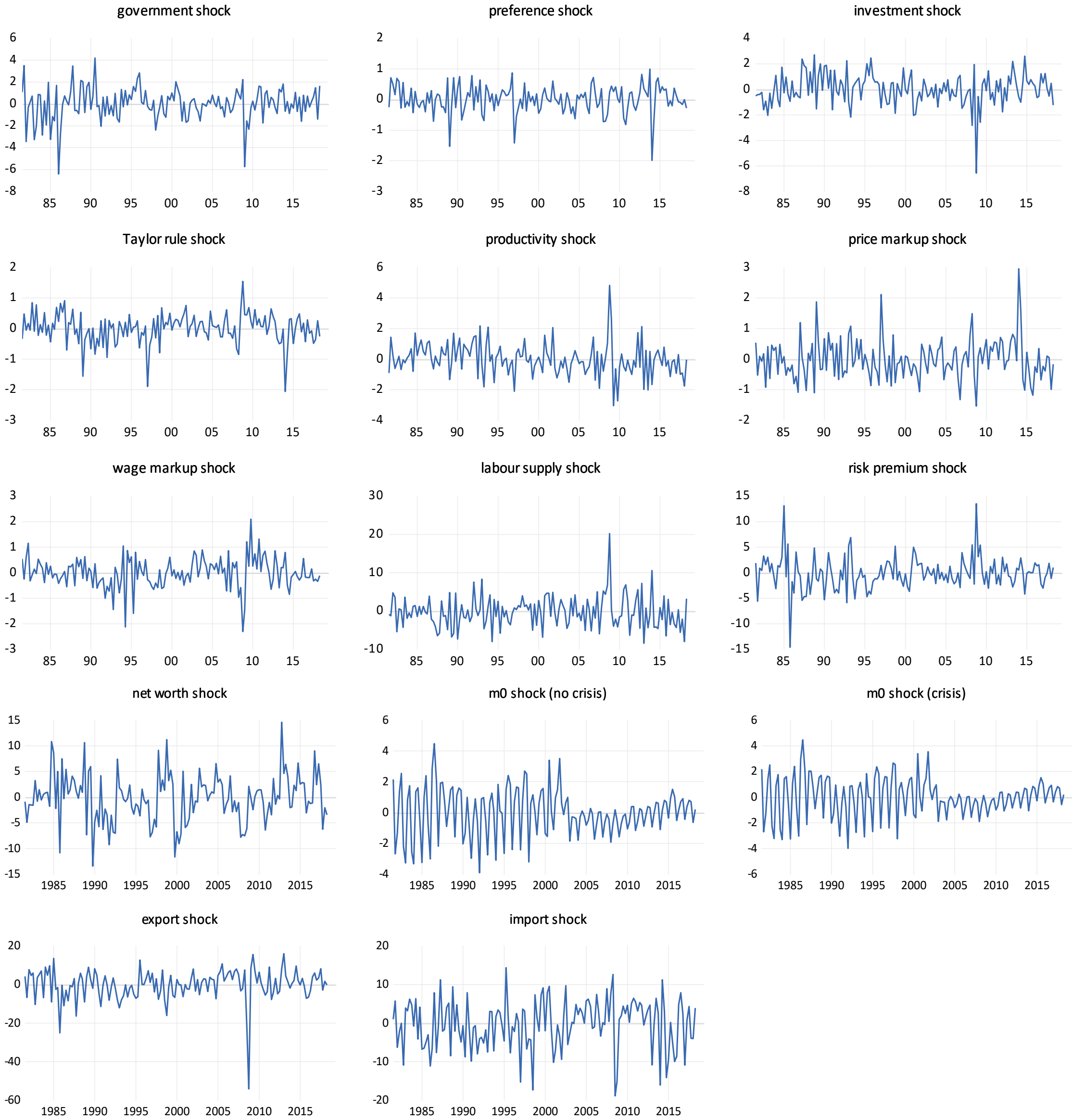


Figure 5.4.2 Model implied residual histories

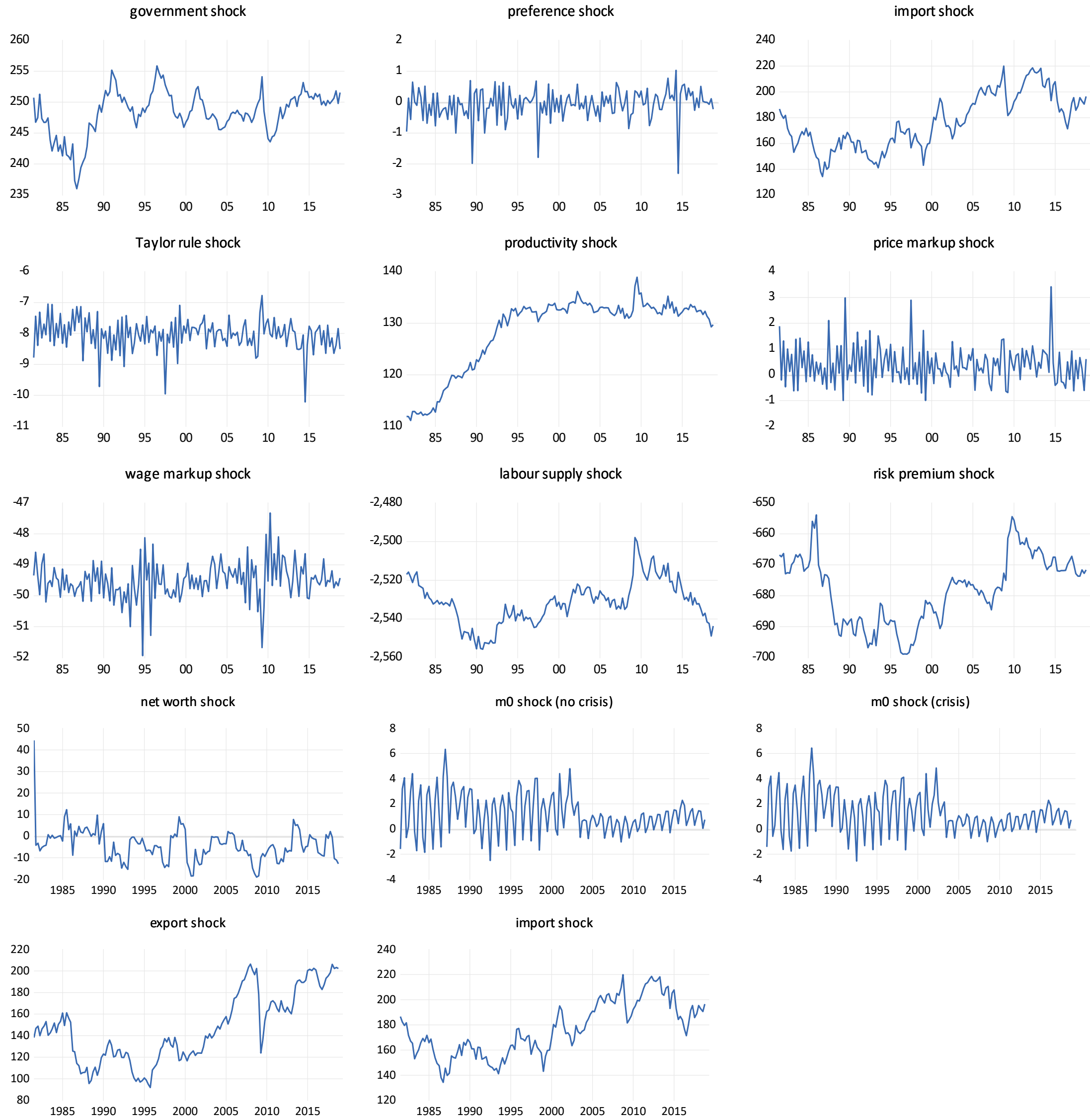


Table 5.4.1 Testing the Null Hypothesis of Non-stationarity

	ADF				PP				KPSS (reject IM >critical)			
	level	level and trend	difference		level	level and trend	difference		level	level and trend	difference	
productivity shock	-1.980	-1.370	-13.594***	I(1)(1%)	-1.939	-1.157	-13.608***	I(1)(1%)	1.025	0.344	0.311+++	I(1)(1%)
Preference shock	-17.230***	-17.756***	-8.856***	S(1%)	-17.360***	-18.913***	-79.724***	S(1%)	0.708+++	0.024	0.070	S(1%)
Export shock	-1.437	-2.982	-8.133***	I(1)(1%)	-1.188	-2.316	-8.232***	I(1)(1%)	0.872	0.136++	0.107	TS(5%)
Government spending shock	-2.783**	-3.188*	-13.238***	S(5%)	-2.888**	-3.368*	-13.234***	S(5%)	0.356++	0.112	0.051	S(5%)
Import shock	-1.834	-3.695**	-6.442***	TS(5%)	-1.834	-3.635**	-12.139***	TS(5%)	1.071	0.160+++	0.100	TS(1%)
Investment shock	-6.445***	-6.426***	-13.833***	S(1%)	-11.885***	-11.848***	-88.630***	S(1%)	0.138+++	0.138	0.475	S(1%)
Labour supply shock	-2.223	-2.412	-13.750***	I(1)(1%)	-3.04**	-4.214***	-15.679***	S(5%)	0.483+++	0.156	0.133	S(1%)
M0 shock no crisis	-3.917***	-4.418***	-13.142***	S(1%)	-5.872***	-6.498***	-29.071***	S(1%)	0.505+++	0.070	0.092	S(1%)
M0 shock crisis	-3.839***	-4.360***	-13.080***	S(1%)	-5.951***	-6.850***	-28.860***	S(1%)	0.538+++	0.067	0.089	S(1%)
Net worth shock	-4.959***	-5.171***	-14.738***	S(1%)	-4.901***	-5.092***	-24.607***	S(1%)	0.477+++	0.122	0.233	S(1%)
Price markup shock	-5.175***	-5.186***	-17.960***	S(1%)	-20.248***	-20.277***	-64.778***	S(1%)	0.115+++	0.053	0.138	S(1%)
Taylor rule shock	-3.759***	-4.005***	-16.210***	S(1%)	-15.309***	-15.572***	-47.388***	S(1%)	0.256+++	0.082	0.131	S(1%)
Risk premium shock	-1.719	-2.037	-8.000***	I(1)(1%)	-1.869	-2.166	-10.820***	I(1)(1%)	0.427++	0.245	0.150	S(5%)
Wage markup shock	-7.450***	-7.746***	-13.614***	S(1%)	-14.762***	-14.968***	-68.793***	S(1%)	0.387++	0.107	0.033	S(5%)

Note: For the Augmented Dickey-Fuller (ADF) test and Phillips–Perron test , statistic with ***, ** and * indicate a rejection of the unit root process at 10%, 5% and 1% significant level respectively. For the KPSS test, statistic with ***, ** and * indicate a rejection of the stationary process at 10%, 5% and 1% significant level respectively.

Table 5.4.2 Statistic properties of shocks

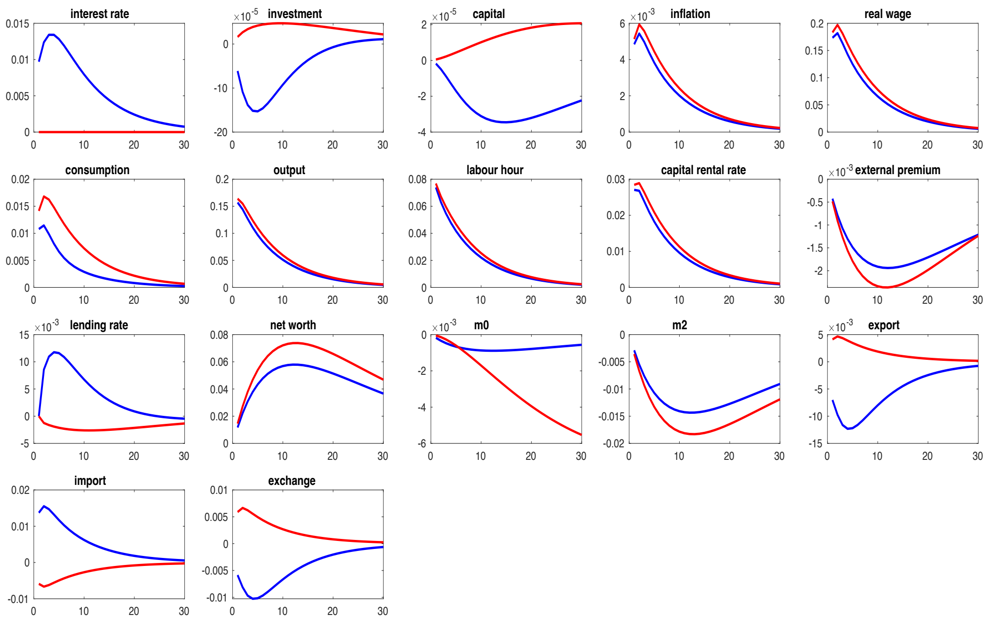
Description		AR coefficients		Standard deviation	
$\varepsilon_{g,t}$	Government spending shock	ρ_g	0.810	η_t^g	1.490
$\varepsilon_{g,t}$	Response of exogenous spending to productivity development	ρ_{pg}	0.381	/	/
$\varepsilon_{b,t}$	Preference shock	ρ_b	0.291	η_b^g	0.659
$\varepsilon_{i,t}$	Investment shock	ρ_i	0.040	η_t^i	0.451
$\varepsilon_{r,t}$	Taylor rule shock	ρ_r	0.555	η_t^r	0.489
$\varepsilon_{a,t}$	Productivity shock	ρ_a	0.591	η_t^a	1.235
$\varepsilon_{p,t}$	Price mark-up shock	ρ_p	0.216	η_t^p	0.599
$\varepsilon_{w,t}$	Wage mark-up shock	ρ_w	0.780	η_t^w	1.051
$\varepsilon_{l,t}$	Labour supply shock	ρ_l	0.968	η_t^l	4.103
$\varepsilon_{pr,t}$	Risk premium shock	ρ_{pr}	0.016	η_t^{pr}	3.210
$\varepsilon_{nw,t}$	Net worth shock	ρ_{nw}	0.095	η_t^{nw}	4.704
$\varepsilon_{moc,t}$	Quantitative easing shock (no crisis)	ρ_{moc}	0.107	η_t^{moc}	1.540
$\varepsilon_{mc,t}$	Quantitative easing shock (crisis)	ρ_{mc}	0.933	η_t^{mc}	1.544
$\varepsilon_{ex,t}$	Export shock	ρ_{ex}	0.868	η_t^{ex}	8.018
$\varepsilon_{im,t}$	Import shock	ρ_{im}	0.981	η_t^{im}	6.394
C_t^f	Foreign consumption shock	ρ_{cf}	0.914	η_t^{cf}	6.968
R_t^f	Foreign interest rate shock	ρ_{rf}	0.810	η_t^{rf}	0.163

5.5 Impulse response function

Response to a government spending shock

Table 5.5.1 shows the impact of a positive government spending shock on the economy. When there is no crisis, the increase in government spending revitalizes labour and wage levels to a certain extent, thereby promoting consumption and output, and further triggering moderate inflation. However, the increase in nominal interest rates caused by the expansion of fiscal policy suppressed inflation. In addition, the net worth of entrepreneurs increases with the increase in output and employment. Under the BGG financial accelerator mechanism, credit spreads decrease. However, the decline in credit spreads is not enough to counter the rise in deposit interest rates, which affects the economy's lending capacity, thus forming a crowding-out effect on private investment. Also, the figure shows that the relationship between expansionary fiscal policy and money supply is negatively correlated. This is because the change in the money supply does not rely on and adapt to fiscal policy, and it depends more on the setting from the central bank. For the part in an open economy, the increase in real interest rates must be balanced by the appreciation of the domestic currency, thereby encouraging imports and hindering exports. Under the ZLB crisis, the Taylor rule failed, and the monetary authorities lost the ability to use the Taylor rule to increase nominal interest rate to stabilize the economy. The near-zero nominal interest rate and positive inflation make the real interest rate fall, thereby better stimulating consumption, investment, output, real wages, etc. Coupled with the significant increase in the net worth of entrepreneurs, credit expansion is reduced, and investment can be further expanded. In addition, positive inflation leads to domestic currency depreciation, which stimulates exports and discourages imports.

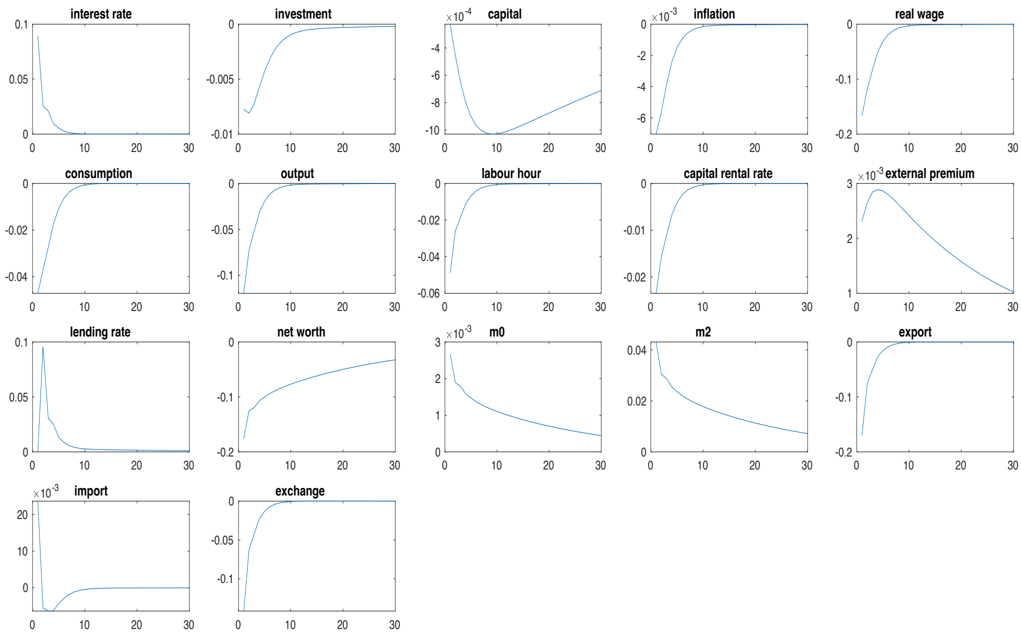
Figure 5.5.1 Impulse Response to a Positive Government Spending Shock



Response to a monetary policy shock

Table 5.5.2 depicts the IRFs under the impact of the positive Taylor rule (i.e., tightening monetary policy) when the economy is not suffering from a crisis. The standard Taylor rule transmission mechanism suggests that the contractionary monetary policy under Taylor's rule increases nominal interest rates, which further inhibits borrowing, investment and consumption, thereby reducing aggregate demand. Downward pressure on the demand side forced entrepreneurs to cut the labour force, increasing the unemployment rate and falling real wages. With the sluggish economic environment, the output gap causes a tightening of inflation, which results in an increase in the capital borrowing rate of entrepreneurs and a decrease in the return on investment. The net worth of entrepreneurs is also compressed, which further restricts investment. In addition, deflation and high nominal interest rates increase the value of the domestic currency, thereby stimulating imports while also negatively impacting exports.

Figure 5.5.2 Impulse Response to a Positive Monetary Policy Shock

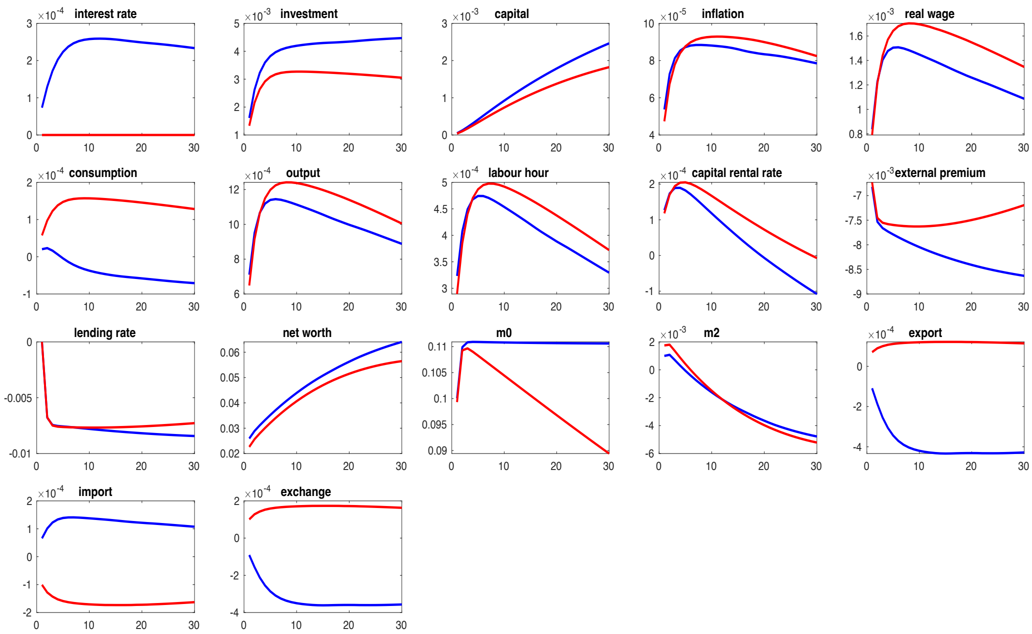


Response to Quantitative Easing shock

Table 5.5.3 shows the impact of M0 growth under the normal case and under the zero lower bound. It can be observed that the responses of the increase in M0 to each variable are basically the same. The M0 growth lowers the risk premium and then pushes the demand for investment up. This promotes more output and higher wages, making more workers attracted by higher wages, further stimulating labour supply, and increasing consumption. Inflation is then generated. In the foreign sector, domestic currency in the economy without ZLB appreciates because real interest rates raises; thereby, a lower exchange rate encourages imports and hinders exports. Under ZLB crisis, the higher money supply led to the higher real exchange rate, and currency depreciation makes exports more competitive and imports less attractive.

Moreover, without crisis, the central bank uses Taylor's rule to raise interest rates in response to inflation. This restricts the growth of consumption. Under ZLB, the growth in inflation lowers the real interest rate and stimulates consumption even more strongly. Therefore, the contraction effect of traditional monetary policy is not fully reflected, and M0 has a good performance in stimulating the economy when the risk premium is reduced in a crisis. We can conclude that quantitative easing policy is effective in this model under both normal and crisis regimes.

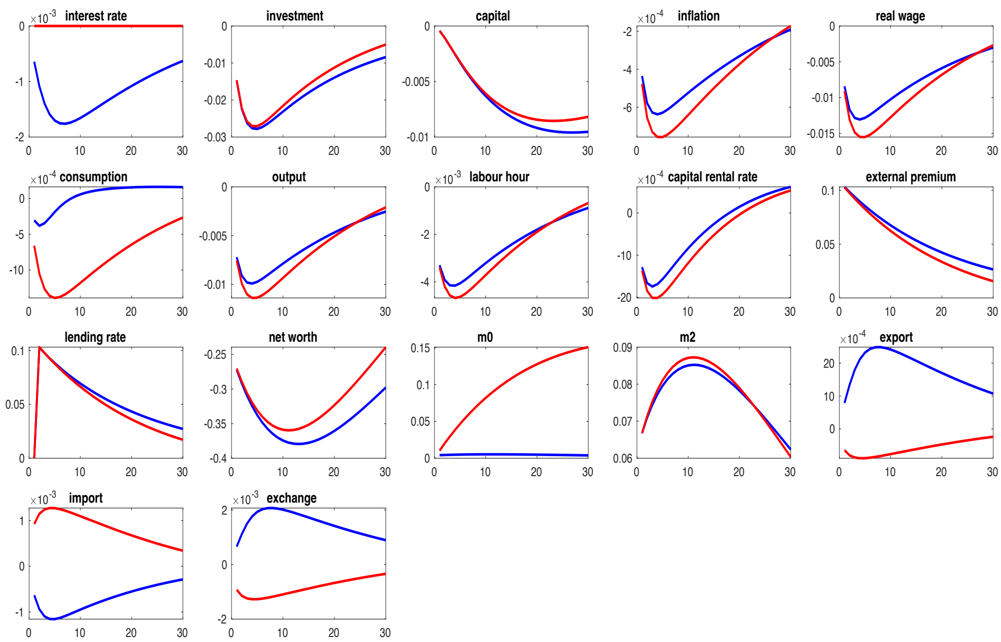
Figure 5.5.3 Impulse Response to a Positive Quantitative Easing Shock



Response to an external financial premium shock

Table 5.5.4 shows the impact of a positive external financial premium shock. First, the increase in external financial premium raises the cost of borrowing and directly negatively impacts investment. In addition, the increase in borrowing cost declines Tobin's Q, which directly affects the net present value of entrepreneurs and indirectly affects the investment demand. Furthermore, the decline in net present value further intensifies the balance sheet effect; that is, lower net worth reduces entrepreneurs' shares in capital expenditure financing, asking them to borrow at a higher premium over the deposit rate. In this way, a negative cycle is formed. The increase in financial premiums ultimately suppresses aggregate domestic demand through reductions in investment, capital, consumption, net worth, and output. In an economy without crisis, the central bank stimulates the economy by lowering nominal interest rates. When the crisis came, the central bank used quantitative easing to increase the money supply to offset the negative impact of the positive premium shock on the economy. Comparing the performance of variables in different economic environments, we can find that non-traditional monetary policy performs better even in crisis, especially on variables from the demand side. In the open economy, there is a significant currency depreciation to restore the uncovered interest rate parity, which stimulates the demand for exports and reduces the demand for imports.

Figure 5.5.4 Impulse Response to a Positive External Financial Premium Shock



5.6 Variance decomposition

In this section, we will use the variance decomposition to explore what drives fluctuations of the Japanese economy and find out the importance of each shock in explaining the variation of the variable. Based on the model estimation, Table 5.6.1 shows the variance decomposition of model shocks for interest rate, inflation, consumption, output, and exchange rate. In general, productivity shocks and export shocks play an essential role in the five economic variables. First, productivity shock contributes more than 50% to inflation, consumption, output and exchange rate, and its effect on inflation variance can even reach as high as 82%. Second, as Japan's economic development has been highly dependent on exports in recent decades, the contribution of export shocks on interest rate accounted for 75.8%, and the contribution rate to the variance of the other four variables is also very high. Finally, we found that more than 90% of changes in interest rates, inflation, consumption, output, and exchange rate can be explained by export and productivity shocks.

To take a closer look, neither the Taylor rule used under the standard economy nor the unconventional monetary policy (i.e., here is QE) used under the ZLB policy have brought relatively great help to the economy. The former is due to Japan's long-term use of the ZLB policy, which makes the Taylor rule invalid and does not contribute too much to economic fluctuations; the latter can be explained as QE has not been able to inject new vitality into the economy in a real sense. Compared with monetary policy, fiscal policy has a higher degree of explanation. About 1.5% of the output variance is affected by government spending shocks. In addition, the severe malformation of Japan's financial structure and the imperfect development of subsequent structural reforms have led to insufficient contribution of external financing premiums shock to the variance of variables.

Moreover, flexible wage shock has also been identified as a driving factor for volatility, especially for interest rate fluctuations, with a ratio of 4.13%. For Japan being an open economy, the volatility brought by the import shock is significant. It contributed 9.38% to the interest rate variance, 7.7% to the output variance, and approximately 2% to the remaining three variances.

Table 5.6.1 Variance Decomposition of Key Variables

	interest rate	inflation	consumption	output	exchange rate
government shock	0.3171	0.0421	0.0565	1.4894	0.0153
preference shock	0.0004	0.0002	0.0403	0.0042	0.0000
investment shock	0.0001	0.0000	0.0000	0.0004	0.0000
taylor rule shock	0.0254	0.0001	0.0034	0.0023	0.0038
productivity shock	9.8348	82.0327	65.8184	65.5933	57.1934
price markup shock	0.0258	0.0468	0.0044	0.0023	0.0018
wage markup shock	0.3171	0.0420	0.0564	1.4887	0.0153
labour supply shock	4.1323	1.0298	2.3716	0.0930	0.8328
risk premium shock	0.0815	0.0109	0.0013	0.1940	0.0088
net worth shock	0.0601	0.0085	0.0017	0.1367	0.0055
m0 shock	0.0000	0.0000	0.0000	0.0001	0.0000
export shock	75.8162	14.7477	30.5862	23.2949	39.0098
import shock	9.3893	2.0393	1.0597	7.7006	2.9134
	100.0000	100.0000	100.0000	100.0000	100.0000

6. Policy Analysis

In this chapter we look at the policy issue of how the Japanese economy could have been better managed through monetary and fiscal policy. We begin by examining the implications for economic stability of the status quo, as estimated in our baseline model. This consists of a Taylor Rule when interest rates are above the zero bound and a QE policy for the monetary base when interest rates are at the zero bound (in 'crisis' periods). We find that the economy is very turbulent with a high variance of both output and inflation. This is connected with the prevalence of zero bound episodes when inflation moves around sharply including into deflation. This behaviour is not well controlled by the QE policy, essentially because its power is highly attenuated by the flattening of interest rates at all maturities. We show below some illustrative simulations. One can see how interest rate variation is suppressed by long zero bound episodes; and how inflation fluctuates beyond monetary control, with destabilising output consequences.

It seems clear from this that to succeed in stabilising the economy, it is necessary to suppress the zero lower bound. Unfortunately, monetary policy cannot do this, since the bound hits when demand is weak, and this weakness perpetuates the bound. In this situation, monetary tightening and the raising of rates would simply weaken demand further, pushing rates back down to the bound. Monetary loosening, of which there has been a preponderance in recent years, pushes the economy deeper into the bound by lowering long term interest rates.

Therefore, fiscal policy needs to be brought into the picture to push up demand until the zero bound is eliminated. To achieve this, strong fiscal expansion needs to be uncompromising and unresponsive to rising debt. In recent decades this has not been the case; every so often the government has become alarmed by the rising debt/GDP ratio and sharply raised the consumption tax, so reversing its fiscal thrust and plunging the economy back into weakness. Yet there has never been a solvency issue with public debt, as is evident from the zero rates at which long debt has persistently been sold, at apparently very high ratios. The reason is plain: Japanese households have a high savings rate and have always been willing to buy. Furthermore, there has never been any doubt that taxes could be raised if necessary to pay any likely interest burden, so guaranteeing solvency.

We also consider as an optimising policy a strong Taylor rule targeting Nominal GDP, which is effectively a combination of a price level target with the usual output gap response. The PLT element in effect provides strong forward guidance, since an inflation deviation provokes a long-lasting interest rate response. On the fiscal side we consider a fiscal policy that suppresses the ZLB. Of course this implies a much stronger fiscal response to the output gap than in current Japanese policy. We implement it as a fiscal policy that is 'whatever it takes' to prevent interest rates falling to the bound. We compute what this public spending amount must be as whatever the demand residual would be for demand to equal supply potential at the ZLB. Besides this suppressant policy we also consider a straightforward strong fiscal response to the output gap.

6.1 The fiscal policy results under different policy regimes

In this section we show how the economy performs in response to full stochastic simulations of all the economy's shocks under different fiscal policy regimes. Three different fiscal policy regimes are shown as follows.

- Baseline policy regime:

$$\mathrm{g_t} = \rho_g \mathrm{g_{t-1}} + \rho_a e_t^a + e_t^g$$

Where $\mathrm{g_t}$ is the government expenditure shock; e_t^a is the the productivity i.i.d innovation; e_t^g is the government expenditure i.i.d innovation.

- Suppressing fiscal policy regime:

$$\mathrm{g_t} = \rho_g \mathrm{g_{t-1}} + \rho_a e_t^a + e_t^g + f_t$$

Where f_t is a fiscal shock pushing interest rate out of the ZLB.

- Strong fiscal feedback policy regime:

$$\mathrm{g_t} = \rho_g \mathrm{g_{t-1}} + \rho_a e_t^a + e_t^g - \theta(y_t - \overline{\mathrm{y_t}})$$

Where $\overline{\mathrm{y_t}}$ is the base run output, $(y_t - \overline{\mathrm{y_t}})$ is the output gap. $\theta = 1$.

Table 6.1 summarises the resulting variances of key variables for each fiscal rule. According to these estimates, we find that strong fiscal feedback policy has best performance, which greatly reduces the variance of each variable. In particular, the variance of output drops sharply from 101.83 under baseline policy to 4.68 under strong fiscal feedback policy. The following figures demonstrate some typical bootstrap simulations to support the findings in table 6.1.1.

Table 6.1.1Variance of simulations

Variance	Baseline	ZLB-suppressing Fiscal policy	Strong Fiscal feedback policy
	(M policy:non-crisis Taylor rule +crisis[ZLB] QE)	**(non-crisis Taylor Rule + fiscal shock)**	**(non-crisis Taylor Rule +crisis [ZLB] QE +strong fiscal feedback in both models)**
Var (output)	101.83	34.40	4.68
Var (inflation)	1.13	0.60	0.76
Var (interest rate)	1.52	0.74	1.86
Var (consumption)	2.03	1.54	0.72

Figure 6.1.1 Examples of some output simulations

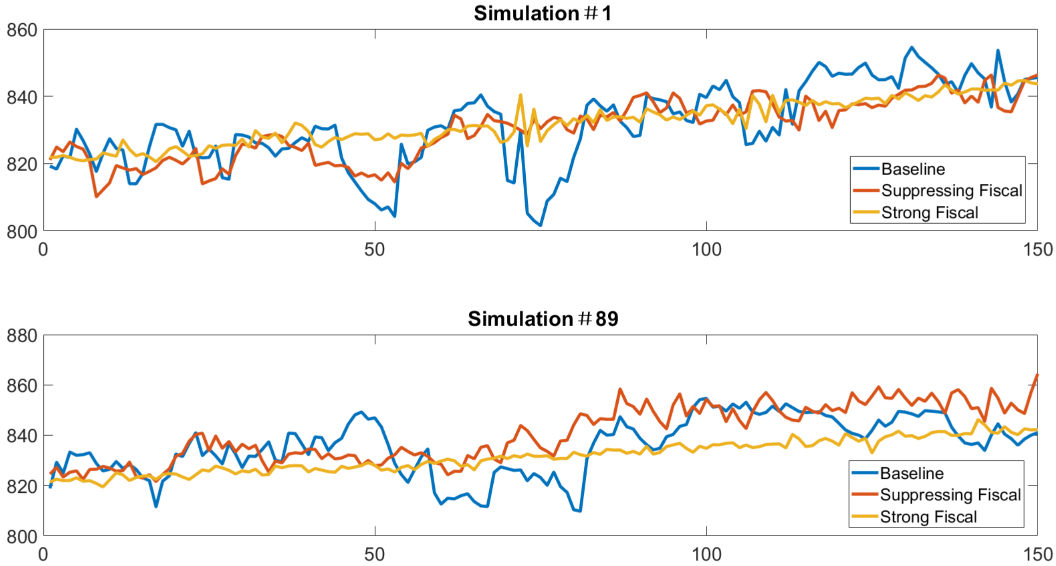


Figure 6.1.2 Simulations of output and government spending under different policy regimes

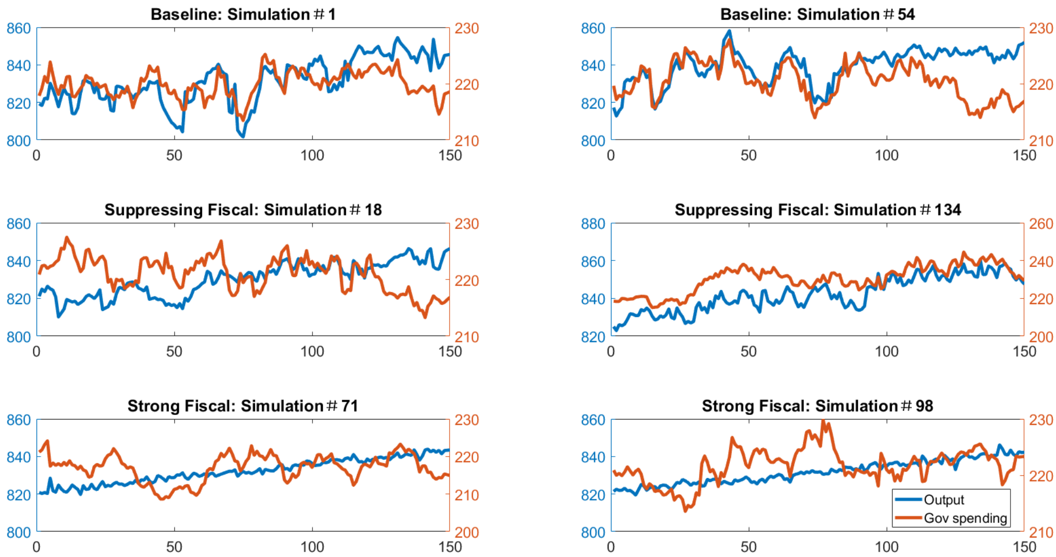
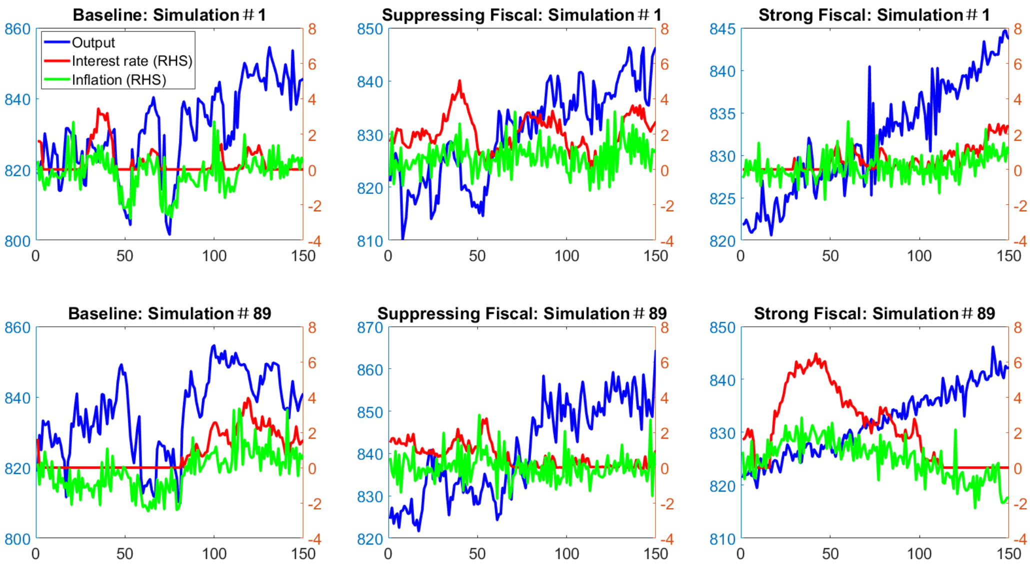


Figure 6.1.3 Simulations of output, interest rate and inflation under different policy regimes



6.2 A Nominal GDP targeting policy- does it improve matters?

We show below finally the results of combining the Fiscal ZLB-suppression regime with Nominal GDP targeting in monetary policy, as follows:

$$r_t = 0.692r_{t-1} + 2.745(y_t + p_t - \overline{y_t} - \overline{p}) + er_t$$

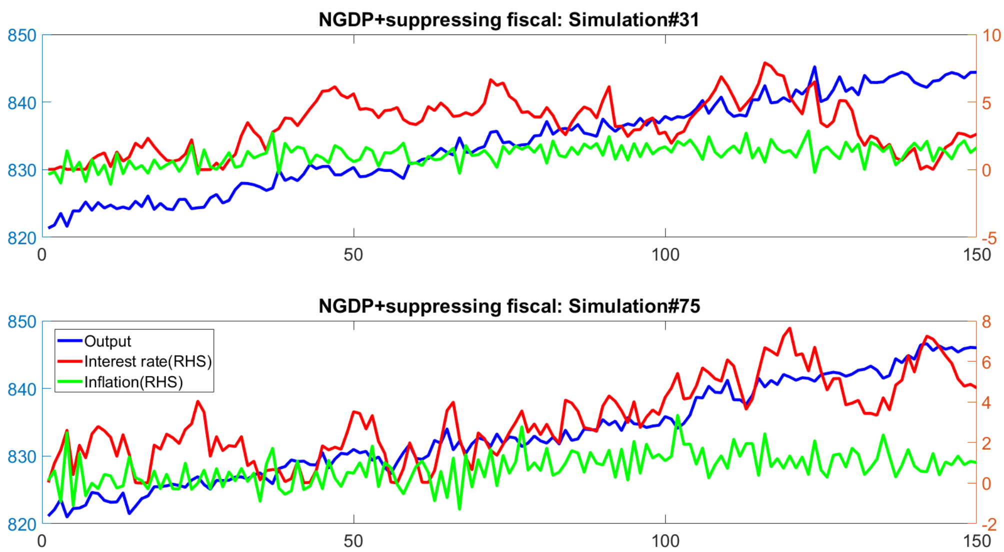
Where $\overline{\mathrm{p}} = 0$ and $\overline{\mathrm{y}_{\mathrm{t}}}$ is the base run output.

From table 6.2.1, it can be seen that in some respects the Fiscal ZLB-suppression regime with Nominal GDP targeting in monetary policy improves on the best of the previous regimes considered, which was the baseline monetary regime with a strong fiscal response. It keeps the inflation variance low while further reducing output variance. However, it does so at the expense of very high interest rate variation which induces much higher consumption variance, the best indicator of household utility. When all these things are taken into account, the best regime is the current monetary one - a Taylor Rule with QE under the ZLB, allied to a strong fiscal response. More evidence are shown in following charts (see Figure 6.2.1 and 6.2.2).

Table 6.2.1 Variance of simulations

Variance	**Strong Fiscal feedback policy with Taylor rule**	**ZLB-suppressing Fiscal policy with nominal GDP interest rate targeting policy**
Var (output)	4.68	3.79
Var (inflation)	0.76	0.72
Var (interest rate)	1.86	3.20
Var (consumption)	0.72	2.81

Figure 6.2.1 Examples of some output simulations



6.3 The effect on debt ratios: is there a solvency problem?

What we have found implies that the introduction of strong fiscal feedback policy is the key to macro stability in Japan. In this section- Table 6.3.1- we review the effect of such an active fiscal policy on the public debt/GDP ratios, to see if there is any threat to government solvency. We accumulate the deficits that result from fiscal policy as follows:

$$D_t = D_{t-1} + G_t$$

Where D_t denotes Debt and $D_0 = 0$. G_t denotes the government spending residual. Debt/Output ratio is calculated as: e^{D_t}/e^{y_t}. We can that even though active fiscal policy, whether suppressant or straightforward, pushes the public debt ratio to high levels of around 300% in particular simulated years, average debt ratios are not out of line with those in the data which plainly caused no solvency issues; nor did the particular year maximum in the data of nearly 300% in 2020. Hence the evidence here suggests there is no solvency problem.

Table 6.3.1 Debt -Output Ratio under different policy regimes

		Simulations		
Debt / Output Ratio	**Sample Data**	**Baseline**	**Suppressing Fiscal**	**Strong Fiscal**
Average	141.61*	147.39	151.07	153.07
Max	266 (2020)	292.30	338.41	343.56
Min	51 (1980)	5.21	5.35	4.97

Note: Debt is the government spending residual accumulated for 150 period.

Source: Ministry of Finance, Japan

6.4 Policy regime results

We have explored policy regimes involving stronger fiscal policy, whether totally suppressing the ZLB or simply strongly responding to the output gap. We have looked at combining these both with a standard Taylor Rule and a rule targeting Nominal

GDP; we left the existing QE rule in the ZLB in place for the strong fiscal rule, but it was omitted as irrelevant with a fiscal rule suppressing the ZLB.

The optimal regime is one where together with the current monetary policy regime there is simply a strong fiscal response, with no attempt to suppress the ZLB but with QE following the existing rule. Here the variance of output is brought down very strongly while that of inflation is brought down substantially from the baseline case. Also, the variance of consumption is lowest. The debt ratio does not move out of the region found in the data, where markets have not reacted with solvency fears.

6.5 Policy conclusions

What we find in this thesis is that a robust fiscal policy has been the key missing ingredient in Japanese policy. Effectively the government has resiled from it periodically in a temporary panic over a high debt ratio. However, this debt ratio is essentially disregarded by markets because the Japanese have a huge appetite for buying this debt with their high savings, while being assumed to be willing to raise taxes if in the future it needed to be stabilised. The fiscal resiling above- usually in the form of sudden rises in the consumption tax- has prevented fiscal policy from pushing interest rates up from the ZLB and so stabilising output and inflation with combined monetary and fiscal instruments.

Chapter 7 Conclusion

The motivation behind this research is that the Japanese economy has been in long-term stagnation since 1990. Under a financial crisis, the few available policy options such as QE have performed well in stimulating the economy and coping with deflationary pressures, but they appear weak in the face of the Japanese economy. This study has investigated how Japan's economic behaviour is affected by different monetary and fiscal policies. We build up a medium DSGE model of a small open economy. The model includes the financial transmission mechanism and captures the dynamic response on quantitative easing, which works through the bank lending channel. The ZLB constraint divides the model into two regimes. Under a standard regime (without ZLB binding), the model chooses conventional monetary policy to adjust nominal interest rates, but if the economy stays in the ZLB situation, it turns to QE automatically.

We have used indirect inference to estimate and test this model, and its powerful capability guarantees the robustness of the policy analysis results. Our finding shows that both the standard Taylor Rule and fiscal expansion have a significant effect on stimulating the economy. But variance decomposition analysis found that fiscal policy had a relatively high proportion of impact on the fluctuations of the primary macroeconomic variables in the sample period. To explore the effective policy for Japan, we provide alternative policies to stabilize Japan's economy. On the fiscal side, we first consider a ZLB-suppressing fiscal policy that always uses fiscal expansion to avoid interest rates falling into the ZLB; we also provide a strong active fiscal policy that simply strongly responds to the output gap. We then combine each policy with the standard Taylor rule and the rule for nominal GDP targeting. The simulation results show that the optimal regime is to have a strong fiscal response togethering with the current monetary policy system, instead of trying to suppress the ZLB, but following the existing rules for quantitative easing. Compared with the baseline policy, the variance of output and inflation under the control of the optimal policy combination has dropped significantly. Also, the variance of consumption is lowest. Furthermore, we found that the debt ratio does not exceed the maximum threshold so that Japanese markets have not responded to solvency concerns facing such a high debt ratio. We can conclude that a robust fiscal policy has been the key missing ingredient in Japanese policy.

Appendices

A-1: Model in Log-linearised Form

All model equations are log-linearized around the long-run trends or balanced growth path, the model is consistent with a balanced steady state growth path driven by deterministic labour-augmenting technological progress. The derivation of the linearization process mainly refers to Smert and Wouter (2007). The linearization equations are listed below.

Consumption Euler equation:

$$\widetilde{C_t} = \frac{\frac{\lambda}{\gamma}}{1+\frac{\lambda}{\gamma}}\widetilde{C_{t-1}} + \frac{1}{1+\frac{\lambda}{\gamma}}E_t\widetilde{C_{t+1}} + \frac{(\sigma_c - 1)\frac{w_* N_*}{C_*}}{\left(1+\frac{\lambda}{\gamma}\right)\sigma_c}\left(\widetilde{N_t} - E_t\widetilde{N_{t+1}}\right)$$

$$-\left(\frac{1-\frac{\lambda}{\gamma}}{\left(1+\frac{\lambda}{\gamma}\right)\sigma_c}\right)\left(\widetilde{r_t} - E_t\widetilde{\pi_{t+1}}\right) + \varepsilon_{b,t}$$

Real uncovered interest parity

$$\widetilde{Q_t} = E_t\widetilde{Q_{t+1}} + \widetilde{r_t^f} - \widetilde{r_t}$$

Investment Euler

$$\widetilde{I_t} = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\widetilde{I_{t-1}} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}E_t\widetilde{I_{t+1}} + \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi}\widetilde{q_t} + \varepsilon_{i,t}$$

Tobin Q

$$\widetilde{q_t} = \frac{1-\delta}{1-\delta+MPK^*}E_t\widetilde{q_{t+1}} + \frac{MPK^*}{1-\delta+MPK^*}E_t\widetilde{MPK_{t+1}} - E_t\widetilde{R_{t+1}^k}$$

Capital accumulation equation

$$\widetilde{K_t} = \left(\frac{1-\delta}{\gamma}\right)\widetilde{C_{t-1}} + \left(1-\frac{1-\delta}{\gamma}\right)E_t\widetilde{C_{t+1}} + \left(1-\frac{1-\delta}{\gamma}\right)(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi\varepsilon_{i,t}$$

Aggregate production equation

$$\widetilde{Y_t} = \phi\left[\alpha\frac{1-\Psi}{\Psi}\widetilde{MPK_t} + \alpha K_{t-1} + (1-\alpha)\widetilde{N_t} + \varepsilon_{a,t}\right]$$

Labour demand equation

$$\widetilde{N_t} = -\widetilde{w_t} + \left(1+\frac{1-\Psi}{\Psi}\right)\widetilde{MPK_t} + \widetilde{K_{t-1}}$$

Credit premium

$$E_t\widetilde{R_{t+1}^k} - (\widetilde{r_t} - E_t\widetilde{\pi_{t+1}}) = \chi\left(\widetilde{q_t} + \widetilde{K_{t+1}} - \widetilde{nw_{t+1}}\right) - \psi\widetilde{m_t} + \xi_t + \varepsilon_{pr,t}$$

Net worth

$$\widetilde{nw}_t = \frac{K}{N}\left(\widetilde{R_t^k} - \widetilde{E_{t-1}R_t^k}\right) + E_{t-1}\widetilde{R_t^k} + \theta\widetilde{n_{t-1}} + \varepsilon_{nw,t}$$

Entrepreneurs' consumption

$$\widetilde{C_t^e} = \widetilde{nw}_t$$

Capital services equation

$$K_t^s = K_{t-1} + z_t$$

Capital utilisation equation

$$z_t = \left(1 + \frac{1-\Psi}{\Psi}\right)\widehat{MPK}_t$$

Market clearing condition

$$\widetilde{Y}_t = \frac{C}{Y}\widetilde{C}_t + \frac{C^e}{Y}\widetilde{C_t^e} + \frac{I}{Y}\widetilde{I}_t + MPK^*K_y\frac{1-\Psi}{\Psi}MPK_t + \frac{EX}{Y}\widetilde{EX}_t - \frac{IM}{Y}\widetilde{IM}_t + g_t$$

The real balance of payments:

$$b_t^f = \left(1 + r_t^f\right)b_{t-1}^f + \frac{p_t^d}{Q_t}\frac{ex}{y}ex_t + \frac{p_t^d}{Q_t}\frac{ex}{y}\frac{1}{\omega}Q_t - \frac{im}{y}im_t$$

The import goods demand in Taylor expansion:

$$\widetilde{IM}_t = \widetilde{C}_t - \eta\widetilde{Q_t} + \varepsilon_t^{im}$$

The export goods demand in Taylor expansion:

$$\widetilde{EX}_t = \widetilde{C_t^F} + \eta^F\frac{1}{\omega}\widetilde{Q_t} + \varepsilon_t^{ex}$$

Price setting

$$\pi_t = \omega_p\left(\frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}E_t\pi_{t+1} + \frac{\iota_p}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\pi_{t-1} - \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\left(\frac{\left(1-\beta\gamma^{(1-\sigma_c)}\zeta_p\right)\left(1-\zeta_p\right)}{\zeta_p\left(1+(\phi_p-1)\epsilon_p\right)}\right)\left(\alpha MPK_t + (1-\alpha)\widetilde{w_t}\right) - \varepsilon_{a,t}\right) - \varepsilon_t^p\right) + \left(1-\omega_p\right)\left(\alpha MPK_t + (1-\alpha)\widetilde{w_t} - \varepsilon_{a,t}\right)$$

Wage setting

$$\pi_t = \omega_p \left(\frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}\iota_p} E_t w_{t+1} + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p} w_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} E_t \pi_{t+1} \right.$$

$$- \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}} \pi_t - \frac{\iota_w}{1+\beta\gamma^{(1-\sigma_c)}} \pi_{t-1}$$

$$- \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} \left(\frac{\left(1-\beta\gamma^{(1-\sigma_c)}\zeta_w\right)(1-\zeta_w)}{\zeta_w(1+(\phi_w-1)\epsilon_w)} \right) \left(w_t - \sigma_l N_t \right.$$

$$\left. \left. - \left(\frac{1}{1-\frac{\lambda}{\gamma}} \right) \left(\widetilde{C_t} - \frac{\lambda}{\gamma} \widetilde{C_{t-1}} \right) \right) + \varepsilon_{w,t} \right)$$

$$+ (1-\omega_w) \left(\sigma_l N_t - \left(\frac{1}{1-\frac{\lambda}{\gamma}} \right) \left(\widetilde{C_t} - \frac{\lambda}{\gamma} \widetilde{C_{t-1}} \right) - (\pi_t - E_{t-1}\pi_t) \right.$$

$$\left. + \varepsilon_{w,t} \right)$$

Taylor rule

$$\widetilde{R_t} = \rho \widetilde{R_{t-1}} + (1-\rho)\left\{ r_\pi \widetilde{\pi_t} + r_Y \widetilde{Y_t} \right\} + r_{\Delta Y}\left(\widetilde{Y_t} - \widetilde{Y_{t-1}} \right) + er_t, for\ r_t > 0$$

M0

$$\widetilde{\Delta m_t} = \psi_1 \Delta\widetilde{M}_t + errm_{2t}, for\ r_t > 0$$

$$\Delta\widetilde{m_t} = \psi_2 \left(\widetilde{R_t^k} - R^k \right) + errm_{2t}, for\ r_t \leq 0$$

M2

$$\widetilde{M_t} = (1-v-\mu)\widetilde{K_t} + \mu\widetilde{m_t} - vn\widetilde{w}_t$$

A-2: Stochastic Shock Processes

We set up 14 shocks including two exogenous variables, foreign consumption, and foreign interest rate. The shock process is listed as following:

Government spending shock (market clearing equation)

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \rho_{ga} \varepsilon_t^a + \eta_t^g$$

Preference shock (consumption Euler equation)

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$$

Productivity shock (production function)

$$A_t - A_{t-1} = \rho_a (A_{t-1} - A_{t-2}) + \eta_t^a$$

Investment shock (Investment Euler equation)

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$$

Monetary policy shock (Taylor rule equation)

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$$

Price mark-up shock (Hybrid inflation rate equation)

$$\lambda_{p,t} = \rho_p \lambda_{p,t-1} + \eta_t^p$$

Wage mark-up shock (Hybrid wage equation)

$$\lambda_{w,t} = \rho_w \lambda_{w,t-1} + \eta_t^w$$

External finance premium shock (External finance premium equation)

$$\varepsilon_t^{pr} = \rho_{pr} ln\varepsilon_{t-1}^{pr} + \eta_t^{PR}$$

Net worth shock (Net Worth equation)

$$\varepsilon_t^{nw} = \rho_{nw} ln\varepsilon_{t-1}^{nw} + \eta_t^{nw}$$

Money supply shock (M0 equation with crisis)

$$\varepsilon_t^{m0} = \rho_{m0} \varepsilon_{t-1}^{m0} + \eta_t^{m0}$$

Export demand shock (Export demand equation)

$$\varepsilon_t^{ex} = \rho_{ex} ln\varepsilon_{t-1}^{ex} + \eta_t^{ex}$$

Import demand shock (Import demand equation)

$$\varepsilon_t^m = \rho_m ln\varepsilon_{t-1}^m + \eta_t^m$$

Exogenous foreign consumption process

$$C_t^f = \rho_{cf} C_{t-1}^f + \eta_t^{cf}$$

Exogenous foreign interest rate process

$$R_t^f = \rho_{rf} R_{t-1}^f + \eta_t^{rf}$$

A-3: Data source

NAME	CODE	SOURCE	DESCRIPTION
C	JPCNPER.B	Carbinet Office	personal consumption expenditure
I	JPINVCH.B	Carbinet Office	total fixed capital + the change in inventories
Y	JPGDP...B	Carbinet Office	gross domestic product
L	JPQHO005E	OECD	hours worked*total employment/total population
PI	N/A	N/A	CPI deflator
WAGE	LCEAMN01JPM661S	FRED	Hourly Earnings: Manufacturing for Japan
R	IR3TCD01JPM156N	FRED	3-Month or 90-day Rates and Yields: for Japan
K	N/A	N/A	derived from equation
Q	N/A	N/A	derived from equation
MPK	N/A	N/A	derived from equation
NW	JAPDOWA	NIKKEI	Market capitalization of listed companies
S	lending-deposit	N/A	the difference between lending rate and deposit rate
RK	JAPSTPM	Bank of Japan	prime lending short-term rate
M0	JPM0....A	Bank of Japan	money base m0
M2	JPM2....A	Bank of Japan	money supply m2
EX	JPEXPGDSA	Ministry of Finance	total export
IM	JPIMPGDSA	Ministry of Finance	total import
EXCH	JPXTW..RF	Bank of Japan	real effective exchange rate index, 2015=100
BF	JPOCFC%GQ	Oxford Economics	Net foreign account/ GDP
CF	WEIGHTED	Refinitiv	world export in goods and services
RF	WEIGHTED	Refinitiv	3-Month or 90-day Rates and Yields: Interbank Rates for the United States, Percent, Quarterly, Not Seasonally Adjusted

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