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Key Points:

- The likelihood of joint occurrence of storm tide and intense rainfall is calculated using bivariate copulas
- Multivariate non-exceedance probability of rainfall and storm tide was higher using the Survival Kendall (SK) definition when compared to the "AND" definition
- Copulas analysis combined with multivariate non-exceedance probability showed there were periods of temporal yet cyclical high intensities

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The Devil Is in the Tail Dependence: An Assessment of Multivariate Copula-Based Frameworks and Dependence Concepts for Coastal Compound Flood Dynamics

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Abstract A hurricane event can often produce both intense rainfall and a storm tide that can cause a major compound flooding threat to coastlines. This paper examined applications of multivariate copula-based time series models using data observed during Hurricane Irma (2017) along the coastlines of Florida, Georgia, and South Carolina, United States. Multivariate time series models were developed using bivariate copulas wherein storm tide and rainfall data were modeled using LOWESS-based autoregressive moving average (ARMA). n samples of observed data were then synthesized using a Monte Carlo approach in which the empirical copula and the parametric estimate of the copula were obtained to approximate two-sided p-values using the Rosenblatt probability integral transform method. Analysis suggested that proper selection of the underlying LOWESS-based ARMA model was the crucial aspect for modeling compound flooding wherein Archimedean, Elliptical, and Extreme Value copulas all offered consistent flexibility in terms of dependence modeling. As a backdrop to compound flood probabilities, this research also outlined both theoretical and applied frameworks for the calculation of non-exceedance probabilities in a multidimensional environment using classical isofrequency probability assumptions for the "AND" (a bivariate joint probability) and Survival Kendall definitions. Random realizations from storm copulas combined with multivariate non-exceedance probability definitions ultimately showed there were periods of temporal yet cyclical high intensities that lasted 1–2 hr. Lastly, a discussion is presented on the broader application of the proposed methodology within the field of engineering design and risk management which may serve as a catalyst for the continued research in compound flooding.

Plain Language Summary When a storm tide and intense rainfall simultaneously co-occur in coastal areas, the potential for flooding is often much greater than from either independent event. Understanding how to assess the probability of these compound events is important in planning for and managing flood risks in coastal communities. This study investigated the temporal dynamics of these two phenomena during a landfalling hurricane across the southeast United States using a multivariate copula-based time series model. The analysis revealed that storm tide dynamics were more accurately captured in the proposed model when compared to rainfall observations, although temporal rainfall was reasonably described by the model. This study highlights the temporal multivariate probabilistic approach needed to cope with compound flood risk assessment. All outcomes outlined herein are based on the theory of multivariate copula-based dependence models with three application sites that illustrated the proposed methods.

1. Introduction

Hurricane events generally pose a threat to the coastlines across the globe (e.g., Klotzbach et al., 2018; Peduzzi et al., 2012; Phillips et al., 2018). During such events, coastal drainage systems may be adversely affected by a combination of multiple hazards due to their destructive power of storm surge and intense rainfall. This is commonly referred to as a hydrological compound or multivariate event (Leonard et al., 2014; Moftakhari et al., 2017; Wahl et al., 2015; Zscheischler et al., 2020) and represents a combination of physical weather phenomena that are spatiotemporally correlated (Gori et al., 2020). The simultaneous occurrence or co-occurrence of intense rainfall and storm tides can lead to or exacerbate the potential for compound flooding in low-lying coastal areas (e.g., Klotzbach et al., 2021). Understanding the probability of these compound events and the processes driving them is essential to mitigate the associated high-impact flood risks (Wahl et al., 2015).

Over the last two decades copulas have become a more popular tool to construct the joint distribution of a variety of hydrological extremes, such as intense rainfall (e.g., Salvadori & De Michele, 2006; Serinaldi & Kilsby, 2014), runoff/discharge (e.g., De Michele et al., 2005; Favre et al., 2004; Genest et al., 2007; Ghizzoni et al., 2010; Karmakar & Simonovic, 2009; Shiau et al., 2006; Volpi & Fiori, 2012), droughts (e.g., Aghakouchak, 2014; Kao & Govindaraju, 2010; Salvadori & De Michele, 2015; Serinaldi, 2016; Shiau, 2006), and tidal surge (e.g., Tu et al., 2018; Xu et al., 2014). These studies focused on applications of copulas to characterize the nature of a single hydrologic/hydraulic extreme event variable on an annual basis (i.e., block maxima).

There has been a recent attempt to redraw the attention on the application of copulas for compound flood events and the risk associated with a given return period. To name a few, Svensson and Jones (2004) provided an early application of copulas theory to compound flood assessment when sea surge, river flow, and precipitation co-occurred in the south and west Britain. Kew et al. (2013) applied copulas to the Rhine delta compound flooding events by exploring the joint occurrence of storm surge and extreme discharge. Concurrently, Zheng et al. (2013) quantified the dependence between extreme rainfall and storm surge in the coastal zone in Australia. Wahl et al. (2015) provided the first application of copulas theory to compound flood assessment across United States' coastlines where storm surge and extreme precipitation co-occurred simultaneously. The theoretical basis of copulas for compound flood assessment was strengthened by Salvadori et al. (2016) with an introduction to hazard scenarios, a probabilistic approach of multivariate occurrences, accounting for risk and failure. An excellent application of Salvadori et al. (2016) is given by Moftakhari et al. (2019) with a bivariate copula study of joint return periods for stream flow and ocean water level for Newport Bay, CA. Compound flood heights were generated by considering a bivariate definition of the hazard scenarios where both streamflow and ocean water level are extreme (an "AND" scenario) or where at least one of those variables is extreme (an "OR" scenario). Gori et al. (2020) and Zhang and Wang (2021) are the latest applications of copulas for tropical storm-driven compound flood studies.

These applications yield success when the practitioner is interested in the likelihood of the joint occurrence of two (or more than two) extreme events defined by static point estimates (e.g., 10-year peak stage and 24-hr precipitation or 100-year extreme wave, intense rainfall, and storm surge). However, it might be difficult to justify these approaches in the case of hurricanes when the timing of coincident peaks and temporal interdependence throughout a compound flood event, particularly between rainfall and storm tide, is paramount and can significantly affect the design of infrastructure in low-lying coastal areas. For example, coastal systems are dynamic in nature and require the use of unsteady hydrodynamic models, and therefore require dynamic boundary conditions (e.g., hourly rainfall and storm tide). When using dynamic coastal boundary conditions, there are unlimited choices on the timing of rainfall and storm tide, thus leaving engineers and planners to assume a "conservative" estimate for design. In many cases, designers may artificially set the rainfall hyetograph timing such that peak rainfall intensity occurs at high storm tide or mid storm tide rising. Although simple, and frequently used in infrastructure design, could we consider the dynamic dependence between rainfall and storm tide to develop a multivariate probabilistically based design metrics for completing coastal hydrodynamic assessments? To answer this question and fill the knowledge gap, this research presents three case studies wherein multivariate copula-based models were developed to capture the multivariate temporal dependence of tidal-rainfall relationships observed during a landfalling hurricane in the southeast United States.

The objective of this paper is to present and use a multivariate copula-based dependence framework for a time series assessment that is suitable to deal with (a) temporal dependence between rainfall and storm tide at a single site during a landfalling hurricane and (b) the concept of multivariate non-exceedance probabilities such as "AND" (a bivariate joint probability) and Survival Kendall (SK) for assessing the temporal evolution of probabilities observed during a hurricane event. Herein, we focused on applications of copulas for estimation of continuous joint marginals extended to the case in which each univariate marginal can be a mixture of an absolutely continuous random variable and a discrete random variable, all of which were measured with hourly and/or sub-hourly time scales from a singular hurricane event. It should be noted that due to the paucity of data, n samples of the hurricane data were synthesized using a Monte Carlo (MC) approach by randomly sampling sets of storm tide and rainfall time series.

This paper is organized as follows. The study region and data used in this research are described in Section 2. Methodology and mathematical structures of multivariate copula-based time series, parameter estimates, statistical performance, and techniques for evaluating multivariate non-exceedance probabilities are outlined in

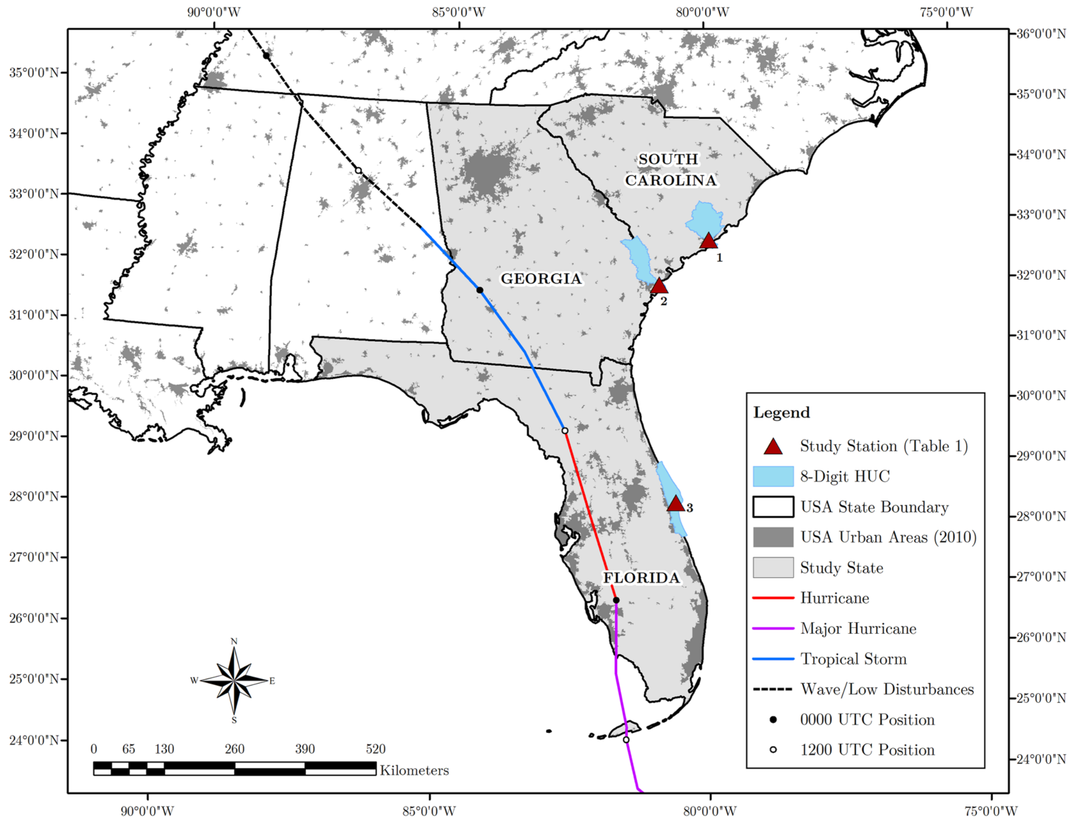

Figure 1. Hurricane Irma study area with specific study sites and their associated hydrologic unit code (HUC).

Section 3. Results are discussed in Section 4 using three applications with different study sites. A detailed discussion and conclusion of the study, its potential use in flood risk management of coastal drainage systems, and critical guidelines to address limitations of the proposed methodology for use in evaluating coastal compound flooding are discussed in Section 5.

2. Study Area and Data

This study focused on exploring the temporal joint occurrence of rainfall and storm tide observed during Hurricane Irma (September 2017). The focus was on the Southeast Atlantic coastlines of Florida (FL), Georgia (GA), and South Carolina (SC) where Hurricane Irma caused severe damage and widespread flooding (see Figure 1). Irma's sustained winds (i.e., ~75–110 mph) brought rising storm surge and intense rainfall alongside the ever-rising high tide stages in Cape Canaveral, FL, Savannah GA, and Charleston, SC. As a result, severe damage was experienced along these coastlines. For example, many areas including Miami were reported to have more than 1 m of inland flooding (BBC News, 2017). Also reports show that the Charleston Harbor experienced its third highest stage of approximately 2.07 m NAVD 88 (~6.79 feet NAVD 88), causing the historic city to experience severe flooding (Yan et al., 2017). Following these reports and based on data availability, this study focused on Cape Canaveral, FL where 15-min rainfall data from Pinebrook Canal (the United States Geological Survey; USGS) and 15-min storm tide data from Port Manatee (the National Oceanic and Atmospheric Administration; NOAA) were considered. Savannah was selected as a representative study location in Georgia with hourly rainfall data and hourly storm tide data from Fork Pulaski (NOAA) and the Savannah tide gauge (NOAA), respectively. Representative hourly rainfall and storm tide data for coastal South Carolina was collected from gauging stations at the Charleston International Airport (NOAA) and the Cooper River (NOAA), respectively.

Representative hourly/sub-hourly rainfall and storm tide data recorded during the hurricane at coastal gauging stations managed by USGS and NOAA were carefully examined. Storm tide data used for the analysis presented herein represents the combined water levels due to the astronomical tide, storm surge, and limited wave setup. A

Table 1
Summary of Hydrological Study Locations, Their Names, Their Associated Variables, and IDs

Study station	Location	Station names	Rainfall ID	Tide ID
1	Cape Canaveral, FL	Pinebrook Canal and Port Manatee	02308870	8726384
2	Savannah, GA	Savannah and Fork Pulaski	72207003822	8670870
3	Charleston, SC	Charleston International Airport and Cooper River Entrance	72208013880	8665530

summary of the stations used in this study and their representative identification (ID) numbers are presented in Table 1. Each data set was evaluated and analyzed using geostatistical techniques, such as inverse distance weighting (IDW) and Thiessen polygon, to ensure spatial continuity and representativeness of compound flooding data within the study area.

3. Methodology

The temporal dependence structure of hourly/sub-hourly rainfall and the storm tide observed during Hurricane Irma was analyzed with applications of two-dimensional multivariate copula-based dependence models. It was assumed that flooding in the study region was a compound coastal flood event caused by heavy rainfall and storm tides wherein only coincident time series observed during the event were considered. Multivariate copula-based dependence modeling requires data representative of an independent and identically distributed (*iid*) sample. Because observed hurricane data is not representative of a stationary process, the development of a multivariate copula-based dependence model involved a two-step process. First, the temporal dependence of each variable was investigated separately to fit an appropriate time series model of the observations. Once an appropriate time series model was selected for each variable, residuals were computed. The second step was to perform traditional copula modeling of the residuals, assuming residuals represented an *iid* sample.

Two-dimensional copulas described in subsequent sections were implemented to link univariate marginal probability distributions of rainfall and storm tide residuals to the joint distribution of rainfall and storm tide residuals. This allowed the data dependence structure to be modeled in a stochastic context and an investigation of tail dependence and the evolution of exceedance probabilities observed during Hurricane Irma. The suitability of constructed copulas and joint distributions were evaluated using the multi-dimensional Cramér-von Mises test, root mean square error (RMSE), and the Nash-Sutcliffe model efficient coefficient (NSE) between the observed event and model results. The Akaike information criterion (AIC) was also used to evaluate performance between copulas. The method of maximum pseudo-likelihood was implemented to estimate copula dependence parameters independent of univariate marginals such that inadequate selection of marginal distributions would not affect the copula selection process. We refer readers to Genest et al. (1995) for more information on this methodology. For comparative purposes, we also applied the *tau* inversion method, as described by Nelsen (2006), to estimate copula performance. However, this method was not implemented in the final multivariate models presented herein. A summary of the hurricane copula-based time series development process is summarized and presented in Figure 2.

3.1. Time Series Modeling

This study implemented stationary multivariate copula-based time series modeling of data observed during Hurricane Irma. Such copula-based models have been widely used in economics and finance (e.g., Bonhomme & Robin, 2009; Chen & Fan, 2006; Heinen & Rengifo, 2007; Patton, 2009). Recently, these same models have found use in the field of hydrology (e.g., Arya & Zhang, 2017; Sugimoto et al., 2016; Zhang & Singh, 2019). Herein, we adopted the assumption that hourly/sub-hourly observations of rainfall and storm tide follow an autoregressive moving average (ARMA) process. ARMA models have been successfully applied to understand and predict hydrologic time series (e.g., Burlando et al., 1993; McMichael & Hunter, 1972) since first popularized by Box and Jenkins (1970) and have found an integral part in multivariate copula-based modeling (Zhang & Singh, 2019).

In general form, the ARMA (p, q) model of a time series is given by

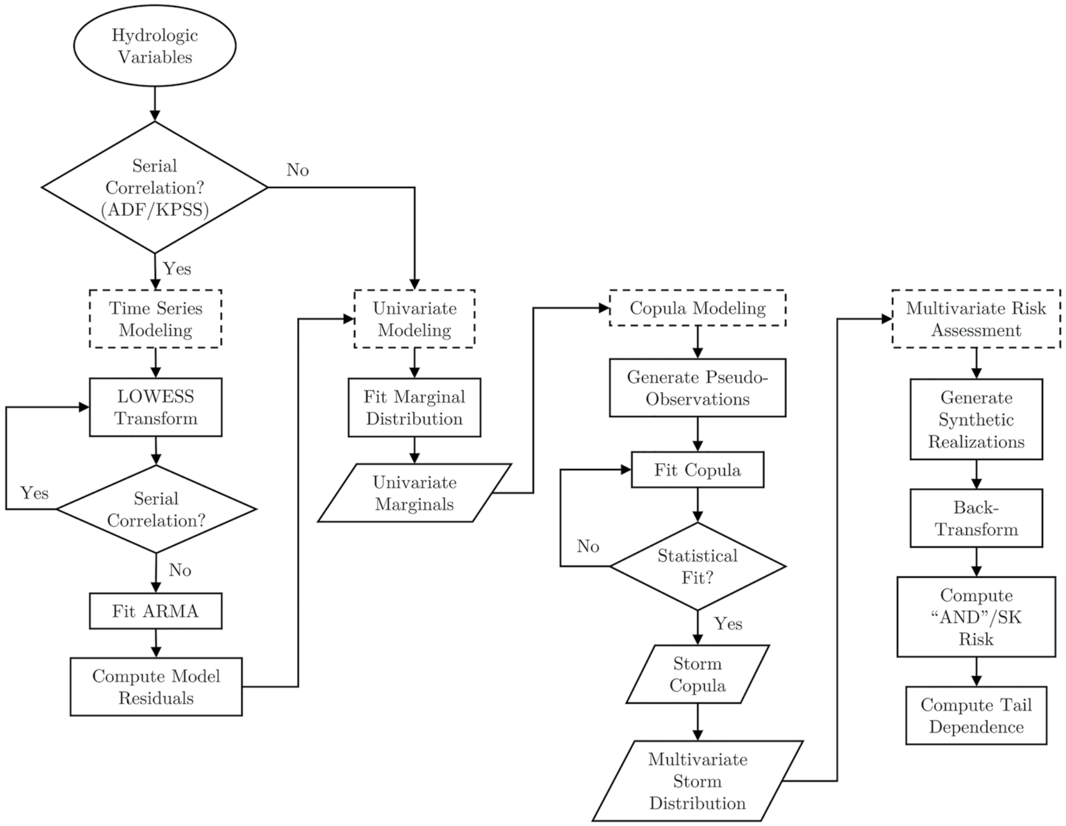

Figure 2. Storm modeling and multivariate probabilistic assessment process.

$$X_t = c + \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t \tag{1}$$

where X_t is the stochastic variable at time t, p and q represent the number of autoregressive and moving average terms, respectively, ϕ_i and θ_i are the autoregressive and moving average coefficients, respectively, X_{t-i} and ε_{t-i} are the lagged response and residual at time $t - i$, and ε_t is the model error at time t.

A fundamental assumption of the ARMA process outlined in Equation 1 is that the stochastic variable is representative of a Gaussian stationary process with error terms representing white noise. However, observed rainfall and storm tide time series were not representative of a Gaussian stationary process and required transformation prior to ARMA fitting. Transformation to a stationary process was achieved by detrending, that is, fitting LOWESS (Cleveland, 1979; Cleveland & Devlin, 1988) curves to each observed time series and using their inherited residuals. Using a smoothing method to isolate and remove the trend in a time series, so that the remaining residual time series exhibits characteristics of stationarity, is a common practice in time series analysis (Cryer & Chan, 2008). Shumway and Stoffer (2019) suggest the LOWESS approach, originally proposed by Cleveland (1979). In the context of ice velocity time series, Derkacheva et al. (2020) compared three methods of smoothing (i.e., moving averages, cubic splines, and LOWESS), and found that LOWESS yielded the best solution for their data. Lawrance (2013) used a LOWESS method to smooth a volatility function related to financial time series. Gumbricht (2016) used LOWESS to smooth a soil surface wetness curve, noting that the LOWESS smoothing "removes the effects of erratic rainfall events and noise" and is "more robust for identification of wetness phenology." Crawford et al. (2017) smoothed the CO_2 time series using LOWESS in R, which accounted for both the stochastic shocks caused by storms and regular anomalies in the data.

The presence and removal of serial correlation in LOWESS residuals were confirmed using the autocorrelation function, partial autocorrelation function, augmented Dickey-Fuller (ADF) test, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The detrending procedure, by which a smooth curve was fit

Table 2
Selected Copulas, Their Functions, and Parameter Space

Copula	Copula function $C(u_1, u_2)$	Parameter (θ) range
Gumbel	$\exp\left[-\left\{(-\log u_1)^{\theta} + (-\log u_2)^{\theta}\right\}^{1/\theta}\right]$	$[1, \infty)$
Clayton	$\left(u_1{}^{-\theta} + u_2{}^{-\theta} - 1\right)^{-1/\theta}$	$[-1, \infty)/\{0\}$
Frank	$-\frac{1}{\theta}\log\left[1 + \frac{(e^{-\theta u_1}-1)\cdot(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right]$	$(-\infty, \infty)/\{0\}$
Joe	$1 - \left[(1-u_1)^{\theta} + (1-u_2)^{\theta} - (1-u_1)^{\theta}\cdot(1-u_2)^{\theta}\right]^{1/\theta}$	$[1, \infty)$
Normal	$\phi_{\theta}{}^{n}\left(\phi^{-1}(u_1), \phi^{-1}(u_2)\right)$	$(-1, 1)$
Student's t	$T_{\theta,\nu}\left(t_{\nu}{}^{-1}(u_1), t_{\nu}{}^{-1}(u_2)\right)$	$(-1, 1)$
Galambos	$u_1 u_2 \exp\left\{\left[(1-u_1)^{-\theta} + (1-u_2)^{-\theta}\right]^{-1/\theta}\right\}$	$(0, \infty)$
Hüsler-Reiss	$\exp\left[\phi\left\{\frac{\theta}{2} + \frac{1}{\theta}\log\left(\frac{\log u_2}{\log u_1}\right)\right\}\log u_1 + \phi\left\{\frac{\theta}{2} + \frac{1}{\theta}\log\left(\frac{\log u_1}{\log u_2}\right)\right\}\log u_2\right],$	$(0, \infty)$
Tawn	$u_1 u_2\left(-\theta\frac{\log(u_1)\log(u_2)}{\log(u_1 u_2)}\right)$	$(0, 1)$
t-EV	$\exp\left[T_{\nu+1}\left\{-\frac{\theta}{b} + \frac{1}{b}\left(\frac{\log u_2}{\log u_1}\right)^{1/\nu}\right\}\log u_1 + T_{\nu+1}\left\{-\frac{\theta}{b} + \frac{1}{b}\left(\frac{\log u_1}{\log u_2}\right)^{1/\nu}\right\}\log u_2\right];\ b^2 = \frac{1-\theta^2}{\nu+1}$	$(-1, 1)$

Note. Notes on additional variables. $\phi =$ univariate normal cumulative distribution function; $\phi^{-1} =$ inverse of the univariate normal cumulative distribution function; $\phi_{\rho}{}^{n} =$ joint cumulative distribution function for the n-dimensional normal distribution; $T_{\nu} =$ cumulative distribution function of a Student random variable with ν degrees of freedom; $T_{\theta,\nu} =$ n-dimensional Student's t distribution with ν degrees of freedom and correlation matrix θ.

to the series and the time series analysis proceeded on the residuals of the fit, produced a residual series which was judged stationary by common tests of stationarity. However, the proposed method cannot transform the non-Gaussian rainfall field into a Gaussian process, which is a limitation of this framework and may lead to the introduction of epistemic uncertainty (i.e., bias). Finally, ARMA models were fit to these residuals to develop *iid* observations of the observed event for direct use in copulas presented in Section 3.1.

3.2. Copula Modeling

A copula (also commonly referred to as a copula function) is a multivariate joint probability distribution function that captures the entire scale-free dependence structure between two or more dependent hydrological variables (Hao & AghaKouchak, 2014; Salvadori & De Michele, 2007; Serinaldi & Grimaldi, 2007). Sklar's (1959) theorem provides the theoretical framework for the application of copulas and the dependence between two or more random variables, $X_1, \ldots, X_n$. According to the theorem, the joint (i.e., multivariate) cumulative distribution function (CDF) is given by

$$H(x_1, \ldots, x_n) = \mathbb{P}(X_1 \le x_1, \ldots, X_n \le x_n). \quad (2)$$

The copula function, C, can then be defined such that joint CDFs of the random variables can be expressed as

$$H(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)). \quad (3)$$

Equation 3 provides the basis for evaluating the dependence structure between the univariate distribution functions of random variables through the copula function. Any multivariate CDF can be written in copula form, and if the marginal distributions are continuous, the copula is unique (Sklar, 1959).

While a variety of copulas have been applied in hydrology, the Archimedean, Elliptical, and Extreme Value are well-known tools to model the dependence structure among multiple hydrological extremes. These copulas have been widely used in hydrology (e.g., Favre et al., 2004; Grimaldi & Serinaldi, 2006a; Grimaldi & Serinaldi, 2006b; Salvadori & De Michele, 2010; Bracken et al., 2018; among others). Table 2 presents a summary of copula functions and parameter space for the selected Archimedean, Elliptical, and Extreme Value copulas.

3.2.1. Archimedean Copulas

Archimedean copulas have been largely used in hydrology because of their flexibility and simplicity (Grimaldi & Serinaldi, 2006b; Saad et al., 2015). As provided by Nelsen (2006), an n-dimensional Archimedean copula takes the form

$$C(u_1,\ldots,u_n)=\varphi^{-1}\left[\sum_{i=1}^{n}\varphi(u_i)\right] \tag{4}$$

where φ is a continuous generator (i.e., function) such that $\varphi(1)=0$, φ is strictly decreasing on [0,1], φ is convex on [0,1], and $(u_1,\ldots,u_n)\in[0,1]^n$.

Among the well-known Archimedean copulas, the Clayton (Clayton, 1978), Frank (Frank, 1979), Gumbel, and Joe (1997) one-parameter copulas were considered in this research since they have been frequently used in hydrological applications (e.g., Favre et al., 2004; Grimaldi & Serinaldi, 2006a; Volpi & Fiori, 2012). Table 2 presents the Archimedean copulas considered herein, and their functions and parameters. The Gumbel and Joe Archimedean copulas are limited to positive dependence only, while the remaining can model both positive and negative dependence structures. Readers are referred to Nelsen (2006) for more details on Archimedean copulas. It is important to note that the Gumbel copula is considered both an Archimedean and Extreme Value copula.

3.2.2. Elliptical Copulas

The choice of an Elliptical copula arises when the dependence structure among correlated variables is best suited with Elliptical distributions. n-dimensional Elliptical copulas can be generalized as

$$C(u_1,\ldots,u_n)=\phi\left(\phi^{-1}(u_1),\ldots,\phi^{-1}(u_n)\right) \tag{5}$$

where ϕ is a suitable multivariate distribution and ϕ^{-1} is the inverse of the univariate marginal distribution. Elliptical copulas are symmetric and capable of positive and negative dependence modeling. For many years, Elliptical copulas have been extensively used in the fields of economics and finance (e.g., Ortobelli et al., 2002; Owen & Rabinovitch, 1983; Pradier, 2011) and have become increasingly popular in modeling the dependence structure among hydrologic variables (e.g., Bracken et al., 2018; Moazami & Golian, 2017; Song & Singh, 2010). Herein, two popular families of Elliptical copulas were considered: Normal (i.e., Gaussian) and Student's t (see Table 2).

3.2.3. Extreme Value Copulas

The family of Extreme Value copulas naturally represents the joint behavior of random variables derived in classical Extreme Value theory, which makes their use in hydrology very appealing and quite convenient (Gudendorf & Segers, 2010). For the n-dimensional case, the Extreme Value copula is represented as

$$C(u_1,\ldots,u_n)=\exp\left\{\left(\sum_{j=1}^{n}\log u_j\right)A\left(\frac{\log u_1}{\sum_{j=1}^{n}\log u_j},\ldots,\frac{\log u_n}{\sum_{j=1}^{n}\log u_j}\right)\right\} \tag{6}$$

where A is the Pickands dependence function, which is a convex function that satisfies $\max(t,1-t)\le A(t)\le 1$ for all $t\in[0,1]$ (Beirlant et al., 2006). Henceforth, successful application of an Extreme Value copula lies with proper selection of the appropriate Pickands dependence function.

Here, we implemented parametric forms of the Pickands dependence functions to formulate the Galambos, Hüsler-Reiss, Tawn, and t-EV Extreme Value copulas (see Table 2). Each of these copulas is limited to positive dependence, while the t-EV is capable of modeling both positive and negative dependence. As previously mentioned, it is important to note that the Gumbel copula represents both Archimedean and Extreme Value copulas (i.e., Gumbel-Hougard) and that was also implemented.

The Galambos copula is typically limited to medium-to-large dimensional problems (Mathieu & Mohammed, 2012), while the Hüsler-Reiss and extremal-t (i.e., t-EV) Extreme Value copulas do not suffer from this drawback. The Tawn copula was also implemented in this study, which is an asymmetric extension of the

Gumbel copula. Theoretical background and derivations of Extreme Value copulas can be found in the works of Demarta and McNeil (2005), Beirlant et al. (2006), and Mathieu and Mohammed (2012).

3.2.4. Univariate Marginal Distributions

Construction of univariate distributions to describe residual time series were required for copula modeling. Several parametric distributions were fit to rainfall and storm tide residual time series, including the Generalized Extreme Value (GEV), Gumbel, and normal. Final distributions were selected based on graphical inspection of probability plots (e.g., quantile plots). For all three applications, rainfall residuals were represented by the GEV distribution while storm tide residuals were represented by the normal distribution.

3.2.5. Parameter Estimation

The method of maximum pseudo-likelihood was used to estimate copula parameters. This approach was selected given the nature of the semiparametric method wherein copula parameters are selected based on empirical marginal probabilities rather than fitted parametric marginals. Using this method, the empirical probability is given in terms of the Weibull plotting formula by (Zhang & Singh, 2019).

$$\hat{F}_i(x) = \frac{1}{n+1}\sum_{j=1}^{n} 1(X_{ij} \leq x), \quad i = 1, \ldots, d. \tag{7}$$

Using marginal probabilities obtained from Equation 7, the pseudo-log-likelihood function can be maximized to provide optimal copula parameter(s) expressed as (Zhang & Singh, 2019)

$$\log L(\boldsymbol{\theta}) = \sum_{j=1}^{n} \ln\left[C\left(\hat{F}_1(x_{1j}), \ldots, \hat{F}_d(x_{dj}); \boldsymbol{\theta}\right)\right]. \tag{8}$$

Copula parameters estimated using Equation 8 are independent of the parametric marginal distribution. Hence, misidentification of marginal distributions does not entirely limit the accuracy of the selected copula.

3.2.6. Statistical Performance

Throughout this study, we primarily applied standard statistics, that is, the Cramér-von Mises test statistic, to characterize the significance of the fitted copula model. The major strength of this test lies in the fact that the asymptotic distribution of its test statistic is completely independent of the null-hypothesis CDF. The Cramér-von Mises test is defined by Genest et al. (2009) as

$$S_n = \int_{[0,1]^d} C_n(\boldsymbol{u})^2 \, dC_n(\boldsymbol{u}). \tag{9}$$

In particular, the Cramér-von Mises test statistic was derived by considering a 1,000-sample MC approach in which the empirical copula and the parametric estimate of the copula obtained via the method of maximum pseudo-likelihood were used to approximate two-sided p-values using the Rosenblatt probability integral transform method (Rosenblatt, 1952). Readers are referred to Genest and Rémillard (2008), Genest et al. (2009), and Genest et al. (2011) for more detailed information.

The Akaike information criterion (AIC) (Akaike, 1974) was a secondary copula selection criterion used to relatively evaluate the balance between copula accuracy (i.e., pseudo-likelihood) and simplicity (i.e., number of model parameters, m) given by

$$\mathrm{AIC} = -2\log L(\boldsymbol{\theta}) + 2m. \tag{10}$$

3.3. Multivariate Probability and Tail Dependence Assessment

3.3.1. Multivariate Non-Exceedance Probabilities

Multivariate non-exceedance probabilities using copula-based dependence models were estimated using classical isofrequency probability assumptions for the "AND" (bivariate joint probability) and Survival Kendall (SK) cases based on a d-dimensional Kendall distribution (see Salvadori et al., 2013, 2016). These metrics are commonly

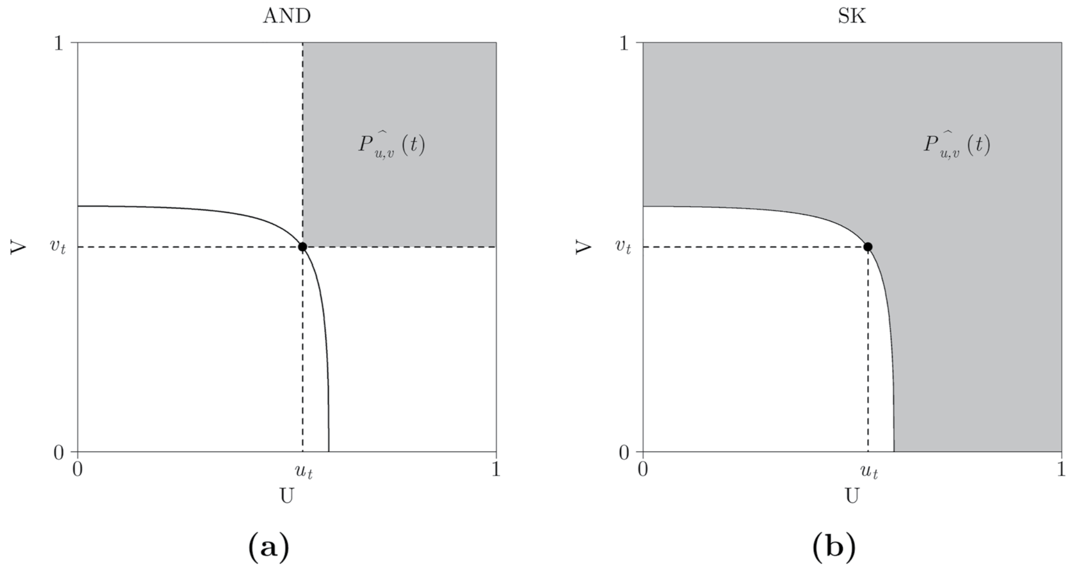

Figure 3. Multivariate non-exceedance probability definitions using the "AND" (a) case and SK (b) case.

applied in a return period context using annual maximum/minimum or peaks-over-threshold data series and have found use in drought and water resources research around the globe (e.g., AghaKouchak et al., 2014; De Michele et al., 2013; Salvadori et al., 2011). Rather than focus on historical data to develop return period relationships and compare to Irma's observed data, we focus solely on the multivariate temporal non-exceedance probabilities using storm copulas outlined in the previous sections. As a result, the non-exceedance probabilities presented herein represent event-specific frequency curves and are not necessarily representative of long-term region-specific multivariate frequency curves.

The non-exceedance probability for the "AND" case is classified as the probability of observing each event (e.g., storm tide and intense rainfall) exceeding given magnitudes. For the bivariate applications presented herein, this probability is given in copula terms as

$$P(X \geq x \cap Y \geq y) = 1 - u - v + C(u, v). \tag{11}$$

Kendall's measure, K_c, has become a popular tool for evaluating multivariate probabilities using copulas wherein K_c represents the Kendall distribution of copula C (Genest & Rivest, 1993) given in bivariate terms as

$$K_c(t) = P(C(u, v) \leq t) = t - \int_t^1 \frac{\partial}{\partial u} C\left(u, v_{u,t}\right) \tag{12}$$

where $0 < t \leq 1, v_{u,t} = C_u^{-1}$, and $C_u(v) = C(u, v)$. From the definition of K_c in Equation 12, risk managers can define "safe" and "dangerous" regions, known as multivariate hazard scenarios, as $K_c\left(t^*\right) > t^*$ and $1 - K_c\left(t^*\right) < 1 - t^*$ where t^* is selected as the critical probability level $C(u, v) = t^*$ (Salvadori et al., 2011). However, as pointed out by Salvadori et al. (2013), the "safe" region may be unbounded, whereas the SK's measure is bounded, thereby providing a better multivariate risk measure. Accordingly, SK's measure, $\overline{K}_c$, for a bivariate copula can be written as

$$\overline{K}_c(t) = P(C(u, v) \leq t) = 1 - K_c(t), \tag{13}$$

which represents the exact probability that the given event, t, happens given any random realization of the subject coastal multivariate event under question (Salvadori et al., 2007). Figure 3 visually illustrates multivariate non-exceedance probability definitions using the "AND" and SK cases.

3.3.2. Tail Dependence

The tail dependence coefficient is a statistic that summarizes how individual extreme events tend to occur simultaneously (Mathieu & Mohammed, 2012). In the case of flood risk management, statisticians often parameterize the strength of the dependence of the variables using an upper tail dependence coefficient to investigate the

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conditional effects between the variables of interest (Schmid & Schmidt, 2007) wherein the upper tail dependence coefficient is given by

$$\lambda_U = \lim_{u\to 1^-} P\left(Y > F_Y^{-1}(u) | X > F_X^{-1}(u)\right) = \lim_{u\to 1^-} \frac{1-2u+C(u,u)}{1-u} \in [0,1]. \tag{14}$$

Several parametric measures of dependence have been examined and developed in the past (e.g., Capéraà et al., 1997; Coles et al., 1999; Frahm et al., 2005; Joe et al., 1992). However, each of these parametric techniques are developed from the underlying basis of the fitted copula (e.g., diagonals along the copula), thereby relying on the choice of the copula. These shortfalls were overcome by studying the strength of upper tail dependence among the copula models using a nonparametric approach by considering the empirical copula function. We focused on two commonly applied upper tail dependence coefficients: ρ_U^{SS} (Schmid & Schmidt, 2007) and λ_U^{CFG} (Capéraà et al., 1997; Frahm et al., 2005).

Conditional versions of correlation coefficients are commonly applied nonparametric approaches to investigating the strength of dependence in the tails of multivariate distributions (Croux & Dehon, 2010; Durante et al., 2014). Schmid and Schmidt (2007) introduced such an approach based on conditional versions of Spearman's *rho*, λ_U^{SS}. Given the following d-dimensional conditional version of Spearman's *rho*

$$\rho_p(C) = \frac{\int_{[0,p]^d} C(\boldsymbol{u})d\boldsymbol{u} - \left(p^2/2\right)^d}{p^{d+1}/(d+1) - \left(p^2/2\right)^d} \quad \text{with} \quad 0 < p \le 1, \tag{15}$$

the coefficient of multivariate upper tail dependence, λ_U^{SS}, can be defined by

$$\lambda_U^{SS}(C) = \lim_{p\to 1^-} \rho_p(C) = \lim_{p\to 0^+} \frac{d+1}{p^{d+1}} \int_{[0,p]^d} C\left(\overline{\boldsymbol{u}}\right) d\overline{\boldsymbol{u}} \quad \text{with} \quad 0 \le \rho_U^{SS} \le 1, \tag{16}$$

where $\overline{\boldsymbol{u}}$ represents the d-dimensional vector of residual marginal probabilities $(1-\boldsymbol{u})$. For the analysis presented herein, we limited λ_U to pairwise dependence (i.e., $d = 2$).

Under the assumption that the empirical copula function is approximated as a limiting form of an Extreme Value copula and based on the work of Capéraà et al. (1997), Frahm et al. (2005) defined a closed form upper tail dependence coefficient (λ_U^{CFG}) which has gained popularity in tail dependence modeling using the copula framework. In this approach, λ_U^{CFG} is defined as

$$\lambda_U^{CFG} = 2 - 2\exp\left[\frac{1}{n}\sum_{i=1}^{n} \log\left\{\sqrt{\log\left(\frac{1}{U_i}\right)\log\left(\frac{1}{V_i}\right)}/\log\left(\frac{1}{\max(U_i,V_i)^2}\right)\right\}\right]. \tag{17}$$

4. Applications

4.1. Florida Analysis

Dependence modeling for Florida data was done in two steps. First, correlation pairs among the observed random variables for each location were evaluated for temporal correlation (i.e., serial dependence). New and uncorrelated random variables were then generated by fitting a LOWESS trend model to storm tide and rainfall time series and then using ARMA (1,0) and ARMA (0,1) models to model the residuals for the LOWESS fits for storm tide and rainfall time series, respectively. The second step of dependence modeling was to fit copula models to the ARMA residuals.

Pearson (ρ_p), Spearman (ρ_s), and Kendall (τ) correlation coefficients, the most popular nonparametric measures of dependence, were used to measure the initial dependence structure and suitability of copula modeling. Figures 4a and 4b present pairwise plots of observed (serial correlated) and residual (non-serially correlated) time series, respectively, for Cape Canaveral storm variables available during the event. Figures 4c and 4d present the observed and simulated time series using the fitted ARMA models.

Observed rainfall and storm tide co-occurrences at Cape Canaveral resulted in slightly negative correlation coefficients, all between approximately −0.1 and −0.2 (Figure 4a). This is primarily because rainfall intensities

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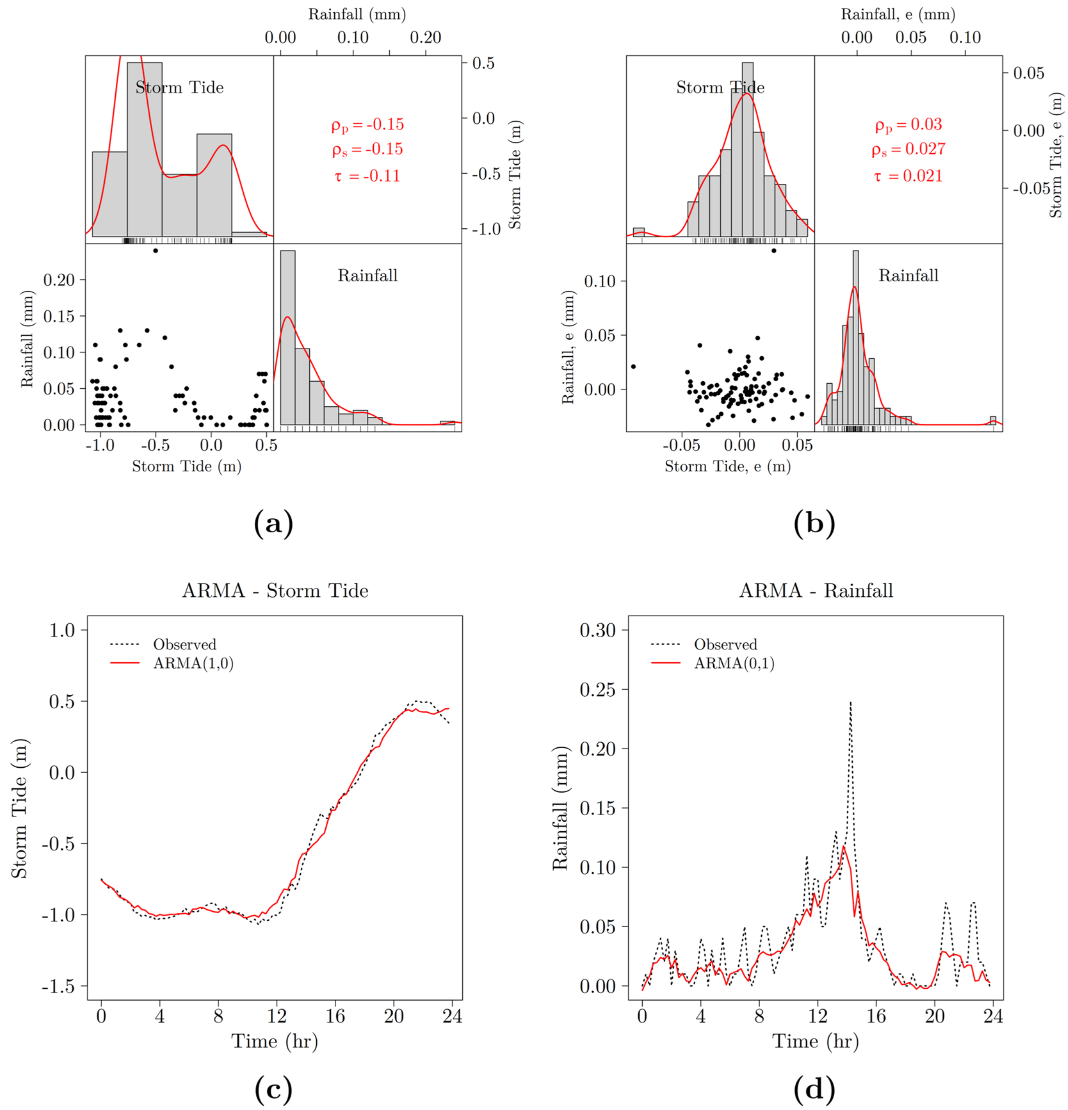

Figure 4. Event correlation/frequency structure for Cape Canaveral, FL for (a) observed data and (b) time series model residuals; and autoregressive moving average (ARMA) model fitting for (c) storm tide and (d) rainfall time series. Lower left panels of (a, b) represent pairwise correlation plots between storm variables. Diagonal panels of (a, b) represent the frequency of each associated variable with the best fit indicated by red lines. Upper right panels of (a, b) contain the computed correlation coefficient between bivariate variables for: (1) Pearson (ρ_p); (2) Spearman (ρ_s); and (3) Kendall (τ).

started to decrease as storm tide levels increased. Correlation coefficients were significantly less when comparing ARMA residual time series (Figure 4b) and more likely to represent the independent stochastic nature of observed storm tide and rainfall time series. Although independence was observed in ARMA residuals, copula modeling was still implemented for testing since a correlation coefficient of 0 is not always representative of independence. In the case that a low correlation coefficient is observed in storm tide and rainfall time series and there is a lack of dependence, the resulting copula would be classified as an independent copula and would still encapsulate the underlying probabilistic nature of each time series. Rather than using a non-dependent univariate model of both time series for independence and a copula model for dependent time series, this approach could potentially allow the development of a single universal multivariate copula-based model for coastal boundary conditions under both dependence and independence scenarios.

Table 3 presents fitting performance and estimated parameters for the copulas applied to Cape Canaveral. Two-sided p-values indicated there was not sufficient evidence to reject the null hypothesis of dependence for the tested copulas at the 5% significance level (i.e., p-values > 0.05), thereby suggesting tested copulas could

Table 3
Copula Fitting Parameters and Statistical Fitting Results for Cape Canaveral, FL

	Maximum pseudo-likelihood		Tau inversion						
Copula	Parameter value	S_n *p*-value	Parameter value	S_n *p*-value	AIC	RMSE	NSE	λ_U^{SS}	λ_U^{CFG}
Gumbel	1.000	0.330	1.001	0.337	2.001	0.013	0.997	0.011	0.008
Clayton	−0.046	0.261	−0.047	0.246	1.869	0.015	0.996	0.010	−0.015
Frank	0.168	0.619	0.167	0.598	1.929	0.012	0.998	0.016	0.028
Joe	1.000	0.406	1.001	0.386	2.000	0.013	0.997	0.011	0.009
Normal	−0.022	0.190	−0.022	0.181	1.963	0.014	0.997	0.009	−0.005
Student's *t*	−0.001	0.336	0.000	0.374	1.724	0.013	0.997	0.034	0.023
Galambos	0.013	0.354	0.019	0.326	2.000	0.013	0.997	0.012	0.009
Hüsler-Reiss	0.121	0.335	0.121	0.339	2.000	0.013	0.997	0.012	0.008
Tawn	0.002	0.389	0.001	0.355	2.004	0.013	0.997	0.014	0.009
t-EV	−0.998	0.341	−0.999	0.323	2.000	0.013	0.997	0.012	0.009

Note. Statistical significance represents approximate two-sided *p*-values from 1,000 MC simulations.

encapsulate the interdependent relationships of rainfall and storm tide time series observed at Cape Canaveral. Parameter values and two-sided *p*-values were nearly identical when comparing results between the maximum pseudo-likelihood method and *tau* inversion method. Suitability of copulas was apparent in this application although tail dependence was nearly inexistent. Although such flooding could be modeled with copulas, tails of rainfall and storm tide residuals did not appear to stochastically coincide when considering the hurricane event window.

All copulas performed relatively similar when comparing two-sided *p*-values. As indicated by computed correlation coefficients, all but the Frank and Hüsler-Reiss copulas resulted in parameters near independence. Tabular fitting results did not conclusively help to identify a "best" fit copula to describe the rainfall and storm tide time series from Irma since all copulas were generally within the same level of performance. Of the tested copulas, the Frank, Joe, and Normal were selected to further evaluate dependence modeling and applicability of the methodology presented herein. Herein, the Normal copula was always selected as a baseline for testing and analysis.

Diagonal curves and contour levels plots for the Frank, Joe, and Normal copulas for Cape Canaveral are presented in Figure 5. Each model generally showed good performance in predicting the measured probability structure between rainfall and storm tide. In all three cases, the model showed some instability along the diagonals (i.e., $u_1 = u_2$) and contour levels, but otherwise, very consistent results were produced. Overall, all three copulas produced nearly identical results for both diagonals curves and contour levels without any practical difference. However, based on the results presented in Table 3, the Frank copula outperformed the Normal copula by approximately 225% (i.e., two-sided *p*-values). Hence, inspection of tabular fitting for copula dependence results does not appear to solely justify the adequacy of Archimedean copulas over Elliptical copulas.

Observed time series and reconstructed randomizations of the event are presented in Figure 6. Rainfall simulations reasonably followed the same pattern as the observed time series in terms of the expected range of values given the natural randomness associated with hourly rainfall. All copulas generated nearly indistinguishable results, with only microscale differences among the three copulas. In all three cases, storm tide simulations bound observed time series well throughout the entire simulation period. Rainfall simulations bound observed time series generally well through approximately hour 10. However, rainfall simulations are not able to accurately predict the observed peak intensity around hour 14 or intensities toward the end of the event (i.e., > hour 20).

Figure 6 presents the univariate viewpoint of the predictive capability to mimic observed rainfall and storm tide time series from the three multivariate copula models, along with relative rRMSE and relative bias (rB) estimates in decimal form. Overall, rRMSE and rB statistics were reasonable for each storm tide model while much more error and moderate bias were apparent in the rainfall models. A combined, multivariate viewpoint of such simulations is presented in Figure 7 and arguably demonstrates the overarching fit of each copula. Each copula resulted in similar scattering, although the Frank and Joe copulas both resulted in similar two-sided *p*-values, while the

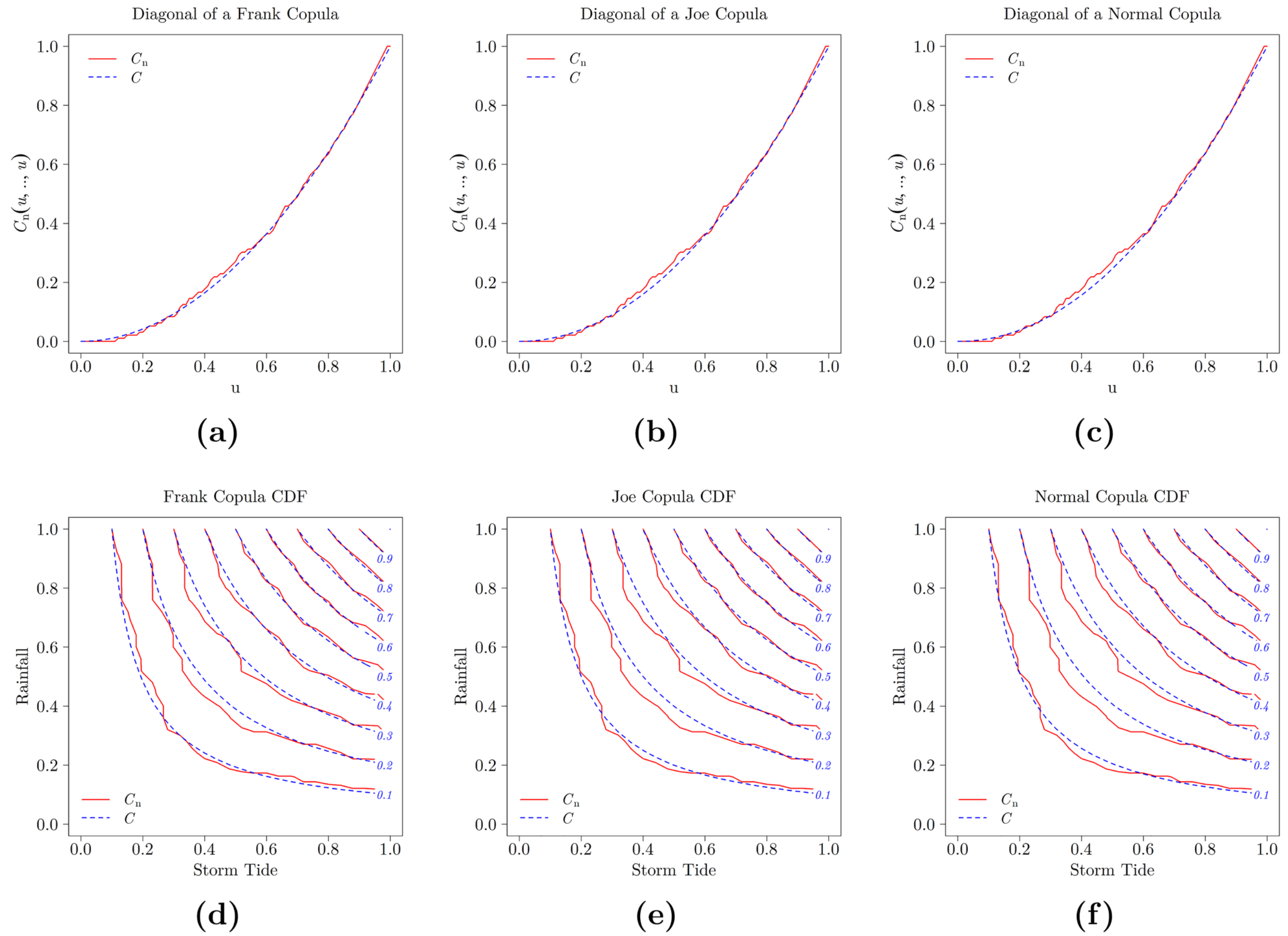

Figure 5. Diagonal curves and empirical and theoretical cumulative distribution function (CDF) contours of the Frank (a, d), Joe (b, e), and Normal (c, f) copulas for the Cape Canaveral, FL hurricane model. Empirical curves are shown as solid red while theoretical curves are represented as dashed blue.

Normal copula was significantly lower (Table 3). Hence, all three copulas seem to be comparable regardless of significant fit test results.

Computed multivariate non-exceedance probability levels are presented in Figure 7 using "AND" and SK definitions. Each copula produced practically identical results. The most noticeable result was that SK non-exceedance probabilities were always higher for any given storm tide and rainfall state throughout the hurricane event. This result substantiates the need to consider an SK measure over the traditional "AND" measure. However, as indicated by Salvadori et al. (2016), the SK definition does not necessarily have a direct interpretation when compared to the "AND" definition in terms of designing structures and should only be used for planning or preliminary assessments. Hence, further research is required to properly compare multivariate statistics of compound events as estimated between "AND" and SK definitions to truly understand the value and differences between the two measures.

A noteworthy result presented in Figure 7 is that the most intense storm tide and rainfall co-occurrence was significantly underpredicted by all three multivariate copula-based dependence models. The methods adopted herein generally performed well during lower intensity periods observed in Florida, but more testing is required to better encapsulate peak intensities. As a result, these methods may not necessarily provide a direct method or accurate results for all regions and data.

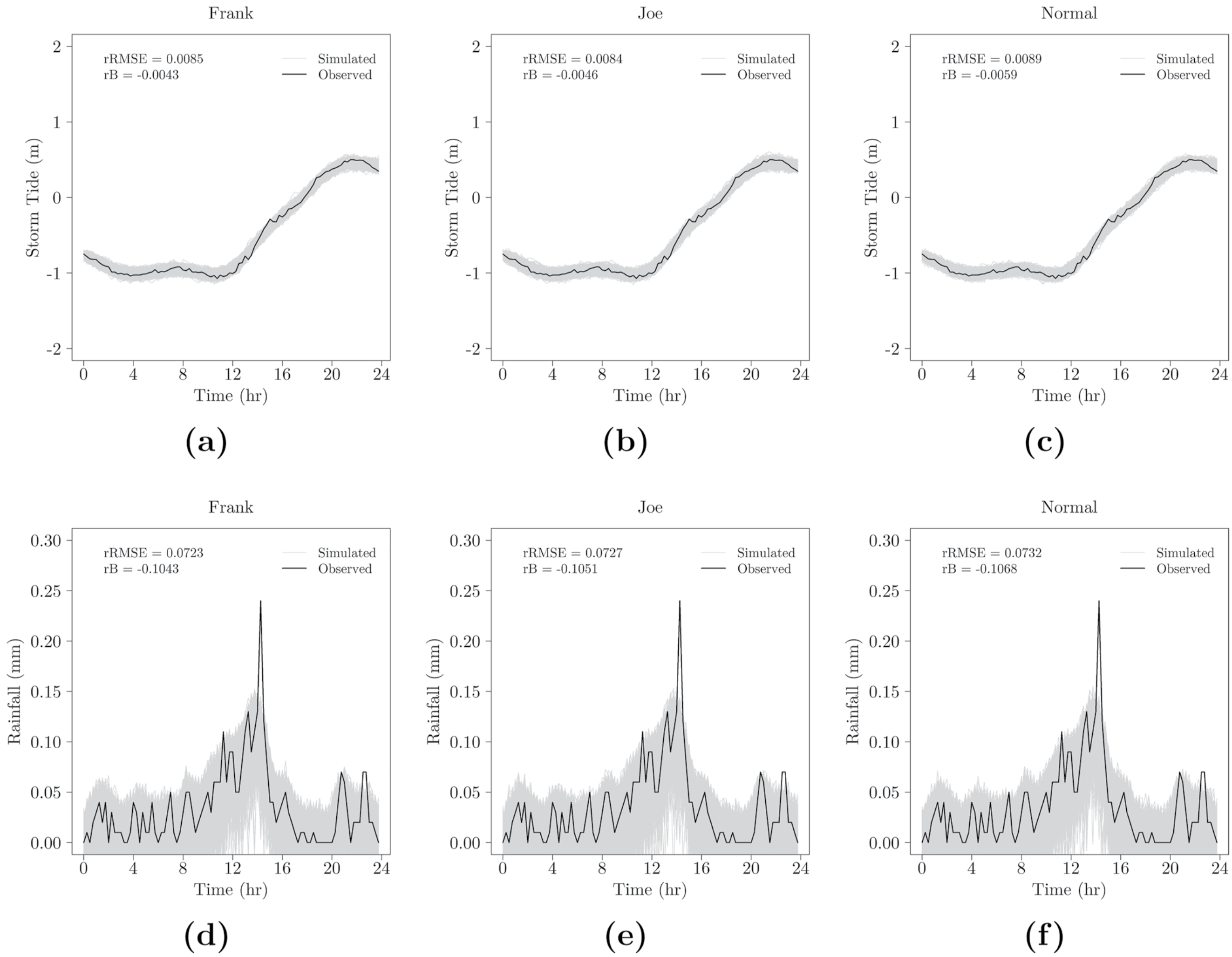

Figure 6. Time series observations and autoregressive moving average (ARMA) model simulations of Cape Canaveral, FL multivariate Irma distribution for Frank (storm tide/rainfall) (a, d), Joe (storm tide/rainfall) (b, e), and Normal (storm tide/rainfall) (c, f) copulas. Descriptive statistics regarding model fits are presented in the form of relative root mean square error (rRMSE) and relative bias (rB).

4.2. Georgia Analysis

Our assessment of the distribution tails of Hurricane Irma in the spatial context of Georgia was focused on data near Savannah. Recorded storm tide and coincident rainfall for Savannah are presented in Figure 8a, along with Pearson, Spearman, and Kendall correlation coefficients, all around 0.20. Hourly storm tide and hourly rainfall time series were positively correlated and suggested an upward-moving dependence with increasing intensity (i.e., peak surge). Fitting a LOWESS trend model and then ARMA (0,2) model to the LOWESS residuals for both the observed storm tide and rainfall produced very similar correlation coefficients when comparing Spearman and Kendall values. However, the computed Pearson correlation coefficient dropped to nearly zero. Unlike the Florida application, the deconstructed time series at Savannah appeared to retain the same basic underlying correlation structure very well, with minor deviations.

None of the copulas implemented herein were rejected at a significance level $\alpha = 0.05$ and 1,000 Monte Carlo simulations (see Table 4). This is a similar result seen in the Cape Canaveral application except a significantly higher acceptance rate (i.e., all above 50% level) which can be attributed to the better correlation measures observed for Savannah (i.e., Kendall's tau of 0.13 as opposed to 0.02). Extreme Value copulas generally produced the most significant fit, followed by Archimedean (outside the Clayton copula) and Elliptical copulas, respectively. Extreme value copulas satisfied significant smoothness assumptions on the copula and convexity assumptions

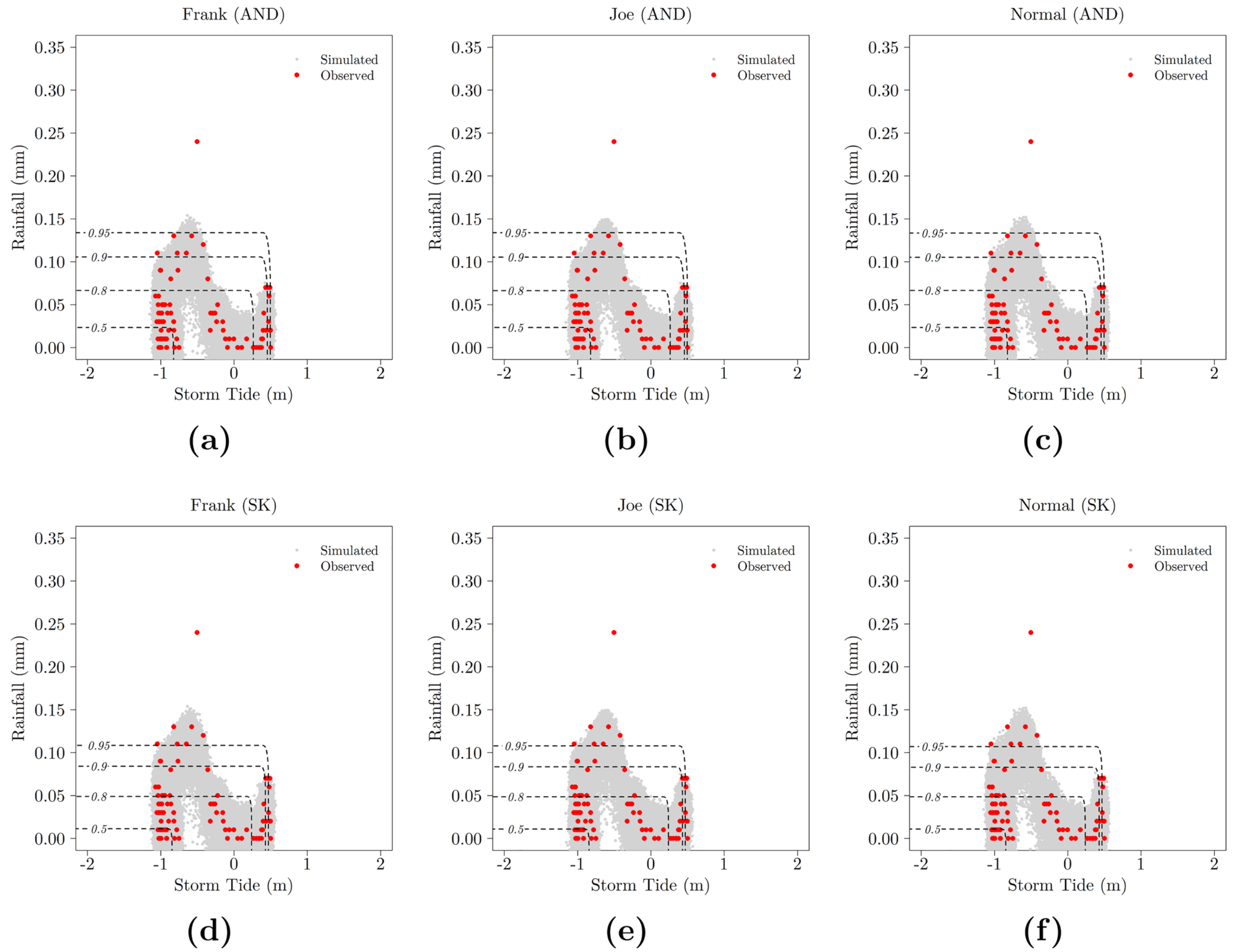

Figure 7. Observations versus simulations and probability level curves of Cape Canaveral, FL multivariate distribution under the hypothesis of Frank ("AND"/SK) (a, b), Joe ("AND"/SK) (b, e), and Normal ("AND"/SK) (c, f) copulas.

on an analog of Pickands dependence function. Although hard to verify analytically, the Extreme value copulas provided better approximations of the true parametric Pickands dependence function for sample sizes n. Results of the *tau* inversion method corresponded well with the maximum pseudo-likelihood method and did not indicate any copula outliers in terms of performance.

Tail dependence was present in the copulas fitted for Savannah when considering both the λ_U^{CFG} estimator and λ_U^{SS} coefficients. In terms of p-values, Savannah copulas were more arguably fitting than Cape Canaveral and ultimately resulted in much more significant dependence measures in ARMA residuals (i.e., Kendall's *tau*). In this case, increased tail dependence may be an indicator of the suitability to describe coincident rainfall and storm tide time series observed in Savannah during Irma.

Gumbel and Hüsler-Reiss copulas were among the top performers and were selected for further analysis, while the Normal copula was evaluated as a baseline. The performance of these copulas was further evaluated based on results from Figure 9 which presents a comparison between the empirical copula function (dashed red) and the theoretical copula function (solid blue). Figures 9a–9c present results in a one-dimensional view frame by considering the diagonal of the copula quantile functions (i.e., $u = v$ in two-dimensional space), while Figures 9d–9f present joint level curves for the copula functions.

Differences among the three selected copula functions were not as apparent as one might expect based on the range of computed p-values and choice of copulas, a similar result seen in the Florida analysis. All empirical

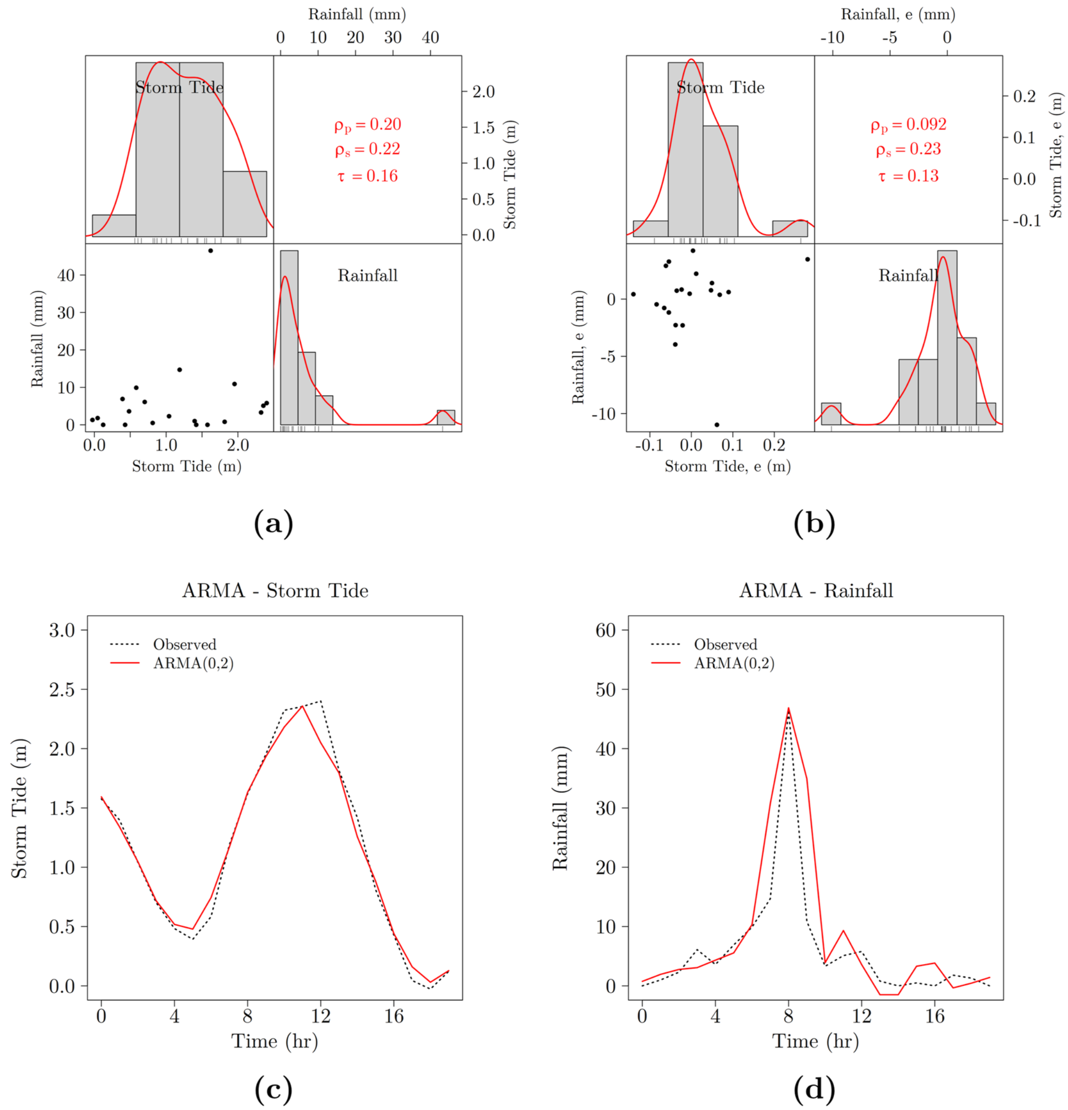

Figure 8. Event correlation/frequency structure for Savannah, GA for observed data (a) and time series model residuals (b) and autoregressive moving average (ARMA) model fitting for storm tide (c) and rainfall (d) time series. Lower left panels of (a, b) represent pairwise correlation plots between storm variables. Diagonal panels of (a, b) represent the frequency of each associated variable with the best fit indicated by red lines. Upper right panels of (a, b) contain the computed correlation coefficient between bivariate variables for: (1) Pearson (ρ_p); (2) Spearman (ρ_s); and (3) Kendall (τ).

curves produced reasonably comparable results when evaluating theoretical curves, with each copula generally capturing the full range of dependence and symmetry in its dependence structure. Differences in level curves for Savannah and Cape Canaveral are noteworthy. Correlation coefficients and two-sided p-values were significantly lower for Cape Canaveral. However, empirical copula contours for Cape Canaveral better represented theoretical curves at all levels, producing an overall "more appealing" visual fit. This result is merely a response due to the larger sample size of the Cape Canaveral application. Given that our interest was focused on multivariate copula-based dependence modeling of a singular coincident event, large sample sizes may not always be physically available, particularly for hurricane events. For example, a typical coastal thunderstorm may only last hours while a hurricane event could span up to 24 hr. Therefore, the adequacy of such copula models should be reserved until a thorough inspection has been made from all perspectives.

Copula models were combined with ARMA forecasting techniques to synthesize storm data by randomly sampling sets of storm tide and rainfall time series and then reconstruct to the same scale/dimension of observed time series (Figure 10). Much like the Cape Canaveral application, the Savannah application resulted in simulated

Table 4
Copula Fitting Parameters and Statistical Fitting Results for Savannah, GA

	Maximum pseudo-likelihood		*Tau* inversion						
Copula	Parameter	S_n *p*-value	Parameter	S_n *p*-value	AIC	RMSE	NSE	λ_U^{SS}	λ_U^{CFG}
Gumbel	1.237	0.814	1.238	0.810	0.642	0.033	0.966	0.306	0.254
Clayton	0.259	0.333	0.262	0.347	1.711	0.038	0.949	0.025	0.125
Frank	1.509	0.765	1.508	0.777	0.959	0.034	0.962	0.058	0.180
Joe	1.393	0.795	1.394	0.789	0.443	0.033	0.967	0.402	0.266
Normal	0.289	0.658	0.290	0.674	1.010	0.034	0.962	0.122	0.210
Student's *t*	0.290	0.665	0.290	0.665	1.010	0.034	0.962	0.121	0.209
Galambos	0.511	0.824	0.510	0.822	0.568	0.033	0.966	0.316	0.262
Hüsler-Reiss	0.898	0.844	0.899	0.854	0.519	0.033	0.966	0.328	0.270
Tawn	0.442	0.677	0.441	0.677	0.811	0.033	0.964	0.259	0.225
t-EV	0.711	0.790	0.712	0.804	0.586	0.038	0.973	0.318	0.262

Note. Statistical significance represents approximate two-sided *p*-values from 1,000 MC simulations.

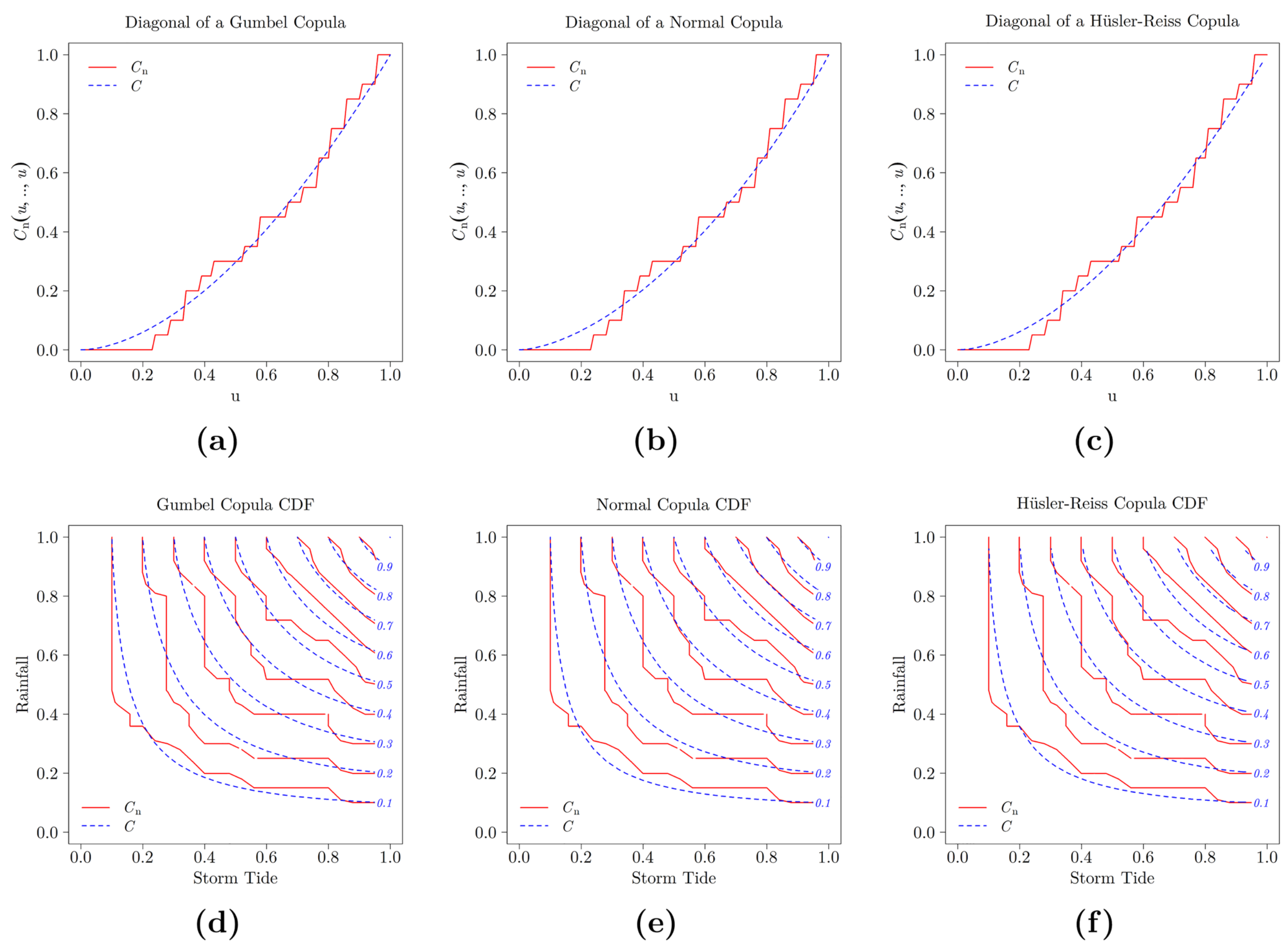

Figure 9. Diagonal curves and contour plots for the empirical and theoretical cumulative distribution functions (CDFs) for Gumbel (a, d), Normal (b, e), and Hüsler-Reiss (c, f) copulas of the Savannah, GA. Empirical curves are shown as solid red while theoretical curves are represented as dashed blue.

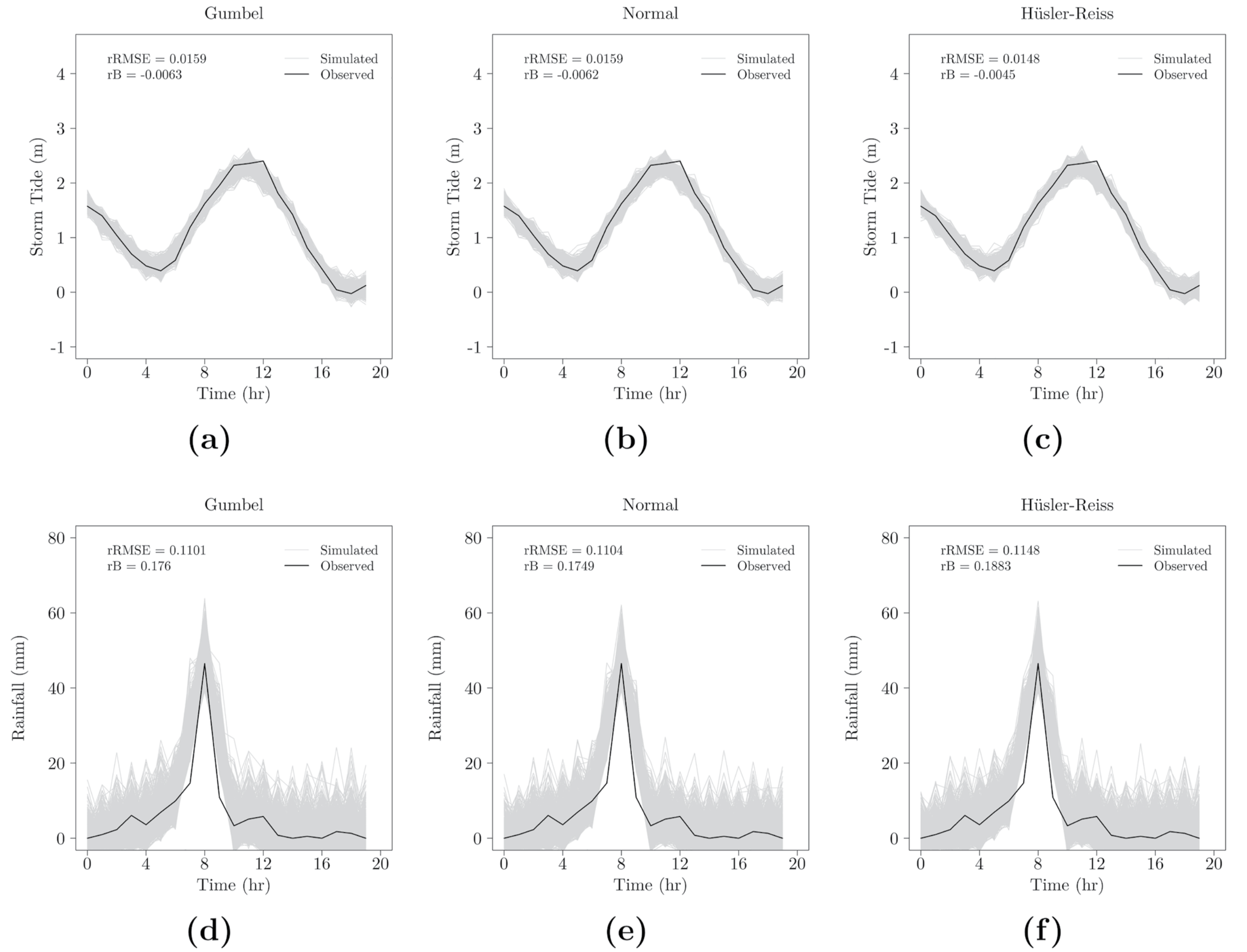

Figure 10. Time series observations and autoregressive moving average (ARMA) model simulations of Savannah, GA multivariate distribution for Gumbel (storm tide/rainfall) (a, d), Normal (storm tide/rainfall) (b, e), and Hüsler-Reiss (storm tide/rainfall) (c, f) copulas. Descriptive statistics regarding model fits are presented in the form of relative root mean square error (rRMSE) and relative bias (rB).

storm tide time series which well-bounded observed time series with relatively low error (rRMSE) and bias (rB). However, in this case, simulated peak rainfall intensity was consistently higher than observed, whereas Cape Canaveral was always lower than observed. Although the Savannah rainfall models produced a more consistent peak intensity, moderately high error (rRMSE) and bias (rB) were apparent. Variability in simulated to observed storm tide time series was much lower when compared to rainfall time series. Again, this result is similar to Cape Canaveral and is representative of the difficulty in modeling the natural stochastic variability in observed rainfall with ARMA time series models.

Figures 11a–11f show the comparison of simulated two-dimensional multivariate copula model results and the observed/measured values using both "AND" and SK definitions. Parallel to previous figures, two-dimensional non-exceedance probability plots highlight very similar results among the copula models. Clustering of the simulated data was within the observed values and tended to be attributed to less than the 80% probability level. However, in all three copulas, whether "AND" or SK, there was a short time window of high intensity well beyond the average storm duration.

Differences in predicted non-exceedance probabilities for "AND" and SK definitions were more apparent for Savannah. For example, observed peak storm intensity registered approximately at the 90% non-exceedance threshold for the "AND" definition. On the other hand, the SK definition registered at nearly the 95% threshold.

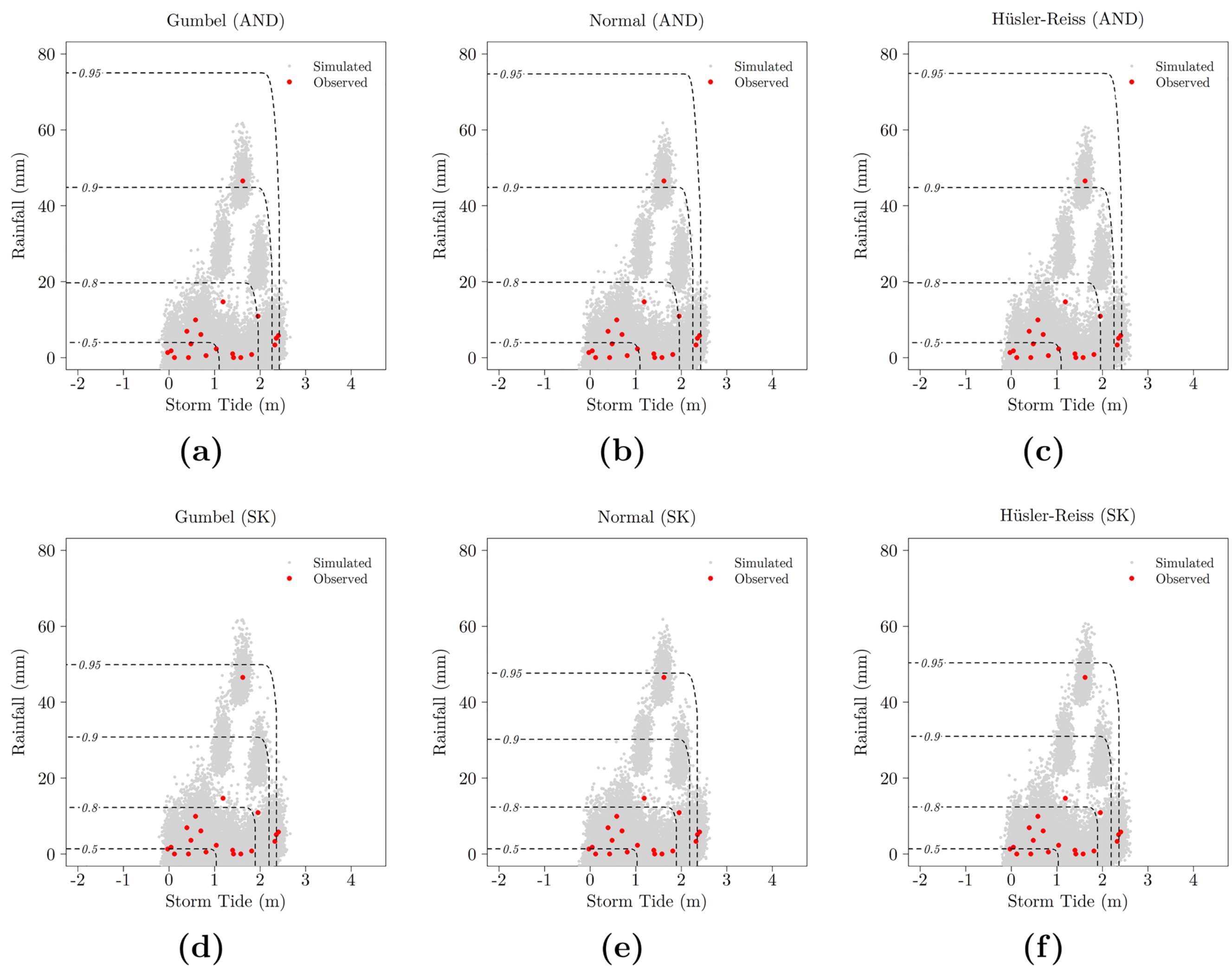

Figure 11. Observations versus simulations and probability level curves of Savannah, GA multivariate distribution under the hypothesis of Gumbel ("AND"/SK) (a, b), Normal ("AND"/SK) (b, e), and Hüsler-Reiss ("AND"/SK) (c, f) copulas.

This result corroborates the need for understanding implications of one multivariate non-exceedance probability measure over another. For example, in this case, one may consider the SK definition to provide a sense of resilient design when compared to "AND." However, one measure over the other and a single multivariate hurricane event analysis cannot provide an all-encompassing viewpoint for coastal infrastructure design. Instead, a much broader analysis is needed to understand the true design and risk setting.

4.3. South Carolina Analysis

Our analysis for South Carolina focuses on the Lowcountry where the extent of Irma's radius impacted much of the Charleston region. The occurrence of hourly rainfall and storm tide levels during the peak of the storm is presented in Figure 12. The bulk of the high storm tide occurred during peak rainfall intensities (Figure 12a), thus resulting in a positive correlation structure when evaluating observed time series. In this case, Pearson, Spearman, and Kendall coefficients were greater than zero and very close to 0.50, which clearly indicates some positive dependence was present for the storm tide and rainfall interaction. LOWESS trend models were fit to storm tide and rainfall series, and ARMA (2,0) and ARMA (3,1) models were fit on the LOWESS residuals, to generate *iid* samples for both the storm tide and rainfall, respectively. Overall, the ARMA models described the observed time series quite well after the trend in the mean was accounted for. Examination of ARMA residuals showed the correlation pairs were still positively dependent although slightly less than the observed time series (Figure 12b).

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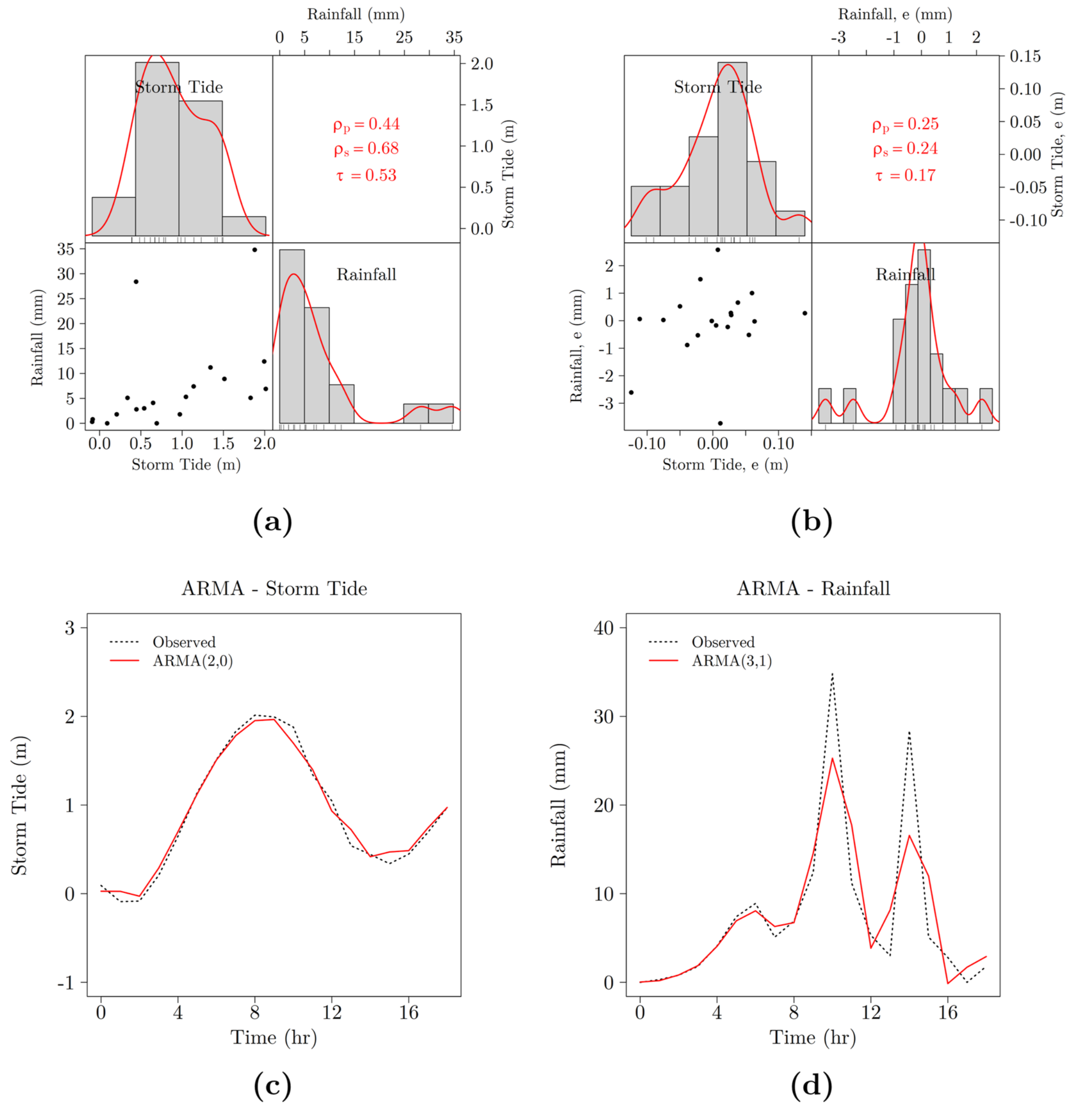

Figure 12. Event correlation/frequency structure for Charleston, SC for observed data (a) and time series model residuals (b) and autoregressive moving average (ARMA) model fitting for storm tide (c) and rainfall (d) time series. Lower left panels of (a, b) represent pairwise correlation plots between storm variables. Diagonal panels of (a, b) represent the frequency of each associated variable with the best fit indicated by red lines. Upper right panels of (a, b) contain the computed correlation coefficient between bivariate variables for: (1) Pearson (ρ_p); (2) Spearman (ρ_s); and (3) Kendall (τ).

Results of the bivariate copula modeling for South Carolina are presented in Table 5, with the goodness of fit statistics. The Cramér-von Mises two-sided p-values clearly show the adequacy of copula fitting to the event observed in Charleston. Elliptical copulas demonstrated the highest statistically sufficient modeling capacity with p-values around 0.80 followed by Archimedean and Extreme Value copulas. Both Archimedean and Extreme Value copulas had inconsistent goodness-of-fit statistics with two-sided p-values ranging from 0.20 to approximately 0.70. However, like the previous applications, each of the tested copulas was able to encapsulate the dependence structure with statistically reliable results (i.e., two-sided p-values>0.05), with generally only the Joe and Tawn copulas producing significance levels below 0.50. In this case, the Pickands dependence function is sufficiently smooth, however the approximation rate could be improved using larger sample size.

Positive tail dependence was present and both coefficients, λ_U^{SS} and λ_U^{CFG},were in agreement for most of the tested copulas. The strength of residual tail dependence was similar to the Georgia application. In each of these applications, increasingly higher performance was followed by increased measures of tail dependence. Although tail dependence in residuals does not necessarily reveal tendencies in raw observations, such dependence does

Table 5
Copula Fitting Parameters and Statistical Fitting Results for Charleston, SC

	Maximum pseudo-likelihood		Tau inversion						
Copula	Parameter	S_n *p*-value	Parameter	S_n *p*-value	AIC	RMSE	NSE	λ_U^{SS}	λ_U^{CFG}
Gumbel	1.188	0.477	1.188	0.436	1.336	0.025	0.988	0.256	0.214
Clayton	0.547	0.634	0.547	0.632	0.290	0.026	0.986	0.040	0.225
Frank	1.514	0.687	1.515	0.657	0.963	0.023	0.989	0.060	0.182
Joe	1.200	0.199	1.199	0.188	1.729	0.026	0.987	0.244	0.161
Normal	0.337	0.796	0.337	0.783	0.693	0.026	0.987	0.150	0.243
Student's *t*	0.337	0.777	0.337	0.789	0.693	0.026	0.987	0.149	0.244
Galambos	0.462	0.544	0.462	0.543	1.181	0.025	0.988	0.274	0.229
Hüsler-Reiss	0.853	0.631	0.851	0.602	1.089	0.026	0.987	0.299	0.247
Tawn	0.309	0.229	0.309	0.235	1.589	0.025	0.988	0.179	0.160
t-EV	0.515	0.436	0.513	0.450	1.276	0.035	0.974	0.269	0.225

Note. Statistical significance represents approximate two-sided *p*-values from 1,000 MC simulations.

help with improved time series reconstruction using realizations drawn from fitted copulas, a result presented later in this section. For this application, the Normal, Student's *t*, and Frank represent the top three rated copulas, respectively, and were selected for further analysis.

Figure 13 presents copula diagonals and two-dimensional copula level curves for empirical and theoretical models using the method of maximum pseudo-likelihood. Diagonal curve results were very similar, with better agreement between empirical and model curves. Level curves presented in Figures 13d–13f confirmed the diagonal copula results. Similar to the previous applications, diagonals were generally the more statistically unstable domain among the copula functions. Although, instabilities along the diagonals (i.e., $u = v$) were more apparent in the contour plots, propagation of the copula functions toward the upper tails improved the theoretical contour results, showing better agreement with measured data (i.e., empirical curves). Differences among non-exceedance probability curves did not appear to be significant, with each model performing adequately in terms of predicting the upper tail dependency of the hurricane event. Nonetheless, the Frank copula produced a 13% less statistically significant fit when comparing computed two-sided *p*-values.

Realizations of reconstructed rainfall and storm tide time series from each copula model and univariate marginal distributions (i.e., residual time series distributions) are presented in Figure 14. Both storm tide and rainfall simulations generally bounded observed time series, with storm tide simulations producing a much narrower band. Rainfall simulations presented more variability but still provided a reasonable prediction of observations with significantly lower error (rRMSE) and bias (rB) estimates. However, rainfall peak intensities at hours 10 and 12 were consistently underpredicted. Overall, all three copulas generally spanned the range of observed rainfall intensities. The highly variable and stochastic nature of hourly rainfall resulted in a less defined range of simulated rainfall curves, although this result should be expected in most stochastic rainfall simulations.

A multivariate representation of the results is presented in Figure 15 along with estimated non-exceedance probability curves. As expected, each copula model produced relatively little-to-no differences in the upper tail examining results for both "AND" and SK definitions. In this case, most of the event was below the 80% threshold, with only two observations beyond the 95% level. Results showed positive dependence with a clear trend in increasing intensity while simultaneously increasing both rainfall and storm tide. As with previous results and applications, the "AND" definition produced higher non-exceedance probability levels given the same rainfall and storm tide.

5. Discussion and Conclusions

This paper examined a probabilistically consistent framework suitable for modeling coastal multivariate time series. We take the traditional multivariate copula-based dependence approach often used in applied statistics wherein explanatory variables are drawn from the same population class (i.e., similar units of measurements)

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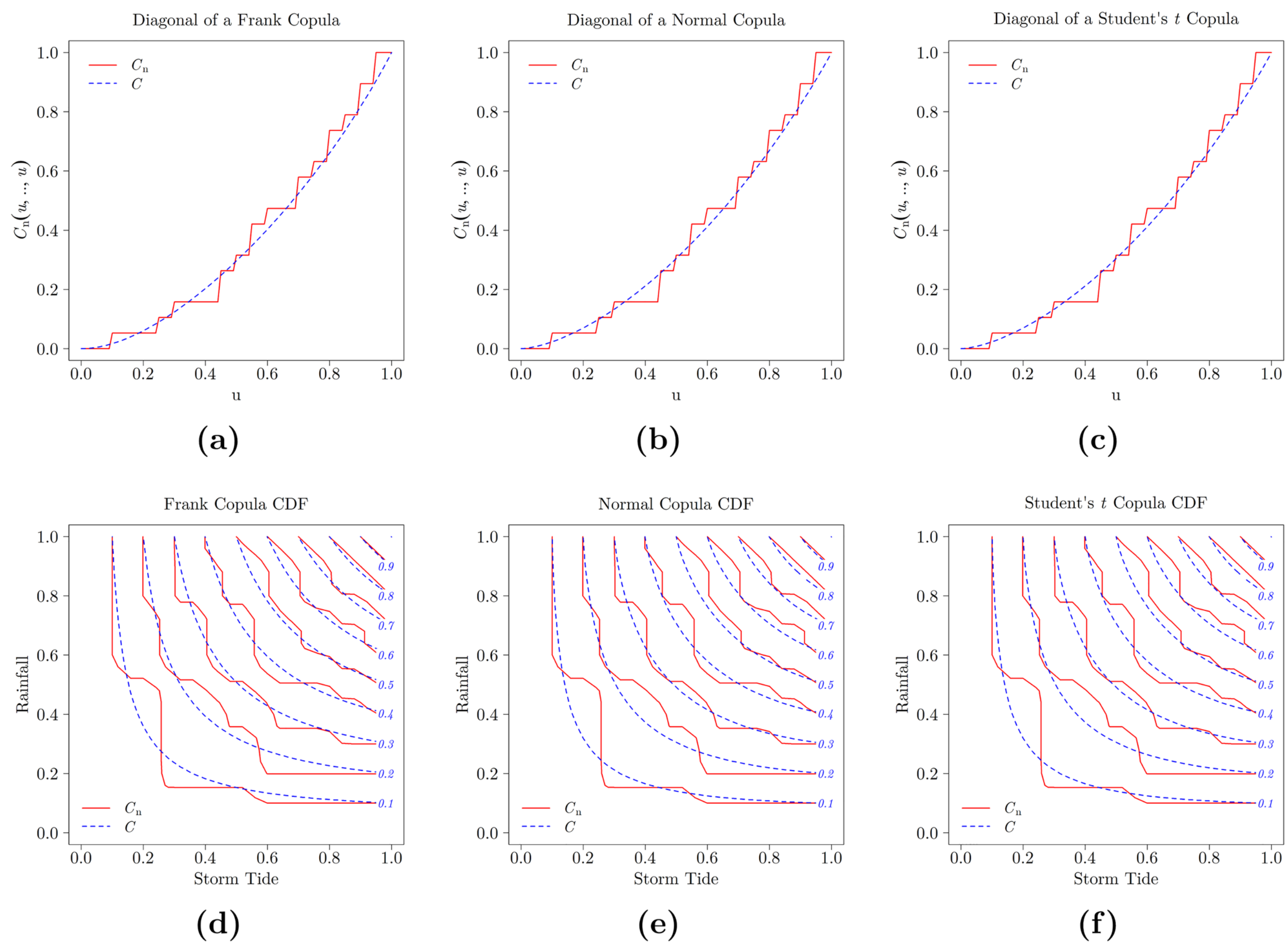

Figure 13. Diagonal curves and contour plots for the empirical and theoretical cumulative distribution functions (CDFs) for Frank (a, d), Normal (b, e), and Student's *t* (c, f) copulas of the Charleston, SC. Empirical curves are shown as solid red while theoretical curves are represented as dashed blue.

and expand to a storm tide and rainfall application using Hurricane Irma data (September 2017) as a case study in Florida, George, and South Carolina, United States. Due to the presence of observed serial correlation, LOWESS-based ARMA time series models were implemented to generate residuals that represented *iid* samples of the observed data. Apparent dependencies (i.e., Spearman, Pearson, and Kendall's *tau*) between rainfall and storm tide time series were reduced in all applications after removing serial correlation. Archimedean, Elliptical, and Extreme Value copulas were then fit using ARMA residuals and the method of maximum pseudo-likelihood in which copula parameters were estimated independent of the univariate marginals. n samples of the observed data were then synthesized using an MC approach in which the empirical copula and the parametric estimate of the copula were obtained to approximate two-sided p-values using the Rosenblatt probability integral transform method.

Each application progressively exhibited the power of a copula-based dependence model to describe multivariate hydrologic variables observed during Hurricane Irma. An important component in developing a suitable multivariate copula-based model to describe dependencies in the observed event was the selection of an appropriate time series model. In the Florida application, observed time series exhibited negative correlation while residuals represented near independence with correlation coefficients near zero. Unlike the other two applications, Florida did not have a recurring astronomical tide. Nevertheless, the Florida model still performed well although observed peaks were significantly underestimated. Georgia and South Carolina applications resulted in decreased residual dependence when compared to observed time series but still retained the observed positive dependence. Both Georgia and South Carolina's simulated peak rainfall and storm tide were in better agreement with observations under Extreme Value copulas assumption compared to Florida. Under the assumption of an Extreme Value

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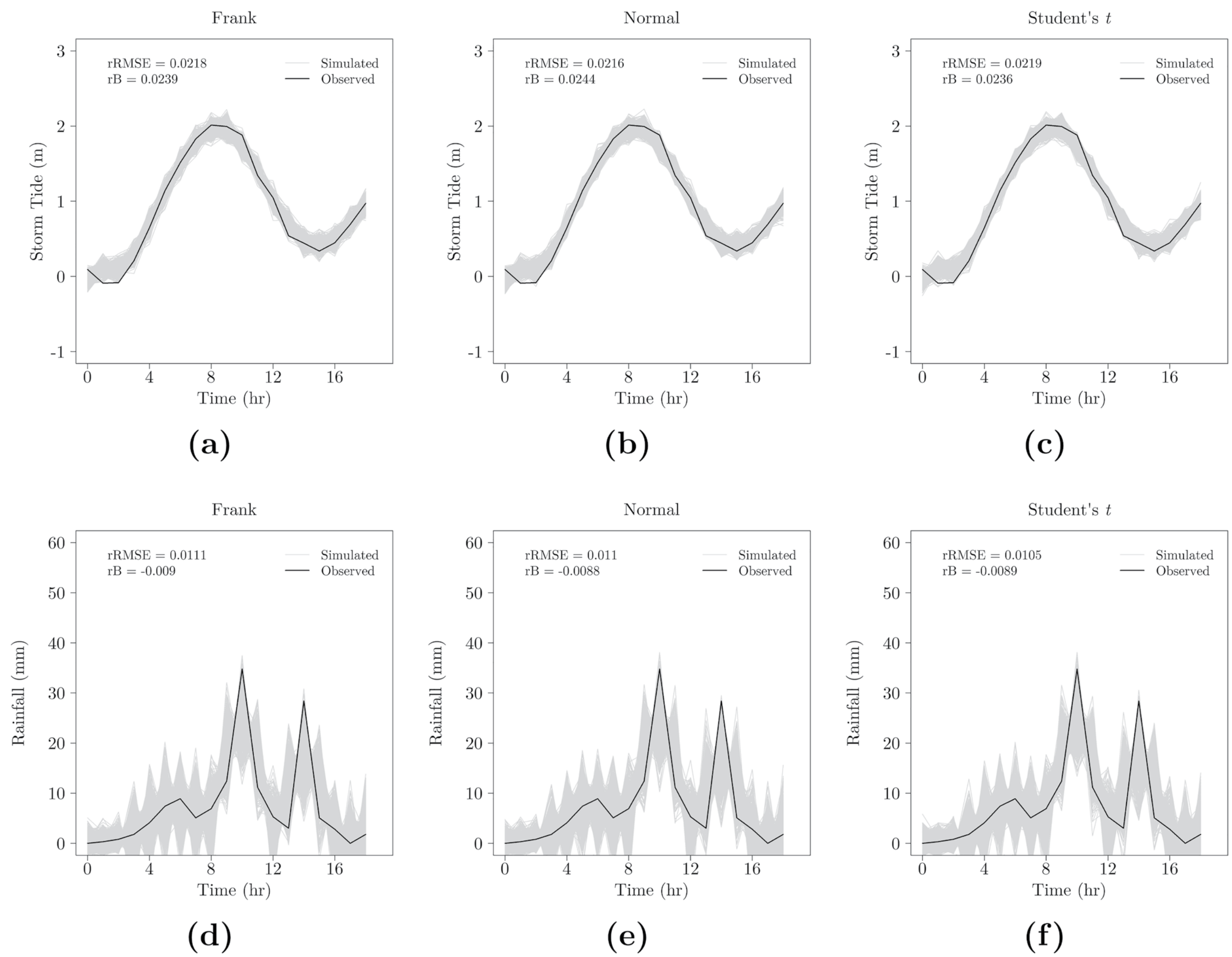

Figure 14. Time series observations and autoregressive moving average (ARMA) model simulations of Charleston, SC multivariate distribution for Frank (storm tide/rainfall) (a, d), Normal (storm tide/rainfall) (b, e), and Student's *t* (storm tide/rainfall) (c, f) copulas. Descriptive statistics regarding model fits are presented in the form of relative root mean square error (rRMSE) and relative bias (rB).

copula, the Pickands dependence function was estimated which was consistent against alternatives with left tail decreasing and relatively satisfied strong smoothness assumptions on the copula. In many cases, the Pickands dependence function can yield an estimator with a substantially larger error if the coefficient of tail dependence is small. This is the reason why the advantages of the asymptotically optimal weight function only start to play a role for rather large sample sizes, certainly larger than the sample and data used in this research.

Temporal dynamics of rainfall and storm tide time series observed during Hurricane Irma were investigated using "AND" and SK non-exceedance probability definitions. All three applications' observed rainfall and storm tide co-occurrences fell within 50% and 80% non-exceedance probability levels. However, there were at least 1–2 hr of combined peak intensities above the 95% threshold in Florida, Georgia, and South Carolina. This is typically the case with smaller compound flooding events in coastal areas wherein high-intensity rainfall and peak surges are short-lived. Although multivariate non-exceedance probabilities of observed rainfall and storm tide at any given hour were generally higher using the SK definition when compared to the "AND" definition, a direct comparison on this implication is not as straightforward (Salvadori et al., 2016) and further research should be conducted.

Multivariate copula-based dependence models developed as part of this study proved the ability to model hourly/sub-hourly rainfall and storm tide data of Hurricane Irma event with the presence of autocorrelation by using ARMA time series models. However, it is important to point out that a multivariate copula-based

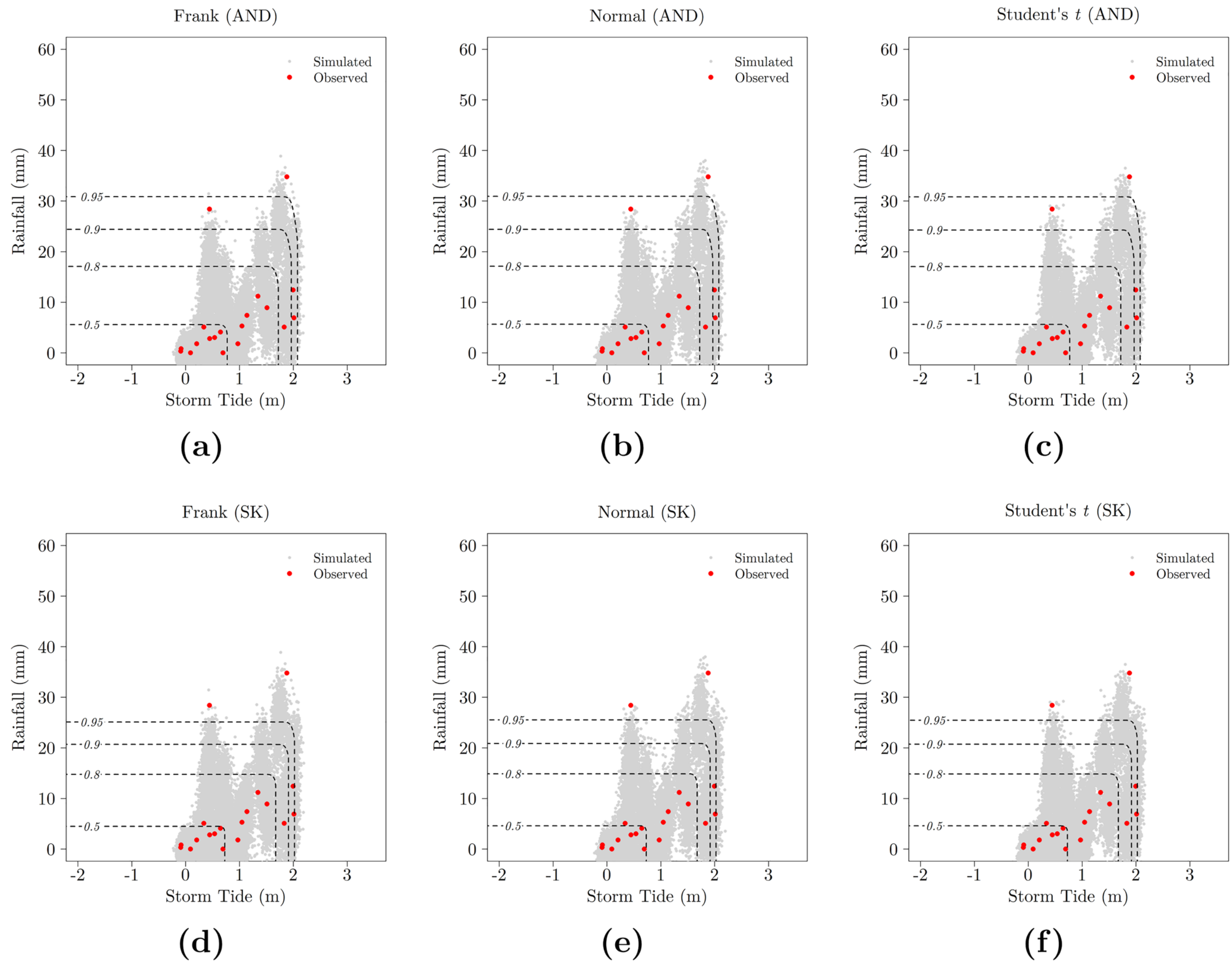

Figure 15. Observations versus simulations and probability level curves of Charleston, SC multivariate distribution under the hypothesis of Frank ("AND"/SK) (a, b), Normal ("AND"/SK) (b, e), and Student's *t* ("AND"/SK) (c, f) copulas.

model developed based on numerous individual storms/hurricanes would warrant more conclusive results on the long-term inter-storm dependency that can be utilized as a more useful tool for risk managers. For instance, consider an event-based copula model as a set of random variables (e.g., peak storm tide, peak rainfall, and storm duration). Although the individual model may be composed of possibly small sample sizes, the estimated copula model of a large sample size of multivariate extreme events can be considered an *iid* sample of random variables approaching the underlying representation of the extreme multivariate event population. This extension of the methodology presented herein could help begin to address the variability and impacts of compound coastal flooding and better assess the risk of such events occurring at a given location as stated by AghaKouchak et al. (2020).

Although the framework and results presented herein are promising, the proposed methodology has several limitations that should be mentioned. First, ARMA models assume stationary conditions, and, although transformations from a stationary to non-stationary process are possible, epistemic uncertainty is inevitable. Such bias presents itself in several of the rainfall models herein. As a result, alternative time series methods to emulate observed rainfall data, such as multifractals (e.g., Schertzer & Lovejoy, 1987; Veneziano & Langousis, 2010), should be considered in future research. Second, copulas are stationary by nature. Transformation of non-stationary time series to stationary time series and then incorporating into a stationary copula may introduce additional uncertainty. Henceforth, caution should be exercised when using copulas for design and policy decisions. Third, although the path of a hurricane was not considered herein, it is a critical component of modeling multivariate flooding in coastal environments. The hurricane path and other hurricane components such as radius of maximum winds and forward speed could provide additional insight and clarity in coastal compound flood dynamics and assessment.

Development of a storm copula to describe the long-term probabilistic nature of rainfall and storm tide (or astronomical tide) time series is no different than techniques similarly applied to dimensionless cumulative rainfall curves implemented during the design and evaluation of stormwater facilities (e.g., Huff, 1967; Oppel & Fischer, 2020). Indeed, Grimaldi and Serinaldi (2006b) presented methods using copulas to generate such design hyetographs and dimensionless rainfall curves, substantiating the applicability of copulas for synthetic design events. The broader impacts of a copula-based dependence model could provide a mechanism to approach multivariate time series modeling on hourly/sub-hourly scales in which parameters are dependent and exhibit serial correlation. Such the case is storm tide and rainfall, as shown in these applications. Development and implementation of a temporal tidal-rainfall relationship for a given annual exceedance probability would be beneficial in enacting policy, supporting decision-making needs, co-producing knowledge (see Sanders et al., 2020), and establishing design standards by which engineers and planners can quantitatively and more accurately assess the risk of compound flooding for aging and vulnerable infrastructure within coastal communities.

A multivariate dynamic design storm would help reduce subjective aspects (people's risk perception, see Kellens et al., 2013) in evaluating coastal flood risk and design of stormwater management facilities by "leveling the playing field." For example, the City of Charleston, South Carolina (i.e., peninsular Charleston) is vulnerable to compound flooding during high storm tides (or typical astronomical high tides) and the onset of rainfall since a substantial portion of the drainage network is below average tide elevations. As a result, a typical analysis for an infrastructure or development project may simulate such an event using continuous rainfall and a static mean storm tide. Obviously, this may not be the worst-case condition, and as such many engineers may model the system with a recurring tide in which the peak runoff occurs at mid-tide rising. However, there is no common method from a probabilistic perspective that represents the underlying risk observed or expected. But obviously, the time when peak rainfall and the storm tide co-occur simultaneously (the tails dependency) was when the maximum risk and damage might be occurred that the authors refer to as a "devil" condition. However, in the case of Hurricane Irma, "the devil" condition was short and temporary with two high-intensity 1-hr windows of concurrent rainfall and storm tide conditions.

Although, copulas are flexible mathematical tools that can support different configurations in terms of marginal fitting distribution. More research is needed to evaluate the historical evolvement of hurricane-driven compound floods, and certainly with longer datasets than what were used in this research. The applications presented herein are exploratory in nature and primarily aimed at hypothesizing stochastic dynamics between rainfall and storm tide time series observed during Hurricane Irma, thereby the methodology and results may have limited application in the field of compound flood hazards assessment. To compute the likelihood of hurricane-driven compound flood events more precisely, the knowledge of multivariate probabilities can be potentially combined with the physical processes that generate and shape the patterns and magnitude of hurricane events. This will offer a framework where copula dependence structure describing the interdependency among different hydrologic variables can be motivated by physical properties of the hurricane generating process. This combined approach can provide critical boundary conditions and precise return periods for hydrodynamic design models. These boundary conditions can also be used to analyze stormwater and riverine models and assess the risk associated with future hurricanes or intense storms (e.g., Farve et al., 2004; Grimaldi & Serinaldi, 2006a; Kao & Govindaraju, 2007; Salvadori et al., 2013; Zhang & Singh, 2012).

We recognize that point measurement techniques are both simple and yet statistically robust. However, simple stochastic risk models may not provide an appropriate answer during compound flood crises that span multiple hours or even days. As a result, we need more comprehensive models to understand both the stochastic and dynamic (i.e., spatiotemporal) nature of compound flood events as recently pointed out by Gori et al. (2020) and AghaKouchak et al. (2020). This call is not for "less mathematics (less dynamics)" involved in compound flood risk calculations, but rather for a better understanding of the physics and then the necessary mathematics (dynamics) involved (e.g., Samadi et al., 2021). This will encourage us to keep vigilant and communicate in a forceful way those actuarial, hydrological, and risk findings which are of societal importance/concern while keeping in mind that these types of extreme events are no longer unexpected or surprising.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

All data used to support this project may be accessed at Zenodo (https://doi.org/10.5281/zenodo.4306779).

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