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Does the impact of private education on growth differ at different levels of credit market development?

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Abstract

Using an overlapping generations model, we show that the impact of private financing of education on growth depends on credit market development, being positive when credit markets are adequately developed but negative if sufficiently low levels of credit market development occur alongside relatively high private financing intensities. Employing cross-country data, we find that reduced-form growth relationships are statistically significant and robust under various controls and samples. We also lay out conditions under which economies with missing credit markets are dynamically efficient and outperform, in terms of growth, economies with complete credit markets. The latter may explain large cross-country differences in savings and growth, while facilitating the evaluation of policies on financing education.

KEYWORDS
credit market development, economic growth, private education

JEL CLASSIFICATION
I25, O16, O41

1 | INTRODUCTION

Education is widely thought to be a driver of long-run development and economic growth. Empirical work, however, has reached mixed conclusions about the impact of education on growth (Hanushek & Woessmann, 2010). This raises some important questions. Does the type of education – public versus private – matter for the impact of education on growth? What are the consequences of not having well-developed credit markets for financing education in many developing economies? In this paper, we study the impact of private transfer arrangements between parents and children on human capital and economic growth, bearing in mind that public education is not always easily accessible. Whereas the implications of public education and related government interventions have been widely studied in the literature, less is known about the consequences of private education, that is, private financing of education that is arranged within families. Such informal relationships are of particular importance in developing economies where access to credit is limited, as emphasized in Banerjee and Duflo's *Poor Economics* (Banerjee & Duflo, 2011). Following this lead, this paper provides new insights on the relationship between private financing of education and economic growth and explores its implications for dynamic efficiency.

We start out by documenting cross-country differences in private financing of education, relating it to a newly constructed index of credit market development. The latter incorporates three indicators of credit provision and financial development. The evidence indicates that most parents pay for the education of their children while the intensity of private financing of education varies significantly across countries, with high levels of intensity in regions such as East and South Asia and low levels in western economies such as the European Union (EU) and the United States. Here, the intensity of private financing of education is defined as the ratio to household consumption, thus correcting for the relative size of different economies. Countries in East and South Asia such as China and India with high private financing intensities are also characterized by high economic growth rates and less developed credit markets, whereas economies in the western world, such as the US and the EU economies, are characterized by low private financing intensities, moderate growth rates and more developed credit markets.

We set out an overlapping generations model to explain these observations. In the model, private financing of education matters for economic growth, but the impact on growth varies at different levels of credit market development. We subsequently estimate the reduced-form growth relationships implied by the model and find that they are empirically supported after controlling for public education and a host of other variables. In the model, growth is endogenous, and parents make transfers to children, which they may use to acquire education. Parental transfers play an important role because the young would like to invest in human capital but face incomplete credit markets for educational loans. Parents find it optimal to make such transfers because they derive utility from the amount transferred to their children. If parents choose consumption and transfers optimally, then the share of parental transfers in household consumption is related to the parental joy-of-giving parameter, or parental transfer motive; we therefore refer to this share as the intensity of private financing.[1]

We demonstrate that, at adequately high levels of credit market development, the relationship between the intensity of private financing and economic growth is positive, whereas at low levels of credit market development, the relationship is negative, as long as the intensity exceeds a certain threshold. We also show that economies where markets for education loans are absent may have higher growth rates than economies with complete credit markets and a dynamically efficient balanced growth path. Low growth rates and dynamic inefficiency are

linked to high ratios of physical to human capital, which occur owing to either over-saving or under-borrowing. In particular, we show that economies with missing credit markets exhibit higher growth rates than economies with complete credit markets, either when both the intensity of parental transfers and intergenerational persistence of human capital are relatively low or when they are both relatively high. We also show conditions under which an economy with borrowing constraints exhibits higher growth than an otherwise identical economy in which credit is not rationed.

A meaningful role for private education is present in our model owing to credit market imperfections. As demonstrated by Boldrin and Montes (2005), if young individuals cannot borrow to finance educational investment and parental giving is ignored, then the competitive equilibrium in an endogenous growth economy is dynamically inefficient and involves stagnation. To alleviate the dynamic inefficiency and restore growth, they propose government intervention, accompanied by a pension scheme. Our findings suggest that such intervention is not necessary to restore dynamic efficiency if the intensity of parental transfers exceeds a certain threshold. We show that if private financing is sufficiently high, then in-family transfers can substitute for missing credit markets for education loans, enabling dynamic efficiency along the balanced growth path. Furthermore, while economies with missing credit markets are dynamically efficient at sufficiently high levels of the private financing intensity, identical economies with complete markets are not. We also clarify that dynamic inefficiency of the competitive equilibrium allocation may occur even in the absence of capital over accumulation because a low intensity of parental transfers leads to an under-provision of human capital for future generations. This result arises because human capital has positive externalities and, hence, from a social perspective, may be underinvested in by the young.

Our results provide a possible explanation for the high growth rates of several emerging East Asian economies with high levels of parental investment in education but relatively undeveloped credit markets. For instance, Seth (2002) discusses the history and cultural roots of *education fever*, the very high levels of parental investment in education in South Korea – and similarly high levels of parental investment in education have been noted in other East Asian economies (BBC, 2013; Economist, 2013). Such differences in parental investment across countries may be a factor behind the mixed findings in the empirical literature on borrowing constraints and growth: Jappelli and Pagano (1994) conclude that borrowing constraints promote growth, whereas De Gregorio (1996) reaches the opposite conclusion. Our results suggest that, at certain levels of the intensity of parental transfers, over-investment in physical capital reduces growth relative to the growth of an otherwise identical but credit-constrained economy. Our results are also consistent with the observation, highlighted in Coeurdacier et al. (2015), that savings in emerging Asian markets with high growth rates are higher than savings in advanced economies with lower growth rates. Thus, the mechanisms we highlight may be a factor behind cross-country differences in saving and growth that the literature has hitherto had difficulty explaining.

Our paper is related to a large literature on credit market development and economic growth. Galor and Zeira (1993) show that the combination of credit market imperfections and initial wealth differentials can drive persistent differences in economic development through the impact on human capital accumulation. Much of the literature has focused specifically on borrowing constraints and growth. Using an endogenous growth model, Jappelli and Pagano (1994) show that borrowing constraints can raise economic growth owing to higher accumulation of physical capital that drives productivity growth. This result depends on the absence of human capital investment. If there are borrowing constraints which hinder investment in

human capital, then the positive relationship between credit constraints and growth may be reversed (De Gregorio, 1996), though this need not be the case (De la Croix & Michel, 2007; Kitaura, 2012). Here, we show that parental financing of education may have important implications for this debate: its effects on economic growth are not monotonic but rather depend crucially on the level of credit market development.

There is also a literature on different types of education and growth. De La Croix and Monfort (2000) show that theoretically public financing of education may raise growth relative to private financing. Unlike the present paper, they do not study the interplay between credit market development, growth and dynamic efficiency or allow the possibility of parental transfers to children under private financing. We also go beyond their work through a reduced-form estimation that allows for the possibility that both private and public education influence growth. Both Yakita (2004) and Horii et al. (2008) study private education, with the former studying subsidies to education debt and the latter, direct limits on the availability of higher education. By comparison, the present paper allows *parental* financing of education in the presence of credit constraints, which allows us to ask how intensity of financing affects growth at different levels of credit market development. This seems to be both a relatively unexplored topic and one that is highly relevant, given that such financing may alleviate the impact of undeveloped credit markets for education loans.

Our estimates of the model reduced-form growth relationships confirm the prediction of a positive relationship between intensity and economic growth at relatively high levels of credit market development and a negative relationship at relatively low levels of credit market development.[2] This result is robust to various controls and sample sizes. By comparison, our estimates suggest that public education, as measured by the share of government education expenditures in GDP, is statistically insignificant for explaining growth.

The remainder of the paper proceeds as follows. Section 2 provides cross-country empirical evidence on private financing of education at different levels of credit market development, while Section 3 introduces the economic environment and derives a number of results relating to growth and dynamic efficiency. Section 4 estimates the reduced-form relationship between growth and the intensity of private financing of education as implied by the model. Finally, Section 5 concludes. All proofs are in the Appendix unless otherwise indicated.

2 | PRIVATE FINANCING OF EDUCATION

Private financing of education is quite diverse across countries. Most commonly, it takes the form of parental transfers toward children's education. Survey data from HSBC's *Value of Education* study indicates that 86% of parents are paying for their children's education, while 84% of those with a child in university or college are paying towards their education.[3] According to the survey, 63% of parents pay for private tuition for their children, the highest percentages being reported in developing and East Asian countries and the lowest in developed countries. The survey also shows that the highest proportions (22–39%) of university students paying their own education costs reside in developed countries, while the lowest (<1–5%) reside in developing and East Asian countries.[4] Unfortunately, detailed cross-country data on parental transfers towards education is not widely available.[5]

A broader measure of private financing of education – namely total household expenditures on education – makes it possible to study a larger sample of countries. To facilitate cross-country comparison, we use data on household consumption expenditures on education relative to total household consumption, and refer to this as the 'intensity of household expenditures on

education'.[6] This intensity varies significantly across regions, with relatively high intensities in East Asia and the Pacific, South Asia and the Middle East and relatively low intensities in North America and the European Union. In countries such as the United States, the United Kingdom and Australia, where access to education (even in public institutions) is costly, there are well-developed credit markets for education loans. More generally, public and private education loans for tertiary education are quite common in a number of countries.

To relate intensity of household expenditures on education to the degree of credit market development, we construct a discrete-value credit index, $Credit_i$, that measures the extent of credit provision of country i. Here, credit refers to all credit transactions and not only to those available for education financing. High values of $Credit_i$ indicate a more developed credit market. The credit index may take on four different values, depending on whether certain thresholds for credit indicators are met.[7] Table 1 displays the intensity of household expenditures on education (*int*) and the credit index for the 74 countries in our sample.

The mean intensity of household expenditures on education is 2.4%, with a standard deviation of 1.8%. Countries with a credit index equal to unity constitute 35% of the sample, while countries with a zero credit index constitute 22% of the sample. The intensity of household expenditures on education varies significantly across countries, and it is notable that the vast majority of countries with highly developed credit markets (equal to unity) are characterized by relatively low intensities of household expenditure on education. The majority of countries with very high intensities of education expenditures are characterized by imperfect credit markets, as reflected in a credit index less than 1. As expected, the intensity levels in China and South Korea are extremely high; however, we also see very high intensities in some economies in Africa, the Middle East and south of the North America region (e.g. Ghana, Uganda, Jordan and Mexico), and this finding is supported by data from the HSBC *Value of Education* survey.[8] Interestingly, most countries with a zero credit index – indicating minimal credit market development – have non-negligible intensity levels, which in some cases are higher than the sample average. These results suggest that private financing of education may be important in such low-credit economies.

Motivated by the above observations, we build an overlapping generations model of endogenous growth in which private financing of education matters for economic growth but has a differential impact on growth at different levels of credit market development.[9] We subsequently estimate the reduced-form growth relationships implied by the model and find that they are empirically supported (see Section 4). Here we control for a host of possible determinants of growth highlighted in the empirical literature, including provision of public education, which we approximate by the share of government expenditures on education in GDP. We emphasize the latter because access to public education could minimize the need of private financing of education. For instance, countries such as Denmark, Finland, France and Norway offer low or no tuition fees and provide students access to generous public subsidies for higher education. We find however that the cross-country correlation between the intensity of household expenditures on education and the share of government expenditures on education in our sample is close to zero, and in our regressions the intensity of *private financing* of education is a statistically significant factor of growth, whereas the share of government expenditure on education is found to be statistically insignificant.

3 | ECONOMIC ENVIRONMENT

We consider an OLG economy, populated by agents who live for three periods. The basic set up is similar to Boldrin and Montes (2005), but we extend their model by allowing children to

TABLE 1 Intensity of household education, expenditures (*int*) and credit index

Country	Int.	Credit	Country	Int.	Credit	Country	Int.	Credit	Country	Int.	Credit
Albania	0.51	0.33	Bangladesh	3.78	0.33	Benin	2.59	0	Bolivia	3.54	0.66
Brazil	3.49	1	Bulgaria	1.86	0.66	Burkina Faso	1.86	0	Burundi	0.77	0
Cambodia	1.90	0.33	Cameroon	3.23	0	Chad	0.35	0	China	7.03	0.66
Colombia	1.63	0.66	Congo D.	2.76	0	Congo Republic	2.12	0	Egypt	3.52	0.33
Gabon	1.75	0	Ghana	8.29	0.33	Guatemala	2.42	0.33	Guinea	0.97	0
India	3.54	0.66	Indonesia	2.90	0.66	Jamaica	4.62	0.33	Jordan	5.63	0.66
Madagascar	2.06	0	Malawi	1.22	0.33	Mali	0.58	0	Mauritius	2.70	1
Mexico	8.34	0.66	Mongolia	2.59	0.33	Nepal	3.22	0.66	Nicaragua	3.14	0.33
Niger	0.74	0	Pakistan	1.92	0	Peru	2.12	0.33	Philippines	3.62	0.66
Romania	0.89	0.33	Rwanda	2.50	0.33	Senegal	0.09	0	Sierra Leone	2.26	0
South Africa	2.85	0.66	Sri Lanka	2.14	0.66	Tanzania	2.08	0.33	Thailand	0.14	0.66
Togo	1.79	0	Turkey	1.22	0.66	Uganda	5.61	0.33	Vietnam	4.38	0.66
Zambia	3.29	0	Saudi Arabia	2.5	1	Australia	4.4	1	Canada	1.5	1
Japan	2.2	1	USA	2.4	1	South Korea	6.7	1	Belgium	0.4	1
Denmark	0.8	1	Germany	0.8	1	Ireland	3.5	1	Greece	2.4	1
Spain	1.5	1	France	0.9	1	Italy	1.0	1	Cyprus	2.6	1
Luxembourg	0.9	1	Hungary	1.5	0.66	Malta	1.4	1	Netherlands	0.7	1
Austria	0.7	1	Poland	1.2	1	Portugal	1.3	1	Finland	0.4	1
Sweden	0.3	1	UK	1.5	1						

Note: The first 49 countries listed are GCD sample; the remaining 25 countries are Eurostat sample. *Int* is measured in %.

receive transfers from their parents that may be used for education investment or as a financial investment opportunity, and we also consider households facing a borrowing limit. Within each generation, the agents are homogeneous, and population increases at the rate $n > -1$. For simplicity, we assume that agents survive to parenthood (middle age) and old age with probability one, that is, there is no early mortality.[10] An agent draws utility from consumption, $c_{t,m}$, when middle aged; consumption, $c_{t+1,o}$, when of old age; and the amount of transfers, ω_t, that the agent when middle aged provides to each of their children.[11] The consumption of the young is assumed to be incorporated in the consumption of the middle-aged – that is, their consumption such as food and other subsidence is provided by parents.[12] The lifetime utility of a young agent born in period $t-1$ is defined as $U(c_{t,m}, c_{t+1,o}, \omega_t) = u(c_{t,m}) + \beta u(c_{t+1,o}) + \gamma u(\omega_t)$, where $\beta > 0$, $\gamma > 0$ and $u(\cdot)$ is an increasing and twice differentiable function with $u''(\cdot) < 0$.[13] Young agents born in period $t-1$ are endowed with $h_{t-1}^y > 0$ units of human capital that are invested in the production of next period human capital, along with additional resources denoted by $d_{t-1} > 0$. An agent's human capital, $h_t > 0$, is produced using a smooth, homogeneous of degree 1 function $h(d_{t-1}, h_{t-1}^y)$. Aggregate output Y_t is produced in a perfectly competitive market consisting of a large number of homogeneous firms, each producing output using human and physical capital according to a smooth, concave and constant returns to scale production function F. The latter enables us to write output of the representative firm as $Y_t = F(H_t, K_t)$, where $H_t > 0$ and $K_t > 0$ correspond to aggregate human and physical capital, respectively. Firms maximize profits by taking as given the price of human capital, w_t, and the price of physical capital R_t^k, while the price of output is normalized to unity. This implies that w_t and R_t^k correspond to the marginal products of human and physical capital, respectively.

There is a perfectly competitive financial market which serves as an intermediary between agents and firms, enabling them to borrow and invest at the same gross interest rate, R_t, as in Boldrin and Montes (2005). No arbitrage suggests that the gross interest rate, R_t equals the return on physical capital R_t^k. Specifically, a young individual born in period $t-1$ will either borrow $\bar{b}_{t-1} > 0$ from the financial market if the optimal investment in human capital, d_{t-1}, exceeds parental transfers, that is, $\bar{b}_{t-1} = d_{t-1} - \omega_{t-1}$, or save (invest) $-\bar{b}_{t-1} > 0$ if parental transfers exceed the optimal investment in human capital, that is, $-\bar{b}_{t-1} = \omega_{t-1} - d_{t-1}$. A middle-aged individual saves s_t for their retirement in the financial intermediary and receives return R_t. Firms rent capital k_t from the financial intermediary to produce output, while each young individual may borrow or invest $\bar{b}_{t-1}$ via the financial intermediary as described above. It is assumed that one unit of investment corresponds to one unit of physical capital, that is, $I_{t-1}^k = K_t$.[14] Following Coeurdacier et al. (2015), we model credit market imperfections as limiting the young's borrowing to a fraction $0 < \lambda \leq 1$ of their present value labour income: $\bar{b}_{t-1} \leq \lambda w_t h_t / R_t$. We appeal here to the fact that access to credit and loan size may be rationed by lenders and depend in part on the borrower's prospective earnings. This constraint nests three cases: (i) the case of non-zero but limited access to credit; (ii) the case of no credit where $\lambda = 0$ and the borrowing constraint binds; (iii) the case of complete financial markets where λ is large enough that the constraint does not bind.[15]

In the second period of their life, a middle-aged individual in elastically supplies labour in a perfectly competitive labour market at the wage rate w_t, per unit of human capital, and receives the revenue from their investment in the financial market or pays off the loan of the previous period at the gross interest rate R_t. Then, they make further personal consumption-saving decisions and a transfer to their children. In particular, the middle-aged agent transfers ω_t to each of their $1 + n$ children and saves s_t in the financial market for their retirement.[16] Since agents within each generation are homogeneous, the aggregate savings of the middle aged, the

aggregate borrowing (saving) of the young and the aggregate human capital can be written as $S_t = (1 + n)^{t-\text{first}}$, $\overline{B}_t = (1+n)^t \overline{b}_t$ and $H_t = (1+n)^{t-1}h_t$, respectively. The total assets held by financial intermediaries must be equal to the total liabilities recorded in their balance sheets, that is, $S_t = B_t + K_{t+1}$, where $B_t = \overline{B}_t$. The latter expressed per middle-aged individual is $s_t = b_t + k_{t+1}$, where $b_t = (1+n)\overline{b}_t$, $k_{t+1} = (1+n)\widetilde{k}_{t+1}$ and $\widetilde{k}_{t+1}$ is physical capital per middle-aged individual in period $t+1$. Given that F is homogeneous of degree one, input prices are a function of $x_t = \widetilde{k}_t / h_t$, that is, $w_t = f(x_t) - x_t f'(x_t)$ and $R_t = f'(x_t)$, where $f(x_t) = F(1, x_t)$. Notice that the production function can be expressed in terms of output per middle-aged agent of period t, that is, $y_t = F\left(h_t, \widetilde{k}_t\right)$. Finally, in old age, the agent consumes all their wealth. It follows that the budget constraints, respectively, of an agent in middle age and old age are the following: $c_{t,m} + s_t + R_t \overline{b}_{t-1} + (1+n)\omega_t = w_t h_t$ and $c_{t+1, o} = R_{t+\text{first}}$.

Then, the problem for an agent born in period $t-1$ is:

$$\max_{d_{t-1}, s_t, \omega_t} \{u(w_t h_t - s_t - R_t(d_{t-1} - \omega_{t-1}) - (1+n)\omega_t) + \beta u(R_{t+1}s_t) + \gamma u(\omega_t)\}$$

$$\text{s.t.} h_t = h(d_{t-1}, h^y_{t-1}), \overline{b}_{t-1} \equiv d_{t-1} - \omega_{t-1} \leq \lambda w_t h_t / R_t.$$

The optimality conditions are:

$$u'(c_{t,m}) = \beta R_{t+1} u'(c_{t+1,o})$$

$$u'(c_{t,m})(1+n) = \gamma u'(\omega_t)$$

$$u'(c_{t,m})[w_t h_1(d_{t-1}, h^y_{t-1}) - R_t] = \overline{\mu}_t[1 - \lambda w_t h_1(d_{t-1}, h^y_{t-1})/R_t]$$

$$\overline{\mu}_t\left(\overline{b}_{t-1} - \lambda w_t h_t / R_t\right) = 0,$$

where $\overline{\mu}_t \geq 0$ is the Lagrange multiplier on the borrowing constraint and subscript i of a function indicates the partial derivative of the function with respect to its ith argument.

In the case of complete markets, $\overline{\mu}_t = 0$ for all $t \geq 0$, as the borrowing constraint does not exist, while in the case of no-credit market, $s_t \equiv k_{t+1}$, d_{t-1} is no longer a choice variable since it coincides with ω_{t-1} which is chosen by parents. Hereafter, to avoid unnecessary repetition, whenever we refer to no-credit market for young agents, we will simply use no-credit market. Note that, when the borrowing constraint binds ($\overline{\mu}_t > 0$), there is a wedge between the marginal return to education $wh_1(.)$ and the marginal cost of credit R. That is, agents would like to borrow to invest in education up to the point where the net return is zero, but a binding credit limit prevents the full education investment from taking place.

Equilibrium: *Given initial conditions d_{-1}, b_{-1}, h_0, $k_0 > 0$, an equilibrium consists of sequences of prices $\{R_t, w_t\}_{t=0}^{\infty}$ and quantities$\{d_t, \overline{b}_t, h_{t+1}, k_{t+1}, \omega_t\}_{t=0}^{\infty}$ such that the optimality conditions, the resource constraint, $F\left(h_t, \widetilde{k}_t\right) = c_{t,m} + \frac{c_{t,o}}{1+n} + s_t + (1+n)\omega_t$, and the balance sheet of financial intermediaries, $s_t = b_t + k_{t+1}$, hold for all $t \geq 0$.*

Definition 1. *A balanced growth path (BGP) is an equilibrium (or social planner allocation) in which there are constants x, g and g_y such that, $\frac{K_t}{H_t} = x$, $\frac{K_t}{K_{t-1}} = \frac{H_t}{H_{t-1}} = \frac{Y_t}{Y_{t-1}} = 1 + g_y$, and $\frac{k_t}{k_{t-1}} = \frac{y_t}{y_{t-1}} = \frac{h_t}{h_{t-1}} = 1 + g$, where $1 + g_y = (1+n)(1+g)$.*

3.1 | Parametric model

In the analysis that follows, we consider the following parametric version of the economy: $u(\theta) = \ln(\theta)$, $F\left(h_t, \widetilde{k}_t\right) = Ah_t^{\delta}\widetilde{k}_t^{1-\delta}$, $h(d_{t-1}, h_{t-1}^y) = B(d_{t-1})^{\zeta}(h_{t-1}^y)^{1-\zeta}$, $h_{t-1}^y = \mu h_{t-1}$, with $A \geq 1$, $B \geq 1$, $\mu > 0$, $\delta \in (0,1)$ and $\zeta \in (0,1)$. Note that the exponent $1 - \zeta$ determines the intergenerational persistence of human capital or, equivalently, the elasticity of human capital production with respect to past (i.e. parents') human capital, while δ determines the intensity of human capital in the production of output.

The optimality condition with respect to ω_t indicates that γ corresponds to the intensity of parental transfers: $\gamma = \frac{(1+n)\omega_t}{c_{t,m}}$. Manipulating the optimality conditions, it can be shown that physical and human capital accumulation under each credit regime take the form $k_{t+1} = \widetilde{\Psi}(\gamma) = \Psi A^{-1} y_t$ and $h_{t+1} = \Phi h_t^{1-\zeta(1-\delta)}\widetilde{k}_t^{\zeta(1-\delta)}$, and $x_t = \widetilde{k}_t/h_t$ is given by[17]:

$$x_t = \left(\frac{\Psi}{\Phi(1+n)}\right)^{\sum_{i=0}^{t-1}(1-\delta)^i(1-\zeta)^i} x_0^{(1-\delta)^t(1-\zeta)^t}, \tag{1}$$

where the coefficients Ψ and Φ depend on the state of the credit market as follows:

	Complete markets	No credit	Binding borrowing constraint
$\Psi =$	$\frac{(1-\delta)[\delta\beta(1-\zeta)+\gamma]A}{[1-\delta(1-\zeta)](1+\beta+\gamma)}$	$\frac{\beta\delta A}{(1+\beta+\gamma)}$	$\frac{\beta\delta(1-\delta)(1-\lambda)A}{(1+\beta+\gamma)[1-\delta(1-\lambda)]}$
$\Phi =$	$\left(\frac{B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\delta\zeta\Psi}{(1-\delta)(1+n)}\right)^{\zeta}$	$\left[\frac{B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\gamma\Psi}{\beta(1+n)}\right]^{\zeta}$	$\left[\frac{B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\Psi[\gamma[1-\delta(1-\lambda)]+\beta\delta\lambda]}{\beta(1-\delta)(1+n)}\right]^{\zeta}$

Taking the limit of Equation (1) as $t \to \infty$ and noting that $x_0 > 0$, we have $\lim_{t\to\infty} x_t = x = \left(\frac{\Psi}{\Phi(1+n)}\right)^{\frac{1}{\delta+\zeta(1-\delta)}}$. That is, the economy converges on the BGP ratio of human to physical capital for all initial conditions – that is, x is a globally stable steady state. Then, it can be shown that, at the BGP,

$$1 + g = \left(\frac{\Psi}{1+n}\right)^{\frac{\zeta(1-\delta)}{\delta+\zeta(1-\delta)}}\Phi^{\frac{\delta}{\delta+\zeta(1-\delta)}}.$$

In the above economy, balanced growth and long run welfare are intimately linked. If we compare growth at some rate $g > 0$ to growth at a higher rate $g' > g$, then lifetime utility U_t is guaranteed to be higher in the long run (i.e. for sufficiently large t) in the high-growth case g'. This result holds for the simple reason that both consumptions and parental transfers are financed out of output per-person, which will grow to be larger in the higher-growth economy. However, higher growth does not imply that resources are allocated efficiently, so it is also instructive to study dynamic efficiency.

A laissez-faire BGP is dynamically efficient if investment cannot be re-allocated between physical and human capital in such a way that there is a strictly positive gain in welfare for generations living on the existing or the new BGP. Following Del Rey and Lopez-Garcia (2013) and Del Rey and Lopez-Garcia (2017), we assume that the social planner preserves the functional form of individual preferences, while treating generations equally across time. Along the BGP,

the objective of the social planner is to pick stationary values for $\widehat{c}_m = c_{t,m}/h_t$, $\widehat{c}_o = c_{t+1,o}/h_t$ and $\widehat{\omega} = \omega_t/h_t$ that maximize the utility function, given by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega}) = u(\widehat{c}_m) + \beta u(\widehat{c}_o) + \gamma u(\widehat{\omega})$, subject to the balanced growth version of the resource constraint, as demonstrated in the proof of proposition 2. Then dynamic (in)efficiency of the BGP is formally defined as follows.

Definition 2. *A* laissez-faire *BGP is dynamically inefficient if a reduction in x strictly increases the welfare, as measured by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$, of generations living on the existing or a new BGP. Otherwise, the BGP is dynamically efficient.*[18]

3.2 | Results

We now compare different credit regimes and present the main results relating to growth rates and dynamic efficiency in a series of propositions. In what follows, we distinguish the growth rates of the economies with incomplete credit markets from the growth rate of the economy with complete credit market by denoting the growth rate of the economy with no credit market by $\overline{g}$ and the growth rate of the economy with a binding borrowing constraint by $\overline{\overline{g}}$. The propositions are expressed in terms of coefficient γ, which coincides with the intensity of parental transfers under optimal choices; see Section 3.1.

Along the BGP, for any $0 < \lambda < \zeta$, there exists $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \gamma^{bin}\} \neq \emptyset$ such that the borrowing limit is binding only if $\gamma \in \Omega^{bin}$, while for any $\lambda \geq \zeta$, $\Omega^{bin} = \emptyset$ and the borrowing limit does not bind.[19] Hence, for a high enough level of the intensity of parental transfers ($\gamma \geq \gamma^{bin}$), the young will receive large enough parental transfers that the borrowing limit is slack. Then the first-order conditions collapse to those under complete markets, and hence the economy is on the complete markets BGP with $g = \overline{\overline{g}}$. As long as the intergenerational persistence of human capital, $1 - \zeta$, is lower than threshold $(1 - \lambda)$, the borrowing constraint binds if and only if the intensity of parental transfers is relatively small.

It is also straightforward to show that, on the complete markets BGP, there exists $\gamma^* > 0$ such that, when $\gamma < \gamma^*$, then $\overline{b}_t > 0$, while when $\gamma > \gamma^*$, then $\overline{b}_t < 0$, and when $\gamma = \gamma^*$, then $\overline{b}_t = 0$, where $\gamma^* \equiv \beta \delta \zeta (1 - \delta)^{-1}$. Given the characteristics of the economy, the latter establishes the condition under which young agents borrow from the credit market (i.e. $\overline{b}_t > 0$) to fund their education. If parents are motivated to give to their children more than the threshold value, the children receive more than they would like to invest in education, and place what is left over as savings in the financial intermediary. Notice that the threshold γ^{bin}, below which the borrowing constraint binds, is strictly smaller than the threshold γ^*, below which the young borrow to fund their education under complete markets. This implies that, even if $\lambda < \zeta$, for any $\gamma \in \{\gamma > 0; \gamma^{bin} < \gamma < \gamma^*\}$ the borrowing constraint does not have any effect on young agents, as they can borrow the same amount they would have borrowed if markets were complete.

Proposition 1. *(Growth rates) For any γ_1 and γ_2 such that $0 < \gamma_1 < \gamma_2$, the* laissez-faire *BGP implies that.*

(i) *for the complete markets economy, $g(\gamma_1) < g(\gamma_2)$;*
(ii) *for the no-credit economy, there exists $\gamma^{\overline{g}} > \gamma_2$ such that $\overline{g}(\gamma_1) < \overline{g}(\gamma_2) < \overline{g}(\gamma^{\overline{g}})$, while for any $\gamma_3 > 0$ and $\gamma_4 > 0$ that satisfies $\gamma^{\overline{g}} < \gamma_3 < \gamma_4$, $\overline{g}(\gamma^{\overline{g}}) > \overline{g}(\gamma_3) > \overline{g}(\gamma_4)$;*
(iii) *for the binding borrowing limit economy, $\overline{\overline{g}}(\gamma_1) < \overline{\overline{g}}(\gamma_2)$ and $\lim_{\gamma \to \gamma^{bin}} \overline{\overline{g}}(\gamma) \equiv g(\gamma^{bin})$.*

Proposition 1 (i) and (iii) state that the growth rate is strictly increasing in the intensity of parental transfers when credit markets are complete and when households face a binding borrowing limit; the reason is that higher γ implies higher ratios of human to physical capital. By 1 (i) and 1(iii) the growth rate is strictly increasing when the economy switches from a binding borrowing limit to a non-binding one. Proposition 1(ii) indicates that when a credit market is absent, the growth rate of the BGP is strictly increasing in the intensity of parental transfers, as long as γ is below threshold $\gamma^{\bar{g}}$, and strictly decreasing otherwise. The latter is due to the fact that, in the absence of financial markets, the young are unable to invest the excess funds, owing to high γ, in the financial market, and indirectly channel them to physical capital. This is not the case in the presence of a borrowing limit because the constraint is non-binding at those levels of γ.

Proposition 2. *The* laissez-faire *BGP is dynamically efficient if.*

(i) *markets are complete and* $1 \leq R/(1+n)(1+g) \leq \gamma_R^c$, *where* $\gamma_R^c \geq 1$ *which is equivalent to* $\gamma \in \Omega^c \equiv \{\gamma > 0 : 0 < \gamma_1^c \leq \gamma \leq \gamma_2^c < \infty\} \neq \emptyset$, *as long as* $\delta < (1 + \beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$;

(ii) *there is no credit market and* $1 \leq R/(1+n)(1+\bar{g}) \leq \delta\gamma_R^c + 1$, *where* $\gamma_R^c > 0$ *which is equivalent to* $\gamma \in \Omega^{in} \equiv \{\gamma > 0; \gamma \geq \gamma^{in}\} \neq \emptyset$, *where* $\gamma^{in} < \infty$;

(iii) *there is a borrowing limit which is not binding and the conditions of part (i) hold.*

Proposition 2 (i) confirms the standard result that the competitive equilibrium allocation is dynamically inefficient if the capital ratio exceeds the value required by the Golden Rule (case $\gamma > \gamma_2^c$); see Diamond (1965). However, Proposition 2(i) also clarifies that dynamic inefficiency may occur even in the absence of capital overaccumulation, owing to underinvestment in human capital by the young (case $\gamma < \gamma_1^c$) who ignore its positive externalities.[20]

In particular, Proposition 2(i) establishes that the BGP is dynamically efficient under complete markets if the intensity of parental transfers lies between two thresholds. Contrary to the benchmark OLG model with physical capital (see Diamond, 1965), the condition $R \geq (1 + n)(1 + g)$ is not sufficient for dynamic efficiency. The case where $\gamma > \gamma_2^c$ is standard in the sense that dynamic efficiency is ruled out because $R < (1+n)(1+g)$ – that is, an over accumulation of physical capital means that $x = \tilde{k}/h$ exceeds the maximum efficient level. Intuitively, this is because if the intensity of parental transfers is sufficiently high, the young receive transfers in excess of their desired educational investment and invest the remainder in the financial intermediary, where it earns income that finances investment in physical capital by the middle-aged individual saving for their retirement. The case where $\gamma < \gamma_1^c$ is more interesting in the sense that dynamic efficiency fails despite the fact that $R \geq (1+n)(1+g)$. Note that although the ratio of physical to human capital is not too high, the relative level of human capital is low enough that the welfare of all generations living on the BGP can be increased via a reallocation of investment from physical to human capital that keeps $R \geq (1+n)(1+g)$. This is because the competitive equilibrium has a tendency to underinvest in human capital, as young individuals ignore the positive externality of their educational investment on the human capital of future generations (cf. Caballe, 1995).

Proposition 2(ii) establishes that the laissez-faire BGP is dynamically efficient under no credit market if the intensity of parental transfers exceeds a certain threshold. Intuitively, since the young cannot save in the absence of a credit market, transfers in excess of their desired

educational investment must be devoted to education. As a result, large enough devotion to parental transfers ensures that that the level of human capital is not too low for externality reasons, while the ratio of physical to human capital is reduced so that $R \geq (1 + n)(1 + g)$, and hence physical capital is not overaccumulated. This result suggests that economies with sufficiently high levels of parental transfer will accumulate capital in an efficient manner despite the absence of credit markets. Hence government intervention coupled with a pension scheme, as proposed by Boldrin and Montes (2005), is not needed to ensure dynamic efficiency if the intensity of parental transfers is sufficiently high.

Finally, Proposition 2(iii) indicates that, when the borrowing limit binds, the laissez-faire BGP is always dynamically inefficient. The intuition is simply that the competitive equilibrium has a tendency to underinvest in human capital for externality reasons, and a binding borrowing limit worsens this situation because it forces the young to underinvest in human capital even more. We now consider the BGP growth rates of the economies with a borrowing constraint, no credit market and complete credit markets.

Proposition 3. *(Relative growth rates – no credit) There are thresholds* $0 < \widetilde{\zeta} < \infty$, $1 < \overline{\gamma} < \infty$ *and* $0 < \underline{\gamma} < 1$, *where* $\underline{\gamma} < \gamma^* < \overline{\gamma}$ *such that*:

    (a) *If* $\zeta < \widetilde{\zeta}$, *then* (i) $\mathbf{g} = \overline{\mathbf{g}}$ *if* $\gamma = \gamma^*$ *or* $\gamma = \overline{\gamma}$; (ii) $\mathbf{g} < \overline{\mathbf{g}}$ *if* $\gamma^* < \gamma < \overline{\gamma}$; (iii) $\mathbf{g} > \overline{\mathbf{g}}$ *if* $\gamma < \gamma^*$ *or* $\gamma > \overline{\gamma}$.
    (b) *If* $\zeta > \widetilde{\zeta}$, *then* (i) $\mathbf{g} = \overline{\mathbf{g}}$ *if* $\gamma = \underline{\gamma}$ *or* $\gamma = \gamma^*$; (ii) $\mathbf{g} < \overline{\mathbf{g}}$ *if* $\underline{\gamma} < \gamma < \gamma^*$; (iii) $\mathbf{g} > \overline{\mathbf{g}}$ *if* $\gamma < \underline{\gamma}$ *or* $\gamma > \gamma^*$.
    (c) *If* $\zeta = \widetilde{\zeta}$, *then* (i) $\mathbf{g} = \overline{\mathbf{g}}$ *if* $\gamma = \gamma^*$; (ii) $\mathbf{g} > \overline{\mathbf{g}}$ *if* $\gamma < \gamma^*$ *and if* $\gamma > \gamma^*$.

Proposition 3 lays out conditions on the intensity of parental transfers, γ, and the degree of intergenerational persistence of human capital, $1 - \zeta$, such that an economy with a missing credit market outperforms or underperforms, in terms of growth along the BGP, an otherwise identical economy with complete financial markets. The proposition first shows that it is possible for the growth rate under no credit markets to exceed that under complete markets and, second, that whether it does depends on the intensity of parental transfers in relation to the degree of intergenerational persistence of human capital. Hence, the absence of credit markets need not imply lower economic growth, contrary to some conventional wisdom on the impact of credit markets.[21]

Notice that an economy with a missing credit market for education loans outperforms, in terms of growth, an otherwise identical economy with complete credit markets if the level of intergenerational persistence of human capital, $1 - \zeta$ and the level of the intensity of parental transfers γ are both relatively high, or if they are both relatively low.[22] Note that threshold $\widetilde{\zeta}$ is positively related to the intensity of the use of labour in production. It follows that, as the degree of labour intensity (i.e. δ) increases, an economy with a missing credit market outperforms the growth of an economy with complete markets for a wider range of (relatively high) values of the intensity of parental transfers.[23]

Proposition 3 may help to explain the mixed results of empirical analyses of the effects of borrowing constraints on growth. For instance, while Jappelli and Pagano (1994) found that borrowing constraints are associated with higher growth, the opposite result was reached by De Gregorio (1996). Further, Proposition 3 offers a possible explanation for the high growth rates experienced by several Asian economies with high intensity of investment in children's education (e.g. China, South Korea, India) and some credit market development.

Figure 1 provides a numerical illustration of the results in Propositions 1 to 3. Here, we set $\beta = 0.3$, $\zeta = 0.60$, $\delta = 2/3$, $\mu = 1$ and $n = 1/2$, while the scaling parameters A and B are set at 10 and 2.5. As indicated by Proposition 1, the growth rate under complete markets increases monotonically with γ, while in the no-credit case, growth initially increases with γ before reaching a maximum and declining (left panel). Notice that the growth rate when the credit market is absent exceeds the growth rate of the economy with complete markets for a range of γ values, as indicated in Proposition 3. Since $\zeta > \widetilde{\zeta}$ under the above calibration, we are in case (b) of Proposition 3: the growth rates intersect when $\gamma = \underline{\gamma} \approx 0.075$ and when $\gamma = \gamma^* = 0.36$, and for $\gamma \in (\underline{\gamma}, \gamma^*)$ the growth rate is higher under the no-credit market. Intuitively, an economy that lacks a credit market may have a higher growth rate than an otherwise identical one with a developed credit market because, for a range of values of the intensity of parental transfers, the missing credit market prevents under accumulation of human capital relative to physical capital. This result seems to resemble the cases of several developing countries with largely undeveloped credit markets and moderate-to-high parental transfers that do not have low growth rates. We discuss potential policy implications of the results in section 3.4.

The right panel of Figure 1 indicates the range of γ values for which the complete markets and no credit BGP are dynamically efficient; note that the solid part of the curves corresponds to a dynamically efficient BGP. On the y-axis is the ratio $\frac{R}{(1+n)(1+g)}$, which must exceed unity for the BGP to be dynamically efficient. However, as stressed above, the condition $\frac{R}{(1+n)(1+g)} \geq 1$ is necessary but not sufficient for dynamic efficiency. As Proposition 2 suggests, the complete markets laissez-faire BGP is dynamically efficient only if γ lies between γ_1^c (≈ 0.35) and γ_2^c (≈ 3.2), while the corresponding BGP of the economy without a credit market is always dynamically efficient as long as γ is greater or equal than threshold $\gamma_1^{in} \approx 0.1$. When the intensity of parental transfers is low, the BGP of both economies is dynamically inefficient. Conversely, when the intensity of parental transfers is high ($> \gamma_2^c$), the BGP of the no-credit market economy is dynamically efficient, whereas the BGP of the complete market economy is not.

We now compare the growth rate in the case of a borrowing limit $\bar{\bar{g}}$ with that under unrestricted credit g. Recall that the borrowing limit is summarized by the parameter λ.
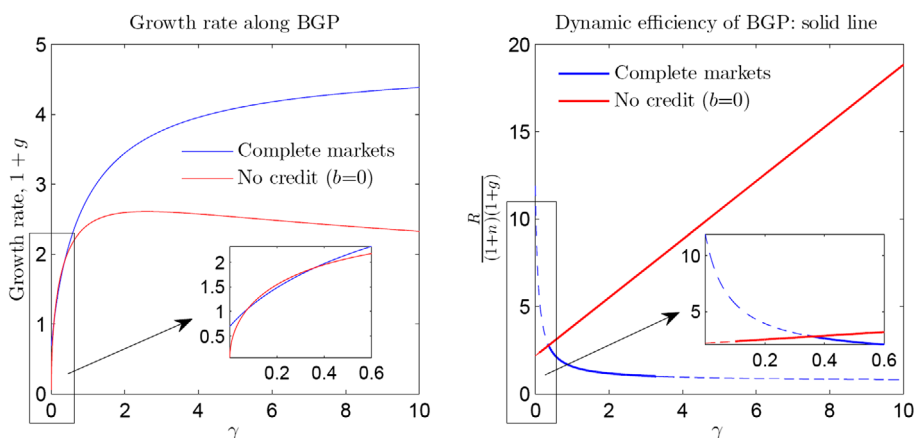

FIGURE 1 Growth and dynamic efficiency: complete and missing credit market [Colour figure can be viewed at wileyonlinelibrary.com]

Proposition 4. *(Relative growth rates – borrowing limit) For $\lambda < \zeta$, there are thresholds $0 < \widetilde{\lambda} < \infty$, $0 < \widetilde{\zeta}^* < \infty$ and $0 < \lambda_\gamma < \infty$ such that:*

(a) *If $\lambda \geq \widetilde{\lambda}$, then (i) $\mathbf{g} = \overline{\overline{\mathbf{g}}}$ if $\gamma \geq \gamma^{bin}$; (ii) $\mathbf{g} < \overline{\overline{\mathbf{g}}}$ if $\gamma < \gamma^{bin}$.*

(b) *If $\lambda < \widetilde{\lambda}$, then*

- *for $\zeta \leq \widetilde{\zeta}^*$, (i) $\mathbf{g} = \overline{\overline{\mathbf{g}}}$ if $\gamma \geq \gamma^{bin}$; (ii) $\mathbf{g} > \overline{\overline{\mathbf{g}}}$ if $\gamma < \gamma^{bin}$,*
- *for $\zeta > \widetilde{\zeta}^*$, there exists $\gamma_2 \in \Omega^{bin}$ as long as $\lambda < \lambda_\gamma$, such that (i) $\mathbf{g} = \overline{\overline{\mathbf{g}}}$ if $\gamma = \gamma_2$ or $\gamma \geq \gamma^{bin}$; (ii) $\mathbf{g} < \overline{\overline{\mathbf{g}}}$ if $\underline{\gamma_2} < \gamma < \gamma^{bin}$; (iii) $\mathbf{g} > \overline{\overline{\mathbf{g}}}$ if $\gamma < \underline{\gamma_2}$.*

Proposition 4 states that the BGP of an economy with a binding borrowing limit has higher growth than an otherwise identical economy with complete credit markets either when restrictions are loose or when they are tight, as long as the intergenerational persistence of human capital, $1 - \zeta$, is below certain thresholds. This is because the degree of credit market tightness, in combination with the level of intergenerational persistence of human capital, is such that young individuals' borrowing to invest in human capital is limited, which keeps the ratio of physical to human capital at a level that generates higher growth.

Both De Gregorio (1996) and Aghion et al. (2010) suggest that the relationship between borrowing constraints and growth may be negative. However, there are cases of countries with relatively undeveloped credit markets exhibiting high rates of economic growth. For instance, economic growth in China has proceeded at a fast rate despite the absence of a highly developed credit market. Proposition 4 provides a possible explanation for these observations that relates the restrictions to borrowing with the intergenerational persistence of human capital (determined by ζ) and the intensity of parental financing γ.[24]

Figure 2 provides a numerical illustration of the results in Proposition 4. The calibration is the same as previously, except that to illustrate two different cases we consider two values for ζ and fix the borrowing constraint parameter λ at 0.1. In Case I, we set $\zeta = 0.60$, which corresponds to case (b) of Proposition 4. As expected, growth is higher for $\gamma < \gamma^{bin}$ (=0.25) in the economy with borrowing limit and coincides with the complete markets growth rate for
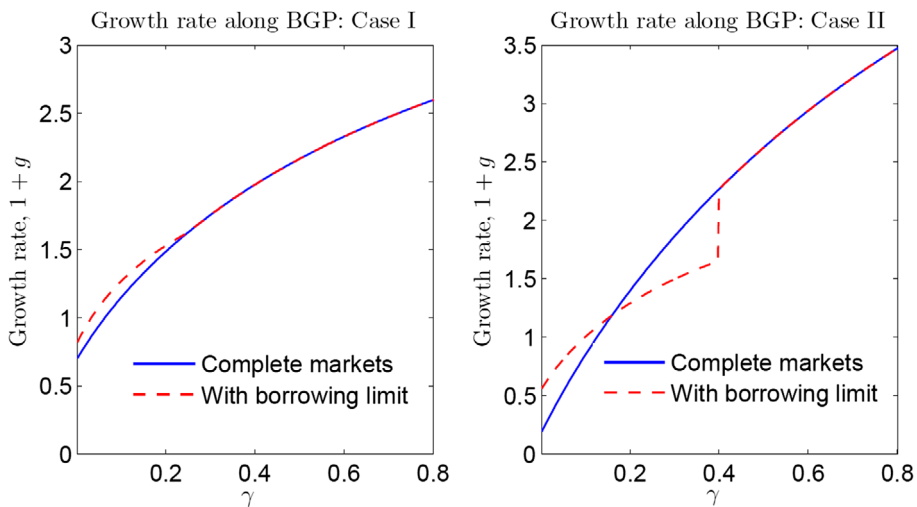

FIGURE 2 Growth under complete markets and borrowing limit [Colour figure can be viewed at wileyonlinelibrary.com]

$\gamma \geq \gamma^{bin}$. In Case II, we set $\zeta = 0.90$, giving the final sub-case of Proposition 4. The growth rate starts out higher in the economy with borrowing limit but then falls below the growth rate in the complete market economy. Once $\gamma \geq \gamma^{bin}$ ($=0.40$), the growth rates in the complete market and borrowing limit cases coincide because the borrowing constraint is slack. The economy with the binding borrowing constraint is always dynamically inefficient, consistent with Proposition 2(iii).

3.3 | Relation to the literature

Several studies have examined the relationship between borrowing constraints, the savings rate and economic growth. Jappelli and Pagano (1994) and De La Croix and Michel (2002) consider OLG models without a parental transfer motive and show that borrowing constraints raise saving (in capital) and economic growth. This finding is consistent with the observation of Coeurdacier et al. (2015) that emerging Asian economies exhibit higher growth rates and savings than those of advanced economies. De Gregorio (1996), on the other hand, shows that once investment in human capital is introduced in the OLG model, it is possible that borrowing constraints may lower growth by increasing the incentive to work, rather than study, when young. Our model suggests that the positive effect of borrowing constraints on savings and growth holds even in the presence of human capital investment as long as the intensity of parental transfers is at relatively low levels. Specifically, the results in Propositions 3 and 4 indicate that $\overline{g} > g$ and $[s/y]^{NC} > [s/y]^{CM}$ if $\gamma < \gamma < \gamma^*$ and $\zeta > \widetilde{\zeta}$, and that, for any $\gamma \in \Omega^{bin}$, $\overline{\overline{g}} > g$ and $[s/y]^{LA} > [s/y]^{CM}$ if $\lambda \geq \widetilde{\lambda}$ or $\lambda < \widetilde{\lambda}$ and $\zeta > \widetilde{\zeta}^*$ and $\gamma > \gamma_2$, where s/y is the savings rate. Moreover, for any $\gamma \in \Omega^{bin}$, we have $[s/y]^{NC} > [s/y]^{LA} > [s/y]^{CM}$ along the BGP, that is, greater credit market imperfection corresponds to a higher savings rate. These results are related to investment in physical capital, which is increasing with γ when markets are complete and decreasing with γ when markets are incomplete. Thus, when the intensity of parental transfers is low, that is, $\gamma < \gamma^*$, and markets are complete, investment in physical capital is low relative to investment in physical capital in the economy with incomplete markets. The latter leads to a lower savings rate under complete markets, even if the middle-aged individuals invest a strictly positive amount to the credit market.[25] The model prediction that credit market imperfections raise saving rates is consistent with the empirical findings of Bandiera et al. (2000) and Loayza et al. (2000).

3.4 | Discussion

Thus far we have provided theoretical results on credit market development, intensity of parental financing and economic growth. What, if any, are the policy implications of these results (assuming supportive empirical analysis as shown in the next section)?

First, our results suggest that for given credit market development, an increase in the intensity of private financing will raise long-run growth unless we are in the no-credit economy. In the latter case, long-run growth is guaranteed to increase in γ only if we start at a sufficiently low value of γ; beyond this threshold, growth will decrease (see Proposition 1 and Figure 1). Thus, one interpretation of our results is that an increase in the intensity of private financing will raise lon-grun growth if (i) credit market development is above a minimal level, or (ii) if credit is largely absent but the extent private financing is low.

On the other hand, for a given intensity of parental financing, a discrete increase in credit market development (e.g. through an increase in λ) has mixed effects on long-run growth. In particular, long-run growth can be higher in a zero-credit economy, provided the intensity γ is not too large (see Proposition 3 and Figure 1, left panel). On the other hand, if borrowing limits are relaxed so that they no longer bind, then this is not unambiguously beneficial for growth either: the impact on long-run growth depends on the value of the intergenerational persistence of human capital, as determined by ζ (see Figure 2 and Proposition 4).

From a growth policy perspective, these results seem to suggest that parental financing should be encouraged (or at least not discouraged) in economies with some credit market development, while in those economies with minimal credit, the policy implications are not clearcut. In particular, in such economies (which are more likely to be developing – see Table 1) a growth-beneficial policy cannot easily be stated without precise information on the extent of private financing and the critical threshold at which the marginal impact of private financing on growth turns negative; unfortunately, such precise information is unlikely to be available to policymakers in practice.

Could credit policy help here? The answer based on theory seems to be yes. Note that, if a minimal-credit economy were to increase both credit market development and intensity of private financing sufficiently, then its long-run growth rate would increase, corresponding to the region in Figure 1 and Proposition 3 where the long-run growth rate is highest under complete markets. Of course, such a policy would be a long-term project and difficult to implement; many successive governments would be required to support it (or at least not get in the way) and obstacles such as war, disease or poverty could obstruct financial development or hinder attempts to encourage saving that can finance children's education.

It is also worth noting that, even if these obstacles are avoided, the transition to the new long-run growth path might not be attractive. In this vein, Kitaura (2012) argues that the transition following a relaxation of a binding borrowing constraint need not be beneficial for early generations even if long-run growth increases, whereas Hatcher (2022) pinpoints the specific political economy ramifications: the initial middle-aged generation are guaranteed to experience a welfare gain, but subsequent generations may experience welfare losses until the economy gets sufficiently close to the new balanced growth path. The intuition is that an expansion of education loans crowds out physical capital in favour of human capital (raising expected future returns for the initial middle-aged individuals) but also induces a fall in the savings rate of subsequent generations that can more than offset any gains from higher interest rates for intermediate generations (because a larger return is earned on a smaller quantity of savings).

Hence, on the one hand, a myopic government has political economic incentives to support expansions of credit: this way it will please current voters. A government with a longer-term perspective on the other hand – as would be needed to implement the dual policy we are considering – might find the welfare losses to some subsequent generations unpalatable and not follow through. Note that, while the above papers study a version of our model with $\gamma = 0$ (no parental transfers), it is easy to show that the same result applies in the present model where $\gamma > 0$, such that parental transfers are incorporated in the analysis.[26]

4 | REDUCED-FORM ESTIMATION

In this section, we evaluate the model's predictions regarding the relationship between growth and the intensity of private education expenditures. The growth equation implied by the model

can be written in logarithmic form as $g \approx ln(1 + g) = \Delta_1 + \Delta_2 ln\Psi(\gamma) + \Delta_3 ln\Phi(\gamma)$, where Δ_1, Δ_2 and Δ_3 are functions of structural parameters, while $\Psi(\gamma)$ and $\Phi(\gamma)$ are both nonlinear functions of γ, that is, the intensity of parental transfers. Here we denote the growth rate by g in all cases. As stated in Proposition 1, the growth rate g is strictly increasing in γ in all cases except no credit market (namely, when $\gamma \in \Omega^{in}$). In the latter case, growth is decreasing with γ only when γ is above threshold $\gamma^{\bar{g}} \equiv \delta(1-\delta)^{-1}(1+\beta)$, and increasing only at relatively low levels of γ which are below the threshold. It is straightforward to show that the same signs of the relationships between growth and the intensity of parental transfers hold between growth and the ratio of household expenditures on education to parental consumption, denoted by γ_{EX}, since the two measures are always strictly positively related[27]:

$$\gamma_{EX} \equiv \frac{(1+n)d_t}{c_{t,m}} = \gamma\frac{d_t}{\omega_t} = \begin{cases} \dfrac{\delta\zeta(\beta+\gamma)}{[1-\delta(1-\zeta)]} & \text{if markets are complete} \\ \gamma & \text{if there is no credit market} \\ \dfrac{\gamma^2[1-\delta(1-\lambda)]+\beta\delta\lambda\gamma}{[1-\delta(1-\lambda)](1+n)} & \text{if the borrowing limit binds.} \end{cases}$$

Empirically, however, it is difficult to obtain cross-country data on parental consumption which is the denominator of γ_{EX}. Proposition 5 enables us to replace γ_{EX} and thus γ in the growth equation with the intensity of household expenditures on education, int, defined in section 2. It is straightforward to show that $int = \alpha_{c_m}\gamma_{EX}$, where $\alpha_{c_m} > 0$ is the share of parental consumption in total consumption, which is also a function of γ.[28]

Proposition 5. *For any γ_1 and γ_2 such that $0 < \gamma_1 < \gamma_2$, $int(\gamma_1) < int(\gamma_2)$.*

Proposition 5 demonstrates that int is strictly increasing in γ, which implies that the sign response of growth to γ or γ_{EX} is exactly the same as the sign response of growth to int. This allows us to evaluate the model's predictions regarding the impact of the intensity of private financing of education on growth using int instead of γ_{EX} or γ in the growth equation. Therefore, the growth equation is now written as $g \approx \Delta_1 + \Delta_2 ln\Psi(int) + \Delta_3 ln\Phi(int)$, where $\Psi(int)$ and $\Phi(int)$ are now functions of int.

Estimating the highly non-linear relationship between growth and the intensity using the exact functional form of the growth equation with a relatively short sample raises the problem of identification of the three models. We therefore proceed with an alternative approach that utilizes the discrete-valued credit index introduced in section 2. We isolate the combined effect of the two variables of interest, the country's intensity of household expenditures on education, int_i and $Credit_i$, on the country's growth rate, g_i, in function $G(int_i, Credit_i)$, such that $G \in \mathbb{G}$, where $\mathbb{G}$ is a set of functions such that $G(0, Credit_i) = 0$ for any $G \in \mathbb{G}$ and $Credit_i \geq 0$, as the model suggests.[29] Other country-specific factors which potentially affect the country's growth rate are included in vector $\mathbf{Z_i}$. Our cross-country growth regression is:

$$g_i = G(int_i, Credit_i) + \boldsymbol{\beta}'\mathbf{Z_i} + \epsilon_i. \tag{2}$$

We have considered various specifications for G that include square and interaction terms between int_i and $Credit_i$, which can capture not only the sign of the response but also the shape of the relationship between the intensity of household expenditures on education at different

levels of credit market development. Among these specifications we selected a specification that provides a good fit of the regression to the data, as measured by the adjusted R^2 and the F-statistic, and does not reject the null of no misspecification of the Ramsey RESET test. The function we use is:

$$G(int_i, Credit_i) = \alpha_0 + \alpha_1 int_i + \alpha_2 int_i \times Credit_i. \tag{3}$$

Although this function does not include non-linearities other than the interaction term, it is a parsimonious specification that can capture a change of the sign of the impact of the intensity on growth. We consider various versions of $\mathbf{Z_i}$, similar to mainstream literature; see for example, Knack and Keefer (1995), Acemoglu et al. (2001) and Sachs and Warner (2001). In particular, we considered versions of $\mathbf{Z_i}$ which include the growth rate of population, the share of public expenditure on education in GDP, the per capita level of real GDP in 1970 (as control for initial conditions), colonial dummies, legal origin dummies, indexes for political stability and property rights, measures of latitude and democracy, an indicator of natural resource rents, and regional dummies. Detailed information and descriptive statistics of all the variables used in the empirical analysis can be found in the Appendix.[30]

We estimate (3) with ordinary least squares and each column of Table 2 corresponds to either a different variation of $\mathbf{Z}_i$ or sample. In specification (I), $\mathbf{Z}_i$ includes controls of legal and colonial origins that correspond to dummy variables, as well as regional dummies. Since legal and colonial dummies are all statistically insignificant, we only report the joint significance level (i.e. p-value) of the corresponding F-statistic. In specification (II), $\mathbf{Z}_i$ excludes legal, colonial and regional dummies. Specifications (III) and (IV) are the same as specifications (I) and (II), respectively, with the addition of democracy and property rights indicators. Specification (V) is the same as specification (I), excluding countries with growth rate outliers, which slightly reduces the sample. Specifications (VI) and (VII) are the same as specification (I), with the difference that the former is estimated using data only from the Global Consumption Database (GCD), while the latter is estimated by excluding countries with fewer than 5 observations to compute average public expenditure on education.[31]

Table 2 shows two main results. First, the intensity of household expenditures on education is statistically significant for cross-country growth rates, both on its own and through its interaction with credit market development. Second, the effect of the interaction term is always positive while the effect of the intensity on its own is negative. These results hold under various controls and sample sizes.[32]

The negative coefficient on the intensity of household expenditures on education combined with the positive coefficient on the interaction term captures (i) the non-monotonicity of the growth rate with respect to intensity and (ii) the change in the slope of the growth rate with respect to intensity when the credit index switches from low (high) to high (low). Specifically, the positive coefficient on the interaction term captures the positive effect of the intensity of household expenditures on education as the credit market develops, while the negative coefficient on intensity captures the negative effect of the latter when the credit market is undeveloped (Proposition 1). Our estimates suggest a positive relationship between the intensity and growth for countries with a credit index of 2/3 or 1 (i.e. countries with developed credit markets) and a negative relationship for countries with a credit index of 1/3 or less (i.e. countries with less developed credit markets).[33] For the full sample of 74 countries, the credit index is below the threshold for 32 countries, which make 43% of the countries in the sample. In the

TABLE 2 Reduced-form estimation of Equation (2): $G(int, Credit)$ follows Equation (3)

	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
Intercept	10.06***	9.93***	10.77***	9.76***	8.42***	7.66***	9.96***
	(2.05)	(1.32)	(2.29)	(1.43)	(1.93)	(2.61)	(2.12)
Int	−0.36***	−0.36***	−0.32**	−0.33**	−0.33***	−0.46**	−0.33**
	(0.13)	(0.13)	(0.15)	(0.13)	(0.12)	(0.19)	(0.14)
Int × *Credit*	0.78***	0.90***	0.73***	0.85***	0.59***	0.84***	0.72***
	(0.18)	(0.16)	(0.19)	(0.17)	(0.18)	(0.29)	(0.17)
$ln(y_{1970})$	−0.89***	−0.89***	−1.03***	−0.91***	−0.75***	−0.47	−0.90***
	(0.19)	(0.15)	(0.26)	(0.19)	(0.18)	(0.29)	(0.22)
Public expenditure on education	−0.05	−0.06	−0.07	−0.06	−0.08	0.03	−0.08
	(0.13)	(0.11)	(0.14)	(0.11)	(0.13)	(0.18)	(0.15)
Political stability	0.31	0.24	0.31	0.15	0.35	0.48	0.10
	(0.24)	(0.19)	(0.26)	(0.20)	(0.21)	(0.32)	(0.26)
Latitude	−0.90	1.21	−0.85	1.06	−0.29	−3.23	0.71
	(1.52)	(1.07)	(1.56)	(1.11)	(1.40)	(2.27)	(1.74)
n	−0.69***	−0.77***	−0.75***	−0.83***	−0.57**	−0.65*	−0.68***
	(0.26)	(0.19)	(0.31)	(0.21)	(0.23)	(0.37)	(0.27)
Natural resource rents	0.03	0.03	0.04	0.03	0.03	0.03	0.02
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)
East Asia & Pacific	1.40***		1.41***		1.23***	1.73***	1.18**
	(0.54)		(0.56)		(0.48)	(0.67)	(0.58)
Europe & Central Asia	1.20		1.27		1.22	1.37	0.81
	(0.85)		(0.89)		(0.77)	(1.48)	(0.87)
America & Caribbean	0.79		0.81		0.40	0.09	0.52
	(0.58)		(0.60)		(0.52)	(0.74)	(0.59)
Middle East & North Africa	1.76**		2.14**		1.71***	1.66	1.79**
	(0.74)		(0.93)		(0.65)	(1.25)	(0.75)
South Asia	1.09*		1.10		1.14*	1.67*	0.43
	(0.65)		(0.67)		(0.59)	(0.85)	(0.74)
Democracy			0.03	−0.03			
			(0.07)	(0.06)			
Property rights			0.06	0.11			
			(0.11)	(0.10)			
p-Value for legal var.	0.75		0.78		0.95	0.55	0.74
p-Value for colonial var.	0.71		0.89		0.54	0.65	0.77
R^2_{adj}	0.64	0.60	0.61	0.58	0.55	0.60	0.61
F-statistic	7.78	14.85	6.41	11.13	5.46	4.95	6.23
R^2_{adj} when $G_1 \equiv \alpha_1$	0.52	0.42	0.51	0.42	0.47	0.50	0.47
	0.52	0.36	0.50	0.35	0.48	0.51	0.47

(Continues)

TABLE 2 (Continued)

	(I)	**(II)**	**(III)**	**(IV)**	**(V)**	**(VI)**	**(VII)**
R^2_{adj} when $G \equiv 0$							
Obs	74	74	73	73	69	49	65

Note: Dependent variable: Average growth rate of real GDP per person 1970–2010. Specifications I–VII differ in terms of the explanatory variables included in $\mathbf{Z_i}$ and/or the sample (see above discussion).

***, ** and * correspond to 1%, 5% and 10% significance levels, respectively.

case of regression 6 which uses data from GCD (primarily developing countries), 65% of the countries are below the credit threshold.[34]

To examine further the significance of the intensity of private financing of education and its interaction with credit market development in explaining cross-country growth rates, we re-estimate regressions (1)–(7) first, by setting *Credit* = 0 (i.e. $G_1 \equiv \alpha_1$) and, second, by excluding both variables from the regressions (i.e. $G \equiv 0$). As shown in the bottom two rows of Table 2, the goodness-of-fit of the regressions, as measured by the adjusted R^2, reduces significantly under the alternative specifications, indicating that the intensity of private financing of education and its interaction with credit development are important in explaining cross-country growth rates.

Among the factors included in **Z**, we find that only a small subset of them is statistically relevant in explaining the cross-country variation in growth rates. Our estimates suggest that both the growth rate of population and initial per capita real GDP in 1970 have a negative association with cross-country growth. As we would expect based on the 'convergence hypothesis', the lower the initial level of output is, the higher the growth rate. The only other factors that are found to be statistically relevant for cross-country growth are regional dummies such as the East Asia and Pacific dummy, the Middle East and North Africa dummy and the South Asia dummy. In contrast to the private financing intensity, the share of government expenditure on education does not show up as statistically significant.

What is the quantitative relevance of the point estimates in Table 2? To illustrate the quantitative implications of the intensity of household expenditures on education, consider the point estimates of specification I, which has the best fit among all specifications of Table 2. As noted, the (total) coefficient on the intensity of parental transfers is positive as long as the credit market is sufficiently developed, where the latter translates into the credit index being above a threshold (proposition 1). For illustration purposes we consider (i) a credit index of either 1/3 (low) or 2/3 (intermediate) (the sample mean is 0.56 – see Table A1) and (ii) a unit increase in the intensity (share) of 1% (the sample mean is 2.38% and the standard deviation is 1.78%). Then, according to the point estimates of Table 2 (Column I), if a country with a relatively undeveloped credit market has intensity of education expenditure higher by 1%, this would be associated with growth lower by 0.10% per annum (p.a.) in the low-credit case and higher by 0.15% p.a. in the case of intermediate credit market development. These quantitative impacts are non-trivial and highlight the non-monotonic relationship between intensity and growth at different levels of credit market development. As remarked in Section 3.4, these results suggest that countries with little credit market development may need to raise intensity alongside credit market development for a positive overall effect on growth.

5 | CONCLUSION

In this paper, we have explored the effects of the intensity of private financing of education on growth and dynamic efficiency at various levels of credit market development. We motivated our analysis with empirical observations which show that the intensity of private financing of education varies substantially across countries, exhibiting higher levels of intensity in East and South Asia, where countries such as China, Korea and India also exhibit high GDP growth rates, and lower levels of intensity in western countries, such as the EU and the US, which are characterized by well-developed credit markets and moderate growth rates.

We present an OLG model of endogenous growth which predicts that the relationship between the intensity of parental transfers and growth is negative at sufficiently high levels of intensity when credit markets for education loans are absent, but otherwise is positive. We also establish conditions for which missing or imperfect credit markets increase economic growth and do not hinder dynamic efficiency. We show that, as long as the parental transfer intensity is sufficiently high, government intervention along with a pension scheme, as proposed by Boldrin and Montes (2005), is not necessary for an economy with a missing credit market to achieve dynamic efficiency. Our main finding is that the impact of parental financing of education for growth is not monotonic but depends crucially on the extent of credit market development. We thus argue that the intensity of parental financing of education may be a factor behind cross-country differences in saving and growth which the literature has hitherto had difficulty explaining.

We assess the growth implications of the model using a measure of the intensity of household expenditures on education. After controlling for various potential factors associated with economic growth, our estimates from a reduced-form type of equation indicate that the negative relationship between the intensity and growth at relatively low levels of credit market development, and the positive relationship elsewhere, are statistically significant and robust under different controls and samples.

Our statistically significant results for the intensity of private education contrast with those for public expenditure on education, as proxied by the share of government spending on education in GDP. The latter is not found to be a statistically significant factor of cross-country growth differences. We view our analysis as a first step in documenting cross-country differences in private financing of education and assessing their implications for economic growth. Future work could investigate in more detail the exact mechanisms through which private financing of education might facilitate long-run economic development, as well as addressing the important question of why such large cross-country differences exist in the first place.

DATA AVAILABILITY STATEMENT

The data used in this research are from publicly available data sources: Growth is measured using the average growth rate of real GDP per capita for the period 1970–2010 (in % p.a.) computed from Penn World Table 9. The logarithm of per-capita real GDP in 1970, ln(y1970), is expressed in 2011 USD and is also obtained from Penn World Table 9. The intensity of household expenditures on education is computed using data from the World Bank's Global Consumption Database 2010 (GCD), Eurostat (code: nama 10 co3 p3, 2010) and EU (2015).[35] The share of public expenditures on education on GDP (Public Expenditure Edu.) was computed using data from the World Bank Development Indicators.[36] The Political Stability index is an average of the years 1996 to 2000, and was obtained from the World Bank's Worldwide Governance Indicators (WGI). Data for the Latitude index is obtained from the World Cities database

at simplemaps.com. The growth rate of population corresponds to the average population growth rate (% p.a.) for the period 1970–2010 in Penn World Table 9. Data for natural resource rents are obtained from the Databank of the World Bank's World Development Indicators. The democracy index corresponds to the average of democracy score for the period 1970–1994, as used by La Porta et al. (1999) and constructed by Jaggers and Gurr (1996). The information about the legal origin of countries (legal origin dummy variable) is obtained from La Porta et al. (2008). Where available, the colonial origin of countries is based on Grier (1999) or Acemoglu et al. (2001). In cases where countries were colonized by multiple European nations, we followed the classification in Grier (1999); if a classification was not available, we identify the most recent colony using the CIA World Factbook 2018. The credit index (Credit) was constructed using information from World Bank Financial Inclusion Database (FINDEX). Our data can be found at https://github.com/MCHatcher/Private edu growth.

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ENDNOTES

[1] Examples of the bequest version of parental transfer (or "warm glow") preferences include Yaari (1964) and Galor and Zeira (1993). Here, we focus on inter-vivos transfers made by middle-aged parents to their children of education age. Using US data on social benefits, Mukherjee (2018) finds that parents view resource transfers to their children as a normal good, without expectations for reciprocal caregiving. Although the sample is based on elderly parents, it is indicative of the tendency of parents to provide to their children.

[2] In estimation we proxy the intensity of parental transfers with the intensity of household expenditures on education, which we show impacts growth in the same direction in our model and is available for a large sample in the data.

[3] All information reported from the survey is reproduced with permission from The Value of Education Foundations for the Future, published in 2016 & 2017 by HSBC Holdings. The survey represents the views of 8,481 parents from a diverse sample of 15 countries and territories, covering all continents.

[4] The highest percentages of parents paying for education are in China (93%), Indonesia (91%), Egypt (88%), Hong Kong (88%), India (83%), Singapore (82%) and Malaysia (81%), and the lowest are in France (32%), Canada (31%), Australia (30%) and the UK (23%). The highest proportions of university students paying their own education costs are in Canada (39%), USA (37%) and Australia (22%), and the lowest are in Egypt (<1%), India (1%), Hong Kong (4%) and Singapore (5%).

[5] The US Consumer Expenditure Survey and the UK Student Income and Expenditure Survey provide panel data on parental transfers for children's education, but cross-country data is very limited. Studies such as Zissimopoulos and Smith (2009) and Alessie et al. (2014) provide evidence on cross-country inter-vivos parental transfers which are not solely focused on education and involve adult children.

[6] The data is from Eurostat and the Global Consumption Database (GCD), where education expenditures cover educational services. Details are provided in the appendix.

[7] More information on the construction of the credit market index can be found in the appendix.

[8] Data from the HSBC *Value of Education* survey also show that parental transfers toward children's education, scaled by average per capita household consumption, are larger in Asian and Middle Eastern economies such as China, India, Indonesia, Egypt, the United Arab Emirates and Hong Kong than in countries of the western world such as France, Canada and the United Kingdom.

[9] In our model, parental transfers to children and investment in education are positively related on an equilibrium path. Therefore, we can refer to education as "financed by parents" or "privately financed" in contrast to a public finance approach. Any borrowing by the young also finances education, but we consider credit market specifications where this channel is "shut down" or subject to imperfections (credit rationing).

[10] A fixed probability of death less than 1 would have a similar role to the discount factor β.

[11] The literature distinguishes between (i) bequests (transfers made upon death) and (ii) inter-vivos transfers, which are made between living people (as in our model). The latter are a non-trivial fraction of total transfers from parents to children (see for example, Cox & Raines, 1985; Gale & Scholz, 1994).

[12] Following Boldrin and Montes (2005), we assume that lifetime utility is unaffected by consumption when young; this reflects the fact that most children are financially dependent on their parents. It still makes sense, however, for the young to make an education decision (human capital investment) to the extent that children can choose how much to work at school and have some influence on their human capital accumulation.

[13] An alternative way to model the provision of parents to their children is to assume that the utility function of the children is an argument of the utility function of the parents (see Barro, 1974). However, Rangazas (2000) finds such a model is inconsistent with the data.

[14] Full depreciation of physical capital is a reasonable assumption, as each period in the model may correspond to 30–40 actual years.

[15] Parameter λ may also reflect rationing of credit by lenders owing to the expectation that some creditors may be left unable to repay debts owing to morbidity. In our model, $\lambda = 1$ corresponds to the maximum amount that an agent could repay when middle-aged. Hence, we choose this as the upper bound.

[16] Since we are interested in transfers that can be used for children's education, we abstract from bequests which arrive too late to be used for this purpose.

[17] Notice that $k_{t+1} = \Psi h_t^\delta \tilde{k}_t^{1-\delta}$ and $h_{t+1} = \Phi h_t^{1-\zeta(1-\delta)} \tilde{k}_t^{\zeta(1-\delta)}$ imply that $x_t = \frac{\Psi}{\Phi(1+n)} x_{t-1}^{(1-\delta)(1-\zeta)}$. Then the latter can be solved backwards to get (1).

[18] As shown in the proof of proposition 2, the social planner will set the investment variable $\hat{d} = \hat{\omega}$. Unlike in Del Rey and Lopez-Garcia (2017), $\hat{d}$ becomes an argument of the utility function just like $\hat{c}_y$ and $\hat{c}_o$, and so it is treated as such.

[19] The first optimality condition implies that $\bar{\mu}_t > 0$ either if (i) $w_t h_1(.) > R_t$ and $\lambda w_t h_1(.) < R_t$ or (ii) $w_t h_1(.) < R_t$ and $\lambda w_t h_1(.) > R_t$. However, only (i) is feasible because (ii) implies that $\lambda > 1$ and thus $c_{t,\,m} = -s_t + (1-\lambda) w_t h_t - (1+n)\omega_t < 0$, which is not possible. Then, using the other two optimality conditions, (i) further implies that $\bar{\mu}_t > 0$ only if $\underline{\gamma}^{bin} \equiv \frac{\lambda\delta\beta(\zeta-1)}{1-\delta(1-\lambda)} < \gamma < \frac{\delta\beta(\zeta-\lambda)}{1-\delta(1-\lambda)} \equiv \overline{\gamma}^{bin}$. Since $\underline{\gamma}^{bin} < 0$ while $\gamma > 0$, only $\gamma < \overline{\gamma}^{bin}$ is relevant for a binding borrowing constraint. It follows that, if $\lambda < \zeta$, then $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \overline{\gamma}^{bin}\} \neq \emptyset$.

[20] The result that $R \geq (1+n)(1+g)$ is necessary but not sufficient for dynamic efficiency holds even without warm glow preferences – see Del Rey and Lopez-Garcia (2017).

[21] See, for example, the references cited by Law and Singh (2014).

[22] Case a.ii indicates that, at a range of high intensities of parental transfers γ, young agents do not exploit the relatively high intergenerational persistence in human capital – as would a social planner – to generate more human capital for future generations, but rather choose to save part of the parental transfer, leading to over-accumulation of physical capital. This excess saving by the young is avoided when they are prevented from accessing credit. Case b.ii indicates that at a range of low levels of γ, the young use the financial market to borrow a relatively large amount to complement the parental transfer in investing in human capital. Excess borrowing that leads to a lower ratio of physical to human capital than the growth-maximizing one is avoided when the young are prevented from accessing the credit market.

[23] Proposition 3 (b.ii) can be compared with the finding of De La Croix and Michel (2007), that the maximum growth rate is achieved in a borrowing constrained regime as long as the elasticity of earnings to education is high enough. In our model, the elasticity of earnings to education corresponds to $\delta\zeta$. The proposition indicates that a high growth rate in the constrained economy ($\overline{g} > g$) can be achieved when ζ is high, which also implies that the elasticity is high. Likewise, a necessary condition of achieving a high growth rate in the constrained economy is that ζ must be high because the threshold $\tilde{\zeta}$ is high. The latter can be high only when δ is high, which implies an even higher elasticity. All these hold as long as γ is sufficiently small.

[24] Using the expressions for the growth rates along the BGP, it is straightforward to show that, at low values of γ, the growth rate in the economy with a missing credit market is strictly greater than the growth rate in the economy with limited access to credit.

[25] The negative relationship between investment in physical capital and γ under no credit market is explained by the fact that the credit market adds another channel for investment in physical capital – that is, if the amount transferred by parents is in excess of the optimal level, the young invest the excess amount in the credit market, which is then directed to investment in physical capital. When this channel is missing, relative investment in physical capital falls as the level of γ increases.

[26] Note that, in the parametric model with $\gamma > 0$, we have $\omega_t = \frac{\gamma}{1+n}c_{t,m}$ and hence $U_t = (1+\gamma)\ln(c_{t,\,m}) + \beta\ln(c_{t+1,\,o}) + constants$ and $(1+\gamma)c_{t,m} + s_t + R_t\bar{b}_{t-1} = w_t h_t$ (by the budget constraint of the middle-aged). Following the same steps as in Hatcher (2022), it is then straightforward to show the result.

[27] The ratio of household expenditures on education to parental consumption captures cross-country differences in γ, which influences the model BGP. Note that $\gamma = \frac{(1+n)\omega_t}{c_{t,m}}\frac{d_t}{d_t}$ can be rewritten as $\gamma = \gamma_{EX}\frac{\omega_t}{d_t}$.

[28] The ratio of total household expenditures on education to total household consumption in our model can be written as $int = \frac{(1+n)^{t-1}(1+n)d_t}{(1+n)^{t-1}c_{m,t} + (1+n)^{t-2}c_{o,t}} = \frac{(1+n)c_m}{(1+n)c_m + c_o}\frac{(1+n)d}{c_m} = \alpha_{c_m}\gamma_{EX}$.

[29] The model indicates that for zero household expenditure on education, credit has no effect on growth because production is trivially zero. Our regression specification is consistent with this implication of the model (see Equation (3)) because credit is not included as a separate regressor.

[30] The sample could have been extended to 92 if countries whose data begin in 1990 were included. The latter however would have caused inconsistencies with the rest of the countries whose data series are longer and also with the initial condition of the per capita level of output which is used as control variable.

[31] Note that regression (VI) consists almost entirely of developing economies.

[32] We have conducted additional robustness checks, including alternative definitions of the credit index with thresholds adjusted up or down by one-tenth, and we found similar results to those in Table 2.

[33] The threshold value of the credit index for specifications I–VII ranges between 0.39 and 0.55 while the average threshold value across all specifications is 0.43. Note that at sufficiently low levels of the intensity where, according to the theory, the sign of the response of the growth rate to intensity is the same across the three models, the models cannot be identified using (3). Owing to data limitations, our focus is only on estimating the differences in the slope of the credit index rather than identifying the three models across the full range of values of the intensity of household expenditures on education.

[34] In regressions (5) and (7), respectively 42% and 43% of the countries are below the credit threshold.

[35] Both GCD and Eurostat are based on the COICOP Education classification, which covers education services. Data for 49 countries were obtained from the Global Consumption Database (2010) and the rest from Eurostat. The Eurostat data correspond to 2010, except for Australia, Japan, South Korea, the United States (all 2012) and Canada and Saudi Arabia (2013); see EU (2015). Consumption data of these last six countries also appear in an earlier version of the same publication (see EU, 2013) which uses data from the early and mid-2000s. Using these consumption shares (or an average) does not affect our conclusions.

[36] The data correspond to averages for years 1995–2010, except for the Democratic Republic of Congo (average 1986–88, 2010). Missing values are excluded.

[37] First, we found the city/region in the database with the maximum absolute value of the latitude (Alert, Canada). We then normalized country latitude values by dividing the absolute value of their latitude (based on capital city) by that of Alert. The data is obtained from the World Cities database at simplemaps.com

[38] The data is obtained from the Databank of the World Bank's World Development Indicators. Any missing values are excluded from the calculation of averages.

[39] Percent (age 15+ years) of respondents who report borrowing any money from a bank or another type of financial institution in the past 12 months. An average value is computed based on years 2011 and 2014.

[40] Percent (age 15+ years) respondents who report having a credit card. Average value based on years 2011 and 2014.

[41] Domestic credit to the private sector (WDI) includes financial resources provided to the private sector, including loans, purchases of nonequity securities and trade credits and other accounts receivable, that establish a

claim for repayment. For some countries these claims include credit to public enterprises. Average value from 1990 to 2010 is used.

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APPENDIX A: DATA DESCRIPTION AND CREDIT INDEX CONSTRUCTION

Growth is measured using the average growth rate of real GDP per capita for the period 1970–2010 (in % p.a.) computed from Penn World Tables. The logarithm of per capita real GDP in 1970, $ln(y_{1970})$, is expressed in 2011 USD and is also obtained from Penn World Tables. The intensity of household expenditures on education, *int*, is defined as the share of consumption on education in total household consumption. It is computed using data from the Global Consumption Database 2010 (GCD), Eurostat (code: nama_10_co3_p3, 2010) and EU (2015).[35] The share of public expenditures on education on GDP (Public Expenditure Edu.) was computed using data from the World Bank Development Indicators.[36] The Political Stability index is an average of the years 1996 to 2000, which is obtained from the World Bank's Worldwide Governance Indicators (WGI) – its values range from −2.5 to +2.5.

The Latitude index, which is a measure of distance from the equator, is rescaled to lie between 0 and 1.[37] The growth rate of population is the average population growth rate (% p.a.) for the period 1970–2010 in Penn World Tables. Natural resource rents is the average value of

TABLE A1 Descriptive statistics

	Mean	Median	Max	Min	Standard Deviation
g	1.82	1.82	6.47	−2.77	1.55
Int	2.38	2.10	8.34	0.09	1.78
Credit	0.56	0.67	1	0	0.39
$ln(y_{1970})$	8.37	8.21	10.88	6.26	1.12
Public expenditure on education (%)	4.10	4.02	8.02	1	1.34
Political stability	−0.01	0.02	1.63	−2.67	1.04
Latitude	0.32	0.29	0.73	0.004	0.21
Natural resource rents	5.37	2.50	38.19	0	7.62
Property rights	5.81	5	9	1	2.18
Democracy	4.12	2.20	10	0	4.05
n[a]	1.71	1.86	4.02	−0.34	1.05

Dummy variables: % in the sample					
Region	**%**	**Legal origin**	**%**	**Former colony**	**%**
East Asia & Pacific	13.5	English	28.4	UK	22.9
Europe & C. Asia	29.7	French	55.4	France	20.2
America & Caribbean[b]	13.5	German	12.2	Europe (other)	17.6
Middle East & North Africa	5.4	Scandinavian[c]	4.1	Others[c]	39.2
South Asia	6.7				
Sub-Saharan Africa[c]	31.1				

[a]The growth rate of population is measured in percent (as is growth in real GDP per person).
[b]Rather than defining a separate North America group consisting of only the United States and Canada, we created an America & Caribbean group, defined as Latin America and the Caribbean (World Bank definition) plus the United States and Canada.
[c]Excluded categories in the regressions.

the sum of oil rents, natural gas rents, coal rents (hard and soft), mineral rents and forest rents for the period 1990–2002.[38] The democracy index corresponds the average of democracy score for the period 1970–1994, as used by La Porta et al. (1999) and constructed by Jaggers and Gurr (1996). Lower values indicate a less democratic environment.

The information about the legal origin of countries (legal origin dummy variable) is obtained from La Porta et al. (2008). Where available, the colonial origin of countries is based on Grier (1999) or Acemoglu et al. (2001). In cases where countries were colonized by multiple European nations, we followed the classification in Grier (1999); if a classification was not available, we identified the most recent colony using the CIA World Factbook 2018. Allowing countries to be classified as colonies of multiple European nations in our sample does not affect the main conclusions.

The credit index (*Credit*) was constructed using data from World Bank Financial Inclusion Database (FINDEX). It is based on three indicators of credit provision and financial development: (1) borrowed from a financial institution in the past year[39]; (2) credit card ownership[40]; (3) domestic credit to the private sector/GDP (average 1990–2010, %).[41]

The credit index is calculated as Credit Index$_i \equiv 1 - \frac{1}{3}\sum_{j=1}^{3} D_{i,j}$, where $D_{i,j}$ is a dummy variable equal to 1 if indicator j in country i is below a chosen threshold for the given indicator. Our chosen baseline thresholds were, respectively, 6%, 14%, and 25%.

APPENDIX B: PROOFS

Proof of Proposition 1. (i) Using the growth equation at the BGP and the functional forms of $\Psi(\gamma)$ and $\Phi(\gamma)$, it is straightforward to show that g is monotonically increasing in γ by taking the derivative of g with respect to γ.

(ii) It follows as part in (i). Note that, contrary to the case of complete markets, the ratio of investment in physical capital to output, $\overline{\Psi}(\gamma)$, is a monotonically decreasing function of γ.

(iii) $\frac{\partial \overline{\overline{g}}}{\partial \gamma} = \zeta[\varphi(\gamma) - 1]$, where $\varphi(\gamma) = \frac{\delta[1-\delta(1-\lambda)](1+\beta+\gamma)}{\gamma\frac{[1-\delta(1-\lambda)]+\beta\delta\lambda}{[1-\delta(1-\lambda)]+\beta[1-\delta(1-\zeta)]}}$ is strictly decreasing in γ. Then, since $\zeta < 1 - \delta(1-\zeta)$, $\varphi(\gamma)|_{\gamma=\gamma^{bin}} = \frac{[1-\delta(1-\lambda)]+\beta[1-\delta(1-\zeta)]}{\beta\zeta}$ is strictly greater than unity, which implies that for any $\gamma < \gamma^{bin}$, $\varphi(\gamma) > \varphi(\gamma)|_{\gamma=\gamma^{bin}}$. It follows that $\frac{\partial \overline{\overline{g}}}{\partial \gamma} > 0$ for any $\gamma \in \Omega^{bin}$. It is also straightforward to show that $\lim_{\gamma \to \gamma^{bin}} \frac{1+g(\gamma)}{1+\overline{\overline{g}}(\gamma)} = 1$. □

Proof of Proposition 2. (i) Since $s_t = b_t + k_{t+1} = (1+n)\overline{b}_t + k_{t+1} = (1+n)d_t - (1+n)\omega_t + k_{t+1}$, savings per efficient labour at the balanced growth path reduce to $\widehat{s} = (1+n)\widehat{d} - (1+n)\widehat{\omega} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$, where $\widehat{d} = d_t/h_t = \widehat{\omega} + \widehat{\overline{b}}$, $\widehat{\omega} = \omega_t/h_t$, $\widehat{\overline{b}} = \overline{b}/h_t$ and $B\mu^{1-\zeta}\widehat{d}^{\zeta} = 1+g$. The balanced growth version of the resource constraint is obtained by dividing all terms of (6) by h_t: $Ax^{1-\delta} = \widehat{c}_m + \frac{\widehat{c}_o}{B\mu^{1-\zeta}\widehat{d}^{\zeta}}\frac{1}{1+n} + (1+n)\widehat{d} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$. The social planner maximizes $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$ subject to the latter. Note that the $\widehat{\overline{b}}$ terms cancel out in the BGP resource constraint, which implies that the planner will set $\widehat{d} = \widehat{\omega}$, and so the planner's choice variables reduce to $\{\widehat{c}_m, \widehat{c}_o, x, \widehat{d}\}$. The optimal conditions of the social planner can be summarized as follows:

$$(1-\delta)Ax^{-\delta} = (1+n)(1+g), \tag{A.1}$$

$$\frac{\gamma\widehat{c}_m}{\widehat{d}} = -\frac{\zeta\widehat{c}_o}{(1+n)(1+g)\widehat{d}} + (1+n) + \frac{\zeta x(1+n)(1+g)}{\widehat{d}}, \tag{A.2}$$

$$\widehat{c}_o = \beta(1+n)(1+g)\widehat{c}_m, \tag{A.3}$$

where (A.1) is the optimal condition for x, (A.2) is implied by the optimal conditions for $\widehat{c}_m$ and $\widehat{d}$ and (A.3) is implied by the optimal conditions for $\widehat{c}_o$ and $\widehat{c}_m$. According to definition 2, the BGP is dynamically inefficient if a reduction in x induces a strictly positive change in either $\widehat{c}_m$ or $\widehat{c}_o$ or $\widehat{d}$ of current generations as

well as generations of transient periods. Using the BGP resource constraint, it can be shown that $\frac{\partial \hat{c}_m}{\partial x}\big|_{\hat{c}_o,\hat{d}} = (1-\delta)Ax^{-\delta} - (1+n)B\mu^{1-\zeta}\widehat{d}^{\zeta} \equiv R - (1+n)(1+g)$, $\frac{\partial \hat{c}_o}{\partial x}\big|_{\hat{c}_y,\hat{d}} = (1+n)(1+g)[R-(1+n)(1+g)]$ and $\frac{\partial \hat{d}}{\partial x}\big|_{\hat{c}_y,\hat{c}_o} = \frac{R-(1+n)(1+g)}{\Phi}$, where Φ is equal to the right hand side of (A.2). Condition $R \geq (1+n)(1+g)$ ensures that $\frac{\partial \hat{c}_m}{\partial x}\big|_{\hat{c}_o,\hat{d}} \geq 0$ and $\frac{\partial \hat{c}_o}{\partial x}\big|_{\hat{c}_y,\hat{d}} \geq 0$, when they are evaluated at the laissez-faire equilibrium. The only case where $R \geq (1+n)(1+g)$ may not be sufficient for dynamic efficiency of the BGP is when $R > (1+n)(1+g)$ and $\Phi < 0$, where Φ is evaluated at the laissez-faire equilibrium. $\Phi < 0$ only if $R/(1+n)(1+g) > \gamma/\delta\zeta\beta$. Therefore, a sufficient condition for dynamic efficiency of the laissez-faire BGP is $1 \leq R/(1+n)(1+g) \leq \gamma/\delta\zeta\beta$, for $\gamma/\delta\zeta\beta > 1$. If the left inequality holds but the right does not, then the BGP is not dynamically efficient because $R/(1+n)(1+g) \geq 1$ implies that $R/(1+n)(1+g) > \gamma/\delta\zeta\beta$ for $\gamma/\delta\zeta\beta < 1$. Since $R/(1+n)(1+g) = (1-\delta)A\Psi^{-1}$, using the BGP laissez-faire condition, $x = \Psi^{1/\delta}/[(1+n)(1+g)]^{1/\delta}$, the sufficient condition for dynamic efficiency of the BGP, in terms of γ, is $\gamma_1^c \equiv \frac{A\zeta\beta\delta(1-\delta)}{\Psi} \leq \gamma \leq \frac{[1-\delta(1-\zeta)](1+\beta) - \delta\beta(1-\zeta)}{\delta(1-\zeta)} \equiv \gamma_2^c$ for any $\gamma \geq \delta\zeta\beta$, where $\gamma_2^c > 0$ only if $\delta < (1+\beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$.

(ii) In the economy with no credit market, we denote Φ and Ψ with $\overline{\Phi}$ and $\overline{\Psi}$, respectively. Then, following the proof of part (i), using the functional form of the optimal $\hat{d}$, at the laissez-faire BGP with no-credit market, the functional form of $\overline{\Psi}$, and replacing g with $\overline{g}$, it is straightforward to show that $\overline{\Phi} < 0$ only if $R/(1+n)(1+\overline{g}) > \delta\gamma_R^c + 1$, where $\gamma_R^c = \gamma/\delta\zeta\beta$. Therefore, the sufficient condition for dynamic efficiency of the laissez-faire BGP of the economy with no-credit market is $1 \leq R/(1+n)(1+\overline{g}) \leq \delta\gamma_R + 1$. Since $R/(1+n)(1+\overline{g}) = (1-\delta)A\overline{\Psi}^{-1}$, the left-hand side of the inequality reduces to $\gamma \geq \gamma_1^{in} \equiv \frac{\beta\delta - (1-\delta)(1+\beta)}{1-\delta}$, while the right-hand side reduces to $\gamma \geq \gamma_2^{in} \equiv \frac{\zeta[1-\delta(1+\beta)]}{1-\zeta(1-\delta)}$. It follows that the laissez-faire BGP of the economy with no-credit market is dynamically efficient if $\gamma \in \Omega^{in} \equiv \{\gamma > 0; \gamma \geq \gamma^{in}\} \neq \varnothing$, where $\gamma^{in} = max\{\gamma_1^{in}, \gamma_2^{in}\}$.

(iii) In the economy with a binding borrowing constraint, we denote Φ and Ψ with $\overline{\overline{\Phi}}$ and $\overline{\overline{\Psi}}$, respectively. Following the proof of part (i), using the functional form of the optimal $\hat{d}$, at the laissez-faire BGP with a binding borrowing constraint, the functional form of $\overline{\overline{\Psi}}$, and replacing g with $\overline{\overline{g}}$, it is shown that $\overline{\overline{\Phi}} < 0$ only if $\frac{R}{(1+n)(1+\overline{\overline{g}})} > \frac{\gamma[(1-\delta)(\zeta\beta+\gamma)+(\beta+\gamma)\delta\lambda]}{\zeta\beta[\gamma(1-\delta)+(\beta+\gamma)\delta\lambda]} \equiv \overline{\gamma}_R$. Therefore, the sufficient condition for dynamic efficiency of the laissez-faire BGP is $1 \leq \frac{R}{(1+n)(1+\overline{\overline{g}})} \leq \overline{\gamma}_R$ (notice that, when $\lambda = 0$, which implies that $\overline{b} \leq 0$, $\overline{\gamma}_R = \delta\gamma_R + 1$). Since $\frac{R}{(1+n)(1+\overline{\overline{g}})} = \frac{(1-\delta)A}{\overline{\overline{\Psi}}}$, the latter inequality can also be written as $\overline{\gamma}_1^{in} \equiv \beta\frac{\delta(1-\lambda)}{1-\delta(1-\lambda)} - (1+\beta) \leq \gamma \leq \frac{\gamma\delta(1-\lambda)}{\zeta[1-\delta(1-\lambda)]}$ $\frac{\beta[\delta\lambda+(1-\delta)\zeta]+\gamma[1-\delta(1-\lambda)]}{\beta\delta\lambda+\gamma[1-\delta(1-\lambda)]} - (1+\beta) \equiv \overline{\gamma}_2^{in}$. Since $\overline{\gamma}_2^{in}$ is increasing in γ and $\gamma \in \Omega^{bin}$, the upper limit of the inequality for dynamic efficiency is $\overline{\gamma}_2^{in}\big|_{\gamma=\gamma^{bin}} = \beta\frac{\delta(\zeta-\lambda)(1-\lambda)}{\zeta[1-\delta(1-\lambda)]^2} - (1+\beta)$. Thus, dynamic efficiency is possible when $\gamma \in \Omega^{bin}$ as long as, (i) $\zeta \geq \lambda/\delta(1-\lambda) \equiv \overline{\zeta}_{low}$, which implies that $\overline{\gamma}_2^{in}\big|_{\gamma=\gamma^{bin}} \geq \overline{\gamma}_1^{in}$, (ii) $\beta\left[\frac{\delta(\zeta-\lambda)(1-\lambda)}{\zeta[1-\delta(1-\lambda)]^2} - 1\right] > 1$, which implies that $\overline{\gamma}_2^{in}\big|_{\gamma=\gamma^{bin}} > 0$. In addition, dynamic efficiency under a binding constraint requires $\gamma^{bin} \geq \overline{\gamma}_1^{in}$, which implies that $\beta\delta\frac{(1-\zeta)+(1-\lambda)}{1-\delta(1-\lambda)} \leq 1$.

The latter along with (ii) imply that $\beta_{low} \equiv \frac{\zeta[1-\delta(1-\lambda)]^2}{\delta(\zeta-\lambda)(1-\lambda)-\zeta[1-\delta(1-\lambda)]^2} < \beta \le \frac{1-\delta(1-\lambda)}{\delta[(1-\zeta)+(1-\lambda)]} \equiv \beta_{high}$. Then, it must be the case that $\beta_{high} > \beta_{low}$, which holds only if $\zeta < \frac{\delta(\zeta-\lambda)(1-\lambda)-\zeta[1-\delta(1-\lambda)]^2}{\delta[1-\delta(1-\lambda)][(1-\zeta)+(1-\lambda)]} \equiv \zeta_{high}$. The latter implies that $\delta(\zeta-\lambda)(1-\lambda) - \zeta[1-\delta(1-\lambda)]^2 > 0$, since $\zeta > 0$, which then reduces to $\zeta > \frac{\delta\lambda(1-\lambda)}{\delta(1-\lambda)-[1-\delta(1-\lambda)]^2} \equiv \zeta_{low}$, where $\delta(1-\lambda) > [1-\delta(1-\lambda)]^2$. Since $\zeta_{low} > \overline{\zeta}_{low}$, $\beta_{high} > \beta_{low}$ when $\zeta_{low} < \zeta < \zeta_{high}$. Note that ζ_{high} can be either monotonically decreasing or monotonically increasing, depending on the values of the parameters. Since $\lim_{\zeta\to 0^+}\zeta_{high} < 0$, if ζ_{high} is monotonically decreasing in ζ, then dynamic efficiency is impossible under a binding borrowing constraint as $\zeta > 0$. The only feasible case of dynamically efficient BGP with a binding constraint is when ζ_{high} is monotonically increasing in ζ. Then, since $\lambda < \zeta$, under a binding constraint, it must be that $\lambda < \lim_{\zeta\to 1^-}\zeta_{high} = \frac{\delta(1-\lambda)^2-[1-\delta(1-\lambda)]^2}{\delta(1-\lambda)[1-\delta(1-\lambda)]}$, which can be rewritten as $\phi_1(x) \equiv 1 - x + (1-\lambda)x^2 < 2(1-\lambda)x \equiv \phi_2(x)$, where $x = \delta(1-\lambda)$. Note that $\phi_1(x)$ is a convex function that reaches a minimum at $x = 1/2(1-\lambda)$ while $\phi_2(x)$ is a linear function, with a positive slope, that passes through the origin. Since $\phi_1(0) = 1$, there are two intersection points between $\phi_1(x)$ and $\phi_2(x)$ that lie in the positive area of x. Specifically, $x_{1,2} = 1 + \frac{1\pm\sqrt{4(1-\lambda)^2+1}}{2(1-\lambda)}$. It follows that $\phi_1(x)$ lies below $\phi_2(x)$ only if $x_1 < x < x_2$. Since $x_1 > 1$, there is no $x \equiv \delta(1-\lambda) < 1$ such that $x_1 < x < x_2$. Therefore, dynamic efficiency of the laissez-faire BGP is impossible when the borrowing constraint binds. If the borrowing constraint is slack, the complete markets laissez-faire BGP and associated condition for dynamic efficiency apply – part (i). □

Proof of Proposition 3. Along the BGP, we would like to examine the conditions under which $g = \overline{g}$, $g > \overline{g}$ and $g < \overline{g}$. The latter is equivalent to $\Upsilon_1(\xi) = \Upsilon_2(\xi)$, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ and $\Upsilon_1(\xi) < \Upsilon_2(\xi)$, respectively, where $\xi = \gamma(1-\delta)(\beta\delta\zeta)^{-1}$, $\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta) + \zeta[1-\delta(1-\zeta)]^{-1}\xi$ and $\Upsilon_2(\xi) = \xi^\delta$, using the equations for $(1+g)$ and $(1+\overline{g})$ of sections 3.1 and 3.2.1. Notice that $\Upsilon_1(\xi)$ is a linear and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta)$, while $\Upsilon_2(\xi)$ is a concave and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_2(\xi) = 0^+$ and $\lim_{\xi\to +\infty}\Upsilon_2(\xi) = +\infty$. Since $\lim_{\xi\to 0}\Upsilon_1(\xi) > \lim_{\xi\to 0}\Upsilon_2(\xi)$, $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ do not intersect for any value of ξ if and only if $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. The latter is the case only if $\Upsilon_1(\xi^*) > \Upsilon_2(\xi^*)$, where $\partial\Upsilon_1(\xi^*)/\partial\xi = \partial\Upsilon_2(\xi^*)/\partial\xi$, which implies $\xi^* = [\delta(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. In other words, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ only if $X^{1-\delta} > 1 - \left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right]X$, where $X = [(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. Let the left-hand side of the X inequality be denoted by X_1 and the right-hand side by X_2. Notice that it cannot be the case that $X \le 1$ because that would imply that $\zeta \ge 1$. X_1 is an increasing and concave function of X which starts almost (since $X > 0$) from the origin. X_2 is a linear and decreasing function of X with $\lim_{X\to 0^+}X_2(X) = 1$. It follows that the X inequality may hold only in the region on the right of the intersection point of X_1 and X_2. In this region, $X_2 < 1$ which implies that $\left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right]X > 0$. Given that the term in brackets is negative, the only way the latter holds is when $X < 0$, which cannot hold since $X > 0$. Thus, it cannot be the case that $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. It follows that $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ have at least one, and at most two, intersection

points. At least one of the points of intersection is the point where $\xi = 1$ since $\Upsilon_1(1) = \Upsilon_2(1) = 1$. It can be shown that there are three feasible cases. In case 1, the slope of $\Upsilon_2(\xi)$ is greater than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, that is, $\zeta < \delta(1 + \delta)^{-1}$. Then, there exists $\bar{\xi} > 1$ such that $\Upsilon_1(\bar{\xi}) = \Upsilon_2(\bar{\xi})$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < 1$ and $\xi > \bar{\xi}$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $1 < \xi < \bar{\xi}$. Therefore, under case 1, the relationship between the growth rates can be summarized, as follows: (i) $g = \bar{g}$ if $\xi = 1$ or $\xi = \bar{\xi}$ ($\gamma = \gamma^*$ or $\gamma = \bar{\gamma}$); (ii) $g < \bar{g}$ if $1 < \xi < \bar{\xi}$ ($\gamma^* < \gamma < \bar{\gamma}$); (iii) $g > \bar{g}$ if $\xi < 1$ or $\xi > \bar{\xi}$ ($\gamma < \gamma^*$ or $\gamma > \bar{\gamma}$). In case 2, the slope of $\Upsilon_2(\xi)$ is smaller than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, that is, $\zeta > \delta(1 + \delta)^{-1}$. Then, there exists $\underline{\xi} < 1$ such that $\Upsilon_1(\underline{\xi}) = \Upsilon_2(\underline{\xi})$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < \underline{\xi}$ and $\xi > 1$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $\underline{\xi} < \xi < 1$. Therefore, under case 2, the relationship between the growth rates can be summarized, as follows: (i) $g = \bar{g}$ if $\xi = \underline{\xi}$ or $\xi = 1$ ($\gamma = \underline{\gamma}$ or $\gamma = \gamma^*$); (ii) $g < \bar{g}$ if $\underline{\xi} < \xi < 1$ ($\underline{\gamma} < \gamma < \gamma^*$); (iii) $g > \bar{g}$ if $\xi < \underline{\xi}$ or $\xi > 1$ ($\gamma < \underline{\gamma}$ or $\gamma > \gamma^*$). In case 3, the slope of $\Upsilon_2(\xi)$ is equal to the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, that is, $\zeta = \delta(1 + \delta)^{-1}$. In this case, $\xi = 1$ is the single point of contact between $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ while in all other cases, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$. Therefore, under case 3, the relationship between the growth rates can be summarized, as follows: (i) $g = \bar{g}$ if $\xi = 1$ ($\gamma = \gamma^*$); (ii) $g > \bar{g}$ if $\xi < 1$ and if $\xi > 1$ ($\gamma < \gamma^*$ and if $\gamma > \gamma^*$). Note that the relationships in brackets are due to the fact that for $\bar{\xi}$ and $\underline{\xi}$, there are unique thresholds $\bar{\gamma}$ and $\underline{\gamma}$ such that $\beta\delta\zeta\bar{\xi} = \bar{\gamma}(1 - \delta)$ and $\beta\delta\zeta\underline{\xi} = \underline{\gamma}(1 - \delta)$. Since $\bar{\xi} > 1$ and $\underline{\xi} < 1$, the latter implies that $\bar{\gamma} > 1$, $\underline{\gamma} < 1$ and $0 < \underline{\gamma} < \gamma^* < \bar{\gamma}$. □.

Proof of Proposition 4. Along the BGP, when $\lambda < \zeta$ and $\gamma \in \Omega^{bin}$, we would like to examine the conditions under which $g = \bar{\bar{g}}$, $g > \bar{\bar{g}}$ and $g < \bar{\bar{g}}$. The latter is equivalent to $\overline{\Upsilon}_1(\bar{\xi}) = \overline{\Upsilon}_2(\bar{\xi})$, $\overline{\Upsilon}_1(\bar{\xi}) > \overline{\Upsilon}_2(\bar{\xi})$ and $\overline{\Upsilon}_1(\bar{\xi}) < \overline{\Upsilon}_2(\bar{\xi})$, respectively, where $\bar{\xi} = [\gamma[1 - \delta(1 - \lambda)] + \beta\delta\lambda]/\beta\delta\zeta$, $\overline{\Upsilon}_1(\bar{\xi}) = \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\lambda)[1-\delta(1-\zeta)]} + \left(\frac{\zeta}{(1-\lambda)[1-\delta(1-\zeta)]}\right)\bar{\xi}$ and $\overline{\Upsilon}_2(\bar{\xi}) = \bar{\xi}^\delta$, using the equations for $(1+g)$ and $\left(1+\bar{\bar{g}}\right)$ of sections 3.1 and 3.2.2. Notice that $\overline{\Upsilon}_1(\bar{\xi})$ is a linear and increasing function of $\bar{\xi}$, while $\overline{\Upsilon}_2(\bar{\xi})$ is a concave and increasing function of $\bar{\xi}$, with $\lim_{\bar{\xi}\to 0^+}\overline{\Upsilon}_2(\bar{\xi}) = 0^+$ and $\lim_{\bar{\xi}\to+\infty}\Upsilon_2(\bar{\xi}) = +\infty$. Since $\bar{\xi}$ is a function of γ, $\overline{\Upsilon}_1$ and $\overline{\Upsilon}_2$ can be written as $\overline{\Upsilon}_1(\gamma)$ and $\overline{\Upsilon}_2(\gamma)$. The properties of $\overline{\Upsilon}_1(\bar{\xi})$ and $\overline{\Upsilon}_2(\bar{\xi})$ imply that there might be either zero or, at most, two intersection points between $\overline{\Upsilon}_1(\bar{\xi})$ and $\overline{\Upsilon}_2(\bar{\xi})$. A sufficient condition for no intersection points between $\overline{\Upsilon}_1(\bar{\xi})$ and $\overline{\Upsilon}_2(\bar{\xi})$, that is, $\overline{\Upsilon}_1(\bar{\xi}) > \overline{\Upsilon}_2(\bar{\xi})$ for all values of $\bar{\xi}$, is that $\overline{\Upsilon}_1(\bar{\xi}^*) > \overline{\Upsilon}_2(\bar{\xi}^*)$ for $\bar{\xi}^*$ such that $\partial\overline{\Upsilon}_1(\bar{\xi}^*)/\partial\bar{\xi} = \partial\overline{\Upsilon}_2(\bar{\xi}^*)/\partial\bar{\xi}$. The latter reduces to $\bar{\xi}^* = (\delta(1-\lambda)[1-\delta(1-\zeta)]/\zeta)^{\frac{1}{1-\delta}}$. Thus, $\overline{\Upsilon}_1(\bar{\xi}^*) > \overline{\Upsilon}_2(\bar{\xi}^*)$ implies that $\overline{X}^{\frac{\delta}{1-\delta}} < \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\delta)(1-\lambda)[1-\delta(1-\zeta)]}$, where $\overline{X} = \delta(1 - \lambda)[1 - \delta(1 - \zeta)]/\zeta$. Since, $\overline{X} > 0$, it must be the case that $\lambda < (1 - \zeta)[1 - \delta(1 - \lambda)]$, which also implies that $\zeta < 1 - \delta(1 - \zeta)$ since $\lambda > 0$. Then, it follows that $\lambda < 1 - \delta(1 - \zeta)$ and, thus, $\lambda < (1 - \zeta)[1 - \delta(1 - \lambda)] < (\lambda - \zeta)/\lambda$. Since $\lambda > 0$, the latter can hold only if $\lambda > \zeta$, which cannot be the case since $\lambda < \zeta$. Therefore, it cannot be the case that $\overline{\Upsilon}_1(\bar{\xi})$ and $\overline{\Upsilon}_2(\bar{\xi})$ have no intersection points. In what follows, we focus on the cases where there is either one or two intersection points. At least one of the points of intersection is the point where $\bar{\xi} = 1$ since

$\overline{\Upsilon}_1(1) = \overline{\Upsilon}_2(1) = 1$. Following the proof of proposition 3, it can be shown that there are four feasible cases. In case 1, there is a single intersection point only when the intercept of $\overline{\Upsilon}_1(\overline{\xi})$ is negative, that is, $\lambda \geq (1-\delta)(1-\zeta)[1-\delta(1-\zeta)]^{-1} \equiv \widetilde{\lambda}$. Note that the unique intersection point must be 1. Since $\overline{\xi} = 1$ implies that $\gamma = \gamma^{bin}$, it follows that (i) $g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and (ii) $g < \overline{\overline{g}}$ if $\gamma < \gamma^{bin}$. For cases 2–4, the intercept $\overline{\Upsilon}_1(\overline{\xi})$ is strictly positive, that is, $\lambda < \widetilde{\lambda}$. For case 2, recall that when $\overline{\xi} = 1$, $\gamma = \gamma^{bin}$. Thus, when the slope of $\overline{\Upsilon}_2(\overline{\xi})$ is greater than the slope of $\overline{\Upsilon}_1(\overline{\xi})$ at $\overline{\xi} = 1$, that is, $\zeta < \delta(1-\delta)$ $(1-\lambda)[1-\delta^2(1-\lambda)]^{-1} \equiv \widetilde{\zeta}^*$, then $\overline{\Upsilon}_1(\overline{\xi}) \geq \overline{\Upsilon}_2(\overline{\xi})$ for any $\overline{\xi} \leq 1$ or, equivalently, (i) $g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and (ii) $g > \overline{\overline{g}}$ if $\gamma < \gamma^{bin}$, since the borrowing constraint will not bind if $\gamma \geq \gamma^{bin}$ and the economy will behave as in the case of complete markets. In case 3, the slope of $\overline{\Upsilon}_2(\overline{\xi})$ is smaller than the slope of $\overline{\Upsilon}_1(\overline{\xi})$ at $\overline{\xi} = 1$, that is, $\zeta > \delta(1-\delta)$ $(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. Then, there exists $0 < \xi^* < 1$ such that $\overline{\Upsilon}_1(\underline{\xi}^*) = \overline{\Upsilon}_2(\underline{\xi}^*)$, $\overline{\Upsilon}_1(\overline{\xi}) > \overline{\Upsilon}_2(\overline{\xi})$ for $\overline{\xi} < \xi^*$, and $\overline{\Upsilon}_1(\overline{\xi}) < \overline{\Upsilon}_2(\overline{\xi})$ for $\xi^* < \overline{\xi} < 1$. For any $\overline{\xi} > 1$, the borrowing constraint will not bind, and thus it will behave as in the case of complete markets. It follows that there is γ_2, as long as $\lambda < (1-\zeta)[1-\delta(1-\lambda)] = \lambda_\gamma$, such that $\gamma_2 < \gamma^{bin}$. Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g = \overline{\overline{g}}$ if $\gamma = \gamma_2$ or $\gamma \geq \gamma^{bin}$, (ii) $g < \overline{\overline{g}}$ if $\underline{\gamma}_2 < \gamma < \gamma^{bin}$ and (iii) $g > \overline{\overline{g}}$ if $\gamma < \underline{\gamma}_2$. Finally, in *case 4*, the slope of $\overline{\Upsilon}_2(\overline{\xi})$ is equal to the slope of $\overline{\Upsilon}_1(\overline{\xi})$ at $\overline{\xi} = 1$, that is, $\zeta = \delta(1-\delta)$ $(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. In this case, $\overline{\xi} = 1$ is the single point of contact between $\overline{\Upsilon}_1(\overline{\xi})$ and $\overline{\Upsilon}_2(\overline{\xi})$ while in all other cases, $\overline{\Upsilon}_1(\overline{\xi}) > \overline{\Upsilon}_2(\overline{\xi})$. Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g = \overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and (ii) $g > \overline{\overline{g}}$ if $\gamma < \gamma^{bin}$. $\qquad\square$

Proof of Proposition 5. Using the first-order conditions with respect to $c_{m,t}$ and $c_{o,t+1}$ and the fact that $R_t = (1-\delta)Ax_t^{-\delta}$, $c_{o,t} = \beta(1-\delta)Ax_t^{-\delta}c_{m,t-1}$. Then, using the latter and the notation for complete markets, the share of parental consumption in total consumption at the BGP is written as $\alpha_{c_m} = \frac{(1+n)c_m}{(1+n)c_m + c_o} = \frac{1+n}{1+n+\beta(1-\delta)A\frac{x^{-\delta}}{1+g}}$, where $c_{m,t}/c_{m,t-1} = 1+g$. The functional forms of $1+g$, and x at the BGP, imply that $\frac{x^{-\delta}}{1+g} = \frac{1+n}{\Psi}$ and so the share can be written as $\alpha_{c_m} = \frac{1}{1+\beta(1-\delta)\frac{A}{\Psi}}$, where Ψ is replaced with $\overline{\Psi}$ for the no-credit market model and $\overline{\overline{\Psi}}$ for the model with a binding borrowing constraint. In the complete markets model, $\frac{d(int)}{d\gamma} = \frac{\beta\delta\zeta(\beta+\gamma)[1+\beta-\beta\delta(1-\zeta)]}{[\delta\beta(1-\zeta)+\gamma]^2} + \frac{\alpha_{c_m}\delta\zeta}{1-\delta(1-\zeta)} > 0$. In the no-credit market model, $\frac{d(int)}{d\gamma} = \alpha_{c_m}\left[1 - \frac{(1-\delta)\gamma}{1+(1-\delta)(1+\beta+\gamma)}\right] > 0$. Finally, in the model where the borrowing constraint is binding, $\frac{d(int)}{d\gamma} = \alpha_{c_m}\gamma_{EX}\left[\frac{1}{\gamma} - \frac{1-\delta(1-\lambda)}{\delta[1+(1+\beta+\gamma)[1-\delta(1-\lambda)]]}\right]$. The term in brackets is negative only if $\gamma > \frac{\delta[1+(1+\beta)[1-\delta(1-\lambda)]]}{(1-\delta)[1-\delta(1-\lambda)]}$. The latter and the binding constraint imply that $\frac{\delta[1+(1+\beta)[1-\delta(1-\lambda)]]}{(1-\delta)[1-\delta(1-\lambda)]} < \gamma < \frac{\beta\delta(\zeta-\lambda)}{1-\delta(1-\lambda)}$, which further implies that $\zeta - \lambda > \frac{1+[1-\delta(1-\lambda)](1+\beta)}{1-\delta}$, which is impossible to hold since the right-hand side is less than unity and the left-hand side greater than unity. It follows that $\frac{1}{\gamma} - \frac{1-\delta(1-\lambda)}{\delta[1+(1+\beta+\gamma)[1-\delta(1-\lambda)]]} > 0$, which implies that $\frac{d(int)}{d\gamma} > 0$ for the case of the binding borrowing constraint as well. $\qquad\square$