UK monetary policy in an estimated DSGE model with financial frictions

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Abstract

This paper develops a dual-state monetary DSGE model that accommodates a refined financial accelerator to analyze UK monetary policy. Unconventional monetary policy (QE) is interpreted as expanding the central bank's purchases of bonds using M0 to offset financial disruptions at the ZLB. Within a collateral-augmented costly state verification framework, M0 enters the financial accelerator as the cheapest collateral and reduces the cost of credit. The model is tested and estimated using indirect inference and found to fit the UK data for key variables over 1993–2016. We find that while financial shocks are significant, it is productivity shocks that had slowed down the recovery for 2009–2012. Alternative monetary regimes are evaluated and compared.

1. Introduction

Since 1992Q4, the Bank of England (BoE) has conducted monetary policy mainly by manipulating the level of nominal interest rate within the inflation-targeting framework. Besides, it has also engaged in Open Market Operations (OMOs) via selling or buying short-term government bonds in the open market, to impact commercial banks’ reserves and their ability to lend to the public. The resulting monetary and credit policies have proved to be a success vindicated by the muted economic volatility observed within the period 1993–2007 throughout which the UK had experienced 60 consecutive quarters of GDP growth.

However, the Global Financial Crisis (GFC), ignited by the US subprime mortgage problem, has changed this situation. Not only has it led to a severe economic downturn in the UK such as falling asset prices and declining investment and output, it also caused financial turmoil characterized by a sharply widening credit spread that peaked in 2008Q4 in Fig. 1. Confronted with the deterioration in both financial and real activities, the BoE reacted by cutting the nominal interest rate to near zero in 2009Q1. At the same time, it also embarked on more aggressive bond purchases known as Quantitative Easing (QE), when any further rate cuts were constrained by the Zero Lower bound (ZLB). Shortly after the announcement of QE1, the credit spread narrowed remarkably, and by the end of 2009 it returned to the level it was in early 2007; meanwhile, M0 supply rose substantially, leading to a rapid expansion in BoE’s balance sheet as a by-product. These observations motivate the discussions about the economic mechanisms whereby QE can affect credit spread and economic activities, and also the
development of quantitative monetary models that are able to study unconventional monetary policy in the same manner that existing frameworks can analyze conventional monetary policy.

The purpose of this paper is not to model the way QE works in reality, but whether a model that accommodates a collateral-augmented Bernanke et al. (1999) (BGG) framework to allow for direct credit easing effect can match the UK data. In doing so, we necessarily abstract from the most relevant facts and envisage an economic mechanism through which QE has its impact on the credit supply. To allow for the analysis of the financial market, we embed the BGG financial accelerator in the well-known reference model of Smets and Wouters (2007) (SW (07)). The factor underlying the BGG-type financial frictions is the possibility of non-payment on risky loans. Within the BGG framework, information asymmetry between borrowers (firms) and lenders (commercial banks) and monitoring costs associated with default for banks give rise to a wedge between the lending and deposit rates (credit spread). To add QE to it, we assume that the M0 injected by the central bank via bond purchases is acquired by firms and then used as the cheapest (most liquid) collateral for their loan agreements with banks; M0 thus acts as a powerful agent of credit growth to facilitate banks’ lending by reducing the credit spread for given leverage. To capture the environment since 2009Q1, we additionally specify a crisis state where the nominal interest rate is constrained at the ZLB whilst QE is activated to stabilize the credit spread (credit supply). Our aim is to see whether the evolution of the UK economy over 1993–2016, i.e. with the post-crisis period included, can be plausibly explained by such a dual state model and to assess whether alternative monetary policies that respond to financial market indicators in normal times can prevent the build-up of financial imbalances and enhance macroeconomic stability.

In standard DSGE models, QE can only work via signaling the future lower interest rates (signaling channel) assuming information frictions (see e.g. Eggertsson and Woodford, 2003). Since the GFC, there have been growing attempts to model QE via other channels. In doing so, these models depart from the conventional DSGE framework by adding specific structures. Most studies model QE via the portfolio balance channel by assuming imperfect substitutability between assets of different classes or maturities, so that QE can cause portfolio switches towards riskier assets with its associated adjustment costs affecting the bond yields. Examples in this direction include, among others, Harrison (2012), Harrison (2017), Falagiarda (2014), Ellison and Tischbirek (2014), Sahuc (2016), Priftis and Vogel (2016, 2017), Hohberger et al. (2019, 2020). Meanwhile, another strand of literature has attempted to model QE via the lending/credit channel. Studies in this direction include Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Hilberg and Hollmayr (2013), Le et al. (2016), Del Negro et al. (2017). Within these frameworks, the precondition for QE to have effects is the presence of financial frictions along the lines of Kiyotaki and Moore (1997, 2012), Bernanke et al. (1999), or Iacoviello (2005). Adding financial frictions enables these models to capture disruptions in the financial intermediation that could motivate the central bank’s credit interventions that have been in play since the GFC. It is in this kind of financial turmoil that QE can play its role in facilitating lending. So far, the majority of the DSGE studies on QE have focused on either the Fed Reserve’s (FED’s) or European Central Bank’s (ECB’s) QE programs, whereas very little has been done on the BoE’s QE measures. Accordingly, our paper aims to build a model for the UK to analyze its conventional and unconventional monetary policies. We choose the UK as our object of study since it is a typical medium-sized open economy subject to a lot of exogenous shocks from other parts of the world. Moreover, it has a defined and independent QE policy compared with other large European countries such as Germany and

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**Fig. 1.** Some facts since 2007. Notes: The data are obtained from the BoE and the Federal Reserve Economic Data (FRED). The QE timeline is generated based on Joyce (2013).
France whose policies must be coordinated with the ECB. A small open economy monetary DSGE model built for the UK can be instructive in terms of modeling the monetary policies in other small open economies like Switzerland and Sweden that have pursued similar QE policies since the GFC.

Our approach falls in the subset of literature that studies QE from the credit/lending channel. Compared with typical credit-channel models, we adopt the idea of using cash (M0) as the cheapest collateral, which is a novel mechanism to reduce the credit spread. By augmenting the standard BGG-type contracts with collateral requirements, we model the central bank’s effort to stabilize the financial market via specifying a feedback rule from M0 supply to the credit spread. This proves to fit the UK data including the post-crisis periods in our II test. Given that the UK economy is highly open with its trade openness averaging around 54% over the sample period, we adopt a small one economy setting by assuming the ZLB as an additional channel that has been missing in closed-economy models. Moreover, in studying QE in the presence of an occasionally binding ZLB, we let the ZLB be endogenously determined as a result of shocks’ perturbation instead of the central bank’s commitment to fix the policy rate for a pre-determined duration. Endogenizing the ZLB means we are able to create 1000 pseudo histories in model simulations that could have occurred in the UK where the ZLB can bind at any point for any length of time. In other words, QE is not time-dependent but state-dependent where it is only triggered when the ZLB binds following a sequence of strong adverse shocks and stops when the policy rate escapes from the ZLB as the effect of shocks dies out. We believe this enables the model to provide a more precise and articulated description of the actual economy.

The rest of the paper is structured as follows. Section 2 sets out the DSGE model with emphasis on our modifications to the canonical SW(07) model. Section 3 tests and estimates the model against the UK data using indirect inference whereby we believe this enables the model to provide a more precise and articulated description of the actual economy.

Section 4 discusses the estimated model’s simulated behavior and its impulse response functions to several shocks; it also traces back the major driving forces of economic fluctuations. Section 5 considers three alternative monetary regimes and provides policy implications. Finally, Section 6 concludes.

2. Model micro-foundations

2.1. Households

The model is populated by a continuum of households indexed by \( j \in [0, 1] \). At time \( t \), household \( j \) consumes a composite consumption good \( C_t \) made up of final goods produced domestically \( C^d_t \) and abroad (imported goods) \( IM_t \), supplies a homogenous labor service \( L_t \), and chooses holdings of domestic \( B_t \) and foreign bonds \( B_t^f \) to maximize the following intertemporal utility function:\(^1:\)

\[
\max_{C_t, L_t, B_t, B_t^f} \sum_{j=0}^{\infty} \left( \frac{1}{1 - \sigma_c} (C_{t+j} - \lambda C_{t+j-1}) \right)^{1-\sigma_c} \exp \left( \frac{\sigma_c - 1}{\sigma_c} L_{t+j}^{1-\sigma_c} \right)
\]

where \( \sigma_c \) denotes the inverse of intertemporal elasticity of substitution (degree of risk aversion), \( \sigma_l \) the inverse of labor supply with respect to real wage, and \( \lambda \) the degree of the external habit formation in consumption.

Their budget constraint in real terms is:

\[
C_t + \frac{B_t}{\epsilon_t^b (1 + R_t)} + \frac{S_t B_t^f}{\epsilon_t^{bf} (1 + R_t)} + T_t \leq \frac{B_{t-1}}{P_{t-1}} + \frac{S_{t-1} B_{t-1}^f}{P_{t-1}} + \frac{W_h^b L_t}{P_t} + \frac{Di_v t}{P_t}
\]

where \( P_t \) represents the domestic price level and \( S_t \) the nominal exchange rate. We follow De Walque et al. (2017) to assume that both \( B_t \) and \( B_t^f \) are subject to the same AR(1) preference shock \( \epsilon_t^b \) on financial assets. On the other hand, households receive the return from past positions in bond holdings, wages \( W_h^b \) from providing labor for firms’ production, and dividends \( Di_v t \) distributed by the labor unions. Households’ optimal decisions on \( C_t \) and \( L_t \) are standard and replicate those of SW(07). Combining the optimal conditions in \( B_t \) and \( B_t^f \) yields the following Uncovered Interest Rate Parity (UIP) in logs that pins down the expected movement in the real exchange rate \( Q_t^e \):\(^2:\)

\[
\mathbb{E}_tq_{t+1} = \left( \mathbb{E}_t q_{t+1}^f + \mathbb{E}_t q_{t+1}^s \right) - \left( \mathbb{E}_t q_{t+1}^f - \mathbb{E}_t q_{t+1}^s \right)
\]

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\(^1\) We allow domestic households access to the foreign bonds to capture the financial linkage channel through which world shocks can affect the UK economy via causing movements in the real exchange rate and so trade.

\(^2\) Households’ optimal positions in the bond market are determined by the FOCs from the maximization of Eq. (1) subject to Eq. (2): \( B_t : \frac{\beta_t}{\epsilon_t^b (1 + R_t)} = \beta_{t-1} \frac{\beta_{t-1}}{\epsilon_{t-1}^b (1 + R_{t-1})} \) and \( B_t^f : \frac{\beta_t}{\epsilon_t^{bf} (1 + R_t)} = \beta_{t-1} \frac{\beta_{t-1}}{\epsilon_{t-1}^{bf} (1 + R_{t-1})} \) where \( \beta_t \) is the Lagrange multiplier associated with the budget constraint. Combining these two FOCs and using \( Q_t = S_t \) yield Eq. (3).
Eq. (3) suggests that assuming incomplete asset markets, the differential between domestic and foreign real interest rate is equal to the expected change in the real exchange rate. An example of this is a rise in the domestic real interest rate that violates the UIP must be restored by a current fall in $q_t$ (sterling appreciation) in anticipation of a future rise in $E_t q_{t+1}$.

2.2. Labor intermediaries and hybrid wage-setting

SW(07) assumes a New Keynesian (NK) model where prices and wages are sticky due to Calvo-type pricing in both goods and labor markets. The extent of nominal rigidity, however, is a major area of disagreement. This topic draws attention as it is the nominal rigidities that result in short-run non-neutrality of monetary policy and hence makes room for welfare-enhancing monetary interventions i.e. it gives monetary authorities the leverage it needs to stabilize economy. Le et al. (2011) test this NK model as well as a New Classical (NC) version of it on the US postwar data using indirect inference, and find that both models are strongly rejected as the NK model generates too much nominal rigidity while the NC model generates too little. They also consider the possibility of a weighted model where parts of its economy display nominal rigidities and other parts do not, and find it delivers the suitable amount of nominal rigidities for the US economy. Here we follow Le et al. (2011) in considering a similar hybrid model for the UK where the firms have a production function that combines in a fixed proportion labor from imperfectly competitive markets (unionized labor) with labor from perfectly competitive markets. The hybrid wage-setting is thus given by: \[ W_t = \omega_m^N W^t + (1 - \omega_m^N) W^{2t}, \] where $\omega_m^N$ is the share of unionized labor in the total. The NK version wage-setting $W^t$ follows SW(07), and the NC version wage $W^{2t}$ is set equal to the current expected marginal monetary disutility of work. These wages will then be passed to labor packers (bundlers) who offer above weighted wages for each unit of aggregate labor before them to firms (intermediate goods producers).

2.3. Final goods producers and hybrid price-setting

Final goods producers (retailers) assemble intermediate (wholesale) goods produced by firms; no capital or labor is required in their production. Similar to the hybrid wage-setting, we assume that final output is made up of a fixed proportion of intermediate goods sold in imperfectly competitive markets and the rest sold in perfectly competitive markets. The hybrid price-setting is given by: \[ P_t = \omega_m^N P^t + (1 - \omega_m^N) P^{2t}, \] where $\omega_m^N$ is the share of imperfectly competitive goods market in the total. $P^t$ is set according to the Calvo mark-up equation on marginal costs and $P^{2t}$ at marginal costs. The final goods producers combine these two types of intermediate goods as a bundle which they sell at the above weighted average price.

2.4. Intermediate goods producers (firms) and commercial banks

To introduce financial factors to SW(07), we integrate financial frictions à la BGG into the SW(07) model; the resulting SWBGG model has frequently been employed in studies on the US and EA (e.g. Gilchrist et al., 2009; Gelain, 2010; Del Negro et al., 2013; Villa, 2014; and Merola, 2015). They modify the setup of SW(07), so that firms no longer rent capital from households (who now only invest in bonds or deposits), instead they purchase newly-built capital from capital producers using their net worth $N_t$ with the rest financed by borrowing from banks. The role for banks (financial intermediaries) is thus clear: they take deposits from households at the risk-free deposit rate $R_t$ and convert household savings into lending to firms at the risky loan rate $C_Y$ to finance firms' capital purchases.

2.4.1. Financial frictions and the role of QE

At period $t$, firms purchase capital $K_{t+1}$ at the price $PK_t$ from capital producers, and in period $t+1$, for each unit of capital, they receive a marginal product of capital $RK_{t+1}$ from operating their capital and sell back the undepreciated part $(1 - \delta)$ to capital producers at the price $PK_{t+1}$. Hence in equilibrium, the capital arbitrage condition implies:
\[
E_t[C_{Y_{t+1}}] = E_t \left[ \frac{RK_{t+1} + (1 - \delta)PK_{t+1}}{PK_t} \right]
\]

Eq. (4) states that the expected rate of real return on holding a unit of capital from $t$ to $t + 1$ consists of the marginal product of capital and the capital gain. Turning to the partial equilibrium contracting problem, the interaction between firms and commercial banks closely follows the BGG paper. Financial frictions here arise from information asymmetry between the lender and the borrower and also costly state verification – the lender has to pay a monitoring cost to observe what is produced by the borrower; this only takes place during the default, thereby motivating a non-trivial role for the borrowers’ financial structure. A key relationship known as the “financial accelerator” mechanism thus emerges:
\[
E_t C_{Y_{t+1}} - (\bar{r}_t - E_t \bar{r}_{t+1}) = pm_t = \chi (\bar{p}k_t + \bar{k}_t - \bar{n}_t) + \xi_t^{pm}
\]

where $\chi > 0$ measures the degree of financial frictions, and $pm_t$, the external finance premium (credit spread). Eq. (5) implies that assuming costly monitoring, the optimal contract is the one that charges a premium over the risk-free return to compensate lenders for the additional risk they bear when lending to firms with higher leverage ratios and thus higher default probabilities. With $\chi > 0$, for firms more heavily reliant on external financing and thus associated with higher leverage ratio $\bar{p}k_t$, banks’ willingness to borrow declines, reflected as a rise in $pm_t$ that suggests a further tightening of credit. This kind of
scenario is relatable to the GFC where the crisis started with some adverse financial shocks that increased credit spreads and restrained investment; the ensuing contraction in the real economy deteriorated firms’ balance sheet positions and raised the cost of debt financing even further.

With this background in mind, we introduce QE as credit market interventions by the central bank aimed at narrowing the credit spread to break the vicious cycle of adverse financial feedback loop. We follow Le et al. (2016) to augment the CSV contract with collateral, such that banks require all firms to put up a fraction $\epsilon$ of their net worth as collateral before loan agreement while the liquidity of collateral costs a fraction $\delta$ of its original value. We then propose using M0 (cash) as the cheapest collateral to eliminate this liquidation cost and thus lower the credit spread.\footnote{Compared with more common types of collateral such as real estate (mortgage loans) and cars (auto loans), the virtue of cash collateral lies in its liquidity which means lenders can directly seize cash to recoup their loss; this reduces banks’ liquidation costs when firms default, and increase their willingness to lend.} We assume that during crises, the central bank issues M0 in exchange for deposits (which also yield a return of $\phi$). Firms then wish to acquire as much of this M0 from banks for their future collateral use. The newly created M0 thus finds its way into firms’ balance sheets as the most liquid collateral pledged to banks in the future events of bankruptcy. By varying the supply of M0, the central bank intervenes in the credit market via stabilizing the flow of funds. With M0 acting as the cheapest collateral, the credit spread is reduced for given leverage:

$$\epsilon_t = \chi_t \left( \hat{p} \hat{k}_t + \hat{k}_t - \hat{n}_t \right) - \eta \hat{n}_t^0 + \hat{\epsilon}_t^{pm}$$

where $\vartheta > 0$ captures the credit easing effect of M0 on the loan supply (QE’s role in boosting lending). Firms (entrepreneurs)’ net worth at the end of period $t$ is given by:

$$\hat{n}_t = \hat{n}_{t-1} + \frac{K}{N} \hat{c}_t - \left( \frac{K}{N} - 1 \right) \epsilon_{t-1} \hat{c}_t + \hat{\epsilon}_t^n$$

where $\vartheta$ is the survival rate of firms, and $\hat{\epsilon}_t^n$ the steady-state ratio of capital to net worth. Eq. (7) implies that firms’ net worth is given by the past net worth of the $\vartheta$ fraction of surviving ones plus the leveraged realized return on capital, minus the required payment to the banks.

2.4.2. Production of intermediate goods

Firms produce differentiated intermediate goods by combining capital in operation (determined by the capital utilization $U_t$ selected) and hired labor via a Cobb-Douglas production function: $Y_t = \epsilon_t^a [K_t]^{\gamma_x} [L_t]^{1-\gamma_x} - \mathcal{G}$, where $\xi$ denotes the share of capital in the production function, $K_t = K_{t-1} U_t$ the effective capital used in production, $\mathcal{G}$ the fixed cost that enters as a source of real rigidity, $\gamma_x$ the labor augmenting deterministic growth rate, and $\epsilon_t^a$ the total factor productivity (TFP) shock that follows a nonstationary ARIMA(1,1,0) process. Firms’ problem is to choose the quantity of production factors to maximize their profits.

2.5. Capital producers

Besides intermediate and final goods producers, there also exist perfectly competitive capital producers on the production side. At time $t$, they combine investment goods $I_t$ and the undepreciated capital $(1-\delta)K_{t-1}$ installed at $t-1$ to build new capital to be sold to firms at price $PK_t$. Their problem is to choose $I_t$ to maximize their expected discounted profits

$$\max E_t^{\delta} \sum_{t=0}^{\infty} \delta^t \{PK_t \{ K_t - (1-\delta)K_{t-1} - I_t \} \}$$

subject to the law of motion of capital: $K_t = (1-\delta)K_{t-1} + \epsilon_t^a \{ 1 - F \left( \frac{\hat{c}_t}{\hat{n}_t} \right) \} I_t$, where $F(\cdot)$ is the cost of adjusting investment, and $\epsilon_t^a$ the investment-specific shock that affects the efficiency in transforming investment into new capital.

2.6. The rest of the world

The Smets-Wouters models (SW03, SW07) treat the EA and US as single entities isolated from the rest of the world (RoW), and do not address how the global developments have affected the object of studies. Here we adopt a SOE framework wherein world shocks can affect the UK economy, while home-grown UK-specific shocks have no impact on the wider global economy. UK households have access to foreign bonds $B_s$ and foreign-produced goods and services (UK’s imports $M_s$). The RoW purchases UK-produced goods and services (UK’s exports $E_s$) for foreign consumption. For simplicity, only final goods are considered here. The domestic consumption bundle $C_t$ thus is a composite of home-produced and imported goods represented by an Armington (1969) aggregator of the form:

$$C_t = \left[ \omega \left( C_t^d \right)^{-\varphi} + (1-\omega) \varphi \left( C_t^{im} \right)^{-\varphi} \right]^{1/\varphi}$$

where $C_t^d$ and $C_t^{im}$ are the consumption of home-produced and foreign-produced (imported) final goods, respectively. $\omega$ measures the bias towards (weight of) home-produced goods in the consumption bundle, $\varphi$ the preference error, and $\varphi = \frac{1}{1-\sigma}$ the
elasticy of substitution between domestic and foreign varieties. Maximization of Eq. (8) subject to the expenditure constraint
\[ C_t = p_t^D c_t^D + q_t^M c_t^M \]
yields the UK’s demand for imports:
\[ c_t^M = \tilde{im}_t = \sigma \log(1 - \omega) + \tilde{c}_t + \sigma \tilde{q}_t + \sigma \log \tilde{z}_t. \]  
(9)

By symmetry, the RoW’s demand for its imported goods yields the demand for the UK exports:
\[ \tilde{e}_t = \sigma^f \log(1 - \omega^f) + \tilde{c}_t^f + \sigma^f \tilde{q}_t + \sigma^f \log \tilde{z}_t^f \]  
(10)

where \( \sigma^f, \omega^f, \tilde{c}_t^f \) and \( \tilde{c}_t^f \) (consumption in the RoW bloc) are the foreign equivalents of \( \sigma, \omega, \tilde{c}_t \) and \( \tilde{c}_t^f \) treated as an exogenous AR(1) process. Assuming no capital controls, the balance of payments constraint is satisfied in Eq. (11) which links the net foreign assets position (NFA) with trade balance:
\[ \frac{Q_t^B}{(1 + r_t^f)} - \frac{Q_t^B}{(1 + r_t)} = p_t^D EX_t - Q_t^M M_t = TB_t \]  
(11)

In other words, for a country to have net savings or asset accumulation it must run a trade surplus. Log-linearization of Eq. (11) yields:
\[ \tilde{b}^f_t = \left(1 + \frac{r_t^f}{1 + r_t^f}\right) \tilde{b}^f_{t-1} + \frac{EX_t}{V} (\tilde{e}_t - \tilde{q}_t) - \frac{IM_t}{V} \tilde{im}_t \]  
(12)

where \( \tilde{b}^f \) and \( \tilde{b}^f_{t-1} \) are steady-state ratios. Eq. (12) states that exports and imports together with interest receipts/payments determine the evolution of NFA.

2.7. Government and aggregations

The government conducts conventional and unconventional monetary policies as well as fiscal policy.

2.7.1. Monetary policy

Traditionally, monetary policy is conducted mainly via adjusting the short-term policy rate according to some rules. However, the frequent encounters with the ZLB in most advanced economies since the GFC have played a constraint on such policy, making it imperative to allow for state switches in DSGE modeling. Here we assume that an occasionally binding ZLB divides the model into two states - a normal state with a slack ZLB where interest rate policy is active, and a crisis state with a binding ZLB where the QE tool is active. The nominal interest rate obtainable on either domestic bonds or bank deposits is set according to the Taylor rule and enforced by OMOs. When the notional interest rate solves above the ZLB of 0.025% quarterly, it is set according to the Taylor rule by the central bank to keep inflation in check; meanwhile, the supply of M0 is set to accommodate the broad money supply M2, with the latter derived from firms’ balance sheets. The situation is summarized by the following equation set:

\[
\begin{align*}
\text{For } r_t > 0.025 & \quad \left\{ \begin{array}{l}
\tilde{r}_t = \rho \tilde{r}_{t-1} + (1 - \rho) \left(r_p \tilde{r}_t + r_y \tilde{y}_t + r_n \left(\tilde{y}_t - \tilde{y}_{t-1}\right) + \tilde{c}_t^f 
\end{array} \right. \\
\tilde{m}_0^0 - \tilde{m}_0^{t-1} = \psi_{m2}^{\text{norm}} \left( \tilde{m}_2^0 - \tilde{m}_2^{t-1} \right) + \tilde{e}_t^{\text{pm}} \left(\text{OMOs}\right) \\
\tilde{m}_2^0 - \tilde{m}_2^{t-1} = \left(1 - \frac{\tilde{M}_0}{\tilde{M}_2^{\text{norm}}} + \frac{\tilde{N}_2}{\tilde{M}_2^{\text{norm}}} \right) \tilde{k}_t + \frac{\tilde{M}_0}{\tilde{M}_2^{\text{norm}}} \tilde{m}_2^{t-1} - \frac{\tilde{N}_2}{\tilde{M}_2^{\text{norm}}} \tilde{n}_t
\end{align*}
\]

(13)

where \( \rho \) measures the degree of interest rate smoothing, and \( r_p, r_y, \) and \( r_n \) are Taylor rule’s responses to inflation, output, and changes in output, respectively. \( \psi_{m2}^{\text{norm}} \) denotes the elasticity of M0 to M2. \( \frac{\tilde{M}_0}{\tilde{M}_2^{\text{norm}}} \) and \( \frac{\tilde{N}_2}{\tilde{M}_2^{\text{norm}}} \) are steady-state ratios of M0 and net worth to M2, respectively. The equation for M2 supply is derived from firms’ balance sheets: \( M_2 = \text{cash (M0)} + \text{bank deposits} = \text{capital expenditures in excess of net worth.} \)

When the nominal interest rate solves at or below the ZLB, the Taylor rule is suspended with the interest rate fixed at 0.025, leaving no room for further rate cuts. To boost demand, the central bank embarks on QE financed by the issuance of M0. Households have no use for M0 and will deposit M0 with banks who lend it to firms as the latter is interested in holding as much M0 as possible for future collateral use. M0 thus enters the financial accelerator as the credit agent within a collateral-augmented CSV framework. We consider ZLB episodes as situations where the credit spread widens drastically, and specify a feedback rule from the credit spread to M0 to model the central bank’s QE effort to stabilize the credit market:

\[
\begin{align*}
\text{For } r_t \leq 0.025 & \quad \left\{ \begin{array}{l}
\tilde{r}_t = 0.025% \\
\tilde{m}_2^{t-1} - \tilde{m}_2^0 \psi_{m2}^{\text{crisis}} \left( pim_t - pm_t \right) + \tilde{e}_t^{\text{pm}} \left(\text{QE}\right)
\end{array} \right.
\end{align*}
\]

(14)

\footnote{UK consumers form the Lagrangian \( \mathcal{L} = \left[ \sigma^f (\tilde{c}_t^f)^{\omega} + (1 - \omega) \tilde{c}_t^M (\tilde{c}_t^M)^{\omega} \right]^{\frac{1}{1-\omega}} + \lambda \left[ C_t - p_t^D c_t^D - Q_t^M c_t^M \right] \), where the FOC w.r.t. \( C_t^M \) yields: \( C_t^M = C_t \left[ \frac{\rho}{\tilde{r}^f} \right]^{\frac{\omega}{1-\omega}} \). Further log-linearization yields Eq. (9).}
where $\psi_{PM}^{(\text{crisis})}$ is the elasticity of M0 to the credit spread at the ZLB. Eqs. (14) describe the situation where the ZLB binds following a sequence of adverse shocks: the model switches to the crisis state where the Taylor rule is rendered inoperative; QE is activated to replace OMOs with M0 set contingent on the deviation of the credit spread from its steady-state. Essentially, this allows the central bank to smooth the flows of funds via varying the supply of M0. The wider the credit spread (the more disrupted the financial intermediation), the greater scale of M0 injection is required to bring the credit provision back to normal. We think of this mechanism as properly capturing how the massive expansion of M0 since QE1 first offset the spike in the credit spread and then kept it under control. Once the interest rate solves above the ZLB again as the effect of negative shocks dies out, the economy switches back to the normal state. In our model, QE differs from OMOs in two aspects. First, QE is conducted in crises whereas OMOs in normal times. Second, QE is modeled as a vast expansion of OMOs with its main purpose being to stabilize the credit spread and loan supply. In a later section we discuss and compare the model’s behavior following the OMOs and QE shocks.

### 2.7.2. Fiscal policy

Fiscal policy closely follows that in SW(07), with government spending $G_t$ set exogenously as a time-varying fraction relative to the steady-state output path: $c_t^g = \frac{C}{\hat{Y}}$, where $c_t^g$ follows an AR(1) process that also responds to productivity process: $\hat{c}_t^g = \rho_{\hat{c}}c_{t-1}^g + \eta_t^g + \rho_{\eta}\eta_t^g$. We assume that the government only purchases home-produced final goods for public expenditure. Their budget constraint is of the form: $P_tG_t + B_{t-1} = T_t + \frac{R_t}{1 + \rho_{\eta}}$.

### 2.7.3. Market clearing condition

With the addition of firms’ (entrepreneurs’) consumption and the trade with the RoW, the economy-wide resource constraint becomes:

$$\bar{y}_t = \frac{C}{\bar{Y}}c_t^e + \frac{R_t}{\bar{Y}}k_t + \left(\frac{R_t}{\bar{Y}}k_t - \frac{1}{\psi}\right)i_t + \frac{C}{\bar{Y}}\tilde{c}_t^g + \frac{EX}{\bar{Y}}\tilde{e}_t^g - \frac{1}{\bar{Y}}IM_t + \tilde{\epsilon}_t^g \quad (15)$$

where $\frac{C}{\bar{Y}}, \frac{R_t}{\bar{Y}}, \frac{C}{\bar{Y}}\tilde{c}_t^g, \frac{EX}{\bar{Y}}\tilde{e}_t^g,$ and $\frac{IM}{\bar{Y}}$ are steady-state ratios. Entrepreneurs’ consumption $c_t^e$ equals net worth $n_t$ (in logs), as firms that die in period $t$ will consume their net worth and depart from the scene.

The past quarter-century or so has witnessed the development of DSGE models that have proven useful for monetary policy analyses. Many central banks around the world e.g. the Sveriges Riksbank (central bank of Sweden), the Norge Bank (central bank of Norway), the Bank of England, the European Central Bank, and the Federal Reserve have employed DSGE models in formulating their monetary policies (Dotsey, 2013). Compared with more traditional tools like VARs which impose very little restrictions, the strengths of DSGE models lie in its ability to integrate key intertwined economic components and formalize these interrelationships, thus offering coherent frameworks of where the restrictions come from (Tovar, 2009). Moreover, DSGE models are considered immune to the famous “Lucas’s critique” as they are micro-founded and contain deep structural parameters thought to be invariant to policy changes. Because of the stability of these parameters, DSGE models are frequently used for counterfactual experiments to explore the effects of alternative policies. However, as almost always, there are caveats that ought to be acknowledged in using DSGE models for policy evaluation and forecasting. First, they make strong assumptions on rational expectations which are not always borne out by the data, and there are numerous cases where the rational expectations break down. One might argue that even if the individuals are not always rational, it is expected their irrationalities would cancel each other out at the aggregate level. This view is supported by Shaikh (2016) which shows different micro behavior can result in the same macro outcome. However, in general, it is hard to justify that the aggregation in model with rational expectations are always sound. The second relevant issue is that of model misspecification. The restrictions imposed by DSGE models are sometimes at odds with data. For example, the statistical rejection of intertemporal Euler equation (see e.g. Christiano et al., 2010). The same growth rate of key variables like consumption, investment, output and wages as implied by the models are inconsistent with data. Del Negro and Schorfheide (2009) show an example of how model misspecification can cause an incorrectly designed policy. Further, the empirical work with DSGE model is typically carried out with macro aggregates revised and released afterwards usually with a few months’ time lag, while in real life policy makers act on constantly updating real-time data. The difference between the real-time data and the final aggregates available to the public is of particular relevance in analysing crises when the consequences of sudden disturbances are hard to predict in the first place. Finally, issues concerning parameter estimation sometimes arise. For example, estimates obtained from the Bayesian method may only reflect the prior belief if the data is not informative enough to update the prior imposed on parameters. As such, identification of the parameters becomes an issue and very little is actually known about the parameters. Having said that, we should not refrain from using DSGE models for monetary policy analyses, as first they are quite competent in providing an articulated description of the economy especially in identifying various shocks that perturb the economy, and second, we rely on the indirect inference method for model testing and parameter estimation which has proven to have low bias in small samples and also high power in rejecting false parameters, so enabling analysis of robustness to estimation errors.

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5. For a further discussion on this, see Haldane and Turrell (2018).
3. Model testing and estimation

We employ 19 quarterly UK macroeconomic series for 1993Q1-2016Q4. The sample starts in 1993 as the UK has officially adopted inflation targeting since 1992Q4 after the European Exchange Rate Mechanism exit. It ends in 2016 as the data for the nominal interest rate is discontinued from 2017Q2. See Table B1 for variable construction and data sources. We employ simulation-based Indirect Inference (II) for model testing and estimation. The method was first proposed by Smith (1993) and developed later in Minford et al. (2009). The II method essentially first tests the model against a three-equation VAR of interest, inflation, and output on some calibrated parameter values, and then allows those parameters to be varied flexibly to the values that minimize the distance between the (fitted model-generated) simulated and actual data. It is dubbed “indirect” as it involves comparing the (model-generated) simulated data with the actual data not directly but via the lens of an auxiliary model in the form of VAR or VARX. By estimating the auxiliary model on both the simulated and the actual data, we aim to summarize their data behavior using some descriptive features represented by the estimated VAR coefficients. In effect, we end up with 1000 sets of VAR coefficients from the simulated data and 1 set of VAR coefficients from the actual data. The distance between the two is measured as a Wald percentile to show where in the Wald distribution of the simulated data the Wald statistic for the actual data lies. The Wald statistic is then transformed into a normalized t-statistic by adjusting the mean and sample size of the chi-square distribution so that the 95% point is 1.645. In a model evaluation (II test), if $t \leq 1.645$ (i.e., the model is able to generate estimates of the auxiliary model similar to those obtained from the actual data), we accept the null hypothesis that the model can fit the data. II method, by contrast, is not a formal test of whether the model can fit the data. II estimation. To carry out II estimation, we first generate many sets of randomized coefficients around their calibrated values and then substitute them one by one into the model to find their resulting t-statistics. This process is repeated until we find the optimal set that minimizes the t-statistic ($t < 1.645$). Here we choose a VARX (vector autoregression with exogenous variables) as our auxiliary model. The parameter vector $\beta$ contains 12 coefficients in total, so the model will pass the test only if it jointly matches 12 coefficients.

Parameters describing the model’s dynamics are calibrated to the US estimates from Le et al. (2016) and subject to II testing and estimation. Table 1 summarizes the values for starting calibrations and estimates. The t-statistic from the calibrated model is 1.9794 (bottom section), exceeding the threshold of 1.645. Thus we carry out II estimation using the simulated annealing algorithm (SAA) to find the global minima. The optimal estimates are chosen to minimize the t-statistic to 0.971. It is the estimated model that our following analyses will be based on. The model is perturbed by 15 structural shocks which provide sources of volatility. Model-implied shock histories are plotted in Fig. B2.

4. Model analysis

4.1. Model simulated behavior

We let the ZLB on the nominal interest rate be endogenously determined by the shocks instead of imposing it for a pre-determined duration as the commitment of monetary authorities; this allows us to evaluate the frequency and duration for the ZLB episodes in our simulations. Fig. 2 plots some examples of the simulated nominal interest rate versus the actual data. Inspection reveals that the nasty periods with a binding ZLB can be relevant quite often. Simulations #193, #599, and #800 show examples of the interest rate hitting the ZLB not just once but repeatedly. Moreover, for some extreme scenarios such as Simulation #671, the ZLB can be binding for nearly 70% of the time; such a protracted stay at the ZLB might occur when sequences of strong adverse shocks perturb the economy. The pseudo histories we have created with our model emphasize the frequency and severity of ZLB episodes that could have occurred and prove the necessity of specifying the “crisis state” whereby we study the model’s responses at the ZLB.

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6 We choose II over the more widely used Bayesian method as 1) we aim to test the model with regime switches as a whole; 2) the Bayesian method essentially maximizes the likelihood function conditional upon the data, but is not a formal test of whether the model can fit the data. II method, by contrast, is a test that evaluates the model’s ability to replicate key aspects of data features. If the model can pass the II test with key macroeconomic variables involved, then it can be used to evaluate alternative policies that may improve these variables’ behavior.

7 $W = (\beta^* - \hat{\beta})\Omega^{-1}(\beta^* - \hat{\beta})$ and $\Omega = \text{cov}(\beta^* - \hat{\beta})$, where $\beta^*$ is the VAR parameters estimated on the actual data, and $\hat{\beta}$ the average of 1000 sets of VAR parameters estimated on the simulated data.

8 $t = 1.645 \left(\frac{2^{0.5b - 0.25k - 1}}{\sqrt{2^{0.5b - 0.25k - 1}}}ight)$, where $W^p$ is the Wald statistic on the actual data, $W^{95}$ the Wald statistic for the 95th percentile of the simulated data, and $k$ the number of parameters in $\beta$.

9 The VARX takes the following matrix form where non-stationary processes (i.e. TFP shocks and foreign bonds) are included to generate stationary errors:

\[
\begin{bmatrix}
Y_t \\
\pi_t \\
r_t
\end{bmatrix}
= B
\begin{bmatrix}
Y_{t-1} \\
\pi_{t-1} \\
r_{t-1}
\end{bmatrix}
+ C
\begin{bmatrix}
T \\
\epsilon_t \\
\epsilon_{q_t}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_{a_{T_t}} \\
\epsilon_{a_{q_t}}
\end{bmatrix}
\text{where } B = \begin{bmatrix}
\theta_{y} & \theta_{\pi} & \theta_{r} \\
\theta_{y} & \theta_{\pi} & \theta_{r} \\
0 & \theta_{r} & \theta_{r}
\end{bmatrix}
\text{(16)}
\]

10 $\beta = \begin{bmatrix}
\theta_{y} & \theta_{\pi} & \theta_{r} & \theta_{\pi} & \theta_{r} & \theta_{r} & \text{var}(\epsilon_t) & \text{var}(\epsilon_t) & \text{var}(\epsilon_t)
\end{bmatrix}$ where the first 9 coefficients describe the data dynamics and the last 3 capture the size of variations.

11 Parameters held fixed throughout our estimation are summarized in Table B2.
Moreover, due to the distinct mechanisms of M0 supply modeled in two states, we see materially different monetary responses under state switches. In the normal state, conventional interest rate policy is active and M0 policy (OMOs) is only supportive. In the crisis state, interest rate policy is non-operative at the ZLB whilst M0 policy (QE) is active in keeping the...
credit spread in check. Simulation #417 in Fig. 3 shows the case where the policy rate had stayed out of the ZLB for the entire sample period. In the absence of ZLB situations, the main policy rate was approximated by the Taylor rule that responded to current developments; M0 expanded moderately via OMOs to accommodate the rising M2. While the economy stayed in the “normal state”, the growths of M0 and M2 were steady and slow. By contrast, in Simulation #293 the interest rate hit the ZLB in 2001 and had stayed there ever since; meanwhile, QE was triggered with M0 becoming the main tool to stabilize the financial market by targeting the credit spread. The structural break in M0 captured the transition from the normal state where OMOs operated via moderate gilt purchase to enforce the prescribed policy rate, to the crisis state where OMOs were replaced by QE that involved a massive expansion of gilt purchase to stabilize the credit spread. Finally, Simulation #996 shows the routes in and out of the ZLB and their resulting shifts in the monetary responses. As with Simulation #293, the interest rate hit the ZLB in 2005, resulting in a sharp rise in the M0 growth. It however escaped from the ZLB at a later date in 2010, so the economy switched back to the normal state with M0 growth subsequently leveling off. The normal interest rate once again became the primary instrument for monetary policy, as was the case in the pre-QE days.

4.2. Impulse response functions

To shed light on some key transmission mechanisms at work, we employ impulse response functions (IRFs) for a few shocks of interest to us.

4.2.1. Impulse responses to a positive government spending shock

Fig. 4 shows the IRFs following a fiscal expansion shock under both states. Under the normal state, we see the expansion in government spending raises labor, wage, consumption, and output. This generates inflationary pressure that is kept in check by a rise in the policy rate. Firms’ net worth goes up, driving down the credit spread via the financial accelerator mechanism. The real lending rate rises and crowds out investment. UIP is violated owing to the rising real interest rate, and hence must be restored by a sterling appreciation (a drop in Q), in anticipation of a future depreciation. This leaves the UK production less competitive in the global market, encouraging imports and discouraging exports. Net exports fall, so the country is running a current account deficit, with its holdings of foreign bonds decreased (NFA position deteriorates). Under the crisis state, the boost in labor, wage, consumption, and output is more pronounced with the Taylor rule response shut off. This raises the impact fiscal multiplier from 1.25 to 1.5 - a result in line with previous findings by Eggertsson (2011), Christiano et al. (2011), and Hills and Nakata (2018) who report greater fiscal multipliers at the ZLB relative to the non-ZLB case. The significant rise in net worth reduces credit spread even more compared to the normal state, and as a result, the real lending rate decreases, stimulating the investment instead of crowding it out. Absent a Taylor rule response, the decreasing real interest rate depreciates the domestic currency. This raises exports and reduces imports, improving the NFA position.

4.2.2. Impulse responses to a positive Taylor rule shock

Fig. 5 shows the IRFs following a positive Taylor rule (contractionary monetary policy) shock in the standard non-crisis context. We see it raises both the nominal and real interest rates, reducing consumption and output. Confronted with lower aggregate demand, firms cut back on workforces and wages. Net worth drops in accordance with the fall in output and employment. This drives up the credit spread and the real lending rate. The price of the existing capital stock falls, which then discourages investment. Lower net worth causes distress borrowing from banks, and thus an accommodating rise in M0. There is a sterling appreciation against the foreign currency to restore the UIP, which lowers the demand for exports and raises the demand for imports as the UK-produced goods become more expensive. The country runs a current account deficit and NFA falls.

4.2.3. Impulse responses to a positive M0 shock

Regarding the effectiveness of BoE’s QE in stabilizing the UK’s financial market and boosting the wider economy, there exists some empirical evidence from non-DSGE studies. Joyce et al. (2011) use a structural VAR (SVAR) model and show that the BoE’s QE1 raised the real UK GDP by 1.5–2%, and inflation by around 0.75–1.5 pps. Breeden et al. (2012) employ a macro term structure model and find that QE1 was effective in reducing the UK’s long-term bond yields (via the portfolio-rebalance channel); however, its broader impact on the real activity (production, retail sales, and the claimant count) remains controversial. Kapetanios et al. (2012) resort to 3 types of VAR models and show that QE1 had a peak effect of around 1.5% on the UK’s real GDP and a peak effect of around 1.25 pps on its annual CPI inflation. Lyonnet and Werner (2012) utilize a general-to-specific econometric modeling methodology and report a stable relationship between the M4 lending growth and the nominal GDP growth in the UK. Weale and Wieladek (2016) use a BVAR model to show that a QE shock that is equivalent to bond purchases worth 1% of nominal GDP can raise the UK real GDP and CPI by 0.25% and 0.32% respectively. Churm et al. (2018) assess the macroeconomic effects of BoE’s QE2 using simulations from a BVAR, finding that QE2 boosted the UK GDP by around 0.5–0.8%, inflation by 0.6 pp at its peak. Our model assumes that QE plays its role via reducing the lending-deposit spread; this would provide the stimulus for the economy at the ZLB, raising the output and inflation. Now we proceed to check if this is indeed the case from our estimated model.

Fig. 6 shows the IRFs following a positive M0 shock that approximates the central bank’s conduct of OMOs (under the normal state) or QE (under the crisis state). In the non-ZLB situation, the effect of a rising M0 is to lower the credit spread
via M0’s role as the cheapest collateral; this raises the amount of credit and triggers an investment boom. Labor, wage, consumption, and output all rise, generating inflationary pressure and subsequently a rise in the policy rate via the Taylor rule response. The real interest rate rises slightly, causing a sterling appreciation; this in turn decreases net exports and deteriorates the NFA position.

At the ZLB, the M0 expansion also reduces the credit spread and real lending rate. There are more substantial expansions in labor, wage, consumption, investment, and output, as the “leaning against the wind” offsetting mechanism to the stimulus is shut off. Inflation and output are boosted even more compared with the normal scenario. The QE shock is modeled as an AR (1) process so that the expansion of the BoE’s balance sheet peaks immediately upon announcement; it then declines quickly before the complete exit within 8 quarters (2 years). It is clear that in response to M0 shocks of the same size (which correspond to bond purchases of the same scale), QE is able to generate much greater stimulus compared to OMOs, although they both induce hump-shaped responses in most variables with the maximum impacts occurring in the 5th quarter (1.25 years) before declining in a quasi-linear way. Turning to the international channel, we see that a stimulus to M0 at

Fig. 3. A selection of simulations for interest rate (R), M0 and M2. Notes: The simulated interest rate (blue solid line) is plotted along the left axis; the simulated M0 growth (red dash-dotted line) and M2 growth (red dotted line) are plotted along the right axis.

Fig. 4. IRFs to a fiscal expansion shock. Notes: The time intervals on the x-axis are quarters; the units on the y-axis are percentage deviations, except for the interest rate, inflation, credit spread, real lending rate and net foreign assets that are reported as percentage-point deviations. The last plot in the bottom row depicts the fiscal expansion shock used to perturb the estimated model, creating the distinct responses under the normal state (blue solid line) and the crisis state (red dash-dotted line).
the ZLB increases the inflation expectation, and reduces the domestic real interest rate; this causes a depreciation of the home currency to restore UIP where a devalued pound sterling improves the UK’s price competitiveness in international trade. Exports increase while imports decrease given that trade flows are sufficiently price-elastic (the elasticity of substitution between home and foreign-produced goods is set high at 2.74). Overall, QE depreciates the home currency and strengthens the demand for assets denominated in foreign currency, leading to an improvement in the NFA position. Now we see that the predictions of our model on QE are broadly consistent with the non-DSGE evidence for the UK - QE raises inflation, output, and holdings of foreign assets via depreciating the domestic currency.

**Fig. 5.** IRFs to a contractionary monetary policy shock. Notes: The time intervals on the x-axis are quarters; the units on the y-axis are percentage deviations, except for the interest rate, inflation, credit spread, real lending rate and net foreign assets that are reported as percentage-point deviations. The last plot in the bottom row depicts the Taylor rule shock used to perturb the estimated model, creating the responses under the normal state (blue solid line).

**Fig. 6.** IRFs to an OMO or a QE shock. Notes: The time intervals on the x-axis are quarters; the units on the y-axis are percentage deviations, except for the interest rate, inflation, credit spread, real lending rate and net foreign assets that are reported as percentage-point deviations. The last plot in the bottom row depicts the M0 shock used to perturb the estimated model, creating the distinct responses under the normal state (blue solid line) and the crisis state (red dash-dotted line).
Nowadays, conventional monetary policy mainly involves setting a target for the short-term policy rate and achieving the target by adjusting the supply of central bank money via OMOs. Through manipulating the key policy rate, the central bank in effect adjusts the market liquidity to pursue its primary objective of price stability in the medium term. Before the GFC, it was believed to be a reliable way of squeezing liquidity to curb inflation during upturns and injecting liquidity to the system during downturns. We model conventional monetary policy in two parts: steering the level of nominal interest rate when it solves above the ZLB and enforcing the prescribed rate by OMOs. Fig. 5 shows the transmission mechanism of the former (the core element) where it primarily works through adjusting the interest rate to influence aggregate demand. The effects on consumption, investment, and output are rather short-lived and last no more than 15 quarters. For most variables, the maximum impacts occur at the first quarter following the shock. This tool, however, only functions when the economy is not trapped in the ZLB. When the adverse disturbance is strong and persistent to the extent that the notional policy rate needs to be brought down below the ZLB, further rates cutting is no longer available. This is when the central bank can only resort to unconventional monetary policy for additional monetary stimulus. Generally speaking, unconventional policy tools are a broad range of measures designed to improve financial conditions by targeting the cost of credit. In our model, unconventional measures are defined as expanding the supply of M0 to reduce the credit spread and stimulate the flow of funds available to intermediate goods producers for their capital purchases. Compared with traditional rate cuts, the design of QE takes into account the economy’s financial structure and directly targets the credit spread. We see from Fig. 6 that QE causes immediately a massive expansion of M0; this pumps liquidity into the financial system and reduces the external cost of financing via acting as the cheapest collateral. The stimulus provided by QE is more long-lasting and creates hump-shaped responses in most variables. The boosting effects do not die out at the end of 30-quarter horizon. Given the similarities and differences in the responses to Taylor rule and QE shocks, it is tempting to ask if non-standard tools like QE would be needed even in non-ZLB situations if the flow of funds is severely impaired. This is the question we aim to answer in Section 5.

4.2.4. Impulse responses to a positive credit spread shock

Fig. 7 shows the IRFs following a positive credit spread shock under both states. The rise in the credit spread can be due to unexpected disturbances in the financial intermediation or a tightening of macroprudential regulation, both of which in effect raise the cost of borrowing and absorb the lending capacity in the economy. Under the normal state, we see that a widening credit spread raises the real lending rate and keeps it lifted for the following years. This results in long-lasting reductions in investment, labor, wage, consumption, and output, with the troughs occurring around 5 quarters (1.25 years) after the impulse. The dampened economic activity implies downward pressure on inflation, and a drop in the nominal deposit rate. Domestic currency depreciates, which encourages exports and discourages imports. Under the crisis state, the dampening impact is not counteracted by any rate cuts; M0 policy becomes the active one to stabilize the credit market at the ZLB, and thus we see a rapid expansion of M0. The permanent nominal interest rate peg implies a rising real interest rate and hence a pound sterling appreciation; this causes the opposite movements in imports, exports, and NFA relative to the normal state.

4.3. Historical decomposition of key variables

To gauge shocks’ relative contributions to the model dynamics for our sample period (i.e. which shocks have mattered the most for key variables’ fluctuations, and when), we employ a historical shock decomposition in this section. We start by feeding the marginal effect of each of the 15 shocks to the model’s BGP. The total effect is the sum of all shocks’ marginal contributions and approximates the historical data with its trend stripped out.

Fig. 8 shows the estimated historical contribution of shocks to the UK GDP growth. We see that throughout the sample period, a large share of the GDP fluctuations was driven by productivity, government spending, labor supply and world shocks. Among them, productivity shocks have been the dominant force behind both the pre-crisis boom and also the bust. Labor supply shocks were another major driving force on the supply side. They contributed negatively up until the GFC, but after that they have had a positive impact on the recovery. On the demand side, we see government spending and investment shocks were two major players; however, the effect of the former has falled over time while that of the latter has grown since 2000s. Financial shocks (credit spread and net worth shocks) have become increasingly important since the pre-crisis boom; unsurprisingly, they played a large role in the serious dip during the GFC. The feeble recovery for 2009–2012 was mainly haunted by the productivity slump - this is consistent with the evidence that the UK had experienced a more dramatic fall in productivity than any other leading western economies. Financial shocks only play a minor role in hindering the recovery, while it is the persistent and weak productivity that had held back the economic recovery between 2010–2012, although the effects of their drag have waned since 2013.

Fig. 9 shows that productivity, preference, labor supply and world shocks were the main driving forces behind the consumption fluctuations. In the 1990s, consumption was dominated by preference, productivity, and to a lesser extent labor

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12 Besides QE, unconventional policy measures implemented in the UK also include enhanced liquidity support (long-term repos, discount window facility, and the Special Liquidity Scheme (SLS)), actions to address dysfunctional markets (e.g. corporate asset purchase), and Funding for Lending Scheme (FLS). We limit our discussion to QE here.

13 The rationale for modeling macroprudential regulation via the credit spread channel is that financial regulation works through forcing up (down) the cost of funds, so as to constrain (encourage) risky lending. However, as we do not have a variable for the regulatory instrument, it is only modeled here as an exogenous error process included as part of the credit spread shocks instead of an active policy.
supply shocks. The expansion in the early noughties was explained by a combination of world, labor supply and productivity shocks. Following the Lehman brother bankruptcy, the contraction in consumption was mainly due to negative and persistent productivity shocks. A large part of its pickup since 2013 appears to have been driven by a waning of the drag from the productivity slump. Financial shocks have played a negligible role in consumption fluctuations while labor supply shocks have become increasingly important and contributed to the slow recovery between 2009 and 2012.

Fig. 10 shows that the nominal interest rate was dominated by the Taylor rule, price mark-up, and productivity shocks, with a smaller contribution from world and labor supply shocks. Productivity shocks were once the dominant force behind

![Graph showing historical shock decomposition for output.](image)

**Fig. 8.** Historical shock decomposition for output. Notes: The world shocks include the export demand, import demand, foreign consumption and foreign real interest rate shocks; the financial shocks include the credit spread and net worth shocks. The periods marked in the figure are: 1. The pre-crisis credit boom (2000Q1-2007Q2); 2. The Great Recession (2008Q2-2009Q2); and 3. The sluggish recovery (2010Q1-2012Q4). Bars above (below) the horizontal axis represent positive (negative) shock contributions to the BGP; units on the vertical axis correspond to percentage deviations from trend growth.
its fluctuations from the mid-1990 to the early 2000s; however, they played little to no role in explaining interest rate volatility since the crisis. On the other hand, world and labor supply shocks have grown increasingly important since 2008. In contrast to output and consumption fluctuations, the effect of productivity shocks has dwindled since the mid-1990s and become almost negligible in the post-crisis episodes. Price markup shocks were once dominant in the 1990s and become less important in the pre-crisis boom; it has become one of main driving forces again since 2015. Overall, we see growing influence of world shocks since 2008 while that of demand-side shocks (preference, investment and government spending) has remained subdued.

5. Optimal policy and welfare

A broad consensus has formed since the GFC around the idea of incorporating financial stability considerations into monetary policy analyses. This came about because the standard Taylor rule does not respond much to credit conditions. Permitted by inflation targeting, credit growth in the UK was elevated in the periods leading up to the crisis. During the episodes of “irrational exuberance” (2000Q1 - 2007Q2), signs of credit overheating such as a lower-than-average credit spread associated with the overly lax lending standards were largely ignored by policymakers. Regarding how to best pre-empt the next financial crisis, two views have emerged in the literature. The first one states that credit stability concerns ought to be the second objective of conventional interest rate policy in addition to inflation control. The reason is that compared to more-specific credit interventions that exclusively target the credit spread, interest rate policy is able to get in all of the cracks (Stein, 2013). The one thing that the government, commercial banks, and depositors have in common is that they all face the same policy rate R; to the extent that it affects the market risk appetite, so that changes in interest rate may reach corners of the economy that more targeted credit policy tools cannot.

Taylor et al. (2008) first propose an augmented Taylor rule that responds to variations in the LIBOR-OIS spread. Later work by Curdia and Woodford (2010) compares a spread-adjusted rule with a credit volume-adjusted rule, finding the former more beneficial for welfare gains and also more robust to alternative assumptions. Gilchrist and Zakrajsek (2012) examine a spread-augmented rule and a spread shock-augmented rule in response to financial disturbances and adverse financial news, concluding that the former dampens the economic fluctuations more substantially. The second view, however, argues that interest rate policy is too blunt a tool to be routinely used for containing financial risks (see e.g. Bernanke, 2012; Yellen, 2014; and Svensson, 2015; Svensson, 2018). According to them, more specific credit interventions should be deployed while interest rate policy should stay focused on price stability; the suc-
cessful conduct of monetary and financial policies hence requires each to have its own instrument. Badarau and Popescu (2014) find that augmenting the Taylor rule with a financial stability indicator (credit-to-GDP ratio) brings little to no gain in welfare, as the trade-off appears when one instrument (nominal interest rate) is employed for multiple goals. Carrillo et al. (2021) compare a spread-augmented Taylor rule regime, with a dual rule regime that supplements the standard Taylor rule with a financial rule to target credit spread exclusively, and find the latter yields smaller welfare costs.

Following the aforementioned literature, we specify three alternative regimes that modify the baseline regime BR’s normal state setup: a spread-augmented regime, a spread shock-augmented regime, and a dual rule regime. The crisis state specification for the alternative regimes are the same as for the BR. All four regimes are compared and ranked according to two welfare criteria.

5.1. Spread-augmented regime

For the credit spread-augmented regime (AR_{pm}), we inject into the baseline Taylor rule a term that responds to the credit spread. The monetary authorities now adjust the short-term interest rate in reaction to developments in inflation, output gap, and the credit spread which serves as the financial market indicator:

\[
\hat{r}_t = \rho \hat{r}_t + (1 - \rho) (r_p \hat{r}_t + r_y \hat{y}_t - r_{pm} \hat{m}_t) + r_{Hy} (\hat{y}_t - \hat{y}_{t-1}) + \xi_t
\]

where \(0 < r_{pm} < 1\) measures the magnitude of spread adjustment. The negative sign before \(r_{pm}\) suggests that \(r_t\) should be raised (lowered) relative to what the BR would prescribe when the spread is below (above) the average. \(r_{pm} = 0\) corresponds to the baseline case where the monetary policy does not directly respond to variations in the credit market. In Section 4 we have examined the model’s responses under the BR to a positive credit spread shock that approximates a credit policy tightening. Now we look at its responses under the AR_{pm} to a negative credit spread shock (or equivalently, a positive credit supply shock) that approximates a loosening of credit standards relatable to the pre-crisis credit boom.

Fig. 11 shows the IRFs under the BR and AR_{pm} (with different degrees of spread adjustment). We see that under the BR, a credit boom featuring a persistent lower-than-normal credit spread reduces the real lending rate, and raises the price of capital, investment, and output. As we increase \(r_{pm}\) from 0 to 0.01, 0.03, and 0.04, \(r_t\) is raised more compared to the BR, which in turn partially offsets the stimulus on inflation and output; this is consistent with the rationale that in response to a credit boom (crunch), spread-adjusted rule implies a higher (lower) policy rate than the baseline rule would otherwise suggest.

\[\text{Carrillo et al. (2021) consider a financial rule implemented via subsidies (to lenders) whose amount varies with the credit spread; this incentivizes lending when the credit spread rises.}\]
Note that when \( r_{pm} \) is increased to 0.04, consumption and inflation no longer rise, but fall. The responses in the credit spread, real lending rate, and investment are virtually the same regardless of the value of \( r_{pm} \). To find the optimal value of \( r_{pm} \), we search over its calibrated parameter space and keep the other coefficients fixed. Through simulation experiment, the optimal \( r_{pm} \) is found to be 0.035.

5.2. Spread shock-augmented regime

Gilchrist and Zakrajsek (2012) show that because of the endogenous response of asset price in the financial accelerator, a positive credit spread shock can cause a rise in the spread variable that exceeds the size of the shock itself. This motivates us to investigate if responding to the exogenous component of credit spread (i.e. the spread shock \( e_{pm}^t \)) can better stabilize the economy. In doing so, we consider an AR\(_{pm}\) where we replace the spread variable \( pm_t \) in AR\(_{pm}\) with the spread shock \( e_{pm}^t \):

\[
\hat{r}_t = \hat{p}_t \hat{r}_t + (1 - \rho) (r_{ps} \hat{p}_t + r_y \hat{y}_t - r_{pm} e_{pm}^t) + r_{y} (\hat{y}_t - \hat{y}_{t-1}) + \epsilon_t^{e_{pm}}
\]

Likewise, we also look at the model’s IRFs to a credit boom shock under the AR\(_{pm}\). For better comparison, the set of elasticities for \( e_{pm}^t \) are maintained the same as for \( pm_t \). Fig. 12 shows IRFs under the BR and the AR\(_{pm}\) (with different magnitudes of the reaction coefficient \( r_{pm} \)). In contrast to Gilchrist and Zakrajsek (2012) that report a much weaker dampening effect with the shock-augmented rule compared to the spread-augmented one, our result shows little to no disparity between the two. Following the credit stimulus, both the AR\(_{pm}\) and AR\(_{pm}\) raise \( r_t \) more than does the BR; by responding to certain credit market indicators, they enhance the offsetting mechanism to restrain the expansionary effects via strengthened Taylor rule responses. From simulation experiment, we find the optimal degree of response to be \( r_{pm} = 0.03 \).

5.3. Dual rule regime

Finally, we consider a dual rule regime (DRR) wherein we complement the normal state of the BR with a powerful M0 rule that responds to the credit spread, such that the counteractive M0 rule is activated in both states. There are two instruments (dual rules) in the normal state with each pursuing its own objective - the Taylor rule (R rule) promotes price stability via stabilizing inflation whilst the M0 rule targets financial stability via stabilizing the credit spread. Previously we have established M0’s credit easing effect during crises (when spread rises) via its role as the cheapest collateral. Now we explore if this M0 policy can work the opposite way - whether a M0 rule in non-ZLB situations can counteract the credit stimulus when the spread falls below normal due to a relaxation of regulatory policy.\(^{16}\) The M0 rule in normal times takes the same form as for crises:

\(^{16}\) This is also referred to as “monetary reform” in Le et al. (2016).
m_0 - m_{t-1} = \beta^{\text{(normal)}} \left(p_{t-1} - p_{t} \right) \tag{19}

Fig. 13 shows IRFs to a negative spread shock that triggers a credit boom under the BR and the DRR (with varying strengths of the M0 rule). We see that the benefit from using a counteractive M0 rule in normal times is also substantial. Unlike the AR_pm and AR_{pm}/C15 that fail to offset the stimulus in the credit spread, real lending rate, capital, and investment, under the DRR these variables return to the equilibrium quicker than under the BR. Further, we find that increasing reaction coefficients cause appreciably different responses across regimes. Under either AR_{pm} or AR_{pm}/C15, raising r_{pm} or r_{pm}/C15 affects the model’s initial responses whereas under the DRR, different magnitudes of the response parameter \#_{\text{normal}} lead to the same initial responses; the stronger cushioning effects under more powerful M0 rules emerge over time and grow wider over a 7-year horizon. From simulations, we find the optimal degree of response to be \#_{\text{normal}} = 0.45.

5.4. Comparison of regimes

We summarize in Table 2 the optimal elasticities that minimize the sum of key variables’ variances for each regime; HP filter was employed to remove the trends before variance calculations. Moreover, we also utilize a welfare measure similar to Lucas (1987) for regime evaluation and comparison. Lucas (1987) defines the cost of fluctuations as the percentage by which the average consumption that an individual has to be compensated to be indifferent between the certain path of consumption and the volatile one. Here we measure the welfare improvement under any alternative regimes compared to the BR by a compensation parameter \( s \) which measures the percentage by average the BR consumption has to be increased for the representative household to be indifferent between staying in the BR and moving to any alternative regimes.

From Table 2 we see that welfare cost (households’ utility) is reduced (increased) under any alternative regimes. Specifically, the DRR yields the best outcome in minimizing the sum of variances of output and inflation; it is also associated with the biggest compensation parameter. The DRR is followed by AR_{pm} and finally AR_{pm}/C15. Moreover, violation of the Tinbergen rule appears costly as the two-instrument regime DRR clearly outperforms one-instrument regimes. While augmenting the Taylor rules with responses to credit market conditions is definitely welfare improving compared to not responding at all, using a separate financial rule to target the credit spread proves better at taming fluctuations and maximizing welfare.

Fig. 14 plots 3 selected examples of simulated output under the 4 regimes. Qualitatively similar results are obtained across regimes; the reason is that in all regimes policymakers adjust policy stances according to some feedback rules that

\footnote{According to the Tinbergen rule, policymakers with multiple economic targets to achieve need to have at least one tool for each target; violation of the Tinbergen rule will undermine the efficiency of policies, if the achievement of one target precludes the achievement of others. This rule of thumb applies here because under the AR_{pm} and AR_{pm}/C15, we rely on a single instrument R to tackle two sources of inefficiencies: price instability induced by staggered pricing and financial instability from costly state verification.}
react to certain measures of the economy’s current state to bring them closer to the targets. The built-in credit stabilizer in the DRR keeps the financial market in check regardless of the state of the economy—it constrains (encourages) the credit supply before (after) the crisis when the credit spread is low (high). By taming the credit cycle both before the crisis and after, the M0 rule dampens the boom and bust cycles of the broader economy. Given that the DRR can be seen as a generalization of QE, in that using a counteractive M0 rule to regulate credit provision is implemented in both states. This provides insights into how policymakers should coordinate conventional and unconventional monetary policies in the pursuit of price and financial stability. See also Ellison and Tischbirek (2014) and Quint and Rabanal (2017) who documented evidence of benefits from using the unconventional tools in normal times.

6. Concluding remarks

In this paper, we try to capture the key elements relevant to analyzing the BoE’s credit market interventions since the GFC. Our purpose is not to model the way QE works in reality, but whether a variant of the SW(07) model that accommodates a collateral-augmented BGG framework can match the UK data. We model QE’s credit easing effect via the bank lending channel which has been less frequently investigated, and in doing so we envisage a novel transmission channel whereby M0 injection (via QE) can reduce the credit spread via acting as the most liquid collateral for firms’ loan agreements. Through indirect inference testing and estimation, we find a set of coefficients that can explain the UK key variables’ behavior for Fig. 13. A comparison of IRFs for the baseline regime vs. dual rule regime in response to a negative credit spread shock. Notes: The time intervals on the x-axis are quarters; the units on the y-axis are percentage deviations, except for interest rate, inflation, credit spread, real lending rate and net foreign assets that are reported as percentage-point deviations. The baseline regime is compared with the dual rule regime under 3 values of M0-adjustment parameter in response to a credit boom shock.

Table 2
Welfare analysis comparison for the various regimes.

<table>
<thead>
<tr>
<th></th>
<th>BR</th>
<th>AR_pmin</th>
<th>AR_pmin</th>
<th>DRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal elasticities</td>
<td>r_pmin</td>
<td>—</td>
<td>—</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>r_pmin</td>
<td>—</td>
<td>0.030</td>
<td>—</td>
</tr>
<tr>
<td>var(output)</td>
<td>1.6200</td>
<td>1.4490</td>
<td>1.3307</td>
<td>0.9573</td>
</tr>
<tr>
<td>var(inflation)</td>
<td>0.3543</td>
<td>0.3822</td>
<td>0.3811</td>
<td>0.5350</td>
</tr>
<tr>
<td>var(output)+var(inflation)</td>
<td>1.9743</td>
<td>1.8312</td>
<td>1.7118</td>
<td>1.4923</td>
</tr>
<tr>
<td>Compensation parameter τ</td>
<td>—</td>
<td>0.02156</td>
<td>0.02162</td>
<td>0.0560</td>
</tr>
</tbody>
</table>

Note: compensation parameter τ is calculated from $U(\beta + \tau ALT) = U(\beta ALT)$, where $\tau ALT = [AR_pmin, AR_pmin, DRR]$.

react to certain measures of the economy’s current state to bring them closer to the targets. The built-in credit stabilizer in the DRR keeps the financial market in check regardless of the state of the economy—it constrains (encourages) the credit supply before (after) the crisis when the credit spread is low (high). By taming the credit cycle both before the crisis and after, the M0 rule dampens the boom and bust cycles of the broader economy. Given that the DRR can be seen as a generalization of QE, in that using a counteractive M0 rule to regulate credit provision is implemented in both states. This provides insights into how policymakers should coordinate conventional and unconventional monetary policies in the pursuit of price and financial stability. See also Ellison and Tischbirek (2014) and Quint and Rabanal (2017) who documented evidence of benefits from using the unconventional tools in normal times.18

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18 Ellison and Tischbirek (2014) study the central bank’s asset purchases in a DSGE model, finding that unconventional monetary policy in the form of long-term government bonds purchase could be a valuable addition to the conventional interest rate policy even in non-ZLB situations. Quint and Rabanal (2017) use an estimated non-linear DSGE model and show that asset purchases of the government and corporate bonds should be employed in conjunction with conventional interest rate policy even when the nominal interest rate normalize.
1993–2016. Simulations show that the ZLB on the nominal interest rate can bind much more often than previously thought. Shock decompositions reaffirm the long-standing issue of the “productivity puzzle”. While financial shocks matter, it is the productivity and world shocks that had held back the recovery between 2009–2012, although the effect of their drag has been muted since 2013. We also experiment with three alternative monetary regimes where monetary authorities react to credit conditions in normal times. Simulations show that a dual rule regime with a counteractive M0 rule implemented in both states excels at minimizing business cycle fluctuations, although any type of adjustment (under alternative regimes), if of a suitable magnitude, can improve the model’s responses to financial market variations. Through attenuating the impact of shifts in credit supply and smoothing the flow of funds in booms and recessions alike, targeted credit interventions merit a place in the policymakers’ toolkit even in non-ZLB situations. Our work is built upon Le et al. (2016) for the US and is extended here to an open economy setting to study UK monetary policy. We think this UK exercise can provide insights into studying the behavior of other small open economies such as Japan which has also pursued QE policies in the context of the zero lower bound since the 2000s.\footnote{For example, Fang (2021) reports results of a similar model for Japan in studying the policies for reviving the Japanese economy.}

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**Appendix A. Appendices**

**A.1. Log-linearized model list**

Consumption Euler equation:

\[
\hat{c}_t = \left( \frac{1}{1 + \frac{\gamma}{\delta}} \right) \hat{c}_{t-1} + \left( \frac{1}{1 + \frac{\gamma}{\delta}} \right) E_t \hat{c}_{t+1} + \left( \frac{\sigma_c - 1}{\sigma_c} \right) \frac{w_t}{\sigma_c} \left( \frac{1 - \frac{\gamma}{\delta}}{1 + \frac{\gamma}{\delta}} \right) \left( \hat{r}_t - E_t \hat{r}_{t+1} \right) - \frac{1 - \frac{\gamma}{\delta}}{\sigma_c} \left( \hat{r}_t - E_t \hat{r}_{t+1} \right) + \hat{e}^p_t \tag{A1} \]

Investment Euler equation:

\[
\hat{i}_t = \left( \frac{1}{1 + \beta_i - (1 - \sigma_i)^\gamma} \right) \hat{i}_{t-1} + \left( \frac{\beta_i(1 - \sigma_i)}{1 + \beta_i(1 - \sigma_i)} \right) E_t \hat{i}_{t+1} + \left[ \frac{1}{(1 + \beta_i(1 - \sigma_i))\gamma} \right] \rho_k \hat{r}_t + \hat{e}^i_t \tag{A2} \]
Credit spread is reduced for given leverage via injection of M0:

\[ \varepsilon_t \rho \hat{p}_t + \hat{k}_t - \hat{n}_t \]

(A11)

Conventional and unconventional monetary policies in the baseline and the alternative regimes:

\[
\text{BR} = \begin{cases} 
\hat{r}_t = \rho \hat{r}_t + (1 - \rho) (r^p \hat{t}_t + r^y \hat{y}_t) + r^\Delta (\hat{y}_t - \hat{y}_{t-1}) + \hat{\varepsilon}_t & \text{For } r_t > 0.025 \\
\hat{m}_t^0 - \hat{m}_t^{0a} = \psi^{\text{normal}} (\hat{m}_t^0 - \hat{m}_t^{0a}) + \hat{\varepsilon}_t^{m0} & \text{(Normal state)} \\
\hat{m}_t^2 = (1 - \frac{MO}{M2} + \frac{N}{M2}) \hat{k}_t + \frac{MO}{M2} \hat{m}_t^0 - \frac{N}{M2} \hat{n}_t & \text{(a)} \\
\hat{m}_t^0 - \hat{m}_t^{0c} = \psi^{\text{crit}} (\hat{m}_t^{0c} - \hat{m}_t^{0c}) + \hat{\varepsilon}_t^{m0} & \text{(Crisis state)} \\
\hat{r}_t = 0.025 & \text{For } r_t \leq 0.025 \\
\hat{m}_t^0 - \hat{m}_t^{0a} = \psi^{\text{crit}} (\hat{m}_t - \hat{m}_t^{0a}) + \hat{\varepsilon}_t^{m0} & \text{(b)} 
\end{cases}
\]
Evolution of foreign bonds satisfies the balance of payment constraint:

\[
\dot{y}_t = \rho \dot{r}_t + (1 - \rho) (r_p \dot{r}_t + r_p \dot{y}_t - r_{pm} \dot{e}_{im}) + r_{sh} (\dot{y}_t - \dot{y}_{t-1}) + \dot{e}_t^{y}
\]

(Normal state)

\[
\dot{m}_t^0 - \dot{m}_{t-1}^0 = \omega^{(normal)} (\dot{m}_t^0 - \dot{m}_{t-1}^0) + \dot{e}_{im}
\]

\[
\dot{m}_t^m = (1 - \frac{m_0}{M}) \hat{k}_t + \frac{M_0}{M} \dot{m}_t^0 - \frac{N}{N^2} \hat{n}_t
\]

\[
\text{(A13)}
\]

\[
\text{For } r_t > 0.025 \quad \dot{r}_t = \rho \dot{r}_{t-1} + (1 - \rho) (r_p \dot{r}_t + r_p \dot{y}_t - r_{pm} \dot{e}_{im}) + r_{sh} (\dot{y}_t - \dot{y}_{t-1}) + \dot{e}_t^{y}
\]

\[
\dot{m}_t^0 - \dot{m}_{t-1}^0 = \omega^{(normal)} (\dot{m}_t^0 - \dot{m}_{t-1}^0) + \dot{e}_{im}
\]

\[
\dot{m}_t^m = (1 - \frac{m_0}{M}) \hat{k}_t + \frac{M_0}{M} \dot{m}_t^0 - \frac{N}{N^2} \hat{n}_t
\]

\[
\text{(A14)}
\]

\[
\text{For } r_t \leq 0.025 \quad \dot{r}_t = 0.025
\]

\[
\dot{m}_t^0 - \dot{m}_{t-1}^0 = \omega^{(crisis)} (\dot{m}_t^0 - \dot{m}_{t-1}^0) + \dot{e}_{im}
\]

\[
\text{(A15)}
\]

\[
\text{Export demand:}
\]

\[
e^{x_t} = \sigma^2 \log(1 - \omega^F) + \ddot{c}_t^x + \sigma^2 \dot{q}_t + \dot{e}_t^x
\]

\[
\text{(A16)}
\]

\[
\text{Import demand:}
\]

\[
im_t = \sigma \log(1 - \omega) + \dot{c}_t - \sigma^{2} \dot{q}_t + \dot{e}_t^m
\]

\[
\text{(A17)}
\]

\[
\text{Expected movement in the real exchange rate satisfies the uncovered interest rate parity:}
\]

\[
\dot{e}_t - \dot{e}_{t-1} = (\dot{r}_t - \dot{r}_{t-1}) - (\dot{r}_t^f - \dot{r}_{t-1}^f)
\]

\[
\text{(A18)}
\]

\[
\text{Evolution of foreign bonds satisfies the balance of payment constraint:}
\]

\[
\dot{b}_t^f = (1 + \dot{r}_t^f) \dot{b}_{t-1}^f + \frac{EX}{Y} (\dot{e}_t - \dot{q}_t) - \frac{IM}{Y} \dot{i}_t
\]

\[
\text{(A19)}
\]

\[
\text{Aggregate resource constraint:}
\]

\[
\dot{y}_t = \frac{C}{Y} \dot{c}_t + \frac{l}{Y} \dot{i}_t + \left( R^k \dot{k}_t - \frac{1}{\psi} \right) r_{mk} + \frac{C}{Y} \dot{c}_t^f + \frac{EX}{Y} (\dot{e}_t - \dot{q}_t) - \frac{IM}{Y} \dot{i}_t + \dot{e}_t^x
\]

\[
\text{(A20)}
\]

\[
\text{NOTE: hatted variables denote deviations from the BGP. Variables in block capitals without time subscripts are steady states.}
\]

\[
\text{Exogenous processes:}
\]

\[
\text{Government spending shock: } \dot{e}_t^g = \rho_s \dot{e}_{g-1} + \eta_t^g + \rho_{sg} \eta_t^f
\]

\[
\text{Investment-specific shock: } \dot{e}_t^i = \rho_i \dot{e}_{i-1} + \eta_t^i
\]

\[
\text{Productivity shock: } \dot{e}_t^p = \dot{e}_{p-1} + \rho_p ( \dot{e}_{p-1} - \dot{e}_{i-1} ) + \eta_t^p
\]

\[
\text{Wage mark-up shock: } \dot{e}_t^{wm} = \rho_{wm} \dot{e}_{wm-1} + \eta_t^{wm}
\]

\[
\text{Credit spread shock: } \dot{e}_t^{pm} = \rho_{pm} \dot{e}_{pm-1} + \eta_t^{pm}
\]

\[
\text{M0 shock: } \dot{e}_t^{m0} = \rho_{m0} \dot{e}_{m0-1} + \eta_t^{m0}
\]

\[
\text{Import demand shock: } \dot{e}_t^{im} = \rho_{im} \dot{e}_{im-1} + \eta_t^{im}
\]

\[
\text{Foreign real interest rate shock: } \dot{r}_t^{f(real)} = \rho_{f(real)} \dot{r}_{t-1}^{f(real)} + \eta_t^{f(real)}
\]

\[
\text{Preference shock: } \dot{e}_t^p = \rho_p \dot{e}_{p-1} + \eta_t^p
\]

\[
\text{Taylor rule shock: } \dot{e}_t^i = \rho_i \dot{e}_{i-1} + \eta_t^i
\]

\[
\text{Price mark-up shock: } \dot{e}_t^p = \rho_p \dot{e}_{p-1} + \eta_t^p
\]

\[
\text{Labor supply shock: } \dot{e}_t^{ws} = \rho_{ws} \dot{e}_{ws-1} + \eta_t^{ws}
\]

\[
\text{Net worth shock: } \dot{e}_t^w = \rho_w \dot{e}_{w-1} + \eta_t^{w}
\]

\[
\text{Export demand shock: } \dot{e}_t^{ex} = \rho_{ex} \dot{e}_{ex-1} + \eta_t^{ex}
\]

\[
\text{Foreign consumption shock: } \dot{e}_t^c = \rho_c \dot{e}_{c-1} + \eta_t^c
\]
### A.2. Tables and figures

Tables B1,B2,B3 and Figs. B1,B2.

#### Table B1
Model variable construction and data sources.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Definition and description</th>
<th>Source¹</th>
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<tbody>
<tr>
<td>R</td>
<td>Nominal interest rate</td>
<td>1-monthTreasuryBillRate</td>
<td>BoE</td>
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<tr>
<td>l</td>
<td>Investment</td>
<td>TotalGrossFixedCapitalformation</td>
<td>ONS</td>
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<tr>
<td>PK</td>
<td>Price of capital</td>
<td>Derived from capital arbitrage equation</td>
<td>Calculation</td>
</tr>
<tr>
<td>k</td>
<td>Capital stock</td>
<td>Derived from capital accumulation equation</td>
<td>Calculation</td>
</tr>
<tr>
<td>π</td>
<td>Inflation</td>
<td>Percentage change in GDP deflator</td>
<td>FRED</td>
</tr>
<tr>
<td>W</td>
<td>Wage</td>
<td>WagesandSalaries - Employeesocialcontributions - Incomefromselfemployment</td>
<td>ONS</td>
</tr>
<tr>
<td>C</td>
<td>Consumption</td>
<td>Householdfinalconsumption</td>
<td>ONS</td>
</tr>
<tr>
<td>Y</td>
<td>Output</td>
<td>RealGrossdomesticproduct</td>
<td>ONS</td>
</tr>
<tr>
<td>L</td>
<td>Labor</td>
<td>TotalActualWeeklyHoursWorked</td>
<td>ONS</td>
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<tr>
<td>RK</td>
<td>Marginal product of capital</td>
<td>Derived from labor demand equation</td>
<td>Calculation</td>
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<td>PM</td>
<td>Credit spread</td>
<td>1-monthLIBOR - 3-monthTreasuryBillRate</td>
<td>FRED, BoE</td>
</tr>
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<td>CY</td>
<td>Real cost of credit</td>
<td>1-monthLIBOR - one – periodaheadinflation</td>
<td>FRED</td>
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<td>N</td>
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<td>Yahoo Finance</td>
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<td>M0</td>
<td>Monetary base</td>
<td>Currentaccountdeficit</td>
<td>FRED</td>
</tr>
<tr>
<td>M2</td>
<td>Broad supply of money</td>
<td>Currentaccountdeficit</td>
<td>FRED</td>
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<tr>
<td>EX</td>
<td>Exports</td>
<td>Exportsforgoodsandservices</td>
<td>FRED</td>
</tr>
<tr>
<td>IM</td>
<td>Imports</td>
<td>Importsforgoodsandservices</td>
<td>FRED</td>
</tr>
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<td>Q</td>
<td>Real exchange rate</td>
<td>Sterlingeffectiveexchangerate</td>
<td>BoE</td>
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<td>Bf</td>
<td>Net foreign assets position</td>
<td>Accumulatedcurrentaccountdeficits</td>
<td>ONS</td>
</tr>
</tbody>
</table>

2GDP deflator is constructed using “implied GDP deflator at market prices” (ONS: QNA), normalized so that QNA(2010Q1 = 100); population index is constructed using “Population aged 16+” (ONS: MGSL), normalized so that MGSL (2010Q1)=1.

¹BoE, FRED, and ONS are short for the Bank of England, Federal Reserve Economic Data, and Office for National Statistics, respectively.

#### Table B2
Parameters held fixed throughout our investigation.

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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<td>β</td>
<td>Quarterly discount factor</td>
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<td>SW(07)</td>
</tr>
<tr>
<td>δ</td>
<td>Quarterly depreciation rate</td>
<td>0.025</td>
<td>SW(07)</td>
</tr>
<tr>
<td>γ</td>
<td>Common quarterly trend growth rate</td>
<td>1.004</td>
<td>SW(07)</td>
</tr>
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<td>ε_p</td>
<td>Kimball aggregator curvature in the goods market</td>
<td>10</td>
<td>SW(07)</td>
</tr>
<tr>
<td>ε_w</td>
<td>Kimball aggregator curvature in the labor market</td>
<td>10</td>
<td>SW(07)</td>
</tr>
<tr>
<td>θ</td>
<td>Survival rate of entrepreneurs</td>
<td>0.97</td>
<td>BGG, Villa (2014)</td>
</tr>
<tr>
<td>ω</td>
<td>Bias towards domestically produced goods in consumption bundle</td>
<td>0.5</td>
<td>Dong et al. (2019)</td>
</tr>
<tr>
<td>c₀^F</td>
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Fig. B1. Key quarterly UK data. Notes: The grey shaded band indicates the UK recession measured as two consecutive quarters of negative GDP growth; the yellow shaded band corresponds to the pre-crisis credit boom where $PM < PM^*$ (steady-state value is the data average for 1993–2016). Variables are in natural logs except for the nominal interest rate, inflation and credit spread which are already expressed in percentages.

Fig. B2. Model implied shock histories. Notes: The grey shaded bar indicates the recession. To derive the model implied shock histories, we first plug the estimated coefficients into the model to calculate the differences (residuals) between the data and model equations; these residuals are then regressed on their past values to derive the shocks (innovations) that perturb the economy in each period. Foreign real interest rate and consumption shocks are the errors from the AR regressions of data.

References


Taylor, J.B. et al. (2008). Monetary policy and the state of the economy. testimony before the committee on financial services, us house of representatives, volume 26.


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