Volt-VAr Optimization of a Low Voltage Distribution Network in Nigeria

Ye-Obong Udoakah$^{\diamond}$, Adedoyin Inaolaji$^{\oplus}$, Liana Cipcigan$^{\diamond}$, and Sumit Paudyal$^{\oplus}$
$^{\diamond}$Cardiff University, UK; $^{\oplus}$Florida International University, USA
Emails: udoakahyn1@cardiff.ac.uk, ainao003@fiu.edu, cipciganlm@cardiff.ac.uk, spaudyal@fiu.edu

***Abstract*—Volt-VAr optimization (VVO) is important in a distribution system as the performance of the entire network depends on the voltage profile to a very large extent. The deployment of renewable energy sources, particularly photovoltaic (PV) systems, is usually achieved at the medium voltage (MV) and low voltage (LV) levels of distribution feeders and may exacerbate the challenges associated with maintaining the voltage profile within the pre-defined limits. For such a PV-rich system, VVO can be carried out such that the inverter-based PV systems can actively participate in voltage regulation and optimization by providing flexible reactive power support. This paper addresses the voltage concerns associated with high PV penetration by implementing a distribution grid optimal power flow (DOPF) problem on a realistic LV distribution network in Kano, Nigeria. The current injection-based I-V DOPF formulation is used to model the grid and the VVO utilizes the reactive power of the PV inverter to minimize the active power losses in the network. The results demonstrate the ability of the VVO to perform voltage regulation and can serve as a viable technique for mitigating voltage violation issues in the Nigerian grid.**

***Index Terms*—Volt-VAr optimization, LV Nigerian distribution grid, optimal power flow, voltage regulation.**

I. INTRODUCTION

Increasing oil prices and resultant threats of energy boycotts by countries majorly depending on fossil fuel by-products for their power generation has intensified the need for more renewable deployment and energy sufficiency. Such deployment of renewable energy sources (RES), particularly photovoltaic (PV) systems, and its subsequent integration into the distribution network (DN) is causing the power system, which was conventionally designed for a unidirectional transfer of power to be subtly reconfigured to cater for a bidirectional transfer of power. This RES integration is usually achieved at the medium voltage (MV) and low voltage (LV) levels of distribution feeders and can culminate into the violation of DN "statutory low voltage (LV)" steady state voltage limits. Furthermore, the high penetration level of RES may result in frequent operation of traditional Volt-VAr control devices such as step voltage regulators, load tap changers and capacitor banks. These devices are limited in their ability to respond to fast voltage variation triggered by a high fluctuation in the generated power from PV. On the other hand, inverter-based PVs are fast-acting and have the capability of providing flexible reactive power support for voltage regulation as indicated in IEEE-1547 standard [1]. Hence, for such a PV-rich system, inverters present opportunities for Volt-VAr optimization (VVO) techniques to be carried out in the DN.

Nigeria currently has an energy mix which mainly comprises of thermal and hydro power sources. Still, the Nigerian power system continues to suffer recurring challenges associated with a high gap between electricity generation and demand despite being significantly endowed with renewable energy potentials [2]. In the Renewable Energy Master Plan (REMP), the Energy Commission of Nigeria (ECN) proposed to increase the supply of renewable electricity from 13% of total electricity generation in 2015 to 23% and 36% by 2025 and 2030 respectively [3]. Despite the numerous reforms proposed by successive governments however, several studies have revealed that the power distribution at the MV and LV level is marred with various voltage control challenges culminating in frequent outages experienced by consumers without any prior notification in most cases [4, 5]. Specifically, the results of the study performed by [6] on the simulation and analysis of an LV DN of a real Nigerian feeder did show that at certain loading levels, the acceptable voltage limits are being exceeded. The frequency of blackout events occasioned by such voltage collapse in the power system has therefore raised renewed interest on the subject of VVO.

VVO is typically formulated as a distribution grid optimal power flow (DOPF) problem. As opined by [7], the objective of DOPF is to determine the steady state operating points which minimize a specified objective while meeting demand and also satisfying the physics of the network and other operational constraints. Several studies have investigated various voltage control measures on the medium and low voltage levels and have proposed several optimization solution methods. For instance, a proprietary Algorithm of the Innovative Gunner (AIG) algorithm was used in [8] to achieve voltage control in a medium-voltage network. Compared to the traditional approach, the proposed optimization technique reported significant improvement in the voltage performance of the feeders as well as reduction of losses. In a related study, using the particle swarm optimization (PSO) method, a novel voltage control strategy was proposed in [9] to minimize power loss within a voltage optimization model. The results indicated that the proposed method was capable of solving the voltage violation issues, while also reducing the power loss and reasonably improving the voltage performance. These methods in [8, 9], however, are based on heuristics and are incapable of providing viable solutions that deliver the optimal solution. In contrast

This work is supported by the Decarbonising Transport through Electrification (DTE) Network and funded by EPSRC grant reference EP/S032053/1. Corresponding Author, Email: udoakahyn1@cardiff.ac.uk.

to heuristic methods, mathematical optimization methods are able to consistently provide a guarantee on the optimality of the solution [10–12].

Given the power utilization challenges at the MV and LV distribution level reported by different studies with respect to the Nigerian grid [4–6], this paper presents a VVO model that is capable of providing a localised solution to the LV DN in Nigeria. The VVO is achieved using a mathematical DOPF formulation that is based on a current injection-based I-V formulation. The resulting formulation is non-convex and can be formulated as a nonlinear programming (NLP) problem. The NLP formulation is exact and accurate representation of the DOPF. Analysis of the VVO model is carried out for multi-period cases and is considering the injection and absorption of reactive power of PV inverters as controllable resources. Although DOPF is well-studied in the literature, its application in the VVO of a real LV DN in Nigeria has not been investigated to the best of the authors' knowledge. This paper therefore aims to fill this knowledge gap by applying a VVO model to address the voltage concerns in a realistic DN in Nigeria.

The rest of the paper is organized as follows. Section II presents the model and topological description of the LV Nigerian distribution feeder under examination. In Section III, the VVO model is presented. Section IV presents the test system setup and case studies, while the relevant results and discussion of the study is presented in Section V. The conclusion of this work and the future research direction is captured in section VI.

II. NETWORK MODEL AND TOPOLOGICAL DESCRIPTION

The various levels of voltage transition in a typical DN in Nigeria is represented in Fig. 1. At the MV end of the line diagram, the network is tied to the grid system on a 3-phase single line circuit and power is transmitted to most of the LV distribution substation from the 11kV feeder through the 11kV overhead lines. At the 11kV substations, the voltage level is reduced to 0.415kV using appropriate transformer ratings determined by the local population size and anticipated load demand.

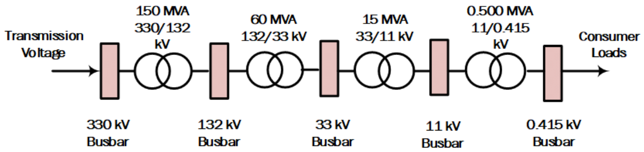

Fig. 1: Line diagram indicating voltage transformation from transmission to distribution level.

The section of the DN modelled for this study showing the 33/11kV distribution lines is shown in Fig. 2. A detailed description of the model is provided in our related work [6]. The DN is modelled using the network parameters from Kano central region in Nigeria as depicted in Fig. 3. The DN which consists of 7 of 11kV feeders serves Kano central region having 35km span of overhead conductors. Feeder 1 which has been chosen for this study has a total of about 1,908 connected consumers with overhead lines stretch of up to 1,055m. The transformer substation of interest (4^{th} Transformer) has a total of 408 connected consumers.

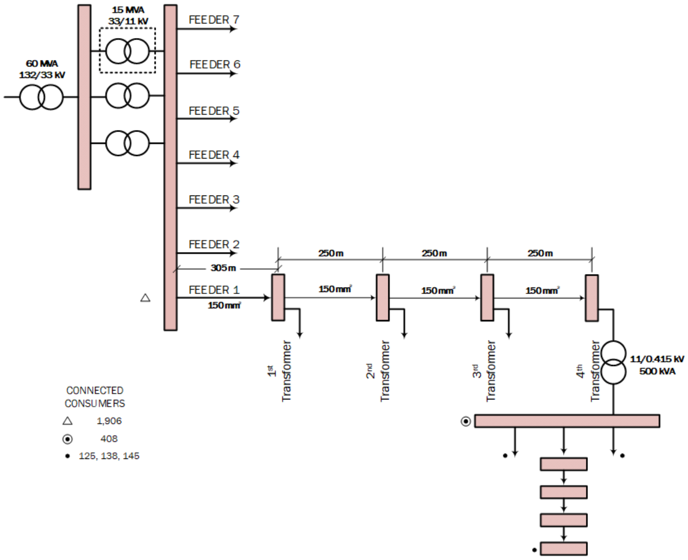

Fig. 2: Model of a typical DN in Nigeria

.

Feeder 1 supplies four different transformer substations labelled as 1^{st} transformer to 4^{th} transformer respectively. The 11/0.415kV, 500kVA transformer substation labelled as 4^{th} transformer is one of the four substations investigated in this study. The nodal representation of each of the three arms of the DN labeled as circuits A, B and C respectively indicating some of the highlighted parameters is shown in Fig. 4 to Fig. 6.

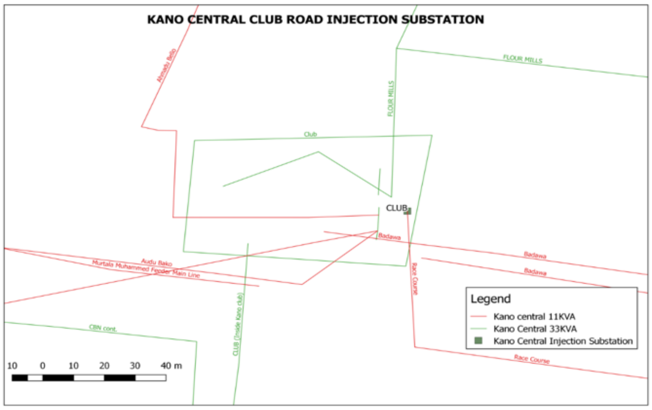

Fig. 3: Schematic showing MV distribution at Kano Central Injection Substation.

The investigated arms of the LV network are assigned different node numbers with the various nodes having different number of consumers with varying daily load profiles (see Fig. 4 to Fig. 6). For circuit C, Nodes 1 to 14 have residential consumers while nodes 15 and 16 have commercial consumers. Different consumers were classified into different housing types namely: residential and commercial. The average length

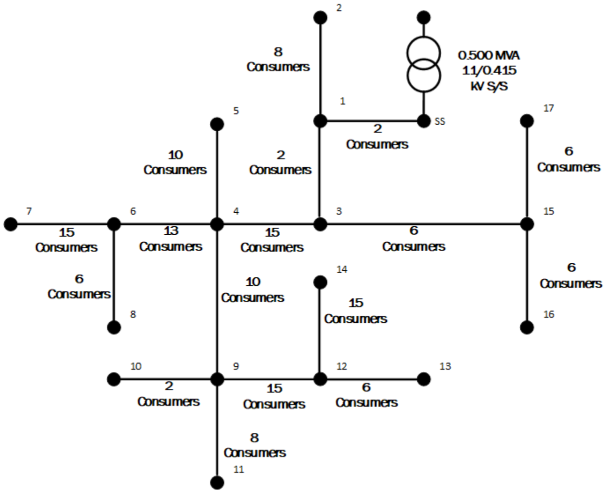

Fig. 4: The nodal representation of circuit A of the DN.

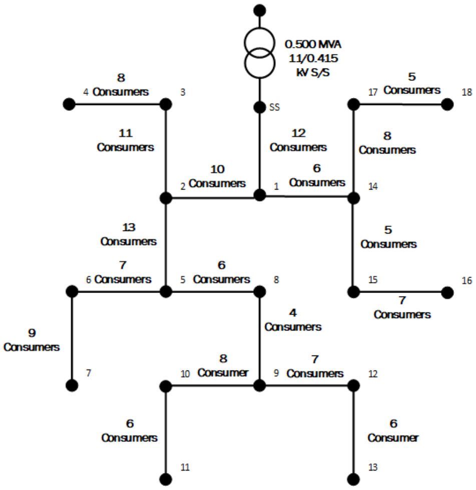

Fig. 5: The nodal representation of circuit B of the DN.

for circuits A, B and C are 2,295m, 1,620m and 990m, respectively. Further analysis indicates that the loads across the various circuits are not uniformly or evenly distributed. The number of consumers on circuits A, B C are 145, 138 and 125, respectively.

III. VVO FORMULATION

A DN is represented by an undirected graph consisting of a set of $\mathcal{N}$ nodes, and a set of $\mathcal{E}$ branches. Nodes are indexed by *i= 0,1,2,...,n*, where *n* is the total number of nodes in the network, and $\mathcal{N}' = \mathcal{N} \backslash \{0\}$. j is defined as an alias of i. Node 0 is the slack bus, representing the substation node with voltage fixed to the nominal value. $t \in \mathcal{T}$ denotes the time index, where $t = 1, 2, ..., T$ for T time intervals.

Let $V_{i,t} = V^{re}_{i,t} + \mathbf{j}V^{im}_{i,t}$ denote the complex voltage at node i during time t, and $|V_{i,t}| = V_{i,t}$. Let $I_{i,t} = I^{re}_{i,t} + \mathbf{j}I^{im}_{i,t}$ denote the complex current injected at node i during time t. Let $s_{i,t} = p_{i,t} + \mathbf{j}q_{i,t}$ represent the net complex powers injected at node i during time t, which is the sum of generation from substation and PV minus load demand from consumers.

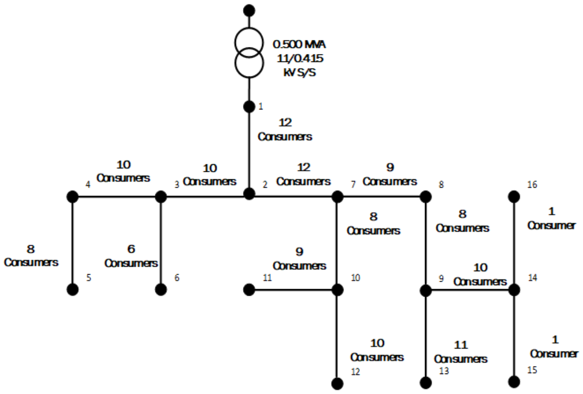

Fig. 6: The nodal representation of circuit C of the DN.

$G_{ij} + \mathbf{j}B_{ij}$ represents the $(ij)th$ element of the bus admittance matrix.

The I-V DOPF model [13] is used as the base grid model. It expresses the power flow equations in terms of the current-voltage relationships where the network flows are linear and the nonconvexities appear in the constraints relating bilinear terms in the load model [13]. The VVO for the distribution grid is therefore modelled as in (1)-(8).

$$OF_1 = \min \sum_{i \in \mathcal{N}} \sum_{t \in \mathcal{T}} p_{i,t}. \tag{1}$$

$$OF_2 = \min \sum_{i \in \mathcal{N}'} \sum_{t \in \mathcal{T}} (|V_{i,t}| - |V_0|)^2. \tag{2}$$

Subject to the following constraints
Current injection constraints:

$$I^{re}_{i,t} = \sum_{j:(i,j) \in \mathcal{E}} V^{re}_{j,t} G_{ij} - V^{im}_{j,t} B_{ij}, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{3}$$

$$I^{im}_{i,t} = \sum_{j:(i,j) \in \mathcal{E}} V^{re}_{j,t} B_{ij} + V^{im}_{j,t} G_{ij}, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{4}$$

Power injection constraints:

$$p_{i,t} = V^{re}_{i,t} I^{re}_{i,t} + V^{im}_{i,t} I^{im}_{i,t}, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{5}$$

$$q_{i,t} = V^{im}_{i,t} I^{re}_{i,t} - V^{re}_{i,t} I^{im}_{i,t}, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{6}$$

PV model:

$$(q^G_{i,t})^2 \leq (s^G_i)^2 - (p^G_{i,t})^2, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{7}$$

Voltage bounds:

$$\underline{V_{i,t}}^2 \leq |V_{i,t}|^2 \leq \overline{V_{i,t}}^2, \quad \forall i \in \mathcal{N}, \ \forall t \in \mathcal{T} \tag{8}$$

The VVO given in (1)-(8) is an NLP problem. The first objective, OF_1, minimizes the total active power loss in the DN, which is equivalent to the sum of injections and modelled as (1). The second objective, OF_2, is the minimization of total voltage deviation in the DN, which can be represented as (2). The I-V DOPF equations in (3)-(6) serve as the underlying grid model. The reactive power of the inverter is limited by its nameplate rating and active power rating as given in (7). Customer voltages are constrained to operate within the pre-specified voltage bounds by (8).

IV. DISTRIBUTION SYSTEM SETUP AND CASE STUDIES

The VVO formulation is performed on the LV distribution feeder located in Kano, Nigeria by considering single-phase sections of feeders A, B and C as described in Section II. The goal of the VVO is to control inverter reactive power resources in the DN to regulate the system voltages. The aggregate 1-hour resolution of the time-series of the actual active and reactive power load demand over an entire day is shown Fig. 7(a). A 100% PV penetration is considered, where all customers have PV systems operating at maximum power point tracking (MPPT), each with 4kW (average installation size in Nigeria [14]). Each PV system is oversized such that the inverter's apparent power capacity is 10% above the PV active power rating. The PV generation is based on the normalized data shown in Fig. 7b.

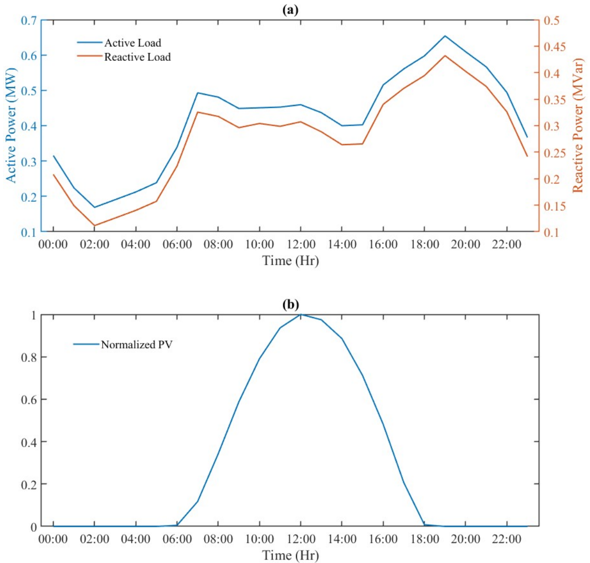

Fig. 7: Load and PV profiles: (a) aggregate active and reactive power load, and (b) normalized daily performance of PV.

For the simulation, the feeder circuit head voltage is set as the phase secondary voltage of the substation transformer, which is 240 kV (1 p.u.). According to the Nigerian Electricity Supply Industry (NESI) [15], the operational voltages of residential customers connected to the distribution system should be within the range of $+/-6\%$ of the nominal system voltage, i.e. between 230.4 V (0.94 p.u.) and 254.4 V (1.06 p.u). The goal of the VVO is therefore to control the reactive power of the PV systems connected on the distribution system in order to regulate the nodal voltages within this limit.

Three case studies are considered in this study. In the Case-1 scenario, no VVO is carried out and the PV reactive power is not utilized. During Case-2, the VVO optimally dispatches the reactive power of the PV system by minimizing active power losses, i.e OF_1. During Case-3, the VVO optimally dispatches the reactive power of the PV system by minimizing the voltage deviation of the customer voltages from the nominal system voltage, i.e OF_2.

V. SIMULATION RESULTS AND DISCUSSION

Here, we compare the performance of the network during the three scenarios considered. Fig. 8 shows the minimum and maximum nodal voltages of the distribution feeder over the entire day. During Case-1, the high PV penetration causes the voltage upper limit to be violated at 12 noon by 0.12% as indicated by Fig. 8(a). Additionally, the voltage lower limit is violated by 0.23% during a period of high demand around 7 PM. The application of the VVO during Case-2 and Case-3, however, leads to a significant improvement in the voltage profile as all the customer voltages are within the pre-specified limit as shown in Fig. 8(b) and Fig. 8(c) respectively. Notice that the PV systems are operating at MPPT during the control case (Case-2 and Case-3), and so there is no curtailment of PV active power. The inverter therefore participates in VVO to regulate the system voltages and mitigate voltage violations through injection and absorption of the reactive power of the inverter. Hence, allowing very high penetrations of residential PV systems into the Nigerian grid will require the utilization of the reactive power capability of the PV inverter in order to avoid undervoltage caused by critical load demands and overvoltage caused by high PV generation without the need to curtail the PV active power.

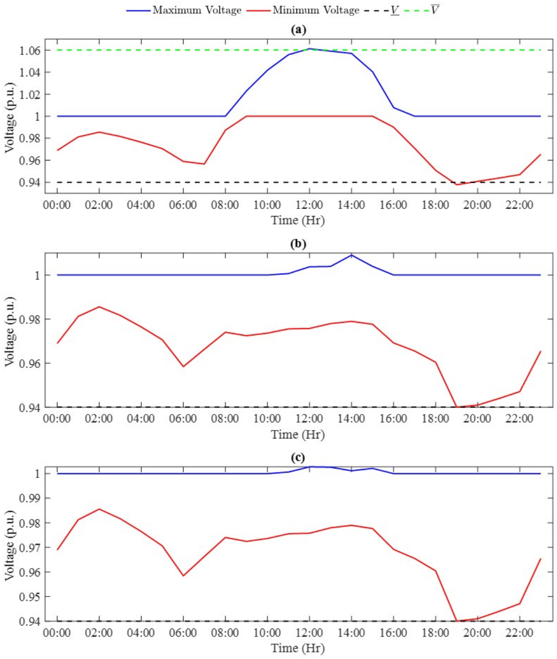

Fig. 8: Minimum and maximum voltage profiles for (a) Case-1, (b) Case-2, (c) Case-3.

The active and reactive power dispatch at the substation, the total reactive power dispatch from the PV systems, the total active power loss and the total voltage deviation over the entire day are shown in Table I. During Case-1, the high penetration of PV systems in the network causes a reverse active power flow (indicated by the negative sign) of 1.328 MW to the substation, while there is an injection of 7.439 MVAr of reactive power from the substation. Such reverse power flow is undesirable and could be hazardous to the network equipment.

Utilizing the reactive power resources of the inverter during both Case-2 and Case-3 eliminates the reverse power flow associated with high PV penetration such that there is an injection of 7.281 MW from the substation. A net total of 1.359 MVAr and 1.301 MVAr respectively are utilized from the inverter to regulate the voltage within the specified limits. This subsequently reduces the substation reactive power by 19.8% and 18.9% during Case-2 and Case-3 respectively. Consequently, the stress on the distribution feeder is reduced. Lastly, it is worth noting that the total active power loss reduces from 0.437 MW to 0.172 MW with the application of VVO during both Case-2 and Case-3. Similarly, the deviation of customer voltages from the nominal system voltage is reduced by 33.4% during the VVO application (Case-2 and Case-3). Hence, the VVO improves the feeder voltage profile and also reduces the active power losses in the network. Such an improvement is beneficial as it enhances the power quality provided to the customers.

TABLE I: Substation Power, Total Reactive Power Dispatch, Total Active Power Loss and Total Voltage Deviation.

	Case-1	Case-2	Case-3
Substation active power (MW)	-1.328	7.281	7.281
Substation reactive power (MVAr)	7.439	5.839	5.898
Total PV reactive power (MVAr)	-	1.359	1.301
Total active power loss (MW)	0.437	0.172	0.172
Total voltage deviation (pu)	0.772	0.514	0.514

VI. CONCLUSION

This paper analysed and addressed the voltage concerns resulting from critical load demand and high PV penetration in a real LV Nigerian DN by implementing a DOPF model for a VVO application. The need for this DOPF model is motivated by the growing integration of PV systems in distribution networks, coupled with an increase in load demand in the Nigerian grid. Using a current injection-based IV DOPF formulation, the VVO was carried out such that the inverter-based PV systems can actively participate in voltage regulation by providing flexible reactive power support and the operating conditions of the LV DN were analyzed under three scenarios. By implementing VVO, total active power loss was reduced from 0.437 MW to 0.172 MW, while the deviation of customer voltages from the nominal system voltage was reduced by 33.4%. The simulation results demonstrate the ability of the VVO to perform voltage regulation and maintain the customer voltage levels within the acceptable limits without any need for curtailing the active power of the PV system.

VII. ACKNOWLEDGMENT

The sponsorship of TETFund Nigeria, and the efforts made by Uduak Akpanedet and other staff of the Kano distribution company in providing the relevant network data is acknowledged and appreciated.

REFERENCES

[1] “IEEE standard for interconnection and interoperability of distributed energy resources with associated electric power systems interfaces,” *IEEE Std 1547-2018 (Revision of IEEE Std 1547-2003)*, pp. 1–138, 2018.

[2] Y.-O. N. Udoakah and M. D. Umoh, “Sustainably meeting the energy needs of nigeria: The renewable options,” in *2014 IEEE International Energy Conference (ENERGYCON)*. IEEE, 2014, pp. 326–332.

[3] “The Energy Commission of Nigeria (ECN),,” https://www.iea.org/policies/4974-nigeria-renewable-energy-master-plan.

[4] M. O. Oseni, “An analysis of the power sector performance in nigeria,” *Renewable and Sustainable Energy Reviews*, vol. 15, no. 9, pp. 4765–4774, 2011.

[5] E. G. Saturday, “Nigerian power sector: A new structure required for effective and adequate power generation, transmission and distribution,” *Global Journal of Engineering and Technology Advances*, vol. 7, no. 1, pp. 006–018, 2021.

[6] Y.-O. Udoakah, H. B. Sonder, and L. Cipcigan, “Low voltage distribution network simulation and analysis for electric vehicle and renewable energy integration,” in *2021 IEEE Power & Energy Society Innovative Smart Grid Technologies Conference (ISGT)*. IEEE, 2021, pp. 1–5.

[7] W. A. Bukhsh, A. Grothey, K. I. McKinnon, and P. A. Trodden, “Local solutions of the optimal power flow problem,” *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4780–4788, 2013.

[8] P. Kacejko and P. Pijarski, “Optimal voltage control in mv network with distributed generation,” *Energies*, vol. 14, no. 2, p. 469, 2021.

[9] Q. Meng, R. Che, and S. Gao, “Reactive power and voltage optimization control strategy in active distribution network based on the determination of the key nodes,” in *IOP Conference Series: Materials Science and Engineering*, vol. 199, no. 1. IOP Publishing, 2017, p. 012101.

[10] K. Turitsyn, P. Sulc, S. Backhaus, and M. Chertkov, “Options for control of reactive power by distributed photovoltaic generators,” *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1063–1073, 2011.

[11] M. Farivar, R. Neal, C. Clarke, and S. Low, “Optimal inverter var control in distribution systems with high PV penetration,” in *Proc. IEEE Power and Energy Society general meeting*. IEEE, 2012, pp. 1–7.

[12] X. Su, M. A. Masoum, and P. J. Wolfs, “Optimal PV inverter reactive power control and real power curtailment to improve performance of unbalanced four-wire lv distribution networks,” *IEEE Transactions on Sustainable Energy*, vol. 5, no. 3, pp. 967–977, 2014.

[13] M. B. Cain, R. P. O’neill, A. Castillo *et al.*, “History of optimal power flow and formulations,” *Federal Energy Regulatory Commission*, vol. 1, pp. 1–36, 2012.

[14] “Solar Report Nigeria,,” https://www.rvo.nl/sites/default/files/2021/06/Solar-Report-Nigeria.pdf.

[15] “The Distribution Code,,” https://nerc.gov.ng/index.php/library/documents/func-startdown/625/.