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#### Developing Analytical Model for Nighttime Cooling of Internal Thermal Mass

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#### 12 Abstract

Nighttime mechanical ventilation of internal building thermal mass has the potential to create 13 energy flexibility by shifting peak cooling demand. The research on nighttime cooling of internal 14 thermal mass is limited due to lacking simplified models by considering mass dimension and real-15 world discrete climate data like outdoor air temperatures. This study develops an analytical model 16 for nighttime cooling of internal thermal mass with a constant air change rate and hourly varied 17 air temperatures. The analytical model is verified by both numerical, conduction transfer function 18 methods, and ANSI/ASHRAE Standard 140-Case 600. The analytical model is applied to quantify 19 20 the free cooling energy storage in 48 selected U.S. cities in different climate zones and in the 16 climate zones of California. Among the 48 cities, the maximum free cooling energy storage is 21 reported in Santa Fe, NM with a total free cooling energy storage of 19.1 kWh m<sup>-2</sup> a<sup>-1</sup> and a net 22 free cooling energy storage of 3.88 kWh m<sup>-2</sup> a<sup>-1</sup>. Coastal regions in California are not suitable for 23 24 nighttime ventilation of internal thermal mass. The maximum total free cooling energy storage in California achieves 27.5 kWh m<sup>-2</sup>  $a^{-1}$ , while the maximum net free cooling energy storage is 6.11 25 kWh  $m^{-2} a^{-1}$ . The analytical model has a potential to be integrated into whole building energy 26 simulation software to improve the calculation of the effect of internal thermal mass. 27

Key words: energy flexibility; sensible heat storage; CFD; thermal inertia; night cooling; night
 ventilation.

# 30 **1. Introduction**

The U.S. has committed to joining the global efforts to reach net-zero emissions by 2050 in the 31 32 26<sup>th</sup> Conference of the Parties (COP26). A key pathway toward carbon neutrality is to increase the use of renewable energy sources. The intermittency and fluctuation of onsite renewable energy 33 sources may lead to imbalance between supply and demand. Demand-side energy flexibility with 34 35 intermediate energy storage is considered a promising solution to alleviate the supply and demand mismatch [1]. Buildings can play an important role to create demand-side energy flexibility by 36 37 transforming massive structures into thermal mass to store thermal energy [2]. Thermal mass has the capacity to absorb sensible heat during the daytime and release it during the nighttime. The 38 time lag of heat absorption and release shifts the peak thermal load in time and creates energy 39 flexibility [3]. The amount of energy flexibility created by thermal mass alone might be relatively 40 small due to its varied ability to shift heating and cooling demand in different climates [4]. 41

42 The capacity of thermal mass to shift cooling demand depends on climates. In cold climates,

thermal mass on walls might increase building cooling demand in summer [5]. In warm and mild 43 climates, thermal mass on walls is reported to have low influence on energy demand reduction [6]. 44 In hot climates, high thermal mass of insulated concrete wall increases cooling energy demand as 45 46 high ambient temperature causes more heat stored during daytime than that released during nighttime [7]. An effective measure to increase the heat release from thermal mass is nighttime 47 cooling [8]. Cool outdoor air can be directly induced to flush the thermal mass and create a heat 48 sink to absorb space heat gains via convection and radiation in the following daytime. Nighttime 49 50 cooling of thermal mass has the potential to reduce the need for compressor-based cooling energy demand [9]. The efficiency of nighttime cooling is most effective when night-time temperatures 51 are below 20°C, summer diurnal temperature swings are between 15-20°C, and maximum daytime 52 temperature is between 30 and 36°C [10]. A review [11] of related research undertaken between 53 1997 and 2017 reveals that most work has been done to investigate the energy demand reduction 54 of nighttime cooling of external thermal mass on walls. The external thermal mass is exposed to 55 varied outdoor climate conditions and the heat sink generated in the nighttime might be filled too 56 rapidly in the morning to shift the peak cooling load in the afternoon. One way to expand the 57 lifetime of the heat sink is to use internal thermal mass on the floor. The presence of internal 58 59 thermal mass can increase the building time constant by up to 42%, which is calculated as the time needed to reach 63.2% of the temperature change between two steady states and measures the 60 ability of thermal mass to time-shift cooling load [12]. A question remains unanswered is: what is 61 the capacity of nighttime cooling of internal mass to reduce energy demand in different U.S. 62 climates? 63

Limited research on nighttime cooling of internal thermal mass is attributed to the lack of 64 simplified but accurate models to quantify the cooling demand reduction under realistic climate 65 conditions [13,14]. In-situ measurements provide the most realistic quantification of the cooling 66 demand reduction of internal thermal mass. Experiments carried out in two real-scale buildings 67 showed that the use of thermal mass reduced the cooling energy demand by 67% to 75% during 68 69 14 days with only slight diurnal temperature variations [15]. However, measuring cooling energy storage is challenging due to difficulties in placing sensors in thermal mass. In most experiments, 70 only air temperatures are measured and it is difficult to separate the influence of thermal mass from 71 72 other impact factors [16]. Whole building energy simulation software like EnergyPlus [17] and 73 REGCAP [18] are suitable to investigate the transient behavior of internal thermal mass and nighttime ventilation. However, in EnergyPlus, internal thermal mass is either simplified as 74 75 interior construction materials with an effective area or lumped into the zone air with a capacitance multiplier. The oversimplification might lead to overestimation of energy storage of internal 76 77 thermal mass that behaves superficially energy efficient. In REGCAP, the internal thermal mass 78 is described with an empirical relationship that scales with floor area. The empirical relationship 79 is based on the concept of the building time constant and does not consider the effect of thermal properties. In most whole building energy simulation software, the thermal mass dimension is 80 81 completely ignored [19]. Reduced-order models are developed to consider details of internal thermal mass and nighttime cooling. Thermal quadrupole approach [20] can be implemented to 82 calculate thermal energy storage in internal thermal mass. The approach is based on the analytical 83 solution of one-dimensional heat transfer in an infinitely large solid plane, and the boundary 84 conditions are limited to sinusoidal signals. Numerical methods such as finite element [21] or finite 85 difference [22] can be used to quantify the energy demand reduction with internal thermal mass 86 87 under realistic temperature boundary conditions. The drawback of numerical methods is the requirement of a small time-step to ensure numerical stability, which increases the computational 88

time [23]. A case study of monthly temperature variation can take up to days to complete the 89 90 computation. Another approach that is essentially similar to numerical methods is the RC thermal model [24]. A reduced-order RC (2R2C) model was adopted to study the performance of internal 91 92 thermal mass and results showed that 'building as battery' efficiency is between 40% and 80% depending on the insulation level [25]. A limitation of the 2R2C model is the short prediction time 93 span of up to 12 h. An alternative reduced-order model is to directly derive analytical solutions to 94 the time-dependent heat conduction equation in the internal thermal mass [26]. The state-of-the-95 art analytical models are limited to simplified boundary conditions. A typical example is to use a 96 sinusoidal function to approximate real-world diurnal outdoor air temperature variations [27]. The 97 sinusoidal approximation cannot reflect the actual complexity of time-varying temperature 98 boundary conditions [4], for example, the nighttime air temperatures are hourly-varied values and 99 the daytime air temperature is the constant setpoint value. To deal with the difference in the 100 nighttime and daytime temperature boundary conditions, Yang and Li [28] developed nighttime 101 and daytime analytical models separately by expressing the outdoor temperature as a sinusoidal 102 profile with a frequency of 24 h. The analytical model is developed for zero-dimensional thermal 103 mass during a period of 24 h. It is in urgent need to develop a simplified mathematical model for 104 105 nighttime cooling of internal thermal mass to consider mass details and real weather data. The question is how to develop the model by considering mass dimension and real-world weather data, 106 for example, discrete (hourly varied) air temperatures? 107

In summary, much research has been done to investigate the nighttime cooling of external thermal 108 mass on walls. The research gap is to understand the potential of nighttime cooling of internal 109 thermal mass to reduce cooling demand in different climates. An urgent need is to develop a 110 simplified mathematical model for nighttime cooling of internal thermal mass by considering mass 111 dimension and real-world weather data, e.g., discrete air temperatures in a span of a year. The 112 detailed objectives of this study include: 1) developing a one-dimensional analytical model for 113 nighttime cooling of internal thermal mass with annually discrete air temperature boundary 114 115 conditions; the model will be validated against numerical simulations, traditional conduction transfer function (CTF) method [29], and ANSI/ASHRAE Standard 140-2017 [30]; 2) quantifying 116 cooling demand reduction with nighttime cooling of internal thermal mass in different climates. 117 This study will provide a new method to assess the efficiency of nighttime ventilation of internal 118 119 thermal mass to create energy flexibility [31] and enhancing the utilization of renewable energy [32]. The validated analytical solution could also be used for model predictive control of thermal 120 mass [33] and potentially be integrated into whole building energy simulation software. 121

#### 122 **2.** Materials and methods

The nighttime cooling in this study refers to a system that supplies unconditioned outdoor air into 123 the space to directly cool the concrete floor slab that is the internal thermal mass (Fig. 1a). During 124 the nighttime period, the supply air temperature is hourly varied. In the following daytime period, 125 the slab acts as a heat sink to absorb part of the space heat gains via convection and radiation (Fig. 126 1b); the air temperature distribution in the space is uniform and equal to a constant setpoint 127 temperature [28]. The daytime air conditioning and nighttime ventilation are provided by a 128 constant air volume system. The heat transfer in the slab is one-dimensional along the vertical 129 direction [34] and the bottom of the slab is well insulated. The slab is a single-layer material. The 130 heat transfer between the slab and the air through radiation and convection is described by a total 131 heat transfer coefficient. 132



133Fig. 1. (a) Nighttime cooling of internal thermal mass and (b) daytime air conditioning. T is the thermal mass134temperature field;  $T_o$  is the nighttime hourly-varied supply air temperature;  $T_{set}$  is the daytime setpoint135temperature of the conditioned space.

The heat balance for a building with nighttime cooling of internal thermal mass can be described by the following equations presented by Yang and Li [28]:

$$Q_{cl} + Q_{wr} + Q_{win,cond} + Q_{win,sol} + Q_i + Q_{inf} + Q_{itm} = 0$$
(1)

$$\rho_a c_a q_v (T_o - T_{room}) + Q_{itm} + Q_{wr} + Q_{win,cond} + Q_i + Q_{inf} = 0$$
(2)

Equation (1) describes the heat balance during the daytime, where  $Q_{cl}$  is the cooling demand (W), 138  $Q_{wr}$  is the conduction cooling load through walls and roof (W),  $Q_{win,cond}$  is the conduction cooling 139 load through windows (W), Q<sub>win.sol</sub> is the cooling load due to solar heat gain through windows (W), 140  $Q_i$  is the cooling load due to internal heat gains (W),  $Q_{inf}$  is the infiltration heat gain (W), and  $Q_{itm}$ 141 is the cooling load due to heat absorption or release from the internal thermal mass (W). The room 142 143 air and the internal thermal mass exchange heat through convection and radiation. The total heat absorption or release from thermal mass is calculated as  $h_i A_i (T_{surf} - T_{room})$ , where  $h_i$  is the total heat 144 transfer coefficient above the internal thermal mass (W m<sup>-2</sup> K<sup>-1</sup>),  $A_i$  is the total area of the internal 145 thermal mass (m<sup>2</sup>),  $T_{surf}$  is the surface temperature of the internal thermal mass (K), and  $T_{room}$  is 146 147 the air temperature in the room (K). Equation (2) describes the heat balance during the nighttime and the first term is the supplied nighttime cooling, where  $\rho_a$  and  $c_a$  is the density (kg m<sup>-3</sup>) and 148 specific heat (J kg<sup>-1</sup> K<sup>-1</sup>) of air, respectively. The nighttime ventilation rate is  $q_V$  (m<sup>3</sup> s<sup>-1</sup>). 149

150 The temperature field in the internal thermal mass is described by the following equation with 151 Robin boundary conditions:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial v^2} \tag{3}$$

$$-\lambda \frac{\partial T}{\partial y}\Big|_{y=L} = h_i [T(y=L,t) - T_r(t)]$$
<sup>(4)</sup>

$$\lambda \frac{\partial T}{\partial y}\Big|_{y=0} = 0 \tag{5}$$

Equation (3) assumes one-dimensional heat conduction in the internal thermal mass, where *T* is the slab temperature field (K), *y* is the vertical distance from the bottom of the internal thermal mass (m), *t* is time (s),  $\rho$  and *c* is the density (kg m<sup>-3</sup>) and specific heat (J kg<sup>-1</sup> K<sup>-1</sup>) of the internal thermal mass, respectively. Equation (4) describes the heat exchange between the internal thermal mass and the supply air temperature  $T_r(K)$ , where,  $\lambda$  is the thermal conductivity (W m<sup>-1</sup> K<sup>-1</sup>) of the internal thermal mass, and *L* is the total slab thickness (m).  $T_r$  is a combination of unconditioned nighttime outdoor air temperatures and the daytime setpoint temperature of the conditioned space.  $T_r$  equals to the hourly varied temperature ( $T_o$ ) of the outdoor air that is supplied to directly cool the slab in the nighttime period and is the setpoint temperature of the space in the daytime period ( $T_{set}$ ) (Fig. 2).  $T_r$  can be described using the step function:

$$T_{r}(t) = [H(t) - H(t - t_{1})] T_{r,1} + [H(t - t_{1}) - H(t - t_{2})] T_{r,2} + \cdots$$

$$+ [H(t - t_{M-2}) - H(t - t_{M-1})] T_{r,M-1} + H(t - t_{M-1})T_{r,M}$$
(6)

$$H(t) = \begin{cases} 0 & t \le 0\\ 1 & t > 0 \end{cases}$$
(7)



162

163

#### Fig. 2. Hourly varied air temperature.

Equation (5) describes a perfect insulation of the bottom of the internal thermal mass. The initial temperature of the internal thermal mass is set equal to the initial air temperature ( $T_{set}$ ) and the effect of the initial temperature can be eliminated by periodic calculations.

#### 167 **2.1** Analytical model for nighttime cooling of internal thermal mass

The challenge to derive the analytical solution of equation (3) is the inhomogeneous and timedependent boundary condition (6) that has discrete free-stream temperatures  $T_r$ . When separation of variables is applied, the first derivative of  $T_r$  will be introduced into the equation system. Since  $T_r$  is discrete and varies hourly in this study, its first derivative requires the use of Dirac delta function:

$$\frac{dT_r}{dt} = \delta(t)T_{r,1} + \sum_{m=1}^{M-1} \delta(t - t_m) \left[ T_{r,m+1} - T_{r,m} \right]$$
(8)

$$\delta(t) = \begin{cases} 0 & t \neq 0\\ 1 & t = 0 \end{cases}$$
(9)

where, M is the total number of days for nighttime cooling. The analytical solution can be soughtby applying shifting function method [35], separation of variables, Fourier series expansion and

variation of parameters (VOP) method [26] to the equations (4-9) normalized with:

$$\theta = \frac{T - T_{set}}{T_{set}}, \quad \tau = \frac{\alpha t}{L^2}, \quad \eta = \frac{y}{L}, \quad Bi = \frac{h_i L}{\lambda}, \quad \theta_r = \frac{T_r - T_{set}}{T_{set}}$$
(10)

- where  $\alpha$  is the thermal diffusivity (m s<sup>-2</sup>) and equals to  $\lambda/\rho c$ , *Bi* is the Biot number that measures
- 177 the ratio of the thermal resistance in the internal thermal mass to the convection of nighttime
- 178 cooling at the surface of the internal thermal mass. The detailed derivation is given in Appendix
- 179 A. The final analytical model is established for nighttime mechanical ventilation of building
- 180 thermal mass with discrete (hourly varied) temperature boundary conditions:

$$\theta = \sum_{n=1}^{\infty} \cos(\beta_n \eta) \left\{ \frac{S_n}{\beta_n^2} \left[ \theta_{r,1} \left( e^{-\beta_n^2 (\tau_M - \tau_1)} - e^{-\beta_n^2 \tau_M} \right) + \sum_{m=1}^{M-1} \theta_{r,m+1} \left( e^{-\beta_n^2 (\tau_M - \tau_{m+1})} - e^{-\beta_n^2 (\tau_M - \tau_m)} \right) \right] - W_n \left[ \theta_{r,1} e^{-\beta_n^2 \tau_M} + \sum_{m=1}^{M-1} (\theta_{r,m+1} - \theta_{r,m}) e^{-\beta_n^2 (\tau_M - \tau_m)} \right] \right\} + \frac{\eta^2 - 1}{2} Bi \theta_r$$
(11)

#### 181 **2.1.1 Model verification with numerical simulation**

The analytical model is verified and validated with numerical simulation, the traditional CTF 182 method and ANSI/ASHRAE Standard 140-2017. This section verifies the time-dependent 183 184 temperature variations in a concrete slab as internal thermal mass using the finite-volume method [36]. The verification process is targeted to represent a case as general as possible. The thermal 185 mass is assumed to be on the floor and has a thickness (L) of 0.3 m. The material density is 2500 186 kg m<sup>-3</sup>. The thermal conductivity is 1.5 W m<sup>-1</sup> K<sup>-1</sup>. The specific heat is 750 J kg<sup>-1</sup> K<sup>-1</sup>. The nighttime 187 ventilation rate is 1.5 h<sup>-1</sup>. The hourly air temperatures are TMY3 dry bulb values between 19:00 188 and 07:00 hours in the night on November 13<sup>th</sup> in Long Island of California. The daily air 189 temperature is the room setpoint temperature of 24°C that is also the initial temperature of the 190 internal thermal mass. 191



Fig 3. Temperature profiles at different hours: symbols and solid lines represents the results of the numerical simulation and the analytical model, respectively.

- 192 The finite-volume simulation was performed with the open source CFD code OpenFOAM 7 and
- the laplacianFoam solver was used to solve the heat transfer in solids. The cell distance was chosen as L/100 uniformly based on both grid independence and sufficient resolution to compare with the
- analytical solution [26]. The time derivative was discretized with the first-order implicit scheme.
- 196 The time step was chosen to satisfy the condition:  $F_o(1+\xi Bi) \leq 0.5$ , where  $F_o = \lambda \Delta t / (\rho c \Delta^2 x)$ ,  $\xi$  is
- 197 0 and 1 for internal and boundary nodes, respectively,  $\Delta t$  is time step and  $\Delta x$  is cell distance. The
- time step is determined to be 3 seconds. The Laplacian term was discretized by the central-
- 199 differencing scheme. The boundary and initial conditions have the form of (4-5).

- Fig. 3 compares the temperature profiles obtained from the finite-volume method and the
- analytical model. The temperature profiles are shown along the thickness of the internal thermal
- 202 mass at different hours. The theoretical model predicts the same temperature profiles as the
- numerical simulation. Both approaches show that the surface temperature (at 0.3 m) of the
- thermal mass is reduced by  $0.2^{\circ}$ C after 1 hour and  $0.9^{\circ}$ C after 12 hours. The bottom temperature
- 205 of the slab starts to decrease after more than 3 hours and drops by  $0.3^{\circ}$ C after 12 hours.

## 206 2.1.2 ANSI/ASHRAE Standard 140 - Case 600 for model validation

- 207 ANSI/ASHRAE Standard 140 [30] is widely used in the building simulation community to
- validate models. Standard 140 documents the simulated energy demand of a thermal zone using
- 209 different building energy simulation tools. In this study, Case 600 is presented for model
- validation. Case 600 is a lightweight cubic zone (6 m  $\times$  8 m  $\times$  2.7 m) with two windows (3 m  $\times$  2
- m each) on the south wall. All details including materials and climate can be found in the section
   5.2 in Standard 140-2017. Construction material properties are summarized in Appendix B. The
- 212 U-value of the windows is  $2.10 \text{ W m}^{-2} \text{ K}^{-1}$ . The internal heat gains are 200 W during all 24 hours
- per day for the full year. The internal heat gains are assumed 100% sensible and 60% radiative.
- The infiltration is 0.5 ACH. TMY3 weather data for Denver is used for the case. The setpoint
- temperature for cooling is 27°C. When nighttime cooling is applied to Case 600, the floor timer
- is changed to the concrete slab in section 2.1.1. Figure 4 compares the annual hourly integrated
- 218 peak cooling loads predicted by the model in this study with those obtained from other two
- building simulation tools. The peak cooling load is 6.50 kW predicted in this study is in very
- close agreement with simulation results of other tools.



## 221 222



# 223 **2.2.** Nighttime cooling of internal thermal mass in different climates

The validated analytical model is used to study the capacity of nighttime cooling of internal 224 225 thermal mass to reduce cooling demand in different climates. The sources of the hourly weather data are Typical Meteorological Year 3 (TMY3) and California Climate Zones 2 (CTZ2) data sets 226 227 that also provide climate data for EnergyPlus. Forty-eight TMY3 and sixteen CTZ2 climate data are chosen to investigate the energy performance of nighttime mechanical ventilation of internal 228 thermal mass in cities located in forty-eight U.S. states and across the sixteen climate zones in 229 California, respectively. Filter criteria are applied to the climate data to ensure that it is beneficial 230 231 to apply nighttime cooling: 1) the nightly average relative humidity is in the range of 20-70% [10];

232 2) average nightly temperatures are lower than 18  $^{\circ}$ C and the average monthly diurnal temperature

- range are higher than  $7^{\circ}C$  [10]; 3) Nighttime ventilation is activated from 21:00 hours in the night
- to 07:00 hours in the following day. The months for nighttime ventilation are picked based on

these three criteria. The hourly temperature data between 7:00 and 21:00 in TMY3 and CTZ2 is

replaced with the setpoint temperature 24°C. Figure 2 illustrates a sample temperature profile for the first 21 hours of a 24-h period. The first 3-day data is duplicated in the beginning of the weather

- file to eliminate the influence of the assumption of uniform temperature distribution in the thermal
- 239 mass.

240 The heat removed from the internal thermal mass during the period of 10-h nighttime ventilation

is used to indicate the capacity to shift and reduce cooling demand and defined as the total freecooling energy:

$$Q_{s} = \rho c_{p} H T_{init} \left\{ \sum_{n=1}^{\infty} \frac{\sin\beta_{n}}{\beta_{n}} \Phi_{n}(\tau_{J}) - \sum_{n=1}^{\infty} \frac{\sin\beta_{n}}{\beta_{n}} \Phi_{n}(\tau_{K}) + \frac{1}{3} Bi \left[ \theta_{r}(\tau_{K}) - \theta_{r}(\tau_{J}) \right] \right\}$$
(12)

where, J and K express each start and end hour of nighttime ventilation for all months. Total free 243 cooling energy is the sum of all positive values of free cooling energy storage  $(O_3)$ . The total free 244 245 cooling days are the number of nights with positive values of free cooling energy and also used as a capacity indicator. When mechanical ventilation is used, there will be fan energy consumption. 246 247 Therefore, two more capacity indicators are defined by considering fan energy consumption. The net free cooling energy is the sum of all positive values of the difference between free cooling 248 energy storage and fan energy consumption. The net free cooling days are the number of nights in 249 which free cooling energy is larger than fan energy consumption. 250

#### 251 **2.2.1 Estimation of mechanical fan energy consumption**

An accurate calculation of fan energy consumption requires the detailed information of the 252 mechanical system including pressure drop and airflow rates that vary from building to building 253 and from system to system. Such detailed information is often hard to obtain, and simplified 254 methods are needed to estimate fan energy consumption in a more general fashion. An alternative 255 to characterize fan energy performance is to use the metric of fan efficacy that is defined as the 256 power required to transport a fixed volume of air (W h m<sup>-3</sup>) [10]. Springer et al. [10] summarized 257 the fan efficacy of a central fan system. The total area of the internal thermal mass is assumed to 258 be 200 m<sup>2</sup>. The total volume flow rate is 4800 m<sup>3</sup> h<sup>-1</sup> to maintain a high ventilation rate of 8 ACH 259 that is average value for commercial buildings like schools according to CIBSE Guide B2 [37]. 260 The total volume flow rate leads to a fan efficacy around  $0.5 \text{ W h m}^{-3}$ . 261

#### 262 **2.2.2 Validation of total free cooling energy**

The total free cooling energy calculated by equation (12) is validated by the time-dependent heat fluxes at the slab surface calculated using CTF method [38]. The CTF method is based on Z-

transform to compute solutions of transient heat transfer. It relates the current heat flux to past and

present values of interior and exterior boundary temperatures (air or surface temperatures) as well

as heat flux history. The heat flux (q') at the slab" surface at the present time (t) is given by

$$q_{t}^{"} = X_{0}T_{es,t} + \sum_{m=1}^{M_{es}} X_{m}T_{es,t-m\Delta t} - Y_{0}T_{is,t} - \sum_{m=1}^{M_{is}} Y_{m}T_{is,t-m\Delta t} + \sum_{m=1}^{M_{q}} Z_{m}q_{t-m\Delta t}^{"}$$
(13)

where,  $X_m$  (W m<sup>-2</sup> K<sup>-1</sup>),  $Y_m$  (W m<sup>-2</sup> K<sup>-1</sup>),  $Z_m$  are conduction transfer function coefficients for exterior 268 temperatures  $(T_{es})$ , interior temperatures  $(T_{is})$ , and past heat fluxes, respectively. In this study, the 269 exterior temperatures are the above-mentioned hourly air temperatures.  $\Delta t$  is the time step. A 270 difficult and complex task is to determine the conduction transfer coefficients. Seem [39] 271 developed a detailed procedure for calculating CTF coefficients for one-dimensional slabs. The 272 procedure is coded with Python to generate CTF coefficients and calculate heat flux at the slab 273 surface. The Python code is verified by calculating CTF coefficients and heat fluxes at the exterior 274 and interior surface of a single-layer material presented in the thesis of Seem. When CTF method 275 is applied, it is necessary to assume past values of exterior temperatures, interior temperatures, and 276 heat fluxes for the initial calculation. Then the calculation is iterated on the periodical air 277 temperatures until a steady periodic solution is reached. In this study, the heat fluxes during the 278 third 24 hours are used to verify the analytical model. 279

280

Table 1 Heat flux (W m<sup>-2</sup>) at the surface of the slab

hour	CTFM	Analytical	Relative difference
19	4 780	4 725	0.01151
20	4 973	4 936	0.007440
21	5 123	5.092	0.006051
22	5.349	5.320	0.005422
23	5.713	5.684	0.005076
0	6.010	5.981	0.004825
1	6.375	6.345	0.004706
2	6.064	6.041	0.003793
3	5.830	5.810	0.003431
4	5.534	5.516	0.003253
5	5.376	5.360	0.002976
6	5.220	5.204	0.003065
7	-1.005	-0.9578	0.04697
8	-0.9321	-0.9100	0.02371
9	-0.8932	-0.8793	0.01556
10	-0.8666	-0.8572	0.01085
11	-0.8470	-0.8406	0.007556
12	-0.8320	-0.8278	0.005048
13	-0.8202	-0.8177	0.003048
14	-0.8107	-0.8094	0.001603
15	-0.8029	-0.8025	0.0004982
16	-0.7963	-0.7966	0.0003767
17	-0.7906	-0.7915	0.001138
18	-0.7855	-0.7868	0.001655

281

Table 1 compares the heat flux calculated with CTF methodand the analytical method. In general, a very good agreement is found between the two methods. The relative difference in the heat flux ranges from 0.00003767 to 0.04697. The maxima occurs when the exterior air temperature is switched from TMY3 dry bulb temperature (14.8°C) at 7:00 to the room setpoint temperature (24°C) at 8:00. In other words, the maximum difference in heat flux is due to the maximum exterior temperature difference between the previous and current hour. The CTF coefficients are obtained

by spatially discretizing the slab into 51 resistances and 50 capacitances. The limited number of

resistance-capacitance finite-difference discretization would inevitably result in accuracy issues.
 The good agreement between the two methods indicate that the analytical model might be

instrumental to optimize the CTF coefficients when the exterior temperatures have large gradients.

#### 292 **3. Results**

#### 293 **3.1.** Analytical model to assess nighttime cooling of internal thermal mass in Case 600

The analytical model is applied to evaluate the potential of nighttime cooling of the internal 294 thermal mass in Case 600. Fig.5 shows the temperature profiles at different locations in the internal 295 thermal mass for 216 hours from June 2<sup>nd</sup> to June 11<sup>th</sup> in Denver. The line y=0.30m represents the 296 temperature variations of the floor surface, whereas the line y=0.05m represents a location near 297 the adiabatic boundary. The grev line represents the time-dependent temperature variations of the 298 299 room air. During the nighttime period (21:00-06:00), the room air temperature is calculated with equation (2). During the daytime period (07:00-20:00), the room air temperature is assumed to be 300 the constant setpoint temperature (27°C). In the second full-day period in Fig.5, the room air 301 temperature differs by 9.8°C during the nighttime, whereas the temperature variation of thermal 302 mass surface is 2.4°C. The room air temperature cannot recover to the setpoint temperature 27°C 303 304 at 08:00, but it is in the vicinity of 25°C that will not cause thermal discomfort. The lowest room air temperature is 13.7°C at 06:00 due to the lowest outdoor air temperature in the same hour. The 305 thermal mass surface temperature reaches to its lowest value 19.8°C at 06:00 but the thermal mass 306 temperature near the adiabatic surface reaches its lowest value 21.5°C in 4 hours later. The 307 maximum temperature difference between the daytime and nighttime period is 3.7°C on the floor 308 surface and 1.3°C at a depth of 0.15 m below the surface. The time-variation of the thermal mass 309 temperature at depths of 0.15 m below the surface is only 33% of that on the surface. Therefore, a 310 slab thickness of 0.15 m is optimum in terms of storing cooling energy during nighttime ventilation. 311 The analytical model shows a great performance to calculate the dynamic temperature variations 312

of the internal thermal mass under hourly varied and discrete air temperatures.



314

315

Fig. 5. Temperature variation at different locations of thermal mass in Case 600.

The daily cooling demand of Case 600 with and without nighttime cooling of internal thermal mass is shown in Fig.6. For the entire season requiring cooling, nighttime cooling of internal thermal mass can reduce daily cooling demand. The total cooling demand reduction is 1.2 MWh and about 26% of the annual cooling demand. The average daily cooling demand reduction is 7.3

- kWh with a standard deviation of 2.4 kWh. The maximum reduction is 13.9 kWh and about 30%
- 321 of the daily cooling demand. The maximum peak demand reduction is 1.4 kWh. For a specific day
- 322 like June 3<sup>rd</sup>, the hourly peak demand (3.2 kWh) occurs at 15:00. Nighttime cooling of internal
- thermal mass can reduce peak cooling demand by 706.2 Wh that covers the peak cooling load of
- 324 640.5 W due to conduction heat gains through walls, roof and windows, internal and infiltration
- 325 heat gains.



326

- Fig. 6. Total cooling demand for Case 600 with and without nighttime cooling of internal thermal mass.
- 328 **3.2.** Capacity of nighttime cooling of internal thermal mass in different U.S. climates



329 330

Fig 7. Total and net free cooling days in different U.S. cities.

331 The analytical model is applied to evaluate the potential of nighttime mechanical ventilation of building internal thermal mass by calculating total and net free cooling days in different U.S. cities 332 (Fig 7). In general, the number of total free cooling days decreases from southern states to northern 333 states. This is associated to the change in the climate zones with latitude. In the southern U.S. 334 335 region with hot-dry or hot humid climate, the average value of annual total and net free-coolingdays is 160 and 28, respectively. In the middle U.S. region with mixed humid climate, the average 336 337 value of annual total and net free-cooling-days is 121 and 24, respectively. In the northern U.S. regions with cold climate, the average value of annual total and net free-cooling-days is 98 and 32, 338 339 respectively.





Fig. 8. a) Total and b) net free cooling energy storage in different U.S. cities.

341 For a region with hot-dry climate, the range of annual total free-cooling-days is from 117 to 182, while the range of annual net free-cooling-days is from 24 to 36. For example, the total free cooling 342 days are 117 in Las Vegas, AZ, but the net energy saving is achieved for only 36 days. This is due 343 to high average nighttime temperature of 20°C during the ventilation period. The total and net free 344 cooling days are 117 and 24, respectively, in Long Beach, CA. For a region with hot-humid climate, 345 the range of annual total free-cooling-days is from 131 to 222, while the range of annual net free-346 347 cooling-days is from 3 to 47. For example, the total free cooling days are 222 in Orlando, FL, while net energy saving is achieved in 47 of those days. The total and net free cooling days are 348 349 179 and 3, respectively, in Honolulu, HI. The small number of net free cooling days in Honolulu is due to the high average nighttime temperature of 23°C during the ventilation period. For a region 350 with marine climate, two cities are selected to demonstrate the potential of nighttime mechanical 351 ventilation of internal thermal mass. In Seattle, WA, the total free cooling days are only 61, and 352 net cooling storage is achieved in 50% of those nights. In Portland, OR, the total free cooling days 353 354 increase to 119, but the net cooling storage is achieved in only 48 nights. For a region with mixed humid climate, total free cooling days range from 95 to 135, while most of the net free cooling 355 days are below 30. Among the 15 tested cities in the mixed-humid climate zone, only Charleston, 356 WV and Kansas City, MO have net cooling days of more than 30 days. In eastern coastal cities 357 such as New York, the net free cooling days are only 12. For a region with cold or very cold 358 climate, the total free cooling days range from 60 to 150, while the net free cooling days range 359 360 from 15 to 68. Cheyenne, WY has least net free cooling days of 15 due to too cold nighttime

361 temperature. Santa Fe, NM has 68 net free cooling days and shows most promising to flush internal



362 thermal mass with nighttime mechanical ventilation.

363 Fig. 9. a) California climate zone. b) total and net free cooling days, c) total and d) net free cooling energy 364 storage in different California climate zones.

365 The analytical model is further applied to quantify the total and net free cooling energy storage in 48 different U.S. cities (Fig. 8). The value of the total and free cooling energy storage in each city 366 367 is used to contour the corresponding state. Compared to total free cooling days, the total free cooling energy storage does not show a decreasing trend from southern states to northern states. 368 For all the three cities (Yuma, Long Beach and Las Vegas) with hot-dry climate, the total free 369 cooling energy storage is above 10 kWh m<sup>-2</sup> per year, while the fan energy consumption is more 370 than 80% of the total free cooling energy. For a region with hot-humid climate, the stored total 371 free cooling energy ranges from 6.82 to 15.3 kWh m<sup>-2</sup> per year, while the net free cooling energy 372 accounts for less than 26%. In Honolulu, HI, the net free cooling energy is nearly 0 kWh m<sup>-2</sup> per 373 year. High total free cooling energy does not necessarily lead to high net free cooling energy. For 374 example, the total free cooling energy in Orlando, FL, is 15.3 kWh m<sup>-2</sup> per year that is higher than 375

14.7 kWh m<sup>-2</sup> per year in Jackson, MS. On the contrary, the net free cooling energy in Orlando is 376 2.48 kWh m<sup>-2</sup> per year and lower than 3.74 kWh m<sup>-2</sup> per year in Jackson. In Portland and Seattle 377 with marine climate, the average total cooling energy is 10.4 kWh m<sup>-2</sup> per year, while the fan 378 379 energy consumption accounts for more than 90%. For a region with mixed humid climate, the minimum total (5.86 kWh m<sup>-2</sup> a<sup>-1</sup>) and net (0.39 kWh m<sup>-2</sup> a<sup>-1</sup>) free cooling energy storage is 380 reported in New York. Birmingham, AL, has the maximum net free cooling energy storage of 2.24 381 kWh m<sup>-2</sup> per year, while the maximum total free cooling energy storage of 12.3 kWh m<sup>-2</sup> per year 382 is found Charleston, WV. For a region with cold or very cold climate, the total free cooling energy 383 ranges from 6.08 to 19.1 kWh m<sup>-2</sup> per year, while the net free cooling energy ranges from 0.34 to 384 3.88 kWh m<sup>-2</sup> per year. The maximum total and net free cooling energy are reported in Santa Fe, 385 NM. Comparing to Santa Fe, Rapid City, SD, has a lower total free cooling energy of 13.7 kWh 386  $m^{-2}$  per year, but the net free cooling energy of 3.11 kWh  $m^{-2}$  per year is more similar. 387

Finally, the analytical model is applied to elaborate the free cooling energy storage in different 388 California climate zones (Fig. 9). In general, it is not suitable to apply nighttime ventilation of 389 internal thermal mass in coastal regions including climate zone CZ01, CZ03, CZ06 and CZ07. The 390 total free cooling days is determined as zero in CZ01. Despite an average total free cooling energy 391 of 12.4 kWh m<sup>-2</sup> per year in CZ03, CZ06 and CZ07, the average net free cooling energy of 0.62 392 kWh m<sup>-2</sup> per year indicates fan energy consumption accounts for 95%. In regions further into the 393 land, nighttime ventilation of internal thermal mass shows better performance. The total free 394 cooling days range from 116 to 206, while the net free cooling days range from 55 to 126. The 395 total free cooling energy storage is above 17.2 kWh m<sup>-2</sup> per year for these climate zones. Especially 396 in CZ02, the total free cooling energy storage achieves 27.5 kWh m<sup>-2</sup> per year, while the net free 397 398 cooling energy is  $6.11 \text{ kWh m}^{-2}$  per year.

#### 399 **4. Discussion**

The research on nighttime cooling of internal thermal mass is limited due to lacking simplified 400 models by considering mass dimension and real-world discrete climate data like hourly outdoor 401 air temperatures. The analytical model in this study is developed to evaluate the performance of 402 nighttime cooling of internal thermal mass with discrete (hourly varied) air temperature boundary 403 404 conditions. It has a potential to be integrated into whole building energy simulation software, such as EnergyPlus that sets internal thermal mass either as an effective area or a capacitance multiplier 405 406 for the zonal air. EnergyPlus predicts hourly zonal air temperature that can be coupled with the analytical model. The temperature of the zonal air and the internal thermal mass can be updated 407 by iteration method. The fan energy consumption in this study is roughly estimated with fan 408 efficacy curve that affects the accuracy on quantifying net free cooling energy storage. Once the 409 analytical model is integrated into EnergyPlus, the fan energy can be simulated in details and the 410 411 estimation accuracy of the net free cooling energy can also be improved. The analytical model has some advantages over existing simplified models and the methods dealing with internal thermal 412 mass in whole building energy simulation software. For example, the simplified model [28] 413 414 predicts the same temperature variation of room air and thermal mass due to the ignorance of thermal mass dimension. The analytical model shows that the lag in temperature variation between 415 thermal mass and room air can be up to 4 hours (Fig.5). The ignorance of internal thermal mass 416 417 dimension like EnergyPlus multiplier method may lead to exaggerate the efficiency of internal thermal mass for energy storage and such overestimation can be overcome by the integration of 418 the proposed analytical model in this study. The standalone analytical model can also be used to 419

quantify the free cooling energy storage with hourly temperature data without knowing the details
of building components, construction materials and etc. The limitation of the analytical model is
the requirement of constant ACH. The present form can be used only for constant-air-volume
system. Through the analysis of the total free cooling energy storage under different U.S. climates,
it will be very useful to further explore the performance of nighttime ventilation of internal thermal
mass with both hourly varied ventilation rates and air temperatures.

The analytical model is then applied to evaluate the performance of nighttime mechanical 426 ventilation of building internal thermal mass in different U.S. climate zones by total free cooling 427 days, net free cooling days, total free cooling energy storage and net free cooling energy. Total 428 429 free cooling days might be a good index to indicate energy flexibility. Total free cooling days are above 60 days per year for all the 48 tested cities. It means that nighttime mechanical ventilation 430 of internal thermal mass can be designed to shift peak cooling demand for at least two months. 431 The dependency of total free cooling days on climate zones is not obvious (Fig. 7). It indicates that 432 analyzing only climate data is not adequate to evaluate the energy flexibility of nighttime cooling. 433 Therefore, total free cooling energy can be further used to quantify the energy flexibility. The total 434 free cooling energy is above 10 kWh m<sup>-2</sup> per year for 27 cities. Liu et al. [40] reported that the 435 energy demand of zero energy buildings should be less than 15 kWh m<sup>-2</sup> per year. Therefore, 436 design of nighttime mechanical ventilation and internal thermal mass is able to provide more than 437 438 67% useful energy demand for those type of buildings in the 27 cities. Nighttime cooling of 439 internal thermal mass is indeed promising in terms of energy flexibility for many U.S. climate zones. Net free cooling days might be a good index to indicate energy savings. The difference in 440 net free cooling days is small between cities in various U.S. climate zones. Net free cooling days 441 range from 10 to 40 days for most cities. Santa Fe has the largest number of net free cooling days 442 of 68 days per year and Honolulu has the smallest number of net free cooling days of only 3 days 443 444 per year. Hence, applying nighttime mechanical ventilation of internal thermal mass is not an ideal solution for energy saving purpose. Energy saving can be further quantified by net free cooling 445 energy. The maximum net free cooling energy in the 48 cities is 3.88 kWh m<sup>-2</sup> per year. If the 446 maximum energy demand of zero energy buildings is still used as reference, a maximum energy 447 saving of 26% can be achieved. A comparison between the total and net free cooling energy under 448 different U.S. climates enlightens that nighttime natural ventilation of internal thermal mass might 449 be a more suitable alternative for energy saving. Different locations in the same state might be in 450 451 different climate zones, for example, California has 16 climate divisions. Free cooling days and energy storage in a single city of a state is not adequate to reveal the general pattern of the nighttime 452 mechanical ventilation of internal thermal mass under different U.S. climates. Therefore, the free 453 cooling days and energy storage are quantified for all the 16 climate zones in California. The more 454 455 detailed calculation offers new insights on nighttime mechanical ventilation of internal thermal mass and the performance becomes better from coast regions further into the land. The maximum 456 457 total and net free cooling energy in California are about 1.5 times of those in Santa Fe.

The development of the analytical model is to implement the evaluation of capacity of the nighttime cooling of the internal thermal mass to reduce cooling demand in different U.S. climates. One main purpose to manage energy demand is to create energy flexibility for increasing the integration of renewable energy. The energy flexibility created by nighttime cooling of internal thermal mass is demonstrated using ANSI/ASHRAE Standard 140 – Case 600 in Denver climate. The shifting efficiency ( $\eta_{shifting}$ ) proposed by Dréau and Heiselberg [31] is adopted here to assess the energy flexibility. Fig.10 shows the shifting efficiency of the nighttime cooling of internal thermal mass for Case 600. The efficiency lower than one means that not all stored cooling energy are used in the same day but shifted to the following day. The average efficiency during the cooling demand season is 0.9 with a standard deviation of 0.3. The range of the efficiency is 0.4 ~ 2.2 and a similar range is also reported by Dréau and Heiselberg [31]. It is observed that the maximum efficiency is 2.2 followed by 12 days with an average efficiency 0.7, which shows the capacity of the nighttime cooling of the internal thermal mass to create energy flexibility.







#### 473 **5.** Conclusions

474 The research on nighttime cooling of internal thermal mass is limited due to lacking simplified models by considering mass dimension and real-world discrete climate data like outdoor air 475 476 temperatures. This study develops a one-dimensional analytical model for nighttime mechanical ventilation of internal thermal mass in buildings with a fixed air change rate, and constant air 477 temperature in the daytime period but hourly varied air temperatures in the nighttime period. 478 479 Temperature profiles for a general case obtained from the analytical model is confirmed by the results from time-dependent numerical simulations with the same boundary and initial conditions. 480 The validated model is used to explore the free cooling energy storage of internal thermal mass 481 during nighttime flush, and the main observations are: 482

1) Nighttime mechanical ventilation of internal thermal mass can be designed to shift peak cooling demand for at least two months for most locations. This is indicated by the total free cooling days, which are above 60 days per year for all the 48 cities tested. Among these U.S.
cities in different climate zones, the maximum free cooling energy storage (19.1 kWh m<sup>-2</sup> a<sup>-1</sup>) is reported in Santa Fe, NM. Furthermore, the total free cooling energy is above 10 kWh m<sup>-2</sup>
per year for 27 selected cities, suggesting that they are suitable to apply nighttime cooling of internal thermal mass to shift peak cooling demand.

2) The energy savings by adopting nighttime mechanical ventilation of internal thermal mass is not exceptional when fan energy consumption is taken into account. Net free cooling days range from 10 to 40 days for different cities. Honolulu has the smallest number of net free cooling days with only 3 days per year. Santa Fe has the largest number of net free cooling days of 68 days per year. Santa Fe also has the maximum net free cooling energy of 3.88 kWh

 $m^{-2} a^{-1}$ . It is essential to reduce fan energy consumption to improve energy saving potential of 495 internal thermal mass. 496

- 3) Free cooling energy quantified for the 16 climate zones in California shows that coastal regions 497 498 are not suitable for nighttime mechanical ventilation of internal thermal mass. Large variation among different regions demonstrate that ventilation strategies need to be customized for 499
- different climate zones. The maximum total free cooling energy storage in California achieves 500 27.5 kWh m<sup>-2</sup> a<sup>-1</sup>, while the maximum net free cooling energy is 6.11 kWh m<sup>-2</sup> a<sup>-1</sup>.
- 501

The analytical model developed is convenient to examine the efficacy of nighttime mechanical 502 ventilation of internal thermal mass to create energy flexibility. In addition to quantification of free 503 cooling energy storage shown in this study, this model has the potential to be integrated into whole 504 505 building energy simulation software to improve the efficiency and accuracy of evaluating the impact of internal thermal mass on indoor air temperature. 506

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#### 510 **Appendix A. Derivation of the analytical model**

511 Equation (4) can be normalized as:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} \tag{A1}$$

$$-\frac{\partial\theta}{\partial\eta}\Big|_{\eta=1} = Bi[\theta(\eta=1,\tau) - \theta_r(\tau)]$$
(A2)

$$\left. \frac{\partial \theta}{\partial \eta} \right|_{\eta=0} = 0 \tag{A3}$$

$$\theta(\eta, 0) = \theta_r(0) = 0 \tag{A4}$$

- 512 Shifting function method is applied to find the solution to equation (A1) with an inhomogeneous
- 513 Robin boundary condition (A2). The solution takes the following form:

$$\theta = \Psi(\eta, \tau) + u(\tau)v(\eta) \tag{A5}$$

- where  $v(\eta)$  is a shifting function and  $u(\tau)$  contains time-dependent boundary values. The following 514
- equations are obtained by substituting equation (A5) into equation (A1-3): 515

$$\frac{\partial \Psi}{\partial \tau} + \frac{du}{d\tau} = \frac{\partial^2 \Psi}{\partial \eta^2} + u \frac{d^2 v}{d\eta^2}$$

$$\eta = 1: \qquad -\frac{\partial \Psi}{\partial \eta} - u \frac{dv}{d\eta} = Bi(\Psi + uv - \theta_r) \qquad (A6)$$

$$\eta = 0: \qquad \frac{\partial \Psi}{\partial \eta} + u \frac{dv}{d\eta} = 0$$

To make the Robin boundary condition homogeneous, *u* and *v* take the following form: 516

$$u = Bi\theta_r \tag{A7}$$

$$v = \frac{\eta^2 - 1}{2} \tag{A8}$$

517 The one-dimensional heat equation is recast as

$$\frac{\partial\Psi}{\partial\tau} = \frac{\partial^2\Psi}{\partial\eta^2} + Bi\theta_r - \frac{1}{2}Bi(\eta^2 - 1)\frac{d\theta_r}{d\tau}$$
(A9)

- 518 Equation (A9) can be solved by using Separation of Variables and Fourier series expansion. The 519 separated function for space dimension satisfies homogeneous boundary conditions. The solution 520 tables the following formula
- takes the following form:

$$\Psi = \sum_{n=1}^{\infty} \cos(\beta_n \eta) \,\Phi_n(\tau) \tag{A10}$$

521 where the eigenvalues  $\beta_n$  are the roots of the transcendental equation

$$\beta_n \sin\left(\beta_n\right) = Bi\cos\left(\beta_n\right) \tag{A11}$$

- 522 Since the initial values of equation (A9) are zero,  $\Phi$  can be determined by using the variation of
- 523 parameters (VOP) method [26] and the principle of orthogonality:

$$\Phi_{n} = \frac{1}{R_{n}} \int_{0}^{1} \Psi \cos\left(\beta_{n}\eta\right) d\eta$$

$$R_{n} = \int_{0}^{1} \cos^{2}\left(\beta_{n}\eta\right) d\eta = \frac{\beta_{n}^{2} + Bi^{2} + Bi}{2(\beta_{n}^{2} + Bi^{2})}$$
(A12)

- 524 The essence of VOP is to differentiate equation (A12) with respect to time and replace the partial
- 525 derivative with the right-hand side of equation (A9). An ordinary differential equation (ODE) for 526  $\Phi$  is obtained after carrying out the integration:

$$\frac{d\Phi_{n}}{d\tau} = -\beta_{n}^{2}\Phi_{n} + S_{n}\theta_{r} - W_{n}\frac{d\theta_{r}}{d\tau}$$

$$S_{n} = \frac{Bi\sin\left(\beta_{n}\right)}{R_{n}\beta_{n}}$$

$$W_{n} = \frac{Bi(\cos\beta_{n} - \sin\beta_{n}/\beta_{n})}{R_{n}\beta_{n}^{2}}$$
(A13)

527 Employing the integral properties of Dirac delta function, the ODE can be expressed in the 528 following summation form:

$$\Phi_{n} = \frac{S_{n}}{\beta_{n}^{2}} \left\{ \theta_{r,1} \left( e^{-\beta_{n}^{2}(\tau_{M} - \tau_{1})} - e^{-\beta_{n}^{2}\tau_{M}} \right) + \sum_{m=1}^{M-1} \theta_{r,m+1} \left( e^{-\beta_{n}^{2}(\tau_{M} - \tau_{m+1})} - e^{-\beta_{n}^{2}(\tau_{M} - \tau_{m})} \right) \right\}$$
(A16)  
$$- W_{n} \left\{ \theta_{r,1} e^{-\beta_{n}^{2}\tau_{M}} + \sum_{m=1}^{M-1} (\theta_{r,m+1} - \theta_{r,m}) e^{-\beta_{n}^{2}(\tau_{M} - \tau_{m})} \right\}$$

529 Substituting Equation (A7-8) (A10) (A16) into Equation (A5), the final analytical model for

nighttime cooling of internal thermal mass is obtained as equation (11).

# 531 Appendix B. Simulation details of Case 600

Material (Inside to Outside)	λ W m <sup>-1</sup> K <sup>-1</sup>	ր kg m <sup>-3</sup>	specific heat J kg <sup>-1</sup> K <sup>-1</sup>	Thickness m
Exterior Wall				
Plasterboard	0.16	950	840	0.012
Fiberglass quilt	0.04	12	840	0.066
Wood siding	0.14	530	900	0.009
Floor				
Timber	0.14	650	1200	0.025
Insulation	0.04	-	-	1.003
Roof				
Plasterboard	0.16	950	840	0.010
Fiberglass quilt	0.04	12	840	0.1118
Roof deck	0.14	530	900	0.019

532

 Table B1 Thermal properties of construction materials

533

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