The cost-channel of monetary transmission under positive trend inflation*

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Abstract

We study the dynamics of the cost-channel of monetary transmission mechanism using a New-Keynesian model with nonzero steady state inflation. We show that the effects of cost-channel depend almost entirely on the level of trend inflation in the economy. These results are demonstrated analytically and numerically by focusing on the impact of monetary policy shocks on the response of inflation and output.

Keywords: cost-channel; trend inflation

JEL classification: E12, E31, E32, E52

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1 Introduction

The baseline New-Keynesian model suggests that changes in monetary policy primarily impact the economy through the demand channel. However, monetary policy may affect the economy through other channels as well. Several studies demonstrate that monetary policy also works through a cost-channel that directly affects the marginal cost of production, giving rise to an active literature (see, Barth and Ramey (2001), Dedola and Lippi (2005), Gaiotti and Secchi (2006), Ravenna and Walsh (2006), Chowdhury et al. (2006), Rabanal (2007), Tillmann (2008)).[1] For instance, Abo-Zaid (forthcoming) finds the cost-channel to matter significantly for determining the size of the government spending multiplier.

At the same time, most work-horse macroeconomic models (Galí (2015); Smets and Wouters (2007)) that are used to incorporate the cost-channel are approximated around a zero trend inflation steady state. While convenient, the assumption rarely applies in practise. From 2000 to 2019, World inflation averaged close to 4 percent, with inflation in Advanced economies and Emerging market and developing economies approximately 2 percent and 6 percent, respectively.[2] Central banks too target a positive rate of inflation.

We analyze the cost-side theory when the New-Keynesian model is log-linearized around a nonzero inflation steady state, based on Ascari and Sbordone (2014). Following Tillmann (2008), we assume that firms borrow money from financial intermediaries to pay the factor of production (labor) prior to the sale of output. Our analytical and numerical results suggest that trend inflation weakens the cost-channel effects on the response of output and inflation to a monetary policy shock. Essentially, the optimizing firms set higher prices to account for the positive trend inflation. As trend inflation rises, firms weigh future inflation more relative to the current cyclical conditions. Since the cost-channel enters the marginal cost of the firm, the rising trend inflation reduces the effect of the cost-channel on inflation and output. The mechanism is sharpened when monetary policy is more inflation averse.

The model is presented in section 2, section 3 outlines the results, and section 4 concludes.

[1] For a theoretical treatment, see Blinder (1987), Fuerst (1992), Christiano and Eichenbaum (1992), Christiano et al. (1997) and (Farmer, 1984, 1988a,b).

[2] Inflation rates are calculated as the percent change in average consumer prices using data from the World Economic Outlook Database, April 2020. ‘World’ consists of 194 countries with 39 Advanced economies, and 155 Emerging market and developing economies.

2 The Model Economy

2.1 The Demand Side

Households seek to maximize the following contemporaneous and separable utility function:

$$\max_{C_t, N_t, B_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \tau_t \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

where C_t is the quantity consumed of the differentiated goods, N_t denotes the labor hours, and τ_t is an exogenous labor supply shock. The parameter $\beta \in (0, 1)$ is the discount factor, σ is the relative risk aversion parameter, and φ represents the inverse of the Frisch labor elasticity. Maximization of the utility is subject to a sequence of flow budget constraints:

$$\frac{B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{B_t (1 + i_t)^{-1}}{P_t} \tag{2}$$

where P_t is the price of the consumption good, W_t denotes the nominal wage, B_t represents the quantity of one-period nominally risk-less discount bonds purchased in period t which mature in period $t + 1$ with price i_t, and T_t represents the lump-sum transfers. The representative household maximizes the expected utility over the choice variables C_t, N_t, and B_t subject to the budget constraint yielding the following optimal conditions:

$$\frac{1}{C_t^{\sigma}} = \beta E_t \left[\frac{(1 + i_t)}{\pi_{t+1}} \frac{1}{C_{t+1}^{\sigma}} \right] \tag{3}$$

$$\frac{W_t}{P_t} = \tau_t N_t^{\varphi} C_t^{\sigma} \tag{4}$$

2.2 The Supply Side

In each period t, a final good, Y_t, is produced by perfectly competitive firms using intermediate goods $Y_{i,t}$ produced by a monopolistically competitive firm i. In equilibrium, the aggregate consumption equals the production of final goods:

$$C_t = Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

where ε indicates the elasticity of substitution among the intermediate goods. The optimal demand for intermediate inputs is equal to $Y_{i,t} = (P_{i,t}/P_t)^{-\varepsilon} Y_t$. Moreover, the production function of the intermediate good's producer is given as follows:

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha} \tag{6}$$

where A_t is an exogenous process for the level of technology that is assumed to be stationary. We assume that firms borrow funds from financial intermediaries to finance their wage bill (Brückner and Schabert, 2003; Christiano et al., 2005; Rabanal, 2007; Ravenna and Walsh, 2006; Tillmann, 2008; Phaneuf et al., 2018). More specifically, we assume that the firms borrow the funds at the market interest rate μi_t to finance their wage bill. In this environment, total cost and marginal cost, in real terms, are respectively:

$$TC_{i,t} = w_t(1+\mu i_t)\left(\frac{Y_{i,t}}{A_t}\right)^{\frac{1}{1-\alpha}} \quad (7)$$

$$MC_{i,t} = \frac{A_t^{\frac{1}{\alpha-1}}}{1-\alpha} w_t(1+\mu i_t) Y_{i,t}^{\frac{\alpha}{1-\alpha}} \quad (8)$$

where $w_t = W_t/P_t$ is the real wage and the parameter $\mu \geq 0$ measures the relationship between the short-term interest rate (policy rate) and the market interest rate, i.e., it captures the extent of interest rate pass-through; the strength of the "cost-channel" or "supply-side" of the monetary transmission mechanism.[3] When $\mu = 0$, the model collapses to the standard New-Keynesian model with positive trend inflation where interest rates have no bearing on inflation through the supply-side, i.e., the benchmark model presented in Ascari and Sbordone (2014).

Intermediate firms re-set prices every period based on Calvo (1983): in each period a firm can re-optimize its nominal price, denoted by $P^*_{i,t}$, with fixed probability $1-\theta$. The problem of the firm i, which sets its price at time t subject to the demand constraint, is as follows:

$$\max_{P^*_{i,t}} E_t \sum_{j=0}^{\infty} \theta^j D_{t,t+j} \left[\frac{P^*_{i,t}}{P_{t+j}} Y_{i,t+j} - w_{t+j}(1+\mu i_{t+j}) \left(\frac{Y_{i,t+j}}{A_{t+j}} \right)^{\frac{1}{1-\alpha}} \right]$$
$$\textbf{s.t.}\;\; Y_{i,t+j} = \left(\frac{P^*_{i,t}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \quad (9)$$

where $D_{t,t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_0}$ is the stochastic discount factor and λ_{t+j} denotes the marginal utility of consumption in period $t+j$. Let $\Pi_{t,t+j}$ indicate the cumulative inflation between periods t and $t+j$:

$$\Pi_{t,t+j} = \begin{cases} 1 & \text{if } j = 0 \\ \pi_t \times \pi_{t-1} \times \ldots \times \pi_{t+j} & \text{if } j > 0 \end{cases} \quad (10)$$

where $\pi_t = P_t/P_{t-1}$ is the inflation rate. The optimal condition for price can be given as:

$$\left(p^*_{i,t}\right)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{\psi_t}{\phi_t} \quad (11)$$

[3] The existing literature also presents an alternative interpretation for the parameter μ, representing the fraction of firms that borrow funds (Rabanal, 2007) or the fraction of the wage bill that is financed through borrowing (Phaneuf et al., 2018). Either of these interpretations imply that $\mu \in [0,1]$. However, interpreting μ as the strength of the cost-channel, as in Tillmann (2008), leads to the possibility of μ being greater than one (Chowdhury et al., 2006).

where $p^*_{i,t} = P^*_{i,t}/P_t$ and:

$$\begin{aligned}\psi_t &= E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} w_{t+j} \left(1+\mu i_{t+j}\right) \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{\frac{1}{1-\alpha}} \left(\frac{1}{\Pi_{t,t+j}}\right)^{\frac{-\varepsilon}{1-\alpha}} \\ \phi_t &= E_t \sum_{j=0}^{\infty} \theta^j \beta^j \lambda_{t+j} \left(\frac{1}{\Pi_{t,t+j}}\right)^{1-\varepsilon} Y_{t+j}\end{aligned} \tag{12}$$

These can be written recursively as follows:

$$\begin{aligned}\psi_t &= \lambda_t w_t \left(1+\mu i_t\right) \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} + \theta\beta E_t \left[\pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \psi_{t+1}\right] \\ \phi_t &= \lambda_t Y_t + \theta\beta E_t \left[\pi_{t+1}^{\varepsilon-1} \phi_{t+1}\right]\end{aligned} \tag{13}$$

In the Calvo price setting framework, the prices are staggered because firms optimizing prices at different periods will set different prices which results in a distribution of prices for any given period t. The aggregate price level is given by:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P^*_{i,t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{14}$$

This can be rewritten as:

$$p^*_{i,t} = \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} \tag{15}$$

Price dispersion results in an inefficiency loss in aggregate production. The labor demand is given as:

$$N_t^d = \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} di = s_t \left(\frac{Y_t}{A_t}\right) \tag{16}$$

Schmitt-Grohé and Uribe (2007) show that s_t can be rewritten as:

$$s_t = (1-\theta)(p^*_{i,t})^{-\varepsilon} + \theta\pi_t^{\varepsilon} s_{t-1} \tag{17}$$

Schmitt-Grohé and Uribe (2007) also prove that s_t is bounded below at one, and represents the resource costs (or inefficiency losses) due to the relative price dispersion under the Calvo mechanism: the higher the s_t, the more labor is needed to produce a given level of output. Equations (3), (4), (11), (13), (15), (16), and (17) constitute the complete non-linear system.

2.3 The log-linearized model

Log-linearizing equations (3), (4), (11), (13), (15), (16), and (17), and a standard monetary policy rule that responds to inflation and output gives the following model:

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - \sigma^{-1}\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) \tag{18}$$

$$\widehat{w}_t = \varphi\widehat{n}_t + \sigma\widehat{y}_t + \widehat{\tau}_t \tag{19}$$

$$\left(1 + \frac{\varepsilon\alpha}{1-\alpha}\right)\widehat{p}^*_{i,t} = \widehat{\psi}_t - \widehat{\phi}_t \tag{20}$$

$$\begin{aligned}\widehat{\psi}_t = {} & \left(1 - \theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}\right)\left[\widehat{w}_t + \mu\widehat{i}_t - \frac{1}{1-\alpha}\widehat{a}_t + \left(\frac{1}{1-\alpha} - \sigma\right)\widehat{y}_t\right] \\ & + \theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}E_t\left[\frac{\varepsilon}{1-\alpha}\widehat{\pi}_{t+1} + \widehat{\psi}_{t+1}\right]\end{aligned} \tag{21}$$

$$\widehat{\phi}_t = (1-\sigma)\left(1 - \theta\beta\bar{\pi}^{\varepsilon-1}\right)\widehat{y}_t + \theta\beta\bar{\pi}^{\varepsilon-1}E_t\left[(\varepsilon-1)\,\widehat{\pi}_{t+1} + \widehat{\phi}_{t+1}\right] \tag{22}$$

$$\widehat{p}^*_{i,t} = \frac{\theta\bar{\pi}^{\varepsilon-1}}{1 - \theta\bar{\pi}^{\varepsilon-1}}\widehat{\pi}_t \tag{23}$$

$$\widehat{n}_t = \widehat{s}_t + \frac{1}{1-\alpha}\left(\widehat{y}_t - \widehat{a}_t\right) \tag{24}$$

$$\widehat{s}_t = -\frac{\varepsilon}{1-\alpha}\left(1 - \theta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}\right)\widehat{p}^*_{i,t} + \frac{\varepsilon}{1-\alpha}\theta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}\widehat{\pi}_t + \theta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}\widehat{s}_{t-1} \tag{25}$$

$$\widehat{i}_t = \phi_r\widehat{i}_{t-1} + (1-\phi_r)(\phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t) + \upsilon_t \tag{26}$$

where the policy parameters, ϕ_π and ϕ_y, measure the response of the nominal interest rate to inflation and output, respectively. The parameter ϕ_r represents the degree of interest rate smoothing and υ_t denotes an exogenous shock to the monetary policy. The log-linearized model consists of the equations (18)-(26). Equations (19), (20), and (24) can be used to substitute out the variables $\widehat{w}_t$, $\widehat{p}^*_{i,t}$, and $\widehat{n}_t$. Moreover, equations (20)-(22) can be used to get rid of $\widehat{\phi}_t$. This reduces the model to a 5-equation system with three exogenous shocks to the model - technology, labor supply, and monetary policy - that are specified as AR(1) processes, $\zeta \in (a, \tau, \upsilon)$: $\zeta_t = \rho_\zeta\zeta_{t-1} + \epsilon_{\zeta,t}$ and $\epsilon_{\zeta,t} \sim$ i.i.d. $N(0,1)$.

3 Results

3.1 Analytical Results

We use the method of undetermined coefficients to derive the analytical solution of the model, relying on the following simplifying assumptions: log preference in consumption ($\sigma = 1$), indivisible labor ($\varphi = 0$), constant returns to scale ($\alpha = 0$), and no persistence in the monetary policy rule ($\phi_r = 0$). Under these assumptions and in the presence of only the monetary policy shock, the model can be rewritten as:

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - \widehat{i}_t + E_t\widehat{\pi}_{t+1} \tag{27}$$

$$\widehat{\psi}_t = (1-\theta\beta\bar{\pi}^{\varepsilon})\left(\widehat{y}_t + \mu\widehat{i}_t\right) + \theta\beta\bar{\pi}^{\varepsilon}E_t\left(\varepsilon\widehat{\pi}_{t+1} + \widehat{\psi}_{t+1}\right) \tag{28}$$

$$\widehat{\pi}_t = \kappa(\bar{\pi})\left(\widehat{y}_t + \mu\widehat{i}_t\right) + b_2(\bar{\pi})E_t\widehat{\pi}_{t+1} + b_1(\bar{\pi})\left(\bar{\pi}-1\right)E_t\widehat{\psi}_{t+1} \tag{29}$$

$$\widehat{i}_t = \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t + v_t \tag{30}$$

$$v_t = \rho_v v_{t-1} + \epsilon_{v,t} \tag{31}$$

where:

$$\kappa(\bar{\pi}) = \frac{\left(1-\theta\bar{\pi}^{\varepsilon-1}\right)\left(1-\theta\beta\bar{\pi}^{\varepsilon}\right)}{\theta\bar{\pi}^{\varepsilon-1}}; \quad b_1(\bar{\pi}) = \beta\left(1-\theta\bar{\pi}^{\varepsilon-1}\right); \quad b_2(\bar{\pi}) = b_1(\bar{\pi})\left[\varepsilon(\bar{\pi}-1) + 1 + \frac{\theta\bar{\pi}^{\varepsilon-1}}{1-\theta\bar{\pi}^{\varepsilon-1}}\right]$$

Guess the solution as $\widehat{y}_t = \kappa_y v_t$, $\widehat{\pi}_t = \kappa_\pi v_t$, and $\widehat{\psi}_t = \kappa_\psi v_t$. Imposing the proposed solution to equations (27)-(30) and solving for the undetermined coefficients of output (κ_y) and inflation (κ_π) gives the following:

$$\kappa_y = \frac{-\mu\rho_v\Omega_1 - \Omega_2}{\left[\left\{(1-\rho_v)\,\mu - 1\right\}\phi_\pi + \rho_v\left(\mu\phi_y + 1\right)\right]\Omega_1 + \left(1-\rho_v+\phi_y\right)\Omega_2} \tag{32}$$

$$\kappa_\pi = \frac{\left[1-\mu\left(1-\rho_v\right)\right]\Omega_1}{\left[\left\{(1-\rho_v)\,\mu - 1\right\}\phi_\pi + \rho_v\left(\mu\phi_y + 1\right)\right]\Omega_1 + \left(1-\rho_v+\phi_y\right)\Omega_2} \tag{33}$$

where:

$$\Omega_1 = 1-\theta\beta\bar{\pi}^{\varepsilon-1}\rho_v; \quad \Omega_2 = \frac{1}{\kappa(\bar{\pi})}\left[\theta\beta\bar{\pi}^{\varepsilon}\rho_v\left\{\varepsilon\rho_v\left(\bar{\pi}-1\right)b_1(\bar{\pi}) + 1 - b_2(\bar{\pi})\rho_v\right\} - 1 + b_2(\bar{\pi})\rho_v\right]$$

It is obvious from equations (32) and (33) that both the cost-channel and trend inflation impact the responses of inflation and output to a monetary policy shock. We impose an additional simplifying assumption of $\rho_v = 0$ to gauge the direction of the impact.[4]

Proposition 1. Suppose that $\sigma = 1$, $\varphi = 0$, $\alpha = 0$, $\phi_r = 0$, and $\rho_v = 0$. Then, we have the following:[5]

1. $\kappa_y < 0$ for $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$.
2. $\kappa_\pi < 0$ for $\mu \in [0,1)$, $\kappa_\pi = 0$ for $\mu = 1$, and $\kappa_\pi > 0$ for $\mu \in (1, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$.
3. $\frac{\partial\kappa_y}{\partial\mu} < 0$ and $\frac{\partial\kappa_\pi}{\partial\mu} > 0$ for for $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$.
4. $\frac{\partial\kappa_y}{\partial\bar{\pi}} < 0$ and $\frac{\partial\kappa_\pi}{\partial\bar{\pi}} > 0$ for $\mu \in [0,1)$, $\frac{\partial\kappa_y}{\partial\bar{\pi}} = \frac{\partial\kappa_\pi}{\partial\bar{\pi}} = 0$ for $\mu = 1$, and $\frac{\partial\kappa_y}{\partial\bar{\pi}} > 0$ and $\frac{\partial\kappa_\pi}{\partial\bar{\pi}} < 0$ for $\mu \in (1, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$.

[4] In the sub-section on numerical analysis, we consider the case with some persistence in the monetary policy shock.

[5] Other possible values for μ can be considered. However, in general, we would expect $\mu \geq 0$ (Tillmann, 2008; Rabanal, 2007; Phaneuf et al., 2018). Additionally, based on the standard values of the parameters, $\mu > 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi}$ would require μ to be at least greater than 5 which may not be realistic because for $i_t = 3\%$ this suggests the market interest rate of $\mu i_t > 15\%$.

5. $\frac{\partial^2 \kappa_y}{\partial \mu \partial \bar{\pi}} > 0$ and $\frac{\partial^2 \kappa_\pi}{\partial \mu \partial \bar{\pi}} < 0$ for $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$.

Proof: Under the assumptions of $\sigma = 1$, $\varphi = 0$, $\alpha = 0$, $\phi_r = 0$, and $\rho_v = 0$, the undetermined coefficients in equations (32) and (33) can be rewritten as:

$$\kappa_y = -\frac{1}{1 + \phi_y + \kappa(\bar{\pi})\,(1-\mu)\,\phi_\pi}; \qquad \kappa_\pi = -\frac{\kappa(\bar{\pi})\,(1-\mu)}{1 + \phi_y + \kappa(\bar{\pi})\,(1-\mu)\,\phi_\pi}$$

Since $\theta\beta\bar{\pi}^{\varepsilon} < 1$ and $\theta\bar{\pi}^{\varepsilon-1} < 1$, it must be the case that $\kappa(\bar{\pi}) > 0$.[6] Since $\kappa(\bar{\pi}) > 0$, we have $1 + \phi_y + (1-\mu)\phi_\pi\kappa(\bar{\pi}) > 0$ for $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$. Therefore, $\kappa_y < 0$ for $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$ and the sign of κ_π is determined based on the sign of $1-\mu$ in the numerator which is positive for $\mu < 1$, zero for $\mu = 1$, and negative for $\mu > 1$.

Note that in the expressions for κ_y and κ_π, only $\kappa(\bar{\pi})$ is a function of $\bar{\pi}$. Therefore, the first order derivatives of the coefficients, with respect to $\bar{\pi}$ and μ, can be written as:

$$\frac{\partial \kappa_y}{\partial \bar{\pi}} = \frac{(1-\mu)\,\phi_\pi}{[1 + \phi_y + (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]^2}\frac{d\kappa(\bar{\pi})}{d\bar{\pi}}; \qquad \frac{\partial \kappa_\pi}{\partial \bar{\pi}} = -\frac{(1-\mu)\,(1+\phi_y)}{[1 + \phi_y + (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]^2}\frac{d\kappa(\bar{\pi})}{d\bar{\pi}}$$
$$\frac{\partial \kappa_y}{\partial \mu} = \frac{-\kappa(\bar{\pi})\phi_\pi}{[1 + \phi_y + \kappa(\bar{\pi})\,(1-\mu)\,\phi_\pi]^2}; \qquad \frac{\partial \kappa_\pi}{\partial \mu} = \frac{\kappa(\bar{\pi})\,(1+\phi_y)}{[1 + \phi_y + \kappa(\bar{\pi})\,(1-\mu)\,\phi_\pi]^2}$$

The denominator of the above derivatives are positive and $d\kappa(\bar{\pi})/d\bar{\pi} < 0$. So, the signs of $\frac{\partial \kappa_y}{\partial \bar{\pi}}$ and $\frac{\partial \kappa_\pi}{\partial \bar{\pi}}$ depend on the sign of $1-\mu$. Therefore, when $\mu < 1$, we have $\frac{\partial \kappa_y}{\partial \bar{\pi}} < 0$ and $\frac{\partial \kappa_\pi}{\partial \bar{\pi}} > 0$, when $\mu = 1$ the numerator becomes zero so that $\frac{\partial \kappa_y}{\partial \bar{\pi}} = \frac{\partial \kappa_\pi}{\partial \bar{\pi}} = 0$, and when $\mu > 1$ we have $\frac{\partial \kappa_y}{\partial \bar{\pi}} > 0$ and $\frac{\partial \kappa_\pi}{\partial \bar{\pi}} < 0$. On the other hand, $\frac{\partial \kappa_y}{\partial \mu} < 0$ and $\frac{\partial \kappa_\pi}{\partial \mu} > 0$ regardless of the values of the cost-channel (μ).

The cross-derivatives with respect to μ can be written as follows:

$$\frac{\partial^2 \kappa_y}{\partial \mu \partial \bar{\pi}} = -\frac{\phi_\pi\,[1 + \phi_y - (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]}{[1 + \phi_y + (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]^3}\frac{d\kappa(\bar{\pi})}{d\bar{\pi}}; \quad \frac{\partial^2 \kappa_\pi}{\partial \mu \partial \bar{\pi}} = \frac{(1+\phi_y)\,[1 + \phi_y - (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]}{[1 + \phi_y + (1-\mu)\,\phi_\pi\kappa(\bar{\pi})]^3}\frac{d\kappa(\bar{\pi})}{d\bar{\pi}}$$

Note that $1 + \phi_y - (1-\mu)\,\phi_\pi\kappa(\bar{\pi})$ is positive as long as $\mu > 1 - \frac{1+\phi_y}{\phi_\pi\kappa(\bar{\pi})}$. However, based on the standard values of the parameters, we have $\frac{1+\phi_y}{\phi_\pi\kappa(\bar{\pi})} > 1$. Since $\mu \in [0, 1 + \frac{1+\phi_y}{\kappa(\bar{\pi})\phi_\pi})$, we get $\frac{\partial^2 \kappa_y}{\partial \mu \partial \bar{\pi}} > 0$ and $\frac{\partial^2 \kappa_\pi}{\partial \mu \partial \bar{\pi}} < 0$, which completes the proof. ■

Proposition 1 shows that output responds negatively for all values of μ whereas inflation responds negatively when $\mu < 1$, positively when $\mu > 1$, and shows no response when $\mu = 1$. The effects of trend inflation and cost-channel on the response of output and inflation to a contractionary monetary policy shock are further elaborated below:

1. **Trend Inflation Effects:** For output, higher levels of trend inflation increases and decreases the magnitude of its response when $\mu < 1$ and $\mu > 1$, respectively. The response of inflation weakens as trend inflation increases when $\mu \neq 1$. When $\mu = 1$, the

[6] This also suggests that $d\kappa(\bar{\pi})/d\bar{\pi} < 0$.

response of output and inflation are unaffected by changes in trend inflation. Notice that the impact of trend inflation on the response of output and inflation are identical to the findings of Ascari and Sbordone (2014) for $\mu < 1$.

2. **Cost-Channel Effects:** An increase in the strength of the cost-channel increases the response of output in absolute terms. The response of inflation weakens when $\mu < 1$ and increases when $\mu \geq 1$ as the cost-channel becomes stronger.

3. **Cross Effects:** As μ increases, the responsiveness of output and inflation to trend inflation becomes flatter when $\mu < 1$ and steeper when $\mu \geq 1$. In contrast, higher levels of trend inflation weakens the cost-channel effect on the response of output and inflation, which is the main result of this paper.

Intuitively, with positive trend inflation, intermediate firms will set higher prices to try to offset the erosion of relative prices and profits that trend inflation automatically creates. Expectation of forward-looking terms are progressively multiplied by larger discount factors. This means that optimal price setting under trend inflation affects future economic conditions more than short-run cyclical variations including marginal costs, and price setting firms become more forward-looking. Since the cost-channel enters the marginal cost function, trend inflation progressively works to dampen its effects.

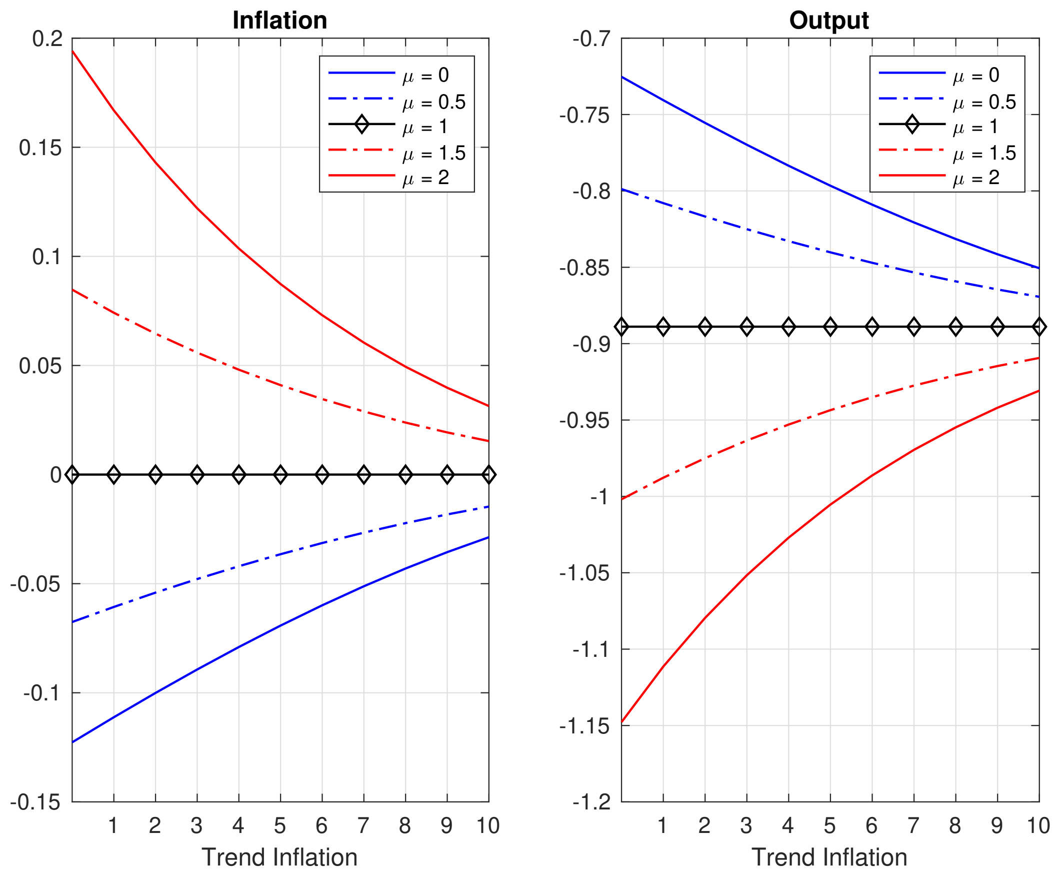
3.2 Numerical Analysis

We combine the solution presented in equations (32) and (33) with calibrated values of the parameters and trend inflation. For this exercise, we continue to assume the values assumed in the previous section.[7] Values for the strength of the cost-channel are taken as $\mu \in \{0, 0.5, 1, 1.5, 2\}$ to encompass the various estimated values in the literature (Tillmann, 2008; Rabanal, 2007).

Figure 1 generates the values for equations (32) and (33) capturing the response of inflation ($\widehat{\pi}_t$) and output ($\widehat{y}_t$) when values of trend inflation and the cost-channel are varied. It is immediately clear that the response of inflation and output changes in the presence of the cost-channel, confirming the analytical results. Looking first at the zero trend inflation case, with no cost-channel ($\mu = 0$), inflation displays a negative response to a contractionary monetary policy shock (since interest rates will rise as v_t rises). When $\mu = 1$, trend inflation has no impact on the response of output and inflation. On the other hand, for $\mu > 1$, the response of inflation to a contractionary monetary policy shock quickly becomes positive. However, high levels of trend inflation considerably dampen the effect of the cost-channel on inflation and output, as also demonstrated analytically.

[7] The values considered are: $\beta = 0.9925$, $\varepsilon = 10$, $\sigma = 1$, $\varphi = 0$, $\alpha = 0$, $\phi_r = 0$, $\rho_v = 0$, $\phi_\pi = 1.5$ and $\phi_y = 0.5$.

Figure 1: Cost-channel under positive trend inflation

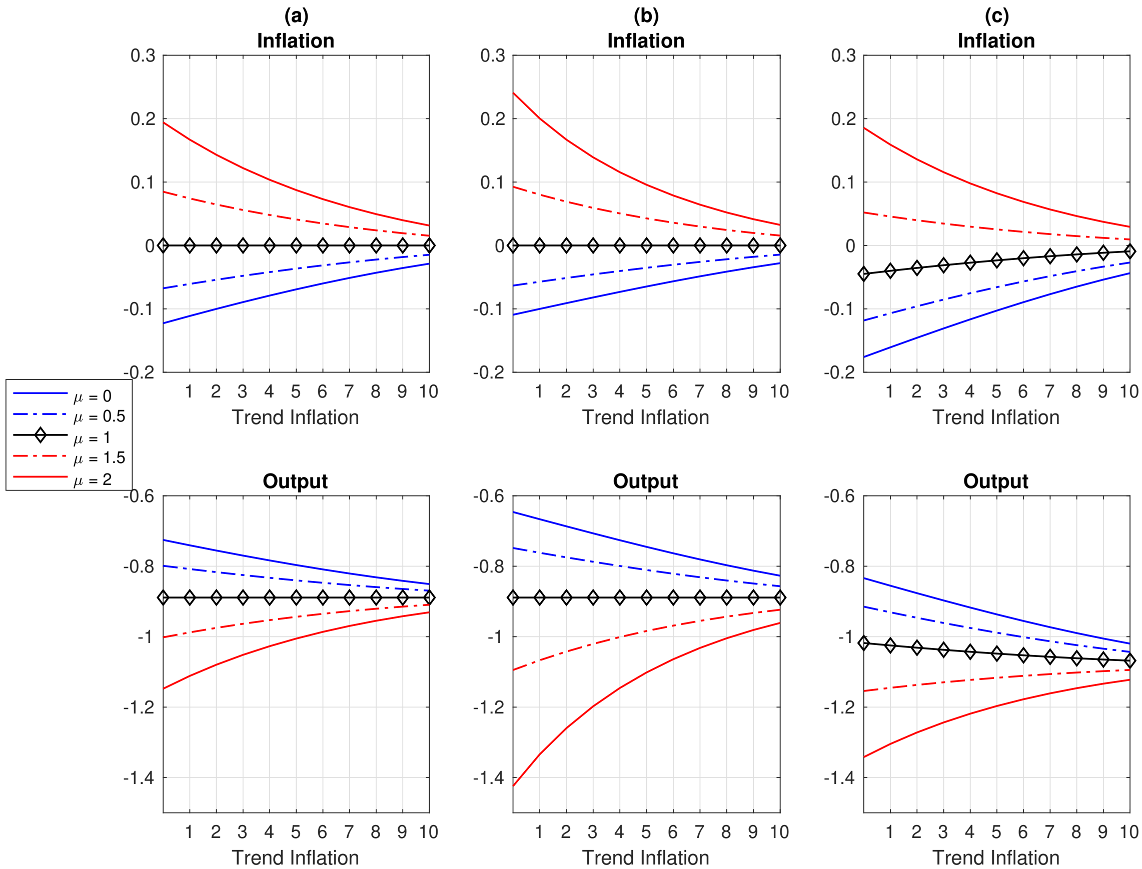


Note: The figure plots the responsiveness of inflation (κ_π) and output (κ_y) to a contractionary monetary policy shock based on a calibrated version of the solution presented in equations (32) and (33).

In Figure 2, we present the baseline case ($\rho_v = 0$, $\phi_y = 0.5$, and $\phi_\pi = 1.5$) in panel (a), the case of higher responsiveness to inflation in the monetary policy rule ($\rho_v = 0$, $\phi_y = 0.5$, and $\phi_\pi = 2.5$) in panel (b), and the case of some persistence in the monetary policy shock ($\rho_v = 0.2$, $\phi_y = 0.5$, and $\phi_\pi = 1.5$) in panel (c).[8] Comparing panels (a) and (b), we observe that inflation aversion sharpens the cost-channel effect which is more pronounced for the response of output and for higher values of the cost-channel, μ. Moreover, panel (c) shows that the introduction of persistence in the monetary policy shock does not affect the main result of the paper.

[8]Persistence in the monetary policy shock ($\rho_v = 0.20$) corresponds to the mid-point of the estimated value in Smets and Wouters (2007).

Figure 2: Cost-channel under positive trend inflation, inflation aversion, and monetary policy persistence



Note: The figure plots the responsiveness of inflation (κ_π) and output (κ_y) to a contractionary monetary policy shock based on a calibrated version of the full model solution presented in equations (32) and (33). Panel (a) presents the baseline case ($\rho_v = 0$, $\phi_y = 0.5$, and $\phi_\pi = 1.5$), panel (b) allows for a relatively larger monetary policy response to inflation ($\rho_v = 0$, $\phi_y = 0.5$, and $\phi_\pi = 2.5$), and panel (c) allows for monetary policy shock persistence ($\rho_v = 0.2$, $\phi_y = 0.5$, and $\phi_\pi = 1.5$).

4 Conclusion

This paper contributes to an active literature that has examined the impact of the cost-channel effects of monetary transmission mechanism in New-Keynesian models. Different from this literature, we focus on the behaviour of the cost-channel in a model which is log-linearized around a nonzero inflation steady state. Using analytical and numerical techniques, we show that the cost-channel is stronger under low levels of trend inflation but is significantly dampened in the presence of high trend inflation. We find that inflation aversion sharpens these effects.

References

Abo-Zaid, S. (forthcoming): "The Government Spending Multiplier in a Model with the Cost Channel," *Macroeconomic Dynamics.*

Ascari, G. and A. M. Sbordone (2014): "The macroeconomics of trend inflation," *Journal of Economic Literature*, 52, 679–739.

Barth, M. J. and V. A. Ramey (2001): "The cost channel of monetary transmission," *NBER macroeconomics annual*, 16, 199–240.

Blinder, A. S. (1987): "Credit Rationing and Effective Supply Failures," *The Economic Journal*, 97, 327–352.

Brückner, M. and A. Schabert (2003): "Supply-side effects of monetary policy and equilibrium multiplicity," *Economics Letters*, 79, 205–211.

Calvo, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 12, 383–398.

Chowdhury, I., M. Hoffmann, and A. Schabert (2006): "Inflation dynamics and the cost channel of monetary transmission," *European Economic Review*, 50, 995–1016.

Christiano, L. J. and M. Eichenbaum (1992): "Liquidity Effects and the Monetary Transmission Mechanism," *American Economic Review*, 82, 346–353.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1997): "Sticky price and limited participation models of money: A comparison," *European Economic Review*, 41, 1201–1249.

——— (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, 113, 1–45.

Dedola, L. and F. Lippi (2005): "The monetary transmission mechanism: Evidence from the industries of five OECD countries," *European Economic Review*, 49, 1543–1569.

Farmer, R. (1984): "A new theory of aggregate supply," *The American economic review*, 920–930.

——— (1988a): "Money and contracts," *The Review of Economic Studies*, 55, 431–446.

Farmer, R. E. (1988b): "What is a liquidity crisis?" *Journal of Economic Theory*, 46, 1–15.

Fuerst, T. S. (1992): "Liquidity, loanable funds, and real activity," *Journal of Monetary Economics*, 29, 3–24.

Gaiotti, E. and A. Secchi (2006): "Is there a cost channel of monetary policy transmission? An investigation into the pricing behavior of 2,000 firms," *Journal of Money, Credit and Banking*, 2013–2037.

Galí, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press.

Phaneuf, L., E. Sims, and J. G. Victor (2018): "Inflation, output and markup dynamics with purely forward-looking wage and price setters," *European Economic Review*, 105, 115–134.

Rabanal, P. (2007): "Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model," *Journal of Economic Dynamics and control*, 31, 906–937.

Ravenna, F. and C. E. Walsh (2006): "Optimal monetary policy with the cost channel," *Journal of Monetary Economics*, 53, 199–216.

Schmitt-Grohé, S. and M. Uribe (2007): "Optimal simple and implementable monetary and fiscal rules," *Journal of monetary Economics*, 54, 1702–1725.

Smets, F. and R. Wouters (2007): "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American economic review*, 97, 586–606.

Tillmann, P. (2008): "Do Interest Rates Drive Inflation Dynamics? An Analysis of the Cost Channel of Monetary Transmission," *Journal of Economic Dynamics and Control*, 32, 2723–2744.