Arithmetics of research specialization

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Abstract
In hiring decisions, universities explicitly reward focusing on a specific field. I model the use of research specialization (focusing) in hiring as a signal of ability. Without explicit reward for focusing, candidates who focus are more likely to be able. However, if job market rewards focusing, less able candidates who would otherwise be indifferent between focusing or not, start focusing, which leads to smaller likelihood of observing an able candidate among those who focus than among those who do not. Specialization works as an effective ability signal only when generation of good ideas is highly likely for all ability levels.

KEYWORDS
job market, research, specialization

JEL CLASSIFICATION
A11, D4, I23, J4

“Jack of all trades, master of none, but oftentimes better than master of one.” Proverb

Some researchers choose to specialize in a field, while some prefer to work in separate, frequently unrelated fields. For freshly minted Economics PhD candidates, who are often without any publications at the time of their job market interviews, specialization in working papers becomes of some significance during the hiring process. Some argue that those who specialize
must be better than those who do not because it is difficult to work well in multiple fields; others argue the opposite for the very same reason. In my model, an environment without an explicit reward for specialization leads to a greater probability of hiring a good candidate if one selects a candidate with a specialized Curriculum Vitae (CV). However, if there are market benefits to specialization, such as a preference for hiring candidates with a specialized CV, it is possible that candidates without specialization are more likely to be better than those who do specialize.

The market outcomes in equilibrium, such as rewarding candidates for acting in a specific way, should be consistent with the relatively higher likelihood of being good in a field, conditional on acting in that specific way. The job market can reward for specialization in the field only if those who specialize in the field are more likely to be better than those who do not specialize, and vice versa. In this paper, I show that there is a significant area of parameter values that leads to an inconsistency between incentives and results.

**Definition 1. Adverse outcome** is an environment where

- *rewarding* for focusing leads to lower the expected ability of those who focus;
- *while not rewarding* for focusing leads to the higher expected ability of those who focus.

## 1 IDEA-GENERATING PROCESS

There is a population of measure 1 of job market candidates. A proportion \( \lambda \) of candidates are *good* at topic 1, the same \( \lambda \) proportion of candidates are *good* at topic 2, and being good at topic 1 is not correlated with being good at topic 2. Every candidate is endowed with two paper ideas in each topic, and being good at either topic means that ideas in this topic are *good* with probability \( p \), while not being good at this topic means ideas in this topic are *good* with probability \( \alpha p \), with \( 1 > p > 0 \) and \( 1 > \alpha > 0 \). Every candidate is characterized by a six-dimensional binary type; that is, two bits denote whether the candidate is good in each topic, and four more record the goodness of candidates’ paper ideas. Overall, there are \( 2^6 = 64 \) types in the economy.

Each candidate chooses paper ideas to work on that constitute that candidate’s CV. In this choice, candidates are mostly motivated by eventual publication, so they prefer to work on good ideas rather than on not good ideas. Candidates choose two paper ideas to work on. Candidates’ preferences about which ideas to work on are lexicographic in respect to idea quality and expected monetary benefits: candidates choose to work on good ideas, and are only motivated by monetary benefits when they are indifferent between multiple ideas.

If candidates have too many good ideas (say, three), or too few (say, one), and the market does not reward them for specialization, candidates select ideas to work on at random. So, the candidate with three good ideas will pick two good ones at random, and the candidate with one good idea will pick one good idea and one not good idea at random. If the job market rewards for specialization, only those who have only one good idea in each topic will not specialize; those with four and no

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1. Chicago faculty (na): “Make sure that you have a well-defined field, or at least make it appear as if you have one.” Levine (na): “Describe one or two research projects that you would like to work on next. Most of your ideas should hang together, as if you were writing a book or two,” but “At least one idea should be distinct from your thesis.” Cawley (2019): “… the four most important pieces of advice regarding the job market. 1) Know where you fit in the discipline of economics; in particular, know: a. In what fields of economics you will specialize.”

2. Rationalization of this outcome could be that the costs of working on three ideas are prohibitively high, whereas someone working on only one idea is believed to be not good in both/all topics.
good paper ideas will select the topic of specialization at random. The candidate who works on both paper ideas on topic $i$ is *focusing* on topic $i$.

Table 1 provides strategies for job market candidates. A candidate with one good idea in Topic 1 and no good ideas in Topic 2 behaves similarly to a candidate who has no good ideas in Topic 1 and one good idea in Topic 2, so some possible outcomes are omitted. If a candidate has only one good idea, they will focus for certain if there is a reward for this, but if there is no reward, they will work on the good idea and will select one not good idea at random. Since there is one idea that will make them focused, and two ideas that will make them not focused, the chance that they will focus if there is no reward is one-third. The same reasoning applies to other options: the only type that cannot be convinced to focus is the one with one good idea in Topic 1 and one good idea in Topic 2. Our parameters, $p$, $\alpha$, and $\lambda$, will govern the conditional expectations.

To calibrate the model, one would need to define what constitutes a good idea. I use a publication in one of the Top 5 journals as a measure of success in the Economics profession for a recent PhD graduate. Conley and Önder (2014) count AER equivalents 6 years after graduation, and report that among the top-30 Economics PhD programs, only Princeton and Rochester programs show more than 20% of their graduates with a publication record of more than one AER equivalent, giving an estimate from above $\lambda = 0.2$. Baghestanian and Popov (2014) provide an estimate of $p$. They estimate the chance to publish in Top-5 for the Top 100 of RePEc ranking economists, whereas these economists were at the beginning of their academic careers. Even for this very competent set of economists, the chance to publish in the Top 5 is at best 30% (the chance decreases if the graduate is graduating from a less competitive school or if the PhD is obtained in later years, when publishing became more competitive), so I take $p \leq 0.3$. Heckman and Moktan (2018) report that having a second Top 5 publication is critical for tenure in competitive American schools. If top American schools are using a tenure decision rule with 5% Type I error, and not good ideas are obfuscated by the tenure-track professors, then one can solve for $\alpha$ from

$$0.05 > P[\text{not good}|2 \text{ good ideas}] = \frac{(1 - \lambda)(\alpha p)^2}{(1 - \lambda)(\alpha p)^2 + \lambda p^2} = \frac{0.8}{0.8 + 0.2(1/\alpha)^2} \Rightarrow \alpha < 0.1147,$$

and

$$0.05 < P[\text{not good}|1 \text{ good idea}] = \frac{(1 - \lambda)(\alpha p)^1}{(1 - \lambda)(\alpha p)^1 + \lambda p^1} = \frac{0.8}{0.8 + 0.2(1/\alpha)} \Rightarrow \alpha > 0.01316.$$

The reader can use other rationales and independently verify the coherence of their calibration with my assertion that the boundaries on the parameters that I obtain are not too restrictive.
Lowering $\lambda$ or Type I error lowers the implied $\alpha$ thresholds.

2 \ MANIPULATING THE REWARD

If the reward for focusing is so large that everyone has decided to focus, everyone focuses, and there is no meaningful signal in focusing. Therefore, I contrast a zero-reward-for-focusing policy and a policy that rewards focusing enough to switch from one idea to another without a loss of quality.

The next two results consider the difference

$$D[\text{policy}] = P[\text{Good in either topic}|\text{Focus}] - P[\text{Good in either topic}|\text{Not Focus}].$$

Using the definition of conditional expectations, it can be written as

$$\frac{P[\text{Good in either topic and Focus}]P[\text{Not Focus}] - P[\text{Good in either topic and Not Focus}]P[\text{Focus}]}{P[\text{Focus}]P[\text{Not Focus}]}.$$  \hspace{1cm} (1)

Observe that the sign of the numerator of this fraction is the sign of $D[\text{policy}]$, which will end up being a polynomial and therefore easier to analyze than the whole fraction.

\textbf{Result 1.} \textit{If there is no monetary reward for focusing, the probability for an agent to be good at any topic conditional on information that agent focusing is higher than the probability for an agent to be good at any topic if that agent does not focus; the adverse outcome as defined in Definition 1 is observed for every ($\alpha, p, \lambda) \in (0, 1)^3$.}

\textit{Proof.} The sign of the numerator of the fraction (1) is proportional\(^4\) to:

$$\propto D_1 = \lambda(1 - \lambda)^3(1 - \alpha)^2p^2 > 0.$$

Therefore, the focusing population on average is more likely to be good in at least one topic than the nonfocusing population. \hfill \square

The driving mechanism is straightforward. It is unlikely that a candidate who is not good in anything has two good ideas in the same topic, which means that a not good candidate is unlikely to focus. On the other hand, those who are good at both topics are also unlikely to focus: the chance to focus for a candidate with three or four good ideas is only one-third. Perhaps, a greater reward for candidates who focus will motivate good candidates to focus more?

\textbf{Result 2.} \textit{If there is a monetary benefit for focusing, the probability for an agent to be good at any topic conditional on focusing is lower than the probability for an agent to be good at any topic if that agent does not focus as long as $p(1 + \alpha) < 1$.}

\(^4\)Hereafter proportionality means omitting clearly positive multiplicands such as $\alpha$ and $(1 - \alpha)$. \url{http://sergeyvpopov.github.io/files/Arithmetics.zip}. A companion set of Matlab files can help verify calculations.
Proof. The sign of the top of the fraction (1) is proportional to
\[\propto D_2 = \lambda(1 - \lambda)^2(1 - \alpha)p^2(p(1 + \alpha) - 1) \left(2\alpha + \lambda - \alpha \lambda - \lambda p - 2\alpha^2 p + \alpha^2 \lambda p\right).\]
The last bracket is positive:
\[
2\alpha + \lambda - \alpha \lambda - \lambda p - 2\alpha^2 p + \alpha^2 \lambda p = \alpha - \alpha^2 p + [\alpha + \lambda - \alpha \lambda] - p[\lambda + \alpha^2 - \alpha^2 \lambda] = \\
> \alpha - \alpha^2 p + [1 - (1 - \alpha)(1 - \lambda)] - p[1 - (1 - \alpha^2)(1 - \lambda)] > \\
> 1 - (1 - \alpha)(1 - \lambda) - [1 - (1 - \alpha^2)(1 - \lambda)] = (1 - \lambda)\alpha(1 - \alpha) > 0.
\]
Therefore, the adverse outcome, when the nonfocusing population on average is more likely to be good in at least one topic than the focusing population, can be observed if \(p(1 + \alpha) < 1\).

Indeed, monetary remuneration for focusing can help, but this motivator also stimulates candidates who have no good ideas to focus on. If there is a significant amount of those candidates (\(p\) is small enough), the informational benefit of a better environment for focusing for those who are good becomes dominated by the abundance of those who are not good but now have the incentives to focus. The only ones who do not focus, when there is a premium for doing so, are those who have a good idea in each field. For small \(p\), these are likely to be candidates who are good in at least one field. On the other hand, if \(\alpha p\) is high, not-focusers are likely to be not good in either field, because high \(\alpha p\) means that even those who are not good in both topics have a good chance in getting one idea in both topics, whereas those who are good in a field have a good chance of getting two ideas in the same field and focus on that field.

Frequently, job search advertisements explicitly call for people who work in a specific field. While the first two results are informative for those who want to hire a candidate who is good at something, the next two results are informative for those who want to hire a candidate who is good in a specific topic.

The next two results consider the difference:
\[
D[\text{policy}] = P[\text{Good in T1|Focus on T1}] - P[\text{Good in T1|Not Focus}].
\]
Using the definition of conditional expectations, it can be written as
\[
\frac{P[\text{Good in T1 and Focus on T1}]P[\text{Not Focus}]}{P[\text{Focus on Topic 1}]P[\text{Not Focus}]} - \frac{P[\text{Good in T1 and Not Focus}]P[\text{Focus on T1}]}{P[\text{Focus on Topic 1}]P[\text{Not Focus}]}.
\]

Result 3. If there is no monetary reward for focusing, the probability for an agent to be good at topic \(i\), conditional on focusing on topic \(i\), is higher than the probability for an agent to be good at topic \(i\) if that agent does not focus.

Proof. The numerator of (2) is proportional to
\[\propto D_3 = p(1 - \alpha)(1 - \lambda)Z(\alpha, p, \lambda),\]
where \( Z(\alpha, p, \lambda) = z(\alpha, p)\lambda^2 - (3(1 - \alpha)p + z(\alpha, p))\lambda + (1 - \alpha)p + (1 - \alpha)p + 1, \)

where \( z(\alpha, p) = -2\alpha^3 p^4 + \alpha^3 p^3 + 4\alpha^2 p^4 - \alpha^2 p^3 + 2\alpha^2 p^2 - 2\alpha p^4 - \alpha p^3 - 4\alpha p^2 + p^3 + 2p^2. \)

\( z(\alpha, p) \) is positive: if one tries to solve \( z(\alpha, p) = 0 \) in terms of \( p \), the solution would be \( p^*(\alpha) = \frac{1 + \sqrt{\alpha^2 + 18\alpha + 1}}{4\alpha} \), which is a decreasing function with respect to \( \alpha \), and at \( \alpha = 1 \), it is equal to \( p^*(1) = \frac{1}{2} + \sqrt{1.25} > 1. \) Therefore, in the space of \((p, \alpha) \in (0, 1)^2\), \( z(\alpha, p) \) has a strictly positive sign. Since the value of \( z(\alpha, p) \) at \((0.5, 0.5)\) is 0.1563, we deduce that \( z(\alpha, p) > 0 \) everywhere at \((\alpha, p) \in (0, 1)^2\). Therefore, in terms of \( \lambda \), \( Z(\cdot) \) is a U-shaped parabola:

\[
Z(\alpha, p, \lambda = 0) = (1 - \alpha)p + (1 - \alpha)p + 1 > 0,
\]

\[
Z(\alpha, p, \lambda = 1) = -3(1 - \alpha)p + (1 - \alpha)p + (1 - \alpha)p + 1 = 1 - (1 - \alpha)p + \alpha(1 - p)p.
\]

The minimum of \( Z(\alpha, p, \lambda) \) for a given \( \alpha \) and \( p \) is at

\[
\lambda^*(\alpha, p) = \frac{z(\alpha, p) + 3(1 - \alpha)p}{2z(\alpha, p)}.
\]

We will now establish that \( \lambda^*(\alpha, p) > 1 \). This will be true if

\[
z(\alpha, p) < 3(1 - \alpha)p \Rightarrow -p(1 - \alpha)(\alpha^2 p^2 + 2\alpha p^3 - 2\alpha^2 p^3 + 2\alpha p^2 - p^2 - 2p + 3) < 0.
\]

Since the minimum of the parabola is at \( \lambda^* > 1 \), there is no change in the value of \( Z(\alpha, p, \lambda) \) from positive to negative for \( \lambda \in (0, 1) \), and therefore, the positivity of the value of \( Z(\cdot) \) at the borders means positivity everywhere inside \((\alpha, p, \lambda) \in (0, 1)^3\). \( \square \)

**Result 4.** If there is a monetary benefit for focusing, the probability for an agent to be good at topic \( i \), conditional on focusing on topic \( i \), is lower than the probability for an agent to be good at topic \( i \) if that agent does not focus if \( p \) is small enough; or if \( \alpha p \) is small enough when \( \lambda \) is small enough.

**Proof.** The numerator of (2) is proportional to

\[
\alpha D_4 = -p^2(\alpha + \lambda - \alpha \lambda - \lambda p - \alpha^2 p + \alpha^2 p^2)^2 Z(\alpha, p, \lambda),
\]

where \( Z(\alpha, p, \lambda) \) is a quadratic equation with respect to \( \lambda \). The determinant of that quadratic equation is

\[
8p^3(1 - \alpha)^2 \left( -2\alpha^2 p^3 + 4\alpha^2 p^2 - \alpha^2 p^2 + 4\alpha p^2 - 6\alpha p + 2\alpha - p + 2 > 0. \right.
\]

This means that \( Z(\alpha, p, \lambda) \) might change its sign with a change in \( \lambda \).
For a given pair of $\alpha$ and $p$, the extremum is at
\[
\hat{\lambda} = \frac{-\alpha^3 p^3 - \alpha^2 p^3 + 2\alpha^2 p^2 + \alpha p^2 - 2\alpha p + 1}{p(1-\alpha)(\alpha^2 p^2 + 2\alpha p^2 - 2\alpha p + p^2 - 2p + 2)}
\]
\[
= 1 + \frac{\alpha^2 p^2 + (p-1/2)^2 + 3/4 > 0}{(1-p)\left(\alpha p^2 - p + p^2 + 1\right)} > 0
\]
which means that there is at most one root of $Z(\alpha, p, \lambda)$ as a function of $\lambda$ when $\lambda \in (0, 1)$, and that there is at most one change of the sign as $\lambda$ changes from 0 to 1.

At $\lambda = 0$, $Z(\alpha, p, \lambda = 0) = 2\alpha^4 p^4 - 4\alpha^3 p^3 + 4\alpha^2 p^2 - 4\alpha p + 1$. Observe that this can be rewritten as
\[
2t^4 - 4t^3 + 4t^2 - 4t + 1 \text{ where } t = \alpha p.
\]
This equation has two roots, and only $t^* = 0.3281$ is relevant. When $\alpha p < t^*$, $Z(\alpha, p, \lambda = 0)$ is positive; to the right, it is negative.

At $\lambda = 1$, $Z(\alpha, p, \lambda = 0) = 2p^4 - 4p^3 + 4p^2 - 4p + 1$. Observe that this equation is identical to (3). When $p < t^*$, $Z(\alpha, p, \lambda = 1)$ is positive; to the right, it is negative.

Since there is at most one switch of the sign with respect to $\lambda$, values of $(\alpha, p)$ that yield a positive value of $D_4$ at $\lambda = 0$ and at $\lambda = 1$ must yield the same value in the interim. Further, those values of $(\alpha, p)$ that yield a negative value of $D_4$ at $\lambda = 0$ and a positive value at $\lambda = 1$ must feature a $\lambda^*(\alpha, p)$ such that the value of $D_4$ is negative at $\lambda < \lambda^*(\alpha, p)$ and positive at $\lambda > \lambda^*(\alpha, p)$. □

The intuition of Results 3 and 4 mirrors the intuition for the Results 1 and 2; the difference in the results is limited to the appropriate pool of focused candidates. For Results 1 and 2, candidates are deemed focused if a candidate focuses on either of two topics; for Results 3 and 4, if a candidate focuses on the wrong topic, that candidate is not considered. Figure 1 illustrates the change in the shape of the cutoff.

3 | DISCUSSION

In my model, I abstract away from the variation in the quality of ideas, limiting my distribution of quality to a binary form. This is relevant to some settings (e.g., tenure decisions appear to treat publications in the Top 5 differently from publications in other journals, see Heckman and Moktan, 2018). Meanwhile, in other settings, some might find it acceptable to sacrifice a small difference in the quality of an idea to focus on a field for monetary gain. It is possible to reformulate the model taking the quality differentials into account, in the spirit of the Olszewski (2020) model for instance. However, the main conclusion will remain the same; that is, that the premium for specialization applies to good candidates and to not good candidates alike, and, if there is a number of not good candidates, this will lead to an adverse outcome. Moreover, in my model, the total quantity of good ideas being worked on is socially optimal in both scenarios, but if one introduces a continuous measure of the idea quality, some authors will work on worse
FIGURE 1  Adverse outcome as defined in Definition 1 when there is a market reward for specializing. ⋆ denotes the calibration exercise: $\lambda = 0.2$, $p = 0.3$, $\alpha = 0.1147$. Dotted line on Figure 1(b) represents a cutoff for $\lambda = 0.2$: any combinations of $(\alpha, p)$ below it demonstrate adverse outcome when $\lambda = 0.2$, and any combinations above never lead to an adverse outcome.

ideas if specialization is encouraged, creating welfare losses, and providing an additional reason to avoid premia for specialization.

In my model, I assume that being good in one field is not a barrier to being good in another field. While the true correlation might go one way or the other, manipulating this correlation\(^5\) does not appear to affect the result strongly. Indeed, providing a focusing bonus works mostly for those who have a number of good ideas (who are likely to be good at both fields), and for those who do not have a number of good ideas (who are not likely to be good at both fields). If there are only a few people who are good in both fields, then there is no point forcing them to focus. Conversely, if there are no candidates who are good in only one field, but there are some candidates who are good in both fields, those who do not focus when they are paid a premium for focusing are likely to be good in both fields.

Limiting the quantity of ideas in each topic by two is somewhat arbitrary. A natural extension is to assume a Poisson distribution for the quantity of good ideas and assume an unlimited supply of not good ideas. Then the number of working papers that a CV contains can be endogenized: it needs to be such that more papers do not signal increased ability, because otherwise candidates without good ideas will emulate productive ones. Conditional on working on $X$ papers, however, the economics of stimulating specialization will remain the same, albeit the thresholds might change.

The reward I model in the main section is not specified, but implicitly assumed to be unconditional on the unobservable ability of the candidate. Some rewards do not have to be explicitly connected to ability, but consider a tenure track contract—a candidate is more likely rewarded for

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\(^5\) https://docs.google.com/spreadsheets/d/1wKIUA-BL8eoAwKaGncPzJ0X68weZ0paCjzgKQ6AnI1AE/edit?usp=sharing I provide a Google Sheets worksheet that contains formulas to obtain conditional probabilities studied in the Results section. Readers can make a copy to try other idea-generating processes, or introduce correlations.
focusing with a contract that pays off handsomely if a candidate produces two good papers in the topic they are focusing in, with a limited time to deliver. Among candidates who have two good ideas, one in two different topics, only candidates who are good in at least one topic will choose to focus and disregard a good idea in another topic, because they believe that they will be able to produce another good idea in the allotted time. The reward for doing so needs to be not too large, so that candidates who are not good in either topic do not want to focus foregoing their good idea.

In my model, specialization per se does not improve productivity. It could be the case that it does—researchers read more, understand the field better, network, and so on—and then this “learning by specializing” would make specialization a natural signal of better productivity. For the interaction that I describe, namely, a market of freshly minted Economics PhD job market candidates, the benefits of specialization might not have enough time to bear fruit in the quality of job market CV yet. Generically, this could manifest in a shift of the thresholds of Figure 1, but this idea deserves further deeper research.

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REFERENCES