PATENT LICENSING AND R&D IN THE PRESENCE OF FIRM HETEROGENEITY

By

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Abstract

Technological development is one of the main determinants of economic growth in the long run. In general, it is achieved by the research and development (R&D) activity and then diffusion of the new technology. This thesis examines both R&D stage and technology diffusion stage from different angles to identify how innovation affects firm competition. Most of the studies on patent licensing and R&D are based on symmetric firms, but we more often see asymmetric firms in the realistic situation. Therefore, firm heterogeneity is an important feature of this thesis. The first main chapter focuses on the diffusion stage of new technology in the presence of asymmetric licensees. That is, given the innovation has already taken place, this chapter aims to find how the asymmetric cost structure affects the most profitable strategy for an outside innovator. Moreover, it also finds that it depends on the type of innovation whether the large firm or small firm has higher incentive to pay for the license. The second main chapter focuses on R&D stage and the firm heterogeneity is characterized by both asymmetric initial production cost and asymmetric R&D spillover rate. Therefore, it aims to find how the firm heterogeneity affects the R&D choices and subsequent market structure. The result shows that the one-way spillover rate drives the small firm to invest more in R&D so that the cost gap is narrowed. The third chapter focuses on the R&D stage in the presence of uncertainty in a multi-period game. It finds the uncertainty of R&D under long-term competition makes two initially symmetric firms grow technological gap instead of evolving neck to neck. Therefore, although the model starts from symmetric firms, it captures asymmetric cases due to uncertain R&D outcomes.

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Chapter 1

Introduction

Economic theory finds that the technological development is one of the main determinants of economic growth in the long run. The technological development can be regarded as an overall process of invention, innovation and diffusion of the technology. To be specific, the innovation's outcomes are commonly divided into two types, either a creation of a good that is new to market, referred as product innovation, or an improvement to the existing product, referred as process innovation. Furthermore, research and development (R&D) activity is the main tool to achieve the innovation. The R&D choice of firms, or more in general, the agents are endogenous and depend on the characteristics of the economy, especially the market structure and the institutions. Once the innovation is created, the innovator owns the property right and then aims to maximize returns on the innovation, that is, to find the most profitable way to diffuse the technology. This thesis considers both stages of technological development and obtains certain results.

The remainder of this thesis is divided in three main chapters, followed by conclusions.

Chapter 2 which is entitled "Licensing versus selling with firm heterogeneity" focuses on the diffusion stage of new technology. Most of the existing literature considers patent licensing as the only commercial policy available to innovators and determines the most profitable licensing contract consisting of fixed fee, per-unit royalty and their combinations. However, Tauman and Weng (2012) first consider the possibility that an outside innovator could sell the patent right to one of the potential licensees and it is more profitable than the directly licensing strategy, which provides another option to diffuse the new innovation. The main difference between traditional patent licensing and selling is whether the ownership of the patent right is transferred. The buyer of the patent right has the subsequent option to license the new technology to other firms with their preferred licensing strategy.

In a recent study, Sinha (2016) finds that selling strategy is the most profitable way for an outside innovator to transfer the new innovation under Cournot competition with asymmetric firms. Although the existing literature shows that the selling strategy is more profitable, it has not shown why it is the case. With the selling strategy, the buyer first obtains the innovation by paying a fixed fee and then sub-license it to its rival. That is to say, both firms finally get the new technology but through different contracts and in different order. I introduce a fully flexible discriminatory licensing scheme in an asymmetric Cournot duopoly model, that is, the outside innovator takes full advantage of the firm heterogeneity and offers different contracts to the firms either simultaneously or sequentially and compare it with the selling strategy to determine what the most profitable way is for the outside innovator and whether licensing replicates the selling strategy. As mentioned before, there are two common types of innovation, process or product innovation, which are analyzed in this chapter respectively. The result shows that selling strategy is the most profitable way for the outside innovator for both types of innovation. The superiority comes from the ability to promise the buyer the best possible outcome while at the same time threaten with the worst outcome. However, the discriminatory licensing strategy cannot achieve it due to the lack of commitment power.

In addition, I also consider whether the more efficient or less efficient firm has higher incentive to acquire the license. The results differ between the types of innovation. The more efficient firm has higher incentive to pay for the license if it is a process innovation while it has lower incentive if it is a product innovation. Since the process innovation reduces the marginal cost by the same magnitude for both firms, the more efficient firm benefits more by using the new technology and it is threatened by a situation where the less efficient firm may catch up with it. Therefore, the fear of losing its leader's position provides the more efficient firm with higher incentive. However, this is not the case for the product innovation. Since both firms will produce at the same marginal cost after adopting the product innovation, the less efficient firm in fact reaps larger cost reduction, which provides higher profit incentive to pay for the license.

Another contribution of this chapter is the welfare analysis. The existing literature mainly aims to find the privately optimal strategy, that is, the most profitable way for the innovator. But whether the privately optimal strategy is also socially optimal remains unclear. Therefore, I provide a welfare analysis to check it. The results show that the discriminatory licensing strategy is socially optimal if the initial cost asymmetry is not too large in case of process innovation. As for the product innovation, the discriminatory licensing is always socially optimal.

Chapter 3 which is entitled "R&D behaviors with firm heterogeneity" focuses on the innovation stage of the new technology, which is achieved by R&D activity. The main research question comes from a controversial topic that whether large firms or small firms spend more on R&D. The majority of theoretical research states that large firm engages more in R&D than small firm (Pavitt et al., 1987; Scherer, 1991; Cohen and Klepper, 1996a). Therefore, the cost gap between firms is getting even larger. But some recent empirical studies show that small firms spend more on R&D in some industries (Arora, Fosfuri, and Gambardella, 2001; Revillan and Fernandez, 2012a; Figueroa and Serrano, 2013).

Starting from an asymmetric situation, which is characterized by difference in initial cost of production, firms conduct R&D to improve their costs. Due to the asymmetric setting, firms' ability to absorb the external knowledge, that is, R&D spillover, differs as well. In this chapter, I assume the situation where only the large firm's R&D spills over to the small firm but not vice versa, to see how the one-way spillover influences the R&D behaviors and further the cost gap. To be specific, this chapter aims to determine whether the one-way spillover is a possible factor to explain that the small firm spends more on R&D and whether the one-way spillover is able to stop the divergence between the firms so that the cost gap is narrowed. Furthermore, the chapter provides a welfare analysis to determine whether the increasing or decreasing cost gap benefits the consumers and the society.

The main results of this chapter are summarized as follows. Firstly, the one-way spillover rate is a possible factor which explains why the small firm chooses higher R&D intensity than the large firm and the cost gap is therefore narrowed. With relatively small asymmetry, the small firm is able to even catch up with the large firm and it benefits both the consumers and the total producer surplus. However, from the welfare point of view, it is not always better that the gap between firm narrows. If the initial cost gap is relatively significant, the society prefers to even broaden the gap further.

Chapter 4 which is entitled "Dynamic R&D competition with stochastic outcomes" also focuses on the R&D activity. R&D is known as a risky, costly and continuous process and firm heterogeneity is one of the key points of the previous chapters, therefore the research question of this chapter comes from whether the R&D competition with uncertainty in a multi-period game makes two initially symmetric firms diverge. Uncertainty is characterized by stochastic outcomes, that is, the firm may succeed or fail to make a progress given its R&D choice. Moreover, the multi-period game is another feature of this chapter. The incentives to invest in R&D might vary as the game proceeds, depending on the position of the firm in the game relative to its rival and relative to the end of the game (Zizzo, 2002). Therefore, the aims of this chapter are to see how the firms' incentives to engage in R&D are affected in a multi-period game with uncertain outcomes and how the technological difference between the firms evolves.

In this chapter, firms are initially symmetric, which means the technological difference is zero at the beginning and compete for finite number of periods. In each period, each firm chooses its own R&D level simultaneously and the probability of success increases with investment. The technological difference is captured by the difference between the total number of successes of firm i and those of its rival. Moreover, I assume that the payoff of each firm in each period depends on its number of successes. Therefore, when a firm makes its own decision on R&D in each period, it is not only maximizing the periodic profit, but also takes into account how the R&D investment in this period affects the following periods. More precisely, the history determines the choice and the corresponding outcomes.

The main analytical results are obtained for the simplest 2-period game. Focusing on the same period, the firm invests the most if it is one period ahead of its rival, followed by the situation where the firms are at the same technological level, and then the firm invests the least if it is one period behind its rival. Furthermore, due to its stochastic nature, the game has various end outcomes with different probabilities. I classify the end outcomes according to probabilities of being symmetric and asymmetric. The result shows that the probability of firms being asymmetric rises continuously as the game proceeds, implying that two firms are diverging instead of evolving neck to neck. Considering the R&D investment over time, I find that the less periods there are remaining, the less investment the firm wants to conduct. It is intuitive that the firm knows that being a leader is more profitable than being a follower and the incremental profit is increasing with the number of leads, therefore at the beginning of the game, it has the highest incentive to invest in R&D in order to become the leader. Additionally, all the analytical results obtained from the 2-period game can be extended to the T-period game, which are proved in a numerical way.

Chapter 5 is the conclusion, which summaries the main findings, limitations and possible extensions of this study.

Chapter 2

Licensing versus selling with firm heterogeneity

2.1 Introduction

Research of patent licensing between innovator and licensees is relatively extensive. The licensing contracts through fixed fee, per-unit royalty and their combinations for both outside and inside innovators are extensively studied in the existing literature. If the innovator is a third party outside of the market, it has been generally shown that fixed fee licensing is optimal (see Katz and Shapiro, 1986; Kamien and Tauman, 1986; Kamien et al., 1992; Stamatopolous and Tauman, 2009); whereas per-unit royalty contract is optimal when the innovator is an insider, that is, one of the producers in the market (see Wang, 1998; Kamien and Tauman, 2002; Wang and Yang, 2004).

However, Tauman and Weng (2012) first consider the possibility that an outside innovator could sell the patent right to one of the potential licensees, which provides another option to profit from technology transfer. The buyer of the patent right has the subsequent option to license the new technology to other firms with their preferred licensing strategy. The main difference between traditional patent licensing and selling is whether the ownership of the patent right is transferred. Following patent sale, the innovator loses all control of the property right and transfers the ownership to the buyer, which is able to license it further to its rivals. The relevant study of selling the patents rights is a relatively new area of research and is not well understood.[1]

In a general Cournot framework with an outside innovator and several symmetric potential licensees, Tauman and Weng (2012) find that selling the innovation can actually be strictly more profitable to the outside innovator than direct licensing strategy. This result is noteworthy, because it provides a more profitable opportunity for the innovator by selling the new technology rather than licensing. In a recent study, Sinha (2016) analyzes selling strategy of an outside innovator under Cournot competition for asymmetric firms and finds that selling strategy is still the most profitable way to transfer the new innovation to the more efficient firm which then licenses it to its rival by per-unit royalty contract. More recently, Banerjee and Poddar (2019) focus on a similar question but in a different framework. They consider a differentiated product price competition in a spatial framework (i.e. a linear city model) and get more general and robust result, which show that selling the innovation to a potential licensee (regardless of the cost efficiency) is always strictly better than any licensing strategy irrespective of the size of the innovations (drastic or non-drastic). From the standpoint of an inside innovator, Niu (2019) compares licensing policy to selling policy and shows that the choice of licensing or selling depends on the size of the initial cost difference.

Although the exisitng theoretical results show the superiority with respect to patent sale in innovator's profit, there is little empirical evidence in real world. By contrast, the majority of empirical papers related to technology transfer focuses on patent licensing. Rostoker (1984) reports that a two-part tariff contract is used 46% of the cases, while a pure per-unit royalty and a fixed fee license account for 39% and 13%, respectively. By employing Spanish data, Macho-Stadler et al. (1996) finds that 59.3% of the contracts have only royalty payments, 27.8%

[1]For theoretical studies on patent sale, see Sinha (2016), Banerjee and Poddar (2019) and Niu (2019).

have fixed fee payments only, and 12.9% include both fixed and royalty payments. More recently, Yanagawa and Wada (2000) find that about 48% of the contracts s from US firms to Japanese firms involve two-part tariff. From these empirical data, it is found that in reality, different types of contracts are offered among all the licensing contracts and the combinations of fixed fees and royalties are commonly observed as well. In the discriminatory licensing analysis part, I do find that any of the licensing strategies mentioned before can be the profit maximising contract depending on the circumstances. Thus the result on patent licensing is consistent with the empirical findings. Besides, recent empirical papers show the evidence of patent sale. Serrano (2010) makes use of data on the transfer of the ownership of patents, which is from US Patent and Trademark Office (USPTO). And mentions that "13.5% of all granted patents are traded at least once over their life cycle".

More often than not, we observe asymmetries among firms. Therefore, I employ asymmetric Cournot duopoly model in this paper, that is, two potential licensees differ in their initial production costs. Specifically, the lower cost firm is more technologically advanced and produces more than the higher cost firm, therefore it occupies a larger market share in the pre-innovation stage. In other words, two potential licensees are not on equal terms in the market. Therefore, if the innovator announces that it does not exclusively offer the license, it is reasonable that the outside innovator designs a unique contract to each firm according to its production scales and technical levels, simultaneously or sequentially. For example, a technological company owns an outstanding application. Both Android and Apple want to get the license to release this application. Therefore the company has the right to choose when and how to offer the license to any of the platforms. As for patent sale, it can be regarded as acquitision, which provides an effective way for the firm to access innovation. Since engaging in its own R&D requires considerable time, energy and capital, and carries a high risk of failure, firms can be motivated to instead seek innovation externally, through M&A.

Given the existing results with respect to selling strategy, a natural question arises, why selling strategy is more profitable than the traditional licensing strategy and it has not been shown in the previous papers. With patent sale, both firms finally get the new technology but through different contracts and in different order. In order to imitate the process without losing property right, I introduce a fully flexible discriminatory licensing scheme in an asymmetric Cournot duopoly model, that is, the outside innovator takes full advantage of the firm heterogeneity and offers different contracts to the firms either simultaneously or sequentially and compare it with the selling strategy to determine what the most profitable way is for the outside innovator and whether the discriminatory licensing replicates the selling strategy. In addition, the existing literature mainly aims to find the privately optimal strategy, that is, the most profitable way for the innovator. But whether the privately optimal strategy is also socially optimal remains unclear. Therefore, I provide a welfare analysis to check whether licensing or selling strategy provides a larger social welfare.

This chapter extends the analysis of Sinha (2016) and Banerjee and Poddar (2019). Sinha (2016) compares uniform licensing with selling strategy in an asymmetric Cournot duopoly market and shows the superiority of patent sale, while welfare analysis is not considered in his paper. On the other hand, Banerjee and Poddar (2019) consider discriminatory licensing and welfare analysis but with a linear city model. They assume the market is matured, i.e., the demand is not growing, and the products are well differentiated, which is more observed in most developed countries. Also, in the linear city model, the firms compete in prices. On contrary, firms produce homogeneous product and compete in quantities in this paper and the Cournot model is used to model competition in homogeneous products. Besides, this paper focuses on the technology transfer and patent licensing in the standard Cournot framework where the demand is elastic. Thus, this paper fills the gap in the existing literature by considering both discriminatory licensing and welfare analysis in an asymmetric Cournot duopoly market.

This chapter contributes to the closed literature in the following four aspects. First, it further shows the superiority of patent sale. However, the empirical literature shows little evidence of patent sale. The majority of innovation transfers are via licensing. So based on heterogeneous firms, I introduce a fuller picture of the intellectual property strategy of the outside patentee in an asymmetric duopoly market which allows the innovator to award different licenses to potential licensees simultaneously or sequentially. Second, this study provides a welfare analysis to see whether the privately optimal strategy is also socially optimal or not. Third, the previous papers are generally from the standpoint of the innovator. In this chapter, I also compare the incentives of the potential licensees (i.e. asymmetric firms) to pay for the license, that is, whether the more efficient or less efficient firm has higher incentive to acquire the license. Finally, two types of innovation are analysed in this chapter. One is the step-eliminating innovation, which generates the same size of cost reduction for both firms. The other one is new process innovation, which helps the firms adopting a new process with the same marginal cost. In other words, this chapter basically covers the most common types of innovation.[2]

The result shows a consistent ranking of different commercial strategies available to the innovator for both types of innovations, that is, patent sale is more profitable than the simultaneous licensing which in turn is better than the sequential licensing. For both types of innovations, the optimal sequence for the innovator is the one which leads to the larger post-innovation cost difference and the follower always has incentive to pay more for the license. Therefore, the outside innovator always offers a fixed fee contract to the follower to extract this larger payment.

The rest of the paper is organized as follows. Section 2.2 describes the model and the structure of the licensing game. Section 2.3 characterizes the optimal

[2] In general, the innovation's outcomes are commonly divided into two types, either a creation of a good that is new to market, or an improvement to the existing product.

discriminatory licensing and compares it with patent sale based on the step-eliminating innovation. Section 2.4 proceeds in the same manner as section 2.3 but for the new process innovation. This is followed by the welfare analysis in section 2.5. Finally section 2.6 concludes.

2.2 The Model

There are two types of players in the model; producers and an innovator.

Producers. Consider a Cournot duopoly with ex-ante asymmetric firms, firm 1 and firm 2. They produce a homogenous product and compete in quantities. Firm 1's initial marginal costs of production is c_1 and firm 2's is c_2, where $c_1 < c_2$. In other words, firm 1 is relatively more efficient than firm 2 initially. The distinct initial production cost is where the firm heterogeneity lies in, which may lead to different incentives for acquiring a new technology. The inverse demand function is given by $p = a - Q$, where p is price of the product, $Q = q_1 + q_2$ is the aggregate quantities produced in the market and a is a positive demand intercept.

Innovator. There is an outside innovator (e.g. an independent research lab) that owns a cost-reducing innovation. Two types of innovation are considered in this paper. One is called common step-eliminating innovation which helps the producers to decrease their marginal costs by x, thereby reducing the initial marginal cost to $c_i - x$. x is also known as the size of the innovation. The other type of innovation is new process innovation which helps firms to replace their previous production process with a brand new process with marginal cost c, where $0 < c < c_1 < c_2$. These inequalities indicate that the new innovation is useful for both the more efficient and less efficient firm.

Finally, the objective of the innovator is to maximize the payoff from its innovation by various available strategies. To be specific, the innovator has the option to design a discriminatory licensing game to both firms or, alternatively, it can sell the innovation to any single firm. In this chapter, I exclude the R&D decision of the innovator and assume instead that the technology has been discovered.

Discriminatory licensing game

As I mentioned before, if the outside innovator employes selling strategy, both firms finally get the new technology. Therefore, in order to be comparable with patent sale, I assume that the outside innovator offers two licenses to both firms in the discriminatory licensing game.

The game structure is as follows. In the first stage of the discriminatory licensing game (see Figure 3.2.1), the outside innovator offers two distinct two-part tariff contracts to both firms simultaneously or sequentially. In the second stage, the firms decide whether to accept or reject its own contract. If simultaneous licensing is chosen by the innovator, two firms decide whether to accept or reject it simultaneously[3]. If sequential licensing is chosen, the leader firm decides whether to accept or reject it, then the follower firm observes the leader firm's choice and makes its own decision. In the third stage, the firms engage in Cournot duopoly competition. As for the selling game (see Figure 2.2), the innovator decides to sell its innovation to one of the two firms. If the innovation is sold, the new patent holder has the option to license it further to the rival. Both games are solved by backward induction.

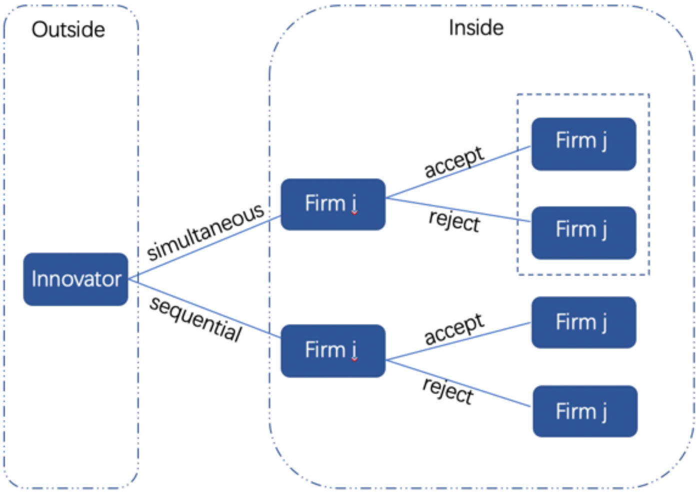
In the post-innovation stage, suppose firm's marginal cost vector is (c'_i, c'_j) and let q_i denote firm i's Cournot equilibrium output and π_i denote the corresponding operative profit, $i, j = 1, 2$ and $i \neq j$,

$$q_i = \frac{a - 2c'_i + c'_j}{3} \tag{2.1}$$

$$\pi_i = (q_i)^2 \tag{2.2}$$

[3]The loop represents that firm j does not know whether firm i chooses accepting or rejecting, that is, two firms play a simultaneous game.

Figure 2.1: The structure of discriminatory licensing game



2.3 Common step-eliminating innovation

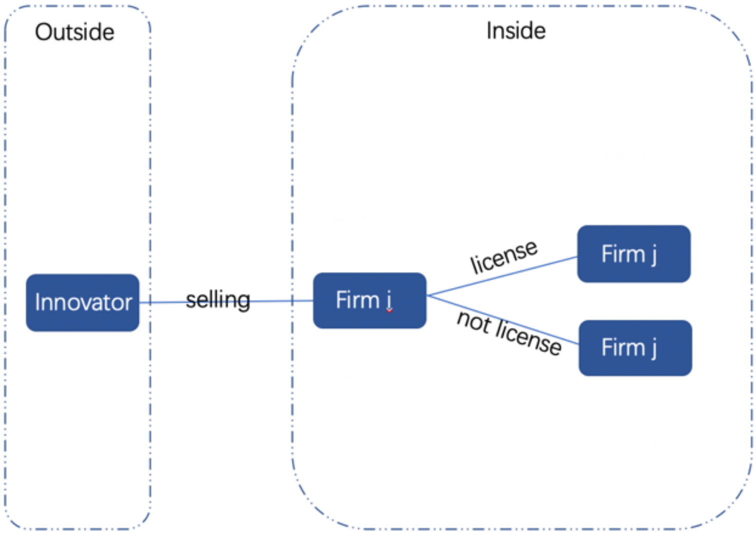
In case of common step-eliminating innovation, I assume that both firms produce a positive output in the equilibrium irrespective of the outcome of the licensing or selling game, which is guaranteed by the following assumptions.

Assumption 2.1 $a > 2c_2$

Assumption 2.2 $0 < x < c_1$

Firms produce at the marginal cost $c_i - x$ if they use the innovation. For a meaningful licensing game, both firms should have positive marginal costs of production after adopting the innovation. Therefore the size of innovation should be $c_1 - x > 0$ and then $c_2 - x > 0$ automatically holds. In addition, firms are assumed to be active, which means the innovation is 'non-drastic'. That is to say, neither of the firms with the new innovation could charge a monopoly price and drive the other firm with the older technology out of the market. Thus, in this framework, as long as the equilibrium output of firm 2 is still positive when firm 1 exclusively gets access to the new innovation, it ensures the innovation is

Figure 2.2: The structure of selling game



non-drastic. It gives the upper bound for the size of innovation $x < a - 2c_2 + c_1$, which is satisfied given assumptions 2.1 and 2.2.

2.3.1 Simultaneous discriminatory license

Consider the case where the outside innovator offers two unique two-part tariff contracts (F_1, r_1) for firm 1 and (F_2, r_2) for firm 2 simultaneously, where $F_i \geqslant 0$ denotes the fixed fee paid to the outside innovator, and $0 \leqslant r_i \leqslant x$ denotes the per-unit royalty rate, with $i = 1, 2$.

The normal form of the game can be expressed as,

Table 2.1: The normal form of simultaneous game with step-eliminating innovation

		Firm 2	
		Accept	*Reject*
Firm 1	*Accept*	$\pi_1^{aa} - F_1, \pi_2^{aa} - F_2$	$\pi_1^{ar} - F_1, \pi_2^{ra}$
	Reject	$\pi_1^{ra}, \pi_2^{ar} - F_2$	π_1^{rr}, π_2^{rr}

where the first and second terms in supercript denote the firm i's choice, and the rival firm j's choice, respectively, with a for accepting, r for rejecting. With different pairs of post innovation marginal costs (c'_1, c'_2), by plugging into Eq.(2.1) and (2.2), the relevant variables in Table 2.1 can be calculated as follows,

$$\pi_i^{aa} = (q_i^{aa})^2 = \frac{(a - 2c_i + c_j - 2r_i + r_j + x)^2}{9} \tag{2.3}$$

$$\pi_i^{ar} = (q_i^{ar})^2 = \frac{(a - 2c_i + c_j - 2r_i + 2x)^2}{9} \tag{2.4}$$

$$\pi_i^{ra} = (q_i^{ra})^2 = \frac{(a - 2c_i + c_j + r_j - x)^2}{9} \tag{2.5}$$

$$\pi_i^{rr} = (q_i^{rr})^2 = \frac{(a - 2c_i + c_j)^2}{9} \tag{2.6}$$

Assume that any firm would accept the license if it is weakly better off than not accepting it. In other words, as long as firm i's profit if it accepts the license is equal or greater than if it rejects given the strategy of its rival, firm i would accept the license.

In case of patent sale, the buyer has the option to license it further to the rival or not. If the buyer is not permitted to license further, the selling strategy is equivalent to the discriminatory licensing when the outside innovator only offers one license. Sinha (2016) finds that the buyer of the patent would always license the technology to its rival and none of them has any incentive to withhold the technology after buying the patent. That is to say, if (Accept,Reject) or (Reject,Accept) is the Nash equilibrium, it cannot be more profitable for the innovator. Therefore, I consider (Accept, Accept) as the unique Nash equilibrium of the game, and this is the case if and only if,

$$\pi_1^{aa} - F_1 \geqslant \pi_1^{ra}$$

$$\pi_1^{ar} - F_1 \geqslant \pi_1^{rr}$$

$$\pi_2^{aa} - F_2 \geqslant \pi_2^{ra}$$

$$\pi_2^{ar} - F_2 \geqslant \pi_2^{rr}$$

Solving the above system of inequalities gives,

$$F_1 \leqslant \pi_1^{aa} - \pi_1^{ra} \leqslant \pi_1^{ar} - \pi_1^{rr}$$

$$F_2 \leqslant \pi_2^{aa} - \pi_2^{ra} \leqslant \pi_2^{ar} - \pi_2^{rr}$$

Therefore in this situation, the outside innovator can charge the profit difference of firm i between it accepts and rejects the contract given its rival firm always accepts the contract, i.e., $F_i = \pi_i^{aa} - \pi_i^{ra}$, $i = 1, 2$.

From the outside innovator's perspective, the objective is to maximize the total revenue R_{ST} from simultaneous licensing to both firms by choosing royalty rates r_1 and r_2, which is detemined as follows,

$$\begin{aligned}
R_{ST}(r_1, r_2) =& F_1 + F_2 + r_1 \cdot q_1^{aa} + r_2 \cdot q_2^{aa} \\
=& \frac{(a - 2c_1 + c_2 - 2r_1 + r_2 + x)^2}{9} - \frac{(a - 2c_1 + c_2 + r_2 - x)^2}{9} \\
+& \frac{(a - 2c_2 + c_1 - 2r_2 + r_1 + x)^2}{9} - \frac{(a - 2c_2 + c_1 + r_1 - x)^2}{9} \\
+& r_1(\frac{a - 2c_1 + c_2 - 2r_1 + r_2 + x}{3}) + r_2(\frac{a - 2c_2 + c_1 - 2r_2 + r_1 + x}{3})
\end{aligned}$$

where F_1 and F_2 are from the previous result. After simplification,

$$\begin{aligned}
\max_{r_1, r_2} R_{ST}(r_1, r_2) =& -\frac{2r_1^2}{9} + \frac{r_1(-a + 2c_1 - c_2 - 2r_2 + 3x)}{9} \\
& -\frac{2r_2^2}{9} + \frac{r_2(-a + 2c_2 - c_1 + 3x)}{9} + \frac{4x(2a - c_1 - c_2)}{9}
\end{aligned}$$

$$s.t.\ 0 \leqslant r_1 \leqslant x$$

$$0 \leqslant r_2 \leqslant x$$

Using the Kuhn-Tucker conditions to solve the following Lagrangian function,

$$L(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = R_{ST}(r_1, r_2) + \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3(x - r_1) + \lambda_4(x - r_2)$$

The critical points of L are the solutions $(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ to the following system of equations,

$$
\begin{aligned}
&\frac{\partial L}{\partial r_1} = \frac{3x - a + 2c_1 - c_2 - 2r_2 - 4r_1}{9} + \lambda_1 - \lambda_3 = 0 \\
&\frac{\partial L}{\partial r_2} = \frac{3x - a + 2c_2 - c_1 - 2r_1 - 4r_2}{9} + \lambda_2 - \lambda_4 = 0 \\
&\lambda_1 \geqslant 0,\ r_1 \geqslant 0,\ \lambda_1 r_1 = 0 \\
&\lambda_2 \geqslant 0,\ r_2 \geqslant 0,\ \lambda_2 r_2 = 0 \\
&\lambda_3 \geqslant 0,\ x - r_1 \geqslant 0,\ \lambda_3(x - r_1) = 0 \\
&\lambda_4 \geqslant 0,\ x - r_2 \geqslant 0,\ \lambda_4(x - r_2) = 0
\end{aligned}
\tag{2.7}
$$

This system of equations defines different pairs of optimal royalty rates r_i^*. To sum up, the following lemma characterizes the optimal simultaneous discriminatory licensing contract.

Lemma 2.1 *When the outside innovator offers two distinct contracts to the firms simultaneously, the optimal licensing policy depends on the size of innovation,*
1) if $0 < 3x \leqslant a - 2c_2 + c_1$*, only fixed fees are charged.*
2) if $a - 2c_2 + c_1 < 3x \leqslant a - 5c_1 + 4c_2$*, the innovator offers a fixed fee contract to firm 1 and a two-part tariff contract to firm 2.*
3) if $3x > a - 5c_1 + 4c_2$*, the innovator offers different two-part tariff contracts to firms.*

From Lemmas 2.1, it is found that if x goes up, that is, as the innovation gets better at cost cutting, the optimal contract for each firm changes from a fixed-fee to a two-part tariff. The economic intuition can be explained as follows. There are two main effects of patent licencing. The first effect is the overall efficiency gain in the industry and the second effect is the increase in competition between the two firms (Poddar and Sinha, 2010). For a relatively small cost reduction, the post-innovation cost gap relative to the market size approximately remains the same as in the pre-innovation stage, that is, the competitive effect can be ignored. Overall efficiency gain can be appropriated by the optimal fixed fee.

As the size of cost reduction increases, the market share of less efficient firm increases, which tends to reduce the industry profit. The outside innovator now would like to charge a royalty for the less efficient firm to enlarge the effective cost difference (including the royalty). But it is not always profit maximising as the effective cost difference increases. There is a trade-off between total industry profit and royalty revenue. Therefore for a sufficiently large cost reduction, the optimal strategy for the outside innovator is also charging a royalty for the more efficient firm, which has a competition-reducing effect in the market.

2.3.2 Sequential discriminatory license

Now consider the case when the outside innovator is offering two unique two-part tariff contracts (F_i, r_i) to firm i (referred as the leader) and then (F_j, r_j) to firm j (referred as the follower) in a sequence, $i, j = 1, 2$ and $i \neq j$. Recall that $0 \leqslant r_i \leqslant x$. Compared with the previous simultaneous discriminatory license, the difference is that both contracts cannot be designed simultaneously at the beginning of the game. The optimal contract for the follower is conditional on the decision of the leader, which means that it may differ when the leader accepts or rejects its own contract. Basically, I assume a lack of commitment by the innovator. Hence, the game should be solved by backward induction. The optimal contract for the follower should be designed subject to the leader's decision at first and then find the optimal contract for the leader.

Consider the case where the innovator first asks firm 1 and then firm 2. Since firm 2 can observe firm 1's decision, the incentive to get the innovation is likely to be distinct, therefore the outside innovator is able to design different contracts for firm 2 depending on the choice of firm 1.

Firstly, when firm 1 accepts the contract, the optimal strategy for firm 2 can be designed as follows. If firm 2 accept the contract as well, firm 2's profit is π_2^{aa} in Eq.(2.3). Otherwise, its profit turns to π_2^{ra} in Eq.(2.5). The maximum fixed fee charged for firm 2 is the profit difference between it accepts or rejects. Therefore, the maximum fee that the innovator could charge is $F_2 = \pi_2^{aa} - \pi_2^{ra}$ from firm 2. Then the objective of outside innovator is to maximize the revenue from firm 2 (related to r_2) instead of the joint revenue, that is, $F_2+r_2 \cdot q_2^{aa}+r_1 \cdot q_1^{aa}$ by choosing r_2. It is easy to check that within the range $0 \leqslant r_2 \leqslant x$, $\frac{\partial(F_2+r_2 \cdot q_2^{aa}+r_1 \cdot q_1^{aa})}{\partial r_2} < 0$, and $\frac{\partial^2(F_2+r_2 \cdot q_2^{aa}+r_1 \cdot q_1^{aa})}{\partial r_2^2} < 0$. Therefore in this case the optimum r_2 will be set at 0 which is the lower bound of r_2. Thus, when firm 1 accepts the contract, the optimal strategy is to offer a fixed fee contract with $F_2 = \pi_2^{aa} - \pi_2^{ra} = \frac{4x(a-2c_2+c_1+r_1)}{9}$ to firm 2, which takes r_1 as given.

Then, when firm 1 rejects the contract, the innovator can design another strategy for firm 2. If firm 2 obtains the contract exclusively, firm 2's profit is π_2^{ar} in Eq.(2.4). Otherwise, no one gets the cost reduction, which remains in the pre-innovation stage leading to firm 2's profit at π_2^{rr} in Eq.(2.6). Therefore, the maximum fee that the innovator could charge is $F_2 = \pi_2^{ar} - \pi_2^{rr}$ from firm 2. The innovator now only maximizes the revenue from firm 2, $F_2 + r_2' \cdot q_2^{ar}$, by choosing r_2'. Similarly, note that the total payoff of the innovator is a concave function and decreasing of r_2' given the restriction $0 \leqslant r_2' \leqslant x$. Then the maximum revenue is attained when r_2' is zero. Thus, the optimal strategy is still to offer a fixed fee contract to firm 2 if firm 1 rejects the contract.

From the above analysis, the optimal mode for firm 2 is independent of firm 1's decision. Whether firm 1 accepts or rejects its own contract, the outside innovator always offers a fixed fee contract to firm 2 with different fees though. Hence, the following lemma holds.

Lemma 2.2 *If the outside innovator offers one license to a firm, a fixed fee contract would always give the outside innovator maximum payoff.*[4]

The intuition behind Lemma 2.2 is that the potential licensee (i.e., the follower) will receive the highest profit when it can take the full advantage of the

[4]It also holds if firm 1 is the follower.

cost reduction no matter what the leader has chosen. This larger payoff can be extracted by the outsider innovator with a fixed fee. Any combination of fixed fee and royalty payment would reduce the amount of surplus subject to the participation constraint of the follower.

The maximum revenue from firm 2 would always be a contract by means of fixed fee whether firm 1 accepted or rejected previously. Hence, solving backwards gives firm 1's optimal contract. Given firm 2's contract (i.e. $r_2 = 0$), firm 1's payoff remains identical whether it accepts or rejects its own contract at the end of the game. The maximum fixed fee charged for firm 1 is the profit difference between it accepts or rejects. If both firms accept the contract, firm 1's payoff is π_1^{aa} in Eq.(2.3). While if firm 1 rejects then given firm 2 accepts, firm 1's no-acceptance payoff is π_1^{ra} in Eq.(2.5). Therefore, the maximum willingness of firm 1 to pay for the license will be $F_1 = \pi_1^{aa} - \pi_1^{ra}$. Recall firm 2's willingness is $F_2 = \frac{4x(a-2c_2+c_1+r_1)}{9}$. Now, the outside innovator will maximize the total payoff R_{ij} by choosing r_i. The subscript ij, $i = 1,2$ and $i \neq j$ denotes the sequence.

$$
\begin{aligned}
R_{12}(r_1) =& F_1 + r_1 \cdot q_1^{aa} + F_2 \\
=& \frac{(a-2c_1+c_2-2r_1+x)^2}{9} - \frac{(a-2c_1+c_2-x)^2}{9} \\
&+ r_1\left(\frac{a-2c_1+c_2-2r_1+x}{3}\right) + \frac{4x(a-2c_2+c_1+r_1)}{9}
\end{aligned}
$$

After simplification,

$$
R_{12}(r_1) = -\frac{2r_1^2}{9} + \frac{r_1(-a+3x+2c_1-c_2)}{9} + \frac{4x(2a-c_1-c_2)}{9}
$$

One can easily check that $R_{12}(r_1)$ is a concave function of r_1, and the unconstrained maximization with respect to r_1 of the above function yields the interior soluton,

$$
\hat{r_1} = \frac{3x-(a-2c_1+c_2)}{4}
$$

Note that $0 \leqslant r_1 \leqslant x$. The following lemma charaterises the optimal licensing contract if the innovator first asks firm 1 and then firm 2.

Lemma 2.3 *Consider the sequence when the outside innovator first offers a contract to firm 1 then firm 2. The optimal licensing policy depends on the size of innovation,*

1) when $0 < 3x \leqslant a - 2c_1 + c_2$*, only fixed fees are charged and the innovator's payoff is* $R_{12} = \frac{4x(2a-c_1-c_2)}{9}$*;*

2) when $3x > a - 2c_1 + c_2$*, the innovator offers a two-part tariff contract to firm 1 and a fixed fee contract to firm 2, and its payoff is* $R_{12} = \frac{4c_1^2+c_2^2+(-4a-20x-4c_2)c_1+(2a-38x)c_2+a^2+58ax+9x^2}{72}$.

Next, change the sequence. Considering the innovator first asks firm 2 whether it accepts the contract (F_2, r_2), and then firm 1 (F_1, r_1).

Following the same logic as before, the innovator would always offer a fixed fee contract to the follower, that is firm 1, with $F_1 = \frac{4x(a-2c_1+c_2+r_2)}{9}$ in this case. The total payoff of the innovator turns out to be,

$$R_{21}(r_2) = -\frac{2r_2^2}{9} + \frac{r_2(-a+3x+2c_2-c_1)}{9} + \frac{4x(2a-c_1-c_2)}{9}$$

One can easily check that R_{21} is a concave function of r_2, and the unconstrained maximization with respect to r_2 of the above function yields the interior soluton,

$$\hat{r_2} = \frac{3x-(a-2c_2+c_1)}{4}$$

With the restriction $0 \leqslant r_2 \leqslant x$, the following lemma charaterises the optimal licensing contract if the innovator first asks firm 2 and then firm 1.

Lemma 2.4 *Consider the sequence when the outside innovator first offers a contract to firm 2 then firm 1. The optimal licensing policy depends on the size of innovation,*

1) when $0 < 3x \leqslant a - 2c_2 + c_1$*, only fixed fees are charged and the innovator's payoff is* $R_{21} = \frac{4x(2a-c_1-c_2)}{9}$*;*

2) when $3x > a - 2c_2 + c_1$*, the innovator offers a fixed fee contract to firm 1 and a* *two-part tariff contract to firm 2, and its payoff is* $R_{21} = \frac{4c_2^2+c_1^2+(-4a-20x)c_2+(2a-38x-4c_2)c_1+a^2+58ax+9x^2}{72}$.

The intuition behind Lemmas 2.3 and 2.4 can be explained as follows. The outside innovator maximises the total payoff by choosing the leader's royalty rate. Rewrite the maximisation problem using a general form,

$$R_{ij}(r_i) = \pi_i^{aa}(r_i) - \pi_i^{ra} + r_i q_i^{aa}(r_i) + F_j(r_i)$$

The optimal r_i^* is implicitly determined by,

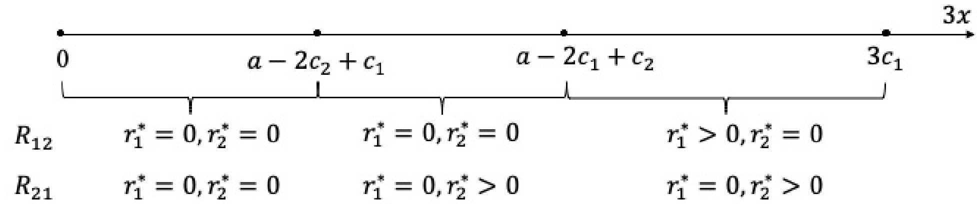
$$\frac{dR_{ij}(r_i)}{dr_i} = \frac{\partial \pi_i^{aa}(r_i)}{\partial r_i} + q_i^{aa}(r_i) + r_i \frac{\partial q_i^{aa}}{\partial r_i} + \frac{\partial F_j(r_i)}{\partial r_i} = 0$$

The first term captures the marginal losses in the leader's profit, which is negative. The second and third terms capture the marginal gains/losses from the royalty payment. And the last term captures the marginal gains from the follower's fixed fee, which is positive. Therefore the equilibrium royalty rate is determined by the overall balance of these gains and losses. Lemmas 2.3 and 2.4 show that the optimal contract moves from a fixed-fee contract to a two-part tariff contract for the leader firm. From Lemma 2.2, the follower is always offered a fixed fee contract, that is, the follower's effective cost is reduced by the size of innovation. If the cost reduction is relatively small, i.e., the initial cost asymmetry is relatively large, the outside innovator wants to keep the cost asymmetry and also provides a fixed fee contract to the leader so that it can extract the increment in the industry profit. On the contrary, if the cost reduction is relatively large, i.e., the initial cost asymmetry is relatively small, the outside innovator charges a royalty to the leader to make the cost asymmetry larger.

Now we are in position to compare the total payoff of the innovator between different sequences. The optimal contracts with different sequences are shown in Figure 2.3. By comparing the total payoffs of the innovator, $R_{21} \geqslant R_{12}$ always holds irrespective of the size of innovation given the assumptions. Therefore,

Proposition 2.1 *For a step-elimination innovation, if the outside innovator offers two unique contracts to the inside firms in a sequence, it does better by first licensing to the less efficient firm followed by the more efficient firm.*

Figure 2.3: Optimal contract with different sequences



Proof: I divide into three parts according to the size of innovation $3x$, which is shown in Figure 2.3.

Part 1. $0 < 3x \leqslant a - 2c_2 + c_1$

The optimal contracts for firms are identical irrespective of the sequence, and therefore the total revenue of the innovator is identical.

Part 2. $a - 2c_2 + c_1 < 3x \leqslant a - 2c_1 + c_2$

$R_{12} - R_{21} = -\frac{(a-2c_2+c_1-3x)^2}{72} < 0$

Part 3. $3x > a - 2c_1 + c_2$

$R_{12} - R_{21} = \frac{(c_2-c_1)(2a-c_1-c_2-6x)}{24}$

It is obvious that $(2a - c_1 - c_2 - 6x)$ is decreasing in x, therefore the maximum value is attained when x is at the lower bound,

$$(2a - c_1 - c_2 - 6x)\bigg|_{x=\frac{a-2c_1+c_2}{3}} = 3(c_1 - c_2) < 0$$

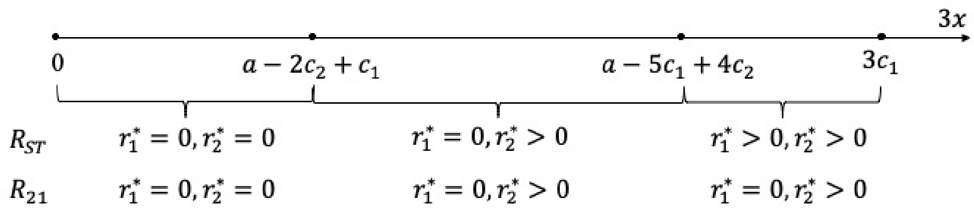
Since $c_2 - c_1 > 0$ given the assumption, $R_{12} - R_{21}$ for the range of values in Part 3 is always negative. ■

The intuition behind proposition 2.1 is as follows. Since firm's profit exhibits convexity in its own marginal cost, the more efficient firm generates a larger profit increment than the less efficient firm given the same cost reduction. And the increment can be extracted by the outside innovator with a fixed fee. Therefore, a fixed fee contract for the more efficient firm ($r_1 = 0$) provides a higher payoff for the outside innovator. Besides, $r_2 > 0$ makes firm 1 more competitive. The sequence in which the innovator first asks firm 2 and then firm 1 achieves this outcome. The selling strategy in Sinha (2016) states that the outside innovator is better off by selling the innovation to the more efficient firm and then license to the less efficient firm. Although the sequence is not identical, the principle is consistent that the more efficient firm (firm 1) obtains the contract by means of fixed fee.

Now, the optimal contracts under simultenous and sequential discriminatory licensing are designed respectively (See Figure 2.4). Accordingly, I compare the payoffs of the outside innovator between those two ways to establish a more complete fully flexible licensing scheme.

Proposition 2.2 *If the outside innovator offers two licenses to the inside firms, simultaneous licensing is more profitable than the sequential licencing.*

Figure 2.4: Optimal contract under simultaneous and sequential license



Proof: If $0 < 3x \leqslant a - 5c_1 + 4c_2$, the optimal contracts are identical with simultaneous licensing and sequential licensing (sequence 21). If $3x > a - 5c_1 + 4c_2$, the revenue with sequential licensing is maximised with restricted by $r_1 = 0$ while that with simultaneous licensing is maximised without the restriction. Therefore, sequential license is nothing but simultaneous licensing given $r_1 = 0$. Thus simultaneous license is superior. ■

It is interesting to see that simultaneous licensing is more profitable than the sequential licensing. Since the outside innovator always offers a fixed fee contract to the follower irrespective of the leader's choice, the leader's profit remains the same under both types of licensing when it rejects the contract while its rival accepts. That is to say, when the leader rejects, the threat point does not increase although the innovator designs another contract for its rival. Therefore, the sequential licensing can be regarded as a special case of simultaneous licensing, because the innovator cannot credibly commit to $r_1 > 0$.

The fully flexible discriminatory licensing scheme is established now. From the inside firm's perspective, the other aim of this chapter is to compare the incentives of asymmetric firms to obtain the license. Suppose (F_i, r_i) is the contract of firm i that the outside innovator offers, the incentive here is the willingness to pay for the license in its totality and should, therefore, include both the fixed and variable part, i.e., $F_i + r_i \cdot q_i$, where F_i denotes the fixed fee and $r_i \cdot q_i$ denotes the royalty payment. By comparing the incentives of firm 1 and 2, I have the following proposition.

Proposition 2.3 *For the step-eliminating innovation, the more efficient firm always has incentive to pay more for the license than the less efficient firm.*

Proof: See appendix. ■

The intuition for proposition 2.3 is as follows. Due to the asymmetric setting, firm 1 which has a lower marginal cost, produces more and owns a larger market share before the innovation. Since the outside innovator announces to offer two licenses to the firms, given its rival accepts the offer, firm 1 is threatened by the situation where firm 2 may catch up with it, leading to a relatively lower market share than before. Whereas it is not the case for the less efficient firm. Firm 2 initially occupies a lower market share and given its rival obtains the cost reduction, it faces either staying the same size of cost gap or even increasing the comparative cost disadvantage. The results illustrate that the fear of losing its leader position provides the more efficient firm with a higher incentive to pay for the license. That is, the more efficient firm has more to lose than the less efficient firm. It is analogous of another proposition, called the efficiency effect which states that the incumbent has a stronger incentive to innovate than the entrant. Besides, it also implies that the innovator nevertheless wants to license to both firms, since only the large firm threatened by losing its higher market share has incentive to pay more for the license.

2.3.3 Selling game

I now compare the above fully flexibility licensing game with the selling game where the innovator wants to sell the innovation to one of the firms by charging a fixed fee and the buyer firm can then license the new technology to its rival with any licensing strategy. Note that the ownership of the innovation is transferred from the innovator to another entity under selling policy, therefore it can be only sold to one potential buyer and then the innovator loses all control of the innovation in contrast to it can be licensed to both firms under licensing. If one firm rejects the selling contract, then it goes to the other firm. This issue in this structure has been analyzed in detail by Sinha (2016) and I invoke his result in the analysis here.

Results (Sinha (2016)): In case the outside innovator decides to sell the property rights in the first stage of the game, then the optimal choice is to set a fixed price for the patent and the more efficient firm buys it. The innovator receives the payoff $R_s = x(a - c_1)$. The complete selling game is in the first stage, the more efficient firm buys the patent at a fixed fee (i.e., $r_1 = 0$), and then in the second stage, the buyer further licenses it to the less efficient firm at a royalty rate which is the same as the size of cost reduction (i.e., $r_2 = x$). Both firms get the cost reduction at the end.

Since the purpose is to identify whether the selling revenue can be replicated by the fully flexible licensing scheme, I now compare the payoffs of the outside innovator between the complete optimal discriminatory licensing scheme and patent sale to get the result. The following proposition states the optimal mechanism for maximizing the value of the new innovation for the outside innovator in an asymmetric Cournot duopoly market for a common step eliminating innovation.

Proposition 2.4 *The patent sale is strictly better than patent discriminatory licensing to the firms for a common step eliminating innovation in asymmetric Cournot duopoly market.*

Proof: See appendix. ■

From the above analysis, it shows that the fully flexible discriminatory licensing is worse than patent selling game from the outside patentee's view. With the selling policy, the post-innovation marginal costs turns to $(c_1 - x, c_2)$. Although both firms get the cost reduction, since the per-unit royalty for firm 2 is equal to the size of cost reduction, the post marginal cost of firm 2 remains the same as beginning c_2, while firm 1 gets the cost reduction without royalty payment, which leads to the possible largest cost asymmetries $c_2 - c_1 + x$. In terms of the nature of asymmetric Cournot duopoly, the total industry profit increases as the asymmetry between two firms increases, therefore the outside innovator is able to extract the largest increment of total profits. From another point of view, the superiority of selling policy comes from the ability to promise firm 1 the best possible outcome ($r_2 = x$) while at the same time threaten with the worst ($r_2 = 0$), which brings the highest possible F_1. Hence, the selling policy takes full advantage of the asymmetry of the producers.

However, with licensing policy, the post-innovation marginal costs turns to $(c_1 - x + r_1, c_2 - x + r_2)$. Since the outside innovator simultaneously offers two contracts to the inside firms, it cannot achieve the same as selling policy due to commitment problems. From the optimal discriminatory licensing strategy, the post cost asymmetry is smaller than that in selling policy, therefore the total industry profit is not as high as that in selling policy. Hence, the outside innovator's revenue with licensing is less than the selling revenue.

The result further shows that the superiority of patent sale for the step-eliminating innovation is even stronger than previously shown in Sinha (2016)'s paper. While as mentioned before, patent sale transfers the ownership of the patent right from the outside innovator to the buyer. If the outside innovator tends to hold the property right such that it can further conduct R&D to update the innovation, this paper proposes a more flexible licensing scheme. And I compare the total payoff between discriminatory licensing in this paper and uniform licensing game in Sinha (2016)'s work and find that it is indeed better than uniform one due to less restricted.

2.4 New process innovation

I proceed to find the optimal discriminatory licensing scheme of the new process innovation for the outsider patentee in the same manner as I have done for the common step-eliminating innovation in the previous section. In case of new process innovation, I assume that the new technology with marginal cost c is more advanced than the original technology of both firms such that $0 < c < c_1 < c_2$, so the difference in pre-innovation costs is wiped out in the post-innovation stage and any firm that uses the new technology would have the same marginal cost c. Compared with step-elimination innovation which reduces the marginal costs by the same size, the new process innovation leads to different magnitudes of cost reduction for the firms, that is to say, the less efficient firm reaps a higher cost reduction if the new technology is applied. For the more efficient firm, the cost reduction would be $x_1 = c_1 - c$ and for the less efficient firm, the cost reduction would be $x_2 = c_2 - c$, which naturally indicates that $x_1 < x_2$. Therefore the results may differ from those with step-eliminating innovation.

Assumption 2.1 ($a > 2c_2$) guarantees that both firms are still active in the post-innovation stage even when firm 1 exclusively gets the access to the new technology and turns to be more efficient, which illustrates that the new process innovation considered in this chapter is non-drastic.

2.4.1 Simultaneous discriminatory licensing

Consider the case where the outside innovator offers two unique two-part tariff contracts (F_1, r_1) to firm 1 and (F_2, r_2) to firm 2 simultaneously, where $F_i \geqslant 0$ denotes the fixed fee paid to the outside innovator, and $0 \leqslant r_i \leqslant c_i - c$ denotes the per-unit royalty rate, with $i = 1, 2$. Assume that any firm would accept the license if it is weakly better off than not accepting it.

Table 2.2: The normal form of simultaneous game with new process innovation

		Firm 2	
		Accept	*Reject*
Firm 1	*Accept*	$\pi_1^{aa} - F_1, \pi_2^{aa} - F_2$	$\pi_1^{ar} - F_1, \pi_2^{ra}$
	Reject	$\pi_1^{ra}, \pi_2^{ar} - F_2$	π_1^{rr}, π_2^{rr}

where

$$\pi_i^{aa} = \frac{(a - c - 2r_i + r_j)^2}{9} \tag{2.8}$$

$$\pi_i^{ar} = \frac{(a - 2c + c_j - 2r_i)^2}{9} \tag{2.9}$$

$$\pi_i^{ra} = \frac{(a - 2c_i + c + r_j)^2}{9} \tag{2.10}$$

$$\pi_i^{rr} = \frac{(a - 2c_i + c_j)^2}{9} \tag{2.11}$$

Following the same logic as before, the outside innovator can at most charge the profit difference of firm i between it accepts and rejects the contract given its rival firm always accepts its own contract, i.e., $F_i = \pi_i^{aa} - \pi_i^{ra}$.

From the outside innovator's perspective, the objective is to maximize the total revenue R_{ST} from licensing to both firms by choosing royalty rates r_1 and r_2, which is detemined as follows,

$$
\begin{aligned}
R_{ST} =& F_1 + F_2 + r_1 q_1^{aa} + r_2 q_2^{aa} \\
=& \frac{(a-c-2r_1+r_2)^2}{9} - \frac{(a-2c_1+c+r_2)^2}{9} + \\
& \frac{(a-c-2r_2+r_1)^2}{9} - \frac{(a-2c_2+c+r_1)^2}{9} + \\
& r_1(\frac{a-c-2r_1+r_2}{3}) + r_2(\frac{a-c-2r_2+r_1}{3})
\end{aligned} \tag{2.12}
$$

After simplification,

$$
\begin{aligned}
\max_{r_1,r_2} R_{ST}(r_1,r_2) =& -\frac{2r_1^2}{9} + \frac{r_1(-a-3c+4c_2-2r_2)}{9} \\
& -\frac{2r_2^2}{9} + \frac{r_2(-a-3c+4c_1)}{9} \\
& + \frac{a(4c_1+4c_2-8c)}{9} + \frac{4c(c_1+c_2)}{9} - \frac{4(c_1^2+c_2^2)}{9}
\end{aligned}
$$

$$s.t.\ 0 \leqslant r_1 \leqslant c_1 - c$$

$$0 \leqslant r_2 \leqslant c_2 - c$$

Using the Kuhn-Tucker conditions to solve the following Lagrangian function,

$$L(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = R_{ST}(r_1, r_2) + \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3(c_1 - c - r_1) + \lambda_4(c_2 - c - r_2)$$

The critical points of L are the solutions $(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ to the following system of equations,

$$
\begin{aligned}
&\frac{\partial L}{\partial r_1} = \frac{4c_2 - a - 3c - 2r_2 - 4r_1}{9} + \lambda_1 - \lambda_3 = 0 \\
&\frac{\partial L}{\partial r_2} = \frac{4c_1 - a - 3c - 2r_1 - 4r_2}{9} + \lambda_2 - \lambda_4 = 0 \\
&\lambda_1 \geqslant 0,\ r_1 \geqslant 0,\ \lambda_1 r_1 = 0 \\
&\lambda_2 \geqslant 0,\ r_2 \geqslant 0,\ \lambda_2 r_2 = 0 \\
&\lambda_3 \geqslant 0,\ c_1 - c - r_1 \geqslant 0,\ \lambda_3(c_1 - c - r_1) = 0 \\
&\lambda_4 \geqslant 0,\ c_2 - c - r_2 \geqslant 0,\ \lambda_4(c_2 - c - r_2) = 0
\end{aligned} \tag{2.13}
$$

This system of equations defines different pairs of optimal royalty rates r_i^*. Recall the size of new marginal cost satisfies $0 < c < c_1 < c_2$. To sum up, the following lemma characterizes the optimal simultaneous discriminatory licensing contract with a new process innovation.

Lemma 2.5 *When the outside innovator offers two distinct contracts to the firms simultaneously, the optimal licensing policy depends on the initial cost asymmetry and the magnitude of new marginal cost,*

For relatively small inital cost asymmetry, i.e., $c_2 < \frac{a+3c_1}{4}$*,*

1) when $-a + 4c_2 < 3c < 3c_1$*, only fixed fees are charged;*

2) when $-a + 8c_1 - 4c_2 < 3c < -a + 4c_2$*, the innovator offers a two-part tariff contract to firm 1 and a fixed fee contract to firm 2;*

3) when $0 < 3c < -a + 8c_1 - 4c_2$*, the innovator offers different two-part tariff contracts to firms.*

For relatively large initial cost asymmetry, i.e., $c_2 > \frac{a+3c_1}{4}$*,*

4) when $3(a + 4c_1 - 4c_2) < 3c < 3c_1$*, the innovator offers a per-unit royalty contract to firm 1 and a fixed fee contract to firm 2;*

5) when $0 < 3c < 3(a + 4c_1 - 4c_2)$*, the innovator offers a two-part tariff contract to firm 1 and a fixed fee contract to firm 2.*

For relatively small initial cost asymmetry, the result shows a consistent move similar to Lemma 2.1 that the optimal contract changes from a fixed fee contract to a two-part tariff contract for each firm as c decreases, i.e., cost reduction increases. Whereas for relatively large initial cost asymmetry, it is not the case.

2.4.2 Sequential discriminatory licensing

Now, the outside innovator offers two distinct contracts (F_1, r_1) to firm 1 and (F_2, r_2) to firm 2 in a sequence. Recall that $0 \leqslant r_1 \leqslant c_1 - c$ and $0 \leqslant r_2 \leqslant c_2 - c$.

Following the same logic as I have done with common step-eliminating innovation, firstly, consider the case where the innovator first asks firm 1 and then firm 2. Since firm 2 can observe firm 1's decision, the incentive to get the innovation is likely to be distinct, therefore the outside innovator is able to design a different contract for firm 2 depending on the choice of firm 1.

From Lemma 2.2, it is found that the follower's optimal mode is independent of the leader's decision, which indicates the outside innovator would always offer a fixed fee contract to firm 2, i.e., $r_2 = 0$. If firm 1 accepts, the maximum willingness of firm 2 to pay for the license is $F_2 = \pi_2^{aa} - \pi_2^{ra} = \frac{4(c_2-c)(a-c_2+r_1)}{9}$, which treats r_1 as given.

Although the outside innovator is fully flexible to design a new contract for firm 2 if firm 1 rejects, firm 1's payoff remains identical at the end of the game, that is to say, the threat point of firm 1 does not change. Thus the maximum fixed fee charged for firm 1 is the same as that in the simultaneous licensing with $r_2 = 0$. If both firm accept the contract, firm 1's payoff is π_1^{aa} in Eq.(2.8). While if firm 1 rejects then given firm 2 accepts, firm 1's no-acceptance payoff is π_1^{ra} in Eq.(2.10). Therefore, the maximum fixed fee changed for firm 1 is $F_1 = \pi_1^{aa} - \pi_1^{ra} = \frac{(a-c-2r_1)^2}{9} - \frac{(a-2c_1+c)^2}{9}$. Now, the outside innovator will maximize the total payoff R_{12} by choosing r_1,

$$
\begin{aligned}
R_{12}(r_1) =& F_1 + r_1 \cdot q_1^{aa} + F_2 \\
=& \frac{(a-c-2r_1)^2}{9} - \frac{(a-2c_1+c)^2}{9} + \\
& r_1 \cdot \left(\frac{a-c-2r_1}{3}\right) + \frac{4(c_2-c)(a-c_2+r_1)}{9}
\end{aligned}
\tag{2.14}
$$

One can easily check that $R_{12}(r_1)$ is a concave function of r_1, and the unconstrained maximization with respect to r_1 of the above function yields the interior soluton,

$$\hat{r}_1 = \frac{4c_2 - a - 3c}{4}$$

Now depending on the parameter configurations $0 \leqslant r_1 \leqslant x$, which results in different optimal royalty r_1^*. To summarize the above cases, the following lemma charaterises the optimal licensing contract if the innovator first asks firm 1 and then firm 2.

Lemma 2.6 *Consider the sequence that the outside innovator first offers a contract to firm 1 then firm 2. The optimal licensing policy depends on the initial cost asymmetry and the magnitude of new marginal cost,*

For relatively small inital cost asymmetry, i.e., $c_2 < \frac{a+3c_1}{4}$*,*

1) when $-a + 4c_2 < 3c < 3c_1$*, only fixed fees are charged;*

2) when $0 < 3c < -a+4c_2$*, the innovator offers a two-part tariff contract to firm 1 and a fixed fee contract to firm 2;*

For relatively large inital cost asymmetry, i.e., $c_2 > \frac{a+3c_1}{4}$*,*

3) when $3(a + 4c_1 - 4c_2) < 3c < 3c_1$*, the innovator offers a per-unit royalty contract to firm 1 and a fixed fee contract to firm 2;*

4) when $0 < 3c < 3(a+4c_1-4c_2)$*, the innovator offers a two-part tariff contract to firm 1 and a fixed fee contract to firm 2.*

Then, change the sequence where the outside innovator first asks firm 2 and then firm 1. Following the same steps as before, the optimal contract for firm 1 is a fixed fee contract with $F_1 = \frac{4(c_1-c)(a-c_1+r_2)}{9}$and then the outside innovator maximises the total revenue by choosing r_2,

$$\begin{aligned} R_{21}(r_1) =& F_2 + r_2 \cdot q_2^{aa} + F_1 \\ =& \frac{(a-c-2r_2)^2}{9} - \frac{(a-2c_2+c)^2}{9} + \\ & r_2 \cdot (\frac{a-c-2r_2}{3}) + \frac{4(c_1-c)(a-c_1+r_2)}{9} \end{aligned} \tag{2.15}$$

The unconstrained maximization with respect to r_2 yields the interior soluton $\hat{r}_2 = \frac{4c_1-a-3c}{4}$. Depending on the magnitude of $\hat{r}_2$, the following lemma charaterises the optimal licensing contract if the innovator first asks firm 2 and then firm 1.

Lemma 2.7 *Consider the sequence that the outside innovator first offers a contract to firm 2 then firm 1. The optimal licensing policy depends on the magnitude of new marginal cost,*

1) when $3c > -a + 4c_1$*, only fixed fees are charged;*

2) when $3c < -a + 4c_1$, the innovator offers a fixed fee contract to firm 1 and a two-part tariff contract to firm 2.

Now, I determine the sequence which provides a higher total payoff. The optimal contracts with different sequences are shown in Figure 2.5 and 2.6 based on different initial cost asymmetries. Figure 2.5 illustrates the situation where the initial cost gap is relatively small, $c_2 < \frac{a+3c_1}{4}$ while Figure 2.6 shows the situation where the initial cost gap is relatively large, $c_2 > \frac{a+3c_1}{4}$. By comparing the total payoffs of the innovator, $R_{12} \geqslant R_{21}$ always holds given the assumptions. Therefore,

Figure 2.5: Sequential license for a new process innovation$(a - 4c_2 + 3c_1 > 0)$

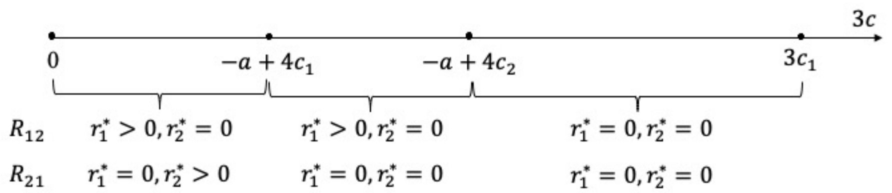
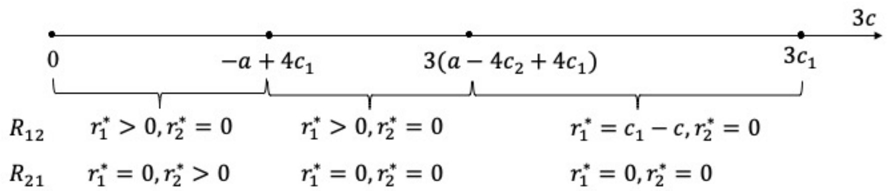


Figure 2.6: Sequential license for a new process innovation$(a - 4c_2 + 3c_1 < 0)$



Proposition 2.5 *For a new process innovation, if the outside innovator offers two unique contracts to the inside firms in a sequence, it does better by first licensing to the more efficient firm following by the less efficient firm.*

Proof: See appendix. ■

Compared with the step-eliminating innovation (stated in Proposition 2.1), the optimal sequence is opposite. The less efficient firm experiences a higher cost reduction than the more efficient firm if they both use the new technology, that is to say, the less efficient firm benefits more and therefore a fixed fee contract for the less efficient firm provides a larger payoff for the outside innovator, which is achieved by the sequence of first firm 1 followed by firm 2.

Now, the optimal contracts under simultenous and sequential discriminatory license are designed respectively (See Figure 2.7 and 2.8). Accordingly, I compare the payoffs of the outside innovator between those two ways to establish the complete fully flexible licensing scheme.

Figure 2.7: Optimal contract under simultaneous and sequential license($a - 4c_2 + 3c_1 > 0$)

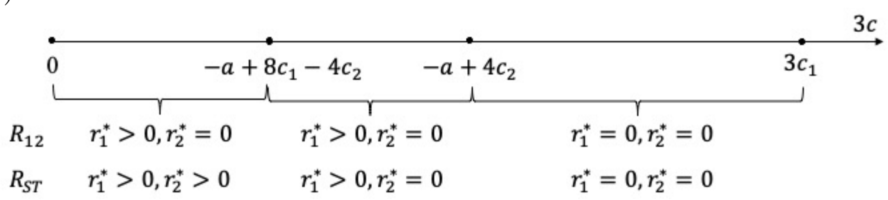
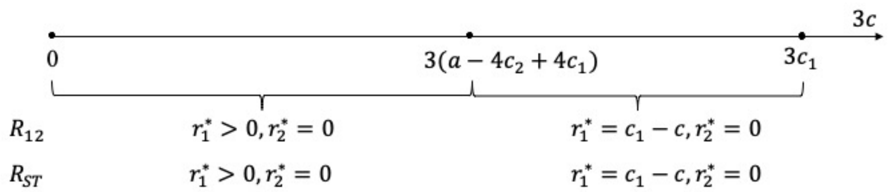


Figure 2.8: Optimal contract under simultaneous and sequential license($a - 4c_2 + 3c_1 < 0$)



Proposition 2.6 *If the outside innovator offers two licenses to the inside firms, simultaneous license is better than the sequential licence.*

Proof: Sequential licensing is nothing but simultaneous licensing given $r_2 = 0$. Thus simultaneous licensing is better. ■

The fully flexible discriminatory licensing scheme is established now. Similarly, from the inside firm's perspective, whether the more efficient firm or the less efficient firm has a higher incentive to pay for the new technology is identified as follows.

Proposition 2.7 *For the new process innovation, no matter what types of contracts that the outside innovator offers to the inside firms, the less efficient firm always has a higher incentive to pay for the license than the more efficient firm.*

Proof: See appendix. ■

Compared with the proposition 2.3 which states that the more efficient firm always has a higher incentive to pay for the license for a step-elimination innovation, the result for a new process innovation is opposite. The intuition is that with the new process innovation, the less efficient firm gains a larger cost reduction compared with the more efficient firm, and the cost disadvantage is wiped out, which provides a higher profit incentive for the less efficient firm to pay for the license.

2.4.3 Selling game

Sinha (2016)'s Result: For the new process innovation, when the patent is sold by the outside patentee then it directly sells the technology to the more efficient firm at a fixed price in the first stage of the game and the buyer firm in the second stage further licenses the technology to the less efficient firm at a per-unit royalty rate $r_2 = c_2 - c$. And thus, the outside patentee receives $R_s = \frac{(a-2c+c_2)^2}{9} - \frac{(a-2c_1+c)^2}{9} + (c_2 - c) \cdot \frac{(a-2c_2+c)}{3}$.

Under both types of innovation, the more efficient firm buys the patent. The reason is deeply associated with the nature of asymmetric Cournot duopoly in the product market. When the efficient firm buys the patent, it creates larger cost asymmetry leading to higher industry profit. This higher profit is mostly captured by the more efficient firm and therefore, it wants to pay more for the patent.

Hence, by comparing the payoffs from patent sale and fully flexible discriminatory licensing for the new process innovation, I have the following proposition,

Proposition 2.8 *The patent sale is strictly better than patent discriminatory licensing for a new process innovation in an asymmetric Cournot duopoly market.*

Proof: See appendix. ■

Similarly, the outside innovator obtains the largest payoff by patent sale. It follows a similar logic as the step-eliminating innovation. With the patent sale, the post innovation marginal costs becomes (c, c_2). Although the less efficient firm gets the license for the new technology, its effective marginal cost remains the same as that in the pre-innovation stage since the per-unit royalty is equal to the size of cost reduction. It is equivalent to the situation where the more efficient firm exclusively gets the license and captures an even larger market share, making the largest possible industry profits.

However, with licensing policy, the post innovation marginal costs turn to $(c + r_1, c + r_2)$. The comparative cost advantage of firm 1 decreases, making firm 1 occupy a relatively smaller market share in equilibrium compared with selling policy. Therefore the industry profit is not as high as that in patent sale. The total revenue that the outside innovator can extract from the industry profit increment therefore is lower.

To sum up, both types of innovations show a consistent ranking of revenues from patent sale, simulataneous and sequential licensing. The patent sale is indeed the most profitable policy for the outside innovator for both step-eliminating innovation and new process innovation. That is to say, patent sale is privately optimal for the innovator and the following section provides a comparative analysis to identify whether patent sale is socially optimal or not.

2.5 Welfare analysis

From the previous section, it is found that patent sale provides the largest revenue for the outside innovator. However, whether patent sale is also socially optimal is unclear in the existing literature. In this section, I compare discriminatory license and patent sale from the welfare point of view to fill the research gap. The welfare analysis is conducted in two parts, consumer surplus and total surplus.

2.5.1 Consumer surplus

Suppose that the vector of post marginal costs is (c_1', c_2'). From Eq.(2.1) and (2.2), the industry output Q is,

$$Q = \frac{2a - (c_1' + c_2')}{3}$$

where $(c_1' + c_2')$ is the sum of firms' marginal costs. With linear demand function and constant marginal cost, the consumer surplus (CS) is,

$$CS = \int_0^Q p(Q)dQ - pQ = \frac{1}{2}Q^2$$

Therefore the maximization of consumer surplus is equivalent to maximising the industry output, and maximising the industry output is equivalent to minimising the sum of firms' marginal costs.

For step-eliminating innovation, the post-innovation marginal cost turns to $(c_1 - x + r_1^*, c_2 - x + r_2^*)$ in the case of discriminatory licensing, and $(c_1 - x, c_2)$ in the case of patent sale. With the optimal r_1^* and r_2^*, it is found that the sum of the marginal costs is lower in the case of discriminatory licensing (i.e., $c_1 + c_2 + r_1^* + r_2^* - 2x < c_1 + c_2 - x$). Similarly, for new process innovation, the post-innovation marginal cost is $(c + r_1^*, c + r_2^*)$ in the case of discriminatory licensing, and (c, c_2) in the case of patent sale. It is also the case that the sum of marginal costs is lower with discriminatory licensing (i.e., $2c + r_1^* + r_2^* < c + c_2$).

Therefore, for any type and any size of innovation, $Q_L > Q_S$, where L for license and S for sale. Thus, $CS_L > CS_S$. In particular, consumer surplus is the largest when the outside innovator offers fixed fee contracts to both firms. Whereas consumer surplus of patent sale is the smallest which illustrates it does not benefit the consumers. It is interesting to see that consumer surplus is greater with the licensing than patent sale. From the previous section, patent sale is found to be the most profitable way for the outside innovator, in other words, firms spend more with buying the patent but less with licensing. Therefore firms with licensing pass less on to the consumers, which benefits the consumers.

2.5.2 Total surplus

The total surplus (TS) in this game is the sum of consumer surplus, producer's net profits and the innovator's total payoff. The innovator's payoff is a transfer from the welfare point of view, which means the total surplus should either include the innovator's payoff, or equivalently, only the producer's operative profits, i.e., net profits before their payments to the innovator. Therefore the total production costs matter. Since the post-innovation cost structure is different for two types of innovation, I analyse them seperately.

Step-eliminating innovation

The firms produce at different marginal costs in the last stage. The total surplus (TS) can be calculated by,

$$TS = a(q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2 - (c_1 - x)q_1 - (c_2 - x)q_2$$

Combined with the equilibrium quantities, the equilibrium social welfare can be determined.

Proposition 2.9 *For a step-eliminating innovation, the equilibrium social welfare in the case of discriminatory licensing is larger than that in the case of patent sale if the initial cost asymmetry is relatively small, i.e.,* $c_2 < \frac{a+4c_1}{5}$.

Proof: See appendix. ■

For the patent sale, firm 1 buys the contract by paying a fixed fee while firm 2 pays for the license by means of royalty which is equal to x. It leads to the largest possible cost asymmetry in the post innovation stage, and the more efficient firm (firm 1) occupies an even larger market share. While for the discriminatory licensing, the post cost asymmetry is smaller than that of patent sale. Previous section established that the equilibrium industry output associated with license is larger than that in the case of patent sale. This is an advantage of license over patent sale from the perspective of social welfare. In contrast, the disadvantage of license over patent sale is that it brings an equilibrium in which the less efficient firm (firm 2) occupies a relatively larger market share compared with the patent sale. And this disadvantage will become more serious when the initial cost asymmetry between firms increases. Accordingly, when the initial cost asymmetry is relative small, social welfare under discriminatory licensing can be larger than that under patent sale.

It is noteworthy that the ranking of patent sales and licensing is ambiguous in the literature. Banerjee and Poddar (2019) conduct a welfare analysis between licensing and selling with an outside patentee and two potential licensees under Hotelling's linear city model and find that selling of innovation is not only privately optimal, it is also socially optimal. In contrast, Niu (2019) considers the innovation transfer from an inside innovator and shows that the social welfare under licensing can be larger than that under selling when the initial cost gap is not too large, which is consistent with my results.

New process innovation

The firms produce at identical marginal costs, c, in the last stage. The total surplus (TS) can be calculated by,

$$\begin{aligned} TS =& a(q_1 + q_2) - \frac{1}{2}(q_1 + q_2)^2 - c(q_1 + q_2) \\ =& (a - c)Q - \frac{1}{2}Q^2 \end{aligned}$$

Take derivative with respect to the total output Q,

$$\frac{dTS}{dQ} = a - c - Q = p - c > 0$$

which immediately follows that the total surplus is increasing in total output Q. Since the total output is higher with discriminatory licensing, for a new process innovation, the discriminatory license is better than the patent sale, not only beneficial for the consumers but also for the whole society. Thus although the patent sale is privately optimal, it is not socially optimal.

2.6 Conclusions

The commonly studied way to transfer the new technology in existing literature is patent licensing, while Tauman and Weng (2012) first propose another possible way which is patent sale. The difference between them lies in whether or not ownership of the innovation is transferred. Sinha (2016) states that patent sale is strictly better than patent licensing. However, patent licensing in his paper is either offering one contract or two identical contracts to the firms. But with patent sale, two firms obtain the new technology by different contracts sequentially in fact. Therefore, I consider the case where the outside innovator takes full advantage of the firm heterogeneity and designs different contracts to the firms simultaneously or even in a sequence, and compare the total payoff with patent sale.

The results further shows the superiority of patent sale in an asymmetric Cournot duopoly market for both step-eliminating innovation and new process innovation. Furthermore, I proceed with the welfare analysis which is absent in Sinha (2016)'s paper. For the step-eliminating innovation, the welfare under patent licensing can be larger than that under patent sale if the initial cost asymmetry is not too large. And for the new process innovation, patent licensing is always better off than patent sale from welfare point of view. That is to say, although patent sale is privately optimal, it is not socially optimal.

There are some possible extensions to this study. First, Assumption 4.1 restricts that the innovation analysed in this study to be non-drastic. That is, each firm is still active in the post innovation stage. Therefore, it can be relaxed to consider drastic innovation later. Second, I compare the fully flexible licensing scheme with patent sale from the standpoint where the innovation is owned by an outside innovator which is not a producer in the market. Therefore, whether the patent sale is strictly better for an inside innovator remains questionable. Discussions on this issue can be found in Fan et al. (2018) and Niu (2019). Third, the difference between licensing and patent sale is whether the ownership of the property rights is transferred or not. Therefore licensing can be a multi-period game while selling is a one-off business. The model can be extended to cover the R&D stage afterwards and then verify whether the patent sale is still strictly better off. Fourth, it would be an interesting empirical test to verify if indeed the large (or more efficient) firms spend more to buy a license than the small (or less inefficient) firms for a step-eliminating innovation and opposite for a new process innovation. Finally, if the flexible licensing scheme further allows the value of royalty rate to be negative, it may replicate the selling strategy or even be more profitable.

Appendix A

Lemma 2.1

Case 1 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0, 0, \frac{a-2c_1+c_2-3x}{9}, \frac{a-2c_2+c_1-3x}{9}, 0, 0)$

This solution is feasible if $0 < 3x \leqslant a - 2c_2 + c_1$. In this case, the optimal royalty rates are $r_1^* = 0$, $r_2^* = 0$. The outside innovator offers a fixed fee contract with $F_i = \frac{4x(a-2c_i+c_j)}{9}$ to firm i. And the total payoff is $R_{ST}(0, 0) = \frac{4x(2a-c_1-c_2)}{9}$.

Case 2 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0, \frac{3x-(a-2c_2+c_1)}{4}, \frac{a-5c_1+4c_2-3x}{18}, 0, 0, 0)$

This solution is feasible if $a - 2c_2 + c_1 < 3x \leqslant a - 5c_1 + 4c_2$. In this case, the optimal royalty rates are $r_1^* = 0$, $r_2^* = \frac{3x-(a-2c_2+c_1)}{4}$. The outside innovator offers a fixed fee contract with $F_1 = \frac{x(a-3c_1+2c_2+x)}{3}$ to firm 1 and a two-part tariff contract with $F_2 = \frac{(a-2c_2+c_1+x)(5a-10c_2+5c_1-3x)}{36}$ to firm 2. And the total payoff is $R_{ST}(0, r_2^*) = \frac{c_1^2+4c_2^2+(-38x+2a-4c_2)c_1-(4a+20x)c_2+a^2+58ax+9x^2}{72}$

Case 3 Interior solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (\frac{3x-(a-5c_1+4c_2)}{6}, \frac{3x-(a-5c_2+4c_1)}{6}, 0, 0, 0, 0)$

This solution is feasible if $3x > a - 5c_1 + 4c_2$. In this case, the optimal royalty rates are $r_1^* = \frac{3x-(a-5c_1+4c_2)}{6}$, $r_2^* = \frac{3x-(a-5c_2+4c_1)}{6}$. The outside innovator offers a two-part tariff contract with $F_i = \frac{(3x+a-5c_i+4c_j)(2a-7c_i+5c_j)}{27}$ to firm i, and the total payoff is $R_{ST}(r_1^*, r_2^*) = \frac{7c_1^2+7c_2^2+(-a-21x-13c_2)c_1-(a+21x)c_2+a^2+42ax+9x^2}{54}$.

Lemma 2.3

Case 1 If $0 < 3x \leqslant a - 2c_1 + c_2$, $\hat{r_1} \leqslant 0$.

The optimal royalty rate $r_1^* = 0$. The outside innovator offers fixed fee contracts to both firms with $F_i = \frac{4x(a-2c_i+c_j)}{9}$. The total payoff of the outside innovator therefore is $R_{12} = \frac{4x(2a-c_1-c_2)}{9}$.

Case 2 If $3x > a - 2c_1 + c_2$, $0 < \hat{r_1} < x$.

In this case, the optimal royalty is $r_1^* = \hat{r}_1 = \frac{3x-(a-2c_1+c_2)}{4}$. The outside innovator offers a two-part tariff contract to firm 1 with $F_1 = \frac{(5(a-2c_1+c_2)-3x)(a-2c_1+c_2+x)}{36}$ and a fixed fee contract with $F_2 = \frac{x(a-3c_2+2c_1+x)}{3}$ to firm 2. The total payoff of the innovator is $R_{12} = \frac{4c_1^2+c_2^2+(-4a-20x-4c_2)c_1+(2a-38x)c_2+a^2+58ax+9x^2}{72}$.

Lemma 2.4

Case 3 If $0 < 3x < a - 2c_2 + c_1$, $\hat{r_2} < 0$.

The optimal royalty rate $r_2^* = 0$. The outside innovator offers fixed fee contracts to both firms as well, which is identical as case 1 in previous sequence. The total payoff is $R_{21} = \frac{4x(2a-c_1-c_2)}{9}$.

Case 4 If $3x > a - 2c_2 + c_1$, $0 < \hat{r_2} < x$.

In this case, the optimal royalty rate is $r_2^* = \hat{r}_2 = \frac{3x-(a-2c_2+c_1)}{4}$. The outside innovator offers a fixed fee contract to firm 1 with $F_1 = \frac{x(a-3c_1+2c_2+x)}{3}$ and a two-part tariff contract to firm 2 with $F_2 = \frac{(a-2c_2+c_1+x)(5(a-2c_2+c_1)-3x)}{36}$. The total payoff of the innovator is $R_{21} = \frac{4c_2^2+c_1^2+(-4a-20x)c_2+(2a-38x-4c_2)c_1+a^2+58ax+9x^2}{72}$.

Lemma 2.5

If $a - 4c_2 + 3c_1 > 0$, i.e., $c_2 < \frac{a+3c_1}{4}$,

Case 1 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (0, 0, \frac{a+3c-4c_2}{9}, \frac{a+3c-4c_1}{9}, 0, 0)$

This solution is feasible if $-a + 4c_2 < 3c < 3c_1$. In this case, the optimal royalty rates are $r_1^* = 0$, $r_2^* = 0$. The outside innovator offers fixed fee contracts to both firms with $F_i = \frac{4(c_i-c)(a-c_i)}{9}$. The total payoff is $R_{ST}(0,0) = \frac{4(a+c)(c_1+c_2)-4(c_1^2+c_2^2)-8ac}{9}$.

Case 2 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (\frac{4c_2-a-3c}{4}, 0, 0, \frac{a+3c+4c_2-8c_1}{18}, 0, 0)$

This solution is feasible if $-a + 8c_1 - 4c_2 < 3c < -a + 4c_2$. In this case, the optimal royalty rates are $r_1^* = \frac{4c_2-a-3c}{4}$, $r_2^* = 0$. The outside innovator offers a two-part tariff contract with $F_1 = \frac{(a-c-4c_2+4c_1)(5a-4c_1-4c_2+3c)}{36}$ to firm 1 and a fixed fee contract with $F_2 = \frac{(c_2-c)(a-c)}{3}$ to firm 2. The total payoff is $R_{ST}(r_1^*, 0) = \frac{a^2+(-58c+32c_1+24c_2)a+9c^2+(32c_1+8c_2)c-32c_1^2-16c_2^2}{72}$.

Case 3 Interior solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (\frac{8c_2-a-3c-4c_1}{6}, \frac{8c_1-a-3c-4c_2}{6}, 0, 0, 0, 0)$

This solution is feasible if $0 < 3c < -a + 8c_1 - 4c_2$. In this case, the optimal royalty rates are $r_1^* = \frac{8c_2-a-3c-4c_1}{6}$, $r_2^* = \frac{8c_1-a-3c-4c_2}{6}$. The outside innovator offers two-part tariff contracts to both firms with $F_i = \frac{2(a-2c_j+c_i)(a-8c_j+10c_i-3c)}{27}$ to firm

i. The total payoff is $R_{ST}(r_1^*, r_2^*) = \frac{a^2+9c^2+(20a+12c-16c_2)c_1+(20a+12c)c_2-8c_1^2-8c_2^2-42ac}{54}$.

If $a - 4c_2 + 3c_1 < 0$, i.e.,$c_2 > \frac{a+3c_1}{4}$,

Case 4 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (c_1 - c, 0, 0, \frac{a+c-2c_1}{9}, \frac{c-a+4c_2-4c_1}{9}, 0)$

This solution is feasible if $3(a + 4c_1 - 4c_2) < 3c < 3c_1$. In this case, the optimal royalty rates are $r_1^* = c_1 - c$, $r_2^* = 0$. The outside innovator offers a per-unit royalty contract to firm 1 and a fixed fee contract with $F_2 = \frac{4(c_2-c)(a-c_2+c_1-c)}{9}$ to firm 2. The total payoff is $R_{ST}(c_1 - c, 0) = \frac{c^2+(3a+5c+4c_2)c_1+4ac_2-7ac-6c_1^2-4c_2^2}{9}$.

Case 5 Corner solution

$(r_1, r_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = (\frac{4c_2-a-3c}{4}, 0, 0, \frac{a+3c+4c_2-8c_1}{18}, 0, 0)$

This solution is feasible if $0 < 3c < 3(a + 4c_1 - 4c_2)$. In this case, the optimal royalty rates are $r_1^* = \frac{4c_2-a-3c}{4}$, $r_2^* = 0$. The outside innovator offers a two-part tariff contract with $F_1 = \frac{(a-c-4c_2+4c_1)(5a-4c_1-4c_2+3c)}{36}$ to firm 1 and a fixed fee contract with $F_2 = \frac{(c_2-c)(a-c)}{3}$ to firm 2. The total payoff is $R_{ST}(r_1^*, 0) = \frac{a^2+(-58c+32c_1+24c_2)a+9c^2+(32c_1+8c_2)c-32c_1^2-16c_2^2}{72}$.

Lemma 2.6

If $a - 4c_2 + 3c_1 > 0$, i.e.,$c_2 < \frac{a+3c_1}{4}$,

Case 1 When $3c \geqslant -a + 4c_2$, $\hat{r}_1 \leqslant 0$

The optimal royalty rate is $r_1^* = 0$. Thus, the outside innovator offers two diffrent fixed fee contracts to firms with $F_i = \frac{4(c_i-c)(a-c_i)}{9}$. The total payoff is $R_{12} = \frac{4(a+c)(c_1+c_2)-4(c_1^2+c_2^2)-8ac}{9}$.

Case 2 When $0 < 3c < -a + 4c_2$, $0 < \hat{r}_1 < c_1 - c$

The optimal royalty rate $r_1^* = \hat{r}_1 = \frac{4c_2-a-3c}{4}$. Thus, the outside innovator offers a two-part tariff contract with $F_1 = \frac{(a-c-4c_2+4c_1)(5a-4c_1-4c_2+3c)}{36}$ to firm 1 and a fixed fee contract with $F_2 = \frac{(c_2-c)(a-c)}{3}$ to firm 2. The total payoff is $R_{12} = \frac{a^2+(-58c+32c_1+24c_2)a+9c^2+(32c_1+8c_2)c-32c_1^2-16c_2^2}{72}$.

If $a - 4c_2 + 3c_1 < 0$, i.e.,$c_2 > \frac{a+3c_1}{4}$,

Case 3 when $3c \geqslant 3(a - 4c_2 + 4c_1)$, $\hat{r}_1 \leqslant c_1 - c$.

The optimal royalty rate $r_1^* = c_1 - c$. Thus the outside innovator offers a per-unit royalty contract to firm 1 with $r_1^* = c_1 - c$ and a fixed fee contract to firm 2 with $F_2 = \frac{4(c_2-c)(a-c_2+c_1-c)}{9}$. The total payoff is $R_{12} = \frac{c^2+(3a+5c+4c_2)c_1+4ac_2-7ac-6c_1^2-4c_2^2}{9}$.

Case 4 when $0 < 3c < 3(a - 4c_2 + 4c_1)$, $0 < \hat{r}_1 < c_1 - c$

The optimal royalty rate $r_1^* = \hat{r}_1 = \frac{4c_2-a-3c}{4}$, which is identical as case 2.

Lemma 2.7

Case 5 If $3c > -a + 4c_1$, $\hat{r}_2 < 0$.

The optimal royalty rate is $r_2^* = 0$. The outside innovator offers two distinct fixed fee contracts to both firms which is completely identical as case 1. $R_{21} = \frac{4(a+c)(c_1+c_2)-4(c_1^2+c_2^2)-8ac}{9}$.

Case 6 If $3c < -a + 4c_1$, $0 < \hat{r}_2 < c_2 - c$.

The optimal royalty rate $r_2^* = \hat{r}_2 = \frac{4c_1-a-3c}{4}$. Thus, the outside innovator offers a fixed fee contract to firm 1 with $F_1 = \frac{(c_1-c)(a-c)}{3}$ and a two-part tariff contract to firm 2 with $F_2 = \frac{(a-4c_1+4c_2-c)(5a-4c_1-4c_2+3c)}{36}$. The total payoff is $R_{21} = \frac{a^2+(-58c+32c_2+24c_1)a+9c^2+(32c_2+8c_1)c-32c_2^2-16c_1^2}{72}$.

Appendix B

Proof of Proposition 2.3: Let I_i to denote firm i's incentive to pay for the license.

$$I_1 = F_1 + r_1 q_1 = \frac{4(x-r_1^*)(a-2c_1+c_2-r_1^*+r_2^*)}{9} + r_1(\frac{a-2c_1+c_2-2r_1^*+r_2^*+x}{3})$$

$I_2 = F_2 + r_2 q_2 = \frac{4(x-r_2^*)(a-2c_2+c_1-r_2^*+r_1^*)}{9} + r_2(\frac{a-2c_2+c_1-2r_2^*+r_1^*+x}{3})$

Given the size of cost reduction x, the optimal royalty rates are different. Recall r_i^* in Lemma 2.1.

For $r_1^* = 0$, $I_1 - I_2 = \frac{2r_2^{*2}}{9} + \frac{r_2^*(a-2c_2+c_1+5x)}{9} + \frac{4x(c_2-c_1)}{3} > 0$.

For $r_1^* = \frac{3x-(a-5c_1+4c_2)}{6}$, $r_2^* = \frac{3x-(a-5c_2+4c_1)}{6}$, $I_1 - I_2 = \frac{(c_2-c_1)(2a-c_1-c_2+42x)}{18} > 0$.

Therefore, $I_1 > I_2$ always holds. ■

Proof of Proposition 2.4: Recall for any size of cost reduction x, the revenue of patent sale is $R_s = x(a-c_1)$. And the optimal contract under fully flexible licensing scheme is shown in Fig.4.

Part 1. $0 < 3x \leqslant a - 2c_2 + c_1$

$$R_s - R_{ST} = \frac{x(a-5c_1+4c_2)}{9} > 0 \tag{2.16}$$

Part 2. $a - 2c_2 + c_1 < 3x \leqslant a - 5c_1 + 4c_2$

$$R_s - R_{ST} = \frac{-9x^2 - c_1^2 - 4c_2^2 + (-34x - 2a + 4c_2)c_1 + (4a + 20x)c_2 - a^2 + 14ax}{72} \tag{2.17}$$

One can see that the difference follows a quadratic and concave function of x, therefore plugging the higher bound of $x = \frac{a-5c_1+4c_2}{3}$ to obtain the minimum of the first derivative,

$$\left.\frac{d(R_s - R_{ST})}{dx}\right|_{x=\frac{a-5c_1+4c_2}{3}} = \frac{2a-c_1-c_2}{18} > 0$$

Therefore, the difference is always increasing with $3x \in (a-2c_2+c_1, a-5c_1+4c_2]$. The minimum difference is obtained at the lower bound,

$$\left.R_s - R_{ST}\right|_{x=\frac{a-2c_2+c_1}{3}} = \frac{(a-5c_1+4c_2)(a-2c_2+c_1)}{27} > 0$$

Therefore, Eq.(2.17)>0 always holds.

Part 3. $3x > a - 5c_1 + 4c_2$

$$R_s - R_{ST} = \frac{-9x^2 - 7c_1^2 - 7c_2^2 + (a - 33x + 13c_2)c_1 + (a + 21x)c_2 - a^2 + 12ax}{54} \tag{2.18}$$

Similarly, the difference follows a quadratic and concave function of x, therefore as long as the differences is not negative at either the lower or higher bound of x, the difference is always positive. Firstly, set x at the lower bound $x = \frac{a-5c_1+4c_2}{3}$,

$$D_1 \equiv R_s - R_{ST}\Big|_{x=\frac{a-5c_1+4c_2}{3}} = \frac{23c_1^2 + 5c_2^2 + 2a^2 - 20ac_1 + 16ac_2 - 26c_1c_2}{54} \quad (2.19)$$

Take partial derivative with respect to c_1,

$$\frac{\partial D_1}{\partial c_1} = \frac{-10(a - 2.3c_1 + 1.3c_2)}{27} < 0$$

It shows that the difference is decrasing of c_1, thus it attains the minimum when c_1 tends to the higher cost c_2. Hence, plugging the value $c_1 = c_2$, the minimum of equation (19) is,

$$D_1\Big|_{c_1=c_2} = \frac{(a-c_2)^2}{27} > 0$$

Therefore, $D_1 > 0$ always holds.

Given the assumptions, the higher bound $x = c_1$ and $a - 2c_2 + c_1 > c_1$. If the difference in Eq.(2.18) at $x = a - 2c_2 + c_1$ is not negative, it is sufficient to show the difference at $x = c_1$ is positive. Thus, set $x = a - 2c_2 + c_1$,

$$D_2 \equiv R_s - R_{ST}\Big|_{x=a-2c_2+c_1} = \frac{-49c_1^2 - 85c_2^2 + 2a^2 - 38ac_1 + 34ac_2 + 136c_1c_2}{54} \quad (2.20)$$

Take partial derivative with respect to c_1,

$$\frac{\partial D_2}{\partial c_1} = \frac{-19(a - \frac{68}{19}c_2 + \frac{49}{19}c_1)}{27}$$

The sign of partial derivative depends on the sign of $(a - \frac{68}{19}c_2 + \frac{49}{19}c_1)$. With $3x > a - 5c_1 + 4c_2$ and $x < c_1$, $3c_1 > a - 5c_1 + 4c_2$ must hold in this case, that is, $a < 8c_1 - 4c_2$. Since $a > c_2$ in assumption 2.1, a is feasible when $2c_2 < 8c_1 - 4c_2$, that is, $c_1 > \frac{3}{4}c_2$. Therefore,

$$a - \frac{68}{19}c_2 + \frac{49}{19}c_1 > 2c_2 - \frac{68}{19}c_2 + \frac{49}{19}c_1 = \frac{49}{19}c_1 - \frac{30}{19}c_2 > \frac{49}{19} * \frac{3}{4}c_2 - \frac{30}{19}c_2 = \frac{27}{76}c_2 > 0$$

Therefore, $\frac{\partial D_2}{\partial c_1} < 0$, which shows that the difference is decreasing in c_1, thus it attains the minimum when c_1 tends to the higher limit c_2. Hence, plugging the value $c_1 = c_2$, the minimum of equation (2.20) is,

$$D_2\Big|_{c_1=c_2} = \frac{(a-c_2)^2}{27} > 0$$

Therefore, $D_2 > 0$ always holds.

Since the difference follows a quadratic and concave function of x, it is positive at both the lower and higher bound of x in this part. Therefore Eq.(2.18) is always positive. ■

Proof of Proposition 2.5:

Fig.5: If $a - 4c_2 + 3c_1 > 0$,

Part 1. $0 < 3c < -a + 4c_1$

$$R_{12} - R_{21} = \frac{(c_1 - c_2)(a - 2c_2 - 2c_1 + 3c)}{9} \tag{2.21}$$

$c_1 - c_2 < 0$ given the assumption. $a - 2c_2 - 2c_1 + 3c$ is increasing in c, thus the maximum is attained at the higher bound $c = \frac{-a+4c_1}{3}$,

$$(a - 2c_2 - 2c_1 + 3c)\Big|_{c=\frac{-a+4c_1}{3}} = 2(c_1 - c_2) < 0$$

Thus, Eq.(2.21)>0 always holds.

Part 2. $-a + 4c_1 < 3c < -a + 4c_2$

$$R_{12} - R_{21} = \frac{(a - 4c_2 + 3c)^2}{72} > 0 \tag{2.22}$$

Part 3. $3c > -a + 4c_2$

The optimal contracts for firms are identical irrespective of the sequence, and therefore the total revenue of the innovator is identical, i.e., $R_{12} = R_{21}$.

Fig.6: If $a - 4c_2 + 3c_1 < 0$,

Part 1. $0 < 3c < -a + 4c_1$

Eq.(2.21) still applies, which is positive.

Part 2. $-a + 4c_1 < 3c < 3(a - 4c_2 + 4c_1)$

Eq.(2.22) still applies, which is positive.

Part 3. $3c > 3(a - 4c_2 + 4c_1)$

$$R_{12} - R_{21} = \frac{(c - c_1)(a + c + 2c_1 - 4c_2)}{9} \tag{2.23}$$

$c - c_1 < 0$ given the assumption. $a + c + 2c_1 - 4c_2$ is increasing in c, thus the maximum is attained at the higher bound $c = c_1$,

$$(a + c + 2c_1 - 4c_2)\Big|_{c=c_1} = a - 4c_2 + 3c_1 < 0$$

under this situation.

Thus, Eq.(2.23)>0 always holds. ■

Proof of Proposition 2.7: Let I_i denote firm i's incentive to pay for the license.

$I_1 = F_1 + r_1 q_1 = \frac{4(c_1-c-r_1^*)(a-c_1-r_1^*+r_2^*)}{9} + r_1(\frac{a-c-2r_1^*+r_2^*}{3})$

$I_2 = F_2 + r_2 q_2 = \frac{4(c_2-c-r_2^*)(a-c_2-r_2^*+r_1^*)}{9} + r_2(\frac{a-c-2r_2^*+r_1^*}{3})$

Given the marginal cost of new technology c, the optimal royalty rates are different. Recall r_i^* in Lemma 2.5.

For $r_2^* = 0$, $I_1 - I_2 = -\frac{2r_1^{*2}}{9} + \frac{r_1^*(-a+5c)}{9} + \frac{c_2(-4a-4c-4r_1^*)}{9} + \frac{4c_2^2}{9} + \frac{4c_1(a+c-c_1)}{9}$.

Take derivative with respect to r_1, $\frac{d(I_1-I_2)}{dr_1} = \frac{(5c-4c_2-4c_1-a)}{9} < 0$. Therefore the difference when $r_2^* = 0$ is a decreasing function of r_1 given the Assumptions. It attains the maximum when $r_1^* = 0$,

$$I_1 - I_2\Big|_{r_1^*=0} = \frac{4(c_1 - c_2)(a + c - c_1 - c_2)}{9} < 0$$

For $r_1^* = \frac{8c_2-a-3c-4c_1}{6}, r_2^* = \frac{8c_1-a-3c-4c_2}{6}$, $I_1 - I_2 = \frac{4(c_1-c_2)(a+c_1+c_2-3c)}{9} < 0$.

Therefore, $I_1 < I_2$ always holds. ■

Proof of Proposition 2.8:

Fig.7 If $a - 4c_2 + 3c_1 > 0$,

Part 1. $0 < 3c < -a + 8c_1 - 4c_2$

$$R_s - R_{ST} = \frac{-a^2 - 9c^2 + (4a + 12c - 16c_2)c_1 + (10a + 18c)c_2 - 16c_1^2 - 22c_2^2 - 12ac}{54} \tag{2.24}$$

One can see that the difference follows a quadratic and concave function of c, therefore as long as the difference (2.24) is not negative at either the lower or higher bound of c, the difference is always positive. Firstly, set c at the lower bound $c = 0$,

$$D_1 \equiv R_s - R_{ST}\bigg|_{c=0} = \frac{-a^2 + (4a - 16c_2)c_1 + 10ac_2 - 16c_1^2 - 22c_2^2}{54} \tag{2.25}$$

Take partial derivative with respect to c_1,

$$\frac{\partial D_1}{\partial c_1} = \frac{2(a - 8c_1 + 4c_2)}{27} < 0$$

It shows that the difference is decrasing of c_1, thus it attains the minimum when c_1 tends to the higher limit c_2. Hence, plugging the value $c_1 = c_2$, the minimum of equation (2.25) is,

$$D_1\bigg|_{c_1=c_2} = -\frac{1}{54}a^2 - \frac{11}{27}c_2^2 + \frac{7}{27}ac_2$$

which is a concave function of a. Since a belongs to $(2c_2, 4c_2)$ under this situation, then evaluate the mininum of D_1 at the two extreme values. At the upper limit $a = 4c_2$, $D_1 = \frac{c_2^2}{3}$, which is positive. At the lower limit $a = 2c_2$, $D_1 = \frac{c_2^2}{27}$, which is also positive. Therefore, $D_1 > 0$ always holds.

Then, set c at the upper bound $c = \frac{-a+8c_1-4c_2}{3}$,

$$D_2 \equiv R_s - R_{ST}\bigg|_{c=\frac{-a+8c_1-4c_2}{3}} \tag{2.26}$$

Take partial derivative with respect to a,

$$\frac{\partial D_2}{\partial a} = \frac{2(a - 4c_1 + 3c_2)}{27} > 0$$

It shows that the difference is increasing of a, thus it attains the minimum when a tends to the lower limit $4c_2 - 3c_1$. Hence, plugging the value to obtain the minimum of equation (2.26) is,

$$D_2\bigg|_{a=4c_2-3c_1} = \frac{(c_2 - c_1)^2}{3} > 0$$

Therefore, $D_2 > 0$ always holds. Thus, for the parameter configurations $0 < 3c < -a + 8c_1 - 4c_2$, R_s is strictly larger than R_{ST}.

Part 2. $-a + 8c_1 - 4c_2 < 3c < -a + 4c_2$

$$R_s - R_{ST} = \frac{-9c^2 - a^2 - 24c_2^2 + (16a + 32c)c_2 - 14ac}{72} \tag{2.27}$$

The payoff difference follows a quadratic and concave function of c, therefore evaluating it at the two extreme values. At the upper limit $c = \frac{-a+4c_2}{3}$, $R_s - R_{ST} = \frac{(a-c_2)^2}{27} > 0$. At the lowest possible limit $c = 0$ which is even smaller than the lower limit for this part, $R_s - R_{ST} = \frac{-a^2-24c_2^2+16ac_2}{72}$, which is a concave function of a and it is maximised at $a = 8c_2$, which is larger than the range of a. Therefore the payoff difference turns to the minimum $\frac{c_2^2}{18} > 0$ when $a = 2c_2$.

Thus, the difference (27) is always positive for the parameter configurations $-a + 8c_1 - 4c_2 < 3c < -a + 4c_2$.

Part 3. $-a + 4c_2 < 3c < 3c_1$

$$R_s - R_{ST} = \frac{(a - c_2)(c_2 - c)}{9} > 0 \tag{2.28}$$

given $c < c_1 < c_2 < a$.

Fig.8: If $a - 4c_2 + 3c_1 < 0$, i.e., $c_2 > \frac{a+3c_1}{4}$

Part 1. $0 < 3c < 3(a - 4c_2 + 4c_1)$

$$R_s - R_{ST} = \frac{-9c^2 - a^2 - 24c_2^2 + (16a + 32c)c_2 - 14ac}{72} \tag{2.29}$$

The payoff difference is a concave function of c. Broaden the range of c in this part to $c \in (0, c_1)$. Now evaluate the difference (29) at the two extreme values. At the lower limit $c = 0$, the difference is proved to be positive in previous section. At the higher limit $c = c_1$, the difference becomes $\frac{-9c_1^2-a^2-24c_2^2+(16a+32c_1)c_2-14ac_1}{72}$, which is increasing of a under this situation. Thus the minimum is attained $-\frac{c_1^2}{8} + \frac{c_1c_2}{18} + \frac{c_2^2}{18}$ when $a = 2c_2$. The expression is increasing of c_2 and $c_2 > \frac{3}{2}c_1$, therefore the minimum is $\frac{c_1^2}{12} > 0$.

Thus, the payoff difference (2.29) is positive.

Part 2. $3(a-4c_2+4c_1) < 3c < 3c_1$

$$R_s - R_{ST} = \frac{-c^2 + 2c_1^2 - c_2^2 + (a-c-4c_2)c_1 + (a+5c)c_2 - 2ac}{9} \tag{2.30}$$

Following the same way as above. In terms of c, at the lower limit $c = 0$, $R_s - R_{ST} = \frac{a(c_1+c_2)-4c_2c_1+2c_1^2-c_2^2}{9}$, which is increasing in a, therefore it attains the minimum of $\frac{(c_2-c_1)^2+c_1^2}{9} > 0$ when $a = 2c_2$. At the higher limit $c = c_1$, $R_s - R_{ST} = \frac{(c_2-c_1)(a-c_2)}{9} > 0$ given $c_1 < c_2 < a$. Thus, the payoff difference (2.30) is positive. ■

Proof of Proposition 2.9: The equilibrium total surplus TS^* is calculated by,

$$TS^* = a(q_1^* + q_2^*) - \frac{1}{2}(q_1^* + q_2^*)^2 - (c_1 - x)q_1^* - (c_2 - x)q_2^*$$

where $q_1^* = \frac{a-2c_1+c_2-2r_1^*+r_2^*+x}{3}$ and $q_2^* = \frac{a-2c_2+c_1-2r_2^*+r_1^*+x}{3}$, which depend on the optimal royalty rates r_1^* and r_2^*.

For the optimal discriminatory licensing policy, the optimal royalties r_1^* and r_2^* depend on the size of cost reduction x. For the patent sale, $r_1^* = 0$ and $r_2^* = x$.

Firstly, consider the case when $r_1^* = 0$. Take derivative with respect to r_2,

$$\frac{dTS^*}{dr_2} = \frac{-(a-5c_2+4c_1+r_2+x)}{9}$$

If $(a-5c_2+4c_1) > 0$, i.e., $c_2 < \frac{a+4c_1}{5}$, $\frac{dTS^*}{dr_2} < 0$ (sufficient condition), which indicates the equilibrium total surplus is a decreasing function of r_2. Recall $r_2 \in [0, x]$, therefore the patent sale where $r_2^* = x$ provides the lowest social welfare when $r_1^* = 0$.

Then, consider the case when $r_1^* = \frac{3x-(a-5c_1+4c_2)}{6}$ and $r_2^* = \frac{3x-(a-5c_2+4c_1)}{6}$, which is the optimal solution when $3x > a - 5c_1 + 4c_2$. Let D_{TS} denote the difference between the equilibrium total surplus TS_L^* and TS_S^*,

$$D_{TS} = TS_L^* - TS_S^* = \left(\frac{2\,a}{27} + \frac{25\,c_1}{54} - \frac{29\,c_2}{54}\right)x + \frac{491\,{c_1}^2}{648} + \frac{(-20\,a - 962\,c_2)\,c_1}{648} + \frac{491\,{c_2}^2}{648} - \frac{5\,ac_2}{162} + \frac{5\,a^2}{162}$$

which shows that D_{TS} is a linear function of x. Set x at the lower bound

$x = \frac{a-5c_1+4c_2}{3}$,

$$D_{TS}\Big|_{x=\frac{a-5c_1+4c_2}{3}} = \frac{(2a-c_1-c_2)(2a-4c_2+c_1)}{72} > 0$$

Then set x at the upper bound $x = c_1$,

$$D_{TS}\Big|_{x=c_1} = \frac{791\,{c_1}^2}{648} + \frac{(28\,a - 1310\,c_2)\,c_1}{648} + \frac{5\,a^2}{162} - \frac{5\,ac_2}{162} + \frac{491\,{c_2}^2}{648}$$

Take partial derivative with respect to a, $\frac{\partial D_{TS}|_{x=c_1}}{\partial a} = \frac{10a-5c_2+7c_1}{162} > 0$, therefore the minimum is attained when $a = 2c_2$ (Assumption 2.1), which equals $\frac{791\,{c_1}^2}{648} - \frac{209\,c_2\,c_1}{108} + \frac{59\,{c_2}^2}{72} > 0$. Thus, $D_{TS} > 0$ always holds. That is to say, for $x > \frac{a-5c_1+4c_2}{3}$, the equilibrium total surplus under licensing is always larger than that under patent sale irrespective of the initial cost asymmetry.

To sum up, for any size of cost reduction, the equilibrium total surplus under licensing is strictly larger than that under patent sale if $c_2 < \frac{a+4c_1}{5}$. ■

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Chapter 3

R&D behaviours with Firm heterogeneity

3.1 Introduction

Starting from the pioneering works of Schunmpeter (1942) and Arrow (1962), the relationship between firm size and innovation has been highly discussed in the literature. The early literature studies what happens when all firms are either large or small, but neglect the realistic situation when firms of different sizes interact. Given the asymmetric setting, the conventional presumption (Pavitt et al.,1987; Scherer, 1991; Cohen and Klepper, 1996a; Cockburn and Henderson, 2001) is that the low-cost firm (referred as the large firm) engages more in R&D than the high-cost firm (referred as the small firm). In that way, the cost gap between firms is getting even larger. However, a wave of empirical studies hold a different view that small firms act as the engines of technological change and innovative activity, at least in certain industries. Therefore, it naturally leads to the research question: what factors may work against the trend so that the small firms spend more on R&D and the gap is narrowed by R&D.

By employing the United States data, Acs and Audretsch (1988, 1990) find that the small firms exhibit apparently more innovative activity than do large firms in some industries, for example, plastics products and electronic components. For UK, the result in Pavitt et al. (1987) is consistent with the analysis of R&D performing firms in the USA. Small firms produce more than 45% of all innovations in some sectors (e.g., machinery and instruments). Moreover, Kleinknecht and Reijnen (1991) argue that there must be an undercounting of small firms R&D in official dataset of Netherlands, and state that their findings have implications for the reliability of data on R&D in small business in other OECD member countries. Similarly, Patel and Pavitt (1995) also claim that there is a serious limitation with R&D statistics in that they reflect very imperfectly the technological contribution of small firms while very often small firms do not even figure in the R&D data. Furthermore, R&D statistics considerably overestimate the contribution of large firms to R&D and there is a sizable variation between large and small firms across different industries.

As for theoretical research, a few papers examine the factors that influence R&D investment decision with different firm sizes. Rosen (1991) highlights the importance of considering both R&D project riskiness and R&D choices, and concludes that the large firm prefers to choose a safer R&D project and invests more than its smaller rival while small firms produce a disproportionate share of major innovations. Poyago-Theotoky (1996) focuses on two different functional forms how R&D affects initial costs of production. One is the additive cost function which yields that the high cost firm spends less on R&D than the low cost firm, and the other one is the multiplicative case which obtains a totally different outcome characterized by the high cost firm spending more than its low cost rival but not being able to overcome the initial cost gap. Yin and Zuscovitch (1998) focus on product and process innovations, and find that the large firm invests more in process innovation, while the small one is willing to spend more resources to search for new products. Barros and Nilssen (1999) explore two dimensions of asymmetry characterized by R&D efficiency and R&D productivity with asymmetric firm sizes. They find that the *ex ante* high cost firm may end up as the low cost firm because of differences in R&D technology even if it conducts less R&D than the *ex ante* low cost firm.

One of the possible factor considered in this chapter is R&D spillover. Firm heterogeneity is a key feature of this chapter, which is explicitly formulated by assumptions that two firms have different initial marginal costs of production. The model consists of a two-stage game where the first stage is R&D competition and then Cournot production stage. Furthermore, spillover is an important feature of R&D. To be specific, it implies that firms are able to benefit from other firm's R&D flow without payment. Due to the asymmetric setting, firms' ability to absorb the external knowledge is likely to differ as well.

The existing literature provides the rationale for asymmetric spillovers based on two dimensions. First, the literature pertaining to the learning organization (Fiol and Lyles 1985; Stata 1989) reveals that firms within the same industry do not necessarily have the same capacities to learn. In the case of the innovative firm (e.g. Raychem), it was observed that people at the cutting edge of their field do not think anyone can teach them anything. Moreover, rivals may differ in their absorptive capacity as the result of already existing differences in the knowledge base and organizational firm characteristics. Second, for industries characterized by high level of R&D (e.g. electronics, cars, medicine etc.), firms in less developed regions produce goods that are of significantly lower quality than that in developed regions (Zigic 1998; Baniak 2009; Taba 2016). Firms with lower quality products aim to develop a production of high quality goods to capture a bigger part of the market and make higher profits, but the high quality producers are not interested in absorbing technology of rival firms since it is 'old' for them. In addition, Singh (2007) and Knot et al. (2008) support the asymmetric spillover hypothesis by providing empirically significant results.

De Bondt and Henriques (1995) propose that firms are likely to have a different ability to absorb the technological information and know-how in newly created industries and asymmetries are likely to be the rule rather than the exception. Therefore, they analyze the effect of asymmetric spillovers between two agents. The main difference in their model is that firms firstly choose to announce a leader or a follower role before conducting R&D and conclude that the leader is the firm that absorbs the large spillovers in the unique equilibrium. Lukatch and Plasmans (2000) make an extension to d'Aspremont and Jacquemin (1988)'s model, which focuses on an asymmetric environment in terms of different size and R&D efficiency of two agents. They assume that the advantage in size can directly be translated into the advantage in cost, which means that the larger firm with a lower marginal cost of production also has a lower per unit marginal cost of research. Petit, Sanna-Randaccio and Sestini (2012) analyze how identical firms' R&D investment decisions are affected by asymmetries in know-how management capabilities and show that a better ability to manage knowledge flows incentivizes the firm to invest more in R&D. Ishikawa and Shibata (2021) examine the effects of asymmetric spillovers on R&D investment in an oligopoly market. The main result is an increase in spillover asymmetry between firms increases the difference in R&D investments between asymmetric and symmetric spillovers. The recent literature with spillover rate mainly focuses on the comparison between noncooperative and cooperative R&D in a symmetric setup. Thus how asymmetric spillover affects R&D and then firm size is not well understood.

In this chapter, I assume an extreme case of asymmetric spillovers where only the large firm's R&D spills over to the small firm but not vice versa (i.e., one-way spillover), to see how it fluences the R&D behaviors and further the cost gap. One-way spillover is firstly proposed by Amir and Wooders (1997), which consider a uni-directional and stochastic spillover process whereby know-how may flow only from the more R&D intensive firm to its rival in a binomial framework: with probability β full spillover occurs and with $1-\beta$ zero spillover occurs. They aim to show that the ex-ante identical firms engage in different R&D levels and therefore end up with different sizes. Following Amir and Wooders (1997), Tesoriere (2005) compares the sequential and simultaneous play at the R&D stage, in order to assess the role of one-way spillover in stimulating or attenuating endogenous firm asymmetry.

Perhaps surprisingly, when considering the symmetry or asymmetry in firm size and spillovers, the existing literature mostly covers the following conditions: symmetric firms with symmetric spillovers, symmetric firms with asymmetric spillovers, asymmetric firms with symmetric spillovers, but not asymmetric firms with asymmetric spillovers. The extant literature has so far devoted little attention to how asymmetric spillover rate influences the relationship between firm size and R&D intensities, therefore this chapter aims to fill the gap. Due to the *ex ante* asymmetry in firm size, the large firm with lower marginal cost is initially more advanced in technology than its rival. Therefore it is likely that only the small firm is able to imitate the large firm to some extent, which implies the one-way spillover only from the large firm to the small one. By employing one-way spillover, this chapter aims to determine how it affects the R&D levels and how the interaction of R&D further affects the cost gap. In other words, it provides a theoretical basis that the one-way spillover is a possible factor to explain that the small firm spends more on R&D shown from the empirical evidence and the one-way spillover is able to stop the divergence between the firms so that the cost gap is narrowed. Furthermore, this chapter provides a welfare analysis to see whether the society prefers a larger cost gap or smaller cost gap, which is absent in the existing literature.

The main results of this chapter are summarized as follows. Firstly, the one-way spillover rate is a possible factor which yields that the small firm chooses higher R&D intensities than the large firm and the cost gap is therefore narrowed. With relatively small asymmetry, the small firm is able to catch up with the large firm and it benefits both the consumers and the total welfare. However, from the welfare point of view, it is not always better that the gap between firm narrows. If the initial cost gap is relatively significant, the society prefers to even broaden the gap further.

This chapter contributes to the literature in the following aspects. First, it considers the asymmetry described by both firm size and spillover rate, which is more in line with realistic situation. Second, it provides policy statements about whether the post-R&D market structure benefits the consumers and the society.

The rest of the paper is organized as follows. Section 3.2 describes the benchmark model and introduces the one-way spillover rate into the model, and characterizes the equilibrium R&D. Section 3.3 provides some comparative statics and main results. Section 3.4 provides a welfare analysis. Concluding remarks are provided in Section 3.5.

3.2 The Model

3.2.1 The benchmark model

The benchmark follows a standard two-stage duopoly model of the existing R&D literature. Consider an industry consisting of two firms producing a homogenous product with different initial marginal costs c_i, playing the following two-stage game. In the first stage, firms simultaneously choose a process R&D level (or R&D intensity), x_i, with $x_i \in (0, c_i)$, $i = 1, 2$, which results in the post-R&D marginal cost $c_i' = c_i - x_i > 0$. That is to say, each firm chooses its autonomous cost reduction level (R&D output) in this model. The research cost to firm i associated with the cost reduction x_i follows a quadratic equation which is given by,

$$r(x_i) = \frac{\gamma x_i^2}{2} \tag{3.1}$$

which implies $r(x_i) \geqslant 0$, $r'(x_i) > 0$. Additionally, $\gamma > 0$ is an inverse measure of the efficiency of R&D.

In the second stage, after observing the new marginal costs, firms compete in the product market by choosing quantities, q_i, which naturally leads to a Cournot duopoly competition, facing a linear inverse demand, $p = a - Q$, where $Q = q_1 + q_2$ is the total industry output. The two-stage game yields a subgame-perfect Nash equilibrium solved by backward induction.

Assume that demand is high enough relative to the initial marginal cost to ensure that the Cournot competition yields a unique pure Nash equilibrium where both firms are active in the product market for all possible R&D levels that they may undertake, that is,

Assumption 3.1 *$a > 2c_i$, for $i = 1, 2$*

Assumption 3.1 guarantees that the Nash equilibrium output of each firm in the Cournot competition is positive, which means both firms are willing to invest in R&D and still be active in the market after the R&D stage.

The *ex-ante* asymmetric firm size is a key feature of the model, which is reflected by having different initial marginal cost c_i. Firm 1 is assumed to be larger than firm 2, that is, $c_1 < c_2$. Since two asymmetric firms are active in the market, the initial cost gap cannot be quite large. Introduce λ which equals to the difference between firms' marginal cost of production to measure the absolute cost gap. The initial cost gap therefore is $\lambda = c_2 - c_1$. Assumption 3.1 implies that the range of λ, $0 < \lambda < \frac{a-2c_1}{2}$, holds before engaging in R&D.

Solve the game backwards. In the final production stage, the marginal cost of firm i is $c_i - x_i$. Firm i maximises its operative profit π_i given by, $i, j = 1, 2$ and $i \neq j$,

$$\pi_i = (p - (c_i - x_i)) \cdot q_i = (a - q_i - q_j - c_i + x_i) \cdot q_i \tag{3.2}$$

The Cournot equilibrium output is,

$$q_i = \frac{a - 2c_i + c_j + 2x_i - x_j}{3} \tag{3.3}$$

and the corresponding equilibrium profit is,

$$\pi_i = (q_i)^2 \tag{3.4}$$

The total industry output is,

$$Q = \frac{2a - c_i - c_j + x_i + x_j}{3} \tag{3.5}$$

Since the consumer surplus is $CS = \frac{Q^2}{2}$, it is obvious that R&D level has a positive effect on the total output, and consequently, on the consumer welfare.

In the first stage, firms choose their R&D levels simultaneouly. Taking into account how the firms would choose to produce, firm i chooses x_i in order to maximise its overall profit Π_i,

$$\Pi_i = \pi_i - r(x_i) = (q_i)^2 - \frac{\gamma x_i^2}{2} \tag{3.6}$$

The best response function of x_i given x_j is,

$$x_i(x_j) = \frac{4(a - 2c_i + c_j - x_j)}{9\gamma - 8} \tag{3.7}$$

This shows us that the R&D level x_j is strategic substitute for x_i.

The second-order condition requires $9\gamma - 8 > 0$. And the stability condition that the best response functions cross correctly, requires $\gamma > \frac{4}{3}$ where the best response functions cross correctly (Henriques 1990). Therefore, I make the following assumption to limit the R&D efficiency parameter γ.

Assumption 3.2 $\gamma > \frac{4}{3}$

The equilibrium R&D level is,

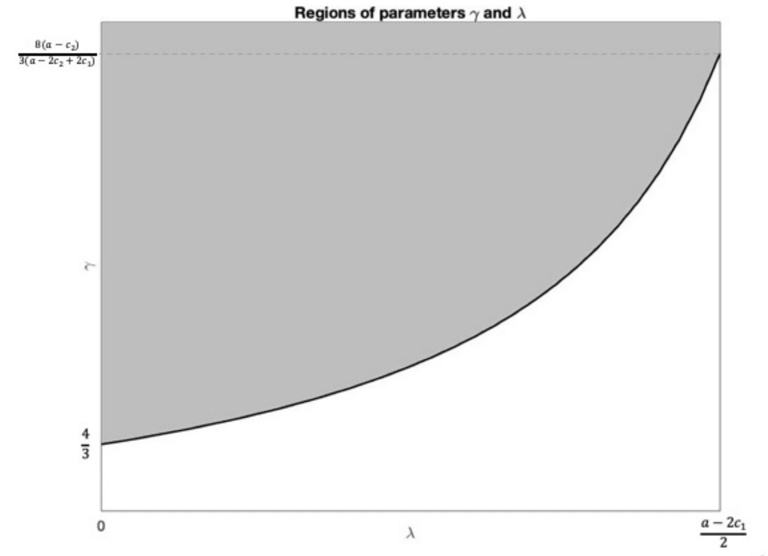
$$x_i^b = \frac{4[(3\gamma - 4)a + 3\gamma c_j + (4 - 6\gamma)c_i]}{(3\gamma - 4)(9\gamma - 4)} \quad (i, j = 1, 2 \textit{ and } i \neq j) \tag{3.8}$$

Since I consider the situation where both firms are willing to invest in R&D, both x_1^b and x_2^b must be positive. Assumptions 3.1 and 3.2 guarantee that x_1^b is positive.[1] To guarantee that x_2^b is also positive, I make the following additional assumption.

[1]The proof is shown in Appendix

Assumption 3.3 $\gamma > \frac{4(a-c_2)}{3(a-2c_2+c_1)}$

Since $\lambda = c_2 - c_1$, combining with assumption 3 gives the relationship between parameters λ and γ where both firms invest in R&D, i.e. $\gamma > \frac{4(a-c_2)}{3(a-c_2-\lambda)}$ (see Fig.1).



Therefore, the difference between x_1^b and x_2^b is,

$$\triangle x = x_1^b - x_2^b = \frac{4\lambda}{3\gamma - 4} > 0 \tag{3.9}$$

Remark 3.1 *The lower the initial marginal cost, the higher the equilibirum R&D level.*

$\frac{\partial x_i}{\partial c_i} < 0$, $\frac{\partial x_i}{\partial c_j} > 0$

From the above analysis, it is shown that the optimal research level of the large firm is always higher than that of the small firm. Thus, the cost gap between firms is broadened by the activities at the R&D stage. Moreover, with higher intial cost gap λ, the difference between the large and small firm's R&D is even larger.

The idea behind the result is intuitive to comprehend. Since the operative profit function exhibits convexity in intial marginal cost, i.e., $\frac{\partial \pi_i}{\partial c_i} < 0$, $\frac{\partial^2 \pi_i}{\partial c_i^2} > 0$, the marginal profit from a reduction in own costs for the initial low cost firm is greater than that for the high cost firm. And the cost reduction in the model is the R&D level chosen by the firm. Hence, a low cost firm always faces a larger incentive effect to spend on R&D that will reduce its cost of production relative to a high cost firm. If both firms get the same cost reduction, then the low cost firm will produce more so that to increase its profit by more than the high cost firm. Thus the low cost firm has a stronger incentive to spend on cost-reducing R&D. The incentive effect for the low cost firm exceeds the same incentive for the high cost firm. Therefore the conventional presumption that a low cost firm will be more willing to spend on R&D comes through, which results in an outcome of persistent dominance with the large firm moving ahead and the gap between firms growing even larger.

In the following, I introduce a one-way spillover rate into the model to see how it affects the R&D intensities and therefore affects the heterogeneity.

3.2.2 A one-way spillover

Suppose now the small firm is able to absorb the technological infromation from the large firm to some extent. That is to say, the large firm's R&D flows to the small firm with a spillover rate β, which forms a part of the effective cost reduction for the small firm without payment, but not the other way around. It is realistic due to the *ex ante* asymmetric position. Compared with the benchmark setting, the modified model illustrates how the spillover rate influences the R&D levels with firm heterogeineity and accordingly how the cost gap varies. X_i is the firm's effective cost reduction (effective R&D output).

$$X_1 = x_1$$

$$X_2 = x_2 + \beta x_1$$

with $\beta \in [0, 1]$. In this model, it is large firm's R&D output that spillovers to the small firm.[2] When $\beta = 0$, it is identical to the benchmark setting.

Therefore, the post-R&D marginal costs for firm i is $c_i' = c_i - X_i, i = 1, 2$.

Suppose that the research cost follows the same quadratic equation which is shown in Eq (3.1). I follow the same steps as in the benchmark model to solve the model by backward induction.

In the production stage, maximising the operative profit π_i yields the Cournot equilibirum outputs which are given by,

$$q_1 = \frac{a - 2c_1 + c_2 + (2 - \beta)x_1 - x_2}{3} \tag{3.10}$$

$$q_2 = \frac{a - 2c_2 + c_1 + (2\beta - 1)x_1 + 2x_2}{3} \tag{3.11}$$

Therefore, the total industry output is,

$$Q = \frac{2a - c_1 - c_2 + x_1 + x_2 + \beta x_1}{3} \tag{3.12}$$

At the R&D stage, each firm chooses its own R&D level to maximise overall profit, Π_i, which leads to the following best response functions for R&D levels,

$$x_1(x_2) = \frac{2(2 - \beta)(a - 2c_1 + c_2 - x_2)}{9\gamma - 2(\beta - 2)^2} \tag{3.13}$$

$$x_2(x_1) = \frac{4(a - 2c_2 + c_1 + (2\beta - 1)x_1)}{9\gamma - 8} \tag{3.14}$$

$9\gamma > 2(\beta - 2)^2$ and $9\gamma > 8$ guarantee that the SOCs are satisfied and the objective functions are strictly concave with respect to own R&D level. $9\gamma > 2(2-\beta)(3-\beta)$ follows from the stability condition proposed by Henriques (1990), which requires the best response functions to cross correctly. It is calculated from $\left|\frac{dx_1}{dx_2}\right| < 1$ using the best response functions given by Equations (3.13) and (3.14).

[2] In contrast, for another common used model (introduced by Kamien et al, 1992), the firms' decision variable is R&D investment y_i for firm i. Firm's unit cost reduction is $f(Y_i)$, where f is an R&D production function and Y_i is firm i's effective R&D investment, which is determined by the combined individual R&D expenditure and a spillover part. Given y_1, y_2, firm i's effective R&D investment is $Y_i = y_i + \beta y_j (i, j = 1, 2$ and $i \neq j)$. Thus, in this context, it is expenditures or R&D inputs that spillover to rivals. A thorough comparison of input versus output spillovers is given by Amir (2000).

Assumption 3.2 ($\gamma > \frac{4}{3}$) guarantees that the restrictions for γ implied by SOCs and stability condition hold for all $\beta \in [0, 1]$.

Therefore, the asymmetric Nash equilibrium research levels are,

$$x_1^* = 2(2-\beta)\frac{A}{B} = \frac{4A}{B} - 2\beta \cdot \frac{A}{B} \tag{3.15}$$

$$x_2^* = 4\frac{[C+D]}{B} = \frac{4C}{B} + 4 \cdot \frac{D}{B} \tag{3.16}$$

where

$$A = (3\gamma - 4)a + (4 - 6\gamma)c_1 + 3\gamma c_2 > 0$$

$$B = (9\gamma - 4)(3\gamma - 4) - 2\beta\,[3\gamma(\beta - 4) + 4] > 0$$

$$C = (3\gamma - 4)a + (4 - 6\gamma)c_2 + 3\gamma c_1 > 0$$

$$D = 2\beta\,[(3 - \beta)a - (2 - \beta)c_1 - c_2] > 0$$

Given the assumptions, the signs of A, B, C, D are all positive.[3] Therefore the equilibria x_1 and x_2 are indeed interior and positive.

3.3 Comparative statics

To make the notation clear, the equilibrium R&D levels (3.15) and (3.16) yield the total R&D level $TX^* = x_1^* + x_2^*$, the effective cost reductions X_1^* for firm 1 and $X_2^* = x_2^* + \beta x_1^*$ for firm 2, which further results in the total cost reductions $TC^* = X_1^* + X_2^*$. Then, I proceed to the following comparative statics and propositions. Lemma 3.1 characterizes how the one-way spillover rate affects the equilibrium R&D level and the corresponding equilibrium output.

Lemma 3.1 *The effects of change in spillover rate on the equilibrium R&D levels and the corresponding equilibrium output are,*

$\frac{dx_1^*}{d\beta} < 0$, $\frac{dx_2^*}{d\beta} > 0$; $\frac{dq_1^*}{d\beta} < 0$, $\frac{dq_2^*}{d\beta} > 0$

[3] The sign of A, B, C, D are proved in Appendix.

Proof: Taking first derivatives of equilibrium R&D levels with respect to β,

$$\frac{dx_1^*}{d\beta} = -\frac{2A}{B} + [2(2-\beta)A \cdot (-B^{-2} \cdot B'(\beta))] \tag{3.17}$$

with $B'(\beta) = -12\gamma(\beta - 2) - 8$. The sign of (17) depends on the sign of $B'(\beta)$. Since $B''(\beta) = -12\gamma < 0$, the minimum $B'(\beta) = B'(1) = 12\gamma - 8 > 0$ given the assumptions. Therefore, $\frac{dx_1^*}{d\beta} < 0$.

$$\begin{aligned} \frac{dx_2^*}{d\beta} &= -\frac{4C}{B^2} \cdot B'(\beta) + (\frac{4D'(\beta)}{B} - \frac{4DB'(\beta)}{B^2}) \\ &= -\frac{48\left[(6\beta - \frac{15}{2})\gamma + (\beta-2)^2) \cdot ((a - 2c_1 + c_2)\gamma - \frac{4}{3}(a - c_1)\right]}{B^2} \end{aligned} \tag{3.18}$$

Given the assumtpions and $\beta \in [0, 1]$, $(6\beta - \frac{15}{2})\gamma + (\beta - 2)^2$ is increasing with β. When $\beta = 1$, It is equal to $1 - \frac{3}{2}\gamma$ which is less than 0. Thus, $(6\beta - \frac{15}{2})\gamma + (\beta - 2)^2$ is always negative. In addition, $(a - 2c_1 + c_2)\gamma - \frac{4}{3}(a - c_1)$ is always positive given the assumptions.

Therefore, $\frac{dx_2^*}{d\beta} > 0$.

Recall Equation (3.10) and (3.11), the equilibrium output is given by,

$$q_1^* = \frac{a - 2c_1 + c_2 + (2-\beta)x_1^* - x_2^*}{3}$$

$$q_2^* = \frac{a - 2c_2 + c_1 + (2\beta - 1)x_1^* + 2x_2^*}{3}$$

Totally differentiate with respect to β,

$$\begin{aligned} \frac{dq_1^*}{d\beta} =& \frac{\partial q_1}{\partial x_1} \cdot \frac{dx_1}{d\beta} + \frac{\partial q_1}{\partial x_2} \cdot \frac{dx_2}{d\beta} + \frac{\partial q_1}{\partial \beta} \\ =& \frac{1}{3}\left[(2-\beta) \cdot \frac{dx_1^*}{d\beta} + (-\frac{dx_2^*}{d\beta}) + (-x_1^*)\right] \end{aligned}$$

Since $2 - \beta > 0$, $\frac{dx_1^*}{d\beta} < 0$ and $\frac{dx_2^*}{d\beta} > 0$, $\frac{dq_1^*}{d\beta} < 0$.

$$\begin{aligned} \frac{dq_2^*}{d\beta} =& \frac{\partial q_2}{\partial x_1} \cdot \frac{dx_1}{d\beta} + \frac{\partial q_2}{\partial x_2} \cdot \frac{dx_2}{d\beta} + \frac{\partial q_2}{\partial \beta} \\ =& \frac{1}{3}\left[(2\beta - 1) \cdot \frac{dx_1^*}{d\beta} + 2\,\frac{dx_2^*}{d\beta} + (2x_1^*)\right] \end{aligned}$$

Since $\frac{dx_2^*}{d\beta} > 0$, the sign of $\frac{dq_2^*}{d\beta}$ depends on the remaining part. Set $L(\beta) \equiv (2\beta - 1) \cdot \frac{dx_1^*}{d\beta} + 2x_1^*$.

$$\frac{dL(\beta)}{d\beta} = \frac{1296\gamma\left[\frac{27\gamma^2}{2} + (9\beta^2 - 22.5\beta - 3)\gamma + \beta^3 - 6\beta^2 + 12\beta - 2\right]}{-B^3} \cdot \frac{\left[(a - 2c_1 + c_2)\gamma - \frac{4}{3}(a - c_1)\right](\gamma - \frac{8}{9})}{-B^3}$$

Given the assumptions, $(\gamma - \frac{8}{9}) > 0$, $\left[(a - 2c_1 + c_2)\gamma - \frac{4}{3}(a - c_1)\right] > 0$ and $B > 0$.

$\left[\frac{27\gamma^2}{2} + (9\beta^2 - 22.5\beta - 3)\gamma + \beta^3 - 6\beta^2 + 12\beta - 2\right]$ is increasing in $\gamma > \frac{4}{3}$. When $\gamma = \frac{4}{3}$, it equals $b^3 + 6[(b - \frac{3}{2})^2 + \frac{3}{4}] > 0$. Therefore, $\frac{dL(\beta)}{d\beta} < 0$. The minimum $L(\beta)$ is attained at $\beta = 1$, $L(1) = \frac{54(\gamma - \frac{8}{9})((a-2c_1+c_2)\gamma - \frac{4}{3}(a-c_1))}{(9\gamma-4)^2(3\gamma-2)} > 0$.

Therefore, $\frac{dq_2^*}{d\beta} > 0$. ■

From the equilibrium R&D levels shown in (3.15) and (3.16), the first term captures the incentive effect explained in section 2, i.e., $\frac{4A}{B} > \frac{4C}{B}$ due to $c_1 < c_2$. That is, the incentive effect drives the large firm to engage more in R&D than the small firm. However, the second term containing β reflects the impact of spillover on R&D levels, named spillover effect. It is obvious that the sign of the second term in Eq (3.15) is negative while that in Eq (3.16) is positive, which implies that the spillover effect favors the high cost firm but harms the low cost firm. Therefore whether the large or small firm has higher total incentive to engage in R&D depends on the relative magnitude of two effects.

From Lemma 3.1, the spillover rate β has an opposite effect on the R&D levels chosen by two firms. It is intuitive that the large firm decrases the R&D level when its research output spills over to its rival to some extent. Since the large firm's output and the corresponding profit in equilibrium decreases with spillover, the firm has a lower marginal benefit from R&D, which leads to a lower equilibrium R&D level in the presence of spillover.

On the other hand, it is interesting to see that the small firm invests more in R&D with higher spillover. It chooses a higher R&D level even though it can benefit from the large firm by taking "free ride" without any own R&D.

From Lemma 3.1, it immediately follows that $x_1 - x_2$ is decreasing with β. If $\beta = 0$, as in the benchmark model, the large firm invests more than the small firm, $x_1^* > x_2^*$. While if $\beta = 1$, firm 1's equilibrium R&D level becomes minimum and firm 2's becomes maximum, which are given by,

$$x_1^*\Big|_{\beta=1} = \frac{2\left[3\gamma(a-2c_1+c_2)-4(a-c_1)\right]}{(9\gamma-4)(3\gamma-2)}$$

$$x_2^*\Big|_{\beta=1} = \frac{4\left[3\gamma(a-2c_2+c_1)+2(c_2-c_1)\right]}{(9\gamma-4)(3\gamma-2)}$$

If and only if the maximum x_2^* is larger than the minimum x_1^*, there is an intersection between $x_1^*(\beta)$ and $x_2^*(\beta)$ as β increases. With any β above the intersection, the small firm invests more. Then the following result is proceeded.

Proposition 3.1 *There is always a sufficiently high spillover parameter such that the small firm chooses to invest more than the large firm if (1) the initial cost gap is not too large (i.e., $\lambda < \frac{a-c_1}{5}$); (2) the initial cost gap is relatively large (i.e., $\frac{a-c_1}{5} < \lambda < \frac{a-c_1}{3}$) and the R&D efficiency is relatively small (i.e., $\frac{4(a-c_2)}{3(a-2c_2+c_1)} < \gamma < \frac{4(a-2c_1+c_2)}{3(5c_2-4c_1-a)}$).*

Proof: The difference between x_2^* and x_1^* at $\beta = 1$ is,

$$(x_2^* - x_1^*)\Big|_{\beta=1} = \frac{6\gamma(a-5c_2+4c_1)+8(a-2c_1+c_2)}{(9\gamma-4)(3\gamma-2)}$$

where $(9\gamma-4)(3\gamma-2) > 0$ and $8(a-2c_1+c_2) > 0$ given the assumptions.

If $a - 5c_2 + 4c_1 > 0$, i.e., $\lambda < \frac{a-c_1}{5}$, the above difference must be positive.

If $a - 5c_2 + 4c_1 < 0$, i.e., $\lambda > \frac{a-c_1}{5}$, that the above difference larger than zero requires $\gamma < \frac{4(a-2c_1+c_2)}{-3(a-5c_2+4c_1)}$. ■

Proposition 3.1 implies that the one-way spillover indeed drives the small firm to invest more than the large firm under certain conditions. It does not violate the assumption that the knowledge spills over from more R&D intensive firm to less one since the direction depends on the pre-R&D cost structure rather than the size of R&D investment. Moreover, the situation is analogous of leapfrogging which refers to the ability of developing or less developed countries to make a "quick jump in economic development". Only if the initial cost is large enough (i.e. $\lambda > \frac{a-c_1}{5}$) and the R&D efficiency is relatively low (i.e., $\gamma > \frac{4(a-2c_1+c_2)}{-3(a-5c_2+4c_1)}$), the incentive effect strictly dominates the spillover effect so that the large firm has higher incentive to engage in R&D even though its R&D fully flows to its rival.

Figure 3.1 shows the equilibrium R&D levels for each firm with $\beta = 0$ and $\beta = 1$ as the initial cost gap λ changes. In the benchmark setting (dashed lines), it is obvious that firm 1 always spends more than firm 2 and the difference increases as the cost gap increases. With $\beta = 1$, the figure shows that as long as the initial cost gap λ is not too big (around 22), firm 2 has higher incentive to engage in R&D than firm 1.

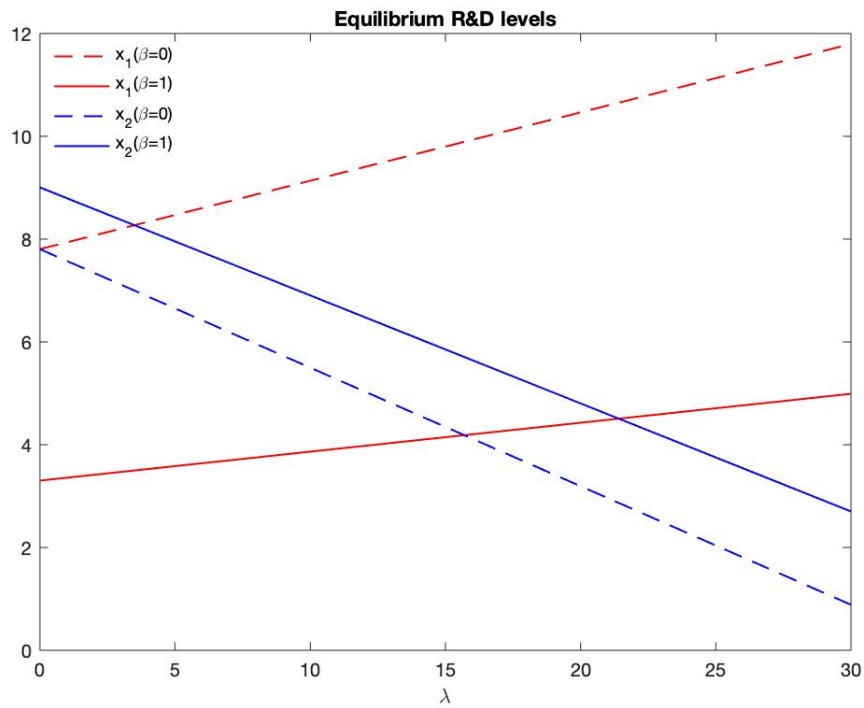


Figure 3.1: Equilibrium R&D levels ($a = 100, c_1 = 20, \gamma = 5$)

In Poyago-Theotoky (1996)'s multiplicative case, which assumes that R&D affects the cost of production in a multiplicative way, she considers the incentive effect and the effectiveness factor, and finds that the high cost firm always spends more on R&D and produces a larger R&D output than the low cost firm irrespective of the size of initial cost gap. It indicates that the effective factor always dominates the incentive effect. While the relative magnitude of spillover effect proposed in this chapter and incentive effect depends on the initial cost gap.

Due to one-way spillover, the effective cost reduction of the small firm consists of both its autonomous R&D output and the know-how from the large firm. Next, I identify the effect of β on the effective cost redution for firm 2.

Lemma 3.2 *The effective cost reduction for the small firm increases with β.* $\frac{dX_2^*}{d\beta} > 0$.

Proof: Totally differentiate $X_2^* = x_2^* + \beta x_1^*$ with respect to β,

$$\begin{aligned}\frac{dX_2^*}{d\beta} =& \frac{dx_2^*}{d\beta} + \beta \cdot \frac{dx_1^*}{d\beta} + x_1^* \\ =& \frac{72\gamma\,(1-\beta)(\beta + \frac{9\gamma}{2} - 3)\left[(a - 2c_1 + c_2)\gamma - \frac{4}{3}(a - c_1)\right]}{B^2} > 0\end{aligned}$$

given the assumptions and $c_1 < c_2$. ■

From Proposition 3.1, when $\beta = 1$, the small firm still chooses lower R&D levels than the large firm if the initial cost gap is quite large. However, how the post-R&D cost gap varies depends on the relative magnitude of the effective cost reduction instead of R&D level between firms. From Lemma 3.2, the effective cost reduction for the small firm monotically increases with β, therefore it must be greater than that for the large firm for some β.

With the one-way spillover, the post-R&D cost gap $c_2' - c_1' = (c_2 - X_2) - (c_1 - X_1) = (c_2 - c_1) + (X_1 - X_2)$. Therefore, comparing the post-R&D cost gap with the initial cost gap is equivalent to identifying the sign of $X_1 - X_2$. From

Lemmas 3.1 and 3.2, $\frac{dX_1}{d\beta} < 0$ and $\frac{dX_2}{d\beta} > 0$. Therefore, it immediately follows that the post-R&D cost gap $c_2' - c_1'$ is decreasing with β.

$$\frac{d(c_2' - c_1')}{d\beta} = \frac{dX_1}{d\beta} - \frac{dX_2}{d\beta} < 0$$

Compared with the initial cost gap $c_2 - c_1$, with $\beta = 0$, the benchmark model shows that the post-R&D cost gap is widened. With $\beta = 1$, the post-R&D cost gap is the smallest given by,

$$c_2' - c_1' = (c_2 - c_1) + (x_1 - (x_2 + x_1)) = (c_2 - c_1) - x_2$$

It is obviously smaller than the initial cost gap due to the fact that the small firm fully absorbs the R&D from the large firm. Since the cost gap is monotonically decreasing with the spillover rate, there must exist a threshold $\overline{\beta}$ where the cost gap between firms narrows for any $\beta > \overline{\beta}$.

Proposition 3.2 *The cost gap is narrowed by R&D if β exceeds $\overline{\beta} \in (0, 1)$ where*

$$\begin{aligned}\overline{\beta} =& \frac{1}{6\gamma(a - 2c_1 + c_2)} \cdot (9\gamma(a - 2c_1 + c_2) + 4(c_1 - c_2) \\ &- \sqrt{81\gamma^2(a - \frac{5}{3}c_2 + \frac{2}{3}c_1)(a - 2c_1 + c_2) - 24\gamma(c_1 - c_2)(a - 2c_1 + c_2) + 16(c_1 - c_2)^2)}\end{aligned} \tag{3.19}$$

Proof: The condition of narrowing the gap is equivalent to that firm 2's effective cost reduction is larger than firm 1's. Setting $X_2 > X_1$, one can get the threshold $\bar{\beta} \in (0, 1)$.

■

Proposition 3.2 shows the condition where the cost gap is narrowed, but it does not show which firm ends up with lower cost. Next, I focus on the condition where the small firm catches up with the large firm, that is, the small firm ends up with a lower cost.

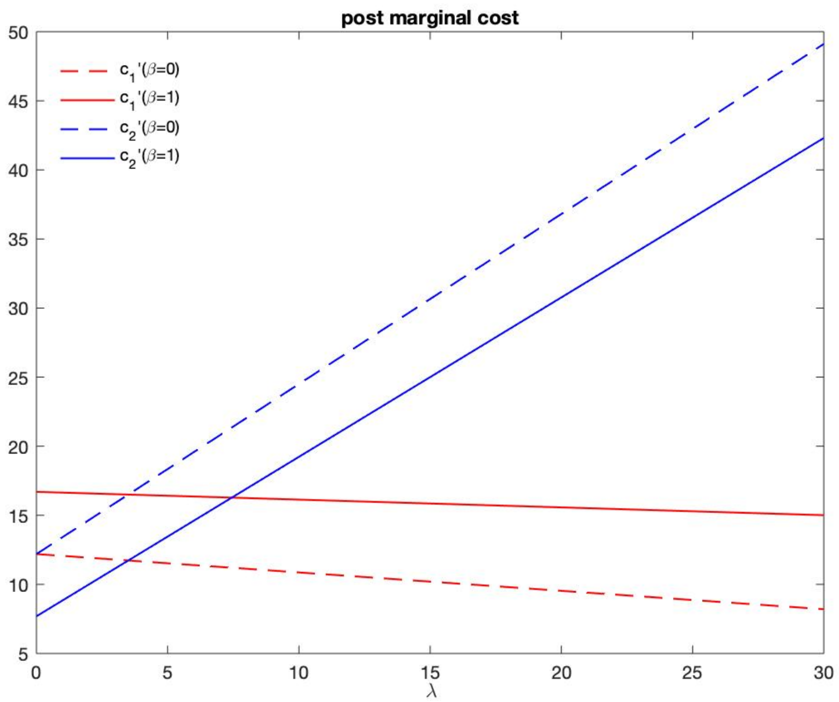
When $\beta = 1$, we know that the cost gap reaches its minimum, which is,

$$(c_2' - c_1')\Big|_{\beta=1} = \frac{6\gamma\left[\frac{9(c_2-c_1)(\gamma-\frac{4}{9})}{2} - (2a - c_1 - c_2)\right]}{(9\gamma - 4)(3\gamma - 2)}$$

If it is no more than zero, the small firm is able to catch up with the large firm, which yields,

$$\gamma \leqslant \frac{2(2a + c_2 - 3c_1)}{9(c_2 - c_1)}$$

Figure 3.2: marginal cost of production after R&D stage
$(a = 100, c_1 = 20, \gamma = 5)$



Combined with Assumption 3.3, such a γ exists only when the initial cost gap is relatively small. Figure 3.2 illustrates the new marginal cost of each firm with $\beta = 0$ and $\beta = 1$ as the initial cost gap λ increases. One can see that with full spillover rate, the initial high cost firm ends up as a lower cost firm only when λ is relatively small (around 8). Then, the following proposition is obtained.

Proposition 3.3 *If* $\frac{4(a-c_2)}{3(a-2c_2+c_1)} < \gamma \leqslant \frac{2(2a-3c_1+c_2)}{9(c_2-c_1)}$*, which requires the initial cost gap being relatively small, the small firm is able to catch up with and even reverse the position with the large firm when* $\beta = 1$*. Otherwise, the small firm never catches up with the large firm.*

In Poyago-Theotoky (1996)'s multiplicative case, although the high cost firm spends more on R&D, it still ends up with a higher cost than its rival, hence it is not able to overcome the initial asymmetry regardless of the initial size of the cost gap. While with one-way spillover, it is possible that the initial high cost firm ends up as the larger firm.

After analysing the individual R&D level, I proceed to the comparative statics of total R&D levels.

Lemma 3.3 *The total R&D levels chosen by two firms decrease with* β*.*
$\frac{dTX^*}{d\beta} < 0$

Proof: Totally differentiate $TX^* = x_1^* + x_2^*$ with respect to β,

$$\frac{dTX^*}{d\beta} = -\frac{36[(a-2c_1+c_2)\gamma - \frac{4}{3}(a-c_1)](\frac{9\gamma^2}{2} + (\beta^2+4\beta-10)\gamma + \frac{4(\beta-2)^2}{3})}{B^2}$$

Given the assumptions, $[(a-2c_1+c_2)\gamma - \frac{4}{3}(a-c_1)] > 0$. The sign of $\frac{dTX^*}{d\beta}$ depends on the remaining part in the numerator. Set $M(\beta) \equiv \frac{9\gamma^2}{2} + (\beta^2 + 4\beta - 10)\gamma + \frac{4(\beta-2)^2}{3}$.

$$\frac{dM(\beta)}{d\beta} = (2\gamma + \frac{8}{3})\beta + 4(\gamma - \frac{4}{3}) > 0$$

given the assumptions. Therefore, the minimum M is attained at $\beta = 0$, $M(0) = \frac{(9\gamma-8)(3\gamma-4)}{6} > 0$.

Therefore, $\frac{dTX^*}{d\beta} < 0$.

■

By decomposing the effect of β on total R&D levels, since $\frac{dx_1^*}{d\beta} < 0$ while $\frac{dx_2^*}{d\beta} > 0$, Lemma 3.3 indicates that β has a larger effect on the equilibrium R&D for the large firm than that for the small one. The increment in the small firm's level of R&D does not compensate for its competitor's lower investment in response to a change in the spillover parameter. Therefore, the total research levels chosen by two firms are the highest when there is no spillover, which is the sum of equilibrium R&D in the benchmark setting. As for the total R&D expenditure, since the research cost follows a quadratic form of R&D output, how the total R&D expenditure changes with β is not equivalent to the changes of total R&D levels.

Totally differenciate the total R&D expenditure with respect to β.

$$\frac{d[r(x_1)+r(x_2)]}{d\beta} = \gamma\left(x_1\frac{dx_1}{d\beta} + x_2\frac{dx_2}{d\beta}\right)$$

If the initial cost gap is relatively large, the total R&D expenditure decreases with β due to $x_1^* > x_2^*$ and Lemma 3.3. While if the initial cost gap is not that large, it is easy to check that $\left.\frac{d[r(x_1)+r(x_2)]}{d\beta}\right|_{\beta=0} < 0$ and $\left.\frac{d[r(x_1)+r(x_2)]}{d\beta}\right|_{\beta=1} > 0$, which indicates that the total R&D expenditure first decreases and then increases with β. The smaller the initial cost gap is, the faster the small firm increases its R&D level due to spillover. It is intuitive that when the initial cost gap is small, it is more likely for the small firm to catch up with the large firm, therefore the effect of knowledge spillover is more apparent. Hence, although the total R&D level decreases, the increment of small firm's research cost is much greater than the reduction of large firm's cost so that the total R&D expenditure increases when the spillover is large.

Lemma 3.4 *The total cost reductions of the firms due to R&D increase with β for $\beta \in (0, \hat{\beta})$ where $\hat{\beta} = \frac{\sqrt{(3\gamma-2)(3\gamma-4)}-(3\gamma-4)}{2} < \frac{1}{2}$:*
$\frac{dTC^*}{d\beta} > 0$ *with $\beta \in (0, \hat{\beta})$.*

Proof: Totally differentiate $TC^* = X_1^* + X_2^*$ with respect to β and set it equal to zero,

$$\frac{dTC^*}{d\beta} = -\frac{108\gamma[(a-2c_1+c_2)\gamma - \frac{4}{3}(a-c_1)][(3\beta-\frac{3}{2})\gamma+\beta^2-4\beta+2]}{B^2} = 0$$

where $[(a-2c_1+c_2)\gamma-\frac{4}{3}(a-c_1)] > 0$ given the assumptions. The roots of the above derivative are $\hat{\beta} = \frac{\sqrt{(3\gamma-2)(3\gamma-4)}-(3\gamma-4)}{2} > 0$, $\hat{\beta}_2 = \frac{-\sqrt{(3\gamma-2)(3\gamma-4)}-(3\gamma-4)}{2} < 0$. Since $\beta \in [0,1]$, $\hat{\beta}_2$ is excluded.

$$\begin{aligned}\hat{\beta}-\frac{1}{2} =& \frac{\sqrt{(3\gamma-2)(3\gamma-4)}-(3\gamma-3)}{2} \\ =& \frac{\sqrt{9\gamma^2-18\gamma+8}-\sqrt{9\gamma^2-18\gamma+9}}{2} < 0\end{aligned}$$

Therefore, $\frac{dTC^*}{d\beta} > 0$ with $\beta \in (0, \hat{\beta})$ where $\hat{\beta} < \frac{1}{2}$. ■

Decompose the effect of β on the total cost reduction.

$$\frac{d(X_1^*+X_2^*)}{d\beta} = \frac{d(x_1^*+x_2^*)}{d\beta} + \beta\frac{dx_1^*}{d\beta} + x_1^*$$

From Lemma 3.3, the first term $\frac{d(x_1^*+x_2^*)}{d\beta} < 0$. From Lemma 3.1, the second term $\beta\frac{dx_1^*}{d\beta} < 0$. The result shows that x_1^* (the only positive term) is large enough only when the spillover is small enough ($\beta < \hat{\beta}$), because the whole expression is positive.

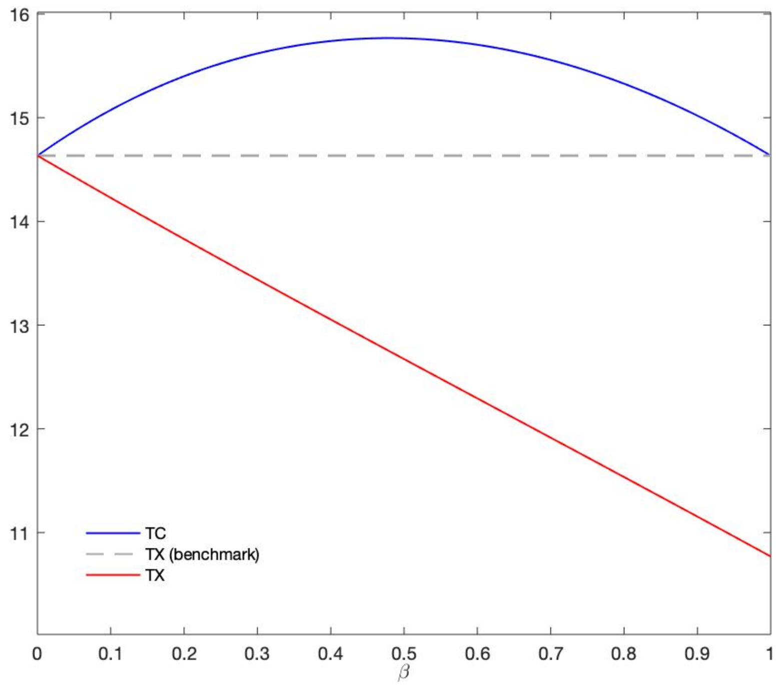
Compared with the symmetric spillovers, the total cost reduction optimal spillover rate $\hat{\beta} = \frac{1}{2}$. From the total industry output shown in Eq.(3.12), maximising the total industry output Q is equivalent to maximising the total cost reduction $x_1+x_2+\beta x_1$ and moreover, maximising consumer surplus is also equivalent to maximising the total industry output Q. Lemma 3.4 shows that with small spillover, i.e., $\beta < \frac{1}{2}$, the industry output increases so that the price of product decreases. Therefore, by introducing asymmetry in terms of both initial costs and spillover rates, from the consumer's point of view, spillover benefits consumers as long as it is a slightly smaller spillover β than in the symmetric case.

Proposition 3.4 *Any spillover rate β increases the total cost reduction with less R&D levels. i.e., $TC(\beta) \geqslant TC(0)$ and $TX(\beta) \leqslant TX(0)$ for all β.*

Proof:

Lemma 3.3 states that the total R&D levels decreases with β (the red line in Figure 3.3) while lemma 3.4 shows the concavity of total cost reduction in β

Figure 3.3: Comparison between total cost reductions and total R&D levels ($a = 100, c_1 = 20, c_2 = 30, \gamma = 5$)



(the blue curve in Figure 3.3) with $TC(1) = TC(0) = \frac{8a-4c_1-4c_2}{9\gamma-4}$ for all parameter values. The dashed line denotes the total R&D levels, i.e., the total cost reductions in the benchmark ($\beta = 0$). It is evident that for any spillover, the total cost reduction is higher than that without spillover but the total R&D levels are lower than those without spillover.

■

The total cost reduction is a concave function of $\beta \in (0, 1)$. Surprisingly, the total cost reduction when $\beta = 0$ is identical with that when $\beta = 1$ even though firms are asymmetric. That is to say, when the large firm's R&D fully spills over to its rival, the increment in the effective cost reduction for firm 2 exactly offsets the decrease in cost reduction for firm 1, which means given any spillover rate, the total cost reduction with spillover is larger than without spillover. In addition, Lemma 3.3 shows that the total R&D levels are dcreasing in β, that is, the total R&D costs are decreasing in β. Combining Lemmas 3.3 and 3.4, Proposition 3.3 indicates that the one-way spillover decreases the R&D costs while always increases the total cost reduction, industry outputs, and consumer surplus, compared with the benchmark model.

Consider the equilibrium overall profits, which is the difference between the operative profit in the second stage and the R&D cost in the first stage. Since the large firm decreases the R&D level, and further decreases the equilibrium output, both the operative profits and R&D costs decrease. Conversely, both the operative profits and R&D costs increase for the small firm. That is to say, β has same directional effects on these two parts for each firm while the effects are opposite between the firms. Therefore, how the equilibrium overall profit varies depends on which part is stronger. The result is shown as follows.

$$\begin{aligned}\Pi_1^* =& \pi_1^* - \frac{\gamma x_1^{*2}}{2} \\ =& \left(\frac{a - 2c_1 + c_2 + (2-\beta)x_1^* - x_2^*}{3}\right)^2 - \frac{\gamma x_1^{*2}}{2}\end{aligned} \tag{3.20}$$

$$\begin{aligned}\Pi_2^* =& \pi_2^* - \frac{\gamma x_2^{*2}}{2} \\ =& \left(\frac{a - 2c_2 + c_1 + (2\beta-1)x_1^* + 2x_2^*}{3}\right)^2 - \frac{\gamma x_2^{*2}}{2}\end{aligned} \tag{3.21}$$

Lemma 3.5 *The effects of change in spillover rate on equilibrium overall profits is,* $\frac{d\Pi_1^*}{d\beta} < 0$, $\frac{d\Pi_2^*}{d\beta} > 0$.

Proof: Totally differentiate (3.20) and (3.21) with respect to β,

$$\frac{d\Pi_1}{d\beta} = \frac{\partial \Pi_1}{\partial x_1} \cdot \frac{dx_1}{d\beta} + \frac{\partial \Pi_1}{\partial x_2} \cdot \frac{dx_2}{d\beta} + \frac{\partial \Pi_1}{\partial \beta}$$

where the first term is zero in equilibrium.

$$\frac{\partial \Pi_1}{\partial x_2} = \frac{2(a - 2c_1 + c_2 + (2-\beta)x_1 - x_2)}{9} \cdot (-1) < 0$$

$$\frac{\partial \Pi_1}{\partial \beta} = \frac{2(a - 2c_1 + c_2 + (2-\beta)x_1 - x_2)}{9} \cdot (-x_1) < 0$$

$$\frac{dx_2}{d\beta} > 0$$

Therefore, $\frac{d\Pi_1}{d\beta} < 0$.

$$\begin{aligned}\frac{d\Pi_2}{d\beta} =& \frac{\partial \Pi_2}{\partial x_2} \cdot \frac{dx_2}{d\beta} + \frac{\partial \Pi_2}{\partial x_1} \cdot \frac{dx_1}{d\beta} + \frac{\partial \Pi_2}{\partial \beta} \\ =& 0 + \frac{2(a - 2c_2 + c_1 + (2\beta - 1)x_1 + 2x_2)}{9} \cdot \left[(2\beta - 1) \cdot \frac{dx_1^*}{d\beta} + 2x_1^*\right]\end{aligned}$$

where $\frac{2(a-2c_2+c_1+(2\beta-1)x_1+2x_2)}{9} > 0$ given the assumptions. I defined $L(\beta) \equiv (2\beta - 1) \cdot \frac{dx_1^*}{d\beta} + 2x_1^*$ in the proof of Lemma 3.1 and proved it larger than zero.

Therefore, $\frac{d\Pi_2}{d\beta} < 0$. ■

Lemma 3.5 shows that the effect of β on the profits is stronger than that on the R&D costs for both firms. Moreover, similar as the individual R&D levels and Cournot outputs, β has opposite effects on individual firm's overall profits, therefore how the total producer surplus changes depends on which one dominates. Since the producer surplus follows a fourth degree polynomial equation with respect to β, it is hard to identify the sign of the first derivative. Therefore, I use a numerical solution to get the following observations[4].

Observation 1 If $a - 4c_2 + 3c_1 < 0$, i.e., $\lambda > \frac{a-c_1}{4}$, the producer surplus always decreases with β. That is to say, the effect on the large firm dominates that on the small firm.

Observation 2 if $a - 5c_2 + 4c_1 > 0$, i.e., $\lambda < \frac{a-c_1}{5}$, the spillover that maximises the producer surplus is larger than $\frac{1}{2}$. That is to say, the increment of small firm's overall profit is larger than the loss of large firm's as long as β is not too high.

3.4 Welfare Analysis

From the previous analysis, the threshold spillover rate that narrows the cost gap $\bar{\beta}$, and maximises the total cost reduction $\hat{\beta}$ are obtained. This section

[4]The values of the parameters are calibrated to meet all the restrictions placed in this analysis. I also try to prove them analytically which is shown in appendix.

focuses on the spillover rate that maximises the total welfare and compares these three measures. Because one might consider that an ideal spillover rate from the society's point of view would reduce the cost gap and the dominance of the larger firm. Here I test this logic. Therefore, this section aims to look at the issue of how R&D affects the heterogeneity, then given that increasing or decreasing heterogeneity, whether it is necessarily good compared to what comes from the welfare analysis, in order to provide the policy implication. To be specific, the question is whether it is better from the welfare point of view that the firms become more identical or that the gap even widens.

To find the socially optimal spillover rate, define the social welfare in the usual way as the sum of firms' profit and consumer surplus,

$$W(x_1, x_2, q_1, q_2) = \frac{1}{2}(q_1 + q_2)^2 + q_1^2 + q_2^2 - \frac{\gamma x_1^2}{2} - \frac{\gamma x_2^2}{2} \tag{3.22}$$

Given the Cournot equilibrium in the production stage of the game and the equilibrium R&D levels in the R&D stage of the game, one can write social welfare as a function of the spillover rate by substituting the Cournot outputs (Eq.(3.10) and (3.11)) into $W(x_1, x_2, q_1, q_2)$, which yields,

$$W(\beta) = \frac{1}{B^2}\left[54\gamma\left(W_1\beta^4 - W_2\beta^3 + W_3\beta^2 + W_4\beta + W_5\right)\right] \tag{3.23}$$

where

$$
\begin{aligned}
W_1 =&(\gamma - \frac{16}{27})(a-c_1)^2 \\
W_2 =&6(a - \frac{2c_1}{3} - \frac{c_2}{3})(\gamma - \frac{16}{27})(a-c_1) \\
W_3 =&(-\frac{13a^2}{3} + (\frac{13c_1}{3} + \frac{13c_2}{3})a - \frac{c_1^2}{3} - \frac{c_2^2}{3} - \frac{11c_1c_2}{3})\gamma^2 \\
&+ (17a^2 + (-\frac{64c_1}{3} - \frac{38c_2}{3})a + \frac{32c_1c_2}{3} + \frac{16c_1^2}{3} + c_2^2)\gamma \\
&- \frac{22a^2}{27} + (\frac{32c_1}{3} + \frac{160c_2}{27})a - \frac{128c_1c_2}{27} - \frac{80c_1^2}{27} - \frac{16c_2^2}{27} \\
W_4 =&(\frac{40a^2}{3} + (-\frac{31c_1}{3} - \frac{49c_2}{3})a + \frac{11c_1c_2}{3} + \frac{10c_1^2}{3} + \frac{19c_2^2}{3})\gamma^2 \\
&+ (-\frac{224a^2}{9} + (\frac{212c_1}{9} + \frac{236c_2}{9})a - \frac{100c_1c_2}{9} - \frac{56c_1^2}{9} - \frac{68c_2^2}{9})\gamma \\
&+ \frac{256}{27}(a - \frac{c_1}{2} - \frac{c_2}{2})^2 \\
W_5 =&6(\gamma - \frac{4}{9})((a^2 + (-c_1 - c_2)a + \frac{11c_1^2}{8} - \frac{7c_1c_2}{4} + \frac{11c_2^2}{8})\gamma^2 \\
&+ (-\frac{8a^2}{3} + (\frac{8c_1}{3} + \frac{8c_2}{3})a - \frac{13c_1^2}{6} + \frac{5c_1c_2}{3} - \frac{13c_2^2}{6})\gamma \\
&+ \frac{16a^2}{9} + (-\frac{16c_1}{9} - \frac{16c_2}{9})a + \frac{8c_1^2}{9} + \frac{8c_2^2}{9})
\end{aligned}
$$

The problem of the social planner is now to choose the spillover rate β that maximizes total welfare, as given by Eq (3.23), that is,

$$\beta^* = \underset{\beta}{\operatorname{argmax}}\, W(\beta). \tag{3.24}$$

Owing to the difficulty of figuring out a fourth degree polynomial equation, and since the aim is to compare the magnitudes among cost gap threshold $\overline{\beta}$, total cost reduction optimal $\hat{\beta}$ and socially optimal β^*, it is not necessary to solve for the closed form solution of socially optimal β^*. Therefore I used a numerical solution in order to show how the total welfare varies with β and the comparison.

Before the simulation, the parameters should be calibrated. In addition to the restrictions on parameters implied by SOCs and the stability condition which are shown in Assumptions, there are two important conditions in the problem setup that must be satisfied. These conditions are: $Q \leqslant a$, and $x_2 + \beta x_1 \leqslant c_2$, which include endogenous variable and therefore it is impossible to adjust the parameters before solving the problem in order to make the above conditions hold.

Therefore, I run the series of simulations and sensitivity tests to figure out which set of parameters will satisfy the above two conditions. In the simulations, I have fixed a, the parameter of the demand function capturing the size of the market at 100. In addition, γ, the R&D efficiency parameter is set at 5. Changing these parameter values does not lead to qualitatively different results, as both a and γ have a scaling effect only. Furthermore, I have fixed the initial marginal cost of firm 1, which is the large firm, at $c_1 = 20$. Firm 2's initial marginal cost c_2 varies from 20 to 50, that is, $\lambda \in [0, 30]$. Table 3.1 explains the simulation input parameters. Specifically, I assume a difference value of $\lambda = 10$ to represent 'small asymmetry' in initial marginal cost and a difference value of $\lambda = 30$ to indicate 'large asymmetry'. I derive the following set of results.

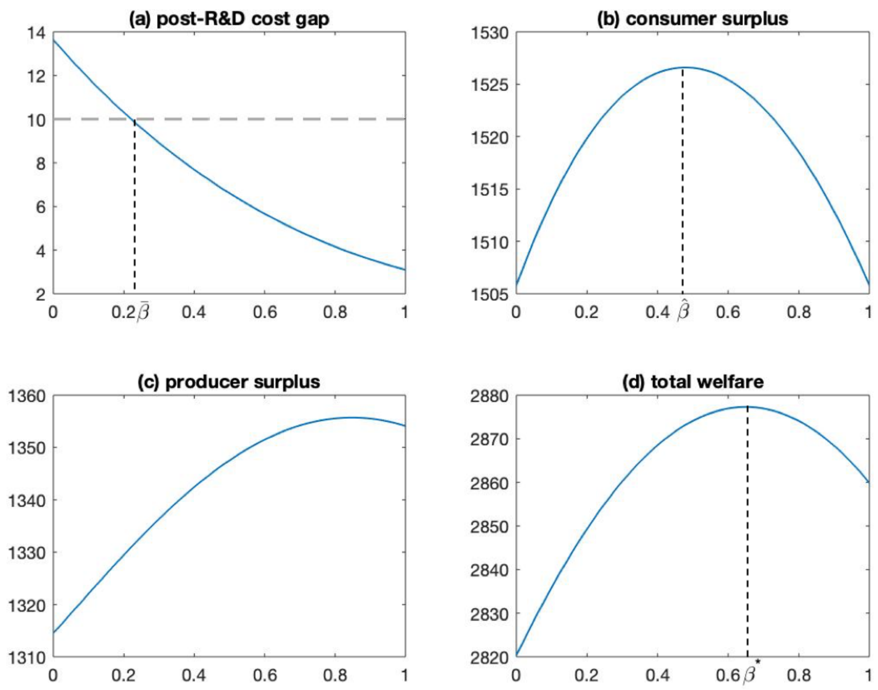
Table 3.1: Simulation input parameters

Parameters	Values	Explanations
c_1	20	firm 1's initial marginal cost
c_2	$c_1 + \lambda$	firm 2's initial marginal cost
λ	[0,30]	initial cost gap
β	[0,1]	spillover rate
γ	5	R&D efficiency
a	100	market size

Result 1 If the initial cost gap is relatively small (i.e., $\lambda = 10$), $\overline{\beta} < \hat{\beta} < 0.5 < \beta^*$.

Figure 3.4 shows how the post-R&D cost gap, the consumer surplus, the producer surplus and the total welfare change with β when the initial cost gap is $\lambda = 10$. In Figure 3.4(a),the grey dashed line denotes the initial cost gap level.

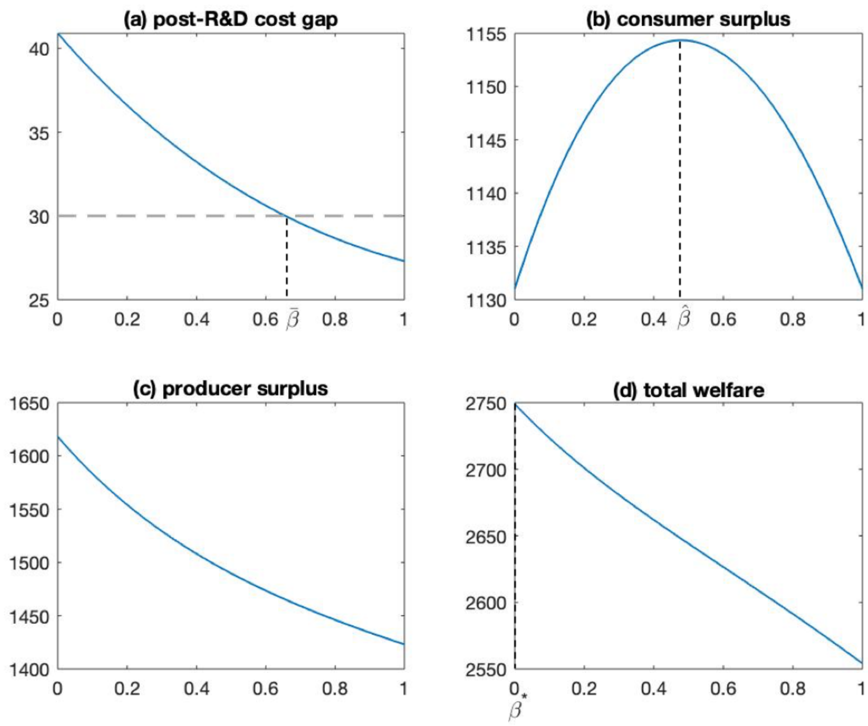
Figure 3.4: Optimal β with small asymmetry



It is obvious that for any $\beta > \bar{\beta}$ where $\bar{\beta} = 0.22$, the post-R&D cost gap is narrowed by R&D. Figure 3.4(b) implies that the total output Q is increasing and then decreasing in β. Equation (3.12) tells that maximising the total output is equivalent to maximising the total cost reduction TC and Lemma 3.4 shows the total cost reduction is maximised at $\hat{\beta} < \frac{1}{2}$. Therefore, $\hat{\beta}$ which is the optimal total cost reduction spillover rate also maximises the consumer surplus. Figure 3.4(c) illustrates that the producer surplus also exhibits a concavity in β.

Since both consumer surplus and producer surplus exhibit a concavity in β, the total welfare must follow concavity which is shown in Figure 3.4(d), and it is maximised at $\beta^* = 0.65$. Compared with the benchmark ($\beta = 0$), the consumer surplus, producer surplus and total welfare are always higher for any β. The intuition is as follows. Due to the one-way spillover, the effective cost reduction decreases for the large firm while increases with β for the small firm. If the initial

Figure 3.5: Optimal β with large asymmetry



cost gap is relatively small, the increment of the small firm's profit is greater than the loss of large firm's profit due to the convexity of the profit function in its own cost of production. At the same time, the consumer surplus is increasing and total R&D costs are decreasing, all these elements work together and lead to a higher total welfare.

The comparison between the three measures ($\overline{\beta} < \hat{\beta} < \beta^*$) implies that the cost gap decreases in socially optimal and total cost reduction optimal spillover rate, which means that if the intial cost gap is relatively small, it is a good thing for both consumers and the society that the firms become more similar. That is to say, the society would prefer firms to be more competitive if the gap between them is not too large.

Result 2 If the initial cost gap is relatively large (i.e., $\lambda = 30$), $\beta^* < \hat{\beta} < 0.5 < \bar{\beta}$.

Figure 3.5 shows how the post-R&D cost gap, the consumer surplus, the producer surplus and the total welfare change with β when the initial cost gap is $\lambda = 30$. Similarly, in Figure 3.5(a), the grey dashed line denotes the initial cost gap level. It is obvious that for any $\beta > \bar{\beta}$ where $\bar{\beta} = 0.66$, the post-R&D cost gap is narrowed by R&D. Figure 3.5(b) implies that the consumer surplus is maximised at the spillover rate that maximises the total cost reduction, which is $\hat{\beta} < \frac{1}{2}$. However, figure 3.5(c) illustrates that the producer surplus is decreasing with β.

Figure 3.5(d) shows that the spillover is always detrimental for the social welfare (i.e., $\beta^* = 0$) if the initial cost gap is already significant. It is intuitive that since the initial cost gap is large, even if the large firm's R&D fully flows to the small firm, the gap between them is still relatively large. Due to the one-way spillover, the effective cost reduction decreases for the large firm while increases for the small firm. The decrement to the large firm's productive profit is much higher than the increment to the small firm's so that the overall profits are decreasing with spillover. Although the consumer surplus is increasing and total R&D costs are decreasing, it cannot compensate the loss of overall profits, therefore the total welfare declines with β.

The comparison between the three measures ($\beta^* < \hat{\beta} < \bar{\beta}$) implies that the cost gap increases in socially optimal and total cost reduction optimal spillover rate, which means that if the intial cost gap is already significant, it is not a bad thing for both consumers and the society that the gap between firms further widens. From the society's perspective, the policy that can affect the spillover rate would not be to minimise the cost gap, since that is not socially optimal.

3.5 Conclusions

This chapter has investigated the properties of a two-stage R&D model that departs from the standard setting by adopting firm heterogeneity. The majority of existing literature states that the large firm is more innovative so that the gap between them even widens. However, empirical studies show that the small firm may invest more than the large firm in some industries. Therefore, I mainly focus on the possible factor that drives the small firm to invest more in R&D to explain the empirical data and how the factor affects the R&D choices of asymmetric firms and the subsequent market structure.

One factor considered in this chapter is the one-way spillover rate where only the large firm's R&D flows to the small firm due to *ex ante* asymmetric setting. I find that it indeed increases the small firm's incentive to invest in R&D even though it can benefit from the large firm by taking free ride, and the cost gap between them is narrowed. Moreover, from the welfare point of view, I find that if the initial cost asymmetry is relatively small, the society prefers the firms to be even more similar. While if the initial cost asymmetry is already significant, it is not a bad thing for both consumers and the society that the gap further widens.

There are some possible extensions of this study. First, since the one way spillover plays a detrimental role for the large firm, it is interesting to see whether it is in the best interest of the large firm to guard the spillover rate β to make sure it is not too large by paying a guarding fee if β were to be a choice variable for the large firm. Second, each firm chooses its R&D output which spills over from the large firm to the small firm in this paper. Another commonly used model is when each firm chooses R&D input (i.e. investment) and there is investment spillover. Whether the same results hold and the comparison between approaches is a possible extension. Third, the one-way spillover is actually an extreme case of two-way asymmetric spillovers between the asymmetric firms. The model can be extended to employ two-way asmmetric spillovers and get more general results.

Finally, since the most employed R&D cost function $\frac{\gamma x^2}{2}$ indicates that the firms spend the same amount of money to achieve the cost reduction x irrespective of the firm size. I think it is not realistic since it is more complicated for the more advanced firm to enhance its technology further. Therefore, a modified R&D cost function which is negatively related to the initial cost c_i might be another possible factor to explain why the small firm invests more than the large firm. I have tried to organise such a R&D cost function which is shown in appendix. But it still needs a further analysis.

Appendix

Proof of the sign of equilibrium R&D levels:

1) Benchmark model

Remind the equilibrium R&D level in Eq.(3.8),

$$x_1^b = \frac{4[(3\gamma - 4)a + 3\gamma c_2 + (4 - 6\gamma)c_1]}{(3\gamma - 4)(9\gamma - 4)}$$
$$x_2^b = \frac{4[(3\gamma - 4)a + 3\gamma c_1 + (4 - 6\gamma)c_2]}{(3\gamma - 4)(9\gamma - 4)}$$

The denominator $(3\gamma - 4)(9\gamma - 4) > 0$ given assumption 3.2. Since $c_1 < c_2$, $x_1^b > x_2^b$ hold. For x_2^b, to make $(3\gamma - 4)a + 3\gamma c_1 + (4 - 6\gamma)c_2 = 3\gamma(a - 2c_2 + c_1) - 4(a - c_2) > 0$, γ should be larger than $\frac{4(a-c_2)}{3(a-2c_2+c_1)}$. Therefore I make assumption 3.3.

Hence, both x_1^b and x_2^b are positive given assumptions.

2) In the presence of one-way spillover

Remind the equilibrium R&D level in Eq.(3.15) and (3.16),

$$x_1^* = 2(2 - \beta)\frac{A}{B} = \frac{4A}{B} - 2\beta \cdot \frac{A}{B}$$
$$x_2^* = 4\frac{[C + D]}{B} = \frac{4C}{B} + 4 \cdot \frac{D}{B}$$

where

$$
\begin{aligned}
A =&(3\gamma-4)a+(4-6\gamma)c_1+3\gamma c_2\\
B =&(9\gamma-4)(3\gamma-4)-2\beta\left[3\gamma(\beta-4)+4\right]\\
C =&(3\gamma-4)a+(4-6\gamma)c_2+3\gamma c_1\\
D =&2\beta\left[(3-\beta)a-(2-\beta)c_1-c_2\right]
\end{aligned}
$$

For A,

$$
\begin{aligned}
A =&(3\gamma-4)a+(4-6\gamma)c_1+3\gamma c_2\\
=&(3\gamma-4)\left(a-\frac{6\gamma-4}{3\gamma-4}c_1+\frac{3\gamma}{3\gamma-4}c_2\right)\\
=&(3\gamma-4)\left(a-2c_2+c_1+\frac{9\gamma-8}{3\gamma-4}(c_2-c_1)\right)>0
\end{aligned}
$$

For B,

$(9\gamma-4)(3\gamma-4)>0$ given the assumptions. $[3\gamma(\beta-4)+4]$ is obviously decreasing in γ. Therefore the maximum is attained at $\gamma=\frac{4}{3}$, which is negative $(4\beta-12<0)$. Therefore, $B>0$.

For C,

$$
\begin{aligned}
C =&(3\gamma-4)a+(4-6\gamma)c_2+3\gamma c_1\\
=&3\gamma(a-2c_2+c_1)-4(a-c_2)
\end{aligned}
$$

$C>0$ requires $\gamma>\frac{4(a-c_2)}{3(a-2c_2+c_1)}$, which is made in assumption 3.3.

For D, given assumption 3.1, $a>2c_2$ and $c_1<c_2$,

$[(3-\beta)a-(2-\beta)c_1-c_2]>[(3-\beta)\cdot 2c_2-(2-\beta)c_2-c_2]>0$. Therefore, $D>0$.

Hence, the equilibrium R&D level for both the large firm and the small firm is positive.

■

Decreasing returns R&D cost function

The factor in this section considers the R&D cost function, the quadratic function in Eq. (3.1), which is most utilized in the existing R&D literature. It is associated to the R&D intensity x_i and the R&D efficiency γ. Based on asymmetric firms, Lukatch and Plasmans (2000) considers asymmetric R&D efficiencies. They assumes that the large firm has a lower marginal cost of production and also a lower per unit marginal cost of research, i.e., $c_1 < c_2$ and $\gamma_1 < \gamma_2$ hold simultaneously. That is to say, the large firm's research expenditure is even lower than the small firm's in order to realize a same cost reduction, which further favors the large firm to engage in R&D. However, this is contradict to the reality. It is much more complicated for the large firm to do the research and enhance its technology further than the small one, which means that the research costs function should be negatively related to the initial marginal cost c_i. That is to say, with the same research intensity x_i, the resarch cost for the large firm is higher than that for the small one. Accordingly, reconstruct the research costs function $r(c_i, x_i)$ which is characterized by decreasing returns due to different initial cost,

$$r(c_i, x_i) = \frac{\gamma(x_i + z - c_i)^2}{2} - \frac{\gamma(z - c_i)^2}{2} \tag{3.25}$$

where $r(x_i) \geqslant 0$, $r'(x_i) > 0$, $r'(c_i) < 0$. z is a constant slightly larger than c_i. Following the same process as before, the best response function of x_i given x_j is,

$$x_i(x_j) = \frac{4(a - 2c_i + c_j - x_j) - 9\gamma(z - c_i)}{9\gamma - 8} \tag{3.26}$$

Similarly, the second-order condition requires $\gamma > \frac{8}{9}$. And the stablitiy condition requires $\gamma > \frac{4}{3}$.

The equilibrium research level is,

$$x_i^e = \frac{4(a - c_i) - 9\gamma(z - c_i)}{9\gamma - 4} \tag{3.27}$$

The difference between x_1^e and x_2^e is,

$$x_1^e - x_2^e = c_1 - c_2 < 0 \tag{3.28}$$

which is always smaller than zero, that is, $x_1^e < x_2^e$. It indicates that the small firm always has higher incentives to engage in R&D than the large one irrespetive of γ, compared with the previous case in which the research costs function is only related to research level x_i. The cost gap after R&D stage turns to $c_2' - c_1' = 0$. It indicates that the small firm just catches up with the large firm and two post symmetric firms engage in Cournot competition with the same marginal costs in the production stage.

The results point towards the necessity of empirical work to help us ascertain the exact nature of R&D cost functions.

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Chapter 4

Dynamic R&D competition with stochastic outcomes

4.1 Introduction

Research and development (R&D) activity has regarded as the fundamental engine to create innovations. The key features of R&D activity are risky(uncertain), costly and continuous. This chapter presents a model of R&D process in which there is both R&D uncertain outcome and dynamic interaction between competitors as the game proceeds. There are two main approaches in the existing literature how the uncertainty of R&D process is modelled. One is characterized by technological uncertainty, which emphasizes how quickly a new technology arrives, in other words, the length of time required for a successful innovation. (Loury, 1979; Dasgupta and Stiglitz, 1980; Lee and Wilde, 1980; Malueg and Tsutsui, 1997; Erkal and Piccinin, 2010). The other one, adopted in this chapter, assumes stochastic outcomes, that is, whether the firm succeeds or fails to make a progress given an investment in R&D (Aoki, 1991; Choi, 1993; Matsumura, 2003; Horner, 2004; Xing, 2018). As for dynamic interaction considered in this chapter, it refers to how the R&D choice of firms interact in a multi-period game.[1] Since most of the existing literature focuses on a single period game, it is not quite

[1]Dynamic interaction can be either a multi-period game or a repeated game. I focus on a multi-period game in this chapter since firms make a decision in each period based on the outcome of last period.

consistent with the common feature of R&D process in the realistic situation. The incentives to invest in R&D might vary as the game proceeds, according to the position of a firm in the game relative to its rival and relative to the end of the game (Zizzo, 2002).

As I mentioned, R&D is an uncertain, costly and continuous process. The majority of the existing literature focuses on one or the other of R&D features, while this paper combines two important and common features of R&D process to bridge the research gap. The aim of this chapter is to see how the firms' incentives to engage in R&D are affected in a multi-period game in the presence of uncertainty. Moreover, firms are assumed identical at the beginning of the competition, which means there is no technological difference between firms. Due to the uncertain outcomes and multi-period competition, this chapter also aims to identify how the technological difference between the firms evolves.

Broadly speaking, the theoretical models on R&D competition are divided into the patent race, tournament[2] and the incremental investment models[3]. Patent race model, which is widely employed to study the R&D competition, assumes that only the winner of the race will be awarded a prize. Most patent race literature focuses on investigating the effects of technological uncertainty. In those papers, the technological uncertainty is modelled as an exponential process. Early works show that firms decide on their R&D investment once and forever, due to the memoryless feature of the exponential distribution (Loury, 1979; Lee and Wilde, 1980; Malueg and Tsutsui, 1995). Reinganum (1981) allows that firm's effort also to depend on the rival's knowledge levels, and shows that firms increase their R&D efforts as time goes on. Malueg and Tsutsui (1997) adopt Reinganum's (1981) setup, with the addition that firms have imperfect knowledge about the exponential distribution governing the arrival timing of the new technology. They

[2]Patent race is firstly introduced by Gibert and Newbery (1982). Baye and Hoppe (2003) establish the strategic equivalence between tournament and patent race games, therefore the tournament approach is ignored here.

[3]Incremental investment model is first proposed by Athey and Schmutzler (2001).

find that firms reduce their R&D efforts as time passes without success since they are more pessimistic about the final success. Considering dynamic patent race without uncertainty, that is, strategic interaction, Fudenberg et al (1983) and Harris and Vickers (1985) propose a strong result of 'ϵ-preemption', that is, a small ϵ advantage at the start of the race is sufficient to ensure that the follower drops out from the race immediately. Fershtman and Markovich (2010) focus on the firms' asymmetric ability in R&D and use a two-firm multistage R&D race model to analyze the effect of patents, imitations and licensing arrangements on the speed of innovation.

Later work tried to combine both technological uncertainty and dynamic setup to achieve a better characterization of patent race model. Harris and Vickers (1987) develop two models. The first one is a 'tug-of-war', where a prize is won by the first player to achieve a given lead over its rival. In the model, the player that is ahead, makes greater efforts than the follower. The second model is a 'multi-stage race', where the winner is the first of the two players to make a given number of advances. In this model, the leader makes greater efforts than the follower if the leader is close to success in the sense of having no more than two stages to go. Efforts are greater when the gap between the competitors diminishes. Zizzo (2002) designs an experimental test of Harris and Vickers' model, but the experiment provides only limited support for the theory. It illustrates that tied competitors invest more when nearer to the end of the race and also, leaders did not invest more than followers.

The other model of R&D competition, incremental investment model, emphasizes the non-exclusivity of the new technology and non-uniqueness of the prize. Specifically, firms in the same industry may be located at different technological levels along a certain direction, in which technological progress is achieved step by step with multiple prizes. Each competitor can independently acquire technological progress, that is to say, the winner is no longer unique, but a follower cannot overtake the leader with a single innovation. This type of competition coincides with one of the main features in this paper, multi-period game. Athey and Schmutzler (2001) build a model of dynamic oligopolistic competition with ongoing investment, but with deterministic R&D process. They provide circumstances under which the leading firm tends to invest more so that the gap between them widens. With stochastic R&D process, Aoki (1991) and Horner (2004) present a model of dynamic competition between two firms that repeatedly engage in R&D in an endless game. Aoki (1991) focuses on the dynamic stochastic R&D and proposes a basic idea without characterizing the equilibrium. The intuitive result with stochastic R&D is that a leader may choose not to invest if the probability of success is small, thereby favoring the follower. Horner (2004) builds a model in which the R&D investment choice is restricted to two levels, either Higher effort level or Low effort level. Besides, in every period, the payoff is restricted to two constant values, R for the leader and $-R$ for the follower. He derives the Markov perfect equilibria and finds that firms exert the high level effort under two distinct circumstances: while sufficiently ahead, to outstrip their rival and secure a durable leadership; while behind, to regain leadership and prevent the situation from worsening to the point where their rival outstrips them.

In this chapter, I employ incremental investment framework in order to capture dynamic nature of R&D, and combine it with stochastic R&D outcomes, to capture the uncertainty. Following Horner (2004), I define the state of the game, i.e., position, as the difference between the total number of successes of firm i and those of its rival, which also represents the technological difference. There are 3 main differences from Horner (2004)'s paper. First, two initially symmetric firms compete for a finite number of periods, which means the deadline of the game is exogenous. The model is more applicable to markets where there is an end-period, for example, development of Covid vaccine. Besides, it is more related to some government contracts which sets the deadline for a successful innovation, therefore firms are asked to engage in R&D in a finite time horizon. Second, in each period, both firms simultaneously choose their own R&D level which is a continuous variable instead of discrete levels as in Horner (2004), and the probability of success increases with the investment. Third, the payoff of each firm in each period depends on the relative position. That is, I assume the payoff is an increasing function in position, which implies that the firm that is more advanced earns more. While Horner (2004) merely assumes that the payoff for the leader and follower is two constant values. Since the innovation is a risky, costly and continuous process, it cannot be simply modelled as a repeated game for such a market structure. When a firm makes its own decision on R&D in each period, it is not only maximizing the periodic profit, but also takes into account how the R&D investment in this period affects the following periods. More precisely, the history determines the choice and the corresponding outcomes. The model covers both symmetric and asymmetric situations[4] and captures both rivalry and dynamic interactions[5].

The main analytical results are obtained for the simplest 2-period game. Focusing on the same period, this chapter ranks the equilibrium R&D investments in different situations. That is, the firm invests the most if it is one lead ahead of its rival, followed by the situation where the firms are at the same technological level, and then the firm invests the least if it is one lag behind its rival. Furthermore, the corresponding equilibrium profit shows the same ranking as the R&D investments. It is consistent with Grossman and Shapiro (1985) and it is analogous of another proposition in the R&D literature, according to which the firm with lower initial costs in a Cournot duopoly has a higher incentive than its rival to engage in R&D. Due to the stochastic property, the game has various end outcomes with different probabilities, which shows that the probability of firms being asymmetric rises continuously as the game proceeds, implying that two firms are diverging instead of evolving neck to neck.

[4]Symmetric(asymmetric) situation denotes the firm's technological level is identical(different).

[5]Considering each period, different firm's R&D choice interacts. Considering periods over time, R&D choice of each period interacts.

Considering the R&D investment over time, I find that the less periods are remaining, the less investment the firm wants to conduct, irrespective of the leader or follower. That is to say, the firm in the beginning of the game invests the most and its investment decreases as the game goes on. It is intuitive that the firm knows that being a leader is more profitable than being a follower and the incremental profit is increasing with the number of leads, therefore at the beginning of the game, it has the highest incentive to invest in R&D in order to become the leader.

Furthermore, all the analytical results obtained from the 2-period game can be extended to the T-period game. Besides, some numerical results are obtained for the T-period game, which mainly focus on the time paths of R&D investments and technological difference, and the effect of the key exogeneous parameter, α, which denotes the revenue multiplier from successful innovation. The numerical simulation shows the consistency with 'the joint-profit effect' and 'the endpoint effect'.[6] That is to say, the leader with more leads ahead of its rival has higher incentive to invest in R&D since the joint profit increases with larger technological difference. Moreover, since firms know the finishing line of the game, it relaxes the investment incentive near or at the finishing line. Furthermore, α presents different effects on the leader and follower's R&D investment, because it always increases the leader's incentive to engage in R&D while may decrease the follower's. However, from the industry's perspective, the total R&D increases with α. Finally, the numerical result shows that although the probability of firms being asymmetric indeed increases continuously as the game proceeds, the technological difference between firms is not as big at the end of the game. Specifically, the revenue multiplier slows the divergence.

The structure of the rest of the chapter is as follows. Section 4.2 describes the

[6]The joint-profit effect and endpoint effect are firstly proposed in Budd, Harris and Vickers (1993), which identify how market structure evolves in a dynamic competition between two asymmetric firms

model and sections 4.3 presents the main analytical results. Section 4.4 contains the numerical simulation and further results. Section 4.5 concludes and make suggestions for future research.

4.2 The Model

There are two *ex ante* symmetric firms in the industry, firm 1 and firm 2.[7] They play a T-period R&D competition game, indexed by $t = 1, 2, ..., T$. Time is discrete and $T > 1$. In each period, both firms simultaneously choose their own R&D expenditure, or investment levels. For each firm, there are two possible R&D outcomes, success or failure. Therefore, there are three possible cases in every period, where both firms succeed, one firm succeeds while the other fails and both fail. A firm can be in one of three positions, even, ahead or behind.

The R&D outcome is assumed to be independent across periods and firms, which means the probability of success only depends on its current R&D choice. In particular, it does not depend on its rival's strategy. Following Choi (1993), let $p(x_t^i)$ be the probablity of success for firm i if it invests x_t^i in R&D. The properites of the probability function are stated in Assumption 4.1.

Assumption 4.1 *(i)* $p(0) = 0,\ \lim_{x\to\infty} p(x_t^i) = 1$*;*
(ii) $p'(x_t^i) \geqslant 0$ *with* $p'(0) = \infty$ *and* $p'(\infty) = 0$*;*
(iii) $p''(x_t^i) \leqslant 0$

Part (i) of Assumption 4.1 states that if firm invests nothing, it will definitely fail and all probabilities belong to the interval [0,1). Parts (ii) and (iii) state that the probability of success increases with R&D expenditure but at a decreasing rate. Assumption 4.1 guarantees an interior solution of the model.

Position is meant to capture the overall situation till period t. Following Horner(2004), I define the position of the game in period t, n, as the difference

[7] Or use $i, j = 1, 2$ and $i \neq j$ to denote either of these firms in the rest of the chapter.

between the total number of successes of firm i and those of its rival, where $n = -(t-1), -(t-2), .., 0, ...t-2, t-1$. To be specific, in every period t, each player observes all outcomes and choices up to and including period $t-1$, and then makes its own choice on R&D investment $x_t^i(n)$ based on its position n. At the beginning of the game, n is equal to zero. Overall, firm i has more successes in period t than does firm j if and only if n is positive. To be specific, the following analysis is from firm i's perspective. Firm i acts as the leader, which is n leads ahead of its rival in period t whenever $n > 0$, while firm i acts as the follower, which is $|n|$ lags behind its rival whenever $n < 0$. When $n = 0$, both firms are located at the same techonology level.

In every period, the net profit of each firm is the difference between the revenue, which depends on the firm's position, and the cost, which is equivalent to its R&D expenditure. The more advanced the firm is, the higher revenue it earns. Therefore, I assume that the revenue is an increasing function in n. For the firm who is n leads ahead of its rival, the revenue is assumed to be $\alpha^n \pi$, while the revenue of its rival is $\alpha^{-n}\pi$ ($\alpha > 1$), where α denotes the revenue multiplier from successful innovation. Naturally, if both firms are at the same technology level, the revenue is identical at π. In the related R&D literature, when considering both R&D stage and production stage, the R&D cost function contains a parameter capturing the R&D efficiency[8]. Since production stage is ignored in this model, α can be regarded as the measurement of R&D efficiency indirectly. Larger α implies more efficient R&D, therefore it brings higher marginal revenue.

Moreover, it is assumed that the aggregate revenue $(\alpha^n + \alpha^{-n})\pi$ increases in n as well. First, it is consistent with Cournot model where the profit of the firm with successful cost reduction innovation increases and that of its rival decreases while the joint profits increase. Second, we try to keep the model as tractable as possible so that α is the only exogenous variable is included in the revenue

[8]For example, in Chapter 3, R&D cost function is $r(x) = \frac{\gamma x^2}{2}$ if firm chooses R&D output x, where γ is a measurement of R&D efficiency.

function.

Besides, an additional successful unilateral innovation increases the aggregate revenue. In other words, increasing marginal revenue is assumed in the model, which may occur in certain industries or stages of a company's development, especially some emerging technology industry like artificial intelligence. Aldrich and Martinez (2001) states that at the early stage of a company, when it successfully engages in technological innovation, incremental marginal returns arise as investment in R&D increases to solve more technical challenges and improve the competitiveness of products or services.

Assume the discount factor $\delta = 1$. In each period, firm i maximises the overall value, which includes both the current period profit and the expected profits of future periods. Given R&D choices $x_t^i(n)$ and $x_t^j(n)$, the corresponding probability of success and failure are denoted by $p_t^i(n)$, $q_t^i(n)$ for firm i and $p_t^j(n)$, $q_t^j(n)$ for firm j, where $q = 1 - p$. The current profit of firm i is denoted by $\pi_t^i(n)$, and the overall value is denoted by $V_t^i(n)$. The game yields a subgame perfect equilibrum which is solved by backward induction.

In every period t, given the outcomes up to and including period $t-1$, firm i at position n maximises the overall value, $i, j = 1, 2$ and $i \neq j$.

$$\max_{x_t^i(n)} V_t^i(n) = \pi_t^i(n) + E(V_{t+1}^i(n)) \tag{4.1}$$

where

$$\begin{aligned} \pi_t^i(n) = &\left[p_t^i(n)p_t^j(n) + q_t^i(n)q_t^j(n)\right] \alpha^n \pi \\ &+ p_t^i(n)q_t^j(n)\alpha^{n+1}\pi + q_t^i(n)p_t^j(n)\alpha^{n-1}\pi \ - x_t^i(n) \end{aligned} \tag{4.2}$$

$$\begin{aligned} E(V_{t+1}^i(n)) = &\left[p_t^i(n)p_t^j(n) + q_t^i(n)q_t^j(n)\right] V_{t+1}^i(n) \\ &+ p_t^i(n)q_t^j(n)V_{t+1}^i(n+1) + q_t^i(n)p_t^j(n)V_{t+1}^i(n-1) \end{aligned} \tag{4.3}$$

Notice that in the last period where $t = T$, $E(V_{t+1}^i(n)) = E(V_{t+1}^j(n)) = 0$.

Due to the dynamic setting and rivalry interaction, the game structure is rather complicated, therefore the following analysis starts from the simplest 2-period game and then extend to T-period game.

4.3 Analytical Results

4.3.1 2-period game

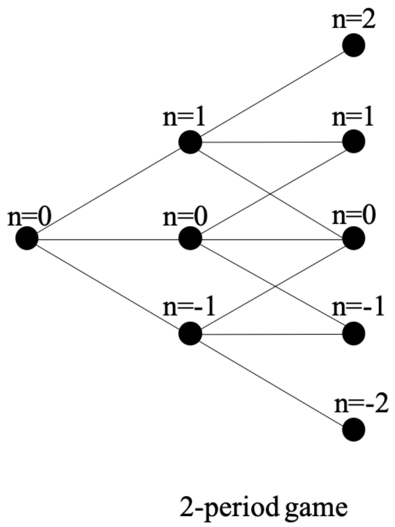


Figure 4.1: 2-period game structure

Set $T = 2$. Figure 4.1 shows the structure of a 2-period game. At the beginning of the first period, firms that start symmetric ($n = 0$), simultaneously choose to invest x_1^i and x_1^j in R&D, and then both observe the outcomes. Given the position after period 1 ($n = -1$, 0 or 1), both firms simultaneously choose $x_2^i(n)$ and $x_2^j(n)$ and then the profits are realized. The game yields a subgame perfect Nash equilibrium solved by backward induction.

Period 2 In period 2, both firms have observed the research outcomes of period 1 and the profits have been realized. According to various outcomes, the firm chooses different R&D investments to maximise profits. To make the notations clear, let x_2^{iS}, x_2^{iL} and x_2^{iF} denote firm i's R&D investment when firms are symmetric, firm i is the leader and firm i is the follower after period 1, respectively.

Case 1 Both firms succeeded or failed in period 1, which means they are still located at the same technology level, i.e. $n = 0$. Firm i maximises,

$$\pi_2^{iS} = \Big[p(x_2^{iS})p(x_2^{jS}) + q(x_2^{iS})q(x_2^{jS})\Big]\,\pi + p(x_2^{iS})q(x_2^{jS})\ \alpha\pi + q(x_2^{iS})p(x_2^{jS})\frac{\pi}{\alpha} - x_2^{iS} \tag{4.4}$$

The optimal x_2^{iS*} satisfies the first order condition,

$$p'(x_2^{iS*})\ \pi \left[p(x_2^{jS})\ +\ \alpha(1 - p(x_2^{jS}))\ -\ p(x_2^{jS})\ \frac{1}{\alpha} - (1 - p(x_2^{jS}))\right]\ =\ 1 \tag{4.5}$$

In case 1, as firms are symmetric, the equilibrium R&D expenditures are identical. $x_2^{iS*} = x_2^{jS*}$

Case 2 Firm i succeeded while the rival firm j failed in period 1, which means firm i is one lead ahead of firm j, i.e. $n = 1$. Firm i maximises,

$$\pi_2^{i\,L} = \Big[p(x_2^{i\,L})p(x_2^{j\,F}) + q(x_2^{i\,L})q(x_2^{j\,F})\Big]\,\alpha\pi + p(x_2^{i\,L})q(x_2^{j\,F})\alpha^2\pi + q(x_2^{i\,L})p(x_2^{j\,F})\pi - x_2^{i\,L} \tag{4.6}$$

For firm j which failed in period 1, it maximises

$$\pi_2^{j\,F} = \Big[p(x_2^{i\,L})p(x_2^{j\,F}) + q(x_2^{i\,L})q(x_2^{j\,F})\Big]\,\frac{\pi}{\alpha} + p(x_2^{i\,L})q(x_2^{j\,F})\frac{\pi}{\alpha^2} + q(x_2^{i\,L})p(x_2^{j\,F})\pi - x_2^{j\,F} \tag{4.7}$$

The equilibrium x_2^{iL*} and x_2^{jF*} satisfy the first order conditions,

$$p'(x_2^{iL*})\ \pi \cdot \alpha \left[p(x_2^{j\,F})\ +\ \alpha(1 - p(x_2^{j\,F}))\ -\ \frac{1}{\alpha} \cdot p(x_2^{j\,F}) - (1 - p(x_2^{j\,F}))\right] = 1 \tag{4.8}$$

$$p'(x_2^{jF*})\ \pi \cdot \frac{1}{\alpha} \left[p(x_2^{i\,L})\ +\ \alpha(1 - p(x_2^{i\,L}))\ -\ \frac{1}{\alpha} \cdot p(x_2^{i\,L}) - (1 - p(x_2^{i\,L}))\right] = 1 \tag{4.9}$$

Due to Assumption 4.1, the second-order condition is always satisfied because of the concavity assumption on the probability function. Besides, it guarantees that the positive interior solution exists since $p'(0) = \infty$. Hence, the first-order conditions implicitly define the best response functions of firm i and j, $x_2^{iL*}(x_2^{jF})$

and $x_2^{jF*}(x_2^{iL})$. The intersection of the two reaction functions give the Nash equilibrium in R&D expenditures (x_2^{iL*}, x_2^{jF*}).

Case 3 Firm i failed while the rival firm j succeeded in period 1, which means firm i is one lag behind firm j, i.e. $n = -1$. Case 3 is omitted since firm i with $n = -1$ is equivalent to the situation where firm j with $n = 1$ in Case 2. Both cases 2 and 3 denote the situation where the technological difference is one after period 1.

One of the aims is to identify how R&D investments vary with different positions n. Rewrite the FOCs and organize a system of equations $F(\alpha, n, x_2^i, x_2^j) = 0$ and $G(\alpha, n, x_2^i, x_2^j) = 0$ to analyze the effects of n on the equilibrium R&D expenditures $x_2^i(n)$ and $x_2^j(n)$ using implicit function theorem. Notice that although n is a discrete variable due to its economic implication, but functions F and G are monotonic functions in n, therefore they are differentiable in n in order to obtain the comparative statics.

$$F(\alpha, n, x_2^i, x_2^j) = p'(x_2^i)\ \pi \cdot \alpha^n \left[p(x_2^j)\ +\ \alpha(1 - p(x_2^j)) - \frac{1}{\alpha} \cdot p(x_2^j) - (1 - p(x_2^j))\right] - 1 \tag{4.10}$$

$$G(\alpha, n, x_2^i, x_2^j) = p'(x_2^j)\ \pi \cdot \frac{1}{\alpha^n} \left[p(x_2^i)\ + \alpha(1 - p(x_2^i)) - \frac{1}{\alpha} \cdot p(x_2^i) - (1 - p(x_2^i))\right] - 1 \tag{4.11}$$

If $n = 0$, the solution to the system of Equations (4.10) and (4.11) defines the symmetric equilibrium x_2^{iS*}. If $n = 1$, it gives the equilibrium x_2^{iL*} in case 2 and if $n = -1$, the equilibrium x_2^{iF*} in case 3. Therefore, sign$\left\{\frac{\partial x_2^{i*}}{\partial n}\right\}$ induces the ranking for the equilibrium R&D expenditures between the cases.

Take partial derivatives with respect to n, x_2^i, x_2^j for both functions,

$$\frac{\partial F}{\partial x_2^i} = p''(x_2^i)\ \pi \cdot \alpha^n \cdot (\alpha - 1) \cdot (\frac{1-\alpha}{\alpha} p(x_2^j) + 1) < 0 \tag{4.12}$$

$$\frac{\partial F}{\partial x_2^j} = p'(x_2^i)\ \pi \cdot \alpha^{n-1} \cdot (\alpha - 1) \cdot (1-\alpha) \cdot p'(x_2^j) < 0 \tag{4.13}$$

$$\frac{\partial G}{\partial x_2^i} = p'(x_2^j)\ \pi \cdot \frac{1}{\alpha^{n+1}} \cdot (\alpha - 1) \cdot (1-\alpha) \cdot p'(x_2^i) < 0 \tag{4.14}$$

$$\frac{\partial G}{\partial x_2^j} = p''(x_2^j)\ \pi \cdot \frac{1}{\alpha^n} \cdot (\alpha - 1) \cdot (\frac{1-\alpha}{\alpha} p(x_2^i) + 1) < 0 \tag{4.15}$$

$$\frac{\partial F}{\partial n} = p'(x_2^i)\ \pi \cdot (\alpha - 1) \cdot (\frac{1-\alpha}{\alpha} p(x_2^j) + 1) \cdot \alpha^n \ln \alpha > 0 \tag{4.16}$$

$$\frac{\partial G}{\partial n} = -p'(x_2^j)\ \pi \cdot (\alpha - 1) \cdot (\frac{1-\alpha}{\alpha} p(x_2^i) + 1) \cdot \alpha^{-n} \ln \alpha < 0 \tag{4.17}$$

Due to Assumption 4.1 and $\alpha > 1$, the signs of Equations (4.12) to (4.27) are easy to check.

The determinant of Jacobian matrix is,

$$J = \begin{vmatrix} \frac{\partial F}{\partial x_2^i} & \frac{\partial F}{\partial x_2^j} \\ \frac{\partial G}{\partial x_2^i} & \frac{\partial G}{\partial x_2^j} \end{vmatrix} = \frac{\partial F}{\partial x_2^i} \cdot \frac{\partial G}{\partial x_2^j} - \frac{\partial F}{\partial x_2^j} \cdot \frac{\partial G}{\partial x_2^i} \tag{4.18}$$

The stability condition, which requires that the best response functions cross correctly (Henriques, 1990), is calculated from $\left|\frac{\partial x_2^i}{\partial x_2^j}\right| < 1$. From (4.10) and (4.11), according to implicit function theorem,

$$\left|\frac{\partial x_2^i}{\partial x_2^j}\right| = \left|-\frac{\frac{\partial F}{\partial x_2^j}}{\frac{\partial F}{\partial x_2^i}}\right| = \frac{\frac{\partial F}{\partial x_2^j}}{\frac{\partial F}{\partial x_2^i}} < 1$$

$$\left|\frac{\partial x_2^j}{\partial x_2^i}\right| = \left|-\frac{\frac{\partial G}{\partial x_2^i}}{\frac{\partial G}{\partial x_2^j}}\right| = \frac{\frac{\partial G}{\partial x_2^i}}{\frac{\partial G}{\partial x_2^j}} < 1$$

which imply $\frac{\partial F}{\partial x_2^j} > \frac{\partial F}{\partial x_2^i}$ and $\frac{\partial G}{\partial x_2^i} > \frac{\partial G}{\partial x_2^j}$. Therefore, J is strictly positive. The sufficient conditions for the stability conditions to hold are,

$$p_i' p_j' (1-\alpha) > p_i''((1-\alpha)p_j + \alpha) \tag{4.19}$$

$$p_j' p_i' (1-\alpha) > p_j''((1-\alpha)p_i + \alpha) \tag{4.20}$$

Besides, firms' R&D investment choices are strategic substitutes due to $\frac{\partial x_2^i}{\partial x_2^j} < 0$.

Then the derivatives $\frac{\partial x_2^{i*}}{\partial n}$, $\frac{\partial x_2^{j*}}{\partial n}$ can be solved from,

$$\frac{\partial x_2^{i*}}{\partial n} = -\frac{1}{J}\begin{vmatrix} \frac{\partial F}{\partial n} & \frac{\partial F}{\partial x_2^j} \\ \frac{\partial G}{\partial n} & \frac{\partial G}{\partial x_2^j} \end{vmatrix} = -\frac{1}{J}\left(\frac{\partial F}{\partial n}\cdot\frac{\partial G}{\partial x_2^j} - \frac{\partial F}{\partial x_2^j}\cdot\frac{\partial G}{\partial n}\right) \tag{4.21}$$

$$\frac{\partial x_2^{j*}}{\partial n} = -\frac{1}{J}\begin{vmatrix} \frac{\partial F}{\partial x_2^i} & \frac{\partial F}{\partial n} \\ \frac{\partial G}{\partial x_2^i} & \frac{\partial G}{\partial n} \end{vmatrix} = -\frac{1}{J}\left(\frac{\partial F}{\partial x_2^i}\cdot\frac{\partial G}{\partial n} - \frac{\partial F}{\partial n}\cdot\frac{\partial G}{\partial x_2^i}\right) \tag{4.22}$$

The signs of (4.21) and (4.22) depend on the term in parentheses. It is obvious that the term in parentheses of (4.21) is always negative and that of (4.22) is positive. Thus, we have the following Lemma.

Lemma 4.1 $\frac{\partial x_2^{i*}}{\partial n} > 0$, $\frac{\partial x_2^{j*}}{\partial n} < 0$, *that is,* $x_2^{iL*} > x_2^{iS*} > x_2^{iF*}$

It shows that the equilibrium x_2^{i*} increases with the leads ahead. Firm i invests the most in period 2 if it is the only one who succeeded in period 1. Conversely, it invests the least in period 2 if only the rival succeeded in period 1. Since the revenue is assumed to be $\alpha^n\pi$, the marginal benenit from a given fixed level of R&D is increasing with n. At the same time, the marginal cost of exerting effort is the same. Therefore the firm with more leads would like to exert more efforts in R&D. It is consistent with Grosman and Shapiro (1985). Moreover, it is analogous of another proposition in the R&D literature, which states that the firm with lower initial costs in a Cournot duopoly has a higher incentive than its rival to engage in cost-reducing R&D.

Next, consider how the equilibrium profits of firm i in period 2 changes as n changes. The corresponding equilibrium profits of firm i is,

$$\begin{aligned}\pi_2^i(n) = {} & p(x_2^i)\ p(x_2^j)\ \alpha^n\pi + \ p(x_2^i)\ (1-p(x_2^j))\ \alpha^{n+1}\pi + \\ & (1-p(x_2^i))\ p(x_2^j)\ \alpha^{n-1}\pi\ + (1-p(x_2^i))\ (1-p(x_2^j))\ \alpha^n\pi - x_2^i\end{aligned} \tag{4.23}$$

Totally differentiating eq.(4.23) with respect to n gives,

$$\frac{d\pi_2^i}{dn} = \frac{\partial\pi_2^i}{\partial x_2^i}\frac{\partial x_2^{i*}}{\partial n} + \frac{\partial\pi_2^i}{\partial x_2^j}\frac{\partial x_2^{j*}}{\partial n} + \frac{\partial\pi_2^i}{\partial n} \tag{4.24}$$

The first and second terms on the right-hand side represent the indirect effects, which are the change in profit due to the induced change in its own and rival firm's R&D investments. Since in the Nash equilibrium the R&D investment of firm i is already at optimum, the first term is identially zero. The third term shows the direct effect of a change of the number of leads on profits. Since profits increase with every single success, the third term is always positive.

Now identify the sign of the second term $\frac{\partial \pi_2^i}{\partial x_2^j}$,

$$\begin{aligned}\frac{\partial \pi_2^i}{\partial x_2^j} &= p(x_2^i)p'(x_2^j)\alpha^n\pi - p(x_2^i)p'(x_2^j)\alpha^{n+1}\pi + (1-p(x_2^i))p'(x_2^j)\alpha^{n-1}\pi - (1-p(x_2^i))p'(x_2^j)\ \alpha^n\pi \\ &= p(x_2^i)p'(x_2^j)\alpha^n\pi\ (2-\alpha-\frac{1}{\alpha}) + p'(x_2^j)\ \alpha^n\pi\ (\frac{1}{\alpha}-1)\end{aligned}$$

Since $\alpha > 1$, $\frac{\partial \pi_2^i}{\partial x_2^j}$ is always negative. Therefore, according to eq.(4.24), the equilibrium profit is an increasing function of the position n.

Lemma 4.2 $\frac{\partial \pi_2^{i*}}{\partial n} > 0$, $\frac{\partial \pi_2^{j*}}{\partial n} < 0$. *That is to say,* $\pi_2^{iL*} > \pi_2^{iS*} > \pi_2^{iF*}$.

The intuition behind Lemma 4.2 is that since the leader firm after period 1 invests more in R&D than the follower firm, and the probability of success increases with the R&D expenditure and the revenue increases with more successes, the leader firm is more likely to win in the next period and ends up with higher revenue. Although the leader spends more on R&D, the expected revenue is even larger which leads to higher corresponding profits.

In addition, α is another key exogenous parameter which has effects on R&D investment in this model. Consider how α affects the equilibrium R&D investments and the corresponding profits. The analytical results focus on the symmetric case where $n = 0$, which defines the symmetric equilibrium outputs, x_2^{S*} and π_2^{S*}. The partial derivative of eq.(4.10) when $n = 0$ with respect to α is,

$$\frac{\partial F}{\partial \alpha} = p'(x_2^i)\ \pi \cdot \left[1 - p(x_2^j) + \frac{p(x_2^j)}{\alpha^2}\right] > 0 \tag{4.25}$$

Using implicit function theorem,

$$\frac{dx_2^{S*}}{d\alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial x_2^i} + \frac{\partial F}{\partial x_2^j}} > 0$$

It shows that the equilibrium R&D investment in symmetric case is an increasing function of the revenue multiplier α. Unsurprisingly, larger revenue increment provides higher incentive to engage in R&D if firms are at the same technological level.

$$\frac{d\pi_2^{S*}}{d\alpha} = \frac{\partial \pi_2^i}{\partial x_2^i}\frac{dx_2^{iS*}}{d\alpha} + \frac{\partial \pi_2^i}{\partial x_2^j}\frac{dx_2^{jS*}}{d\alpha} + \frac{\partial \pi_2^i}{\partial \alpha}$$

with

$$\frac{\partial \pi_2^i}{\partial \alpha} = p(x_2^i)(1-p(x_2^j))\,\pi - \frac{p(x_2^j)(1-p(x_2^i))\,\pi}{\alpha^2} > 0$$

The first term in the RHS is zero since x_2^i is at optimum. The second term is negative while the third term is positive. Therefore the sign depends on whether the direct effect dominates the indirect effect or conversely. Although α increases the R&D investment, its impact on profits is ambiguous.

Period 1 According to different outcomes of period 1, solving period 2 problem gives the different equilibrium R&D investments x_2^{iS*}, x_2^{iL*}, x_2^{iF*}, and the corresponding equilibrium profits π_2^{iS*}, π_2^{iL*} and π_2^{iF*}. Moving backward to period 1, firm i chooses x_1^i to maximise not only the periodic profit π_1^i, but also taking into account the expected profits in period 2, $E(\pi_2^i)$.

$$V_1^i = \pi_1^i + E(\pi_2^i) \tag{4.26}$$

where

$$\pi_1^i = \left[p(x_1^i)p(x_1^j) + q(x_1^i)q(x_1^j)\right]\pi + p(x_1^i)q(x_1^j)\ \alpha\pi + q(x_1^i)p(x_1^j)\ \frac{\pi}{\alpha} - x_1^i \tag{4.27}$$

$$E(\pi_2^i) = \left[p(x_1^i)\ p(x_1^j) + q(x_1^i)q(x_1^j)\right]\pi_2^{i*S} + \ p(x_1^i)q(x_1^j)\ \pi_2^{iL*} + q(x_1^i)\ p(x_1^j)\ \pi_2^{iF*} \tag{4.28}$$

The equilibrium x_1^{i*} satisfies the first order condition,

$$\frac{dV_1^i}{dx_1^i} = \frac{\partial \pi_1^i}{\partial x_1^i} + \frac{\partial E(\pi_2^i)}{\partial p(x_1^i)} \cdot \frac{dp(x_1^i)}{dx_1^i} = 0 \tag{4.29}$$

where

$$\begin{aligned} \frac{\partial E(\pi_2^i)}{\partial p(x_1^i)} &= p(x_1^j)\ \pi_2^{iS*} + (1 - p(x_1^j))\ \pi_2^{iL*} - p(x_1^j)\ \pi_2^{iF*} - (1 - p(x_1^j))\ \pi_2^{iS*} \\ &= p(x_1^j)(\pi_2^{iS*} - \pi_2^{iF*}) + (1 - p(x_1^j))(\pi_2^{iL*} - \pi_2^{iS*}) \end{aligned} \tag{4.30}$$

Due to the ranking of the equilibrium profits in period 2, $\frac{\partial E(\pi_2^i)}{\partial p(x_1^i)} > 0$. Therefore, evaluating the FOC of (4.29) at $x_1^i = x_2^{iS*}$ yields,

$$\left.\frac{dV_1^i}{dx_1^i}\right|_{x_1^i = x_2^{iS*}} = 0 + \frac{\partial E(\pi_2^i)}{\partial p(x_2^S)} \cdot \frac{dp(x_2^S)}{dx_2^S} > 0 \tag{4.31}$$

The first term of eq.(4.29) is equivalent to the first order condition of the symmetric case in period 2 which implicitly defines x_2^{iS*} (i.e., eq.(4.5)). The second term which is positive tells that the expected profits of period 2 increase with the R&D investment in period 1. In addition, it is easy to check the second-order condition is always satisfied (negative). Therefore the equilibrium R&D investment in period 1 x_1^{i*} is larger than that in period 2 of symmetric case x_2^{iS*}.

Proposition 4.1 *If firms are at the same technology level, each firm has higher incentive to invest in R&D in period 1 than in period 2, i.e.,* $x_1^{i*} > x_2^{iS*}$.

The intuition behind Proposition 4.1 is as follows. Due to the multi-period nature of the game and stochastic R&D outcomes, firm's R&D choice in period 1 affects not only period 1's profits, but also the expected profits of period 2. Since being a leader gives the most while being a follower is the worst in period 2, the higher the probability of success, the more the expected profits in period 2, which gives further incentives for the firm to invest in period 1.

Due to the stochastic R&D outcomes, there are 5 different positions for a specific firm at the end of the 2-period game, which are shown in Figure 4.1. To determine whether the firms are evolving neck to neck or diverging, one could calculate the probability of each possible outcome. Let P_t be the probability of being symmetric at the end of period t. At the beginning of the game, both firms are at the same technology level, that is, $P_0 = 1$. After period 1, the probability of firms still being symmetric, that is, both firms succeed or fail in period 1, is $P_1 = p(x_1)^2 + (1 - p(x_1))^2$. Then at the end of the game, i.e., the end of period 2, there are three possible paths leading to the position where $n = 0$. The probability is,

$$P_2 = (p(x_1)^2 + (1-p(x_1))^2) \cdot (p(x_2^S)^2 + (1-p(x_2^S))^2) + 2 \cdot p(x_1)(1-p(x_1)) \cdot p(x_2^F)(1-p(x_2^L))$$

Proposition 4.2 *The probability of firms being symmetric decreases as the game proceeds, i.e., $P_0 > P_1 > P_2$.*

Proof: It is obvious that,

$$\begin{aligned} P_1 - P_0 =& p(x_1)^2 + (1 - p(x_1))^2 - 1 \\ =& -2p(x_1)(1 - p(x_1)) < 0, \ as \ 0 < p(x_1) < 1 \end{aligned}$$

$$\begin{aligned} P_2 - P_1 =& (p(x_1)^2 + (1 - p(x_1))^2) \cdot (p(x_2^s)^2 + (1 - p(x_2^s))^2 - 1) \\ &+ 2 \cdot p(x_1)(1 - p(x_1)) \cdot p(x_2^F)(1 - p(x_2^L)) \\ =& (p(x_1)^2 + (1 - p(x_1))^2) \cdot (-2 \cdot p(x_2^s)(1 - p(x_2^s))) \\ &+ 2 \cdot p(x_1)(1 - p(x_1)) \cdot p(x_2^F)(1 - p(x_2^L)) \end{aligned}$$

as $x_2^F < x_2^s < x_2^L$ and $p'(x) > 0$, $p(x_2^F) < p(x_2^s) < p(x_2^L)$. Therefore,

$$p(x_2^s)(1 - p(x_2^s)) > p(x_2^F)(1 - p(x_2^L)) > 0$$

Besides,

$$(p(x_1)^2 + (1 - p(x_1))^2) - 2 \cdot p(x_1)(1 - p(x_1)) = 4(p(x_1) - \frac{1}{2})^2 \geqslant 0$$

Combine the above two terms,

$$P_2 - P_1 < 0$$

■

4.3.2 T-period game

To make the results more general, following is the analysis for the T-period game. The last period's problem in the T-period game is similar to the last period's problem in the 2 period game of the last section. The only difference is the range of position n, from 3 possible values in 2-period game to $2T-1$ possible values, where $n = -(t-1), -(t-2), ..0, ..t-2, t-1$. Recall the FOCs of maximisation problems for the final period are as follows,

$$F(\alpha, n, x_T^i, x_T^j) = p'(x_T^i)\ \pi \cdot \alpha^n \left[p(x_T^j)\ +\ \alpha(1-p(x_T^j)) - \frac{1}{\alpha} \cdot p(x_T^j) - (1-p(x_T^j)) \right] - 1 \tag{4.32}$$

$$G(\alpha, n, x_T^i, x_T^j) = p'(x_T^j)\ \pi \cdot \frac{1}{\alpha^n} \left[p(x_T^i)\ + \alpha(1-p(x_T^i)) - \frac{1}{\alpha} \cdot p(x_T^i) - (1-p(x_T^i)) \right] - 1 \tag{4.33}$$

Using the implicit function theorem, it was shown that $\dfrac{dx_T^{i*}(n)}{dn} > 0$ and $\dfrac{dx_T^{j*}(n)}{dn} < 0$ in the final period, which further implies that the corresponding equilibrium profit $\dfrac{d\pi_T^{i*}(n)}{dn} > 0$ and $\dfrac{d\pi_T^{j*}(n)}{dn} < 0$.

Next, consider the R&D investments in other intermediate periods which are not analysed in the 2-period game. Moving back to the period $t = T-1$, firm i maximises the total value by choosing $x_{T-1}^i(n)$,

$$\begin{aligned} V_{T-1}^i(n) =& \pi_{T-1}^i(n) + E(\pi_T^i(n)) \\ =& \left[p_{T-1}^i(n) p_{T-1}^j(n) + q_{T-1}^i(n) q_{T-1}^j(n) \right] \left[\alpha^n \pi + \pi_T^{i*}(n) \right] \\ &+ p_{T-1}^i(n) q_{T-1}^j(n) \left[\alpha^{n+1} \pi + \pi_T^{i*}(n+1) \right] \\ &+ q_{T-1}^i(n) p_{T-1}^j(n) \left[\alpha^{n-1} \pi + \pi_T^{i*}(n-1) \right] - x_{T-1}^i(n) \end{aligned} \tag{4.34}$$

The rival firm j's objective is,

$$\begin{aligned}V^j_{T-1}(n) =&\pi^j_{T-1}(n) + E(\pi^j_T(n))\\ &= \left[p^i_{T-1}(n)p^j_{T-1}(n) + q^i_{T-1}(n)q^j_{T-1}(n)\right]\left[\alpha^{-n}\pi + \pi^{j*}_T(n)\right]\\ &+p^i_{T-1}(n)q^j_{T-1}(n)\left[\alpha^{-(n+1)}\pi + \pi^{j*}_T(n+1)\right]\\ &+q^i_{T-1}(n)p^j_{T-1}(n)\left[\alpha^{-(n-1)}\pi + \pi^{j*}_T(n-1)\right] - x^j_{T-1}(n)\end{aligned} \tag{4.35}$$

The equilibrium $x^{i*}_{T-1}(n)$ and $x^{j*}_{T-1}(n)$ satisfy,

$$\begin{aligned}\frac{\partial V^i_{T-1}(n)}{\partial x^i_{T-1}(n)} &= p^{i'}_{T-1}(n)\left\{p^j_{T-1}(n)\left[\alpha^n\pi + \pi^{i*}_T(n) - \alpha^{n-1}\pi - \pi^{i*}_T(n-1)\right]\right.\\ &\left.+(1-p^j_{T-1}(n))\left[\alpha^{n+1}\pi + \pi^{i*}_T(n+1) - \alpha^n\pi - \pi^{i*}_T(n)\right]\right\} - 1 = 0\end{aligned} \tag{4.36}$$

$$\begin{aligned}\frac{\partial V^j_{T-1}(n)}{\partial x^j_{T-1}(n)} &= p^{j'}_{T-1}(n)\left\{p^i_{T-1}(n)\left[\alpha^{-n}\pi + \pi^{j*}_T(n) - \alpha^{-(n+1)}\pi - \pi^{j*}_T(n+1)\right]\right.\\ &\left.+(1-p^i_{T-1}(n))\left[\alpha^{-(n-1)}\pi + \pi^{j*}_T(n-1) - \alpha^{-n}\pi - \pi^{j*}_T(n)\right]\right\} - 1 = 0\end{aligned} \tag{4.37}$$

Similar to 2-period game, implicit function theorem for a system of equations gives comparative statics results on how $x^{i*}_{T-1}(n)$ and $x^{j*}_{T-1}(n)$ respond to n. Let H and Z represent the first-order conditions given in (4.42) and (4.43).

$$\frac{\partial x^{i*}_{T-1}}{\partial n} = -\frac{\frac{\partial H}{\partial n}\frac{\partial Z}{\partial x^j_{T-1}} - \frac{\partial H}{\partial x^j_{T-1}}\frac{\partial Z}{\partial n}}{\frac{\partial H}{\partial x^i_{T-1}}\frac{\partial Z}{\partial x^j_{T-1}} - \frac{\partial H}{\partial x^j_{T-1}}\frac{\partial Z}{\partial x^i_{T-1}}} \tag{4.38}$$

The denominator of the above expression can be interpreted as a stability condition and, hence, is positive. Besides, it is easy to check $\frac{\partial H}{\partial n} > 0$, $\frac{\partial Z}{\partial n} < 0$ and $\frac{\partial Z}{\partial x^j_{t-1}} < 0$.

$$\frac{\partial H}{\partial x^j_{T-1}} = p^{i'}_{T-1}(n)p^{j'}_{T-1}(n)\cdot M \tag{4.39}$$

where

$$\begin{aligned}M =&\alpha^n\pi + \pi^{i*}_T(n) - \alpha^{n-1}\pi - \pi^{i*}_T(n-1) - (\alpha^{n+1}\pi + \pi^{i*}_T(n+1) - \alpha^n\pi - \pi^{i*}_T(n))\\ =&(2\alpha^n\pi - \alpha^{n-1}\pi - \alpha^{n+1}\pi) + (2\pi^{i*}_T(n) - \pi^{i*}_T(n-1) - \pi^{i*}_T(n+1))\end{aligned} \tag{4.40}$$

where $(2\alpha^n\pi - \alpha^{n-1}\pi - \alpha^{n+1}\pi) = \alpha^{n-1}\pi\cdot(2\alpha - 1 - \alpha^2) < 0$ due to $\alpha > 1$.

The sign of $(2\pi^{i*}_T(n) - \pi^{i*}_T(n-1) - \pi^{i*}_T(n+1))$ expresses whether the increment of equilibrium profits follows an increasing rate in n. The numerical result in the next section shows that the gain being one more lead ahead is more than the loss being one less lead ahead, that is, $(2\pi_T^{i*}(n) - \pi_T^{i*}(n-1) - \pi_T^{i*}(n+1)) < 0$.

Therefore, $\frac{\partial H}{\partial x_{t-1}^j} < 0$ implies that the numerator of eq.(4.44) is negative, which further shows $\frac{\partial x_{T-1}^{i*}(n)}{\partial n} > 0$ and conversely, for the rival firm $\frac{\partial x_{T-1}^{j*}(n)}{\partial n} < 0$.

Moving to the period $t = T-2$, it follows that the maximisation problem has the same structure of period $T-1$ due to the iteration. Therefore, I have the following result.

Proposition 4.3 *In each period t, the firm's R&D investment is increasing in the number of leads, i.e., $\frac{dx_t^{i*}(n)}{dn} > 0$.*

Since the revenue function is assumed to be $\alpha^n\pi$, the gain from an additional innovation starting from location n is $\alpha^n(\alpha-1)\pi$, whereas starting from location $n-1$, it is $\alpha^{n-1}(\alpha-1)\pi$ which is smaller than $\alpha^n(\alpha-1)\pi$. Therefore it drives the increase in the firm's R&D investment with its location.

Next, consider how the equilibrium total value in period t responds to the position n. Totally differentiating eq.(4.40) with respect to n gives,

$$\frac{dV_t^i}{dn} = \frac{\partial V_t^i}{\partial x_t^i}\frac{\partial x_t^{i*}}{\partial n} + \frac{\partial V_t^i}{\partial x_t^j}\frac{\partial x_t^{j*}}{\partial n} + \frac{\partial V_t^i}{\partial n} \tag{4.41}$$

The first term on the RHS is zero since in the Nash equilibrium the R&D investment of firm i is already at optimum. The third term shows the direct effect of a change of the number of leads on profits. Since profits increase with every single success, the third term is always positive. One can check $\frac{\partial V_t^i}{\partial x_t^j} < 0$ and from the previous analysis, $\frac{\partial x_t^{j*}}{\partial n} < 0$, which implies the second term is positive. Therefore, the overall effect of a change in position n on total value is positive.

Proposition 4.4 *In each period, firm i's total value increases in the number of leads, i.e., $\frac{dV_t^{i*}(n)}{dn} > 0$.*

In addition, I analyze how the R&D investment changes across periods while keeping the position fixed.

First, it follows the same logic to compare the equilibrium R&D investment in each period $x_t^{i*}(n)$ with that in the final period $x_T^{i*}(n)$ as to compare x_1^* with x_2^{S*} in the 2-period game.

$$\frac{dV_t^i(n)}{dx_t^i(n)} = \frac{\partial \pi_t^i(n)}{\partial x_t^i(n)} + \frac{\partial E(\pi_{t+1}^i(n))}{\partial x_t^i(n)} = 0 \tag{4.42}$$

The first term has the same expression as the first order condition of the final period problem $\frac{\partial \pi_T^i(n)}{\partial x_T^i(n)}$, which implicitly defines the equilibrium R&D investment of the final period $x_T^{i*}(n)$. One can check the second term $\frac{\partial E(\pi_{t+1}^i(n))}{\partial x_t^i(n)}$ is always positive due to the ranking of equilibrium profits at different positions ($\frac{\partial \pi_t^i(n)}{\partial n} > 0$) and Assumption 4.1. Therefore, the first term must be negative which implies that $x_t^{i*}(n) > x_T^{i*}(n)$.

When comparing the equilibrium R&D investments in any two consecutive periods, I have the following result. The complete proof is shown in the Appendix.

Proposition 4.5 *For the same position, the firm decreases the investment as the game proceeds, i.e.,* $x_{t-1}^{i*}(n) > x_t^{i*}(n)$.

4.4 Numerical Simulations

In order to illustrate the equilibrium properties of the T-period model, I further provide some results of a numerical simulation. In this part, I assume that the probability function is given by,

$$p(x_t^i) = \frac{x_t^i}{1 + x_t^i} \tag{4.43}$$

It has the following properties: $p(0) = 0$, $\lim_{x\to\infty} p(x) = 1$, $p'(x_t^i) = \frac{1}{(1+x_t^i)^2} > 0$ and $p''(x) = -\frac{2}{(1+x_t^i)^3} < 0$.

Before starting the simulation, the parameters in the model need to be calibrated. Because (4.49) does not satisfy assumption $p'(0) = \infty$, we need to choose parameter values that ensure that the firms are active for all T periods. In this model, the total number of periods T is exogenous. In the analytical analysis, I make Assumption 4.1 which then implies that the firms are active during the whole game regardless of parameters α and π, which means that both firms are willing to invest in R&D in each period. In the analytical part, it is guaranteed by Assumption 4.1 regardless of parameters α and π. However, it is relatively hard to come up with a simple probability function which perfectly satisfies Assumption 4.1. The probability function in (4.49) does not ensure that the solution to the FOCs is an interior solution without restrictions on α and π. π is a scale parameter, which denotes the revenue if both firms are at the same technology level, therefore changing the value of π does not lead to qualitatively different results. While α denotes the revenue multiplier from successful innovation, or a measurement of R&D efficiency. Therefore, I fix π sufficiently large at $\pi = 1000$ and mainly focus on the effect of α. Table 4.1 summarizes the simulation parameters.

Hence, the numerical results focus on the following issues:

• The time paths of the R&D investment and technological difference, given $T = 10$;

• The effect of α, which denotes the revenue multiplier, where $\alpha \in (1, 2)$.

Table 4.1: Simulation parameters

Parameters	Values	Explanations
α	(1,2)	revenue multiplier
T	10	Total number of periods
π	1000	Benchmark revenue
t	1,2,..,10	Each period
x_i	Endogenous	Firm i's R&D investment

4.4.1 Time paths of the R&D investments and values

First, I show that the analytical results obtained from the 2-period game extend to the T-period game.

In the 10-period game, Figure 4.2 shows the individual equilibrium R&D

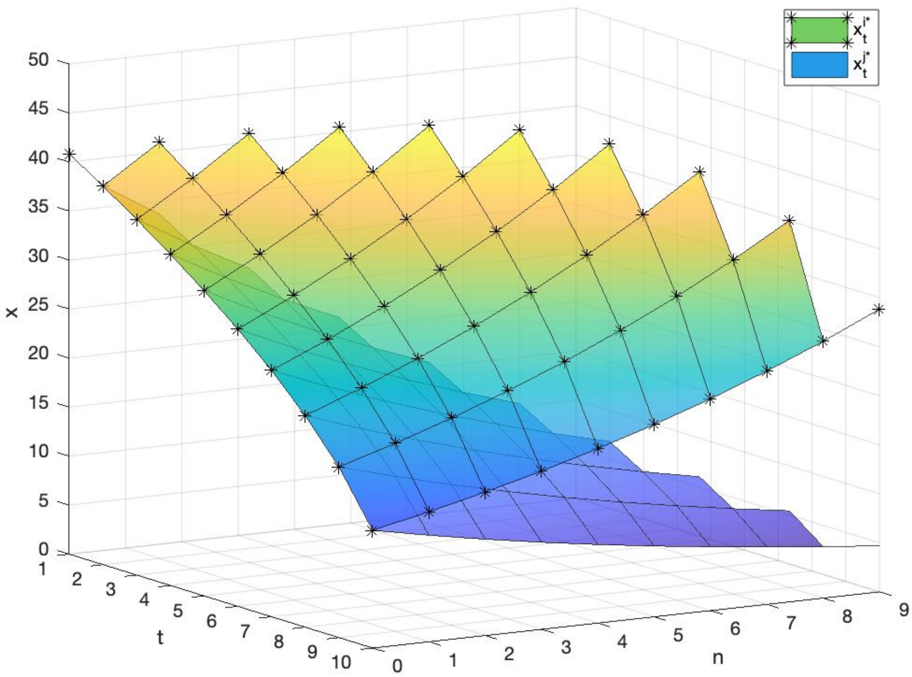


Figure 4.2: Equilibrium R&D investment in period t at position $n(\alpha = 1.2, T = 10)$

investments of firms i and j in each period t at each position n. Since firm i with $n < 0$ is identical to firm j with $n > 0$, Figure 4.2 is drawn where firm i acts as the leader (shown with $*$) and firm j acts as the follower, that is, $n > 0$, which is enough to capture all cases.

The firm's R&D investment trajectories exhibit following features:

i) in each period t, the leader invests more than the follower;

ii) in each period t, the more leads firm i is ahead, the more it invests. Moreover, it increases at an increasing rate;

iii) at each position n, the R&D investment decreases as the game proceeds.

The time paths of R&D investments resonates with the results in previous chapters. Chapter 2 shows that the leader has higher incentive to pay for the license for a process innovation. Both indicate that in general, the gap between

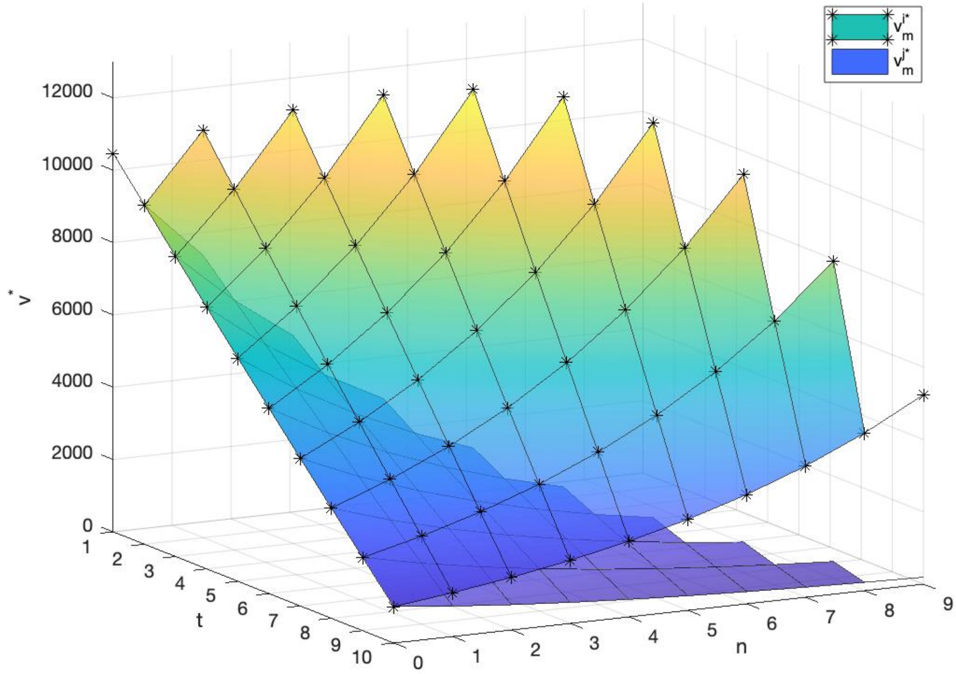


Figure 4.3: Equilibrium value in period t at position $n(\alpha = 1.2, T = 10)$

firms tends to widen, although chapter 3 provides a possible factor (i.e. asymmetric spillover rate) which works against the trend.

The corresponding equilibrium value of the firm experiences a similar pattern as shown in Figure 4.3. The results are consistent with the 'joint-profit' effect proposed in Budd, Harris and Vickers (1993), which states that the leader tends to make greater investments if joint profits from the product market are higher when the gap between firms grows. Although the product market is ignored in this model, the joint profits $\alpha^n + \frac{1}{\alpha^n}$ increase with n, which denotes the technological difference. Moreover, it is also consistent with the asymmetric Cournot duopoly, where the total industry profit is increasing as the asymmetry between two firms increases. The intuition is that being a leader is more profitable than being a follower and the firm can benefit from being the leader more if there are more periods remaining in the game. Therefore, at the beginning of the game, the firm

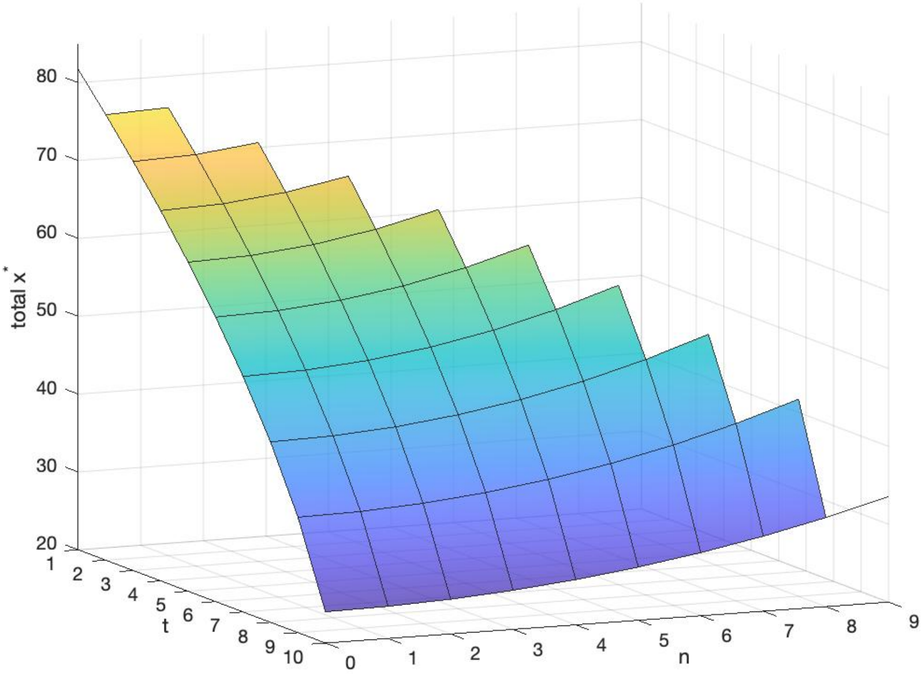


Figure 4.4: Total R&D investments in period t at position $n(\alpha = 1.2, T = 10)$

has the highest incentive to invest in R&D in order to become the leader and as the game continues, the firm knows that it is much closer to the finishing line, and the incentive therefore decreases. It is analogous with the statement proposed by Budd, Harris and Vickers (1993). They point out the endpoint effect that relaxes efforts at or near the endpoint.

Figure 4.4 illustrates the trajectory of total R&D investment. Since both the leader and follower decrease the individual R&D investment over time, it is obvious that the total R&D investment decreases as the game proceeds. Moreover, Figure 4.4 shows that in each period t, the total R&D investment increases as the technological difference increases. To be specific, with bigger technological difference between the firms, the leader will invest more than the follower will reduce it. We know from the previous analysis, the technological difference n and time path t have opposite effects on the R&D investments, however, Figure

4.4 shows that the total R&D investment reaches the peak at the beginning of the game (t=1), which illustrates that the endpoint effect is stronger than the joint-profit effect if α is not too large.

4.4.2 The effect of α on R&D investments

Figure 4.5 and 4.6 show how leader's (firm i) and follower's (firm j) R&D investments vary in each period as α changes, respectively. The black bold line represents the R&D investment if firms are symmetric, that is $n = 0$, in each period and it is obviously increasing with α, which is proved in the previous section. And they are identical in Figure 4.5 and 4.6. Specifically, in Figure 4.5, all other curves above the black one represent the leader's R&D investment at position $n = 1, 2, ..., t-1$ respectively. For example, in period 3, the top curve represents x_3^{i*} when $n = 2$, and the middle one is x_3^{i*} when $n = 1$. Figure 4.5 shows that the leader firm invests more with higher α and with more steps ahead, i.e. larger n, the effect of α is more significant. It is not surprising that the leader's incentive to engage in R&D increases since the increment of profits grows with α.

By contrast, in Figure 4.6, all curves below the black one represent follower's R&D at position $n = 1, 2, .., t-1$. For example, in period 3, the bottom curve represents x_3^{j*} when $n = 2$, and the middle one is x_3^{j*} when $n = 1$. If the follower is not much behind, it increases the R&D investment with higher α, while as the game goes on, if the firm is behind by more lags, it shows initially increasing and then decreasing trend with α. It is intuitive that if the follower is a few lags behind, the probability that it gets closer to the leader or even catches up with the leader is higher than that with much more lags behind. Therefore, the incentive to engage in R&D increases in α. However, with more lags behind, the probability to catch up with the leader is even lower with higher α.

From the industry's perspective (shown in Figure 4.7), the total R&D investment increases with α, which means the effect of α on leader's R&D dominates that on follower's. Futhermore, the effect of α is even stronger when the techno-

Figure 4.5: Leader's R&D investments with changing α

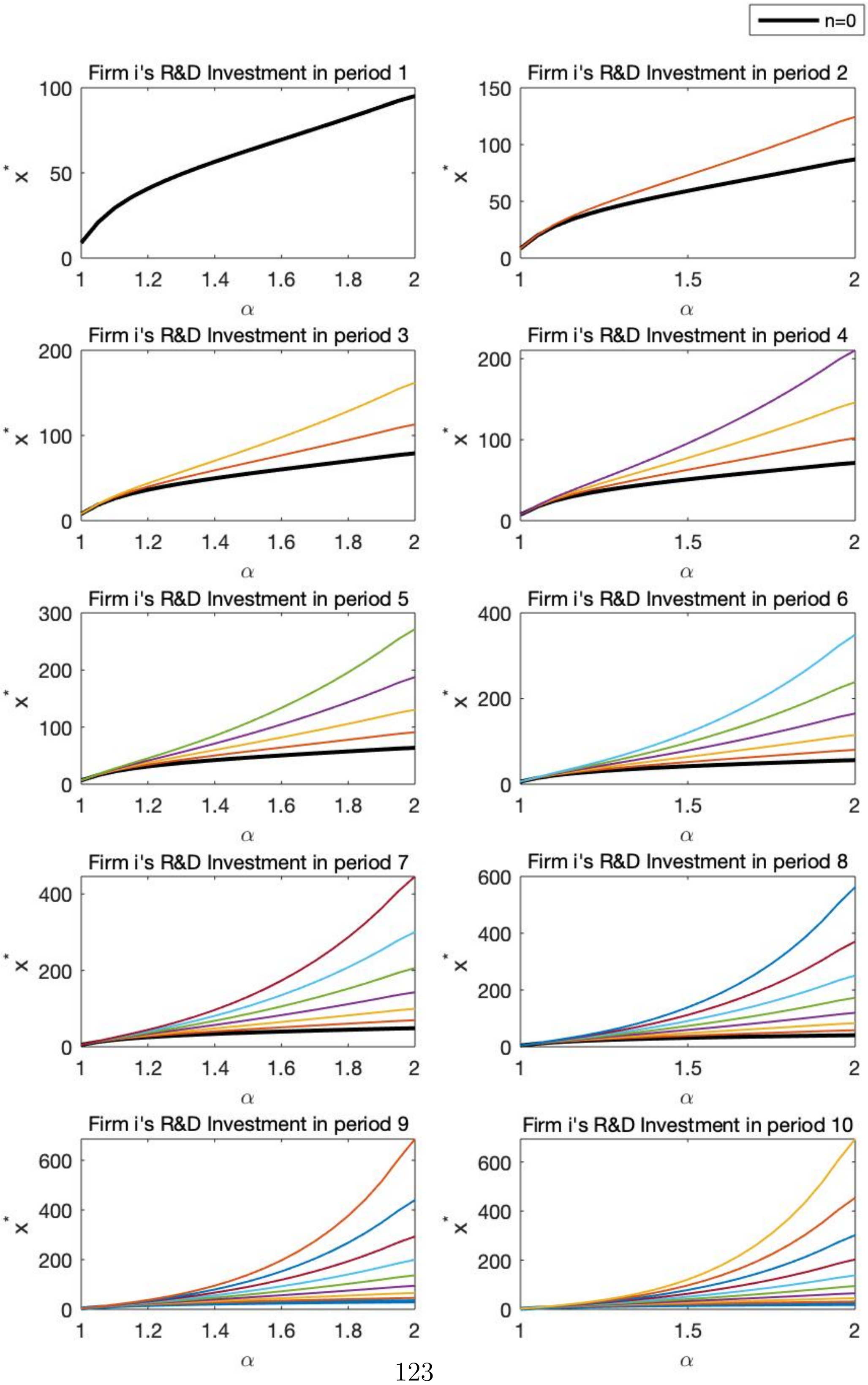


Figure 4.6: Follower's R&D investments with changing α

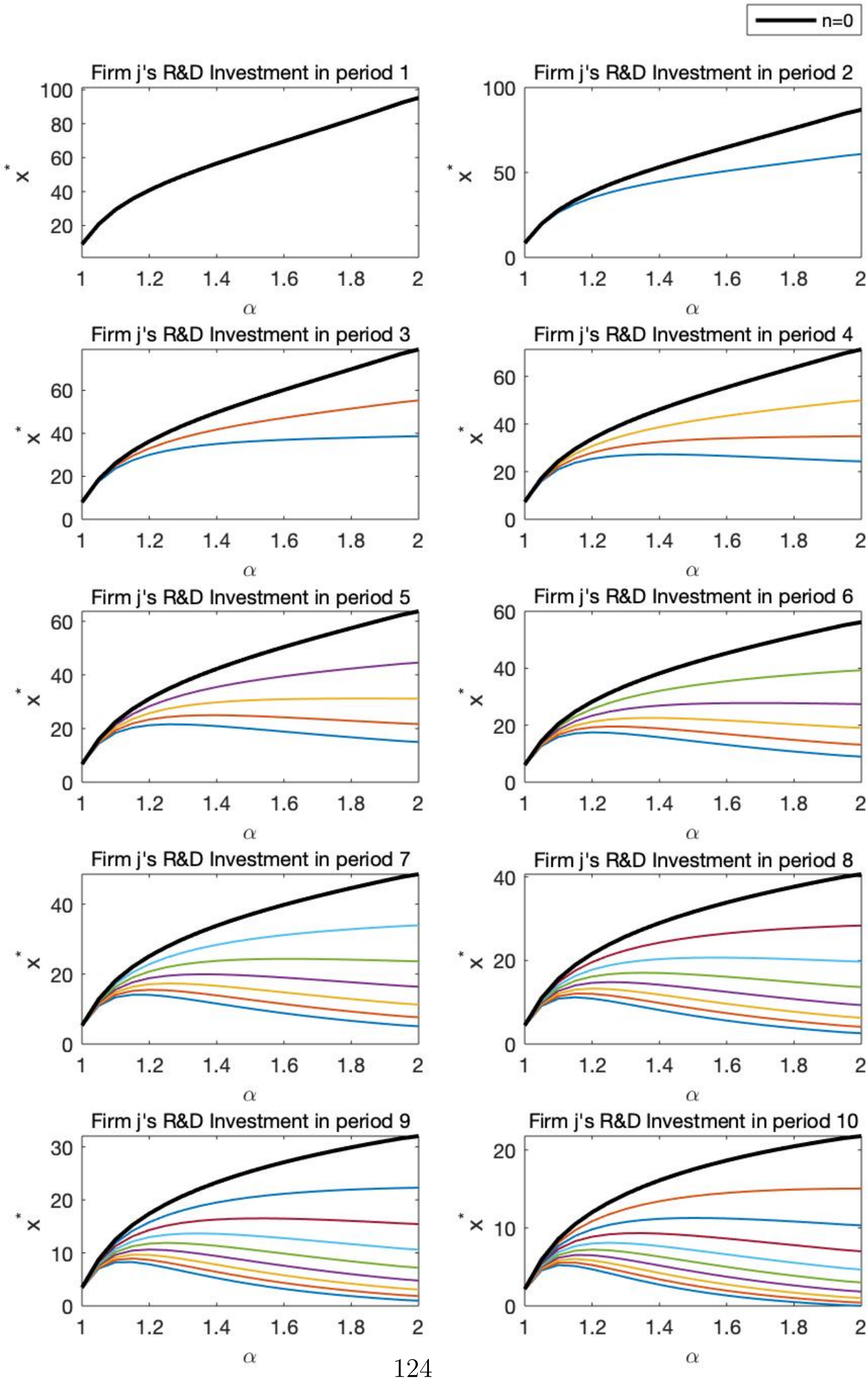
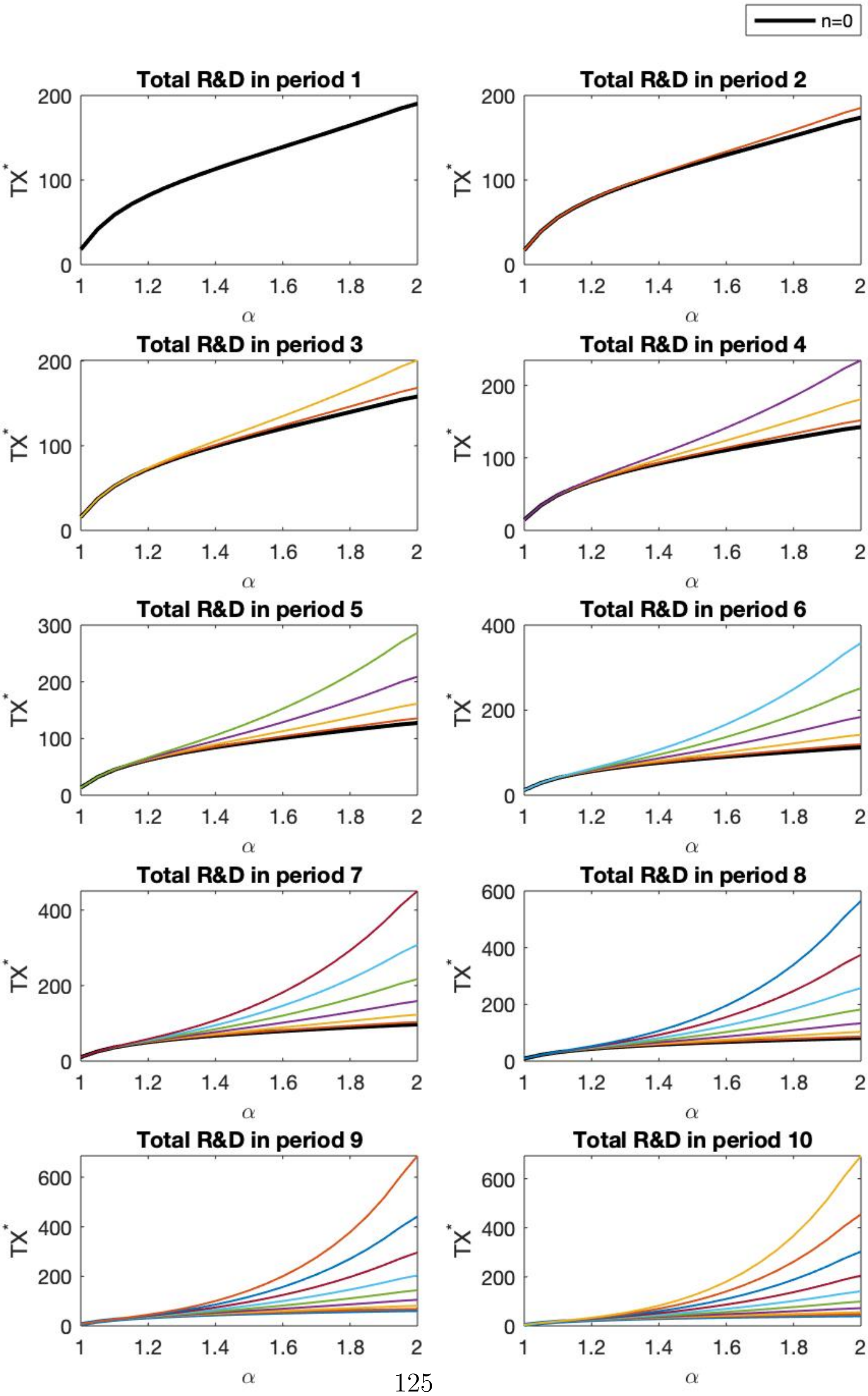


Figure 4.7: Total R&D investment with changing α



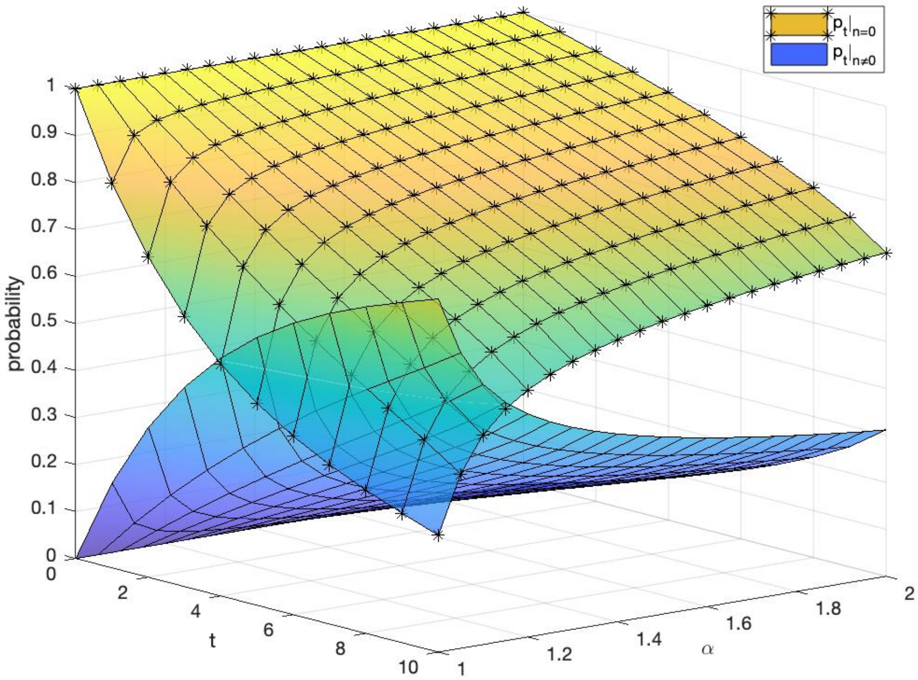
logical difference is larger.

4.4.3 Time paths of the technological difference and the effect of α

One of the aims is to identify how the technological difference between two initially identical firms evolves with stochastic R&D outcomes in a multi-period game. Is it the case that two firms are neck to neck or they are diverging as the game proceeds? Therefore, the main issues are to determine a measurement of technological difference and divergence. In each period t, n is defined as the difference between the total number of successes of firm i and those of its rival, where $n = -(t-1), -(t-2), .., 0, ..t-2, t-1$. The situation where firm i is n leads ahead is equivalent to where firm i is $|n|$ lags behind from the whole industry's perspective. That is to say, $|n|$ expresses the technological difference between firms. As per divergence, due to the stochastic outcomes and dynamic game, although there are $t+1$ possible outcomes, i.e., $|n| = 0, 1, ..., t$, at the end of each period t, many paths could lead to one of the outcomes. As the game goes on, since the possible values of n increase likewise, it is hard to normalize the divergence. Let $p_t\mid_n$ express the probability of technological difference $|n|$ in period t. Therefore $p_t\mid_{n\neq 0}$ is employed to measure the divergence when considering different α. Notice $p_0\mid_{n=0} = 1$ and $p_{10}\mid_n$ therefore denotes the probability of different possible outcomes at the end of the 10-period game.

Figure 4.8 shows that the probability of being symmetric (shown with $*$) continuously decreases as the game proceeds. It is consistent with the expectation that firms who start symmetric become more asymmetric with time. In addition, Figure 4.8 shows that the probability of being symmetric at the end of the game increases with α. To be specific, with lower value of α (around 1.2), $p_t\mid_{n\neq 0}$ is higher than $p_t\mid_{n=0}$, which means firms are more likely to end up with being asymmetric. By contrast, with relatively higher value of α, although the probability being asymmetric increases as the game continues, they are more likely to

Figure 4.8: Time paths of the technological difference with changing α ($T = 10$)



be symmetric at the end. Since α represents the revenue multiplier, the intuition is that with higher α, the firm knows that it can earn more with a single success, which gives larger incentive to invest in R&D so that the probability of success becomes larger. But if both increase their investment, they are more likely to be symmetric at the end. In general, it indicates that although the probability being asymmetric increases, the technological difference between firms is not as big at the end of the game.

4.5 Conclusions

In this chapter, I mainly focus on two research questions, how dynamic competition and stochastic outcome of R&D affect firms' equilibrium R&D investment and how the technological difference between two initially identical firms evolves over time. The model is different from the traditional patent race model. The model in this chapter allows firms to invest in every period and both are likely to obtain the technology progress in every period. The key endogenous parameter, position n, captures the history and I assume the revenue depends on the position. I find that the leader has higher incentive to invest than the follower and the more leads the firm is ahead, the more investment it makes. As the game proceeds, both the leader and follower decrease R&D. That is to say, the position n and time path t show opposite effects on R&D investment. Which one dominates depends on the exogenous parameter α. As for the evolution of technological difference between firms, since firms start from a symmetric position, the probability being asymmetric indeed increases continuously. However, since I assume both firms are active during the whole game, the technological difference is not so big at the end of the game.

There are some possible ways to extend this chapter in future research. First, the probability of success is assumed to be independent across periods in this model. More realistically, if the firm invests significantly in the previous periods but fails, then it may succeed in the upcoming period by investing a little. Therefore, the probability of success depends on cumulative R&D. Second, if a firm is allowed to exit the market, the end of game should be modeled as being endogenous instead of exogenous. Therefore, relaxing Assumption 4.1 to coincide with the endogenous end of the game is another possible extension. Third, it might be interesting to know if we can infer from R&D investment data whether the firm has succeeded or not.

Appendix

Proof of Proposition 4.3

Let $x^i_m(n)$ and $x^j_m(n)$ denote the R&D investment of firm i and its rival firm j in period m, where $m = 1, 2, ..., t$ and $n = -(m-1), -(m-2), ., 0, .., m-2, m-1$, and the corresponding probability of success and failure are denoted by $p^i_m(n)$,

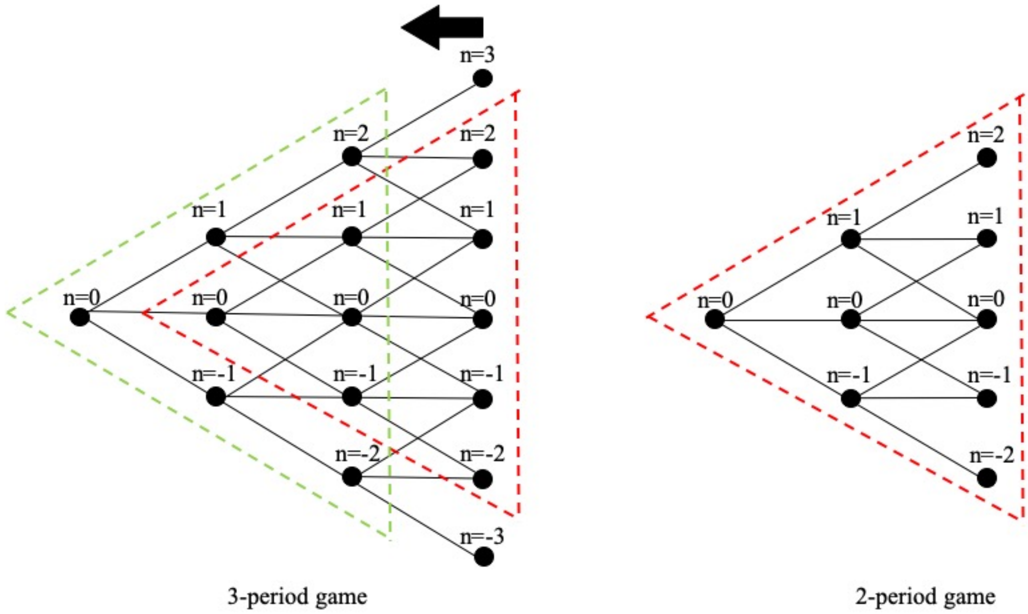


Figure 4.9: The structure of 2-period and 3-period game

$q^i_m(n)$ for firm i and $p^j_m(n)$, $q^j_m(n)$ for firm j, where $q = 1 - p$. In addition, the total payoff of firm i and j in period m is denoted by $V^i_m(n)$ and $V^j_m(n)$.

The firms are symmetric at the beginning, that is, $n = 0$. Suppose that there are t periods left in the game. Take any contingent plan of investments for firm i: $(x^i_1, \vec{x}^i_2, \ldots, \vec{x}^i_t)$. $\vec{x}^i_2$ contains three different investment levels, which are $x^i_2(n = -1), x^i_2(n = 0), x^i_2(n = 1)$, depending on if firm i won, tied, or lost in period 1. In general, $\vec{x}^i_t$ contains $2t - 1$ different investment levels, depending on what happened in the previous $t - 1$ periods. So does p^i_m.

Considering a 2-period game, where $t = 2$, the total payoff is,

$$V^i_1 \mid_{t=2} = \pi_1 + E(\pi_2) \tag{4.44}$$

where

$$\begin{aligned}
\pi_1 =&(p_1^i p_1^j + q_1^i q_1^j)\pi + p_1^i q_1^j \cdot \alpha\pi + q_1^i p_1^j \cdot \alpha^{-1}\pi - x_1^i \\
E(\pi_2) =&(p_1^i p_1^j + q_1^i q_1^j) \cdot \big[(p_2^i(0)p_2^j(0) + q_2^i(0)q_2^j(0))\pi + p_2^i(0)q_2^j(0) \cdot \alpha\pi + q_2^i(0)p_2^j(0) \cdot \alpha^{-1}\pi - x_2^i(0)\big] \\
&+p_1^i q_1^j \cdot \big[(p_2^i(1)p_2^j(1) + q_2^i(1)q_2^j(1))\alpha\pi + p_2^i(1)q_2^j(1) \cdot \alpha^2\pi + q_2^i(1)p_2^j(1) \cdot \pi - x_2^i(1)\big] \\
&+q_1^i p_1^j \cdot \big[(p_2^i(-1)p_2^j(-1) + q_2^i(-1)q_2^j(-1))\alpha^{-1}\pi + p_2^i(-1)q_2^j(-1)\pi + q_2^i(-1)p_2^j(-1)\alpha^{-2}\pi - x_2^i(-1
\end{aligned} \tag{4.45}$$

The equilibrium R&D investments in a 2-period game include $x_1^{i*}, x_2^{i*}(n = 0), x_2^{i*}(n = 1), x_2^{i*}(n = -1)$, which satisfy,

$$\frac{\partial V_1^i \mid_{t=2}}{\partial x_1^{i*}} = \frac{\partial \pi_1}{\partial x_1} + \frac{\partial E(\pi_2)}{\partial x_1} = 0 \tag{4.46}$$

$$\frac{\partial V_1^i \mid_{t=2}}{\partial x_2^{i*}(n=0)} = (p_1^i p_1^j + q_1^i q_1^j) \cdot \frac{\partial \pi_2^i(n=0)}{\partial x_2^{i*}(n=0)} = 0 \tag{4.47}$$

$$\frac{\partial V_1^i \mid_{t=2}}{\partial x_2^{i*}(n=1)} = p_1^i q_1^j \cdot \frac{\partial \pi_2^i(n=1)}{\partial x_2^{i*}(n=1)} = 0 \tag{4.48}$$

$$\frac{\partial V_1^i \mid_{t=2}}{\partial x_2^{i*}(n=-1)} = q_1^i p_1^j \cdot \frac{\partial \pi_2^i(n=-1)}{\partial x_2^{i*}(n=-1)} = 0 \tag{4.49}$$

Next, consider a 3-period game, where $t = 3$. Let $y_1^i, \vec{y}_2^i, \vec{y}_3^i$ denote the contingent plans for 3-period game.

$$V_1^i \mid_{t=3} = \pi_1 + E(\pi_2) + E(\pi_3) \tag{4.50}$$

$$\begin{aligned}
\pi_1 =&(p_1^i p_1^j + q_1^i q_1^j)\pi + p_1^i q_1^j \cdot \alpha\pi + q_1^i p_1^j \cdot \alpha^{-1}\pi - y_1^i \\
E(\pi_2) =&(p_1^i p_1^j + q_1^i q_1^j) \cdot \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j)\pi + \vec{p}_2^i\vec{q}_2^j \cdot \alpha\pi + \vec{q}_2^i\vec{p}_2^j \cdot \alpha^{-1}\pi - \vec{y}_2^i\big] \\
&+p_1^i q_1^j \cdot \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j)\alpha\pi + \vec{p}_2^i\vec{q}_2^j \cdot \alpha^2\pi + \vec{q}_2^i\vec{p}_2^j \cdot \pi - \vec{y}_2^i\big] \\
&+q_1^i p_1^j \cdot \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j)\alpha^{-1}\pi + \vec{p}_2^i\vec{q}_2^j \cdot \pi + \vec{q}_2^i\vec{p}_2^j \cdot \alpha^{-2}\pi - \vec{y}_2^i\big] \\
E(\pi_3^i) =&(p_1^i p_1^j + q_1^i q_1^j) \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j) \cdot \pi_3^i(0) + \vec{p}_2^i\vec{q}_2^j \cdot \pi_3^i(1) + \vec{q}_2^i\vec{p}_2^j \cdot \pi_3^i(-1)\big] \\
&+p_1^i q_1^j \cdot \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j) \cdot \pi_3^i(1) + \vec{p}_2^i\vec{q}_2^j \cdot \pi_3^i(2) + \vec{q}_2^i\vec{p}_2^j \cdot \pi_3^i(0)\big] \\
&+q_1^i p_1^j \cdot \big[(\vec{p}_2^i\vec{p}_2^j + \vec{q}_2^i\vec{q}_2^j) \cdot \pi_3^i(-1) + \vec{p}_2^i\vec{q}_2^j \cdot \pi_3^i(0) + \vec{q}_2^i\vec{p}_2^j \cdot \pi_3^i(-2)\big]
\end{aligned} \tag{4.51}$$

with $\pi_3^i(n) = (\vec{p}_3^i\vec{p}_3^j + \vec{q}_3^i\vec{q}_3^j)\alpha^n\pi + \vec{p}_3^i\vec{q}_3^j \cdot \alpha^{n+1}\pi + \vec{q}_3^i\vec{p}_3^j \cdot \alpha^{n-1}\pi - \vec{y}_3^i$.

Suppose the 3-period game's equilibrium R&D investments in the first two periods are $y_1^{i*}, y_2^{i*}(n=-1), y_2^{i*}(n=0), y_2^{i*}(n=1)$, which satisfy

$$\frac{\partial V_1^i \mid_{t=3}}{\partial y_1^{i*}} = \frac{\partial \pi_1}{\partial y_1^i} + \frac{\partial E(\pi_2^i)}{\partial y_1^i} + \frac{\partial E(\pi_3^i)}{\partial y_1^i} = 0 \tag{4.52}$$

$$\frac{\partial V_1^i \mid_{t=3}}{\partial y_2^{i*}(n=0)} = (p_1^i p_1^j + q_1^i q_1^j) \cdot \frac{\partial \pi_2^i(n=0)}{\partial y_2^i(n=0)} + \frac{\partial E(\pi_3^i)}{\partial p_2^i} \cdot \frac{dp_2^i}{dy_2^i} = 0 \tag{4.53}$$

$$\frac{\partial V_1^i \mid_{t=3}}{\partial y_2^{i*}(n=1)} = p_1^i q_1^j \cdot \frac{\partial \pi_2^i(n=1)}{\partial y_2^i(n=1)} + \frac{\partial E(\pi_3^i)}{\partial p_2^i} \cdot \frac{dp_2^i}{dy_2^i} = 0 \tag{4.54}$$

$$\frac{\partial V_1^i \mid_{t=3}}{\partial y_2^{i*}(n=-1)} = q_1^i p_1^j \cdot \frac{\partial \pi_2^i(n=-1)}{\partial y_2^i(n=-1)} + \frac{\partial E(\pi_3^i)}{\partial p_2^i} \cdot \frac{dp_2^i}{dy_2^i} = 0 \tag{4.55}$$

Firstly, compare Eq.(4.53)(4.54)(4.55) with (4.59)(4.60)(4.61) respectively.

$$\frac{\partial E(\pi_3^i)}{\partial p_2^i} \mid_{n=0} = (p_1^i p_1^j + q_1^i q_1^j) \cdot \left[p_2^j \left(\pi_3^i(0) - \pi_3^i(-1)\right) + q_2^j \left(\pi_3^i(1) - \pi_3^i(0)\right)\right] > 0$$

$$\frac{\partial E(\pi_3^i)}{\partial p_2^i} \mid_{n=1} = p_1^i q_1^j \cdot \left[p_2^j \left(\pi_3^i(1) - \pi_3^i(0)\right) + q_2^j \left(\pi_3^i(2) - \pi_3^i(1)\right)\right] > 0$$

$$\frac{\partial E(\pi_3^i)}{\partial p_2^i} \mid_{n=-1} = q_1^i p_1^j \cdot \left[p_2^j \left(\pi_3^i(-1) - \pi_3^i(-2)\right) + q_2^j \left(\pi_3^i(0) - \pi_3^i(-1)\right)\right] > 0$$

From the previous Proposition, the equilibrium profits in the last period follow $\frac{\partial \pi_3^i(n)}{\partial n} > 0$, which induces that $\frac{\partial E(\pi_3^i)}{\partial p_2^i}$ are always positive. Combined with $p'(y) > 0$, all the second terms in Eq.(4.59)(4.60)(4.61) are positive. Therefore, every equilibrium R&D investment in the second period $y_2^{i*}(n=0), y_2^{i*}(n=1), y_2^{i*}(n=-1)$, is larger than the corresponding $x_2^{i*}(n=0), x_2^{i*}(n=1), x_2^{i*}(n=-1)$.

Next, compare y_1^{i*} with x_1^{i*}. Eq.(4.52) with (4.58). The first term is the same, which implicitly defines the equilibrium R&D investment when firms are symmetric in the last period.

$$\frac{\partial E(\pi_2^i)}{\partial x_1^i} = p_1'^i (p_1^j(\pi_2(0) - \pi_2(-1)) + q_1^j(\pi_2(1) - \pi_2(0))) > 0 \tag{4.56}$$

$$\frac{\partial E(\pi_2^i) + E(\pi_3)}{\partial y_1^i} = p_1'^i (p_1^j(V_2(0) - V_2(-1)) + q_1^j(V_2(1) - V_2(0))) > 0 \tag{4.57}$$

Then the key issue is to determine the magnitude between $\pi_2(n) - \pi_2(n-1)$ with $V_2(n) - V_2(n-1)$. It is equivalent to compare $\frac{d\pi_2^*}{dn}$ with $\frac{dV_2^*}{dn}$.

$$\pi_2^{i*}(n) = (p_2^i p_2^j + q_2^i q_2^j)\alpha^n \pi + p_2^i q_2^j \cdot \alpha^{n+1}\pi + q_2^i p_2^j \cdot \alpha^{n-1}\pi - x_2^i$$

$$V_2^{i*}(n) = (p_2^i p_2^j + q_2^i q_2^j)(\alpha^n \pi + \pi_3^{i*}(n)) + p_2^i q_2^j \cdot (\alpha^{n+1}\pi + \pi_3^{i*}(n+1)) + q_2^i p_2^j(\alpha^{n-1}\pi + \pi_3^{i*}(n-1)) - y_2^i$$

Both $\pi_2^{i*}(n)$ and $\pi_3^{i*}(n)$ are the equilibrium payoffs of the last period in 2-period and 3-period game respectively. Therefore they are identical. Substitute π_3^{i*} with π_2^{i*},

$$\begin{aligned}\mathit{\Delta} \equiv V_2^{i*}(n) - \pi_2^{i*}(n) =&(p_2^i p_2^j + q_2^i q_2^j)(\alpha^n \pi + \pi_2^{i*}(n)) + p_2^i q_2^j \cdot (\alpha^{n+1}\pi + \pi_2^{i*}(n+1)) \\ &+ q_2^i p_2^j(\alpha^{n-1}\pi + \pi_2^{i*}(n-1)) - y_2^i - \pi_2^{i*}(n)\end{aligned}$$

Totally differentiate $V_2^{i*}(n) - \pi_2^{i*}(n)$ with respect to n,

$$\frac{d\mathit{\Delta}}{dn} = \frac{\partial \mathit{\Delta}}{\partial y_2^i}\frac{dy_2^i}{dn} + \frac{\partial \mathit{\Delta}}{\partial y_2^j}\frac{dy_2^j}{dn} + \frac{\partial \mathit{\Delta}}{\partial n} \tag{4.58}$$

$$+\frac{\partial \mathit{\Delta}}{\partial \pi_2^{i*}(n)} \cdot \pi_2'^{i*}(n) + \frac{\partial \mathit{\Delta}}{\partial \pi_2^{i*}(n+1)} \cdot \pi_2'^{i*}(n+1) + \frac{\partial \mathit{\Delta}}{\partial \pi_2^{i*}(n-1)} \cdot \pi_2'^{i*}(n-1) \tag{4.59}$$

The first term in (4.64) is zero and the second and third terms are positive.

$$\begin{aligned}(16) \equiv&(p_2^i p_2^j + q_2^i q_2^j - 1) \cdot \pi_2'^{i*}(n) + p_2^i q_2^j \cdot \pi_2'^{i*}(n+1) + q_2^i p_2^j \cdot \pi_2'^{i*}(n-1) \\ =&p_2^i q_2^j \cdot (\pi_2'^{i*}(n+1) - \pi_2'^{i*}(n)) - q_2^i p_2^j \cdot (\pi_2'^{i*}(n) - \pi_2'^{i*}(n-1))\end{aligned}$$

Due to the convexity of the equilibrium payoff π_2^{i*} in parameter n,

$$\pi_2'^{i*}(n+1) - \pi_2'^{i*}(n) > \pi_2'^{i*}(n) - \pi_2'^{i*}(n-1) > 0$$

In addition,

$$p_2^i q_2^j - q_2^i p_2^j = p_2^i - p_2^j \geqslant 0 \ for \ n \geqslant 0$$

Therefore, (4.65) > 0, which immediately follows that $\frac{d\mathit{\Delta}}{dn} > 0$, that is, $\frac{dV_2^*}{dn} > \frac{d\pi_2^*}{dn}$, which further illustrates that $\frac{\partial E(\pi_2^i)}{\partial x_1^i}$ in Eq.(4.62) is smaller than $\frac{\partial E(\pi_2^i)+E(\pi_3)}{\partial y_1^i}$ in Eq.(4.63). According to Eq.(4.52) and (4.58), $\frac{\partial \pi_1}{\partial x_1}$ is therefore larger than $\frac{\partial \pi_1}{\partial y_1}$, which means y_1 moves even furthur away from the maximization point in the last period than x_1, that is, $y_1 > x_1$.

Moreover, if the firms are still symmetric after period 1 in a 3-period game, then it follows the same situation as the beginning of a 2-period game. That is to say, $y_2^{i*}(n=0) = x_1^{i*}$, $y_3^{i*}(n=0) = x_2^{i*}(n=0)$, $y_3^{i*}(n=1) = x_2^{i*}(n=1)$ and $y_3^{i*}(n=-1) = x_2^{i*}(n=-1)$.

To summarize the proof above, one can get,

$$y_1^{i*} > y_2^{i*}(n=0) > y_3^{i*}(n=0)$$

$$y_2^{i*}(n=1) > y_3^{i*}(n=1)$$

$$y_2^{i*}(n=-1) > y_3^{i*}(n=-1)$$

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Chapter 5

Conclusion

This thesis has provided a fuller picture of technology development, from the R&D stage to the technology diffusion stage. All the research questions are motivated by the features of R&D activity and market structure. At the R&D stage, I studied how asymmetric firms' R&D choices are affected by the one-way spillover and accordingly, how the cost gap between firms varies. Moreover, by employing the uncertain outcome of R&D, I studied how firms' R&D choices are affected in a multi-period game and how the technological difference between initially identical firms evolves. After a successful innovation, the innovator now is willing to maximise the return from this innovation. I introduced a fully flexible discriminatory licensing scheme for the outside innovator in an asymmetric duopoly market and determined the privately optimal and socially optimal strategy.

The whole story can be explained as follows. Assume there are two firms with the same technological level in the market. If they engage in R&D activities themselves and try to make technological progress, the uncertainty of R&D under long-term competition leads to a growing technological gap between two initially symmetric firms. Now the small firm tries to stop the divergence and narrow the gap with the large firm. I found that the one-way spillover is a possible factor which drives the small firm to invest more in R&D so that the small firm may catch up with the larger firm. If the firms just buy the license from an outside innovator, the small firm has higher incentive to pay for the license if it is a product innovation while less incentive if it is a process innovation.

The three chapters analyse the R&D stage and diffusion stage separately. Specifically, chapter 2 excludes the R&D stage of the innovator and assumes instead that the technology has already been discovered. Chapter 3 and 4 only focus on R&D stage under different environments. One possible extension is to combine two stages and determine the optimal R&D choice and the corresponding optimal licensing strategy to maximise the payoff for either the inside innovator or outside innovator.

In addition, the innovation in chapter 2 is owned by an outside innovator which is not a producer in the market. While the R&D activity in chapter 3 and 4 is done by the producers and therefore if it succeeds, they act as the inside innovator. Another possible extension is to investigate under what conditions a firm prefers to just act as an outside innovator without production and under what conditions the firm prefers to be an inside innovator, which provides a platform to link those two roles.

Another limitation of this thesis is that firms are always active in the market, regardless of whether they obtain the license or not in chapter 2, or whether they successfully engage in R&D in chapter 3 and 4. This assumption can be relaxed to allow a firm to exit the market.