Could the UK's Monetary Policy be Improved by Targeting Nominal GDP?
A Model of the UK with State-Dependent Price/Wage Contracts

by

Haixia Chen

A Thesis Submitted in Fulfilment of the Requirements for

the Degree of Doctor of Philosophy of

Cardiff University

Thesis Supervisors

Prof. Patrick Minford

Prof. Vo Phuong Mai Le

Dr. David Meenagh

June 2023

Abstract

There is substantial evidence from both macro and micro data that the durations of price/wage contracts vary with the state of the economy, particularly with inflation. This thesis investigates whether there is macro-level evidence of state-dependent price/wage contracts in the UK and examines the policy implications of a UK macroeconomic model in the presence of state-dependent variation. Two versions of the price/wage-setting framework are considered in this thesis: a fixed price/wage contract duration framework and a state-dependent duration framework. Each framework is incorporated into an open economy Dynamic Stochastic General Equilibrium (DSGE) model. It combines the New Keynesian and New Classical models into a hybrid model, incorporates financial friction, and includes several new developments in the wake of the recent Great Financial Crisis, including allowing for zero lower bound and quantitative easing. The fixed duration and state-dependent models are tested and estimated over the whole sample period 1955-2021 for the UK macroeconomic data using a simulation-based Indirect Inference method. The main findings of this thesis are: 1) the fixed duration model fits the behaviour of the UK data for the inflation targeting era 1992-2021, but not for the whole sample period 1955-2021; 2) The state-dependent model fits the behaviour of the UK historical data well over the whole sample period 1955-2021, implying that the state-dependent price/wage contract framework improves the fit of the DSGE model to the macroeconomic data. Furthermore, the durations of price/wage contracts fluctuate with the state of the economy (especially inflation) throughout the whole sample period; 3) A nominal GDP targeting rule together with a fiscal backstop to prevent zero lower bound can reduce the chances of economic crisis and stabilise the UK macroeconomy. Therefore, it outperforms the Taylor Rule regime.

Acknowledgements

Completing this thesis has been the most significant challenge I have faced. Without the help and support of the following people, I would never complete this thesis.

First and foremost, I would like to express my warm and sincere thanks to my supervisor, Professor Patrick Minford, who made my PhD studies at Cardiff University as well as this thesis possible and gave me the opportunity to pursue this exciting research topic. I am incredibly grateful for his insightful guidance, valuable advice and detailed comments on my thesis. His dedication and passion for academic research, as well as his kindness and great patience with students, have always inspired me.

I would also like to express my warm and sincere thanks to my supervisor, Professor Vo Phuong Mai Le, for her kindness, positive encouragement, valuable suggestions, kind support, helpful comments on the thesis and help with modelling and theoretical questions.

I would also like to express my warm and sincere thanks to my supervisor, Dr. David Meenagh, for his kindness, great patience and insightful guidance. In particular, I would like to thank him for his willingness to sacrifice his time to help me with the questions for data and programming, and to comment on the empirical results at each stage as well as the whole thesis. Without his support, this thesis would not be possible.

I would like to thank the Julian Hodge Institute of Applied Macroeconomics for providing me with a scholarship. I would also like to thank all members of Cardiff Business School for providing a friendly research environment and teaching opportunities over the past years. I would like to thank my friends and colleagues for their help and support, Dr. Zheyi Zhu, Dr. Juyi Lyu, Dr. Ziqing Wang, Dr. Yuji Fang, Dr. Zhiqi Zhao, Xinxing Chen, Soma Housein, Aohan Gao, Jiachen Yi, Yuanli Guo, Feng Wang and many other friends and colleagues whose names are not mentioned here due to word limitations.

Lastly and most importantly, I would like to give special thanks to my family, especially my parents, for their endless love, immense support, optimistic encouragement and continuous understanding. I dedicate this thesis to my amazing parents.

Table of Contents

List of Tables

List of Figures

Chapter 1 Introduction

Background and Motivation

Micro-founded models of price and wage-setting behaviour are essential for understanding the dynamics of macroeconomic variables and for assessing the performance of alternative monetary policy regimes. It is widely debated whether price/wage adjustments are a response to changes in economic conditions or whether this time is exogenous. New Keynesian models now dominate the modern applied macroeconomic models and assume that the durations of price/wage contracts are fixed. However, the classical theory emphasises that prices/wages are flexible and that its contract equivalents are state-contingent contracts; thus, agents could achieve optimal outcomes. There is substantial evidence in both macro and micro data that the durations of price/wage contracts fluctuate with the state of the economy, and in particularly with inflation. However, because of menu costs, it may be optimal for agents to ignore small shocks and keep prices/wages unchanged for some duration, since in this case the cost of changing prices/wages could be greater than the cost of the shocks. But the cost of shocks they would ignore in this way and the duration they would be willing to maintain prices/wages constant would be state-dependent. However, when the cost of not responding to the shocks is higher than the cost of the shocks, then it would be optimal for the agents to adjust the prices/wages in response to the shocks. Therefore, this is different from the classical assumption of fully state-dependent contracts and fully flexible prices/wages. In other words, prices and wages in general are not fully flexible in the presence of the menu costs. In this thesis, this hypothesis of state-dependent price/wage rigidity will be examined.

This thesis is inspired by a series of US research by Le et al. (2011, 2016, 2021). Le et al. (2011) estimate a hybrid DSGE model with fixed price/wage durations by using the method of Indirect Inference. They find that the fixed price/wage duration model fits the behaviour of the US data for the period 1984-2004 but is rejected by the behaviour of the data for the whole post-war period 1947-1984. They suggest that the failure of their model to pass the test for the full post-war sample may be attributed to changes in wage/price-setting behaviour over time in response to a fluctuating macro environment, as the model assumes fixed price/wage durations. Le et al. (2016) extend the hybrid DSGE model of Le et al. (2011) to allow for financial friction, Zero Lower Bound (ZLB) constraint and quantitative easing (QE). The extended model fits the behaviour of the US data well from the Great Moderation to 2011 by using Indirect Inference. However, this fixed duration model still does not fit the data behaviour for the period 1947-1984. Therefore, Le et al. (2021) extend their Le et al. (2016) model to include state-dependence in price/wage durations and then re-estimate the model for the full post-war period. They find that the model with this extension can match the data behaviour well for the full sample period. Moreover, there are strong NK periods during the Great Moderation and flexible price (hereafter flexprice) periods during the Great Inflation and the Great Recession.

Research Questions and Contributions

Inspired by Le et al. (2011, 2016, 2021), in this thesis the state-dependent variation is incorporated by building a hybrid Dynamic Stochastic General Equilibrium (DSGE) model in which a fraction of goods markets is assumed to have flexprice while the rest have sticky prices; similarly, a fraction of labour markets sets wages flexibly whilst the remainder face nominal rigidities. The hybrid state-dependent model assumes that the fraction of flexible firms and unions is state-dependent and is related to the past inflation. For a fixed prices/wages duration model, the fraction of flexible sectors is assumed to be fixed. This thesis tests and estimates both the fixed duration and the state-dependent duration models on unfiltered UK macroeconomic data over the period 1955Q1-2021Q1 using a simulation-based Indirect Inference method. The state-dependent model is then used to assess the performance of an alternative monetary policy regime. The following are the main questions to be addressed in this thesis: 1) Whether the hybrid DSGE model of Le et al. (2016) with fixed shares of sticky and flexprice sectors can explain the behaviour of the UK data over the inflation targeting era 1992-2021 and the whole sample period 1955-2021; 2) Whether there is macro-level evidence of state dependence in the UK and whether a state-dependent price/wage framework can improve the fit of the DSGE model to the macroeconomic data over the period 1955-2021; 3) How monetary policy in the UK could be used to stabilise the economy in the presence of state-dependent variation. The main findings and contributions of this thesis are that the model that takes the state-dependence into account improves the fit of the model to macroeconomic data over the full sample period 1955-2021 in comparison to the fixed duration model. Furthermore, an alternative policy regime, i.e. an interest rate policy targets nominal GDP together with a fiscal backstop to prevent ZLB, improves the macroeconomy stability in the UK. To the best of my knowledge, this thesis is the first macro-level empirical study of the state-dependent price/wage contracts in the UK.

Outline

Chapter 2 provides a review of the literature on state-dependent price/wage adjustments at the micro- and macro-levels. Micro-level empirical studies in different countries and various data episodes have shown state-dependent price/wage adjustments, especially state-dependent pricing. However, only a few macro-level studies have incorporated state-dependent price and wage contracts in DSGE models, all of which have focused on the US. Therefore, there is a gap in empirical research on state-dependence at the macro-level in the UK.

Chapter 3 builds a small open economy DSGE model based on the well-known Smets and Wouters (2007) (hereafter SW07) model. It extends the SW07 model by merging the New Keynesian (NK) and New Classical (NC) models into a hybrid model as in Le et al. (2011) and by incorporating the financial friction of the Bernanke et al. (1999) (hereafter BGG), in addition to several new developments in Le et al. (2016) in the wake of the recent Great Financial Crisis (GFC): allowing for ZLB and QE. Furthermore, the model is extended to the open economy, with trade treated as in Armington (1969) and assuming uncovered interest rate parity (UIP).

A hybrid model merging the NK and NC models could improve the fit of the model to the features of the economy. The reasoning behind the hybrid model is that the economy consists of product sectors with more price rigidity and other sectors with more flexible prices, which reflect the degree of competition in these sectors. The labour market is similar, as in the goods market, certain sectors may be more/less dominated by competition, resulting in greater/less wage flexibility. The hybrid model assumes that parts of the firms/unions enjoy prices/wage flexibility, while others have sticky prices/wages. Regarding the financial fraction, financial sector shocks play an important role in explaining aggregate fluctuations (see examples including but not limited to: Gilchrist et al., 2009; Christiano et al., 2010; Fahr et al., 2011; Jermann and Quadrini, 2012; Hubrich et al., 2013; Caldara et al., 2016). However, one of the criticisms of the standard NK DSGE models is that they lack an adequate financial sector and hence fail to explain an important source of macroeconomic fluctuations. We therefore incorporate the model with the banking/financial accelerator mechanism of the BGG model. In addition, due to the GFC, the Bank of England (BoE) cut its bank rate to near zero with the aim of boosting spending and employment. However, the ZLB constraint limited the scope for any further rate decreases. As a result, the BoE engaged in multiple rounds of massive QE programmes to inject liquidity into the banking system to ease the credit condition. Our model incorporates these new developments since the onset of the GFC by allowing for the presence of ZLB and QE. We achieve this by using a switchable model consisting of two regimes: in the normal regime, the Taylor Rule operates normally and the supply of M0 is set to accommodate broad money M2; while in the crisis regime, nominal interest rate reaches the threshold level or below and is effectively bounded by the ZLB. As a result, the Taylor Rule is suspended and replaced with an exogenous low bound. Meanwhile, the idea of using cash as the cheapest collateral is included to inject the QE element, and M0 becomes the main tool to target external financing premium. Once the model moves away from the ZLB, the model switches back to the normal regime with the Taylor Rule operating again. Furthermore, to take into account the fact that the UK trade to GDP ratios are ranges from 41-63% over 1970-2021[1], we extend the model to a small open economy with trade treated as in Armington (1969) and assuming UIP.

Chapter 4 discusses the construction of the macroeconomic dataset used in this thesis and proposes a set of starting calibrations. It constructs a quarterly and unfiltered UK macroeconomic dataset from 1955Q1 to 2021Q1 for our empirical analysis. This long sample period includes the stagflation in the 1970s, the exchange rate mechanism crisis, the 2008 financial crisis, the 2016 EU referendum, and the first year of the coronavirus pandemic. In addition, a set of starting calibrations will be proposed. In later chapters, it will be used to initially simulate the model. Then the simulated annealing algorithm will randomly search across this starting calibration

[1] Data Source: The World Bank.

within a selected range to find the optimal parameter set that minimises the distance between simulated and actual data.

Chapter 5 introduces the method of Indirect Inference used to test and estimate the model. In contrast to most macroeconomic studies that use the popular Bayesian technique, this thesis employs a simulation-based Indirect Inference method to test and estimate the model. One problem with the Bayesian method is that it heavily relies on the prior information about the macroeconomy. However, this prior information is usually not fully informed. Regarding the Indirect Inference method, it is based on the notion that testing an economic model is not about whether it is true or false, as the current DSGE models are a simplified form of reality that could not 'truly' reflect the real economy. It therefore tests whether a DSGE model is 'pseudo-true' for the data of interest rather than 'literally true'. In other words, the Indirect Inference test measures how well the sample data are represented by an approximately true DSGE model. The basic approach of the Indirect Inference test is to compare the performance of the auxiliary model estimated from simulated data with the performance of the auxiliary model estimated on the actual data. The simulated data is derived from a structure model with given values of its parameters. The auxiliary model usually takes the form of a cointegrated vector autoregressive with exogenous variables (VARX), which can be further approximated to a VAR. The comparison uses a statistical criterion which is based on a Wald test for the differences between the estimates of the auxiliary model obtained from the simulated data with those obtained using the actual data. Therefore, a correct model should yield sensible simulated data and VAR estimates based on these data and they should not considerably differ from the actual data and VAR estimates based on the actual data, respectively. The set of parameters that minimises the distance between the estimates based on the simulated data and the estimates based on the actual data is the optimal choice of set for the structural model. The choices of the auxiliary model, i.e. the choices of the key variables involved, determine the aspects we would like to emphasize. The advantage of using Indirect Inference is that it is less biased than Bayesian method as the latter would be biased for the priors when they are not fully specified.

Chapter 6 investigates whether the hybrid model we developed in Chapter 3 with fixed shares of sticky and flexible price sectors can fit the behaviour of the UK macroeconomic data over the sample period of 1992Q4-2021Q1. This sample period is chosen because the BoE has officially adopted an inflation targeting since 1992Q4. We would like to explore to what extent the fixed duration could explain how the UK data behaved in the inflation targeting era. The results of the Indirect Inference estimation show that the fixed duration model fits the UK data behaviour in this sample period with a P-value of 0.076, implying that the model failed to be rejected at 95%. Chapter 6 also examines the dynamics of the estimated fixed duration model through the impulse responses of endogenous variables to structure shocks under the non-crisis and crisis regimes. The findings in this chapter are highly consistent with those of Lyu et al. (2023) for the UK and Le et al. (2016) for the US.

Chapter 7 extends the data sample period to 1955Q1 to check whether there is any fixed duration set-up that can fit the behaviour of the UK data for this much longer sample period of 1955-2021. In the UK, there was a fixed-but-adjustable exchange rate regime under the Bretton Wood system from October 1949 to May 1972. This poses a problem for our test and estimation for this full sample period 1955-2021. We therefore address this problem by adding an exchange rate target to the Taylor Rule equation in the model for the period of 1955Q1-1972Q2 (referred to as the pre-1972 DSGE model) and turning it into a standard Taylor Rule for the period 1972Q3-2021Q1 (referred to as the post-1972 DSGE model). The results of the Indirect Inference test and estimation show that the fixed duration model does not fit the full sample period, which is consistent with the findings of Le et al. (2011, 2016) for the US. This may be due to distinct economic environments across the full sample period, such as the stagflation crisis in the 1970s and the Great Moderation, so that wage/price behaviour changed in response to fluctuations in macro environment. The results of our UK model over the full sample period can be used as a benchmark to judge the improvements due to state-contingency over the whole sample period in Chapter 8.

In Chapter 8, we integrate the state-dependent price/wage contracts into the hybrid DSGE model we built in Chapter 3 to have a state-dependent DSGE model to investigate whether there is macro-level evidence of state-dependent price/wage durations and whether the state-dependence framework can improve the fit of the macro model to macroeconomic data. To embed state-dependent variations, we assume that the fraction of firms with flexible prices and the fraction of unions with flexible wages are state-dependent rather than fixed, and that they are based on an increasing function of past inflation. The model uses the fixed exchange rate policy regime for the period 1955Q1-1972Q2 and then switches to the inflation targeting regime for the period 1972Q3-2021Q1. The Indirect Inference method is used to test and estimate the state-dependent model on the UK data for the whole sample period 1955Q1-2021Q1. The results show that the state-dependent model fits the behaviour of the historical UK data well over the whole sample period, suggesting that the state-dependent price/wage contract framework improves the fit of the DSGE model to macroeconomic data. Furthermore, the durations of price/wage contracts fluctuate with the state of the economy (inflation) throughout the full sample. Our findings are broadly consistent with those of Le et al. (2021) for the US.

Chapter 9 examines the performance of an alternative policy regime in the state-dependent model, i.e. an interest rate policy targeting nominal GDP together with a fiscal backstop to prevent ZLB, relative to the baseline regime of the Taylor Rule in normal times and QE in times of crisis, in terms of macroeconomic stability. The failure of conventional monetary policy at ZLB and the slow recovery from the Great Recession have reinvigorated the interest of policymakers and economists in targeting nominal GDP. In addition, price/output volatility is high during ZLB episodes and cannot be controlled by unconventional monetary policy QE in the case of state-dependent contracts, as volatility in inflation leads to fluctuations in contracts duration (or stickiness levels) which further exacerbates inflation volatility. We therefore let the nominal GDP targeting regime to be supplemented by a fiscal intervention to prevent ZLB, this fiscal backstop helps nominal interest rates to escape ZLB when it occurs. We assess the performance of nominal GDP targeting together with the fiscal backstop in stabilising the economy relative to the baseline regime. The criteria we consider are 1) how many crises are likely to occur under each policy regime, as a measure of the ability of policy regime to avoid crises, and 2) welfare cost, which is calculated as a weighted sum of the cycle variances of output and inflation. Our simulation results suggest that the nominal GDP targeting rule together with the fiscal backstop reduces the chances of economic crisis and stabilises the UK macroeconomy. Our results are much in line with the findings of Le et al. (2021) for the US. Finally, Chapter 10 concludes this thesis.

Chapter 2 Literature Review

The prevailing paradigm of monetary economics, the NK model, includes the Calvo (1983) time-dependent price and wage stickiness. Under a Calvo contract, a homogeneous firm/labour union has a fixed probability of changing its price/wage in each period. Alternative time-dependent contracts, such as the Taylor (1980) contracts, include staggered contracts, in which the firm/labour union determines its price/wage under a contract of a fixed duration. These time-dependent models assume that the timing of price/wage decisions is exogenous, implying that the frequency of price/wage change is constant. However, a growing number of studies question the empirical validity of time-dependent models, and interest in state-dependent models has increased, because the time-dependent contracts are inconsistent with the evidence from microeconomic level data. In state-dependent models, the decision to change prices/wages depends on the state of the economy. Theoretically, if nominal stickiness is derived from rational decisions, such as menu costs which are costs associated with relabelling or changing prices. In that case, firms change prices when the benefits of changing prices outweigh the menu costs; this makes the degree of stickiness 'state-dependent', i.e. the frequency of price adjustment is influenced by shocks under menu cost rigidity.

Microeconomic Evidence on State-dependent Pricing

The nature of price setting has a crucial implication for monetary policy. In state-dependent pricing models, firms' decisions to adjust prices are a response to changes in economic conditions; therefore, the probability of price change varies endogenously over time. From the monetary policy perspective, whether price setting is time-dependent or state-dependent, that is a critical question. The effectiveness of monetary policy to stabilise business cycles is dependent on the flexibility of the price level. If prices are highly flexible (state-dependent), they will absorb shocks and largely dampen the impact of monetary shocks, and if prices are highly sticky (time-dependent), monetary shocks will have a pronounced effect on output and stabilise business cycles.

In recent years, an increasing amount of empirical microeconomic level research in different countries and across different data episodes have shown state-dependent pricing; see Bhattarai and Schoenle (2014) and Nakamura et al. (2018) for the US, Wulfsberg (2016) and Nilsen et al. (2018) for Norway, Dedola et al. (2021) for Denmark, Rudolf and Seiler (2021) for Switzerland, Alvarez et al. (2018) for Argentina, Konieczny and Skrzypacz (2005) for Poland, Gagnon (2009) for Mexico. Table 2-1, sourced from Le et al. (2021), summarizes findings on the state-dependent pricing in some literature; it shows that contract duration varies very significantly with inflation.

There is also some UK micro-data level evidence of state-dependent pricing. Bunn and Ellis (2012) were the first to use UK consumer prices data from ONS to investigate the behaviour of individual prices. They used two databases: the microdata to construct official UK CPI data and a database of supermarket prices. They found that the probability of a price change did not remain constant over time; it varied between years and months of the year and varied according to the time since the last price change. The authors' plot of the probability of price change in different periods is shown in Figure 2-1. They stated that there was evidence of a correlation between the probability of monthly price increase and the headline inflation rate over the period 1996-2006, with a correlation coefficient of 0.6 between these two series. However, there was less evidence of a link between the probability of prices decreasing and aggregate inflation.

The research work by Zhou and Dixon (2019) investigated the price-setting behaviour underlying the price rigidity in the UK using CPI and PPI (Producer Price index) micro data during the Great Moderation period of 1996-2007, reporting that prices were indeed fixed for average durations, but they were state-dependent. Dixon et al. (2020) extended the UK CPI microdata period to 2013 (1996-2013) and examined the effect of the GFC on firm's pricing behaviour. They found strong evidence of a relationship between inflation and the frequency of price change, with inflation tending to increase the frequency of price change, mainly by increasing the frequency of price increase; a 1%-point rise in annual inflation increases the monthly frequency of price change about 0.5% points. During the Great Recession, the changes in pricing behaviour were mostly explained by the changes in inflation and VAT.

Table 2-1: A Summary of Findings on the State-dependent Pricing in the Literature

	Country	Duration in high inflation	Duration in low inflation
Nakamura et al.(2018)	USA	6.6 months (1978-1983)	9.9 months (1988-2014)
Alvarez et al. (2019)	Argentina	1 week	4.5 months
Wulfsberg (2016)	Norway	6.7 months (1975-1989)	12.3 months (1990-2004)
Konieczny and Skrzypacz (2005)	Poland	1.7 months	3.3 months
Gagnon (2009)	Mexico	3.1 months (1995-1997)	6.6 months (2000-2003) 7.0 months (2003-2004)

Source: Le et al. (2021)

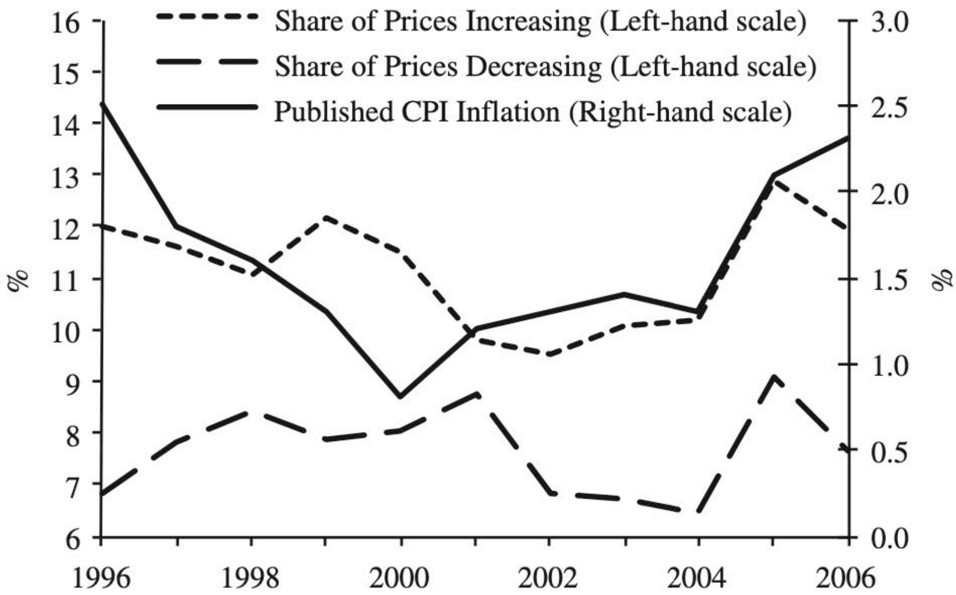

Figure 2-1: Headline Inflation and Percentage of CPI Prices Changing Each Month in the UK

Source: Bunn and Ellis (2012)

Petrella et al. (2018) also examined the ONS micro price data underlying the UK CPI, covering the period of 1996:M2 to 2017:M8. Using the ONS microdata, they reported the frequency of price adjustments and the average size of price changes (see Figure 2-2), which shows a considerable degree of positive co-movement between price changes and inflation. For the frequency of price adjustments, there is some evidence of a link between it and inflation. The authors then estimated a generalized Ss model to condense a broad cross-sectional of information on micro price fluctuations into a measure of price flexibility[2]. They found that state-dependence plays a crucial role in price setting. When inflation is high and volatile, the extensive

[2] The generalized Ss model was developed by Caballero and Engel (2007).

margin of price adjustment (adjustments driven by shocks rather than pre-determined price adjustments) becomes prominent.

More recently, Davies (2021) studied a large-scale micro-dataset of 41 million UK consumer prices to provide facts on price-setting behaviour over the period 1988:M1-2020:M12. This sample period includes the ERM (Exchange Rate Mechanism) crisis, the 2008 GFC and the 2016 EU referendum, as well as the coronavirus pandemic. The author found that state-dependent models, rather than time-dependent pricing models, are consistent with the behaviour of UK firms. In terms of pricing, the coronavirus pandemic has a more severe impact than the GFC, with a surge in the frequency of price change and both upward and downward price movements.

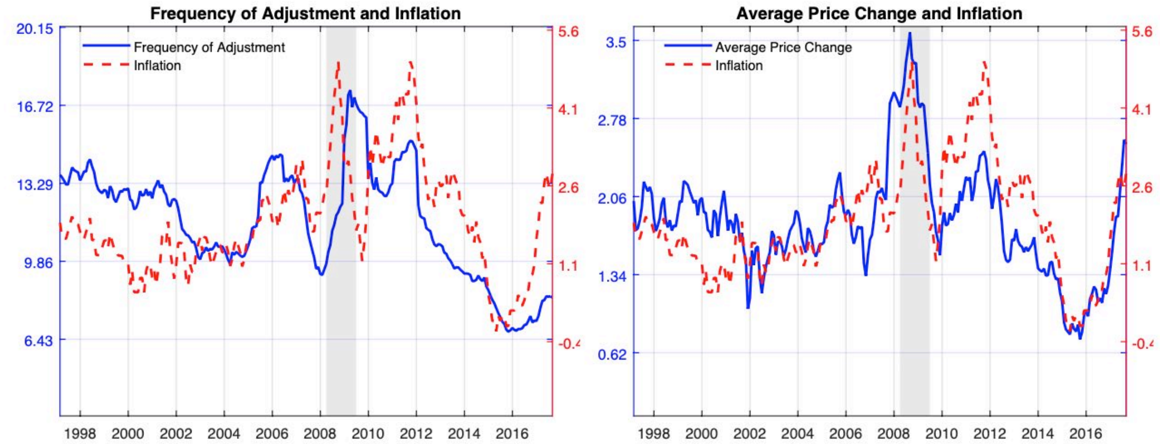

Figure 2-2: Frequency of Adjustment and Average Price Changes in the UK

Source: Petrella et al. (2018)

Microeconomic Evidence on State-dependent wage

In the NK framework, nominal wage rigidity plays a key role in explaining changes in employment and output. The timing of wage change in the NK model is exogenous, meaning that it is unaffected by policy or the state of the economy. From the monetary policy perspective, it is crucial to distinguish whether wage-setting is state-dependent or time-dependent as they have different implications for the level of monetary non-neutrality. When wages are set to be time-dependent, the response of output and employment to monetary shocks is pronounced. On the other hand, state-dependent wage contracts indicate a less pronounced impact of monetary policy on employment and output, as part of the shock is absorbed through wages, and thus has a larger impact on wage adjustment.

In principle, state-dependent wage models are based on the idea that the fixed costs of renegotiating employment contracts prevent frequent wage adjustments and that these fixed costs imply that the probability and the magnitude of wage change vary with the state of the economy.

For empirical studies, compared to state-dependent pricing, there is less empirical research on investigating state-dependent wage adjustments. Still, there is some micro-level evidence of the state-dependent wage changes. Sigurdsson and Sigurdardottir (2016) examined administrative micro data from the Icelandic labour market over the period 1998-2010 and found evidence of time-dependent wage change, but also strong evidence of state-contingency, as the probability of wage increase responds to both cumulated inflation and unemployment in current and past wage spells, in addition to an increase in the frequency of nominal wage cuts following a large macroeconomic shock. Grajales et al. (2019) studied administrative data at the employee level for the period of 2006-2021 for the Netherlands and found a mixture of time- and state-dependent wage behaviour. They reported that aggregate macroeconomic variables (inflation and unemployment) were important determinants of the probability of wage adjustment. More recently, Grigsby et al. (2021) investigated the extent of wage rigidity in the US using micro data from 2008 to 2016; they found evidence of time-dependent wage adjustment, but also strong evidence of state-dependence, as there were downward wage adjustments during the Great Recession. In the UK, the BoE conducted a wage-setting survey over the period 2010-2013, finding that roughly 30% of firms directly and explicitly linked wage changes to inflation, implying that there may be some downward real wage rigidity (Millard and Tatomir, 2015).

Macroeconomic Evidence

The large body of empirical micro-level evidence on state-dependent price and wage changes has motivated a growing number of recent macro-level studies that attempt to reproduce the micro evidence of state-dependence using macroeconomic models.

Gasteiger and Grimaud (2022) constructed an NK model with state-dependent price-setting frequency. They assumed that the decision to change prices depends on an analysis of expected costs and benefits modelled by a discrete choice process;

hence a firm will decide to reset its price optimally only when its expected benefits outweigh its expected costs. According to the model, pricing flexibility increases during expansions and decreases during recessions. They had four main findings: the augmented NK model 1) is consistent with price setting frequency based on micro data; 2) fits the observed dynamics of inflation and output well; 3) is able to explain the dynamics of inflation to a significant extent through discount factor and monetary shocks, as well as the endogenous price setting frequency; also 4) improves the macroeconomic time series fit of the NK model for the US sample period of 1959-2019.

However, the work of Gasteiger and Grimaud (2022) only included state-dependent pricing and assumed time-dependent wage setting. As previously stated, the micro-level evidence suggests that the frequency of wage adjustment also varies with the macroeconomic environment. A model with a state-dependent price setting but a time-dependent wage setting might not be able to accurately measure the impact of monetary changes on real variables (Costain et al., 2019) and could lead to false conclusions. To the best of our knowledge, only a few studies have included state-dependent price adjustment and state-dependent wage adjustment together in DSGE models, and they all focused on the US (see Takahashi, 2017; Costain et al., 2021; Le et al., 2021).

Takahashi (2017) developed a DSGE model that incorporates both state-dependent prices and wages. In this model, the state-dependent pricing framework is based on the stochastic menu cost model of Dotsey et al. (1999). The state-dependent wage setting is endogenously subject to fixed wage adjustment costs that are stochastic and heterogeneous across households, endogenously generating staggered nominal wage adjustments. The author calibrated the distribution of wage setting cost to match the US data on the proportion of wages that remained unchanged for a year and found that the state-dependent wage setting model produces similar responses to monetary shocks as the time-dependent model.

Costain et al. (2021) studied the impact of monetary shocks in a DSGE model that incorporates a state-dependent price/wage setting based on a control cost model, where price and wage decisions are costly and are random variables, combining monopolistic competition for goods and labour with nominal rigidity as a result of costly decisions. Thus, in their model, price/wage setters are subject to control costs and make optimal decisions about when and how to rest their prices/wages. The cost increases with the precision of price/wage decisions. By calibrating the microdata evidence of the frequency of price/wage changes to the DSGE model in which durations depend on inflation. They found that sticky wages play a critical role in the creation of monetary non-neutrality as the sticky-wage-only version of their model can produce almost as much non-neutrality as the version with both wage and price stickiness. Furthermore, the model with both sticky prices and wages has a larger real effect of monetary shocks than the model with only price stickiness.

However, the studies by Takahashi (2017) and Costain et al. (2021) both used micro data from a stable inflation sample period– the Great Moderation. This may be the reason why their macro models turn out to be quite similar to the SW07 model (Le et al., 2021).

Our study is inspired by the works of Le et al. (2011, 2016, 2021). Le et al. (2011) estimated a hybrid DSGE model with fixed price/wage duration for the period 1984-2004 by using the method of Indirect Inference. They found that the weight on the NK sector was about 0.99 for both wages and prices, which means that the flexprice sector's weight was almost zero; the Calvo parameters of not changing prices/wages were about 0.71. Le et al. (2016) extended their sample to 2011 (1984-2011) to include the GFC period and extended the model by allowing for a bank sector, ZLB and QE to capture the feature of the data in the wake of the GFC. They found that the weights on the flexprice sector for wages and prices increased significantly from near zero to 0.56 and 0.91, respectively. Additionally, the Calvo parameter of not changing wages decreased from 0.71 to 0.63 and the Calvo parameter of not changing prices increased from 0.71 to 0.97. The very high Calvo parameter for prices implies that only the most sticky sub-sector remained in the NK sector. Their findings imply that the durations of price and wage adjustments vary with the stochastic environment. Notably, their fixed price/wage duration model was rejected by the behaviour of data for the whole post-war period. They stated that this failure could be attributed to changes in wage/price-setting behaviour over time in response to a fluctuating macro environment.

Therefore, Le et al. (2021) extended their Le et al. (2016) DSGE model by adding state-dependent price and wage settings. They found that their state-dependent DSGE model did well in matching the behaviour of the US economy for the whole post-war period 1959-2017, whereas their fixed duration model failed to match it for the pre-1984 period. Moreover, during the Great Moderation, they found strong NK periods; during the Great Inflation and the Great Recession, they found flexprice periods. When they had the state-dependence in the model, the unconventional monetary policy QE cannot work well and cannot stabilise the economy on its own in the presence of the ZLB. Therefore, they brought in fiscal policy to prevent the occurrence of ZLB and found that nominal GDP targeting together with the fiscal backstop to prevent ZLB can achieve a high degree of price stability and avoid large cyclical fluctuations in output. In Le et al. (2016), however, their fixed price/wage duration DSGE model showed that monetary policy was still effective even at the ZLB.

Motivated by the above micro- and macro-level evidence, this thesis investigates the state-dependence at the macro-level in the UK by incorporating Le et al.'s (2021) state-dependent price/wage framework into an open economy DSGE model for the UK. To the best of my knowledge, this is the first empirical macro-level study of both state-dependent price and wage contracts in an open economy context. The contribution of our state-dependent DSGE model is twofold:

1) To fill the gap in macro-level research on state-dependent price/wage contracts by investigating whether there is macro-level evidence to corroborate micro-level evidence of state-dependence in the UK.

2) To examine improvements to the fixed duration model due to the state-dependence.

Chapter 3 The Model

This chapter presents a small open economy DSGE model for the UK with fixed shares of sticky price/wage sectors, which is the basis for the state-dependent price/wage duration model. The model consists of two blocks: a UK block built on Le et al. (2016), which extends the SW07 model to reflect developments in the monetary scene in the wake of GFC; and a simple world block in the spirit of Lyu et al. (2023), Gali and Monacelli (2005), containing the exchange rates, foreign bonds, exports and imports.

More specifically, this fixed duration model is based on the well-known SW07 model, which was derived from Christiano et al. (2005). It captures consumer habit-persistence, capital adjustment costs, variable capacity utilisation, price/wage setting using Calvo contracts plus indexation, and interest rate setting via Taylor Rule. The model extends the basic framework of SW07 by combining the NK and NC models into a hybrid model by assuming that part of goods/labour markets is perfectly competitive with flexible price/wage setting, while the rest is imperfectly competitive with sticky prices/wages, as proposed by Le et al. (2011). Moreover, the model incorporates the banking sector of the BGG model and includes several new developments from Le et al. (2016) following the Great Financial Crisis, including allowing for the ZLB constraint and QE. In terms of extensions to an open economy, since the UK's export sectors account for about 20-31% of GDP over the period 1970-2020[3], and its average share of global GDP is 3.77% over the period 1980-2020[4], the UK macro environment can be influenced by the rest of the world in this small open economy framework and any UK-specific shocks have no effect on the global economy. Therefore, the impact of the domestic economy on foreign variables is negligible, such as foreign interest rates and prices. These variables are treated as exogenous. Furthermore, trade is treated as a single-version of the Armington (1969) model, which is widely used in open economy models (see Adolfson et al., 2005, 2007; Gali and Monacelli, 2005; Meenagh et al., 2010; Feenstra et al., 2014; Dong et al., 2019; Minford and Meenagh, 2019; Lyu et al., 2021, 2023; Zhao et al., 2022). In addition, this open economy model implies Uncovered Interest Rate Parity (UIP),

[3] Source: The World Bank.
[4] Source: The Global Economy

which is supported by recent empirical studies using data from different countries, including the UK (see Minford et al., 2020; Minford et al., 2021).

The model is made up of the following sectors: households, labour unions, labour packers, intermediate goods producers (entrepreneurs), commercial banks, capital producers, final goods producers (retailers), the government, the central bank and the rest of the world. The log-linearised form of the model and the stochastic process of the model are provided in Appendix B and Appendix C, respectively.

3.1 Households

In this small open economy DSGE model, a representative household faces two optimisation problems. First, the representative household is expected to maximise its utility subject to budget constraints. Second, the household chooses the consumption of domestic goods (C_t^d) and the consumption of foreign goods (C_t^{im}) to maximise its consumption basket (C_t).

3.1.1 Households' Lifetime Utility Maximisation Problem

There is a continuum of households, indexed by j, supplying labour to producers of intermediate goods. It is assumed that households' optimization problem does not include either investment or capital utilisation, but in this small open economy model, households have access to imported goods and foreign bonds. Hence a representative household chooses hours worked $L_t(j)$, foreign bonds $B_t^f(j)$, domestic bonds $B_t(j)$, and consumption $C_t(j)$ consisting of domestic goods and imported goods, to maximise the following utility function:

$$\max_{C_t,L_t,B_t,B_t^f} E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s}(j)-hC_{t+s-1})^{1-\sigma_c}}{1-\sigma_c}\right) \exp\left(\frac{\sigma_c-1}{1+\sigma_l} L_{t+s}(j)^{1+\sigma_l}\right) \tag{3.1}$$

subject to the real terms budget constraint:

$$C_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_{t+s}^b(1+R_{t+s})P_{t+s}} + \frac{S_{t+s}B_{t+s}^f(j)}{\varepsilon_{t+s}^b\left(1+R_{t+s}^f\right)P_{t+s}} + T_{t+s} \leq \frac{W_{t+s}(j)L_{t+s}(j)}{P_{t+s}}$$

$$+\frac{B_{t+s-1}(j)}{P_{t+s}} + \frac{S_{t+s}B_{t+s-1}^f(j)}{P_{t+s}} + \frac{Div_{t+s}}{P_{t+s}} \tag{3.2}$$

On the right-hand side of the budget constraint, the household earns nominal wages $W_{t+s}L_{t+s}$ through supplying labour to intermediate goods producers, dividends Div_{t+s} distributed from the labour unions and returns from the past position in bonds holdings.

On the left-hand side, total income is used to consume C_t, re-invest in domestic and foreign bonds, and pay lump sum taxes T_t (T_t can be considered as subsidies if it is negative). β is the discount factor. h captures external habit formation. σ_c (σ_l) is the inverse of the intertemporal substitution elasticity between consumption (labour hours). P_t is consumer price index. R_t and R_t^f are nominal riskless rates on domestic and foreign bonds, respectively. S_t is the nominal exchange rate; hence, $Q_t = \frac{P_t^*}{P_t} S_t$, where Q_t is the real exchange rate and P_t^* is the general foreign price level. $P_t^* \cong P_{f,t}^*$ as we assume that exports from the UK have little impact on the rest of the world, where $P_{f,t}^*$ is the price of imported goods in foreign currency. B_t is one-period bonds expressed on a discount basis and can also be considered as deposits in commercial banks or as domestic government debt. ε_t^b is an exogenous premium in return to bonds, which is subject to both domestic and foreign bonds, and it follows an AR(1) process: $ln\varepsilon_t^b = \rho_b ln\varepsilon_{t-1}^b + \eta_t^b$, $\eta_t^b \sim N(0, \sigma_b)$.

In equilibrium, each household will make the same choices for consumption, working hours, and domestic and foreign bonds. The optimization problem is:

$$\max_{C_t, L_t, B_t, B_t^f} E_t \sum_{s=0}^{\infty} \beta^s \left\{ \left[\left(\frac{(C_{t+s}(j) - hC_{t+s-1})^{1-\sigma_c}}{1-\sigma_c} \right) exp\,(\frac{\sigma_c - 1}{1+\sigma_L} L_{t+s}(j)^{1+\sigma_L}) \right] \right.$$
$$- \lambda_t \left[C_{t+s}(j) + \frac{B_{t+s}(j)}{\varepsilon_{t+s}^b (1+R_{t+s}) P_{t+s}} + \frac{S_{t+s} B_{t+s}^f(j)}{\varepsilon_{t+s}^b (1+R_{t+s}^f) P_{t+s}} + T_{t+s} \right.$$
$$\left. \left. - \frac{W_{t+s}(j) L_{t+s}(j)}{P_{t+s}} - \frac{B_{t+s-1}(j)}{P_{t+s}} - \frac{S_{t+s} b_{t+s-1}^f(j)}{P_{t+s}} - \frac{Div_{t+s}}{P_{t+s}} \right] \right\}$$

The optimal decisions are determined by the first order conditions (FOCs), they are (dropping the j index):

$$\partial C_t: \quad (C_t - hC_{t-1})^{-\sigma_c} exp\left(\frac{\sigma_c - 1}{1+\sigma_L} {L_t}^{1+\sigma_L}\right) = \lambda_t \tag{3.3}$$

$$\partial L_t: \quad \left(\frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c}\right) (\sigma_c - 1) {L_t}^{\sigma_L} exp\left(\frac{\sigma_c - 1}{1+\sigma_L} {L_t}^{1+\sigma_L}\right) = -\lambda_t \frac{W_t}{P_t} \tag{3.4}$$

$$\partial B_t: \quad \lambda_t = \beta \varepsilon_t^b (1+R_t) E_t \frac{P_t \lambda_{t+1}}{P_{t+1}} \tag{3.5}$$

$$\partial B_t^f: \quad \lambda_t \frac{S_t}{\varepsilon_t^b (1+R_t^f) P_t} = \beta E_t \lambda_{t+1} \frac{S_{t+1}}{P_{t+1}} \tag{3.6}$$

Where λ_t is the Lagrange multiplier. Households' optimal decisions on consumption and working hours are the same as in SW07.

The optimal conditions with respect to C_t (equation 3.3) and B_t (equation 3.5) lead to **the consumption Euler equation**:

$$E_t\left[\beta\frac{(C_{t+1}-hC_t)^{-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_L}L_{t+1}{}^{1+\sigma_L}\right)}{(C_t-hC_{t-1})^{-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_L}L_t{}^{1+\sigma_L}\right)}(1+R_t)\varepsilon_t^b\frac{P_t}{P_{t+1}}\right]=1 \tag{3.7}$$

Combining the optimal conditions with respect to B_t (equation 3.5) and B_t^f (equation 3.6) implies that:

$$\frac{1+\mathrm{R_t^f}}{(1+\mathrm{R_t})\mathrm{S_t}}=\frac{1}{\mathrm{E_t S_{t+1}}} \tag{3.8}$$

Substituting $Q_t=\frac{P_t^*}{P_t}S_t$ into equation 3.8 yields:

$$\frac{(1+\mathrm{R_t^f})P_t^*}{(1+\mathrm{R_t})Q_tP_t}=\mathrm{E_t}\frac{P_{t+1}^*}{Q_{t+1}P_{t+1}} \tag{3.9}$$

which can be log-linearised to find **the real uncovered interest parity condition (RUIP)**:

$$E_tq_{t+1}-q_t=[r_t-(\mathrm{E_t}p_{t+1}-p_t)]-[r_t^f-(\mathrm{E_t}p_{t+1}^*-p_t^*)] \tag{3.10}$$

Since $E_t\pi_{t+1}^{cpi}=\mathrm{E_t}p_{t+1}-p_t$ and $\mathrm{E_t}\pi_{t+1}^f=\mathrm{E_t}p_{t+1}^*-p_t^*$, **the linearisation of RUIP** can be re-written as:

$$E_tq_{t+1}-q_t=(r_t-E_t\pi_{t+1}^{cpi})-(r_t^f-\mathrm{E_t}\pi_{t+1}^f) \tag{3.11}$$

The RUIP states that the relative expected change in the real exchange rate is equal to the difference in real interest rates between two countries. For example, if the condition does hold, then investors are indifferent to either of the two currency cash deposits (e.g., Pound sterling and US dollar); any excess return on the sterling must be balanced out by some anticipated loss from the devaluation of the sterling against the US dollar. On the contrary, any shortfalls in the return on sterling deposits must be balanced out by expected gains from the pound appreciation against the US dollar. If the RUIP does not hold, then currency arbitrage or Forex arbitrage can be used to generate risk-free profits.

Although there is some negative empirical evidence on the UIP condition, however, it is supported by recent empirical studies with data from different countries (see Minford et al., 2020; Minford et al., 2021). Minford et al. (2020) integrated UIP into a full DSGE model and tested the model as a whole using the Indirect Inference method to include the US, Europe and the rest of the world. They found that UIP was accepted in the test, while certain commonly-used single-equation tests would be strongly biased towards the hypothesis's rejection. The findings of Minford et al. (2020) contrast with those of Burnside (2019), which rejected the UIP relation for a dozen pairs of industrialised economies based on single-equation tests. Furthermore, Minford et al. (2021) revisited the evidence on UIP through a comprehensive assessment of the findings of Minford et al. (2020), applying a full-model and the method of Indirect Inference on the ten country currency pairs (including the British Pound) assessed by Burnside. They found that UIP was generally accepted (including the UK case) as part of a full-world DSGE model, avoiding the bias involved in the single-equation tests. Their unbiased method offers strong and rigorous evidence supporting UIP and suggests that previous evidence of UIP rejection may be due to bias in single-equation regression tests.

3.1.2 Choice of Optimal Consumption Basket

This small open economy model assumes that trade is broadly treated as in the Armington model (see Armington,1969; Adolfson et al., 2005, 2007; Meenagh et al., 2010; Feenstra et al., 2014; Dong et al., 2019; Minford and Meenagh, 2019; Zhao et al., 2022; and Lyu et al., 2021, 2023). Its interpretation here is specialised as follows. In this model, the Armington aggregator simply demonstrates that there is an all-purpose home good and an all-purpose foreign good, and they are differentiated according to their country of origin and combined together to create a consumer bundle, regardless of whether it is a production or retail bundle. It shows that there is a bundle of production and retail and all sorts of goods. It is a bundle of home value-added and foreign value-added components and covers a broad range of goods. The consumer then purchases the consumer bundle- notice that this bundle can take a variety of forms, such as differing combinations of outputs used as inputs to a final good together with home distributive service output[5]. The composite consumption good can be expressed as a constant elasticity of substitution (CES) index following Armington:

$$C_t \equiv [(1-\omega)^{\frac{1}{\sigma}}(C_t^d)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}}(\eta_t)^{\frac{1}{\sigma}}(C_t^{im})^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{3.12}$$

[5] The Armington procedure aggregates home and foreign contributions to final consumption, which are traded, and how these are assumed to break down in detail is not detailed. Different ways of assuming how this detail are achieved are consistent with the procedure.

where C_t^d and C_t^{im} are the indices for consumption of domestically produced goods and imported goods respectively, they are given by:

$$C_t^d \equiv \left(\int_0^1 C_t^d(k)^{\frac{\varepsilon-1}{\varepsilon}} dk\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.13}$$

$$C_t^{im} \equiv \left(\int_0^1 C_t^{im}(k)^{\frac{\varepsilon-1}{\varepsilon}} dk\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.14}$$

where $k \in [0, 1]$ denotes the good variety; $\varepsilon > 1$ is the elasticity of substitution between varieties produced within any given country; ω is the weight on imported goods in the consumption bundle, $(0 < \omega < 1)$; η_t is a shock to the demand for imported goods and can be viewed as a preference error; σ is the elasticity of substitution between domestic and imported varieties of goods.

The consumer price index and the indices for domestically produced goods and imported goods are defined following Gali and Monacelli (2005):

the CPI price index is:

$$P_t \equiv \left[(1-\omega)\left(P_t^d\right)^{1-\sigma} + \omega\left(P_t^f\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3.15}$$

the price index of domestically produced goods is:

$$P_t^d \equiv \left(\int_0^1 P_t^d(k)^{1-\varepsilon} dk\right)^{\frac{1}{1-\varepsilon}} \tag{3.16}$$

the price index for imported goods (expressed in domestic currency) is:

$$P_t^f \equiv \left(\int_0^1 P_t^f(k)^{1-\varepsilon} dk\right)^{\frac{1}{1-\varepsilon}} \tag{3.17}$$

Assuming the law of one price holds following Gali and Monacelli (2005), then

$$P_t^f = S_t P_{f,t}^* \tag{3.18}$$

where S_t is nominal exchange rate, $P_{f,t}^*$ is foreign price index of imported goods in foreign currency.

Domestic households split their purchases between home and foreign goods, and their expenditure on domestic and foreign goods is constrained by the total value of the Armington bundle, with a consumption constraint of:

$$P_t C_t = P_t^d C_t^d + P_t^f C_t^{im} \tag{3.19}$$

Equation (3.19) can be rewritten as:

$$C_t = \frac{P_t^d}{P_t} C_t^d + \frac{P_t^f}{P_t} C_t^{im} \tag{3.20}$$

$$C_t = \frac{P_t^d}{P_t} C_t^d + \frac{S_t P_{f,t}^*}{P_t} C_t^{im} \tag{3.21}$$

Since it is assumed that exports from the UK have little impact on the rest of the world, $P_t^* \cong P_{f,t}^*$ and $Q_t = \frac{S_t P_t^*}{P_t} \cong \frac{S_t P_{f,t}^*}{P_t} = \frac{P_t^f}{P_t}$, where P_t^* is the general foreign price index and Q_t is the real exchange rate[6]. Hence, following Meenagh et al. (2010), Dong et al. (2019), Minford and Meenagh (2019), Zhao et al. (2022) and Lyu et al. (2021, 2023), the consumption constraint is rewritten as:

$$C_t \cong p_t^d C_t^d + Q_t C_t^{im} \tag{3.22}$$

where $p_t^d \equiv \frac{P_t^d}{P_t}$ is the domestic price relative to the general price level. $Q_t \cong \frac{P_{f,t}^*}{P_t} S_t = \frac{P_t^f}{P_t}$, it can be seen as a unit free measure of the foreign price in domestic currency relative to domestic general price level. Intuitively, a rise in Q_t can be seen as a real exchange rate depreciation, as it implies a real devaluation of domestic goods and an increase in the competitiveness of domestic exports.

Households' optimal consumption basket problem is to determine how the consumption bundle should be split between domestic and foreign varieties to maximise the composite consumption utility (equation 3.12) subject to the expenditure constraint (equation 3.22):

$$C_t \equiv [(1-\omega)^{\frac{1}{\sigma}} (C_t^d)^{\frac{\sigma-1}{\sigma}} + \omega^{\frac{1}{\sigma}} (\eta_t)^{\frac{1}{\sigma}} (C_t^{im})^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad \text{subject to} \quad C_t = p_t^d C_t^d + Q_t C_t^{im}$$

The first order conditions with respect to C_t^d and C_t^{im} imply:

$$C_t^d = (1-\omega)(p_t^d)^{-\sigma} C_t \tag{3.23}$$

$$C_t^{im} = \omega \eta_t (Q_t)^{-\sigma} C_t \tag{3.24}$$

Equation 3.24 reflects the optimal choice of demand for imports. **The linearisation of demand for world imports** is given by:

$$im_t = ln\omega - \sigma q_t + c_t + e_t^{im} \tag{3.25}$$

where e_t^{im} represents import demand shock. Demand for imports depends positively on consumption and negatively on real exchange rate. This import demand equation is consistent with those of Dong et al. (2019), Minford and Meenagh (2019), Zhao et al. (2022) and Lyu et al. (2023).

[6] $P_t^* \cong P_{f,t}^*$ has been widely assumed in small open economy models (see GalÌ and Monacelli, 2005).

Given equation 3.24, it is intuitively that a symmetric equation exists for the rest of the world, describing foreign demand for domestic goods or export demand (see Dong et al., 2019; Zhao et al., 2022; Lyu et al., 2023):

$$EX_t = \omega^f (Q_t)^{\sigma f} C_t^f \eta_t^f \tag{3.26}$$

where f is the foreign country index. A depreciation of the real exchange rate (a rise in Q) induces an increase in the competitiveness of domestic exports.

The log-linearised form of export demand is:

$$\mathrm{ex_t} = ln\omega^f + \sigma^f \mathrm{q_t} + \mathrm{c_t^f} + e_t^{ex} \tag{3.27}$$

Demand for exports depends positively on the real exchange rate and foreign consumption. Foreign consumption is assumed to be as an exogenous variable, given by a first-order autoregressive process:

$$c_t^f = \rho_c^f c_{t-1}^f + \eta_t^{cf} \tag{3.28}$$

By substituting equations 3.23 and 3.24 into 3.22, we obtain an expression between p_t^d and Q_t:

$$1 = (1-\omega)\left(p_t^d\right)^{1-\sigma} + \omega\eta_t (Q_t)^{1-\sigma} \tag{3.29}$$

The linearisation form is:

$$lnp_t^d = -\frac{\omega}{(1-\omega)(1-\sigma)}\eta_t - \frac{\omega}{1-\omega}\mathrm{q_t} + constant \tag{3.30}$$

The difference between the consumer price index and the price index for domestically produced goods implies a wedge between the real consumer wage and real producer wage. Recalling that $p_t^d \equiv \frac{P_t^d}{P_t}$, thus, the wage wedge in log-linearised form can be expressed as $lnp_t^d = w_t^c - w_t^h$. By combining it with equation 3.30, we obtain (see Minford and Meenagh, 2019): [7]

[7] Using the assumption in Minford (2015), $-\frac{\omega}{(1-\omega)(1-\sigma)}\eta_t + constant = 0$, we obtain equation 3.31. Another method to derive the wedge between the real consumer wage and real producer wage is that, since $W_t^h = \frac{W_t}{P_t^d}$, in log-linearised form we have

$$w_t^h = w_t - p_{d,t} = (w_t - p_t) + \left(p_t - p_{d,t}\right) = w_t^c + \frac{\omega}{1-\omega}\mathrm{q_t}$$

Where w_t is log nominal aggregate wage, $p_t - p_{d,t} = \frac{\omega}{1-\omega}\mathrm{q_t}$ is derived using equations 3.35 and 3.41.

$$w_t^h = w_t^c + \frac{\omega}{1-\omega}\mathrm{q_t} \quad (3.31)$$

where w_t^h is the real producer wage, w_t^c is the real consumer wage and q_t is real exchange rate.

We assume that there is no capital control, the balance of payments constraint is expressed as:

$$B_{t+1}^f - B_t^f = R_t^f B_t^f + \frac{p_t^d EX_t}{Q_t} - IM_t \quad (3.32)$$

Domestic Inflation and CPI Inflation

We follow Gali and Monacelli's (2005) definition of domestic inflation and CPI inflation. The effective terms of trade between the home and foreign country are defined as:

$$TOT_t = \frac{P_t^f}{P_t^d} \quad (3.33)$$

in log-linearised form:

$$tot_t = p_{f,t} - p_{d,t} \quad (3.33)$$

Similarly, the log-linearization of the CPI index $P_t \equiv \left[(1-\omega)\left(P_t^d\right)^{1-\sigma} + \omega\left(P_t^f\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is:

$$p_t \equiv (1-\omega)p_{d,t} + \omega p_{f,t}$$
$$p_t = p_{d,t} + \omega\ tot_t \quad (3.35)$$

Home inflation is defined as the rate of change in home price; then the relationship between home inflation and CPI inflation is:

$$\pi_t^{cpi} = \pi_t^h + \omega\ \Delta tot_t \quad (3.36)$$

where π_t^{cpi} is CPI inflation and π_t^h is home inflation. This equation makes the gap between the two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportional given by the openness index ω.

Assuming that the law of one price holds, $P_t^f = S_t P_{f,t}^* \cong S_t P_t^*$, the following log-linearised equation can be derived following Gali and Monacelli (2005):

$$p_{f,t} = s_t + p_t^* \quad (3.37)$$

Combining equation 3.37 with equation 3.33:

$$tot_t = s_t + p_t^* - p_{d,t} \tag{3.38}$$

Next, we derive a relationship between the real exchange rate and the terms of trade. The real exchange rate in log-linearised form is:

$$q_t = s_t + p_t^* - p_t \tag{3.39}$$

Combining equation 3.38 and equation 3.39 yields:

$$q_t = tot_t + p_{d,t} - p_t \tag{3.40}$$

Substituting equation 3.35 into equation 3.40 yields:

$$q_t = tot_t - \omega\, tot_{t-1}$$

$$tot_t = \frac{1}{1-\omega} q_t \tag{3.41}$$

By substituting the equation 3.41 into equation 3.36, we obtain **an expression for CPI inflation in terms of home inflation and the percent change in real exchange rate**:

$$\pi_t^{cpi} = \pi_t^h + \frac{\omega}{1-\omega} \Delta q_t \tag{3.42}$$

where Δq_t is change in real exchange rate. This equation makes the gap between the two measures of inflation proportional to the percent change in real exchange rate.

3.2 Intermediate Labour Unions (Hybrid Wage Setting)

Households provide their homogenous labour to intermediate labour unions, which allocate and differentiate labour services and set wages subject to a Calvo (1983) rule. These differentiated labour services $\mathrm{L_t(l)}$ are aggregated into final labour input $\mathrm{L_t}$, which is used by intermediate goods producers.

$$\mathrm{L_t} = \left(\int_0^1 \mathrm{L_t(l)}^{\frac{\varrho_{w,t}-1}{\varrho_{w,t}}} \,\mathrm{dl} \right)^{\frac{\varrho_{w,t}-1}{\varrho_{w,t}}} \tag{3.43}$$

Where $\varrho_{w,t}$ is an exogenous process and is assumed to follow an exogenous AR(1) process.

The household labour supply decision is given by the following FOC:

$$\left(\frac{(C_t - hC_{t-1})^{1-\sigma_c}}{1-\sigma_c}\right) (\sigma_c - 1) L_t{}^{\sigma_L} exp\left(\frac{\sigma_c - 1}{1+\sigma_L} L_t{}^{1+\sigma_L}\right) = -\lambda_t \frac{W_t}{P_t} \tag{3.46}$$

It gives the marginal rate of substitution between working hours and consumption, which is the real wage desired by the households.

Calvo Wage Setting

We assume that labour unions set wages according to the Calvo wage-setting rule: each period, a fraction of labour unions $(1-\xi_w)$ have the opportunity to re-adjust their wages, which means that the remaining unions ξ_w cannot adjust their wages. For those unions that can adjust, the problem is to choose an optimal wage $W_t^{\#}(l)$ that maximizes wage income. For those unions that cannot adjust, they set wages with partial indexation to the CPI inflation rate in the previous period and the steady state value: $W_t(l)=\gamma(\pi_{t-1}^{cpi})^{l_w}(\pi_*^{cpi})^{1-l_w}W_{t-1}(l)$, where l_w is the partial wage indexation coefficient. Thus, the wage of a household l in period t is:

$$W_t(l)=\begin{cases} W_t^{\#}(l) & \text{with prob } (1-\xi_w) \\ \gamma(\pi_{t-1}^{cpi})^{l_w}(\pi_*^{cpi})^{1-l_w}W_{t-1}(l) & \text{with prob } \xi_w \end{cases} \tag{3.47}$$

Thus, the aggregate wage index expression is:

$$W_t=\left[\xi_w[\left(\gamma(\pi_{t-1}^{cpi})^{l_w}(\pi_*^{cpi})^{1-l_w}\right)W_{t-1}(l)\,]^{1-\varrho_{w,t}}+(1-\xi_w)W_t^{\#}(l)^{1-\varrho_{w,t}}\right]^{\frac{1}{1-\varrho_{w,t}}} \tag{3.48}$$

Hybrid Wage Setting

Le et al. (2011) used a Wald test based on the Indirect Inference method to test the SW07 model and found that the model was rejected for the whole post-war sample. They also tested an NC version in which prices and wages were completely flexible, and the Indirect Inference test rejected this NC version as well. Therefore, they proposed a hybrid version, a weighted average of the NK and NC equations, which is closer to the behaviour of data for the sample period 1984-2004.

In order to match the model to the behaviour of the data, we follow Le et al. (2011) and use a hybrid model, which is a combination of imperfectly competitive and perfectly competitive markets. We assume that a fixed fraction of labour ω_w comes from imperfectly competitive markets and a fraction of labour $(1-\omega_w)$ comes from competitive markets with wage flexibility. The reasoning behind the hybrid model is that the economy consists of product sectors with more price rigidity and other sectors with more flexible prices, which reflect the degree of competition in these sectors. The labour market is similar, certain sectors may be more/less dominated by competition, resulting in greater/less wage flexibility.

Thus, **the wage setting equation** in the hybrid model is assumed to be a weighted average of the corresponding NK and NC equations:

$$\mathrm{W}_{\mathrm{t}}^{\mathrm{hybrid}} = \omega^{\mathrm{w}}\mathrm{W}_{\mathrm{t}}^{\mathrm{NK}} + (1-\omega^{\mathrm{w}})\mathrm{W}_{\mathrm{t}}^{\mathrm{NC}} \tag{3.49}$$

where $\mathrm{W}_{\mathrm{t}}^{\mathrm{NK}}$ is equation 3.48, which is set according to the Calvo wage setting rule. $\mathrm{W}_{\mathrm{t}}^{\mathrm{NC}}$ is the wage in perfectly competitive markets, which is set equal to the current expected marginal disutility of work.

3.3 Final Goods Producers

Final domestic goods producers (retailers) buy intermediate goods $Y_t(i)$ from intermediate goods producers (entrepreneurs) at the price of $P_t^d(i)$ and combine them into composite final domestic goods Y_t. No capital or labour is required in the production of the final goods, and the final domestic goods producers take their final domestic goods price, P_t^d, and the price of the intermediate goods, $P_t^d(i)$, to be as given and beyond their control in a perfectly competitive market. The final good Y_t is a composite made of a continuum of intermediate goods $Y_t(i)$ as in Kimball (1955):

$$Y_t = \left(\int_0^1 (Y_t(i))^{\frac{\varrho_{p,t}-1}{\varrho_{p,t}}} di\right)^{\frac{\varrho_{p,t}}{\varrho_{p,t}-1}}; \varrho_{p,t} > 1 \tag{3.50}$$

The profit maximisation problem of the final good producers is:

$$\underset{Y_t, Y_t^i}{\mathrm{Max}}\, P_t^d Y_t - \int_0^1 P_t^d(i) Y_t(i)\, di \qquad s.t. \qquad Y_t = \left(\int_0^1 (Y_t(i))^{\frac{\varrho_{p,t}-1}{\varrho_{p,t}}} di\right)^{\frac{\varrho_{p,t}}{\varrho_{p,t}-1}}$$

The FOCs give the optimal demand for intermediate goods input, which depends negatively on their relative price:

$$Y_t(i) = \left(\frac{P_t^d(i)}{P_t^d}\right)^{-\varrho_{p,t}} Y_t \tag{3.51}$$

Equations 3.50 and 3.51 imply the following relationship between the price of the final good and the price of the intermediate good:

$$P_t^d = \left(\int_0^1 \left(P_t^d(i)\right)^{1-\varrho_{p,t}} di\right)^{\frac{1}{1-\varrho_{p,t}}} \tag{3.52}$$

As in the hybrid wage setting of sub-section 3.2, we assume that the final goods are made up of a fixed fraction ω^{p} of intermediate goods $Y_t(i)$ sold in imperfectly competitive markets, and a fraction of $(1-\omega^{\mathrm{p}})$ of $Y_t(i)$ sold in perfectly competitive markets. Thus, the hybrid final domestic goods are:

$$Y_t^{hybrid} = \omega^{\mathrm{p}} \left(\int_0^1 (Y_t(i))^{\frac{\varrho_{p,t-1}}{\varrho_{p,t}}} di \right)^{\frac{\varrho_{p,t}}{\varrho_{p,t-1}}} + (1-\omega^{\mathrm{p}}) \int_0^1 (Y_t(i))^{NC} di \qquad (3.53)$$

The hybrid domestic price-setting is:

$$(P_t^d)^{hybrid} = \omega^p (P_t^d)^{NK} + (1-\omega^p)(P_t^d)^{NC} \qquad (3.54)$$

where $(P_t^d)^{NK}$ is the aggregate home price in imperfectly competitive markets and set by intermediate goods producers according to the Calvo price setting rule; $(P_t^d)^{NC}$ is the home price in competitive markets, which is equal to the marginal cost.

Thus, **the hybrid home-inflation** is given by:

$$(\pi_t^h)^{hybrid} = \omega^p (\pi_t^h)^{NK} + (1-\omega^p)(\pi_t^h)^{NC} \qquad (3.55)$$

Where $(\pi_t^h)^{NK}$ is home inflation in imperfectly competitive markets and $(\pi_t^h)^{NC}$ is home inflation in competitive markets.

3.4 Intermediate Goods Producers and Commercial Banks

Following Le et al. (2016), we integrate the BGG financial friction into the SW07 model. Entrepreneurs act as intermediate goods producers; they still produce intermediate goods but now buy capital from capital producers instead of rent from households. To purchase capital, they have to borrow from commercial banks, which absorb households' savings at the deposit rate R_t and lend these savings to intermediate goods producers at the commercial lending rate CY_{t+1}. As in the BGG model, the demand side of the credit (entrepreneurs) faces financial friction. Following Le et al. (2011), entrepreneurs are assumed to supply intermediate goods at prices set partly in imperfectly competitive markets and partly in perfectly competitive markets; similarly, they hire labour at wages determined in a mixture of perfectly and imperfectly competitive labour markets.

The activities of intermediate goods producers determine 1) the production of intermediate goods, 2) the Calvo price setting, 3) loan contracts, 4) entrepreneurial net worth, and 5) the level of capital utilisation.

3.4.1 Production of Intermediate Goods

A representative entrepreneur employs labour and purchases installed capital inputs to produce intermediate goods using constant returns to scale production technology. It follows the Cobb-Douglas technology:

$$Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha [\gamma^t L_t(i)]^{1-\alpha} - \gamma^t \phi \tag{3.56}$$

where i is the intermediate goods sector index. K_t^s and L_t are effective capital inputs and aggregate labour inputs, respectively. α is the share of capital in the production. ϕ is one plus the fixed costs in production. γ^t is the labour-augmenting deterministic growth rate in the economy. ε_t^a is total factor productivity, which is assumed to be non-stationary and follows an ARIMA (1,1,0) process, $ln\varepsilon_t^a = ln\varepsilon_{t-1}^a + \rho_a(ln\varepsilon_t^a - ln\varepsilon_{t-1}^a) + \eta_t^a$.

The entrepreneur's profit function is given by:

$$\pi_t(i) = P_t^d(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) \tag{3.57}$$

where $P_t^d(i)$ is the intermediate output price. $P_t^d(i)Y_t(i)$ is the revenue of the firm from selling intermediate goods. $R_t^k K_t^s(i)$ is cost of capital. W_t is the nominal cost of labour.

The entrepreneur chooses the amount of effective capital input (K_t^s) and labour input $L_t(i)$ to maximise his profits:

$$\max_{L_t^i, K_t^s(i)} P_t^d(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i)$$

$$s.t \quad Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha [\gamma^t L_t(i)]^{1-\alpha} - \gamma^t \Phi R_t^k$$

First order conditions yield:

$$\partial K_t^s(i) \colon \; \Theta_t^i \gamma^{(1-\alpha)t} \alpha \varepsilon_t^a \left(\frac{L_t(i)}{K_t^s}\right)^{1-\alpha} = R_t^k \tag{3.58}$$

$$\partial L_t(i) \colon \; \Theta_t^i \gamma^{(1-\alpha)t} (1-\alpha) \varepsilon_t^a \left(\frac{L_t(i)}{K_t^s}\right)^{-\alpha} = W_t \tag{3.59}$$

where Θ_t^i is the Lagrange multiplier associated with the production function. By re-arranging equation 3.59, we obtain the nominal marginal cost (MC_t) and **the real marginal cost** deflated by home price (mc_t) as:

$$MC_t = \frac{(R_t^k)^\alpha (W_t)^{1-\alpha}}{\varepsilon_t^a (1-\alpha)^{1-\alpha} \alpha^\alpha} \tag{3.60}$$

$$mc_t = \frac{(R_t^k)^\alpha (\frac{W_t}{P_t^d})^{1-\alpha}}{\varepsilon_t^a (1-\alpha)^{1-\alpha} \alpha^\alpha} \tag{3.61}$$

Combining equation 3.58 and equation 3.59 gives the nominal capital-labour ratio (K_t^s) and **the real capital-labour ratio** deflated by home price (k_t^s) are:

$$K_t^s = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k} L_t \tag{3.62}$$

$$k_t^s = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k P_t^d} L_t \tag{3.63}$$

where $\frac{W_t}{P_t^d}$ is the real producer wage.

3.4.2 Calvo Price Setting

Calvo Price Setting

Each of the domestic entrepreneur is subject to nominal price rigidities according to Calvo (1983). In each period, only a fraction $(1-\xi_p) \in [1,0]$ of entrepreneurs can re-optimise their prices, while the remainder cannot re-optimise their prices and thus set their prices following the partial indexation rule $P_t^d(i) = (\pi_{t-1}^h)^{l_p} (\pi_*^h)^{1-l_p} P_{t-1}^d(i)$, where l_p is the partial price indexation coefficient. For those who can re-optimise, they choose an optimal price $[P_t^d(i)]^\#$ that maximises the present value of their expected future total profits. Thus, the price set by the entrepreneur i in period t is:

$$P_t^d(i) = \begin{cases} [P_t^d(i)]^\# & with\ prob\ (1-\xi_p) \\ (\pi_{t-1}^h)^{l_p} (\pi_*^h)^{1-l_p} P_{t-1}^d(i) & with\ prob\ \xi_p \end{cases} \tag{3.64}$$

For those who are allowed to re-optimise their prices, they have the following optimisation problem:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \frac{\lambda_{t+s} P_t}{\lambda_t P_{t+s}} [P_{t+s}^d(i)\, Y_{t+s}(i) - MC_{t+s} Y_{t+s}(i)] \tag{3.65}$$

$$\text{s.t to the demand for intermediate goods equation}$$

$$Y_{t+s}(i) = \left(\frac{P_{t+s}^d(i)}{P_{t+s}^d}\right)^{-\varrho_{p,t}} Y_{t+s} \tag{3.66}$$

where λ_{t+s} is the Lagrange multiplier associated with the budget constraint for households, $\frac{\beta^s \lambda_{t+s} P_t}{\lambda_t P_{t+s}}$ is the nominal discount factor for the entrepreneurs.

Thus, the aggregate domestic price evolves according to:

$$P_t^d = \left[\xi_p[(\pi_{t-1}^h)^{l_p}(\pi_*^h)^{1-l_p} P_{t-1}^d(i)\,]^{1-\varrho_{p,t}} + (1-\xi_p)([P_t^d(i)]^{\#})^{1-\varrho_{p,t}}\right]^{\frac{1}{1-\varrho_{p,t}}} \quad (3.67)$$

$$P_t^d = (P_t^d)^{NK} \quad (3.68)$$

This P_t^d is the aggregate home price in the imperfectly competitive market. When prices are fully flexible, the firms will set the price of the goods they produce equal to the marginal cost, thus

$$(P_t^d)^{NC} = MC_t \quad (3.69)$$

Hybrid Price Equation Setting

As discussed in the hybrid wage equation setting, we use a hybrid model, which is a combination of imperfectly competitive and perfectly competitive markets. The hybrid model assumes that a fraction of goods markets set prices flexibly, while the rest has sticky prices. We assume that the share of imperfect competition can be different between the labour and goods markets. Thus, in the goods market, we assume that final goods are made up of a fixed fraction ω_p of intermediate goods $Y_t(i)$ from the imperfectly competitive market and a fraction of $(1-\omega_p)$ of $Y_t(i)$ from the perfectly competitive market.

The aggregate domestic price of the hybrid model is then assumed to be a weighted average of the corresponding NK and NC equations:

$$(P_t^d)^{hybrid} = \omega^p (P_t^d)^{NK} + (1-\omega^p)(P_t^d)^{NC} \quad (3.70)$$

where $(P_t^d)^{NK}$ is the aggregate domestic price in the imperfectly competitive market (equation 3.68) which is set according to the Calvo price setting rule. $(P_t^d)^{NC}$ is the domestic price in the competitive market, which is equal to the marginal cost (equation $(P_t^d)^{NC} = MC_t$).

3.4.3 Financial Friction and the Role of QE

In this section, the model incorporates the bank sector of the BGG model. Moreover, two assumptions are introduced to BGG: first, banks require entrepreneurs to provide a certain amount of collateral, c, as part of their net worth; second, the cost of recovering this collateral is a percentage δ of its initial value. The idea of using cash as cheap collateral is then included to inject the QE element.

The Financial Friction of the BGG Model

In the BGG model, at time t, entrepreneurs purchase newly installed capital K_{t+1} from capital producers at price $\mathrm{P}_{\mathrm{t}}^{\mathrm{k}}$ to be used in production in period t+1. At time t+1, entrepreneurs obtain revenue from the marginal product of capital R_{t+1}^{k}, and gain from selling undepreciated capital $(1-\delta)$ to capital producers at price P_{t+1}^{k}. In equilibrium, the capital arbitrage condition implies:

$$E_t(CY_{t+1}) = E_t\left[\frac{R_{t+1}^{k}+(1-\delta)P_{t+1}^{k}}{P_t^k}\right] \quad (3.71)$$

Where CY_{t+1} is the expected marginal rate of real return on capital, which is equal to the expected cost of external funds in equilibrium.

In order to finance capital expenditure, entrepreneurs need external loans to supplement their net worth as a source of funding, and they borrow external funding from banks in the form of bank loans which are repaid at time t+1. Thus, entrepreneurs finance their capital purchases with internal funds and external loans from commercial banks. The contract between the borrower (entrepreneur) and lender (bank) follows a 'Costly State Verification' framework.

We briefly review the three main parts of the BGG model here, and then we will extend the BGG to include collateral and money. A description of the BGG set-up is provided in appendix A.

1) <u>A bankruptcy threshold at which firms will choose to default</u>

$$\overline{\omega}(1+CY_{t+1})K_{t+1}P_t^k = Z_{t+1}Borr_{t+1} = Z_{t+1}(K_{t+1}P_t^k - NW_{t+1}) \quad (3.72)$$

where CY_{t+1} is the expected marginal rate of return on capital. $K_{t+1}P_t^k$ is the value of capital. $Borr_{t+1}$ is the amount of external borrowing. NW_{t+1} is the entrepreneur's net worth. Z_{t+1} is the gross non-default loan rate. $\overline{\omega}$ is the threshold value of the idiosyncratic shock, at which the entrepreneur is indifferent between default and non-default. As described in equation 3.72, the entrepreneur's gross return at time t+1 is equal to the loan payment when ω is at its threshold value $\overline{\omega}$.

2) <u>Bank's zero profit condition</u>

$$[1-F(\bar{\omega})]Z_{t+1}Borr_{t+1}+(1-\mu)\int_0^{\bar{\omega}}\omega(1+CY_{t+1})K_{t+1}P_t^k dF(\omega)=\left(1+R_{t+1}^{real}\right)Borr_{t+1}$$
$$=\left(1+R_{t+1}^{real}\right)(K_{t+1}P_t^k-NW_{t+1}) \quad (3.73)$$

where $F(\bar{\omega})$ is the probability of default, and μ is the auditing cost. R_{t+1}^{real} is the real risk-free rate. The left-hand side of equation 3.73 is the aggregate expected return on the loan to the entrepreneur, which is the sum of return obtained in the case of non-default (the first term on the left-hand side) and the return gained in the event of default (the second term). The right-hand side is the bank's real opportunity cost.

3) The optimal contracting problem; entrepreneur's maximisation of expected return subject to 1) and 2)

$$Lev_t=\frac{K_{t+1}P_t^k}{NW_{t+1}}=v(PM_t)=v\left(\frac{1+CY_{t+1}}{1+\frac{R_t}{\pi_{t+1}^{cpi}}}\right) \quad (3.74)$$

Where $PM_t=\frac{1+CY_{t+1}}{1+R_{t+1}^{real}}$ is defined as external finance premium and $Lev_t=\frac{K_{t+1}P_t^k}{NW_{t+1}}$ is defined as the leverage ratio. In addition, according to Le et al. (2013), the external finance premium is also affected by an exogenous premium shock ε_t^{prem}, which can be thought of as shocks to the credit supply, such as changes in the efficiency of the financial intermediation process. The log-linearised form of the external finance premium consistent with Le et al. is:

$$pm_t=E_t cy_{t+1}-r_t^{real}=\chi(qq_t+k_t-n_t)+e_t^{prem} \quad (3.75)$$
$$pm_t=E_t cy_{t+1}-(r_t-E_t\pi_{t+1})=\chi(qq_t+k_t-n_t)+e_t^{prem} \quad (3.76)$$

Modifications to the BGG model to allow for the effects of QE

In the years following the GFC, there have been a few significant advances in the monetary domain. Firstly, central banks' near-zero lending rates to banks drove the ZLB issue. Secondly, conventional monetary policy was ineffective in boosting the economy in the wake of the GFC; as a response, the BoE introduced an unconventional method (QE) in 2009. Aggressive QE aimed at injecting liquidity into the banking system and stimulating extra credit creation. Therefore, this model is extended to accommodate the facts of our sample period by allowing for the presence of QE and ZLB. To inject QE element, the idea of using cash as the cheapest collateral is included. We discuss the introduction of QE in this section, with the ZLB problem being discussed in sub-section 3.6 (monetary and fiscal policies).

Introduction of QE

In the BGG model, entrepreneurs do not provide collateral. In reality, it is common for banks to require a certain amount of collateral from the firms they lend to. We therefore follow Le et al. (2016) and extend the BGG model by assuming that banks require entrepreneurs to provide a certain amount of collateral, c, as part of their net worth. Moreover, the cost of recovering this collateral is a percentage δ of its initial value, where δ corresponds to the depreciation rate in the SW07 model. Thus, the bank sector of the BGG model is modified according to these two assumptions.

Recall the original bankruptcy threshold (equation 3.72):

$$\overline{\omega}(1+CY_{t+1})K_{t+1}P_t^k = Z_{t+1}Borr_{t+1} = Z_{t+1}(K_{t+1}P_t^k - NW_{t+1})$$

With the introduction of collateral, the entrepreneur's total capital purchase value becomes $K_{t+1}P_t^k + cNW_{t+1}$, and the external borrowing becomes $K_{t+1}P_t^k + cNW_{t+1} - NW_{t+1}$. Thus, **in the presence of collateral, the bankruptcy threshold becomes**:

$$\overline{\omega}(1+CY_{t+1})K_{t+1}P_t^k + cNW_{t+1} = Z_{t+1}Borr_{t+1} = Z_{t+1}(K_{t+1}P_t^k + cNW_{t+1} - NW_{t+1}) \tag{3.77}$$

Equation 3.77 shows that at threshold $\overline{\omega}$, the entrepreneur's gross return at time t+1 plus the value of the collateral equals to the total loan payment.

We combine this new bankruptcy threshold equation with the leverage ratio $Lev_t = \frac{K_{t+1}P_t^k}{NW_{t+1}}$ to solve for the non-default loan rate:

$$Z_t = \frac{\overline{\omega}(1+CY_{t+1})Lev_t + c}{Lev_t - 1 + c} \tag{3.78}$$

Recall the original bank's zero profit condition (equation 3.73):

$$[1-F(\overline{\omega})]Z_{t+1}Borr_{t+1} + (1-\mu)\int_0^{\overline{\omega}} \omega(1+CY_{t+1})K_{t+1}P_t^k dF(\omega) = \left(1+R_{t+1}^{real}\right)Borr_{t+1}$$

we denote $f(\omega)$ as the probability density function of ω, hence $f(\omega)d\omega = dF(\omega)$; the probability of non-default is $[1-F(\overline{\omega})] = \int_{\overline{\omega}}^{\infty} f(\omega)d\omega$. We also denote $G(\overline{\omega}) =$

$\int_0^{\overline{\omega}} \omega f(\omega)d\omega$, which is the bank's share of profits if the entrepreneur defaults. The bank's expected gross share of profits is $\Gamma(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega + \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega$, which can be rewritten as

$$\Gamma(\overline{\omega}) = G(\overline{\omega}) + \overline{\omega}[1 - F(\overline{\omega})] \tag{3.79}$$

Substituting $G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega$ and $f(\omega)d\omega = dF(\omega)$ into equation 3.73 gives:

$$[1 - F(\overline{\omega})]Z_{t+1}Borr_{t+1} + [(1-\mu)G(\overline{\omega})(1 + CY_{t+1})K_{t+1}P_t^k] = \left(1 + R_{t+1}^{real}\right)Borr_{t+1} \tag{3.80}$$

With the introduction of collateral, the bank's zero profit condition becomes:

$$[1 - F(\overline{\omega})]Z_{t+1}Borr_{t+1} + [(1-\mu)G(\overline{\omega})(1 + CY_{t+1})K_{t+1}P_t^k + cNW_{t+1}F(\overline{\omega})(1-\delta)] = \left(1 + R_{t+1}^{real}\right)Borr_{t+1} \tag{3.81}$$

where $cNW_{t+1}F(\overline{\omega})(1-\delta)$ is the recovery of collateral minus its liquidation cost δ in the event of default. The left-hand side of the equation is the aggregate expected return on the loan to the entrepreneur, which is the sum of the return received in the case of non-default (the first term) and the return in the case of default (the second term). The right-hand side is the bank's real opportunity cost.

Substituting equation 3.77 into equation 3.81 gives:

$$[1 - F(\overline{\omega})]\overline{\omega}(1 + CY_{t+1})K_{t+1}P_t^k + \left[(1-\mu)G(\overline{\omega})(1 + CY_{t+1})K_{t+1}P_t^k + cNW_{t+1}\left(1 - \delta F(\overline{\omega})\right)\right] = \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1} + cNW_{t+1}) \tag{3.82}$$

Substituting equation 3.79 into equation 3.82 and divide both sides by NW_{t+1} gives:

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})](1 + CY_{t+1})Lev_t = \left(1 + R_{t+1}^{real}\right)(Lev_t - 1) + c[R_{t+1}^{real} + \delta F(\overline{\omega})]$$

$$Lev_t = \frac{\left(1+R_{t+1}^{real}\right) - c[R_{t+1}^{real} + \delta F(\overline{\omega})]}{\left(1+R_{t+1}^{real}\right) - \Theta(\overline{\omega})(1 + CY_{t+1})} \tag{3.83}$$

where $\Theta(\overline{\omega}) = \Gamma(\overline{\omega}) - \mu G(\overline{\omega})$ is the bank's net share of profits. Equation 3.83 is the banks' leverage offer curve, which defines Lev_t in terms of $\overline{\omega}$. It has an upward slope and is convex in $(1 - \Gamma(\overline{\omega}))$ space, see Figure 3-1.

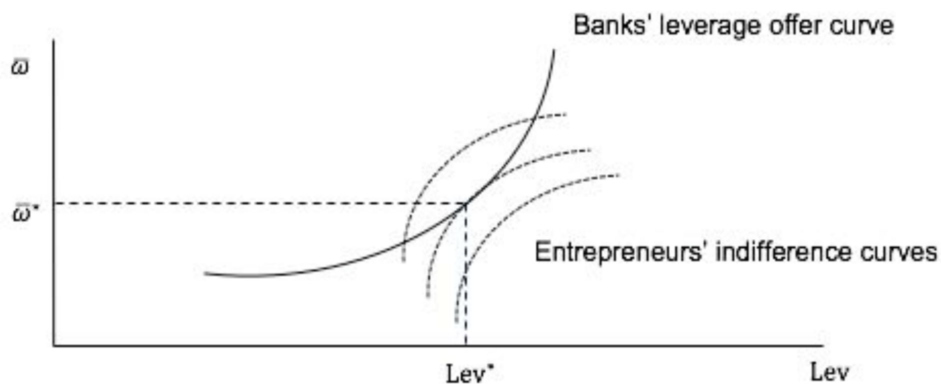

Figure 3-1: The Optimal Contract for ($\bar{\omega}^*$, L^*)

With the collateral present, the entrepreneur's expected return, relative to the cost of funds, is:

$$\int_{\bar{\omega}}^{\infty} \frac{\{\omega(1+CY_{t+1})K_{t+1}P_t^k + cNW_{t+1} - Z_{t+1}(K_{t+1}P_t^k - NW_{t+1} + cNW_{t+1})\}dF(\omega)}{NW_{t+1}(1+R_{t+1}^{real})} \tag{3.84}$$

We are only interested in the return in the non-default case as the entrepreneur's return is zero in the event of default. The numerator of equation 3.84 is the overall return in the non-default case, which is equal to the return on capital $\omega(1+CY_{t+1})K_{t+1}P_t^k$ plus the value of collateral cNW_{t+1} minus the debt payment $Z_{t+1}(K_{t+1}P_t^k - NW_{t+1} + cNW_{t+1})$; the denominator is the cost of funds. Equation 3.84 can be simplified as:

$$\frac{1+CY_{t+1}}{1+R_{t+1}^{real}} Lev_t[1-\Gamma(\bar{\omega})] \tag{3.85}$$

From equation 3.85 we know that the entrepreneur's expected return is unaffected by the introduction of collateral because the term cNW_{t+1} is cancelled out.

The optimal contracting problem is to maximise the entrepreneur's expected return (equation 3.85) subject to the bank's leverage offer curve (equation 3.83) with respect to $(\bar{\omega}, Lev)$. Solving this problem gives the entrepreneur's optimal choice of $\bar{\omega}$ as the solution of:

$$\left[(1+R_{t+1}^{real}) - c(R_{t+1}^{real} + \delta F(\bar{\omega}))\right][1+R_{t+1}^{real} - \Omega'(1+CY_{t+1})] = \left[\frac{c\delta F'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})}\right][1+ R_{t+1}^{real} - \Theta(\bar{\omega})(1+CY_{t+1})] \tag{3.86}$$

where $\Omega' = \frac{\Theta'(\bar{\omega})}{\Gamma'(\bar{\omega})} + \left[1 - \frac{\Theta'(\bar{\omega})}{\Gamma'(\bar{\omega})}\right] \Theta(\bar{\omega}) \approx 1$. We re-express the entrepreneur's optimal choice (equation 3.86) in terms of the bank's leverage as:

$$Lev_t[1 + R_{t+1}^{real} - \Omega'(1 + CY_{t+1})] = \frac{c\delta F'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})} \tag{3.87}$$

We now have two equations in $(\bar{\omega}, L)$ space, from the entrepreneurs' optimum (equation 3.87) and the banks' leverage offer (equation 3.83). We take the total differential of this two-equation system in $dLev, d\bar{\omega}, d\delta$ and dCY, and analyse the comparative static properties of changes around the equilibrium. We will evaluate the derivatives at the equilibrium for $\delta = 0$ as the heavily monetised collateral with δ close to zero in the setup. $lnLev = k_t - nw_t$ is determined and δ is determined by the provision of M0 as a substitute for illiquid collateral. Hence, Lev and δ are treated as exogenous to this BGG banking sector which then solves for $\bar{\omega}$ and CY. These two elements ($\bar{\omega}$ and CY) are internal to the banking sector and unobservable in the public domain.

The total differential for equation 3.87 yields:

$$[1 + R_{t+1}^{real} - \Omega'(1 + CY_{t+1})]dLev_t + Lev_t(-\Omega')dCY$$

$$= (derivative = 0)\bar{\omega} + \left[\frac{-cF'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Gamma'(\bar{\omega})}\right] d\delta \tag{3.88}$$

The total differential for equation 3.83 (the banks' leverage offer curve) yields:

$$dLev = Lev_t \left[\frac{\Theta'(\bar{\omega})(1+CY_{t+1})}{(1+R_{t+1}^{real})-\Theta(\bar{\omega})(1+CY_{t+1})}\right] d\bar{\omega} + Lev_t \left[\frac{\Theta(\bar{\omega})}{(1+R_{t+1}^{real})-\Theta(\bar{\omega})(1+CY_{t+1})}\right] dCY +$$
$$Lev_t \left[\frac{-cF(\bar{\omega})}{(1+R_{t+1}^{real})-\Theta(\bar{\omega})(1+CY_{t+1})}\right] d\delta \tag{3.89}$$

We are interested in the effect of δ on the equilibrium values of $\bar{\omega}, CY$ and Z ($\frac{dCY}{d\delta}, \frac{d\bar{\omega}}{d\delta}$ and $\frac{dZ}{d\delta}$).

From equation 3.88 we have:

$$\frac{dCY}{d\delta} = \frac{cF'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{Lev_t\Omega'[1-F(\bar{\omega})]} > 0 \tag{3.90}$$

From equation 3.89 we have:

$$\frac{d\bar{\omega}}{d\delta} = \frac{cF(\bar{\omega})}{Lev_t\Theta'(1+CY_{t+1})}\left[1 - \frac{F'(\bar{\omega})}{F(\bar{\omega})}\frac{\Theta(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Omega'[1-F(\bar{\omega})]}\right] > 0 \quad (3.91)$$

where $\left[1 - \frac{F'(\bar{\omega})}{F(\bar{\omega})}\frac{\Theta(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Omega'[1-F(\bar{\omega})]}\right]$ is positive, see the appendix of Le et al. (2016).

Finally, we have:

$$\frac{dZ}{d\delta} = \frac{Lev_t}{Lev_t-1+c}\left[\frac{F(\bar{\omega})}{\Theta'(\bar{\omega})}\left(1 - \frac{F'(\bar{\omega})}{F(\bar{\omega})}\frac{\Theta(\bar{\omega})}{\Omega'}\frac{[1-\Gamma(\bar{\omega})]}{[1-F(\bar{\omega})]}\right) + \frac{\bar{\omega}F'(\bar{\omega})[1-\Gamma(\bar{\omega})]}{\Omega'[1-F(\bar{\omega})]}\right] > 0 \quad (3.92)$$

on the assumption that $\frac{d\bar{\omega}}{d\delta} > 0$.

So far, we have verified that $\frac{d\bar{\omega}}{d\delta} > 0, \frac{dZ}{d\delta} > 0$ and $\frac{dCY}{d\delta} > 0$.

Now, we introduce the idea of using cash as cheap collateral for loans as it is completely liquid and riskless. Entrepreneurs hold cash as collateral, and this can be recovered directly, with no verification costs and no loss of value. Thus, it eliminates the cost δ and then in turn lowers credit spreads. We assume that the central bank issues cash M0 to households through QE in exchange for the bonds they hold; households have no use for M0 and will deposit all M0 with commercial banks, which then lend M0 to the entrepreneurs to hold as collateral, see Figure 3-2.

The model captures the effect of M0 on the credit premium via its effect on the cost of liquidating collateral δ:

$$prem_t = E_t cy_{t+1} - \left(r_t - E_t\pi_{t+1}^{cpi}\right) = \chi(qq_t + k_t - n_t) - \vartheta m_t^0 + e_t^{prem} \quad \textbf{(3.93)}$$

Where ϑ is the elasticity of the premium to M0 through its collateral role. Equation 3.93 shows that monetary policy can affect the risk premium on bank lending to entrepreneurs by varying the supply of M0. According to Le et al. (2016), the injection of M0 eliminates the liquidation cost of collateral; therefore an increase in the supply of M0 will be reflected in a lower credit premium, which will translate into a lower commercial lending rate.

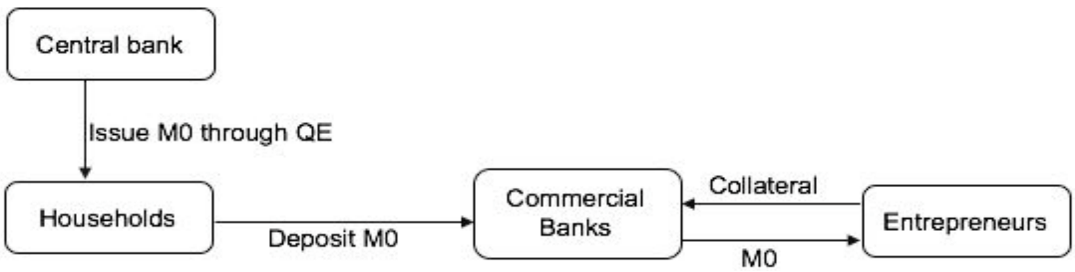

Figure 3-2: The Introduction of QE

3.4.4 Net Worth

We assume that the probability of an entrepreneur surviving to the next period is θ, implying an expected lifetime of $\frac{1}{1-\theta}$. This assumption is designed to capture the phenomena of continual births and deaths of firms and to prevent entrepreneurs from accumulating sufficient wealth to fully fund new capital. Hence, they will need to borrow to finance their targeted investment expenditures in excess of net worth. Each entrepreneur also has a constant probability $(1-\theta)$ of dying from the market in the current period, and if an entrepreneur goes bankrupt, that entrepreneur will consume his remaining resources.

The evolution of an entrepreneur's net worth is defined as:

$$NW_{t+1} = \theta V_t \tag{3.94}$$

where V_t represents the value of the entrepreneur's equity.

The net worth of surviving entrepreneurs evolves according to:

$$NW_{t+1} = \varepsilon_t^{nw}\theta[CY_t P_{t-1}^k K_t - E_{t-1}CY_t(P_{t-1}^k K_t - NW_t)] \tag{3.95}$$

where ε_t^{nw} is a shock to the value of the entrepreneur's equity, which follows an AR(1) process: $ln\varepsilon_t^{nw} = \rho_{nw} ln\varepsilon_{t-1}^{nw} + \eta_t^{nw}$. CY_t is the ex-post real return on capital held at time t. $E_{t-1}CY_t$ is the ex-post cost of borrowing. Equation 3.95 states that the net worth of surviving entrepreneur is equal to the ex-post real return on capital investment minus the cost of borrowing in period t-1.

Entrepreneurs who die from the economy in period t consume their remaining resources. **The bankrupt entrepreneurs' consumption** is:

$$C_t^e = (1-\theta)NW_t \tag{3.96}$$

3.4.5 Level of Capital Utilisation

Intermediate goods producers purchase capital from capital producers and then choose an optimal level of capital utilisation Z_t. The amount of effective capital that capital producers can sell to intermediate goods producers is:

$$K_t^s(i) = Z_t(i)K_{t-1}(i) \tag{3.97}$$

Thus, the income from buying capital services is $R_t^K Z_t(i)K_{t-1}(i)$, and the cost of changing capital utilisation is $\psi(Z_t)K_{t-1}(h)$, where $\psi(Z_t)$ is the adjustment cost of capital utilisation.

The optimal capital utilisation is determined by the following maximisation problem:

$$\max_{Z_t} \sum_{S=0}^{\infty} \beta^s \lambda_{t+s}[R_t^K Z_t(i)K_{t-1}(i) - \psi(Z_t)K_{t-1}(i)] \tag{3.98}$$

s.t.

$$K_t(h) = (1-\delta)K_{t-1}(h) + \varepsilon_t^i \left[1 - S(\frac{I_t(h)}{I_{t-1}(h)})\right] I_t(h) \tag{3.99}$$

The FOC determines **the optimal capital utilisation**:

$$\partial Z_t: \quad R_t^K = \psi'(Z_t) \tag{3.100}$$

3.5 Capital Producers

At the end of each period, capital producers buy existing capital $(1-\delta)K_{t-1}$ from intermediate goods producers and combine it with investment I_t to produce new capital K_t, which they then sell to intermediated goods producers at price P_{t+s}^k in a perfectly competitive market. **The capital accumulation equation** is:

$$K_t = (1-\delta)K_{t-1} + \varepsilon_t^i \left[1 - S(\frac{I_t}{I_{t-1}})\right] I_t \tag{3.101}$$

where I_t is investment. $\delta \in (0,1)$ is the depreciation rate. $S(\cdot)$ is the investment adjustment cost function as in SW07, with $S(1) = S'(1) = 0, S''(1) > 0$. ε_t^i is the investment-specific shock, following an AR (1) process.

$$\ln\varepsilon_t^i = \rho_i \ln\varepsilon_{t-1}^i + \eta_t^i, \qquad \eta_t^i \sim N(0, \sigma_i)$$

The capital producer's problem is to choose the level of investment that maximises its expected discounted profit, i.e. the difference between the revenue from selling newly produced capital and the cost.

$$\max_{I_t} E_t \sum_{t=0}^{\infty} \beta^t \lambda_t \left[P_t^k(K_t - (1-\delta)K_{t-1}) - I_t\right] \tag{3.102}$$

$$\text{s.t.} \quad K_t = (1-\delta)K_{t-1} + \varepsilon_t^i \left[1 - S(\frac{I_t}{I_{t-1}})\right] I_t \tag{3.103}$$

The FOC with respect to I_{t+s} implies **the optimal demand for investment**:

$$\partial \mathrm{I_t(h)}: \quad 1 = \mathrm{P_t^k \varepsilon_t^i}\left[1 - \mathrm{S}\left(\frac{\mathrm{I_t}}{\mathrm{I_{t-1}}}\right) - \frac{\mathrm{I_t}}{\mathrm{I_{t-1}}}\mathrm{S}'\left(\frac{\mathrm{I_t}}{\mathrm{I_{t-1}}}\right)\right] - \beta\frac{\lambda_{t+1}}{\lambda_t}\mathrm{P_{t+1}^k \varepsilon_{t+1}^i}\frac{\mathrm{I_{t+1}}^2}{\mathrm{I_t}^2}\mathrm{S}'\left(\frac{\mathrm{I_{t+1}}}{\mathrm{I_t}}\right) \qquad (3.104)$$

3.6 Monetary and Fiscal Policies

This section discusses monetary and fiscal policies. Following Le et al. (2016), we present monetary policy separately in normal and crisis regimes as monetary policy tools differ in crisis and non-crisis times. We also introduce ZLB into the model in this section.

Monetary Policy

Central banks conduct monetary policy to affect the amount of money in the economy and the borrowing costs. The BoE utilises two primary instruments for monetary policy. The first is the conventional monetary policy tool, where the BoE sets the bank rate, which is the interest rate at which banks borrow money from them. Secondly, The BoE implements the unconventional policy, QE; through QE, they buy bonds to lower the interest rates on savings and loans.

In this model, we discuss the UK's monetary policy under two scenarios: the normal regime and crisis regime. The monetary policy in the crisis regime incorporates new developments since the onset of the recent GFC: the ZLB constraint and the implementation of QE.

Under a normal regime (where quarterly $r_t > 0.025\%$), the BoE conducts conventional monetary policy according to the Taylor Rule, which sets out how the central bank adjusts short nominal interest rates in response to developments in CPI inflation and output. The log-linearised form of the Taylor Rule is:

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_p \pi_t^{cpi} + r_y y_t\right) + r_{\Delta y}(y_t - y_{t-1}) + e_t^r \qquad (3.105)$$

where ρ reflects the degree of interest rate smoothing. r_p, r_y and $r_{\Delta y}$ measure the response to inflation, output and output gap, respectively.

In the normal regime, we assume that the supply of M0 is set to accommodate broad money M2, which is determined by the entrepreneur's balance sheet. Specifically, M2 is equal to M0 plus deposits from households $(\mathrm{M2} = \mathrm{deposits} + \mathrm{M0})$; deposits from households are equal to the number of loans lent to entrepreneurs

$(\text{deposits} = \text{borrowing})$, and equal to total capital expenditure minus net worth $(\text{deposits} = \text{borrowing} = \text{capital expenditure} - \text{net worth})$. Thus, we have $\text{M2} = \text{capital expenditure} - \text{net worth} + \text{M0}$. The log-linearised equations for M0 and M2 are (see Le et al., 2016; Lyu et al., 2023) :

$$\Delta m_t^0 = \vartheta_1(m_t^2 - m_{t-1}^2) + e_t^{m0,nocrisis} \tag{3.106}$$

$$m_t^2 = (1 + v - \mu)k_t + \mu m_t^0 - v n_t \tag{3.107}$$

where $\text{v} = \frac{\text{NW}}{\text{M2}}$ and $\mu = \frac{\text{M0}}{\text{M2}}$ are the steady-state ratio of net worth to M2 and the steady-state ratio of M0 to M2 respectively, and ϑ_1 is the elasticity of M0 to M2.

The monetary policy framework under the normal regime is summarised as follows:

$$For\ r_t > 0.025\%\ (nocrisis) \begin{cases} M0: \quad m_t^0 = m_{t-1}^0 + \vartheta_1(m_t^2 - m_t^2) + e_t^{m0,nocrisis} \\ r: \quad r_t = \rho r_{t-1} + (1-\rho)\left(r_p \pi_t^{cpi} + r_y y_t\right) + r_{\Delta y}(y_t - y_{t-1}) + e_t^r \\ M2: \quad m_t^2 = (1 + v - \mu)k_t + \mu m_t^0 - v n_t, \qquad \text{v} = \dfrac{\text{NW}}{\text{M2}}, \mu = \dfrac{\text{M0}}{\text{M2}} \end{cases}$$

In the crisis regime (where quarterly $r_t \leq 0.025\%$), in response to the crisis, the BoE cuts the bank rate as low as they could go. However, the nominal interest rate is effectively bounded by zero or close to zero. Thus, when the bank rate is at or below the bound level ($r_t \leq 0.025\%$), the conventional tool reaches its limit; we then suspend the Taylor Rule and replace it with an exogenous low bound ($r_t = 0.025\%$). Then the BoE turns to unconventional monetary policy, and M0 becomes the primary tool to target the credit market with the aim of reducing the premium on given leverage and boosting credit supply. It is worth noting that the model keeps inflation determinacy as interest rate shocks cannot last indefinitely, and Taylor Rule will be operative again if the model moves away from the lower bound at some point.

When ZLB constraint binds, M0 (i.e. QE) targets the credit premium around its steady state in an attempt to restore credit condition back to normal through providing cheap collateral. The M0 supply equation for interest rates at or below the zero bound (crisis regime) is:

$$m_t^0 = m_{t-1}^0 + \vartheta_2(prem_t - prem^*) + e_t^{m0,crisis} \tag{3.108}$$

where ϑ_2 is the elasticity of M0 with respect to premium, and $prem^*$ is the steady-state credit premium. The mechanism works as follows: the more significant the credit spread, the more effort is required to stabilise the credit premium through M0 injection.

The monetary policy under the crisis regime is summarised as (see Le et al., 2016; Lyu et al., 2023):

$$For\ r_t \leq 0.025\%\ (crisis) \begin{cases} M0: & m_t^0 = m_{t-1}^0 + \vartheta_2(prem_t - prem^*) + e_t^{m0,crisis} \\ r: & r = 0.025\% \end{cases}$$

Fiscal Policy

The government finances public expenditure by raising lump-sum taxes and through borrowing from domestic households. Bonds issued in period t-1, $\frac{B_{t-1}}{R_{t-1}}$, repay B_{t-1} at time t. The government budget constraint takes the form of:

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \tag{3.109}$$

where T_t represents the nominal lump-sum tax (or subsidy) that is also included in the household's budget constraint. G is government spending and is determined exogenously as a time-varying component relative to the steady-state output path:

$$\varepsilon_t^g = \frac{G_t}{Y\gamma^t} \tag{3.110}$$

where ε_t^g is the government spending shock affecting the amount of government spending relative to GDP, which is modelled as an AR(1) process and that is also affected by the productivity shock, $\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a$.

3.7 Market Clearing Conditions

Goods market clearing is a necessary condition for closing the model so that the supply of domestic goods should equal the demand for domestic goods. The aggregate resource constraint for the economy is:

$$Y_t = C_t + I_t + G_t + \psi(Z_t)K_{t-1} + C_t^e + EX_t - IM_t \tag{3.111}$$

Movements in relative prices (i.e., real exchange rates, linked to real interest rates through RUIP) ensure that market clearing also holds in value terms.

Since $\varepsilon_t^g = \frac{G_t}{Y\gamma^t}$, we have:

$$Y_t = (C_t + I_t + \psi(Z_t)K_{t-1} + C_t^e + EX_t - IM_t)\varepsilon_t^g \qquad (3.112)$$

The log linearised form is:

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + R_*^K k_y \frac{1-\psi}{\psi} rk_{t-1} + \frac{C^e}{Y}c_t^e + \frac{EX}{Y}ex_t - \frac{IM}{Y}im_t + e_t^g \qquad (3.113)$$

Since the goods market clears and income can only be spent on goods or assets. All asset markets also clear.

In addition, according to Minford and Meenagh (2019), a transversality condition is needed to guarantee that a balanced growth equilibrium is reached because trade deficits (surpluses) cannot be run forever by borrowing (lending) from the rest of the world. The balance of payments is subject to a constraint imposed by the transversality condition, i.e. the change in net foreign assets (the capital account) must be zero in the long-run. At some terminal date T, the equilibrium real exchange rate remains constant, and the cost of servicing the current debt is covered by an equivalent trade surplus.

$$R_T^f B_T^f = -\left(\frac{p_T^d EX_T}{Q_T} - IM_T\right) \qquad (3.115)$$

The terminal condition serves to ensure that the transversality condition is satisfied – equivalent to setting the long run change in net foreign assets and the current account to zero. The numerical solution path must be consistent with the constraints it imposes on the rational expectations. This constraint is related to household borrowing because the government's solvency is secured through other means and firms do not borrow from abroad. When solving the model, the balance of payments constraint is scaled by output, enabling the terminal condition imposes a constant ratio of net foreign assets to GDP in the long run, $\Delta\hat{B}_{t+1}^f = 0$ as $t \to \infty$, where $\hat{B}_{t+1}^f = \frac{B_{t+1}^f}{Y_{t+1}}$.

Labour market clears:

$$L_t^S = L_t^D \qquad (3.116)$$

Chapter 4 Data and Calibration

In this chapter, Section 4.1 discusses the macroeconomic data construction, and Section 4.2 presents a set of starting calibration.

4.1 Macroeconomic Data for the UK

In this thesis, we employ quarterly and unfiltered data from 1955Q1 to 2021Q1 on 19 macroeconomic variables, including consumption, nominal interest rate, output, inflation, investment, labour hours, real wage, real lending rate, export, import, real exchange rate, external finance premium, net foreign bond position, net worth (in logs, net worth is equal to entrepreneurial consumption, see Le et al. (2012)), M0, M2 and two variables for the rest of the world: foreign real interest rate and world consumption. This long sample period includes the stagflation in the 1970s, the exchange rate mechanism crisis, the 2008 financial crisis, the 2016 EU referendum, and the first year of the coronavirus pandemic. Data are mainly from the Office for National Statistics (ONS), BoE, Refinitiv DataStream and Federal Reserve Economic Data (FRED). Data on capital stock, price of capital and capital utilisation are not collected but generated by the model equations. Capital data are derived from the capital accumulation equation; capital price data are generated from the investment Euler equation; and capital utilisation data are computed from labour demand equation. Except for variables in percentages and ratios such as interest rate, all variables are taken to their nature logarithm. The details and plots of the dataset are shown in Table 4-1 and Figure 4-1, respectively.

Generally, macroeconomic data are non-stationary with their frequently random movements which lead to considerable economic uncertainties. A large body of empirical work uses some techniques (such as the Hodrick-Prescott and Band Pass filters) to extract trends from the data to stabilise it. However, the use of these techniques has been criticised for some concerns. For example, they are developed based on the statistical properties of the data rather than the theories of models; hence their application could cause a problem with the models' fit (Davidson et al. 2010). In addition, trends are obtained by smoothing the data series, and anything that does not meet the smoothness criterion, which is arbitrarily specified, will be extracted. Therefore, when focusing on stochastic behaviour, the dynamic properties of the model can be eliminated by using filters.

This thesis will use non-stationary data and retain the stochastic trend in the model, as one of our interests is to observe how the behaviour of the stochastic trend is transferred through the model. Therefore, this thesis will use non-stationary data to test and estimate the model.

4.2 Calibration

To simulate the model, the model parameters are calibrated in accordance with the model's logic and the UK data. There are two groups of parameters: the first group captures the dynamic properties of the model, and the parameters are calibrated using estimates from DSGE models on the US, the euro area and the UK; the second group determines the steady state of the model, the parameters in this group remain fixed. The calibrated parameters of the first and second groups are listed in Table 4-2 and Table 4-3, respectively.

On the household side, the intertemporal elasticity of substitution σ_c and the elasticity of labour supply σ_l are calibrated at 1.39 and 2.33, respectively, following SW07. The external habit formation h is set at 0.58, and the wage indexation ι_w is 0.48, in line with Zhu (2017).

In the firm sector, the share of capital in production, α, is calibrated at 0.3, implying the share of labour in output is 0.7, consistent with Gollin (2002). The degree of price stickiness ξ_p is set at 0.711, and the degree of wage stickiness ξ_w is 0.718; these values are used in Zhu (2017). The proportion of sticky prices ω^p is set equal to 0.09, and the proportion of sticky wages ω^w is set at 0.442, consistent with Le et al. (2016). The price indexation l_p is set equal to 0.2, and it is the estimated result by Zhu (2017). The share of fixed costs in production (+1), ϕ, is 1.083, consistent with Le et al. (2021). The elasticity of capital utilisation ψ and the elasticity of capital adjustment φ are set at 0.54 and 5.74, respectively, in line with Le et al. (2011).

Table 4-1: Model Variable Construction and Data Sources

Variable	Symbol	Definition	Source
Output	y	$\ln\left(\frac{\text{Gross domestic product: CVM}}{\text{Working population}}\right)$	ONS
Consumption	c	$\ln\left(\frac{\text{Household final consumption expenditure: CVM}}{\text{Working population}}\right)$	ONS
Investment	i	$\ln\left(\frac{\text{Total fixed capital formation: CVM} + \text{Changes in inventories: CVM}}{\text{Working population}}\right)$	ONS
CPI Inflation	π^{cpi}	$\frac{\text{CPI}_t - \text{CPI}_{t-1}}{\text{CPI}_{t-1}}$	ONS
Nominal interest rate	r	$\left(\frac{\text{3 months Treasury Bills rate}}{4}/100\right)$	OECD and Financial Times
Labour hours	l	ln (Average actual weekly hours worked per employee)	University of Groningen PWT and ONS
Real consumer wage	w^c	$\ln\left(\frac{\text{Wage and salaries}}{\text{total hours worked} * \text{CPI}}\right)$	ONS
External finance premium	prem	$\left(\frac{\text{banking lending rate} - \text{3 month Treasury bills rate}}{4}/100\right)$	Refinitiv DataStream
Real lending rate	cy	$\left(\frac{\text{banking lending rate}}{4 * 100} - \text{one period ahead CPI inflation}\right)$	Refinitiv DataStream
Entrepreneur net worth	nw	$\ln\left(\frac{\text{FTSE all share index}}{\text{CPI}}\right)$	Refinitiv DataStream
Export	ex	$\ln\left(\frac{\text{Total exports: CVM}}{\text{Working population}}\right)$	ONS
Import	im	$\ln\left(\frac{\text{Total imports: CVM}}{\text{Working population}}\right)$	ONS
Real Exchange rate	q	$\ln\left(\frac{1}{\text{Sterling real effective exchange rate index}}\right)$	BoE, BIS
Net foreign bond position	b^f	Current account balance as per cent of GDP, divided by 100	ONS
Foreign consumption	c^f	World exports-goods & services	World Development Indicators
Foreign price level	p^f	Weighted average of CPI: US(0.6), Germany(0.19) and Japan(0.21)	FRED, Refinitiv DataStream

Real foreign interest rate	r^f	$\frac{\text{Weighted av ., interest rate: US}(0.6), \text{Germany}(0.19) \text{ and Japan}(0.21)}{4 * 100} - \text{one period ahead foreign inflation}$	Refinitiv DataStream
M2	m2	$\ln\left(\frac{\text{M2 money stock, CP}}{\text{CPI} * \text{Working population}}\right)$	FRED, BoE
M0	m0	$\ln\left(\frac{\text{Money supply M0, CP}}{\text{CPI} * \text{Working population}}\right)$	BoE
Capital	k	Derived from capital accumulation equation	Calculation
Price of capital	qq	Derived from investment Euler equation	Calculation
Capital rental rate	rk	Derived from labour demand equation	Calculation
Working population		Total claimant count + Work force jobs	ONS

- ONS, BoE, FRED, OECD, and BIS are short for the Office for National Statistics, the Bank of England, the Federal Reserve Economic Data, the Organization for Economic Co-operation and Development, and the Bank for International Settlement, respectively.
- CVM: Chained Volume Measure; CP: Current Price.
- Working population is used to scale the data as per capita.
- Data period: 1955Q1-2021Q1.

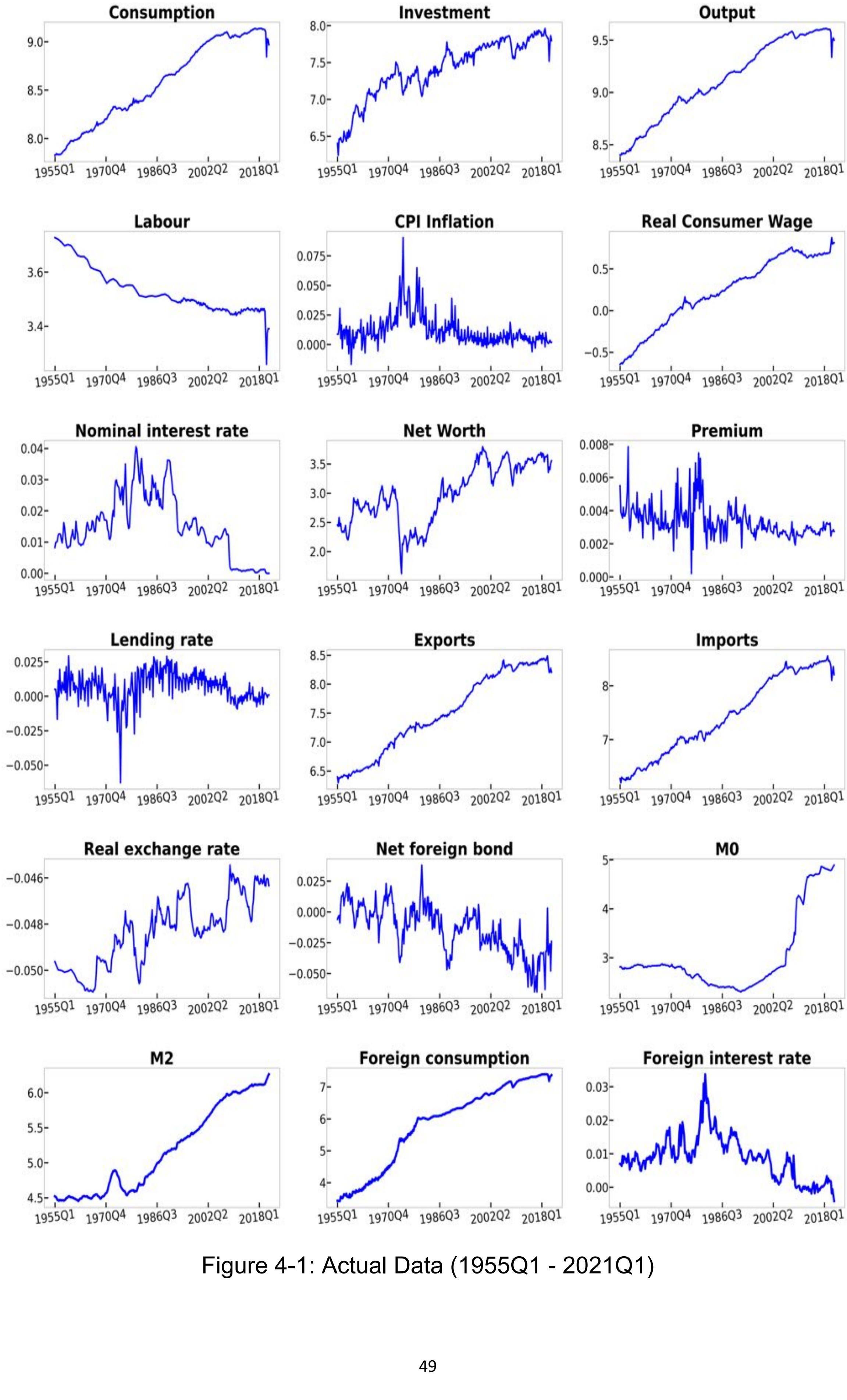

Figure 4-1: Actual Data (1955Q1 - 2021Q1)

On the money market side, parameters are consistent with Le et al. (2016, 2021). The Taylor rule response to output r_y and the response to inflation r_p are 0.019 and 2.375, respectively. The money response to premium ϑ_2 and to credit growth ϑ_1 are 0.059 and 0.052, respectively. On the financial friction, the elasticity of the premium with respect to leverage is 0.04 as in Bernanke et al. (1999), and the elasticity of the premium to M0 ϑ is 0.058, same as in Le et al. (2021).

For fixed parameters, the quarterly discount factor is 0.99, and this is consistent with a 1% steady-state real interest rate. Following SW07, the quarterly capital depreciation rate is 0.025, which implies an annual depreciation rate of 0.1, and the curvature of the Kimball aggregators in the goods and labour market ($\epsilon_w \ and \ \epsilon_p$) are set to be 10. The survival rate of entrepreneurs to the following period is set at 0.97. The preference bias towards the domestic goods ω and the foreign equivalent ω^f are both set to 0.7 as in Meenagh et al. (2010). Following Meenagh et al. and Minford (2015), the elasticity of substitution between the domestic goods and imported goods σ, which is also known as the Armington elasticity, is set at 1 ($\sigma = 1$); the foreign equivalent of σ, σ^f, is set at 0.7.

The steady state ratios are chosen based on UK empirical data. The consumption-to-output ratio $\frac{C}{Y}$ is 0.6367, the investment-to-output ratio $\frac{I}{Y}$ is 0.1148, the government spending-to-output ratio $\frac{G}{Y}$ is 0.20, the export-to-output ratio $\frac{EX}{Y}$ is 0.24, the import-to-output ratio $\frac{IM}{Y}$ is 0.25, the net worth-to-M2 ratio $\frac{\mathrm{NW}}{\mathrm{M2}}$ is 0.24, and the M0-to-M2 ratio $\frac{\mathrm{M0}}{\mathrm{M2}}$ is 0.07.

Table 4-2: Calibration

Description	Symbol	Calibration
Elasticity of capital adjustment	φ	5.740
Intertemporal elasticity of substitution	σ_c	1.390
External habit formation	h	0.580
Probability of not changing wages	ξ_w	0.718
Elasticity of labour supply	σ_l	2.330
Probability of not changing prices	ξ_p	0.711
Wage indexation	ι_w	0.480
Price indexation	l_p	0.200
Elasticity of capital utilisation	ψ	0.540
1+share of fixed costs in production	ϕ	1.083
Taylor Rule response to inflation	r_p	2.375
Interest rate smoothing	ρ	0.730
Taylor Rule response to output	r_y	0.019
Taylor Rule response to change in output	$r_{\Delta y}$	0.210
Share of capital in production	α	0.300
Parameter response of NK weight - wages	ω^w	0.442
Parameter response of NK weight - prices	ω^p	0.090
Elasticity of the premium with respect to leverage	χ	0.040
Money response to premium	ϑ_2	0.059
Elasticity of the premium to M0	ϑ	0.058
Money response to credit growth	ϑ_1	0.052

Table 4-3: Steady State Ratios and Fixed Parameters

Symbol	**Description**	**Value**
Steady state ratios		
$\frac{C}{Y}$	Consumption to output ratio	0.6367
$\frac{I}{Y}$	Investment to output ratio	0.1148
$\frac{G}{Y}$	Government spending to output ratio	0.2000
$\frac{EX}{Y}$	Export to output ratio	0.2400
$\frac{IM}{Y}$	Import to output ratio	0.2500
$\frac{C^e}{Y}$	Entrepreneurial consumption to output ratio	0.0080
$\frac{R_*^K}{Y}$	Capital to output ratio	2.6880
$\frac{K}{N}$	Capital to net worth ratio	2.0000
$\frac{\mathrm{NW}}{\mathrm{M2}}$	Net worth to M2 ratio	0.2400
$\frac{\mathrm{M0}}{\mathrm{M2}}$	M0 to M2 ratio	0.0700
Fixed parameters		
β	Quarterly discount rate	0.9900
δ	Quarterly capital depreciate rate	0.0250
θ	Survival rate of entrepreneurs	0.9700
γ	Quarterly trend growth rate	1.0040
ϵ_w	The curvature of the Kimball aggregator in the labour market	10.000
ϵ_p	The curvature of the Kimball aggregator in the goods market	10.000
σ	Import demand elasticity	1.0000
σ^f	Foreign equivalent of σ	0.7000
ω	Bias for the domestic good	0.7000
ω^f	Foreign equivalent of ω	0.7000

Chapter 5 Methodology: Indirect Inference

This thesis applies a simulation-based Indirect Inference method to test and estimate the structural model, rather than using the most widely used Bayesian method. Section 5.1 introduces the technique of Indirect Inference. Section 5.2 describes the choices of the auxiliary model.

5.1 Indirect Inference Testing and Estimation Process

In recent years, the Bayesian method has been the most popular approach for estimating DSGE models. A problem with the Bayesian method is that it heavily relies on the prior information about the macro economy, as it assumes that both the prior distribution and the model structure are correct. However, this prior information is usually not fully informed. As a result, the risk of biased results could arise from wrong prior choices. Smets and Wouters (2007) estimated their SW07 model by the Bayesian method using data from 1966-2004 and found that it fits the behaviour of the US sample data. However, when using the powerful method of Indirect Inference, the SW07 model was rejected by the behaviour of the sample data over the period 1947-2004 (Le et al., 2011). Another popular method is the Maximum Likelihood Estimation (MLE). The difficulty with MLE is that it is relatively easy to pass. In other words, it has little power against false models. In this thesis, we apply the method of Indirect Inference to test and estimate the DSGE model developed in Chapter 3, as it has been proven to be a powerful method (Minford et al., 2009; Le et al., 2010; Le et al., 2015). Nowadays, there is a growing number of studies using the Indirect Inference method, e.g., Le et al. (2011, 2016, 2021), Guvenen and Smith (2014), Dai et al. (2015), Akcigit and Kerr (2018), Dovonon and Hall (2018), Khalaf and Peraza (2020), and others.

Indirect Inference is a simulation-based approach first introduced by Smith (1993) and later further developed by Le et al. (2011) and Minford et al. (2009) with Monto Carlo. Indirect means choosing an auxiliary model that is independent of the theoretical model (e.g. VAR coefficients) as the lens to generate a description of the data. This method allows the estimation and testing of models of any size and nonlinearity by comparing the performance of an auxiliary model estimated on simulated data with the performance of an auxiliary model estimated on observed data. In addition, it can be used for both stationary and non-stationary datasets. We can use this method to find a set of parameters that makes the behaviour of the auxiliary model on simulated data closest to the behaviour of the auxiliary model based on actual data.

In this thesis, the auxiliary model is represented by a VAR with the exogenous variable model (VARX) following Meenagh et al. (2012). We first simulate the data using the DSGE model with calibrated coefficients and error distribution, then calculate the Wald statistic which is used as a measure of the difference between the simulated data and actual data. If the behaviour of data simulated by the model is very similar to the behaviour of the actual data, it indicates that the model can explain the UK economy, and the model will pass the Wald test. However, if the calibrated model fails the Wald test, we need to search for the optimal coefficient set that minimizes the distance between the estimates of the auxiliary model based on the simulated data with those based on observed data by applying the Indirect Inference estimation.

Indirect Inference Test Procedures

The model is non-linear because of the switching regime due to the ZLB, and the model is solved with nonstationary data, including nonstationary variables (productivity shocks and net foreign assets). Therefore, the model does not converge to a static steady state. For these reasons, the model is solved in FORTRAN using the projection method of Fair and Taylor (1983) and Minford et al. (1984, 1986), whereby rational expectations are solved; at a terminal date *T*, all endogenous variables are at equilibrium steady-state values and net foreign assets are unchanged (current account balance)[8]. The procedure for the Indirect Inference test is as follows:

1. Calculate the residuals of the model based on actual data and calibrated

[8] At the terminal date *T*, the expectations must meet the terminal conditions on the model. These conditions are imposed to guarantee that the simulated paths of the endogenous variables converge to long-run levels at the terminal date, in line with the model's long-run implications (see Minford et al., 1979). Imposing the terminal conditions on the expectations involves solving the equilibrium system sometime in the future, given that shocks have stopped, stationary variables have reached their long-run constant values, and trended variables have maintained constant growth rates. Additionally, the transversality condition must hold to ensure that the net foreign assets are stable and that net international debt does not grow over time.

parameters. Residuals for equations without expectations can be directly backed out from the equations and the data. For equations with expectations, we use lagged endogenous data as instruments. The resulting structural residuals are regarded as the model's error processes and together with exogenous variable processes to create historical shocks that disrupt the model.

2. To generate bootstraps, historical shocks are drawn in an overlapping way by time vector and added to the base run of the model, which is the observed data. Thus, for period 1, given its starting lagged values, a vector of historical shocks is created and added to the base run; the model is solved for period 1, and this solution becomes the lagged variable vector for period 2. Then for period 2, a second vector of historical shocks is created and added to the solution; the model is then solved for period 2 which becomes the lagged variable vector for period 3. This process is repeated until the bootstrapping reaches the entire sample size. Then we subtract the base run from the simulation to find the bootstrap effect of the shocks.

   The bootstraps are added to the balanced growth path (BGP) implied by the model and the deterministic trend terms in the exogenous variables and error processes. To obtain the BGP, the effect of a permanent change in each error/exogenous variable is solved at the terminal condition *T*. The BGP is incorporated into each of the 1000 bootstrap samples used to estimate the VECM auxiliary equation. Then the Wald statistic is calculated to measure the difference between the estimates of the auxiliary model estimated on the simulated data and those obtained from the actual data.

3. Wald statistic is the evaluation criterion of the model that measures the distance between the parameters of the auxiliary model obtained from the simulated data and the parameters based on the actual data.

To calculate the Wald statistic, we first apply the OLS method to calculate the parameter vector of the auxiliary model for the actual and simulated data. The Wald statistic is defined as:

$$WS = \left(\beta^a - \overline{\beta^s(\hat{\theta}_0)}\right)' \Omega^{-1}(\beta^a - \overline{\beta^s(\hat{\theta}_0)})$$

where β^a is the estimate of the parameter vector obtained from the UK actual data; $\overline{\beta^s(\hat{\theta}_0)}$ is the sample average of the estimates of the coefficients in the auxiliary model obtained from s sets of simulated data from the model, conditional on the calibrated or estimated vector of parameters $\hat{\theta}_0$:

$$\overline{\beta^s(\hat{\theta}_0)} = E\left[\beta^i(\hat{\theta}_0)\right] = \frac{1}{s}\sum_{i=1}^{s} \beta^i(\hat{\theta}_0) \qquad s = 1000$$

Ω is the variance-covariance matrix of the distribution of ($\beta^a - \overline{\beta^s(\hat{\theta}_0)}$):

$$\Omega = cov\left(\beta^i(\hat{\theta}_0) - \overline{\beta^s(\hat{\theta}_0)}\right) = \frac{1}{s}\sum_{i=1}^{s} (\beta^i(\hat{\theta}_0) - \overline{\beta^s(\hat{\theta}_0)})\left(\beta^i(\hat{\theta}_0) - \overline{\beta^s(\hat{\theta}_0)}\right)'$$

The Wald statistic measures the difference between the estimates of the auxiliary model estimated on the simulated data and those obtained from the actual data. Therefore, the null hypothesis implies that $\beta^a = \overline{\beta^s(\hat{\theta}_0)}$. Non-rejection of the null hypothesis implies that the model can describe the economy of the sample country because the behaviour of the simulated data is very similar to the behaviour of the actual data. Rejection implies that the model specification is incorrect.

The Wald statistic from the actual data must be smaller than the 95th percentile of the Wald statistic from the simulated data for the model to fit the data with the 95% confidence level (5% significance level). We can convert the Wald result into a normalised t-statistic as follows:

$$t\_statistic = \left(\frac{\sqrt{2WS} - \sqrt{2k-1}}{\sqrt{2WS_i^{0.95}} - \sqrt{2k-1}}\right) \times 1.645$$

where k is the length of β^a (the vector of auxiliary model parameters derived with the actual data). WS is the Wald statistic for the actual data, and $WS_i^{0.95}$ is the Wald statistic for the 95th percentile of the simulated data. $\sqrt{2WS} - \sqrt{2k-1}$ is the Mahalanobis Distance with mean zero and standard deviation of unity; the Mahalanobis Distance is scaled by 1.645, hence when $WS = WS_i^{0.95}$ the t-statistic corresponds to the 95th percentile of the standard normal distribution. For the model to pass the test, the t-statistic must not be greater than the threshold of 1.645. The smaller the t-statistic value is, the better the model matches the actual data.

If the calibrated model fails the test, we reject the null. This implies that the calibrated model cannot explain the UK economy. The problem could arise from incorrect specification or the calibration of the model. Then the optimal set of parameters will be determined by applying the Indirect Inference estimation to minimise the distance between the simulated data and actual data. In this process, we carry out the Simulated Annealing algorithm, in which a search takes place over a wide range around the calibrated parameters (or initial values) by random jumps to search for the best set of parameters that minimises the Wald statistic. More specifically, in the estimation process, we first randomly search for 250 sets of parameters around the calibration within a chosen bound by using the random search method. Second, we test each set of parameters. By doing this, we could find some parameter sets that pass the test, and then we select the optimal set of parameters that can pass the test and produce sensible impulse response functions (IRFs).

5.2 Choice of Auxiliary Model

The solution to the log-linearised DSGE model can be represented as a restricted Vector Autoregressive and Moving Average (VARMA) or an approximate VAR. The DSGE model in this thesis contains non-stationary variables (e.g. foreign bonds) and technology shocks. According to Meenagh et al. (2012), a vector error correlation model (VECM) can be chosen as an auxiliary model in this case because non-stationary residuals in VECM are treated as observed variables.

Following Meenagh et al. (2012), the log-linearised DSGE model can be written in the following form:

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \tag{4.1}$$

where y_t is a $p \times 1$ vector of endogenous variables, $E_t y_{t+1}$ is a $r \times 1$ vector of expected future endogenous variables, x_t is a $q \times 1$ vector of exogenous variables which is assumed driving by:

$$\Delta x_t = a(L)x_{t-1} + d + c(L)\epsilon_t \tag{4.2}$$

The disturbances e_t and ϵ_t both follow an identically independent distribution with a mean zero. x_t and y_t are both non-stationaries. A(L), B(L), C(L), D(L), a(L), b(L) and c(L) are polynomial matrix functions; they all have roots outside the unit circle.

The general solution of y_t is:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \tag{4.3}$$

where f is a vector of constant. Because x_t and y_t are both non-stationary, the solutions for the endogenous variables have the p cointegrating relationships (where Π is a $p \times p$ matrix):

$$y_t = [I - G(1)]^{-1}[H(1)x_t + f] = \Pi x_t + g \tag{4.4}$$

In the long run, the solutions of the model are

$$\bar{y}_t = \bar{\Pi} x_t + g \tag{4.5}$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \tag{4.6}$$

$$\xi_t = \sum_{t=0}^{t-1} \epsilon_{t-s} \tag{4.7}$$

where $\bar{y}_t$ and $\bar{x}_t$ are the long run equilibrium solutions of y_t and x_t, respectively. $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$ contains a deterministic trend $\bar{x}_t^D$=$[1 - a(1)]^{-1}dt$ and a stochastic part $\bar{x}_t^S = [1 - a(1)]^{-1}c(1)\xi_t$. Therefore, the solution of y_t can be rewritten as a cointegrated VECM with a mixed moving average error term (ω_t) by subtracting y_{t-1} on both sides of the equation (4.3):

$$\begin{aligned}\Delta y_t &= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t \\ &= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t \qquad (4.8)\end{aligned}$$

$$\omega_t = M(L)e_t + N(L)\epsilon_t \tag{4.9}$$

The cointegrated VECM can be approximated as a form of VARX:

$$\Delta y_t = K[y_{t-1} - \Pi x_{t-1}] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{4.10}$$

where ζ_t follows i.i.d. with zero mean. As $\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$ and $\bar{y}_t = \bar{\Pi} x_t + g$, we can rewrite the equation (4.10) as

$$\Delta y_t = K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{4.11}$$

In our model, the effect of the trend element in x and the deviation of x from its trend have different effects. To distinguish between these two effects, we follow Meenagh et al. (2012) in using equation (4.11) as our auxiliary model. We estimate equation (4.11) in one stage using OLS method, as Meenagh et al. (2012) proved that this method is extremely accurate.

According to Le et al. (2011), two types of Wald tests can be applied to evaluate the auxiliary model: full Wald and directed Wald tests. The full Wald tests are based on the full set of variables and check the specification of the model in a broad sense.

The directed Wald tests are based on a subset of the variables and focus on only limited properties of the model. Usually, the power of the test grows as more endogenous variables are included in the auxiliary model. However, a broader set of endogenous variables can lead to uniform rejections (Le et al., 2015).

In this thesis, we choose three main macro variables (output, inflation and interest rate) because of 'ideal power' – not so high as to stop a good model from passing, but high enough to reject bad models with high probability. For this Meenagh et al. (2019) show that using a VAR with three variables (it does not matter which three are chosen) is adequate to provide an indirect test with high power for a large model. However, adding more variables would produce excessive power. A VARX (1) with our three variables of interest is the basis for the Wald test, which can take the form of:

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = B \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ r_{t-1} \end{bmatrix} + C \begin{bmatrix} T \\ e_t^a \\ b_{t-1}^f \end{bmatrix} + \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^\pi \\ \epsilon_t^r \end{bmatrix} \tag{4.12}$$

$$B = \begin{bmatrix} \theta_{yy} & \theta_{y\pi} & \theta_{yr} \\ \theta_{\pi y} & \theta_{\pi\pi} & \theta_{\pi r} \\ \theta_{ry} & \theta_{r\pi} & \theta_{rr} \end{bmatrix} \tag{4.13}$$

the VARX(1) includes a matrix B which contains nine OLS estimates of $\theta_{yy}, \theta_{y\pi}, \theta_{yr}, \theta_{\pi y}, \theta_{\pi\pi}, \theta_{\pi r}, \theta_{ry}, \theta_{r\pi}, \theta_{rr}$, deterministic time trend T, non-stationary residual e_t^a, lagged non-stationary foreign assets b_{t-1}^f, and three fitted stationary errors $\epsilon_t^y, \epsilon_t^\pi, \epsilon_t^r$. In the Wald calculation, the parameter vector β contains nine coefficients of matrix B to describe the dynamic properties of the model and data and includes the variances of the three errors to measure the size of variation:

$$\beta = [\theta_{yy}\ \theta_{y\pi}\ \theta_{yr}\ \theta_{\pi y}\ \theta_{\pi\pi}\ \theta_{\pi r}\ \theta_{ry}\ \theta_{r\pi}\ \theta_{rr}\ var(\epsilon_t^y)\ var(\epsilon_t^\pi)\ var(\epsilon_t^r)]' \tag{4.14}$$

The model will only pass the test if it can jointly replicate the data features of output, inflation, and interest rate. This means that in order to pass the test, the model must jointly match the 12 coefficients in β.

Chapter 6 Empirical Results of the Fixed Duration Model for the Inflation Targeting Era 1992-2021

This chapter estimates and tests the hybrid DSGE model we developed in Chapter 3 with fixed shares of sticky and flexible price and wage sectors to explore how well it can match the behaviour of the UK data in the inflation targeting era. In the UK, the BoE has officially adopted an inflation targeting since 1992Q4; therefore, this chapter estimates the fixed duration model using quarterly and unfiltered data from 1992Q4 to 2021Q1.

6.1 Indirect Inference Test and Estimation Results

We tested and estimated the model using the Indirect Inference method on unfiltered (or nonstationary) macroeconomic data. Firstly, we employed the Indirect Inference to test the calibrated fixed price/wage duration model, but the calibrated model failed the test. This may indicate that our calibration was not very good. Secondly, we therefore estimated the model using the Indirect Inference estimation. We performed the Simulated Annealing algorithm in which a search takes place over a wide range around the calibrated parameters (or initial values) by random jumps to search for the optimal choice of parameters that minimises the Wald-statistic (or maximises the p-value).

The empirical results of our calibration and estimation are presented in Table 6-1. The calibrated model did not pass the Wald test with a p-value of 0.004, implying that the calibrated model failed to explain the data behaviour for the inflation targeting era of 1992-2021. Then by using Indirect Inference estimation, the estimated model passed the Wald test with a p-value of 0.076, which implies that the model failed to be rejected at 95%. This suggests that the estimated fixed duration model can fit the behaviour of historical UK data over the inflation targeting era of 1992Q4-2021Q1.

Table 6-2 reports the estimated persistence parameters. Figures 6-1 and 6-2 show the shocks and residuals extracted from the estimated model, respectively.

Table 6-3 shows the VECM parameter estimates, data variance estimates and the model's 95% bounds. Only one of the nine VAR coefficients lies outside its 95% confidence interval. The variances of interest rate and inflation are slightly below the lower bound.

Comparing Estimates to the Literature

As for the estimation results compared to the literature. For the estimates of household utility function, the elasticity of labour supply is 2.678 and the intertemporal elasticity of substitution is 1.510, similar to the estimates by Lyu et al. (2023) for the UK economy. The estimates of the Phillips curve and real wage equation show that the probabilities of not changing wages and prices as well as the indexations of wages and prices are 0.648, 0.942, 0.428 and 0.143, respectively. These estimates are close to those reported by Lyu et al.

Turning to the estimates of the Taylor Rule parameters, the Taylor Rule response to inflation and the response to change in output are 3.469 and 0.045, respectively. These values are higher than those in Lyu et al., implying that monetary policy is more reactive to inflation and change in output over the sample period in this study compared to the findings of Lyu et al. The estimates of the Taylor Rule response to output and interest rate smoothing are 0.014 and 0.672, respectively. They are slightly lower than the estimates of 0.025 and 0.691 reported in Lyu et al., but higher than those of 0.080 and 0.054 by Le et al. (2011) for the US economy. These comparisons suggest that our estimates are reasonable.

Regarding the estimated parameters for monetary response and financial frictions, the estimates of money response to premium and money response to credit growth as well as the elasticity of the premium to M0 are 0.051, 0.048 and 0.066, respectively. They are close to the estimates of Lyu et al. (2023) and Le et al. (2016, 2021). The elasticity of the premium with respect to leverage is estimated at 0.084, which is within the range of 0.03-0.09 found in other studies for advanced economies.

Finally, for the other parameters, the estimated elasticity of capital adjustment is 5.713, lower than the value of 7.254 in Lyu et al., but similar to the estimates found in Meenagh et al. (2019) and SW07. The estimated elasticity of capital utilisation cost of

0.440 is significantly larger than the figure of 0.111 in Lyu et al, suggesting a higher cost associated with changing the utilisation of capital. However, it is lower than the mean of 0.54 in SW07 and the estimate of 0.845 by Aminu (2018) for the UK economy. The estimate of one plus the share of fixed costs in production is 1.228, lower than the estimate by Lyu et al. (2023) but higher than the estimate by Le et al. (2021). The share of capital in production is estimated at 0.368, which is higher than the estimate of 0.185 by Lyu et al., but lower than the calibrated value of 0.385 by Videnova (2016) for the UK economy.

Overall, our estimates for the fixed-duration model over 1992-2021 are close to those in the literature and similar to those by Ly et al. (2023) for the UK economy over the period 1993-2016.

Table 6-1: Calibration and Estimates of the Fixed-Duration Model

		Calibration[9]	**Estimation**
Elasticity of capital adjustment	φ	5.7400	5.7125
Intertemporal elasticity of substitution	σ_c	1.3900	1.5099
External habit formation	h	0.5800	0.8665
Probability of not changing wages	ξ_w	0.7180	0.6483
Elasticity of labour supply	σ_l	2.3300	2.6775
Probability of not changing prices	ξ_p	0.7110	0.9416
Wage indexation	ι_w	0.4800	0.4275
Price indexation	l_p	0.2000	0.1430
Elasticity of capital utilisation	ψ	0.5400	0.4400
1+share of fixed costs in production	ϕ	1.0800	1.2281
Taylor Rule response to inflation	r_p	2.4000	3.4685
Interest rate smoothing	ρ	0.7300	0.6723
Taylor Rule response to output	r_y	0.0190	0.0142
Taylor Rule response to change in output	$r_{\Delta y}$	0.2100	0.0453
Share of capital in production	α	0.3000	0.3677
Proportion of sticky wages	ω^w	0.4000	0.3024
Proportion of sticky prices	ω^p	0.1000	0.0255
Elasticity of the premium with respect to leverage	χ	0.0400	0.0841

[9] We made slight adjustments to the calibration to produce sensible IRFs. In addition, when searching for the optimal parameter set, we searched around this calibration set that produced sensible IRFs.

Money response to premium	ϑ_2	0.0590	0.0508
Elasticity of the premium to M0	ϑ	0.0580	0.0664
Money response to credit growth	ϑ_1	0.0520	0.0482
Wald (Y, π, R)		95.7479	22.5073
P-value		0.0040	0.0760
Transformed t-statistic		5.9567	1.2720

Table 6-2: Estimated AR (1) Coefficients

Shocks	Symbols	AR coefficients
Government Spending shock e_t^g	ρ_g	0.86710542
	σ_{ga}	0.47253022
Preference shock e_t^b	ρ_b	-0.39022188
Investment shock e_t^i	ρ_i	-0.09893808
Taylor Rule shock e_t^r	ρ_r	-0.19544290
Productivity shock e_t^a	ρ_a	-0.67420498
Price mark-up shock e_t^p	ρ_p	-0.45976644
Wage mark-up shock e_t^{wNK}	ρ_w^{nk}	-0.04901604
Labour hours shock $e_t^{w_{NC}}$	ρ_w^{nc}	-0.10635554
Premium shock e_t^{prem}	ρ_{pr}	0.85570170
Net worth shock e_t^{nw}	ρ_{nw}	-0.15451453
Export shock e_t^{ex}	ρ_x	0.76596984
Import shock e_t^{im}	ρ_m	0.69216613
Foreign consumption shock c_t^f	ρ_c^f	0.94431944
Foreign interest rate shock r_t^f	ρ_r^f	0.82082736
M0 shock (crisis) $e_t^{m0,crisis}$	$\rho_{m0,crisis}$	0.30845545
M0 shocks (non-crisis) $e_t^{m0,nocrisis}$	$\rho_{m0,nocrisis}$	0.30753784

Note: σ_{ga} is the response of exogenous spending to productivity development

Table 6-3: VECM Parameters, Data Variance and Model Bootstrap Bounds for Output, Inflation and Interest Rate

	Actual estimate	Lower Bound (2.5th percentile)	Upper Bound (97.5th percentile)	IN/OUT
θ_{yy}	0.6007945	0.5347584	1.0259235	IN
$\theta_{y\pi}$	0.1519883	-1.3992207	0.795265	IN
θ_{yr}	1.1223211	-3.0360946	1.2918210	IN
$\theta_{\pi y}$	0.0032251	0.0105012	0.0992968	OUT
$\theta_{\pi\pi}$	-0.2299627	-0.5938572	-0.1219793	IN
$\theta_{\pi r}$	-0.0404764	-0.1998910	0.7382009	IN
θ_{ry}	0.0012810	-0.0073279	0.0557701	IN
$\theta_{r\pi}$	-0.0063939	-0.3395720	-0.0235946	IN
θ_{rr}	0.9257409	0.4539001	0.9459848	IN
$var(\epsilon_y)$	0.0006363	0.00032297	0.00155926	IN
$var(\epsilon_\pi)$	1.82e-05	1.86e-05	5.38e-05	OUT
$var(\epsilon_r)$	8.66e-07	3.03e-06	2.937e-05	OUT

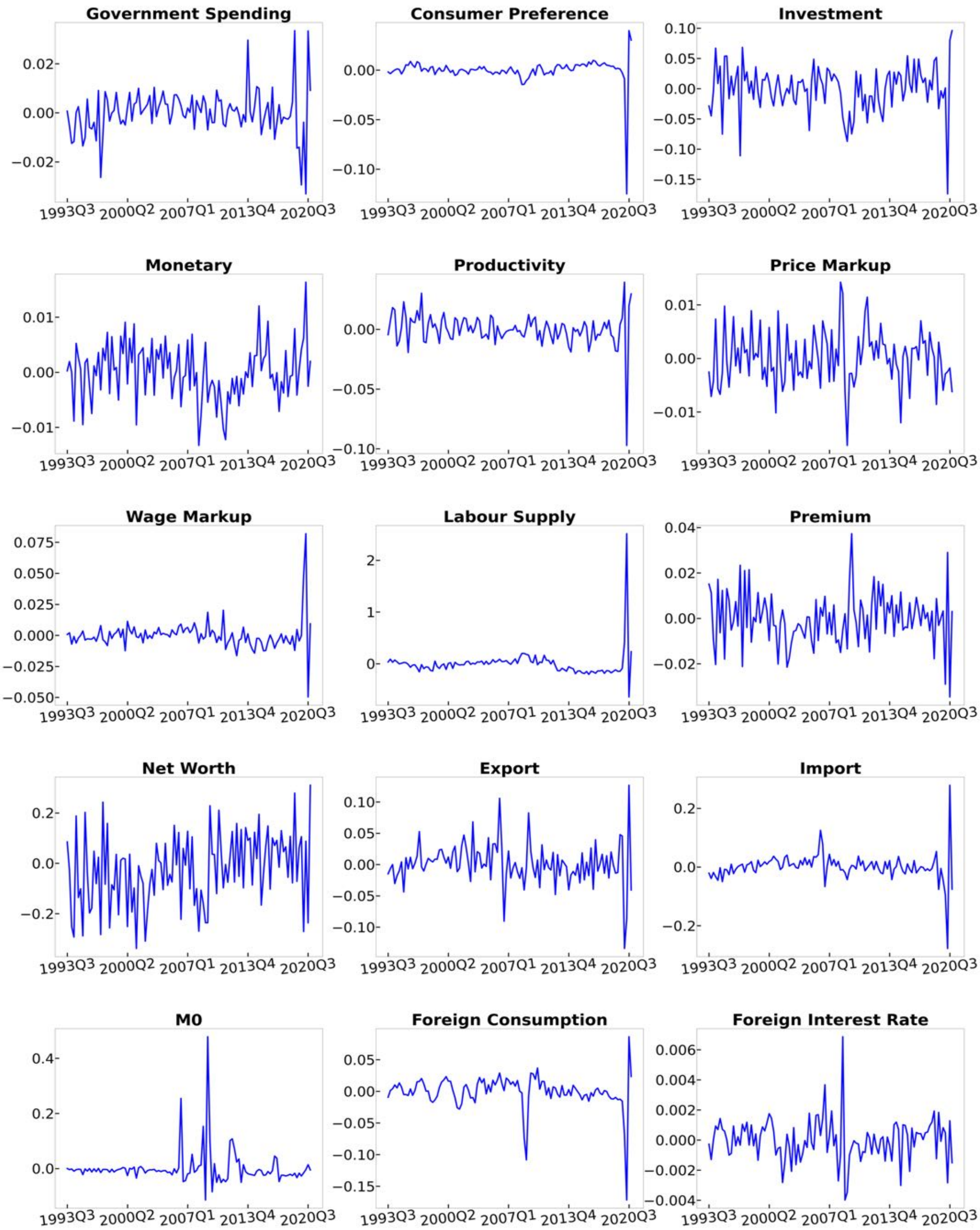

Figure 6-1: Shocks Calculated Using Estimated Parameters (1993Q3-2020Q4)

Note: Figure 6-1 plots model shocks calculated by using estimated parameters. The shocks lost three periods in the beginning from lags and innovations of productivity shock and one at the end from expectations against the actual data.

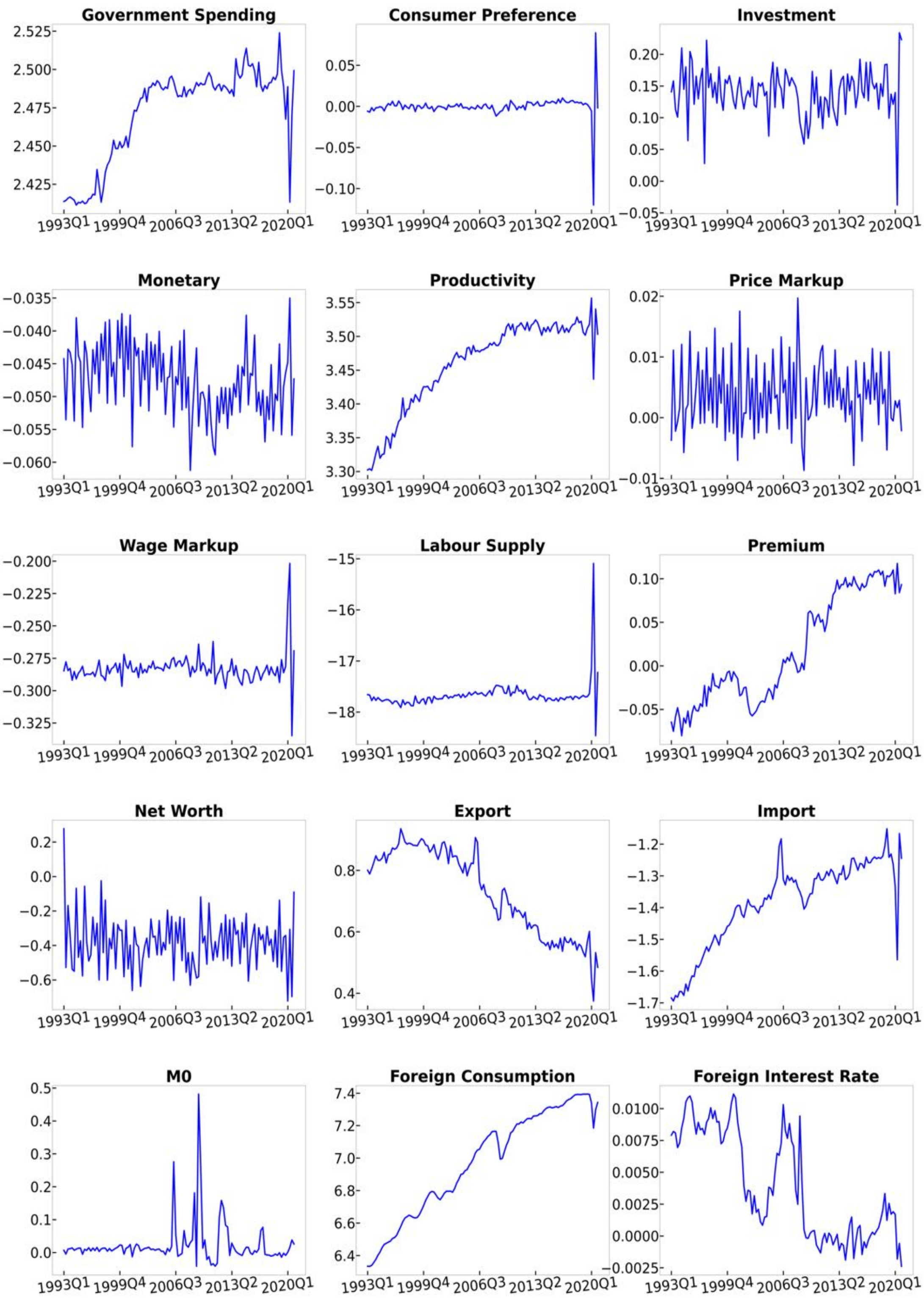

Figure 6-2: Residuals Calculated Using Estimated Parameters (1993Q1-2020Q4)

Note: Figure 6-2 plots model residuals calculated as the difference between the actual data and the equations using estimated parameters. The residuals lost one period in the beginning from lags and one at the end from expectations against the actual data.

6.2 Simulated Behaviour of the Model

Binding ZLB

The ZLB is endogenously determined by the shocks, rather than being imposed for a predetermined duration as the commitment of monetary authorities. This enables us to assess the frequency and duration of ZLB events in our simulations. Figure 6-3 shows some examples of simulated interest rate compared to the actual data, revealing the following two features, which are consistent with the findings of Lyu et al. (2023) [10]:

1) The periods with a ZLB binding can be quite repeatedly; for example, simulations 40 and 308 provide examples of interest rates reaching the ZLB not just once but multiple times.
2) The duration of the ZLB binding could be flexible. There could be extreme cases, as in simulation 242, where the ZLB can be binding for more than half the time, which could be interpreted by the fact that if the model is disturbed by a series of adverse shocks, a prolonged stay at the ZLB could occur.

The pseudo histories generated by our model highlight the frequency and severity of ZLB episodes that could have occurred. This emphasizes the importance of defining the 'crisis state', as it enables us to examine the responses of the model at the ZLB.

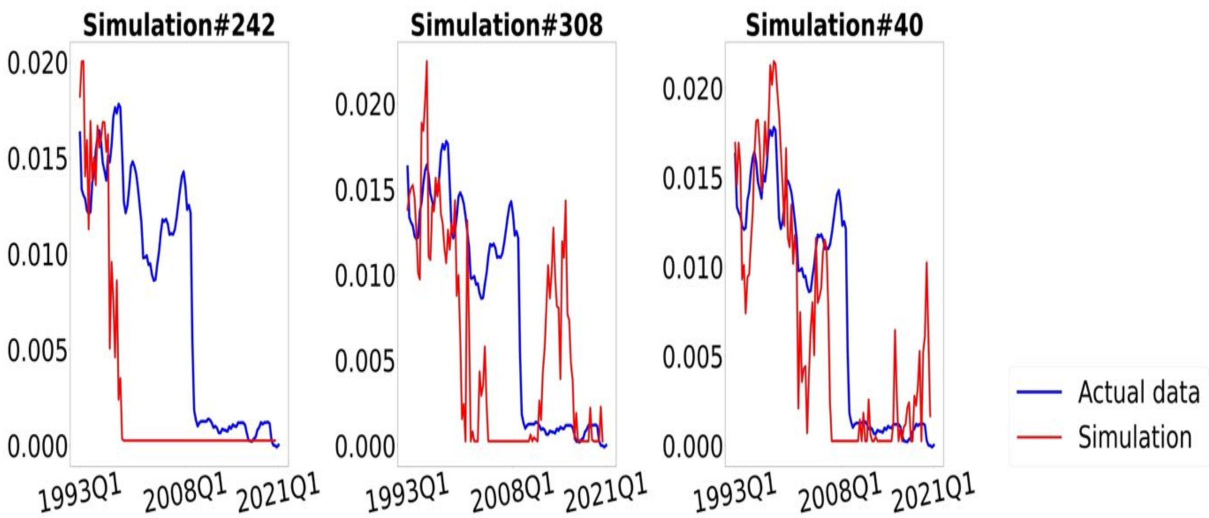

Figure 6-3: A Selection of Simulated Interest Rate Compared to Actual Data

[10] These are pseudo-histories created using bootstrap simulations from the estimated model and its shocks from 1992 to 2021.

Monetary Responses Shift in Non-Crisis and Crisis Regimes

Central banks responded to the Great Recession by cutting their nominal interest rates near zero. A few even allowed their rates to fall to negative values, such as Japan, Denmark, Sweden, Switzerland and the euro zone. That poses a severe obstacle to conventional monetary policy. In the UK, the BoE has adopted a 2% flexible inflation targeting since 1992 but has faced a ZLB issue since 2009. Since the conventional monetary policy was ineffective in boosting the economy during the Great Recession, an unconventional method, QE, was introduced by the BoE in 2009, which aimed at injecting liquidity into the banking system and stimulating more credit creation.

In our model, when the nominal interest rate solves for the bound level or below it ($r_t \leq 0.025\%$), the conventional tool reaches its limit, we then suspend the Taylor Rule and replace it with an exogenous low bound ($r_t = 0.025\%$). During the ZLB, the central bank turns to unconventional monetary policy; therefore, QE becomes the primary tool to target the credit market with the aim of reducing the premium on given leverage and boosting the credit supply. In other words, QE becomes the main policy tool when the ZLB binds.

Figure 6-4 shows several simulation examples to illustrate how monetary responses change in non-crisis and crisis regimes. Simulations 18 and 362 show that when interest rates remain out of the ZLB for almost the entire sample period, the Taylor rule operates at all times, and M0 acts only to accommodate M2. In contrast, in Simulation 488 and Simulation 577, the ZLB binds about half the time, and at these times, the pause in the Taylor Rule triggers QE operations; the central bank operates QE by injecting M0 into the economy. Thus, M0 increases sharply when the ZLB binds. These are consistent with the findings of Lyu et al. (2023).

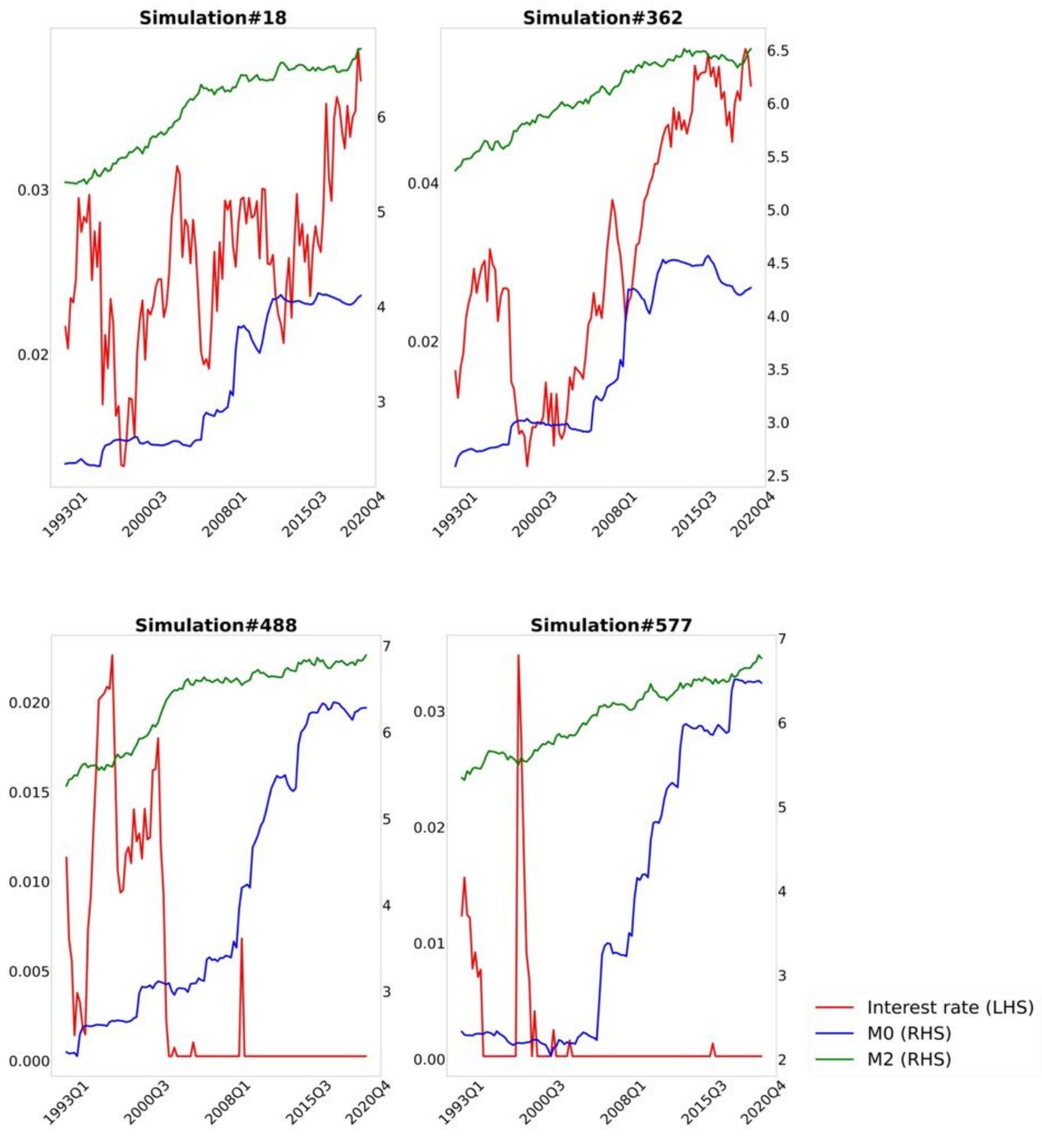

Figure 6-4: ZLB Constraint Triggers QE Operation

6.3 The Causes of Crises

In this subsection, we investigate the nature of economic crises and the causes of economic crises. We define an 'economic crisis' as a significant disruption in GDP growth for at least three years and a 'financial crisis' as an economic crisis accompanied by the ZLB. Based on the model and UK data, we analyse two different scenarios with reference to Le et al. (2016): 1) Standard shock scenarios, which are created by using bootstrap simulations from the model and 'standard shocks' from the period 1992-2007, a period that does not include enormous 'financial shocks'; 2) Financial crisis-inclusive shock scenarios, which are created by bootstrapping shocks from the period of 1992-2021, including financial crisis shocks. The analysis and comparison of these two scenarios led us to the following findings:

1) Economic crises are a normal part of the UK economy. We can see from Table 6-4, under the standard shock scenario, economic crises occur on average once every 28 years; under the financial crisis-inclusive shock scenario, the average economic crisis becomes more frequent: once every 27 years. This implies that crises are a regular outcome of the standard shock scenario and that shocks during financial crises are not a necessary condition for economic recessions. A similar finding has been found for the US, with Le et al. (2016) finding that economic crises occur once every 52 years under the standard shock scenarios, and crisis-inclusive shock scenarios increase the frequency of economic crises once every 48 years. Moreover, Figure 6-5 shows significant drops in output under the standard shock scenario, as in Simulation 115, where there have been several significant falls in output.
2) The fourth column of Table 6-4 reports the ratio of the number of financial crises to the number of economic crises. A financial crisis is defined as an economic crisis accompanied by the ZLB. Thus, the results indicate that when there is an economic crisis, about 57% probability that there will also be a financial crisis. The ratio in the standard shock scenario is very close to the ratio in the financial crisis-inclusive shock scenario. For the US, Le et al. (2016) find that when there is an economic crisis, nearly half the time there is also a financial crisis.
3) Financial shocks are not a necessary condition for triggering financial crises. This is because we have produced financial crises using the standard shock scenarios (see Table 6-4). Including financial shocks only slightly increases the

number of crises.

4) By comparing Figure 6-5 with Figure 6-6, we can see that financial crises prolong the duration of economic crises.

The four findings mentioned above are in line with those of Le et al. (2016). Moreover, these findings are generally consistent with the findings of Stock and Watson (2012) and Boivin et al. (2020). Using a VAR factor analysis, Stock and Watson (2012) find that the same shocks that explained other post-war recessions also explained the 2007 recession, with little evidence of new shocks associated with the 2007 recession and its aftermath. The 2007 recession was caused by more extreme versions of previously experienced shocks. Boivin et al. (2020) find that the economic collapse during the 2007 recession could be explained by particularly large shocks associated with financial disruptions and uncertainty.

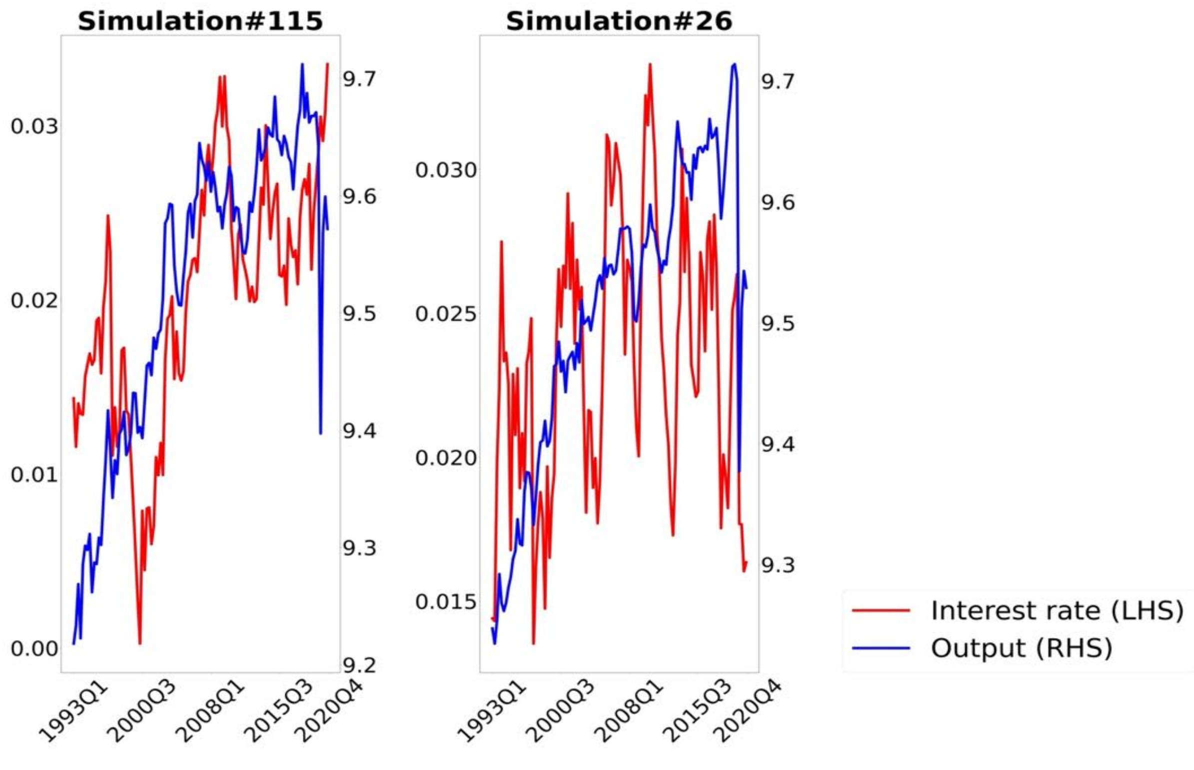

Figure 6-5: Economic Crises Without Financial Crisis

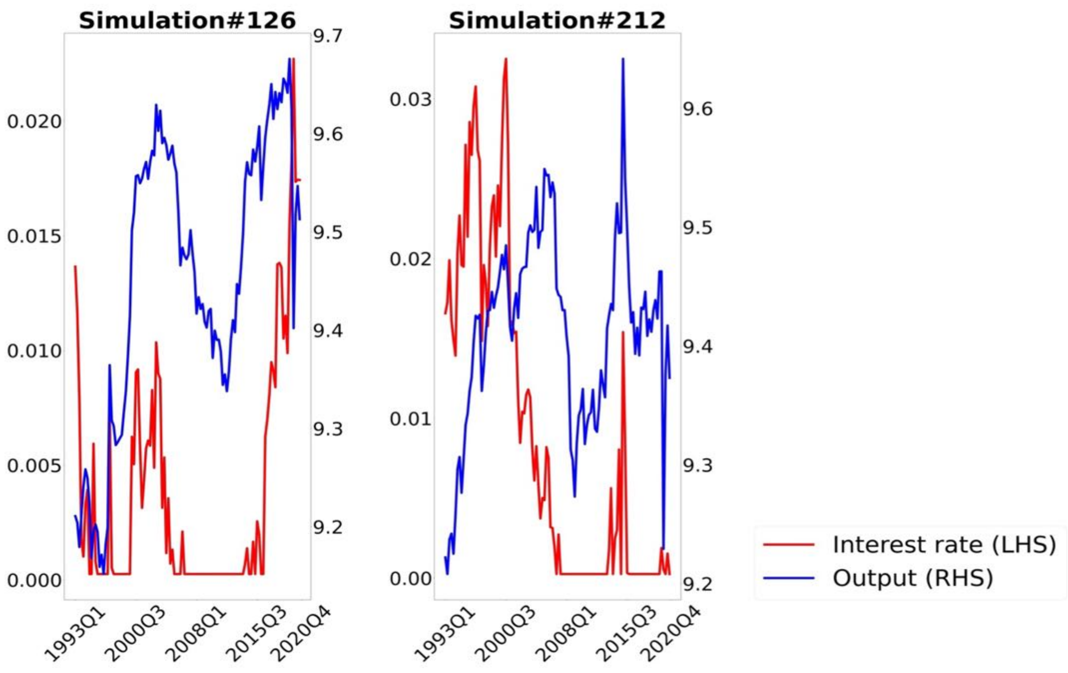

Figure 6-6: Economic Crises With Financial Crisis

Table 6-4: Crises Comparison

	Crises*	Frequency of Crisis	$\frac{\text{number of financial crisis}}{\text{number of economic crisis}}$
Standard shock scenarios	35.11	28.48	0.57
Financial crisis-inclusive shock scenarios	37.64	26.57	0.60

*Expected number of crises per 1000 years.

6.4 Impulse Responses from Estimated Model

This section briefly analyses the dynamics of the estimated fixed price/wage duration model through the impulse responses of the endogenous variables to the structure shocks under the non-crisis and crisis models to illustrate the internal logic of the models. The IRFs show the difference between the base run of the model and the simulated results of the model after a one-off shock. The base run replicates the observed data. These analyses help to assess the validity of the estimated model. In the IRFs figures, wage represents the real consumer wage and inflation is CPI inflation. The y-axis represents the responses to shocks, and the x-axis is the timeline. The blue line represents the non-crisis model, and the red line represents the crisis model.

Taylor Rule Shock

Figure 6-7 shows the IRFs to a positive Taylor Rule (contractionary monetary policy) shock under the normal regime. An increase in the nominal interest rate hits consumption and investment negatively, leading to a reduction in output. In the output market, the reduction in output leads to an output gap, which lowers inflation. In the input market, a higher nominal interest rate decreases labour hours and real wages. In the financial sector, the shock decreases entrepreneurs' net worth, consequently raising external finance premium that further reduces investment. The lower net worth leads to an increase in distressed borrowing, hence M0 raises. In the foreign sector, deflation and a higher nominal interest rate (implying a higher real interest rate) decrease the real exchange rate. Thus, the pound sterling appreciates, which induces import and lowers export as domestic prices are relatively higher than foreign prices. The net foreign bond position also decreases because of a lower net export. The sign of the impulse response (increase or decrease) to a positive Taylor Rule shock is fully consistent with the results of Lyu et al. (2023) for the UK, see Figure 6-8.

Government Spending Shock

Figure 6-9 captures the effects of a temporary rise in government spending under the normal and crisis regimes. Under the normal regime, an expansionary fiscal policy causes an increase in aggregate demand, which pushes up output and labour hours. The firm will raise nominal wages to attract more labour to meet excess output demand. As the expected income rises, so does consumption. Simultaneously, fiscal expansion increases prices and interest rates via the responses of the Taylor Rule. The higher interest rate reduces investment and capital. In the financial sector, the rises in output and labour push entrepreneurs' net worth up, consequently decreasing credit premium. In the money market, fiscal expansion has a negative effect on the money supply, implying that monetary responses tend to offset the expansionary fiscal policy. In the foreign market, real exchange rate falls (domestic currency appreciates) with a higher real interest rate, which decreases exports and increases imports because domestic products are less competitive than foreign ones. The lower net export has a negative impact on net foreign asset accumulation.

In the crisis regime where the nominal interest rate is fixed at the exogenous lower bound, higher inflation decreases real interest rate. As a result, the increase in wage, consumption, output, and labour is higher than that under the normal regime. The higher net worth drives down credit premiums, consequently boosting investment. In the foreign market, the lack of nominal interest rate response and the higher inflation mean a lower real interest rate, which causes a rise in real exchange rate (depreciation of the pound sterling). The depreciation of the pound sterling encourages exports. Imports decline in line with the depreciation of the pound sterling but increase as the domestic consumption demand rises; overall, import rises slightly.

Our response direction of variables to a positive government spending shock is highly consistent with the findings in Lyu et al., see Figure 6-10. In addition, we find that increases in consumption, output, labour and wage are more pronounced in the crisis regime than in the normal regime, implying an increase in the impact of fiscal multiplier at the ZLB relative to the normal regime, which is consistent with the findings of Eggertsson (2011), Christiano et al. (2011), Hills and Nakata (2018) and Lyu et al. (2023).

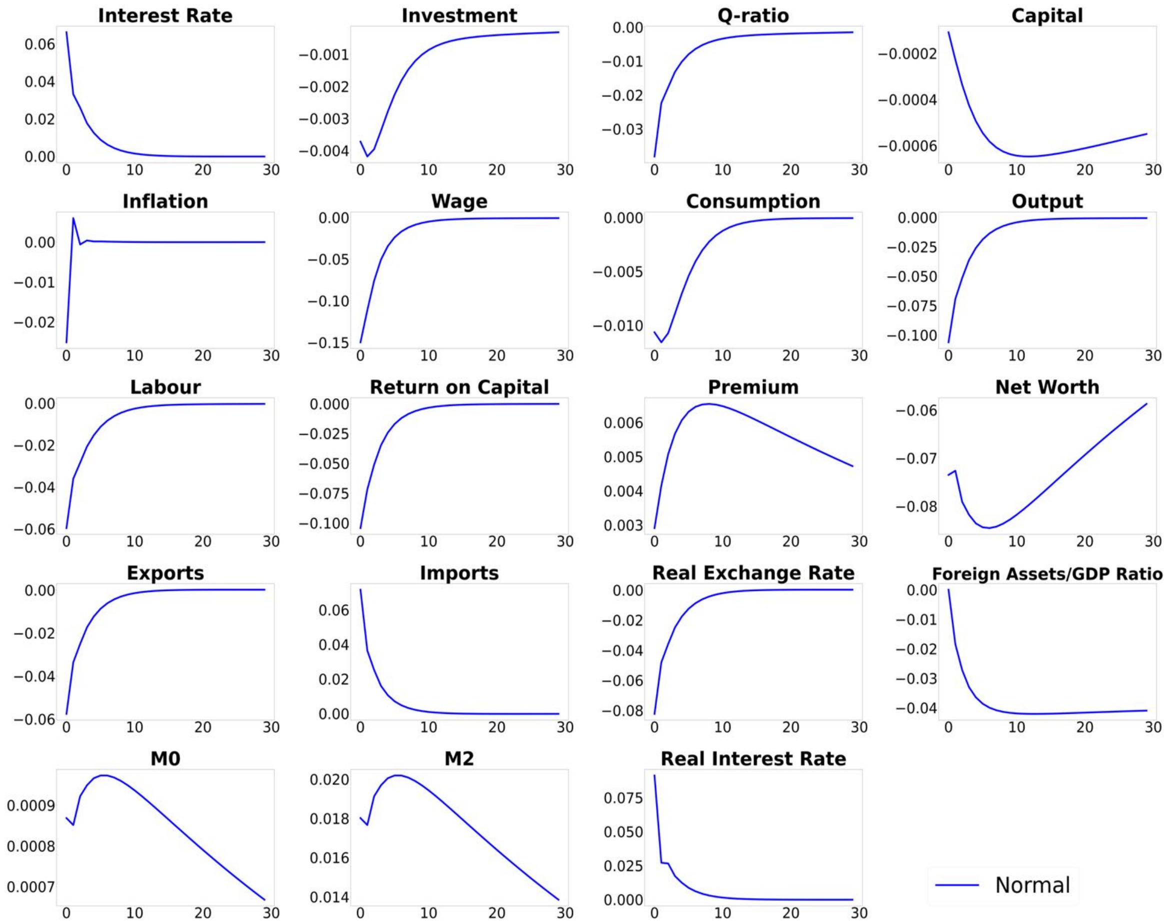

Figure 6-7: IRFs to a Taylor Rule Shock

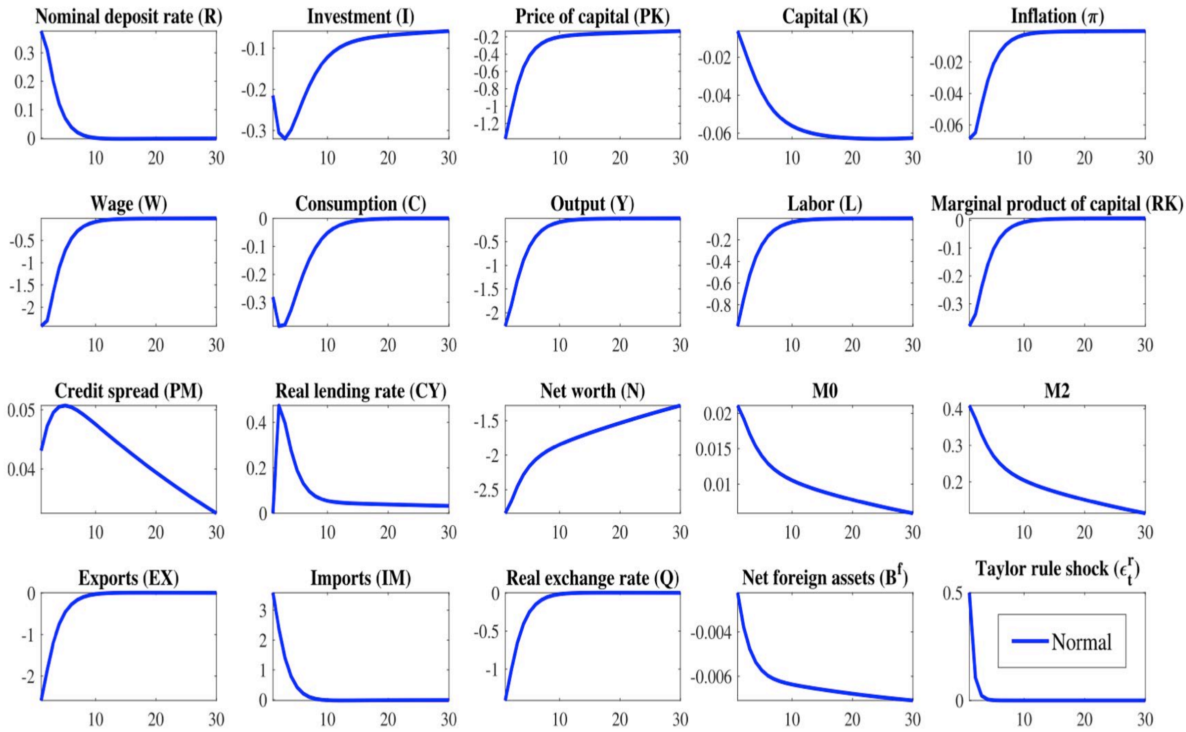

Figure 6-8: Lyu et al.' (2023) IRFs to a Taylor Rule Shock

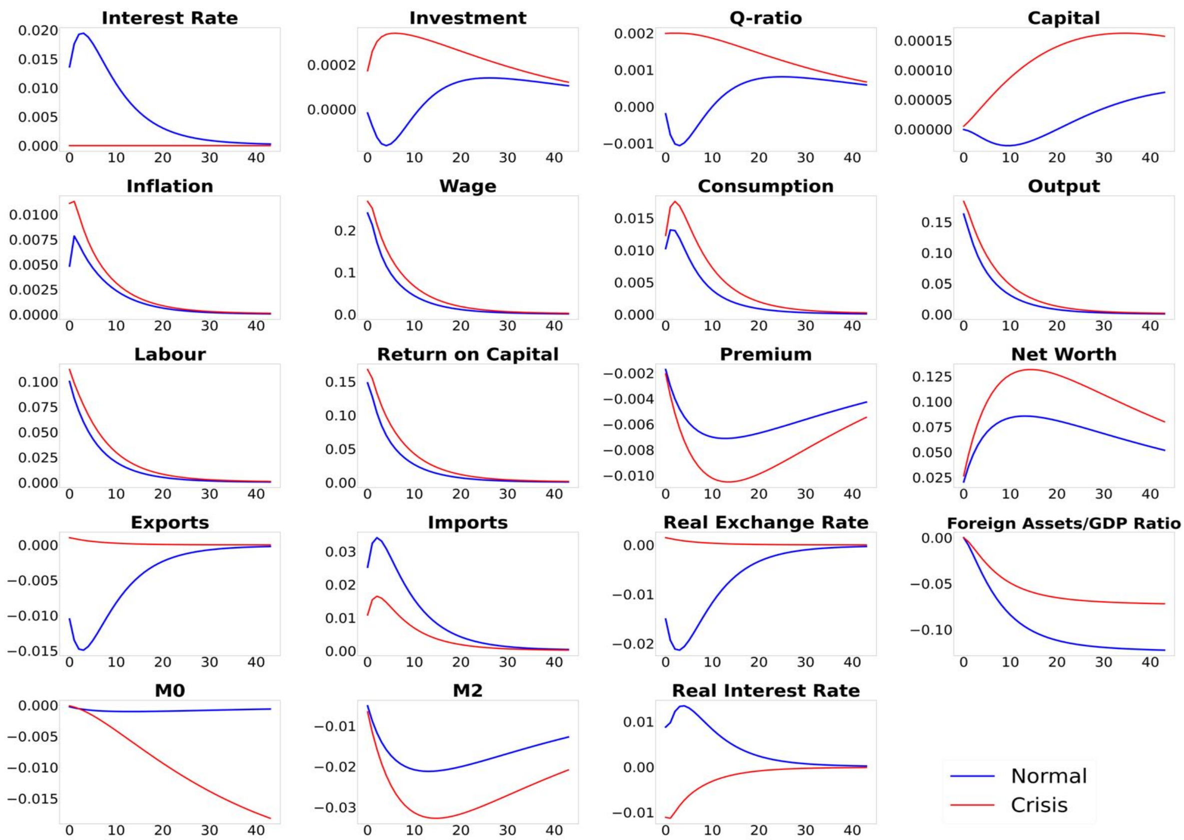

Figure 6-9: IRFs to a Government Spending Shock

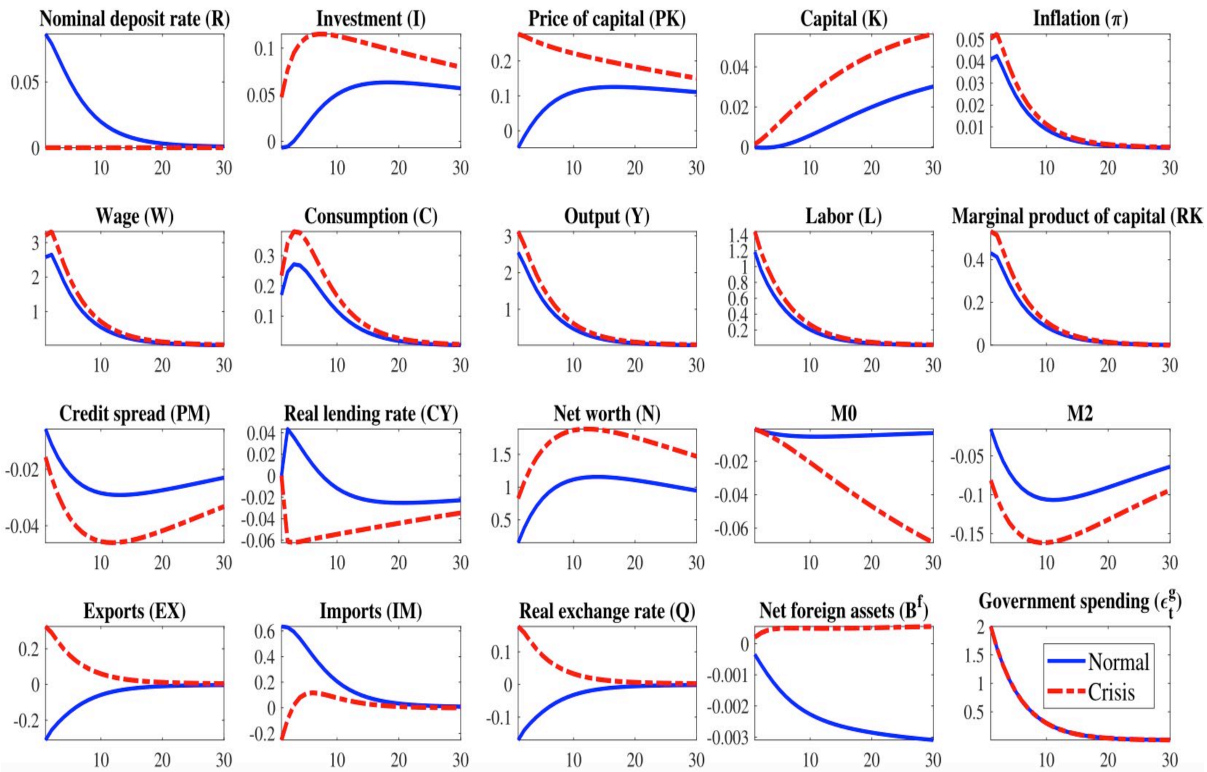

Figure 6-10: Lyu et al.' (2023) IRFs to a Government Spending Shock

External Finance Premium Shock

Figure 6-11 shows the IRFs to a temporary positive credit premium shock under both versions of the model. A higher credit premium implies that the entrepreneur faces a tighter borrowing constraint. Hence investment, capital, the price of capital (Q-ratio), and net worth fall in both regimes, and therefore output, labour hours, real wages, and inflation decrease.

In the normal regime, the nominal interest rate falls to stimulate the economy via the Taylor Rule. In the foreign market, the decrease in the real interest rate drives the real exchange rate up (pound sterling depreciates), affecting exports positively but imports negatively. Net foreign position increases because of a higher net export.

In the crisis regime, the nominal interest rate is at the zero bound, and M0 expansion stimulates the economy. Deflation leads to a rise in the real interest rate, which drives down the real exchange rate (pound sterling appreciated). Thus, export decreases. Import will increase because of the appreciation of the home currency but decrease as domestic consumption decreases; overall, import decreases slightly.

The response direction of the variables to a positive credit premium shock is highly consistent with the results of Lyu et al., see Figure 6-12.

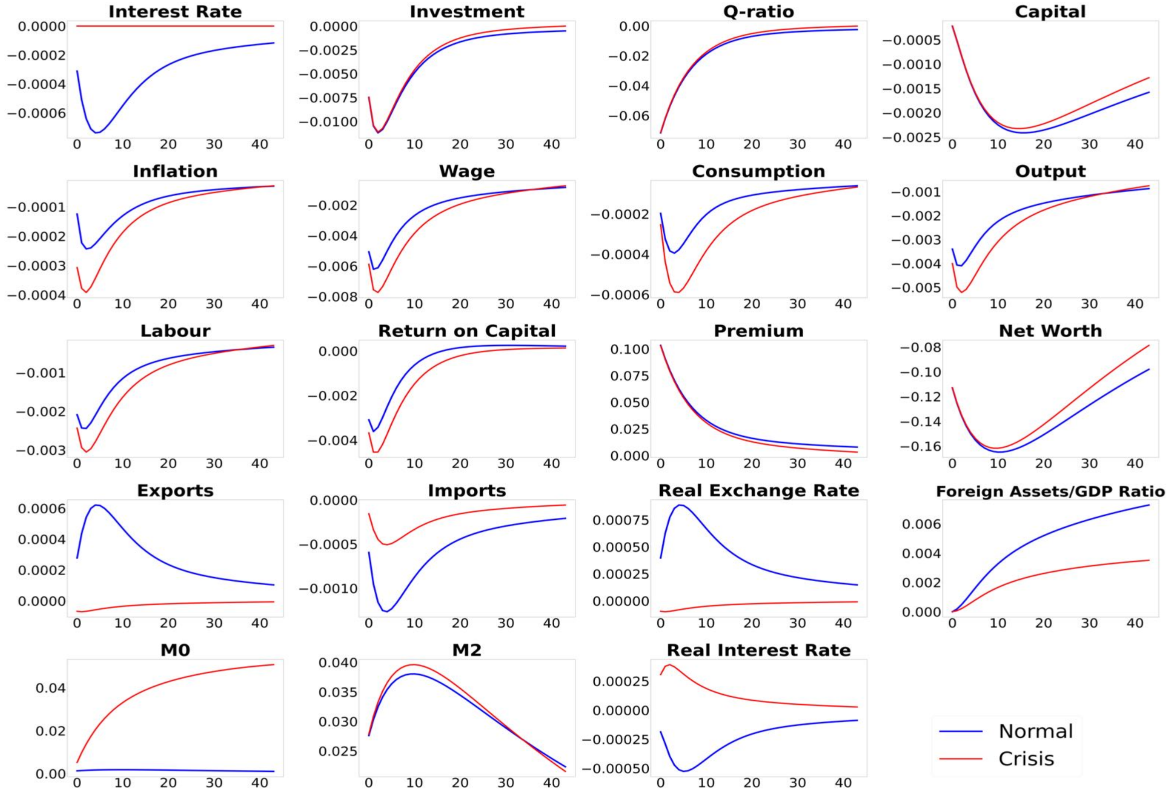

Figure 6-11: IRFs to a Premium Shock

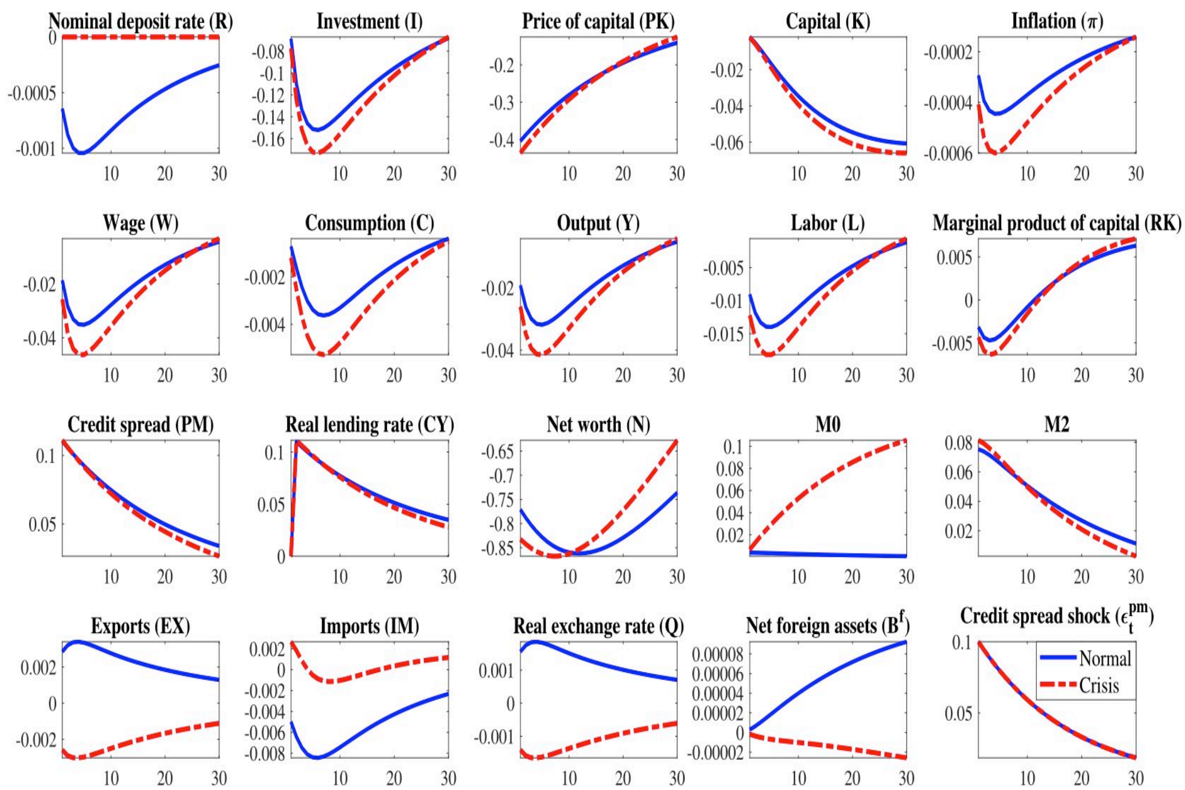

Figure 6-12: Lyu et al.' (2023) IRFs to a Premium Shock

Quantitative Easing Shock

Figure 6-13 shows the IRFs to a positive money supply shock. Generally, the responses of variables are similar under both regimes. An increase in money supply lowers the credit premium and drives up investment, capital, output and consumption. The higher output demand has a positive effect on labour demand. As a result, real wages rise as labour demand increases, leading to an increase in inflation. Under the normal regime, the nominal interest rate rises through the Taylor Rule. In the foreign market, the higher real interest rate results in a lower real exchange rate (pound sterling appreciation), as a result, exports decrease, and imports increase as domestic prices are relatively higher than foreign prices. The net foreign asset position falls due to a lower net export.

Under the crisis regime, the Taylor Rule response is suppressed, but a higher M0 reaction to the premium brings the premium down and boosts the economy. Therefore, the stimulation of M0, with or without the use of the Taylor Rule, has a positive effect on output, suggesting that monetary policy is more than just setting a target interest rate. In the foreign market, the lack of nominal interest rate response and the higher inflation mean a lower real interest rate. As a result, the real exchange rate increases (depreciation of the pound sterling), which encourages export. Import declines as the real exchange rate increases but increases as the domestic consumption demand rises; overall, import rises slightly.

The response direction of the variables to a positive QE shock is highly consistent with the findings in Lyu et al., see Figure 6-14. In addition, the findings regarding the stimulatory effect of QE on the UK economy are in line with the empirical evidence by Bridges and Thomas (2012), Falagiarda (2014), Churm et al. (2018), Kapetanios et al. (2012) and Weale and Wieladek (2014), who all find a positive effect of QE purchases by BoE on output and inflation.

6.5 Conclusion

This chapter tested and estimated the fixed price/wage duration DSGE model developed in Chapter 3 using the Indirect Inference method and found that the model fits the behaviour of the UK data for the inflation targeting era 1992-2021. Additionally, this chapter examined the dynamics of the estimated fixed duration model through the impulse responses of endogenous variables to structure shocks under the non-crisis and crisis models; the estimated model produces sensible IRFs, which are highly consistent with those of Lyu et al. (2023).

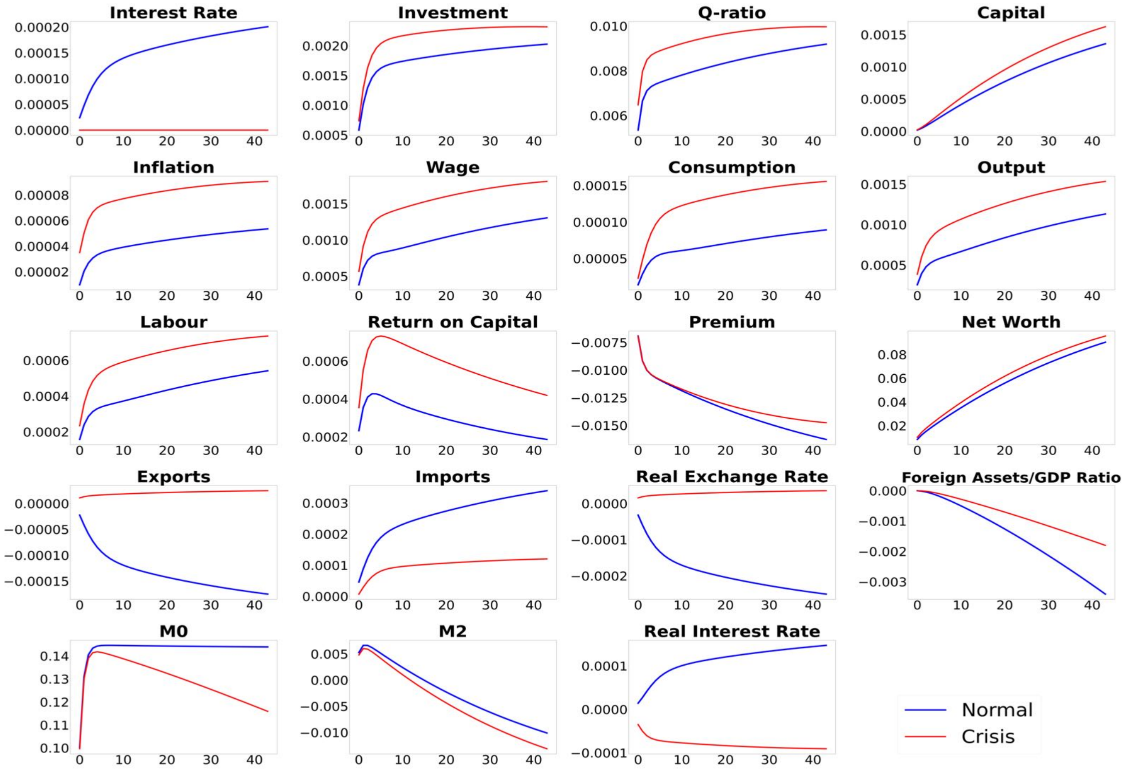

Figure 6-13: IRFs to a Quantitative Easing Shock

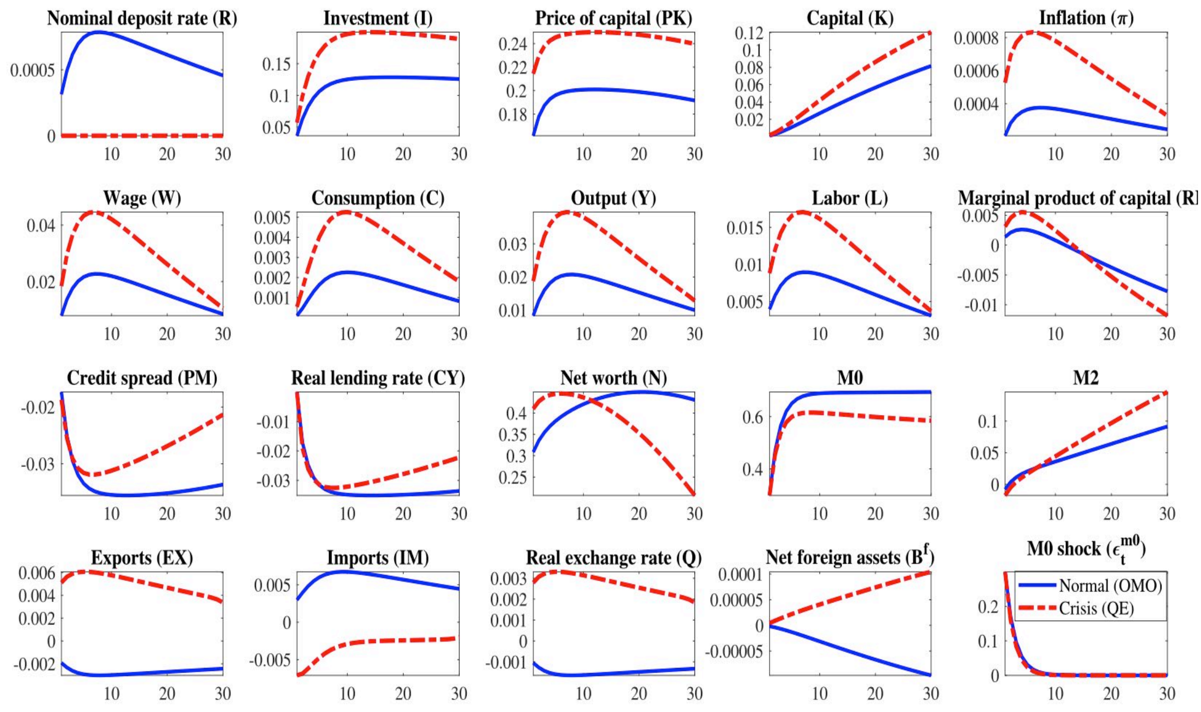

Figure 6-14: Lyu et al.' (2023) IRFs to a Quantitative Easing Shock

Chapter 7 Empirical Results of the Fixed Duration Model for the whole Sample Period 1955-2021

In this chapter, we aim to check whether there is any fixed price/wage duration set-up that can fit the behaviour of the unfiltered UK quarterly data for the whole period of 1955-2021 using the method of Indirect Inference. For the whole US post-war sample period 1947-2004, Le et al. (2011) found that the SW07 model was rejected by the behaviour of the data when using the Indirect Inference method. However, if a fraction of goods markets were assumed to be flexprice while the rest was sticky price; similarly, a fraction of labour markets set wages flexibly whilst the rest faces nominal rigidities. The authors found that the hybrid SW07 model was not rejected for the Great Moderation period 1984-2004 but was rejected for the whole period 1947-2004. Le et al. (2016) extended the hybrid DSGE model developed in Le et al. (2011) to allow for financial friction, ZLB constraint and QE, and then re-estimated it. They found that the model with these extensions can fit the US data behaviour from the Great Moderation through to 2011, a period that includes the GFC and the Great Recession. However, this fixed price/wage duration DSGE model still fails to fit the behaviour of the data from 1947-1984. The failure of the fixed duration model to pass the test for the whole post-war sample period could be attributed to the fact that wage/price-setting behaviour changes over time in response to a fluctuating macro environment. Therefore, Le et al. (2021) extended their Le et al. (2016) model to include state-dependence in price/wage contracts duration and re-estimated the model for the whole post-war period 1959-2017 and found that the model with this extension can indeed match the data behaviour well.

This chapter is inspired by Le et al.'s studies of the US. Our contribution in this chapter is that we extend the UK dataset to the mid-1950s so that this chapter covers a much longer sample period than other similar UK studies to test the fixed price/wage duration model for the period 1955-2021.

7.1 A Method for Testing and Estimating the Full Sample Period

In the UK, there was a fixed exchange rate regime (or the Bretton Woods System) from October 1949 to May 1972. The Bretton Woods System established a framework to maintain a fixed exchange rate against the US dollar. Under this regime, the BoE intervened in the currency market to keep the exchange rate close to the fixed exchange rate target to maintain economic stability.

In June 1972, the UK abandoned the fixed exchange rate regime, and sterling began to float against other currencies. The fixed exchange rate regime creates a problem for our test and estimation for the whole sample period of 1955-2021. We address this by adding an exchange rate target to the Taylor Rule equation for the period of 1955Q1-1972Q2 and turning it into a standard Taylor Rule for the period 1972Q3-2021Q1.

According to Giavazzi and Giovannini (1989), a permanently fixed exchange rate regime differs from a regime where a monetary authority pegs its currency to a numeraire, but is free to correct its exchange rate. They argue that a model in which a monetary authority manages its exchange rate (by pegging it to a numeraire currency and adjusting its peg when necessary) may be relevant to study experiences such as the Bretton Woods system.

For the pre-1972 regime, under the Bretton Wood system of fixed-but-adjustable exchange rates, the model treats as floating throughout with the version of the Taylor Rule for interest rates that has a real exchange rate targeting included. Thus, the central bank could adjust interest rates in response to the deviation of the real exchange rate to its target level. In other words, the pre-1972 model assumes under Bretton Woods that a ‘fixed’ rate is treated as adjustable whenever an adjustment is needed. It is a fixed exchange rate regime but adjustable. Clarida et al. (1998) added a real exchange rate and its target in the Taylor Rules for Italy, France and the UK for the ERM period. Because both the ERM and Bretton Woods systems aimed to maintain a fixed-but-adjustable exchange rate regime, we follow them in adding an exchange rate target to the Taylor Rule, see also Engel and West (2006), Wang and Wu (2009) for examples.

The Taylor Rule equation with an exchange rate target added is:

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_p \pi_t + r_y y_t\right) + r_{\Delta y}(y_t - y_{t-1}) + r_q(q_t - \bar{q}) + e_t^r \qquad (7.1)$$

where q_t is the real exchange rate, $\bar{q}$ is the real exchange rate target, r_q is the Taylor Rule response to the difference between q_t and $\bar{q}$. We use this extended Taylor Rule equation in our model for the period of 1955Q1-1972Q2.

We use the standard Taylor Rule for the sample period of 1972Q3-2021Q1:

$$r_t = \rho r_{t-1} + (1-\rho)\left(r_p \pi_t + r_y y_t\right) + r_{\Delta y}(y_t - y_{t-1}) + e_t^r \qquad (7.2)$$

We employ the Indirect Inference method to test and estimate the UK fixed price/wage duration DSGE model for the whole sample period 1955-2021: using the Taylor Rule with the exchange rate target added for the period 1955-1972 and the standard Taylor Rule afterwards. More specifically:

- Firstly, from the period 1955 to 1972, we use the Taylor Rule equation with the exchange rate target added when calculating shocks and residuals (for simplicity, we refer to these shocks as 'pre-1972 shocks'); From 1972 to 2021, we include the standard Taylor rule equation in our model to calculate post-1972 shocks and residuals.
- Secondly, when simulating the DSGE model for the pre-1972 period, we bootstrap the pre-1972 shocks and use the model including the extended Taylor Rule equation; when simulating the model for the post-1972 period, we use the post-1972 shocks and the model containing the standard Taylor Rule equation. To generate 1000 bootstrapped simulations, we repeat the second step 1000 times.
- Thirdly, we compute the Wald statistic to measure the difference between the simulated and actual data and convert the Wald statistic to the t-statistic. If the t-statistic is less than 1.645, it means that the model can fit the behaviour of the observed data over the period 1955-2021, and vice versa.

The Indirect Inference test and estimation process for the whole sample period is shown in Figure 7-1.

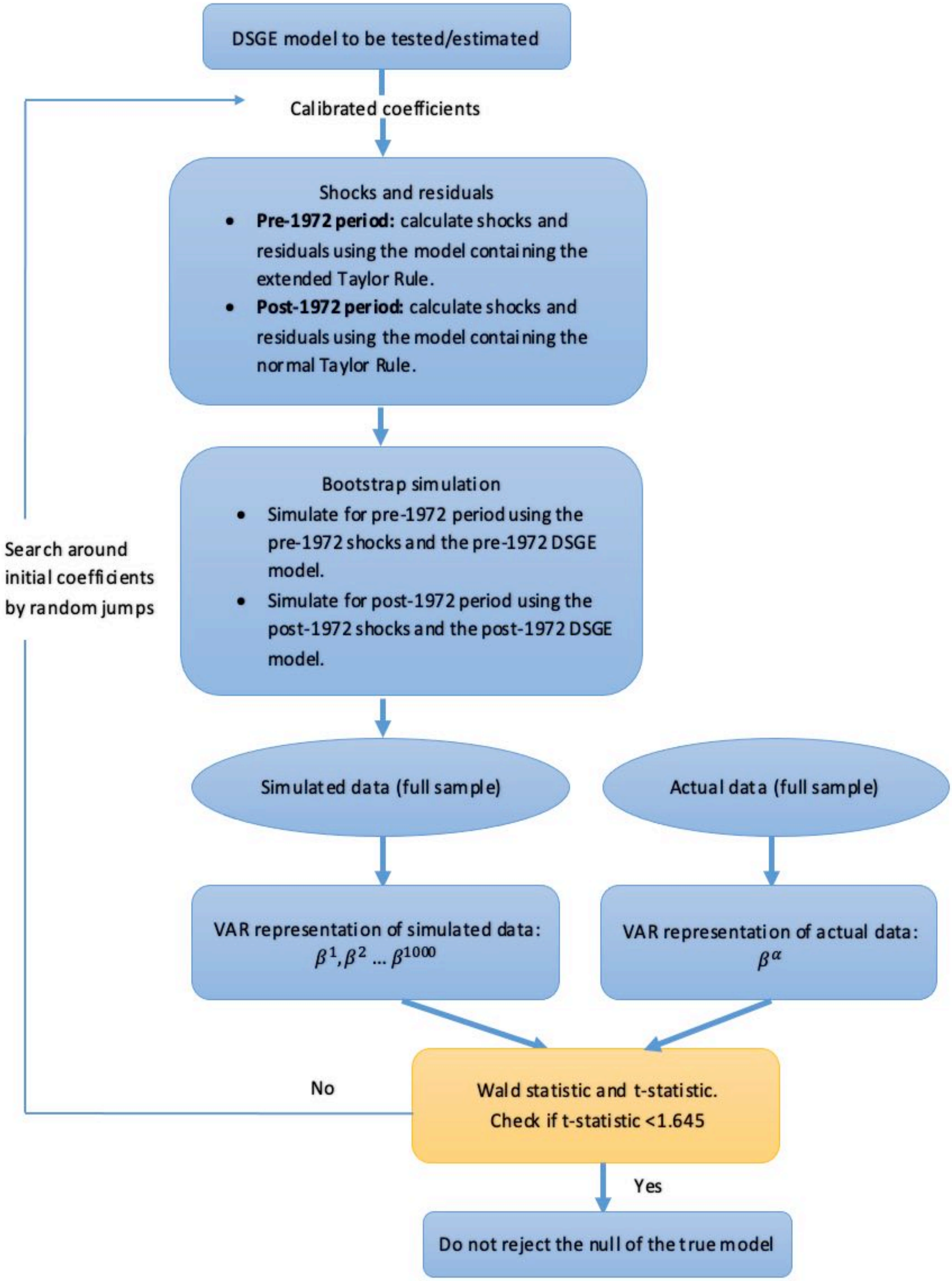

Figure 7-1: The Procedure for the Indirect Inference Test and Estimation for the Full Sample Period

7.2 Indirect Inference Results

In this section, firstly, using the coefficients calibrated from previous studies (see Chapter 4), we test the model for the whole sample period using the Indirect Inference method. Secondly, using these calibrated coefficients as starting coefficients, we attempt to search a set of estimates that allow the model to fit the behaviour of the actual data over the whole sample period, by performing the Simulated Annealing algorithm in which a search takes place over a wide range around the calibrated parameters by random jumps to search for the optimal choice of parameters that minimises the Wald statistic (or maximises the p-value).

We represent our results of the calibrated model for the sample period 1995-2021 in Table 7-1. The calibrated model did not pass the Wald test, as indicated by a p-value of 0.001. This implies that the calibrated model failed to explain the data behaviour over the whole sample period. Additionally, using the Indirect Inference estimation method and calibration coefficients as starting coefficients, we attempted to estimate the model but found that the fixed-duration model did not fit the data behaviour for the whole sample period. This may be because there were more distinct economic environments across the full sample period, such as the stagflation crisis in the 1970s and the Great Moderation; thus, wage/price behaviour changed in response to fluctuations in the macro environment. Our results are consistent with the findings of Le et al. (2011, 2016).

7.3 Impulse Responses from Calibrated Model

As the model does not fit for the long sample period, we use the calibration coefficients to assess the validity of the pre-1972 and post-1972 models. Overall, all the dynamic movements to the structure shocks are similar in the pre-1972 and post-1972 models. Figures 7-2 to 7-7 show IRFs to a positive monetary policy shock, a fiscal expansion shock, a positive productivity shock, a QE shock, a positive export shock, and a positive credit premium shock, respectively[11]. As can be seen from Figures 7-2 to 7-7, the responses of the pre-1971 model are similar to those of the post-1972 model, which are highly consistent with those of Lyu et al. (2023) and Wang (2020), particularly for the key variables- output, inflation and interest rate. Thus, we do not

[11] In the figures, wage represents the real consumer wage and inflation is CPI inflation.

repeat the discussion in this section as we discussed most of them in Chapter 5.

7.4 Conclusion

In this chapter, we examined the fixed price/wage duration model using Indirect Inference over a much longer sample period than in Chapter 6, from 1955 to 2021. In line with the findings in Le et al. (2011, 2016), we found that the fixed-duration model does not fit the behaviour of the UK data over this long sample period.

Table 7-1: Results Based on Calibrations

		Calibration (1955-2021)[12]
Elasticity of capital adjustment	φ	5.7400
Intertemporal elasticity of substitution	σ_c	1.3900
External habit formation	h	0.5800
Probability of not changing wages	ξ_w	0.7180
Elasticity of labour supply	σ_l	2.3300
Probability of not changing prices	ξ_p	0.7110
Wage indexation	ι_w	0.4800
Price indexation	l_p	0.2000
Elasticity of capital utilisation	ψ	0.5400
1+share of fixed costs in production	ϕ	1.0800
Taylor Rule response to inflation	r_p	2.4000
Interest rate smoothing	ρ	0.7300
Taylor Rule response to output	r_y	0.0190
Taylor Rule response to change in output	$r_{\Delta y}$	0.2100
Share of capital in production	α	0.3000
Proportion of sticky wages	ω^w	0.4000
Proportion of sticky prices	ω^p	0.1000
Elasticity of the premium with respect to leverage	χ	0.0400
Money response to premium	ϑ_2	0.0590
Elasticity of the premium to M0	ϑ	0.0580
Money response to credit growth	ϑ_1	0.0520
Taylor Rule response to deviation from the real exchange rate target	r_q	0.1
Wald (Y, π, R)		81.2651
Transformed t-statistic		4.7949
P-value		0.0010

[12] Please see Section 4.2 Calibration. Taylor Rule response to deviation from the real exchange rate target is calibrated to be 0.1, following Engel and West (2006) and Kempa (2018).

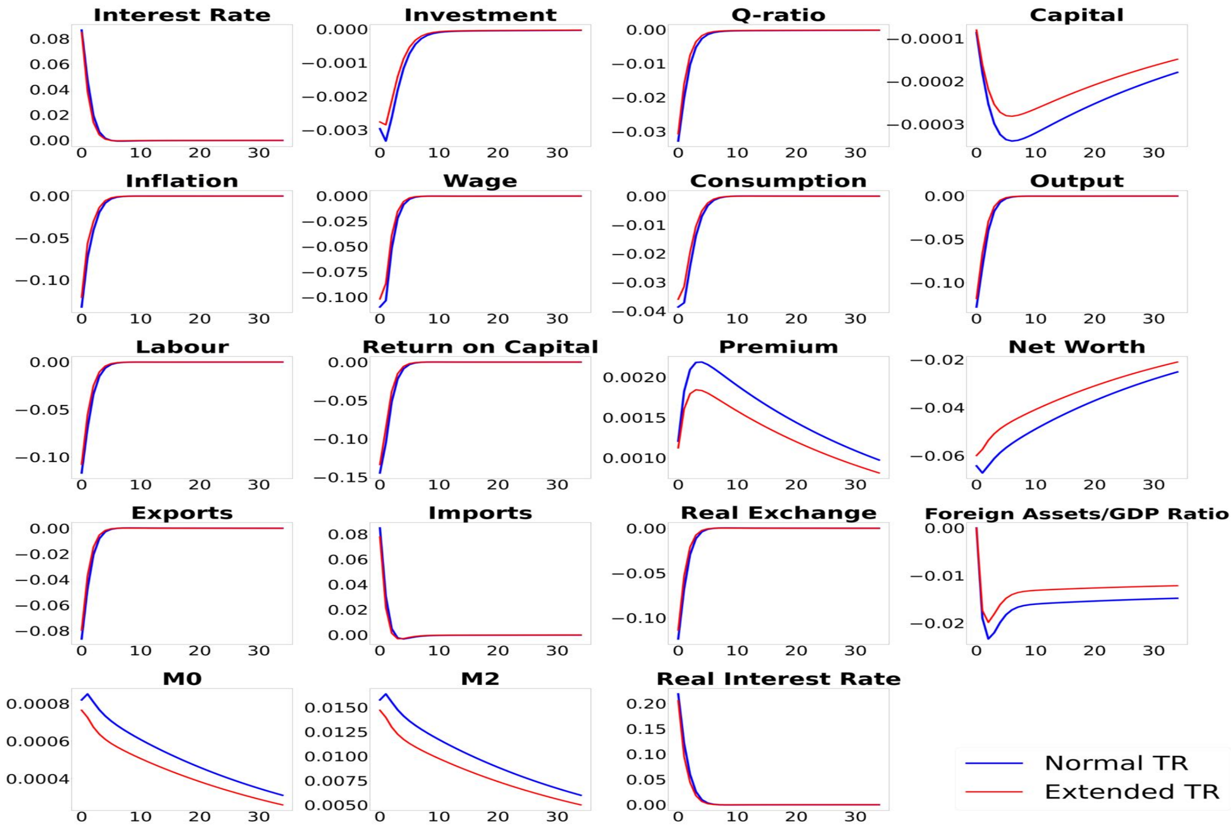

Figure 7-2: IRFs to a Taylor Rule Shock

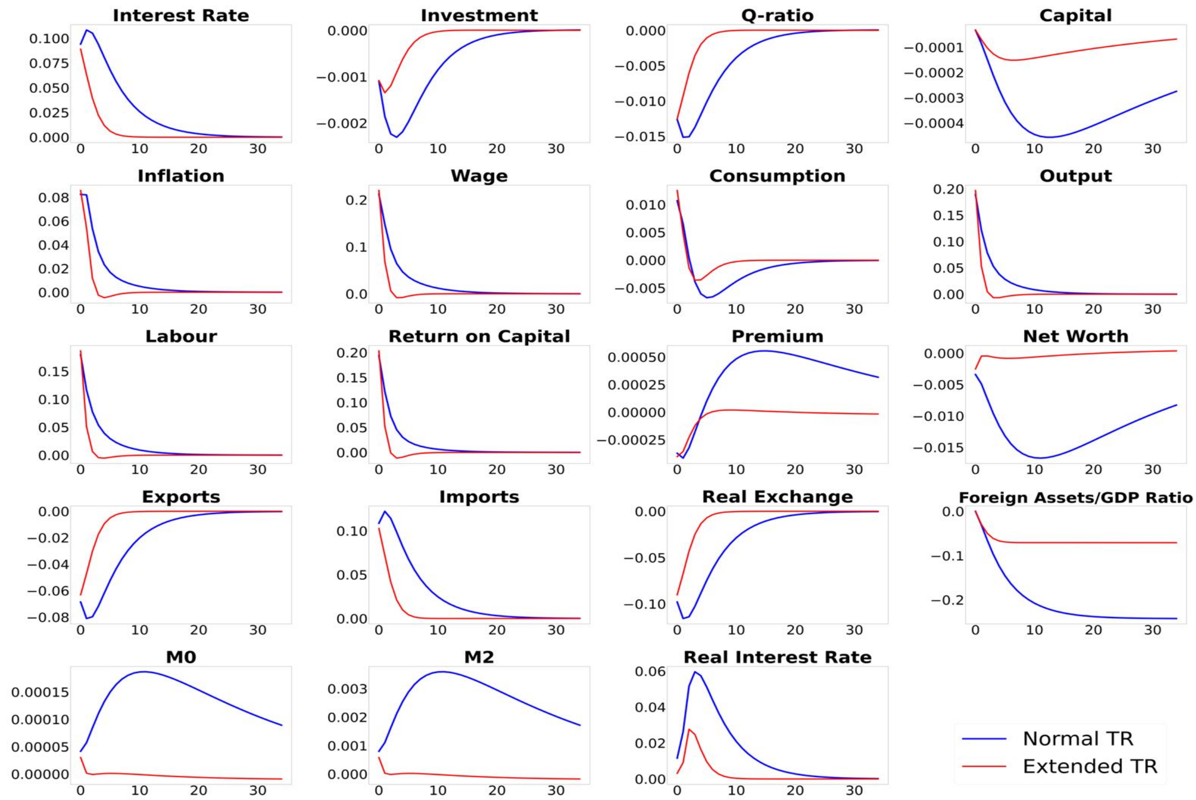

Figure 7-3: IRFs to a Government Spending Shock

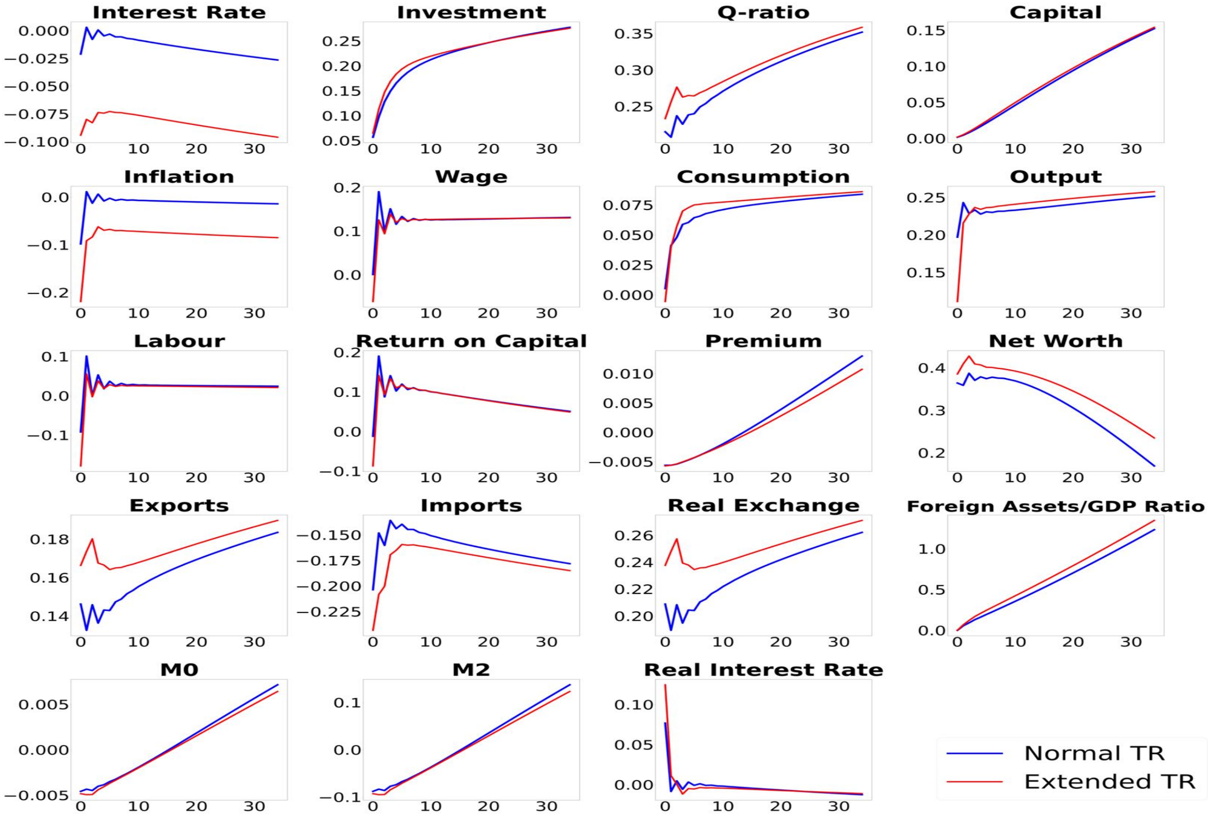

Figure 7-4: IRFs to a Productivity Shock

Note: The non-smooth IRFs for productivity are due to the negative AR (1) coefficient. Thus, we see it fluctuates before settling down.

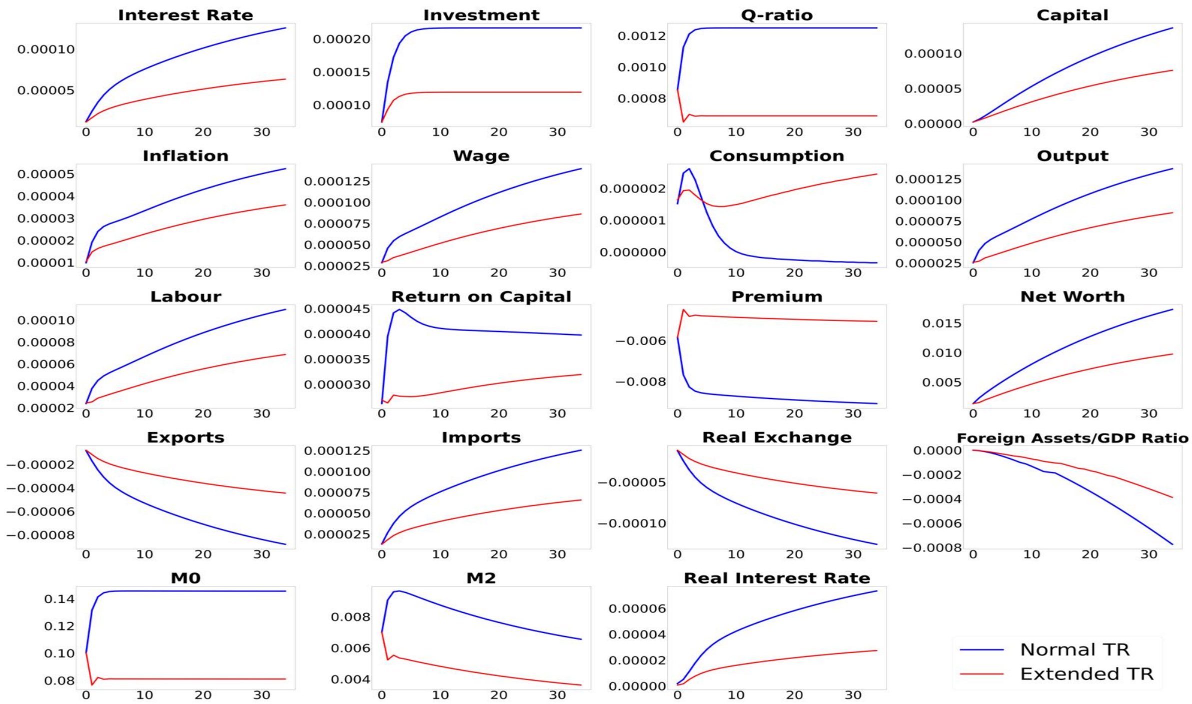

Figure 7-5: IRFs to a M0 Shock

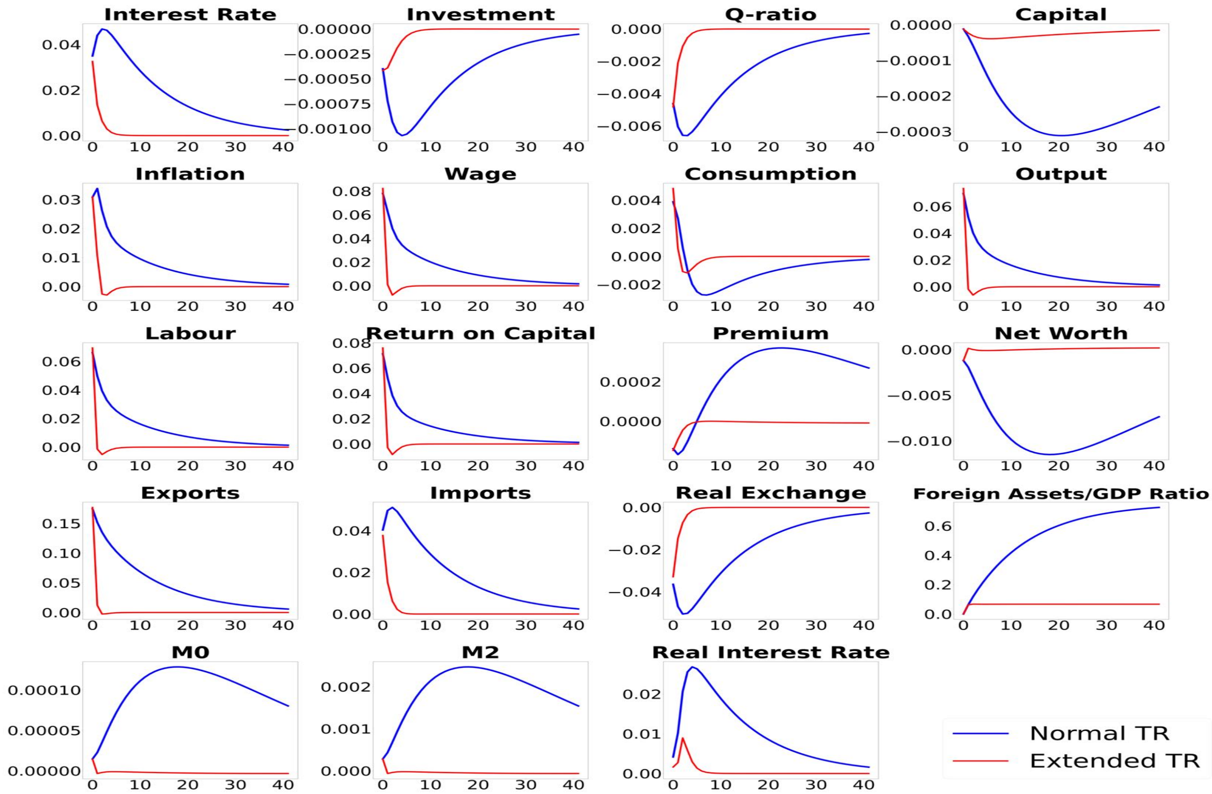

Figure 7-6: IRFs to an Export Demand Shock

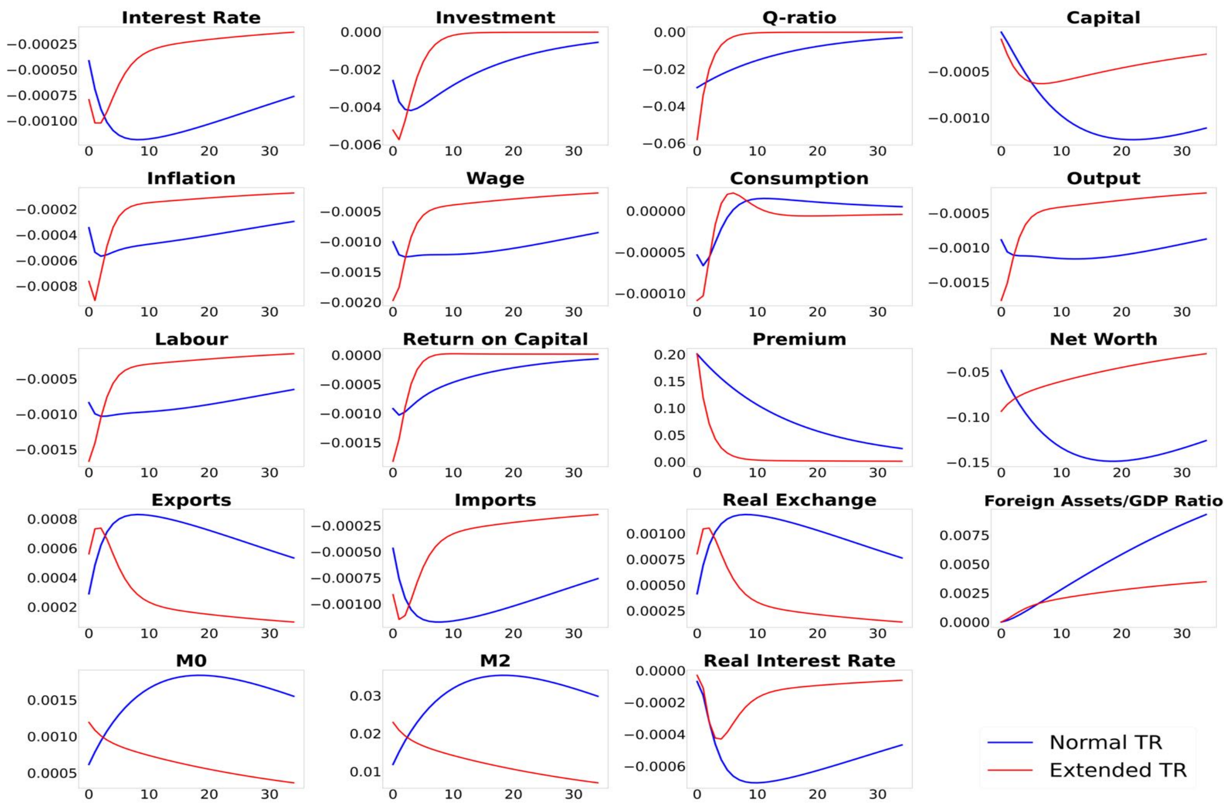

Figure 7-7: IRFs to a Credit Premium Shock

Chapter 8 State-dependent Model

In chapter 3, we built a small open economy DSGE model in which the shares of the sticky and flexprice sectors are fixed, which has several features compared to the standard DSGE model. Firstly, it merges the NK and NC models into a hybrid model. The model assumes that a fraction of goods markets set prices flexibly, while the rest has sticky prices; the labour market is similar. Secondly, to accommodate periods of the GFC, the model is extended by incorporating the financial frictions of the BGG model, allowing for cheap money collateral through QE, and allowing for the ZLB constraint.

Motivated by the micro- and macro-level evidence of state-dependent price/wage duration, this chapter will extend the UK fixed duration model by adding a state-dependent price and wage framework. Therefore, the state-dependent macroeconomic DSGE model will incorporate state-dependence with many of real rigidities in the SW(07), the financial frictions in the BGG model, and the ability to handle the ZLB in Le et al. (2016). The contribution of this chapter is twofold:

1) As macro-level evidence of state-dependence in the UK is surprisingly scarce, in this chapter, we aim to fill this gap by investigating whether there is macro-level evidence to corroborate micro-level evidence of state-dependence.
2) To examine improvements to the UK fixed duration model due to the state-contingency.

In what follows, we first set up a state-dependent macroeconomic model in which both wage and price contracts change endogenously with the state of the economy rather than merely being time-dependent. Specifically, price/wage duration depends on the variance of lagged inflation according to a linear parameter that we estimate using the Indirect Inference method, and inflation in turn depends on durations. Secondly, we apply this state-dependent model to the UK data for the period of 1955-2021 to investigate the state-dependence at the macro level and to examine improvements to the fixed duration model due to the state-contingency. Thirdly, we describe the properties of this state-dependent model. Finally, we conclude this chapter.

8.1 State-dependent Model Setup

In this section, we build up an open economy state-dependent DSGE model by extending the UK fixed price/wage contract duration model to include a state-dependent duration of price/wage setting framework. The fixed duration model is based on the well-known SW(07) model. Following Le et al. (2011, 2016), we extended the SW(07) model to include the hybrid wage and price setting, incorporate the financial frictions of the BGG model, and allow for the ZLB and QE. This extended SW(07) model (or the fixed price/wage duration model) has been derived in Chapter 3.

The fixed price/wage duration hybrid model assumes that a fixed fraction of goods markets is imperfectly competitive, and firms face nominal rigidities, while the rest is perfectly competitive with flexible pricing. Similarly, a fixed fraction of labour markets is imperfectly competitive, and labour unions face nominal rigidities, while the rest of labour markets are perfectly competitive, so labour unions set wages flexibly. To embed state-dependent variation, we assume that the fraction of firms with flexible prices and the fraction of unions with flexible wages are state-dependent rather than fixed and are based on an increasing function of past inflation.

We now define the sticky price/wage sector (or NK sector) as the long duration sector because prices/wages are sticky for more than one quarter, and the flexible price/wage sector as the short duration sector because prices/ wages are constantly changing every quarter. In the fixed price/wage duration model, the long duration sectors have fixed weights, i.e., ω^{p} and ω^{w} are fixed. As a result, the short duration sectors also have fixed weights, $(1-\omega^{p})$ and $(1-\omega^{w})$ are fixed. Furthermore, firms/labour unions change their prices/wages according to the fixed Calvo probabilities. In this chapter, we relax the assumption of fixed duration and assume that the structure of the price/wage duration is state-dependent, which implies that firms and labour unions adjust their prices and wages more frequently in the face of aggregate shocks and therefore shift from the long duration to the short duration in this state-dependent model.

We assume that firms' decisions to change their prices depend on the size of the shock. If the shock is smaller than a particular value, they will choose to stabilise prices in order to ensure their customers against uncertainty. However, if the shock is greater than the critical value, the cost of providing this insurance to customers is greater than the expected benefit it provides; thus, they will update their prices and reset them optimally to respond to the shock. We assume a critical shock size at which the cost of changing prices is just great than the gain from providing consumer insurance. Then firms will only adjust prices when the shock is greater than the critical shock size because if the shock is below the critical value, the cost of changing prices outweighs the benefit. Wage changes are similar.

We assume that the variance of the idiosyncratic cost-shock distribution used by price setters is state-dependent; more specifically, we assume that it is associated with the size of recent inflation shocks, represented by Π and measured by a moving average of inflation. These inflation shocks to general prices generate price shocks in specific markets; if general prices change significantly, then the demand and supply of particular products must also fluctuate. Therefore, recent inflation shocks to the economy, Π, will affect the variance of the cost-shock distribution of all sectors. Then, changes in the variance of the idiosyncratic distribution will affect the Calvo probability of not changing prices. For example, if there is an increase in recent inflation, it will cause a higher variance of the idiosyncratic cost-shock distribution. The higher variance indicates that the critical shock level occurs at a lower percentile of the distribution, as shown in Figure 8-1, and this lower percentile means a lower Calvo probability of not changing prices. Hence, the recent inflation affects the variance of idiosyncratic shock distribution, thereby changing the Calvo probabilities of prices and wages. Wage changes are similar.

If recent inflation rises, the Calvo probability of not changing prices consequently decreases, causing more sectors to become flexprice, which may decrease the Calvo parameters in the remaining sectors (i.e., sticky-price sectors). We describe this as a ‘reduction effect’ on the Calvo parameters in the remaining sectors. On the other hand, there is an ‘abandonment effect’, as the sectors closer to the short duration sector would migrate to it, leaving those sectors with higher Calvo parameters in the sticky sector. This abandonment effect is contrary to the reduction effect. Therefore, the Calvo parameters for the NK sectors may increase, decrease, or remain the same. We estimate the Calvo parameters and other model parameters in the usual way using the Indirect Inference method but allowing for this net response to Π.

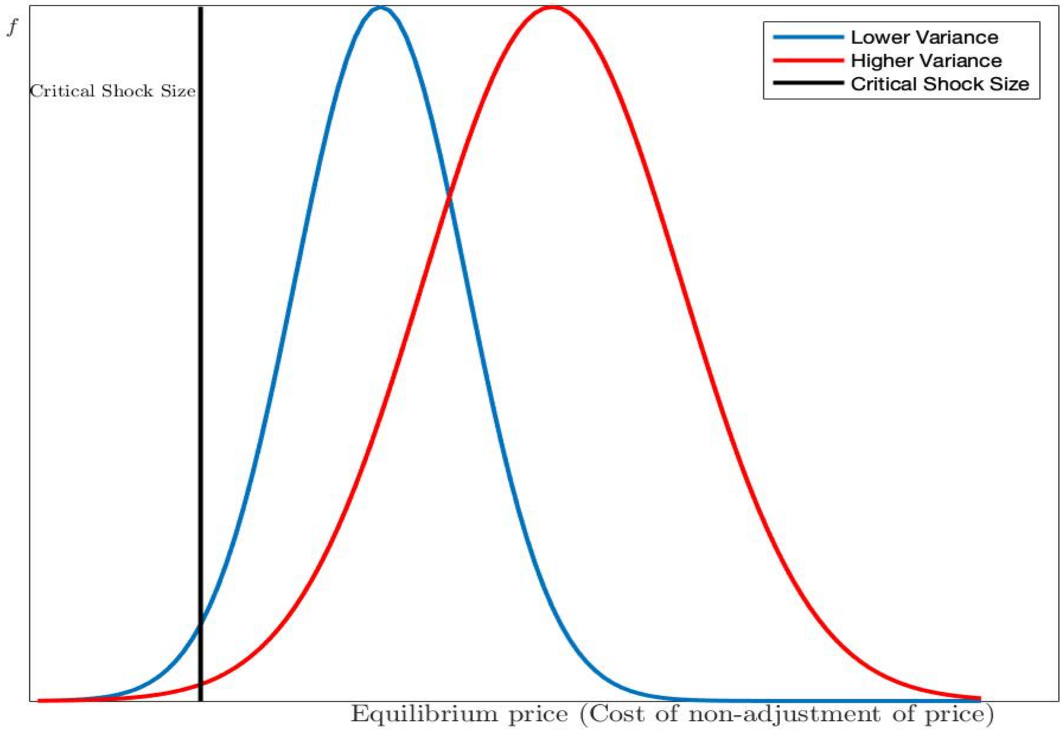

Figure 8-1: Distribution of Idiosyncratic Shocks

The price/wage parameters of the model are now changing continuously in response to historical shocks, implying that this state-dependent model is nonlinear. We use the function proposed by Le et al. (2021) to relate the price/wage parameters to the variance of past inflation:

$$\omega^{i} = \exp\left(-\vartheta^{i}\Pi\right)$$

where $i = p, w$; Π is the square of the moving average of inflation over the past four years; ω^{p} and ω^{w} are proportions of sticky prices and wages, respectively; and ϑ^{p} and ϑ^{w} are parameter responses of NK weights to the variance of inflation for prices and wages, respectively.

ϑ^{p} and ϑ^{w} are determined empirically through the Indirect Inference estimation. The weights on the long duration (or NK) sectors, ω^{i}, are calculated according to the function. This state-dependent price/wage setting is added to the hybrid DSGE model. Therefore, we have a nonlinear DSGE model with shifting-weights.

8.2 Empirical Results

We employ the Indirect Inference method to estimate the state-dependent DSGE model on unfiltered quarterly UK data for the period 1955Q1-2021Q1. The UK adopted a fixed exchange rate regime (or the Bretton Woods System) between October 1949 and May 1972, and sterling began floating against other currencies from June 1972. Therefore, for the period 1955Q1-1972Q2, we estimate the pre-1972 state-dependent DSGE model, which adds an exchange rate target in the Taylor Rule equation; for the period 1972Q3-2021Q1, we then switch to estimating the post-1972 state-dependent model, which includes the standard Taylor Rule.

In Table 8-1, we report the parameter estimates of the state-dependent DSGE model. The estimated state-dependent DSGE model fits the behaviour of the UK data over the period 1955-2021 well, with a p-value of 0.087. However, in Chapter 7 we find that the fixed duration model does not fit this long sample period. This suggests that the state-dependent price/wage framework improves the fit of the DSGE model to macroeconomic data.

Figure 8-2 shows actual inflation data and the square of MA inflation. Figure 8-3 displays the state-dependent NK weights. It shows how the state of the economy (inflation) affects the weights of NK prices and wages. The NK weights on prices and wages fell in the 1970s due to higher inflation, then rose to near one during the Great Moderation.

Table 8-2 shows the VECM parameter estimates, data variance estimates and the model's 95% bounds. Only one of the nine VAR coefficients lies outside its 95% confidence interval. The variance of interest rate data and the variance of output are slightly below their lower bounds.

Table 8-3 reports estimated persistence parameters. Figures 8-4 and 8-5 show the shock histories and residuals calculated from the estimated model for the whole sample period, respectively. The shocks fluctuate around zero, and the residuals are the accumulation of shocks over time.

Comparing Estimates to the Literature

As for the estimation results compared to the literature. In the estimates of household utility function, the elasticity of labour supply is 2.826, which is very similar to the value of 2.865 in the estimated state-dependent DSGE model for the US economy by Le et al. (2021). The intertemporal elasticity of substitution is 1.641, slightly lower than the estimate of 1.700 by Le et al. and somewhat higher than the estimate of 1.565 by Lyu et al. (2023) for the UK economy.

Concerning the estimated parameters of monetary response and financial frictions. The estimates of money response to premium and the elasticity of premium to M0 are 0.070 and 0.0413, respectively. These numbers are not far from those estimated by Lyu et al. (2023) and Le et al. (2016). The money response to credit growth is estimated to be 0.069, about 33% larger than the figures in Lyu et al. (2023) and Le et al. (2021). The elasticity of the premium with respect to leverage is 0.105, which is in line with the mean estimate of 0.105 indicated by De Graeve (2008) based on their posterior sample, and it is also close to the estimate of 0.093 by Fernández and Gulan (2015). However, it is much larger than the estimates of Lyu et al. (2023) and Le et al. (2021), whose findings suggest a lower value of around 0.03.

Turning to the estimates of the Taylor Rule parameters, the estimates of interest rate smoothing and the Taylor Rule response to output are 0.577 and 0.017, respectively. These estimates are close to those of Le et al. (2011). The response of monetary policy to change in output is 0.046, higher than the estimate of 0.025 by Lyu et al. (2023) and 0.019 by Le et al. (2019), but much lower than the estimate of 0.200 in Wang (2020) and 0.242 in Le et al. (2011). The Taylor Rule response to inflation is 3.188, similar to the estimate of 3.49 by Benchimol and Fourçans (2012) for the Eurozone and the estimate of around 3 by Coibio and Gorodnichenko (2011). The Taylor Rule response to the real exchange rate is 0.045, lower than the figure of 0.1 in Engel and West (2006) and Kempa (2018). Overall, the estimates of monetary policy parameters are not far from those in the literature.

As regards the estimates of the Phillips curve and real wage equation, the probabilities of not changing wages and prices are 0.610 and 0.546, respectively. They are not far from the values of 0.635 and 0.746 by Le et al. (2021) as well as 0.670 and

0.596 by Le et al. (2011). The wage indexation is 0.397, similar to the estimate by Lyu et al. (2023). For the price indexation, similar to Cogley and Sbordone (2008) as well as Rabanal and Rubio-Ramírez (2005), we set a search range from 0 and 1. This parameter is estimated to be 0.055, lower than the value of 0.107 by Le et al. (2021). However, Cogley and Sbordone (2005) show that around 78% of estimates of this parameter lie on the lower bound of zero, and 90% are less than 0.15. Del Negro and Schorfheide (2008) also find that it is concentrated around zero, but they require highly auto-correlated mark-up shocks to get this result.

Finally, for the other parameters, the estimate of the elasticity of capital adjustment is 4.487, which is lower than the estimate of 5.835 in Meenagh et al. (2019) as well as 6.881 in Le et al. (2021), but higher than 2.375 in Gerlter et al. (2008). For the elasticity of capital utilisation cost, same as in Punzo and Rivolta (2022), a loose search range is chosen between 0 (utilisation can vary without cost) and 1 (utilisation never changes). The estimate of 0.948 for this parameter is about 12% higher than the estimate by Aminu (2018) for the UK economy and about 17% higher than the value in Casares et al. (2014) for the US economy. It is also larger than most other studies, suggesting a high cost to change capital utilisation. The estimate of 1 plus the share of fixed costs in production is 1.804, not far from the estimate of 1.761 by Le et al. (2016). The share of capital in production is estimated at 0.576, which is higher compared to the figure of 0.385 in Videnova (2016) for the UK economy, as well as in studies for advanced economies, for example, 0.45 in Zhang and Yang (2021) and Freystätter (2011), 0.44 in Ahmad et al. (2013), 3.741 in Christensen and Dib (2008), and 0.4 in Guerrieri et al. (2010). However, it is lower than the estimate of 0.6134 in Soltani et al. (2021) and close to 0.559 in Le et al. (2022). This high value of the capital share in production is the one that best passes the test, meaning that capital plays a crucial role in the production process over the whole sample period in the model. The external habit formation is estimated to be 0.702, similar to the estimates by Lyu et al. (2023) and Le et al. (2016, 2021).

Overall, the estimates of the state-dependent model for the entire sample period are similar to those of the fixed-duration model for 1992-2021 (see Table 8-1). While a few estimates show significant differences compared to the fixed-duration model for the short sample period, these estimates are still consistent with the other literature mentioned above.

Table 8-1: Parameter Estimates (1955Q1-2021Q1)

		State-dependent (1955-2021)	**Fixed-duration (1992-2021)**
Elasticity of capital adjustment	φ	4.4871	5.7125
Intertemporal elasticity of substitution	σ_c	1.6408	1.5099
External habit formation	h	0.7016	0.8665
Probability of not changing wages	ξ_w	0.6101	0.6483
Elasticity of labour supply	σ_l	2.8257	2.6775
Probability of not changing prices	ξ_p	0.5462	0.9416
Wage indexation	ι_w	0.3965	0.4275
Price indexation	l_p	0.0554	0.1430
Elasticity of capital utilisation	ψ	0.9485	0.4400
1+share of fixed costs in production	ϕ	1.8037	1.2281
Taylor Rule response to inflation	r_p	3.1879	3.4685
Interest rate smoothing	ρ	0.5769	0.6723
Taylor Rule response to output	r_y	0.0172	0.0142
Taylor Rule response to change in output	$r_{\Delta y}$	0.0463	0.0453
Share of capital in production	α	0.5765	0.3677
Elasticity of the premium with respect to leverage	χ	0.1048	0.0841
Money response to premium	ϑ_2	0.0700	0.0508
Elasticity of the premium to M0	ϑ	0.0413	0.0664
Money response to credit growth	ϑ_1	0.0692	0.0482
Taylor Rule response to deviation from the real exchange rate target	r_q	0.0445	
Wald (Y, π, R)		**18.3008**	22.5073
P-value		**0.087**	0.0760
Transformed t-statistic		**1.0455**	1.2720

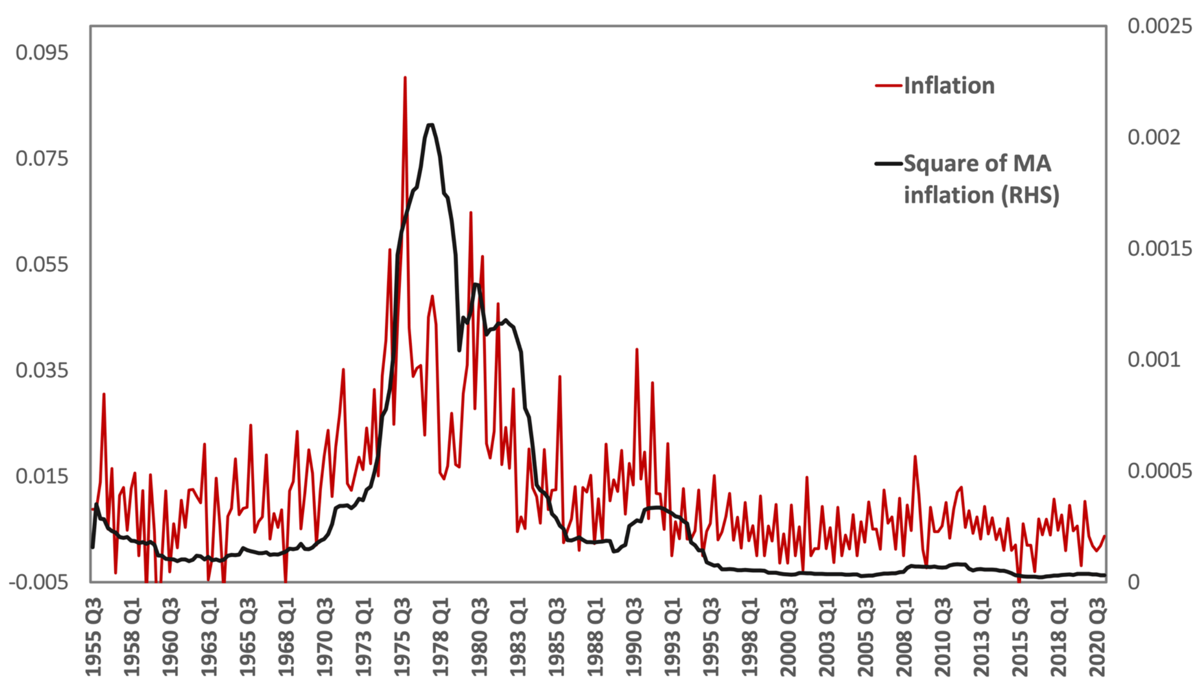

Figure 8-2: Inflation and the Square of MA Inflation

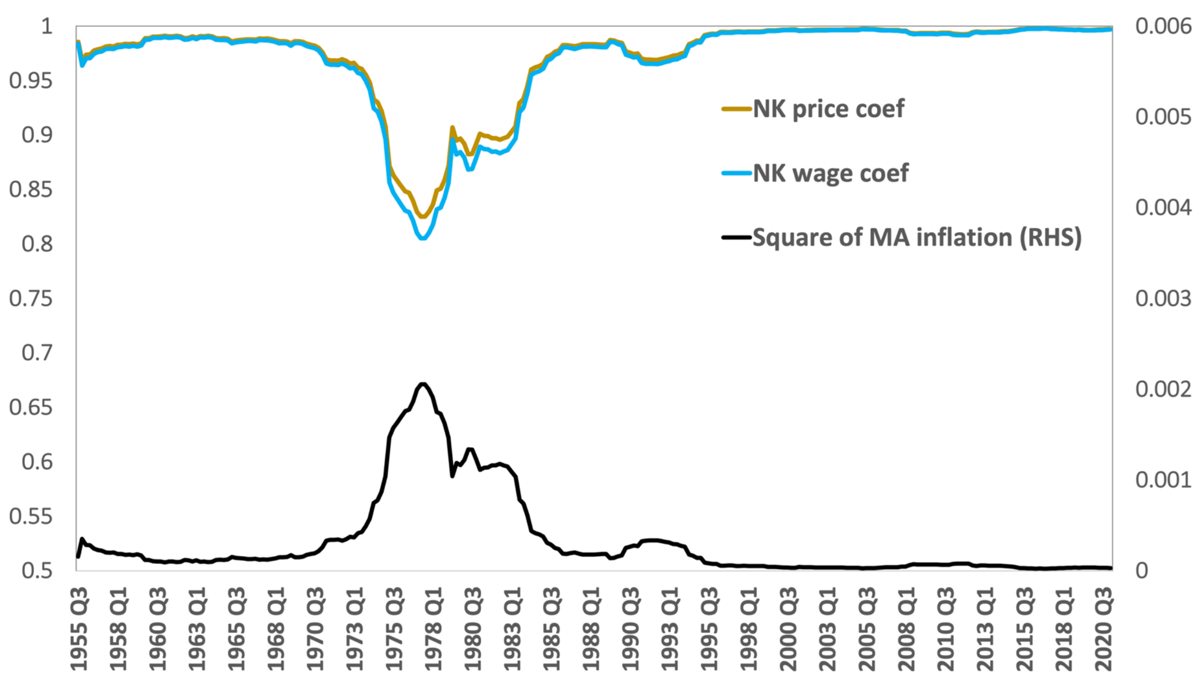

Figure 8-3: Time Varying NK Weights and the Square of MA Inflation

Table 8-2: VECM Parameters, Data Variance and Model Bootstrap Bounds for Output, Inflation, and Interest Rate.

	Actual estimate	Lower Bound (2.5th percentile)	Upper Bound (97.5th percentile)	IN/OUT
θ_{yy}	0.9817	0.7663	0.9493	OUT
$\theta_{y\pi}$	-0.0712	-0.2964	0.4465	IN
θ_{yr}	0.1644	-0.5485	0.1798	IN
$\theta_{\pi y}$	-0.0019	-0.0149	0.0482	IN
$\theta_{\pi\pi}$	0.3572	0.0815	0.8136	IN
$\theta_{\pi r}$	0.1672	-0.2273	0.2336	IN
θ_{ry}	0.0030	-0.0107	0.0495	IN
$\theta_{r\pi}$	0.0008	-0.1596	0.0390	IN
θ_{rr}	0.8936	0.8314	0.9999	IN
$var(\epsilon_y)$	0.000409	0.000578	0.001037	OUT
$var(\epsilon_\pi)$	0.000093	0.000061	0.000177	IN
$var(\epsilon_r)$	4.40e-06	8.39e-06	0.000092	OUT

Table 8-3: Estimated AR (1) Coefficients

Shocks	AR coefficients			
	Symbols	Pre-1972 (1955-1972)	Post-1972 (1972-2021)	Full sample (1955-2021)
Government Spending shock e_t^g	ρ_g	0.35845405	0.80408674	0.77007921
	σ_{ga}	-0.03941629	0.00875456	-0.00005156
Preference shock e_t^b	ρ_b	-0.27826486	-0.37362770	-0.32729780
Investment shock e_t^i	ρ_i	-0.11320271	0.13219215	0.10118243
Taylor Rule shock e_t^r	ρ_r	0.03067440	0.39148606	0.44962626
Productivity shock e_t^a	ρ_a	-0.39405050	-0.37724583	-0.37857038
Price mark-up shock e_t^p	ρ_p	-0.01916309	0.16737299	0.19281609
Wage mark-up shock e_t^{wNK}	ρ_w^{nk}	0.06857690	0.12581522	0.22310822
Labour hours shock $e_t^{w_{NC}}$	ρ_w^{nc}	0.49129847	0.32478658	0.67565103
Premium shock e_t^{prem}	ρ_{pr}	0.60762872	0.77255480	0.92906015
Net worth shock e_t^{nw}	ρ_{nw}	-0.04115083	-0.07801194	-0.10076772
Export shock e_t^{ex}	ρ_x	0.11194501	0.92335509	0.98217348
Import shock e_t^{im}	ρ_m	0.55666848	0.76430205	0.75389997
Foreign consumption shock c_t^f	ρ_c^f	0.62397444	0.92857239	0.99562478
Foreign interest rate shock r_t^f	ρ_r^f	0.68884504	0.86992613	0.92914588
M0 shock (crisis) $e_t^{m0,crisis}$	$\rho_{m0,crisis}$	-0.24621338	0.31279478	0.31180388
M0 shocks (non-crisis) $e_t^{m0,nocrisis}$	$\rho_{m0,nocrisis}$	-0.23263864	0.31126665	0.31146204

$*$ σ_{ga} is the response of exogenous spending to productivity development.

$*$ AR coefficients for the pre-1972 period were calculated by using the estimated pre-1972 model and data from 1955-1972. AR coefficients for the post-1972 period were calculated by using the estimated post-1972 model and data from 1972-2021. AR coefficients for the full sample period were calculated by using the estimated post-1972 model and data from the whole sample period 1955-2021.

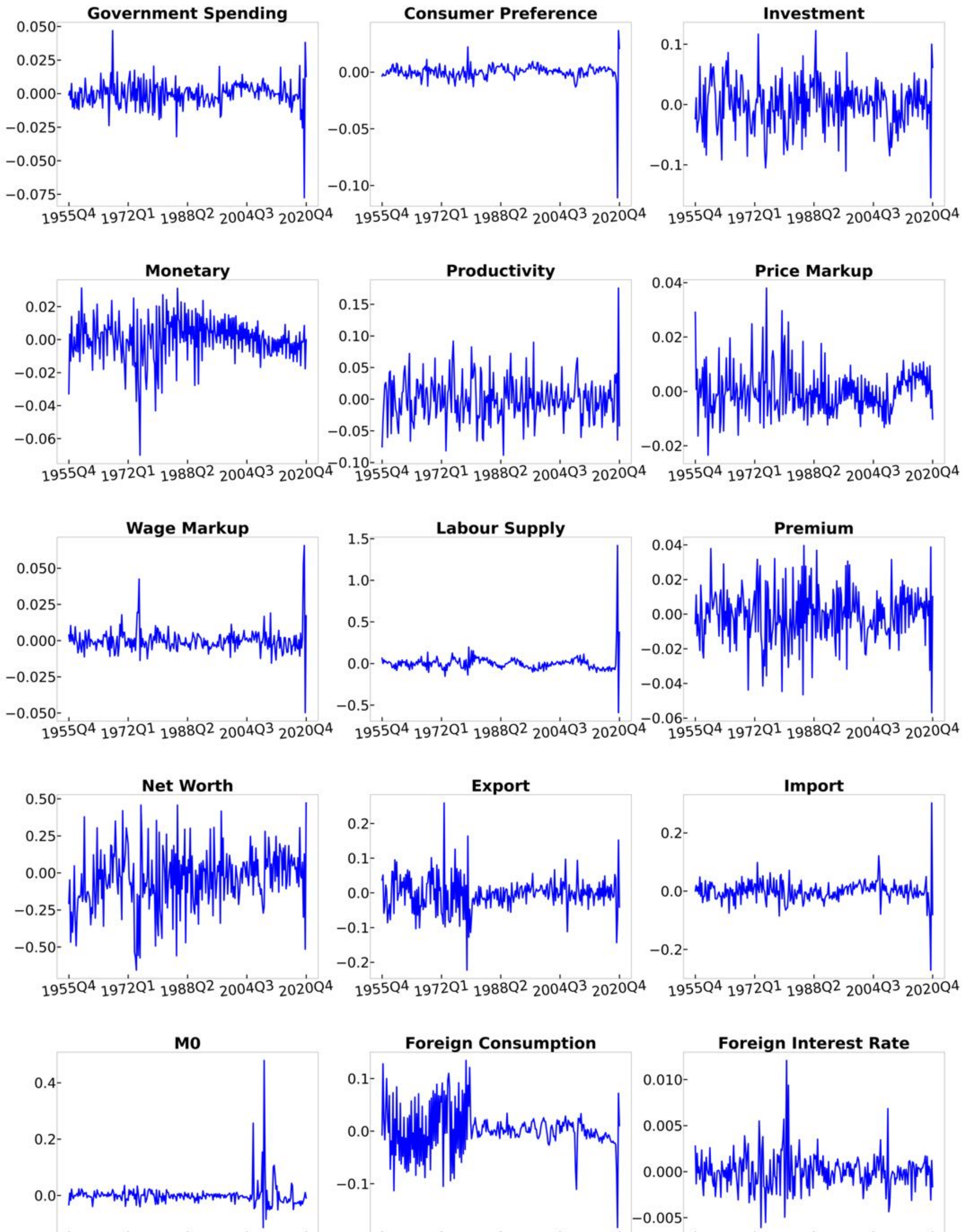

Figure 8-4: Shocks Calculated from Estimated Parameters (1955Q2-2020Q4)

Note: Figure 8-4 plots model shocks for the full sample calculated using estimated parameters. The shocks lost three periods in the beginning from lags and innovations of productivity shock, and one period at the end from expectations against the actual data.

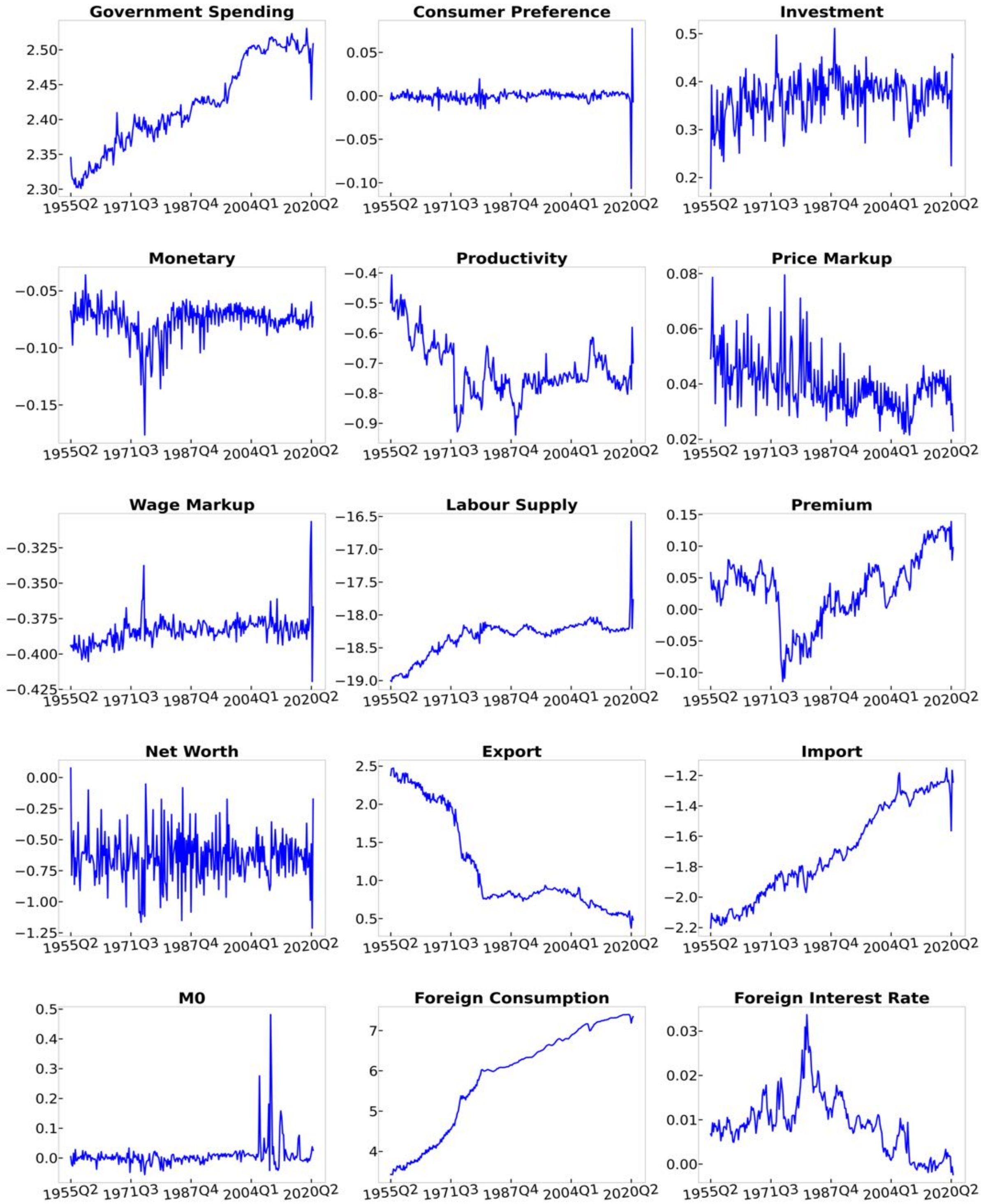

Figure 8-5: Residuals Calculated from Estimated Parameters (1955Q2-2020Q4)

Note: Figure 8-5 plots model residuals for the full sample calculated as the difference between the actual data and the equations by using estimated parameters. The residuals lost one period in the beginning from lags and one at the end from expectations against the actual data.

8.3 Empirical Analysis

8.3.1 Impulse Responses from Estimated Model

This section examines impulse responses of the variables to structure shocks under the NK model and the flexprice model to illustrate the internal logic of the models. The IRFs show the difference between the base run of the model and the simulated results of the model after a one-off shock. The base run replicates the observed data. These analyses help to assess the validity of the estimated model. For the entirely NK model, we set the NK weights to one with corresponding Calvo parameters; for the flexprice model, the NK weights are set to zero. In the IRFs figures, wage represents the real consumer wage and inflation is CPI inflation. The y-axis represents the responses to shocks, and the x-axis is the timeline. The blue line represents the NK model, and the red line represents the flexprice model (FP for short). More IRFs are shown in Appendix D. Here only the IRFs to main shocks, such as Taylor Rule shock, government spending shock, premium shock and productivity shock, are discussed as these are the focus of this thesis.

Taylor Rule Shock (Pure Demand Shock)

Figure 8-6 shows the IRFs to a positive Taylor Rule shock under both the NK and the FP models, which is a pure demand shock. As the figure shows, output fluctuations are more significant under the NK model than the FP model. The explanation for this difference is that under the NK model (with sticky prices), a pure demand shock affects output directly, while inflation does not respond much in the short run and only responds substantially to the resulting output gaps in the medium run. On the other hand, in the FP model (with flexible prices), the Taylor Rule shock disturbs prices because they vary with changes in marginal costs and the output gap; hence it has a limited impact on output as inflation responds quickly to stabilise output. Hence the NK model destabilises output in response to demand disturbances but stabilises inflation through the Calvo framework, while the FP model stabilises output via flexible price adjustments. Therefore, we see in Figure 8-6 that the output response is greater under the NK model than under the FP model, while the inflation response is greater under the FP model versus the NK model. Overall, the FP model implies rapid price responses and smaller real effects of a monetary shock than the NK model: with output, consumption and labour exhibiting smaller and less persistent responses.

The general responses of the variables are similar for both versions of the models. An increase in the nominal interest rate hits consumption and investment negatively, leading to a fall in output. In the output market, the reduction in output leads to an output gap, which lowers inflation. In the input market, a higher nominal interest rate decreases labour hours and real wages. In the financial sector, the shock also decreases entrepreneurs' net worth, consequently raising external finance premium that further reduces investment. The lower net worth leads to an increase in distressed borrowing, hence M0 raises. In the foreign sector, the deflation and higher nominal interest rate (implying a higher real interest rate) decrease the real exchange rate and hence pound sterling appreciates, which induces import and lowers export as domestic prices are relatively higher than foreign prices. The net foreign bond position also decreases because of a lower net export.

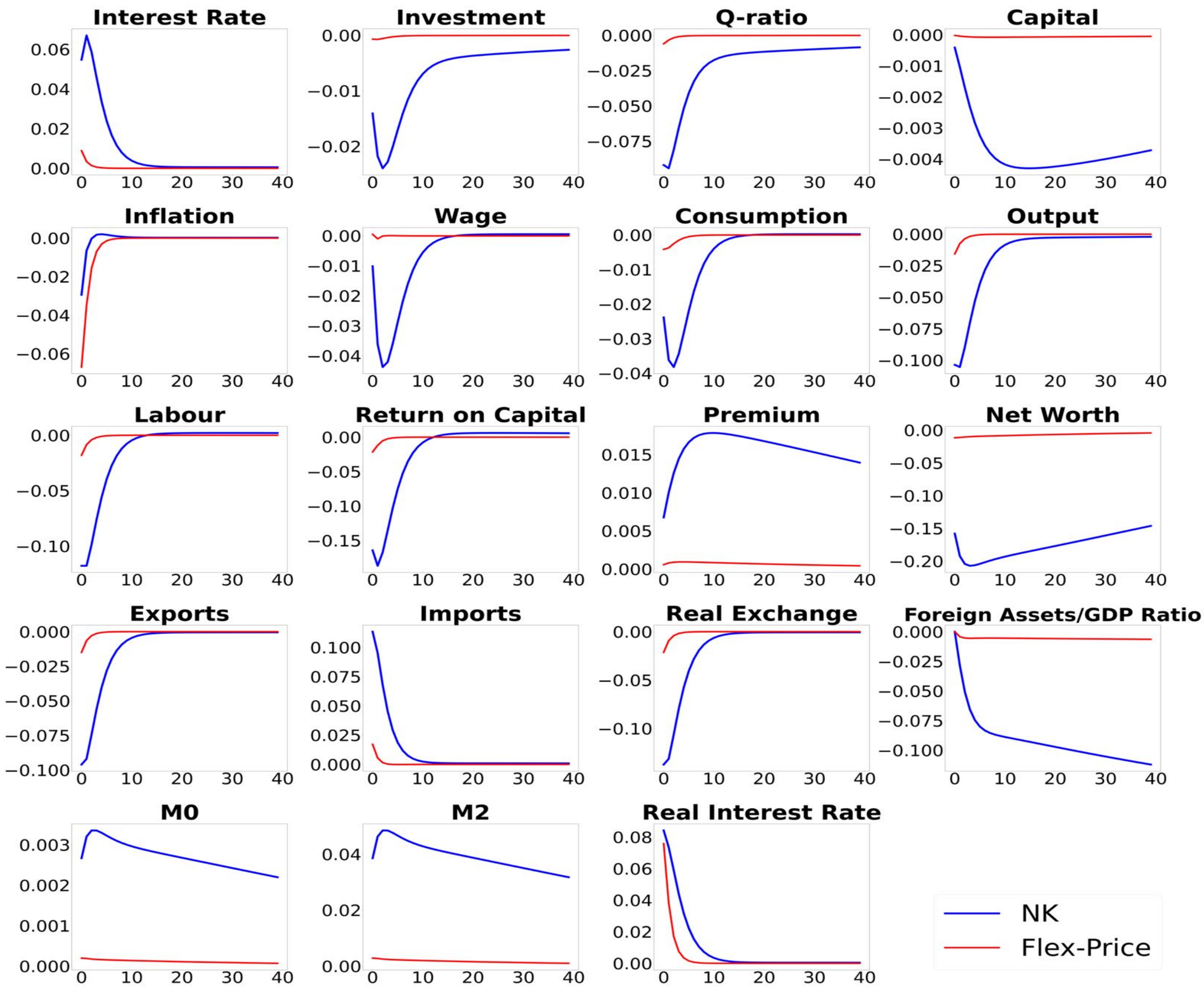

Figure 8-6: IRFs to a Taylor Rule Shock

Government Spending Shock (Pure Demand Shock)

Figure 8-7 captures the effects of a temporary rise in government spending under the NK and the FP models. Government spending shock is a pure demand shock; hence, output fluctuation is higher under the NK model than under the FP model. Under the NK model, because of price rigidity, the positive demand effects of the fiscal shock have a low impact on prices and therefore do not transmit into higher inflation but create output turbulence. Conversely, under the FP model with flexible prices, the positive demand effects cause an increase in the output gap, but the increase in the output gap is transmitted into a higher inflation. Therefore, the output response is greater under the NK model than under the FP model, while the inflation response is greater under the FP model versus the NK model.

In both models, an expansionary fiscal policy causes a rise in aggregate demand, which pushes up output and labour hours. Simultaneously, fiscal expansion increases prices and interest rates via the responses of the Taylor Rule. The higher interest rate reduces investment and capital. In the foreign market, the real exchange rate falls (domestic currency appreciates) with a higher real interest rate; this decreases exports and increases imports because domestic products are less competitive than foreign ones. The lower net export has a negative impact on net foreign asset accumulation.

Although the responses of most variables are similar under both models, there is a difference in the response of consumption. In the NK model, the firm will raise nominal wages to attract more labour to meet excess output demand. As the expected income rises, so does consumption. On the other hand, in the FP model there is a negative impact on consumption, which is in line with the neoclassical model, see for example Baxter and King (1993). With perfect information, government spending is seen as a negative wealth shock to the household, as the household recognises that taxes must be increased either now or in the future to pay for it. The negative wealth effect leads to a decline in consumption and an increase in hours worked.

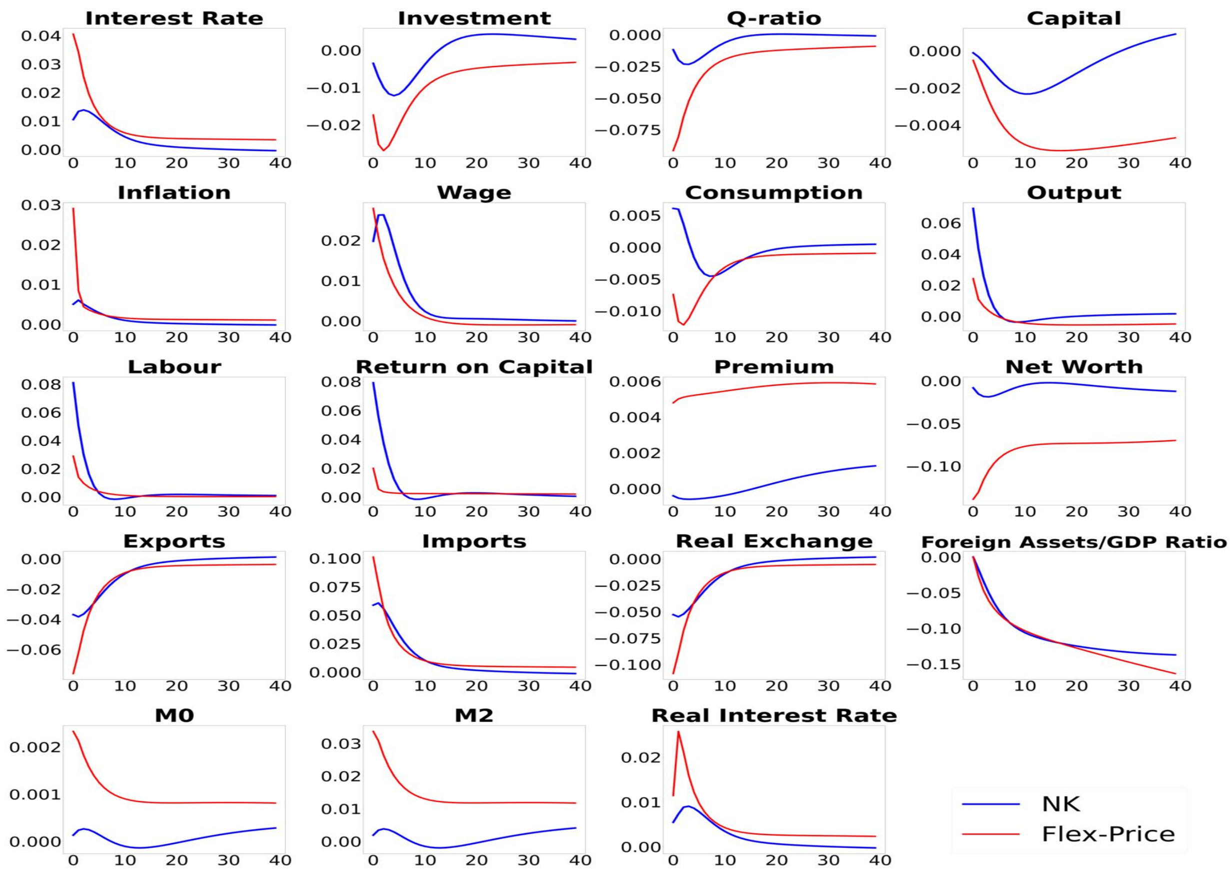

Figure 8-7: IRFs to a Government Spending Shock

External Premium Shock

Figure 8-8 shows the IRFs to a temporary positive credit premium shock under both models. A higher credit premium implies that the entrepreneur faces a tighter borrowing constraint. Therefore, it has a negative impact on the demand. Under the NK model with sticky prices, this demand shock destabilises output and stabilises inflation through the Calvo framework. Under the FP model, the output is stabilised by flexible price adjustments. As a result, the output is more volatile under the NK model than under the FP model, but inflation is less volatile under the NK model than under the FP model.

In general, in both models, a higher credit premium reduces investment, capital, the price of capital (Q-ratio), and net worth, and therefore output, labour hours, real wages and inflation. Nominal interest rate falls to stimulate the economy via the Taylor Rule. Consumption under the NK model falls as wages and labour hours decrease. Consumption under the FP model increases slightly because 1) high deflation under the FP model has a positive effect on consumption, 2) the lower real interest rate has a positive effect on consumption, and 3) labour hours and wages fall less under the FP model than the NK model. In the foreign market, the decrease in the real interest rate drives the real exchange rate up (the pound sterling depreciates), which affects export positively but import negatively. The net foreign position increases because of a higher net export.

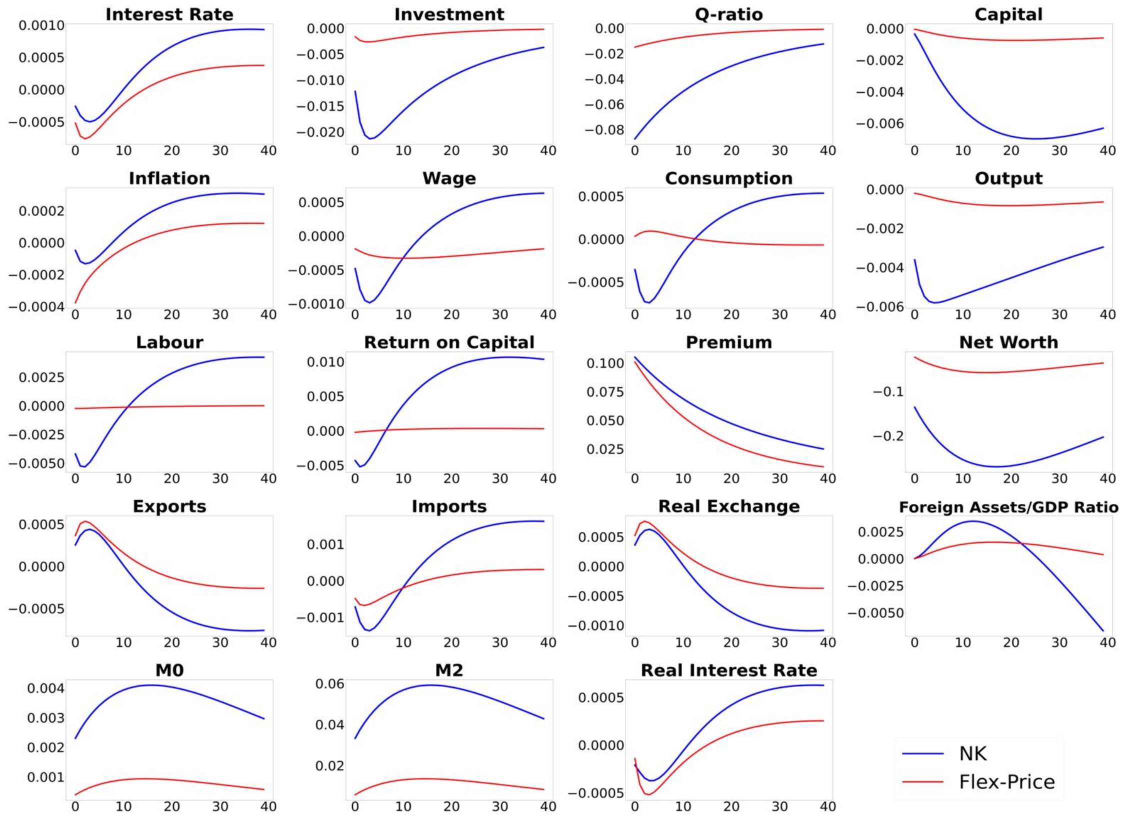

Figure 8-8: IRFs to a Premium Shock

Productivity Shock (Both Supply and Demand Effects Shock)

Figure 8-9 shows the IRFs to a non-stationary positive productivity shock under the NK and FP models. Productivity shocks include both supply-side effects (e.g., technological advances make production more efficient) and demand-side effects (productivity shocks permanently affect income and demand for capital, thereby affecting consumption and investment, respectively). The demand effects lead to higher output fluctuation but lower inflation fluctuation under the NK model than under the FP model, as a higher output gap is transmitted into higher inflation under the FP model. On the other hand, in the NK model, the supply effects have a limited impact on prices due to price rigidity, thus resulting in a weak indirect impact on output via the Taylor Rule. In the FP model, the supply side effects directly affect output supply via the production function, but prices and interest rate react quickly to balance demand and supply.

Therefore, in response to the demand effects, the NK model destabilises output but stabilises inflation through the Calvo framework; the FP model stabilises output via flexible price adjustments. In response to the supply effects, the NK model stabilises both output and inflation, while the FP model stabilises output but destabilises inflation. Thus, for a productivity shock, we see in Figure 8-9 that the output response is greater under the NK model than under the FP model, while the FP model has a greater inflation response relative to the NK model.

In general, the responses of the variables are similar under both models. Since the productivity shock is highly persistent, it has a long-lasting positive impact on output, consumption, investment, and capital. The increase in the supply of goods and the decrease in the marginal cost have a negative impact on inflation. The higher demand for capital drives up the price of capital, which raises the entrepreneur's net worth. Then credit premium decreases, and the negative effect on premium helps to further increase investment. In the labour market, the higher productivity raises real wages and lowers labour hours. In the foreign market, the higher supply of domestic goods decreases the relative price of domestic goods to foreign goods prices. Thus, the real exchange rate rises which indicates the pound sterling experiences a real devaluation. It has a negative impact on imports but makes exports more competitive. Then, a higher net export will increase net foreign asset accumulation.

Output IRFs to Various Shocks

Figure 8-10 summarises output IRFs for different shocks (more IRFs are in Appendix D). We have discussed above that output fluctuation is higher, and inflation fluctuation is lower under the NK model than under the FP model in response to Taylor Rule, government spending, premium and productivity shocks. Apart from labour supply shock, the other shocks all produce greater output fluctuation under the NK model than under the FP model. However, the labour supply shock has no demand effect. Under the NK model, the labour supply shock has almost no effect on wages; hence it does not affect labour and output. Under the FP model, it has only a temporary effect on labour and output.

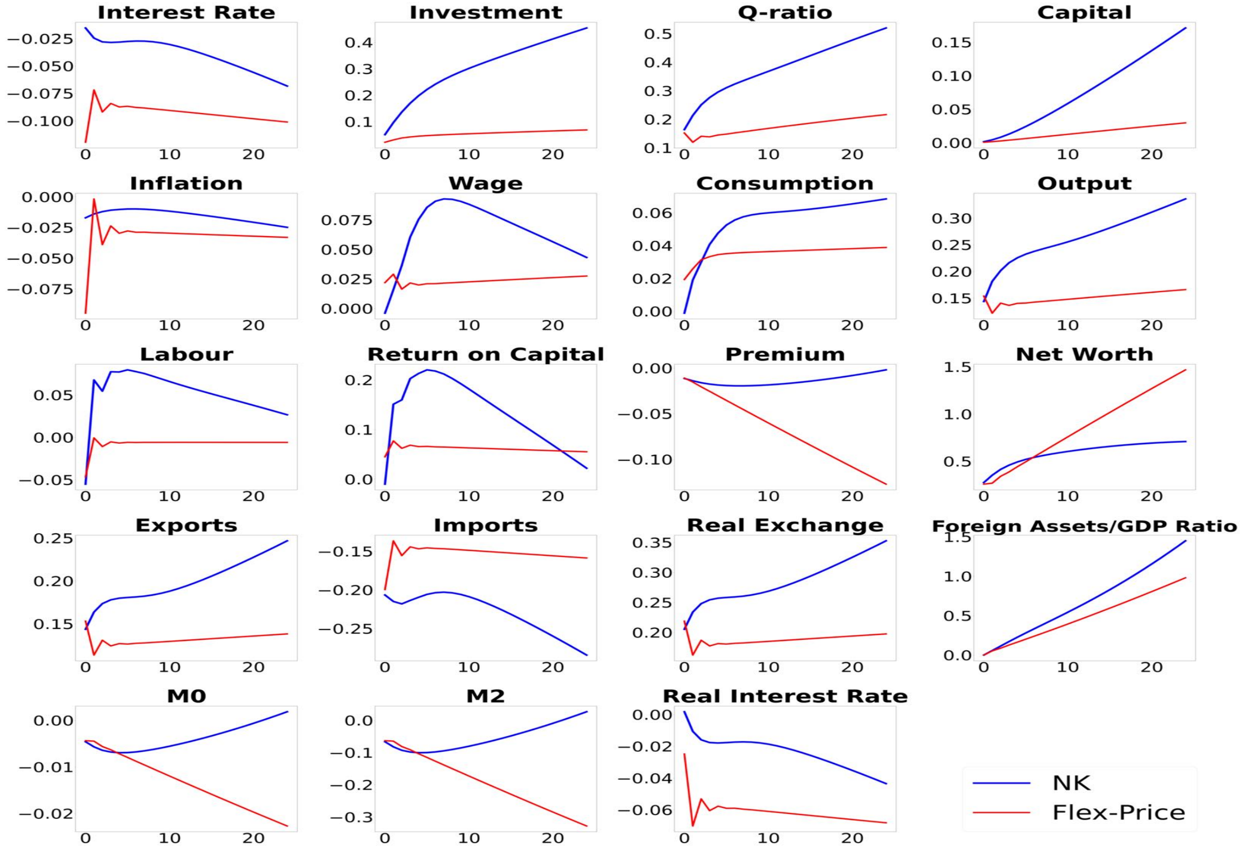

Figure 8-9: IRFs to a Productivity Shock

Note: The non-smooth IRFs for productivity are due to the negative AR coefficient. Thus, we see it fluctuates before settling down.

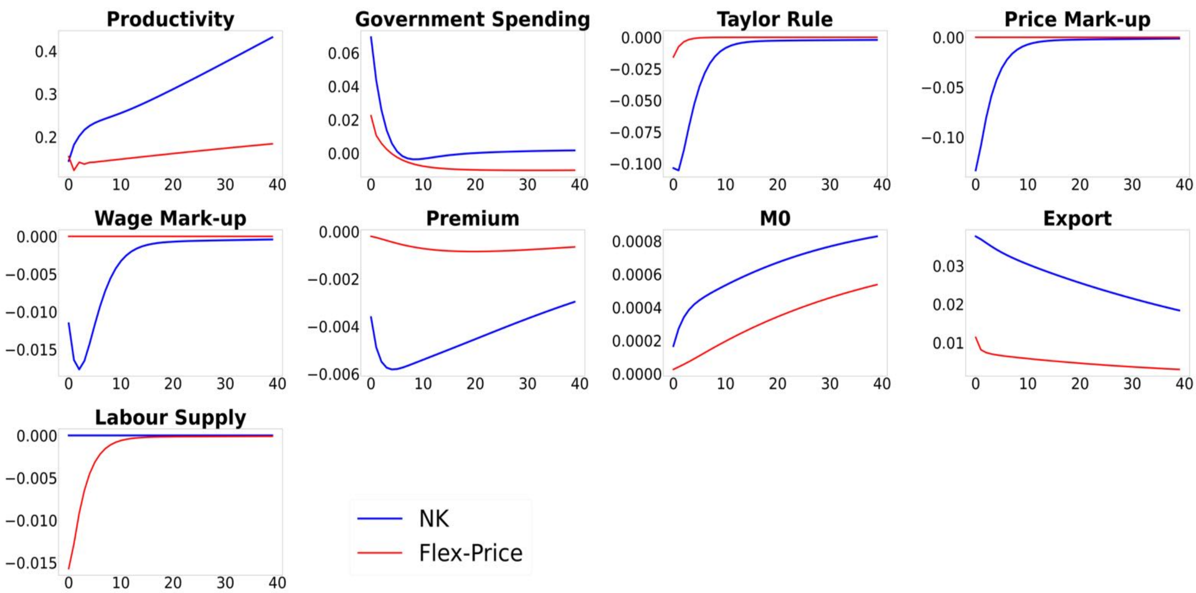

Figure 8-10: Output IRFs to Various Shocks under the NK and FP Models

The impulse response analysis under the NK and FP models in this study is consistent with the findings of Le et al. (2021), and the following similarities can be summarised. First, compared to the FP model, the NK model stabilises output against supply shocks but destabilises it against demand shocks. For inflation, the NK model stabilises inflation through the Calvo framework, while the FP adjusts prices quickly and keeps them related to marginal costs. For example, Figures 8-11 and 8-12 show their IRFs for two pure demand shocks: a Taylor Rule shock and a government spending shock. These figures show that output fluctuations are more significant under NK than under FP, while inflation fluctuations are much smaller. For the productivity shock (see Figure 8-13), with both supply and demand effects, output fluctuations are higher under NK than FP, and inflation fluctuations are lower. Figure 8-14 shows their output IRFs for all the shocks, and for those mainly demand shocks, all show greater output fluctuations under NK than FP. Second, the FP model has a smaller real effect of monetary shocks than the NK model, showing smaller and less persistent responses of output, consumption, wage and labour (see Figure 8-11). Third, the response direction (increase or decrease) of the key variables to shocks in this thesis is consistent with Le et al.'s. It is worth noting that interest rate increases in response to a positive government shock; hence, in the later policy analysis chapter, we add a fiscal backstop to prevent ZLB from happening. In summary, like the state-dependent model of Le et al., our model indicates that a higher inflation fluctuation leads to a higher NC weight, which in turn causes a higher inflation response and a smaller real effect. Therefore, we expect that in the later policy analysis chapter, when considering welfare improvements and economic stability, we will also need to consider the stability of inflation, as in Le et al.

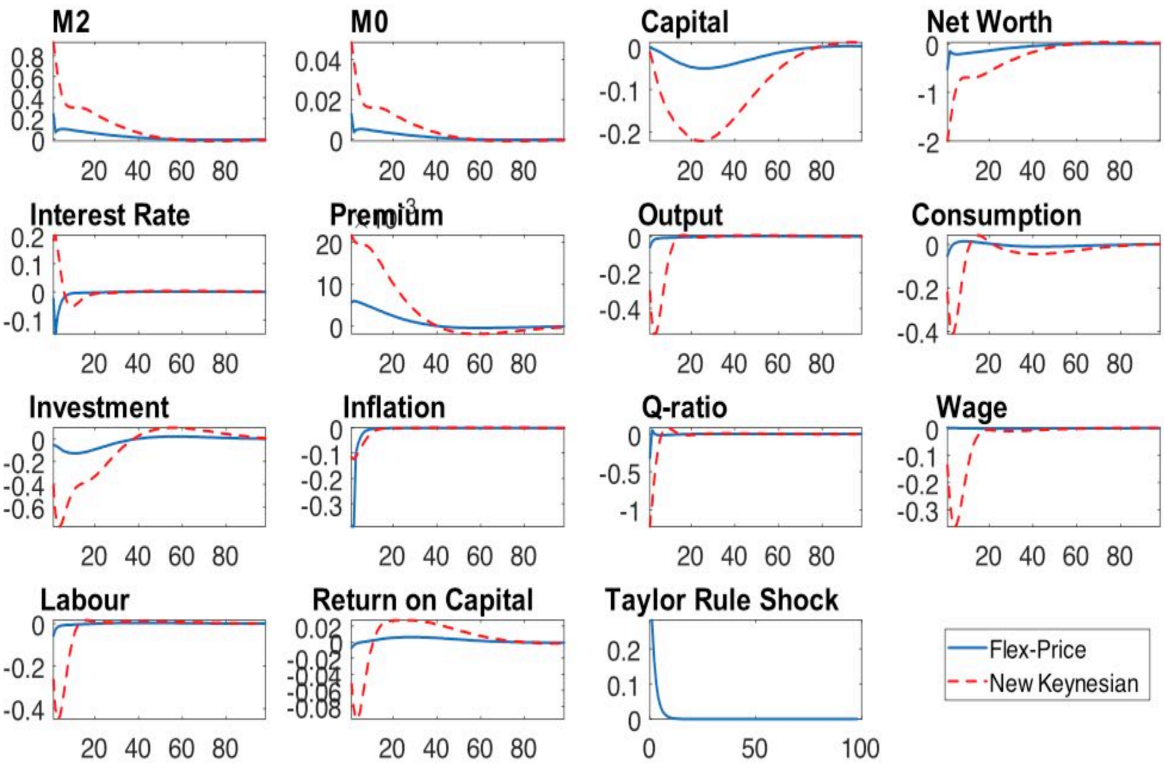

Figure 8-11: Le et al.’ (2021) IRFs to a Taylor Rule Shock

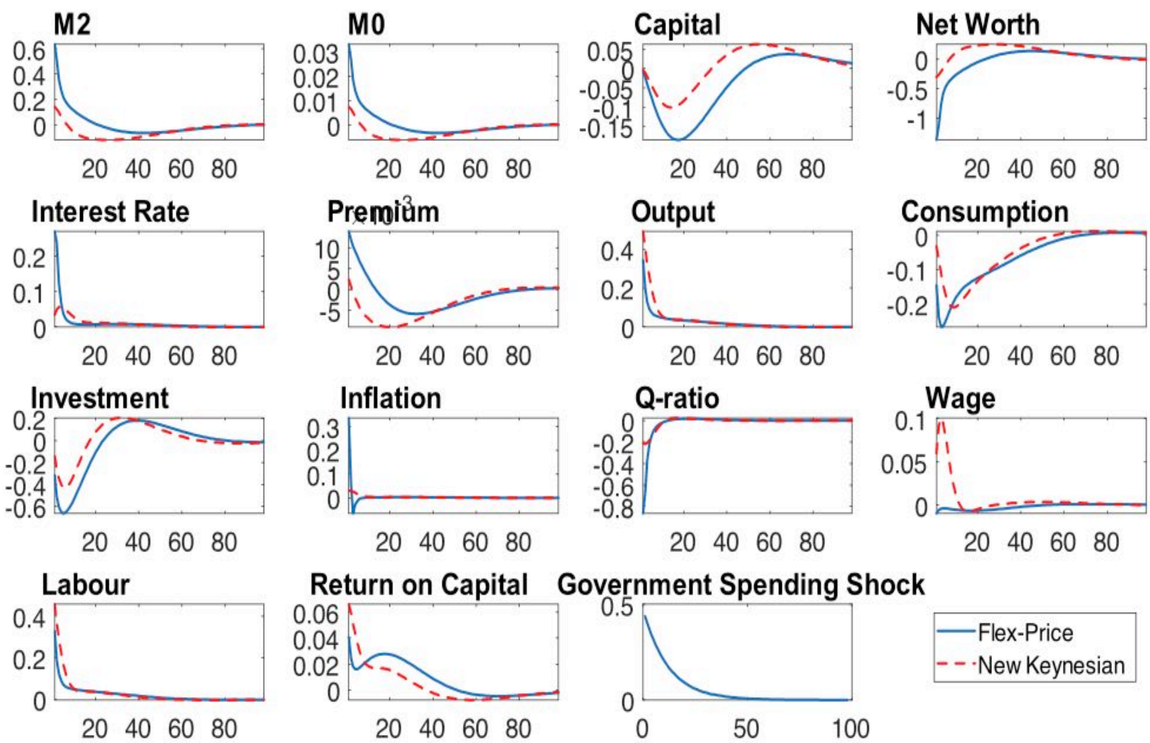

Figure 8-12: Le et al.’ (2021) IRFs to a Government Spending Shock

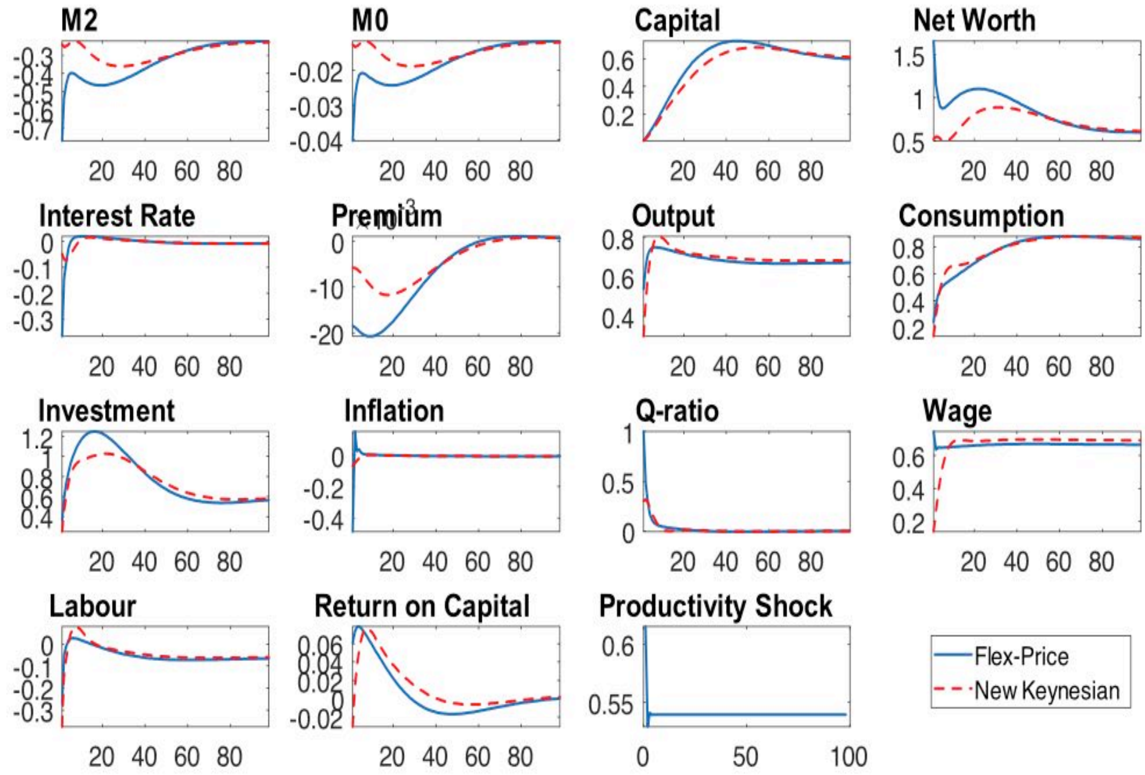

Figure 8-13: Le et al.' (2021) IRFs to a Productivity Shock

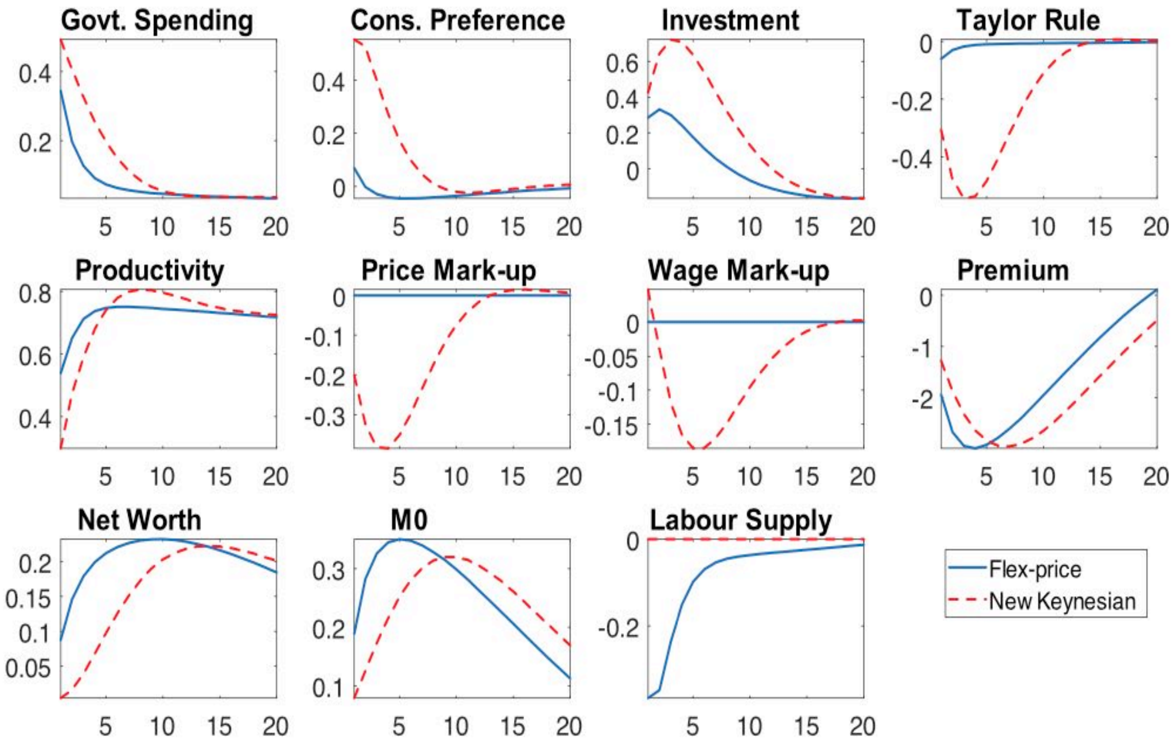

Figure 8-14: Le et al.' (2021) IRFs to Various Shocks under NK and FP Models

8.3.2 Variance Decomposition

To measure the overall model behaviour, we now look at the variance decomposition of the variables for the pre-1972 period and the post-1972 period separately. For the pre-1972 period, the model has an exchange rate target in the Taylor Rule equation, while for the post-1972, the model uses the normal Taylor Rule. We also check the variance decomposition for these two sub-sample periods on different time scales: short-run (S-R) and long-run (L-R). The variance decomposition is calculated under the assumption that shocks are independent of each other to verify the contribution of each shock to the variation of the variables of interest.

Tables 8-4 and 8-5 report each shock's S-R and L-R contributions to the volatility of some key variables for the pre-1972 period, respectively. In terms of output, productivity shock contributes 4.59% in the S-R and a higher proportion of 15.94% in the L-R; Taylor Rule shock explains 7.41% of output fluctuation in the S-R and 7.83% in the L-R. World shocks play a significant role in explaining output fluctuation, from 70.01% in the S-R to 59.82% in the L-R for the pre-1972 period; and from 62.02% in the S-R to 51.52% in the L-R for the post-1972 period (see Tables 8-6 and 8-7). Which is in line with the findings of Lyu (2021) who finds that world shocks account for a significant 46% of output fluctuation in the UK.

When it comes to the nominal interest rate, it is heavily influenced by The Taylor Rule shock, accounting for 52.32% in the S-R and 31.68% in the L-R; price mark-up shock also contributes a significant proportion of 31.68% in the S-R and 17.34% in the L-R; the contribution of productivity shock increases from 3.77% in the S-R to 26.62% in the L-R; and world shocks account for 9.32% in the S-L and 16.93% in the L-R.

In terms of inflation, it is dominated by price mark-up shock with 67.80% in the S-L and 43.21% in the L-R; the contribution of productivity shock increases from 5.89% in the S-R to 26.47% in the L-R; investment shock and Taylor Rule shock together explain about 15% of S-R and L-R inflation volatility; and world shocks explain 8.55% of inflation fluctuation in the S-R and 14.40% in the L-R.

The main difference between the L-R and S-R results is the increased contribution of productivity shocks. This is due to the high persistence of productivity shocks. The results found here are largely consistent with the findings of Le et al. (2021) and Lyu

(2021). Results from the post-1972 model are much the same as those from the pre-1972 model.

8.3.3 Historical Decomposition

Figure 8-11 shows the historical decomposition of output over the period 1955-2021. The total contribution of all shocks is shown by the solid black line. The world shocks have a significant impact on output due to the fact that the UK has a high degree of openness and has been trading extensively with the rest of the world. Historical productivity shocks mainly play a stimulus role in output growth during the Great Moderation and a dampening role during the Great Recession. The impact of Taylor Rule shocks on output has declined over time, especially after the recent financial crisis. We can also see from the graph that the UK entered a period of recession from the third quarter of 2008 onwards, and there was a sharp decline in output during the recent coronavirus pandemic.

8.4 Conclusion

This chapter adds the state-dependent price and wage duration framework into the DSGE model. The state-dependent model is estimated and tested using the Indirect Inference method. We find that the behaviour produced by the state-dependent model is broadly consistent with that produced by Le et al.(2021) and micro level studies in different countries; i.e., prices change more frequently when inflation is higher, as a result, prices respond quickly and less persistently to shocks in high inflation periods. For example, Figure 8-3 shows that the higher the inflation, the smaller the NK weight (or the higher the weight of the FP sector). Moreover, as expected, the flexible pricing case implies rapid price responses and smaller real effects of a monetary shock compared to the NK case: with output, consumption and labour exhibiting smaller and less persistent responses (see Figure 8-6).

The results show that the state-dependent model fits the behaviour of the historical UK data very well over the whole sample period, suggesting that the state-dependent price/wage contract framework improves the fit of the DSGE model to macroeconomic data. This chapter contributes a macro-level evidence of state-dependence in the UK to corroborate micro-level evidence.

Table 8-4: **Short Run** Variance Decomposition **(Pre-1972 DSGE model)**

Shock\variable	Int. Rate	Inv.	Inf.	Wage	Cons.	Output	Hours	Export	Import	Exch. Rate
Govt Spending	1.77	0.05	1.96	5.68	0.28	3.48	1.40	0.40	3.81	12.80
Consumer Pref.	0.01	0.01	0.01	0.22	13.41	0.08	0.03	0.00	1.67	0.04
Investment	0.57	82.55	0.99	1.19	1.05	8.49	2.86	0.03	0.43	1.04
Taylor Rule	52.32	0.00	14.00	1.31	7.44	7.41	2.85	1.22	5.26	39.50
Productivity	3.77	16.63	5.89	27.81	61.05	4.59	59.03	0.59	37.25	2.20
Price Mark-up	31.68	0.00	67.80	8.20	4.01	4.25	1.53	0.68	3.04	22.07
Wage Mark-up	0.41	0.09	0.63	23.52	0.59	0.49	0.26	0.07	0.29	2.35
Labour Supply	0.07	0.02	0.11	2.63	0.09	0.11	0.05	0.02	0.07	0.50
Premium	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
Net Worth	0.08	0.58	0.06	0.32	0.15	1.10	0.40	0.00	0.08	0.09
Quantitative Easing	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Domestic subtotal	90.68	99.96	91.45	70.87	88.06	29.99	68.42	3.02	51.90	80.59
Export Demand	4.16	0.00	3.66	15.82	4.73	29.28	13.47	49.37	3.93	6.41
Import Demand	1.12	0.00	1.00	4.18	1.28	8.69	3.47	0.06	39.46	1.79
Foreign Cons.	2.11	0.00	1.58	8.24	5.89	30.82	14.18	47.26	2.18	1.85
Foreign Int. Rate	1.92	0.03	2.31	0.89	0.04	1.23	0.46	0.29	2.53	9.35
World subtotal	9.32	0.04	8.55	29.13	11.94	70.01	31.58	96.98	48.10	19.41
Total	100.00	100.00	100.00	100.0	100.00	100.00	100.00	100.00	100.00	100.00

*In the pre-1972 DSGE model, an exchange rate target is added to the Taylor Rule equation.

*The values in the table are expressed as percentages.

Table 8-5: **Long Run** Variance Decomposition **(Pre-1972 DSGE model)**

Shock\variable	Int. Rate	Inv.	Inf.	Wage	Cons.	Output	Hours	Export	Import	Exch. Rate
Govt Spending	1.70	0.11	1.70	1.85	0.25	3.14	0.55	0.49	3.08	11.31
Consumer Pref.	0.01	0.01	0.01	0.06	5.19	0.07	0.01	0.00	0.99	0.04
Investment	5.11	26.52	4.32	3.10	4.08	3.92	8.97	0.16	3.28	3.80
Taylor Rule	31.68	0.00	9.17	0.62	3.96	7.83	1.31	1.10	3.35	25.37
Productivity	26.62	71.07	26.47	72.97	78.12	15.94	74.02	12.31	50.95	16.69
Price Mark-up	17.34	0.00	43.21	2.19	1.81	4.04	0.64	0.56	1.82	12.94
Wage Mark-up	0.34	0.20	0.47	6.08	0.43	0.70	0.13	0.08	0.22	1.76
Labour Supply	0.10	0.09	0.12	0.82	0.11	0.23	0.04	0.02	0.07	0.54
Premium	0.00	0.06	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
Net Worth	0.16	1.71	0.11	0.25	0.13	4.24	0.32	0.01	0.14	0.19
Quantitative Easing	0.00	0.06	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
Domestic subtotal	83.07	99.82	85.60	87.93	94.09	40.18	85.99	14.73	63.90	72.64
Export Demand	4.38	0.01	3.70	6.02	2.28	23.62	6.11	46.18	3.72	6.86
Import Demand	0.97	0.01	0.86	1.38	0.56	8.29	1.45	0.07	24.97	1.60
Foreign Cons.	1.47	0.00	1.21	2.41	2.49	24.62	5.74	38.26	1.51	1.38
Foreign Int. Rate	10.10	0.16	8.64	2.27	0.58	3.28	0.72	0.76	5.90	17.52
World subtotal	16.93	0.18	14.40	12.07	5.91	59.82	14.01	85.27	36.10	27.36
Total	100.00	100.00	100.00	100.0	100.00	100.00	100.00	100.00	100.00	100.00

*In the pre-1972 DSGE model, an exchange rate target is added to the Taylor Rule equation.

*The values in the table are expressed as percentages.

Table 8-6: **Short Run** Variance Decomposition **(Post-1972 DSGE model)**

Shock\variable	Int. Rate	Inv.	Inf.	Wage	Cons.	Output	Hours	Export	Import	Exch. Rate
Govt Spending	1.54	0.09	2.02	2.20	0.13	1.85	1.06	0.40	2.53	12.41
Consumer Pref.	0.05	0.00	0.06	0.32	33.89	0.20	0.11	0.00	2.92	0.10
Investment	0.45	37.64	0.91	0.43	0.64	4.97	2.35	0.02	0.18	0.60
Taylor Rule	62.02	0.00	19.19	0.70	6.75	6.48	3.49	1.09	3.31	33.83
Productivity	5.54	61.01	9.29	42.03	37.28	13.46	51.85	32.83	36.76	2.34
Price Mark-up	22.88	0.21	60.27	8.17	8.05	7.77	3.88	1.09	3.28	34.03
Wage Mark-up	0.82	0.51	1.61	26.05	2.19	1.62	1.10	0.20	0.51	6.16
Labour Supply	0.58	0.11	1.06	9.78	0.76	0.87	0.58	0.13	0.41	4.01
Premium	0.00	0.11	0.00	0.00	0.00	0.03	0.01	0.00	0.00	0.00
Net Worth	0.07	0.29	0.06	0.13	0.10	0.71	0.37	0.00	0.04	0.06
Quantitative Easing	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Domestic subtotal	93.94	100.00	94.48	89.82	89.80	37.98	64.80	35.76	49.92	93.53
Export Demand	2.26	0.00	1.82	4.14	5.05	29.18	16.76	40.27	1.24	1.36
Import Demand	1.52	0.00	1.36	2.68	2.60	15.45	8.69	0.04	47.49	1.10
Foreign Cons.	1.81	0.00	1.63	3.31	2.54	16.99	9.54	23.85	0.91	1.46
Foreign Int. Rate	0.47	0.00	0.72	0.04	0.01	0.39	0.21	0.08	0.43	2.55
World subtotal	6.06	0.00	5.52	10.18	10.20	62.02	35.20	64.24	50.08	6.47
Total	100.00	100.00	100.00	100.0	100.00	100.00	100.00	100.00	100.00	100.00

*In the post-1972 DSGE model, the normal Taylor Rule equation is used.

*The values in the table are expressed as percentages.

Table 8-7: **Long Run** Variance Decomposition **(Post-1972 DSGE model)**

Shock\variable	Int. Rate	Inv.	Inf.	Wage	Cons.	Output	Hours	Export	Import	Exch. Rate
Govt Spending	1.08	0.16	1.21	0.42	0.15	1.58	0.37	0.52	1.55	8.87
Consumer Pref.	0.02	0.01	0.03	0.05	7.69	0.16	0.04	0.00	1.34	0.06
Investment	10.21	19.24	9.73	1.62	0.42	5.70	5.00	0.77	3.08	13.21
Taylor Rule	27.28	0.00	8.66	0.20	2.11	6.20	1.40	1.05	1.64	18.07
Productivity	45.25	78.11	47.33	87.64	81.71	21.14	73.06	12.34	63.26	23.46
Price Mark-up	9.07	0.18	26.45	1.41	2.23	6.83	1.41	0.96	1.52	16.55
Wage Mark-up	0.73	0.86	1.07	4.02	1.01	2.58	0.46	0.30	0.46	5.16
Labour Supply	0.58	0.34	0.73	1.81	0.55	1.55	0.33	0.19	0.29	3.31
Premium	0.01	0.23	0.01	0.00	0.00	0.19	0.02	0.00	0.00	0.01
Net Worth	0.10	0.80	0.08	0.06	0.05	2.49	0.25	0.01	0.05	0.10
Quantitative Easing	0.00	0.07	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
Domestic subtotal	94.35	100.00	95.30	97.22	95.92	48.48	82.34	16.13	73.18	88.80
Export Demand	3.53	0.00	2.69	1.85	2.97	28.79	12.28	73.09	1.80	2.55
Import Demand	0.91	0.00	0.81	0.52	0.70	13.62	3.24	0.05	23.61	0.80
Foreign Cons.	0.52	0.00	0.50	0.30	0.35	6.98	1.66	10.31	0.29	0.54
Foreign Int. Rate	0.69	0.00	0.70	0.10	0.06	2.13	0.48	0.42	1.12	7.31
World subtotal	5.65	0.00	4.70	2.78	4.08	51.52	17.66	83.87	26.82	11.20
Total	100.00	100.00	100.00	100.0	100.00	100.00	100.00	100.00	100.00	100.00

*In the post-1972 DSGE model, the normal Taylor Rule equation is used.

*The values in the table are expressed as percentages

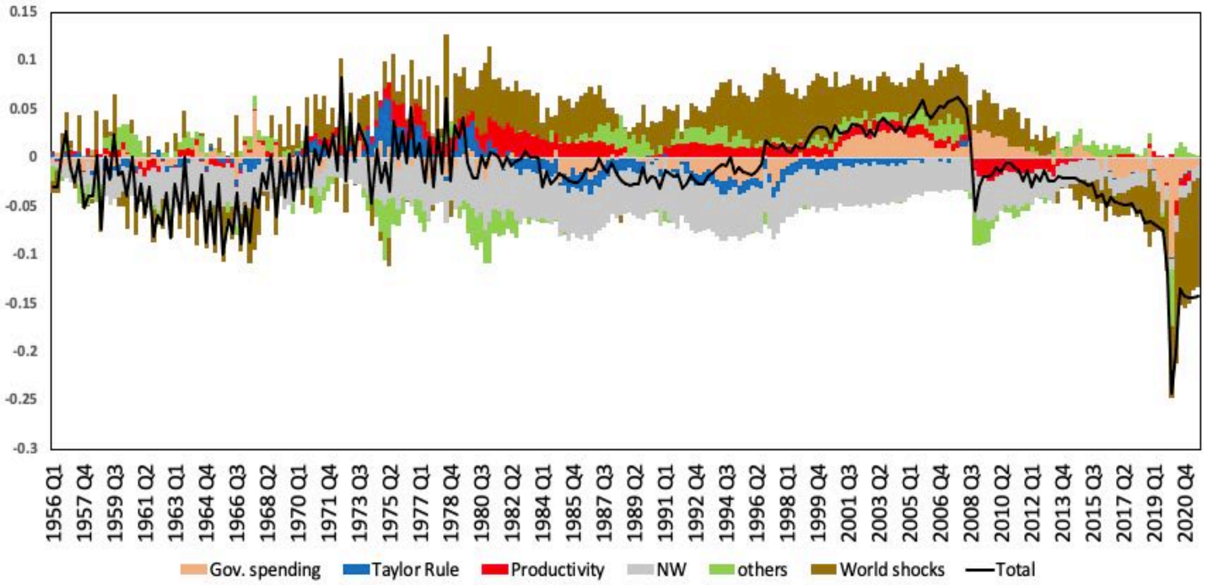

Figure 8-15: Historical Decomposition of Output Over the Whole Sample Period

Chapter 9 Policy Implication

What monetary policy rules should central banks follow? This remains an unsettled question, although considerable literature exists on the subject. This chapter is motivated by the limitations of the traditional monetary policy rule, particularly the narrow operating space in recent years due to the ZLB issue. In this chapter, we examine whether an interest rate policy targeting nominal GDP together with a fiscal backstop to prevent ZLB could improve macroeconomic stability in the UK.

The first section discusses conventional monetary policy, unconventional monetary policy, and monetary policy in the UK. The later section shows how nominal GDP targeting regime works and provides arguments supporting it. Then, we briefly describe why fiscal backstop to prevent the ZLB needs to be brought into the model. The subsequent section presents the empirical investigation and results. Finally, a conclusion of this chapter.

9.1 Conventional and Unconventional Monetary Policies

9.1.1 Inflation Targeting and Quantitative Easing

Since the early 1990s, the prevailing view among central banks and economists has been that the best monetary policy is inflation targeting. Inflation targeting was first introduced by New Zealand in March 1990 and quickly followed by Canada in February 1991 and the UK in October 1992 after its withdrawal from the European ERM. According to Fisher (2019), 67 central banks targeted inflation in 2018: some were just a number, and others were a range or a maximum, with most developed countries having an annual inflation target of around 2%. See Figure 9-1, where Fisher (2019) shows that 20 countries had a target or ceiling of 2% or 2.5% in 2018, including most developed countries (the euro area counts only one).

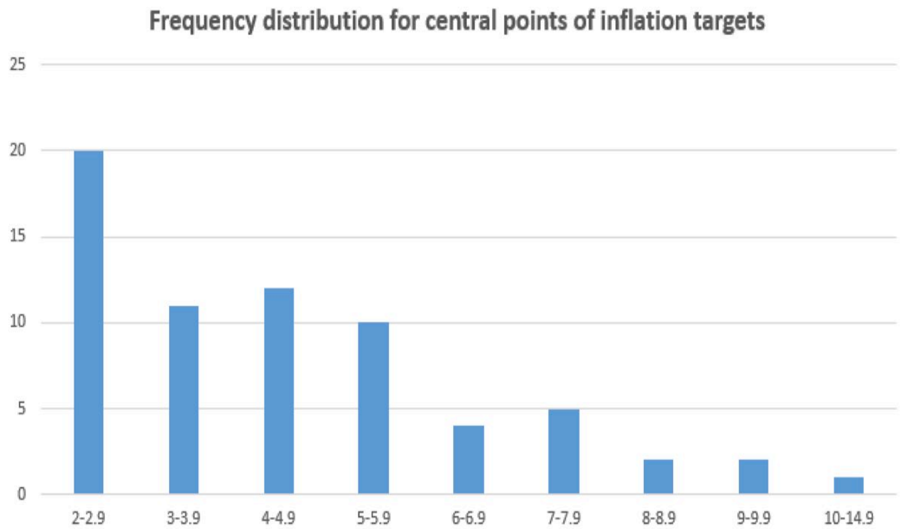

Figure 9-1: The Number of Countries Targeting Each Inflation Range

Source: Fisher (2019)

Traditionally, Inflation-targeting central banks use policy rates to react to changes in inflation and output. For example, during a recession, policymakers would lower the policy rate to stimulate spending with the aim of bringing inflation back to its target and the economy to its desired level. During the Great Moderation, inflation targeting prevailed for almost two decades. It worked relatively well because it was relatively simple to compute and practically enforceable, and helpful in preventing the recurrence of high inflation. However, the recent Great Recession has revealed that the inflation targeting rule is insufficient to cope with big shocks to the economy and foster a robust recovery, and may even result in financial instability and welfare losses because monetary policy is too tight. Several disadvantages of the inflation targeting rule are outlined below:

1. Inflation targeting rule can react well to small demand shocks but has proved less effective in coping with large demand shocks, such as the Great Recession. In addition, inflation targeting fails when there are supply shocks (e.g., technology, oil supply or labour force fluctuations). For example, if a negative supply shock comes such as a reduction in the supply of oil, it would move inflation and output in opposite directions: a rise in inflation and a fall in output. The central bank would tighten monetary policy due to the excessive inflation to bring inflation back to the desired target. However, this would further limit economic activity and plunge the economy into a deeper economic recession at a time when the economy is already struggling to deal with the effects of higher commodity prices (see Figure 9-2).

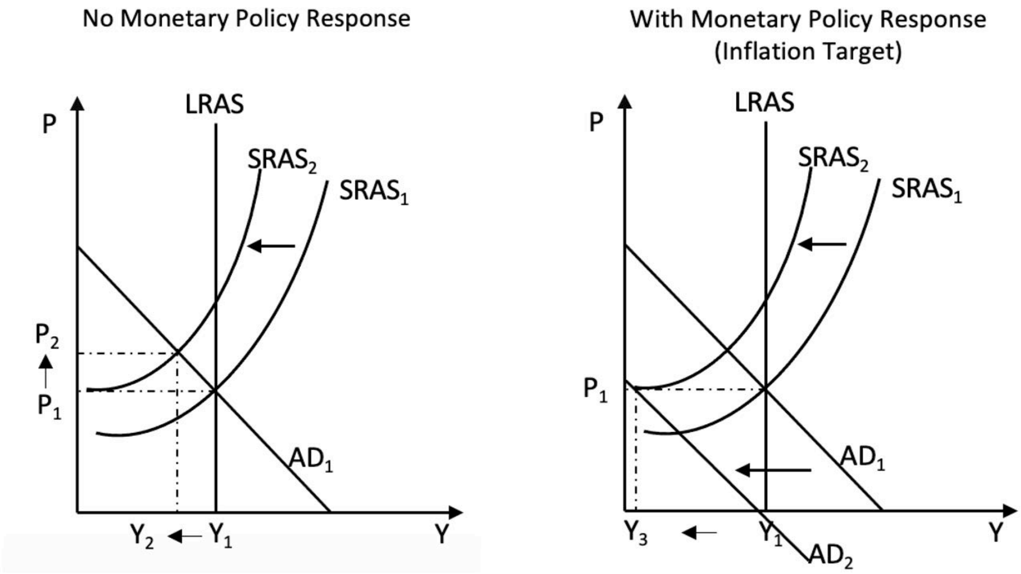

Figure 9-2: Temporary Negative Supply Shock

2. Many central banks responded to the recent Great Recession by cutting their nominal interest rates dramatically to near zero, leaving many countries at ZLB. A few even allowed their interest rates to fall to negative values, such as Japan, Denmark, Sweden, Switzerland and the euro area. With ZLB rates or negative interest rates, central banks can no longer use conventional monetary policy to stimulate the economy, which poses a serious obstacle to conventional monetary policy (see Figure 9-3: conventional monetary policy is effective in

normal times but ineffective when constrained by the ZLB). NK economists argue that there is a liquidity trap in this period.

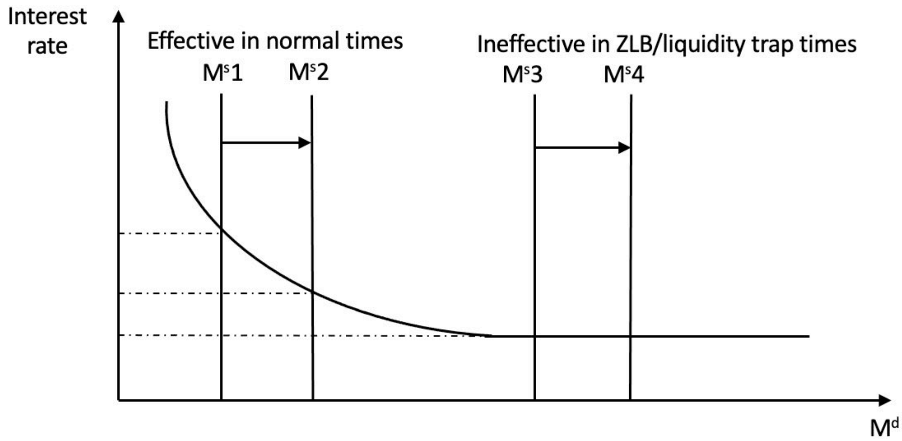

Figure 9-3: Expansionary Conventional Monetary Policy during Normal and ZLB Times

3. During the Great Recession, the inflation fluctuations were relatively mild. However, output fluctuations were large, implying that the current inflation targeting rule has too much effect on inflation stabilisation but too little effect on output stabilisation (Sumner, 2011).

During the Global Financial Crisis, the conventional monetary stimulus methods of most central banks were exhausted as short-term nominal interest rates were stuck at the ZLB. As a result, the failure of the conventional monetary policy triggered the unconventional monetary policy (UMP) tool, QE, to inject liquidity into the banking system and encourage more credit creation, thereby spurring economic activities. QE was introduced through large-scale domestic asset purchase programmes (usually consisting of long-maturity government bonds, private assets, and mortgage-backed securities), as well as liquidity provision and refinancing operations with commercial banks. QE was first implemented by the Bank of Japan in 2001, and then after the GFC, the use of QE became more widespread around the world. For example, the US, the UK, and the Eurozone all engaged in multiple rounds of large-scale asset purchase programmes. In addition to the UMP, several economists like Ball (2014) and Blanchard et al. (2010) advocate the adoption of a higher inflation target such as 4% to ease the monetary policy constraints of the ZLB. However, some studies have found that a high inflation target would cause significant costs and destabilise the macroeconomy; see, for examples, Ascari et al. (2018), Coibion et al. (2012), Ascari and Sbordone (2014) and Kara and Yates (2021). These papers provide a warning to the proposal of targeting inflation at 4%.

9.1.2 An Overview of the UK Monetary Policy since 2009

The UK adopted the inflation targeting in October 1992. During the period 1992-2009, the main instrument of monetary policy in the UK was the BoE's bank rate. From 2009 to the present, the UK's monetary policy experience can be divided into four broad phases: the Great Recession, the EU referendum, the Covid-19 pandemic and the cost-of-living squeeze.

During the Global Financial Crisis, the BoE cut its bank rate sharply from 5% to 0.5% in less than a year (see Figure 9-4) with the aim of achieving the government's 2% inflation target and stimulating economic activity. However, this conventional monetary policy intervention was ineffective in boosting spending and economic growth during the crisis; therefore, further monetary stimulus was needed. However, with a low bank rate of 0.5%, the effectiveness of any additional bank rate cuts would not be significant due to the ZLB constraint. As a result, The BoE began to use the UMP tool (QE) to further stimulate the economy. This corresponded to large-scale asset purchases such as long-term government bonds, which were financed through the issue of newly created, interest-paying reserves. Large-scale purchases of illiquid assets will lower the interest rates on savings and loans, thereby stimulating aggregate economic activity. From 2009 to 2012, the BoE purchased seven rounds of QE, buying a total of £375 billion worth of bonds by July 2021 (House of Commons, 2016).

In August 2016, in response to market uncertainty following the 2016 EU referendum, the BoE cut interest rates from the already low 0.5% to 0.25% (see Figure 9-4) for the first time in seven years since the Global Financial Crisis in order to encourage spending and investment. The BoE also restarted QE with a further injection of £70 billions of asset purchases, including £60 billions of government bonds and £10 billions of corporate bonds, expanding the total QE programme to £445 billions (House of Commons, 2016).

At the start of 2020, in the face of the effects of Covid-19, the BoE cut interest rates from 0.75% to 0.25% on 11 March and then again to 0.1% a few days later on

19 March to support demand, with the new record low rate of 0.1% remaining in place until December 2021 (see Figure 9-4). After cutting interest rates twice, in order to meet its 2% inflation target and decrease the borrowing costs for households and businesses, the BoE made three rounds of asset purchases in March, June and November 2020, adding a total of £450 billion worth of assets. This brought the total value of its QE programme to a peak of £895 billion (including £875 billions of government bonds and £20 billions of corporate bonds), which is equivalent to around 40% of the UK GDP (Bank of England, 2021). Figure 9-6 summarises the BoE's Monetary Policy Interventions between 2009-2021. During Covid-19, the BoE not only purchased a large amount of assets, but the government also implemented a highly expansionary fiscal policy. In most developed countries, fiscal policy has been central to the Covid-19 recovery response.

In 2022, inflationary pressures in the UK increased significantly due to Russia's restrictions on gas supplies to Europe and the UK and the risk of further restrictions. In August 2022, inflation reached 10.1% for the first time in 40 years as food and energy prices continued to soar (see Figure 9-7), signalling the return of the threat of stagflation in the UK for the first time since the 1970s. From December 2021 to August 2022, the UK bank rate was raised six times, from 0.1% to 1.75%, with the aim of bringing inflation back to the 2% target level, particularly in August 2022 when the bank rate was raised by 0.5%, from 1.25% to 1.75% (see Figure 9-4) this was the single-highest interest rate jump since 1995, leaving the bank rate at its highest level since December 2008. In addition to increasing the bank rates, the BoE's QE programme has entered a reversal phase, also known as 'quantitative tightening'. In February 2022, the BoE passively reduced the size of its QE programme by stopping reinvesting maturing assets (Bank of England, 2022b). Furthermore, according to Bank of England (2022b), which was last updated on 19 May 2022, the BoE has begun to consider reducing its asset stock via a combination of passive quantitative tightening and active asset sales.

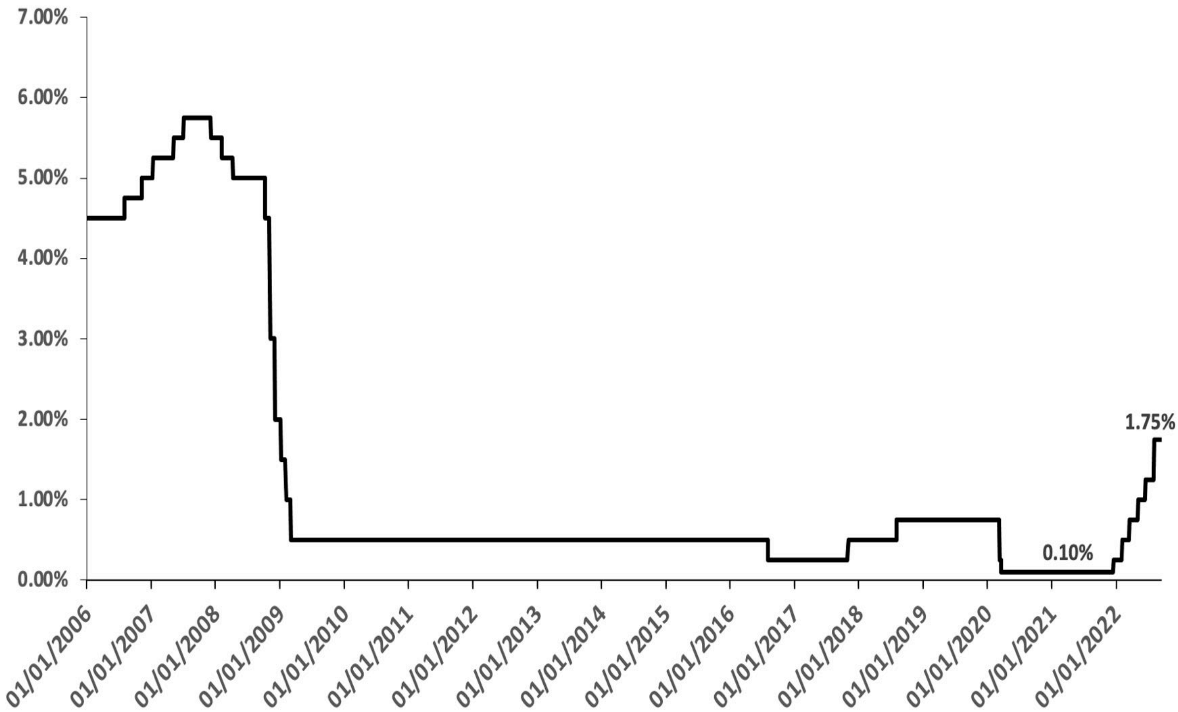

Figure 9-4: UK Official Bank Rate History

Data Source: Bank of England

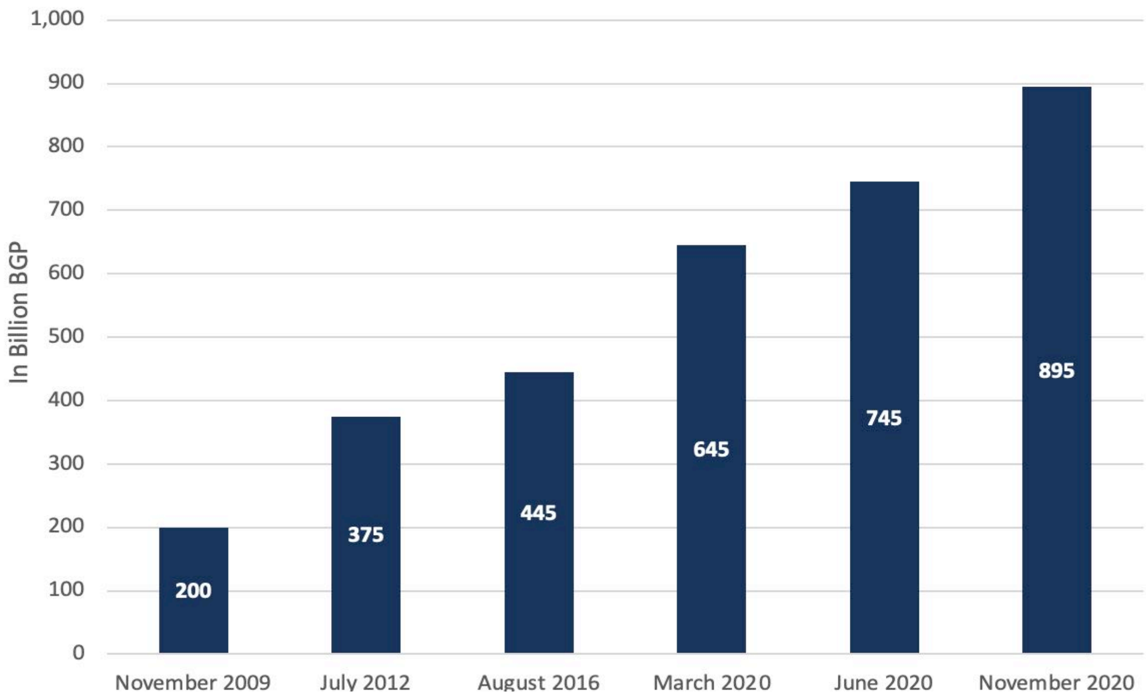

Figure 9-5: BoE Purchases of Bonds Built Up Over the Year

Data Source: Bank of England

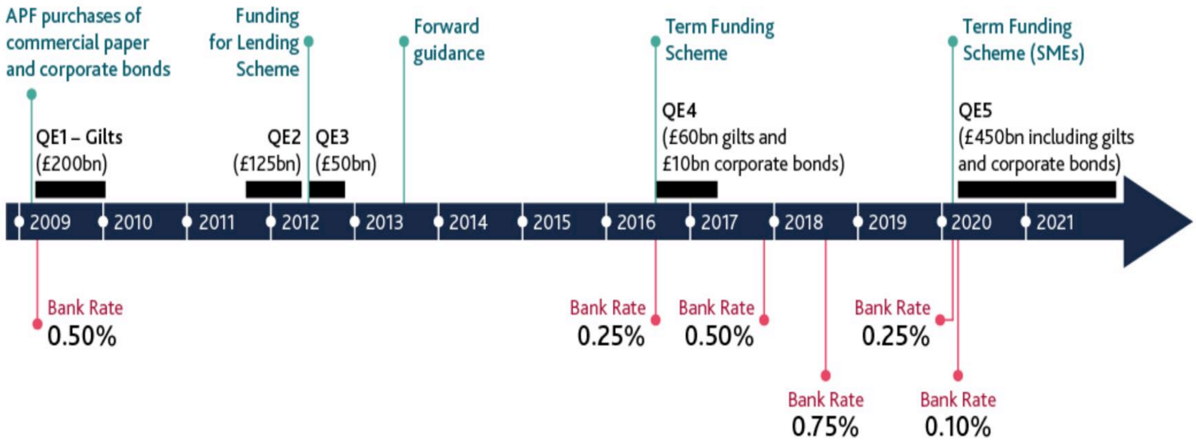

Figure 9-6: The BoE Monetary Policy Interventions Between 2009-2021

Source: Bank of England (2021)

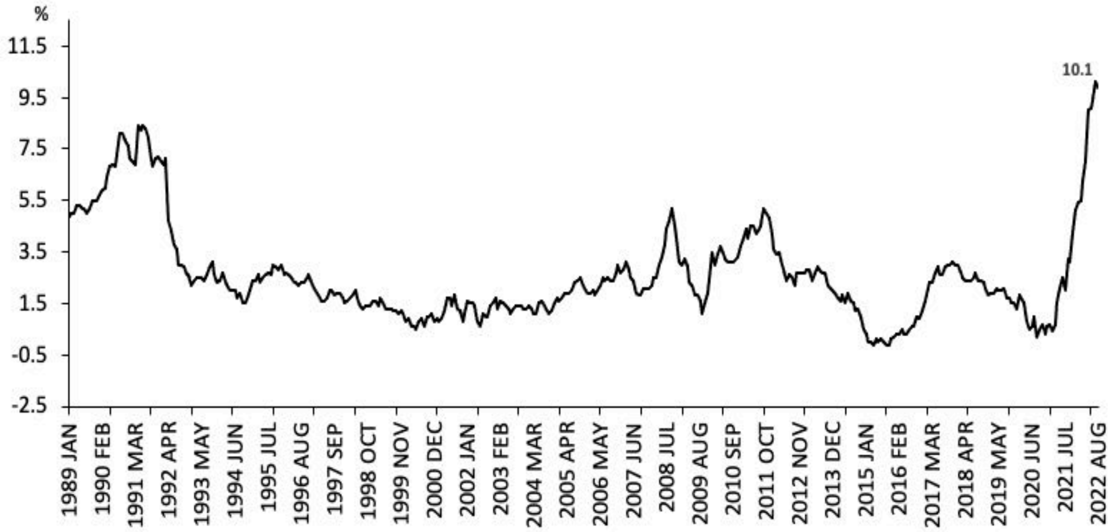

Figure 9-7: UK Consumer Price Inflation

Data Source: Office for National Statistics

9.1.3 Review of Empirical Literature: The Macroeconomic Effects of Unconventional Monetary Policy

Between 1992 and the GFC, the UK's conventional monetary policy showed good macroeconomic outcomes, such as low and stable inflation and low volatility in real GDP growth. However, for most of the period 2009-2022, conventional monetary policy has been constrained by the ZLB. Thus, it has not been effective in boosting the economy in the face of a series of large shocks: the GFC, the EU referendum and the Covid-19 pandemic. In contrast, UMP has improved the BoE's ability to respond to these large shocks by lowering long-term interest rates, and it has created substantial policy 'space' (Bank of England, 2022a). There is a broad consensus in the literature that QE is effective as a tool to stabilise financial markets; see Meier (2009), Breedon et al. (2012), Joyce et al. (2012), Gagnon (2016) and Haldane et al. (2016) for examples in the UK, and see Dell'Ariccia et al.(2018) for a review of the empirical literature on the impacts of QE in the euro area, the UK and Japan. In addition to examining the impact of the UMP on financial markets, a growing literature has attempted to empirically investigate the macroeconomic impact of UMP.

Although the evidence on the macroeconomic impact of the UMP in the previous literature is mixed, in general most agree that UMP instruments have a positive impact on output and inflation, but there is less agreement on the magnitudes of the impact. See, for example, Giannone et al. (2012), Pagliari (2021) and Hohberger et al. (2019) for the euro area, Chung et al. (2011), Chen et al. (2012) and Wu and Xia (2016) for the US, Girardin and Moussa (2011) for Japan. With regard to the UK, there are also a number of empirical studies investigating the effectiveness of QE in supporting the UK macroeconomy.

Several papers have studied the macroeconomic impact of the first round of QE by the BoE (QE1), which started purchasing £200 billions of gilts in March 2009. Kapetanios et al. (2012) examined the output and inflation impacts of the QE1 by using three VAR models with different structural changes: a large Bayesian VAR, a change-point structural VAR, and a time-varying parameter VAR. Their empirical results suggest that without the QE1, real GDP would have fallen more during 2009, and inflation would have become lower or even entered deflation. Their average estimate of the three models suggests that the peak impact of QE on real GDP is likely to be about 1.5% and on annual inflation about 1.25%. Bridges and Thomas (2012) also estimated the impact of the QE1 on inflation and GDP applying a simple money demand and supply framework. They found that QE1 has a positive effect on GDP (peak impact of around 2%) and inflation (peak impact of around 1%) through decreasing yields and raising asset prices to stimulate demand. Falagiarda (2014) built a DSGE model to investigate the impact of the QE1, and the results of their calibrated model indicate that the QE1 has a peak impact on real GDP of 1.25% and on inflation of 0.49%.

Churm et al. (2018) used a Bayesian VAR model to study the second round of QE purchases (QE2) by the BoE between October 2011 and June 2012, when it purchased £175 billion worth of gilts in response to the euro area sovereign crisis. They found a significant positive impact of the QE2 on inflation, which has an inverted U-shape with a peak of 0.6%, and a significant positive impact of the QE2 on GDP growth of around 0.5-0.8%.

Weale and Wieladek (2016) examined the impact of QE programmes on the CPI and real GDP in the UK and the US using a Bayesian VAR model and data from the period March 2009 to May 2014. They found that a QE announcement shock worth of 1% of nominal GDP causes a statistically significant increase in real GDP and CPI of 0.25%(0.58%) and 0.32%(0.62%) in the UK(US), respectively. Table 9-1 summarises the magnitude of the impact of QE on output and inflation in the above literature examining the UK.

However, in contrast to the above literature, Salachas et al. (2018) studied the UMP for the euro area, Japan, the UK and the US by using a VAR model for the period January 1999 to October 2015. Their results for the UK show an upward impact of QE on economic activity; however, there is no evidence regarding its impact on prices. Balatti et al. (2016) studied QE programmes in the US and the UK using a Bayesian VAR model and using UK data from 1975-2015. They found that QE only has a significant impact on financial variables, but not on UK output and inflation.

Overall, most studies have found that QE has a stimulating effect on both output and inflation and is an effective monetary policy tool. However, the reality is that despite massive injections of QE and the ZLB interest rate, recovery from the Great

Recession had been slow in advanced economies including the UK. This has reinvigorated economists' interest in targeting nominal GDP.

In our model, in normal times, interest rates follow the Taylor Rule, and M0 acts only to accommodate M2. However, when interest rates reach the ZLB threshold, the model switches to a crisis regime where the Taylor Rule is suspended and replaced by a lower fixed exogenous rate; meanwhile, QE is incorporated into the model with M0 targeting the credit premium around its steady state to restore credit condition to normal through its role of providing cheap collateral. However, despite the intervention of the QE policy, the volatility of inflation during the ZLB periods appears to be beyond the control of monetary policy (see Figure 9-9 in Section 9.3). For this reason and because of the limitations of the conventional monetary policy rule we mentioned above, we examine an alternative policy rule regime, i.e. an interest rate policy targeting nominal GDP together with a fiscal backstop to prevent ZLB, to see whether such regime can improve macroeconomic outcomes in the UK compared to the Taylor rule with a QE policy regime.

Table 9-1: Output and Inflation Impacts of QE in the UK

	Notes	Output	Inflation
Bridges and Thomas (2012)	Impact of QE1	+2%	+1%
Falagiarda (2014)	Impact of QE1	+1.25%	+0.49%
Churm et al. (2018)	Impact of QE2	+0.5%-0.8%	+0.6%
Kapetanios et al. (2012)	Impact of a 100-basis point reduction in government bond yields	+1.5%	+1.25%
Weale and Wieladek (2014)	Impact of a QE announcement shock worth of 1% of nominal GDP	+0.25%	+0.32%

9.2 Alternative Monetary Policy Rule: Nominal GDP Targeting

Due to the failure of conventional monetary policy at the ZLB and the slow recovery from the Great Recession, market monetarism (which is a macroeconomic theory that suggests central banks adopt nominal GDP level target to stabilise nominal incomes) became popular. There is a rapidly growing literature suggesting that shifting the policy regime to nominal GDP targeting would improve monetary policy compared to the inflation targeting regime (for example Frankel, 1995, 2013, 2014; Sumner, 2011, 2014, 2017, 2018; Hendrickson, 2012; Woodford, 2012; Belongia and Ireland, 2015; Garín et al., 2016; Le et al., 2016, 2021; Sumner and Roberts, 2018; Beckworth and Hendrickson, 2020).

Nominal GDP targeting refers to a rule that targets the level or growth of nominal GDP. It involves the use of a simple feedback rule whereby the central bank adjusts policy rates in response to deviations in nominal GDP from the target. Nominal GDP targeting can be adopted in terms of growth rates and levels. In the case of nominal GDP growth targeting, the central bank seeks to meet a constant growth rate of GDP, and it is a purely forward-looking approach that does not consider past misses. Proponents of nominal GDP growth rate targeting include McCallum (1988; 2015), McCallum and Nelson (1999) and Orphanides (1999), among others. Nominal GDP level targeting is conceptually appealing because the level target, on the other hand, means that policy is history-dependent and must make up for any past overshoots or shortfalls in economic activity to bring nominal GDP back to the fixed path. Proponents of nominal GDP level targeting include Sumner (2011, 2014, 2017, 2018), Hendrickson (2012), Woodford (2012), among others. The version of nominal GDP targeting that has prevailed in recent years is the level targeting rather than the growth rate targeting. A comparison of the level targeting and growth rate targeting is shown in Figure 9-8. In the following discussion, we focus on the nominal GDP level targeting.

There are theoretical reasons and empirical evidence to suggest that nominal GDP targeting could improve the performance of monetary policy compared to inflation targeting. We first present several theoretical advantages of nominal GDP targeting and then review some empirical evidence.

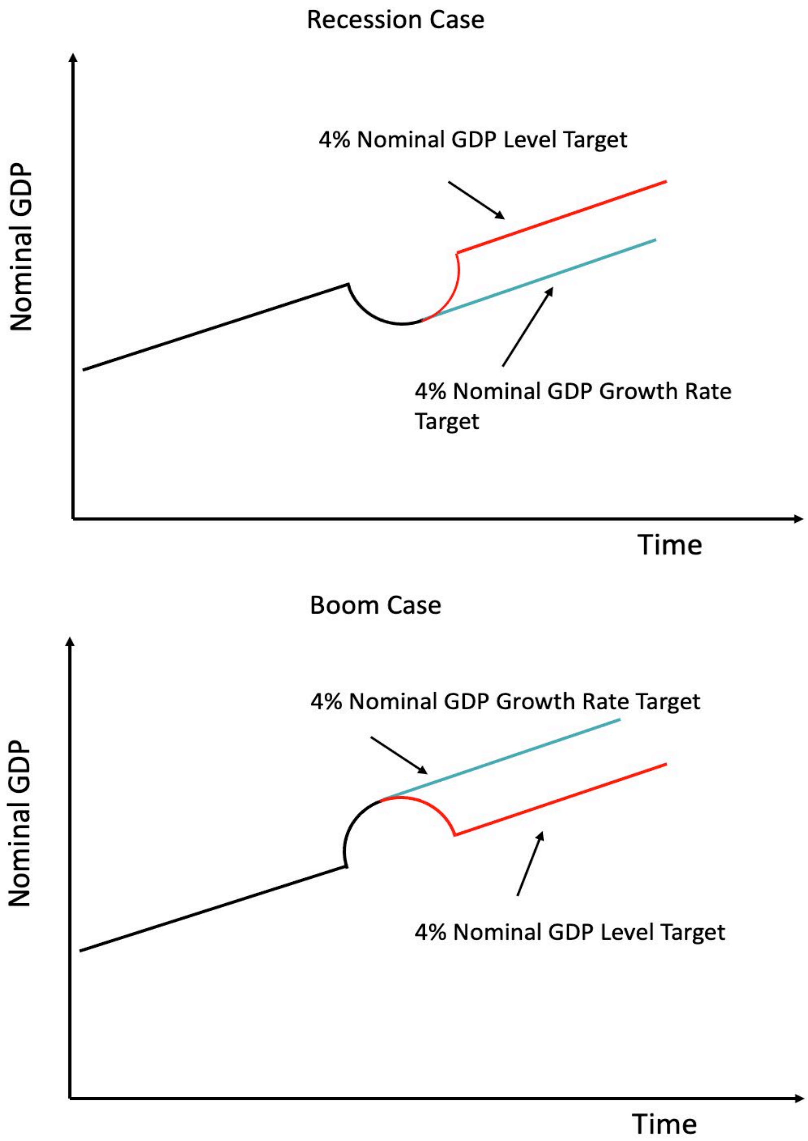

Figure 9-8: Nominal GDP Level and Growth Rate Targets

9.2.1 Theoretical Reasons for Calling for Nominal GDP Targeting

Nominal GDP level target rule, $r_t = \rho_1 r_{t-1} + \rho_y\left(y_t + p_{d,t} - \bar{y}_t - \bar{p}_d\right) + e_t$, where $y_t + p_{d,t} - \bar{y}_t - \bar{p}_d$ is the deviation of nominal GDP from the target, is a combination of a stronger response to output gap $(y_t - \bar{y}_t)$ plus a domestic price level target $\left(p_{d,t} - \bar{p}_d\right)$ in place of an inflation target. The first implies more output stability. The second produces a more persistent response on interest rates to inflation shocks as it gets back to the same level, i.e. it produces a forward guidance effect and is more strongly in stabilising inflation. Theoretically, the relative advantages of nominal GDP targeting versus inflation targeting are as follows:

1) Improving response to demand shocks

   There will be no dilemma under demand shocks; both price and output require the same interest rate response. As a result, nominal GDP targeting will stabilise both inflation and output more strongly due to the persistence/forward guidance effect.

   Nominal GDP targeting will respond to demand shocks by pushing the level of prices and output back to their target levels. This requires action into the future. Nominal GDP targeting can lead to a faster recovery in output to the desired level and correct for past 'miss'. Assuming a recession scenario, a nominal GDP targeting regime would require a more expansionary monetary policy (nominal GDP would be temporarily above the growth rate path) in order to push nominal GDP back to its target level; thus, the policy would have to correct for any past shortfalls in path target. However, the response of an inflation-targeting regime is to provide enough stimulus to bring the inflation rate back to the target and maintain it there. We empirically compared the responses of the nominal GDP target and inflation target to demand shocks; the results show that the nominal GDP target gives more stability to both inflation and output in the presence of merely demand shocks compared to the Taylor Rule (see table 9-2).

   Table 9-2: Stability Comparison in the Case of Only Demand Shocks

	Var(y)	Var(pai)
Taylor Rule	0.0011	8.88e-05
Nominal GDP Targeting	2.04e-04	6.92e-05

2) Improving the response of inflation to supply shocks

   There is a dilemma under the supply shocks: price rises, and output falls. We have shown earlier that one of the setbacks of inflation targeting is inappropriate responses to supply shocks; it has the problem of excessive easing/tightening in response to positive/negative supply shocks. As a result, it would induce more severe booms/recessions when responding to positive/negative supply shocks. In terms of the response of nominal GDP targeting to supply shocks, it responds to both price and output. However, it needs to be checked empirically; it could stabilise inflation but may worsen output. Because the nominal GDP target creates persistence in the interest rate response with a forward guidance effect, this is a powerful stabiliser of current inflation. It could worsen output response by stabilising current inflation more. We check this empirically. The empirical findings demonstrate that for merely supply shocks, the nominal GDP target stabilises inflation but worsens the output response compared to the Taylor Rule (see table 9-3).

Table 9-3: Stability Comparison in the Case of Only Supply Shocks (Productivity Shocks and Labour Supply Shocks)

	Var(y)	Var(pai)
Taylor Rule	3.60e-05	0.0013
Nominal GDP Targeting	4.13e-05	7.22e-04

3) Creating more financial stability

Nominal GDP targeting could improve financial stability as it might prevent defaults. If aggregate income can be kept on a growth path by a nominal GDP targeting regime, then income would not decrease as much during a recession. This would allow people to continue to repay their loans and avoid defaults and bankruptcies.

There are also some criticisms of nominal GDP targeting; for example, nominal GDP data are released quarterly and are usually subject to considerable revisions in some countries. In contrast, inflation data are released more frequently and are rarely revised, so it is easy to see if the target is being met. Therefore, the frequency of release of nominal GDP data and large revisions are the major challenges in implementing the nominal GDP targeting rule. Another criticism is that if a country switches to nominal GDP targeting regime, but the public does not understand or pay any attention to it, then it will not have any effect. However, this argument could be raised against any alternative policy rule.

9.2.2 Empirical Evidence Supporting the Targeting of Nominal GDP

As nominal GDP targeting appears to be theoretically superior, a growing empirical literature has examined the performance of nominal GDP targeting. Beckworth and Hendrickson (2020) compare the variances of inflation and the output gap under the nominal GDP level targeting rule and Taylor Rule using an amended New Keynesian DSGE model, which assumes that the central bank has imperfect information about the output gap and therefore must forecast the output gap based on knowledge of past information. Using a Bayesian method, they estimate the model to fit the US quarterly data over the period 1987Q3-2007Q3. They find that forecast errors in the output gap can cause output fluctuation of 13%. Furthermore, their simulation results suggest that nominal GDP targeting rule improves welfare by stabilising both inflation and the output gap compared to the Taylor Rule.

Garín et al. (2016) use a New Keynesian model with price and wage rigidities to investigate the welfare properties of the Taylor Rule, nominal GDP level targeting, inflation targeting and output gap targeting. The model is estimated using the Bayesian method to fit the US data for the period 1984Q1-2007Q3. They find that nominal GDP targeting significantly outperforms inflation targeting and produces fewer welfare losses than the Taylor Rule, especially when the economy is subject to supply shocks and when wages are stickier than prices. In addition, although there is not much difference in terms of welfare losses between nominal GDP targeting and output gap targeting, nominal GDP targeting is likely to outperform output gap targeting when the central bank has difficulty assessing the output gap in real-time.

Fackler and McMillin (2020) use a different approach from Beckworth and Hendrickson (2020) and Garín et al. (2016) to assess the welfare losses of nominal GDP targeting, inflation targeting and price level targeting. They use a VAR model to analyse the policy rules within a policy framework of constrained discretion. The model is estimated to fit the US quarterly data over the period 1979Q4-2003Q4. They compare the policy rules by using a loss function for the period 2004-2006. Although they use a very different approach, their findings generally support the literature mentioned above; they find that nominal GDP targeting is preferable to either price level targeting or inflation targeting by reducing losses over the simulation period.

The three empirical studies above only cover data from the Great Moderation and do not consider the impact of the GFC. Benchimol and Fourçans (2019) estimate a DSGE model with twelve different monetary policy rules using a Bayesian method and the US data over the whole period 1955-2017 and three different sub-periods:1955-1985, 1985-2007 (the Great Moderation) and 2007-2017 (ZLB). The twelve monetary policy rules include four Taylor-type rules, four nominal GDP growth rules and four nominal GDP level rules. They evaluate the performance of these twelve monetary policy rules and find several results. In terms of fit to actual data, one nominal GDP level targeting rule has the best match with actual data during the Great Moderation and the ZLB periods; one nominal GDP growth targeting rule has the best fit over the whole sample period; and one Taylor-type rule has the best fit over the period 1955-1985. With respect to the current losses of the central bank, which is computed as a weighted sum of the variances of inflation, the output gap, interest rate changes and wage growth, the nominal GDP level targeting rules perform best in all periods except for the Great Moderation when the Taylor-type rules perform better.

Billi (2016, 2020) has some novel findings. Billi (2016) compares the welfare performance of a strict price level targeting with a nominal GDP level targeting in a small NK model that assumes that the central bank operates under optimal discretion and faces the ZLB. The model is calibrated using data from the US for the period 1985Q1-2014Q4. The author finds that if the economy experiences only temporary inflationary shocks, nominal GDP targeting is a better option as it spreads the effects of the shocks on output and prices. However, strict price level targeting could be preferable in the case of persistent demand and supply shocks, as it could trigger policy inertia and enhance the trade-offs faced by the central bank. Billi (2020) employs a New Keynesian model with staggered price and wage setting, with unemployment fluctuations and with ZLB constraint to evaluate the welfare performance of four monetary policy frameworks that include the Taylor Rule, nominal GDP level targeting, employment targeting and the optimal monetary policy with commitment. The author finds that the employment targeting is better if there are supply-side shocks. However, in the presence of the ZLB constraint and demand shocks, nominal GDP targeting performs significantly better than the Taylor Rule and employment targeting, especially when prices are sticky in relation to wages.

Le et al. (2016, 2021) use an extended hybrid DSGE model and an estimation method that differs from the extant literature – Indirect Inference - to evaluate the performance of nominal GDP targeting. Le et al. (2016) merge the NK and NC models into a hybrid model and extend the model by allowing for a bank sector, ZLB and QE. The model is estimated using the Indirect Inference to fit the US data over the period

1984-2011. They use the estimated DSGE model to evaluate alternative monetary policy rules. To compare different rules, they focus on the ability of the rules to reduce the number of crises and welfare costs. The welfare costs are calculated as a weighted sum of variances in consumption and hours of work and a weighted sum of the cycle variances of output and inflation. They find that nominal GDP targeting significantly outperforms the Taylor Rule. In addition, a simple rule that makes M0 respond to credit conditions can improve the stability of the economy. This rule combined with a price level or nominal GDP targeting would further stabilise the economy. Le et al. (2021) extend the Le et al. (2016) model to include state-dependence in price/wage duration and estimate the model for the period 1959-2017 using the Indirect Inference method. They compare nominal GDP targeting with the Taylor Rule using the estimated model and find that with fiscal policy backstop in stopping the ZLB, nominal GDP targeting can provide high levels of price stability and prevent severe output fluctuations.

These US studies generally indicate that nominal GDP targeting could be a desirable alternative to the current monetary policy framework. However, all of these studies use closed economy models. In what follows, we discuss some studies on nominal GDP targeting in the context of open economies.

There are a limited number of empirical studies on nominal GDP targeting in the context of open economies. Bhandari and Frankel (2017) argue that nominal GDP targeting is actually more applicable to developing countries and small open economies, which are subject to larger terms-of-trade shocks and supply shocks. They provide a theoretical model comparing the ability of alternative monetary policy rules to minimize the quadratic loss function of price stability and output stability. Through empirical testing, they find that nominal GDP targeting indeed outperforms other regimes such as inflation targeting. Fang (2021) extends Le et al.’s (2016) model to an open economy for Japan and finds that a fiscal ZLB-suppression regime along with nominal GDP targeting reduces the variance of inflation and output compared to a strong fiscal feedback policy together with the Taylor Rule. However, it does so at the cost of high variance in interest rate and consumption. In the UK context. Hatcher (2016) employs an overlapping generations model to investigate the impact of nominal GDP targeting on the tax burden in the UK. The author finds that nominal GDP targeting reduces the volatility of taxes but raises average taxes compared to inflation targeting. Moreover, the expected tax burden is minimized under nominal GDP targeting. Wang (2020) extends Le et al.'s (2016) model to a UK open economy and finds that nominal GDP targeting is more effective than inflation targeting in reducing welfare losses based on variance in output and inflation, and reducing economic and financial crisis.

Due to the limited number of empirical studies on nominal GDP targeting in small open economies, we also review the literature on price level targeting (PLT)[13]. A nominal GDP target is essentially the same as a PLT as it is effectively a combination of PLT with the output gap response. It should be noted that the PLT in the nominal GDP targeting framework is a domestic price level target rather than a CPI level target. Some studies on PLT in open economy contexts consider a CPI level target, but this could provide a reasonable reference on the domestic PLT, as changes in the CPI are closely related to changes in domestic prices. Thus, we discuss some studies on the effectiveness of PLT compared to inflation targeting in stabilizing the macroeconomy in open economies, particularly in the Canadian and UK contexts. Ortega and Rebei (2006) show that the welfare implications of switching from inflation targeting to PLT or a combination of both are negligible, using an estimated small open economy DSGE model for Canada with traded and non-traded sectors, as well as nominal price/wage rigidities. Coletti et al. (2008) use a two-country (Canada and the US) version of the IMF Global Economy Model (GEM) to show that PLT slightly outperforms inflation targeting in terms of reducing inflation and nominal interest rate volatility, although at the cost of somewhat high output gap variability. Using an estimated Canadian open economy New Keynesian model with imperfect credit markets from Bernanke et al. (1999), Dib et al. (2008) find that PLT significantly reduces distortions in the economy due to nominal debt contracts, as inflation expectations are more stable than under an inflation targeting regime, which in turn reduces real interest rate volatility. Dib et al. (2013) extend this model to include a response to the external finance premium, showing that PLT still outperforms inflation targeting. Coletti et al. (2008) find that PLT outperforms inflation targeting in terms of inflation and output gap stability in a

[13] The central bank's objective under a PLT is to stabilise the aggregate price level around a target price path. For instance, if an inflation shock causes the price level to rise above the target price path, subsequent below-average inflation will be needed to bring the price level back to the target. Under both nominal GDP targeting and PLT, 'bygones are not bygones' as past deviations from the target must be corrected. Both rules have a high degree of history dependence. The PLT element offers strong forward guidance as inflation deviations can lead to a long-lasting interest rate response.

calibrated GEM model for Canada, because terms-of-trade shocks strengthen the PLT as it is a nominal anchor for stabilizing the domestic price level. However, Coletti et al. (2012) find that PLT enhances stability relative to inflation targeting in response to non-energy commodity supply shocks, but not to energy commodity supply shocks and commodity demand shocks.

In the UK context, Batini and Yates (2003) demonstrate that the degree of openness of an economy is critical when comparing inflation targeting and PLT. In their small open economy model for the UK, fluctuations in the real exchange rate affect inflation variability. Due to the UIP condition introducing a channel for managing expectations on economic outcomes, PLT can have a positive effect on inflation stability. However, PLT could deteriorate economic stability relative to inflation targeting, as it increases interest rate volatility, which feeds into greater real exchange rate volatility through the UIP condition. They conclude that the relative benefits of PLT, inflation targeting and the mix of the two depend on particular modelling and policy assumptions. Rysbayeva (2020) employs an open economy DSGE framework for the UK that includes imperfect exchange rate pass-through, and finds similar welfare losses under PLT and different kinds of inflation targeting (core inflation versus CPI inflation). The loss function under PLT is only slightly lower. More recently, Dong et al. (2023) show that PLT improves inflation stability under both UK rational expectations and boundedly rational DSGE models compared to inflation targeting. On the output gap side, PLT increases stability in the rational model but slightly decreases stability in the bounded rational model. For more papers on PLT, please refer to Hatcher and Minford (2014) for a review of theoretical and empirical evidence on PLT.

Overall, the literature above suggests that nominal GDP targeting and PLT could be effective monetary policy frameworks in closed and open economies. However, the relative performance of nominal GDP targeting and inflation targeting may depend on the specific characteristics of the model and the shocks faced by the economy. Given many commonalities in the structure of the economies between the US, Japan and the UK, it may be reasonable to expect that the empirical findings of Le et al. (2016, 2021), Wang (2020) and Fang (2021) could provide a basis for this UK-based study. Thus, we expect the nominal GDP targeting to perform better than inflation targeting in stabilising inflation and output. But empirical investigation is required to confirm this.

9.3 Fiscal Policy

During periods when interest rates are constrained at ZLB, monetary policy no longer has the power to stabilise inflation. In this thesis, inflation is more volatile in the state-dependent model than in the fixed duration model during ZLB episodes. This is because the inability of monetary policy to stabilise inflation during the ZLB periods causes price stickiness to become volatile in the state-dependent model, which further exacerbates price volatility and hence inflation volatility. As a result, ZLB events in the state-dependent model can lead to more significant inflation fluctuations.

Despite the intervention of the QE policy, the volatility of inflation during the ZLB periods appears to be beyond the control of monetary policy. Figure 9-9 shows the volatility of inflation during the ZLB periods, as simulated by the state-dependent model.

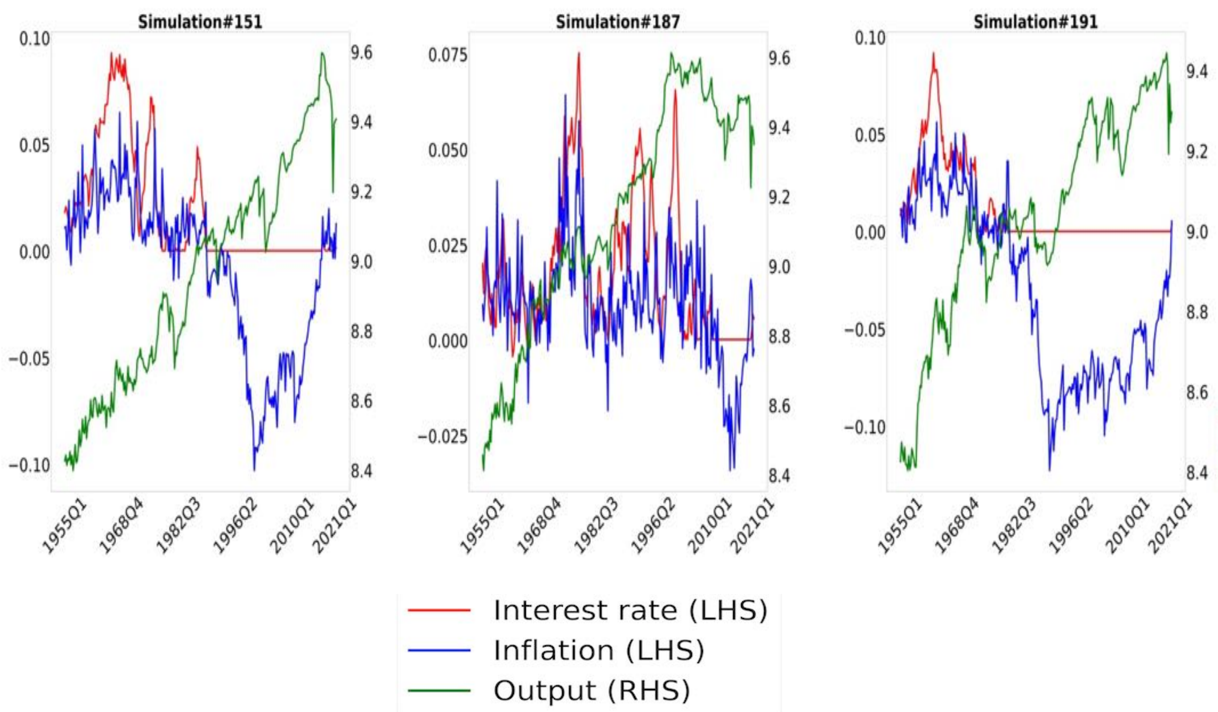

Figure 9-9: Examples of Simulation – Inflation Fluctuations During ZLB

At the ZLB, fiscal policy is essential to stimulate demand because it injects spending directly into the economy. This injection of demand raises economic growth. When facing a liquidity trap, Yao (2021) finds that fiscal stimulus can help the UK economy escape the ZLB and improve social welfare by using a UK open economy DSGE model with financial frictions. Blanchard et al. (2010) use a DSGE model to empirically examine the macroeconomic consequences of fiscal expansionary shocks in four major eurozone economies (France, Germany, Italy, and Spain) and find they can effectively escape from liquidity trap through fiscal policy interventions in the form of government purchases.

There is an increasing recognition that monetary policy alone cannot bring the economy out of the liquidity trap and achieve price and economic stability. Instead, as Keynes (1936) argued, monetary and fiscal policies need to work together to reflate the economy. At the ZLB, fiscal coordination is more essential than ever; please see most recent discussions, including but not limited to Portes and Wren-Lewis (2015), Praščević and Ješić (2019), Bhattarai and Egorov (2016) and Nasir (2020). However, such a fiscal-monetary coordination framework seems challenging to implement in the Euro area as its institutional framework is based on a monetary union without a fiscal union (Coeure, 2015; Hettig and Muller, 2015). In the case of the UK, fiscal-monetary coordination may not encounter this obstacle (Nasir, 2020). Le et al. (2021) and Fang (2021) consider nominal GDP targeting together with a fiscal ZLB-suppression regime in DSGE models for the US and Japan, respectively. They both find that this monetary-fiscal coordination can stabilise inflation and output, while Fang (2021) shows that it comes at the cost of high variance in interest rate and consumption.

Due to significant inflation fluctuations in ZLB events in the state-dependent model, we combine the nominal GDP targeting with fiscal ZLB-suppression to stabilise inflation during ZLB episodes following Le et al. (2021). This fiscal ZLB-suppression rule does not interfere with the stabilising role of monetary policy. Once the monetary policy is constrained by the ZLB, fiscal policy acts as a backstop against the ZLB, bringing back monetary policy by terminating ZLB episodes quickly when they occur. Because our model is similar in structure to those of Le et al. (2021) for the US and Fang (2021) for Japan, we expect lower variance in inflation and output in this UK study under the alternative policy.

In practice, in the face of the Covid-19 Pandemic, governments around the world, particularly in advanced economies, engaged in massive fiscal support programmes aimed at mitigating the health and economic impact. In the UK, in response to the impact of the Covid-19, the BoE cut its bank rate to a new record low rate of 0.1% in March 2020 and purchased a total of £450 billion worth of assets in 2020. Furthermore, the UK government also implemented a highly expansionary fiscal policy, see Keep and Brien (2021) for different estimates of total government spending on the Pandemic.

In the next section, when we examine the macroeconomy effects of nominal GDP targeting rule, we also introduce fiscal policy into the model as a backstop against the occurrence of ZLB to further stabilise inflation during ZLB episodes.

9.4 Empirical Investigation and Results

The objective of this section is to investigate whether a shift to a nominal GDP targeting regime together with the fiscal backstop could improve macroeconomic stability in the UK using our estimated state-dependent model. We examine this question by evaluating the performance of 'a nominal GDP targeting rule with a fiscal backstop' regime relative to our baseline regime (i.e. the Taylor Rule in non-crisis times and a QE policy in crisis times) in terms of stabilising the economy. In the comparison, we consider the following criteria: 1) how many crises are likely to occur under the 'Taylor Rule + QE policy' and the 'nominal GDP targeting rule + fiscal backstop', respectively? This measures the ability of policy to avoid crises; 2) the variance of inflation; 3) the variance of output around a measure of trend output[14]; 4) the welfare cost, which is calculated as a weighted sum of the cycle variances of output and inflation.

Nominal GDP Targeting Rule

To measure whether the implementation of a nominal GDP targeting rule helps to stabilise the economy, in the model we replace the Taylor Rule with the following nominal GDP targeting rule:

$$r_t = \rho_1 r_{t-1} + \rho_y\left(y_t + p_{d,t} - \bar{y}_t - \bar{p}_d\right) + e_t \tag{9.1}$$

Where $\bar{y}_t + \bar{p}_d$ represents the target for nominal GDP, $y_t + p_{d,t}$ is the nominal GDP, $y_t + p_{d,t} - \bar{y}_t - \bar{p}_d$ is the deviation of nominal GDP from the target, $\bar{y}$ follows the real output generated by productivity, $\bar{p}_d$ as steady price level is assumed to be constant

[14] The trend output measure is constructed as the balanced growth path plus the simulated productivity shocks.

and normalised to zero as in Le et al. (2021), and the parameter ρ_y is the partial elasticity of interest rate with respect to the nominal GDP deviation.

Nominal GDP targeting is a combination of a stronger response to $(y_t - \bar{y}_t)$ plus a domestic price level target $\left(p_{d,t} - \bar{p}_d\right)$ in place of an inflation target. The first implies more output stability, while the second creates a more persistent response as it gets back to the same level. This creates an effect from a future response like ‘forward guidance’.

Fiscal Backstop

In our baseline model, when ZLB occurs, the Taylor Rule is suspended and replaced by an exogenous low bound; meantime, a QE policy is incorporated into the model where M0 targets the credit premium around its steady state in an attempt to restore credit condition to normal through its role of providing cheap collateral. Now, to avoid ZLB episodes, we introduce a fiscal backstop; specifically, when ZLB occurs, we add a government spending shock of sufficient size to push the interest rate out of the ZLB.

Recall our baseline policy regime:

$$e_t^g = \rho_g e_{t-1}^g + \sigma_{ga}\eta_t^\alpha + \eta_t^g, \qquad \eta_t^g \sim N(0, \sigma_g^2) \quad \eta_t^a \sim N(0, \sigma_a^2)$$

Where e_t^g is the government spending shock, η_t^g and η_t^α are the government spending and the productivity innovations, respectively.

Fiscal backstop regime is:

$$e_t^g = \rho_g e_{t-1}^g + \sigma_{ga}\eta_t^\alpha + \eta_t^g + f_t, \qquad \eta_t^g \sim N(0, \sigma_g^2) \quad \eta_t^a \sim N(0, \sigma_a^2). \tag{9.2}$$

Where f_t is the fiscal shock that pushes interest rate away from the ZLB. We combine this fiscal backstop (equation 9.2) with the nominal GDP targeting rule (equation 9.1) in the model to investigate whether a shift to the ‘nominal GDP targeting with fiscal backstop’ regime could improve macroeconomic stability in the UK.

Results

In the estimated state-dependent model, we replace the ‘Taylor Rule with QE policy’ regime with the ‘nominal GDP targeting with fiscal backstop’ regime and then bootstrap the model and shocks to compute the average frequency of crises and welfare costs. We find that the rules of the following form can improve the performance of monetary policy:

$$r_t = 0.30r_{t-1} + 1.50(y_t + p_{d,t} - \bar{y}_t - \bar{p}_d) + e_t \tag{9.3}$$

Table 9-2 summarises the average bootstrap simulation results for each monetary regime under the state-dependent model. We have the following findings:

- To measure the ability of each regime to reduce the number of economic crises, we perform 1000 simulations over the sample period and calculate the expected number of crises per 1000 years under each regime. We examine both the number of shallow and deep crises. Shallow (or deep) crises are defined as small (or large) declines in output, where output does not return to its previous peak within five years. The simulations show that the 'nominal GDP targeting with fiscal backstop' regime is slightly better at reducing the number of shallow crises than the 'Taylor Rule with QE' regime but much better at reducing the number of deep crises.
- From a stabilisation perspective, our simulations show that the 'nominal GDP targeting with fiscal backstop' regime yields lower output variance and significantly lower inflation variance compared to the baseline regime. The examples of the bootstrap simulations in Figures 9-10 show that the variance of inflation is substantially lower, and output is somewhat smoother under the 'nominal GDP targeting with fiscal backstop' regime (in Red).
- In terms of welfare costs, the 'nominal GDP targeting with fiscal backstop' regime is associated with a significantly smaller welfare loss than the baseline regime.
- Regarding the degree of price stickiness in the state-dependent model, the 'nominal GDP targeting with fiscal backstop' regime generates a higher average NK price weight level of 0.9874 compared to 0.9126 in the baseline regime, as the lower the volatility of inflation, the higher the level of price stickiness.

Figures 9-10 shows some examples of the bootstrap simulations under different monetary regimes in the state-dependent model. As can be seen, the 'nominal GDPT with fiscal backstop' regime (in Red) prevents the ZLB from occurring. Compared to the ‘Taylor Rule with QE’ regime (in Blue), it largely eliminates the destabilising behaviour in inflation and price/wage durations and smoothes output to some extent. Though this appears to increase interest rates.

As mentioned earlier, in our welfare measures, the variance of output is calculated around the measure of trend output. However, the trend path of real output could not be the true estimate of the flexprice model determined path. Thus, by employing a model-estimated equilibrium output path, we also check the robustness of the welfare measures for the two policy regimes. To get this alternative measure, we combine the balanced growth path with the simulated impact of all model shocks on the output under the flexprice model solution. The comparison of welfare using flexprice model solutions is shown in Table 9-3, where it can be seen that the ‘nominal GDP targeting +fiscal policy’ regime still outperforms the baseline regime.

Results Comparison

The findings on the lower variance in inflation and output under nominal GDP targeting together with fiscal ZLB-suppression generally support the previously mentioned studies by Beckworth and Hendrickson (2020), Benchimol and Fourçans (2019) and Le et al. (2016, 2021) for the US, Wang (2020) for the UK, and Fang (2021) for Japan. The findings on decreasing the chance of crisis are consistent with those of Le et al. (2016, 2021) and Wang (2020).

Our results are broadly consistent with those of Wang (2020) for the UK, who estimates a fixed-duration DSGE model on UK data over 1985-2016 and finds that nominal GDP targeting slightly decreases the frequency of crisis and significantly stabilises output compared to the Taylor Rule. However, the variance of inflation is similar under both rules. In this thesis, inflation variance decreases significantly under the alternative regime as we combine nominal GDP targeting with the fiscal backstop.

Using an estimated open economy DSGE model, Fang (2021) finds that a strong fiscal feedback policy (normal Taylor Rule in non-crisis regime and QE intervention in crisis regime, as well as a strong fiscal feedback rule in both regimes) significantly outperforms a baseline regime (normal Taylor Rule in non-crisis regime and QE intervention in crisis regime) in terms of output, inflation, interest rate and consumption variances. Fang then compares this strong fiscal feedback policy with a combined regime in which nominal GDP targeting is combined with ZLB-suppressing fiscal policy, finding that the latter has lower inflation and output variance. However, it comes at the cost of high variations in both interest rate and consumption. Our results on the lower variance of inflation and output under the nominal GDP targeting-fiscal backstop framework align with Fang's findings for Japan.

To the best of my knowledge, our model is the only open economy DSGE model with all elements of a hybrid price/wage setting, policy regime switching, ZLB, QE and state-dependence. In addition, this study considers a combined policy regime of nominal GDP targeting-fiscal backstop. Le et al. (2021) examine this state-dependent model and this type of combined policy framework for the US as a closed economy, and find that this combined regime can achieve a high price and output stability and avoid large cyclical output fluctuations. In the UK context, we find that their conclusions still hold.

9.5 Conclusion

In this chapter, by using our estimated state-dependent model, we find that nominal GDP targeting rule together with the fiscal backstop can reduce the chances of economic crisis and stabilise the macroeconomy in the UK. Our results are much in line with the findings of Le et al. (2021), who find that an interest rate rule targeting nominal GDP backed by a fiscal backstop is optimal for the US in a state-dependent model.

However, there still remains the practical question of whether this alternative regime can be implemented politically. The nominal GDP target implies keeping interest rates away from normal rates for long periods after inflation has returned to normal; this is vulnerable to time-inconsistency. As for the fiscal backstop, it requires sharp changes in government borrowing which may violate ad hoc fiscal rules. However, these issues are beyond the scope of this thesis here.

Table 9-4: Stability and Crises Comparison

	Var(y)	Var(π)	Welfare[1]	Shallow crises[2]	Deep crises[3]	Av. NK weight wage	Av. NK weight price
Taylor Rule +QE	0.00107	0.00160	0.00128	39.65	30.94	0.9085	0.9126
NGDPT+Fiscal	0.00088	1.04e-04	0.00057	37.86	24.62	0.9831	0.9874

[1] The measurement of welfare costs is based on a weighted resource cost due to price variability and output variability: $welfare = 0.40 * var(\pi) + 0.60 * var(y)$.

[2 & 3] Expected number of crises per 1000 years. Shallow and deep crises are defined as small (0.01) and large (0.15) decreases in output, respectively, where output does not return to its previous peak within five years.

Table 9-5: Welfare Comparison from Output Deviation from Optimum Output under Flexprice Model

	Var(y)[4]	Var(π)	Welfare[5]
Taylor Rule + QE	0.00288	0.00160	0.00206
NGDPT+Fiscal	0.00146	1.04e-04	0.00059

[4] Deviation from optimum output under flexprice model

[5] $welfare = 0.64 * var(\pi) + 0.36 * var(y)$

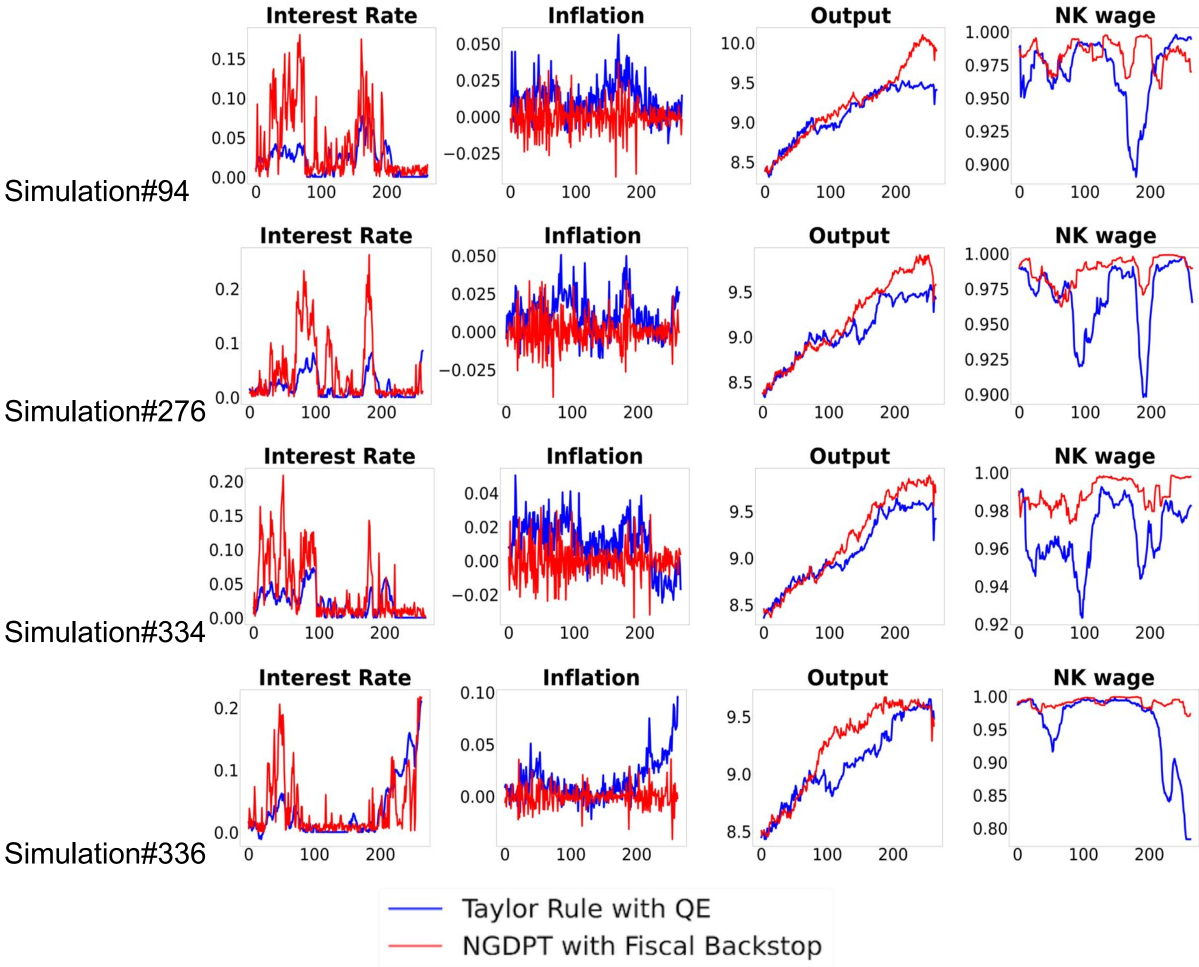

Figure 9-10: Simulation Comparison Between 'Taylor Rule with QE' and 'NGDPT with Fiscal Backstop'

Chapter 10 Overall Conclusion

Most current applied macroeconomic models assume that the durations of price/wage contracts are fixed. However, there is substantial evidence in micro data that the durations fluctuate with the state of the economy, particularly with inflation. This thesis aims to study how the UK's macroeconomic behaviour is affected by state-dependence. To achieve this goal, this thesis developed two models for the UK economy: a hybrid DSGE model with fixed price/wage contract durations and a hybrid DSGE model with state-dependence in price/wage durations. These two models were tested and estimated on UK macroeconomy data by the Indirect Inference method. The following are the main questions addressed in this thesis:

1. Whether the hybrid DSGE model with fixed price/wage duration can explain the behaviour of the UK data over the inflation targeting era 1992-2021 and the long full sample period 1955-2021.
2. Whether there is macro-level evidence of state dependence in the UK and whether the state-dependent price/wage framework can improve the fit of the DSGE model to macroeconomic data for the period 1955-2021.
3. How monetary policy in the UK could be used to stabilise the economy in the presence of state-dependent variation.

Chapter 2 provided a review of the literature on state-dependent price/wage adjustments at the micro- and macro-levels. Chapter 3 built a small open economy DSGE model for the UK economy. The model extends the SW07 model to the small open economy with trade treated as in Armington (1969) and assuming UIP. Moreover, the model merges the NK and NC models into a hybrid model, incorporates the financial friction of the BGG model and includes several new developments in the wake of the GFC including the ZLB and QE. Furthermore, the model switches between two regimes: the normal regime, where the Taylor Rule operates normally and the supply of M0 is set to accommodate M2, and the crisis regime, where the nominal interest rate is at or below the ZLB threshold level. In the crisis regime, the Taylor Rule is suspended and replaced by an exogenous low bound. Meanwhile, a QE policy is incorporated into the model where M0 targets the credit premium around its steady state to restore credit condition to normal through its role of providing cheap collateral. Once the model escapes from the ZLB, the model switches back to the normal regime, and the Taylor Rule resumes operation. Chapters 4 and 5 cover the construction of the dataset, a set of starting calibrations, and an introduction to the testing and estimation method – Indirect Inference.

The empirical studies in Chapter 6 and Chapter 7 tested and estimated the fixed duration model over the UK inflation targeting era 1992-2021 and the full sample period of 1955-2021, respectively. The results of the Indirect Inference estimation suggest that the fixed duration model fits the behaviour of the UK data for 1992-2021. However, we find that it does not fit the whole sample period. This may be because there were more fluctuations in the economic environment across the whole sample period, such as the stagflation crisis in the 1970s and the Great Moderation; thus, wage/price behaviour changed in response to fluctuations in the macro environment. These results are in line with those of Le et al. (2011, 2016).

Therefore, Chapter 8 integrates the state-dependent price/wage contracts into the hybrid DSGE model to examine whether there is macro-level evidence of state-dependent price/wage duration. The results of the Indirect Inference test and estimation for the full sample period 1955-2021 show that the state-dependent model fits the behaviour of the UK historical data well, implying that the state-dependent price/wage contract framework improves the fit of the DSGE model to macroeconomic data compared to the fixed-duration framework. Therefore, it indicates that macroeconomic models should allow for state-dependent price/wage adjustments. Furthermore, the duration of price/wage contracts fluctuates with the state of the economy (inflation) throughout the whole sample.

Chapter 9 investigated the performance of an alternative policy in terms of the ability to avoid crises and decrease welfare cost. The simulations suggest that from the perspective of both criteria, a nominal GDP targeting regime together with a fiscal backstop to prevent the ZLB outperforms the baseline Taylor Rule regime. These findings of the UK policy implications are consistent with the findings of Le et al. (2021) for the US. Thus, like them, we find that an interest rate rule targeting nominal GDP and backed up by a fiscal backstop are optimal. However, there still remains the practical question of whether this alternative regime can be implemented politically. The nominal GDP target implies keeping interest rates away from normal rates for long periods after inflation has returned to normal; this is vulnerable to time-inconsistency. As for the fiscal backstop, it requires sharp changes in government borrowing which may violate ad hoc fiscal rules. However, these issues are beyond the scope of this thesis here.

In summary, the answers to the three main questions in this thesis are:

1. The fixed duration model fits the behaviour of the UK data for the inflation targeting era 1992-2021, but not for the full sample period 1955-2021.
2. The state-dependent model fits the behaviour of the UK historical data well over the whole sample period, implying that the state-dependent price/wage contract framework improves the fit of the DSGE model to macroeconomic data compared to the fixed-duration framework. Furthermore, the durations of price/wage contracts fluctuate with the state of the economy (inflation) throughout the full sample.
3. Our simulation results suggest that the nominal GDP targeting rule together with the fiscal backstop reduces the chances of economic crisis and stabilises the UK macroeconomy; it outperforms the Taylor Rule regime.

Finally, with regard to future work, it could be useful to bring state-dependence into other parameters.

Bibliography

Adolfson, M., Laséen, S., Lindé, J. and Villani. M. 2007. Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics*, *72*(2), pp.481-511.

Adolfson, M., Laséen, S., Lindé, J. and Villani, M. 2008. Empirical properties of closed- and open-economy DSGE models of the euro area. *Macroeconomic Dynamics*, *12*(S1), pp.2-19.

Ahmad, Y., Lo, M. C. and Mykhaylova, O. 2013. Model Equations and Solutions to Accompany Volatility and Persistence of Simulated DSGE Real Exchange Rates.

Akcigit, U. and Kerr, W. R. 2018. Growth through heterogeneous innovations. *Journal of Political Economy*, *126*(4), pp.1374-1443.

Alpanda, S., Kotzé, K. and Woglom, G. 2010. The role of the exchange rate in a New Keynesian DSGE model for the South African economy. *South African Journal of Economics*, *78*(2), 170-191.

Alvarez, F., Beraja, M., Gonzalez-Rozada, M. and Neumeyer, P. 2018. From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models*. *The Quarterly Journal of Economics*, 134(1), pp.451-505.

Aminu, N. 2018. Evaluation of a DSGE model of energy in the United Kingdom using stationary data. *Computational Economics*, *51*(4), pp.1033-1068.

Armington, P. S. 1969. A Theory of Demand for Products Distinguished by Place of Production, IMF Staff Papers, *International Monetary Fund*, Vol. 6, pp.159-178.

Ascari, G. and Sbordone, A. M. 2014. The macroeconomics of trend inflation. *Journal of Economic Literature*, *52*(3), pp.679-739.

Ascari, G., Phaneuf, L. and Sims, E. R. 2018. On the welfare and cyclical implications of moderate trend inflation. *Journal of Monetary Economics*, 99, pp.56-71.

Balatti, M., Brooks, C., Clements, M. P. and Kappou, K. 2016. Did Quantitative Easing Only Inflate Stock Prices? Macroeconomic Evidence from the US and UK. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.2838128.

Bank of England. 2021. *IEO evaluation of the Bank of England's approach to quantitative easing*. [online] www.bankofengland.co.uk. Available at: https://www.bankofengland.co.uk/independent-evaluation-office/ieo-report-january-2021/ieo-evaluation-of-the-bank-of-englands-approach-to-quantitative-easing [Accessed September 15, 2022].

Bank of England. 2022a. *QE at the Bank of England: a perspective on its functioning and effectiveness*. [online] www.bankofengland.co.uk. Available at: https://www.bankofengland.co.uk/quarterly-bulletin/2022/2022-q1/qe-at-the-bank-of-england-a-perspective-on-its-functioning-and-effectiveness [Accessed September 15, 2022].

Bank Of England. 2022b. *Quantitative easing. Bankofengland.co.uk; Bank of England.* https://www.bankofengland.co.uk/monetary-policy/quantitative-easing [Accessed September 15, 2022].

Basu, S. and Kimball, M. 2002. *Long-run labor supply and the elasticity of intertemporal substitution for consumption*. manuscript*, University of Michigan*.

Batini, N. and Yates, A. 2003. Hybrid inflation and price-level targeting. *Journal of Money, Credit and Banking*, 283-300.

Baumeister, C. and Benati, L. 2013. Unconventional Monetary Policy and the Great Recession: Estimating the Macroeconomic Effects of a Spread Compression at the Zero Lower Bound. *International Journal of Central Banking* 9(2), pp.165–212.

Beckworth, D. and Hendrickson, J. 2016. *Nominal GDP Targeting and the Taylor Rule on a Level Playing Field*. George Mason University Mercatus Working paper.

Beckworth, D. and Hendrickson, J. R. 2020. Nominal GDP targeting and the Taylor rule on an even playing field. *Journal of Money, Credit and Banking*, *52*(1), pp.269-286.

Benchimol, J. and Fourçans, A. 2012. Money and risk in a DSGE framework: A Bayesian application to the Eurozone. *Journal of Macroeconomics*, *34*(1), pp.95-111.

Benchimol, J. and Fourçans, A. 2019. Central bank losses and monetary policy rules: A DSGE investigation. *International Review of Economics and Finance*, 61, pp.289-303.

Benchimol, J. and Fourçans, A. 2012. Money and risk in a DSGE framework: A Bayesian application to the Eurozone. *Journal of Macroeconomics*, *34*(1), pp.95-111.

Bernanke, B. S., Gertler, M. and Gilchrist, S. 1999. The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, 1, pp.1341-1393.

Bernanke, B. S. 2000. Japanese monetary policy: a case of self-induced paralysis?. *Japan's financial crisis and its parallels to US experience*, pp.149-166.

Bhandari, P. and Frankel, J. 2017. Nominal GDP targeting for developing countries. *Research in Economics*, *71*(3), 491-506.

Bhattarai, S. and Schoenle, R. 2014. Multiproduct firms and price-setting: Theory and evidence from U.S. producer prices. *Journal of Monetary Economics*, 66, pp.178-192.

Bhattarai, S. and Egorov, K. 2016. Optimal monetary and fiscal policy at the zero lower bound in a small open economy. *Globalization and Monetary Policy Institute Working Paper*, (260).

Bhattarai, S. and Egorov, K. 2016. Optimal monetary and fiscal policy at the zero lower bound in a small open economy. *Globalization and Monetary Policy Institute Working Paper*, (260).

Billi, R. M. 2016. A note on nominal GDP targeting and the zero lower bound. *Macroeconomic Dynamics*, 21(8), pp. 2138–2157. Available at: https://doi.org/10.1017/s136510051500111x.

Billi, R. M. 2020. Unemployment fluctuations and nominal GDP targeting. *Economics Letters*, 188, p. 108970. Available at: https://doi.org/10.1016/j.econlet.2020.108970.

Blanchard, M. O. J., Dell'Ariccia, M. G. and Mauro, M. P. 2010. Rethinking macroeconomic policy (No. 2010/003). *International Monetary Fund*.

Blanchard, O., Dell'Ariccia, G. and Mauro, P. 2010. Rethinking macroeconomic policy. J. Money, Credit Banking 42 (6), 199–215.

Boivin, J. Giannoni, M. P. and Stevanović, D. 2020. Dynamic effects of credit shocks in a data-rich environment. *Journal of Business & Economic Statistics*, *38*(2), pp.272-284.

Boubaker, S., Nguyen, D. K. and Paltalidis, N. 2018. Fiscal policy interventions at the zero lower bound. *Journal of Economic Dynamics and Control*, *93*, pp.297-314.

Breedon, F., Chadha, J. S. and Waters, A. 2012. The financial market impact of UK quantitative easing. *SSRN Electronic Journal.*

Bridges, J. and Thomas, R. 2012. The Impact of QE on the UK Economy – Some Supportive Monetarist Arithmetic. *SSRN Electronic Journal*. doi:10.2139/ssrn.1992942.

Bui, D., Dräger, L., Hayo, B. and Nghiem, G. 2022. The effects of fiscal policy on households during the COVID-19 pandemic: Evidence from Thailand and Vietnam. *World development*, *153*, p.105828.

Buiter, W. H. and Panigirtzoglou, N. 1999. Liquidity traps: how to avoid them and how to escape them.

Bunn, P. and Ellis, C. 2012. Examining the Behaviour of Individual UK Consumer Prices. *The Economic Journal*, 122(558), pp.F35-F55.

Burnside, C. 2019. Exchange rates, interest parity, and the carry trade. In *Oxford Research Encyclopedia of Economics and Finance*.

Busetto, F., Chavaz, M., Froemel, M., Joyce, M., Kaminska, I. and Worlidge, J. 2022. QE at the bank of England: a perspective on its functioning and effectiveness. *Bank of England Quarterly Bulletin Q*, *1*. Available at: https://www.bankofengland.co.uk/quarterly-bulletin/2022/2022-q1/qe-at-the-bank-of-england-a-perspective-on-its-functioning-and-effectiveness [Accessed September 19, 2022].

Céspedes, L. F., Chang, R. and Velasco, A. 2004. Balance sheets and exchange rate policy. *American Economic Review,* vol. 94(4), pp.1183-1193.

Cajner, T., Crane, L. D., Decker, R. A., Grigsby, J., Hamins-Puertolas, A., Hurst, E., and Yildirmaz, A. 2020. *The US labor market during the beginning of the pandemic recession* (No. w27159). National Bureau of Economic Research.

Casares, M. 2007. The New Keynesian model and the Euro area business cycle. *Oxford Bulletin of Economics and Statistics*, *69*(2), pp.209-244.

Casares, M., Moreno, A. and Vázquez, J. 2014. An estimated New-Keynesian model with unemployment as excess supply of labor. *Journal of Macroeconomics*, *40*, pp.338-359.

Chen, H., Cúrdia, V. and Ferrero, A. 2012. The Macroeconomic Effects of Large-scale Asset Purchase Programmes. *The Economic Journal*, 122(564), pp.F289–F315. doi:10.1111/j.1468-0297.2012.02549.x.

Christensen, I. and Dib, A., 2008. The financial accelerator in an estimated New Keynesian model. *Review of economic dynamics*, *11*(1), pp.155-178.

Christiano, L. Eichenbaum, M. Rebelo, S. 2011. When is the government spending multiplier large? *Journal of Political Economy,* 119 (1), 78–121.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 113(1), pp.1–45.

Christiano, L. J., Trabandt, M. and Walentin, K. 2011. Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics & Control*, v. 35, 1999–2041.

Churm, R., Joyce, M., Kapetanios, G. and Theodoridis, K. 2018. Unconventional monetary policies and the macroeconomy: The impact of the UK's QE2 and funding for lending scheme. *The Quarterly Review of Economics and Finance. http:* //doi.org//10.1016/j.qref.2018.10.004.

Clarida, R., Galı, J. and Gertler, M., 1998. Monetary policy rules in practice: Some international evidence. *European economic review*, *42*(6), pp.1033-1067

Coeure, B. 2015. How binding is the zero lower bound? European Central Bank, Speech given at conference, "Removing the Zero Lower Bound on Interest Rates" available at [https://www.ecb.europa.eu/press/key/date/2015/html/sp150519.en.html] accessed at 12th June 2023.

Cogley, T. and Sbordone, A. M. 2008. Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. *American Economic Review*, *98*(5), pp.2101-2126.

Cogley, T. and Sbordone, A. M. 2008. Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. *American Economic Review*, *98*(5), pp.2101-2126.

Coibion, O. and Gorodnichenko, Y. 2011. Strategic interaction among heterogeneous price-setters in an estimated DSGE model. *Review of Economics and Statistics*, *93*(3), pp.920-940.

Coibion, O., Gorodnichenko, Y. and Wieland, J. 2012. The optimal inflation rate in New Keynesian models: should central banks raise their inflation targets in light of the zero lower bound?. *Review of Economic Studies*, *79*(4), pp.1371-1406.

Coletti, D., Lalonde, R. and Muir, D. 2008. Inflation targeting and price-level-path targeting in the global economy model: Some open economy considerations. *IMF Staff Papers*, *55*(2), 326-338.

Costain, J. and Nakov, A. 2017. Logit Price Dynamics. *Journal of Money, Credit and Banking*, 51(1), pp.43-78.

Costain, J., Nakov, A. and Petit, B. 2019. *Monetary policy implications of state-dependent prices and wages*. European Central Bank Working Paper 2272.

Costain, J., Nakov, A. and Petit, B., 2021. Flattening of the Phillips Curve with State-Dependent Prices and Wages. *The Economic Journal*, 132(642), pp.546-581.

Dai, L., Minford, P. and Zhou, P. 2015. A DSGE model of China. *Applied Economics*, 47(59), 6438-6460.

Davies, R. 2021. *Prices and Inflation in the UK-: A New Dataset*. Centre for Economic Performance, London School of Economics and Political Science.

Davidson, J., Meenagh, D., Minford, P. and Wickens, M. 2010. *Why crises happen - nonstationary macroeconomics.* Cardiff Economics Working Papers E2010/13, Cardiff University, Cardiff Business School, Economics Section.

De Graeve, F. 2008. The external finance premium and the macroeconomy: US post-WWII evidence. *Journal of Economic Dynamics and control*, *32*(11), pp.3415-3440.

Dedola, L., Kristoffersen, M. S. and Zullig, G. 2021. The extensive and intensive margin of price adjustment to cost shocks: Evidence from danish multiproduct firms. *Manuscript, April.*

Del Negro, M. and Schorfheide, F. 2008. Forming priors for DSGE models (and how it affects the assessment of nominal rigidities). *Journal of Monetary Economics*, *55*(7), pp.1191-1208.

Dell'Ariccia, G., Rabanal, P. and Sandri, D. 2018. Unconventional monetary policies in the euro area, Japan, and the United Kingdom. *Journal of Economic Perspectives*, 32(4), pp.147–172.

Devereux, M. P. Lane, and J. Xu, 2006. Exchange Rates and Monetary Policy in Emerging Market Economies. Economic Journal, vol. 116(511), pages 478-506.

Dib, A. Gammoudi, M. and Moran, K. 2008. Forecasting Canadian time series with the New Keynesian model. *Canadian Journal of Economics/Revue canadienne d'économique*, *41*(1), pp.138-165.

Dib, A., Mendicino. C. and Zhang. Y. 2013. Price-level targeting rules and financial shocks: the case of Canada. *Economic Modelling*, *30*, 941-953.

DiCecio, R. and Nelson, E. 2007. An estimated DSGE model for the United Kingdom. *Available at SSRN 966310*.

Dixon, H., Luintel, K. and Tian, K. 2020. The Impact of the 2008 Crisis on UK Prices: What We Can Learn from the CPI Microdata1. *Oxford Bulletin of Economics and Statistics*, 82(6), pp.1322-1341.

Doepke, M. and Schneider, M. 2006. Inflation and the redistribution of nominal wealth. *Journal of Political Economy*, *114*(6), 1069-1097.

Dong, X., Minford, P. and Meenagh, D. 2019. How important are the international financial market imperfections for the foreign exchange rate dynamics: A study of the sterling exchange rate. *Journal of International Money and Finance*, *94*, pp.62-80.

Dong, X., Minford, P., Meenagh, D. and Yang, X. 2023. *Bounded Rational Expectation: How It Can Affect the Effectiveness of Monetary Rules in the Open Economy* (No. E2023/4). Cardiff University, Cardiff Business School, Economics Section.

Dotsey, M., King, R. G. and Wolman, A. L. 1999. State-dependent pricing and the general equilibrium dynamics of money and output. *The Quarterly Journal of Economics*, *114*(2), pp.655-690.

Dovonon, P. and Hall, A. R. 2018. The asymptotic properties of GMM and indirect inference under second-order identification. *Journal of econometrics*, *205*(1), pp.76-111.

Dufour, J. M., Khalaf, L. and Kichian, M. 2010. On the precision of Calvo parameter estimates in structural NKPC models. *Journal of Economic Dynamics and Control*, *34*(9), pp.1582-1595.

Eggertsson, G. B. 2011. What fiscal policy is effective at zero interest rates? NBER Macroeconomics Annual 25 (1), 59–112.

Engel, C. and West, K. D. 2006. Taylor rules and the Deutschmark-Dollar real exchange rate. *Journal of Money, Credit, and Banking*, 38, pp. 1175–94.

Fackler, S. J. and McMillin, D. W. 2019. An evaluation of nominal GDP versus Price-level targeting*. Mercatus Center Policy Brief*, pp.1-9.

Fackler, S. J. and McMillin, D. W. 2020. Nominal GDP versus price level targeting: An empirical evaluation. *Journal of Economics and Business*, 109, p. 105890. Available at: https://doi.org/10.1016/j.jeconbus.2019.105890.

Fair, R. C. and Taylor, J. B. 1980. Solution and maximum likelihood estimation of dynamic nonlinear rational expectations models.

Falagiarda, M. 2014. Evaluating quantitative easing: A DSGE approach. *International Journal of Monetary Economics and Finance*, 7(4), p.302.

Fang, Y. 2021. *Policies for reviving the Japanese economy: Small open economy DSGE with indirect inference method* (Doctoral dissertation, Cardiff University).

Feenstra, R. P., Luck, M., Obstfeld, K. and Russ. 2014. *In search of the Armington elasticity*, NBER Working paper No. 20063, April 2014.

Fernández, A. and Gulan, A. 2015. Interest rates, leverage, and business cycles in emerging economies: The role of financial frictions. *American Economic Journal: Macroeconomics*, *7*(3), pp.153-188.

Fisher, P. 2019. *Why do most Central Banks target inflation?*. Warwick Business School. Available at: https://www.wbs.ac.uk/news/why-do-most-central-banks-target-inflation/ [Accessed September 15, 2022].

Frankel, J. 1995. The Stabilising Properties of a Nominal GNP Rule. *Journal of Money, Credit and Banking*, 27(2), pp. 318-334.

Frankel, J. 2013. Nominal-GDP targets, without losing the inflation anchor. *Is Inflation Targeting Dead?*, p.90.

Frankel, J. A. 2014. Nominal GDP targeting for middle-income countries. *Central Bank Review,* 14(3), p.1.

Freystätter, H. 2011. Financial factors in the boom-bust episode in Finland in the late 1980s and early 1990s. *Bank of Finland Research Discussion Paper*, (1).

Gagnon, E. 2009. Price setting under low and high inflation: evidence from Mexico. *The Quarterly Journal of Economics*, 124, pp.1221-63.

Gali, J. and Monacelli, T. 2005. Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies*, *72*(3), 707-734.

Galí, J. and Monacelli, T. 2016. Understanding the gains from wage flexibility: the exchange rate connection. *American Economic Review*, *106*(12), 3829-3868.

Garcıa-Cicco, J. and Kirchner, M. 2016. February. Macroeconomic and Financial Dynamics in Small Open Economies. In *IMFS-CEPR Research Meeting New Methods for Macroeconomic Modelling, Model Comparison and Policy Analysis*.

Garín, J., Lester, R. and Sims, E. 2016. On the desirability of nominal GDP targeting. *Journal of Economic Dynamics and Control*, 69, pp.21-44.

Gasteiger, E. and Grimaud, A. 2022. Price Setting Frequency and the Phillips Curve. *SSRN Electronic Journal*.

Gertler, M., Gilchrist, S. and F. Natalucci. 2007. External constraints on monetary policy and the financial accelerator. Journal of Money, Credit and Banking, vol. 39(2-3), 295-330.

Gertler, M., Sala, L. and Trigari, A. 2008. An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining. *Journal of Money, Credit and Banking*, *40*(8), pp.1713-1764.

Giannone, D., Lenza, M., Pill, H. and Reichlin, L. 2012. The ECB and the interbank market. *The Economic Journal*, *122*(564), pp. 467-486.

Giavazzi, F. and Giovannini, A. 1989. Monetary policy interactions under managed exchange rates. *Economica*, pp.199-213.

Girardin, E. and Moussa, Z. 2011. Quantitative easing works: Lessons from the unique experience in Japan 2001–2006. *Journal of International Financial Markets, Institutions and Money*, *21*(4), pp.461-495.

Gollin, D. 2002. Getting Income Shares Right. *Journal of Political Economy.* vol. 110(2), pp. 458-474.

Grajales, A., Uras, B. and Vellekoop, N., 2019. Rigid Wages and Contracts: Time-Versus State-Dependent Wages in the Netherlands. *SSRN Electronic Journal*.

Grigsby, J., Hurst, E. and Yildirmaz, A. 2021. Aggregate Nominal Wage Adjustments: New Evidence from Administrative Payroll Data. *American Economic Review*, 111(2), pp.428-471.

Guerrieri, L., Gust, C. and López-Salido, J. D. 2010. International competition and inflation: a New Keynesian perspective. *American Economic Journal: Macroeconomics*, *2*(4), pp.247-280.

Guvenen, F. and Smith, A. A. 2014. Inferring labor income risk and partial insurance from economic choices. *Econometrica*, *82*(6), pp.2085-2129.

Haldane, A., Roberts-Sklar, M., Wieladek, T and Young, C. 2016. *QE: the story so far*. Bank of England Staff Working Paper No. 624.

Hatcher, M. and Minford, P. 2014. Inflation targeting vs price-level targeting: A new survey of theory and empirics. *Vox*.

Hatcher, M. 2016. *Nominal GDP targeting and the tax burden* (Discussion Papers in Economics and Econometrics, 1604) Southampton, GB. University of Southampton 35pp.

Hendrickson, J. R. 2012. Nominal Income Targeting and Monetary Stability. *Boom and Bust Banking: The Causes and Cures of the Great Recession*, pp.257-90.

Hettig, T. and Muller, G. J. 2015. Fiscal policy coordination in currency unions (at the zero-lower bound), German Economic Association, Conference paper, Common Currencies, No. F11-V3.

Hills, T. S. and Nakata, T. 2018. Fiscal multipliers at the zero lower bound: the role of policy inertia. *Journal of Money, Credit and Banking*, 50 (1), 155–172.

Hohberger, S., Priftis, R. and Vogel, L. 2019. The macroeconomic effects of quantitative easing in the euro area: Evidence from an estimated DSGE model. *Journal of Economic Dynamics and Control*, 108, 103756. https://doi.org/10.1016/j.jedc.2019.103756

House of Commons. 2016. *Debate Pack - Quantitative Easing*, Number CDP 2016/0166, 14 September 2016. Available at: https://researchbriefings.files.parliament.uk/documents/CDP-2016-0166/CDP-2016-0166.pdf

Jääskelä, J. P. and Nimark, K. 2011. A Medium-Scale New Keynesian Open Economy Model of Australia. *Economic Record*, *87*(276), pp.11-36.

Joyce, M. A., McLaren, N. and Young, C. 2012. Quantitative easing in the United Kingdom: Evidence from financial markets on QE1 and QE2. *Oxford Review of Economic Policy*, 28(4), pp.671–701.

Kapetanios, G., Mumtaz, H., Stevens, I. and Theodoridis, K. 2012. Assessing the Economy-Wide Effects of Quantitative Easing. *SSRN Electronic Journal*, 122(564).

Kara, E. and Yates, T. 2021. A Case against a 4% Inflation Target. *Journal of Money, Credit and Banking*, *53*(5), pp.1097-1119.

Karadi, P. and Reiff, A. 2019. Menu Costs, Aggregate Fluctuations, and Large Shocks. *American Economic Journal: Macroeconomics*, 11(3), pp.111-146.

Keep, M. and Brien, P. 2021. Public spending during the Covid-19 pandemic. *commonslibrary.parliament.uk*. [online] Available at: https://commonslibrary.parliament.uk/research-briefings/cbp-9309/.

Kempa, B. 2018. Taylor Rule Reaction Coefficients And Real Exchange Rate Persistence. *Bulletin of Economic Research*, *70*(1), pp.64-73.

Keynes, J. M. 1936. The General Theory of Employment, Interest and Money. ISBN: 978- 1494854744

Keynes, J. M. 1982 [1932] “Broadcast on state planning”, 14 March, in Activities 1931-1939. Collected Writings Vol. XXI, London: Macmillan, for the Royal Economic Society: 84-92.

Khalaf, L. and Peraza, L. B. 2020. Simultaneous indirect inference, impulse responses and ARMA models. *Econometrics*, *8*(2), p.12.

Koenig, E. F. 2013. Like a Good Neighbor: Monetary Policy, Financial Stability, and the Distribution of Risk. *International Journal of Central Banking*, June, pp.57-82.

Konieczny, J. and Skrzypacz, A. 2005. Inflation and price setting in a natural experiment. *Journal of Monetary Economics*, 52(3), pp.621-632.

Le, V. P. M., Meenagh, D. and Minford, P. 2021. State-dependent pricing turns money into a two-edged sword: A new role for monetary policy. *Journal of International Money and Finance*, 119, p.102496.

Le, V. P. M., Meenagh, D. and Minford, P. 2012. *Do Banks Cause Banking* Crisis?. Cardiff Economics Working Paper E2012/14.

Le, V. P. M., Matthews, K., Meenagh, D., Minford, P. and Xiao, Z. 2022. Regulatory arbitrage, shadow banking and monetary policy in China. *Journal of International Financial Markets, Institutions and Money*, *80*, p.101640.

Le, V. P. M., Meenagh, D. and Minford, P. 2016. Monetarism rides again? US monetary policy in a world of Quantitative Easing. *Journal of International Financial Markets, Institutions and Money*, 44, pp.85-102.

Le, V. P. M., Matthews, K., Meenagh, D., Minford, A.P. and Xiao, Z. 2015. *China’s financial crisis – the role of banks and monetary policy*. Cardiff Economics Working Papers No. E2015/1, Cardiff University.

Le, V. P. M., Meenagh, D., Minford, P. and Ou, Z. 2013. What causes banking crises? an empirical investigation for the world economy. *Open Economic Review,* 21 (4), pp.581–611.

Le, V. P. M., Meenagh, D., Minford, P. and Wickens, M. 2015. *Small sample performance of indirect inference on DSGE models.* Cardiff Economics Working Papers No. E2015/2, Cardiff University.

Le, V. P. M., Meenagh, D., Minford, P. and Wickens, M. 2011. How much nominal rigidity is there in the us economy? testing a new keynesian dsge model using indirect inference. *Journal of Economic Dynamics and Control*, 35(12), pp.2078–2104.

Le, V. P. M., Minford, P. and Wickens, M. 2010. The Puzzles methodology: En route to Indirect Inference?, *Economic Modelling*, 27(6), pp.1417-1428, November.

Lenza, M., Pill, H. and Reichlin, L. 2010. Monetary policy in exceptional times. *Economic policy,* 25(62), pp.295-339.

Lyu, J. 2021. *Formation and effects of UK monetary and macroprudential policies.* PhD Thesis, Cardiff University.

Lyu, J., Le, V. P. M., Meenagh, D. and Minford, P. 2021. Macroprudential regulation in the post-crisis era: Has the pendulum swung too far?. *Journal of International Financial Markets, Institutions and Money*, *74*, 101381.

Lyu, J., Le, V. P. M., Meenagh, D. Minford, P. 2023. UK monetary policy in an estimated DSGE model with financial frictions. *Journal of International Money and Finance.*

Marco, D. N. and Frank, S. 2009. Monetary Policy Analysis with Potentially Misspecified Models. *American Economic Review* 99:4, 1415-1450.

McCallum, B. T. and Nelson, E. 1999. Nominal income targeting in an open-economy optimising model. *Journal of Monetary Economics*. 43 (3), pp.553–578.

McCallum, B.T. 1988. Robustness properties of a rule for monetary policy. *Carnegie-Rochester Conference Series on Public Policy* 29, pp.173–204.

Meenagh, D., Minford, P., Wickens, M. and Xu, Y. 2019. Testing DSGE models by indirect inference: a survey of recent findings. *Open Economies Review*, *30*, pp.593-620.

Meenagh, D., Minford, P. and Wickens, M. 2012. *Testing macroeconomic models by indirect inference on unfiltered data*, Cardiff Economics Working Papers E2012/17, Cardiff University. Cardiff Business School, Economics Section.

Meenagh, D., Minford, P., Nowell, E. and Sofat, P. 2010. Can a real business cycle model without price and wage stickiness explain UK real exchange rate behaviour? *Journal of International Money and Finance*, 29(6), pp.1131–1150.

Meenagh, D., Minford, P., Wickens, M. and Xu, Y. 2019. Testing DSGE models by indirect inference: a survey of recent findings. *Open Economies Review*, *30*(3), pp.593-620.

Meier, A. 2009. Panacea, curse, or nonevent? unconventional monetary policy in the United Kingdom*. SSRN Electronic Journal.*

Millard, S. and Tatomir, S. 2015. *Firms' adjustment during 2010–13: evidence from the Wage Dynamics Survey*. Staff Working Paper No. 568, Bank of England, London.

Minford, L. 2015. *The macroeconomic effects of UK tax, regulation and R&D subsidies: testing endogenous growth hypotheses in an open economy DSGE Model.* PhD Thesis, Cardiff University.

Minford, L. and Meenagh, D. 2019. Supply-side policy and economic growth: A case study of the UK. *Open Economies Review*, *31*, 159-193.

Minford, P., Ou, Z. and Zhu, Z. 2020. Can a small New Keynesian model of the world economy with risk-pooling match the facts?. *International Journal of Finance & Economics*, *26*(2), pp.1993-2021.

Minford, P., Ou, Z. and Zhu, Z. 2021. Is there consumer risk-pooling in the open economy? The evidence reconsidered. *Open Economies Review*, *33*(1), pp.109-120.

Minford, P., Agenor, P. R. and Nowell, E. 1986. A new Classical econometric model of the world economy. Econ. Model. 3 (3), 154–176.

Minford, P., Konstantinos, T. and David, M. 2009. Testing a Model of the UK by the Method of Indirect Inference. *Open Economies Review*, Springer, vol. 20(2), pp.265-291.

Minford, P., Marwaha, S., Matthews, K. and Sprague, A. 1984. The Liverpool macroeconomic model of the United Kingdom. Econ. Model. 1 (1), 24–62.

Minford, P., Matthews, K. and Marwaha, S. 1979. Terminal conditions as a means of ensuring unique solutions for rational expectations models with forward expectations. *Economics Letters*, *4*(2), 117-120.

Nakamura, E., Steinsson, J., Sun, P. and Villar, D. 2018. The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation. *The Quarterly Journal of Economics*, 133(4), pp.1933-1980.

Nasir. M. A. 2021. Zero lower bound and negative interest rates: Choices for monetary policy in the UK. *Journal of policy modeling*, *43*(1), 200-229.

Nilsen, Ø., Pettersen, P. and Bratlie, J. 2018. Time-Dependency in Producers' Price Adjustments: Evidence from Micro Panel Data. *Review of Economics*, 69(2), pp.147-168.

Ortega. E. and Rebei. N. 2006. *The welfare implications of inflation versus price-level targeting in a two-sector, small open economy* (No. 2006-12). Bank of Canada.

Pagliari, M. S. 2021. Does One (unconventional) Size fit all? Effects of the ECB's unconventional monetary policies on the Euro Area Economies. *SSRN Electronic Journal.*

Peña, J. and Prades, E. 2021. Price Setting in Chile: Micro Evidence from Consumer On-Line Prices During the Social Outbreak and Covid-19. *SSRN Electronic Journal*.

Petrella, I., Santoro, E. and Simonsen, L. P. 2019. *Time-varying Price Flexibility and Inflation Dynamics. EMF Research Papers 28*, Economic Modelling and Forecasting Group.

Portes, J. and Wren-Lewis, S. 2015. Issues in the design of fiscal policy rules. *The Manchester School*, *83*, pp.56-86.

Praščević. A. and Ješić. M. 2019. Modeling Macroeconomic Policymakers' Interactions under Zero Lower Bound Environment: The New Keynesian Theoretical Approach. *Journal of Central Banking Theory and Practice*, *8*(1), 5-38.

Punzo, C. and Rivolta, G. 2022. *Money versus debt financed regime: Evidence from an estimated DSGE model*. Dipartimento di Economia e Finanza, Università Cattolica del Sacro Cuore.

Rabanal, P. 2007. Does inflation increase after a monetary policy tightening? Answers based on an estimated DSGE model. *Journal of Economic Dynamics and control*, *31*(3), pp.906-937.

Rabanal, P. and Rubio-Ramírez, J. F. 2005. Comparing New Keynesian models of the business cycle: A Bayesian approach. *Journal of Monetary Economics*, *52*(6), pp.1151-1166.

Richard, D. 2021. *Prices and Inflation in the UK – A New Dataset*. *CEP Occasional Papers 55*, Centre for Economic Performance, LSE.

Rudolf, B. and Zurlinden, M. 2014. A compact open economy DSGE model for Switzerland. *Swiss National Bank Economic Studies*, (8).

Rudolf, B. and Seiler, P. 2022. Price setting before and during the pandemic: evidence from Swiss consumer prices.

Rysbayeva, A. 2020. *Testing small open economy DSGE model with imperfect pass-through using indirect inference*(Doctoral dissertation, Cardiff University).

Salachas, E., Laopodis, N. T. and Kouretas, G. P. 2018. Assessing monetary policies in the Eurozone, US, UK and Japan: new evidence from the post-crisis period. *Applied Economics*, *50*(59), pp.6481-6500.

Schmitt-Grohé, S. and Uribe, M. 2003. Closing small open economy models. *Journal of international Economics*, *61*(1), 163-185.

Sheedy, K. D. 2014. Debt and incomplete financial markets: a case for nominal GDP targeting. *Brookings Papers on Economic Activity*, Spring, pp.301-372.

Sigurdsson, J. and Sigurdardottir, R. 2016. Time-dependent or state-dependent wage-setting? Evidence from periods of macroeconomic instability. *Journal of Monetary Economics*, 78, pp.50-66.

Smets, F. and Wouters, R. 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review* 97, pp.586-606.

Soltani, S., Falihi, N., Mehrabiyan, A. and Amiri, H. 2021. Investigating the Effects of Monetary and Financial Shocks on the Key Macroeconomic Variables, Focusing on the Intermediary Role of Banks Using DSGE Models. *Journal of Money and Economy*, *16*(4), pp.477-500.

Sumner, S. 2011. The case for NGDP targeting - lessons from the Great Depression. *Adam Smith Institute*.

Sumner, S. 2015. Nominal GDP futures targeting. *Journal of Financial Stability*, 17, pp.65-75.

Sumner, S. 2017. Monetary policy rules in light of the great recession. *Journal of Macroeconomics*, 54, pp.90-99.

Sumner, S. and Roberts, E. 2018. The Promise of Nominal GDP Targeting. *Mercatus Policy Primer*.

Sumner, S. B. 2014. Nominal GDP targeting: a simple rule to improve Fed performance. *Cato J.*, *34*, 315.

Takahashi, S. 2017. State dependency in price and wage setting. *International Journal of Central Banking*, 13(1), pp.151-189.

Taylor, J. and Woodford, M. 1999. *Handbook of macroeconomics*. Amsterdam: Elsevier.

Videnova, I. 2016. *Small open economy DSGE model with a banking industry modelled on the financial intermediation approach. Evaluating the performance on UK data* (Doctoral dissertation, Cardiff University).

Wang, J. and Wu, J. J. 2009. *The Taylor Rule and interval forecast for exchange rates*, *Board of Governors of the Federal Reserve System*. Available at: https://www.federalreserve.gov/econres/ifdp/the-taylor-rule-and-interval-forecast-for-exchange-rates.htm (Accessed: 17 May 2023).

Wang, Z. 2020. *Unconventional Monetary Policy of UK: The Study through a SOE-DSGE Model*. PhD Thesis. Cardiff university.

Weale, M. and Wieladek, T. 2016. What are the macroeconomic effects of asset purchases?. *Journal of monetary Economics*, *79*, pp.81-93.

Woodford, M. 2012. Methods of Policy Accommodation at the Interest-Rate Lower Bound(PDF), in *The Changing Policy Landscape*, proceedings of the Jackson Hole

Economic Policy Symposium. Kansas City: Federal Reserve Bank of Kansas City, pp. 185-288.

Woodford, M. 2014. Inflation Targeting: Fix It, Don't Scrap It. *Voprosy Ekonomiki*, (10), pp.44-55.

Wu, J. C. and Xia, F. D. 2016. Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit and Banking*, 48(2-3), pp.253–291. doi:10.1111/jmcb.12300.

Wulfsberg, F. 2016. Inflation and Price Adjustments: Micro Evidence from Norwegian Consumer Prices 1975–2004. *American Economic Journal: Macroeconomics*, 8(3), pp.175-194.

Yao, Y. 2020. *Investigating the effects of fiscal policy in the UK under the zero lower bound on nominal interest rates.* PhD Thesis, Cardiff University.

Zhang, H. and Yang, Y. 2021. Does tourism contribute to real estate boom? A DSGE modeling in small open economy. *Journal of Hospitality & Tourism Research*, *45*(1), pp.257-279.

Zhao, Z., Meenagh, D. and Minford, P. 2022. Should Hong Kong switch to Taylor rule?—Evidence from DSGE model, Applied Economics, 54:50, 5851-5872, DOI: 10.1080/00036846.2022.2053655

Zhou, P. and Dixon, H. 2018. The Determinants of Price Rigidity in the UK: Analysis of the CPI and PPI Microdata and Application to Macrodata Modelling. *The Manchester School*, 87(5), pp.640-677.

Zhu, Z. 2017. *The external finance premium in the UK: a small open economy DSGE model with an empirical indirect inference assessment.* PhD Thesis, Cardiff University.

Appendix A: The Financial Friction of the BGG Model

We incorporate the bank sector of the BGG model into the model. At time t, entrepreneurs purchase newly installed capital $\mathrm{K_{t+1}}$ from capital producers at price $\mathrm{P_t^k}$ to be used in production in period t+1. In order to finance this capital expenditure, entrepreneurs need external loans to supplement their net worth as a source of funding, and they borrow external funding from banks in the form of bank loans which are repaid at time t+1. Thus, entrepreneurs finance their capital purchases with internal funds and external loans from commercial banks. We assume that bank loans come from the domestic market only. At time t+1, an entrepreneur receives the marginal product of capital (R_{t+1}^k) and the gains from selling $(1-\delta)K_{t+1}$ of undepreciated capital to capital producers at price P_{t+1}^k. At the end of period t, the entrepreneur has net worth $\mathrm{NW_{t+1}}$, then the amount of debt is the difference between the expenditures of capital goods and net worth, that is $(\mathrm{K_{t+1}P_t^k - NW_{t+1}})$.

Entrepreneurs' expected marginal rate of return on capital, CY_{t+1}, is defined as:

$$E_t(CY_{t+1}) = E_t\left[\frac{R_{t+1}^k + (1-\delta)P_{t+1}^k}{P_t^k}\right] \quad \text{(A.1)}$$

where E_t is the expectation operator based on the information available at time t. The expected marginal rate of real return on capital, CY_{t+1}, equals to the expected marginal external financing cost. δ is the depreciation rate of capital. $(1-\delta)P_{t+1}^k$ is the value of one unit of capital used in production in period t+1. Equation A.1 shows that the expected marginal rate of return on a unit of capital held from t to t+1 consists of the marginal product of capital and the capital gain.

The Partial Equilibrium Contracting Problem: The contract between the borrower (entrepreneur) and lender (bank) follows a 'Costly State Verification' framework. The lender-borrower relationship is characterised by financial frictions that arise from asymmetric information about the realisation of the project's ex-post returns. We assume that there is no cost for entrepreneurs to observe their output, but commercial banks must pay an audit cost to do so.

In order to borrow, the entrepreneur must sign a debt contract containing the amount of capital expenditure, $K_{t+1}P_t^k$, and the related external borrowing, $Borr_{t+1} = K_{t+1}P_t^k - NW_{t+1}$, before the realisation of idiosyncratic shocks to the return on capital ω. When an idiosyncratic shock occurs, the entrepreneur must decide whether to repay the debt or default after evaluating the outcome of his project, depending on a critical threshold $\overline{\omega}$, at which the entrepreneur is indifferent between default and non-default. If the idiosyncratic shock value is higher than the threshold, the entrepreneur repays the loan and maintains the surplus. However, if the shock value is less than the threshold, the bank audits the loan and recovers the project outcome, and minus the monitoring costs. As a result, the optimal contract could be characterised by the threshold value of the idiosyncratic shock, $\overline{\omega}$, and a gross non-default loan rate, Z_{t+1}. **The bankruptcy threshold $\overline{\omega}$ is defined as** (we drop index i for simplicity):

$$\overline{\omega}(1 + CY_{t+1})K_{t+1}P_t^k = Z_{t+1}Borr_{t+1} = Z_{t+1}(K_{t+1}P_t^k - NW_{t+1}) \qquad \text{(A.2)}$$

where CY_{t+1} is the expected return to capital. $K_{t+1}P_t^k$ is the value of capital. $Borr_{t+1}$ is the amount of external borrowing. NW_{t+1} is the entrepreneur's net worth. As described in equation A.2, the entrepreneur's gross return at time t+1 is equal to the loan payment when ω is at its threshold value $\overline{\omega}$. It is worth noting that the total loanable funds $Borr_{t+1}$ is equal to the household's total deposits in commercial banks. When $\omega \geq \overline{\omega}$, under the optimal contract, the entrepreneur repays the bank the promised amount $Z_{t+1}Borr_{t+1}$ and keeps the surplus $\omega(1 + CY_{t+1})K_{t+1}P_t^k - Z_{t+1}Borr_{t+1}$. When $\omega < \overline{\omega}$, the entrepreneur defaults, the bank pays the auditing cost μ and retains whatever is available, this suggests that the bank's net receipts are $(1 - \mu)\omega(1 + CY_{t+1})K_{t+1}P_t^k$. Thus, the bank's ex-post return can be expressed as:

$$\begin{cases} Z_{t+1}Borr_{t+1} & if\ \omega \geq \overline{\omega} \\ (1-\mu)\omega(1 + CY_{t+1})K_{t+1}P_t^k & if\ \omega < \overline{\omega} \end{cases} \qquad \text{(A.3)}$$

In an optimal contract, the values of $\overline{\omega}$ and Z_{t+1} are determined by the condition that the bank's expected return equals the opportunity of its funds. Since the loan risk is fully diversifiable in this case, the real risk-free rate, R_{t+1}^{real}, is the relevant real opportunity cost of lending for the bank. Therefore, **the loan contract must satisfy the bank's zero profit condition as follows**:

$$[1 - F(\overline{\omega})]Z_{t+1}Borr_{t+1} + (1-\mu)\int_0^{\overline{\omega}} \omega(1 + CY_{t+1})K_{t+1}P_t^k dF(\omega) = \left(1 + R_{t+1}^{real}\right)Borr_{t+1}$$

$$= \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1}) \qquad \text{(A.4)}$$

where $F(\overline{\omega})$ is the probability of default, and μ is the auditing cost. The left-hand side of equation A.4 is the aggregate expected return on the loan to the entrepreneur, which is the sum of return obtained in the case of non-default (the first term on the left-hand side) and the return gained in the event of default (the second term). The right-hand side is the bank's real opportunity cost.

Combining equations A.3 and A.4 eliminates Z_{t+1} and yeilds:

$$\left\{[1 - F(\overline{\omega})]\overline{\omega} + (1-\mu)\int_0^{\overline{\omega}} \omega dF(\omega)\right\}[(1 + CY_{t+1})K_{t+1}P_t^k]$$

$$= \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1}) \qquad \text{(A.5)}$$

Now, we denote $f(\omega)$ as the probability density function of ω, hence $f(\omega)d\omega = dF(\omega)$; the probability of non-default is $[1 - F(\overline{\omega})] = \int_{\overline{\omega}}^{\infty} f(\omega)d\omega$. We also denote $G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega$, which is the bank's share of profits if the entrepreneur defaults. The bank's expected gross share of profits is $\Gamma(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega + \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega$, which can be rewritten as $\Gamma(\overline{\omega}) = G(\overline{\omega}) + \overline{\omega}[1 - F(\overline{\omega})]$. The bank's net share of profits is equal to the gross share of profits minus the monitoring cost, $\Gamma(\overline{\omega}) - \mu G(\overline{\omega})$.

The entrepreneur maximises his expected return subject to equation A.5. The expected return to the entrepreneur is:

$$\int_{\overline{\omega}}^{\infty} \omega(1 + CY_{t+1})K_{t+1}P_t^k dF(\omega) - [1 - F(\overline{\omega})]Z_{t+1}Borr_{t+1} \qquad \text{(A.6)}$$

Substituting equation A.2 into equation A.6 yields:

$$\int_{\overline{\omega}}^{\infty} \omega(1 + CY_{t+1})K_{t+1}P_t^k dF(\omega) - [1 - F(\overline{\omega})]\overline{\omega}(1 + CY_{t+1})K_{t+1}P_t^k \qquad \text{(A.7)}$$

Substituting $f(\omega)d\omega = dF(\omega)$ and $[1 - F(\overline{\omega})] = \int_{\overline{\omega}}^{\infty} f(\omega)d\omega$ into equation A.7 yields:

$$\int_{\overline{\omega}}^{\infty} f(\omega)\omega(1 + CY_{t+1})K_{t+1}P_t^k d\omega - \int_{\overline{\omega}}^{\infty} f(\omega)d\omega\,[\overline{\omega}(1 + CY_{t+1})K_{t+1}P_t^k] \qquad \text{(A.8)}$$

Then **the entrepreneur's expected return can be simplified as**

$$[1 - \Gamma(\overline{\omega})][(1 + CY_{t+1})K_{t+1}P_t^k] \qquad \text{(A.9)}$$

where $1 - \Gamma(\overline{\omega})$ is the entrepreneur's share of profits, and $\Gamma(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega + \overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega$.

Substituting $f(\omega)d\omega = dF(\omega)$, $[1 - F(\overline{\omega})] = \int_{\overline{\omega}}^{\infty} f(\omega)d\omega$ and $G(\overline{\omega}) = \int_0^{\overline{\omega}} \omega f(\omega)d\omega$ into equation A.5 yields:

$$\left\{\overline{\omega}\int_{\overline{\omega}}^{\infty} f(\omega)d\omega + (1-\mu)\int_{0}^{\overline{\omega}} \omega f(\omega)d\omega\right\}[(1+CY_{t+1})K_{t+1}P_t^k]$$
$$= \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1})$$

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})][(1+CY_{t+1})K_{t+1}P_t^k] = \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1}) \quad \text{(A.10)}$$

where $\Gamma(\overline{\omega}) - \mu G(\overline{\omega})$ is the bank's net share of profits.

The optimal contracting problem can be written as maximizing the entrepreneur's expected return (equation A.9) subject to equation A.10:

$$\max_{K,\overline{\omega}}[1 - \Gamma(\overline{\omega})][(1+CY_{t+1})K_{t+1}P_t^k]$$

s.t

$$[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})][(1+CY_{t+1})K_{t+1}P_t^k] = \left(1 + R_{t+1}^{real}\right)(K_{t+1}P_t^k - NW_{t+1})$$

We define $PM_t = \frac{1+CY_{t+1}}{1+R_{t+1}^{real}}$ as external finance premium and $Lev_t = \frac{K_{t+1}P_t^k}{NW_{t+1}}$ as the leverage ratio. The first order conditions are:

$$\partial\overline{\omega}: \quad \lambda = \Gamma'(\overline{\omega}) / \, [\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})] \quad \text{(A.11)}$$

$$\partial Lev_t: \; PM_t = \lambda / \{[1 - \Gamma(\overline{\omega})] + \lambda[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]\} \quad \text{(A.12)}$$

$$\partial\lambda: \; [\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]PM_t Lev_t - (Lev_t - 1) = 0 \quad \text{(A.13)}$$

Since the share of profits to the bank, $\Gamma(\overline{\omega}) - \mu G(\overline{\omega})$, increases on (0, $\overline{\omega}^*$) and decreases on ($\overline{\omega}^*, \infty$), the bank would never pick $\overline{\omega}^* < \overline{\omega}$. We consider the situation $0 < \overline{\omega} < \overline{\omega}^*$, which provides an interior solution.

The first derivative of equation A.11 with respect to $\overline{\omega}$ yields:

$$\lambda'(\overline{\omega}) = \frac{\mu[\Gamma'(\overline{\omega})G''(\overline{\omega}) - \Gamma''(\overline{\omega})G'(\overline{\omega})]}{[\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})]^2} > 0 \;\; \text{for } \overline{\omega} \in (0, \overline{\omega}^*)$$

Taking limits gives:

$$\lim_{\overline{\omega}\to 0} \lambda(\overline{\omega}) = 1, \text{ and } \lim_{\overline{\omega}\to\overline{\omega}^*} \lambda(\overline{\omega}) = +\infty$$

To take the first derivative of equation A.12, we first define $\rho(\overline{\omega}) = \frac{\lambda(\overline{\omega})}{\{[1-\Gamma(\overline{\omega})] + \lambda[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]\}}$ which is the wedge between the expected rate of return on capital and risk-free return on deposit. We combine this with equation A.12 to obtain:

$$PM_t = \frac{\lambda(\overline{\omega})}{[1-\Gamma(\overline{\omega})] + \lambda[\Gamma(\overline{\omega}) - \mu G(\overline{\omega})]} = \rho(\overline{\omega}) \quad \text{(A.14)}$$

The first derivative with respect to $\overline{\omega}$ yields:

$$\rho'(\bar{\omega}) = \rho(\bar{\omega})\frac{\lambda'(\bar{\omega})}{\lambda(\bar{\omega})}\left[\frac{1-\Gamma(\bar{\omega})}{[1-\Gamma(\bar{\omega})]+\lambda[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}\right] > 0 \quad \text{for } \bar{\omega} \in (0, \bar{\omega}^*)$$

Taking limits gives:

$$\lim_{\bar{\omega}\to 0} \rho(\bar{\omega}) = 1 \;\; and \lim_{\bar{\omega}\to\bar{\omega}^*} \rho(\bar{\omega}) = PM_t < \frac{1}{1-\mu}$$

For $PM_t < PM_t^*$, a one-to-one mapping between the optimal cutoff $\bar{\omega}$ and the external finance premium PM_t is guaranteed.

From equation A.13 we can solve for Lev_t

$$Lev_t = 1 + \frac{\lambda[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}{1-\Gamma(\bar{\omega})} = \Psi(\bar{\omega}) \tag{A.15}$$

where we define $\Psi(\bar{\omega}) = 1 + \frac{\lambda[\Gamma(\bar{\omega})-\mu G(\bar{\omega})]}{1-\Gamma(\bar{\omega})}$, taking the derivative we obtain:

$$\Psi'(\bar{\omega}) = \frac{\lambda'(\bar{\omega})}{\lambda(\bar{\omega})}[\Psi(\bar{\omega}) - 1] + \frac{\Gamma'(\bar{\omega})}{1-\Gamma(\bar{\omega})}\Psi(\bar{\omega}) > 0 \quad \text{for } \bar{\omega} \in (0, \bar{\omega}^*)$$

Taking limits gives:

$$\lim_{\bar{\omega}\to 0} \Psi(\bar{\omega}) = 1, \text{ and } \lim_{\bar{\omega}\to\bar{\omega}^*} \Psi(\bar{\omega}) = +\infty$$

Combing equation A.14 and equation A.15 yields:

$$Lev_t = \frac{K_{t+1}P_t^k}{NW_{t+1}} = v(PM_t) = v\left(\frac{1+CY_{t+1}}{1+R_t/\pi_{t+1}^{cpi}}\right) \tag{A.16}$$

where $\mathrm{v}'(\mathrm{PM_t}) > 0$ for $\mathrm{PM_t} \in (0, \mathrm{PM}^*)$, thus PM is an increasing function of the leverage ratio Lev. According to Le et al. (2013), exogenous premium shocks ($\varepsilon_t^{\mathrm{prem}}$) can be thought of as shocks to the credit supply such as changes in the efficiency of the financial intermediation process, also affect the external finance premium. The log-linearised form of the external finance premium consistent with Le et al. is:

$$pm_t = E_t cy_{t+1} - r_t^{real} = \chi(qq_t + k_t - n_t) + e_t^{prem} \tag{A.17}$$

$$pm_t = E_t cy_{t+1} - \left(r_t - E_t \pi_{t+1}^{cpi}\right) = \chi(qq_t + k_t - n_t) + e_t^{prem} \tag{A.18}$$

where the parameter χ is the elasticity of the external finance premium with respect to the leverage ratio. Equation A.17 states that the external finance premium is determined by the wedge between the cost of external funds (cy) and internal funds (r^{real}). An alternative interpretation is the credit spread between the risky return (cy) and riskless return (r^{real}). The equation also implies that the external finance premium rises with the share of the capital investment funded by borrowing.

Appendix B: The Log-linearised Model Listing

We log-linearise the model around the steady-state balanced growth path or long-run trend. Each equation is normalised with an endogenous variable. All variables are in natural logarithms, except those variables already in the form of percentages and ratios. We list all the equations of the linearised DSGE model below:

Consumption Euler Equation: Current consumption c_t is dependent on a weighted average of previous and expected future consumption, as well as on the expected increase in working hours $(l_t - E_t l_{t+1})$, the ex-ante real interest rate $\left(r_t - E_t \pi_{t+1}^{cpi}\right)$ and a disturbance term e_t^b. When $\sigma_c > 1$, hours worked and consumption are complements in utility, and current working hours (l_t) have a positive effect on consumption (c_t), while the expected future working hours $(E_t l_{t+1})$ have a negative effect on consumption (Basu and Kimball, 2002).

$$c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1}) - c_4 \left(r_t - E_t \pi_{t+1}^{cpi}\right) + e_t^b \tag{B.1}$$

$$c_1 = \frac{\frac{h}{\gamma}}{1 + \frac{h}{\gamma}}, \qquad c_2 = \frac{1}{1 + \frac{h}{\gamma}}, \qquad c_3 = \frac{(\sigma_c - 1)\frac{W_* L_*}{C_*}}{(1 + \frac{h}{\gamma})\sigma_c}, \qquad c_4 = \frac{1 - \frac{h}{\gamma}}{(1 + \frac{h}{\gamma})\sigma_c}$$

where h determines the external habit formation of consumption; γ captures the steady-state growth rate; σ_c is the inverse of the intertemporal elasticity of substitution between consumption; $\frac{W_* L_*}{C_*}$ is the steady-state ratio of labour income to consumption; π_t^{cpi} is CPI inflation.

Investment Euler Equation: Investment is positively dependent on past and expected future investment and the real value of the current capital stock (qq_t). The sensitivity of investment (i_t) to the value of the capital stock is decreased by a higher steady-state elasticity of capital adjustment cost (φ). The capital adjustment cost is modelled as a function of the changes in investment rather than its level, which gives extra dynamics to the investment equation and helps capture the hump-shaped responses of investment to numerous shocks, see Christiano, Eichenbaum and Evans (2005).

$$i_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t i_{t+1} + \frac{1}{(1+\beta\gamma^{(1-\sigma_c)})\gamma^2\varphi} qq_t + e_t^i \qquad \text{(B.2)}$$

where φ captures the steady-state elasticity of capital adjustment cost, and β is the discount factor.

Production Function: Output is produced using labour services (l_t) and effective capital (k_t^s).

$$y_t = \phi[\alpha k_t^s + (1-\alpha)l_t + ea_t] \qquad \text{(B.3)}$$

where α is the share of labour in production, and ϕ is one plus the share of fixed costs in production.

Capital Accumulation Equation: The accumulation of installed capital (k_t) depends on investment flows and the relative efficiency of these investment expenditures.

$$k_t = \left(\frac{1-\delta}{\gamma}\right) k_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right) i_t + \left(1 - \frac{1-\delta}{\gamma}\right)\left(\left(1 + \beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi\right) e_t^i \qquad \text{(B.4)}$$

where δ is the depreciation rate.

Current Capital Service: The current capital service used for production (k_t^s) is a function of the capital installed in the previous period (k_{t-1}) and the capital utilisation rate (z_t).

$$k_t^s = k_{t-1} + z_t \qquad \text{(B.5)}$$

Capital Utilisation: The degree of capital utilisation depends positively on the capital rental rate.

$$z_t = \frac{1-\psi}{\psi} rk_t \qquad \text{(B.6)}$$

where ψ is the elasticity of the capital utilisation adjustment cost and is normalized between zero and one. $\psi = 1$ implies that the cost of changing capital utilisation is extremely high, and therefore, the capital utilisation rate remains constant. $\psi = 0$ means that the marginal cost of adjusting capital utilisation is constant so that the marginal product of capital remains constant in equilibrium.

Capital Arbitrage (Tobin' Q) Equation: The current value of capital stock (qq_t) is positively correlated with its expected future value (qq_{t+1}) and expected marginal product of capital (rk_{t+1}) and negatively related to external financing cost (cy_{t+1}). In this model, we assume that entrepreneurs have to borrow from commercial banks to finance their capital expenditures. Thus, cy enters the Tobin's Q equation.

$$qq_t = \frac{1-\delta}{1-\delta+R_*^K} E_t qq_{t+1} + \frac{R_*^K}{1-\delta+R_*^K} E_t rk_{t+1} - E_t cy_{t+1} \quad \text{(B.7)}$$

where R_*^K is the steady-state value of the marginal product of capital.

Demand for Labour: Profit maximisation for the entrepreneur implies that the demand for labour depends negatively on the real producer wage and positively on the marginal product of capital.

$$l_t = -w_t^h + \left(1 + \frac{1-\psi}{\psi}\right) rk_t + k_{t-1} \quad \text{(B.8)}$$

External Finance Premium: The external finance premium depends positively on the amount of capital investment financed by entrepreneurs through borrowing. Moreover, an increase in the supply of M0 lowers the credit premium.

$$prem_t = E_t cy_{t+1} - \left(r_t - E_t \pi_{t+1}^{cpi}\right) = \chi(qq_t + k_t - n_t) - \vartheta m_t^0 + e_t^{prem} \quad \text{(B.9)}$$

where the parameter χ is the elasticity of the external finance premium with respect to the leverage ratio, and ϑ is the elasticity of the premium to M0 through its collateral role.

Net Worth Evolution Equation: Entrepreneurial net worth is given by the surviving entrepreneurs' past net worth (θn_{t-1}) plus their total return on capita (cy_t) minus the expected return (cost of borrowing paid to the banks) on the externally financed portion of capital stock.

$$n_t = \frac{K}{N}(cy_t - E_{t-1} cy_t) + E_{t-1} cy_t + \theta n_{t-1} + e_t^{nw} \quad \text{(B.10)}$$

where $\frac{K}{N}$ is the steady-state ratio of capital to net worth, and θ represents the survival rate of entrepreneurs.

Consumption of Entrepreneurs: Those entrepreneurs who die from the market will consume all their net worth. Thus, the entrepreneur's consumption is equal to the probability of dying from the market $(1-\theta)$ multiplied by net worth. In logarithms, this means that the entrepreneur's consumption varies proportionally to net worth.

$$c_t^e = n_t \tag{B.11}$$

Hybrid Domestic Price Setting (Weighted Home Inflation): Aggregate domestic prices are set to be the weighted average of the corresponding NK and NC equations. In the NK model, home inflation π_t^h is positively correlated with expected future and past home inflation and price mark-up disturbance, and negatively correlated with current price mark-up. When price indexation is zero $(l_p = 0)$, home inflation in the NK model reverts to a purely forward-looking Phillips curve. The NC equation is derived by setting the price mark-up to zero.

$$\boldsymbol{(\pi_t^h)^{NK}}: \quad \pi_t^h = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c} l_p} E_t \pi_{t+1}^h + \frac{l_p}{1+\beta\gamma^{(1-\sigma_c)} l_p} \pi_{t-1}^h - \frac{1}{1+\beta\gamma^{(1-\sigma_c)} l_p}$$

$$\left(\frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_p)(1-\xi_p)}{\xi_p(1+(\phi_p-1)\epsilon_p)}\right)(\alpha r k_t + (1-\alpha) w_t^h - e a_t) + e_t^p \tag{B.12}$$

NC Marginal Product of Labour $\boldsymbol{(\pi_t^h)^{NC}}: r k_t = \frac{1}{\alpha}[-(1-\alpha) w_t^h + e a_t]$ (B.13)

$$(\pi_t^h)^{hybrid} = \omega^p (\pi_t^h)^{NK} + (1-\omega^p)(\pi_t^h)^{NC} \tag{B.14}$$

where ω^p is the fraction of intermediate goods from imperfectly competitive markets (sticky pricing), and $(1-\omega^p)$ is the fraction of intermediate goods from perfectly competitive markets (flexible pricing). In the NK equation, ξ_p is the Calvo probability (level of price stickiness), l_p and ϵ_p are partial price indexation and the Kimball aggregator curvature in the goods market, respectively. $w_t^h = w_t^c + \frac{\omega}{1-\omega} \mathrm{q}_\mathrm{t}$ and $\pi_t^h = \pi_t^{cpi} - \frac{\omega}{1-\omega} \Delta q_t$; see equations B.24 and B.25.

Hybrid Real Consumer Wage Setting: Similarly, the real consumer wage setting equation is set to be a weighted average of the corresponding NK and NC equations. In the NK model, the real consumer wage $(w_t^c)^{NK}$ is given by expected and past real consumer wages, as well as expected, current and past CPI inflation, real consumer wage mark-up and disturbance. When wage indexation is zero (l_w), real wages do not depend on past inflation.

$$\boldsymbol{(w_t^c)^{NK}}: \quad w_t^c = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t w_{t+1}^c + \frac{1}{1+\beta\gamma^{1-\sigma_c}} w_{t-1}^c + \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}} E_t \pi_{t+1}^{cpi}$$

$$-\frac{1+(\beta\gamma^{(1-\sigma_c)})\iota_w}{1+\beta\gamma^{1-\sigma_c}}\pi_t^{cpi}+\frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}}\pi_{t-1}^{cpi}-\frac{1}{1+\beta\gamma^{1-\sigma_c}}\left(\frac{\left(1+\beta\gamma^{(1-\sigma_c)}\xi_w\right)(1-\xi_w)}{\xi_w(1+(\phi_w-1)\epsilon_w)}\right)$$

$$\left(w_t^c-\sigma_l l_t-\left(\frac{1}{1-\frac{h}{\gamma}}\right)\left(c_t-\frac{h}{\gamma}c_{t-1}\right)\right)+e_t^w \quad \text{(B.15)}$$

NC Labour Supply $(\boldsymbol{w_t^c})^{\boldsymbol{NC}}$: $w_t^c=\sigma_l l_t+\left(\frac{1}{1-\frac{h}{\gamma}}\right)\left(c_t-\frac{h}{\gamma}c_{t-1}\right)-\left(\pi_t^{cpi}-E_{t-1}\pi_t^{cpi}\right)+e_t^{w_{NC}}$

(B.16)

$$w_t^{hybrid}=\omega^w(w_t^c)^{NK}+(1-\omega^w)(w_t^c)^{NC} \quad \text{(B.17)}$$

where ω^w is the fraction of labour from imperfectly competitive markets (sticky wages), and $(1-\omega^w)$ is the fraction of labour from perfectly competitive markets with wage flexibility. In the NK equation, ξ_w is the Calvo probability, ι_w and ϵ_w are partial wage indexation and the Kimball labour aggregator, respectively.

Monetary Policy for Normal Regime ($\boldsymbol{r_t>0.025\%}$, non-crisis): In the normal regime, the central bank follows the Taylor Rule by adjusting interest rates (r_t) in response to CPI inflation, output and output gap. The supply of M0 is set to accommodate M2, which is determined by the balance sheet of the entrepreneur.

$$\begin{cases} M0: & m_t^0=m_{t-1}^0+\vartheta_1(m_t^2-m_t^2)+e_t^{m0,nocrisis} \\ Taylor\ Rule: & r_t=\rho r_{t-1}+(1-\rho)\left(r_p\pi_t^{cpi}+r_y y_t\right)+r_{\Delta y}(y_t-y_{t-1})+e_t^r \\ M2: & m_t^2=(1+v-\mu)k_t+\mu m_t^0-vn_t, \qquad \mathrm{v}=\frac{\mathrm{NW}}{\mathrm{M2}}, \mu=\frac{\mathrm{M0}}{\mathrm{M2}} \end{cases} \quad \text{(B.18)}$$

where ρ reflects the degree of interest rate smoothing. r_p, r_y and $r_{\Delta y}$ measure the response to inflation, output, and output gap, respectively. ϑ_1 is the elasticity of M0 to M2.

Monetary Policy for Crisis Regime ($\boldsymbol{r_t\le 0.025\%}$, crisis): In the crisis regime, the Taylor Rule is suspended and replaced by an exogenous low bound ($r=0.025\%$). In addition, M0 (i.e., QE) targets the credit premium around its steady state in an attempt to restore credit condition to normal through its role of providing cheap collateral.

$$\begin{cases} M0: & m_t^0=m_{t-1}^0+\vartheta_2(prem_t-prem^*)+e_t^{m0,crisis} \\ r: & r=0.025\% \end{cases} \quad \text{(B.19)}$$

where ϑ_2 is the elasticity of M0 with respect to premium.

Real Uncovered Interest Rate Parity: Any difference between domestic and foreign real interest rates is offset by expected changes in the real exchange rate.

$$q_t = E_t q_{t+1} + (r_t^f - E_t \pi_{t+1}^f) - (r_t - E_t \pi_{t+1}^{cpi}) \quad \text{(B.20)}$$

Import Equation: Imports depend positively on consumption and negatively on the real exchange rate. Intuitively, an increase in the real exchange rate implies a devaluation of the home currency. Thus, domestic prices are more advantageous than foreign prices, hence discouraging imports.

$$im_t = ln\omega + c_t - \sigma q_t + e_t^{im} \quad \text{(B.21)}$$

where σ is the elasticity of substitution between domestic and imported varieties of goods, and ω is the weight of imported goods in the consumption bundle.

Export Equation: Exports depend positively on foreign consumption and real exchange rate. Intuitively, a depreciation of the home currency encourages exports.

$$ex_t = ln\omega^f + c_t^f + \sigma^f q_t + e_t^{ex} \quad \text{(B.22)}$$

where f is foreign country index.

Net Foreign Assets Evolution: Exports, imports and interest rates determine the evolution of net foreign assets. The balance of payments constraint indicates that the sum of 'the reduction in net foreign assets' (capital account deficit) and 'net exports plus income flows from foreign assets' (the current account surplus) is zero.

$$b_t^f = \left(1 + r_{t-1}^f\right) b_{t-1}^f + \frac{EX}{Y}(ex_{t-1} - q_{t-1}) - \frac{IM}{Y} im_{t-1} \quad \text{(B.23)}$$

where $\frac{EX}{Y}$ and $\frac{IM}{Y}$ are the steady-state export-output ratio and import-output ratio respectively.

CPI Inflation: The gap between the CPI inflation π_t^{cpi} and home inflation π_t^h proportional to the percent change in real exchange rate, with the coefficient of proportional given by $\frac{\omega}{1-\omega}$.

$$\pi_t^{cpi} = \pi_t^h + \frac{\omega}{1-\omega} \Delta q_t \quad \text{(B.24)}$$

The Wedge Between the Real Consumer Wage and Real Producer Wage: The real producer wage w_t^h is related to the real consumer wage w_t^c and the real exchange rate.

$$w_t^h = w_t^c + \frac{\omega}{1-\omega}\mathrm{q_t} \quad \text{(B.25)}$$

Aggregate Resource Constraint: Output is absorbed by consumption, investment, capital utilisation cost, entrepreneurial consumption, and net exports. As in the BGG model, the costs of monitoring are ignored here as they have a negligible effect on the dynamics of the model.

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + R_*^K k_y \frac{1-\psi}{\psi} rk_t + \frac{C^e}{Y}c_t^e + \frac{EX}{Y}ex_t - \frac{IM}{Y}im_t + e_t^g \quad \text{(B.26)}$$

where ψ denotes the elasticity of the cost of capital utilisation. $\frac{C}{Y}, \frac{I}{Y}, \frac{C^e}{Y}, \frac{EX}{Y}$ and $\frac{IM}{Y}$ are the steady-state consumption-output ratio, investment-output ratio, entrepreneurial consumption-output ratio, export-output ratio, and import-output ratio, respectively.

Appendix C: Stochastic Process

The DSGE model contains a full range of structural shocks: eleven domestic and four foreign stochastic processes. These shocks are generally assumed to have an autoregressive structure.

Government spending shock follows an AR(1) process and is also affected by the productivity process:

$$e_t^g = \rho_g e_{t-1}^g + \sigma_{ga}\eta_t^\alpha + \eta_t^g, \qquad \eta_t^g \sim N(0, \sigma_g^2) \quad \eta_t^a \sim N(0, \sigma_a^2)$$

Preference shock follows an AR(1) process:

$$e_t^b = \rho_b e_{t-1}^b + \eta_t^b, \qquad \eta_t^b \sim N(0, \sigma_b^2)$$

Productivity shock follows ARIMA(1,1,0) process:

$$(e_t^a - e_{t-1}^a) = \rho_a (e_t^a - e_{t-1}^a) + \eta_t^a, \qquad \eta_t^a \sim N(0, \sigma_a^2)$$

Investment-specific shock follows an AR(1) process:

$$e_t^i = \rho_i e_{t-1}^i + \eta_t^i, \qquad \eta_t^i \sim N(0, \sigma_i^2)$$

Taylor Rule shock follows an AR(1) process:

$$e_t^r = \rho_r e_{t-1}^r + \eta_t^r, \qquad \eta_t^r \sim N(0, \sigma_r^2)$$

Price mark-up shock follows an AR(1) process (reduced from AEMA(1,1)):

$$e_t^p = \rho_p e_{t-1}^p + \eta_t^p, \qquad \eta_t^p \sim N(0, \sigma_p^2)$$

NK wage mark-up shock follows an AR(1) process (reduced from AEMA(1,1)):

$$e_t^{wNK} = \rho_w^{nk} e_{t-1}^{wNK} + \eta_t^{wNK}, \qquad \eta_t^{wNK} \sim N(0, \sigma_{wNK}^2)$$

NC wage mark-up (labour supply) shock follows an AR(1) process:

$$e_t^{w_{NC}} = \rho_w^{nc} e_{t-1}^{w_{NC}} + \eta_t^{wNC}, \qquad \eta_t^{wNC} \sim N(0, \sigma_{wNC}^2)$$

External finance premium shock follows an AR(1) process:

$$e_t^{prem} = \rho_{pr} e_{t-1}^{prem} + \eta_t^{pr}, \qquad \eta_t^{prem} \sim N(0, \sigma_{prem}^2)$$

Net worth shock follows an AR(1) process:

$$e_t^{nw} = \rho_{nw} e_{t-1}^{nw} + \eta_t^{nw}, \qquad \eta_t^{nw} \sim N(0, \sigma_{nw}^2)$$

Export demand shock follows an AR(1) process:

$$e_t^{ex} = \rho_x e_{t-1}^{ex} + \eta_t^x, \qquad \eta_t^{ex} \sim N(0, \sigma_{ex}^2)$$

Import demand shock follows an AR(1) process:

$$e_t^{im} = \rho_m e_{t-1}^{im} + \eta_t^m, \qquad \eta_t^{im} \sim N(0, \sigma_{im}^2)$$

Exogenous foreign consumption process:

$$c_t^f = \rho_c^f c_{t-1}^f + \eta_t^{cf}, \qquad \eta_t^{cf} \sim N(0, \sigma_{cf}^2)$$

Exogenous foreign interest rate process:

$$r_t^f = \rho_r^f r_{t-1}^f + \eta_t^{rf}, \qquad \eta_t^{rf} \sim N(0, \sigma_{rf}^2)$$

Money supply shock (crisis shock) follows an AR(1) process:

$$e_t^{m0,crisis} = \rho_{m0,crisis} e_{t-1}^{m0,crisis} + \eta_t^{m0,crisis}, \qquad \eta_t^{m0,crisis} \sim N(0, \sigma_{m0,crisis}^2)$$

Money supply shock (non-crisis shock) follows an AR(1) process:

$$e_t^{m0,nocrisis} = \rho_{m0,nocrisis} e_{t-1}^{m0,nocrisis} + \eta_t^{m0,nocrisis}, \qquad \eta_t^{m0,nocrisis} \sim N(0, \sigma_{m0,nocrisis}^2)$$

Appendix D: Impulse Response Functions for Estimated model

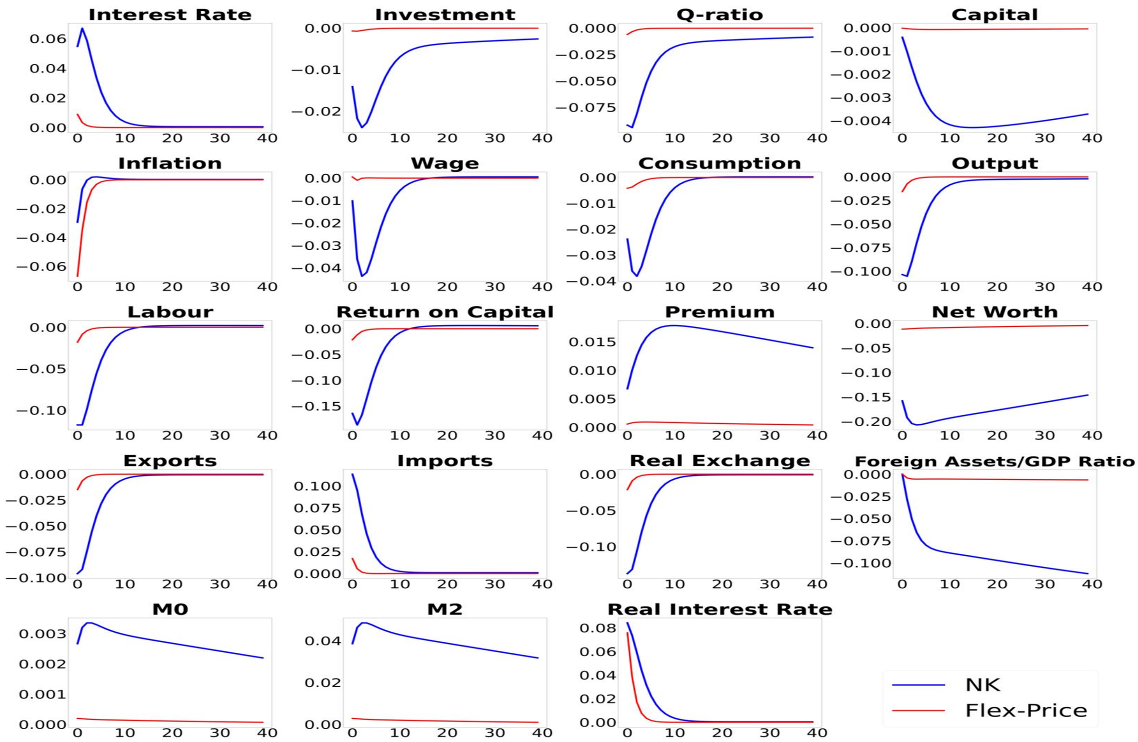

Figure D1: IRFs to a Taylor Rule Shock

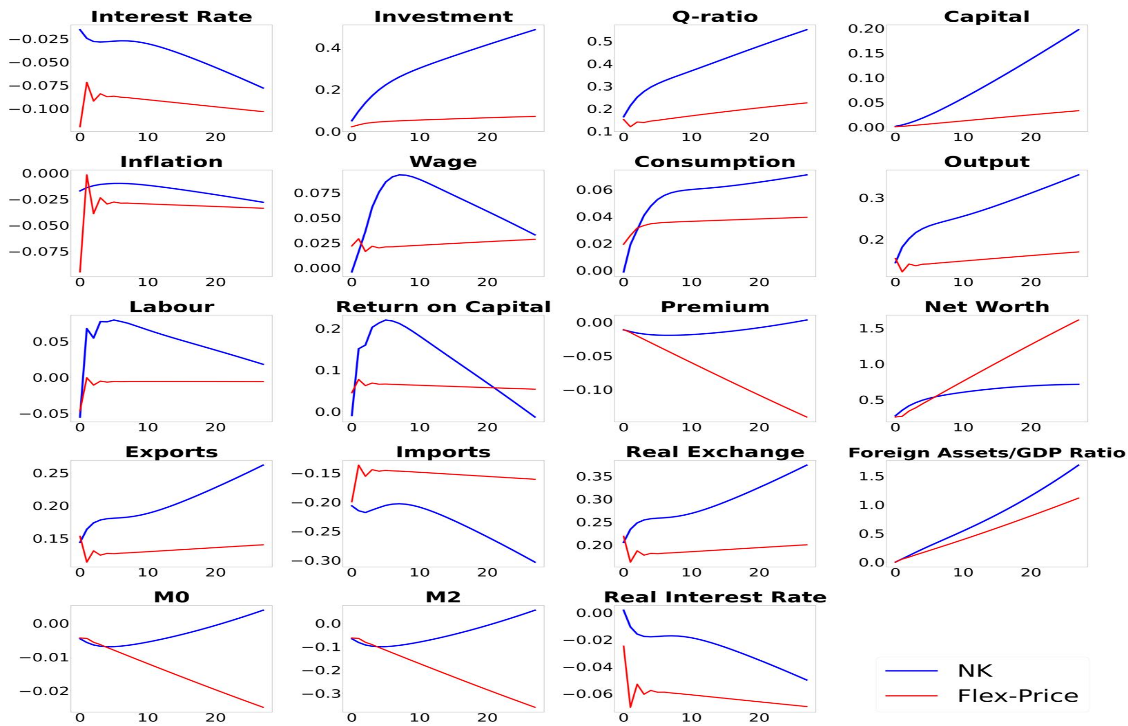

Figure D2: IRFs to a Productivity Shock

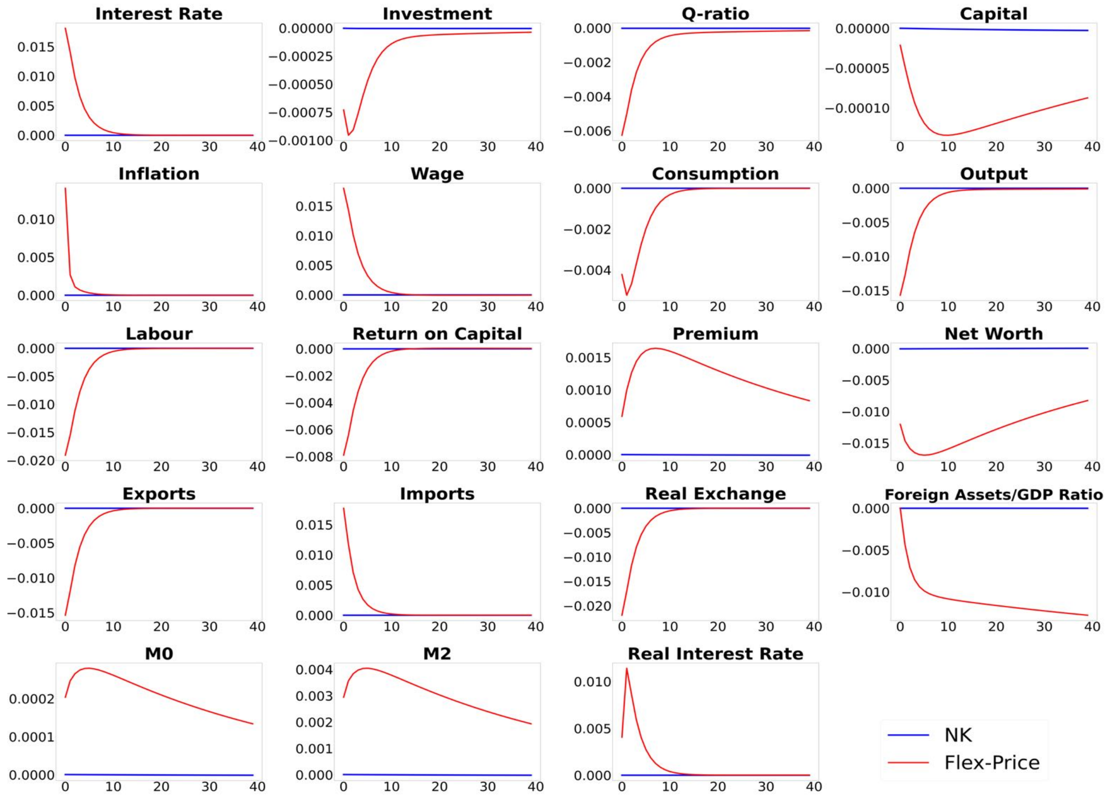

Figure D3: IRFs to a Labour Supply Shock

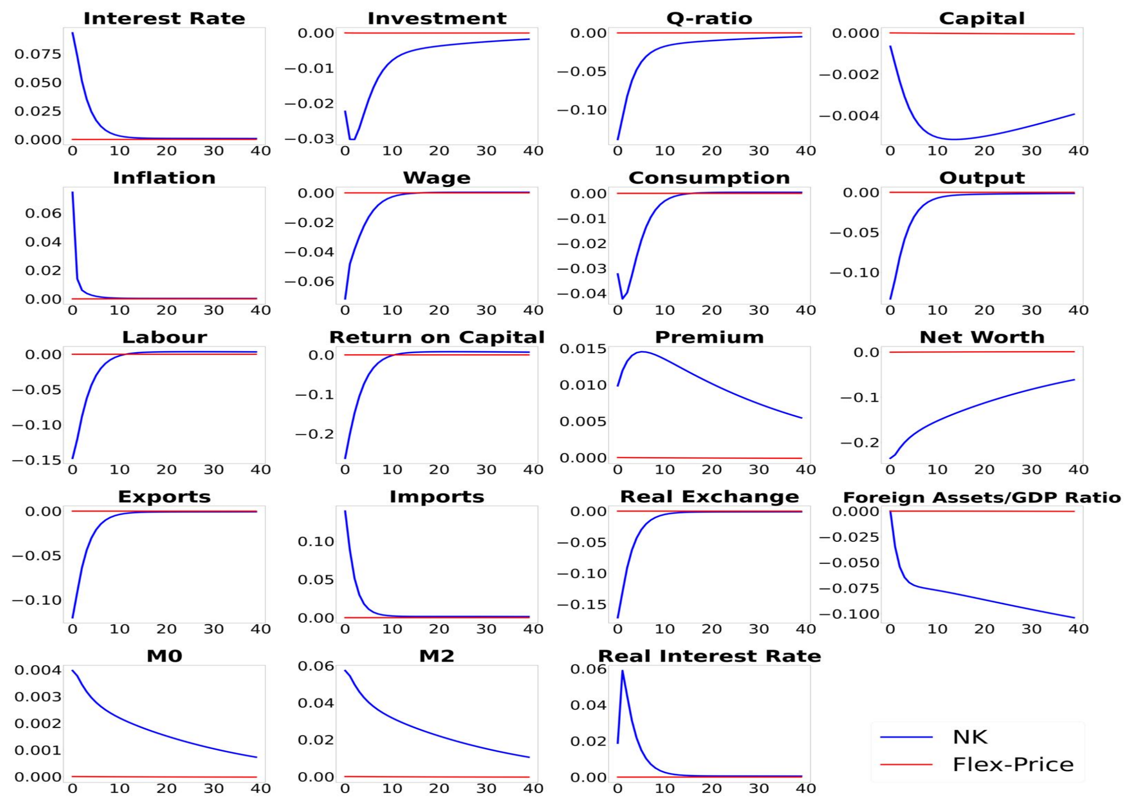

Figure D4: IRFs to a Price Mark-up Shock

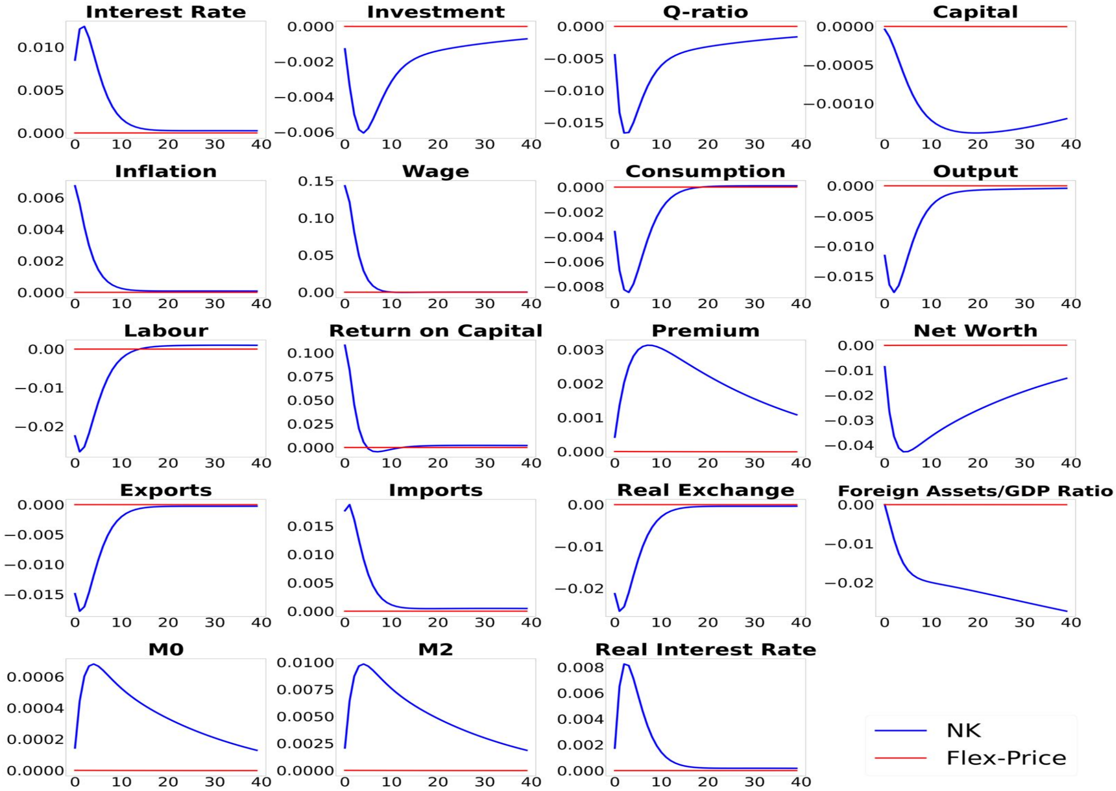

Figure D5: IRFs to a Wage Mark-up Shock

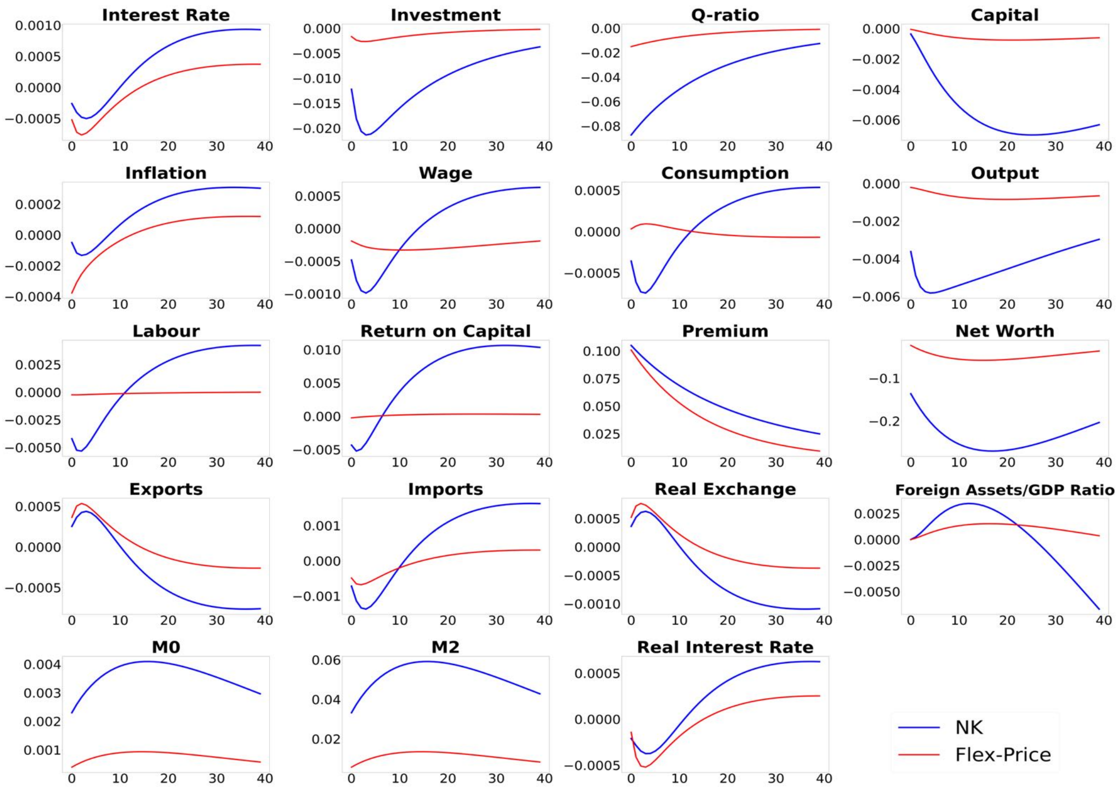

Figure D6: IRFs to a Premium Shock

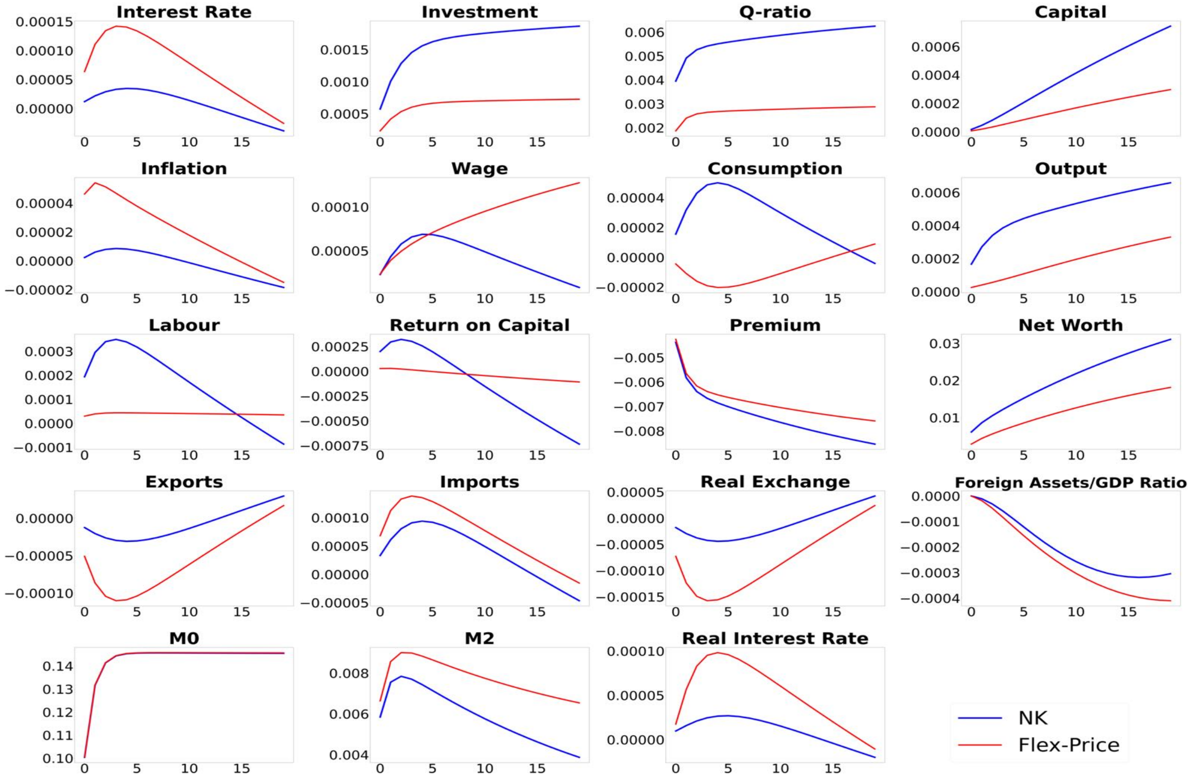

Figure D7: IRFs to a Money Supply Shock

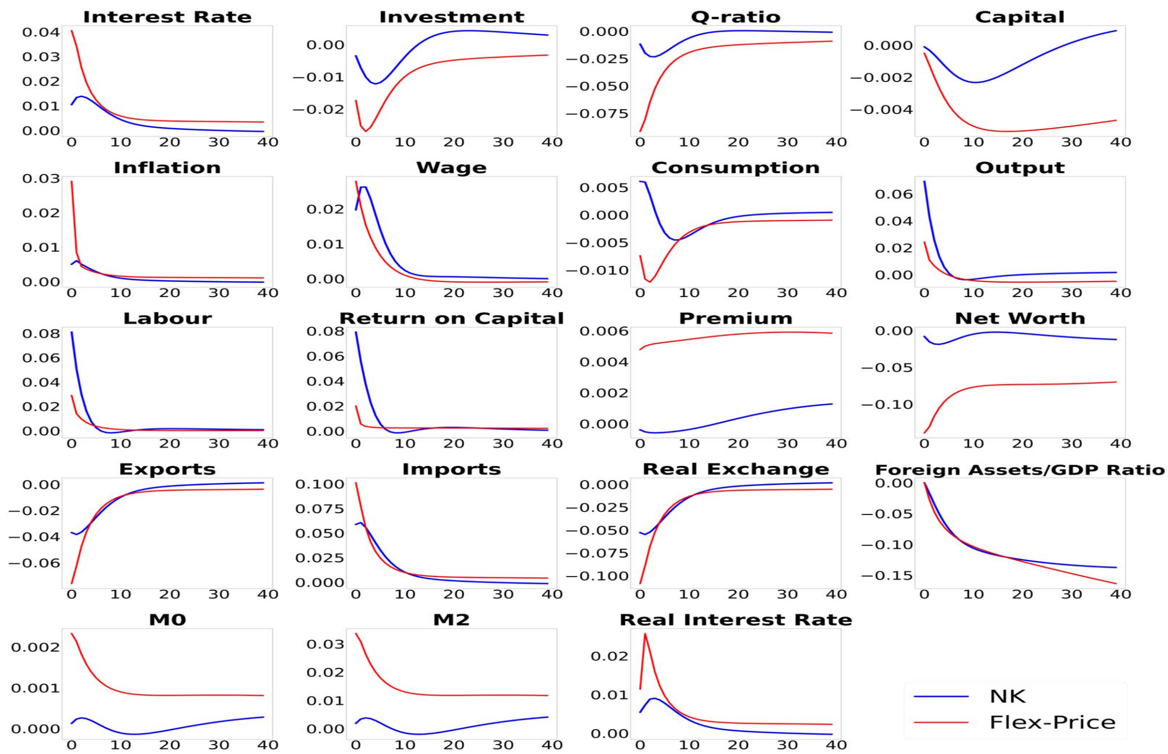

Figure D8: IRFs to a Government Shock