

Resource Misallocation in the Presence of R&D Spillovers*

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Abstract

We study resource misallocation by explicitly modelling R&D input and knowledge spillovers. The effects of R&D and spillovers on firm-level productivity are extensively studied in applied work, but not in the context of resource misallocation. We establish that, in the presence of spillovers, efficient resource allocation requires that more productive firms face higher R&D input prices. Analysing UK firm-level data, we find that the output gains from correcting misallocation are greatly overestimated when spillovers are ignored. Output losses due to capital distortions dominate those from labour and R&D inputs. Adopting a wrong R&D policy could lead to significant output losses.

Keywords: resource misallocation, productivity, R&D spillover, the UK manufacturing firms

JEL Classification Codes: D24, D61, O30, O47

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1 Introduction

In their seminal work, Hsieh and Klenow (2009) provide a theoretical framework for the analysis of resource misallocation across firms.¹ In their model, firms differ in productivity and the efficiency requires that more productive firms employ more of inputs. The efficient allocation will be implemented if profit-maximizing firms face the same effective input prices. However, because of the government policies or the market frictions, the effective prices can differ between the firms. The wedge between the price that the firm pays for an input and its market price is referred to as the distortion. Taking their model to the sample of Chinese and Indian firms, Hsieh and Klenow (2009) show (i) that resource misallocation is indeed prevalent across firms in both countries, and (ii) addressing this misallocation results in substantial output gains. Specifically, Hsieh and Klenow (2009, Table IV) report the gains of 86.6% for China and 127.5% for India in 2005.

This has spurred huge interest in modelling and further understanding the issue of resources misallocation. Consequently, the literature on this issue now is quite considerable, extending the original work of Hsieh and Klenow (2009) in several important directions. For example, Dias et al. (2016) adds intermediate inputs, while Choi (2020) adds energy inputs in the model of Hsieh and Klenow (2009) to make it more comprehensive. Several studies, using somewhat different models, have endogenized firm's productivity by allowing it to depend on the allocation of resources in the economy. For example, in Bello et al. (2011); Ranasinghe (2014); Da-Rocha et al. (2023), firms invest in their productivity, but the amount of investment depends negatively on the size of distortions. Relatedly, in Hopenhayn (2016), distortions affect firm's decision to enter and, hence, the average productivity in the economy.

¹Other early models of resource misallocation are by Hopenhayn and Rogerson (1993) and Restuccia and Rogerson (2008). Surveys by Hopenhayn (2014); Restuccia and Rogerson (2013, 2017) neatly summarize the earlier literature on resource misallocation.

Important though it is, the extant literature does not consider resource misallocation in the presence of externalities, particularly those originating from R&D and knowledge spillovers. However, the impact of R&D and its spillovers on productivity has been extensively studied in applied work. In the words of Griliches (1992, p. S43), “a significant number of . . . studies [are] all pointing in the same direction: R&D spillovers are present, their magnitude may be quite large, and social rates of return remain significantly above private rates.” More recent studies also confirm the importance of R&D and its spillovers; see, for example, Lucking et al. (2019) for the US and Audretsch and Belitski (2020) for the UK.

We aim to bridge this gap. We incorporate the R&D input as the third factor of production, besides capital and labour, and we adopt the standard production function from Griliches (1979, p. 102) which explicitly allows for within-industry R&D spillovers. We compute each industry’s spillover pool (see Section 2 for details) as a geometric average of R&D inputs that are employed by the firms in that industry. We later argue that this measure of spillover could be interpreted as a probability of information sharing, similar to Bloom et al. (2013).

The existence of R&D spillovers introduces externalities in the model, which changes the efficient allocation of resources. While it is still optimal for more productive firms to employ more resources, the optimal allocation of resources becomes more even across the firms when the spillovers are present than when they are not. The intuition for this result is straightforward: the spillover is maximized when R&D input is shared equally between the firms. Because the output now depends on the spillover, the optimal allocation of all inputs, but especially that of R&D input, is less sensitive to the differences in productivities across firms. To induce the firms to choose the right amount of inputs, all firms in the industry must still pay the same labour and capital prices, but the price of R&D input must increase in the firm’s productivity. The latter result has useful policy implications: as long as the firm size

correlates with productivity, any tax incentives that promote R&D, should favour smaller firms. This, for example, is true in the case of the UK tax policy.

We apply our model to the firm-level data in the UK manufacturing sector in 2019 and find some interesting results. First, the output gain from equalizing the effective prices of all firms is substantially overestimated when spillovers are present but ignored. In other words, the model of Hsieh and Klenow (2009) is possibly overestimating the output gains by not considering the knowledge spillover externalities across firms. We find that the output gain is 128.1% when the spillovers are kept fixed at the original level, while the gain is only 76.3% when the spillovers are allowed to change due to the reallocation of R&D input between the firms. Second, as explained above, the efficiency requires that more productive and bigger firms pay higher R&D input prices. When this efficient allocation is implemented, the output gain is 118.8%, which is significantly higher than the gain of 76.3% that is obtained when all firms face the same prices.

Third, we define industry-level total factor productivity (TFP) as the part of aggregate industry output that is not explained either by inputs or the spillover. When reallocating resources between the firms, the output changes because of changes in both TFP and the spillover. When implementing the efficient allocation, we find that most of the output gains come from the increase in TFP. However, it does not mean that the spillover does not play an important role in the output. For instance, if all firms faced the same prices, TFP would be maximized², but the output would not be maximized because of the large drop in the size of spillover. Put differently, the sizeable overestimation of output that we report above, is because of the decrease in the spillover.

Fourth, we study how much different distortions contribute to the output

²Note that in the absence of spillovers, maximizing industry output is equivalent to maximizing industry TFP.

loss. We have three types of distortions in our model, one for each factor of production. Following Hsieh and Klenow (2009); Chen and Irarrazabal (2015); Ryzhenkov (2016), we write the output in terms of variances and covariances of these distortions where the covariances are between distortions and the firm's productivity. In the absence of externalities, the output is maximized when these variances and covariances are zero because it corresponds to the situation when all firms face the same prices. However, with externalities present, the efficient allocation requires that R&D input prices increase in productivity. Hence, the variance and covariance of R&D input distortion are positive in the optimum. When we apply this output decomposition in variances and covariances to the data, we find that the biggest output gain comes from eliminating capital distortions. We also find that if the variance and covariance of R&D input distortion are set equal to zero, then *ceteris paribus*, the output would be lower than it actually was in 2019. It again illustrates the importance of getting the policy right towards R&D.

Finally, we also compare the results across groups of industries and years. We have 22 industries in our sample. We split them into four groups depending on their technological intensity and perform a separate analysis for each group. The results are qualitatively similar across the groups. We also briefly redo the calculations using the data from 2013. We find that R&D input distortions, rather than capital distortions, now play the largest role in the potential output gain. Also, the joint growth of TFP and spillover between 2013 and 2019 has been poor, but we conclude that it cannot be attributed to the spillover.

The rest of paper is organized as follows. Section 2 describes the model, identifies the optimal allocation of resources, and decomposes the output in terms of capital, labour, and R&D input distortions. Section 3 describes the data and calibrates parameters of the model. Section 4 contains the empirical results, while Section 5 concludes. Some of the derivations are relegated to the Appendix.

2 The Model

We take the model of Hsieh and Klenow (2009) and extend it by introducing R&D spillovers. There is a single final good that is produced by a representative firm in a perfectly competitive market. To produce the final good, this firm combines the outputs of industries $s = 1, \dots, S$ using a Cobb-Douglas production technology:

$$Y = \prod_{s=1}^S Y_s^{\theta_s},$$

where Y and Y_s are the quantities of the final good and industry s output, respectively. Production exhibits constant returns to scale, $\prod_{s=1}^S \theta_s = 1$. In the equilibrium, $\theta_s = \frac{P_s Y_s}{PY}$ holds, where P and P_s are the prices of the final good and industry s output, respectively. The final good serves as a numeraire, and so $P = 1$.

The industry s output is also produced by a representative firm in a perfectly competitive market. It combines the differentiated products of firms si , $i = 1, \dots, m_s$, using a CES production technology:

$$Y_s = \left(\sum_{i=1}^{m_s} Y_{si}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (1)$$

where Y_{si} is the quantity of firm si output. ρ is the elasticity of substitution between products of different firms, which is the same for all industries. In the equilibrium,

$$P_{si} = P_s \left(\frac{Y_s}{Y_{si}} \right)^{\frac{1}{\rho}}$$

holds, where P_{si} is the price of firm si output. Without loss of generality, we impose a normalization that $P_s Y_s^{1/\rho} = 1$, and so $P_{si} = Y_{si}^{-1/\rho}$. We will say that firms si , $i = 1, \dots, m_s$ belong to industry s .

Firm si ($s = 1, \dots, S$, $i = 1, \dots, m_s$) produces its output using a Cobb-

Douglas production technology:

$$Y_{si} = B_{si} K_{si}^{\alpha_s} L_{si}^{\beta_s} H_{si}^{\gamma_s} X_s^{\delta_s}, \quad (2)$$

where B_{si} , K_{si} , and L_{si} are firm si total factor productivity (TFP), capital, and labour, respectively. H_{si} stands for the resources that firm si devotes to R&D. We will refer to H_{si} as an R&D input. $\alpha_s > 0$, $\beta_s > 0$, and $\gamma_s > 0$ are industry-specific elasticities. We assume that the production function exhibits constant returns to scale in the capital, labour, and R&D inputs: $\alpha_s + \beta_s + \gamma_s = 1$.

There is ample empirical evidence of R&D spillovers when R&D of one firm affects the productivity of other firms in the industry (see, for example, Bloom et al. (2013); Lucking et al. (2019); Audretsch and Belitski (2020); Ugur et al. (2020) and references therein). R&D spillover, which is the same for all firms in industry s , is captured by $X_s^{\delta_s}$ where X_s is the spillover pool and δ_s is the spillover parameter. Because any changes in the spillover will only come from the changes in the spillover pool, for simplicity, we will occasionally refer to X_s as the spillover.

We assume that the spillover pool is a geometric average of R&D inputs of the firms in industry s :

$$X_s = \prod_{j=1}^{m_s} H_{sj}^{\frac{1}{m_s}}. \quad (3)$$

To motivate the formula, suppose H_{si} stands for the number of scientists employed by firm si . To share some knowledge between all m_s firms, suppose that a team of size m_s , with one scientist from each firm, must be formed. If a scientist of firm si joins the team with a probability of $\frac{H_{si}}{H_s}$ where $H_s = \sum_{j=1}^{m_s} H_{sj}$, then the required team is formed with probability $(X_s/H_s)^{m_s}$. Because we will keep H_s fixed when reallocating resources between the firms in industry s , the spillover is simply an increasing function of the likelihood of information sharing. Bloom et al. (2013, p. 1357) provide a similar motivation when introducing their technological proximity measures.

Furthermore, although R&D spillover depends on H_{si} , we assume that firm si treats X_s as exogenous.

Let r_s , w_s , and q_s denote the rental rate of capital, the wage rate of labour, and the price of R&D input, respectively. We allow the input prices to be industry-specific. Firm si might employ an input at a level where the marginal revenue product of that input is not equalized to its price. We can think that firm si effectively faces prices of $(1 + \tau_{K_{si}})r_s$, $(1 + \tau_{L_{si}})w_s$, and $(1 + \tau_{H_{si}})q_s$ for capital, labour, and R&D input, respectively. We refer to the variables $\tau_{K_{si}}$, $\tau_{L_{si}}$, and $\tau_{H_{si}}$ as distortions.³

Because firm si produces a differentiated product, it possesses a market power, meaning, it faces a downward sloping inverse demand function $P_{si} = Y_{si}^{-1/\rho}$. Firm si chooses inputs to maximize its profit

$$P_{si}Y_{si} - (1 + \tau_{K_{si}})r_sK_{si} - (1 + \tau_{L_{si}})w_sL_{si} - (1 + \tau_{H_{si}})q_sH_{si}$$

subject to the production function in (2) and the inverse demand function. The first order conditions are

$$\frac{\rho - 1}{\rho} \frac{\alpha_s Y_{si}^{\frac{\rho-1}{\rho}}}{K_{si}} = (1 + \tau_{K_{si}})r_s, \quad (4)$$

$$\frac{\rho - 1}{\rho} \frac{\beta_s Y_{si}^{\frac{\rho-1}{\rho}}}{L_{si}} = (1 + \tau_{L_{si}})w_s, \quad (5)$$

$$\frac{\rho - 1}{\rho} \frac{\gamma_s Y_{si}^{\frac{\rho-1}{\rho}}}{H_{si}} = (1 + \tau_{H_{si}})q_s. \quad (6)$$

If there is a change in distortions, say, because of the government intervention, it will lead to reallocation of resources and, consequently, to a change in output. In fact, it is easier to start by defining the new allocation of resources. We only consider the reallocation of resources that keep the

³Occasionally, it will be more convenient to refer to $1 + \tau_{K_{si}}$, $1 + \tau_{L_{si}}$, and $1 + \tau_{H_{si}}$ as distortions.

aggregate industry demand for inputs at the original level. Then, we use (3) to calculate the new spillover pool of each industry and (2) to calculate the new outputs. Finally, using (4)-(6), we recover the new distortions. Note that we will keep the input prices at their original levels.

2.1 The Social Planner's Problem

In this section, we identify a couple of scenarios for allocation of inputs. Later we use the data from British manufacturing industries to evaluate the output gains from implementing these allocations.

Let $K_s = \sum_{i=1}^{m_s} K_{si}$, $L_s = \sum_{i=1}^{m_s} L_{si}$, and $H_s = \sum_{i=1}^{m_s} H_{si}$ be the aggregate quantities of inputs in industry s . Consider the social planner who maximizes Y_s taking the aggregate quantities of inputs in industry s as given. Substituting (2) and (3) into (1) and taking the logarithm, the planner's problem is

$$\max_{\{K_{si}, L_{si}, H_{si}\}_{i=1}^{m_s}} \log Y_s = \frac{\rho}{\rho - 1} \log \left(\sum_{j=1}^{m_s} (B_{sj} K_{sj}^{\alpha_s} L_{sj}^{\beta_s} H_{sj}^{\gamma_s})^{\frac{\rho-1}{\rho}} \right) + \frac{\delta_s}{m_s} \sum_{j=1}^{m_s} \log H_{sj}$$

subject to $K_s = \sum_{i=1}^{m_s} K_{si}$, $L_s = \sum_{i=1}^{m_s} L_{si}$, and $H_s = \sum_{i=1}^{m_s} H_{si}$.

Suppose first that there is no R&D spillover effect: $\delta_s = 0$. Then, the solution to the planner's problem is given by

$$\frac{K_{si}}{K_s} = \frac{L_{si}}{L_s} = \frac{H_{si}}{H_s} = \frac{Y_{si}^{\frac{\rho-1}{\rho}}}{\sum_{j=1}^{m_s} Y_{sj}^{\frac{\rho-1}{\rho}}} = \frac{B_{si}^{\rho-1}}{\sum_{j=1}^{m_s} B_{sj}^{\rho-1}} \quad (7)$$

for all $i = 1, \dots, m_s$. Comparison with (4)-(6) reveals that if we want to implement this solution as the equilibrium outcome, the distortions must be equalized across firms: there exist τ_{K_s} , τ_{L_s} , and τ_{H_s} such that $\tau_{K_{si}} = \tau_{K_s}$, $\tau_{L_{si}} = \tau_{L_s}$, and $\tau_{H_{si}} = \tau_{H_s}$ for all $i = 1, \dots, m_s$. This is the standard case that is considered in the literature when there are no externalities (e.g., Hsieh and Klenow, 2009; Dias et al., 2016; García-Santana et al., 2020).

If we consider the planner's problem in the presence of R&D spillover, the conditions that describe the optimum, are

$$\frac{K_{si}}{K_s} = \frac{L_{si}}{L_s} = \frac{Y_{si}^{\frac{\rho-1}{\rho}}}{\sum_{j=1}^{m_s} Y_{sj}^{\frac{\rho-1}{\rho}}} \quad (8)$$

and

$$\frac{H_{si}}{H_s} = \frac{\gamma_s Y_{si}^{\frac{\rho-1}{\rho}} + \frac{\delta_s}{m_s} \sum_{j=1}^{m_s} Y_{sj}^{\frac{\rho-1}{\rho}}}{(\gamma_s + \delta_s) \sum_{j=1}^{m_s} Y_{sj}^{\frac{\rho-1}{\rho}}} \quad (9)$$

for all $i = 1, \dots, m_s$. These conditions imply that in the optimum, $\tau_{K_{si}} = \tau_{K_s}$ and $\tau_{L_{si}} = \tau_{L_s}$ still hold for all $i = 1, \dots, m_s$, but $\tau_{H_{si}}$ is increasing in Y_{si} .

To understand (9) better, note that when $\delta_s = 0$, (9) together with (8) reduces to the solution with no R&D spillover given in (7). But, when $\gamma_s = 0$, (9) reduces to

$$H_{si} = \frac{H_s}{m_s} \quad (10)$$

for all $i = 1, \dots, m_s$. This is the allocation of R&D input that we would obtain if we maximized the spillover pool X_s or, equivalently, $\sum_{j=1}^{m_s} \log H_{sj}$ subject to $H_s = \sum_{i=1}^{m_s} H_{si}$. That is, R&D spillover is maximized if R&D input is shared equally by the firms. Therefore, we can interpret the expression in (9) as a weighted average of two extreme cases, with the weights being determined by the R&D input parameter, γ_s and the spillover parameter, δ_s .

It appears that there is no closed form solution to the optimal allocation of inputs in terms of the exogenous parameters. Therefore, we will consider an allocation of inputs that while not optimal, is close to it. First, given any allocation of R&D input, H_{si} for $i = 1, \dots, m_s$, let the share of capital and labour that is allocated to firm si , be given by

$$\frac{K_{si}}{K_s} = \frac{L_{si}}{L_s} = \frac{(B_{si} H_{si}^{\gamma_s})^{\frac{\rho-1}{1+\gamma_s(\rho-1)}}}{\sum_{j=1}^{m_s} (B_{sj} H_{sj}^{\gamma_s})^{\frac{\rho-1}{1+\gamma_s(\rho-1)}}} \quad (11)$$

for all $i = 1, \dots, m_s$. If we substitute (11) into (2), it can be shown that

$$\frac{Y_{si}^{\frac{\rho-1}{\rho}}}{\sum_{j=1}^{m_s} Y_{sj}^{\frac{\rho-1}{\rho}}} = \frac{(B_{si} H_{si}^{\gamma_s})^{\frac{\rho-1}{1+\gamma_s(\rho-1)}}}{\sum_{j=1}^{m_s} (B_{sj} H_{sj}^{\gamma_s})^{\frac{\rho-1}{1+\gamma_s(\rho-1)}}}. \quad (12)$$

It then follows from (4)-(5) that the proposed capital and labour allocation in (11) still ensures $\tau_{K_{si}} = \tau_{K_s}$ and $\tau_{L_{si}} = \tau_{L_s}$ for all $i = 1, \dots, m_s$.

Now we turn to determine the allocation of R&D input. In the next section, we decompose the industry output assuming that productivity and distortions are jointly log-normally distributed. Given that decomposition, output is maximized when

$$\begin{aligned} \text{var}(\log(1 + \tau_{H_{si}})) &= \left(\frac{\delta_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)} \right)^2 \text{var}(\log B_{si}), \\ \text{cov}(\log B_{si}, \log(1 + \tau_{H_{si}})) &= \frac{\delta_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)} \text{var}(\log B_{si}). \end{aligned}$$

This implies that the allocation of R&D input H_{si} for all $i = 1, \dots, m_s$ is

$$\frac{H_{si}}{H_s} = \frac{B_{si}^{\frac{\gamma_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}}}{\sum_{j=1}^{m_s} B_{sj}^{\frac{\gamma_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}}}. \quad (13)$$

To see it, combine (13) with (12) and (6). This gives that $1 + \tau_{H_{si}} \propto B_{si}^{\frac{\delta_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}}$ and, hence, the above variance and covariance relationships are indeed satisfied.

The allocation in (13) is only approximately optimal. We can improve on it through the iterative process where we substitute (13) into (12), which we then substitute into (9) to get a new allocation of R&D input:

$$\frac{H_{si}}{H_s} = \frac{\gamma_s B_{si}^{\frac{(\gamma_s + \delta_s)(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}} + \frac{\delta_s}{m_s} \sum_{j=1}^{m_s} B_{sj}^{\frac{(\gamma_s + \delta_s)(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}}}{(\gamma_s + \delta_s) \sum_{j=1}^{m_s} B_{sj}^{\frac{(\gamma_s + \delta_s)(\rho-1)}{\gamma_s + \delta_s + \gamma_s \delta_s(\rho-1)}}}. \quad (14)$$

Although we could continue iteratively producing new allocations of R&D input by using (12) and (9), we view (14) already as a good approximation and use it in the empirical analysis.

To summarize, the allocation of R&D input to firm si is increasing in its productivity B_{si} in (7), (13), and (14). However, compared to (7), H_{si} increases with B_{si} at slower rates in (13) and (14), meaning that in the presence of R&D spillover, it is optimal to allocate R&D input more evenly between the firms than dictated by (7). Because the allocation in (7) corresponds to the situation when all firms face the same R&D input distortion, the optimal allocation requires that more productive firms face higher R&D input distortions than the less productive firms.

Guner et al. (2008) study the output loss due to governmental policies that depend on the firm size. In their model, such policies always lead to resource misallocation. Our model tells that size-dependent policies that affect the use of R&D input, can actually be welfare improving. To keep matters simple, suppose we want to implement the allocation given by (11) and (13). We have found that $\tau_{K_{si}} = \tau_{K_s}$, $\tau_{L_{si}} = \tau_{L_s}$, and $1 + \tau_{H_{si}} \propto B_{si}^{\frac{\delta_s(\rho-1)}{\gamma_s + \delta_s + \gamma_s\delta_s(\rho-1)}}$ must hold for all $i = 1, \dots, m_s$ in this case. Using (11) and (13), the latter can be written as $1 + \tau_{H_{si}} \propto L_{si}^{\frac{\delta_s}{\gamma_s + \delta_s}}$ or

$$\tau_{H_{si}} \approx \log(1 + \tau_{H_{si}}) = \text{const} + \frac{\delta_s}{\gamma_s + \delta_s} \log L_{si}. \quad (15)$$

If we view $\tau_{H_{si}}$ as a tax rate on R&D input that is set by the government, then it should increase with the number of employees but at a decreasing rate.⁴

Currently, firms in the UK are eligible for R&D tax relief and the size of this relief depends on the number of employees. From April 1, 2023, the corporate tax rate is 25%. Firms with less than 500 employees can reduce

⁴Provido that firms do not view the tax rate as dependent on their hiring decisions.

their taxable income by additional 86% of R&D expenditure.⁵ Therefore, compared with the situation of no R&D tax relief at all, the after-tax benefit is 21.5% of R&D expenditure. Firms with 500 employees or more can claim an expenditure credit equal to 20% of R&D expenditure which is subject to the corporate tax. Hence, their after-tax benefit is 15% of R&D expenditure. In short, the R&D tax relief is more generous for the smaller firms. Our model lends support to this policy. Though, according to (15), resource allocation would benefit from an even finer relationship between the firm size and the R&D tax relief.

2.2 Decomposition of Industry Output

In order to gauge how much each type of distortion contributes to the output loss, we now decompose the industry output similar to Hsieh and Klenow (2009); Chen and Irarrazabal (2015); Ryzhenkov (2016) but by accounting for R&D externalities. We assume that B_{si} and $1 + \tau_{I_{si}}$ for $I = K, L, H$ are drawn from a multivariate log-normal distribution. The draws are independent across firms. The variance-covariance matrix of $\log B_{si}$ and $\log(1 + \tau_{I_{si}})$ for $I = K, L, H$ for all i is

$$\Sigma_s = \begin{pmatrix} \sigma_{Bs}^2 & \sigma_{BKs} & \sigma_{BLs} & \sigma_{BHs} \\ \sigma_{BKs} & \sigma_{Ks}^2 & 0 & 0 \\ \sigma_{BLs} & 0 & \sigma_{Ls}^2 & 0 \\ \sigma_{BHs} & 0 & 0 & \sigma_{Hs}^2 \end{pmatrix}$$

where σ_{Bs}^2 stands for $var(\log B_{si})$, and σ_{Is}^2 and σ_{BIs} respectively stand for $var(\log(1 + \tau_{I_{si}}))$ and $cov(\log B_{si}, \log(1 + \tau_{I_{si}}))$ for $I = K, L, H$. Thus, distortions can be correlated with the productivity, but for simplicity, we assume that there is no correlation between the distortions.

⁵To qualify, these firms must additionally have a turnover of under 100 million euros or a balance sheet total under 86 million euros. More information is available at: <https://www.gov.uk/government/collections/research-and-development-rd-tax-relief>.

Let industry s TFP be defined as

$$TFP_s \equiv \frac{Y_s}{K_s^{\alpha_s} L_s^{\beta_s} H_s^{\gamma_s} X_s^{\delta_s}}. \quad (16)$$

Then, the industry output can be expressed as $\log Y_s = \log TFP_s + \delta_s \log X_s + \log (K_s^{\alpha_s} L_s^{\beta_s} H_s^{\gamma_s})$. We show in Appendix A that TFP can be approximated as

$$\begin{aligned} \log TFP_s = & E[\log B_{si}] + \frac{\rho - 1}{2} \sigma_{B_s}^2 - \frac{(\rho - 1) \alpha_s^2 + \alpha_s}{2} \sigma_{K_s}^2 \\ & - \frac{(\rho - 1) \beta_s^2 + \beta_s}{2} \sigma_{L_s}^2 - \frac{(\rho - 1) \gamma_s^2 + \gamma_s}{2} \sigma_{H_s}^2. \end{aligned} \quad (17)$$

(17) implies that industry s TFP is maximal when all firms face the same distortions because then the variances are zero. Similarly, R&D spillover pool can be approximated as

$$\begin{aligned} \log X_s = & \log H_s - \frac{(\rho - 1)^2}{2} \sigma_{B_s}^2 - \frac{(\rho - 1)^2 \alpha_s^2}{2} \sigma_{K_s}^2 \\ & - \frac{(\rho - 1)^2 \beta_s^2}{2} \sigma_{L_s}^2 - \frac{(1 + (\rho - 1) \gamma_s)^2}{2} \sigma_{H_s}^2 + (\rho - 1)^2 \alpha_s \sigma_{BK_s} \\ & + (\rho - 1)^2 \beta_s \sigma_{BL_s} + (\rho - 1) (1 + (\rho - 1) \gamma_s) \sigma_{BH_s}. \end{aligned} \quad (18)$$

(18) says that the spillover is decreasing in the variances of distortions but increasing in their covariances with the productivity parameter. Of course, one cannot have zero variance and positive covariance. That is, if the covariances are positive, then so are the variances. This, in turn, implies that there is a trade off between maximizing industry TFP and R&D spillover.

Given (17) and (18), industry s output is

$$\begin{aligned} \log Y_s = & -\frac{1}{2} (a_{K_s} \sigma_{K_s}^2 + a_{L_s} \sigma_{L_s}^2 + a_{H_s} \sigma_{H_s}^2 \\ & - 2b_{K_s} \sigma_{BK_s} - 2b_{L_s} \sigma_{BL_s} - 2b_{H_s} \sigma_{BH_s}) + const \end{aligned} \quad (19)$$

where

$$\begin{aligned}
a_{Ks} &= \alpha_s + (\rho - 1) \alpha_s^2 + \delta_s (\rho - 1)^2 \alpha_s^2, \\
a_{Ls} &= \beta_s + (\rho - 1) \beta_s^2 + \delta_s (\rho - 1)^2 \beta_s^2, \\
a_{Hs} &= \gamma_s + (\rho - 1) \gamma_s^2 + \delta_s (1 + (\rho - 1) \gamma_s)^2, \\
b_{Ks} &= \delta_s (\rho - 1)^2 \alpha_s, \\
b_{Ls} &= \delta_s (\rho - 1)^2 \beta_s, \\
b_{Hs} &= \delta_s (\rho - 1) (1 + (\rho - 1) \gamma_s),
\end{aligned}$$

and *const* contains all those terms which are independent of distortions. Maximizing (19) w.r.t. $\sigma_{I_s}^2$ and σ_{BI_s} for $I = K, L, H$ subject to the constraint that Σ_s is a positive semi-definite matrix, we find in Appendix B that $\sigma_{K_s}^2 = \sigma_{L_s}^2 = \sigma_{BK_s} = \sigma_{BL_s} = 0$,

$$\begin{aligned}
\sigma_{BH_s} &= \frac{\delta_s (\rho - 1)}{\gamma_s + \delta_s + \gamma_s \delta_s (\rho - 1)} \sigma_{B_s}^2, \\
\sigma_{H_s}^2 &= \left(\frac{\delta_s (\rho - 1)}{\gamma_s + \delta_s + \gamma_s \delta_s (\rho - 1)} \right)^2 \sigma_{B_s}^2.
\end{aligned}$$

This solution corresponds to the allocation of resources given in (11) and (13).

3 Data and Model Calibration

We apply the model to the firm-level data of the UK manufacturing sector. The data come from two datasets provided by the Office for National Statistics (ONS). One is the Annual Business Survey (ONS, 2022a) and the other is the Business Enterprise Research and Development Survey (ONS, 2022b). We take the data on $P_{si} Y_{si}$, K_{si} , L_{si} , and H_{si} directly from the surveys. (Table 7 in Appendix C lists the names of the corresponding variables in the surveys.) We use the value added to measure $P_{si} Y_{si}$ because our model

does not have intermediate goods. We chose the number of scientists and researchers to represent the R&D input, H_{si} . While it is a narrow measure of R&D input, the advantage is that we can take it directly from a survey without the need of estimating it.

Because we only have the data on the total wage bill of firm si , $W_{si} \equiv w_s L_{si} + q_s H_{si}$, we assume that it is split between the non-R&D and R&D employees proportionally to the expenditure on capital, $I_{K_{si}}$ and R&D, $I_{R\&D_{si}}$:

$$\begin{aligned} w_s L_{si} &= W_{si} \frac{I_{K_{si}}}{I_{K_{si}} + I_{R\&D_{si}}}, \\ q_s H_{si} &= W_{si} \frac{I_{R\&D_{si}}}{I_{K_{si}} + I_{R\&D_{si}}}. \end{aligned}$$

The input prices in industry s are the ratio of total industry expenditure on the given input to the aggregate quantity of that input in industry s :

$$\begin{aligned} r_s &= \frac{\sum_{i=1}^{m_s} (P_{si} Y_{si} - W_{si})}{K_s}, \\ w_s &= \frac{\sum_{i=1}^{m_s} w_s L_{si}}{L_s}, \\ q_s &= \frac{\sum_{i=1}^{m_s} q_s H_{si}}{H_s}, \end{aligned}$$

while the output elasticities are the ratio of total industry expenditure on the given input to the aggregate industry revenue:

$$\begin{aligned} \alpha_s &= \frac{r_s K_s}{\sum_{i=1}^{m_s} P_{si} Y_{si}}, \\ \beta_s &= \frac{w_s L_s}{\sum_{i=1}^{m_s} P_{si} Y_{si}}, \\ \gamma_s &= \frac{q_s H_s}{\sum_{i=1}^{m_s} P_{si} Y_{si}}. \end{aligned}$$

Similar to Hsieh and Klenow (2009); Dias et al. (2016); García-Santana et al. (2020), we set the elasticity of substitution equal to $\rho = 3$. It implies

a markup of 50%, which is comparable to the markup estimate of 68% by De Loecker and Eeckhout (2018) for the UK in 2016. Note that because $P_{si} = Y_{si}^{-1/\rho}$, we can obtain the output of firm si from its revenue: $Y_{si} = (P_{si}Y_{si})^{\frac{\rho}{\rho-1}}$.

To calibrate the spillover parameter, δ_s , we assume that it is the same for all industries. We first calculate the firm-level productivity that includes the R&D spillover:

$$A_{si} \equiv B_{si}X_s^{\delta_s} = \frac{Y_{si}}{K_{sj}^{\alpha_s}L_{si}^{\beta_s}H_{sj}^{\gamma_s}},$$

and then, by using the variation in the spillover pool across industries, we estimate the common δ from the regression

$$\log A_{si} = \delta_0 + \delta \log X_s + \epsilon_{si}.$$

OLS produced an estimate of $\delta = 0.12$. The IV estimator with the aggregate industry expenditure on R&D input, $\log q_s H_s$ as the instrumental variable gave a slightly higher, but insignificant estimate of $\delta = 0.17$.⁶ We chose to set $\delta_s = 0.12$ for all s .

If all firms in industry s were symmetric, then the private and social marginal returns would be $\frac{\rho-1}{\rho} \frac{P_s Y_s}{H_s} \gamma_s$ and $\frac{\rho-1}{\rho} \frac{P_s Y_s}{H_s} (\gamma_s + \delta_s)$, respectively. Thus, the social return exceeds the private one by $\frac{\delta_s}{\gamma_s} \cdot 100\%$. As we report in Table 1 below, the average value of γ_s is 0.22 in the sample, implying $\frac{\delta_s}{\gamma_s} \approx 0.55$. There is substantial literature that attempts to estimate the elasticities γ_s and δ_s so as to infer the private and social returns to R&D. However, there does not seem to be a consensus about the relative sizes of these elasticities. For example, Lucking et al. (2019) estimate the ratio of social to private return for the US firms to be around four to one, implying $\frac{\delta_s}{\gamma_s} = 3$. Ugur et al. (2020) conduct a meta-study and instead conclude that the spillover effect is smaller than the own effect of R&D: $\gamma_s = 0.073$, while $\delta_s = 0.069$ for

⁶The 95% confidence interval is [0.02,0.23] in the case of OLS estimator and [-0.05, 0.39] in the case of IV estimator.

knowledge spillovers and $\delta_s = 0.036$ for all spillover types.

Finally, once we have calibrated all the parameters, we can also recover firm-level productivities, B_{si} and distortions, $\tau_{I_{si}}$ for $I = K, L, H$ and all si .

We use the data for 2019 when the economy is still not affected by the Covid-19 pandemic. We exclude all firms with non-positive or missing values of output or any input. To limit the influence of outliers, we also exclude the firms with the capital stock in either the top or bottom 5th percentile. After data cleaning, there are 1759 firms left in our database. These firms belong to 22 different industries. In order to see if the results change systematically with the technological intensity of industries, we follow ONS (2018, p. 20) and group them into low, medium-low, medium-high, and high technology groups. (See Table 8 in Appendix C.) There are 10, 5, 6, and 2 industries in low, medium-low, medium-high, and high technology groups, respectively.

Table 1 contains descriptive statistics of the main variables. An average firm employs 258 workers and 9 researchers, has £5.7m worth of capital, and generates £19m in revenues (value added). Though, large standard deviations imply that there is a lot of heterogeneity among firms. The firms in the low-tech group are on average larger in terms of output as well as in terms of capital and labour employed. If we compare across the groups, then the use of R&D input increases while the use of labour input decreases with the increase in the technological intensity of industry. A similar pattern can be observed if we compare the output elasticity of labour, β_s and that of R&D input, γ_s across the groups. The output elasticity of capital, α_s is relatively stable across the groups and, on average, it is the largest between the output elasticities, followed by the elasticity of labour, while the elasticity of R&D input is the smallest. Though, in the case of high-tech group, the elasticity of R&D input exceeds that of labour. Finally, the prices of capital and labour inputs are relatively stable across the groups, while the price of R&D input decreases quite substantially with the technological intensity of industry: the price of R&D input in the high-tech group is only one third of

Table 1: Descriptive Statistics

Variable	All Industries			Low Tech			Medium-Low Tech			Medium-High Tech			High Tech		
	Mean	Std. Dev.	No. of Obs.	Mean	Std. Dev.	No. of Obs.	Mean	Std. Dev.	No. of Obs.	Mean	Std. Dev.	No. of Obs.	Mean	Std. Dev.	No. of Obs.
$P_{si}Y_{si}$	19245.2	40057.8	1759	34806.5	66708.0	251	16570.7	38416.5	462	16616.33	31724.7	778	16912.7	25612.5	268
K_{si}	5752.1	7537.4	100%	7254.3	8951.8	14%	4871.3	7314.3	26%	5987.81	7424.6	44%	5197.6	6503.3	15%
L_{si}	258.3	457.3		551.7	875.0		229.8	406.9		212.47	287.3		165.7	199.0	
H_{si}	9.01	20.41		5.56	6.35		4.26	11.85		9.29	20.84		19.59	32.22	
α_s	0.43	0.08		0.39	0.09		0.43	0.06		0.43	0.10		0.46	0.05	
β_s	0.35	0.08		0.41	0.06		0.41	0.06		0.33	0.05		0.23	0.02	
γ_s	0.22	0.09		0.19	0.13		0.16	0.09		0.24	0.06		0.31	0.02	
r_s	1.39	0.46		1.76	0.73		1.47	0.30		1.19	0.34		1.47	0.34	
w_s	26.41	4.19		25.69	6.40		29.63	2.98		25.99	3.19		22.76	0.53	
q_s	508.64	248.52		749.11	405.63		646.39	119.88		434.69	139.97		249.03	40.99	

Note: Revenue $P_{si}Y_{si}$, capital K_{si} , wage rate w_s , and the price of R&D input q_s are reported in thousand GBP.

that in the low-tech group.⁷

4 Empirical Results

4.1 Output Gains

Table 2 reports output gains for different allocations of inputs and different technological intensities. Both in columns [1] and [2], the new input allocation is given by eq. (7). It corresponds to the situation when all firms in a given industry face the same distortions and, hence, the same input prices, which is optimal in the absence of spillovers. In column [1], the spillover pool X_s for all s is recalculated according to (3) given the new allocation of R&D input, while in column [2], X_s is kept at the original level so that there are no spillover effects. By comparing columns [1] and [2], the first conclusion that we draw, is that the allocation given by (7) reduces the size of spillovers in the economy. As a result, the output gains are overestimated quite substantially when spillovers are ignored. If we take all manufacturing industries, we find that the output gain is overestimated by around two-thirds or 50 percentage points (pp).

In column [3], the input allocation is given by eqs. (11) and (13), while in column [4], it is given by (11) and (14). In both cases, capital and labour distortions are still equalized across the firms in the same industry, but now R&D input distortions are increasing in firm productivity. Recall that the R&D input allocation in (13) is the one that maximizes the aggregate output of the approximate model in (19). Comparison with column [1] tells that the output gain from a more even distribution of R&D input across firms can be quite large. Thus, for the entire manufacturing sector, we find that the

⁷Note that output gains or losses do not depend on the input prices because the input quantities at the industry level are fixed. If there is any mismeasurement of input prices, then according to (4)-(6), it will only affect the level of distortions. However, the output only depends on the dispersion of distortions, not on their level.

output gain is more than two-fifth or around 34pp.

The results in column [4] are obtained by performing one additional iteration on the R&D input allocation: we take the allocation in (13) and combine it with (12) and (9) to arrive at (14). Although the output gain in column [4] is even higher than in column [3], the increment is much smaller compared to the difference between columns [3] and [1]. (Though, the increment in output gain appears larger in more technology-intensive groups.) Therefore, we will view the results in column [4] as the maximal output gain that the manufacturing industries can achieve in our model. In fact, the output gains in column [4] are similar to the ones reported in column [2]. However, they are based on different assumptions and have different policy implications. The results in column [2] assume that there are no spillovers and it is optimal to charge all firms the same input prices, while the results in column [4] hinge on the presence of R&D spillovers and say that it is optimal to subsidize R&D input for less productive firms. As noted previously, there is extensive empirical evidence supporting the existence of R&D spillovers and so, they need to be taken into account.

Finally, if we compare the output gains by the technological intensity, the largest gains are in the medium-high tech group, followed by the low tech, high tech, and the medium-low tech groups. We can infer from Table 1 that the medium-high tech group produces the largest output and uses most inputs. Thus, it appears that the size of output gain is related to the size of group. The ranking of the remaining three groups, however, is not clear-cut: the low tech group employs more labour, the medium-low tech group employs more capital, and the high tech group employs more R&D input. In the following section, we will explore further if there are any systematic differences in the distortions that these groups face.

Table 2: Output Gains

	[1]	[2]	[3]	[4]
All Industries	76.28	128.09	110.48	118.77
Low Tech	75.17	128.61	116.48	121.18
Medium-Low Tech	43.16	82.31	67.81	73.76
Medium-High Tech	103.21	165.76	143.58	154.70
High Tech	69.08	114.39	92.76	104.86

Note: The output gain is measured as $(Y^*/Y - 1) \cdot 100\%$, where Y is the initial output and Y^* is the new output after reallocation of inputs. In column [1], the input allocation is given by eq. (7); in column [2], it is also given by (7) but keeping X_s for all s at the initial level; in column [3], it is given by (11) and (13); in column [4], it is given by (11) and (14).

4.2 Output Decomposition

We now analyze the sources of potential output gains. In so doing, we separate the output gains due to changes in TFP and R&D spillover and then relate them to the three types of distortions. Thus, Table 3 reports the weighted averages of $\log Y_s$, $\log TFP_s$, $\delta_s \log X_s$, and the variances and covariances that appear in (19) for the entire manufacturing sector. (With some abuse of notation, for example, $\log Y_s$ stands for $\sum_{s=1}^S \theta_s \log Y_s$. It is analogous for the other variables.) The values in column [1] correspond to the actual allocation of inputs in the economy in 2019. The results in column [4] correspond to the optimal allocation of inputs given by (11) and (14). For the comparison purposes, we also report the values of variables when the allocation of inputs is given by (7), in column [2].

We can make several observations based on Table 3. First, we conclude from column [1] that the firms with higher productivity tend to face higher distortions of all three types because all covariances (σ_{BI_s} for $I = K, L, H$) are positive. This is consistent with the earlier findings in the literature. For instance, Hsieh and Klenow (2009, Table A.1) find it for China and India. Guner et al. (2008) provide examples of size-dependent policies that lead

to positively correlated distortions. Restuccia and Rogerson (2008); Bento and Restuccia (2017) also analyze the consequences of positively correlated distortions. The positive correlation between $\log B_{si}$ and $\log(1 + \tau_{H_{si}})$ can also be seen in Figure 1a.⁸

In column [2], all variance and covariance terms are obviously 0 because all firms in the same industry face identical distortions. The allocation of inputs is such that it maximizes TFP. Therefore, the increase in the (average) log of TFP is higher in column [3] than in column [5]: 0.82 vs. 0.77. However, identical distortions also mean that R&D input is less evenly spread between the firms, which leads to lower spillover. This decrease in spillover is quite substantial in column [3]: almost a third of the increase in the log of TFP goes towards compensating for the decrease in the spillover. A related point is that by equalizing R&D input distortions, *ceteris paribus*, the output would be reduced by approximately 6% (0.22 – 0.28) compared to the one realized in 2019, which shows the importance of getting the policies that affect the allocation of R&D input, right.

If we consider the variance and covariance terms in column [4], clearly the only difference with column [2] is that the terms related to the R&D input distortion are now different from 0. In fact, the absolute values of these terms have actually increased compared to the initial values in column [1]. According to (18), the R&D spillover is increasing in covariances between the distortions and the productivity because it leads to more even use of inputs across the firms. Therefore, higher value of σ_{BH_s} in column [4] compensates for the reduction of σ_{BK_s} and σ_{BL_s} to 0, so that R&D spillover remains approximately at the initial level. Because one now avoids a reduction in spillover as happened in column [3], the resulting output gain in column [5] is larger than in column [3]: 0.78 vs. 0.57.⁹ Figure 1b illustrates the relationship between $\log B_{si}$ and $\log(1 + \tau_{H_{si}})$ when the latter is set optimally.

⁸Scatter plots for the other two distortions are similar and, therefore, omitted.

⁹Note that these changes in the log of output correspond, respectively, to 118.77% and 76.28% reported in Table 2.

Finally, according to column [5], capital distortions are the main source of the output loss: the change in $-\frac{a_{Ks}}{2}\sigma_{Ks}^2 + b_{Ks}\sigma_{BKs}$ is equal to 0.24 compared to the change of 0.06 in the labour distortions and that of 0.02 in the R&D input distortions. Hence, while it is important to foster more even use of R&D input by subsidizing less productive firms, it is also important to ensure that all firms in the same industry face an identical price of capital so that more productive firms use more of capital. Note, however, that the sum of all variance and covariance terms in column [5] is 0.32, giving an output gain of $(e^{0.32} - 1) \cdot 100\% = 37.71\%$. This is the output gain as implied by the decomposition in (19). But the actual output gain from the optimal allocation of inputs is 118.77% as shown in Table 2. Therefore, one should keep in mind that the decomposition in (19) provides only a crude approximation of the output.

Table 3: Output Decomposition for All Manufacturing Industries

	[1]	[2]	[3]=[2]-[1]	[4]	[5]=[4]-[1]
$\log Y_s$	21.99	22.56	0.57	22.77	0.78
$\log TFP_s$	10.80	11.62	0.82	11.57	0.77
$\delta_s \log X_s$	0.18	-0.08	-0.26	0.19	0.01
$-\frac{a_{Ks}}{2}\sigma_{Ks}^2$	-0.41	0	0.41	0	0.41
$-\frac{a_{Ls}}{2}\sigma_{Ls}^2$	-0.17	0	0.17	0	0.17
$-\frac{a_{Hs}}{2}\sigma_{Hs}^2$	-0.22	0	0.22	-0.43	-0.20
$b_{Ks}\sigma_{BKs}$	0.17	0	-0.17	0	-0.17
$b_{Ls}\sigma_{BLs}$	0.11	0	-0.11	0	-0.11
$b_{Hs}\sigma_{BHs}$	0.28	0	-0.28	0.50	0.22

Note: $a_{Is}\sigma_{Is}^2/2$ and $b_{Is}\sigma_{BIs}$ for $I = K, L, H$ are the terms given in (19). The reported values are weighted averages across industries where the weight of industry s is θ_s . In column [1], the input allocation is the one given by the data; in column [2], the input allocation is given by eq. (7); in column [4], it is given by eqs. (11) and (14).

Table 4 provides output decomposition separately for each group of technological intensity. (Now, $\log Y_s$ stands for $\sum_{s \in G} \theta_s \log Y_s / (\sum_{s \in G} \theta_s)$ where

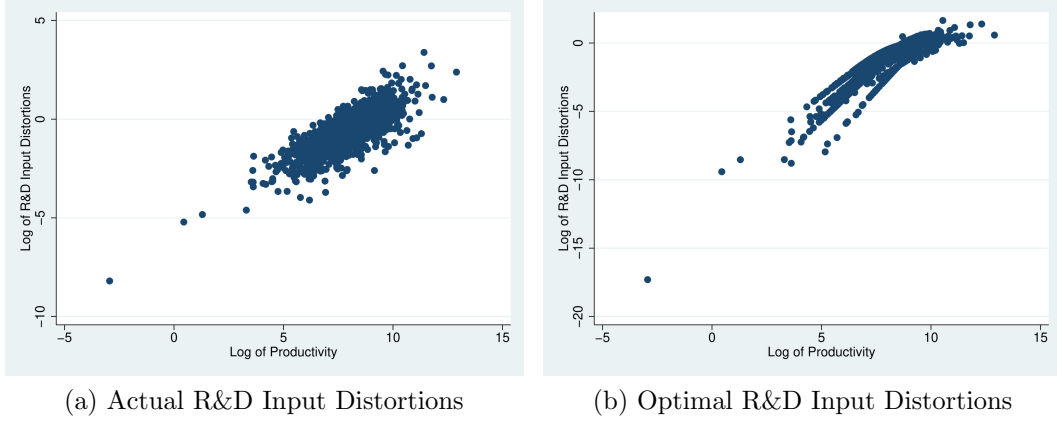


Figure 1: Scatter plots of $\log(1 + \tau_{H_{si}})$ against $\log B_{si}$

G is the group of industries of given technological intensity.¹⁰ It is analogous for the other variables.) The table reports the values of variables for the actual allocation of inputs in 2019 (columns [1], [4], [7], and [10]) and for the optimal allocation of inputs (columns [2], [5], [8], and [11]), as well as their difference (columns [3], [6], [9], and [12]).

Qualitatively, the results for each group are similar to the results for the entire manufacturing sector. For all groups, the output gains due to the optimal allocation of inputs come almost exclusively from the increase in TFP, and the largest gain in TFP occurs in the medium-high tech group. As noted before, this group has the largest number of firms and employs the largest amount of inputs. Between the three types of distortions, the capital distortion contributes the most, while the R&D input distortion contributes the least to the output gain irrespective of the technological intensity of the group. The largest contribution of the capital distortion is in the high tech group: 0.32. The largest contributions of labour and R&D input distortions are both in the low tech group: 0.10 and 0.08, respectively.

Even though the R&D input distortion does not add much to the output

¹⁰The groups are given in Table 8.

Table 4: Output Decomposition by Technological Intensity

	Low Tech			Medium-Low Tech			Medium-High Tech			High Tech		
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
$\log Y_s$	21.84	22.63	0.79	21.61	22.16	0.55	22.31	23.25	0.94	22.04	22.75	0.72
$\log TFP_s$	10.65	11.44	0.79	10.78	11.33	0.55	10.96	11.88	0.92	10.66	11.37	0.71
$\delta_s \log X_s$	0.16	0.16	0.01	0.12	0.12	0.00	0.19	0.20	0.01	0.29	0.30	0.01
$-\frac{a_{Ks}}{2} \sigma_{Ks}^2$	-0.43	0	0.43	-0.40	0	0.40	-0.33	0	0.33	-0.61	0	0.61
$-\frac{a_{Ls}}{2} \sigma_{Ls}^2$	-0.24	0	0.24	-0.18	0	0.18	-0.12	0	0.12	-0.12	0	0.12
$-\frac{a_{Hs}}{2} \sigma_{Hs}^2$	-0.13	-0.30	-0.17	-0.29	-0.49	-0.20	-0.20	-0.37	-0.17	-0.37	-0.75	-0.38
$b_{Ks} \sigma_{BKs}$	0.16	0	-0.16	0.17	0	-0.17	0.13	0	-0.13	0.29	0	-0.29
$b_{Ls} \sigma_{BLs}$	0.14	0	-0.14	0.12	0	-0.12	0.09	0	-0.09	0.10	0	-0.10
$b_{Hs} \sigma_{BHs}$	0.23	0.47	0.25	0.35	0.56	0.21	0.22	0.40	0.18	0.42	0.74	0.32

Note: $a_{Is} \sigma_{Is}^2 / 2$ and $b_{Is} \sigma_{BI_s}$ for $I = K, L, H$ are the terms given in (19). In columns [1], [4], [7], and [10], the input allocation is the one given by the data; in columns [2], [5], [8], and [11], it is given by eqs. (11) and (14); [3]=[2]-[1], [6]=[5]-[4], [9]=[8]-[7], and [12]=[11]-[10].

gain, the terms related to this distortion, $\frac{a_{H_s}}{2}\sigma_{H_s}^2$ and $b_{H_s}\sigma_{BH_s}$, approximately double in the absolute size under the optimal allocation of inputs. This helps to keep the R&D spillover high. If besides capital and labour distortions, we also equalized the R&D input distortions across firms, that is, set $\sigma_{H_s}^2 = \sigma_{BH_s} = 0$, the spillover $\delta_s \log X_s$ would decrease by 0.24 to 0.27 depending on the group (not shown in Table 4). Therefore, any policy that affects the allocation of R&D input, still has an important impact on output.

4.3 Output Decomposition in 2013

We also briefly perform a similar exercise using the data from 2013 to see if the results are qualitatively similar. By this year, the effects of the Global Financial Crisis of 2007-2008 had receded, while the “Brexit” referendum has not yet taken place. The sample for 2013 only contains 1141 firms compared to 1759 firms in the 2019 sample. Table 5 contains the descriptive statistics of the variables. One thing to note is that the firms in the 2013 sample are on average larger both in terms of the output produced and the amount of inputs employed. At the same time, the average output elasticities of capital, labour, and R&D input are quite similar in both years. The estimated value of δ is 0.25 in 2013, which is higher than in 2019.

Table 6 reports the same results as Table 3 but for 2013. Compared to 2019, the R&D spillover and R&D input distortions play a larger role in the output, which is to be expected given that now $\delta_s = 0.25$ instead of 0.12 for all s . Thus, according to column [3], when all variations in distortions are eliminated, now more than half of the TFP gain goes towards compensating for the reduction in the spillover. In column [5], although the increase in TFP still explains most of the output gain arising from the optimal allocation of inputs, the contribution of the spillover $\delta_s \log X_s$ to the output gain is an order of magnitude larger than in Table 3. In terms of individual distortions, the R&D input distortion now contributes twice as much as the capital distortion to the output gain in column [5]: $-\frac{a_{H_s}}{2}\sigma_{H_s}^2 + b_{H_s}\sigma_{BH_s} = 0.27$ vs. $-\frac{a_{K_s}}{2}\sigma_{K_s}^2 +$

Table 5: Descriptive Statistics for the Data in 2013

Variable	Mean	Std. Dev.
$P_{si}Y_{si}$	26932.88	56966.83
K_{si}	7291.67	9252.50
L_{si}	394.29	715.08
H_{si}	12.92	39.73
α_s	0.41	0.09
β_s	0.35	0.11
γ_s	0.25	0.11
r_s	1.53	0.99
w_s	22.15	5.55
q_s	489.45	141.32

Note: Revenue $P_{si}Y_{si}$, capital K_{si} , wage rate w_s , and the price of R&D input q_s are reported in thousand GBP.

$$b_{Ks}\sigma_{BKs} = 0.13.$$

Finally, if we compare column [1] in Tables 3 and 6, we find that $\log TFP_s + \delta_s \log X_s$ has only increased by 0.02 over the six year period. One could ask if this poor performance can be attributed to changes in the R&D spillover. Besides lower δ_s , we find that the average size of X_s has indeed decreased from 4.96 to 4.40 between 2013 and 2019. However, this decrease could be due to changes in the sample composition. As noted, the 2013 sample has less firms but they are, on average, larger. Therefore, we also compute $\frac{X_s}{H_s/m_s}$ (and average it across industries). The maximal value that $\frac{X_s}{H_s/m_s}$ can take is 1, irrespective of H_s and m_s . Therefore, any differences in $\frac{X_s}{H_s/m_s}$ between 2013 and 2019 only depend on how evenly the R&D input is distributed between the firms. We find that $\frac{X_s}{H_s/m_s} = 0.48$ in 2013 and $\frac{X_s}{H_s/m_s} = 0.60$ in 2019. Thus, the R&D input is actually more evenly distributed in 2019 than in 2013. This increase in $\frac{X_s}{H_s/m_s}$ even outweighs the decrease in δ_s , that is, $\delta_s \log \left(\frac{X_s}{H_s/m_s} \right)$ has increased by 0.12 from 2013 to 2019 or approximately 2% a year. Hence, we conclude that the poor performance of $\log TFP_s + \delta_s \log X_s$ is not because of decline in R&D spillovers in the economy.

Table 6: Output Decomposition for All Manufacturing Industries in 2013

	[1]	[2]	[3]=[2]-[1]	[4]	[5]=[4]-[1]
$\log Y_s$	21.89	22.26	0.37	22.78	0.89
$\log TFP_s$	10.56	11.41	0.85	11.33	0.76
$\delta_s \log X_s$	0.40	-0.08	-0.48	0.53	0.13
$-\frac{a_{Ks}}{2}\sigma_{Ks}^2$	-0.54	0	0.54	0	0.54
$-\frac{a_{Ls}}{2}\sigma_{Ls}^2$	-0.16	0	0.16	0	0.16
$-\frac{a_{Hs}}{2}\sigma_{Hs}^2$	-0.40	0	0.40	-0.75	-0.35
$b_{Ks}\sigma_{BKs}$	0.41	0	-0.41	0	-0.41
$b_{Ls}\sigma_{BLs}$	0.20	0	-0.20	0	-0.20
$b_{Hs}\sigma_{BHs}$	0.55	0	-0.55	1.17	0.62

Note: $a_{Is}\sigma_{Is}^2/2$ and $b_{Is}\sigma_{BI_s}$ for $I = K, L, H$ are the terms given in (19). The reported values are weighted averages across industries where the weight of industry s is θ_s . In column [1], the input allocation is the one given by the data in 2013; in column [2], the input allocation is given by eq. (7); in column [4], it is given by eqs. (11) and (14).

5 Conclusion

Motivated by the applied work on R&D spillovers, we introduce them in the model of Hsieh and Klenow (2009) and study their effect on resource (mis)allocation. Because of the externalities associated with R&D spillovers, the efficient allocation of resources requires that more productive firms face higher R&D input distortions and, consequently, higher R&D input prices. We apply the model to the firm-level data in the UK manufacturing sector in 2019 and find that the output gains from eliminating resource misallocation are significantly overestimated if the spillovers are not taken into account. When we express output as a function of factor distortions, we find that the capital distortions are the main contributor to output loss. However, if a wrong policy is adopted towards R&D, the output loss due to R&D input distortions is also significant.

The model we study is static, with an exogenously-given amount of resources. A possible next step is to build a dynamic model where current

decisions determine the future stock of inputs. Whether our conclusions about the optimal allocation of R&D input will continue to hold, is likely to depend on how the accumulation of R&D stock is modelled. If the investment in R&D depends on the aggregate output in the economy, we expect that it is still optimal to favour less productive firms to achieve more even use of R&D input because it maximizes the output. If, instead, the production of new R&D only requires the R&D input, it might be optimal to concentrate the R&D input in more productive firms.

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Appendix

A Decomposition of Industry TFP and Spillover

We start by substituting (4)-(6) into the production function (2) to arrive at

$$Y_{si} = \left(\frac{B_{si}\Gamma_s X_s^{\delta_s}}{\Theta_{si}} \right)^\rho, \quad (20)$$

where

$$\Gamma_s \equiv \frac{\rho - 1}{\rho} \left(\frac{\alpha_s}{r_s} \right)^{\alpha_s} \left(\frac{\beta_s}{w_s} \right)^{\beta_s} \left(\frac{\gamma_s}{q_s} \right)^{\gamma_s}$$

and

$$\Theta_{si} \equiv (1 + \tau_{K_{si}})^{\alpha_s} (1 + \tau_{L_{si}})^{\beta_s} (1 + \tau_{H_{si}})^{\gamma_s}.$$

Given (20), the industry s output in (1) becomes

$$Y_s = \left(\Gamma_s X_s^{\delta_s} \right)^\rho \left(\sum_{i=1}^{m_s} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right)^{\frac{\rho}{\rho-1}}.$$

Also, if we substitute (20) back into (4)-(6) and sum across the firms, input demands by industry s are

$$\begin{aligned} K_s &= \frac{\rho - 1}{\rho} (\Gamma_s X_s^{\delta_s})^{\rho-1} \frac{\alpha_s}{r_s} \sum_{i=1}^{m_s} \frac{1}{1 + \tau_{K_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1}, \\ L_s &= \frac{\rho - 1}{\rho} (\Gamma_s X_s^{\delta_s})^{\rho-1} \frac{\beta_s}{w_s} \sum_{i=1}^{m_s} \frac{1}{1 + \tau_{L_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1}, \\ H_s &= \frac{\rho - 1}{\rho} (\Gamma_s X_s^{\delta_s})^{\rho-1} \frac{\gamma_s}{q_s} \sum_{i=1}^{m_s} \frac{1}{1 + \tau_{H_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1}. \end{aligned}$$

We can use the above expressions for Y_s , K_s , L_s , and H_s to write industry TFP in (16) as

$$TFP_s = \frac{\left(\sum_{i=1}^{m_s} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right)^{\frac{\rho}{\rho-1}}}{\left(\sum_{i=1}^{m_s} \frac{1}{1 + \tau_{K_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right)^{\alpha_s} \left(\sum_{i=1}^{m_s} \frac{1}{1 + \tau_{L_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right)^{\beta_s} \left(\sum_{i=1}^{m_s} \frac{1}{1 + \tau_{H_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right)^{\gamma_s}}.$$

Similar to Chen and Irarrazabal (2015), we now provide an approximation of TFP_s assuming that distortions and firm TFPs are log-normally distributed with the variance-covariance matrix given by Σ_s and that the number of firms in the industry tends to infinity. Let $\mu_{B_s} \equiv E[\log B_{si}]$, $\mu_{\Theta_s} \equiv E[\log \Theta_{si}]$, $\sigma_{\Theta_s}^2 \equiv var(\log \Theta_{si})$, and $\sigma_{B\Theta_s} = cov(\log B_{si}, \log \Theta_{si})$. Then,

$$\begin{aligned} \log TFP_s &= \frac{\rho}{\rho - 1} \log \int \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di - \alpha_s \log \int \frac{1}{1 + \tau_{K_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di \\ &\quad - \beta_s \log \int \frac{1}{1 + \tau_{L_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di - \gamma_s \log \int \frac{1}{1 + \tau_{H_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di, \end{aligned}$$

where

$$\log \int \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di = (\rho - 1) (\mu_{B_s} - \mu_{\Theta_s}) + \frac{(\rho - 1)^2}{2} (\sigma_{B_s}^2 + \sigma_{\Theta_s}^2) - (\rho - 1)^2 \sigma_{B\Theta_s},$$

and

$$\begin{aligned} \log \int \frac{1}{1 + \tau_{I_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} di &= (\rho - 1) (\mu_{B_s} - \mu_{\Theta_s}) - \mu_{I_s} + \frac{(\rho - 1)^2}{2} (\sigma_{B_s}^2 + \sigma_{\Theta_s}^2) \\ &\quad + \frac{1}{2} \sigma_{I_s}^2 - (\rho - 1)^2 \sigma_{B\Theta_s} - (\rho - 1) (\sigma_{BI_s} - \sigma_{\Theta I_s}), \end{aligned}$$

for $I = K, L, H$. Observe that

$$\begin{aligned} \mu_{\Theta_s} &= \alpha_s \mu_{K_s} + \beta_s \mu_{L_s} + \gamma_s \mu_{H_s}, \\ \sigma_{\Theta_s}^2 &= \alpha_s^2 \sigma_{K_s}^2 + \beta_s^2 \sigma_{L_s}^2 + \gamma_s^2 \sigma_{H_s}^2, \\ \sigma_{\Theta I_s} &= \phi_{I_s} \sigma_{I_s}^2, \end{aligned}$$

where $\phi_{I_s} = \alpha_s, \beta_s, \gamma_s$ for $I = K, L, H$, respectively. Substituting it all in the expression for $\log TFP_s$ and noting that $\alpha_s + \beta_s + \gamma_s = 1$, we obtain the expression in (17).

To write $\log X_s$ in terms of variances and covariances of distortions and firm TFPs, we note that

$$\begin{aligned} \log H_s &= \log \int H_{si} di = E[\log H_{si}] + \frac{1}{2} \text{var}(\log H_{si}) \\ &= \log X_s + \frac{1}{2} \text{var} \left(\log \left(\frac{1}{1 + \tau_{H_{si}}} \left(\frac{B_{si}}{\Theta_{si}} \right)^{\rho-1} \right) \right). \end{aligned}$$

Hence,

$$\log X_s = \log H_s - \frac{(\rho - 1)^2}{2} (\sigma_{B_s}^2 + \sigma_{\Theta_s}^2) - \frac{1}{2} \sigma_{H_s}^2 + (\rho - 1)^2 \sigma_{B\Theta_s} + (\rho - 1) (\sigma_{BH_s} - \sigma_{\Theta H_s}).$$

After substituting the expressions for $\sigma_{\Theta_s}^2$, $\sigma_{\Theta H_s}$, and $\sigma_{B\Theta_s} = \alpha_s \sigma_{BK_s} + \beta_s \sigma_{BL_s} + \gamma_s \sigma_{BH_s}$, we arrive at (18).

B Maximizing Industry Output

We want to maximize (19) subject to the constraint that Σ_s is a positive semi-definite matrix. A symmetric matrix is positive semi-definite if and only if all of its principal minors are nonnegative:

$$\sigma_{K_s}^2 \geq 0, \quad (21)$$

$$\sigma_{L_s}^2 \geq 0, \quad (22)$$

$$\sigma_{H_s}^2 \geq 0, \quad (23)$$

$$\sigma_{B_s}^2 \sigma_{K_s}^2 - \sigma_{BK_s}^2 \geq 0, \quad (24)$$

$$\sigma_{B_s}^2 \sigma_{L_s}^2 - \sigma_{BL_s}^2 \geq 0, \quad (25)$$

$$\sigma_{B_s}^2 \sigma_{H_s}^2 - \sigma_{BH_s}^2 \geq 0, \quad (26)$$

$$\sigma_{B_s}^2 \sigma_{K_s}^2 \sigma_{L_s}^2 - \sigma_{BK_s}^2 \sigma_{L_s}^2 - \sigma_{BL_s}^2 \sigma_{K_s}^2 \geq 0, \quad (27)$$

$$\sigma_{B_s}^2 \sigma_{K_s}^2 \sigma_{H_s}^2 - \sigma_{BK_s}^2 \sigma_{H_s}^2 - \sigma_{BH_s}^2 \sigma_{K_s}^2 \geq 0, \quad (28)$$

$$\sigma_{B_s}^2 \sigma_{L_s}^2 \sigma_{H_s}^2 - \sigma_{BL_s}^2 \sigma_{H_s}^2 - \sigma_{BH_s}^2 \sigma_{L_s}^2 \geq 0, \quad (29)$$

$$\sigma_{B_s}^2 \sigma_{K_s}^2 \sigma_{L_s}^2 \sigma_{H_s}^2 - \sigma_{BK_s}^2 \sigma_{L_s}^2 \sigma_{H_s}^2 - \sigma_{BL_s}^2 \sigma_{K_s}^2 \sigma_{H_s}^2 - \sigma_{BH_s}^2 \sigma_{K_s}^2 \sigma_{L_s}^2 \geq 0. \quad (30)$$

There are four more constraints $\sigma_{K_s}^2 \sigma_{L_s}^2 \geq 0$, $\sigma_{K_s}^2 \sigma_{H_s}^2 \geq 0$, $\sigma_{L_s}^2 \sigma_{H_s}^2 \geq 0$, $\sigma_{K_s}^2 \sigma_{L_s}^2 \sigma_{H_s}^2 \geq 0$, but they are automatically satisfied if the other constraints are satisfied.

We minimize

$$a_{K_s} \sigma_{K_s}^2 + a_{L_s} \sigma_{L_s}^2 + a_{H_s} \sigma_{H_s}^2 - 2b_{K_s} \sigma_{BK_s} - 2b_{L_s} \sigma_{BL_s} - 2b_{H_s} \sigma_{BH_s}$$

subject to (21)-(30) where the coefficients a_{I_s} and b_{I_s} for $I = K, L, H$ are given right after (19). As it turns out, the constraint qualifications are not satisfied in this problem and, therefore, the Kuhn-Tucker conditions are not necessary conditions. We identify the solution in several steps.

First, suppose the solution is such that $\sigma_{BK_s}^2 > 0$, $\sigma_{BL_s}^2 > 0$, and $\sigma_{BH_s}^2 > 0$. But then from (24)-(26), $\sigma_{K_s}^2 > 0$, $\sigma_{L_s}^2 > 0$, $\sigma_{H_s}^2 > 0$. Furthermore, the

only constraint that binds is (30). To see it, suppose for example that (24) binds. But then from (30), $\sigma_{BLs}^2 \sigma_{Ks}^2 \sigma_{Hs}^2 + \sigma_{BHs}^2 \sigma_{Ks}^2 \sigma_{Ls}^2 = 0$, a contradiction. The same applies if we assume that any other constraint from (25) to (29) binds. Hence, we solve the following problem:

$$\begin{aligned} \min L = & a_{Ks} \sigma_{Ks}^2 + a_{Ls} \sigma_{Ls}^2 + a_{Hs} \sigma_{Hs}^2 - 2b_{Ks} \sigma_{BKs} - 2b_{Ls} \sigma_{BLs} - 2b_{Hs} \sigma_{BHs} \\ & - \lambda \left(\sigma_{Bs}^2 \sigma_{Ks}^2 \sigma_{Ls}^2 \sigma_{Hs}^2 - \sigma_{BKs}^2 \sigma_{Ls}^2 \sigma_{Hs}^2 - \sigma_{BLs}^2 \sigma_{Ks}^2 \sigma_{Hs}^2 - \sigma_{BHs}^2 \sigma_{Ks}^2 \sigma_{Ls}^2 \right). \end{aligned}$$

The Kuhn-Tucker conditions are

$$\frac{\partial L}{\partial \sigma_{Ks}^2} = a_{Ks} - \lambda \sigma_{Ls}^2 \sigma_{Hs}^2 \left(\sigma_{Bs}^2 - \frac{\sigma_{BLs}^2}{\sigma_{Ls}^2} - \frac{\sigma_{BHs}^2}{\sigma_{Hs}^2} \right) = 0, \quad (31)$$

$$\frac{\partial L}{\partial \sigma_{Ls}^2} = a_{Ls} - \lambda \sigma_{Ks}^2 \sigma_{Hs}^2 \left(\sigma_{Bs}^2 - \frac{\sigma_{BKs}^2}{\sigma_{Ks}^2} - \frac{\sigma_{BHs}^2}{\sigma_{Hs}^2} \right) = 0, \quad (32)$$

$$\frac{\partial L}{\partial \sigma_{Hs}^2} = a_{Hs} - \lambda \sigma_{Ks}^2 \sigma_{Ls}^2 \left(\sigma_{Bs}^2 - \frac{\sigma_{BKs}^2}{\sigma_{Ks}^2} - \frac{\sigma_{BLs}^2}{\sigma_{Ls}^2} \right) = 0, \quad (33)$$

$$\frac{\partial L}{\partial \sigma_{BKs}} = -b_{Ks} + \lambda \sigma_{BKs} \sigma_{Ls}^2 \sigma_{Hs}^2 = 0, \quad (34)$$

$$\frac{\partial L}{\partial \sigma_{BLs}} = -b_{Ls} + \lambda \sigma_{BLs} \sigma_{Ks}^2 \sigma_{Hs}^2 = 0, \quad (35)$$

$$\frac{\partial L}{\partial \sigma_{BHs}} = -b_{Hs} + \lambda \sigma_{BHs} \sigma_{Ks}^2 \sigma_{Ls}^2 = 0, \quad (36)$$

$$\frac{\partial L}{\partial \lambda} = \sigma_{Bs}^2 - \frac{\sigma_{BKs}^2}{\sigma_{Ks}^2} - \frac{\sigma_{BLs}^2}{\sigma_{Ls}^2} - \frac{\sigma_{BHs}^2}{\sigma_{Hs}^2} = 0. \quad (37)$$

If one plugs (37) into (31)-(33), one gets that

$$\begin{aligned} a_{Ks} - \lambda \sigma_{Ls}^2 \sigma_{Hs}^2 \frac{\sigma_{BKs}^2}{\sigma_{Ks}^2} &= 0, \\ a_{Ls} - \lambda \sigma_{Ks}^2 \sigma_{Hs}^2 \frac{\sigma_{BLs}^2}{\sigma_{Ls}^2} &= 0, \\ a_{Hs} - \lambda \sigma_{Ks}^2 \sigma_{Ls}^2 \frac{\sigma_{BHs}^2}{\sigma_{Hs}^2} &= 0. \end{aligned}$$

Substituting (34)-(36) into the above expressions, one obtains that

$$\begin{aligned}\frac{\sigma_{BKs}}{\sigma_{Ks}^2} &= \frac{a_{Ks}}{b_{Ks}}, \\ \frac{\sigma_{BLs}}{\sigma_{Ls}^2} &= \frac{a_{Ls}}{b_{Ls}}, \\ \frac{\sigma_{BHs}}{\sigma_{Hs}^2} &= \frac{a_{Hs}}{b_{Hs}}.\end{aligned}$$

However, from (34)-(36), we also have that

$$\begin{aligned}\frac{b_{Ks}}{b_{Ls}} &= \frac{\sigma_{BKs}\sigma_{Ls}^2}{\sigma_{BLs}\sigma_{Ks}^2}, \\ \frac{b_{Ks}}{b_{Hs}} &= \frac{\sigma_{BKs}\sigma_{Hs}^2}{\sigma_{BHs}\sigma_{Ks}^2},\end{aligned}$$

which imply

$$\begin{aligned}\frac{\sigma_{BKs}}{\sigma_{Ks}^2} &= \frac{b_{Ks}}{b_{Ls}} \frac{\sigma_{BLs}}{\sigma_{Ls}^2} = \frac{b_{Ks}}{b_{Hs}} \frac{\sigma_{BHs}}{\sigma_{Hs}^2}, \\ \frac{a_{Ks}}{b_{Ks}} &= \frac{b_{Ks}}{b_{Ls}} \frac{a_{Ls}}{b_{Ls}} = \frac{b_{Ks}}{b_{Hs}} \frac{a_{Hs}}{b_{Hs}}.\end{aligned}$$

However, these equalities cannot be satisfied simultaneously. It follows that the solution cannot be such that $\sigma_{BKs}^2 > 0$, $\sigma_{BLs}^2 > 0$, and $\sigma_{BHs}^2 > 0$ hold.

Second, suppose the solution is such that $\sigma_{BKs}^2 > 0$ and $\sigma_{BLs}^2 > 0$, while $\sigma_{BHs}^2 = 0$. From (24)-(25), $\sigma_{Ks}^2 > 0$ and $\sigma_{Ls}^2 > 0$ hold. Further, inspection of (21)-(30) tells that $\sigma_{Hs}^2 > 0$ does not help to relax any of the constraints, while the objective is decreasing in σ_{Hs}^2 . Therefore, $\sigma_{Hs}^2 = 0$ must hold. Then, (26) and (28)-(30) are all automatically satisfied. Of the remaining constraints, (24)-(25) and (27), only the last will bind. To see it, suppose for example that (24) binds. But then from (27), $\sigma_{BLs}^2\sigma_{Ks}^2 = 0$, a contradiction. The same applies if we assume that (25) binds. Hence, we solve the following

problem:

$$\begin{aligned} \min L = & a_{Ks}\sigma_{Ks}^2 + a_{Ls}\sigma_{Ls}^2 - 2b_{Ks}\sigma_{BKs} - 2b_{Ls}\sigma_{BLs} \\ & - \lambda \left(\sigma_{Bs}^2\sigma_{Ks}^2\sigma_{Ls}^2 - \sigma_{BKs}^2\sigma_{Ls}^2 - \sigma_{BLs}^2\sigma_{Ks}^2 \right). \end{aligned}$$

Using similar steps as before, we find the solution where

$$\begin{aligned} \frac{\sigma_{BKs}}{\sigma_{Ks}^2} &= \frac{b_{Ks}}{b_{Ls}} \frac{\sigma_{BLs}}{\sigma_{Ls}^2}, \\ \frac{a_{Ks}}{b_{Ks}} &= \frac{b_{Ks}}{b_{Ls}} \frac{a_{Ls}}{b_{Ls}}. \end{aligned}$$

However, this equality is not again satisfied. It follows that the solution cannot be such that $\sigma_{BKs}^2 > 0$, $\sigma_{BLs}^2 > 0$, and $\sigma_{BHs}^2 = 0$ hold. By symmetry, the same is true if only $\sigma_{BKs}^2 = 0$ or $\sigma_{BLs}^2 = 0$ holds.

Third, the remaining case is when only one of the covariances is strictly positive. Thus, suppose that $\sigma_{BKs}^2 > 0$, while $\sigma_{BLs}^2 = 0$ and $\sigma_{BHs}^2 = 0$. From (24), $\sigma_{Ks}^2 > 0$ holds. Further, inspection of (21)-(30) tells that $\sigma_{BLs}^2 > 0$ or $\sigma_{Hs}^2 > 0$ does not help to relax any of the constraints, while the objective is decreasing in σ_{BLs}^2 and σ_{Hs}^2 . Therefore, $\sigma_{BLs}^2 = \sigma_{Hs}^2 = 0$ must hold. Then, (25)-(30) are all automatically satisfied. The only constraint that we need to take into account is (24), which will bind. If it did not, we could decrease the objective by decreasing σ_{Ks}^2 . Hence, we solve the following problem:

$$\min L = a_{Ks}\sigma_{Ks}^2 - 2b_{Ks}\sigma_{BKs} - \lambda \left(\sigma_{Bs}^2\sigma_{Ks}^2 - \sigma_{BKs}^2 \right).$$

The solution is

$$\begin{aligned} \sigma_{BKs} &= \frac{b_{Ks}}{a_{Ks}}\sigma_{Bs}^2, \\ \sigma_{Ks}^2 &= \left(\frac{b_{Ks}}{a_{Ks}} \right)^2 \sigma_{Bs}^2. \end{aligned}$$

Evaluating the objective at this solution, gives that

$$-\frac{b_{K_s}^2}{a_{K_s}}\sigma_{B_s}^2.$$

Similar analysis would apply if we assumed that either only $\sigma_{BL_s}^2 > 0$ or only $\sigma_{BH_s}^2 > 0$. To decide which of these covariances is strictly positive, we need to compare $\frac{b_{K_s}^2}{a_{K_s}}$, $\frac{b_{L_s}^2}{a_{L_s}}$, and $\frac{b_{H_s}^2}{a_{H_s}}$, and pick the largest. One can verify that $\frac{b_{H_s}^2}{a_{H_s}}$ takes the largest value if either $0 \leq \delta_s \leq \max\{1, \rho\}$ or $\rho \geq 2$ holds, both of which are satisfied by our calibration of the model. We conclude that the output is maximized when $\sigma_{K_s}^2 = \sigma_{L_s}^2 = \sigma_{BK_s} = \sigma_{BL_s} = 0$,

$$\begin{aligned}\sigma_{BH_s} &= \frac{\delta_s(\rho - 1)}{\gamma_s + \delta_s + \gamma_s\delta_s(\rho - 1)}\sigma_{B_s}^2, \\ \sigma_{H_s}^2 &= \left(\frac{\delta_s(\rho - 1)}{\gamma_s + \delta_s + \gamma_s\delta_s(\rho - 1)}\right)^2\sigma_{B_s}^2.\end{aligned}$$

C Data Description

This section contains the tables omitted from the main text. Table 7 lists the data sources, while Table 8 classifies industries by their technological intensity.

Table 7: Data Sources

Variable in the model	Dataset	Variable in the dataset	Label
$P_{si}Y_{si}$	ABS	wq613	Approximate Gross Value Added (aGVA) at basic prices (£,000)
K_{si}	ABS	wq599	Total value of all stocks at the end of the year (£,000)
$L_{si} + H_{si}$	ABS	empment	IDBR employment at time of sample selection
H_{si}	BERD	emp_sci	Number of scientists, researchers
W_{si}	BERD	slries	Salaries and wages (£,000)
$I_{K_{si}}$	ABS	wq522	Value of total net capex (excluding NYIP) (£,000)
$I_{R\&D_{si}}$	BERD	intram	Total in-house capital and non-capital expenditure for performing R&D (£,000)

Table 8: Industry Classification by Technological Intensity

	Code	SIC07 2-digit Level Code
Low Technology	10	Manufacture of food products
	11	Manufacture of beverages
	13	Manufacture of textiles
	15	Manufacture of leather products
	16	Manufacture of wood products, except furniture
	17	Manufacture of paper products
	18	Printing and reproduction of recorded media
	31	Manufacture of furniture
	32	Other manufacturing
Medium-Low Technology	22	Manufacture of rubber and plastic products
	23	Manufacture of other non-metallic mineral products
	24	Manufacture of basic metals
	25	Manufacture of fabricated metal products
	33	Repair and installation of machinery and equipment
Medium-High Technology	19	Manufacture of coke and petroleum
	20	Manufacture of chemicals
	27	Manufacture of electrical equipment
	28	Manufacture of machinery and equipment
	29	Manufacture of motor vehicles
High Technology	30	Manufacture of other transport equipment
	21	Manufacture of pharmaceutical products
	26	Manufacture of computer, electronic and optical products