

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/162963/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Lin, Junyi, Huang, Hongfu, Li, Shanshan and Naim, Mohamed M. 2023. On the dynamics of order pipeline inventory in a nonlinear order-up-to system. *International Journal of Production Economics* 266 , 109061. 10.1016/j.ijpe.2023.109061

Publishers page: <https://doi.org/10.1016/j.ijpe.2023.109061>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.





1 **1. Introduction and contributions**

2 System dynamics plays a critical role in influencing supply chain performance, especially given  
3 the current volatile market environment (Spiegler and Naim 2017). Dynamic characteristics,  
4 particularly the bullwhip effect (Lee et al. 1997), are considered the main source of business  
5 disruption (Wang and Disney 2016). This effect refers to a phenomenon in which low variation  
6 in market demand causes significant changes in upstream production for suppliers and has  
7 associated costs, such as ramping up/down of machines, hiring and firing of staff and  
8 increased/decreased inventory levels (Lin et al. 2017).

9 An important cause of bullwhip is the inappropriate control of *order pipeline inventory*  
10 (Serman 1989; John et al. 1994; Disney and Towill 2005; Springer and Kim 2010). Order  
11 pipeline inventory refers to the work-in-process (WIP)/in-transit inventory after order  
12 placement by buyers but before delivery of them. It has been experimentally and empirically  
13 shown that decision makers often ignore, underestimate or overestimate order pipeline  
14 inventory (Serman 1989; Croson and Donohue 2006; Croson et al. 2014; Udenio et al. 2017),  
15 leading to supply chain instability and high bullwhip effect. Top companies such as Intel and  
16 AstraZeneca have reported a number of cases where managing pipeline inventory information  
17 remains a significant challenge for supply chain practitioners (Hopp and Spearman 2001; Lin  
18 et al. 2018).

19 Theoretically, it has been proven that order pipeline control can be achieved by  
20 incorporating an order pipeline adjustment feedback loop via comparison of the desired and  
21 actual pipeline inventory in the ordering process (John et al. 1994; Lin et al. 2017). However,  
22 a key challenge is the setting of *desired order pipeline inventory* in such a feedback loop  
23 (Serman 2000). Most studies have treated the desired pipeline inventory as a function of  
24 forecasted demand and estimated lead times in a linear order-up-to (OUT) policy (e.g., Disney  
25 et al. 2006; Udenio et al. 2017; Udenio et al. 2022) or constrained OUT system (e.g., Wang et  
26 al. 2012; Wang et al. 2014; Ponte et al. 2017; Disney et al. 2020). This ignores another  
27 recognised setting method, as experimentally reported by Serman (2000), in which the decision

1 maker may consider not only estimated demand and lead times but also on-hand stock  
2 fluctuations as the desired pipeline setting (Sterman 2000).

3 Springer and Kim (2010) and Lin et al. (2018) assessed the impact of different desired  
4 pipeline settings on system dynamics performance. However, the two studies assumed their  
5 supply chains were completely *linear*. In supply chain system structures, nonlinearity can  
6 naturally occur through the existence of physical and economic constraints; for instance,  
7 capacity and returns constraints in manufacturing and shipping processes (Spiegler et al. 2016,  
8 Ponte et al. 2017). Although the constrained capacity may be relaxed by adopting an  
9 outsourcing strategy, linearity assumptions may ignore an important constraint in practice: the  
10 buyer (e.g., manufacturer) is often not allowed to freely return its excess inventory to suppliers.  
11 Mathematically, this means the order placed on the supplier cannot be negative (Wang et al.  
12 2012; Wang et al. 2014). It has been demonstrated that a forbidden returns policy plays a key  
13 role in influencing the dynamics of inventory systems—sometimes even a dominant role  
14 (Nagatani and Helbing 2004; Wang et al. 2014; Lin et al. 2020). When free-returns assumptions  
15 are removed, complex dynamic behaviours are revealed. More importantly, oscillations  
16 generated internally by the system itself, rather than by the external environment, may arise.

17 Motivated by practical observations and research gaps, our aim is to analytically explore  
18 the impact of the desired order pipeline setting—with due consideration of visibility and hence  
19 estimate of the lead time—on the bullwhip phenomenon when both free returns and forbidden  
20 returns scenarios are considered. Our key contributions are as follows:

- 21 1. We analytically assess the dynamics of different desired order pipeline inventory policies  
22 when both free and forbidden returns between manufacturer and supplier are considered.  
23 This study extends the analysis of pipeline inventory dynamics from the traditional linear  
24 OUT system to a generalised nonlinear proportional OUT system with a returns constraint.  
25 An analysis in the time domain and frequency domain allows us to assess the dynamic  
26 behaviour of the supply chain system in responding to demand volatility and seasonality.
- 27 2. We analytically derive a bullwhip expression in the time and frequency domains in which  
28 the corresponding ‘shock’ and ‘seasonal’ customer demands are assumed. In particular,

1 for frequency analysis, we derive bullwhip as a function of the demand frequency and the  
2 supply chain system structure, highlighting the importance of jointly considering the  
3 endogenous system feedback control structure and exogenous demand characteristics.  
4 Mathematically approximate closed-form results are derived to predict the propagation of  
5 order fluctuations in the nonlinear forbidden returns context.

6 The rest of the paper is structured as follows. Section 2 reviews the relevant literature and  
7 highlights research gaps. Section 3 presents the returns-forbidden-based OUT models with  
8 different desired order pipeline settings. Section 4 introduces the main analysis methods  
9 adopted in this study with the detailed dynamics analysis undertaken in Section 5 and 6.  
10 Extensive numerical simulation is conducted in Section 7 and all results and corresponding  
11 managerial implications are discussed in Section 8. A final discussion and conclusion can be  
12 found Section 9.

13

## 14 **2. Literature review**

15 We review two themes in the literature: 1) order pipeline inventory dynamics and 2) inventory  
16 dynamics with return-forbidden constraints.

17

### 18 *2.1. Research on order pipeline inventory dynamics*

19 The bullwhip effect (Lee et al. 1997)—also known as demand information amplification or the  
20 Forrester effect (Forrester 1961)—is frequently observed in industries (Chen and Lee 2012).  
21 Based on Zotteri’s (2013) classification, there are three main streams of research relating to  
22 bullwhip: theoretical, empirical and natural experimental. The first focusses on its causes and  
23 potential solutions, with information transparency as a possible remedy (e.g., Chen et al. 2000;  
24 Cachon and Fisher 2000; Lee et al. 2004). The second stream adopts the well-known Beer Game  
25 or its variants to test hypotheses on causes and potential solutions (Croson and Donohue 2003,  
26 2006; Croson et al. 2014; Cantor and Kator 2012). Finally, natural experimental research offers  
27 empirical evidence for the existence, scope and significance of bullwhip for several industries  
28 (Cachon et al. 2007; Brav and Mendelson 2012; Shan et al. 2014).

1 Our study is in the first, theoretical, stream. We focus on control of order pipeline  
2 information (Disney and Towill 2005; Croson and Donohue 2006; Springer and Kim 2010;  
3 Macdonald et al. 2013; Croson et al. 2014; Lin et al. 2018; Yang et al. 2021). The common  
4 conclusion is that, driven by the lack of pipeline transparency, decision makers tend to ignore  
5 or underestimate the supply pipeline and the associated feedback loop, which creates bullwhip  
6 effect and supply chain instability. Notably, bullwhip exists even when supply pipeline  
7 information is transparent and there is no underestimation (Wu and Katok 2006).

8 Given the order pipeline feedback loop incorporated in the ordering policy decision process,  
9 there are two associated challenges: (1) the estimation of real-time physical delay (e.g.,  
10 production lead times) and (2) the desired pipeline setting methods. For the former, several  
11 researchers (Towill et al. 1997; Disney and Towill 2005; Aggelogiannaki and Sarimveis 2008)  
12 have explored different control methods to improve the adaptability of the supply chain system  
13 in accurately estimating lead times and pipeline inventory.

14 Regarding the second challenge, Serman (2000, p. 714) discussed two desired pipeline  
15 settings for pipeline inventory management experimentally identified in the Beer Game. The  
16 first setting considers the product of constant estimated lead times and demand, whereas for the  
17 second setting, the on-hand inventory adjustment is added into the first pipeline settings. Most  
18 studies have simply adopted the first setting method (e.g., Dejonckheere et al. 2003; Disney et  
19 al. 2016; Ponte et al. 2017; Disney et al. 2020). To the best of our knowledge, only two studies  
20 have explored the impact of these two desired pipeline inventory settings on system dynamics  
21 performance. Specifically, Springer and Kim (2010) named the two mechanisms as static and  
22 dynamic pipeline control, and analytically compared the two settings in a two-stage linear  
23 supply chain system based on a pre-assumed ‘shock’ demand pattern. They found that bullwhip  
24 can be minimised by adopting static pipeline control. However, the inventory dynamics cost  
25 can be minimised by using dynamic pipeline control. Lin et al. (2018) highlighted the similar  
26 impact of these two control methods in a linear assemble-to-order (ATO) system.

27 The above two studies assumed that returns between manufacturer (or buyer) and supplier  
28 are allowed. However, the result derived for a free-returns environment may not be applicable

1 to the forbidden returns environment commonly observed in many industries. This has greatly  
2 limited the applicability of published results and has made it difficult to fully explain  
3 oscillations caused by internal factors (Lin et al. 2020; Lin et al. 2022). In the next section, we  
4 review existing insights about the dynamics of supply chains with returns constraints to further  
5 highlight our research positioning.

## 7 *2.2. Research on inventory dynamics with returns constraints*

8 In supply chain system structures, capacity and order returns constraints are the two most  
9 common nonlinearities present in practice (Lin et al. 2019). Several studies (e.g., Ponte et al.  
10 2017; Spiegler and Naim 2017; Cannella et al. 2018) have explored the impact of capacity  
11 constraints in a traditional OUT policy by setting desired order pipeline inventory as a function  
12 of forecasted demand and estimated lead times. The general conclusion is that the capacitated  
13 supply chains may benefit from improved dynamics performance relative to unconstrained  
14 ones, since the capacity limit acts as a smoothing filter.

15 Also, several studies have explored the impact of returns assumptions on the dynamics of  
16 supply chains. From a control engineering perspective, Wang and Disney (2012) and Wang et  
17 al. (2014) explored the stability boundaries of forbidden returns to identify a set of behaviours  
18 in the unstable region. Also, Wang et al. (2015), Spiegler and Naim (2017) and Lin and Naim  
19 (2019) adopted describing functions (DF) to study the order-constrained supply chains. The  
20 impact of forbidden returns policies on system dynamics performance is also studied by  
21 simulation. Chatfield and Pritchard (2013) showed that the free-returns assumption may  
22 systematically overestimate the bullwhip effect. Dominguez et al. (2015) found that the impact  
23 of returns conditions (returns or no returns) on the bullwhip effect is greatly influenced by  
24 whether a supply network is serial or divergent.

25 However, all previous nonlinear studies set the desired order pipeline inventory as a  
26 function of forecasted demand and estimated lead times based on a traditional OUT policy. We  
27 term this policy as '*reactive pipeline control*' (RPC) policy. It ignores the fact that decision  
28 markers may adjust the desired order pipeline change via additional on-hand inventory

1 fluctuations, as experimentally reported by Sterman (2000). The ‘*proactive pipeline control*’  
 2 (PPC) policy is adopted to define such policy. This motivates us to explore the dynamics of  
 3 different desired order pipeline settings in a nonlinear, forbidden returns environment.

4

5 *2.3. Summary of research gaps*

6 Table 1 summarises the main relevant literature and the focus of this paper based on Sections  
 7 2.1 and 2.2.

8

| <b>Authors</b>   | <b>Type of system</b>   | <b>Desired order pipeline settings</b> | <b>Key objectives</b>   | <b>Findings</b>   |
|--|---|--|---|---|
| Springer and Kim (2010)                                  | Linear unconstrained OUT policy (i.e., returns allowed)                                 | RPC and PPC policies                   | Analytically compare two settings in responding to impulse demand (i.e., a demand shock)  | RPC minimises bullwhip, PPC improves inventory dynamics.  |
| Wang and Disney (2012) and Wang, Disney, and Wang (2014) | Nonlinear constrained OUT policy with forbidden returns                                 | RPC policy                             | Explore the stability boundaries of the forbidden returns OUT system  | Criteria for different types of dynamic behaviour are derived, including convergence, periodicity, quasi-periodicity, chaos and divergence. |
| Chatfield, and Pritchard (2013)                          | Multi-stage nonlinear constrained supply chain with forbidden returns                   | RPC policy                             | Explore the impact of returns assumption on the bullwhip effect in multi-stage supply chain systems   | Returns allowance significantly increases bullwhip in supply chains.  |
| Dominguez et al. (2015)                                  | Nonlinear constrained supply chain network (serial v. divergent) with forbidden returns | RPC policy                             | Investigate the impact of returns conditions (returns v. no returns) on the bullwhip effect under a serial and divergent supply chain network configuration | Returns have a lower impact on bullwhip and transport costs in divergent supply chains than in serial supply chains.                        |

|                          |  |                      |   |  |
|--------------------------|--|----------------------|---|--|
| Wang et al. (2015)       | Nonlinear constrained OUT policy with forbidden returns                          | RPC policy           | Analytically explore the impact of forbidden returns on bullwhip in serial supply chains                                      | Forbidden returns contribute to a reduction in bullwhip.   |
| Spiegler and Naim (2017) | Nonlinear constrained OUT policy with forbidden returns and shipment constraints | RPC policy           | Investigate the effect of non-negative order and shipment constraints on dynamic performance                                  | The root causes of sustained oscillation are identified. Lead and lag compensation strategies are proposed to reduce oscillations.   |
| Lin et al. (2018)        | Linear ATO system (i.e., returns allowed)  | RPC and PPC policies | Assess the impact of main system control parameters on system dynamics performance  | Forecasting and an inventory decoupling point correction policy play a major role in the bullwhip effect for both RPC and PPC policies.  |
| Lin and Naim (2019)      | Nonlinear constrained ATO system with forbidden returns and capacity constraints | RPC policy           | Assess the impact of nonlinear forbidden returns and capacity constraints on system dynamics performance in ATO systems       | Forbidden returns contribute to bullwhip reduction, at the expense of slower inventory recovery speed.   |
| <b>This study</b>        | Nonlinear constrained OUT policy with forbidden returns                          | RPC and PPC policies | Analytically explore the impact of two different desired pipeline settings in a forbidden returns production inventory system | RPC yields slower inventory convergence while PPC produces unwanted oscillations. PPC generates more bullwhip regardless of physical delays and system control parameters. Forbidden returns generate less bullwhip and less oscillation but slow inventory recovery for both RPC and PPC. |

1

Table 1. Summary of relevant literature

2

3

From Table 1, our research position can be clearly highlighted: this is the first study that explores the dynamic implications of different desired pipeline settings in a non-linear forbidden returns supply chain context. Specifically, it has been identified that most researchers

5

1 who focus on the dynamics of a forbidden returns policy simply set the desired order pipeline  
2 inventory as a function of estimated demand and lead times. This may ignore an alternative  
3 desired order pipeline setting where a decision maker may react to a change in order pipeline  
4 inventory by considering their on-hand inventory fluctuation (Sterman 2000; Croson and  
5 Donohue 2006). Although Springer and Kim (2010) examined two pipeline settings in great  
6 detail, we extend their study by relaxing several important assumptions. Specifically, we  
7 examine the impact of forbidden returns policy on dynamic behaviour, thus relaxing their  
8 linearly unconstrained system that cannot represent the practical scenario where returns  
9 between manufacturer and supplier are not allowed. Also, Springer and Kim's (2010) model,  
10 based on Beer Game settings, oversimplified real-world manufacturing system by assuming  
11 known demand patterns, order lost-sales and accurate lead time estimation. We relax these  
12 assumptions by developing a backlogged order-up-to model with forecasting and incorporating  
13 estimated lead time parameters. As the result, our order-up-to model can better reflect a  
14 practical manufacturing inventory replenishment system and offer new insights into order  
15 pipeline policies, and the impact of forecasting and estimated lead times on system dynamics  
16 performance.

17

### 18 **3. Preliminaries**

19 We study a single-echelon supply chain system that represents the production–inventory system  
20 formed by a customer placing orders with a manufacturer (Disney et al. 2020). The customer,  
21 depending on the industry and sourcing strategy, can be a distributor or a retailer. We develop  
22 a stylised system dynamics model and focus on the material and information flows of this  
23 production–inventory system at an aggregate level. Differing from the application of stochastic  
24 theory in studying supply chain dynamics, our model is fundamentally deterministic. This is  
25 because we analyse complicated dynamic behaviour, namely the bullwhip effect driven by  
26 feedback loops, nonlinearities and delays, which depend on various *deterministic cause-and-*  
27 *effect relationships* between variables. The analysis derived from a deterministic model can  
28 assist with long-term strategic planning (e.g., capacity planning, labour expansion, inventory

1 holding) and identifies benchmark system dynamics performance for subsequent disaggregate  
2 dynamic modelling and analysis (Lin et al. 2020). We make several general assumptions as  
3 follows.

4 ***Manufacturing process:*** The manufacturing line produces new products if necessary.  
5 Raw materials from qualified suppliers arrive in a just-in-time manner; that is, no raw material  
6 inventory is held. Further, in line with Disney et al. (2016) and Disney et al. (2020), the  
7 manufacturing process has unlimited capacity and an average lead time. In practical terms, we  
8 can analytically assess the capacity unevenness issue identified in many industries, such as the  
9 semiconductor industry (Karabuk and Wu 2003; Lin et al. 2018), which is driven by reactive  
10 dynamics capacity adjustment. That is, managers reactively adjust production capacity because  
11 they can determine maximum capacity requirement, leading to capacity unevenness.

12 ***Stock points and backlog orders:*** The capacity of all stock points is infinite. Demands that  
13 cannot be fulfilled immediately are backordered and backlog orders are presented by the  
14 negative inventory in our dynamic analysis.

15

16 *3.1. System dynamics model*

17 The generalised OUT policy in a continuous time manner is adopted for inventory  
18 replenishment (Kim and Springer 2008; Lin et al. 2020); the equivalent discrete time model  
19 (Dejonckheere et al. 2003; Udenio et al. 2017) can be considered depending on the real-world  
20 production system—that is, according to whether it is a periodic or a continuous review  
21 (Warburton and Disney 2007). The OUT policy is widely implemented in practice (Wang and  
22 Disney 2016; Udenio et al. 2017; Dominguez et al. 2020). All system notations are illustrated  
23 in Table 2.

24

| Notation for system variables |   | Notation for system parameters     |  |
|-------------------------------|---|------------------------------------|--|
| $r(t)$                        | Receipt rate of new products                        | $\tau_l$                           | Manufacturing lead times                                       |
| $d(t)$                        | Customer demand rate                                | $\hat{\tau}_l$                     | Estimated lead times   |
| $\hat{d}(t)$                  | Forecasted demand rate                              | $\tau_a$                           | Forecast smoothing factor                                      |
| $i(t)$                        | On-hand inventory                                   | $\delta$                           | Adjustment speed for proportional OUT                          |
| $i_c(t)$                      | Inventory error correction rate                     | $\beta$                            | Safety stock   |
| $o(t)$                        | Desired order rate                                  | $a$                                | Exponential smoothing coefficient                              |
| $o_a(t)$                      | Actual order rate (under forbidden returns policy)  | <b>Notation for system metrics</b> |  |
| $w(t)$                        | WIP inventory                                       | $o_s$                              | Order peak ratio in responding to a sustained shock demand     |
| $w_r(t)$                      | RPC policy  | $d_v$                              | Demand variance  |
| $w_p(t)$                      | PPC policy  | $o_p$                              | Order variance ratio in responding to periodic/seasonal demand |
| $w_c(t)$                      | Proportional correction of on-hand inventory errors | $i_v$                              | Inventory variance   |
| $r_o(t)$                      | Reorder point                                       | $\omega_n$                         | Natural frequency  |
|                               |   | $\zeta$                            | Damping ratio  |
|                               |   | $BW$                               | Bullwhip effect  |

1 Table 2. Notation for system variables, parameters and metrics

2

3 Specifically, the desired order quantity,  $o(t)$ , placed with a raw material supplier is  
4 determined by the difference between the reorder point,  $r_o(t)$ , and the total inventory  
5 including  $i(t)$  and  $w(t)$ :

$$6 \quad o(t) = r_o(t) - (i(t) + w(t)),$$

7 (1)

8 where  $o(t)$  aims to bring the total system inventory,  $i(t) + w(t)$ , up to  $r_o(t)$ . As illustrated  
9 in Equation (2),  $r_o(t)$  depends on  $\hat{d}(t)$  during the estimated lead time,  $\hat{\tau}_l$ , which determines  
10 the inventory-offset error (Zhou et al. 2017), plus a constant  $\beta$  (e.g., days, weeks' supply),  
11 although other approaches such as setting the function as  $\hat{d}(t)$  can be considered (Springer  
12 and Kim 2010). Thus:

$$13 \quad r_o(t) = \hat{d}(t) \cdot \hat{\tau}_l + \beta, \quad (2)$$

14 where the simple exponential smoothing (SES) forecasting technique is applied for estimating  
15  $\hat{d}(t)$  smoothed by  $\tau_a$ , following Spiegler and Naim (2017) and Lin et al. (2020):

$$16 \quad \frac{d_{\hat{d}(t)}}{dt} = \frac{d(t) - \hat{d}(t)}{\tau_a}. \quad (3)$$

1 It should be noted that more sophisticated methods, such as exponential smoothing with  
 2 additional seasonality (Udenio et al., 2022), may perform better than SES for some pre-  
 3 determined demand patterns, such as deterministic seasonal demand with known seasonality.  
 4 However, SES is a fairly robust method for model selection errors, and its simplicity, intuition,  
 5 small computational effort and ease of application has been well recognised in practice (Disney  
 6 et al. 2006).

7 By substituting Equation (2) into (1) and rearranging it, we obtain:

$$8 \quad o(t) = \hat{d}(t) + \left( \hat{d}(t) \cdot (\hat{\tau}_l - 1) - w(t) \right) + \beta - i(t), \quad (4)$$

9 where  $\hat{d}(t) \cdot (\hat{\tau}_l - 1)$  is the desired pipeline inventory policy. In our continuous time analysis,  
 10  $\hat{d}(t) \cdot (\hat{\tau}_l - 1)$  becomes  $\hat{d}(t) \cdot \hat{\tau}_l$ , as the delay in the order of events is not required in the  
 11 continuous analysis (Disney et al. 2006) and thus Equation (4) can be re-written as:

$$12 \quad o(t) = \hat{d}(t) + \left( \hat{d}(t) \cdot \hat{\tau}_l - w(t) \right) + (\beta - i(t)). \quad (5)$$

13 The proportional controller,  $\delta, \forall \delta \in (0,1)$ , is added to Equation (5) to represent the  
 14 proportional adjustment of the WIP inventory and on-hand inventory errors. Compared with  
 15 the OUT policy—that is,  $\delta = 1$ —this policy is widely recognised as improving system  
 16 dynamics performance (Wang and Disney 2017):

$$17 \quad o(t) = \hat{d}(t) + \delta \cdot \left( \hat{d}(t) \cdot \hat{\tau}_l - w(t) \right) + \delta \cdot (\beta - i(t)), \quad (6)$$

18 where  $\delta \cdot \left( \hat{d}(t) \cdot \hat{\tau}_l - w(t) \right) = w_c(t)$  and  $\delta \cdot (\beta - i(t)) = i_c(t)$  are the proportional  
 19 correction of on-hand and pipeline inventory errors, respectively.  $i(t)$  is the cumulative level  
 20 between  $r(t)$  and  $d(t)$ ; that is, whereas  $d(t)$  depletes  $i(t)$ ,  $r(t)$  replenishes it. Hence:

$$21 \quad \frac{d_{i(t)}}{dt} = r(t) - d(t), \quad (7)$$

22 where  $r(t)$  is delayed  $o(t)$  because of production lead times ( $\tau_l$ ), which are determined by  
 23 the corresponding  $w(t)$  and  $\tau_l$ . A first-order delay with deterministic  $\tau_l$  is assumed (Udenio  
 24 et al. 2017). This represents the production smoothing element that represents the rate at which  
 25 the production unit adapts to changes in  $o(t)$  (Lin et al. 2017):

$$26 \quad \frac{d_{w(t)}}{dt} = o(t) - r(t), r(t) = \frac{w(t)}{\tau_l}. \quad (8)$$

1 *3.2. Returns policy*

2 We focus on two distinct returns policies between the raw material supplier and the  
3 manufacturer in an OUT-based system: (1) free returns and (2) forbidden returns. Specifically,  
4 the traditional linear OUT policy assumes that returns between the supplier and manufacturer  
5 are allowed; that is, a negative value of  $o_t$  is allowed. This policy is well recognised in  
6 research on the bullwhip effect (Chatfield and Pritchard 2013; Dominguez et al. 2015; Lin and  
7 Naim 2019). However, unless there are quality or warranty issues, returns between  
8 manufacturer and supplier are forbidden in many real-world supply chain systems (Wang et al.  
9 2012; Chatfield and Pritchard 2013; Wang et al. 2015). This policy creates a nonlinear OUT  
10 system:

11 
$$\text{Linear free returns: } o(t) = o_a(t) = r_o(t) - (i(t) + w(t)),$$

12 
$$\text{Nonlinear forbidden returns: } o_a(t) = [r_o(t) - (i(t) + w(t))]^+. \quad (9)$$

13 where  $[r_o(t) - (i(t) + w(t))]^+$  is the maximum operator. By truncating negative values, we  
14 can capture the influence of the forbidden returns policy.

15

16 *3.3. Pipeline policy*

17 Pipeline inventory management plays a key role in the bullwhip effect (Springer and Kim  
18 2010). The target pipeline, as a decision variable, can be set in two distinct ways (Sterman  
19 2000), as shown in Figure 1.

20

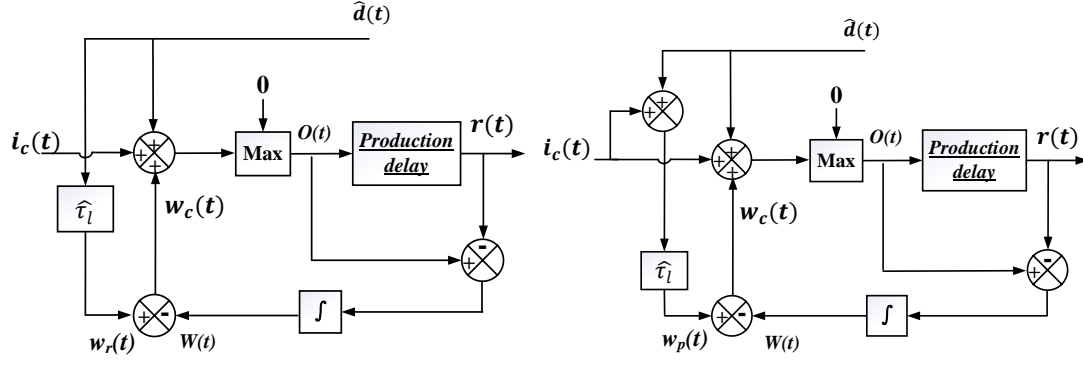


Figure 1a. The RPC policy

Figure 1b. The PPC policy

Figure 1. Block diagram representation of reactive (1a) and proactive (1b) pipeline control policies

First, the production manager at the manufacturer may simply estimate the demand and manufacturing lead times as constants and use the product of these two parameters to determine the desired pipeline. This policy is well recognised in practical OUT-based systems, and we term it as the *Reactive Pipeline Control (RPC)* policy:

$$RPC: w_r(t) = \hat{d}(t) \cdot \hat{\tau}_l. \quad (10)$$

Alternatively, the decision maker may adjust the desired pipeline inventory based on not only estimated demand and lead times but also on on-hand stock fluctuations. In this scenario, a sudden increase in the receipt of the on-hand product above its target inventory may lead the decision maker to reduce the target pipeline stock, although the forecasted demand may remain the same. A lower pipeline stock target would then further lower the order rate for materials. We term this target pipeline setting as the *Proactive Pipeline Control (PPC)* policy:

$$PPC: w_p(t) = [ \hat{d}(t) \cdot \hat{\tau}_l + \delta \cdot (\beta - i(t)) \cdot \hat{\tau}_l ]^+. \quad (11)$$

Note that the maximum operator is added to the dynamic pipeline policy to ensure a non-negative target pipeline; that is, the manufacturer will aim to bring the pipeline inventory to zero if the target level drops below zero. According to these illustrated returns and target pipeline policies, four scenarios for an OUT-based supply chain system can be derived, as shown in Table 3.

| Pipeline policy | Free returns  | Forbidden returns   |
|-----------------|---|---|
| RPC             | $o_t = r_o(t) - (i(t) + w(t))$ $w_r(t) = \hat{d}(t) \cdot \hat{\tau}_l$   | $o_t = [r_o(t) - (i(t) + w(t))]^+$ $w_r(t) = \hat{d}(t) \cdot \hat{\tau}_l$   |
| PPC             | $o_t = r_o(t) - (i(t) + w(t))$ $w_p = [\hat{d}(t) \cdot \hat{\tau}_l + \delta \cdot (\beta - i(t)) \cdot \hat{\tau}_l]^+$ | $o_t = [r_o(t) - (i(t) + w(t))]^+$ $w_p = [\hat{d}(t) \cdot \hat{\tau}_l + \delta \cdot (\beta - i(t)) \cdot \hat{\tau}_l]^+$ |

Table 3. Four scenarios based on different pipeline and returns policies

## 4. Method

### 4.1. Time domain analysis

In the time domain analysis, we consider the transient volatility of the production–inventory system in responding to a shock exogenous demand increase; namely, the Heaviside function,  $d(t) = 0, \forall t < 0, d_t = 1, \forall t \geq 0$ . A demand shock or demand jump (Biçer et al. 2018) is often observed in practice as a cause of supply chain risks, such as the sudden increase in demand for medical care products during the early outbreak period of COVID-19 pandemic (Ivanov 2021). Also, demand shocks can be observed during the early stage of manufacturing innovative products, such as in the case of electric cars and lithium demand shortages (Zhou et al. 2017; Biçer et al. 2018).

Usually, three criteria are used to measure the transient volatility of orders and inventory (Towill et al. 2007): (1) the new equilibrium status of the inventory and order, which is directly linked to the customer service level; (2) the transient order peak overshoot and inventory undershoot for measuring the bullwhip effect cost and inventory backlog orders; and (3) the inventory and order convergence speed and oscillation measured by the natural frequency ( $\omega_n$ ) and the damping ratio ( $\zeta$ ).

### 4.2 Frequency domain analysis

In frequency domain analysis, the deterministic sinusoid, that is,  $d(t) = a \cdot \cos(\omega t) + \beta, \forall \beta, a, \omega \in \mathbb{R}, \beta > a > 0$ , is assumed to be the system demand input.  $a$  is the amplitude of seasonal demand and  $\beta$  is the mean.  $\omega, \forall \omega \in [0, 2\pi]$  is the periodicity of the seasonal

1 component in the demand. For a demand observed every period, the seasonality—defined as  
2 the number of periods within a season—is equal to  $T = \frac{2\pi}{\omega}$  periods. Note that the ‘week’ is  
3 used as the unit for demand cycle time,  $T(\text{weeks}) = \frac{2\pi}{\omega}$ . If the customer demand cycle is  
4 roughly one year,  $T = 52$  weeks; then  $\omega = 0.12$  rads/week.

5 The deterministic sinusoid can represent the seasonal demand (i.e., predictable/seasonally  
6 unadjusted demand data), which is a major source of demand variability (Cachon et al. 2007)  
7 commonly observed in many industries such as fashion (Li et al. 2017) and agri-food (Jonkman  
8 et al. 2019). If the system is linear and time invariant, the steady-state amplification ratio  
9 (AR)—denoted by the ratio between the amplitude of orders and the amplitude of demand—  
10 can be used to measure the dynamic performance of the system (Dejonckheere et al. 2003). We  
11 denote  $o_p(\omega) = \frac{\text{amplitude of orders}}{\text{amplitude of demand}}$  as the AR measure.  $o_p(\omega)$  is a powerful measure of  
12 system dynamics performance because it allows us to detect whether, and to what extent, a  
13 replenishment policy will lead to the order amplification for sinusoid demand with a particular  
14 demand frequency (Udenio et al. 2017). In particular, a robust system can be designed by  
15 exploring the worst-case scenario—that is, the maximum AR (Udenio et al. 2022)—which is  
16 well accepted as a bullwhip measure in frequency domain analysis (Towill et al. 2007; Udenio  
17 et al. 2022)

18 Although real demand patterns are rarely perfectly sinusoid, our interest in the equilibrium  
19 performance is not restricted to the expectation of a sinusoidal demand. This is because any  
20 demand stream can be decomposed into a sum of sinusoids; hence, analysing the relevant  
21 frequency response (FR) plots (i.e., the graphical representation of  $o_p$  as a function of the  
22 demand harmonics with frequencies) provides a preliminary understanding of system  
23 performance with regard to any arbitrary demand pattern based on the amplitude of its  
24 constituent harmonics (Dejonckheere et al. 2003).

25 Another reason for adopting  $o_p(\omega)$  is its close relationship to the bullwhip effect  
26 measure if i.i.d stochastic demand is assumed in a time domain analysis. This means that if the  
27 input of a system consists of a Gaussian stream with zero mean and unity variance, the

1 bullwhip—defined as the ratio of demand variance to order variance (Lee et al. 1997)—is  
 2 proportional to the ‘noise bandwidth’ of the system; that is, the square of the area below its FR  
 3 plot, formally  $BE = \frac{1}{\pi} \int_0^\pi [o_p(\omega)]^2$  (Udenio et al. 2017; Udenio et al. 2022).

4 However, linear techniques are no longer valid in a nonlinear supply chain system in which  
 5 returns between supplier and customer are forbidden, as described in Equation (9). Given that  
 6 such nonlinearity is characterised by discontinuous piecewise linear functions, the DF can be  
 7 applied to analyse the bullwhip effect (Wang et al. 2015; Spiegler and Naim 2017). DF is a  
 8 quasi-linear representation of a nonlinear element subjected to specific input signal forms such  
 9 as bias, sinusoid and a Gaussian process (Spiegler and Naim 2017). Specifically, for a given  
 10 sinusoid demand as input,  $d(t) = A \cdot \cos(\omega t) + B$ , the output  $o_t$  can be approximated as  
 11 follows:

$$o(t) \approx N_A \cdot A \cdot \cos(\omega t + \phi) + N_B \cdot B, \quad (12)$$

13 where  $N_A$  is the amplitude gain,  $N_B$  is the mean gain and  $\phi$  is the phase shift. The objective  
 14 of DF method is to replace the nonlinear component by a gain derived from the effect of the  
 15 input (e.g., sinusoidal input). The Fourier series expansion can be applied to obtain the terms  
 16 of the DF ( $N_A$ ,  $N_B$  and  $\phi$ ):

$$\begin{aligned} o(t) &\approx b_0 + a_1 \cdot \cos(\omega t) + b_1 \cdot \sin(\omega t) + a_2 \cdot \cos(2\omega t) + b_2 \cdot \sin(2\omega t) + \dots \\ &\approx b_0 + \sum_{n=1}^{\infty} (a_n \cdot \cos(n\omega t) + b_n \cdot \sin(n\omega t)), \end{aligned} \quad (13)$$

19 where the Fourier coefficient can be determined by:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} o(t) \cdot \cos(n\omega t) d_{\omega t}, b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} o(t) \cdot \sin(n\omega t) d_{\omega t}, b_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} o(t) \cdot d_{\omega t}. \quad (14)$$

21 To approximate a periodic series, only the first or fundamental harmonic is needed; thus,  
 22 we need to find the first-order coefficient of the Fourier series expansion demonstrated in  
 23 Equation (13):

$$o(t) \approx b_0 + a_1 \cdot \cos(\omega t) + b_1 \cdot \sin(\omega t) = b_0 + \sqrt{a_1^2 + b_1^2} \cos(\omega t + \phi). \quad (15)$$

25 By comparing Equations (15) and (12), we can obtain the gains of the DF as follows:

$$N_A = \frac{\sqrt{a_1^2 + b_1^2}}{A}, N_B = \frac{b_0}{B}, \phi = \arctan\left(\frac{b_1}{a_1}\right). \quad (16)$$

1 In other words, given the sinusoidal input, the output of discontinuous nonlinearity can be  
2 approximated, not only as a function of the inherent system structure and policies, but also as a  
3 function of input properties including amplitude, mean and frequency.

#### 4 5 **5. Analytical results**

6 In this section, we analytically explore the dynamic performance of the OUT system under RPC  
7 and PPC policies. Inventory equilibrium, order and inventory convergency speed and  
8 oscillations, as the performance metrics, are used to measure the dynamic performance of the  
9 system. Regarding the transient order peak (i.e., the bullwhip effect) as analysed in Springer  
10 and Kim (2010), please refer to Appendix B1 for details. Furthermore, following Spiegler et al.  
11 (2016) and Lin and Naim (2019), we assume returns are allowed, to test the system's transient  
12 response under unrestricted conditions. This helps decision makers to understand the resulting  
13 performance of key metrics, e.g. unrestricted bullwhip (peak order), unrestricted inventory  
14 undershoot and oscillation zone, and therefore corresponding capacity (e.g. outsourcing  
15 strategy) and safety stock strategies can be implemented. Note that we consider forbidden  
16 returns in the seasonal demand scenario, because frequent return requests between a supplier  
17 and customers may be observed, given demand fluctuations.

##### 18 19 *5.1. Time domain analysis of a free returns system*

20 Recall Equation (2) in which the reorder point is set as the function of the estimated lead time.  
21  $\hat{\tau}_l$  is assumed to be equal to the actual production lead times to avoid inventory drift; that is,  
22 from a long-term perspective, the phenomenon that inventory levels do not reach the targeted  
23 equilibrium when a sustained shock demand has occurred. We derive an analytical expression  
24 for the new inventory equilibrium via the following proposition:

25 ***Proposition 1:*** *For a supply chain following the proportional OUT replenishment policy,*  
26 *when such a system, initially in equilibrium, is disturbed by a sudden but sustained positive unit*  
27 *step demand shock, the new equilibria of inventory for RPC and PPC are quantified` by:*

$$1 \quad i^{RPC}(t)_{(new)} = (\hat{\tau}_l - \tau_l), \quad (17)$$

$$2 \quad i^{PPC}(t)_{(new)} = \frac{(\hat{\tau}_l - \tau_l)}{1 + \delta \hat{\tau}_l}. \quad (18)$$

3 *Proof.* See Appendix A1

4 It is clear that, for both RPC and PPC systems, the inventory drift—the permanent  
 5 inventory error from the target inventory ( $\beta = 0$ )—occurs if  $\hat{\tau}_l \neq \tau_l$ . A positive value of  $\hat{\tau}_l -$   
 6  $\tau_l$ , driven by the overestimation of actual production lead time, will result in excess inventory  
 7 and increased inventory holding costs. Conversely, an underestimation in lead times ( $\hat{\tau}_l - \tau_l <$   
 8  $0$ ) negatively impacts customer service levels due to possible stock-out issues. For the RPC  
 9 system—that is, Equation (17)—the result is well known, as per Disney and Towill (2005).

10 However, interestingly, comparing Equations (17) and (18), an additional  $(1 + \delta \hat{\tau}_l)$  is  
 11 incorporated in the denominator of the PPC system. This indicates that the inventory drift for  
 12 the PPC system, if  $\hat{\tau}_l \neq \tau_l$ , is completely different from that for the RPC system. If an  
 13 overestimation of actual production lead time occurs ( $\hat{\tau}_l - \tau_l > 0$ ), the PPC system always  
 14 generates less inventory drift than that in the RPC system, while the opposite result can be  
 15 obtained if  $\hat{\tau}_l - \tau_l < 0$ . Furthermore, inspection of Equation (18) shows that the proportional  
 16 controller,  $\delta$ , negatively affects the final inventory drift if  $\hat{\tau}_l \neq \tau_l$ . This means the traditional  
 17 OUT policy ( $\delta = 1$ ) may increase the inventory holding cost and reduce the customer service  
 18 level in the PPC system.

19 To assess the convergence speed and oscillation level of the supply chain system, we  
 20 evaluate  $\omega_n$  and  $\zeta$  under RPC and PPC policies via following proposition:

21 **Proposition 2.1.** *For a supply chain system initially in equilibrium that is disturbed by a*  
 22 *sudden but sustained unit step demand shock, if the system is controlled by an OUT with RPC*  
 23 *policy (i.e., the RPC system),  $\omega_n$  and  $\zeta$  of orders and inventory are determined by:*

$$24 \quad \omega_n(\text{order}) = \sqrt{\frac{\delta}{\tau_a}}; \zeta(\text{order}) = \frac{1}{2} \sqrt{\frac{1}{\delta \tau_a} + \delta \tau_a} + 2, \quad (19)$$

$$25 \quad \omega_n(\text{inventory}) = \sqrt{\frac{\delta}{\tau_l}}; \zeta(\text{inventory}) = \frac{1}{2} \sqrt{\frac{1}{\delta \tau_l} + \delta \tau_l} + 2. \quad (20)$$

1        **Proposition 2.2.** *If such a system is controlled by an OUT with PPC policy (i.e., the PPC*  
 2 *system),  $\omega_n$  and  $\zeta$  of orders and inventory are identical and can be determined by:*

$$3 \quad a) \omega_n (\hat{\tau}_l \neq \tau_l) = \sqrt{\frac{\delta + \delta^2 \hat{\tau}_l}{\tau_l}}; \zeta (\hat{\tau}_l \neq \tau_l) = \frac{(1 + \delta \tau_l)}{2} \sqrt{\frac{1}{\delta \tau_l (1 + \delta \hat{\tau}_l)}}, \quad (21)$$

$$4 \quad b) \omega_n (\hat{\tau}_l = \tau_l) = \sqrt{\frac{\delta}{\tau_l} + \delta^2}; \zeta (\hat{\tau}_l = \tau_l) = \frac{1}{2} \sqrt{\frac{1}{\delta \tau_l} + 1}. \quad (22)$$

5        *Proof.* See Appendix A2.

6        There are several observations for *Proposition 2.1*. First, from Equation (19),  $\omega_n$  increases  
 7 in  $\delta$  and decreases in  $\tau_a$ , indicating that the quick forecasting adjustment or the adjustment  
 8 of inventory error can increase order convergence speed. When either  $\delta$  or  $\tau_a$  increases,  $\zeta$   
 9 increases as well, which results in a decline in the number of oscillations. Note that  $\zeta \geq 1$  is  
 10 satisfied for all positive values of  $\delta$  and  $\tau_a$ , which means that orders in the RPC system  
 11 always produce overdamped behaviour and are guaranteed to be stable. Regarding inventory  
 12 performance shown in Equation (20), the inventory recovery speed increases as  $\tau_l$  decreases  
 13 or  $\delta$  increases, suggesting a short lead time or a quick inventory error adjustment can bring  
 14 inventory up to target quickly (Lin et al. 2018).

15        *Proposition 2.2* suggests that estimated production lead times ( $\hat{\tau}_l$ ) play an important role  
 16 in influencing the convergence speed and oscillatory behaviour of the PPC system. Equation  
 17 (21) shows that  $\hat{\tau}_l > \tau_l$  leads to increased  $\omega_n$  but decreased  $\zeta$ , while the opposite results can  
 18 be derived if  $\hat{\tau}_l < \tau_l$ . This finding is consistent with empirical and experimental findings  
 19 (Sterman 1989; Croson and Donohue 2006) that the inaccurate estimation of lead times—such  
 20 as overestimation of  $\hat{\tau}_l$ —will result in high convergence speed to the new equilibrium at the  
 21 expense of an increase in the number of oscillations. Furthermore, if  $\hat{\tau}_l = \tau_l$  can be achieved  
 22 (Disney and Towill 2005), the impact of  $\delta$  on  $\omega_n$  and  $\zeta$  is similar to that in the  
 23 corresponding RPC system, whereas the increase of  $\tau_l$  may decrease  $\omega_n$  and  $\zeta$ , leading to  
 24 poor dynamic behaviour (Lin et al. 2017).

25        Last, by inspecting Equations (20) and (22) and assuming  $\hat{\tau}_l = \tau_l$ , we have the following  
 26 properties to compare the dynamics of order and inventory under different pipeline policies:

1        **Property 1.** Under identical system policy settings, the convergence speed of orders in the  
2        PPC system exceeds that in the RPC system if  $\tau_a < \frac{\tau_l}{(1+\delta\tau_l)}$ . The PPC-based system may  
3        produce more oscillations than the RPC-based system if  $\tau_l > \frac{\tau_a}{(1+\delta\tau_a+\delta\tau_a^2)}$ .

4        **Property 2:** Under identical system policy settings, the convergence speed of inventory in  
5        the PPC system always exceeds that in the RPC system, whereas the PPC-based system may  
6        produce more oscillatory behaviour than the RPC-based system.

7        From *Properties 1* and *2*, PPC policy can yield quick convergence at the expense of  
8        possible additional unwanted oscillation. Notably, the RPC system cannot generate oscillatory  
9        behaviour as  $\zeta > 1$ , whereas the PPC system can avoid oscillation only if  $\delta$  is small ( $\delta > \frac{1}{3\tau_l}$ ),  
10        which is not a realistic setting in a production environment with a long physical lead time. Thus,  
11        managers need to consider system trade-off design by using the analytical expressions of  $\omega_n$   
12        and  $\zeta$  to predict the system transient behaviour and the corresponding cost of the supply chain  
13        dynamics when expecting a customer demand shock.

14

## 15        5.2. Frequency domain analysis of a forbidden returns system

16        If demand is characterised by periodic or similar seasonal patterns, a negative value of  $o(t)$   
17        (i.e., the manufacturer's desire to return excess raw material to the supplier) can be frequently  
18        observed (Wikner et al. 2017). In this section, we assess the effects of two pipeline control  
19        policies on dynamic performance under forbidden returns policies.

20        First, we introduce the *Proposition 3.1* to compare the amplitude ratio in free- and  
21        forbidden returns supply chains:

22        **Proposition 3.1.** For a supply chain controlled by proportional RPC- and PPC-OUT  
23        policies in responding  $d(t) = A \cdot \cos(\omega t) + B, \forall B, A, \omega \in \mathbb{R}, B > A > 0$ , the amplitude of  
24         $o_t$  ( $A_{o(t)}$ ) will grow exponentially under the linear forbidden returns policy and can be  
25        measured by:

$$26 \quad A_{o(t)}(RPC) = A \cdot \sqrt{\frac{\delta^2 + (\omega(1 + \delta\tau_a + \delta\tau_l))^2}{(\omega^2 + \delta^2)(1 + \omega^2\tau_a^2)}}, \quad (23)$$

$$A_{o(t)}(PPC) = A \cdot (1 + \delta\tau_l) \sqrt{\frac{(\omega^2(1 + \delta\tau_a)^2 + \delta^2)(1 + \omega^2\tau_l^2)}{(1 + \omega^2\tau_a^2)(\omega^2 + \delta^2 + 2\delta^3\tau_l + (\omega^4 - \omega^2\delta^2 + \delta^4)\tau_l^2)}}. \quad (24)$$

**Proposition 3.2.**  $A_{o(t)}$  is stabilised and bounded in the proportional RPC- and PPC-OUT system with forbidden returns policy. The order amplitude ratio can be approximated as:

$$o_p(\text{forbidden returns}) \approx \frac{N_A \cdot A_{o(t)}}{A}, \quad (25)$$

$$N_A = \frac{\frac{B \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2}}}{A_{o(t)}} + \text{Cos}^{-1}\left(-\frac{B}{A_{o(t)}}\right)}{\pi}, \quad (26)$$

where  $N_A$  is the amplitude gain based on the DF method.

*Proof.* See Appendix A3.

*Proposition 3.1* suggests that the selection of order pipeline policies to avoid high amplitude ratio is determined by the inherent system structure ( $\tau_l$ ), system control parameters ( $\delta$  and  $\tau_a$ ) and external demand characteristics  $\omega_c$ . By obtaining the first-order derivative of  $A_{o(t)}(RPC)$  and  $A_{o(t)}(PPC)$  with respect to  $\tau_l$  and  $\omega$ , the order amplitude ratios controlled by the RPC and by PPC policies are monotonously increasing in  $\tau_l$  and decreasing in  $\omega$ , suggesting that the amplitude ratio increases as the physical lead times increase, and decreases as the demand frequency increases.

Also, by differentiating  $N_A$  with respect to  $o(t)$  and  $\beta$  in Equation (26), we show that

$$\text{the value of } N_A \text{ decreases as } o(t) \text{ increases and } \lim_{o(t) \rightarrow \infty} \frac{\frac{B \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2}}}{A_{o(t)}} + \text{Cos}^{-1}\left(-\frac{B}{A_{o(t)}}\right)}{\pi} = \frac{1}{2}; \text{ that is,}$$

$N_A \in (\frac{1}{2}, 1]$ . Moreover,  $N_A$  increases as  $B$  increases. Two corresponding managerial implications can be identified. First, the manufacturer's order variance tends to be stabilised and eventually bounded as half of the variance of customer demand under the forbidden returns policy. Although it is unlikely that  $A_{o(t)}$  will reach the infinite level in our two-stage downstream supply chain (i.e., a manufacturer and a customer), it is clear that the bullwhip effect can be alleviated in the forbidden returns system regardless of the desired pipeline control policy.

1 We can conclude that this effect may not exist if (1) the average customer demand is  
 2 sufficiently low and (2) the frequency of the customer demand fluctuation is sufficiently high.  
 3 Correspondingly, managers may adopt a bullwhip effect avoidance strategy by reducing the  
 4 average demand per order or increasing the customer purchase frequency. It should be noted  
 5 that Equations (23), (24) and (26) form a complete analytical expression in predicting the  
 6 bullwhip effect in the forbidden returns supply chain system when both RPC and PPC control  
 7 policies are considered. Finally, we note the following property to approximate the average  
 8 level of orders and inventory under the forbidden returns policy.

9 **Property 3.** *Compared with the linear forbidden returns system, the forbidden returns*  
 10 *policy leads to an increase in the mean of  $o(t)$  and  $i(t)$ . The new mean ( $B_n$ ) can be measured*  
 11 *by:*

$$12 \quad B_n = N_B \cdot B = \frac{A_{o(t)} \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2} + \beta \cdot \cos^{-1}\left(-\frac{B}{A_{o(t)}}\right)}}{\pi}. \quad (27)$$

13 *Proof.* We know that the equilibrium of order in the linear system eventually equals the  
 14 mean of demand in the long-term steady state condition. However, owing to forbidden returns,  
 15 from Equation (12), we know the mean of order is determined by the product of mean gain  
 16 ( $N_B$ ) and mean of demand ( $B$ ).  $N_B$ , based on the DF method in the proof of *Proposition 3*

17 (Appendix A3), can be obtained as follows:  $N_B = \frac{A_{o(t)} \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2} + \beta \cdot \cos^{-1}\left(-\frac{B}{A_{o(t)}}\right)}}{B \cdot \pi}$ .

18 It is easy to see  $N_B \in [1, \infty)$  as  $A_{o(t)}$  increases, which means the equilibrium of order is  
 19 higher than that in the linear system under the steady-state condition. The mean of  $i(t)$  will  
 20 be higher than that in the linear system owing to the increased mean of orders.

21 Furthermore, we introduce the following proposition to evaluate the dynamic oscillations  
 22 and recovery in forbidden returns supply chain systems:

23 **Proposition 4.** *When a forbidden returns supply chain system that is initially in*  
 24 *equilibrium and is then disturbed by a sustained shock plus periodic demand adopts*

1 (1) proportional OUT replenishment with the RPC policy, the inventory and order of  $\omega_n$   
 2 and  $\zeta$  are identical and are determined by:

$$3 \quad \omega_n = \sqrt{\frac{N_A \delta}{\tau_l}} \quad \zeta = \frac{1}{2} \sqrt{\frac{1}{N_A \delta \tau_l} + N_A \delta \tau_l + 2}, \quad (28)$$

4 (2) proportional OUT replenishment with the PPC policy, the inventory and order of  $\omega_n$   
 5 and  $\zeta$  are identical and are determined by:

$$6 \quad \omega_{n(\hat{\tau}_l \neq \tau_l)} = \sqrt{\frac{N_A(\delta + \delta^2 \hat{\tau}_l)}{\tau_l}} \quad \zeta(\hat{\tau}_l \neq \tau_l) = \frac{(1 + N_A \delta \tau_l)}{2} \sqrt{\frac{1}{N_A \delta \tau_l (1 + \delta \hat{\tau}_l)}}, \quad (29)$$

$$7 \quad \omega_{n(\hat{\tau}_l = \tau_l)} = \sqrt{N_A \left( \frac{\delta}{\tau_l} + \delta^2 \right)} \quad \zeta(\hat{\tau}_l = \tau_l) = \frac{1}{2} \sqrt{\frac{1}{N_A \delta \tau_l} + 1} \quad (30)$$

8 where  $N_A = \frac{B \cdot \sqrt{1 - \frac{B^2}{A_o(t)^2}} + \cos^{-1}\left(-\frac{B}{A_o(t)}\right)}{\pi} \in \left(\frac{1}{2}, 1\right]$  is the amplitude gain based on DF  
 9 approximation.

10 *Proof.* Please see Appendix A5

11 There are several insights. First, similar to the linear system result, the convergence speed  
 12 generated by the PPC-based system always exceeds that generated by the RPC-based system,  
 13 at the expense of producing more oscillatory behaviour in a nonlinear forbidden returns system.

14 Second, by comparing linear analytical results, i.e. Equations (19)–(22), it is easy to find

15  $\omega_n(\text{nonlinear}) < \omega_n(\text{linear})$  for both RPC- and PPC-based systems. These are  $\sqrt{\frac{N_A \delta}{\tau_l}} <$

16  $\sqrt{\frac{\delta}{\tau_l}}$  and  $\sqrt{N_A \left( \frac{\delta}{\tau_l} + \delta^2 \right)} < \sqrt{\frac{\delta}{\tau_l} + \delta^2} \quad \forall N_A \in \left(\frac{1}{2}, 1\right]$ . This means that the incorporation of a

17 forbidden returns policy leads to a slow convergence speed, e.g. a slower inventory recovery

18 speed, comparing to the linear forbidden returns system. Furthermore, by inspecting  $\zeta$ , we can

19 conclude that  $\zeta(\text{nonlinear}) > \zeta(\text{linear})$  for both policies. That is,  $\zeta =$

20  $\frac{1}{2} \sqrt{\frac{1}{N_A \delta \tau_l} + N_A \delta \tau_l + 2} > \zeta = \frac{1}{2} \sqrt{\frac{1}{\delta \tau_l} + \delta \tau_l + 2}$  and  $\frac{1}{2} \sqrt{\frac{1}{N_A \delta \tau_l} + 1} > \frac{1}{2} \sqrt{\frac{1}{\delta \tau_l} + 1} \quad \forall N_A =$

21  $\left(\frac{1}{2}, 1\right]$ . This result implies that the forbidden returns policy can mitigate oscillations to a greater

22 extent than the linear forbidden returns system. Last, regarding the nonlinear forbidden returns

23 system, both  $\omega_n$  and  $\zeta$  increase as  $N_A$  or  $a_i$  increase. This result suggests that a decrease in

1 the customer order amplitude not only reduces the cost of the bullwhip effect (Dominguez et al.  
2 2015; Lin and Naim 2019), but can also improve the transient dynamic performance of the  
3 system. Using these analytical results, supply chain planning managers may carefully evaluate  
4 different customer demand characteristics to determine the best coordination strategy with their  
5 suppliers.

6

## 7 **6. Numerical study**

8 In this section, we conduct extensive numerical experiments (Matlab) to further explore the  
9 dynamics of the proportional OUT system under RPC and PPC policies. Descriptions of the  
10 experimental design are presented in Table 4. As introduced in Section 3, convergence speed,  
11 oscillations, order peak and order amplitude ratio are used as performance metrics in  
12 simulations. Four experimental factors, including proportional controller ( $\delta$ ), exponential  
13 smoothing factor ( $\tau_a$ ), system lead times ( $\tau_l$ ) and demand frequencies ( $\omega$ ) cover the main  
14 system parameters in the OUT policy. The baseline system parameter settings follow  $\tau_a =$   
15  $2\tau_l = 8$  (Disney et al. 2000), while we vary  $\tau_a$  and  $\tau_l$  between 2 and 32 to understand the  
16 impact of different smoothing level and lead times on the dynamics of RPC and PPC systems.  
17 The baseline for the proportional controller,  $\delta$ , is set as 0.5, while its value ranges between 0.1  
18 and 0.9 to explore the influence of slow and fast inventory correction speed on dynamic  
19 performance. Furthermore, three demand frequencies, 0.1, 0.5 and 0.9 rad/week with 0.5  
20 rad/week as baseline are chosen to represent the different types of product characterised by low,  
21 medium and high demand frequencies, following Lin et al. (2022). Given that we have four  
22 experimental factors, three with five levels and one with three levels, the total number of  
23 experiments is 78.

24

| <b>Performance measure</b>  |                       |
|---|-----------------------|
| Convergence speed   | $\omega_n$            |
| Oscillation   | $\zeta$               |
| Peak step response (Bullwhip)   | $o_s$                 |
| Order amplitude ratio   | $o_p$                 |
| <b>Experimental factor</b>  |                       |
| $\delta$  | 0.1, 0.3, 0.5, 0.7, 1 |
| $\tau_a$  | 2, 4, 8, 16, 32       |
| $\tau_l$  | 2, 4, 8, 16, 32       |
| $\omega$ (rad/week)   | 0.1, 0.5, 0.9         |
| <b>System parameter baseline setting</b>                                    |                       |
| $\tau_a = 8, \hat{\tau}_l = \tau_l = 4,$                                    |                       |
| $\delta = 0.5, \omega = 0.1 \text{ rad/week}$                               |                       |
| <b>System input setting (Demand)</b>  |                       |
| Seasonal demand: $d(t) = \cos(\omega t) + 1,$                               |                       |
| Step demand: $d(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ |                       |

1 Table 4. Experimental design

2 *6.1 Verification*

3 We verify the analytical results for AR in the nonlinear forbidden returns system  
4 (*Proposition 3*) shown in Table 5. In general, our analytical results are reasonably accurate,  
5 although some differences between simulation and analytical results can be observed because  
6 the AR in the nonlinear system is an approximation based on the DF method.

7

| <b>Amplitude ratio under seasonal demand (nonlinear forbidden returns system)</b> |              |               |              |               |          |              |               |              |               |
|---|--------------|---------------|--------------|---------------|----------|--------------|---------------|--------------|---------------|
| $\delta$  | RPC<br>(Ana) | RPC<br>(Simu) | PPC<br>(Ana) | PPC<br>(Simu) | $\tau_a$ | RPC<br>(Ana) | RPC<br>(Simu) | PPC<br>(Ana) | PPC<br>(Simu) |
| 0.1   | 1.23         | 1.14          | 1.29         | 1.19          | 2        | 1.17         | 1.11          | 1.11         | 1.07          |
| 0.3   | 1.25         | 1.18          | 1.17         | 1.13          | 4        | 1.2          | 1.14          | 1.13         | 1.09          |
| 0.5   | 1.21         | 1.16          | 1.15         | 1.11          | 8        | 1.21         | 1.16          | 1.15         | 1.11          |
| 0.7   | 1.19         | 1.15          | 1.12         | 1.07          | 16       | 1.18         | 1.13          | 1.14         | 1.09          |
| 0.9   | 1.18         | 1.13          | 1.11         | 1.06          | 32       | 1.12         | 1.07          | 1.11         | 1.07          |

8 Table 5. Simulation verification for bullwhip under shock and seasonal demand  
9 (Ana: analytical result; Simu: simulation result)

10

11

1 6.2. Amplitude ratio and order peak analysis

2 In this section, we systematically assess the effects of demand frequency ( $\omega$ ) on AR under  
3 seasonal demand, and the impact of forecasting policy ( $\tau_a$ ), physical lead times ( $\tau_l$ ) and the  
4 proportional controller ( $\delta$ ) on order peak under shock demand.

5 Figure 2 reports the simulation results. Specifically, Figure 2a plots AR as the function of  
6 demand frequency in response to seasonal demand. The figure shows that this effect has a  
7 concave U-shaped relationship with demand frequency in both the RPC and PPC systems, in  
8 which the AR level increases as the demand frequency increases and reaches peak level around  
9  $\omega = 0.3\text{rad/week}$ . Notably, on comparing the PPC- and RPC-based systems, we find that the  
10 difference in AR is small for the low-demand frequency range—that is, 0.1–0.3 rad/week—  
11 whereas the PPC system generates significantly higher bullwhip effect than the RPC system for  
12 the media and high-demand spectrum.

13 Figure 2b presents the impact of  $\delta$  on order peak; i.e. the bullwhip effect measure under  
14 shock demand. Specifically, an increase in the inventory proportional controller,  $\delta$ , increases  
15 the bullwhip effect in both the PPC and RPC systems. In particular, this effect significantly  
16 increases as  $\delta$  increases in the PPC-based system, which suggests that the RPC strategy should  
17 be adopted if the inventory error should be corrected quickly in response to a shock demand.  
18 However, the bullwhip effect decreases as  $\delta$  increases in both the PPC- and RPC-based  
19 nonlinear systems, due to the incorporation of the nonlinear forbidden returns policy in which  
20 the increases of desired order amplitude (because of an increase in  $\delta_t$ ) reduce the value of  $N_a$ ,  
21 leading to decreases in the bullwhip effect level. In other words, the forbidden returns policy  
22 contributes to reducing the bullwhip effect (Dominguez et al. 2015).

23 Figure 2c shows the impact of lead times on order peak, or bullwhip. As expected, an  
24 increase in lead times increases the bullwhip effect for both systems, consistent with previous  
25 RPC-related studies (Chen et al. 2000; Towill et al. 2007). Also, as derived in *Proposition 3*, if  
26 the PPC policy is adopted, the increase of bullwhip effect is much more significant than in the  
27 RPC-based system as the lead times increase. This finding, similarly observed by Springer and

1 Kim (2010) and Lin et al. (2018), suggests that to avoid a strong bullwhip effect, a PPC strategy  
2 should not be considered for systems with long lead times.

3 Last, the impact of  $\tau_a$  on the bullwhip effect is shown in Figure 2d. Increases in  $\tau_a$  lead  
4 to decreases in the bullwhip effect in both linear RPC and PPC systems. This finding is  
5 consistent with that of previous studies (Chen et al. 2000; Zhang 2004; Lin et al. 2017; Lin and  
6 Naim 2019), showing that a gradual adjustment to forecasts can reduce the bullwhip effect,  
7 although adopting this strategy may reduce the inventory convergence speed. However, if  
8 nonlinear forbidden returns are considered, the impact of  $\tau_a$  on the bullwhip effect is very  
9 limited, although a concave U-shaped relationship with demand frequency is seen for the  
10 nonlinear RPC and PPC systems.

11

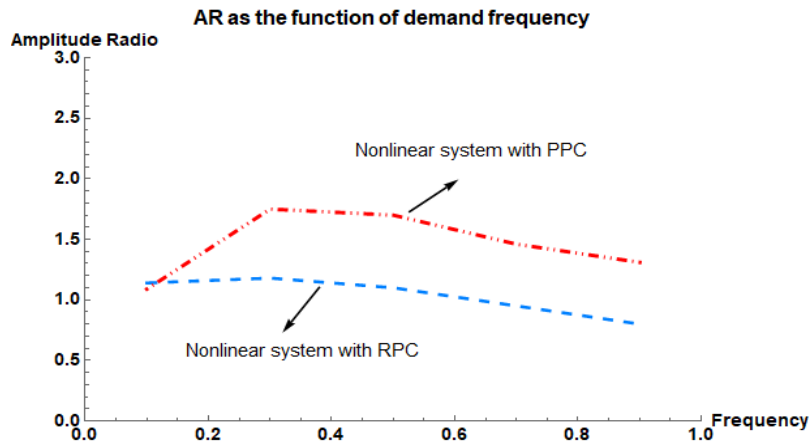


Figure 2a. Impact of  $\omega$  on amplitude ratio

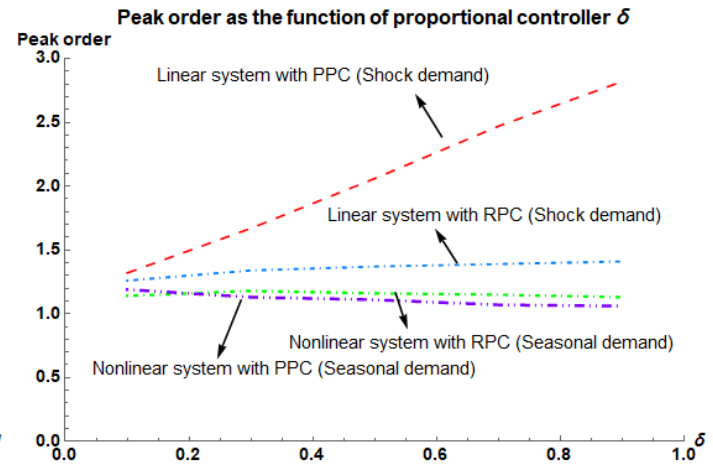


Figure 2b. Impact of  $\delta$  on order peak

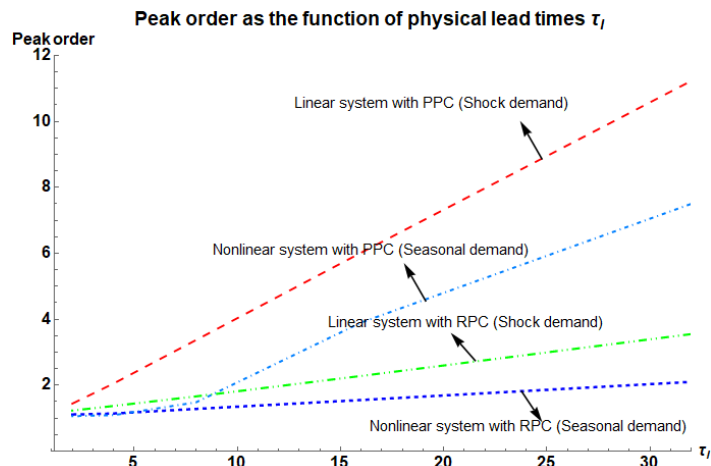


Figure 2c. Impact of  $\tau_l$  on order peak

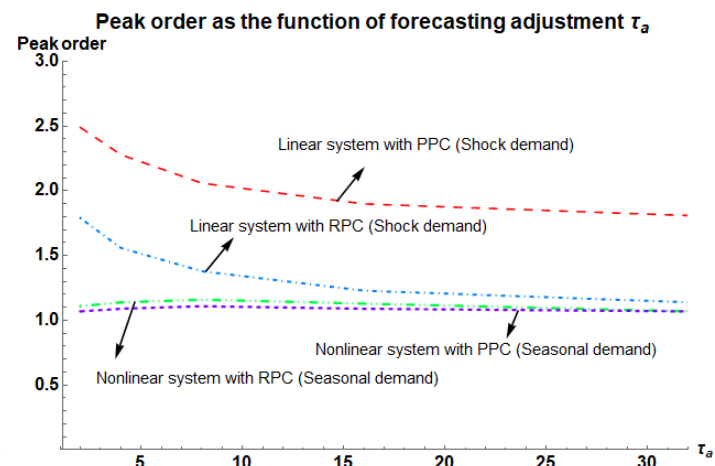


Figure 2d. Impact of  $\tau_a$  on order peak

Figure 2. Amplitude ratio and order peak as a function of  $\omega$ ,  $\tau_a$ ,  $\tau_l$  and  $\delta$  in response to shock and seasonal demand under RPC and PPC policies

### 6.3. Inventory dynamic analysis for step demand

In this section, we explore the dynamic performance of finished goods inventory under the RPC and PPC policies. Figures 3a and 3b reported the impact of  $\delta$  on  $i(t)$ . Overall, the inventory convergence speed increases for both RPC- and PPC-based systems with an increase in  $\delta$ . Moreover, the PPC-based system always *outperforms* the RPC-based system with lower undershoot and higher inventory convergence speed for the same  $\delta$ . This result verifies *Proposition 2*. Furthermore, for both RPC- and PPC-based systems, the dynamic performance of finished goods inventory can be dramatically improved from  $\delta = 0.1$  to  $\delta = 0.3$ , whereas the improvement is limited if  $\delta$  is increased further. This result suggests that a slow proportional correction speed is undesirable in the proportional OUT system because of the high stock-out possibility and slow recovery speed.

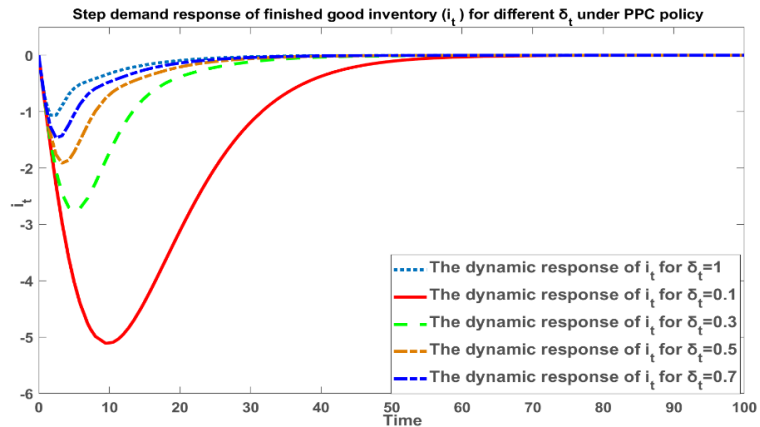
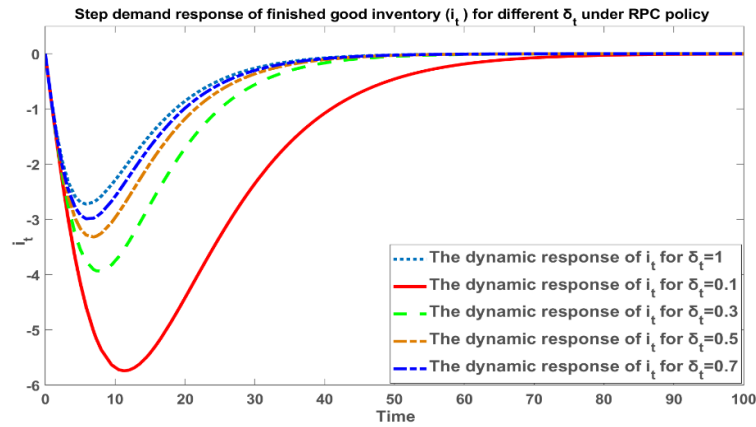
Also, an increase in  $\tau_a$  reduces inventory convergence speed, as shown in Figures 3c and 3d. That is, a greater weight on historical forecasting than on current demand leads to a decrease in the system speed response to a sudden demand change. This is consistent with Chen et al. (2000), Zhang (2004), Lin et al. (2018) and Lin and Naim (2019). However, for the same smoothing coefficient, the system response speed with the PPC policy is significantly greater than that with the RPC-based system. In particular, the system with the PPC policy generates oscillations for  $\tau_a = 2$ , but no oscillations can be identified for the RPC-based system. These results verify prior analytical (Springer and Kim 2010; Lin et al. 2018) and empirical results (Sterman 2000) showing that incorporation of on-hand stock fluctuation feedback, as part of the target order pipeline settings, generates a faster system response than the method that only forecast and lead times are considered target pipeline settings.

Finally, Figures 3e and 3f show the impact of  $\tau_l$  on the dynamics of  $i(t)$ . It can be seen that the PPC-based system significantly outperforms the corresponding RPC system in terms of recovery speed and the undershoot of finished goods inventory. This result verifies *Proposition 2* that  $\omega_n(PPC)\sqrt{\frac{\delta}{\tau_l} + \delta_t^2}$  is always larger than  $\omega_n(RPC) = \sqrt{\frac{\delta}{\tau_l}}$ . This is particularly the case when lead times are long (large values of  $\tau_l$ ). Furthermore, Figure 3f

1 shows  $\tau_l$  has a limited role in influencing convergence speed in the PPC-based system

2 (recall  $\omega_n(PPC) = \sqrt{\frac{\delta}{\tau_l} + \delta_t^2}$ ).

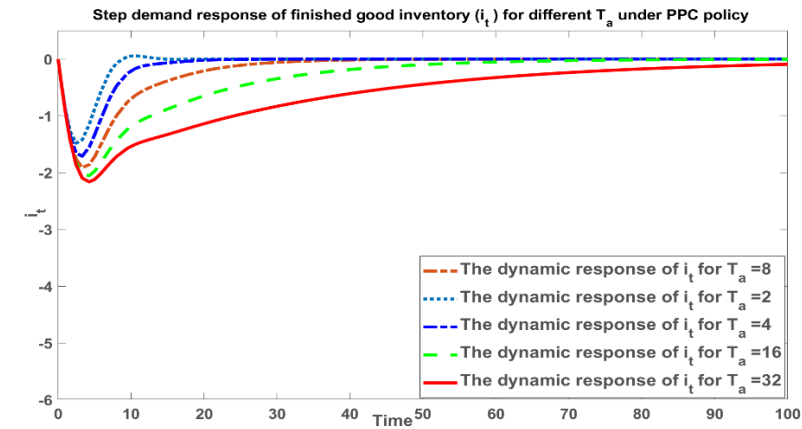
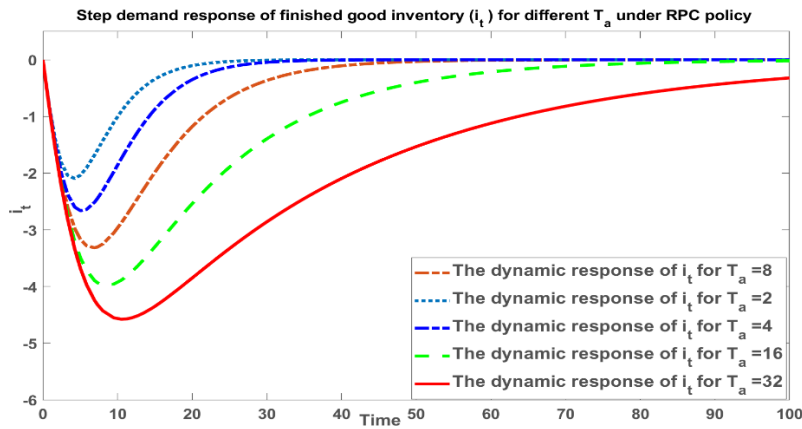
3



1  
2

Figure 3a. Impact of proportional controller ( $\delta$ ) under RPC policy

Figure 3b. Impact of proportional controller ( $\delta$ ) under PPC policy



3  
4

Figure 3c. Impact of forecast smoothing ( $\tau_a$ ) under RPC policy

Figure 3d. Impact of forecast smoothing ( $\tau_a$ ) under PPC policy

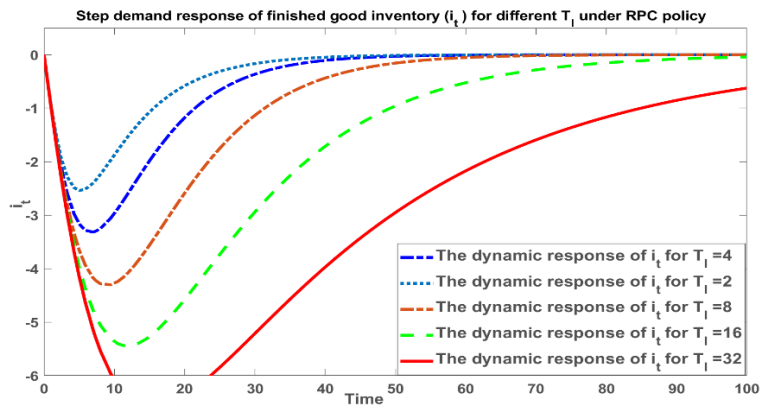


Figure 3e. Impact of physical lead times ( $\tau_l$ ) under RPC policy

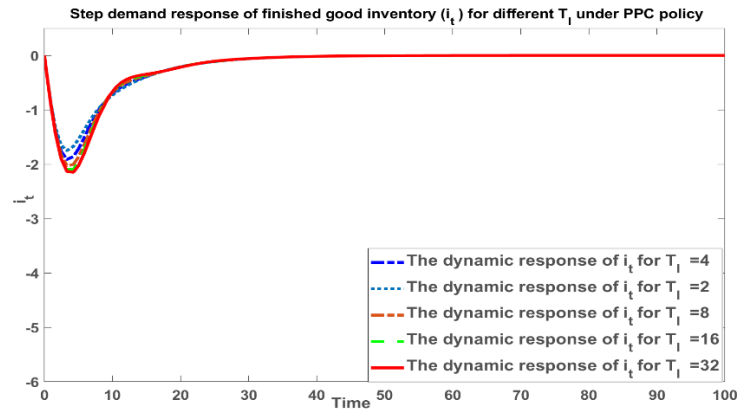


Figure 3f. Impact of physical lead times ( $\tau_l$ ) under PPC policy

Figure 3. Impact of  $\delta$  and  $\tau_a$  on finished goods inventory in responding to step demand increase under RPC and PPC policies

1  
2  
3  
4

#### 1 6.4. Dynamic analysis for seasonal demand

2 Figure 4 reports the dynamics of  $o(t)$  and  $i(t)$  in response to seasonal demand under  
3 both RPC and PPC policies. Note that Figure 4a shows the FR plot for the linear RPC and PPC  
4 systems, while the forbidden returns policy is incorporated in both systems to generate the  
5 dynamic behaviour reported in Figures 4b–4e.

6 From Figure 4a, we can make several observations. First,  $o_p$  shows a concave U-shape for  
7 both systems. The demand frequency that generates peak  $o_p$ , or worst-case amplification  
8 (Udenio et al. 2022), is 0.15 rad/week in the RPC system ( $o_p = 1.2$ ) and 0.7 rad/week in the  
9 PPC system ( $o_p = 2.4$ ). Second, it is clear that the PPC system underperforms relative to the  
10 RPC system by generating significantly higher  $o_p$  for most demand frequencies. The only  
11 exception is for low-demand frequencies of 0–0.2 rad/week where the RPC system generates  
12 higher  $o_p$ . Finally,  $o_p < 1$  for most demand frequencies in the RPC system, which means the  
13 RPC system can filter most demand frequencies to avoid the bullwhip effect. However, this is  
14 not the case for the PPC system where  $o_p > 1$  for demand frequencies of 0–1.65 rad/week.

15 When the nonlinear forbidden returns is incorporated, as shown in Figures 4b–4e, a  
16 decrease in demand frequencies leads to a change in order and inventory dynamics from linear  
17 to nonlinear behaviour in the RPC system. For example, the system is completely linear for  
18  $\omega = 0.9$  rad/week where the mean of  $o(t)$  and  $i(t)$  can be predicted by using the final  
19 value theorem (see Appendix A1). However, the PPC-based system generates nonlinear  
20 behaviour for all simulated frequencies, resulting in increased mean of  $o(t)$  and  $i(t)$ , as  
21 shown in Figure 4d. This simulation result also verifies *Property 3*. As illustrated in the linear  
22 FR,  $o_p(PPC) > 1$  for all simulated frequencies. Therefore, the PPC system generates  
23 nonlinear behaviour driven by the forbidden returns policy; that is, a negative value of orders  
24 is prevented. We can conclude that the PPC policy underperforms to the RPC policy in terms  
25 of its higher amplitude ratio and inventory variance for most of the demand frequencies. The  
26 only exceptional case is  $\omega = 0.9$  rad/week for which the bullwhip effect and inventory  
27 variance in the PPC-based system are lower than those in the RPC-based system.

1

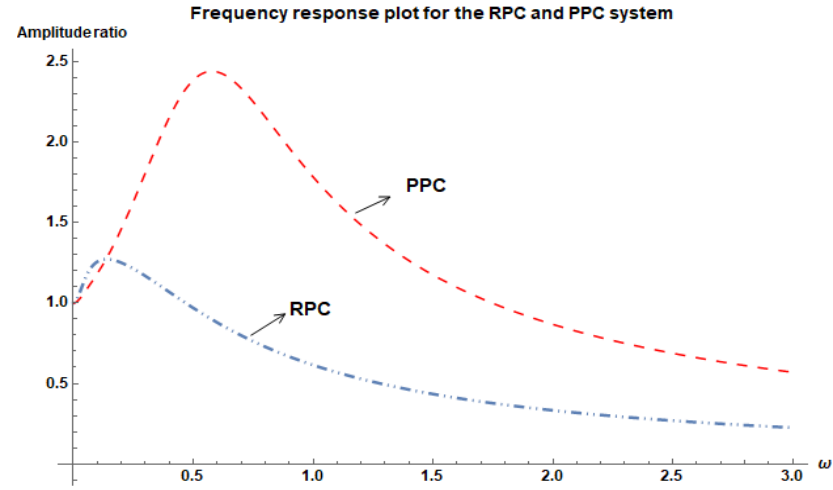


Figure 4a. FR plot for order amplitude ratio

2

3

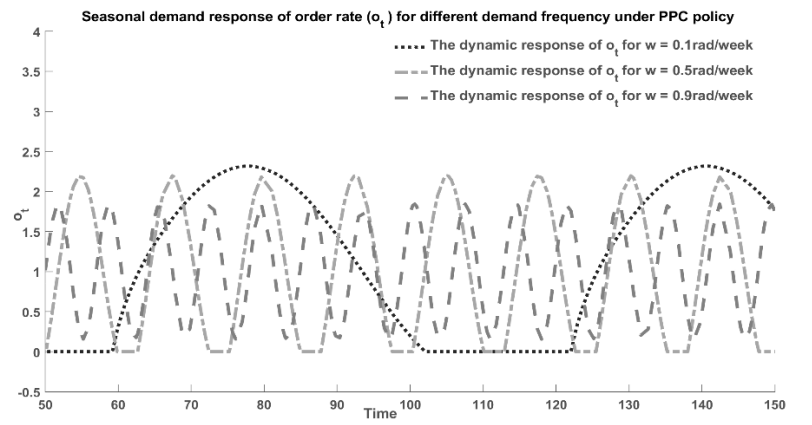


Figure 4b. The impact of  $\omega$  on order dynamics under RPC policy

4

5

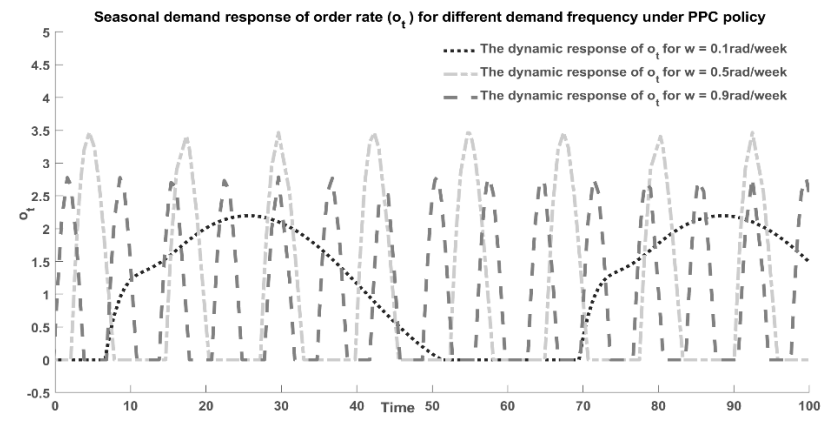
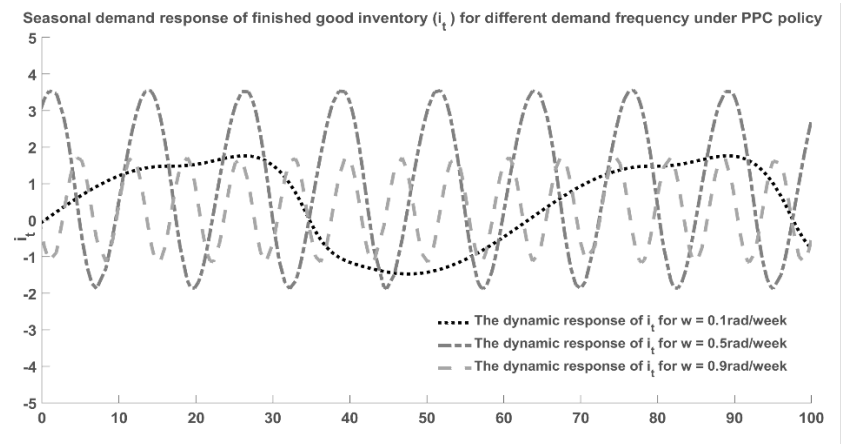
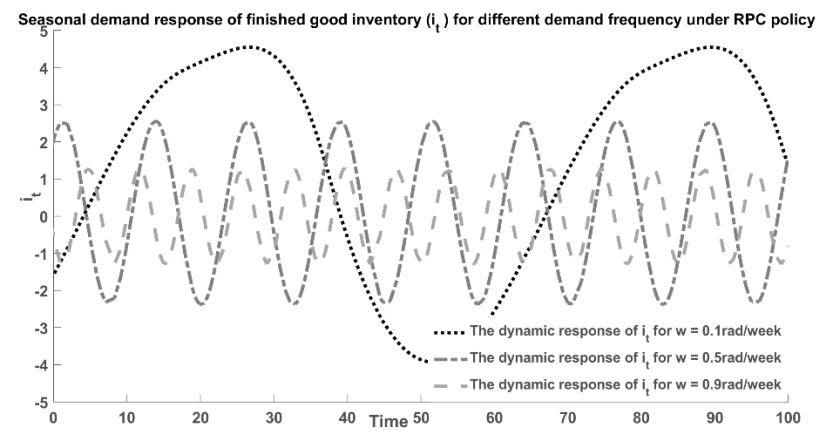


Figure 4c. The impact of  $\omega$  on order dynamics under PPC policy



1

2 Figure 4d. The impact of  $\omega$  on finished good inventory dynamics under RPC policy Figure 4e. The impact of  $\omega$  on finished good inventory dynamics under PPC policy.

3

Figure 4. The impact of  $\omega$  on order and finished good inventory in responding to seasonal demand under RPC and PPC policies.

### 6.5. RPC and PPC policies comparison under fixed bullwhip and inventory variance

This section compares the adjustment of control policies for RPC and PPC controlled systems, including inventory proportional controller ( $\delta$ ) and exponential forecasting smoothing factor ( $\tau_a$ ), for given dynamic performance metrics. This helps decision makers to understand how their system parameters need to be adjusted to achieve a targeted dynamic output. We explore the dynamic response of the proportional OUT system with normally distributed stochastic demand process with mean  $\mu = 3$  and variance  $\sigma^2 = 1$ ,  $d(t) \sim (\mu, \sigma^2)$ . Bullwhip ( $BW$ ) and inventory variance ( $IV$ ), calculated as variance ratios between order/inventory and demand, are adopted as the performance metrics. The former is directly related to the capacity-related production cost and the latter is linked to the customer service level and inventory holding cost (Disney et al. 2021). We vary  $\delta$  or  $\tau_a$  to reach the desired dynamic performance with baseline settings for  $\delta$  and  $\tau_a$  as 0.3 and 8, respectively, while other system parameter settings can be found in Table 4. Figure 5 shows the simulation results. Note that the numerical values shown in Figure 5 represent the values of the other metric generated for one fixed performance metric; for instance, the values shown in Figure 5.1 refer to the inventory variance generated for a target bullwhip.

Specifically, under each target bullwhip (0.5–2) and inventory variance (2–3.5), the RPC system requires a significantly larger value of  $\delta$  than that in the PPC system (Figures 5a and 5b), meaning inventory correction speed is tuned to a fast value in the RPC-based system. In particular, for target  $BW=2$  or  $IV=2$ , the unconventional setting, i.e.,  $\delta > 1$ , is required if an RPC is adopted in which an over-ordering strategy is needed to achieve target dynamic performance. In contrast, if the PPC policy is adopted, the required adjustment to  $\delta$  is in the range of 0.1–0.3 for targeted  $BW$  and 0.1–0.4 for targeted  $IV$ —that is, a slower inventory correction behaviour is observed. Furthermore, if target  $BW$  is high (e.g.,  $BW=2$ ), the corresponding  $IV$  generated in the PPC-based system (2.16) is lower than that in the RPC system (2.27). However, for low target  $IV$  (2–2.5), the PPC system achieves lower  $BW$  than the RPC system. This means if a company prioritises customer service level, a win–win solution can be achieved by adopting the PPC policy to minimise both bullwhip and inventory variance. However, if bullwhip cost is paramount, the RPC with lower  $BW$  and  $IV$  is more advantageous than the PPC strategy.

Figures 5c and 5d show the required adjustment in  $\tau_a$  for target  $BW$  and  $IV$ . Overall, for the same target  $BW/IV$ , the RPC system requires a smaller  $\tau_a$  than the PPC system. Notably, for a low  $BW$  (0.5) and high  $IV$  (3.5) target, the PPC system requires an unconventional setting of  $\tau_a$ —that is, a very large value of  $\tau_a$ —to greatly smooth demand. Furthermore, from the numerical values shown in Figures 5c and 5d, it can be concluded that, under all fixed  $BW/IV$

1 and proportional inventory adjustment ( $\delta$ ), the RPC outperforms the PPC strategy by generating  
2 a lower  $IV/BW$ .  
3

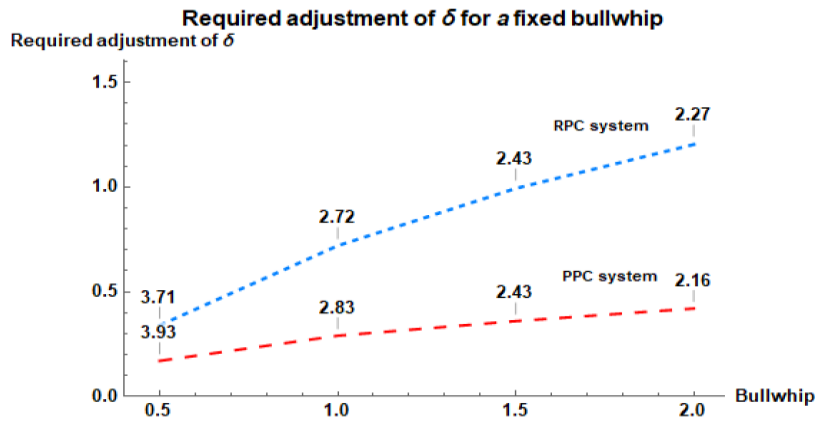


Figure 5a. Required adjustment to  $\delta$  for given bullwhip

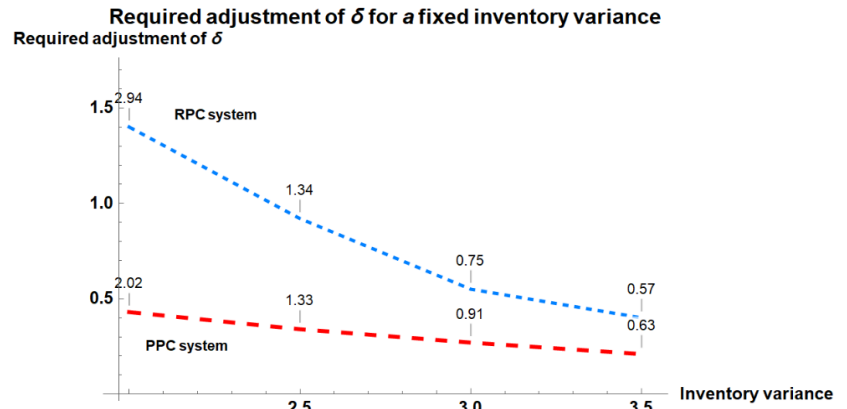


Figure 5b. Required adjustment to  $\delta$  for given inventory variance

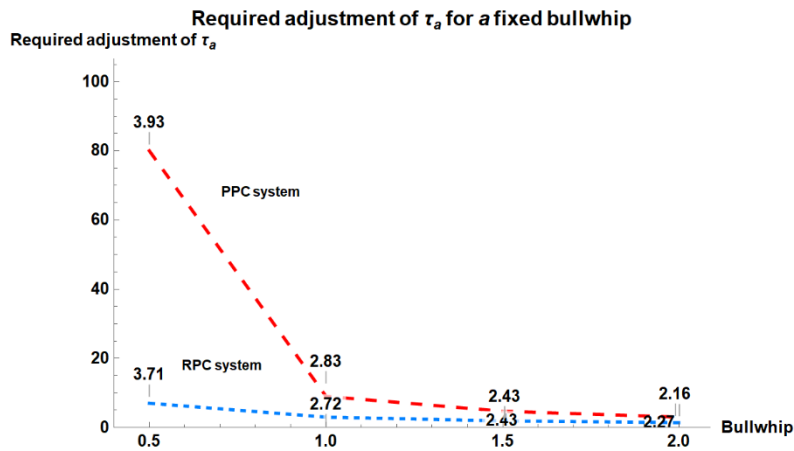


Figure 5c. Required adjustment of  $\tau_a$  for given bullwhip

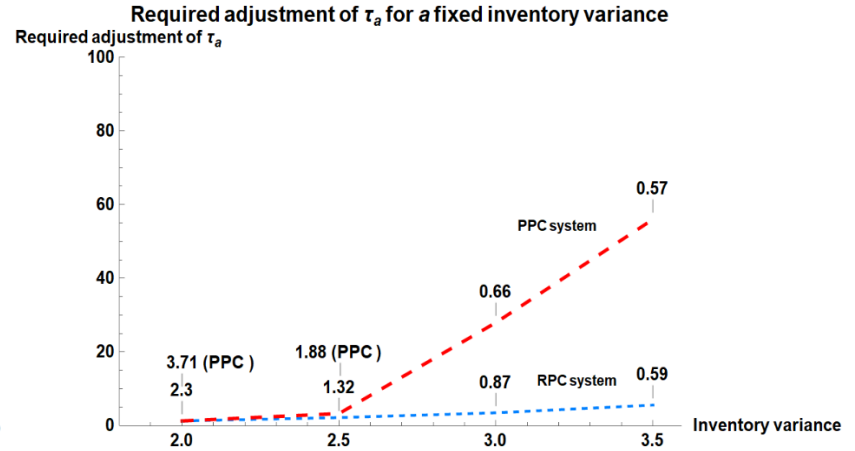


Figure 5d. Required adjustment of  $\tau_a$  for given inventory variance

Figure 5. The required adjustment to  $\delta$  and  $\tau_a$  for fixed bullwhip and inventory variance under RPC and PPC policies

1  
2  
3  
4  
5

## 1 7. Summary and managerial implications

2 We summarise in Table 6 our main analytical and simulation results.

3

| System dynamics analysis                             |   |   | Main results   |
|--|---|---|--|
| <i>Time domain analysis (step demand)</i>            | Inventory equilibrium                       | RPC   | $i^{RPC}_{new}(t) = (\hat{\tau}_l - \tau_l)$   |
|  |   | PPC   | $i^{PPC}_{new}(t) = \frac{(\hat{\tau}_l - \tau_l)}{1 + \delta\hat{\tau}_l}$  |
|  | Convergence speed and oscillation           | RPC   | $\omega_n(o_t) = \sqrt{\frac{\delta}{\tau_a}}; \zeta(o(t)) = \frac{1}{2} \sqrt{\frac{1}{\delta\tau_a} + \delta\tau_a + 2}$   |
|  |   | PPC   | For $\hat{\tau}_l \neq \tau_l$<br>$\omega_n(o(t)) = \sqrt{\frac{\delta + \delta^2\hat{\tau}_l}{\tau_l}}; \zeta(o(t)) = \frac{(1 + \delta\tau_l)}{2} \sqrt{\frac{1}{\delta\tau_l(1 + \delta\hat{\tau}_l)}}$<br>For $\hat{\tau}_l = \tau_l$<br>$\omega_n(i(t)) = \sqrt{\frac{\delta}{\tau_l} + \delta^2}; \zeta(i(t)) = \frac{1}{2} \sqrt{\frac{1}{\delta\tau_l} + 1}$                               |
| Order peak (bullwhip effect)                         |   | 1. $o_{s(PPC)} > o_{s(RPC)}$ , suggesting the PPC-based system always generates more bullwhip than the RPC-based system under the same system structure and parameter settings.<br>2. $o_{s(RPC)}$ is independent of $\tau_l$ , while $o_{s(PPC)}$ is dependent on both $\tau_l$ and $\hat{\tau}_l$ . |  |
| <i>Frequency domain analysis (sinusoidal demand)</i> | Nonlinear dynamic oscillations and recovery | RPC   | The inventory and order $\omega_n$ and $\zeta$ are identical:<br>$\omega_n = \sqrt{\frac{N_A\delta}{\tau_l}}; \zeta = \frac{1}{2} \sqrt{\frac{1}{N_A\delta\tau_l} + N_A\delta\tau_l + 2}$  |
|  |   | PPC   | The inventory and order $\omega_n$ and $\zeta$ are identical and for $\hat{\tau}_l \neq \tau_l$ : $\omega_n = \sqrt{\frac{N_A(\delta + \delta^2\hat{\tau}_l)}{\tau_l}}$<br>$\zeta = \frac{(1 + N_A\delta\tau_l)}{2} \sqrt{\frac{1}{N_A\delta\tau_l(1 + \delta\hat{\tau}_l)}}$  |
|  |   |   | For $\hat{\tau}_l = \tau_l$ :<br>$\omega_n = \sqrt{N_A\left(\frac{\delta}{\tau_l} + \delta^2\right)}; \zeta = \frac{1}{2} \sqrt{\frac{1}{N_A\delta\tau_l} + 1}$  |
| Amplitude ratio (AR)                                 |   |   | 1. AR for both policies is determined by inherent system structure ( $\tau_l$ ), system control parameters ( $\delta$ and $\tau_a$ ) and external demand characteristics $\omega$ .<br>2. There exists a cross-over demand frequency; that is: $\omega_c$ for $o_p(RPC) = o_p(PPC)$ , so that $o_p(RPC) > o_p(PPC) \forall \omega < \omega_c$ , and $o_p(RPC) < o_p(PPC)$ if $\omega > \omega_c$ . |

|                            |   |   |
|----------------------------|---|---|
| <i>Simulation analysis</i> | AR and order peak (bullwhip)  | <ol style="list-style-type: none"> <li>1. If the sinusoid demand is assumed, AR shows a concave U-shaped relationship with demand frequency in both RPC and PPC systems.</li> <li>2. If the sinusoid demand is assumed, the forbidden returns policy contributes to the AR reduction in RPC and PPC systems.</li> <li>3. As the lead time increases, the increase of bullwhip in the PPC system is much more significant than that in the RPC system.</li> </ol>  |
|                            | Inventory dynamics  | <ol style="list-style-type: none"> <li>1. If a step demand is assumed, the PPC-based system always <i>outperforms</i> the RPC-based system in terms of the lower undershoot and the higher inventory convergence speed for the same <math>\delta_t</math> and <math>\tau_l</math>.</li> <li>2. If the sinusoid demand is assumed, the PPC policy underperforms relative to the RPC policy in terms of its higher level of bullwhip effect and inventory variance, for <math>\omega = 0.1, 0.3, 0.5</math> and <math>0.7</math>.</li> <li>3. The only exceptional case is <math>\omega = 0.9</math>, where the bullwhip effect and inventory variance in the PPC-based system are lower than those in the RPC system.</li> </ol> |
|                            | Control policy adjustment for target dynamics performance (i.i.d normally distributed demand) | <ol style="list-style-type: none"> <li>1. For a given <math>BW/IV</math> and fixed <math>\tau_a</math>, <math>\delta</math> is set to a larger value for RPC than for PPC. RPC outperforms relative to the PPC strategy if bullwhip reduction is the priority, while the PPC should be chosen if customer service level is important.</li> <li>2. For a given <math>BW/IV</math> and fixed <math>\delta</math>, <math>\tau_a</math> is set to a larger value for PPC than RPC, meaning a more smoothing forecasting strategy should be adopted in the PPC-based system. Also, RPC outperforms the PPC strategy with correspondingly lower <math>IV/BW</math>.</li> </ol>  |

Table 6. Summary of the main analytical and simulation results

From Table 6, we can derive the following managerial implications:

1. If *customer service level* is the top priority for a company in responding to a shock demand increase, the firm should adopt the PPC strategy, setting the desired order pipeline inventory based on forecasted demand and on-hand inventory fluctuations. Compared to the RPC policy, high customer service levels including less likelihood of stock-out and quicker inventory recovery speed can be achieved in the PPC system. However, the company may need to pay more bullwhip-related costs induced by the PPC policy, such as the cost of ramping up/down machines, hiring/firing people and fluctuating inventory holding cost. Thus, a trade-off cost-benefit analysis is needed before determining desired order pipeline inventory policies.
2. If *bullwhip cost reduction* is the top priority for a company in responding to a sudden increase in demand, the traditional RPC strategy should be adopted. However, the RPC system may lead to low customer service levels driven by low stock levels and slow stock recovery speed during the early period of the sudden demand change. To avoid

- 1 poor customer service, more safety stock and/or an outsourcing strategy may be  
2 implemented, thereby increasing operation-related costs (e.g., inventory holding cost).
- 3 3. The *forbidden returns policy* contributes to the reduction in bullwhip-related costs at  
4 the expense of increased inventory dynamics costs in both RPC and PPC systems. For  
5 instance, the inventory holding cost (increased mean inventory) and inventory variance  
6 (increased stock-out possibility) can be increased, while the inventory recovery speed  
7 will decrease. Managers thus need to consider the cost structure to minimise the total  
8 cost, i.e. bullwhip + inventory dynamics cost under a forbidden returns policy in the  
9 supply chain.
- 10 4. *The impact of demand characteristics on bullwhip cost reduction.* Beside selecting  
11 appropriate decision parameters to avoid bullwhip, there is a need to consider customer  
12 demand characteristics given purchasing frequency plays an important role in  
13 influencing bullwhip costs. It is recommended that decision makers use the ‘cross-over  
14 frequency’ and FR plot to visualise how order amplitude ratio changes with a change  
15 in customer demand frequency. In this way, a corresponding marketing and sales  
16 strategy—for example, promotion and incentive strategies to increase/decrease in  
17 customer purchase frequency—may be implemented.
- 18 5. *The impact of lead time.* The PPC policy is *not* recommended if the production lead  
19 time is long, as bullwhip and related costs are significantly higher than in the RPC-  
20 based system. This result is applicable to both allowable and forbidden returns contexts.

## 21 22 **8. Conclusions**

23 In this paper, we study dynamic behaviour under two desired order pipeline inventory policies:  
24 (1) the RPC, where the target order pipeline is set as a function of forecast and estimated lead  
25 times; and (2) the PPC, where the target order pipeline is determined not only by the product of  
26 forecast and lead times, but also by on-hand inventory adjustment. We develop a system  
27 dynamics model representing a one-echelon production–inventory control system replenished  
28 by the proportional OUT policy. The forbidden returns policy is considered to capture the real-  
29 world returns restriction phenomenon between customers and suppliers.

30 Overall, the choice between PPC and RPC is determined by the system’s inherent structure  
31 and customer demand characteristics. The RPC can be chosen if the market environment is  
32 relatively stable and the system is characterised by long physical lead times, such as those often  
33 encountered in the defence industry (Goltsos et al. 2019). Conversely, if the system faces a  
34 volatile environment, such as the characteristics of fast-moving consumer goods industry  
35 including short lead times and high inventory sensitivity, the PPC strategy may be selected.  
36 Also, the relative cost weighting of bullwhip and inventory variance should be considered as  
37 one of main factors for the order pipeline policy choice. RPC should be adopted if minimising

1 bullwhip costs is the priority, while PPC outperforms RPC when inventory related cost (e.g.  
2 high requirement of customer service level) is prioritised.

3 We also explored the adjustment of system parameter settings for targeted bullwhip or  
4 inventory variance under i.i.d stochastic demand. The decision maker in the RPC system needs  
5 to set a much faster inventory correction speed and higher forecasting weight on current demand  
6 than that in the PPC system to achieve the same targeted bullwhip. Similar insights can be  
7 identified for a targeted inventory variance: a fast inventory correction speed and quick reaction  
8 to current demand for the forecasting are needed in the RPC system. In terms of RPC and PPC  
9 policy choice, decision makers should adopt the RPC strategy for a given bullwhip target and  
10 inventory/forecasting policy, as the RPC system outperforms PPC by generating less inventory  
11 variance. However, for a target inventory variance and forecasting policy, the PPC strategy with  
12 less bullwhip is better than the RPC policy. In contrast, decision makers should consider the  
13 RPC policy to generate less bullwhip for a targeted inventory variance and inventory control  
14 policy.

15 We contribute to the systematic comparison of two order pipeline strategies using bullwhip  
16 and inventory variance measures, and identify several future research directions. First, the  
17 capacity constraint, assumed as unlimited in our study, can be further studied to reflect capacity-  
18 constrained supply chain systems. In addition, cost functions based on system dynamics can be  
19 developed to optimise system control policies, and the results derived from the one-echelon  
20 production–inventory system can be extended to multi-echelon supply chain systems.  
21 Additional empirical research should be conducted to verify analytical/modelling research  
22 findings and update industrial practice regarding order pipeline control. Last, future research  
23 might explore analytical approaches to estimate stochastic lead times, given that accurate lead  
24 time estimation greatly influences bullwhip under the RPC and PPC systems.

25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37

1 **Reference**

- 2 1. Aggelogiannaki, E., Doganis, P., & Sarimveis, H. (2008). An adaptive model predictive  
3 control configuration for production–inventory systems. *International Journal of*  
4 *Production Economics*, 114(1), 165-178.
- 5 2. Biçer, I., Hagspiel, V., & De Treville, S. (2018). Valuing supply chain responsiveness under  
6 demand jumps. *Journal of Operations Management*, 61(1), 46-67.
- 7 3. Bray, R. L., & Mendelson, H. (2012). Information transmission and the bullwhip effect: An  
8 empirical investigation. *Management Science*, 58(5), 860-875.
- 9 4. Cachon, G. P., & Fisher, M. (2000). Supply chain inventory management and the value of  
10 shared information. *Management Science*, 46(8), 1032-1048.
- 11 5. Cachon, G. P., Randall, T., & Schmidt, G. M. (2007). In search of the bullwhip  
12 effect. *Manufacturing & Service Operations Management*, 9(4), 457-479.
- 13 6. Chatfield, D. C., & Pritchard, A. M. (2013). Returns and the bullwhip effect. *Transportation*  
14 *Research Part E: Logistics and Transportation Review*, 49(1), 159-175.
- 15 7. Chen, F., Ryan, J. K., & Simchi - Levi, D. (2000). The impact of exponential smoothing  
16 forecasts on the bullwhip effect. *Naval Research Logistics*, 47(4), 269-286.
- 17 8. Chen, L., & Lee, H. L. (2012). Bullwhip effect measurement and its  
18 implications. *Operations Research*, 60(4), 771-784.
- 19 9. Croson, R., & Donohue, K. (2006). Behavioral causes of the bullwhip effect and the  
20 observed value of inventory information. *Management Science*, 52(3), 323-336.
- 21 10. Croson, R., & Donohue, K. (2003). Impact of POS data sharing on supply chain  
22 management: An experimental study. *Production and Operations management*, 12(1), 1-11.
- 23 11. Croson, R., Donohue, K., Katok, E., & Serman, J. (2014). Order stability in supply chains:  
24 Coordination risk and the role of coordination stock. *Production and Operations*  
25 *Management*, 23(2), 176-196.
- 26 12. Dejonckheere, J., Disney, S. M., Lambrecht, M. R., & Towill, D. R. (2003). Measuring and  
27 avoiding the bullwhip effect: A control theoretic approach. *European Journal of*  
28 *Operational Research*, 147(3), 567-590.
- 29 13. Disney, S. M., & Towill, D. R. (2005). Eliminating drift in inventory and order based  
30 production control systems. *International Journal of Production Economics*, 93, 331-344.
- 31 14. Disney, S. M., Farasyn, I., Lambrecht, M., Towill, D. R., & Van de Velde, W. (2006).  
32 Taming the bullwhip effect whilst watching customer service in a single supply chain  
33 echelon. *European Journal of Operational Research*, 173(1), 151-172.
- 34 15. Disney, S. M., Towill, D. R., & Warburton, R. D. (2006). On the equivalence of control  
35 theoretic, differential, and difference equation approaches to modeling supply  
36 chains. *International Journal of Production Economics*, 101(1), 194-208.

- 1 16. Disney, S. M., Maltz, A., Wang, X., & Warburton, R. D. (2016). Inventory management for  
2 stochastic lead times with order crossovers. *European Journal of Operational*  
3 *Research*, 248(2), 473-486.
- 4 17. Disney, S. M., Ponte, B., & Wang, X. (2021). Exploring the nonlinear dynamics of the lost-  
5 sales order-up-to policy. *International Journal of Production Research*, 59(19), 5809-5830.
- 6 18. Dominguez, R., Cannella, S., Ponte, B., & Framinan, J. M. (2020). On the dynamics of  
7 closed-loop supply chains under remanufacturing lead time variability. *Omega*, 97, 102106.
- 8 19. Dominguez, R., Cannella, S., & Framinan, J. M. (2015). On returns and network  
9 configuration in supply chain dynamics. *Transportation Research Part E: Logistics and*  
10 *Transportation Review*, 73, 152-167.
- 11 20. Dominguez, R., Ponte, B., Cannella, S., & Framinan, J. M. (2019). On the dynamics of  
12 closed-loop supply chains with capacity constraints. *Computers & Industrial*  
13 *Engineering*, 128, 91-103.
- 14 21. Ivanov, D. (2021). *Introduction to supply chain resilience: management, modelling,*  
15 *technology*. Springer Nature.
- 16 22. Jakšič, M., & Rusjan, B. (2008). The effect of replenishment policies on the bullwhip effect:  
17 A transfer function approach. *European Journal of Operational Research*, 184(3), 946-961.
- 18 23. Jonkman, J., Barbosa-Póvoa, A. P., & Bloemhof, J. M. (2019). Integrating harvesting  
19 decisions in the design of agro-food supply chains. *European Journal of Operational*  
20 *Research*, 276(1), 247-258.
- 21 24. Karabuk, S., & Wu, S. D. (2003). Coordinating strategic capacity planning in the  
22 semiconductor industry. *Operations Research*, 51(6), 839-849.
- 23 25. Kim, I., & Springer, M. (2008). Measuring endogenous supply chain volatility: Beyond the  
24 bullwhip effect. *European Journal of Operational Research*, 189(1), 172-193.
- 25 26. Lee, H. L., Padmanabhan, V., & Whang, S. (1997). Information distortion in a supply chain:  
26 The bullwhip effect. *Management Science*, 43(4), 546-558.
- 27 27. Li, J. C., Zhou, Y. W., & Huang, W. (2017). Production and procurement strategies for  
28 seasonal product supply chain under yield uncertainty with commitment-option contracts.  
29 *International Journal of Production Economics*, 183, 208-222.
- 30 28. Lin, J., Naim, M. M., Purvis, L., & Gosling, J. (2017). The extension and exploitation of  
31 the inventory and order based production control system archetype from 1982 to  
32 2015. *International Journal of Production Economics*, 194, 135-152.
- 33 29. Lin, J., Spiegler, V. L., & Naim, M. M. (2018). Dynamic analysis and design of a  
34 semiconductor supply chain: a control engineering approach. *International Journal of*  
35 *Production Research*, 56(13), 4585-4611.
- 36 30. Lin, J., & Naim, M. M. (2019). Why do nonlinearities matter? The repercussions of linear

- 1 assumptions on the dynamic behavior of assemble-to-order systems. *International Journal*  
2 *of Production Research*, 57(20), 6424-6451.
- 3 31. Lin, J., Naim, M. M., & Spiegler, V. L. (2020). Delivery time dynamics in an assemble-to-  
4 order inventory and order based production control system. *International Journal of*  
5 *Production Economics*, 223, 107531.
- 6 32. Lin, J, Zhou, L, Spiegler, V., Naim, M. M, Syntetos, A, 2022. Push or Pull? The impact of  
7 ordering policy choice on the dynamics of a hybrid closed-loop supply chain. *European*  
8 *Journal of Operational Research*, 300 (1), pp.282-295,
- 9 33. Nagatani, T., & Helbing, D. (2004). Stability analysis and stabilization strategies for linear  
10 supply chains. *Physica A: Statistical Mechanics and its Applications*, 335(3-4), 644-660.
- 11 34. Niranjana, T. T., Wagner, S. M., & Aggarwal, V. (2011). Measuring information distortion in  
12 real-world supply chains. *International Journal of Production Research*, 49(11), 3343-  
13 3362.
- 14 35. Ponte, B., Wang, X., de la Fuente, D., & Disney, S. M. (2017). Exploring nonlinear supply  
15 chains: the dynamics of capacity constraints. *International Journal of Production*  
16 *Research*, 55(14), 4053-4067.
- 17 36. Shan, J., Yang, S., Yang, S., & Zhang, J. (2014). An empirical study of the bullwhip effect  
18 in China. *Production and Operations Management*, 23(4), 537-551.
- 19 37. Spiegler, V. L., & Naim, M. M. (2017). Investigating sustained oscillations in nonlinear  
20 production and inventory control models. *European Journal of Operational*  
21 *Research*, 261(2), 572-583.
- 22 38. Spiegler, V. L., Naim, M. M., Towill, D. R., & Wikner, J. (2016). A technique to develop  
23 simplified and linearised models of complex dynamic supply chain systems. *European*  
24 *Journal of Operational Research*, 251(3), 888-903.
- 25 39. Springer, M., & Kim, I. (2010). Managing the order pipeline to reduce supply chain  
26 volatility. *European Journal of Operational Research*, 203(2), 380-392.
- 27 40. Sterman, J. D. (1989). Modeling managerial behavior: Misperceptions of feedback in a  
28 dynamic decision making experiment. *Management Science*, 35(3), 321-339.
- 29 41. Sterman, J. (2000). *Business dynamics*. Irwin/McGraw-Hill c2000.
- 30 42. Towill, D.R., Evans, G. N., & Cheema, P. (1997). Analysis and design of an adaptive  
31 minimum reasonable inventory control system. *Production Planning & Control*, 8(6), 545-  
32 557.
- 33 43. Towill, D. R., Zhou, L., & Disney, S. M. (2007). Reducing the bullwhip effect: Looking  
34 through the appropriate lens. *International Journal of Production Economics*, 108(1-2),  
35 444-453.
- 36 44. Udenio, M., Vatamidou, E., Fransoo, J. C., & Dellaert, N. (2017). Behavioral causes of the  
37 bullwhip effect: An analysis using linear control theory. *IIE Transactions*, 49(10), 980-

- 1 1000.
- 2 45. Udenio, M., Vatamidou, E., & Fransoo, J. C. (2022). Exponential smoothing forecasts:  
3 Taming the Bullwhip Effect when demand is seasonal. *International Journal of Production*  
4 *Research*, 1-18.
- 5 46. Wang, X., Disney, S. M., & Wang, J. (2012). Stability analysis of constrained inventory  
6 systems with transportation delay. *European Journal of Operational Research*, 223(1), 86-  
7 95.
- 8 47. Wang, X., Disney, S. M., & Wang, J. (2014). Exploring the oscillatory dynamics of a  
9 forbidden returns inventory system. *International Journal of Production Economics*, 147,  
10 3-12.
- 11 48. Wang, Z., Wang, X., & Ouyang, Y. (2015). Bounded growth of the bullwhip effect under a  
12 class of nonlinear ordering policies. *European Journal of Operational Research*, 247(1),  
13 72-82.
- 14 49. Wang, X., & Disney, S. M. (2016). The bullwhip effect: Progress, trends and  
15 directions. *European Journal of Operational Research*, 250(3), 691-701.
- 16 50. Wang, X., & Disney, S. M. (2017). Mitigating variance amplification under stochastic lead-  
17 time: the proportional control approach. *European Journal of Operational*  
18 *Research*, 256(1), 151-162.
- 19 51. Warburton, R. D., & Disney, S. M. (2007). Order and inventory variance amplification: The  
20 equivalence of discrete and continuous time analyses. *International Journal of Production*  
21 *Economics*, 110(1-2), 128-137.
- 22 52. Wikner, J., Naim, M. M., Spiegler, V. L., & Lin, J. (2017). IOBPCS based models and  
23 decoupling thinking. *International Journal of Production Economics*, 194, 153-166.
- 24 53. Wu, D. Y., & Katok, E. (2006). Learning, communication, and the bullwhip effect. *Journal*  
25 *of Operations Management*, 24(6), 839-850.
- 26 54. Yang Y, Lin, J., Liu, G., & Zhou, L. (2021). The behavioural causes of bullwhip effect in  
27 supply chains: a systematic literature review. *International Journal of Production*  
28 *Economics* (1), 108120.
- 29 55. Zhang, X. (2004). The impact of forecasting methods on the bullwhip effect. *International*  
30 *Journal of Production Economics*, 88(1), 15-27.
- 31 56. Zhou, L., Disney, S., & Towill, D. R. (2010). A pragmatic approach to the design of  
32 bullwhip controllers. *International Journal of Production Economics*, 128(2), 556-568.
- 33 57. Zhou, L., Naim, M. M., & Disney, S. M. (2017). The impact of product returns and  
34 remanufacturing uncertainties on the dynamic performance of a multi-echelon closed-loop  
35 supply chain. *International Journal of Production Economics*, 183, 487-502.
- 36 58. Zotteri, G. (2013). An empirical investigation on causes and effects of the Bullwhip-effect:  
37 Evidence from the personal care sector. *International Journal of Production*

2  
3 **Appendix A. Proof of Propositions**

4 *A1. Proof of Proposition 1*

5 To derive the inventory equilibrium for both RPC and PPC systems, we need to obtain the  
6 inventory transfer function in relation to demand.

7 Specifically, based on Equations (1) - (8), we re-write the following equations in describing  
8 the dynamic structure of the linear production-inventory system replenished by proportional  
9 OUT policy:

$$10 \quad o(t) = o_a = \hat{d}(t) + \delta \cdot (\hat{w}(t) \cdot \hat{\tau}_l - w(t)) + \delta \cdot (\beta - i(t)), \quad (A1)$$

11 where

$$12 \quad i(t) = \int_0^{\infty} (r(t) - d(t)) dt, \quad (A2)$$

$$13 \quad w(t) = \frac{1}{\tau_l} \int_0^{\infty} (o(t) - r(t)) dt, \quad (A3)$$

$$14 \quad \hat{d}(t) = \frac{1}{\tau_a} \int_0^{\infty} (d(t) - \hat{d}(t-1)) dt, \quad (A4)$$

15 and

$$16 \quad \text{RPC: } \hat{w}(t) = w_r = \hat{d}(t) \cdot \hat{\tau}_l \quad \text{PPC: } \hat{w}(t) = w_p = \hat{d}(t) \cdot \hat{\tau}_l + \delta \cdot (\beta - i(t)) \cdot \hat{\tau}_l. \quad (A5)$$

17 Using the Laplace transform, i.e.  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ ,  $\forall 0 \leq t < \infty$ , we have the  
18 corresponding following Equations in Laplace domain:

$$19 \quad i(t) = \frac{1}{s} (r(t) - d(t)), \quad (A6)$$

$$20 \quad w_t = \frac{1}{s} (o(t) - r(t)), \quad (A7)$$

$$21 \quad \hat{d}(t) = \frac{1}{1 + \tau_a s} d(t), \quad (A8)$$

$$22 \quad r(t) = \frac{1}{1 + \tau_l s} o(t). \quad (A9)$$

23 For simplicity without losing generality we assume  $\beta = 0$  (Zhou et al. 2017). Substituting  
24 Equation (A5)-A(9) to (A1), the dynamic response of  $o(t)$  and  $i(t)$  with RPC and PPC  
25 policies in responding  $d(t)$  in Laplace domain can be derived:

$$27 \quad \frac{o(t)^{RPC}}{d(t)} = \frac{(s + \delta(1 + s\tau_a) + s\delta\hat{\tau}_l)}{(s + \delta)(1 + s\tau_a)}, \quad (A10)$$

$$28 \quad \frac{o(t)^{PPC}}{d_t} = \frac{(s + \delta + s\delta\tau_a)(1 + s\tau_l)(1 + \delta\hat{\tau}_l)}{(1 + s\tau_a)(s^2\tau_l + s(1 + \delta\tau_l) + \delta + \delta^2\hat{\tau}_l)}, \quad (A11)$$

$$1 \quad \frac{i(t)^{RPC}}{d(t)} = \frac{\delta(\hat{\tau}_l - \tau_l) - s(\tau_a + \tau_l) - s\tau_a\tau_l(\delta - s)}{(s + \delta)(1 + s\tau_a)(1 + s\tau_l)}, \quad (A12)$$

2

$$3 \quad \frac{i(t)^{PPC}}{d(t)} = \frac{\delta\hat{\tau}_l - (s + \delta)\tau_l - s\tau_a(1 + (s + \delta)\tau_l)}{(1 + s\tau_a)(s^2\tau_l + s(1 + \tau_l\delta) + \delta + \delta^2\hat{\tau}_l)}. \quad (A12)$$

4 To obtain the new inventory equilibrium in responding a step demand increase ( $d_s = \frac{1}{s}$ ),

5 the Final Value Theorem (FVT), i.e.  $\lim_{s \rightarrow 0} s \frac{i(t)}{d(t)}$  for both RPC and PPC systems:

6

$$7 \quad \lim_{s \rightarrow 0} s \frac{i(t)^{RPC}}{d(t)} = s \cdot \frac{1}{s} \cdot \frac{\delta(\hat{\tau}_l - \tau_l) - s(\tau_a + \tau_l) - s\tau_a\tau_l(\delta - s)}{(s + \delta)(1 + s\tau_a)(1 + s\tau_l)} = \hat{\tau}_l - \tau_l, \quad (A13)$$

$$8 \quad \lim_{s \rightarrow 0} s \frac{i_t(PPC)}{d_t} = s \cdot \frac{1}{s} \cdot \frac{\delta\hat{\tau}_l - (s + \delta)\tau_l - s\tau_a(1 + (s + \delta)\tau_l)}{(1 + s\tau_a)(s^2\tau_l + s(1 + \tau_l\delta) + \delta + \delta^2\hat{\tau}_l)} = \frac{(\hat{\tau}_l - \tau_l)}{1 + \delta\hat{\tau}_l}. \quad (A14)$$

9

## 10 A2. Proof of Proposition 2

11 The damping ratio and natural frequency of orders can be found based on the second order  
12 polynomial of denominator of the transfer function, i.e.  $s^2 + 2 \cdot \omega_n \cdot \zeta \cdot s + \omega_n^2$ .

13 By inspecting Equation (A9) and (A10), we can derive the  $\omega_n$  and  $\zeta$  of orders under  
14 different pipeline control policies:

$$15 \quad \text{For RPC: } \omega_n = \sqrt{\frac{\delta}{\tau_a}}; \quad \zeta = \frac{1}{2} \sqrt{\frac{1}{\delta\tau_a} + \delta\tau_a + 2}, \quad (A15)$$

$$16 \quad \text{For PPC: } \omega_n (\hat{\tau}_l \neq \tau_l) = \sqrt{\frac{\delta + \delta^2\hat{\tau}_l}{\tau_l}}; \quad \zeta (\hat{\tau}_l \neq \tau_l) = \frac{(1 + \delta\tau_l)}{2} \sqrt{\frac{1}{\delta\tau_l(1 + \delta\hat{\tau}_l)}}, \quad (A16)$$

$$17 \quad \omega_n (\hat{\tau}_l = \tau_l) = \sqrt{\frac{\delta}{\tau_l} + \delta^2}; \quad \zeta (\hat{\tau}_l = \tau_l) = \frac{1}{2} \sqrt{\frac{1}{\delta\tau_l} + 1}. \quad (A17)$$

18 Similarly, we can derive the  $\omega_n$  and  $\zeta$  of inventories for both RPC and PPC systems by  
19 inspecting the denominator of Equation (A11) and (A12). It can be seen that by comparing  
20 Equation (A11) and (A9), a new term  $(1 + s\tau_l)$  is added in the denominator of transfer  
21 function, while the denominator of Equation (A12) and (A10) are identical. As the result, the  
22  $\omega_n$  and  $\zeta$  of inventory and order under PPC policy are identical, while we can obtain the  
23 following  $\omega_n$  and  $\zeta$  of inventory under RPC policy as follows:

$$24 \quad \text{For RPC: } \omega_n = \sqrt{\frac{\delta}{\tau_l}}; \quad \zeta = \frac{1}{2} \sqrt{\frac{1}{\delta\tau_l} + \delta\tau_l + 2}, \quad (A18)$$

25

1 *A3. Proof of Proposition 3.1 and 3.2*

2 If  $d(t) = A \cdot \cos(\omega t) + B, \forall B, A, \omega \in \mathbf{R}, \beta > a > 0$  is assumed, its Laplace form can be  
3 written as

$$4 \quad F(s) = \int_0^{\infty} e^{-st} (A \cdot \cos(\omega t) + B) dt = \frac{A \cdot s}{s^2 + \omega^2} + \frac{B}{s}. \quad (A19)$$

5 The dynamic response of  $o(t)^{RPC}$  in responding  $d(t)$  in Laplace domain, based on  
6 Equation (A9) can be written as,

$$7 \quad o(t)^{RPC} = \frac{(s + \delta(1 + s\tau_a) + s\delta\hat{\tau}_l)}{(s + \delta)(1 + s\tau_a)} \cdot \left( \frac{A \cdot s}{s^2 + \omega^2} + \frac{B}{s} \right). \quad (A20)$$

8 We apply the inverse Laplace transform of Equation (A20) to obtain dynamic response of  
9  $o(t)^{RPC}$  in relation to  $d(t)$  in time domain:

$$10 \quad o(t)^{RPC} = \frac{A \left( \begin{array}{l} e^{-t\delta} \delta^3 (1 + \omega^2 \tau_a^2) (\tau_a + \hat{\tau}_l) - e^{-\frac{t}{\tau_a}} (\omega^2 + \delta^2) (1 + \delta \hat{\tau}_l) + (1 - \delta \tau_a) ((\omega^2 + \delta^2) \cos(\omega t) \\ + \omega^2 \delta (\delta \cos(\omega t) + \omega \sin(\omega t)) \tau_a^2 + \omega \delta (\omega \cos(\omega t) - \delta \sin(\omega t)) \hat{\tau}_l \\ + \omega^2 (\delta \cos(\omega t) + \omega \sin(\omega t)) \tau_a (1 + \delta) \end{array} \right)}{(\omega^2 + \delta)(1 - \delta \tau_a)(1 + \omega^2 \tau_a^2)} + B. \quad (A21)$$

11 For a long-time response in equilibrium,  $e^{-\frac{t}{\tau_c}} = e^{-t} = 0$ . Equation (A21) can be re-  
12 arranged as:

$$13 \quad o(t)^{RPC} = \frac{A \left( (1 - \delta \tau_a) \left( \begin{array}{l} (\omega^2 + \delta^2) \cos(\omega t) + \omega^2 \delta (\delta \cos(\omega t) + \omega \sin(\omega t)) \tau_a^2 + \\ \delta (\omega \cos(\omega t) - \delta \sin(\omega t)) \hat{\tau}_l + \omega^2 (\delta \cos(\omega t) + \omega \sin(\omega t)) \tau_a (1 + \delta \hat{\tau}_l) \end{array} \right) \right)}{(\omega^2 + \delta)(1 - \delta \tau_a)(1 + \omega^2 \tau_a^2)} + B. \quad (A22)$$

14 Which can be simplified as:

$$15 \quad o(t)^{RPC} = A \sqrt{\frac{\delta^2 + (\omega(1 + \delta \tau_a + \delta))^2}{(\omega^2 + \delta^2)(1 + \omega^2 \tau_a^2)}} \cos \left( \omega t - \tan^{-1} \left( \frac{\omega + \omega \tau_a}{1 - \omega^2 \tau_a} \right) \right. \\ 16 \quad \left. + \tan^{-1} \left( \omega + \omega(\hat{\tau}_p + \tau_a) \right) \right) + B, 0 \leq t < \infty. \quad (A23)$$

17 The  $o_p(RPC)$ , measured by amplitude ratio, can be derived

$$18 \quad o_p(RPC) = \frac{A \cdot \sqrt{\frac{\delta^2 + (\omega(1 + \delta \tau_a + \delta \hat{\tau}_l))^2}{(\omega^2 + \delta^2)(1 + \omega^2 \tau_a^2)}}}{A} = \sqrt{\frac{\delta^2 + (\omega(1 + \delta \tau_a + \delta \hat{\tau}_l))^2}{(\omega^2 + \delta^2)(1 + \omega^2 \tau_a^2)}}, \quad (A24)$$

19 and  $A_{o(t)}(RPC)$  can be derived:

$$20 \quad A_{o(t)}(RPC) = A \cdot o_p(RPC) = A \cdot \sqrt{\frac{\delta^2 + (\omega(1 + \delta \tau_a + \delta \hat{\tau}_l))^2}{(\omega^2 + \delta^2)(1 + \omega^2 \tau_a^2)}}. \quad (A25)$$

21 Similarly, the bullwhip of a PPC-based system can be derived using the same procedure  
22 shown above. So we prove the *Proposition 3.1*

23 For the forbidden return system, we have  $o_a(t) = [o(t)]^+$ , where  $[o(t)]^+$  is the  
24 maximum operator. By truncating negative value we can capture the influence of forbidden

1 return. So, we can re-write the above forbidden return policy as the following piecewise linear  
2 function

$$3 \quad o_a(t) = \begin{cases} o(t) & |o(t) > 0 \\ 0 & |o(t) < 0 \end{cases} . \quad (A26)$$

4 For a given seasonal demand  $d(t) = A \cdot \cos(\omega t) + B, \forall B, A, \omega \in R, B > A > 0$ , using  
5 describing function approach,  $o_a$  can be approximated as:

$$6 \quad o_a(t) \approx N_A \cdot A_{o(t)} \cdot \cos(\omega t + \phi) + N_B \cdot B, \quad (A27)$$

7 where  $N_A, N_B$  and  $\phi$  are amplitude gain, mean gain and phase shift,  $A_{o_t}$  is the  
8 amplitude of  $o(t)$  in responding  $d(t)$ . Amplitude ratio,  $o_p$ , is measured by

$$9 \quad o_p = \frac{\text{amplitude of } o_a}{\text{amplitude of } d_t} = \frac{N_A \cdot A_{o(t)}}{A}. \quad (A28)$$

10  $N_A$  can be approximated, based on Equation (12) – (16) in the main paper by

$$11 \quad N_A = \frac{\sqrt{a_1^2 + b_1^2}}{A_{o(t)}}, \quad (A29)$$

12 where

$$14 \quad a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} o(t) \cdot \cos(\omega t) d_{\omega t}, \quad (A30)$$

$$15 \quad b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} o(t) \cdot \sin(\omega t) d_{\omega t}, \quad (A31)$$

16 Using Mathematica<sup>®</sup>, we can derive the  $N_A = \frac{B \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2}}}{A_{o(t)} + \text{Cos}^{-1}\left(-\frac{B}{A_{o(t)}}\right)}$ . So we obtain  $o_p$   
17 as follow:

$$18 \quad o_p = \frac{N_A \cdot A_{o(t)}}{A} = \frac{\frac{B \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2}}}{A_{o(t)} + \text{Cos}^{-1}\left(-\frac{B}{A_{o(t)}}\right)} \cdot A_{o(t)}}{A}. \quad (A32)$$

19

20 So we prove the *Proposition 3.2*

21

#### 22 A5. Proof of Proposition 4

23 Based on *Proposition 3*, we have  $o_a(t) = [o(t)]^+ = N_A \cdot o(t)$ , where  $N_A =$

24  $\frac{B \cdot \sqrt{1 - \frac{B^2}{A_{o(t)}^2}}}{A_{o(t)} + \text{Cos}^{-1}\left(-\frac{B}{A_{o(t)}}\right)}$ . By replacing  $o(t) = \frac{o_a(t)}{N_A}$ , we can derive the transfer function of  $o_a(t)$

25 and  $i(t)$  in relation to  $d(t)$  based on Equations (A1) and (A5)- (A8):

$$1 \quad \frac{o_a^{RPC}(t)}{d(t)} = \frac{(s + \delta(1 + s\tau_a) + s\delta\hat{\tau}_l)(1 + s\tau_l)}{(1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta}{\tau_l})}, \quad (A33)$$

$$2 \quad \frac{o_a^{RPC}(t)}{d(t)} = \frac{((\delta + \delta^2\hat{\tau}_l)(1 + s\tau_a) + s(1 + \hat{\tau}_l\delta))(1 + s\tau_l)}{(1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta(1 + \delta\hat{\tau}_l)}{\tau_l})}, \quad (A34)$$

$$3 \quad \frac{i(t)^{RPC}}{d(t)} = \frac{(s + \delta(1 + s\tau_a) + s\delta\hat{\tau}_l)N_A - (1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta}{\tau_l})}{s(1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta}{\tau_l})}, \quad (A35)$$

$$4 \quad \frac{i(t)^{RPC}}{d(t)} = \frac{((\delta + \delta^2\hat{\tau}_l)(1 + s\tau_a) + s(1 + \hat{\tau}_l\delta)) - (1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta(1 + \delta\hat{\tau}_l)}{\tau_l})}{s(1 + s\tau_a)(s^2 + s(N_A\delta + \frac{1}{\tau_l}) + \frac{N_A\delta(1 + \delta\hat{\tau}_l)}{\tau_l})}. \quad (A36)$$

5 From Equation (A34) and (A36), we can observe  $\hat{\tau}_l$  only appears in the numerator in both  
6 order and inventory under RPC policy. Thereby, damping ratio and natural frequency of  
7 inventory and orders are identical and remain the same for  $\hat{\tau}_l \neq \tau_l$  and  $\hat{\tau}_l = \tau_l$ . By inspecting  
8 the second order polynomial of denominator of the transfer function, i.e.  $(s^2 + s(N_A\delta + \frac{1}{\tau_l}) +$   
9  $\frac{N_A\delta}{\tau_l})$ , we can derive  $\omega_n$  and  $\zeta$  of order and inventory for the RPC controlled system:

$$10 \quad \omega_n = \sqrt{\frac{N_A\delta}{\tau_l}} \quad \zeta = \frac{1}{2} \sqrt{\frac{1}{N_A\delta\tau_l} + N_A\delta\tau_l + 2}. \quad (A37)$$

11 However, if PPC policy is adopted in the nonlinear forbidden return system,  $\hat{\tau}_l$  is observed  
12 in both numerator and denominator of the transfer function shown in Equation (A34) and (A36).  
13 As a result, we consider two conditions when  $\hat{\tau}_l \neq \tau_l$  and  $\hat{\tau}_l = \tau_l$ . Similar to the derivation  
14 for the RPC-controlled system, we can obtain  $\omega_n$  and  $\zeta$  for the PPC-controlled system  
15 below:

$$16 \quad \omega_n(\hat{\tau}_l \neq \tau_l) = \sqrt{\frac{N_A(\delta + \delta^2\hat{\tau}_l)}{\tau_l}} \quad \zeta(\hat{\tau}_l \neq \tau_l) = \frac{(1 + N_A\delta\tau_l)}{2} \sqrt{\frac{1}{N_A\delta\tau_l(1 + \delta\hat{\tau}_l)}}, \quad (A38)$$

$$17 \quad \omega_n(\hat{\tau}_l = \tau_l) = \sqrt{N_A\left(\frac{\delta}{\tau_l} + \delta^2\right)} \quad \zeta(\hat{\tau}_l = \tau_l) = \frac{1}{2} \sqrt{\frac{1}{N_A\delta\tau_l} + 1}. \quad (A39)$$

18 So we prove the *Proposition 6*.  
19

## 20 Appendix B: Order peak derivation under RPC and PPC controlled systems

21 This section derives the analytical results of order peak in responding a unit demand shock. A  
22 unit demand shock is also named as Heaviside Function,  $d(t) = 0, \forall t < 0, d(t) = 1, \forall t \geq 0$ ,  
23 its Laplace form can be written as

1 
$$F(s) = \int_0^{\infty} e^{-st} d(t) = \frac{1}{s}. \quad (B1)$$

2

3 The dynamic response of  $o(t)^{RPC}$  in responding  $d(t)$  in Laplace domain can be written as,

4 
$$o(t)^{RPC} = \frac{(s + \delta(1 + s\tau_a) + s\delta\hat{\tau}_l)}{(s + \delta)(1 + s\tau_a)s}. \quad (B2)$$

5 Using inverse Laplace transform of Equation (A20), we can obtain the dynamic response  
6 of  $o(t)^{RPC}$  in time domain:

7 
$$o(t)^{RPC} = 1 + \frac{e^{-t\delta}(-\delta\tau_a - \delta\hat{\tau}_l)}{\delta\tau_a - 1} + \frac{e^{-\frac{t}{\tau_a}(1 + \delta\hat{\tau}_l)}}{\delta\tau_a - 1}. \quad (B3)$$

8 Based on the definition, the order peak of the RPC system, that is,  $o_s^{RPC}$ , in responding a  
9 shock demand can be measured by the ratio between peak of  $o(t)^{RPC}$  and demand ( $d(t) =$   
10 1). So we need to find the maximum of  $o(t)^{RPC}$ . First, we differentiate Equation (A21) with  
11 respect to  $t$ :

12 
$$\frac{o(t)^{RPC}}{dt} = -\frac{e^{-t\delta}(-\delta\tau_a - \delta\hat{\tau}_l)}{\delta\tau_a - 1} - \frac{e^{-\frac{t}{\tau_a}(1 + \delta\hat{\tau}_l)}}{\tau_a(-1 + \delta\tau_a)}. \quad (B4)$$

13 Then, solve the zero gradient of Equation (A22) with  $t$ :

14 
$$t = \frac{\tau_a \left( \ln(-\delta^2\tau_a - \delta^2\hat{\tau}_l) - \ln\left(\frac{1 + \delta\hat{\tau}_l}{\tau_a}\right) \right)}{\delta\tau_a - 1}. \quad (B5)$$

15 Substituting Equation (A23) into (A22) we can obtain the bullwhip of RPC system as  
16 follow:

17 
$$o_s^{RPC} = 1 + \frac{1 + \delta\hat{\tau}_l}{\delta\tau_a} \cdot \left( \frac{1 + \delta\hat{\tau}_l}{\delta^2\tau_a \cdot (\tau_a + \hat{\tau}_l)} \right)^{\frac{1}{\delta\tau_a - 1}}. \quad (B6)$$

18 The order peak of the PPC-based system can be derived using the same procedure shown  
19 above:

20 
$$o_s^{PPC} = \left( 1 + \delta \cdot (\tau_a + \hat{\tau}_l) \cdot \left( \frac{1 + \delta\hat{\tau}_l}{\delta^2\tau_a \cdot (\tau_a + \hat{\tau}_l)} \right)^{1 + \frac{1}{\delta\tau_a - 1}} \right) \left( 1 + \frac{\tau_l\delta}{3} \right). \quad (B7)$$

21 By comparing Equation (B6) and (B7), it is straightforward to observe that  $o_s^{PPC} > o_s^{RPC}$ ,  
22 which indicates that the RPC system always generates less bullwhip effect than the PPC system.  
23 This finding is consistent with that of Kim and Spring (2010). Similarly,  $o_s^{PPC} > o_s^{RPC} > 1$ , for  
24 any system policy control and system physical structure. This result suggests that the bullwhip  
25 effect is unavoidable in the proportional order-up-to system, regardless of pipeline control  
26 policies. Furthermore, regarding Equation (B6), the bullwhip effect is dependent on estimated  
27 lead times ( $\hat{\tau}_l$ ) only. This highlights the importance of monitoring actual lead times in

1 controlling unwanted system dynamics behaviour. The impact of  $\tau_l$  on bullwhip becomes  
2 complex and greatly depends on  $\hat{\tau}_l$  under the RPC system. For example,  $\hat{\tau}_l$  can be estimated  
3 by following a particular distribution centred on the actual lead time, and thus the variability of  
4  $\hat{\tau}_l$  may play an additional role in influencing bullwhip effect (Disney et al. 2016).

5 However, this is not the case for the PPC-controlled system in which  $\tau_l$  positively  
6 influences the bullwhip effect level. Furthermore, given the additional term, that is,  $\left(1 + \frac{\tau_l \delta_t}{3}\right)$ ,  
7 in Equation (B7) compared with Equation (B6), it is interesting to note that an increase of  $\tau_l$   
8 can significantly increase the difference in the bullwhip effect between PPC- and RPC-  
9 controlled systems. In other words, if the physical lead time is long, the bullwhip effect under  
10 the PPC supply chain system is significantly higher than that in the RPC system.

11

12 **Reference:**

13 59. Disney, S. M., Maltz, A., Wang, X., & Warburton, R. D. (2016). Inventory management for  
14 stochastic lead times with order crossovers. *European Journal of Operational*  
15 *Research*, 248(2), 473-486.

16 60. Kim, I., & Springer, M. (2008). Measuring endogenous supply chain volatility: Beyond the  
17 bullwhip effect. *European Journal of Operational Research*, 189(1), 172-193.

18

19