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Joint maintenance and spare-parts inventory models: a review and discussion of practical stock-keeping rules

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It is natural to coordinate spare-parts inventory planning and maintenance. However, work in the former area often neglects part utilization, and work in the latter neglects the fact that effective execution of maintenance schedules is conditioned to the availability of the necessary spare parts. This paper is a call for further integration between the two areas, and to that end, we review the literature on mathematical modelling and analysis of inventory-maintenance-planning. We are not the first to address this issue (though we take a fresh perspective to the problem), but we are the first to complement such review with a discussion of simple stock keeping rules that may be used effectively in practice. We identify a growing gap between modelling and application, between theory and practice, which justifies the presentation of these simple stock keeping rules for the joint planning of inventory and maintenance. Thus, our work should be of interest not only to researchers who are looking for promising avenues for future research but also to practitioners who are seeking to improve inventory-maintenance operations.

Keywords: inventory; maintenance; spare parts; literature review; theory; practice.

1. Context and motivation

The coordination of maintenance-activity planning with spare-parts inventory planning is recognized as a good thing (Van Horenbeek *et al.*, 2013a; De Almeida *et al.*, 2015; Shafiee, 2015; Eruguz *et al.*, 2017; Durugbo, 2020). Such coordination is simply the action in which inventory managers use known maintenance schedules to plan the provision of spare parts for maintenance or in which maintainers use known inventory factors to plan maintenance. The value of coordination depends on particular circumstances, and savings in total maintenance and inventory costs in the region of 5–10% have been reported (Kabir & Olayan, 1996; Sarker & Haque, 2000), although Deshpande *et al.* (2006) claim a much higher value in a case-study.

Nonetheless, much modelling of spare-parts provisioning neglects the maintenance planning perspective, and vice versa. Thus, spare-parts inventory review papers to date tend to focus on the forecasting of demand for spares and to organize the review using forecasting methods (e.g. Boylan & Syntetos, 2010; Hu *et al.*, 2018; Nikolopoulos, 2021) or structure of the spare-parts supply chain (e.g. Basten & van Houtum, 2014), and review papers in maintenance are generally organized by system (engineered object) (Wang, 2002; Nicolai & Dekker, 2008; De Jonge & Scarf, 2020), strategy (Swanson, 2001), or risk (Ding & Kamaruddin, 2015) classifiers. There are exceptions: Kennedy *et al.* (2002) review models of maintenance inventories; Van Horenbeek *et al.* (2013a) classify joint maintenance-inventory optimisation papers; and Driessen *et al.* (2015) describe a framework for maintenance-informed spare-parts supply chain planning. We continue this line of work and put maintenance planning at the heart of inventory modelling for spare parts. Our approach is different however, because we not only use maintenance-policy classes for the classification of the papers that we review but also discuss standard inventory optimality rules that are also near-optimal in maintenance settings. We will show that drawing these two aspects (review and simple stock keeping rules) together is justified and timely because much mathematical modelling of the joint problem is becoming increasingly complicated, so that the gap between theory and practice is growing. A remedy, we think, is to present some simple stock keeping rules for the joint problem that bridge this gap.

The scope of this review is the mathematical modelling and analysis of inventory-maintenance-planning papers, so that this review belongs naturally in this journal (Mamon *et al.*, 2020). Nonetheless, we wish to provide a resource not only for those who conduct research into the modelling of engineering services, but also to practitioners seeking stock-keeping rules for the spare parts of the engineered objects they maintain. Thus, the review comes in two parts: (i) a classification of published papers for researchers, accompanied by some interesting open research problems (Section 2); and (ii) a discussion of practical stock-keeping rules for spare parts (Section 3).

1.1 *Background*

Research that studies both the spare-parts inventory problem and the maintenance problem has a long history. Mitchell (1962) noted that an early warning of failure is a characteristic of some items, and they recommended a specific inventory policy (hold no stock) for these items. Arguably, this work was the first not only to discuss the role of inspection maintenance in inventory-planning, but also to describe maintenance-informed spare-parts inventory planning. Wiggins (1967) and Falkner (1969) went on to study failure at random and hence random demand for spares. Thereafter, several authors considered demand driven by maintenance planning. Thomas & Osaki (1978) considered a simplification of the joint age-replacement, continuous review policy in which initially no stock is held, and one unit of stock is ordered on failure or when the operating unit reaches a particular age, whichever occurs sooner. Acharya *et al.* (1986) optimized co-incident, periodic block-replacement and inventory-review instants, where the commonality of the block-replacement interval and the inventory-review period keeps the analysis tractable. In a continuous review framework, it is mathematically convenient to partner continuous inventory review with age-based replacement, and an early work here is Kabir & Olayan (1996).

There was a considerable lapse in time until the development of models that use the maintenance schedule to improve forecasts of the demand for spares. Wang & Syntetos (2011) pioneered this approach. For a more recent paper on this topic, see Zhu *et al.* (2020). The use of base-information in inventory planning (Van der Auweraer *et al.*, 2019) is similarly asset-focused rather than spares-focused, although less specific about the maintenance policy. Base-information, for example, relates to the number, ages, and usages of engineered objects that may require spares (Dekker *et al.*, 2013). The next practical step in this direction is inventory planning that is informed by maintenance. Thus, Romeijnders *et al.* (2012) forecast the number of repairs required rather than the number of spares required per period and simultaneously models the number of spares used per repair. Operations plans can also inform spare-parts requirements (see e.g. Topan *et al.*, 2020).

On the other hand, there are models that generalize a maintenance policy using inventory as a justification for a generalization rather than explicitly modelling inventory itself. For example, Van Oosterom *et al*. (2014) model postponement of replacement (of a part) in an inspection-maintenance model, with a stock-out as a motivating example of postponement. Quality of spares (e.g. Scarf & Cavalcante, 2012; Cavalcante *et al*., 2018) is a related issue. These might be labelled as inventory-informed maintenance models.

1.2 *Scope and contribution*

The distinction between inventory planning informed by maintenance, joint planning, and maintenance planning informed by inventory provides a natural classification. However, the first and last of these three classes have vague boundaries, and so the assignment of papers to these classes is difficult. Therefore, we focus on papers in which there is modelling of the joint planning of maintenance and spare-parts inventory. Then, naturally, we use the characteristics of maintenance policies as one classifier and the characteristics of inventory policies as another. This classification is not the first review, but the breadth and depth of the papers that we classify is new, not least because the field is growing rapidly as the latest maintenance policy models are being extended to consider spare-parts inventory. Furthermore, this paper rebalances the review-literature on spare-parts inventory modelling by focusing on maintenance in the first place. We think it is important to do so because inventories of spare parts exist for maintenance.

Note, research that studies both production inventory (materials planning) and maintenance (e.g. Rezg *et al*., 2005; Peng & Van Houtum, 2016) is about a different set of problems that are outside our scope. Nonetheless, it is frequently difficult to distinguish papers that study this combination of materials-inventory and maintenance from papers that study spare-parts-inventory and maintenance, given that convenient search terms are *inventory* and *maintenance,* and that papers can at face value appear to be writing about the one when meaning the other.

Repair-kit problems (e.g. Prak *et al*., 2017; Saccani *et al*., 2017; Karabağ *et al*., 2020) are also out of scope. In such problems, engineers typically perform daily tours along various job sites, and carry in their vehicles a kit that consists of one or more units of several items that can be required to complete the service jobs. The repair-kit problem aims to find the optimal set of items to be stocked in the vehicle, and the order strategies for the different items. Additive manufacturing of spare parts for maintenance, a developing field (see e.g. Lolli *et al*. 2022), is also out of scope.

There are good reasons why review papers to date on inventory planning for spares take the forecasting perspective. Principally there are two. Firstly, most research on spare-parts inventory modelling is about forecasting—one need only compare the number of references in Hu *et al*. (2018) with the number of citations in our paper here to appreciate this point—and the authors of the review papers tend to be forecasters themselves. Secondly, inventory managers may ignore maintenance planning. They may be third-party inventory providers, for example, in which case the maintenance requirements of customers are unknown to them. They only perceive demand for spares, and this demand may be aggregated over a heterogeneous set of customers. When maintenance planning and inventory control are functions within the same organization, there would seem to us to be good reason that maintenance planning should drive the management of spare-parts inventory. Thus, whether one should organize the literature according to forecasting method or maintenance policy will depend on one's perspective. In this paper, we adopt the perspective of a maintainer who holds spare parts. This maintainer may be the operator, the OEM, or a third-party provider to whom maintenance and spares parts inventory are sub-contracted. These three are the maintenance-inventory networks discussed in literature that takes a broad view of engineering services (e.g. Basten & van Houtum, 2014; Arts *et al*., 2019). Furthermore, the decision-maker with this viewpoint may be interested to forecast requirements for spare parts based on historic demand or use known maintenance plans to determine future requirements for spares. This finer perspective will be determined broadly by how many systems the decision-maker maintains (the fleet size). We will focus then on engineered objects for which it is practical to simultaneously plan maintenance and inventory.

Maintainers who manage spare-parts inventories would likely argue that much of the research that coordinates maintenance planning and inventory planning is difficult to use in practice. This gap between industry and academia (Cavalieri *et al.*, 2008; Bacchetti & Saccani, 2012; Syntetos *et al.*, 2016; Siponen *et al.*, 2019) is not confined to inventory control and maintenance management, although Pintelon & Parodi-Herz (2008) argue that the gap is closing. Therefore, in our paper, we do more than just review the literature. We also present some practical stock-keeping rules for spare parts that are informed by maintenance planning, some of which are new and some existing but taken from a different context. In this way, we attempt to write a paper that is not only useful for academics in the fields of maintenance modelling and inventory modelling, but also directly useful to maintainers and inventory-keepers in industry.

Given the two-part approach (review, useful inventory rules) that we discuss above, the paper is organized as follows. Section 2 is the literature review, with subsections that describe the specifics of our classification and how the classification hierarchy is organized (2.1), discuss the methodology (2.2), classify the papers (2.3) (using the hierarchy described in Section 2.1) and discuss some interesting open research problems. Then, Section 3 considers near-optimal rules in maintenance settings, and discusses how these are motivated by the review in Section 2. We conclude the paper in Section 4.

2. The review

2.1 *Classification design*

For our classification, we will use a matrix structure, with rows corresponding to maintenance policy, and columns corresponding to inventory policy. Then, papers will be assigned to the elements of the matrix. This will allow the reader to understand:

1. the effort expended on a particular class of problems;
2. the temporal character of that effort because we shall list papers by date of publication;
3. where there has been little or no effort and hence opportunities for new research;
4. natural linkages between maintenance and inventory policies.

On this last point, for example, some classes are sparse or even empty because the joining of a particular maintenance policy with a particular inventory policy may be unnatural. An example is inspection-maintenance and a continuous-review inventory policy because typically inspections are planned and periodic.

The maintenance policy classes and inventory policy classes that we use have a practical basis. Nonetheless, empirical evidence about the types of maintenance policies that are used in industry is patchy. The most thorough extant study is by Alsyouf (2009) in which Swedish manufacturing industries are surveyed. Among the 185 companies in the sample, 50% of maintenance activity is planned maintenance activity, 37% is unplanned maintenance activity and the remaining 13% concerns the planning (management) of those activities, although 28% of the companies in the sample indicated that they had no maintenance strategy, which itself suggests that they did no maintenance planning.

Of the planned maintenance, about one third of activity is determined by OEM recommendations, one third is condition-based, and in the remainder modelling and experience are used to plan maintenance in equal measure. Other studies consider single cases. In railway track maintenance, 30% is failure-based maintenance (unplanned) and 70% is planned maintenance (Espling & Kumar, 2008). In a reverse-osmosis desalination plant in California, 50% of maintenance is unplanned although the operator is determined to reduce this to 25% (Van Rooij & Scarf, 2019). Of the other 50%, a major part is filter cleaning, which is planned on-condition (Van Rooij *et al.*, 2022). In a cement factory in Saudi Arabia, 75% is planned maintenance (Shafeek, 2012). In marginal oil fields in Nigeria, time-based preventive maintenance dominates and constitutes up to 40% of net operational expenses (Adenuga *et al.*, 2022). Furthermore, over time, maintainers have increasingly adopted planned, preventive maintenance policies (Pintelon & Parodi-Herz, 2008) and more so for safety-critical systems (Liyanage, 2008).

Consequently, empirical studies suggest three classes for maintenance policies: (i) unplanned; (ii) planned, preventive maintenance; and (iii) condition-based maintenance. In the first, maintenance is reactive and unscheduled and typically failure based (and often called corrective maintenance in practice and in the literature). We label this as failure-based maintenance (FBM). However, unplanned maintenance might be opportunistic, wherein an unplanned stoppage of a plant provides an opportunity for maintenance of some systems or units within the plant. New models and papers are emerging in this area. Therefore, we include the class of opportunity-based maintenance policies. We label this class CBM-O, for a reason we shall explain shortly. Planned, preventive maintenance is classically referred to as time-based maintenance, with actions specified well in advance in scheduled work-programs. System age or calendar time or loading cycles may determine the moment of a particular maintenance action for a particular system or unit, and so ‘time-based’ is something of a misnomer. However, we shall use the term time-based maintenance (TBM) and divide this into two sub-classes, age-based and block (time) based replacement, TBM-A and TBM-B, respectively. In the first, maintenance of a unit is scheduled on the basis of the age of the unit. In the latter, maintenance of a unit is scheduled periodically (e.g. every T days) regardless of the age of the unit. In condition-based maintenance, maintenance actions are dynamic, with typically the condition or state or estimated remaining life of a unit determining the appropriate moment for maintenance. Furthermore, condition-based maintenance can be simplified, wherein periodic monitoring might reveal whether an item is good or defective or even failed if the item is typically on stand-by. This is the class of models we call inspection maintenance (TBM-I), noting that inspections might be scheduled on the basis of system age or calendar time. We regard opportunity-based maintenance (CBM-O) as a derivative of condition-based maintenance (CBM) because, although the trigger for a maintenance intervention on a unit is unscheduled, and thus in a sense unplanned, the action (e.g. repair or replacement) is typically determined by the condition of the unit at the intervention. Condition-based maintenance can also be extended to plans that take account of both item condition and production schedule. Thus, both the condition of a unit and the demand for the function of the unit impact on the maintenance decision. This we call production-based maintenance with its own class (CBM-P). As shall be seen, papers in this area are sparse but emerging, because it is at the forefront of developments in the CBM field (De Jonge & Scarf, 2020). This class should not be confused with production-inventory planning (materials planning), which is out of scope. Production-based maintenance is, specifically, maintenance planning that prioritizes production, so that maintenance stops are planned for periods when production is not interrupted, e.g. maintaining a wind turbine when there is no wind (Yürüşen *et al.*, 2020).

Reviews papers on maintenance (e.g. Wang, 2002; De Jonge & Scarf, 2020) use similar classifications, although maintenance terminology is evolving, e.g. condition-based-, proactive-, predictive-maintenance. Thus, condition-based maintenance has evolved to mean a reactive intervention carried out once the condition of a unit crosses some boundary of acceptability, whereas predictive-maintenance forecasts when a unit will ‘fail’ eventually, and plans proactively. Nonetheless, such forecasts are based on the condition of a unit. Therefore, for simplicity, we group predictive-maintenance with condition-based maintenance in the class we label CBM-C.

We use empirical evidence to justify our maintenance classes above, and we consider this a necessity because this review is maintenance focused. Nonetheless, this leads to a classification that coincides with the historical development of maintenance models, from FBM (oldest) through to CBM-P (newest). Therefore, we think it is justifiable to take a chronological perspective on spare-parts inventory models, so that our classes are (i) initial provisioning policies (first buy), (ii) continuous review policies and (iii) periodic review policies. Class (iv), final provisioning (last buy), is a natural additional class, because matters of obsolescence, sustainability and the circular economy are topical. Thus, the age of the system ought to determine its maintenance requirements (Dwight *et al.*, 2012) and in turn its spare-parts requirements. Other classes could be used, such as the structure/hierarchy of the inventory. However, for example, single or multi-level inventory classes would be very unbalanced, since there has been little work that integrates maintenance planning in a multi-level inventory framework. We found three papers that are exceptions. Costantino *et al.* (2013) model a hierarchical inventory with random demands for spares, but in which maintenance intervention determines the level of repair and hence the precise level of spare-part demand (unit or component). Sleptchenko & van der Heijden (2016) jointly optimize the redundancy of components in a k-out-of-n system and the inventory of these components and the spare parts for the components (two levels). Sheng & Prescott (2019) consider a hierarchical inventory and cannibalization of parts in an involved simulation model. Nonetheless, the scarcity of papers that jointly consider hierarchical inventory and maintenance planning indicates that either mathematical modelling is too difficult in this case or there is considerable scope for new contributions.

Another potential classifier is demand-rate. Fast moving spare parts, such as consumables and spares for consumer products, lie at the one end of the demand-rate spectrum. However, there is less scope for maintenance planning to interact with inventory planning for these items. This is because, if the maintainer is the inventory keeper (e.g. machine tools), then established stock-keeping rules are effective because demand is not likely to be intermittent, and if the inventory keeper is a third-party logistics provider (e.g. automotive spare parts; Johnston *et al.*, 2011), the maintenance policy will not be in their control, although demand could be lumpy (Boylan & Syntetos, 2021). At the other end of the spectrum are very slow-moving parts. These are parts for which the likely future demand rate, taken over any period, is (almost) zero. These are usually parts for near-obsolete systems (see e.g. Mellal, 2020). Slow-moving parts (see, e.g., Klein Haneveld & Teunter, 1997) would be an intermediate class. The vast majority of papers that we review would lie in this middle class, so this classification is not very useful. This stated, another class could be added here, namely negative demands. These arise, for example, when mature or retiring systems are cannibalized (Diallo *et al.*, 2017), spare parts are recycled (Fisher & Brennan, 1986; Fisher, 1990; Sheng & Prescott, 2017; Dreyfuss *et al.*, 2018; Dreyfuss & Stulman, 2018; Gayialis *et al.*, 2022), or more generally in circular supply chains for life-extension of engineered objects (Fontana *et al.*, 2021; Wakiru *et al.*, 2021).

Finally, another classifier that we might use considers demand, but its nature rather than rate. We might classify models and hence papers as to whether demand (for spare parts) is largely deterministic or stochastic. However, in this case, the nature of demand is broadly determined by the nature of the maintenance policy. For example, inspection maintenance implies stochastic demand, because while the inspection time may be known in advance with certainty, the demand for spare parts will not (the purpose of inspection in a sense is to determine whether a spare part is required), although of course there may be some deviation from the schedule (Alotaibi *et al.*, 2020). Demand is determined by the states of parts inspected, which may become known only at the moment of inspection. Thus, there is imperfect advanced demand information (Poppe *et al.*, 2017; Van der Auweraer *et al.*, 2019). Also, a positive inspection (defect found) may not imply an immediate demand—replacement may be postponed (e.g. Van Oosterom *et al.*, 2014; Berrade *et al.*, 2017). On the other hand, time-based maintenance typically implies deterministic demand because replacements are scheduled, and failures are necessarily rare. We do not use this classification because of this strong association between type of demand and the maintenance policy.

Thus, to summarize, we classify papers by type of policy, for both maintenance and inventory, and there are practical and conventional reasons for doing so. Moreover, the perspective of the review is that of a maintainer and stock-keeper who wishes to co-plan maintenance and inventory for systems, young or old or retiring, that generate positive or negative demands for spare parts that are neither consumables nor fast-moving.

Next, we discuss our review methodology briefly: which journals we use; how far back we go; which databases we use. We cannot claim that this review is exhaustive, but we do claim that all the important papers to date in the field (that contain an element of spare-parts inventory modelling and an element of maintenance modelling) are reviewed.

2.2 *Methodology*

We used a keyword search. We did not restrict the date range. This is because we aimed to not miss the opportunity to take an historic perspective. Also, the simple stocking rules typically in early works were important to us for the practitioner element of this paper. We did restrict the scope of the journals. Only those in the fields of operational research, industrial engineering, reliability, and engineering management were considered. Specifically, we searched with the rule: find articles with at least one word in {‘maintenance’, ‘reliability’} *and* at least one word in {‘inventory’, ‘provisioning’, ‘spare parts’, ‘spares’, ‘spare parts’, ‘cannibalisation’}. For robustness, we used two search platforms: Scopus because it provides broader coverage than Web of Science, and Google Scholar because it allows fast downstream-searching (of articles that cite a discovered article). As the draft of our article evolved, we repeated the search to find the most recent articles. Also, we cross-checked our list (of articles found) against citations in recent, related review papers and citations in the very latest research papers.

Then, discovered articles were designated as in or out of scope. This procedure was sometimes necessarily subjective and based on reading an article. Thus, only coherent papers were included, not least because incoherent papers defy classification. Some papers are about maintenance and the manufacturing and distribution of spare parts (e.g. Frazzon *et al.*, 2014) and so are out of scope. As previously discussed, papers concerned with production-inventory (materials planning) and maintenance (of which there are many) are also out of scope. A paper was not classified if the planning-policy is too vaguely stated (e.g. Cheng & Tsao, 2010) or not specified at all. Thus, for example, in Romeijnders *et al.* (2012), component repair information is an important part of their methods, but specific maintenance policies are not studied. This provides a further argument for not considering forecasting-based methods in this review, as relevant papers do not explicitly consider a specific maintenance policy. As another example, Huiskonen (2001) uses qualitative maintenance-related information for inventory planning. On the other hand, in some papers, specific inventory policies are not studied. Spares-criticality studies are cases in point (e.g. Molenaers *et al.*, 2012; Antosz & Ratnayake, 2019). Papers that use simulation as the solution methodology (e.g. Lynch *et al.*, 2013; Alrabghi & Tiwari, 2016; Sharma *et al.*, 2017; Wakiru *et al.*, 2019) are often similarly difficult to classify. For brevity and accessibility, conference proceedings papers have been omitted.

Some papers study many models (e.g. Panagiotidou, 2014; Zahedi-Hosseini *et al.*, 2017) and could be assigned to more than one class. We only list a paper once in the classification table and choose what we consider to be the most relevant class for it.

2.3 *Classification of papers*

The outcome of our classification of 116 papers is Table 1. Note, the abbreviations used there for the maintenance policy (classification) imply a higher-level classification of maintenance polices into unplanned (FBM), planned (TBM-) and condition-based (CBM-). Also, papers that model a one-component system (or multiple, independent copies of a one-component system) are distinguished from papers that model a multi-component system. While in practice all systems are multi-component, in modelling, the notion of a one-component system often provides a useful simplification. In our classification, we regard a model as multi-component only if the components interact in a way that is structural, economic, or stochastic (see Dekker *et al.*, 1996), or logistical, but only if the logistical interaction is other than through a shared inventory of spare parts. Further, these dependencies may exist at the level of a unit or at the level of a fleet (of units) (Petchrompo & Parlikad, 2019).

2.4 *Discussion*

Let us pick up here some important points from the literature review. Note, we will not be giving a detailed commentary on what is in all papers, as it would be very repetitive and not particularly interesting. Table 1 does address the four main deliverables discussed in the beginning of Section 2. It does also allow further interesting practical insights and research implications to emerge, which are discussed in this sub-section.

Firstly, we broadly review the mathematical methods used in the reviewed papers as follows.

There are typically two stages in the mathematical analysis of the joint optimization of maintenance and inventory policies. Stage 1 is the formulation of an objective criterion that depends on model parameters and decision variables (e.g. maintenance interval, review period, order quantity, etc.). Stage 2 is the optimization, that is, the search for the best policy. Some broad patterns emerge:

1. FBM models (top row of Table 1) are approached using a range of methods. Early initial provisioning policies consider an analysis of costs over a finite horizon and optimization is straightforward because the decision space is small. The exceptions are Eruguz *et al.* (2018), which formulates the problem as a Markov decision process, and Marseguerra *et al.* (2005), which uses simulation. In the other FBM classes, with large decision spaces, a range of solution approaches are used: integer programming; mixed integer programming; dynamic programming. Particularly interesting is Demgne *et al.* (2017), which uses a very general piecewise deterministic Markov process (Davis, 1984).

2. Models in the TBM-A and TBM-B classes nearly always formulate a cost-rate using renewal-reward theory. Then, models with large decision spaces, which are more recent, are typically optimized using a stochastic search technique e.g. the genetic algorithm. Exceptions are Ruiz *et al.* (2020) and Abderrahmane *et al.* (2022) in TBM-B/periodic review, and Clavareau & Labeau (2009) and Block *et al.* (2019) in TBM-B/final provisioning, which use a finite planning horizon, noting that for the final provisioning problem it is natural to model events over a finite horizon.

3. The advancing sophistication of models is most apparent in the CBM classes. Ross (1969) pioneered the joint decision about condition-based replacement and spares provisioning using a discrete Markov chain with dynamic decision-making over a small decision space: replace

TABLE 1 *Classification by inventory policy and maintenance policy and system (one-component, multi-component)*

	Initial provisioning policy	Continuous review policy	Periodic review policy	Final provisioning policy
FBM	Wiggins (1967) Falkner (1969) Burton & Jacquette (1973) Tavares & Almeida (1983) Eruguz *et al*. (2018) Marseguerra *et al*. (2005) Zammori *et al*. (2020)	Park (1981) Al-Bahi (1993) Walker (1996) Demgne *et al*. (2017) Van Jaarsveld & Dekker (2011) Xie *et al*. (2014) Sleptchenko *et al*. (2018)	Dui *et al*. (2022) Bülbül *et al*. (2019)	
TBM-A	Mine & Kawai (1977) Thomas & Osaki (1978) Kaio & Osaki (1978) Kaio & Osaki (1979) Osaki *et al*. (1981) Sheu *et al*. (1992) Csenki (1998) Dohi *et al*. (2004) Godoy *et al*. (2014)	Kabir & Olayan (1996) Gan *et al*. (2015) Panagiotidou (2020) Chen *et al*. (2013) Jin *et al*. (2015) Yan *et al*. (2023)	Van Horenbeek *et al*. (2013b)	
TBM-B		Chelbi & Ait-Kadi (2001) Diallo *et al*. (2008) Sarker & Haque (2000) Ilgin & Tunali (2007) Shum & Gong (2007) Lynch *et al*. (2013)	Acharya *et al*. (1986) Yoo *et al*. (2001) Aät-Kadi *et al*. (2003) Samal & Pratihar (2015) Yang & Kang (2017) Siddique *et al*. (2018) Zhang *et al*. (2021a) Abderrahmane *et al*. (2022) Alamri & Mo (2022) Brezavscek & Hudoklin (2003) Huang *et al*. (2008) Jiang *et al*. (2015) Basten & Ryan (2019) Ruiz *et al*. (2020)	Clavareau & Labeau (2009) Block *et al*. (2019)

(*Continued*)

TABLE 1 *Continued*

	Initial provisioning policy	Continuous review policy	Periodic review policy	Final provisioning policy
TBM-I	Mitchell (1962) De Smidt-Destombes *et al.* (2007)	Vaughan (2005) Panagiotidou (2014) Yan *et al.* (2020)	Wang (2011) McNamara *et al.* (2017) Zahedi-Hosseini *et al.* (2017) Bjarnason *et al.* (2014) Zahedi-Hosseini *et al.* (2018) Salari & Makis (2020) Vincent *et al.* (2022)	Nguyen *et al.* (2013)
CBM-C	Ross (1969) Kawai (1983) Sung & Kim (1987) Elwany & Gebraeel (2008) Louit *et al.* (2011) Kian *et al.* (2019) De Smidt-Destombes *et al.* (2004) De Smidt-Destombes *et al.* (2009) Yan *et al.* (2022) Zhu *et al.* (2022)	Wang *et al.* (2009) Zhou *et al.* (2012) Liu *et al.* (2013) Wang *et al.* (2015) Cai *et al.* (2017a) Cai *et al.* (2017b) Poppe *et al.* (2017) Nguyen & Medjaher (2019) Zhao *et al.* (2020) Al Hanbali *et al.* (2022) Feng *et al.* (2023) Zheng *et al.* (2024) Olde Keizer *et al.* (2017) Farsi & Zio (2020) De Pater & Mitici (2021) Wang *et al.* (2021)	Wang *et al.* (2008) Li & Ryan (2011) Lin *et al.* (2017) Topan *et al.* (2018) Wang *et al.* (2018) Liu *et al.* (2019) Rodrigues & Yoneyama (2020) Zheng *et al.* (2023) Wang & Zhu (2021)	Wakiru *et al.* (2021) Zhu *et al.* (2022)
CBM-O		Zhang *et al.* (2022) Zhang & Zeng (2017) Zhang *et al.* (2019)	Nguyen *et al.* (2017) Zhang *et al.* (2018) Zhang *et al.* (2021b) Zhu & Zhou (2022) Zhu & Zhou (2023)	
CBM-P	Gan *et al.* (2021)	Rausch & Liao (2010)	Ba *et al.* (2016) Dellagi *et al.* (2020)	

Failure-based maintenance, FBM; age-based, TBM-A; block-replacement, TBM-B; inspection, TBM-I; condition-based, CBM-C; opportunistic, CBM-O; production-based, CBM-P or not and stock an additional spare or not. More recent works essentially consider the same decision space but using prognostic information (a forecast of residual life) and continuous, dynamic decision-making (e.g. Nguyen & Medjaher, 2019; De Pater & Mitici, 2021). Key in these approaches is the notion of a rolling decision-horizon, over which the consequences (typically costs) of decisions are evaluated. An important open issue is the choice of a suitable decision-horizon that balances short- and long-term goals (see e.g. Yatsenko & Hritonenko, 2017).

4. Again with reference to the CBM classes, two broad scenarios are modelled: one in which condition information arises at discrete times, e.g. at inspection points, and the other in which condition information arises continuously, e.g. using connected sensors (Pinciroli *et al*., 2023). For the first scenario, it is natural to review inventory periodically (CBM-C/periodic review). For the second scenario, a continuous review is natural. We might argue that the relevance of the latter class is prevailing over the latter as connectivity of engineered objects extends.

An emerging feature of this analysis of the methodology is the high complexity of the methods for the joint modelling and optimization problem. This is particularly apparent with the emergence of production-based maintenance (CBM-P), in which production (demand for the operation of the system), system health, and spare-parts provisioning are managed simultaneously. This increasing sophistication of models suggests a growing gap between modelling and application, between theory and practice. Thus, arising from our review, there is a case for specifying some simple models of joint inventory and maintenance planning with simple solutions. This is the link between the two parts of this paper: the review; and the simple stock keeping rules. There is perhaps also a case for researchers to revisit earlier, simpler models (e.g. Ross, 1969) for the purpose of building demonstrators.

Our second point is about usefulness. Our review is useful for two reasons: (i) because it indicates gaps in the literature where researchers might focus their future work, and (ii) because we have found that most of the papers we have examined are not useful for practitioners. This is because, generally, the focus of papers is typically on the optimisztion methodology for a specific problem rather than on implementation or managerial insights. This statement is based on carrying out this review and on our collective study of maintenance and inventory modelling over many decades. There is some evidence that much maintenance modelling is ineffective (Da Costa & Cavalcante, 2022). Nonetheless, a survey of practitioners to examine this claim would be timely. Furthermore, papers are typically published without demonstrators being made freely available to readers and no data are provided, although this is improving as journals develop their policies (Boylan, 2016).

Then, in our view, should the practitioner either: use some simple rule (for planning); or build a simulator, a digital twin (DT) (see e.g. Errandonea *et al*., 2020) for their specific problem. In the latter case, the DT might be used to investigate some simple rules to see which work best (offline decision support), or it might be embedded in a control tower (e.g. Gerrits *et al*., 2022) and used to aid decision-making in real time (online decision support), or it could be used as an artificial environment in which to train an autonomous decision-maker (autonomous or semi-autonomous decision support). In Section 3 we help with the former: simple, practical rules. Related to this, our message to graduate students would be that they should focus more on the pathway to implementation of a modelling solution and less on developing newer and more complex models. Equivalently, they might try to solve real problems rather than develop exact solutions to artificial problems.

Next, a point about development of joint policy modelling over time. Early polices considered the simple decision of whether to hold a spare or not, for a unit that may or may not fail, either over an indefinite time scale (FBM) or prior to time (age) T, when a unit would be maintained, and a spare required. In this category are the so called ‘ordering policies’ of Osaki *et al*. that are extensions of

TBM-A, in which the decision maker chooses when to order a spare part for an ageing unit. See Csenki (1998) for a full review of these works. These policies also model the question of when a spare should be put in service, on arrival or on failure? If on arrival, then notice that there is never stock on-hand. Incidentally, these articles are not very well cited, but there are many of them. A nice extension presents itself here, whereby the part is repaired, and repaired spares are held in stock. Then, one must trade off the benefit of repairing against holding cost.

Next, a point about one-component versus multi-component models. TBM-A is natural for one-component systems, but in a multi-component context replacement decisions become unsynchronized. Thus, the 'ordering policies' discussed in the previous paragraph all consider the one-component case. Nonetheless, such asynchronization may be accepted by the maintainer, so that there exist some studies that we have classified as: TBM-A, multi-component.

On natural partnering, CBM-C (the most researched) may partner with continuous inventory review when units are continuously monitored (particularly in the Industry 4.0 era) or with periodic review when units are periodically monitored. Also, there are many more studies of TBM-B with periodic inventory review than continuous inventory review. Since TBM-B is periodic by definition, one might argue that it is unnatural to pair TBM-B with continuous review, so that while analytically approaching this pairing may be mathematically interesting, its practical value may be limited. TBM-A on the other hand is not 'periodic' because a failure of a part resets the schedule for the preventive replacement of that part. The older is the system, the less synchronized are the age-based replacements of the individual parts. For example, in a fleet of commuter trains aged 30 years, the traction motors, if changed-out at age six years, would have moderately heterogeneous ages (Scarf *et al.*, 2009). Thus, to the stock-keeper, demand for parts will look like that for block replacement (TBM-B) in the early life of the system and become more random as the system ages. In the limit, the demand will be pure Poisson. A fixed ordering cost incentivizes synchronization. On the other hand, as age-based replacement is known to be cost-efficient relative to block replacement (Dekker, 2014), asynchronization is preferred in the long run. Thus, this creates an interesting trade-off that to our knowledge has not been studied.

When maintenance is scheduled by time as in block replacement (TBM-B), a sensible inventory policy orders sufficient parts in advance of the next scheduled block replacement so that the parts arrive just-in-time. Additional insurance stock or emergency orders are placed to respond to failures. A more interesting case is when the number of replacements at the scheduled maintenance moment is unknown. This issue arises in periodic inspection policies (TBM-I). Inspections are periodic, but units are replaced only if the inspection is positive (fault found). The idea here is that a unit operates when it is faulty although failure is imminent. Then, it is not clear how many parts to hold or to order just-in-time, and the advance demand information is uncertain. If there is a stock-out, postponement may be possible (Van Oosterom *et al.*, 2014; Berrade *et al.*, 2017) or replacement may be subject to defaulting (Alotaibi *et al.*, 2020). Anyway, determining a best inventory policy may make an interesting research study, although the notion that particular types of parts have a warning period is not a new idea (Mitchell, 1962). The modified-block replacement policy (Berg & Epstein, 1976) also implies uncertain advance demand information because this maintenance policy only replaces older units at the maintenance epochs and the ages of units are revealed only at the maintenance epochs.

We now wish to make two points about inventory policies when maintenance is condition-based (CBM-C). Firstly, for slow moving parts, which may be slow moving because they are high value parts in high value systems, predictive maintenance predicts when a part is required and so inventory requirements can be planned accordingly. Thus, if at time t the estimated remaining useful life (Banjevic, 2009) of a part is $\hat{x}$, so that failure is predicted to occur at time $\hat{x} + t$, then the part might be ordered at time $\hat{x} + t - L$ (provided $L < \hat{x}$). Here, L is the lead-time. If L is very long, this simple stock-keeping rule cannot be used. Then, the lead-time should drive the specification of the predictive maintenance policy and not the remaining-useful-life. This is an open avenue for new research. Secondly, when parts are not slow moving, for example when there are many copies of the same system all subject to monitoring and predictive maintenance, replacements will be asynchronous. Then, to the stock-keeper, maintenance will appear to be unscheduled even though it is most carefully planned. Whence the simple rules that relate to unplanned maintenance (FBM) might be used, wherein the events that require stock arise at random so that Poisson demand is assumed and the rate of demand (or equivalently the mean time between failure), along with the inventory parameters, determines the inventory policy, provided demand is not too low on average. The case of low demand (slow-moving parts) is considered by Klein Haneveld & Teunter (1997). When demand is very low (very slow-moving parts) (see Wall, 2013 for a real example at a steel works), then a binary stock policy might be used (e.g. Tavares & Almeida, 1983) or when there is very little information about the demand rate (Burton & Jacquette, 1973).

Finally, some further under-researched areas are apparent when viewing our classification. Joint maintenance-planning and final provisioning studies are few, and there are opportunities for gains here because inventories of unused parts are wasteful (Boylan & Syntetos, 2021). CBM-P is new when considering maintenance alone, so it is not surprising that there are not many papers that jointly model production-based maintenance and spare-parts inventory. CBM-O and CBM-P share similar characteristics: maintenance is typically scheduled at times when a unit is down. They are however different because in CBM-O such stoppages occur randomly, so that stock must be held (or carried if service agents do maintenance) in the anticipation of demand.

3. Simple stock keeping rules

It has become clear from the literature review is that there is a lack of simple stocking rules for spare parts in maintenance settings. Therefore, in the remainder of the paper, we seek to close this gap, and to propose such rules that lend themselves to development as demonstrators and practical application. We first explain why stocking rules in maintenance settings are more difficult to optimize exactly than in traditional inventory control models (Section 3.1). This discussion also provides insight into when standard inventory optimality rules are also near-optimal in maintenance settings. Thereafter, we will discuss the specific setting that we consider (Section 3.2) and derive near-optimal rules (Section 3.3).

Linking to the classification in the review part of this paper and Table 1 in particular, the rules that we will propose apply in the first place to the continuous review and periodic inventory review policies, as these consider reordering decision without a specific beginning or end. To some degree, they can also be applied to initial provisioning decisions, but they do not apply to final provisioning decisions. Furthermore, the rules are flexible enough to deal with all types of maintenance policies. This will be discussed further after presenting the rules.

3.1 *Maintenance vs standard inventory control*

In a maintenance setting, if spare parts are backordered, then this is likely to delay maintenance actions. So, longer backordering durations imply longer maintenance delays, which, in turn, lead to a lower relative uptime (vs downtime) and so to fewer demands per time unit. Hence, spare-parts stocking rules not only affect holding and backordering (standstill) costs per time unit, but also the purchase costs per time unit.

This is different from (standard) inventory control theory (for consumables), where the inventory policy does not affect future demand. In standard inventory control models with backordering, the number of ordered items per time unit equals the number of demands per unit, and this is not affected by the stocking rule. As a result, when optimizing the stocking rule, purchase costs need not be considered. Moreover, given an order quantity, the number of replenishments per time unit and corresponding replenishment costs also need not be considered. Therefore, the optimal stocking rule can be determined by minimizing the holding and backordering costs per time unit.

Therefore, exact cost minimization of spare-parts stocking policies in a maintenance setting is more complex than cost minimization for inventory systems of consumables. However, if the expected (life)time between maintenance actions is much larger than the replenishment lead time of parts, then the effect of the stocking rule on the purchase costs is expected to be minor. This is all the more so since machine standstills are typically very expensive and so backordering durations for (near) optimal spare-parts stocking rules are expected to be small. Hence, we can still build on known results from inventory control theory to derive approximately optimal spare-parts stocking rules for the maintenance context, which we will do. We remark that we need to generalize and adapt those known results to the maintenance context, as will be clarified in the analysis that follows in Section 3.3. Before doing so, in Section 3.2, we describe the considered maintenance setting in detail.

3.2 *Setting*

In line with both the spare-parts management and the more general inventory control literature, we will consider two ways of modelling the backordering costs. This is either as a cost per backorder per time unit, b_1, or as a cost per backorder (independent of the length of a backorder), b_2. We remark that a combination is also possible and arguably most realistic, but seldom considered. Here, for the sake of exposition, we also consider them separately. It is natural to assume that holding costs, h, are incurred per unit (on hand) and per time unit.

We consider a deterministic replenishment lead time, L. In fact, it is relatively easy to modify the analysis in Section 4.3 to the case of stochastic lead times, under the assumption that orders do not cross; that is, if orders arrive in the same sequence that they are placed. Especially in a spare-parts setting, where specific parts are typically bought from a single supplier, this is realistic. In such a situation, replenishment lead times may well vary, but orders are still unlikely to cross. For a further discussion on the effects of stochastic lead times and order crossing on inventory control, we refer interested readers to Axsäter (2015).

The two most discussed types of stocking rules in the inventory control literature are the so-called (R, Q) and (s, S) policies. Both place orders based on the Inventory Position (IP), calculated as the stock on hand plus stock on order minus backorders. Both place orders if the IP drops to or below an order level (R and s, respectively). They differ in that the former uses a fixed replenishment quantity Q and the latter a variable quantity that brings the IP back up to S. Note that if demands are always for a single item, then both policy types are equivalent when setting $R = s$ and $Q = S - s$. Generally, without the need for a constant replenishment quantity, (s, S) policies typically outperform (R, Q) policies and indeed under relatively mild assumptions the optimal policy is of that type (see Axsäter, 2015). As transpired from our review, they have also been considered most often for spare-parts management. Indeed, a special case with $s = S - 1$ is often considered, which keeps the IP at a constant level, S. This is called a base-stock of order-up-to policy. Spare parts are often slow moving and expensive, and order batching is not needed for such parts, explaining the popularity of the base-stock policy. We will also restrict our analysis in Section 3.3 to this policy, but a similar analysis could be done for the more general policy types.

3.3 *Simple near-optimal spare-parts stocking rules*

A key relation in our analysis and that of inventory control models in general, presented next, relates the inventory level (IL) (on hand minus backorders) to the inventory position (IP). Recall that the IP drives stocking decisions, but the inventory level determines the number of parts on hand or backordered, and thereby the holding or backordering costs.

This key relation is as follows: the inventory level (on hand minus backorders) at any time t, $I_{\mathrm{L}}(t)$, is equal to the inventory position at time $t-L$, $I_{\mathrm{P}}(t-L)$, minus the lead time demand, D_{L}, in period $[t-L,t]$. That is, $I_{\mathrm{L}}(t) = I_{\mathrm{P}}(t-L) - D_{\mathrm{L}}$. The underlying logic is that any orders placed at or before time $t-L$ and so included in $I_{\mathrm{P}}(t-L)$ have arrived by time t and so ‘add to’ $I_{\mathrm{L}}(t)$, that later orders have not yet arrived, and that all demands in period $[t-L,t]$ are ‘taken from’ $I_{\mathrm{L}}(t)$. Note that for the base-stock policy that we consider with a constant IP level of S, the above discussed relation simplifies to $I_{\mathrm{L}}(t) = S - D_{\mathrm{L}}$.

The goal is to find the value of S that (nearly) minimizes the combined holding costs and the backorder costs. Recall that we consider backorder costs that are either per backordered demand per time unit (b_1) or per backorder demand independent of the delay (b_2). The former is arguably the most realistic in maintenance settings as ordering delays cause downtime. However, there may also be situations where the latter type of backorder cost modelling is more suitable, e.g. because any delay asks for replanning or emergency measures/procedures. We will therefore consider both backorder cost formulations and the differences in optimality conditions will also be insightful.

We realize that although the most suitable type of backorder cost can be determined based on general consequences of a shortage of parts, estimating the exact backorder cost parameter (b_1 or b_2) can be very difficult. For instance, in case of redundancy, it is not clear how large backlog costs are, since a delayed maintenance activity implies an increased risk of downtime rather than immediate downtime. As another example, if multiple parts are short but this can be resolved by a single emergency procedure (e.g. flying them in from a central warehouse), then it is difficult to transform emergency costs to single item backorder costs. A detailed analysis of setting backorder costs is relevant and deserves further research, but is outside the scope of this paper.

3.3.1 *Cost per backorder per time unit backordered (b_1).* As remarked in Section 3.1, we build on results from the inventory control literature to derive near optimality conditions. Let us first consider a cost per backorder per time unit backordered, b_1, independent of the length of a backorder. For this case, Axsäter (2015) derives a base-stock level optimality condition but restricted to a Poisson demand process. For standard inventory control systems (of consumables), assuming Poisson demand is natural and indeed this assumption is often made without further discussion. In a maintenance setting, however, it is important to also consider other demand processes, because (i) part of the demand may be connected to planned/preventive maintenance activities and (ii) even demand resulting from corrective maintenance activities may not be memoryless if, for example, maintenance restores the quality of an item to as-good-as-new. Therefore, next, we generalize the result of Axsäter (2015) to an arbitrary demand process.

Consider an arbitrary time t. Using $I_{\mathrm{L}}(t) = S - D_{\mathrm{L}}$, we obtain that the expected cost per time unit is given by

$$C_1(S) = h\sum_{k=1}^{S} \Pr\left(D_{\mathrm{L}} = k\right)(S-k) + b_1\sum_{k=S+1}^{\infty} \Pr\left(D_{\mathrm{L}} = k\right)(k-S).$$

From this, we easily find that

$$\begin{aligned} C_1(S+1) - C_1(S) &= h\Pr\left(D_{\mathrm{L}} \le S\right) - b_1 \Pr\left(D_{\mathrm{L}} > S\right) \\ &= h - \left(h + b_1\right)\Pr\left(D_{\mathrm{L}} > S\right). \end{aligned}$$

This marginal cost is increasing and so the optimal base-stock level is the smallest value of S for which $C_1(S+1) - C_1(S)$ is positive, that is, for which

$$\Pr\left(D_{\mathrm{L}} > S\right) < \frac{h}{h+b_1} = 1/\left(1 + b_1/h\right). \tag{1}$$

3.3.2 *Cost per backorder* (b_2). Next, we consider a cost b_2 per backorder, independent of the length of a backorder. The holding cost per time unit is unchanged from the above analysis with a duration dependent backorder cost. The backorder cost is analyzed differently because the (average) number of backorders at an arbitrary time is no longer relevant. Instead, we observe that an ordered part is used to satisfy a backorder (and not a demand immediately) if and only if the lead time demand (not including the demand that triggered the order, since the IP is raised to S after that demand) is at least S, because then the ordered part was needed before it arrives. Note that the considered part is still late if the lead time demand is exactly S, because the IP dropped to S -1 when it was ordered. So, the probability that an ordered part arrives too late is $\Pr\left(D_{\mathrm{L}} \ge S\right)$. Denoting the demand rate (that is, the number of demands per time unit) byμ, we get that the number of backordered demands per time unit is $\mu \Pr\left(D_{\mathrm{L}} \ge S\right)$with a corresponding cost rate of $b_2\mu \Pr\left(D_{\mathrm{L}} \ge S\right)$. The expected holding cost per time unit was already derived in Section 3.3.1, and hence that the total expected cost per time unit is given by

$$C_2(S) = h\sum\nolimits_{k=1}^{S} \Pr\left(D_{\mathrm{L}} = k\right)(S-k) + b_2\mu \Pr\left(D_{\mathrm{L}} \ge S\right).$$

From this, we obtain

$$C_2(S+1) - C_2(S) = h\Pr\left(D_{\mathrm{L}} \le S\right) - b_2\mu \Pr\left(D_{\mathrm{L}} = S\right).$$

Note that this marginal cost is not necessarily decreasing, implying that the cost function may not be convex in the base-stock-level. To understand this, let us consider a special situation where the lead time demand is likely to either be very small (say less than 3 in regular conditions) or very large (say more than 10). Then base-stock levels of up to 3 or more than 10 make sense, covering for regular conditions only or emergency conditions as well, respectively. However, base-stock levels of 4–9 are clearly suboptimal.

Nevertheless, in most realistic situations, backorders are expensive relative to holding parts on hand, and so the optimal S is expected to be considerably larger than the expected lead time demand. That is, the optimal value for S is expected to be in the right-hand tail of the lead time demand distribution, where the probabilities are typically decreasing. So, for the ‘relevant range’, the marginal cost is in fact expected to be increasing. If so, then the optimal base-stock level is (again) the smallest value of S for which $C_2(S+1) - C_2(S)$ is positive.

Indeed, building on this logic that $\Pr\left(D_{\mathrm{L}} \le S\right)$ is close to one (i.e., we consider the right-hand tail of the lead time demand distribution) for the optimal base-stock level, we can approximate the marginal cost as

$$C_2(S+1) - C_2(S) \approx h - b_2\mu \Pr\left(D_{\mathrm{L}} = S\right).$$

From this, we an approximation for the optimal base-stock level as the smallest value of S for which

$$\Pr\left(D_{\mathrm{L}} = S\right) < \frac{h}{b_2\mu}. \tag{2}$$

Assessing the exact quality of this approximation is beyond the scope of this research. Indeed, it would differ per type of demand process. What is clear is that the approximation improves as the backorder cost increases—for high backorder costs (relative to holding costs), having parts on hand most of the time is optimal, implying that increasing the order-up-to level by one unit means having one more unit on hand for most of the time, and so increases the holding costs by approximately h, as underlies (2).

3.3.3 *Discussion.* Comparing optimality conditions (1) and (2), there are clear similarities, but also striking differences. Both show that, for determining the optimal base-stock level for spare parts in a maintenance context, the holding to backorder cost-ratio matters rather than the two costs separately, which is intuitive. The left-hand side differences reflect that a backorder cost per time unit underlies (1), and a cost per backorder underlies (2). For a backorder cost per time unit, increasing the base-stock level pays off as long as that reduces backorder durations (and thereby the expected number of backorders at an arbitrary time); and so there is a benefit of raising the base-stock level to $S+1$ whenever$\Pr\left(D_{\mathrm{L}} > S\right)$. For a cost per backorder (independent of the duration), there is only a benefit for avoiding backorders completely, and this is only achieved for a specific order if the lead time demand is exactly S; if lead time demand is $S+1$ or more, then the ordered items arrives late even if the IP increased to $S+1$ when it was placed.

Linking to the classification in the review part of this paper, it is clear from the (approximate) analysis underlying (1) and (2) that these conditions apply in the first place to continuous and periodic inventory review policies, as they consider the reordering of spare parts without considering when to start or stop ordering. However, they can also be applied for initial provisioning decisions. After all, besides the relevant cost parameters, all that is needed is an estimate for the (initial) lead-time demand distribution. The difficulty, obviously, is that such an estimate is difficult to obtain without historic demand information. For final provisioning decisions, other rules need to be developed that take the risk of obsolescence into account. Surprisingly, despite the relevance of obsolescence and waste avoidance for inventory systems in general, to the best of our knowledge no simple reordering rules have been developed in the inventory control literature.

As to the different maintenance policy types (see Table 1), the optimality conditions (1) and (2) can, in principle, be applied to all of them, as long as the lead time demand distribution is estimated accordingly. For instance, under a perfect (without unexpected failures) age-based (TBM-A) policy, spare parts can be ordered L periods ahead so that they arrive just-in-time; for condition-based (CBM-C), the lead time demand distribution, and thereby the base-stock level, should be continuously updated based on condition information; etc.

4. Conclusions

We have reviewed the literature on mathematical modelling and analysis of inventory-maintenance-planning. Spare-parts inventories exist to facilitate maintenance, and no maintenance can take place unless there are spare parts is stock. It is obvious that the interface between the two determines the performance of the system, and yet emphasis is being put predominantly on optimizing each constituent part in isolation from the other.

The literature review led to the conclusion that most published work to date is not useful for practitioners. We have then naturally also been concerned with practical implementation and what that entails in terms of relevant rules and their operationalization. We have discussed why stocking rules in maintenance settings are more difficult to optimize exactly than in traditional inventory control models, and subsequently suggested simple near-optimal spare-parts stocking rules that should be valuable in practice. These rules are flexible and can in principle be applied for all types of maintenance strategies (time-based, condition-based, etc.) and for initial provisioning as well as continuous/periodic reordering decisions. They cannot be used for final provisioning decisions; there is a need to develop simple rules for such situations as well, for spare-parts management and also for inventory control in general.

Joint planning of maintenance and spare-parts inventory supposes that maintenance managers and inventory managers are coordinating decisions. In practice, maintenance and inventory planning takes place in different departments. These departments may not share information and may have conflicting objectives. We think therefore that modellers should appreciate this point as they work to develop new models. A further difficulty is the lack of integration of maintenance and inventory computer systems. Thus, there is a role that software manufacturers can play in closing the gap between maintenance optimisation and spare-parts inventory provisioning. ERP packages and manufacturing software have not massively evolved over the years, and they are often judged to be ineffective in handling joint (integrated) maintenance-inventory tasks. Original equipment manufacturers (OEMs) and software manufacturers (SMs) have crucial roles to play in closing the gap between theory and practice in this area. OEMs should, when systems are procured, specify the parts required for specific maintenance activities, and SMs should evolve their systems to encode simple rules, such as those discussed in Section 4.

In reviewing the literature, we have also attempted to identify useful lines of further enquiry, areas where relevant research can be conducted. This is particularly important given the previously discussed lack of simple stocking rules for spare parts in maintenance settings. This obviously impedes progress in practice, where simplicity and robustness are of paramount importance.

Let us close this paper with a final, perhaps repetitive, but very important general point about the value of looking at joint operational problems like the maintenance-inventory one. There is an increasing recognition of the fact that real-world decision making is far more involved than academic studies assume, whereby problems are decomposed into constituent components (in this case maintenance and inventory) and studied separately. The same has been happening with inventory-forecasting, or, until recently, with transportation-inventory, etc. In that respect, approximate simple solutions to real complex problems are much more preferable than exact solutions to problems that are made up (since they are not realistic at all). All three authors of this paper take the view that unless operational researchers are willing to sacrifice elegant proofs for relevance and contribution, progress will be impeded.

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There are no data associated with the work described in the paper.

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