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In-house or outsourcing? The impact of remanufacturing strategies on the dynamics of component remanufacturing systems under lifecycle demand and returns

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Abstract

We consider a component manufacturing and remanufacturing system where, due to the end-of-life warranty, new and after-sales demand must be satisfied. Two kinds of demand exhibit different lifecycle patterns with different scales and a time lag, while a third correlated component return lifecycle with again a different lag and scale, driven by adoption of remanufacturing, is also presented. To achieve supply and demand balance during demand lifecycles, companies need a strategic decision on their remanufacturing: remanufacturing outsourcing strategy (ROS) or remanufacturing in-house strategy (RIS), yet inadequately studied from system dynamics perspective. We developed base-stock system dynamics models and analytically explored the dynamic implications of RIS and ROS remanufacturing strategies under correlated lifecycle demand and returns. Applying z-transform and discrete time simulation, we found that RIS outperforms ROS system including less peak capacity cost, less inventory holding cost and less backlog cost. Also, the bullwhip of the RIS is always less than the ROS system. However, the adoption of the RIS may result longer-lasting manufacturing production and thus lead to a higher cost: an important cost needs to be strategically considered. Thereby, from system dynamics perspective, the component manufacturer needs carefully consider trade-offs between production and inventory costs, as well as their demand lifecycle characteristics to choose the right remanufacturing strategy.

Keyword: *System dynamics, Remanufacturing in-house and outsourcing, Lifecycle demand and returns, Bullwhip effect, z-transform*

1. Introduction

We consider a component manufacturing and remanufacturing system, e.g. car engine production in the automotive industry, where the new sales and after-sales demand (or service demand) of components , as the result of end-of-life (EoL) warranty, must be satisfied by the original equipment manufacturers (OEMs). Two kinds of demand exhibit two different lifecycle patterns with a lag and different scales. Traditionally, a last time buy/batch production is executed at EoL for matching the uncertain after-sales demand during the warranty period (Behfard et al. 2015). However, such decisions can inevitably create very high EoL stocks with high holding costs, which lock up a lot of capital, often have limited lifetimes and can be obsolescent if actual demand is lower than that forecast. On the other hand, if OEMs underestimate the demand for aftermarket support, it could lose sales and disappoint customers (Spengler et al. 2003). Remanufacturing is an attractive alternative solution to simultaneously minimize inventory, maintain high customer service level and create sustainable social and economic value (Jia et al. 2016; Dominguez et al. 2019). Remanufacturing is an EoL recovery option whereby returned products are disassembled, cleaned, inspected, and then are finally reassembled and tested to restore them as-good-as-new products (Dominguez et al. 2019). Given remanufacturing uses returned component as the fed-in materials, there is a third lifecycle, i.e., return component lifecycle, with different time spans and delays.

To achieve supply and demand balance during demand lifecycles, the manufacturer often faces a strategic decision for their remanufacturing production: whether to manage remanufacturing production in-house or outsource to a third-party remanufacturer (TPR), in other words, decide on *remanufacturing in-house strategy (RIS)* or *remanufacturing outsourcing strategy (ROS)* (Martin et al. 2010; Hallak et al. 2021; Liu et al. 2022; Niu et al. 2022). This is particular the case when the dynamics of production and inventory, as result of nonstationary lifecycle-based demand and the return process, should be considered. This is because the costs of inventory and production are dynamically changing due to different phases of lifecycle patterns from market infancy, growth, maturity and saturation/decline (Georgiadis et al. 2006; Östlin et al. 2009; Georgiadis and Athanasiou 2013; Jia et al. 2016). For instance, the manufacturing production cost increases as the component sales move from early growth to the late decline stage of the lifecycle, since economies of scale is achieved during the growth and maturity stages of the lifecycle. Remanufacturing production cost, however, is decreasing in after-sales demand lifecycle, from high remanufacturing cost in the early growth stage, with limited returned component supply, to low remanufacturing cost at later stages of after-sales demand lifecycle, that sees increased returned components (Östlin et al. 2009; Georgiadis and Athanasiou 2010).

Furthermore, short-term stationary demand uncertainty, e.g. bullwhip effect (BE) related costs (Wang and Disney 2016), play a role in influencing remanufacturing strategies decision in CLSCs (Lin et al.2022). However, bullwhip behaviour driven by different remanufacturing strategies is not fully understood (Goltsos et al. 2019). Most of the relevant studies compare the traditional open loop supply chains and CLSCs by adopting in-house remanufacturing strategy (Hosoda et al. 2018; Borja et al. 2019; Lin et al. 2022). This ignores the practical prevalence that remanufacturing can be outsourced to the independent TPR (Zou et al. 2016; Niu et al. 2022). Also, the dynamic performance derived in responding short-term stationary demand patterns, e.g. Independent Identically Distribution (i.i.d.), may not be appliable for the long-term lifecycle demand patterns with returns correlation. This leads to challenges in making the strategic decision about remanufacturing when facing long-term lifecycle demand and correlated return patterns.

Motivated by academic gaps and practical observations, this paper aims to explore the system dynamics of remanufacturing in-house and outsourcing strategies under non-stationary and stationary lifecycle demand. By adapting the industrially recognized base stock policy, two system dynamics models are developed to represent the IRS and ROS systems. We advanced the closed-loop supply chain dynamics literature by analytically comparing the dynamic performance of two main remanufacturing strategies commonly observed in practice. This extends the literature focusing only on the dynamics of the in-house remanufacturing. We analytically derive the impact of system control and non-control parameters on the dynamic performance of both systems, highlighting the trade-offs between production and inventory costs, as well as demand lifecycle characteristics to choose the right remanufacturing strategy.

2. Literature review

Our study is related to two streams of literature: remanufacturing sourcing strategies and remanufacturing dynamics in closed loop supply chains.

2.1. Remanufacturing sourcing strategy in closed loop supply chains

A comprehensive review of closed loop supply chain by Govinda et al. (2015) indicated that outsourcing vs. in-house remanufacturing decision is an important topic. On one hand, manufacturers can invest in-house remanufacturing to provide customers both new and remanufactured products. For instance, Abdulrahman et al. (2015) found Chinese leading auto-part companies prefer in-house remanufacturing than outsourcing. On the other hand, many OEMs prefer to outsource their remanufacturing to an independent remanufacturer who possesses specialized technology and dedicated facilities for remanufacturing. A good example is Caterpillar Remanufacturing, who as a supplier offers remanufacturing products and service for Land Rover (Zou et al. 2016).

A number of researches focus on this issue in recent years gave the significant growth of remanufacturing volume in various industries. Using the industrial quantitative and qualitative data, Martin et al. (2010) suggested intellectual property, operational assets, and remanufacturing frequency are significant drivers of the remanufacturing re-make versus buy decision, while remanufacturing volume, condition, technological uncertainties present less impact. Zou et al. (2016) showed that the OEM obtains higher profits through outsourcing than remanufacturing in-house. Wang et al. (2017) found remanufacturing cost drives the in-house or outsourcing decisions, i.e., a higher fixed cost of in-house remanufacturing favours outsourcing. Hallak et al. (2020) studied the outsourcing and in-house recycling decision via a continuous (r, Q) re-ordering policy for single-item inventory systems with stochastic demand and recycling. They indicated that in-house recycling outperforms outsourcing when the proportion of recovered item is high. Liu et al. (2022) investigated different remanufacturing strategies under selling and renting considerations. They found that manufacturing costs critically impact on selling, renting decision and remanufacturing decisions. Fang et al. (2023) compared two remanufacturing strategies based on TPR's cost and quality advantages. In-house remanufacturing was proved as a better choice than outsourcing if TPRs' advantages and the customers' perception of returned quality are unobvious.

Another stream explored the impact of products (new and remanufactured) or supply chain party competitions on the choice of remanufacturing production strategies. For example, Niu et al. (2022) considered a retailer's make or buy remanufacturing decisions under in-store competition and in-house remanufacturing yield rate uncertainty. The results showed that the remanufacturing technology and customer's valuation play a role in influencing outsourcing decisions. Chen et al. (2022) compared outsourcing and inhouse collection strategy when the dynamic long-term remanufacturing technology development is considered. A game model, including one manufacturer, one remanufacturer and one retailer, was developed and system dynamics was adopted to identify the trade-off in the choice of collection strategies.

To summarize, literature often compares in-house and outsourced remanufacturing strategies by considering static costs and remanufacturing uncertainties. Yet limited study offered the implication of dynamics on the selection of different remanufacturing strategies. Below we review system dynamics literature, focusing on the dynamics of remanufacturing and closed loop supply chains to further highlight research gaps and our contributions.

2.2. Remanufacturing dynamics in closed loop supply chains

The System Dynamics discipline, pioneered by Prof. Jay Forrester (Forrester 1961), aims to facilitate the understanding of system structures and its impact on dynamic behaviour in various systems. In the context of the dynamics of remanufacturing in closed loop supply chains (CLSCs), the authors refer to Goltsos et al (2019) for a comprehensive review. Overall, there are two research streams. First, several works focus on the single-echelon, hybrid manufacturing and remanufacturing systems and analytically explore the impact of system structure and parameters on bullwhip and inventory variance. Starting with Tang and Naim (2004), who investigated a single echelon, push-based hybrid system and explored the impact of different information sharing mechanisms on bullwhip and inventory variance. Under the similar system setting, Zhou and Disney (2006) derived an order variance ratio measure using Åström's method（Åström 2012）. They found that the return rate plays a significant role in influencing bullwhip and inventory variance, while this is not the case for remanufacturing lead times. Hosoda et al. (2015) studied the impact of information sharing, random yield, correlation, and lead times on the dynamics of a single echelon hybrid manufacturing and remanufacturing system when demand and returns are stochastic and correlated. Hosoda et al. (2021) further focused on such a system and investigated the so called "yield rate paradox" phenomenon. Finally, Lin et al. (2022) systematically compared the "push" and "pull" remanufacturing dynamics and suggested that the return rate plays a key role in different remanufacturing production selections.

The second study stream prefers to adopt system dynamics simulation to study complex one-echelon or multi-echelon closed loop supply chains. This includes the exploration of lead time variabilities (Dominguez et al. 2020), return quality (Ponte et al. 2021) and batching effect (Ponte et al. 2022), to name a few. However, both streams assume demand is either a classic deterministic (e.g. step demand) or stochastic (e.g. i.i.d.), thus conclusions derived may not be applicable for the long-term lifecycle-based demand process. To overcome the limitation, Georgiadis et al. (2006) investigated the impact of lifecycles and return patterns of products and developed the optimal policies of collection and remanufacturing capacities in a pure CLSC system. Georgiadis and Athanasiou (2013) further studied such a system with two products under demand and return lifecycle patterns. They proposed the flexible policies to avoid overcapacity issues in return collection and remanufacturing production.

Based on literature review, the research gap can be clearly identified. First, most researches studied the decision of remanufacturing in-house and outsourcings from static cost and uncertainties perspective. This ignored that the system dynamics cost may impact on the remanufacturing production decision. Second, most system dynamics studies for remanufacturing and closed loop supply chains did not explicitly compare two remanufacturing scenarios under lifecycle demand and returns, although the lifecycle demand and returns may greatly impact the remanufacturing and manufacturing production (Georgiadis et al. 2006; Östlin et al. 2009; Georgiadis and Athanasiou 2010; Georgiadis and Athanasiou 2013; Jia et al. 2016). Thus, conclusions derived from those system dynamics studies may not be applicable for remanufacturing in-house or outsourcing decisions with correlated lifecycle demand and returns. Our study aims to fill above research gaps by symmetrically compare dynamic performance of remanufacturing in-house and outsourcing systems with correlated lifecycle demand and returns.

3. Model

3.1. System dynamics models of the component production system

Consider a hybrid component manufacturing and remanufacturing system, where market demand, $d_t(t)$, including new-sales and after-sales demand, i.e. $d_t(t) = d_s(t) + d_w(t)$, are satisfied by serviceable inventory, $i_s(t)$, as shown in Figure 1. All notations of system variables, parameters and metrics in this paper can be found in Table 1. Note that our system dynamics model is assumed to be deterministic to analyse the complex dynamic behaviour (i.e. bullwhip) driven by deterministic cause-and-effect feedback loops, nonlinearities and delays present in the system (Größler et al., 2008; Lin et al. 2022). The deterministic system assumption also fits the practical component remanufacturing system where, different from the product remanufacturing system, supply of manufactured components (e.g. return a failed automotive engine for a replacement) and demand for remanufactured components (based on installed base and failure rate) are relatively predictable (Östlin et al., 2009). We model the hybrid system as a linear discrete-time, discrete-review production inventory system where discrete time, $t = 1, 2 \dots, \infty$. It is assumed in each time period t, there are always two sources to meet $d_t(t)$ during the lifecycle: newly manufactured components, $c_m(t)$, and remanufactured components, $c_r(t)$.

Notation of system variables		**Notation of system parameters**	
d_t	Component total market demand	τ_m	Actual manufacturing production lead time
d_s	Component new-sales demand	τ_r	Actual remanufacturing production lead time
d_w	Component after-sales demand	τ_a	Forecasting smoothing factor
$\widehat{d_t}$	Forecasted market demand	τ_c	Residence time
c_m	Manufacturing completion rate	τ_l	Delay length in responding impulse input
c_r	Remanufacturing completion rate	$\hat{L}$	Estimated lead times
o_m	Manufacturing order rate	L_m	Actual manufacturing lead time
o_r	Remanufacturing order rate	L_r	Actual remanufacturing lead time
i_s	Serviceable inventory	$\hat{L}$	Estimated lead times
g_s	Lifecycle demand	a	Component return rate
ε	i.i.d. demand with zero mean	γ	Proportional rate of demand install base
S	Order up to level	δ_t	Inventory proportional controller
w_t	Pipeline inventory	μ	Exponential smoothing coefficient
IP	System total inventory (serviceable, manufacturing and remanufacturing pipeline inventories)		**Notation of system (metrics)**
		$ITAE$	Integral of Time-Weighted Absolute Error
		BE	Bullwhip effect
		RIS	Remanufacturing in-house strategy
r_r	Component return rate	ROS	Remanufacturing outsourcing strategy

Table 1. Notations of system variables, parameters and metrics.

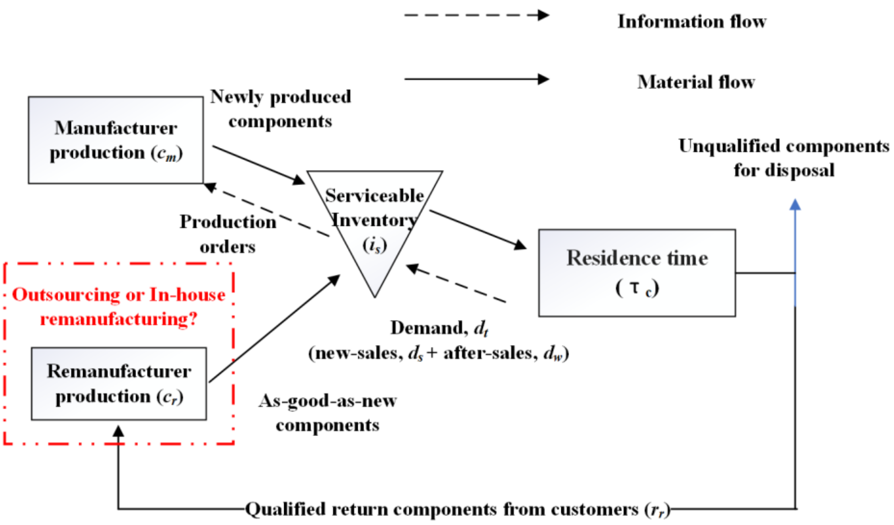

Figure 1. Schematic of the model.

The manufacturer faces a strategic decision about his remanufacturing production: remanufacturing in-house or outsourcing. That is, the manufacturer either remanufactures products in their own production line or outsource remanufacturing production to a qualified TPR who can continuously supply $c_r(t)$. For both options the manufacturer receives same sales and service part demand information from customers due to the characteristics of the component remanufacturing: supply of a qualified component for remanufacturing is directly linked to the after-sales demand for component.

Furthermore, the perfect substitution for manufactured and remanufactured components is assumed, i.e. the remanufactured components are as-good-as-new components. This can be commonly observed in practical closed loop supply chains, such as HP (Nichols 2014; Zhou et al. 2017); PET bottles (Hosada and Disney 2018; Hosoda et al. 2021); service market in automotive industry and automotive engine parts (Niu et al. 2022). Therefore, the inventory balance is assumed as following:

Assumption 1. $d_s(t)$ and $d_w(t)$ can be satisfied by $c_m(t)$ and $c_r(t)$ such that

$$i_s(t) = \sum_{t=1}^{\infty}[c_r(t) + c_m(t) - d_s(t) - d_w(t)]. \quad (1)$$

The replenishment of $i_s(t)$ follows the sequence of 1) receive products, 2) fulfil demand and 3) place order (Hosada and Disney 2018), as shown in Figure 2. Specifically, at the beginning of time period *t*, the manufacturer receives newly produced and remanufactured components, $c_r(t) + c_m(t)$, which are manufacturing and remanufacturing orders, o_m and o_r, placed L_m and L_r ($\forall\, L_m, L_r \in \mathbb{R}^+$) periods ago:

$$c_r(t) = o_r(t - L_r), \quad (2)$$

$$c_m(t) = o_m(t - L_m). \quad (3)$$

where L_m and L_r consist of actual manufacturing (τ_m) and remanufacturing (τ_r) production delays plus a nominal unit delay, i.e. $L_m = \tau_m + 1$ and $L_r = \tau_r + 1$, $\forall\, \tau_m \geq 0, \tau_r \geq 0$,

$\tau_m, \tau_r \in \mathbb{R}^+$, due to the sequence of events: i.e. receive components at the beginning of the review period and place orders at the end of the review period.

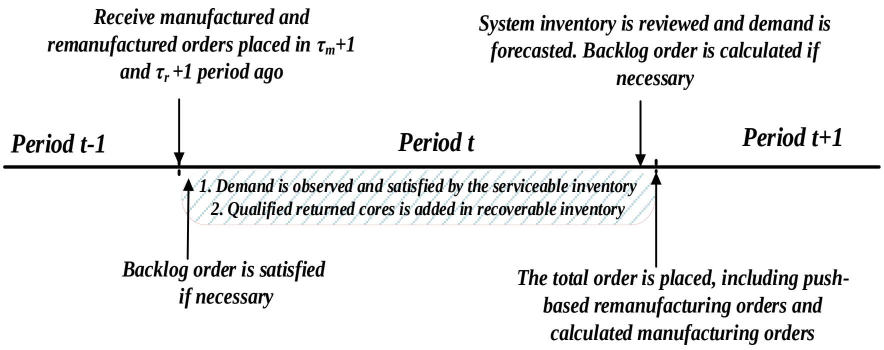

Figure 2. The sequence of events

During period t, market demand $d_t(t) = d_s(t) + d_w(t)$, are observed and satisfied by serviceable inventory, $i_s(t)$. Any unfulfilled demand is backlogged. Finally, at the end of review period, the manufacturer places the order via a *base stock policy*, i.e. place the total order, $o_t(t)$, to reach system total inventory including serviceable inventory (i_s) and work-in-process inventory (w_t), $IP(t) = i_s(t) + w_t(t)$, up to S level:

$$o_t(t) = S(t) - [i_s(t) + w_t(t)], \tag{4}$$

where $S(t)$ is the updated order up to level at each period:

$$S(t) = \widehat{d_t}(t) \cdot \hat{L} + k \cdot \widehat{\sigma_L}. \tag{5}$$

where $\widehat{d_t}(t) \cdot \hat{L}$ is the forecast for market demand ($d_s(t) + d_w(t)$) over estimated lead times, i.e. $\hat{L}$ $(= \widehat{\tau_p} + 1)$. $\widehat{\sigma_L}$ is an estimation of the standard deviation of the demand over $\hat{L}$ periods, and k is a chosen constant to meet a desired service level. To simplify the analysis and simultaneously ensure the safety stock is appropriately considered, we set $k = 0$ and increase $\hat{L}$ by one, i.e. $S(t) = \widehat{d_t}(t) \cdot (\hat{L} + 1)$, following Disney et al. (2006). In practice, this policy is often used in which the extra inventory due to increase value of $\hat{L}$ is represented by the safety stock. Furthermore, we adopted exponential smoothing (ES) forecasting method with smoothing factor τ_a, $\widehat{d_t}(t) = \mu \cdot d_t(t) + (1 - \mu) \cdot \widehat{d_t}(t - 1), \forall\, \mu = \frac{1}{\tau_a + 1} \in [0,1]$. Note that $\hat{L}$ is the decision maker's estimated lead time and the relationship between $\hat{L}$ and actual manufacturing and remanufacturing lead times, i.e., L_m $(= \tau_m + 1)$ and $L_r (= \tau_r + 1)$, should be determined to ensure zero inventory offset, which will be analysed in Section 4.

Two remanufacturing strategies (Borja et al. 2020; Niu et al. 2022) are considered for demand fulfilment as follows:

Scenario 1*: Remanufacturing in-house strategy (RIS). The manufacturer simultaneously produces manufactured and remanufactured components by developing its own remanufacturing line, i.e. the hybrid manufacturing-remanufacturing system follows*

$$RIS\text{: } o_m(t) = o_t(t) - o_r(t), \tag{6}$$

$$RIS: \quad w_t(t) = \sum_{t=1}^{\infty}[o_m(t) - c_m(t) + o_r(t) - c_r(t)]. \tag{7}$$

Scenario 2: *The remanufacturing outsourcing strategy (ROS). The manufacturer outsources remanufacturing to a qualified TPR to receive remanufactured components as part of serviceable inventory.*

$$ROS: \quad o_m(t) = o_t(t), \tag{8}$$

$$ROS: \quad w_t(t) = \sum_{t=1}^{\infty}[o_m(t) - c_m(t)], \tag{9}$$

where Equations (6) and (7) indicate that under RIS strategy the manufacturer has full information visibility for remanufacturing production including remanufacturing orders and pipeline inventory. Such a model is firstly proposed by Tang and Naim (2004) and further studied by Borja et al. (2019). On the other hand, under the ROS the manufacturer only controls their manufacturing production, although remanufactured components from the TPR can be added into serviceable inventory. Regarding remanufacturing production policy in both systems, the following assumption is hold:

Assumption 2. A proportion of returned components, driven by one-to-one exchange policy after-sales demand, i.e. $a \cdot r_r(t), \forall\, a \in (0,1)$, will become qualified recoverable inventory for remanufacturing production such that

$$o_r(t) = r_r(t) = a \cdot d_w(t). \tag{10}$$

Under *Assumption 2*, remanufacturing production in both remanufacturing in-house and outsourcing scenarios follows the push policy in which all qualified recoverable components are batched and pushed into the remanufacturing line immediately after disassembly and testing. Such a policy is well-recognized in studying the dynamics of CLSCs (e.g. Hosoda et al. 2021; Lin et al. 2022), fitting well with sustainability (Hosoda and Disney 2018; Ponte et al. 2019). Note that a is the deterministic proportional rate between returns and qualified recoverable components, although its sensitivity analysis will be conducted in the numerical study (Section 7).

Equation (4) can be re-arranged by substituting $S(t) = \widehat{d_t}(t) \cdot (\hat{L} + 1)$ so that:

$$o_t(t) = \widehat{d_t}(t) + [\widehat{d_t}(t) - i_s(t)] + [\widehat{d_t}(t) \cdot (\hat{L} - 1) - w_t(t)], \tag{11}$$

where an order is placed at each review period, $o_t(t)$, including forecast demand, full on-hand serviceable inventory adjustment and pipeline serviceable inventory adjustment. We can further add a proportional controller, $\delta_t, \forall\, \delta_t \in (0,1]$, to increase the flexibility of inventory error correction, that is, the inventory errors can be proportionally corrected by adding δ_t as a decision parameter in the base stock policy. This policy is also well recognized as the proportional order-up-to policy (POUT), which is the optimal linear replenishment rule for minimizing the weighted sum of order and inventory variance (Boute et al, 2022):

$$o_t(t) = \widehat{d_t}(t) + \delta_t \cdot \left[\widehat{d_t}(t) - i_s(t)\right] + \delta_t \cdot \left[\widehat{d_t}(t) \cdot \left(\hat{L} - 1\right) - w_t(t)\right]. \quad (12)$$

3.2. Lifecycle demand and return models

We model component new sales, $d_s(t)$, as a deterministic lifecycle trend, $g_s(t)$, plus stochastic i.i.d. process with zero mean and σ^2 variance, $\varepsilon(t) \sim (0, \sigma^2)$, that is, $d_s(t) = g_s(t) + \varepsilon(t)$. The term "lifecycle" was originally derived from various product lifecycle management theories and frameworks, which may be summarized as two categories:1) marketing product lifecycle management (M-PLM) and engineering product lifecycle management (E-PLM) (Cao and Folan, 2012). The M-PLM describes the evolution of a product / component measured by its sales volume versus time (Levitt 1965), which can be divided into four phases (infancy, growth, maturity and decline). The E-PLM, on the other hand, refers to the management of the real and complete life of a single product / component – from product conception, through design, production, sale, customer use and service, to, finally, decommissioning.

In our study, we focus on component M-PLM in which we explore how a component's demand (new sales) lifecycle impacts on the dynamics of remanufacturing systems and remanufacturing outsourcing decisions. Drawing from control theory and the z-transform, $G_s(z) = \mathcal{Z}\left[g_s(t)\right] = \sum_{t=0}^{\infty} g_s(t) z^{-t}, z = e^{i\emptyset}$ (Boute et al. 2022), we model $g_s(t)$ as the dynamic response function (i.e. the output) of a discrete linear nth order delay system, $G(z) = \frac{\beta}{(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1})^n}$, for an unit impulse input (also refers to Kronecker delta), $\delta(z) = \sum_{t=0}^{\infty} \delta(t) z^{-t} = 1, \forall\, \delta[t = 0] = 1, \delta[t \neq 0] = 0$. Then we use such output ($G(z)$) as the input to RIS and ROS systems to explore their dynamic behaviour:

$$G_s(z) = G(z) \cdot \delta(z) = \frac{\beta}{(1 + \frac{\tau_l}{n} - \frac{\tau_l}{n} z^{-1})^n}, \quad (13)$$

where n is the order of the delay function, $\tau_l, \forall\, \tau_l \in \mathbb{N}^+$, is the average delay length in responding to the impulse input and $\beta, \forall\, \beta \in \mathbb{R}^+$, is the scaling factor. Note that we use the z-transform to transform the difference equations into an easier-to-solve polynomial equations, although Equation (13) can be written as difference equations in time domain by using the inverse z-transform, $g_s(t) = \mathcal{Z}^{-1}\left[G_s(z)\right] = \frac{1}{2\pi i} \oint G_s(z) z^{t-1} d_z$,

$$g_s(t) = \beta \cdot \frac{n!}{t!\,(n-t)!} \left(-\frac{\tau_l}{n}\right)^t \left(1 + \frac{\tau_l}{n}\right)^{-n-t}. \quad (14)$$

For $n = 1$, $g_s(t)$ is the exponential smoothed response0 0of $\delta(t)$, $g_s(t) = \delta(t) \cdot \beta \cdot \frac{(-\tau_l)^t (1+\tau_l)^{-1-t}}{t!(1-t)!}$. When n approaches infinity, $g_s(t)$ is purely delayed $\delta(t)$, $g_s(t) = \beta \cdot \delta(t - \tau_l)$. Equation (13) with three parameters can capture important dynamic characteristics of the lifecycle process, including lifecycle length from introduction to decline (τ_l), the length of each stage in a lifecycle (τ_l and n) and the maximum value of demand during the lifecycle (β) (Georgiadis and Athanasiou 2010). Also, another benefit is its mathematical traceability given the output, i.e., $d_s(t)$ can be directly input into another linear discrete system for further analysis. This is because for a linear system, any input and similarly, the output, is the superposition of scaled and delayed impulse response functions (Boute et al. 2022).

After a considerable residence time (τ_c), a proportion of failed components are returned for a replacement (new or remanufactured one). That is, the after-sales demand, $d_w(t)$, with a different lifecycle plus $\varepsilon(t)$, $d_w(t) = g_w(t) + \varepsilon(t)$. Empirically, the component after-sales demand is primarily linked to the characteristics of the installed base related to *sales demand* and the *failure rate* of individual components (Östlin et al., 2009). We model quantity and timing dependencies between new-sales demand and after-sales demand, $d_s(t)$ and $d_w(t)$, as a second order delay relationship:

$$d_w(t) = d_s(t) \cdot \gamma \cdot \frac{4{\tau_c}^t \cdot (t+1)}{(2+\tau_c)^{t+1}} \tag{15}$$

where γ , $\forall\gamma \in (0,1)$ is the proportional rate of install base of $d_s(t)$ that will eventually need a replacement, i.e., the deterministic failure rate. This is appropriate for modelling mechanical component failure rates, for instance, for the automotive industry, where the failure rate is approximately deterministic and largely depends on residence time (Östlin et al. 2009). τ_c is the considerable long residence time between the new component sold and the failed one received.

Furthermore, one characteristic of component remanufacturing is that the supply of a qualified component for remanufacturing, $r_r(t)$, is directly linked to the after-sales demand for component, $d_w(t)$, i.e., end-of-use component take-back and remanufacturing scheme (Geyer et al. 2007). Specifically, when a customer supplies a failed component for the manufacturer, in return receives a remanufactured or new component (Östlin et al.2009). Practically, this is found in automotive and toner cartridges component remanufacturing (Junior and Filho, 2015). Therefore, we can model the dependency between $d_w(t)$ and $r_r(t)$ as:

$$r_r(t) = a \cdot d_w(t), \tag{16}$$

where $a\ \forall\ a \in (0,1)$ is the deterministic manufacturable component rate, indicating a proportion of returned component driven by service component demand eventually become qualified recoverable inventory for remanufacturing, while the rests are not recovered, due to, e.g. quality issues, third party collection, user's unwillingness for return (Geyer et al. 2007).

To study the dynamic behaviour of order and system inventories in RIS and ROS systems, we need to obtain the solutions of high-order difference equations represented by Equations (1)-(12) for a given input function, i.e., lifecycle demand and returns. To simplify the algebraic manipulations required, the z-transform is adopted to translate the difference equations into easier-to-solve polynomial equations in z domain as the transfer functions. *Appendix 1* has the detailed transfer functions derivations. Here we show the transfer functions of $o_m(t)$, $i_s(t)$ and $IP(t)$ in relations to market demand, $d_t(t)$, which represent the dynamic behaviour of order and inventories under our modified base stock policy.

For the RIS system:

$$\frac{O_m^{RIS}(z)}{D_t(z)} = \frac{z\left(\delta_t(z + z\tau_a - \tau_a) + (z-1)\left(1 + \delta_t\hat{L}\right)\right)}{(z - 1 + \delta_t)(z + z\tau_a - \tau_a)} - \frac{z\alpha\gamma}{(z + z\gamma + (z-1)\tau_c)}, \quad (17)$$

$$\frac{IP^{RIS}(z)}{D_t(z)} = \left(\frac{\delta_t(z + z\tau_a - \tau_a) + (z-1)\left(1 + \delta_t\hat{L}\right)}{(z - 1 + \delta_t)(z + z\tau_a - \tau_a)} - 1\right)\frac{z}{z-1}, \quad (18)$$

$$\frac{I_S^{RIS}(z)}{D_t(z)} = \left(\frac{z\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)z^{\tau m}} + \frac{z\alpha\gamma(z^{\tau m}-z^{\tau r})}{z^{\tau r+\tau m}(z+z\gamma+(z-1)\tau_c)} - 1\right)\frac{z}{z-1}. \quad (19)$$

For the ROS system:

$$\frac{O_m^{ROS}(z)}{D_t(z)} = \frac{z\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)} - \frac{z\alpha\gamma\delta_t}{z^{\tau r}(z-1+\delta_t)(z+z\gamma+(z-1)\tau_c)}, \quad (20)$$

$$\frac{IP^{ROS}(z)}{D_t(z)} = \left(\frac{\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1)(z-1+\delta_t)(z+z\tau_a-\tau_a)} - 1\right)\left(\frac{z(z-1)\alpha\gamma(\delta_t-1)}{z^{\tau r}(z-1+\delta_t)(z+z\gamma+z\tau_c-\tau_c)}\right), \quad (21)$$

$$\frac{I_S^{ROS}(z)}{D_t(z)} = \left(\frac{z\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)\left(1+\delta_t\hat{L}\right)\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)z^{\tau m}} + \frac{z(z^{\tau m}\alpha\gamma(z-1+\delta_t)-z\alpha\gamma\delta_t)}{z^{\tau r+\tau m}(z-1+\delta_t)(z+z\gamma+(z-1)\tau_c)} - 1\right)\frac{z}{z-1}. \quad (22)$$

4. System dynamics performance measurement

We explore system dynamics performance of orders and inventories in the RIS and ROS systems for a given $d_s(t) = g_s(t) + \varepsilon(t)$. The non-stationary $d_s(t)$ and associated $d_w(t)$ includes a lifecycle trend and a stationary i.i.d. process, and thus traditional stationary mean-variance based analysis, i.e. order and inventory variance analysis, is difficult to be applied for the system dynamics performance. Instead, given the linear system, the dynamic behavior in responding to $g_s(t)$ and $\varepsilon(t)$ can be independently accessed by using different performance measurements. For the non-stationary lifecycle trend, $g_s(t)$, Integral of Time-Weighted Absolute Error (ITAE) (Udenio et al. 2017) is adopted:

$$ITAE = \sum_{t=0}^{\infty} t|\epsilon|, \quad (23)$$

where ϵ represents the absolute error between the actual dynamic response at time t and the equilibrium response. This measure penalizes deviations from the new, or target, equilibrium and introduces a linear penalty for longer-lasting deviations. Thus, both the *amplification* (i.e., how large the order and inventory error are from the target) and the *convergency time* (i.e., how long it takes for the actual order and inventory responding to converge to the equilibrium) of the system play a role in its quantification.

The reason for using ITAE is that convergency time and amplification of order and inventory are directly linked to the strategic related costs such as long-term capacity investment, production and inventory dynamic costs due to economies of scale. Figure 3 gives an example of dynamic behaviour of manufacturing orders in the RIS system. It can be seen that order response with lower ITAE generates high amplification but short convergency time and vice versa. The amplification of orders can indicate the long-term maximum capacity requirement so that the capacity investment or outsourcing decision can be determined (Lin et al. 2018; Ponte et al. 2019). Convergency time of orders is associated with the manufacturing/remanufacturing production lasting time during demand lifecycle, indicating dynamic production on-costs (Georgiadis et al. 2006; Östlin et al. 2009; Georgiadis and Athanasiou 2013; Jia et al. 2016).

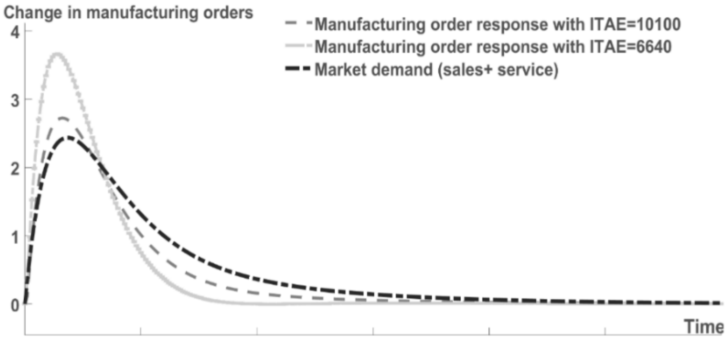

Figure 3. An example for the dynamic response of change in manufacturing orders with different ITAE values. System settings: $\beta = 200, \delta_t = 1, \alpha = 0.5, \gamma = 0.5, \tau_a = 16, \tau_r = 4, \tau_c = 64, \tau_m = 4\ (\mathrm{ITAE} = 10100);\ \tau_m = 16\ (\mathrm{ITAE} = 6640)$

Regarding the i.i.d. process, $\varepsilon(t)$, we can use the variability ratio between demand and manufacturing/remanufacturing orders, i.e. information bullwhip (BE) based on order and demand information, as the dynamic performance of the system. The BE can be directly linked to production costs, as it is a main indicator for capacity variability and inventory variance (Hosoda and Disney 2018; Boute et al. 2022).

$$BE = \frac{var(order)}{var(demand)}. \tag{24}$$

5. System convergency and stability

Before analysing the dynamic performance of the two systems, it is important to understand fundamental properties of dynamic systems including *convergency* and *stability* to ensure system robustness in the face of external disturbance. Regarding system convergence, recall the order up to level, $S(t)$, is determined by the estimated lead time, $\hat{L}$. In traditional open loop supply chains without remanufacturing, the decision marker's estimated lead times are assumed as equal to the actual manufacturing production to ensure inventory and orders converge to the desired new equilibrium in responding to a demand change. Practically, such settings can avoid long-term inventory and orders mismatch in order to maximize customer service levels (Disney and Towill 2005). However, in our system the estimated lead times ($\hat{L}$) are more complicated due to the simultaneous estimation of manufacturing (τ_m) and remanufacturing lead times (τ_r). We explore the convergence of orders and serviceable inventory in RIS and ROS systems via the following propositions.

Proposition 1. *For RIS and ROS systems replenished by the base stock policy,*

1.1. When such systems link initially in equilibrium is disturbed by a lifecycle-based market demand patterns, the order and serviceable inventory in both systems converge to zero regardless of estimated lead times for manufacturing and remanufacturing.

1.2. When such systems link initially in equilibrium is disturbed by a sudden but sustained unit sales demand increase, the new equilibrium of order (O_m^{eq}) and inventory (I_s^{eq}) are:

$$O_m^{eq}(RIS) = O_m^{eq}(ROS) = 1 + \gamma(1-\alpha), \tag{25}$$

$$I_s^{eq}(RIS) = (1+\gamma)\left(\hat{L} - \tau_m\right) + \alpha\gamma(\tau_m - \tau_r), \tag{26}$$

$$I_s^{eq}(ROS) = \frac{\alpha\gamma}{\delta_t} + (1+\gamma)\left(\hat{L} - \tau_m\right) + \tau_m\alpha\gamma. \tag{27}$$

Proof. See Appendix 2.1

Proposition 1.1. shows that both systems' orders and inventories will eventually return to zero, as the lifecycle-based market demand will reduce to zero from growth to decline. From *Proposition 1.2*, if both systems face a sudden but sustained sales demand increase, the new equilibrium of orders is increased as the increase of component failed rate (γ) and the decrease of qualified recoverable inventory rate (α). The new convergency of serviceable inventory, however, is different for different outsourcing strategies. It is straightforward that $I_s^{eq}(RIS) < I_s^{eq}(ROS)$ given positive value of τ_m, γ, α, δ_t and τ_r, indicating that for a fixed $\hat{L}$, in-house remanufacturing strategy always produce less serviceable inventory than that in remanufacturing outsourcing strategy. Also, by comparing Equations (26) and (27), $I_s^{eq}(RIS)$ is independent with δ_t, while τ_r plays no role on $I_s^{eq}(ROS)$, although both $I_s^{eq}(RIS)$ and $I_s^{eq}(ROS)$ increase as the increase of τ_m, γ and α. Finally, we have the following property for avoiding inventory permanent drift caused by the difference between estimated lead times and manufacturing/remanufacturing lead times (Towill and Disney 2005).

Property 1*: The permanent inventory drift for RIS and ROS systems, disturbed by a sudden but sustained demand increase, can be avoided by estimating lead times as*

$$\hat{L}(RIS) = \tau_m + 1 + \frac{\alpha\gamma(\tau_r - \tau_m)}{1+\gamma}, \tag{28}$$

$$\hat{L}(ROS) = \tau_m + 1 - \frac{\alpha\gamma(\tau_m\delta_t + 1)}{(1+\gamma)\delta_t}. \tag{29}$$

Proof. From the modified base stock policy, the safety serviceable inventory is simplified as one period ahead forecasted market demand, $k \cdot \widehat{\sigma_L} = \widehat{d_t}(t)$. As the result, when the system reaches new equilibrium under shock demand increase, $k \cdot \widehat{\sigma_L} = \widehat{d_t}(t) = d_t = 1 + \gamma$, that is, under new equilibrium forecast will equal to market demand including new-sales demand and after-sales demand. The targeted serviceable inventory for both systems $I_s^{eq}(RIS) = I_s^{eq}(ROS) = d_t = 1 + \gamma$. We can then find the solutions of $I_s^{eq}(RIS) = I_s^{eq}(ROS) = 1 + \gamma$ with respect $\hat{L}$ in Equations (26) and (27).

Stability is another fundamental property in supply chain and operations systems. A stable dynamic system yields a bounded output for any bounded input, which is one main objective in designing inventory system facing external disturbance (Wang et al. 2012). In IRS and ROS systems, stability guarantees finite orders and inventories as a response to changes in demand—a pre-condition for any real-life system. *Definition 1* gives formal mathematical necessary condition for the stability of a linear system.

Definition 1 *(Jury, 1964, p. 302). Suppose that* $G(z) = \frac{N(z)}{C(z)}$ *is the transfer function of a linear, time-invariant system and that the denominator* $C(z)$ *has exactly* n *roots* p_i*, namely,* $C(p_i) = 0, i = 1, \dots n$*. We call the roots* p_i *poles of the transfer function, and we say that a system is stable if all poles* p_i *are within the unit circle of the complex plane (*$|p_i| < 1$*), marginally stable if at least one pole is on the unit circle (*$|p_i| = 1$*), and unstable if at least one pole resides outside unit circle (*$|p_i| > 1$*).*

Based on *Definition 1*, the stability of RIS and ROS systems can be determined by inspecting the denominators of Equations (17)-(22). Specifically, $C_{RIS}(O_m)$ and $C_{ROS}(O_m)$ share the same term $(z - 1 + \delta_t)(z + z\tau_a - \tau_a)(z + z\gamma + (z-1)\tau_c)$ in the denominator, while $C_{ROS}(O_m)$ has additional z^{τ_r} in it. Equations (30) - (32) show the denominator of inventory (IP and I_s) for both systems:

$$C_{RIS}(IP) = (z - 1 + \delta_t)(z + z\tau_a - \tau_a)(z - 1), \tag{30}$$

$$C_{ROS}(IP) = (z - 1 + \delta_t)(z + z\tau_a - \tau_a)(z + z\gamma + (z-1)\tau_c)(z-1)z^{\tau_r}, \tag{31}$$

$$C_{RIS}(I_s) = C_{ROS}(I_s) = (z-1)(z - 1 + \delta_t)(z + z\tau_a - \tau_a)(z + z\gamma + (z-1)\tau_c)z^{\tau_r + \tau_m}. \tag{32}$$

Note that for both systems the denominator of inventories transfer functions includes an additional $(z-1)$, i.e., additional root $(z = 1)$, in Equation (32) comparing with the order transfer functions, which can be cancelled out due to the same root $(z = 1)$ appears in the nominator of Equations (18), (19), (21) and (22). Thus, based on Equations (30) – (32), the sufficient condition for the stability of RIS and ROS are derived in following Theorem.

Theorem 1. *The RIS and ROS systems can be guaranteed to be stable or marginally stable if* τ_a*,* τ_c *and* δ_t *satisfy the following conditions:*

i. $0 \le \delta_t \le 2$, and

ii. $\tau_a \geq -0.5$ $(0 < \mu \leq 2)$, and

iii. $-1 \leq \frac{\tau_c}{1+\gamma+\tau_c} \leq 1$.

Proof. See Appendix 2.2

From *Theorem 1*, we can conclude that RIS and ROS systems are stable for conventional system parameter settings, i.e., proportional/full correction of two inventory feedback loop errors with $0 < \delta_t \leq 1$ and common exponential smoothing setting $0 < \mu \leq 1$ $(\tau_a \geq 0)$.

6. Nonstationary performance analysis

The component sales demand process, $d_s(t) = g_s(t) + \varepsilon(t)$, include a nonstationary lifecycle pattern and stationary i.i.d. process. In this section we focus on the dynamic performance of ROS and RIS systems in responding $g_s(t)$. For simplicity without losing generality, scale factor of lifetime sales demand, $\beta = 1$ is assumed for ITAE measurement.

In RIS and ROS systems in responding a lifecycle demand, given the new equilibrium of system inventory and order are zero *(Proposition 1.1)*, ϵ becomes *the actual dynamic response of inventory and orders over time.* Thereby, the dynamic performance of the RIS and ROS systems is quantified by considering dynamic production and inventory costs associated with lifecycle demand and return patterns. i.e., the production and inventory costs are dynamically changed with time from market introduction, growth, maturity and saturation/decline. Formally,

$$ITAE_{O_m} = \sum_{t=0}^{\infty} t|O_m(t)|\,;\ ITAE_{O_r} = \sum_{t=0}^{\infty} t|O_r(t)|\,;\ ITAE_{IP} = \sum_{t=0}^{\infty} t|IP(t)|. \tag{33}$$

We consider the ITAE of manufacturing and remanufacturing orders, as well as the system total inventory, *IP(t)*, including serviceable and work-in-process inventories. The following proposition derives the ITAE for $O_r(t)$, $O_m(t)$ and $IP(t)$.

Proposition 2. *For RIS and ROS systems replenished by the base stock policy:*

2.1. When IRS link initially in equilibrium is disturbed by a lifecycle-based market demand patterns $d_t(t) = d_s(t) + d_w(t)$, *ITAE for* $O_r(t)$, $O_m(t)$ *and* $IP(t)$ *can be quantified:*

$$ITAE^{RIS}{}_{o_r} = \alpha\gamma(\tau_c + \tau_l), \tag{34}$$

$$ITAE^{RIS}{}_{o_m} = (\gamma - \alpha\,\gamma)(\tau_c + \tau_l) - (1 + \gamma)\big(1 + \hat{L}\big) + \tau_l, \tag{35}$$

$$ITAE^{RIS}{}_{ip_t} = \hat{L}(\gamma(\tau_c + \tau_l) + \tau_l) + \frac{(1+\gamma)\left(\tau_a + 1 + \hat{L}(\tau_a\delta_t + 1)\right)}{\delta_t}. \tag{36}$$

2.2. When ROS link initially in equilibrium is disturbed by a lifecycle-based market demand patterns, $d_t(t) = d_s(t) + d_w(t)$, *ITAE for* $O_r(t)$, $O_m(t)$ *and* $IP(t)$ *can be quantified:*

$$ITAE^{ROS}{}_{o_r} = \alpha\gamma(\tau_c + \tau_l), \tag{37}$$

$$ITAE^{ROS}{}_{o_m} = (1+\gamma)\big(\tau_l - \hat{L} - 1\big) - \frac{\alpha\gamma}{\delta_t} - (\tau_r + \tau_c + \tau_l)\alpha\gamma + \gamma\tau_c, \tag{38}$$

$$ITAE^{ROS}{}_{ip_t} = \frac{\alpha\gamma}{\delta_t^2} + \hat{L}\tau_a(1+\gamma) + \gamma\tau_c\left(1+\hat{L}-\alpha\right) + (\tau_l - 1)\left(1+\hat{L}+\gamma+\hat{L}\gamma\right)$$

$$+\frac{(1+\gamma)\left(\tau_a+1+\hat{L}\right)+\alpha\gamma(\tau_c+\tau_l+\tau_r-2)}{\delta_t} - \alpha\gamma(\tau_r - 1 + \tau_l). \qquad (39)$$

Proof. See Appendix 2.3.

From *Proposition 2,* there are several important observations. First, it is straightforward to see that the ITAE of inventory and orders in both systems is irrelevant to the order of lifecycle model function, n. That is, different lifecycle patterns determined by the order of delay function have no impact on system dynamics performance. Also due to ‘push’ remanufacturing policy adopted for both systems, $ITAE^{RIS}{}_{o_r} = ITAE^{ROS}{}_{o_r}$, the $ITAE_{o_r}$ increases in return delay (τ_c) and sales lifecycle length (τ_l). Similarly, component failed rate (γ) and manufacturable component rate (α) are also positively corrected to $ITAE_{o_r}$. That is, long return delay, lifecycle pattern, high return rate and low fail rate result slow convergency speed of remanufacturing production. Given remanufacturing production cost decreases from growth to decline stage of component return lifecycle driven by the increased manufacturable component availability (Östlin et al. 2009), it is important to understanding the return and demand lifecycle patterns and the recoverable component rate before remanufacturing capacity investment (Georgiadis and Athanasiou 2010).

Regarding the RIS system, by inspecting Equations (35) and (36), $ITAE^{RIS}{}_{o_m}$ and $ITAE^{RIS}{}_{ip_t}$ increase with respect to τ_c and τ_l. Thus, a long return delay and demand lifecycle length will cause slow convergency speed of manufacturing production and system total inventories. By differentiating Equation (35) with respect γ, $ITAE^{RIS}{}_{o_m}$ is positively corrected to γ if $(1-\alpha)(\tau_c+\tau_l)-(1+\hat{L})>0$. Given customer consumption and demand lifecycle length are much longer than estimated production lead times, i.e. $\tau_c+\tau_l \gg 1+\hat{L}$, $ITAE^{RIS}{}_{o_m}$ increases in γ if return is not extremely high. Moreover, $ITAE^{RIS}{}_{o_m}$ decreases in $\hat{L}$, while $ITAE^{RIS}{}_{ip_t}$ increases in $\hat{L}$. Given $\hat{L}$ is positively related manufacturing and remanufacturing lead times (*Proposition 1*), the long τ_m and τ_r will decrease the convergency time for manufacturing production during lifecycle at the expense of a high amplification at early stage of lifecycle. This is consistent with statistics study of bullwhip where longer lead times causes high bullwhip for stationary i.i.d. demand process (Disney and Towill 2003; Disney et al. 2006). At the same time, the increased τ_m and τ_r lead to the increased convergency time of system inventories with less transient amplification. Thus, a clear trade-off between production dynamic costs and inventories holding costs should be considered (Lin et al. 2022).

Furthermore, $ITAE^{RIS}{}_{ip_t}$ decreases in δ_t and increases in τ_a. This means that the increased proportional inventory correction speed (large δ_t) and quick forecasting adjustment (small τ_a) can reduce convergency time at the expense of high amplification during early lifecycle stage. Such result can also be observed in traditional open loop supply chains when demand is a step increase (Lin et al. 2018). However, ${ITAE^{RIS}}_{o_m}$ is independent on δ_t and τ_a, suggesting that two main control parameters play no role in influencing convergency speed of manufacturing production. This derives an important insight: if system inventory convergency speed and amplification related costs are the priority to be reduced, inventory and forecasting should be carefully controlled during lifecycle demand, while this is not the case if production costs (e.g., capacity and labour adjustment) become dominant. Finally, manager may consider quick inventory correction and forecasting adjustment (large δ_t and small τ_a) at the growth stage of demand lifecycle, driven by growing market demand and limited remanufacturing (limited returned components), to maintain a high customer service level including both new-sales and after-sales demand. At the mature and decline stage, however, the slow inventory correction and smoothing forecasting strategy are recommended, because of decreased sales demand and increased manufacturable components, to reduce the serviceable inventory holding cost.

For the ROS system, ${ITAE^{ROS}}_{o_m}$ and ${ITAE^{ROS}}_{ip_t}$ increase in τ_c and τ_l. Also, $\hat{L}$ is positively correlated to ${ITAE^{ROS}}_{ip_t}$ and is negatively correlated to ${ITAE^{ROS}}_{o_m}$. However, unlike the IRS system that ${ITAE^{RIS}}_{o_m}$ is independent of δ_t, ${ITAE^{ROS}}_{o_m}$ increases as the increase of δ_t. This illustrates that the increased inventory error adjustment speed, e.g. from $\delta_t = 0.5$ to $\delta_t = 1$, generates less peak amplification at the expense of slower manufacturing convergency speed for the ROS system. This is opposite to bullwhip research in closed loop supply chains where the incorporation of proportional controller can effectively reduce order variance when stationary stochastic demand process is assumed (Hosoda et al. 2015; Hosoda and Disney 2018). However, if the manufacturing convergency speed is the main cost contributor, e.g. manufacturing production cost increases with demand lifecycle time due to the next generation introduced, the proportional inventory adjustment is still recommended.

Similar to the ${ITAE^{RIS}}_{ip_t}$, ${ITAE^{ROS}}_{ip_t}$ is negatively impacted by δ_t, meaning that the total inventory convergency time can be reduced as the increase of inventory correction speed (large value of δ_t), at the expense of high amplification (peak level) during lifecycle dynamics response. Furthermore, the following property compares ${ITAE^{RIS}}_{o_m}$ and ${ITAE^{ROS}}_{o_m}$:

Property 2. *Under the same demand lifecycle characteristics and system parameters,*

$$ {ITAE^{RIS}}_{o_m} - {ITAE^{ROS}}_{o_m} = \alpha\gamma\left(\frac{1}{\delta_t} + \tau_l\right), \tag{40} $$

$$ {ITAE^{RIS}}_{IP_t} - {ITAE^{ROS}}_{IP_t} = (1 + L)(1 + \gamma) - \alpha\gamma\left(1 + \frac{1}{\delta_t^2}\right) - \tau_l - \gamma(1 - \alpha)(\tau_c + \tau_l) + \alpha\gamma\tau_r - \frac{\alpha\gamma(\tau_c+\tau_l+\tau_r-2)}{\delta_t}. \tag{41} $$

From *Property 2*, it is straightforward to see ${ITAE^{RIS}}_{o_m} > {ITAE^{ROS}}_{o_m}$, while

${ITAE^{RIS}}_{IP_t} < {ITAE^{ROS}}_{IP_t}$. This means that the manufacturing orders in RIS system always generate less amplification at the expense of fast convergency speed than that in the ROS systems. The system total inventory in the RIS, however, creates higher amplifications with faster convergency speed than the ROS. It can be concluded that if the RIS strategy is implemented, the manufacturer needs to pay less capacity costs during peak demand lifecycle, although the convergency time of manufacturing production is increased and thereby manufacturing cost is high at the decline stage of order lifecycle. Regarding the inventory cost, the RIS based manufacturer has less concern about the later inventory holding cost (e.g. obsolescence), while the high inventory holding cost during demand peak needs to be paid. The opposite result can be obtained if the ROS strategy is adopted. Such findings also are consistent with empirical observations (Behfard et al. 2015). To concludes for this section, Table 2 summaries the impact of system structure and control parameters on ITAE for both systems.

Performance metrics	**System**	$\hat{L}$	δ_t	γ	τ_a	τ_c	α	τ_l
ITAE for $O_r(t)$	RIS	N	N	↑	N	↑	↑	↑
	ROS	N	N	↑	N	↑	↑	↑
ITAE for $O_m(t)$	RIS	↓	N	D	N	↑	↓	↑
	ROS	↓	↑	↓	N	↑	↓	↑
ITAE for $IP(t)$	RIS	↑	↓	↑	↑	↑	N	↑
	ROS	↑	↓	↑	↑	↑	↑	↑

Table 2. Summary of the impact of system parameters on ITAE and Bullwhip effect (↑: Increases as the increase of the parameter; ↓: Decreases as the increase of the parameter; N: No impact; D: depend on the relations between other parameters)

7. Stationary performance analysis

In this section we focus on the dynamic performance of ROS and RIS systems in responding a stationary i.i.d. process, $\varepsilon(t)$. Given that the ROS and RIS systems are linear, we apply Tsypskin's relation to calculate the bullwhip of manufacturing orders for both systems:

Proposition 3. For the IRS and ROS systems replenished by the base stock policy

When both systems initially in equilibrium are disturbed by i.i.d. stochastic market demand, their variance ratio between manufacturing orders and market demand can be quantified:

$$BE^{RIS} == \frac{\begin{pmatrix} 2+\delta_t\left(3+4\hat{L}+6\tau_a\right)+ \\ \delta_t^2\left(2L\left(1+\hat{L}\right)+\left(1+4\hat{L}\right)\tau_a+2\tau_a^2\right) \end{pmatrix}\left((1+\gamma)^2+(3+4\gamma+\gamma^2)\tau_c+2\tau_c^2\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)(1+\tau_c)(1+2\tau_c)}$$

$$-\frac{\alpha^2\gamma^2\left((1+\gamma)^2+(3+4\gamma+\gamma^2)\tau_c+2\tau_c^2\right)}{(1+\gamma)(1+\gamma+2\tau_c)(1+\tau_c)(1+2\tau_c)}, \qquad (42)$$

$$BE^{ROS} = \frac{\begin{pmatrix} 2+\delta_t\left(3+4\hat{L}+6\tau_a\right)+ \\ \delta_t^2\left(2\hat{L}\left(1+\hat{L}\right)+\left(1+4\hat{L}\right)\tau_a+2\tau_a^2\right) \end{pmatrix}\left((1+\gamma)^2+(3+4\gamma+\gamma^2)\tau_c+2\tau_c^2\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)(1+\tau_c)(1+2\tau_c)}$$

$$-\frac{\alpha^2\gamma^2\delta_t(1+\gamma-\delta_t\tau_c+2\tau_c)\left((1+\gamma)^2+(3+4\gamma+\gamma^2)\tau_c+2\tau_c^2\right)}{(1+\gamma)(2-\delta_t)(1+\gamma+2\tau_c)(1+\gamma+\delta_t\tau_c)(1+\tau_c)(1+2\tau_c)}. \qquad (43)$$

Proof. See Appendix 2.4

By taking the difference between Equations (42) and (43), i.e. $BE^{RIS} - BE^{ROS} =$

$\frac{2\alpha^2\gamma^2(1-\delta_t)\left((1+\gamma)^2+(3+4\gamma+\gamma^2)\tau_c+2\tau_c^2\right)}{(\delta_t-2)(1+\tau_c)(1+2\tau_c)(1+\gamma+2\tau_c)(1+\gamma+\delta_t\tau_c)}, \forall\, \tau_c, \gamma, \delta_t, \alpha \in \mathbb{R}^+, \delta_t \in (0,1]$, it is clear that $BE^{RIS} < BE^{ROS}$ regardless of system structure and control parameters. This means that RIS system always generates less bullwhip than the ROS system. By taking the first order derivative of $\hat{L}$, δ_t, γ, τ_a, τ_c and α, we summarize the impact of system parameters on BE in Table 3.

Performance metrics	**System**	$\hat{L}$	δ_t	γ	τ_a	τ_c	α
Bullwhip Effect	RIS	↑	↑	↑	↓	↓	↓
	ROS	↑	↑	↑	↓	↓	↑

Table 3. Summary of the impact of system parameters on ITAE and Bullwhip effect (↑: Increases as the increase of the parameter; ↓: Decreases as the increase of the parameter; N: No impact)

Based on Table 3, we can conclude that if i.i.d. demand is assumed, both BE^{RIS} and BE^{ROS} increase as the increase of $\hat{L}$, δ_t and γ, and decrease as the increase of τ_a and τ_c. This means that BE^{RIS} and BE^{ROS} increase in long production lead times, fast inventory correction speed, low return yield rate and fast forecasting adjustment of demand. The results are consistent with literature on the dynamics of open loop (Disney and Towill 2003; Disney et al. 2006) and closed loop supply chains (Hosoda et al. 2018; Ponte et al. 2019; Lin et al. 2022). However, the increase of α decreases bullwhip in the RIS system but increases bullwhip in the ROS system. The result is well-supported by Tang and Naim (2004); Ponte et al. (2019); Lin et al. (2022).

Interestingly, both RIS and ROS systems generate less bullwhip as the increase of customer residence time (τ_c), which is opposite to the CLSCs dynamics literature (e.g., Tang and Naim 2004; Zhou et al. 2017; Ponte et al. 2019). This is due to the after-sales demand with long residence time as the filter, will reduce the total demand variance as the input of the RIS and ROS systems, thus reducing the bullwhip effect. Similarly, the increases of component failed rate (γ), as the part of total market demand, will increase system bullwhip due to the increases of demand variance. An important managerial insight thereby can be obtained: from system dynamics perspective, the CLSCs systems may prioritize the component durability design (i.e. increases the residence time and decrease the failed rate) to reduce the bullwhip effect. This is opposite to other remanufacturing systems where decision makers, who aim to improve system dynamics performance such as bullwhip minimization, may prefer to design incentive policies to improve customer return rate and reduce residence time.

8. Numerical study

In this section we conduct an extensive numerical study for RIS and ROS systems to verify and extend the analytical results. Given the lifecycle-based demand as the input of our remanufacturing systems, we present typical values of critical system parameters including residence time ("the time that a product stays with its customer before its end-of-use" Georgiadis & Athanasiou, 2010) and lifecycle for several real-world component examples, shown in Table 4, to determine the numerical lifecycle models. Also, we adopt the residence index (RI), i.e. the ratio between customer residence time over lifecycle length (Georgiadis & Athanasiou, 2010) to represent different levels of remanufacturability. This is because RI is directly associated with remanufacturing profitability and therefor remanufacturing viability: those components in which RI equals or is greater than 1 is not appropriate for remanufacturing since the residence time is longer than demand lifecycle (Georgiadis et al. 2006; Georgiadis and Athanasiou, 2013).

Based on Table 4, we fix the length of lifecycle to τ_l=136 weeks while varying the value of residence time (τ_c) between 8 and 64 weeks to generate the different values of RI (0.06, 0.12, 0.24, 0.47) to represent different component remanufacturing practice, as shown in Table 5. Note that, consistent with the analytical study given in Sections 4-7, the demand scale factor, $\beta, \forall\, \beta \in \mathbb{R}^+$ is normalized as 1 to simplify the analysis without losing generalization.

Also, based on Table 5, $\tau_m = 8$ and $\tau_r = 4$ are assumed to represent the fact that remanufacturing lead time is longer than manufacturing as commonly observed in practice (Lin et al. 2022; Yang et al. 2023). Return rate (α) varies between 20%-80% to represent different return rates in different remanufacturing industry practices or different return rates in the entire closed loop supply chain. For instance, for the HP closed-loop cartridge recycling program, the sold cartridges are returned to retailers at a return yield=80%, where 50% of them (i.e. 40%) will be used to refill ink and directly resell, while around 25% of them (i.e. 20%) of returned products will be recycled into plastic due to poor quality (Zhou et al. 2017). Similarly, different proportion of installed base, γ values between 20%-80% are selected to represent the different component average failure rates in different remanufacturing industries. Finally, two system control parameters, exponential smoothing level (τ_a) and proportional controller (δ_t) of inventory adjustment, are varied to explore the impact of forecasting and proportional inventory control policies on ITAE and bullwhip effect, in line with remanufacturing dynamics studies (Ponte et al. 2019; Dominguez et al. 2019; Lin et al. 2022)

Component	**Average residence time**	**Lifecycle**	**Residence index**	**Source**
Computer chips	20,000 hrs	80,000 hrs	0.25	Keeble (1998)
Xerox copiers	1 cycle	Up to 7 cycles	0.15-0.5	Charter and Gray (2008)
Automotive parts	100,000 km (20,000 per year assumed)	500,000-620,000 km	0.5-0.62/	Esaki et al. (1992)
Electrical engines	6 years	12 years	0.5	Klausner et al. (1998)
Solar air heating components	4-7 years	9.5-12 years	0.42-0.47	Xing et al. (2007)

Table 4. Typical residence indices for different components, based on Georgiadis and Athanasiou (2013).

System	**Baseline Value**	**Sources/Justifications**

Parameter	(Varying range)	
τ_l	32 (Months) ≈ 136 weeks	This value is adopted to generate the sales demand lifecycle around 136 weeks
n	2 (order of delay function)	The second order delay function is adopted to generate a typical new-sales demand lifecycle with growth, mature and decline stages
τ_c	64 (8,16,32,64) weeks	Represent different residence time and corresponding residence index (Georgiadis and Athanasiou, 2013), see Table 4
τ_m	8 weeks	Zhou et al. (2017); Lin et al. (2022)
τ_r	4 weeks	Zhou et al. (2017); Lin et al. (2022)
γ	60% (20%, 40%, 60%, 80%)	Represent different components failed rate
τ_a	16 (4, 8, 16, 32)	Represent different exponential smoothing level Ponte et al. (2019); Lin et al. (2022)
α	60% (20%, 40%, 60%, 80%)	Percentage of used components eventually returned (Zhou et al. 2017; Dominguez et al. 2019)
δ_t	100% (12.5%, 25%, 50%,100%), proportion of inventory error corrected	Traditional base stock policy with 100% inventory error adjustment ($\delta_t = 1$) is assumed, while the proportional base stock policy ($0 < \delta_t < 1$) can be explored to compare two control strategies in RIS and ROS systems
$\hat{L}$ $(=\widehat{\tau_p}+1)$	8 (2,4,8,16) weeks	Fixed estimated lead times ranging between 2-16 are assumed to explore its impact on dynamic performance, including lead times overestimation and underestimation scenarios.

Table 5. System parameter settings for the numerical study

7.1. Verification and sensitivity analysis

We verify the analytical results presented in Sections 4, 5 and 6. Three main control parameters (δ_t, τ_a and $\hat{L}$) and three system physical structure parameters (τ_c,γ and α) are varied to verify the analytical results. The former investigates the accuracy of analytical result under different decision-making preferences. On the other hand, by varying system physical structure, we can explore system sensitivity under different scenarios.

Tables 6 and 7 show simulation results for the ITAE of orders and inventories in the RIS and ROS systems. Overall, the analytical results are sufficiently accurate. The average gap between simulation and analytical predictions (*Proposition 2*) is 4.78% for orders' ITAE and 2.28% for inventories' ITAE. The simulation also verifies the impact of system control parameters and structure parameters on ITAEs. Specifically, both $ITAE^{RIS}{}_{O_m}$ and $ITAE^{ROS}{}_{O_m}$ decrease in α and $\hat{L}$, while increase in τ_c and γ. Also, $ITAE^{RIS}{}_{O_m}$ is independent of δ_t shown by the unchanged value (43.76) as the increase of δ_t. Furthermore, simulation result verifies that $ITAE^{RIS}{}_{IP_t}$ and $ITAE^{ROS}{}_{IP_t}$ increase in γ, τ_c, τ_a and $\hat{L}$, while decrease in δ_t. It is also proved that α plays no role in changing $ITAE^{RIS}{}_{IP_t}$, while $ITAE^{ROS}{}_{IP_t}$ increases in α. Finally, the simulation verifies the important findings of ITAE comparison (the *Property 2)*:

$ITAE^{RIS}{}_{O_m} > ITAE^{ROS}{}_{O_m}$ and $ITAE^{RIS}{}_{IP_t} < ITAE^{ROS}{}_{IP_t}$.

The impact of parameters on $ITAE$		**Parameter varied**	**Analytical prediction**	**Numerical results**	**Gap**
$\boldsymbol{ITAE_{O_r}}$ $(\boldsymbol{RIS = ROS})$	α	0.2	11.52	11.63	0.95%
		0.4	23.04	23.25	0.91%
		0.6	34.56	34.88	0.93%
		0.8	46.08	46.5	0.91%
	γ	0.2	11.52	11.63	0.95%
		0.4	23.04	23.25	0.91%
		0.6	34.56	34.88	0.93%
		0.8	46.08	46.5	0.91%
	τ_c	8	14.4	14.76	2.50%
		16	17.28	17.63	2.03%
		32	23.04	23.4	1.56%
		64	34.56	34.88	0.93%
$\boldsymbol{ITAE^{RIS}{}_{O_m}}$	α	0.2	63.68	66.7	4.74%
		0.4	52.16	55.09	5.62%
		0.6	40.64	43.46	7.68%
		0.8	29.12	31.8	9.20%
	γ	0.2	28.88	31.15	7.86%
		0.4	34.76	37.3	7.31%
		0.6	40.64	43.46	7.68%
		0.8	46.52	49.6	6.62%
	τ_c	8	27.2	30	10.29%
		16	29.12	31.96	9.75%
		32	32.96	35.8	8.62%
		64	40.64	43.46	7.68%
	$\hat{L}$	2	50.24	53.05	5.59%
		4	47.04	49.85	5.97%
		8	40.64	43.46	6.94%
		16	27.84	30.67	10.17%
	δ_t	0.125	N	43.46	N
		0.25	N	43.46	N
		0.5	N	43.46	N
		1	N	43.46	N
$\boldsymbol{ITAE^{ROS}{}_{O_m}}$	α	0.2	63.08	64.59	2.39%
		0.4	50.96	52.49	3.00%
		0.6	38.84	40.38	3.96%
		0.8	26.72	28.27	5.80%
	γ	0.2	28.28	29.43	4.07%
		0.4	33.56	34.9	3.99%
		0.6	38.84	40.38	3.96%
		0.8	44.12	45.86	3.94%
	τ_c	8	25.4	26.94	6.06%
		16	27.23	28.87	6.02%
		32	31.16	32.71	4.97%
		64	38.84	40.38	3.96%
	$\hat{L}$	2	48.44	51.26	5.82%
		4	45.24	48.06	6.23%
		8	38.84	40.38	3.96%
		16	26.04	28.87	10.94%
	δ_t	0.125	36.32	37.91	4.38%
		0.25	37.76	39.35	4.21%
		0.5	38.48	40.07	4.13%
		1	38.84	40.38	3.96%
Average gap					**4.78%**

Table 6. Simulation verification for the ITAE of manufacturing and remanufacturing orders.

The impact of parameters	**Parameter**	**Analytical**	**Numerical**	**Errors**

on $ITAE$		**varied**	**prediction**	**results**	
$\boldsymbol{ITAE^{RIS}{}_{IP_t}}$	α	0.2	N	971.9	N
		0.4	N	971.9	N
		0.6	N	971.9	N
		0.8	N	971.9	N
	γ	0.2	593.2	601.32	1.37%
		0.4	777.4	786.65	1.19%
		0.6	961.6	971.9	1.07%
		0.8	1145.8	1157.3	1.00%
	τ_c	8	692.8	703.99	1.62%
		16	731.2	742.39	1.53%
		32	808.1	819.19	1.37%
		64	961.6	971.9	1.07%
	δ_t	0.125	1241.6	1240.47	0.09%
		0.25	1081.6	1087.05	0.50%
		0.5	1001.6	1010.3	0.87%
		1	961.6	971.9	1.07%
	$\hat{L}$	2	260.8	262.17	0.53%
		4	494.4	498.77	0.88%
		8	961.6	971.9	1.07%
		16	1896	1918.37	1.18%
	τ_a	4	788.8	799.35	1.34%
		8	846.4	856.9	1.24%
		16	961.6	971.9	1.07%
		32	1192.1	1201.92	0.82%
$\boldsymbol{ITAE^{ROS}{}_{IP_t}}$	α	0.2	1036.8	984.29	5.06%
		0.4	1043.91	996.5	4.54%
		0.6	1045.25	1008.74	3.49%
		0.8	1046.58	1020.97	2.45%
	γ	0.2	633.6	613.58	3.16%
		0.4	835.2	811.63	2.82%
		0.6	1036.8	1008.74	2.71%
		0.8	1238.4	1206.32	2.59%
	τ_c	8	734.4	720.56	1.88%
		16	777.6	764.84	1.64%
		32	864	844.39	2.27%
		64	1036.8	1008.74	2.71%
	δ_t	0.125	1586.44	1554.14	2.04%
		0.25	1268.04	1238.18	2.35%
		0.5	1113.16	1084.5	2.57%
		1	1036.8	1008.74	2.71%
	$\hat{L}$	2	345.6	299.14	13.44%
		4	576	535.55	7.02%
		8	1036.8	1008.74	2.71%
		16	1958.4	1955.23	0.16%
	τ_a	4	864	836.1	3.23%
		8	921.6	893.66	3.03%
		16	1036.8	1008.74	2.71%
		32	1267.2	1238.74	2.25%
Average gap					**2.28%**

Table 7. Simulation verification for the ITAE of system total inventories.

The impact of parameters on bullwhip		**Parameter varied**	**Analytical prediction**	**Numerical results**	**Error**
BE (RIS)	α	0.2	2.3974	2.3794	0.75%
		0.4	2.3972	2.3774	0.83%
		0.6	2.3968	2.3696	1.13%
		0.8	2.3963	2.365	1.31%

	γ	0.2	2.3626	2.3547	0.33%
		0.4	2.379	2.3621	0.71%
		0.6	2.3968	2.3696	1.13%
		0.8	2.416	2.3774	1.60%
	τ_c	8	2.705	2.6058	3.67%
		16	2.5363	2.4761	2.37%
		32	2.4446	2.4096	1.43%
		64	2.3968	2.3696	1.13%
	δ_t	0.125	0.2022	0.2022	0.00%
		0.25	0.3839	0.3821	0.47%
		0.5	0.8326	0.8267	0.71%
		1	2.3968	2.3696	1.13%
	τ_a	4	8.3736	8.3056	0.81%
		8	4.1444	4.1104	0.82%
		16	2.3968	2.3696	1.13%
		32	1.6547	1.6409	0.83%
BE (ROS)	α	0.2	2.3975	2.3849	0.53%
		0.4	2.3977	2.3855	0.51%
		0.6	2.3981	2.3862	0.50%
		0.8	2.3986	2.3872	0.48%
	γ	0.2	2.3628	2.3602	0.11%
		0.4	2.3797	2.3731	0.28%
		0.6	2.3981	2.3862	0.50%
		0.8	2.418	2.3997	0.76%
	τ_c	8	2.7156	2.6743	1.52%
		16	2.5415	2.5105	1.22%
		32	2.4472	2.4271	0.82%
		64	2.3981	2.3862	0.50%
	δ_t	0.125	0.2034	0.2133	4.87%
		0.25	0.3852	0.3947	2.47%
		0.5	0.8338	0.84	0.74%
		1	2.3981	2.3862	0.50%
	τ_a	4	8.3749	8.3015	0.88%
		8	4.1457	4.1175	0.68%
		16	2.3981	2.3862	0.50%
		32	1.656	1.6499	0.37%
Average gap					**1.03%**

Table 8. Simulation verification for the bullwhip.

Table 8 provides simulation verification for *Proposition 3*. It shows our analytical prediction of BE is accurate, i.e., on average 1.03% gap between analytical and simulation results. Also, the impact of system main parameters on BE is verified: the BE for RIS and ROS increases in δ_t and γ, while decreases in τ_a and τ_c. Also, the increase of α decreases bullwhip in the RIS system but increases bullwhip in the ROS system.

There are several additional insights. First, δ_t and τ_a play dominant role in influencing system's bullwhip, while recoverable component rate (α) and component failed rate (γ) have limited impacts. The latter result is opposite to closed loop supply chain dynamics literature that component return rate plays an important role in influencing bullwhip effect, see Lin et al. (2022). This is due to the characteristics of component (re)manufacturing system where market demand consists of new-sales and after-sales demand, which the latter, as part of sales demand input into the system, is exponentially smoothed by α and γ (i.e. first order smoothing correction between new-sales and after-sales demand). Thereby, α and γ involved in the feedforward loop play limited role in influencing the dynamic performance of the system. Second, as predicated by *Proposition 3*, bullwhip effect generated by the RIS system ordering structure is always less than that in the ROS system, suggesting that the RIS system always has benefits from the reduced bullwhip, although such benefits are insignificant based on sensitivity analysis.

We further investigate the impact of two main control policies, τ_a and δ_t, on the dynamic performance of both systems. Note that the analytical results of IP_t, including both serviceable inventory and work-in-process inventory, may produce limited insights on the serviceable inventory holding cost and customer service level. As the result, the dynamic behaviour of serviceable inventory is assessed via extensive simulation in this section shown in Figure 4.

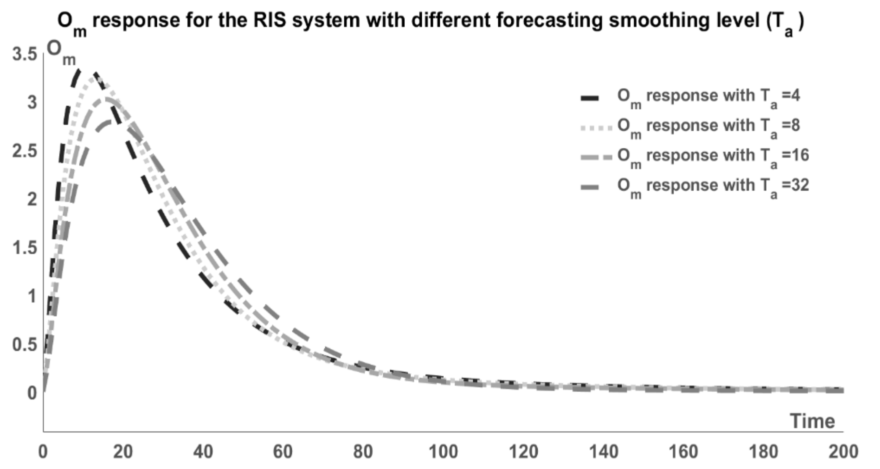

Figure 4.1. RIS o_m response under different τ_a.

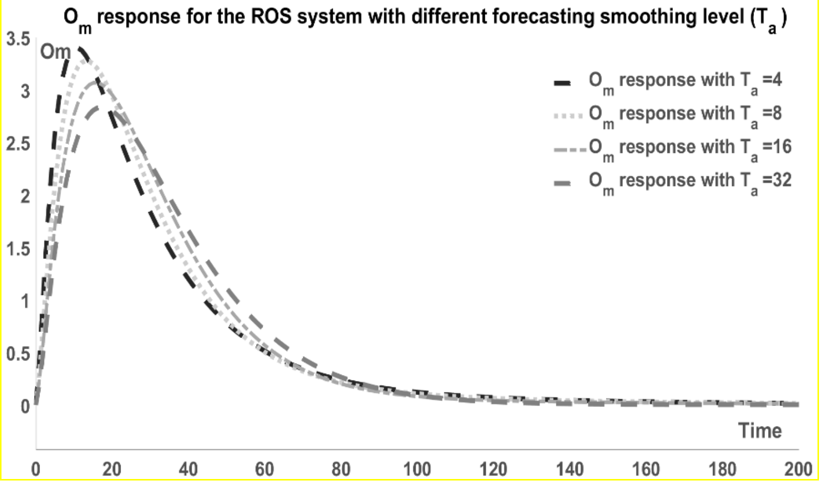

Figure 4.2. ROS o_m response under different τ_a

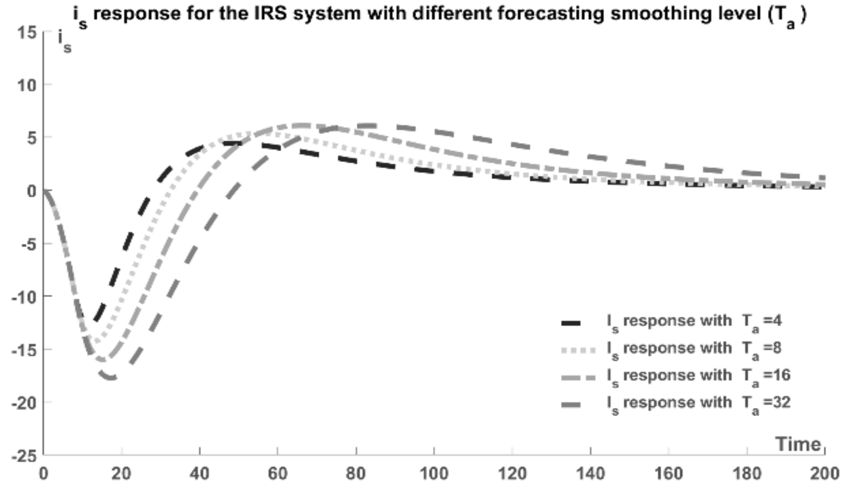

Figure 4.3. RIS i_s response under different τ_a.

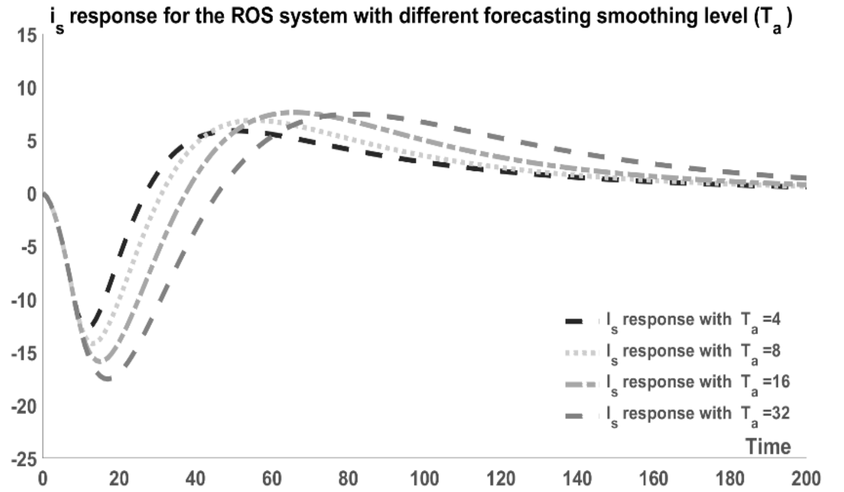

Figure 4.4. ROS i_s response under different τ_a

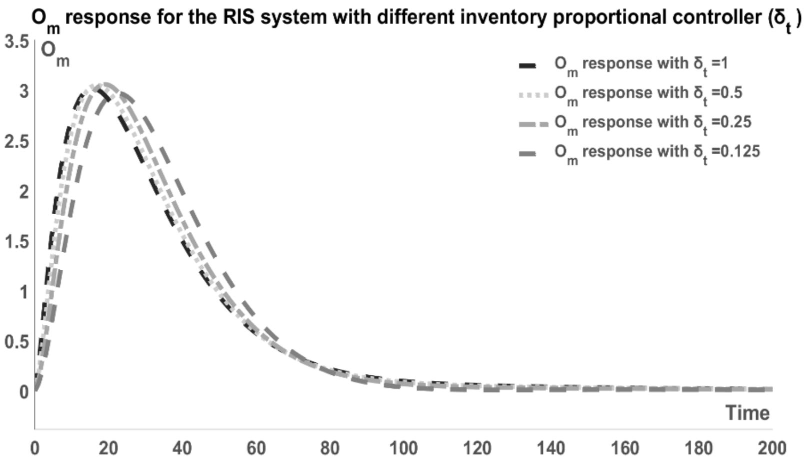

Figure 4.5. RIS o_m response under different δ_t.

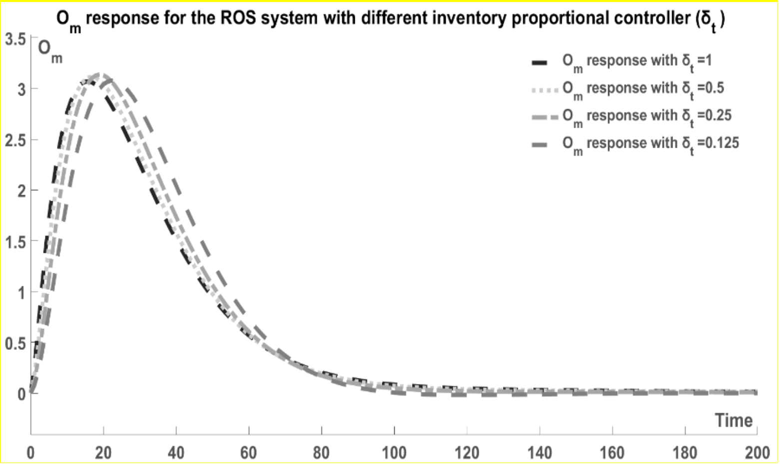

Figure 4.6. ROS o_m response under different δ_t

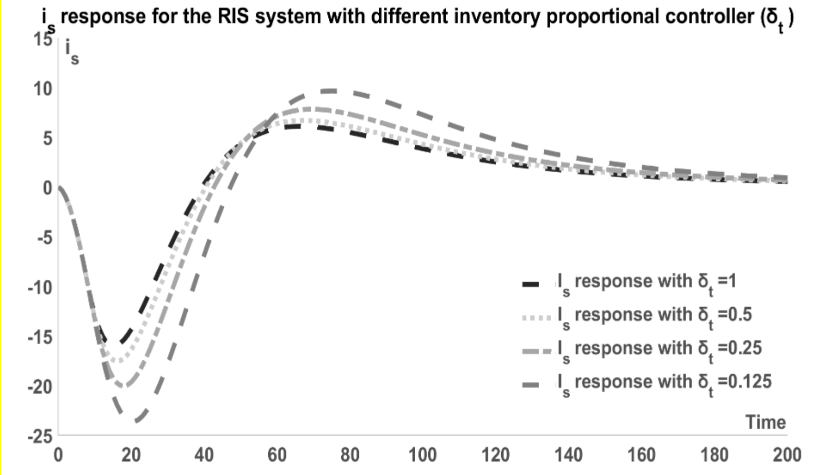

Figure 4.7. RIS i_s response under different δ_t.

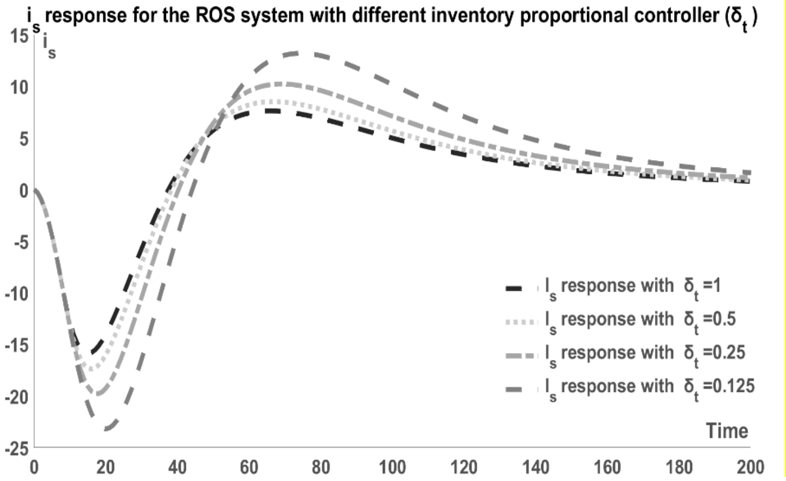

Figure 4.8. ROS i_s response under different δ_t

Figure 4. Dynamic behaviour of manufacturing order and serviceable inventories under different forecasting smoothing and proportional controller level

There are several observations based on Figure 4. First, the increase of τ_a decreases the peak manufacturing orders at the expense of slow convergency speed (shifting manufacturing orders at later stage) during demand lifecycle in both RIS and ROS systems. This verifies the analytical results that $ITAE^{ROS}{}_{o_m}$ and $ITAE^{RIS}{}_{o_m}$ are independent of τ_a. i.e. $ITAE_{o_m}$ remain unchanged because of the opposite change of peak demand and convergency speed as the change of τ_a. Second, slow forecasting smoothing (small value of τ_a) has a negative impact on dynamic performance of i_s by increasing the backlog level and decreasing the inventory recovery speed. This is consistent with Lin et al. (2022). Third, δ_t has a limited impact on the dynamic performance of o_m, although it shows U-shaped relations to manufacturing orders. However, the decreased δ_t negatively impacts the performance of serviceable inventory by increasing the inventory oscillations and reducing the convergency speed.

Comparing RIS and ROS systems, as predicted by *Proposition 2*, $ITAE^{RIS}{}_{o_m} > ITAE^{ROS}{}_{o_m}$ with less peak orders (i.e. less peak capacity requirement) and slow convergency speed under same parameter settings, although such difference is small. Also, $ITAE^{RIS}{}_{i_s} < ITAE^{ROS}{}_{i_s}$ is confirmed, suggesting that the RIS system always produces lower bound (i.e. higher backlog cost) and faster inventory convergency speed than the ROS system.

9. Managerial implications and conclusions

We consider a component remanufacturing system where two remanufacturing strategies are systematically compared from system dynamics perspective: the RIS system and ROS systems. Given the non-stationary lifecycle demand that contains a deterministic lifecycle trend and a stochastic random demand, the ITAE and bullwhip as two performance measurements are adopted in this study. Using lifecycle-based demand as the input of the RIS and ROS system contributes to systematically understand how different remanufacturing outsourcing strategies and main system parameters (e.g. forecasting and inventory control, manufacturing and remanufacturing lead times) impact on the dynamic behaviour of orders and inventory in responding to lifecycle-based demand. The dynamic behaviour of orders and inventory matters, since the associated convergency time and amplification of orders and inventory, measured by ITAE and bullwhip, are directly linked to the related costs such as long-term capacity investment, inventory dynamic costs and remanufacturing cost (Georgiadis et al. 2006; Östlin et al. 2009; Georgiadis and Athanasiou 2013; Jia et al. 2016). Table 9 summarizes all analytical and simulations results in this study.

Specifically, for both systems, the conventional system settings, i.e. $0 \leq \delta_t \leq 1, \tau_a \geq 0$ guarantee the system stability. This demonstrates that the modified base stock policy with proportional controller is robust for demand uncertainties. Second, the ITAE measurement is adopted due to its capabilities in measuring *convergency time* and *amplification* of orders and inventories, which directly link to companies' strategic decisions including long-term capacity requirements, manufacturing and remanufacturing production lifecycles and inventory holding volumes. The corresponding cost implications, e.g. capacity, production and inventory holding costs, can therefore be obtained. It can be concluded that under the same system parameter settings and structures, ${ITAE^{RIS}}_{O_m} > {ITAE^{ROS}}_{O_m}$ and ${ITAE^{RIS}}_{IP_t} < {ITAE^{ROS}}_{IP_t}$. Regarding the stationary bullwhip effect, $BE^{RIS} < BE^{ROS}$ regardless of system structures and control parameters.

The extensive numerical study was conducted to verify the analytical results and further enhance the understanding of both systems. ${ITAE^{RIS}}_{i_s} < {ITAE^{ROS}}_{i_s}$ is confirmed with a significant difference, suggesting that the RIS system always produces a lower bound and faster inventory convergency speed than the ROS system. Furthermore, δ_t and τ_a play a dominant role in influencing system's bullwhip, while recoverable component qualified rate (α) and component failed rate (γ) have limited impact for both systems. Based on all results, we derive the following managerial implications:

System dynamics measurement		**Analytical or simulation results**
Stability		For both systems, stability can be guaranteed if $0 \le \delta_t \le 2, \tau_a \ge -0.5$ $(0 < \mu \le 2)$ and $-1 \le \frac{\tau_c}{1+\gamma+\tau_c} \le 1$
Convergency for IRS and ROS systems		1. If a lifecycle based market demand pattern is assumed, the order and serviceable inventory in both systems converge to zero independent of $\hat{L}$. 2. If a sudden but sustained unit sales demand increase is assumed, the new equilibrium of order $O_m^{eq} = 1 + \gamma - \alpha\gamma$ and inventory $I_s^{eq}(RIS) = (1+\gamma)(\hat{L} - \tau_m) + \alpha\gamma(\tau_m - \tau_r)$ and $I_s^{eq}(ROS) = \frac{\alpha\gamma}{\delta_t} + (1+\gamma)(\hat{L} - \tau_m) + \tau_m\alpha\gamma$
ITAE	$ITAE_{O_r}$	${ITAE^{RIS}}_{O_r} = {ITAE^{ROS}}_{O_r}$, which increases with an increase in γ, τ_c, α and τ_l
	$ITAE_{O_m}$	1. ${ITAE^{RIS}}_{O_m} > {ITAE^{ROS}}_{O_m}$ given the same system parameters 2. ${ITAE^{RIS}}_{O_m}$ increases with τ_c, and τ_l, and decreases with $\hat{L}$ and α. δ_t and τ_a have no impact on ${ITAE^{RIS}}_{O_m}$ 3. ${ITAE^{ROS}}_{O_m}$ increases with τ_c, δ_t and τ_l, and decreases with $\hat{L}, \gamma$ and α. ${ITAE^{RIS}}_{O_m}$ is independent of τ_a
	$ITAE_{IP_t}$	1. ${ITAE^{RIS}}_{IP_t} < {ITAE^{ROS}}_{IP_t}$ given the same system parameters 2. ${ITAE^{RIS}}_{IP_t}$ increases with $\hat{L}, \tau_c, \gamma, \tau_a$and τ_l, while δ_t is negatively correlated to ${ITAE^{RIS}}_{IP_t}$. α has no impact on ${ITAE^{RIS}}_{IP_t}$ 3. ${ITAE^{ROS}}_{IP_t}$ increases with $\hat{L}, \tau_c, \gamma, \alpha, \tau_a$ and τ_l, and decreases in δ_t.
Bullwhip effect (BE)		1. $BE^{RIS} < BE^{ROS}$ regardless of system structure and control parameters 2. Both BE^{RIS} and BE^{ROS} increases with $\hat{L}$, δ_t and γ, and decreases with τ_a, τ_c and α
Verification and sensitivity analysis		1. δ_t and τ_a play a dominant role in influencing the system's bullwhip 2. ${ITAE^{RIS}}_{i_s} < {ITAE^{ROS}}_{i_s}$

Table 9. Summary of main findings for this study.

1. *Lifecycle demand and return effect:* when a nonstationary lifecycle demand and return are introduced as inputs of the system, companies need to consider both strategic-related and operational-related performance, driven by long-term lifecycle and short-term stochastic demand, before deciding on a remanufacturing in-house or outsourcing strategy. For the former, it includes peak capacity required, manufacturing production volume at different stages of the lifecycle, inventory holding volume (peak inventory) and backlog volume (maximum backlog). The latter performance is associated with the short-term orders and inventory fluctuations and thereby corresponding workforce, inventory and capacity adjustments.
2. *System dynamics performance of manufacturing orders:* Under the same system parameter settings, RIS requires less peak production capacity than the ROS system driven by less peak manufacturing orders. However, if both manufacturing and remanufacturing production (in-house or outsourcing) are available for meeting lifecycle market demand, the RIS system requires more manufacturing production volume than that in the ROS system during the decline stage of the demand lifecycle. As a result, a company that adopts RIS or ROS strategies may need different investments at different stages of the demand lifecycle: more capacity investment is need during the growth stage for the RIS strategy, while more manufacturing production cost results at the decline stage for the ROS strategy.
3. *System dynamics performance of serviceable inventory:* the RIS system outperforms the ROS system with less inventory and less backlog levels, thereby less inventory related cost (e.g. inventory holding cost) and higher customer service levels. However, the RIS system is worse than the ROS system with respect to higher pipeline inventory level and slower inventory convergency speed. A clear trade-off between serviceable and pipeline inventory cost can be observed and companies should consider the different remanufacturing strategies based on the total inventory cost consideration.
4. *Bullwhip performance of manufacturing orders:* From the short-term operational level, the RIS system generates less bullwhip than that in the ROS system. For both systems, the bullwhip can be reduced by incorporating a proportional adjustment of serviceable inventory errors and a greater forecasting smoothing value. As a result, less order variation induced costs can be achieved by adopting the RIS strategy, including short-term capacity variation and labour adjustment costs.

Overall, we found that RIS outperforms ROS system, including less peak capacity, less inventory holding and less backlog. This is mainly driven by the less information visibility in the ROS system due to the remanufacturing outsourcing strategy where remanufacturing pipeline inventory and order information cannot be used for updating manufacturing orders. Also, the stationary bullwhip produced by the RIS is always less than the ROS system under the same system parameters, thus less bullwhip cost is possible by choosing the RIS. However, the adoption of the RIS may result slow convergency of manufacturing production, and thus a higher cost can be generated, particularly at the decline stage of demand lifecycle: an important cost needs to be strategically considered. Thereby, from system dynamics perspective, the component manufacturer needs carefully consider the trade-offs between production and inventory performance, and their demand lifecycle characteristics for the appropriate remanufacturing strategy.

There are two main contributions of this study. First, we advanced the closed-loop supply chain dynamics literature by comparing the dynamics of two remanufacturing strategies commonly observed in practice, extending the previous literature focusing solely on the in-house remanufacturing. Second, the impact of corrected lifecycle demand and return patterns on system dynamics performance are analytically assessed. Although several studies investigated the dynamics of CLSCs under lifecycle demand and return via pure simulation (e.g. Georgiadis et al. 2006; Georgiadis and Athanasiou 2010; Georgiadis and Athanasiou 2013), our study is the first analytical work which analytically explores the impact of corrected lifecycle demand and return on system dynamics performance.

Several future research directions can be considered. First, four stage of sales and returns lifecycle, including infancy, growth, maturity and decline stage can be differentiated and its dynamics implications can be independently analyzed to obtain further managerial implications. Second, the multi-echelon closed loop supply chains can be studied to extend the current one-echelon component manufacturing system with different remanufacturing strategies. Third, cost-based objective functions can be developed to focus on the detailed minimization problem of production and inventory costs and thereby derive the corresponding control parameter settings. Finally, remanufacturing pull as an important production strategy can be incorporated into our system dynamics model and compared with push production under different remanufacturing strategies.

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Appendix 1. Derive transfer functions of IRS and ROS systems using $\mathcal{Z}$-transform

1.1. An introduction for $\mathcal{Z}$-transform

Linear discrete-time systems are represented by linear difference equations. System analysis, that is, the dynamic response of the system variables, requires the solution of difference equations for a given input function. This solution can be characterized in the time-domain or, alternatively, in the frequency domain using the $\mathcal{Z}$-transform. The benefit of the latter is that difference equations are transformed into an easier-to-solve polynomial equations. The $\mathcal{Z}$-transform of a discrete time series, $d(t)$, $t = 1, 2, 3 \ldots, \infty$ is defined as

$$\mathcal{D}(z) = \mathcal{Z}\,[d(t)] = \sum_{t=0}^{\infty} d(t) z^{-t} \quad \forall\, z = e^{i\emptyset}. \qquad (A1.1)$$

If the system is linear and time-invariant, the output of the system, denoted by $g(t)$, for any input function, $d(t)$, can be fully described by the impulse response function, which is the solution of difference equations for the unit impulse input function, $\delta(t)$, $\delta[t = 0] = 1, \delta[t \neq 0] = 1$. This is due to the linearity property that any input is the superposition of scaled and delayed impulse function, which the output will be the superposition of scaled and delayed impulse function. Equivalently, the output is the convolution of the input and impulse response:

$$g(t) = \sum_{i=0}^{t} d(t)\delta(t-i). \qquad (A1.2)$$

The $\mathcal{Z}$-transform of the impulse response function is called the transfer function $\Delta(z)$. The convolution, Equation (A1.2), thereby can be transformed into simple multiplication to obtain:

$$G(z) = D(z)\Delta(z). \qquad (A1.3)$$

1.2. Derive the transfer functions of order and inventories for IRS and ROS systems

In this section we derive the transfer function of orders and inventories for the IRS system and ROS systems. Starting from the inventory balance equation for both systems:

$$i^{IRS}{}_s(t) = i_s(t-1) + o_r^{IRS}(t-L_r) + o_m(t-L_m) - d_s(t) - d_w(t), \qquad (A1.4)$$

$$i^{ROS}{}_s(t) = i_s(t-1) + o_r^{ROS}(t-L_r) + o_m(t-L_m) - d_s(t) - d_w(t). \qquad (A1.5)$$

The $\mathcal{Z}$-transform of Equations A1.4 and A1.5 can be presented as:

$$I^{IRS}{}_s(z) = z^{-1} I^{IRS}{}_s(z) + z^{-L_r} O_r^{IRS}(z) + z^{-L_m} O_m(z) - D_s(z) - D_w(z), \qquad (A1.6)$$

$$I^{ROS}{}_s(z) = z^{-1} I^{ROS}{}_s(z) + z^{-L_r} O_r^{ROS}(z) + z^{-L_m} O_m(z) - D_s(z) - D_w(z). \qquad (A1.7)$$

Where Equations (A1.6) and (A1.7) can be re-written as:

$$I^{IRS}{}_s(z) = \left(z^{-L_r} O_r^{IRS}(z) + z^{-L_m} O_m(z) - D_s(z) - D_w(z)\right) \frac{z}{z-1}, \qquad (A1.8)$$

$$I^{ROS}{}_s(z) = \left(z^{-L_r} O_r^{ROS}(z) + z^{-L_m} O_m(z) - D_s(z) - D_w(z)\right) \frac{z}{z-1}. \qquad (A1.9)$$

Equations (A1.8) and (A1.9) show that serviceable inventory transfer function for both systems, $I_s(z)$, can be described as the function of manufacturing ($z^{-L_m}O_m(z)$) and remanufacturing ($z^{-L_r}O_r(z)$) orders and market demand, $D_t(z) = D_s(z) + D_w(z)$ transform.

It should be noted that due to different remanufacturing strategies (in-house and outsourcing) and thereby the level of autonomous, $z^{-L_r}O_r^{ROS}(z)$ and $z^{-L_r}O_r^{IRS}(z)$ are different (Abbey & Guide Jr 2018).

To find the transfer function of orders, the transfer function of system total inventory (IP) including on-hand serviceable inventory and outstanding receipts ($W(z)$) should be derived:

$$IP(z) = I_s(z) + W(z), \tag{A1.10}$$

where $W(z)$ for IRS and ROS systems can be derived:

$$W^{IRS}(z) = \sum_{i=1}^{L_r-1} z^{-i}O_r^{IRS}(z) + \sum_{i=1}^{L_m-1} z^{-i}O_m(z)$$

$$= \left(z^{-1}O_r^{IRS}(z) - z^{-L_r}O_r^{IRS}(z) + z^{-1}O_m(z) - z^{-L_m}O_m(z)\right)\frac{z}{z-1}$$

$$= \frac{1-z^{1-L_r}}{z-1}O_r^{IRS}(z) + \frac{1-z^{1-L_m}}{z-1}O_m(z), \tag{A1.11}$$

$$W^{ROS}(z) = \sum_{i=1}^{L_m-1} z^{-i}O_m(z) = \left(z^{-1}O_m(z) - z^{-L_m}O_m(z)\right)\frac{z}{z-1} = \frac{1-z^{1-L_m}}{z-1}O_m(z). \tag{A1.12}$$

Equations (A1.11) and (A1.12) show that, for the manufacturer, both manufacturing and remanufacturing outstanding receipts are considered for the IRS system, while only manufacturing outstanding receipts are needed as part of order up to decision in the ROS system. This is explained as the remanufacturing is outsourced to the third-party remanufacturer in the ROS system and thereby remanufacturing pipeline inventory information cannot be shared to the manufacturer (Ponte et al. 2021). Thus, the transfer functions of $IP(z)$ for both systems can be derived:

$$IP^{IRS}(z) = \left(z^{-1}O_m(z) + z^{-1}O_r^{IRS}(z) - D_s(z) - D_w(z)\right)\frac{z}{z-1}, \tag{A1.13}$$

$$IP^{ROS}(z) = \left(z^{-1}O_m(z) + z^{-L_r}O_r^{ROS}(z) - D_s(z) - D_w(z)\right)\frac{z}{z-1}. \tag{A1.14}$$

The $\mathcal{Z}$-transform of the base stock policy with proportional controller, $\delta_t, \forall\ \delta_t \in (0,1]$, can be obtained:

$$O_t(z) = \mathrm{S}(z) - IP(z) = \hat{D}(z)\cdot\left(\delta_t\hat{L} + 1\right) - \delta_t \cdot IP(z), \tag{A1.15}$$

where

$$O_t{}^{IRS}(z) = O_m(z) + O_r^{ROS}(z), \tag{A1.16}$$

$$O_t{}^{ROS}(z) = O_m(z). \tag{A1.17}$$

It should be noted that remanufacturing push is adopted for both systems and thereby

$$O_r(z) = a \cdot D_w(z) = a \cdot \frac{z\gamma}{z + z\tau_l - \tau_l} \cdot \frac{\beta}{\left(1 + \frac{\tau_l}{n} - \frac{\tau_l}{n}z^{-1}\right)^n}. \tag{A1.18}$$

Finally, based on Equations (A1.16) - (A1.18), the total orders (O_t) transfer functions for the IRS and ROS systems can be derived:

$$\begin{aligned} {O_t}^{IRS}(z) &= \widehat{D_t}(z) \cdot (\delta_t \hat{L} + 1) - \delta_t \cdot IP(z) \\ &= \widehat{D_t}(z) \cdot (\delta_t \hat{L} + 1) - \delta_t \cdot \frac{z}{z-1}\left(z^{-1}{O_t}^{IRS}(z) - D_t(z)\right) \\ &= \frac{(\delta_t \hat{L} + 1)(z-1)}{z + \delta_t - 1} \cdot \hat{D}(z) + \frac{\delta_t z}{z + \delta_t - 1} \cdot D_t(z), \end{aligned} \quad (A1.19)$$

$$\begin{aligned} {O_t}^{ROS}(z) = {O_m}^{ROS}(z) &= \widehat{D_t}(z) \cdot (\delta_t \hat{L} + 1) - \delta_t \cdot IP(z) \\ &= \widehat{D_t}(z) \cdot (\delta_t \hat{L} + 1) - \frac{\delta_t z}{z-1}\left(z^{-1}O_m(z) + z^{-L_r}O_r^{ROS}(z) - D_t(z)\right) \\ &= \frac{(\delta_t \hat{L} + 1)(z-1)}{z + \delta_t - 1} \cdot \widehat{D_t}(z) + \frac{\delta_t z}{z + \delta_t - 1} \cdot D_t(z) - \frac{\delta_t z^{-L_r+1}}{z + \delta_t - 1} \cdot O_r(z), \end{aligned} \quad (A1.20)$$

where $D_s(z) = \frac{\beta}{\left(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1}\right)^n}$, $D_w(z) = \frac{z\gamma}{z+z\tau_c-\tau_c} \cdot \frac{\beta}{\left(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1}\right)^n}$ and $D_t(z) = D_s(z) + D_w(z)$.

Note that $\widehat{D_t}(z)$ is the forecasting for market demand ($D_t(z)$) and z-transform of exponential smoothing forecasting can be obtained, $\widehat{D_t}(z) = \frac{z}{z+(z-1)\tau_a} D_t(z)$, $\mu = \frac{1}{\tau_a+1} \in [0,1]$, although other forecasting methods such as moving average can be also represented in z domain.

Substituting $D_s(z)$, $D_w(z)$ and $\widehat{D_t}(z)$ into Equations (A1.19) and (A1.20), the orders transfer function of IRS and ROS systems in responding to $D_m(z)$ can be derived:

$$\frac{{O_m}^{IRS}(z)}{D_t(z)} = \frac{z\left(\delta_t(z + z\tau_a - \tau_a) + (z-1)(1 + \delta_t \hat{L})\right)}{(z - 1 + \delta_t)(z + z\tau_a - \tau_a)} - \frac{z\alpha\gamma}{(z + z\gamma + (z-1)\tau_c)}, \quad (A1.21)$$

$$\frac{{O_m}^{ROS}(z)}{D_t(z)} = \left(\frac{z\left(\delta_t(z + z\tau_a - \tau_a) + (z-1)(1 + \delta_t \hat{L})\right)}{(z - 1 + \delta_t)(z + z\tau_a - \tau_a)} - \frac{z\alpha\gamma\delta_t}{z^{\tau_r}(z - 1 + \delta_t)(z + z\gamma + (z-1)\tau_c)}\right). \quad (A1.22)$$

Based on Equations (A1.13) and (A1.14), the transfer functions of IP for IRS and ROS system are derived as follows:

$$\begin{aligned} \frac{IP^{IRS}(z)}{D_t(z)} &= \left(z^{-1}{O_t}^{IRS}(z) - 1\right)\frac{z}{z-1} \\ &= \left(\frac{\delta_t(z + z\tau_a - \tau_a) + (z-1)(1 + \delta_t \hat{L})}{(z - 1 + \delta_t)(z + z\tau_a - \tau_a)} - 1\right)\frac{z}{z-1}, \end{aligned} \quad (A1.23)$$

$$\frac{IP^{ROS}(z)}{D_t(z)} = \left(z^{-1}{O_t}^{ROS}(z) + z^{-L_r}\frac{O_r(z)}{D_t(z)} - 1\right)\frac{z}{z-1}$$

$$= \left(\frac{\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1)(z-1+\delta_t)(z+z\tau_a-\tau_a)} - 1\right)\left(\frac{(z-1)z^{1-\tau_r}\alpha\gamma(\delta_t-1)}{(z-1+\delta_t)(z+z\gamma+z\tau_c-\tau_c)}\right). \quad (A1.24)$$

Finally, given $IP(z) = I_s(z) + W(z)$, the transfer function of I_s in relation to D_t can be derived as follow:

$$\frac{I_s^{IRS}(z)}{D_t(z)} = \left(\frac{z\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)z^{\tau_m}} + \frac{z\alpha\gamma(z^{\tau_m}-z^{\tau_r})}{z^{\tau_r+\tau_m}(z+z\gamma+(z-1)\tau_c)} - 1\right)\frac{z}{z-1}, \tag{A1.25}$$

$$\frac{I_s^{ROS}(z)}{D_t(z)} = \left(\frac{z\left(\delta_t(z+z\tau_a-\tau_a)+(z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)z^{\tau_m}} + \frac{z(z^{\tau_m}\alpha\gamma(z-1+\delta_t)-z\alpha\gamma\delta_t)}{z^{\tau_r+\tau_m}(z-1+\delta_t)(z+z\gamma+(z-1)\tau_c)} - 1\right)\frac{z}{z-1}. \tag{A1.26}$$

Appendix 2. Proof of Propositions and Theorems

2.1 Proof of Proposition 1.

First, the $\mathcal{Z}$ -transform of lifetime-based market demand can be derived, $D_t(z) = \mathcal{Z}\,[d_s(t)] + \,[d_w(t)] = \frac{\beta}{\left(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1}\right)^n} + \frac{\beta}{\left(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1}\right)^n} \cdot \frac{z\gamma}{z+z\tau_l-\tau_l}$. Based on Equations (A1.18-1.22), The convergency of $\mathrm{O_m}(z)$ and $I_s(z)$ for $D_t(z)$ for both systems then can be obtained by exploring the *Final Value Theorem*, i.e. $\lim_{z\to 1} \mathrm{O_m}(z) \cdot (z-1)$. For *Proposition 1.2,* if the new-sales demand d_s, is assumed to be increased, i.e. Heaviside Function, $d_s = 0, \forall\, t < 0, d_s = 1, \forall\, t \geq 0$, its $\mathcal{Z}$-transform is $D_s(z) = \mathcal{Z}\,[d_s] = \sum_{n=0}^{\infty} d_s(t) z^{-n} = \frac{z}{z-1}$. We can then obtain total market demand in z domain, $D_t(z) = \mathcal{Z}\,[d_s(t)] + \,[d_w(t)] = \frac{z}{z-1} + \frac{z}{z-1} \cdot \frac{\gamma}{1+\tau_l-\tau_l z^{-1}}$. The convergency of $O_m(z)$ and $I_s(z)$ for $D_t(z)$ for both systems can be obtained using the same process above.

2.2 Proof of Theorem 1.

Denote the roots of order and inventory transfer functions in IRS and ROS systems as p_i^{IRS} and p_i^{ROS} , $i = 1, \dots n$. Table 1 shows p_i^{IRS} and p_i^{ROS} based on Equations (26)-(28) and *Definition 1* in the main paper.

Denominator of system transfer function	Poles (p_i^{IRS} and p_i^{ROS})
$C_{IRS}(O_m)$	$p_1 = 1 - \delta_t; p_2 = \frac{\tau_a}{1+\tau_a}; p_3 = \frac{\tau_c}{1+\gamma+\tau_c}$
$C_{ROS}(O_m)$	$p_1 = 1 - \delta_t; p_2 = \frac{\tau_a}{1+\tau_a}; p_3 = \frac{\tau_c}{1+\gamma+\tau_c}\ p_4 = 0$
$C_{IRS}(I_s) = C_{IRS}(I_s)$	$p_1 = 1 - \delta_t; p_2 = \frac{\tau_a}{1+\tau_a}; p_3 = \frac{\tau_c}{1+\gamma+\tau_c}\ p_4 = 0; p_5 = 0$

Table 1. Poles for IRS and ROS system.

Under the sufficient and necessary condition of system stability, i.e. $-1 \leq |p_i| \leq 1$, we have $0 \leq \delta_t \leq 2$ and $\tau_a \geq -0.5$. Note that $p_3 = \left|\frac{\tau_c}{1+\gamma+\tau_c}\right| < 1$ holds for positive τ_c (considerable long customer in-use delay) and $\gamma,\ \forall\, \gamma \in (0,1)$, i.e. deterministic fail rate. As the result, p_3 is guaranteed within the unit circle of the complex plane.

2.3. Proof of Proposition 2

To derive the analytical expression of ITAE, the system should be stable, i.e. the convergency of ϵ should exist given $ITAE = \sum_{t=0}^{\infty} t|\epsilon|$. Second, the error term in ITAE, $|\epsilon|$, should always be positive or negative. By inspecting transfer functions of $O_m(t), O_r(t)\ and\ IP\ (t)$ for both IRS and ROS systems, we can ensure their dynamic response are positive for conventional system settings, $0 \leq \delta_t \leq 1$ and $\tau_a \geq 0$.

Due to the high-order difference equations of $O_m(t)\ and\ IP(t)$, it is not possible to analytically derive the ITAE results by using its formal definition and thereby ITAE is traditionally calculated via simulation (Disney and Towill 2003). However, we explore the novel approach (Tsypkin 1964) to derive ITAE by linking the transfer function and difference equations. Specifically, Tsypkin proposed the so-called discrete Laplace transform (DLT) to draw analogies between differential equations and difference equations:

$$D[\hbar(n)] = \mathcal{H}\,[e^{q}] = \sum_{n=0}^{\infty} \hbar(n)e^{-qn}, \qquad (A2.1)$$

where $D[\hbar(n)]$ ($\mathcal{H}\,[e^{q}]$) is the discrete Laplace transform of difference equation, $\hbar(n)$, and it can be seen that Equation (A2.1) is directly equivalent to the definition of $\mathcal{Z}$-transform by letting $z = e^{q}$. Tyspkin (1964) further showed that the sum of the difference equation is equal to the value of the transfer function at $q = 0$, i.e. $z = e^{q} = 1$, formally,

$$\mathcal{H}\,[0] = \lim_{q\to 0} \mathcal{H}\,[q] = \sum_{n=0}^{\infty} \hbar(n). \qquad (A2.2)$$

As the result, if the dynamic response of $O_m(t), O_r(t)\ and\ IP\ (t)$ are always positive, the $ITAE_{O_m}, ITAE_{O_r}$ and $ITAE_{IP}$ can be calculated by *the value of their transfer functions multiplied by time at* $q = 0$*, or, alternatively, the value of* $\mathcal{Z}$ *transformed transfer functions at* $z = 1$

$$\lim_{q\to 0} n\mathcal{H}\,[e^{q}] = \lim_{z\to 1} \mathcal{Z}\,[\hbar(n)] = \sum_{n=0}^{\infty} n\hbar(n). \qquad (A2.3)$$

Using the transfer functions of $O_m(t), O_r(t)\ and\ IP\ (t)$ derived in Appendix 1, that are, Equations (A1.16), (A1.17), (A1.20)- (A1.23), we can analytically derive the ITAE. First, for simplicity without losing generality, we assume the scaler factor of lifetime sales demand, $\beta = 1$ for all ITAE measurement. Regarding remanufacturing orders, IRS and ROS systems have the identical dynamic response given the adoption of remanufacturing push policy. As the result, $ITAE_{O_r}$ should be the same for both systems. First, we can obtain dynamics response of remanufacturing orders by using transfer function multiplied by sales demand $D_s(z)$, i.e. $O_r(z) = \frac{\gamma}{(1+\tau_c-\frac{\tau_c}{z})} \cdot \alpha \cdot D_s(z) = \frac{\gamma}{(1+\tau_c-\frac{\tau_c}{z})} \cdot \alpha \cdot \frac{1}{\left(1+\frac{\tau_l}{n}-\frac{\tau_l}{n}z^{-1}\right)^{n}}$. We then take the inverse z-transform of $O_r(z)$ to obtain time response of $O_r(t)$:

$$O_r(t) = \frac{1}{2\pi i}\oint O_r(z)z^{t-1}d_z = \frac{\alpha\gamma\tau_c^{t}\left(\frac{1}{1+\tau_c}\right)^{1+t}\left(\tau_c(1+\tau_l) - (1+\tau_c)\tau_l\left(\frac{\tau_l+\tau_l\tau_c}{\tau_c+\tau_c\tau_l}\right)^{t}\right)}{(\tau_c-\tau_l)(1+\tau_l)}. \qquad (A2.4)$$

Using Equation (2.4) to multiply t and take its $\mathcal{Z}$-transform:

$$t \cdot O_r(z) = \mathcal{Z}\,[O_r(t)] \cdot t = \frac{z^{2}\alpha\gamma\left(z\tau_l + \tau_c(z + 2\tau_l z - 2\tau_l)\right)}{(z+(z-1)\tau_c)^{2}(z+(z-1)\tau_l)^{2}}. \qquad (A2.5)$$

Finally, $ITAE_{O_r}$ can be derived by letting $z = 1$ for Equation (A2.5):

$$ITAE_{O_r} = \lim_{z\to 1} \frac{z^2\alpha\gamma\left(z\tau_l + \tau_c(z + 2\tau_l z - 2\tau_l)\right)}{(z + (z-1)\tau_c)^2(z + (z-1)\tau_l)^2} = \alpha\gamma(\tau_c + \tau_l). \qquad (A2.6)$$

The above process can be implemented for deriving $ITAE_{O_m}$ and $ITAE_{IP}$ for IRS and ROS systems.

2.4. Proof of Proposition 3

Before deriving the variance ratio between manufacturing order and market demand in IRS and ROS systems, we highlight the Tsypkin relations via following Lemma:

Lemma 1. *Tsypkin (1964). If the stochastic input* d_t *to a linear system with impulse response function* $\delta_t, \forall\ \delta[t=0]=1, \delta[t\neq 0]=1$ *with* $\mathcal{Z}$*-transform* $\Delta(z)=1$*, is an i.i.d random process with variance,* σ_{dt}^2*, then the long-run variance of the output* O_t:

$$\sigma_{O_t}^2 = \sigma_{dt}^2 \sum_{t=0}^{\infty} (\delta_t)^2. \qquad (A2.7)$$

Proof. See Boute et al. (2022).

Note, *Lemma 1* holds regardless of the distribution of d_t; all required is that d_t is i.i.d process. Based on *Lemma 1,* we can calculate the variance ratio between order and demand (i.e. bullwhip effect) by calculating *the sum of the square of the system's impulse response*.

To derive the variance ratio between manufacturing order and market demand for IRS and ROS systems, the corresponding transfer function can be obtained:

$$\frac{O_m^{IRS}(z)}{D_t(z)} = \frac{z\left(\delta_t(z + z\tau_a - \tau_a) + (z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)} - \frac{z\alpha\gamma}{(z + z\gamma + (z-1)\tau_c)}, \qquad (A2.8)$$

$$\frac{O_m^{ROS}(z)}{D_t(z)} = \frac{z\left(\delta_t(z + z\tau_a - \tau_a) + (z-1)(1+\delta_t\hat{L})\right)}{(z-1+\delta_t)(z+z\tau_a-\tau_a)} - \frac{z\alpha\gamma\delta_t}{z^{\tau_r}(z-1+\delta_t)(z + z\gamma + (z-1)\tau_c)}, \qquad (A2.9)$$

where market demand, $D_t(z)$, including sales and service demand, i.e. $D_t(z) = D_s(z) + D_w(z) = D_s(z)(1 + \frac{z\gamma}{z + z\tau_c - \tau_c})$. That is, the only endogenous input of the system is new-sales, $D_s(z)$, while $D_w(z)$ and $D_t(z)$ are dependent on $D_s(z)$. As the result, in order to obtain the variance ratio between manufacturing order rate and endogenous sales demand (i.i.d stationary process), the variance ratio between total market demand $D_t(z)$ and sales demand, $D_s(z)$ should be derived first. We can simply have the transfer function of $D_t(z)$ in relation to $D_s(z)$ as follows:

$$\frac{D_t(z)}{D_s(z)} = 1 + \frac{z\gamma}{z + z\tau_c - \tau_c}. \qquad (A2.10)$$

Take the inverse z-transform of $D_t(z)$ to obtain time response of $d_t(\mathrm{t})$:

$$d_t(\mathrm{t}) = \frac{1}{2\pi i}\oint D_t(z) z^{t-1} d_z = \gamma\tau_c^t \frac{1}{(1+\tau_c)^{1+n}}(1 - h[t]) + \frac{(1+\gamma+\tau_c)h[t]}{1+\tau_c}, \qquad (A2.11)$$

where $h[t]$ is the Unit Step function, i.e. $h[t < 0] = 0$, otherwise $h[t \geq 0] = 1$. Based on Tsypkin relations, the variance ratio between $d_t(\mathrm{t})$ and $d_s(\mathrm{t})$ can be derived:

$$\frac{\sigma_{d_t}^2}{\sigma_{d_s}^2} = \sum_{t=0}^{\infty} \left(d_t(\mathrm{t})\right)^2 = \sum_{t=0}^{\infty} \left(\gamma \tau_c^t \frac{1}{(1+\tau_c)^{1+n}} (1 - h[t]) + \frac{(1+\gamma+\tau_c)h[t]}{1+\tau_c}\right)^2$$
$$= \frac{1 + 2\gamma + \gamma^2 + 3\tau_c + 4\gamma\tau_c + \gamma^2\tau_c + 2\tau_c^2}{(1+\tau_c)(1+2\tau_c)}. \qquad (A2.12)$$

Equation (A2.12) analytically shows the variance ratio between total market demand and sales demand, which is dependent on failed rate (γ) and customer return delay (τ_c). Based on Equations (A2.8) and (A2.9) and apply the process above to obtain the variance ratio between manufacturing order and total market demand:

$$\frac{\sigma_{o_m(IRS)}^2}{\sigma_{d_t}^2} = \frac{\left(2 + \delta_t(3 + 4L + 6\tau_a) + \delta_t^2\left(2L(1+L) + (1+4L)\tau_a + 2\tau_a^2\right)\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)}$$
$$+ \frac{\alpha^2\gamma^2}{(1+\gamma)(1+\gamma+2\tau_c)}, \qquad (A2.13)$$

$$\frac{\sigma_{o_m(ROS)}^2}{\sigma_{d_t}^2} = \frac{\left(2 + \delta_t(3 + 4L + 6\tau_a) + \delta_t^2\left(2L(1+L) + (1+4L)\tau_a + 2\tau_a^2\right)\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)}$$
$$- \frac{\alpha^2\gamma^2\delta_t(1+\gamma-\delta_t\tau_c+2\tau_c)}{(1+\gamma)(2-\delta_t)(1+\gamma+2\tau_c)(1+\gamma+\delta_t\tau_c)}. \qquad (A2.14)$$

Finally, variance ratio, or bullwhip effect, between manufacturing order (o_m) and sales demand (d_s) for both systems can be calculated by multiplying Equations (2.13) and (2.14) with Equation (2.12):

$$BE^{IRS} = \frac{\sigma_{o_m(IRS)}^2}{\sigma_{d_s}^2} = \frac{\sigma_{o_m(IRS)}^2}{\sigma_{d_t}^2} \cdot \frac{\sigma_{d_t}^2}{\sigma_{d_s}^2}$$
$$= \frac{\begin{pmatrix} 2 + \delta_t(3 + 4L + 6\tau_a) + \\ \delta_t^2(2L(1+L) + (1+4L)\tau_a + 2\tau_a^2) \end{pmatrix}\left((1+\gamma)^2 + (3+4\gamma+\gamma^2)\tau_c + 2\tau_c^2\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)(1+\tau_c)(1+2\tau_c)}$$
$$- \frac{\alpha^2\gamma^2\left((1+\gamma)^2 + (3+4\gamma+\gamma^2)\tau_c + 2\tau_c^2\right)}{(1+\gamma)(1+\gamma+2\tau_c)(1+\tau_c)(1+2\tau_c)}, \qquad (A2.15)$$

$$BE^{ROS} = \frac{\sigma_{o_m(ROS)}^2}{\sigma_{d_s}^2} = \frac{\sigma_{o_m(ROS)}^2}{\sigma_{d_t}^2} \cdot \frac{\sigma_{d_t}^2}{\sigma_{d_s}^2}$$
$$= \frac{\begin{pmatrix} 2 + \delta_t(3 + 4L + 6\tau_a) + \\ \delta_t^2(2L(1+L) + (1+4L)\tau_a + 2\tau_a^2) \end{pmatrix}\left((1+\gamma)^2 + (3+4\gamma+\gamma^2)\tau_c + 2\tau_c^2\right)}{(2-\delta_t)(1+2\tau_a)(1+\delta_t\tau_a)(1+\tau_c)(1+2\tau_c)}$$
$$- \frac{\alpha^2\gamma^2\delta_t(1+\gamma-\delta_t\tau_c+2\tau_c)\left((1+\gamma)^2 + (3+4\gamma+\gamma^2)\tau_c + 2\tau_c^2\right)}{(1+\gamma)(2-\delta_t)(1+\gamma+2\tau_c)(1+\gamma+\delta_t\tau_c)(1+\tau_c)(1+2\tau_c)}. \qquad (A2.16)$$

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