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Low-frequency vibration absorption of magnetic quasi-zero-stiffness structures with lever mechanism

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Abstract

Dynamic vibration absorbers (DVAs) are widely employed in diverse engineering systems for their capacity to mitigate structural vibrations. However, traditional absorbers exhibit sensitivity to natural frequencies and present challenges in tuning within the low-frequency range. This study introduces a lever mechanism to enhance the low-frequency vibration absorption performance of magnetic quasi-zero-stiffness structures, resulting in a lever-type DVA (L-DVA). We elucidate the design philosophy behind the proposed L-DVA. An analytical model is derived based on the Lagrange equation and the frequency-response relationship is determined using the harmonic balance method. We conduct numerical and experimental analyses to assess the impact of the lever ratio, mass ratio, frequency ratio, nonlinear stiffness coefficient ratio, and damping ratio on vibration absorption performance. Furthermore, we fabricate a prototype of the L-DVA with an adjustable magnetic quasi-zero-stiffness structure, and the experimental results align with simulation outcomes. The result shows that this study is the ease of customization in the vibration absorption capabilities of magnetic quasi-zero-stiffness structures, achieved by adjusting the lever ratio. An increase in the

1 lever ratio and tip mass effectively shifts the anti-resonant peak to a lower frequency.
2 The decrease in the frequency ratio leads to a reduction in the anti-resonant frequency.
3 This study validates the efficacy and feasibility of utilizing the L-DVA for low-
4 frequency vibration absorption.

5 **Keywords:** vibration absorption; lever mechanism; nonlinear vibration; low-frequency
6 vibration; quasi-zero-stiffness

7 1. Introduction

8 Mechanical vibrations have a significant impact on the accuracy, stability, and
9 reliability of mechanical structures [1-3]. Consequently, various approaches have been
10 developed and applied to mitigate unwanted vibrations. These methods include the use
11 of vibration isolators [4-9], dynamic vibration absorbers (DVAs) [10-13], damping
12 treatments [14], and active control methods [15]. Among these approaches, DVAs have
13 garnered considerable attention ever since Frahm's invention of them in the 1900s [16]
14 DVAs are valued for their ability to reduce undesirable vibrations without
15 compromising the supporting capacity of the structures. To enhance vibration
16 absorption performance, traditional linear DVAs can be optimized by adjusting the
17 natural frequency and damping ratio using fixed-point theory [17] or equal-peak theory
18 [18, 19]. Nishihara and Asami [20, 21] have indicated that optimal DVA achieved when
19 the maximum amplitude response of the primary system is minimized.

20 Various approaches have been developed to enhance vibration absorption
21 performance, including nonlinear DVAs [22-27], nonlinear energy sinks [28-30], and
22 energy harvesting methods [31-35]. For instance, Jo and Yabuno [36] introduced a
23 novel nonlinear vibration absorbers with tunable natural frequency characteristics.
24 They demonstrated that under large excitation amplitudes, the nonlinear restoring force
25 acting on the host system reduced the efficiency of the absorber. Habib et al. [37]
26 devised a nonlinear absorber based on a classical linearly tuned vibration absorber,
27 capable of mitigating nonlinear vibrations in mechanical systems. Feudo et al. [38]
28 proposed a nonlinear magnetic vibration absorber, providing an effective approach for

controlling vibrations in the first vibrational mode. Habib et al. [39] analyzed the characteristics of impulsive vibration dissipation in a nonlinear DVA within a damping oscillator. Sun et al. [40] utilized the equal-peak method to design a nonlinear vibration absorber for suppressing vibrations in nonlinear primary vibration systems. Chang et al. [41] introduced a quasi-zero-stiffness (QZS) DVA with high static stiffness and low dynamic stiffness, effectively mitigating ultralow-frequency vibrations in primary systems. Additionally, inerter-like devices find widespread application in vibration absorption systems [42-44], offering a straightforward means to improve the high-frequency performance of vibration absorber systems without severely compromising low-frequency performance. Qian et al. [45] conducted a study on the exact nonlinear dynamics of a simply-supported beam equipped with a nonlinear spring-inerter-damper vibration absorber. Zhang et al. [46] proposed an archetype of nonlinear energy sinks enhanced using an inerter to suppress mechanical vibrations. More recently, multifunctional energy-harvesting methods that combine vibration absorption and energy-harvesting functions have gained attention. Raj and Santhosh [47] introduced a dual-functional vibration absorber and energy harvesting system, that achieved outstanding absorption performance across a wide frequency range.

Numerous adjustment methods have been proposed to enhance vibration absorption performance [48-54]. Chang et al. [55] introduced a semi-active QZS DVA capable of effectively absorbing vibrations across a broad frequency range, especially ultralow-frequency vibrations. Pu et al. [56] innovatively designed a magnetorheological elastomer torsional vibration absorber to enhance frequency-tuning capabilities. Dogan et al. [57] presented a compact inerter-based dynamic vibration absorber, that can be tuned by adding a small mass. Rasooli et al. [58] suggested a semi-active adaptively tuned vibration absorber, allowing adaptive tuning of the natural frequency within the low-frequency range. Pu et al. [59] developed a novel optimization method for designing a tracking scheme for vibration absorbers. Sun et al. [60] proposed a hybrid damper with adjustable Coulomb and electromagnetic shunt damping, suitable for precise tuning of damping ratios in DVAs. Kakou et al. [61] introduced a nonlinear electromagnetic resonant shunt-tuned mass damper inerter, that

1 integrates multiple methods to achieve nonlinear vibration control.

2 Numerous other mechanisms have been explored to achieve exceptional vibration
3 absorption performance [62]. Zhang et al. [63] introduced an auto-parametric pendulum
4 absorber that combined a pendulum with magnetic stiffness. Their results demonstrated
5 that this configuration outperformed conventional vibration absorbers significantly.
6 Kecik [64] proposed a coupled system based on magnetic levitation, capable of
7 simultaneously reducing vibrations and harnessing energy from a pendulum-tuned mass
8 damper. Sheng et al. [65] presented a DVA incorporating a grounded stiffness element
9 and amplifying mechanism, resulting in a significant reduction in the resonance
10 amplitude and a broadened frequency band. Gomez et al. [66] designed bifilar
11 centrifugal pendulum vibration absorbers, that exhibited enhanced performance in
12 torsional vibration absorption. Mahe et al. [67] conducted an analysis of the stability of
13 centrifugal pendulum vibration absorbers, focusing on rotational mobility.

14 In addition to the previously mentioned mechanisms, lever structures have the
15 capability to amplify the mass effect, a concept widely employed to extend the
16 operational bandwidth of vibration isolators [68-70]. Li and Zhou [71] introduced
17 multiple active lever-type tuned mass dampers, designed to deliver high robustness and
18 effectiveness in vibration absorption. Su et al. [72] proposed a vibrational absorber
19 based on a lever-type inerter. Supported by a flexible auxiliary structure, this innovative
20 absorber demonstrated enhanced vibration absorption performance. Zang et al. [29]
21 introduced a form of nonlinear energy sink that utilizes a lever to reduce the required
22 mass. They constructed a lever-type nonlinear energy sink and energy harvester for
23 combined vibration isolation and energy harvesting [73]. Shen et al. [74] put forward a
24 grounded type DVA featuring a lever component, achieving outstanding control
25 performance with a small auxiliary mass.

26 In **previous investigations of** lever-type vibration absorbers, the lever was
27 commonly **functioned as a mere connector bridging two auxiliary systems, thus**
28 **facilitating vibration absorption.** However, **our study integrates** a lever and additional
29 mass as integral components, **forming a direct and unified** vibration absorber (L-DVA).
30 The proposed L-DVA utilizes a mass-adjustment mechanism instead of a restoring force

to attain adjustable frequency characteristic for low-frequency vibration absorption. Consequently, an important feature of the L-DVA is its ability to modify the anti-resonant frequency, achieved through precise adjustments in the mass ratio and the tip mass. Furthermore, the host structure of the L-DVA is designed to incorporate stiffness nonlinearity, which further enhances vibration absorption performance. An integral part of the host structure is the inclusion of a tunable magnetic QZS component, which adds an additional dimension of adjustability. The frequency of the proposed L-DVA can be simultaneously adjusted by the lever and QZS structures, resulting in superior low-frequency vibration absorption performance.

The remainder of this paper is organized as follows: Section 2 presents the schematic and derives the governing equations of the L-DVA. The vibration absorption performance of the L-DVA is compared and analyzed in Section 3. Section 4 covers the design and experimentation of the L-DVA. Finally, we summarize the conclusions in Section 5.

2. Design of L-DVA

This section outlines the design philosophy of the proposed L-DVA and establishes a theoretical model using Lagrange equations. We then derive dimensionless equations of motion, and obtain the corresponding transmissibility using the harmonic balance method.

2.1. Design philosophy

Fig. 1 shows a schematic and configuration of the L-DVA, which comprises a nonlinear host structure and a lever-type vibration absorber, where x and y denote the relative displacements of the host structure and absorber, respectively. When the L-DVA is subjected to harmonic excitation with amplitude u , the total restoring force of the host structure is given by $F = \sum_{n=1\sim3} k_n x^n$, where k_1 denotes the equivalent linear stiffness coefficient, and k_2 and k_3 are the equivalent nonlinear stiffness coefficients. k_1 is the summation of k_0 and k_{11} , where k_0 is the inherent stiffness coefficient of the host structure, and k_{11} is the first-order stiffness coefficient of the nonlinear force. The mass

of the host structure is denoted as m_1 , and c_1 is the linear damping coefficient.

As shown in Fig. 1(a), the lever-type vibration absorber system contains mass m_2 , tip mass m_3 , linear stiffness k_4 , and linear damping coefficient c_2 . It is assumed that the rotation angle of the lever is small, and the relative displacement of the load plate y and the tip mass z satisfy $z = \alpha y$, where $\alpha = l_2/l_1$ is the lever ratio and the relative motion between m_2 and m_3 is opposite.

Fig. 1(b) illustrate a feasible configuration of the L-DVA, integrating a tunable magnetic QZS structure with a lever-type vibration absorber. The lever-type vibration absorber comprises three linear coil springs, three linear bearings, a base plate, a load plate, and the lever structure. As shown in Fig. 1(c), a mass m_3 is installed on the tip of the lever, and the rod allows for movement to adjust the lever ratio. The proposed L-DVA combines the lever-type vibration absorber with the tunable magnetic QZS structure.

The base plate and load plate of the magnetic QZS structure are interconnected by three coil springs, and three linear bearings are employed to reduce friction and ensure the precision of vertical movement. A tunable nonlinear magnetic spring is comprised of three base permanent magnets (PM^{B} s) and two moving PMs (PM^{M} s) as illustrated in Fig. 1(d). Detailed information regarding the modeling and testing of the tunable magnetic spring can be found in Appendix A.

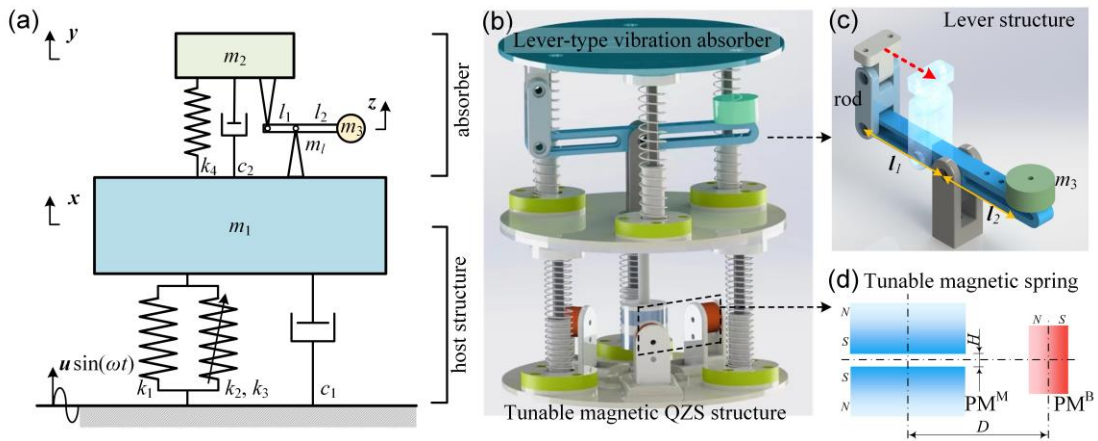


Fig. 1. (a) Schematic, (b) configuration, (c) lever structure, and (d) tunable magnetic spring of the L-DVA system, which comprises a lever-type vibration absorber and a tunable magnetic QZS structure. Three PM^{B} s are symmetrically mounted on the base

plate in the circumferential direction, while two PM^Ms are vertically positioned on the load plate. The distance D and gap H among the PMs can be adjusted to achieve various nonlinear restoring forces. The tunable magnetic QZS structure is subjected to harmonic excitation with the amplitude u . The rod in lever structure allows for movement to adjust the lever ratio $\alpha = l_2/l_1$.

2.2. Theoretical modeling

The absolute displacements of m_1, m_2 and m_3 were $x+u$, $x+y+u$ and $x+u-\alpha y$, respectively. The elastic deformation of a rigid lever can be neglected because of its small rotation angle. Hence, the motion of the tip mass was regarded as a vertical movement. The total kinetic energy of the L-DVA is expressed as:

$$T = \frac{1}{2}m_1(\dot{x}+\dot{u})^2 + \frac{1}{2}m_2(\dot{x}+\dot{y}+\dot{u})^2 + \frac{1}{2}m_3(-\alpha\dot{y}+\dot{x}+\dot{u})^2 + \frac{1}{2}\int_{-l_2}^{l_1}\rho_l(s\dot{\phi}+\dot{x}+\dot{u})^2 ds, \quad (1)$$

where the rotation angle of the lever ϕ is approximated as y/l_1 . The mass of the lever is given as $m_l = \rho_l l_1(1+\alpha)$, where ρ_l is the mass of the lever per unit length.

The potential energy of L-DVA with nonlinear characteristic is simplified as:

$$V = k_1 x^2/2 + k_2 x^3/3 + k_3 x^4/4 + k_4 y^2/2. \quad (2)$$

The total virtual work done by non-conservative forces of L-DVA is

$$\delta W = -c_1 \dot{x} \delta x - c_2 \dot{y} \delta y. \quad (3)$$

Therefore, by applying the Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = Q_\alpha, \quad (4)$$

where $L = T - V$; the governing equations for L-DVA system are derived as:

$$\left(m_1 + m_2 + m_3 + m_l - \frac{m^2}{M}\right)\ddot{x} - \frac{m}{M}(k_4 y + c_2 \dot{y}) + \left(m_1 + m_2 + m_3 + m_l - \frac{m^2}{M}\right)\ddot{u} + (k_1 x + k_2 x^2 + k_3 x^3 + c_1 \dot{x}) = 0, \quad (5)$$

$$1 \quad \left(M - \frac{m^2}{(m_1 + m_2 + m_3 + m_l)} \right) \ddot{y} - \frac{m}{(m_1 + m_2 + m_3 + m_l)} (k_1 x + k_2 x^2 + k_3 x^3 + c_1 \dot{x}) + k_4 y + c_2 \dot{y} = 0. \quad (6)$$

2 where $\ddot{u}$ represents the base acceleration excitation with an amplitude denoted as A .

3 The expressions for m and M are given by

$$4 \quad \begin{cases} m = m_2 - \alpha m_3 + m_l (1 - \alpha) / 2, \\ M = m_2 + \alpha^2 m_3 + m_l (\alpha^2 - \alpha + 1) / 3. \end{cases} \quad (7)$$

5 By defining the following dimensionless parameters:

$$6 \quad \begin{aligned} \tilde{x} &= x/l_2, \tilde{y} = y/l_2, \tilde{u} = u/l_2, \Omega_{10} = \sqrt{k_0/m_1}, \tau = \Omega_{10} t, \Omega = \omega/\Omega_{10}, \\ \mu_1 &= m_2/m_1, \mu_2 = m_3/m_1, \mu_3 = m_l/m_1, \eta_1 = k_{11}/k_0, \eta_2 = k_2 l_2/k_0, \\ \eta_3 &= k_3 l_2^2/k_0, \xi_1 = c_1/(2m_1 \Omega_{10}), \xi_2 = c_2/(2m_1 \Omega_{10}), \varepsilon = k_4/k_0, \tilde{A} = \ddot{u}/(\Omega_{10}^2 l_2), \\ \bar{m} &= \mu_1 - \alpha \mu_2 + \mu_3 (1 - \alpha) / 2, \bar{M} = \mu_1 + \alpha^2 \mu_2 + \mu_3 (\alpha^2 - \alpha + 1) / 3. \end{aligned} \quad (8)$$

7 Eqs. (5) and (6) are rewritten in a nondimensional form as:

$$8 \quad \left(1 + \mu_1 + \mu_2 + \mu_3 - \frac{\bar{m}^2}{\bar{M}} \right) \tilde{x}'' - \frac{\bar{m}}{\bar{M}} (\varepsilon \tilde{y} + 2\xi_2 \tilde{y}') + \left(1 + \mu_1 + \mu_2 + \mu_3 - \frac{\bar{m}^2}{\bar{M}} \right) \tilde{A} + ((1 + \eta_1) \tilde{x} + \eta_2 \tilde{x}^2 + \eta_3 \tilde{x}^3 + 2\xi_1 \tilde{x}') = 0, \quad (9)$$

$$9 \quad \left(\bar{M} - \frac{\bar{m}^2}{(1 + \mu_1 + \mu_2 + \mu_3)} \right) \tilde{y}'' - \frac{\bar{m}}{1 + \mu_1 + \mu_2 + \mu_3} ((1 + \eta_1) \tilde{x} + \eta_2 \tilde{x}^2 + \eta_3 \tilde{x}^3 + 2\xi_1 \tilde{x}') + \varepsilon \tilde{y} + 2\xi_2 \tilde{y}' = 0. \quad (10)$$

10 The harmonic balance method was used to derive the frequency-response
11 relationship of the L-DVA. When the system enters a steady-state response, we have

$$12 \quad \begin{cases} \tilde{x} = a \sin(\Omega \tau) + b \cos(\Omega \tau), \\ \tilde{y} = p \sin(\Omega \tau) + q \cos(\Omega \tau). \end{cases} \quad (11)$$

13 Substituting Eq. (11) into Eqs. (9) and (10), neglecting the higher-order harmonic
14 terms, and balancing the constant terms multiplied by $\sin(\Omega \tau)$ and $\cos(\Omega \tau)$, we
15 obtain,

$$16 \quad \begin{pmatrix} r_1 & r_2 & r_3 & r_4 \\ r_2 & -r_1 & r_4 & -r_3 \\ r_5 & r_6 & r_7 & r_8 \\ r_6 & -r_5 & r_8 & -r_7 \end{pmatrix} \begin{pmatrix} a \\ b \\ p \\ q \end{pmatrix} = \begin{pmatrix} (1 + \mu_1 + \mu_2 + \mu_3 - \bar{m}^2/\bar{M}) \tilde{A} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (12)$$

17 where

$$\begin{cases}
r_1 = 2\xi_1\Omega, \\
r_2 = 1 + \eta_1 + 3\eta_3 X^2/4 - (1 + \mu_1 + \mu_2 + \mu_3 - \bar{m}^2/\bar{M})\Omega^2, \\
r_3 = -2\bar{m}\xi_2\Omega/\bar{M}, \\
r_4 = -\bar{m}\varepsilon/\bar{M}, \\
r_5 = -2\bar{m}\xi_1\Omega/(1 + \mu_1 + \mu_2 + \mu_3), \\
r_6 = -\bar{m}(1 + \eta_1 + 3\eta_3 X^2/4)/(1 + \mu_1 + \mu_2 + \mu_3), \\
r_7 = 2\xi_2\Omega, \\
r_8 = \varepsilon - \bar{M}\Omega^2 + \bar{m}^2\Omega^2/(1 + \mu_1 + \mu_2 + \mu_3),
\end{cases} \quad (13)$$

$$\begin{cases}
X = \sqrt{a^2 + b^2}, \\
Y = \sqrt{p^2 + q^2}.
\end{cases} \quad (14)$$

The amplitude-frequency response equations are expressed in terms of X , Y and Ω , respectively, as

$$X^2 \begin{bmatrix} (r_1^2 + r_2^2)(r_7^2 + r_8^2) + (r_3^2 + r_4^2)(r_5^2 + r_6^2) \\ + 2r_1(r_4r_6r_7 - r_3r_5r_7 - r_3r_6r_8 - r_4r_5r_8) \\ + 2r_2(r_3r_5r_8 - r_3r_6r_7 - r_4r_5r_7 - r_4r_6r_8) \end{bmatrix} \quad (15)$$

$$= \left(1 + \mu_1 + \mu_2 + \mu_3 - \frac{\bar{m}^2}{\bar{M}}\right)^2 \tilde{A}^2 (r_7^2 + r_8^2)$$

$$Y^2 \begin{bmatrix} (r_1^2 + r_2^2)(r_7^2 + r_8^2) + (r_3^2 + r_4^2)(r_5^2 + r_6^2) \\ + 2r_1(r_4r_6r_7 - r_3r_5r_7 - r_3r_6r_8 - r_4r_5r_8) \\ + 2r_2(r_3r_5r_8 - r_3r_6r_7 - r_4r_5r_7 - r_4r_6r_8) \end{bmatrix} \quad (16)$$

$$= \left(1 + \mu_1 + \mu_2 + \mu_3 - \frac{\bar{m}^2}{\bar{M}}\right)^2 \tilde{A}^2 (r_5^2 + r_6^2)$$

Assuming θ is the phase angle between the excitation and acceleration $\tilde{x}''$, the absolute acceleration $\tilde{A}_1$ of m_1 is expressed as

$$\tilde{A}_1 = \tilde{A} \cos(\Omega\tau) + (-X\Omega^2/\tilde{A}) \cos(\Omega\tau + \theta). \quad (17)$$

Consequently, the transmissibility of L-DVA is expressed as:

$$T = |\tilde{A}_1/\tilde{A}| = \sqrt{1 + (X\Omega^2/\tilde{A})^2 - 2(X\Omega^2/\tilde{A})\cos\theta}. \quad (18)$$

3. Numerical simulations

In this section, the vibration absorption performance of the L-DVA was analyzed.

Specifically, the effects of the lever ratio, mass ratio, frequency ratio, nonlinear

coefficient, and damping ratio on the vibration absorption performance were analyzed.

The frequency ratio of the L-DVA is defined as $\beta = \varepsilon / (1 + \eta_1)$, and the nonlinear stiffness coefficient ratio of the host structure is defined as $\lambda = k_3 / k_1$. The damping ratio ζ of L-DVA is defined as $\zeta = \xi_1 / \xi_2$. During the simulation, an excitation with constant acceleration A was applied. The transmissibility is defined in Eq. (18).

The parameters listed in Table 1 are utilized for simulating and validating the accuracy of the L-DVA modeling. The transmissibility from the base to the magnetic QZS structure is depicted in Fig. 2(a) for $A = 0.2$ g. The grey dashed curve represents the transmissibility of a two-degree-of-freedom linear system without a lever, showing suboptimal vibration absorption performance compared to the proposed L-DVA. The solid lines represent the theoretical solutions, while the red pentagrams correspond to the numerical results obtained using the Runge-Kutta method. The theoretical solutions closely match the numerical results, validating the analytical model. Notably, an anti-resonant peak occurs at 0.65, indicating a low anti-resonant frequency for the proposed L-DVA. Consequently, the following sections will focus on the parametric study of the L-DVA.

Table 1 Parameters of L-DVA

Parameters	α	μ_1	μ_2	μ_3	ε	η_1	η_2	η_3	ζ
Value	1	1	0.2	0.01	0.5	-0.5	0	1.25	1

3.1. Lever ratio

The lever ratio is a key parameter in the design of the L-DVA because it directly influences the anti-resonant frequency of the absorber. Fig. 2(b) shows the transmissibility of the magnetic QZS structure with respect to α for $A = 0.2$ g and $\lambda = 1e3$. Other parameters are selected from Table 1. Both the second resonant peak and the anti-resonant peak of the transmissibility shift to left with the increase in α for L-DVA. Although the first resonant peak of the transmissibility decreases slightly, the absorption performance is enhanced in the high frequency region, indicating that the vibration absorption bandwidth of the L-DVA widens with the increase in α . The lever

structure offers a convenient means to fine-tune both the frequency and absorption performance, providing an alternative method for altering the mass of the DVA without introducing additional devices.

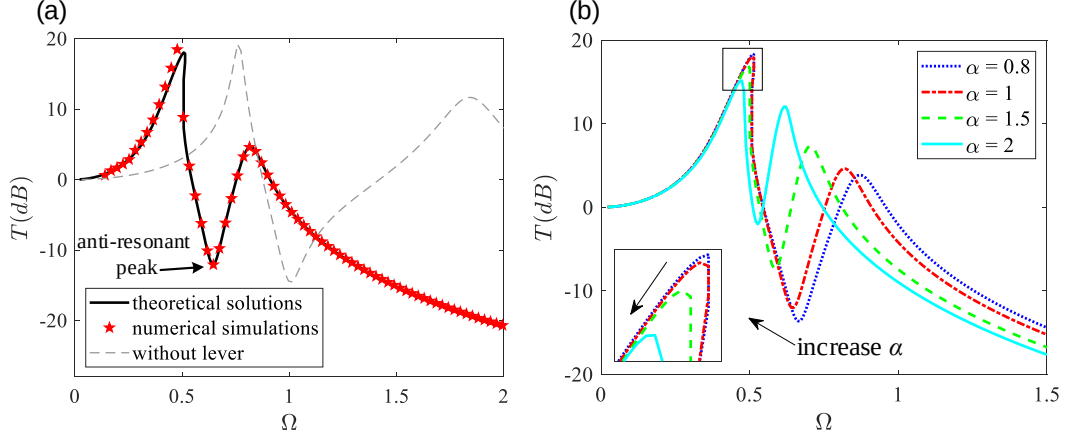


Fig. 2. (a) Transmissibility of the magnetic QZS structure with L-DVA for $\alpha = 1$ and $A = 0.2$ g. The transmissibility of a two degree-of-freedom linear system without a lever is depicted by the gray dotted curve. The solid black line represents the theoretical solutions, while the red asterisks represent the numerical simulations. The theoretical solutions are consistent with the numerical simulations, where an anti-resonant peak occurs at 0.65. (b) Transmissibility of the magnetic QZS structure with L-DVA with respect to α . The first resonant peak of the transmissibility decreases with the increase in α . Additionally, both the second resonant peak and the anti-resonant peak of the transmissibility shift to left as α increases.

3.2. Mass ratio

Fig. 3(a) shows the variation of M_e with respect to μ_1 and α . M_e is defined as $\bar{M} - \bar{m}^2 / (1 + \mu_1 + \mu_2 + \mu_3)$ shown in Eq. (10). It can be observed that M_e increases with the increase in μ_1 under different α . Fig. 3(b) shows the transmissibility of the magnetic QZS structure with L-DVA with respect to μ_1 when $\alpha = 1, \mu_2 = 0.2, \varepsilon = 0.5, \zeta = 1, A = 0.2$ g and $\lambda = 1e3$. Only m_2 varies while the rest of the system parameters remain constant. With the increase in μ_1 , the second resonance peak of L-DVA shifts to a lower frequency, while the corresponding transmissibility improves. The anti-resonant frequency and corresponding transmissibility decrease with an increase in μ_1 , indicating the

outstanding vibration absorption performance of the proposed L-DVA.

Fig. 3(c) shows the variation of M_e with respect to μ_2 and α when $\alpha=1, \mu_1=1, \varepsilon=0.5, \zeta=1, A=0.2$ g and $\lambda=1e3$. On comparing Fig. 3(a) to Fig. 3(c), it was observed that M_e can be tuned efficiently by altering μ_2 . Fig. 3(d) shows the transmissibility of the L-DVA with respect to various values of the tip mass attached to the free-end of the lever. The anti-resonant frequency decreased with an increase in tip mass. Furthermore, the first resonant peak of the L-DVA decreased slightly, and the transmissibility of the L-DVA in the high-frequency region notably declined, indicating the superior vibration absorption performance of the proposed L-DVA.

The lever structure provides a more convenient means of achieving different resonant frequencies. The small tip mass of the L-DVA is amplified to the desired mass through the use of a lever mechanism. Consequently, the proposed L-DVA provides an ideal approach for tuning the frequency of the absorbers.

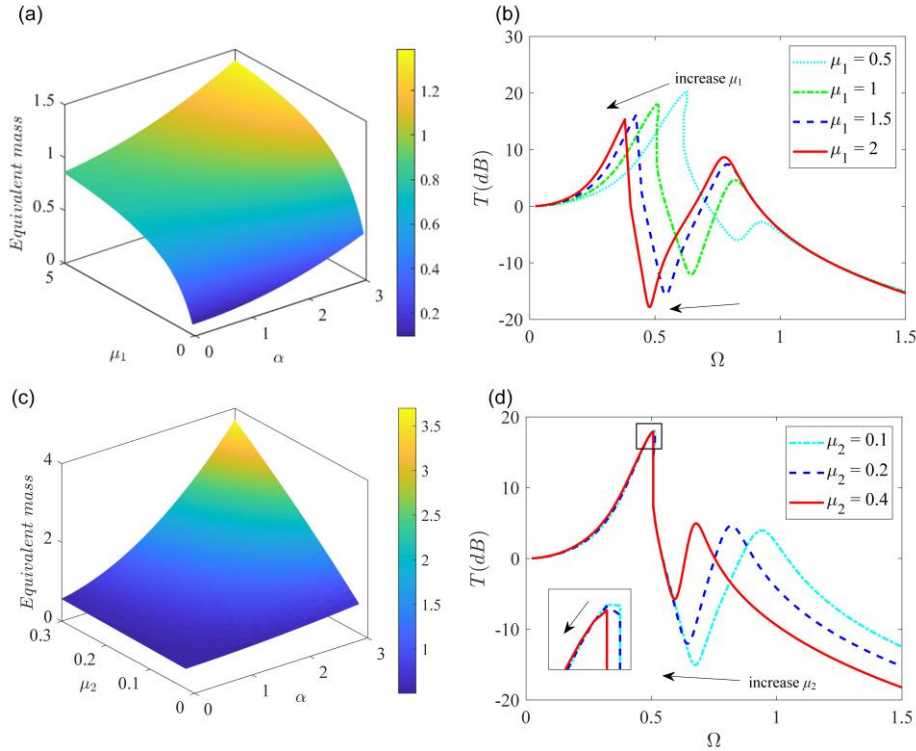
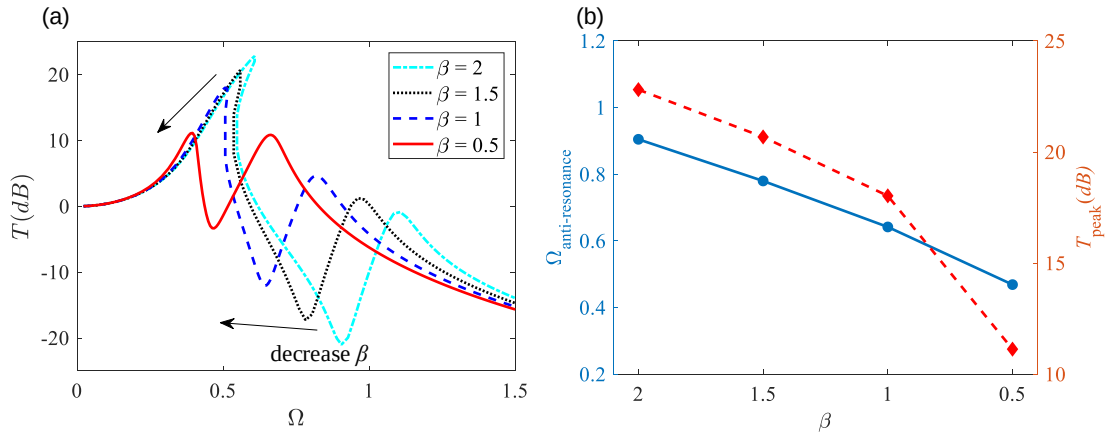


Fig. 3. (a) Variation of M_e with respect to μ_1 and α . (b) Transmissibility of the magnetic QZS structure under different mass ratios μ_1 (0.5, 1, 1.5, 2). The anti-resonant frequency and the corresponding transmissibility decrease with the increase in μ_1 . (c) Variation of M_e with respect to μ_2 and α . M_e can be efficiently adjusted by altering μ_2 . (d)

1 Transmissibility of the magnetic QZS structure under different mass ratios μ_2 (0.1, 0.2,
 2 and 0.4). The anti-resonant frequency of L-DVA decreases with the increase in the tip
 3 mass. The first resonant peak **slightly decreases**, and the transmissibility of L-DVA in
 4 **the** high-frequency region declines.

5 3.3. Frequency ratio

6 Fig. 4(a) shows the transmissibility of L-DVA with respect to different inherent
 7 frequency ratios β for $A = 0.2$ g, $\alpha = 1$, $\mu_1 = 1$, $\mu_2 = 0.2$, $\zeta = 1$, and $\lambda = 1e3$. With the
 8 decrease in β , the first resonant peak decreases significantly, while the anti-resonant
 9 frequency decreases and the second resonant peak increases. Fig. 4(b) represents the
 10 variation of the anti-resonance frequency and peak transmissibility with respect to β .
 11 The vibration absorption performance of L-DVA is strongly influenced by β . Hence, β
 12 should be designed carefully to achieve a better vibration absorption performance.



13 **Fig. 4.** (a) Transmissibility of the magnetic QZS structure under different β (0.5, 1, 1.5,
 14 2) for $A = 0.2$ g. (b) Variation of the anti-resonance frequency and peak transmissibility
 15 with respect to β for $A = 0.2$ g and $\lambda = 1e3$. The line chart embellished with circular
 16 markers signifies anti-resonance frequency, **while** the dashed line decorated with
 17 diamond markers corresponds to the peak transmissibility of the magnetic QZS
 18 structure. With the decrease in the frequency ratio β , the first resonant peak decreases
 19 significantly, while the anti-resonant frequency decreases and the second resonant peak
 20 increases.
 21

3.4. Nonlinear stiffness coefficient ratio

Fig. 5(a) represents the nonlinear resorting force with respect to various nonlinear stiffness coefficient ratios λ . Fig. 5(b) shows the transmissibility curves of L-DVA with respect to λ for $A = 0.2$ g, $\alpha = 1$, $\mu_1 = 1$, $\mu_2 = 0.2$, $\varepsilon = 0.5$, and $\zeta = 1$. The first resonant peak of L-DVA decreases with the increase in λ , but the anti-resonance peaks do not change significantly. We can clearly observe a significant increase in the second resonant peak when λ increases from $1e4$ to $1e5$, accompanied by a hardening characteristic. Consequently, the nonlinear stiffness coefficient ratio of the L-DVA must be carefully designed to achieve the desired vibration absorption performance.

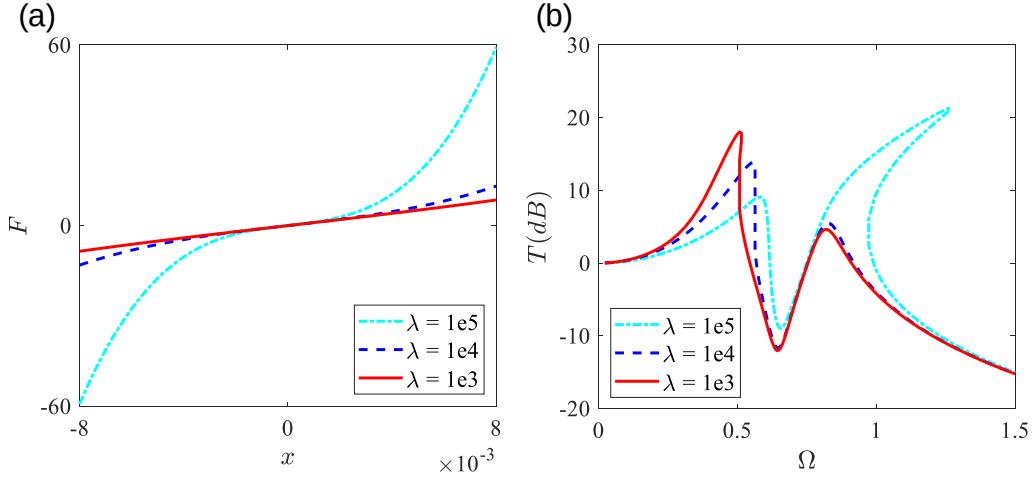
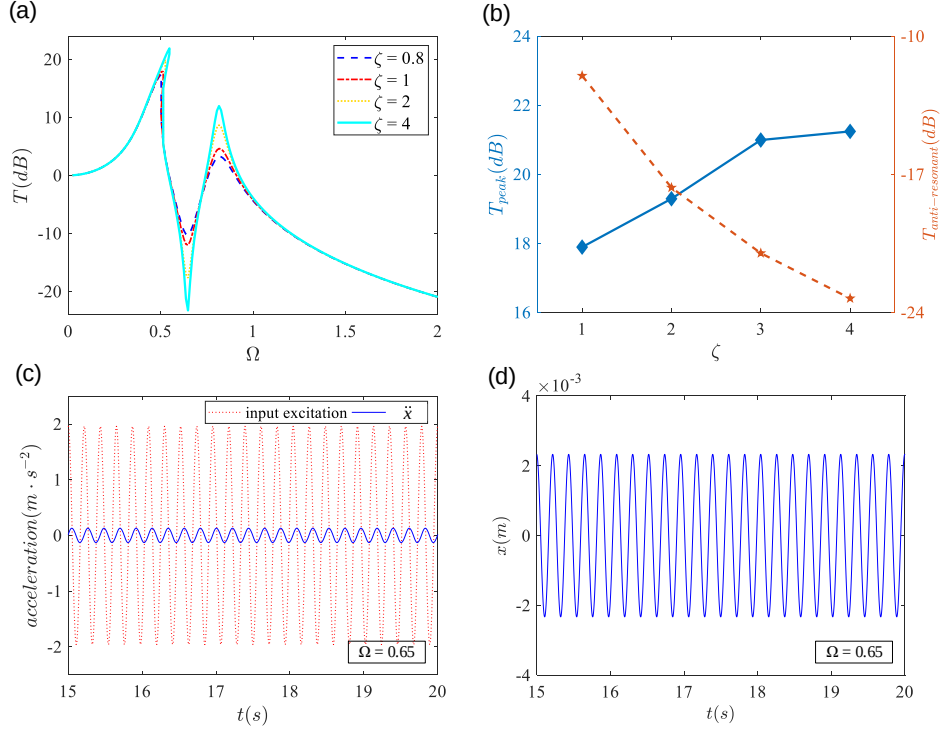


Fig. 5. (a) Resorting force and (b) transmissibility curves of the magnetic QZS structure under different λ for $A = 0.2$ g. Nonlinear stiffness coefficient ratio λ is set as $1e3$, $1e4$, and $1e5$, respectively. The first resonant peak of L-DVA decreases with the increase in λ , while the second resonant peak increases drastically when λ increases from $1e4$ to $1e5$.

3.5. Damping ratio

Fig. 6(a) shows the transmissibility curves of L-DVA with respect to ζ for $A = 0.2$ g, $\alpha = 1$, $\mu_1 = 1$, $\mu_2 = 0.2$, $\varepsilon = 0.5$, and $\lambda = 1e3$. With an increase in ζ , two resonant peaks increase slightly, while the anti-resonant peak decreases. Fig. 6(b) shows the variations of the peak transmissibility and the transmissibility at the anti-resonant peak with respect to ζ . Fig. 6(c) shows the comparison between the input excitation and the output

1 acceleration of the magnetic QZS structure for $\Omega = 0.65$ and $\zeta = 4$. The desired vibration
 2 absorption performance of the proposed L-DVA at the anti-resonant frequency was
 3 realized. Fig. 6(d) shows the corresponding displacement x of the host nonlinear
 4 structure for $\Omega = 0.65$.



5 **Fig. 6.** (a) Transmissibility of the magnetic QZS structure with L-DVA under different
 6 ζ for $A = 0.2$ g. (b) Variation of the peak transmissibility and the transmissibility of the
 7 anti-resonance peak under different ζ . There is a slight increase in the amplitudes of
 8 two resonant peaks as ζ increases, coupled with a decrease in the anti-resonant peak. (c)
 9 Comparison between the input excitation and output acceleration of the magnetic QZS
 10 structure, and (d) the corresponding displacement x for $\Omega = 0.65$ and $\zeta = 4$.

12 4. Experimental verification

13 In this section, an experimental prototype of the L-DVA is presented. Subsequently,
 14 an experimental setup was established to verify the vibration absorption performance
 15 of L-DVA.

16 4.1. Experimental setup

17 A schematic diagram and configuration of the proposed L-DVA are shown in Fig.

1 1, which contains a magnetic QZS structure and a lever-type vibration absorber. During
2 the experiments, a prototype L-DVA integrating the magnetic QZS structure and a
3 lever-type vibration absorber was fabricated, as shown in Fig. 7(a). The geometric
4 parameters of the L-DVA are listed in Table 2.

5 **Table 2.** Geometric parameters of L-DVA

Parameters	Value
Mass of the host structure, m_1 (kg)	0.936
Mass of the absorber, m_2 (kg)	0.331
Mass of the lever, m_l (kg)	0.023
Tip mass, m_3 (kg)	0.08
Stiffness of springs, k_0 (N/m)	2154
Stiffness of springs, k_4 (N/m)	1157
Damping coefficient, c_1 (N•s/m)	2.57
Damping coefficient, c_2 (N•s/m)	2.32
l_1 (mm)	50
l_2 (mm)	50

6
7 Fig. 7(b) shows a photograph of the experimental setup used to validate the
8 theoretical model and vibration absorption performance of the L-DVA. The L-DVA was
9 mounted on an exciter (TIRA 200N). The controller receives the corresponding
10 feedback signal to control the input excitation, which is produced by a signal generator
11 and amplified using a power amplifier. Two accelerometers were mounted on the base
12 plates of the magnetic QZS structure and L-DVA. The excitation acceleration amplitude
13 and operating bandwidth were 0.2 g and [2, 20] Hz, respectively. The sweep speed was
14 set to 60 Hz/min. The initial position parameters of the PMs were adjusted to $D = 0.035$
15 m and $H = 0.01$ m.

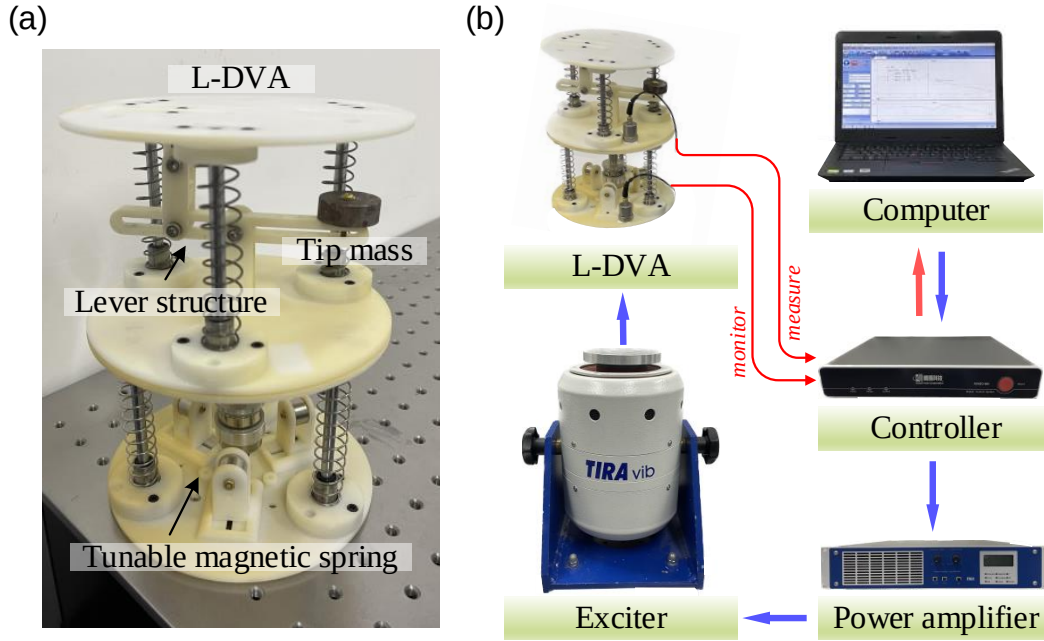


Fig. 7. Photograph of (a) L-DVA, and (b) the experimental setup. L-DVA comprises a tunable QZS structure and a lever-type vibration absorber. The experimental system includes two accelerometers, an exciter, a power amplifier, a controller, a signal processor, the data collector, and a computer.

4.2. Parametric studies

Fig. 8 compares the experimental and simulated transmissibility of the L-DVA concerning α for $A = 0.2$ g. In Fig. 8(a), it can be observed that the first peak transmissibility experiences a slight decrease as α increases from 0.81 to 1, 1.28, and 1.8. This qualitative observation verifies the tuning effect of the lever ratio on the vibration absorption of the L-DVA. Additionally, it is noticeable that M_e decreases with increasing α . The experimental anti-resonance frequency closely aligns with the simulation, confirming the effectiveness of the theoretical models and functionality of the lever mechanism. However, it is worth noting that the resonance peaks of the L-DVA exhibited slight variations, potentially due to complex damping effects and installation errors.

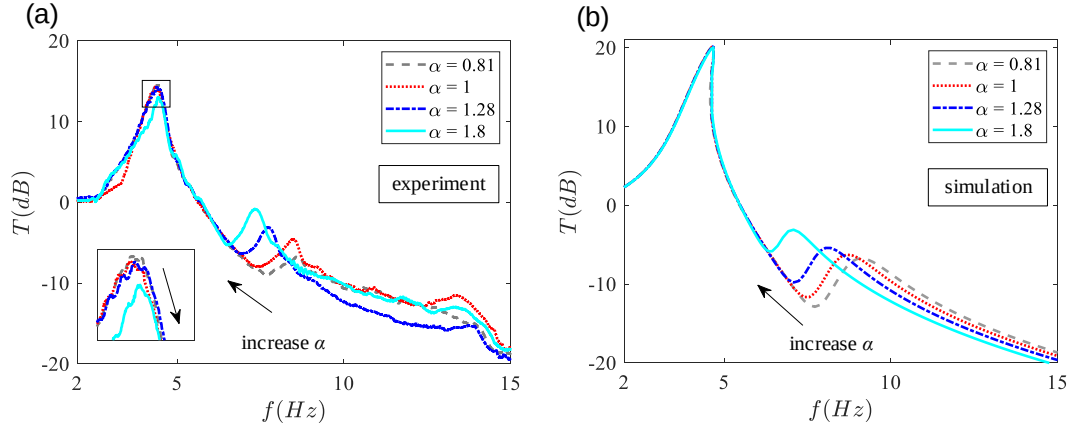
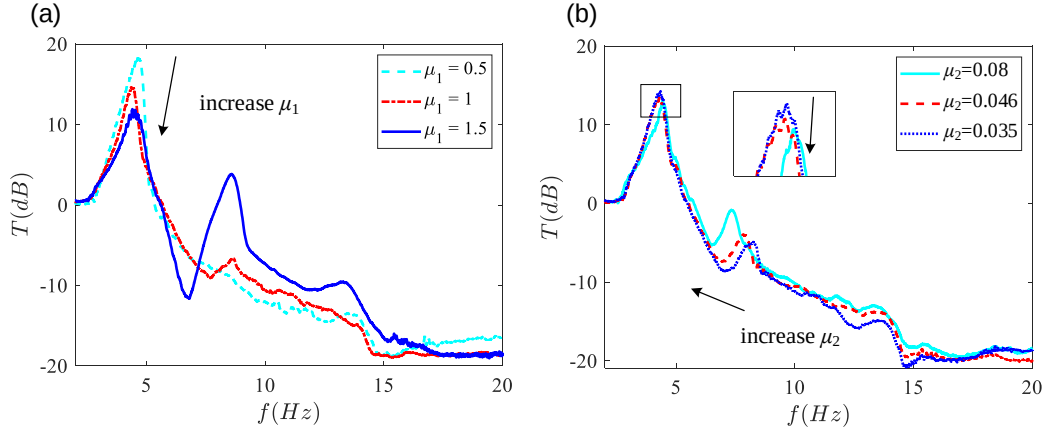


Fig. 8. (a) Experimental and (b) simulated transmissibility of the magnetic QZS structure with respect to different lever ratio for $A = 0.2$ g and $\mu_1 = 1$. The first peak transmissibility exhibits a slight decrease as α increases from 0.81 to 1.8. The experimental anti-resonance frequency is consistent with the simulation.

Fig. 9(a) shows the experimental transmissibility of L-DVA with respect to various mass ratios μ_1 for $A = 0.2$ g and $\alpha = 1$. During the experiment, only load mass m_2 was altered to achieve different mass ratios, while m_1 remained unchanged. The transmissibility of L-DVA is similar to single-degree-of-freedom systems when μ_1 is small. With the increase in μ_1 , both the first resonant peak and the anti-resonance peak experience significant decreases, since altering the value of m_2 directly influences the inherent frequency of L-DVA, which demonstrates that the vibration absorption performance of L-DVA can be effectively tuned with the lever structure compared to the method of using a large additional mass. The latter inevitably increases the mass of the host nonlinear structure.

Fig. 9(b) displays the transmissibility curves of the L-DVA concerning the tip mass of the lever. The first peak transmissibility slightly decreases as μ_2 increases, primarily because the equivalent mass of the L-DVA increases with a higher tip mass. However, in the high-frequency region, the transmissibility of the L-DVA increases with an increase in tip mass, resulting in a reduction in the inherent frequency of the L-DVA. Furthermore, the experimental results closely match the simulation results, highlighting the importance of careful tuning of the tip mass to achieve the desired vibration

1 absorption performance.



2

3 **Fig. 9.** Experimental transmissibility of the magnetic QZS structure under the different
4 mass ratio (a) μ_1 and (b) μ_2 for $A = 0.2$ g and $\alpha = 1$. With the increase in μ_1 , the first
5 resonant peak exhibits a significant decrease. Additionally, the first peak
6 transmissibility slightly decreases with the increase in μ_2 .

7 5. Conclusions

8 In this study, we propose, design, and investigate a lever-type dynamic vibration
9 absorber (L-DVA) aiming to enhance vibration absorption performance by integrating
10 a tunable magnetic QZS structure with a lever-type vibration absorber. The governing
11 equations of the L-DVA are derived using the Lagrange equation, and an analytical
12 model is established based on the harmonic balance method. Then, we conduct a
13 numerical investigation to analyze the effects of various parameters, including the lever
14 ratio, mass ratio, frequency ratio, nonlinear stiffness coefficient ratio, and damping ratio
15 on the L-DVA's vibration absorption performance. Furthermore, we develop, fabricate,
16 and experimentally test an L-DVA prototype, confirming the lever structure's capacity
17 to adjust the absorption bandwidth by altering the lever ratio. The conclusions can be
18 summarized as follows.

19 The capability to adjust the anti-resonant frequency of the L-DVA is achieved
20 through the lever structure. Comparative analyses with a linear two degree-of-freedom
21 system without a lever demonstrate the superior vibration absorption performance of
22 L-DVA. The lever structure facilitates convenient adjustment of the absorption

bandwidth by altering the lever ratio. Increasing the lever ratio and tip mass improves the transmissibility of the L-DVA, validating the effectiveness of the lever mechanism in vibration absorbers.

Enhancing vibration absorption performance is achieved by introducing a tunable nonlinear magnetic spring into the host structure. This adjustability is further complemented by the magnetic QZS structure, contributing to superior vibration absorption performance. The first resonant peak of L-DVA decreases with the increase in the nonlinear stiffness coefficient ratio, but the anti-resonance peaks do not change significantly. Careful design of the nonlinear stiffness coefficient ratio is crucial to achieving the desired vibration absorption performance in the L-DVA. In conclusion, this study offers valuable guidelines for the design, analysis, and optimization of L-DVAs, providing insights into their robustness and wide operating bandwidth.

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Appendix A. The modeling and testing of the tunable magnetic spring

As shown in Fig. 1(d), the three PM^B s were symmetrically mounted on the base plate along the circumferential direction, whereas the two PM^M s were vertically positioned on the load plate. The distance D and gap H between the PMs can be tuned to achieve different nonlinear restoring forces, which provides a means of adjustment to reduce the oscillation of the host structure.

Table A.1 shows the geometric parameters and residual magnetic flux density of the PMs. Considering that all the ring PMs are axially magnetized, the current model is utilized to calculate the nonlinear magnetic force among the PMs. The total nonlinear restoring force F_T equals to the summation of nonlinear magnetic force and linear

restoring force, and F_T is approximated using a third-order polynomial [75].

Fig. A.1(a) presents a comparison between the simulated and experimental magnetic forces, validating the nonlinear magnetic force model. The stiffness of the linear coil springs k_0 is 2154 N/m for $D = 0.035$ m, $H = 0.01$ m as shown in Fig. A.1(b), where the hollow red dots and solid black line represent the theoretical calculation and fit of the restoring forces, respectively.

Table A.1. Geometric and magnetization parameters of the PMs

Parameters	PM ^B	PM ^M
Inner radius (m)	0.002	0.003
Outer radius (m)	0.005	0.01225
Height (m)	0.01	0.008
B_r (T)	1.25	1.18

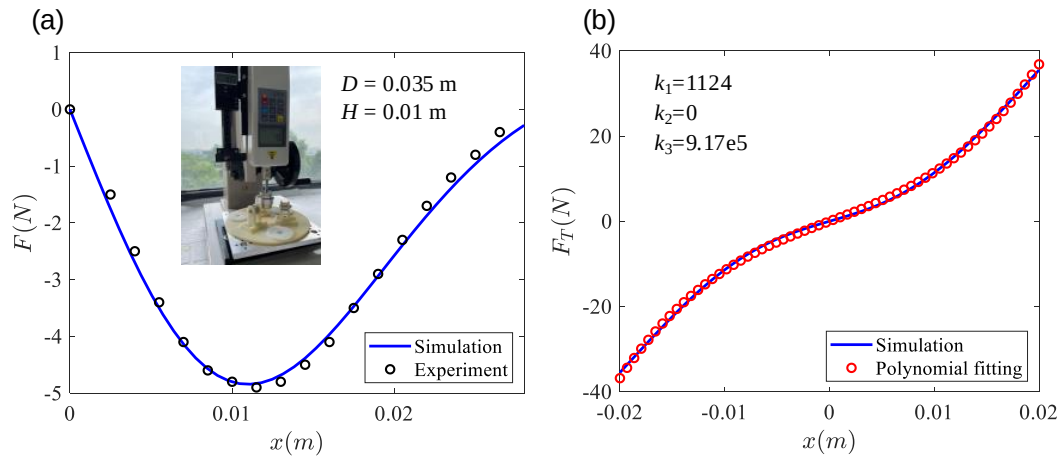


Fig. A.1. Comparison between (a) the simulated and experimental magnetic force for $D = 0.035$ m and $H = 0.01$ m, and (b) the theoretical and polynomial fit of the restoring forces. The fitting parameters are $k_1 = 1124$, $k_2=0$, and $k_3=9.17e5$.

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