

# **News Shocks and the Exchange Rate in a DSGE Model of China**

by

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# Abstract

This thesis explores the dynamics of China's exchange rates using an open economy DSGE model and assesses the effects of news shocks constrained by the signal extraction process. This model, representing a three-country open economy framework with China as the home country, the US as the foreign country, and a rest of the world component functioning as a transfer pot is estimated and evaluated by Indirect Inference method. The sample period spans from 2005Q3 to 2021Q4, encompassing China's shift from a dollar-peg to a managed floating exchange rate regime. We find the model empirically fits the data and the non-stationary productivity shocks are the primary driver of the real exchange rate fluctuations, while monetary shocks surpass productivity shocks in influencing the nominal exchange rate. Subsequently, the model is extended to incorporate news shocks, which are constrained by the signal extraction process that has been overlooked in the literature (Le et al., 2020). However, the incorporation of anticipated shocks diminishes the model's ability to match the data, as indicated by the higher Wald statistic. When news shocks are perfectly anticipated, they do not substantially alter the conclusions drawn from the base model, although the degrees of response differ.

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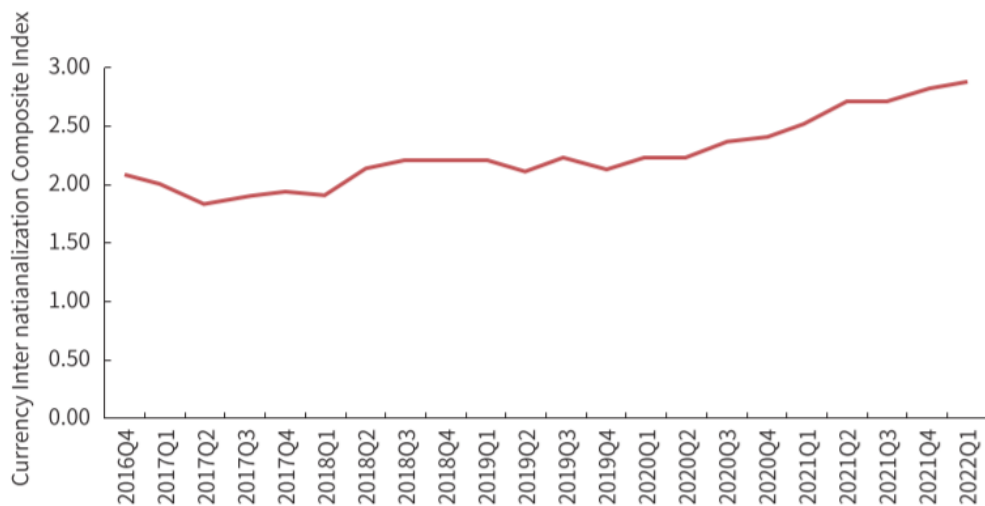


# Chapter 1 Introduction

## 1.1 Background and motivation

Since the initiation of the Renminbi (RMB) exchange rate mechanism reform in July 2005, the flexibility of the RMB exchange rate has been enhanced, confirming the direction towards market-oriented reforms and the internationalization of RMB. Currency internationalization refers to the extensive utilization of a currency beyond the geographical boundaries of its country of origin. The primary objective of the People's Bank of China (PBOC) in intervening in the foreign exchange market has been to mitigate short-term excessive fluctuations in the exchange rate and reduce the risk of medium- to long-term rate imbalances, although the intervention has been criticised by other countries. Following the implementation of the foreign exchange settlement and sales system reform in 2005, the RMB exchange rate mechanism has evolved towards greater flexibility and market orientation by achieving its aim to internationalization.

Figure 1. 1 The trend of the international status of the RMB

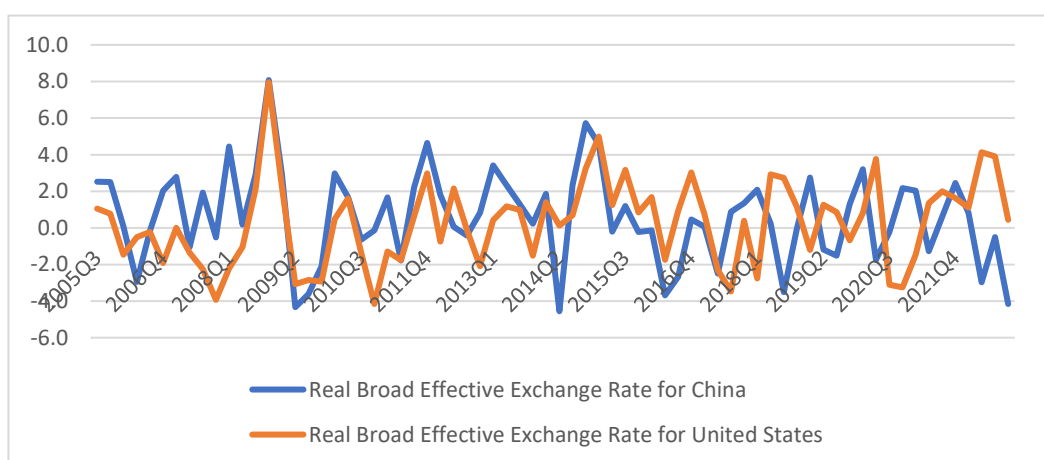


Source: 2022 RMB internationalization report, the People's Bank of China

Post-financial crisis, in June 2010, the PBOC announced the resumption of exchange rate reforms, further expanding the range of exchange rate fluctuations and enhancing the volatility of the RMB. In August 2010, the State Administration of Foreign Exchange initiated market-making trials in the interbank foreign exchange market, further advancing the perfection of the market-maker system. In August 2014, Chinese monetary authorities stated that the central bank would withdraw from regular interventions, enhancing the market's role in the RMB exchange rate formation mechanism. Figure 1.1 depicts a rising trend in RMB internationalization as the index increases, albeit being relatively small compared to other currencies, such as the USD, whose index is over 50 according to the report by the PBOC. The composite index, established by the PBOC, incorporates four key indicators that reflect the international usage of the currency in payment, investment, financing, and reserve holding.

As marketization in the foreign exchange market progresses, exchange rate fluctuations now not only indicate the influence of relevant authorities but also serve as a reflection of a country's economic conditions. Figure 1.2 illustrates the percentage changes in real broad effective exchange rates for both China and the US, revealing comparable levels of fluctuation.

Figure 1. 2 Percentage changes of real broad effective exchange rates



Source: Bank for International Settlements

## 1.2 Research questions and findings

The transition of China to a floating exchange rate policy raises the first research question: Could an open DSGE model fit the data and explain the exchange rate fluctuations? To explore this question, the three-country model inspired by Minford et al. (2021) is adopted, utilizing an open Dynamic Stochastic General Equilibrium (DSGE) model. This model is employed to test whether the model could fit the data and explore the various shocks influencing RMB exchange rate movements in the context of China and the US.

The second research question focuses on examining the impact of news shocks. The idea of incorporating news with signal extraction process into economic modelling is inspired by the work of Le et al. (2020). They propose a methodology to investigate the effects of news shocks when agents can fully anticipate the future shocks. This thesis aims to apply and expand upon these ideas to shed light on the specific effects of news shocks in the context of exchange rate fluctuations.

Indirect Inference is employed to evaluate the models constructed in this thesis instead of the conventional estimation techniques like the Bayesian method. It is first proposed by Smith (1993) and then developed by Gregory and Smith (1991, 1993), Gouriéroux et al. (1993), Gouriéroux and Montfort (1995) and Canova (2007). Minford et al. (2009) and Le et al. (2011) further refine this method using Monte Carlo experiments. This method not only tests models but also can estimate models. The details of Indirect Inference test and estimation procedure are introduced in Chapter 4.

The main findings of this thesis are as following: the three-country open economy model can explain the real data behaviour and it finds the non-stationary domestic productivity shock is the main driver of the fluctuations of real exchange rates. Second, after taking news into consideration, the results

do not change significantly from the base model, indicating the domestic productivity shock is still the main reason for currency movements.

### 1.3 Thesis structure

The remainder of this thesis is structured as follows. Chapter 2 reviews literature on exchange rate determination through uncovered interest rate parity and the development of open economy models. It also provides a comprehensive review of the literature on the implications of news shocks in both closed and open economy models, with a particular focus on how news affects exchange rate movements. Chapter 3 introduces the base three-country open economy model characterized by two symmetric economies and a rest of the world component functioning as a transfer pot. Then it describes the data used in this thesis and the calibration for the base model. Chapter 4 discusses the methodology of Indirect Inference, explaining how it is utilized for testing and estimating the model. It also discusses the small sample properties of this method and the use of non-stationary data. Chapter 5 presents the results of the tests and estimations conducted using the Indirect Inference method based on the real data of the sample period. Chapter 6 adds news shocks to the model to address the second research question. These news shocks are constrained by the signal extraction process. The chapter uses the Indirect Inference method to assess how these additions impact the model's performance. Chapter 7 concludes the findings of this thesis and discusses possible further extensions to the current model.

# Chapter 2 Literature Review

## 2.1 Introduction

In this chapter, we initially take a look at how the exchange rate is determined by uncovered interest parity and the development of open economy models. Subsequently, we survey the existing literature concerning news shocks, followed by a specific focus on news shocks related to exchange rates.

## 2.2 Exchange rates and open economy models

### 2.2.1 The determination of exchange rates

Since the term “purchasing power parity” (PPP) was coined by Cassel (1918), numerous researchers have dedicated their efforts to examining the validity of the probably oldest theory of exchange rate determination. The exchange rate serves as a comprehensive indicator of the intricate relationship between the price level and exchange rates of both domestic and foreign countries. The study of PPP holds importance not only for theoretical research but also for practical purposes. It can be utilized to assess the exchange regime, a country’s balance of payments, and monetary policy. Furthermore, it enables the comparison of international income levels and aids in determining the equilibrium exchange rates.

PPP is not only the theoretical fundamentals of many exchange rate determination models but also indicative of policy. For example, exchange rate reforms are implemented in Africa as the policy makers believe that hypothesis that PPP would hold in long-term. Kargbo (2006) investigates the empirical support for long-run PPP in details then he finds the overwhelming support for the long-run equilibrium. Sarno (1997) analyses the effectiveness of the Exchange Rate Mechanism (ERM) finds the long-run PPP holds in the ERM group and the countries in this group enjoy the increase of exchange

rate stability. PPP can be employed to predict the movements of exchange rates or to compare the national income levels with different countries or to determine whether the currency is overvalued or undervalued.

In the model chapter, the model adopts producer currency pricing (PCP). We assume that the law of one price (LOOP) holds under producer currency pricing. The absolute LOOP assumes that

$$P_{i,t} = S_t P_{i,t}^* \quad i = 1, 2, \dots, N \quad (2.1)$$

where  $P_{i,t}$  represents good  $i$  is priced as domestic currency at time  $t$ ,  $P_{i,t}^*$  denotes the price of good  $i$  is priced as the foreign currency at time  $t$ , and  $S_t$  is the bilateral nominal exchange rate that measures the units of domestic currency in terms of foreign currency at time  $t$ . The absolute LOOP essentially suggests that the price of the same good should be equal across countries when prices are expressed in the same currency. The fundamental argument for the validity of the LOOP is primarily grounded in the concept of frictionless goods arbitrage.

The LOOP has another version of relative version,

$$\frac{P_{i,t+1}^* S_{t+1}}{P_{i,t+1}} = \frac{P_{i,t}^* S_t}{P_{i,t}} \quad i = 1, 2, \dots, N \quad (2.2)$$

The relative version tells that ration between same goods priced in different currencies should be consistent across the time. The absolute LOOP derives the relative LOOP, but not vice versa. There are numerous studies trying to prove the validity of LOOP so that the PPP would hold.

Isard (1977) provides compelling empirical evidence of substantial and persistent deviations from the LOOP by examining classified data for a range of tradable commodities across different countries using the seven-digit Standard International Trade Classification categories. Building on this research, Giovannini (1988) demonstrates similar findings, highlighting that

deviations from the LOP are primarily driven by fluctuations in exchange rates. Engle and Rogers (1996) analyse consumer price index data for nine Canadian cities and fourteen American cities, focusing on fourteen disaggregated consumer price indices. Their objective is to examine the stochastic properties of deviations from the LOOP. The findings reveal that a substantial portion of the price variation among equivalent goods across different cities can be attributed to the geographical distance between them. Notably, the study highlights that price differentials between cities in different countries are significantly larger compared to price differences between cities in the same country that are equally distant. While sticky nominal prices may offer some explanation for this phenomenon, it is important to note that they fail to account for all the observed border effects.

In contrast to many studies that have failed to validate the LOOP, Sarno et al. (2004) provide compelling evidence supporting its validity while considering the influence of transaction costs. By examining data on five major bilateral U.S. dollar exchange rates and nine commodity categories since the 1970s, when the floating exchange rate regime was implemented.

The uncertainty about the future makes the exchange rate fluctuate so that arbitrageurs enter into the forward markets to make profits. However, when investors have full knowledge of future and are risk neutral, the expected change of spot exchange rate would be equal to the interest rate differential. This is the so-called uncovered interest rate parity (UIP) condition. It can be expressed by the following equation

$$i_t - i_t^* = E_t s_{t+1} - s_t \quad (2.3)$$

where  $i_t$  represents the return on a domestic risk-free bond and is also referred to as the nominal interest rate, an asterisk denotes a foreign magnitude, and  $i_t^*$  is the counterpart interest rate for foreign currency

denominated bond.  $s_t$  is the natural logarithm of  $S_t$  and  $E_t$  represents the rational expectation operator based on information at time  $t$ .

Numerous empirical studies on the presence of UIP have been carried out by researchers and there is no consensus yet. Some early empirical studies reject the UIP theory (Cumby and Obstfeld, 1981; Bilson, 1981; Fama 1984). Chinn and Meredith (2004) find the favourable evidence for UIP when they test the long run relationship (5 to 10 years) instead of short-term maturity (twelve months or less) which are tested in other studies (Meese and Rogoff, 1983). In contrast, Chaboud and Wright (2005) discover that UIP holds over extremely brief periods, particularly when the time interval spans just an hour or two. Chinn (2006) replace the rational expectation assumptions with survey data expectation to measure the anticipated change in exchange rate, and the findings support the UIP. Lothian and Wu (2011) support the existence of UIP by analysing a long-time dataset spanning two centuries, covering the UK and US.

Gali (2020) states UIP is assumed in most open economy models in the literature which research the effects of expected changes in interest rate on the exchange rate. Minford et al. (2021) find a world economy model with UIP passes the powerful Indirect Inference test. Liu and Lee (2022) verify the presence UIP hold during the certain subperiods when they investigate the relationship between interest rates and exchange rates in China and the United States. While the US monetary policy significantly influences the exchange rate dynamics between China and the US, fluctuations in the China/US exchange rate exert more profoundly impact the US interest rates compared to those in China. Kim and Cho (2011) argue that the risk-adjusted uncovered interest parity holds when they investigate monthly data from 1994 to 2008 during the period China implemented reform to improve capital market efficiency.



### 2.2.2 Open economy models

Obstfeld and Rogoff (1995) mark a pivotal shift in research from partial equilibrium models to general equilibrium models, which is named as the redux model, so their work is treated as the initiation of new open-economy macroeconomics. They develop a two-country model which integrates monopolistic competition and sticky nominal prices. Unlike traditional frameworks such as the Mundell-Fleming model, monetary models, and the portfolio balance model, this DSGE model is grounded in micro-foundations, providing a more robust analytical foundation. This model assumes homogenous agents and no costs to trade in the domestic and foreign countries, thus not only the LOOP holds, but also the PPP. The incorporation of price stickiness yields real effects of monetary shocks on both output and exchange rates. The impact on exchange rates diminishes as the elasticity of substitution between domestically and foreign-produced goods increases.

However, this two-country model generates permanent impact on the consumption differential and net foreign assets despite a temporary shock, lacking the ability to identify a unique steady state. Ghironi (2006) argues that the indeterminacy observed in incomplete market models does not adequately address how changes in net foreign assets explain the spillover effects and transmission of shocks. Additionally, he highlights that certain variations of the redux model assume the current account remains unresponsive to shocks due to the assumption of a complete financial market with unitary elasticity of substitution between domestic and foreign goods, which are unrealistic. Consequently, he proposes a more realistic two-country model which does not necessitate the unity elasticity of substitution between domestic and foreign goods or complete financial markets. In his model, the world economy returns to steady states following temporary shocks, and net foreign assets play a pivotal role in the international propagation of productivity shocks through the

adoption of an overlapping generations structure with infinitely lived households.

Obstfeld and Rogoff (1995) argue it would be inconsistent to analyse imperfections or rigidities in goods markets while simultaneously assuming that international capital markets are complete. Chari et al. (1997) contrast the impact of monetary shocks under two scenarios: one with complete markets and another where trade is limited to a noncontingent nominal bond denominated in the domestic currency. Their findings suggest that the redux model remains robust in this context. Specifically, the incompleteness of financial markets seems to lead to minimal and likely insignificant differences in the persistence of monetary shocks.

In the baseline redux model, prices are assumed to be set one period ahead, allowing for full adjustment to equilibrium after a single period. Corsetti and Pesenti (2001) argue the validity of the redux analysis may be deemed reasonable only within a specific range of shocks because they think firms may have an incentive to promptly adjust prices instantly after a substantial large shock. Kollmann (2001) explores the responses of exchange rates and prices to monetary shocks with a calibrated open economy model that incorporates predetermined price- and wage-setting and nominal rigidities of the Calvo type price. The results demonstrate that Calvo-type nominal rigidities successfully capture the observed high correlation between nominal and real exchange rates, as well as the smooth adjustment in the price level.

Jang and Okano (2013) investigate the impact of foreign productivity shocks on monetary policy in a symmetric two-country model by varying the level of trade openness. Their calibrated model shows that the effects of productivity shocks exhibit greater persistence with an increased degree of openness. UIP holds in their model while PPP only holds for the intermediate degrees of trade openness.

Ida (2023) examines a theoretical two-country model featuring a complete asset market with state-contingent bonds, holding of the law of one price, and the incorporation of a cost channel. The study reveals that an expanded foreign cost channel plays an influential role in widening worldwide equilibrium indeterminacy, particularly when both home and foreign central banks exhibit strong responses to inflation and the output gap.

While many extensions of the redux two-country model emphasize theoretical innovations, Lubik and Schorfheide (2005) investigate the empirical side by estimating a three-equation New Keynesian two-country model using Bayesian methods and data from the US and the Euro area. In contrast to the redux model, domestic and foreign agents are distinct, and monetary policy rules are differentiated. The model incorporates structural shocks related to monetary policy, technology, and government purchases, along with a non-structural shock capturing deviations from PPP. The estimation reveals a substantial standard deviation of the PPP shock, and as this shock accounts for nearly 90% of exchange rate fluctuations, constraining its effects results in a poorer fit of the model, rendering it unable to adequately explain exchange rate movements.

Walque et al. (2017) construct a medium-sized two-country open economy model for the US and Euro area, distinguishing between oil and non-oil goods. The model integrates local currency pricing, a Calvo price setting, and variable demand elasticity to confine the exchange rate pass-through to wholesale foreign prices as they propose that a constrained exchange rate pass-through is crucial for the model to provide a satisfactory fit for international variables such as the exchange rate. Distribution costs are also incorporated to mitigate the pass-through to retail foreign price. The model encompasses various domestic shocks, including productivity shock, risk premium shock, and UIP shock, as well as open economy shocks such as rest of world demand shock and oil price shock. According to their findings,

short-term fluctuations in output and consumer price inflation are primarily influenced by open economy shocks, whereas long-term movements in the exchange rate and trade balance are predominantly driven by domestic shocks.

Another significant branch of literature focus on a small open economy model. Bergin (2003) employs the maximum likelihood method to estimate and test an intertemporal small open economy model featuring monetary shocks and nominal rigidities. Despite supporting the model's assumption of sticky prices in the currency of the buyer, the model falls short in explaining exchange rate fluctuations.

Gali and Monacelli (2005) establish an influential small open economy model that consists of two countries: one representing the home country as a small open economy, and the other representing the rest of the world. The domestic economy is considered infinitesimally small, thus having no impact on the rest of the world's economic activities. Within this framework, they evaluate three alternative monetary policies for the small open economy and discover that a CPI-based Taylor rule results in lower volatility of the nominal exchange rate compared to a domestic inflation targeting rule.

Lubik and Schorfheide (2007) employ a simplified version of the Gali and Monacelli (2005) model to investigate whether central banks, including those of Australia, New Zealand, the United Kingdom, and Canada, respond to exchange rate movements. Their findings indicate that only the central bank of Canada consistently reacts to changes in the exchange rate.

Himmels and Kirsanova (2018) adapt the small open economy model derived from Gali and Monacelli (2005) by introducing incomplete financial markets characterized by transaction costs influencing portfolio allocation. The modified model anticipates the presence of multiple equilibria under discretionary monetary policy, offering an explanation for the volatilities

observed in the nominal exchange rates of emerging countries that implement a fully credible soft peg policy.

In addition to small open economy models and two-country models, three-country models have been developed. Kollmann et al. (2016) employ a three-region DSGE model to examine the post-crisis recession in the Euro area and the US, taking into account the rest of the world. The study utilizes quarterly data spanning from 1999 to 2014 and shows that a positive region-specific TFP shock leads to a real exchange rate depreciation. This, in turn, prompts a substitution of imports by domestic goods, resulting in a decline in foreign output.

Minford et al. (2021) examine a three-country model involving trade between the US, Europe, and the rest of the world using the Indirect Inference test. They observe contrasting monetary responses to the exchange rate under risk-pooling and UIP conditions. In Minford et al. (2022), they modify the three-country model by incorporating the North and South regions of the eurozone, along with the rest of the world. This modified model captures the behavioural dynamics and interactions among intra-eurozone regional economies.

The reviewed literature primarily assesses the performance of open economy models concerning developed countries or regions. There are some studies exploring the behaviour of China within these frameworks. Bénassy-Quéré et al. (2013) utilizes a two-country DSGE model encompassing China and the US to examine the exchange rate regime. The model proposes UIP does not hold due to the imposition of capital controls in China. The calibrated model suggests that transitioning to a flexible exchange rate regime in China, under different US monetary policies, would lead to varying degrees of global rebalancing. Notably, a flexible regime may trigger an undesirable rebalancing

characterized by exchange rate appreciation and a reduction in the trade surplus.

Zheng and Guo (2013) employ a small open economy model, utilizing data spanning from 1992 to 2011 in China. Their findings indicate that the nominal interest rate responds not only to inflation and the output gap but also to changes in the RMB exchange rate. Moreover, they observe that monetary policy shocks have effects on nominal variances, including inflation and the exchange rate over the long run.

Hsiao et al. (2023) customize a two-country DSGE model featuring the US and China, ensuring that UIP holds within the model. Their empirical analysis, employing Bayesian estimation, focuses on examining the impacts of financial frictions and entrepreneurial risk. However, their study does not emphasize the determination of exchange rates.

## 2.3 News shocks

Expectations of the future have been widely acknowledged as a crucial factor driving fluctuations in economic activities. This notion was originally proposed by Pigou (1927). He argued the level of economic activity and fluctuations in business cycles could be influenced by the changes in the expectations of entrepreneurs and firms regarding future productivity. Improvements of future productivity are kind of good news which encourage entrepreneurs and firms to invest more in capital, hire more labour force, so that aggregate demand increases and overall economy expands. On the other hand, economic contraction with reduced investment and lower employment could occur if pessimistic expectations on future productivity growth were formed. Since Pigou's work highlighted the importance of expectations in driving economic fluctuations, a number of literature has been investigating the empirical effects of news on business cycles.

### **2.3.1 The effects of news shock in the closed economy**

The initial research on news effects primarily focused on investigating whether news related to future total factor productivity (TFP) serves as a significant source of business cycle fluctuations. This series of studies begins with Beaudry and Portier (2006). As stock price is a good candidate of describing the agents' anticipations about future economic conditions, they isolate a news shock from stock prices that represents the innovations in stock prices which are orthogonal to innovations in TFP, and one disturbance from TFP which drives the long-run shifts in TFP. The correlation of the two types of innovations is found to be almost unity which implies that stock market booms precede permanent positive changes in productivity growth, resulting in fluctuations in the business cycle. They demonstrate that news related to future productivity growth rate can account for half of the fluctuations observed in the business cycle within the United States. Jaimovich and Rebelo (2009) further demonstrate that in order to generate the rise in demand for labour, a combination of adjustment costs related to investment and variable capital utilization is required. This is another evidence of news about future TFP explains business cycles. The presence of investment adjustment costs prompts firms to invest in current period when they anticipate the increase in future TFP. They also discover that recessions are not triggered by unfavourable news but by dull news about the future development in technology.

Fujiwara et al. (2011) employ the canonical sticky price DSGE model but find that news shock contributes little to the output fluctuation compared with the contribution of unanticipated shock. The model without news does not gain higher fitness with the addition of news shocks because surprise TFP shock is able to explain nearly fifty percent of total output variance. However, in their findings, news shock does exert a relatively more important influence on business cycles in the US than on those in Japan.

Barsky and Sims (2011) propose an alternative method to identify the news shocks on TFP, which is orthogonal to the TFP innovation. This approach applies principal components with news shocks identified as the first principal component derived from observed TFP. Consequently, it becomes the optimal for explaining the future TFP volatility, allowing the data to guide the identification of news shocks without imposing significant constraints. They find that although news shocks have an important role in accounting for output fluctuations in the medium term, they do not emerge as primary contributions to post-war US recessions and, as a result, lack substantial influence as drivers of business cycles.

Khan and Tsoukalas (2012) employ the Bayesian methods to estimate a DSGE model with several nominal and real frictions like investment adjustment costs and capital utilization costs, and both anticipated and unexpected technology and nontechnology shocks to arrive at a conclusion that surprise shocks have much larger effects over news shocks on the variance of main macroeconomic variables for the post-war period in the US. They find that stationary TFP news shocks have a negligible role in variances of real variables is consistent with Fujiware et al. (2011). The effects of news shocks are dampened by the frictions compared with the model without frictions (Schmitt-Grohe and Uribe, 2012). In terms of news shocks, nontechnology news shocks generate larger variance of all observables than technology news shocks.

Gortz and Tsoukalas (2017) argue that the reason why the TFP news is muted in either RBC models or New Keynesian models is that a financial sector is omitted in these studies. They find the financial sector creates a transmission channel linking real activity with financial markets so that the influence of TFP news shock is amplified. Thus, they employ a two-sector NK DSGE model with incorporation of a financial channel and leverage constraints to conclude consumption-specific TFP news shock accounts for the majority part of



business cycle frequencies in post-Greenspan period. They also demonstrate that the TFP news shock contributes less when it competes with other news shocks, which is consistent with Schmitt-Grohe and Uribe (2012).

Although most of studies about the news shocks focus on TFP shocks, other types of anticipated shocks also matter. Schmitt-Grohe and Uribe (2012) use the DSGE model with full-information estimation approach to identify that news shocks can account for almost half of the aggregate fluctuations of fundamentals. They find four-quarter wage-markup shocks account for 16 percent of the volatility of growth in output and more than half of the variance of employment growth, while the effect of unexpected change in wage markups could be negligible. Government spending shocks explain almost one tenth of fluctuation of output growth where 60 percent of this fraction is due to the anticipated shocks. Although neutral technology shocks account for around one-third of the output growth volatility, surprise shocks are assigned to the entirety of the contribution. However, in their formulation, the influence of anticipated neutral productivity shocks is minor because this shock is suppressed by other shocks.

Milani and Treadwell (2012) disentangle news on future monetary policy shock from explicit central bank communication. They find anticipated shock has much larger and more persistent effect on output and inflation than surprise shock. They test different anticipation horizon equal to 4, 8 and 12 quarters ahead, and news play the most significant role when the horizon is set to be 4 quarters.

Fiscal policy shocks also draw attention from the literature because fiscal measures undergo public deliberation well before their implementation then agents can anticipate it well. Hoon and Phelps (2008) investigate the impacts of upcoming tax and budgetary shocks. By employing a continuous-time model featuring Non-Ricardian agents and incorporating asset price

considerations, they conclude that reduced taxes in the future results in reduced hours worked and output. Born et al. (2013) estimate a New Keynesian DSGE model with anticipated government spending shocks and tax shocks. The fiscal policy explains approximately 15% of output variance but it is largely attributed the government spending shocks and tax shocks contribute little. Consistent with earlier research, the cumulative influence of news shocks accounts for approximately 50% of the output variance.

### **2.3.2 The effects of news shocks in the open economy**

In the previous literature mentioned, all of them test the effects of news shocks in the closed economies. There is another strand of studies that put emphasis on the open economy.

Kamber et al. (2017) are the first to examine the effect of news shocks in four small open economies, the United Kingdom, New Zealand, Canada and Australia. They adopt a small economy model with the addition of financial frictions, which is different from the financial channel in the Gortz and Tsoukalas (2017). Although both financial sectors can amplify the effects of TFP news shock, in the former model with financial frictions, news shocks easing the borrowing restrictions so that output and investment are increased because firms increase labour input into production in anticipation of forthcoming TFP improvements, thus higher dividends are expected with increasing share price. The latter model with financial channel emphasizes on amplification of increasing capital goods demand, which leads to a rise in the capital price and the further shifts in the demand for assets. This small open economy model could replicate the business cycle co-movements well with the help of news about TFP. They also develop a VAR to identify the news shocks using real data, where they identify the news is known to the agents two periods ahead it materializes. As the existence of heterogeneity in the news shocks in different countries, some fundamentals may react to news

shocks differently. In addition, they find real exchange rate appreciates initially in response to the news shock but depreciates as the increased supply of domestically produced goods dampens their relative price when the news is realised.

Arezki et al. (2017) employ a novel and observable method to identify news shocks about future output which is represented by the oil and gas discoveries. In their small open economy model, an additional sector of oil sector is included. The discoveries of oil and gas are the only source of news shock in the model, it can be clearly identified without further estimation in contrast to other literature uses VAR. They argue the TFP news shock is challengeable to be entirely foreseen in advance, so it is suspicious that all agents could perceive it and react to it properly.

### **2.3.3 News shock effects on exchange rate behaviour**

The study of how economic news impacts exchange rates dates back to Mussa (1979) who introduces the trader's expectation into exchange rate determination model and analyses the relationship between macro fundamentals and exchange rate. He argues that based on rational expectation theory, the unanticipated arrival of fundamentals news will change the exchange rate. Since then, a substantial body of research on news shocks model and their impact on exchange rate have been conducted.

Frenkel (1981) is the first to test effects of news, the unanticipated events, that generated by time series methods on exchange rate. He tests five bilateral currencies: U.S dollar/pound, dollar/franc, dollar/DM, pound/DM and franc/DM during 1973 and 1979 and finds that unexpected interest differentials have week effects on change of exchange rate. However, he says there was no such model of the foreign exchange rate model that could explain the variation adequately.

After Frenkel (1981), a series of studies use survey data and time series method to investigate the news effects on exchange rates. Hakkio and Pearce (1985) use survey data about market participants' forecasts of economic announcements. They find exchange rates only react to surprises of money stock but not react to the other economics surprises. Hardouvelis (1988) supports that foreign exchange rate market responds primarily to monetary news. Fatum and Scholnick (2008) point out only the unanticipated part of the monetary policy has a significant influence on exchange rates. They also find the announcement of the monetary policy leads to the instantaneous changes of exchange rate within the same day. However, Fatum and Scholnick (2008) emphasize the actual impact of monetary policy would be underestimated even the hypothesis of monetary policy affecting exchange rates movements would be rejected if participants fail to isolate the surprise part from the actual policy change. Koedijk and Wolff (1996) use monthly survey data of exchange rate expectation that predicted by market participants in order to avoid problems created by econometric techniques. Their results suggest the unexpected interest rate differential, in most cases, has negative impact on exchange rate movements, which means an unanticipated decline in the interest rate differential would weaken the domestic exchange rate, in other words, to increase spot rate.

Although exchange rates react to unanticipated state of the economy, the reaction appears to be quite small even it is significant (Edison, 1997). The macro news variables used by Edison are consumer price index for urban consumers (CPI), total index of industrial production(IP), producer price index for finished goods (PPI), the change in nonfarm payroll employment (NF), growth in nominal retail sales (RS) and the unemployment rate (UN). Like Koedijk and Wolff (1996), Edison (1997) also uses survey data to forecast these announcements. The expectation errors, which is the independent

variable, is determined by the difference of actual and expected macro variables. The dependent variable is the change of the exchange rate between the day of announcement and the next day. The results indicate that exchange rates react to news asymmetrically, but the difference is small; meanwhile the exchange rates do not react to large and small errors differently.

Peramunetilleke and Wong (2002) employ a new method, which uses news headline that is textual information instead of numerical time series data, to forecast intraday exchange rates. Their results show this approach outperforms other methods because textual information could provide not only the effect of the surprise but also the cause of the news.

Evans and Lyons (2008) find an indirect transmission mechanism that macro news can affect currency prices directly and indirectly via order flow and almost two thirds of total effect is transmitted by the flow. Another finding of Evans and Lyons (2008) is that exchange rates do not respond to surprise immediately while Fatum and Scholnick (2008) argue that exchange rates react to news instantaneously.

Ho et al. (2017) compare the effects on the RMB-USD volatility of USD news releases and the RMB news releases and conclude that the RMB news releases have a stronger impact. Moreover, they find the influence of negative news sentiment is much larger than that of positive news sentiment. Ben Omrane and Savaşer (2017) find exchange rate volatility response to news indicators is, on average, larger during expansion periods than that during depression periods. They test different transition indicators: non-farm payroll employment or manufacturing, consumer confidence and housing data, then the results suggest different the crisis thresholds for major FX markets. The new home sales provide a better indication of currency market during financial

crisis as the indicator is more relevant to the evolution of the US financial crisis.

Dornbusch (1976) develops a theory of exchange rates movements with application of rational expectations formation. He constructs the model by employing slow adjustments of goods markets relative to assets markets, perfect capital mobility and rational expectations. In his model, the long-run exchange rate is assumed to be known. By introducing the money market, the relationship between the price level, the spot exchange rate and the long-run exchange rate is established. The relative price of domestic goods depends on exchange rates and domestic price level. After placing restrictions on the formation of expectations, he finds that a monetary expansion will induce an immediate depreciation in the spot rate.

Most existing literatures focus on applied tests by using high frequency data (Andersen et al., 2003; Dominguez and Panthaki, 2006; Evans and Lyons, 2008; Fatum and Scholnick, 2008), few literatures employ macroeconomics model to test news model with exchange rate. Chen and Zhang (2015) use a new open economy macroeconomics model to explore the effect of news shocks in explaining exchange rate movements. They include both anticipated and unanticipated shocks in the model and find anticipated shocks could explain over 40% of exchange rate movements.

Ca'Zorzi et al. (2017) find the open economy model can only forecast the real exchange rate but not nominal exchange rate under the conditions that forecast low volatility and exhibits mean reversion characteristics, while a random walk model has a better performance.

Enders et al. (2011) and Coresetti et al. (2014) find unexpected positive shocks to the TFP leads to an appreciation in exchange rate while Miyamoto and Lan Nguyen (2017) and Ca'Zorzi et al. (2017) provide the opposite evidence that exchange rate depreciates. There is also other research like

Nam and Wang (2015) and Levchenko and Panadalai-Nayar (2020) that exploit various movements of exchange rate: expected TFP shocks tend to lead to appreciation, while surprise shocks result in depreciation. Klein and Linnemann (2021) provide another finding that both domestic expected and unanticipated technology shocks cause the real exchange rate to appreciate.

## 2.4 Conclusion

In this chapter, the review traces two strands of literature. Firstly, how the exchange rate is determined by the uncovered interest rate parity and how the related issues are incorporated into the open DSGE models while few studies empirically explore the exchange rate movements in this framework of RMB currency.

Secondly, how the news is incorporated in the VAR or DSGE models. While most literature emphasizes TFP shocks, other significant shocks like monetary news or demand shocks are often overlooked. While there's consensus on the explanatory power of anticipated TFP shocks for fundamental variables, the impact on exchange rate behaviour remains unclear. Additionally, existing news shock literature predominantly centres around the US, with some attention to the UK and Canada.

The thesis contributes to the literature in twofold. Firstly, it presents an empirical investigation into China's exchange rate fluctuations using a three-country model, where China is the domestic country, the US is the foreign country, and the rest of the world acts as a transfer pot, showing a good fit with the data.

Secondly, it explores the effects of news shocks by incorporating both domestic and foreign shocks into the analysis on China and the US. Each country is exposed to four types of shocks: demand shock, supply shock, productivity shock, and monetary policy shock. Therefore, this study adds

value by examining an open economy model empirically, contributing to the existing literature on news shocks.



# Chapter 3 Benchmark Model

## 3.1 Introduction

This chapter is going to present a three-country open economy model, which is primarily modified from Gali and Monacelli (2005) and Minford et al. (2021), as the basis for this thesis. To address the news effects on exchange rates movements, we incorporate the signal extraction process as presented by Le et al. (2020). A more detailed explanation about news shock is provided in Chapter 6.

Minford et al. (2021) present a New Keynesian model wherein the US and Europe trade with the rest of the world. As their primary focus is on examining the interconnected behaviours of major developed countries, the rest of the world is not modelled in an entirety form and only engages in trade with these major countries. Their model assumes complete financial markets where the bonds are contingent to get risk sharing condition. Empirically, they also find that UIP is valid. The corporate bond rate is used in place of the government bond rate to circumvent the zero lower bound issue.

Therefore, the contribution would be focus on the relationship between China and the United States, which has not been examined by this world economy model. China is treated as the home country while the US is chosen as the foreign country. Subsequently, the rest of the world is viewed collectively to account for the indirect trade between China and the US. For the purposes of clarity in this chapter, we use subscript  $H$  to denote the home country, China, and subscript  $F$  to represent the foreign country, the US. Meanwhile, the economic activities occurred in the foreign country are indicated by an asterisk (\*), whereas domestic economic activities remain unindexed.

## 3.2 The model

In this three-country open economy model, there are four classes of agents in domestic and foreign countries separately: households, intermediate goods producers, final goods producers, and a central bank. Contrary to the real business cycle model, where economic agents optimally respond to exogenous shocks, agents within a Keynesian framework exhibit distinct behaviours due to price rigidity. Consequently, this leads to the distortion of shock transmission channels in an open economy, which is reflected in the spillover between countries. First, we will introduce the domestic households. The home countries and foreign countries are symmetric, which indicates they share the same degree of openness (Jang and Okano, 2013; Minford et al. 2021; Ida, 2023).

### 3.2.1 Domestic households

Each economy is populated by a continuum of identical households. A representative household's utility function, which takes a form of constant relative risk aversion (CRRA) utility function and has an additively separable form, is assumed to be as follows:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \epsilon_t \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi} \right) \quad (3.1)$$

where  $E_0$  is the rational expectation factor conditional on the information available at period 0,  $\beta \in (0,1)$  is the subjective discount factor,  $\epsilon_t$  is the time-preference shock,  $\sigma$  and  $\varphi$  represent the inverse of consumption elasticity and the inverse of labour elasticity, respectively.  $C_t$  denotes the aggregate consumption index of domestic and foreign consumption, and it is defined as:

$$C_t \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (3.2)$$

where  $C_{H,t} \equiv \left[ \int_0^1 C_{H,t}(h)^{\frac{\gamma-1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}}$  is the CES index of consumption of goods produced in home country, where  $h \in [0,1]$  denotes the good variety, and  $C_{F,t} \equiv \left[ \int_0^1 C_{F,t}(f)^{\frac{\gamma^*-1}{\gamma^*}} df \right]^{\frac{\gamma^*}{\gamma^*-1}}$  is the quantity of import of goods produced in foreign country. Parameter  $\eta > 0$  is the degree of substitution between domestic and foreign goods consumed.  $\alpha \in [0,1]$  is the import share which measures the degree of openness. Since we assume two countries are symmetric, so they have the same degrees,  $\alpha = \alpha^*$ .  $\gamma$  and  $\gamma^* > 1$  are the price elasticities of differentiated goods produced in home country and foreign country.

As the model assuming incomplete international financial market in both the domestic and foreign economies, domestic households are able to invest domestic non-contingent bonds  $B_t$  as well as foreign non-contingent bonds  $B_t^f$ . The budget constraint for a home country household is:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + E_t \left( \frac{B_{t+1}}{1 + R_t} \right) + E_t \left( \frac{S_t B_{t+1}^f}{(1 + R_t^*) \Phi_t} \right) \leq W_t N_t + B_t + S_t B_t^f + TR_t \quad (3.3)$$

where  $P_{H,t}$  is the price of domestic goods,  $P_{F,t}$  is the price of imported goods,  $S_t$  is the nominal exchange rate defined as the units of domestic currency per unit of foreign currency (RMB/US dollar),  $R_t$  is the home net nominal interest rates and  $R_t^*$  is the counterpart foreign net nominal interest rate,  $\Phi_t$  is the bond premium which adjusts the interest rate on foreign bonds.  $W_t$  is the nominal wage and  $TR_t$  is the lump-sum transfer.

In equation (3.3), the left-hand side delineates the total income of the representative household, which is allocated for the consumption of both domestic and imported goods ( $P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ ) and for investments in domestic and foreign bonds  $\left( E_t \left( \frac{B_{t+1}}{1+R_t} \right) + E_t \left( \frac{S_t B_{t+1}^f}{(1+R_t^*) \Phi_t} \right) \right)$ . On the right-hand side

of the budget constraint, the household earns nominal wages  $W_t N_t$  by supplying labour to domestic producers, receives returns from the two types of bonds acquired in the preceding period and gets a tax transfer from the government ( $TR_t$  can be considered as lump sum taxes if it is negative).

Given the finite income of households in each period, they prioritize minimizing the consumption cost of goods. This allows them to determine the proportion of consumption of both types of goods before proceeding to maximize their utility function. The households' cost minimization problem with consumption constraint is:

$$\begin{aligned} & \min_{C_{H,t}, C_{F,t}} P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \\ & s. t. \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \geq C_t \end{aligned}$$

They minimize the total expenditure on consumption given prices levels are known. Solving this minimization problem with Lagrange method

The first order conditions with respect to  $C_{H,t}$  and  $C_{F,t}$  are:

$$\frac{\partial \mathcal{L}}{\partial C_{H,t}}: P_{H,t} = \lambda_t \left\{ (1 - \alpha)^{\frac{1}{\eta}} \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_{H,t}^{-\frac{1}{\eta}} \right\} \quad (3.4)$$

$$\frac{\partial \mathcal{L}}{\partial C_{F,t}}: P_{F,t} = \lambda_t \left\{ \alpha^{\frac{1}{\eta}} \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}} C_{F,t}^{-\frac{1}{\eta}} \right\} \quad (3.5)$$

where  $\lambda_t$  is the Lagrange multiplier on the constraint. Using the definition of aggregate consumption index (3.2) and symmetric assumption, the two first order conditions can be written as

$$P_{H,t} = (1 - \alpha)^{\frac{1}{\eta}} \lambda_t C_t^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}}$$

$$P_{F,t} = \alpha^{\frac{1}{\eta}} \lambda_t C_t^{\frac{1}{\eta}} C_{F,t}^{-\frac{1}{\eta}}$$

Simplifying them further we can get the demand for goods produced in domestic and foreign countries,  $C_{H,t}$  and  $C_{F,t}$ , respectively, with the Lagrange multiplier

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{\lambda_t} \right)^{-\eta} C_t \quad (3.6)$$

$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{\lambda_t} \right)^{-\eta} C_t \quad (3.7)$$

Substituting the solutions for  $C_{H,t}$  and  $C_{F,t}$  into the aggregate consumption index (3.2) yields

$$\begin{aligned} C_t &= \left\{ (1 - \alpha)^{\frac{1}{\eta}} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{\lambda_t} \right)^{-\eta} C_t \right]^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left[ \alpha \left( \frac{P_{F,t}}{\lambda_t} \right)^{-\eta} C_t \right]^{\frac{\eta-1}{\eta}} \right\}^{\frac{\eta}{\eta-1}} \\ &= \left[ (1 - \alpha) \left( \frac{P_{H,t}}{\lambda_t} \right)^{1-\eta} + \alpha \left( \frac{P_{F,t}}{\lambda_t} \right)^{1-\eta} \right]^{\frac{\eta}{\eta-1}} C_t \end{aligned} \quad (3.8)$$

Dividing both sides by  $C_t$  and solving for  $\lambda_t$  yields

$$\lambda_t = \left[ (1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv P_t$$

where  $P_t$  is defined as the aggregated price index for consumption or consumer price index (CPI). When the price index for domestic equals the price index for foreign goods, the parameter  $\alpha$  indicates the proportion of domestic consumption replaced by imported goods.

Using the definition of aggregated price index for consumption, the optimal allocation of expenditures between domestic and imported goods are determined by the following equations

$$C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad (3.9)$$

$$C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (3.10)$$

Using (3.9), (3.10) and the definition of aggregated price index we can have the total consumption expenditures by domestic households:

$$\begin{aligned}
P_{H,t}C_{H,t} + P_{F,t}C_{F,t} &= P_{H,t}(1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + P_{F,t}\alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \\
&= [(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}] C_t P_t^\eta \\
&= P_t^{1-\eta} C_t P_t^\eta \\
&= P_t C_t
\end{aligned}$$

Then the budget constraint could be further simplified as:

$$P_t C_t + E_t \left( \frac{B_{t+1}}{1 + R_t} \right) + E_t \left( \frac{S_t B_{t+1}^f}{(1 + R_t^*) \Phi_t} \right) \leq B_t + S_t B_t^f + W_t N_t + TR_t \quad (3.11)$$

The household chooses consumption, domestic and foreign bonds and labour supply to maximize utility function (3.1) subject to the budget constraint (3.11).

First order conditions are as following:

$$\frac{\partial L}{\partial C_t} : C_t^{-\sigma} = \lambda_t^c P_t \quad (3.12)$$

$$\frac{\partial L}{\partial N_t} : N_t^\varphi = \lambda_t^c W_t \quad (3.13)$$

$$\frac{\partial L}{\partial B_{t+1}} : \epsilon_t \lambda_t^c = \beta E_t \epsilon_{t+1} \lambda_{t+1}^c (1 + R_t) \quad (3.14)$$

$$\frac{\partial L}{\partial B_{t+1}^f} : \epsilon_t \lambda_t^c = \beta E_t \epsilon_{t+1} \lambda_{t+1}^c [1 + R_t^*] \Phi_t \frac{S_{t+1}}{S_t} \quad (3.15)$$

where  $\lambda_t^c$  is the Lagrange multiplier. The Euler equation for consumption can be derived by combining (3.12) and (3.14)

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} (1 + R_t) \frac{P_t}{P_{t+1}} \frac{\epsilon_{t+1}}{\epsilon_t} \quad (3.16)$$

Log-linearize (3.16) to get the linearized form

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (R_t - E_t \pi_{t+1} - \bar{r} + E_t \ln \epsilon_{t+1} - \ln \epsilon_t) \quad (3.17)$$

where  $c_t = \ln C_t$ ,  $E_t \pi_{t+1} = E_t \ln P_{t+1} - \ln P_t = E_t p_{t+1} - p_t$  is the expected CPI inflation.  $\bar{r} = \frac{1}{\beta} - 1$  is the steady-state real interest rate. From now on, a

variable in lowercase represents the logarithm of the corresponding uppercase variable.

The intratemporal condition is derived by combining (3.12) and (3.13). This condition represents the marginal rate of substitution between leisure and consumption equal to the real wage.

$$N_t^\varphi C_t^\sigma = \frac{W_t}{P_t} \quad (3.18)$$

Log-linearize (3.18)

$$w_t - p_t = \varphi n_t + \sigma c_t \quad (3.19)$$

Combing the FOCs of bonds holding equations (3.14) and (3.15) to get the uncovered interest rate parity condition (UIP):

$$1 + R_t = (1 + R_t^*) \Phi_t \frac{E_t S_{t+1}}{S_t} \quad (3.20)$$

The equation states that the returns on domestic and foreign bonds are identical when measured in the same currency. The current form of UIP condition guarantees the stationarity of net foreign assets in the long run with the inclusion of a risk premium term. Schmitt-Grohé and Uribe (2003) note that a model featuring incomplete financial markets with only risk-free foreign bonds is not stationary. Thus, it is also a technical reason for introducing a bond premium dependent on foreign bond holding in this world economy model. This risk premium is assumed to be a function of holding of foreign bonds,  $\Phi_t = \exp(-\phi_b(\tilde{B}_t^f - \tilde{B}^f))$ , where  $\tilde{B}_t^f = \frac{S_t B_t^f}{P_t Y_t}$ , following Adolfson et al. (2007), Walque et al. (2017) and Makarski et al. (2022).  $\Phi_t$  is therefore assumed to be strictly decreasing in the relative position and satisfy  $\Phi(0) = 1$ . When domestic households are net lenders, they receive a return lower than the foreign interest rate. Conversely, if they are net debtors, they incur a premium over the interest rate.

Log-linearize the above condition to get uncovered interest parity (UIP):

$$E_t s_{t+1} - s_t = R_t - R_t^* - \widehat{\Phi}_t \quad (3.21)$$

where  $s_t = \ln S_t$  and  $\widehat{\Phi}_t$  is the linearised form and  $\widehat{\Phi}_t = -\phi_b \tilde{b}_t^f$ .

Real exchange rate is defined by  $Q_t = \frac{S_t P_t^*}{P_t}$ . Then log-linearize it, we have

$$q_t = s_t + p_t^* - p_t \quad (3.22)$$

Substitute (3.22) into (3.21)

$$E_t q_{t+1} - q_t = (R_t - E_t \pi_{t+1}) - (R_t^* - E_t \pi_{t+1}^*) + \phi_b \tilde{b}_t^f \quad (3.23)$$

Equation (3.23) is named as the real uncovered interest parity condition (RUIP). It states the relative expected change in the real exchange rate is equal to the real interest rate differential adjusted by net foreign assets position between domestic and foreign country. For example, if the RUIP is valid between the two nations, then households exhibit no preference between investing in either of the two currencies, the Chinese yuan or the US dollar. Should the real return on US bonds exceed that of Chinese bonds, any excess return would be offset by a corresponding depreciation of the US dollar relative to the Chinese yuan.

### 3.2.2 Domestic firms

#### Final Goods Producers

Final goods producers, also referred to as retailer firms, purchase intermediate goods  $Y_t(h)$  from intermediate goods producers or wholesale firms at a price  $P_{H,t}(h)$ . These are then combined to produce composite final goods  $Y_t$ . The final goods producers only require intermediate goods but no labour or other inputs in their production process. Final goods producers operate in a perfectly competitive market thus they take their final domestic goods price,  $P_{H,t}$ , and intermediate goods price,  $P_{H,t}(h)$ , as given. The final goods  $Y_t$  is a form of Dixit-Stiglitz aggregator function, which can be explained as the aggregate domestic output:



$$Y_t = \left( \int_0^1 Y_t(h)^{1-\frac{1}{\gamma}} dh \right)^{\frac{\gamma}{\gamma-1}} \quad (3.24)$$

A representative final goods producer chooses the quantity of input of each intermediate good to maximize its profit

$$\begin{aligned} \max_{Y_t(h)} P_{H,t} Y_t - \int_0^1 P_{H,t}(h) Y_t(h) dh \\ \text{s. t. } Y_t = \left( \int_0^1 Y_t(h)^{1-\frac{1}{\gamma}} dh \right)^{\frac{\gamma}{\gamma-1}} \end{aligned}$$

After taking the FOC with respect to  $Y_t(h)$ , we can derive the demand function for the intermediate good, which is negatively correlated with its relative price

$$Y_t(h) = \left( \frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\gamma} Y_t \quad (3.25)$$

From equations (3.24) and (3.25), we can derive the producer price index (PPI) is given by the following formula

$$P_{H,t} = \left[ \int_0^1 P_{H,t}(h)^{1-\gamma} dh \right]^{\frac{1}{1-\gamma}} \quad (3.26)$$

### Intermediate Goods Producers

A domestic intermediate firm has access to produce a differentiated good with a production function that only use labour as its input

$$Y_t(h) = A_t N_t(h) \quad (3.27)$$

where  $A_t$  is the productivity factor that common to all domestic intermediate firms and  $N_t(h)$  is labour input. The constant return to scale is assumed for the production function which is homogenous of degree 1.

The equilibrium in the labour market is aggregate employment, then we substitute equations (3.25) and (3.27):

$$\begin{aligned}
N_t &\equiv \int_0^1 N_t(h) dh \\
&= \int_0^1 \frac{Y_t(h)}{A_t} dh \\
&= \int_0^1 \frac{\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\gamma} Y_t}{A_t} dh \\
&= \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\gamma} dh
\end{aligned}$$

Let  $d_t \equiv \int_0^1 \left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\gamma} dh$ . Around the perfect foresight steady state,  $d_t$  is up to a second order approximation (Gali and Monacelli, 2005), the derivation is in Appendix 1. Therefore, up to a first order approximation, the aggregate relationship of equation (3.27) in the log-linearized form is given by

$$y_t = a_t + n_t \quad (3.28)$$

Prior to optimizing their profits, firms address the problem of determining their labour needs by solving a cost minimization question. The cost minimization problem for a representative intermediate firm is:

$$\min_{N_t(h)} W_t N_t(h) + P_{H,t} MC_t [Y_t(h) - A_t N_t(h)]$$

where  $W_t$  is the nominal wage,  $MC_t$  is the firm's real marginal cost. The nominal wage and the real marginal cost will be common to all the domestic firms under perfectly competitive labour markets.

The first order condition is given by taking derivative of  $N_t(h)$

$$W_t = P_{H,t} MC_t A_t \quad (3.29)$$

Equation (3.29) indicates the nominal wage is positive correlated with domestic good price, marginal cost and productivity. Log-linearize equation (3.29) to get

$$mc_t = w_t - p_{H,t} - a_t \quad (3.30)$$

The domestic intermediate goods firms produce differentiated goods; thus market is considered imperfectly competitive. In this case, intermediate firms are assumed to set price,  $P_{H,t}(h)$ , subject to the Calvo price fashion. Hence,  $1 - \theta$  fraction of all firms adjust price optimally each period, while the remaining  $\theta$  fraction of firms do not reset the price which is the same as the previous period. The probability of a firm not to reset price during  $t$  and  $t + k$  is  $\theta^k$ . With this probability, each firm's profit at time  $t + k$  will only be affected by the price adjusted at time  $t$ . A representative firm maximizes its discount present profit subject to a downward-sloping demand function by picking  $P_{H,t}(h)$ :

$$\begin{aligned} \max_{\tilde{P}_{H,t}(h)} E_t \sum_{k=0}^{\infty} (\theta)^k \Delta_{t,t+k} \left\{ Y_{t+k}(h) \left[ \frac{P_{H,t}(h)}{P_{H,t+k}} - MC_{t+k} \right] \right\} \\ \text{s. t. } Y_{t+k}(h) = \left( \frac{P_{H,t}(h)}{P_{H,t+k}} \right)^{-\gamma} Y_{t+k} \end{aligned} \quad (3.31)$$

where  $\Delta_{t,t+k} = \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$  is the discount factor. As firms have access to the same production technology and have the same demand elasticities, they will set the same optimal price,  $\tilde{P}_{H,t}(h)$  to maximize their discount profits. Equation (3.32) is the new Keynesian Philips curve which describes the relation between domestic price inflation. See the Appendix 2 of the derivation.

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda \widehat{mc}_t \quad (3.32)$$

where  $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ .

As the consumer price index is defined by

$$P_t \equiv \left[ (1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (3.33)$$

Log-linearize the CPI around the zero-inflation steady state

$$p_t = (1 - \alpha) p_{H,t} + \alpha p_{F,t} \quad (3.34)$$

Then we have the CPI as follows by subtracting  $p_{t-1} = (1 - \alpha)p_{H,t-1} + \alpha p_{F,t-1}$

$$\pi_t = (1 - \alpha)\pi_{H,t} + \alpha(p_{F,t} - p_{F,t-1}) \quad (3.35)$$

As we assume the same model structure of the two countries, the foreign country has the symmetric setting. Thus, the foreign country is treated fairly as the home country that the foreign country has effect on the home country, which stands in contrast to the small open economy (SOE) model constructed by Gali and Monacelli (2005). The SOE model specifies the country modelled is too small to affect others. Conversely, our model emphasizes mutual interdependence, where each country has reciprocal effects on the other. The openness,  $\alpha$ , affects the price in both countries. Analogous to the equation (3.34), CPI of the foreign country is

$$p_t^* = (1 - \alpha)p_{F,t}^* + \alpha p_{H,t}^* \quad (3.36)$$

where  $p_{F,t}^*$  denotes the US prices of goods produced in the US, and  $p_{H,t}^*$  represents the US prices of goods produced in home country, China.

As the law of one price holds, we have

$$P_{H,t} = S_t P_{H,t}^*$$

$$P_{F,t} = S_t P_{F,t}^*$$

However  $P_t = S_t P_t^*$  does not hold unless  $\alpha = \alpha^* = 0.5$ .

Log-linearize LOOP conditions around the zero-inflation steady state are:

$$p_{H,t} = s_t + p_{H,t}^* \quad (3.37)$$

$$p_{F,t} = s_t + p_{F,t}^* \quad (3.38)$$

Substitute equation (3.36) and (3.34) into (3.22) we have

$$q_t = s_t + [(1 - \alpha)p_{F,t}^* + \alpha p_{H,t}^*] - [(1 - \alpha)p_{H,t} + \alpha p_{F,t}] \quad (3.39)$$

Then substitute (3.37) and (3.38) into (3.39) we have

$$p_{F,t} = p_{H,t} + \frac{1}{1-2\alpha} q_t \quad (3.40)$$

Equation (3.40) describes the relationship between domestic goods price and imported goods price.

Substitute (3.40) into (3.35) to substitute out  $p_{F,t}$  and  $p_{F,t-1}$

$$\pi_t = \pi_{H,t} + \frac{\alpha}{1-2\alpha} (q_t - q_{t-1}) \quad (3.41)$$

Substitute (3.41) into (3.32) to get the open-economy New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{m} c_t - \frac{\alpha}{1-2\alpha} [\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] \quad (3.42)$$

### 3.2.3 The trade balance

As the optimal allocation of expenditures between domestic and imported goods are determined by equations (3.9) and (3.10), we could rewrite the total expenditure equation for domestic household as

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \quad (3.43)$$

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} IM_{F,t}^H \quad (3.44)$$

where  $IM_{F,t}^H$  denotes the home import from foreign country, which represents that China's import from the US. As the rest of the world only picks up trade flows of the two main countries with other countries, the consumption of foreign goods in equation (3.44) does not take goods from the rest of the world into account (Minford et al., 2021; Minford et al., 2022).

$$IM_{F,t}^H = C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (3.45)$$

To model the trade spillovers to and from the rest of the world, we follow Le et al. (2013), Minford et al. (2021) and Minford et al. (2022), home import from the rest of the world is assumed to be

$$im_{RoW,t}^H = \nu y_t \quad (3.46)$$

The home import from the rest of the world is assumed to only be affected by the domestic income level, measured by  $v$ , while the exchange rate does not matter. The import from the foreign country is therefore assumed to be dependent on domestic income and real exchange rate

$$im_{F,t}^H = \mu y_t - \psi q_t \quad (3.47)$$

The domestic country's imports from the foreign country are positively influenced by its own income level, denoted by  $\mu$ , as well as negatively with the real exchange rate, represented by  $\psi$ .

Analogous to the home country, the import of foreign country from home country and the rest of the world are

$$im_{H,t}^F = \mu^* y_t^* + \psi^* q_t \quad (3.48)$$

$$im_{RoW,t}^F = v^* y_t^* \quad (3.49)$$

As shown in equation (3.47) and (3.48), the real exchange rate has opposite effects on the imports.

Trade balance of the rest of world economy can be summed as:

$$\Xi im_{RoW,t}^H + (1 - \Xi) im_{RoW,t}^F = \Gamma ex_{RoW,t}^H + (1 - \Gamma) ex_{RoW,t}^F \quad (3.50)$$

where  $\Xi$  and  $\Gamma$  respectively represent the steady-state import and export ratios of the two countries in relation to the rest of the world,  $ex_{RoW,t}^H$  is the home export to the rest of the world, and  $ex_{RoW,t}^F$  is the foreign export to the rest of the world.

The LHS of equation (3.50) measures import of China and the US from the rest of the world, so it can be treated as the output of the rest of the world as the rest of world is assumed to produce nothing but transfer products from one to the other country.

$$y_t^{RoW} = \Xi im_{RoW,t}^H + (1 - \Xi) im_{RoW,t}^F \quad (3.51)$$

where  $y_t^{RoW}$  represents the output of the rest of world.

The world's relative demand for China and US goods is determined by:

$$ex_{RoW,t}^H = ex_{RoW,t}^F + \psi^{RoW} q_t \quad (3.52)$$

where  $\psi^{RoW} q_t$  reflects the effect of real exchange rate on China's global trade share (Minford et al., 2021).

Let  $NX_t$  denote the net exports, then the home country market clearing condition is:

$$Y_t = C_t + NX_t \quad (3.53)$$

Log-linearizing (3.53) gives:

$$\begin{aligned} y_t &= \frac{C}{Y} c_t + \frac{NX}{Y} nx_t \\ y_t &= cc_t + xnx_t \end{aligned} \quad (3.54)$$

where  $c = \frac{C}{Y}$  represents the steady state domestic consumption to output ratio,  $x = \frac{NX}{Y}$  denotes the steady state home country net export to output ratio, and  $C$ ,  $Y$  and  $NX$  are the steady states of the variables.

Net export is defined as

$$\begin{aligned} NX_t &= X_t - M_t \\ &= (IM_{H,t}^F + EX_{RoW,t}^H) - (IM_{F,t}^H + IM_{RoW,t}^H) \end{aligned} \quad (3.55)$$

where  $X_t$  represents the domestic exports and  $M_t$  denotes domestic imports.

Linearize equation (3.55)

$$\begin{aligned} NXnx_t &= Xx_t - Mm_t \\ \frac{NX}{Y} nx_t &= \frac{X}{Y} x_t - \frac{M}{Y} m_t \\ \frac{NX}{Y} nx_t &= \frac{(IM_{H,t}^F im_{H,t}^F + EX_{RoW,t}^H ex_{RoW,t}^H)}{Y} - \frac{(IM_{F,t}^H im_{F,t}^H + IM_{RoW,t}^H im_{RoW,t}^H)}{Y} \\ xnx_t &= m_1 im_{H,t}^F + m_2 ex_{RoW,t}^H - (n_1 im_{F,t}^H + n_2 im_{RoW,t}^H) \end{aligned} \quad (3.56)$$

where  $X_t$  is the home export and  $M_t$  is the home import,  $m_1, m_2, n_1$  and  $n_2$  are steady state ratios of China export to the US to China GDP ratio, China export to the rest of the world to China GDP ratio, China import from the US to China GDP ratio and China import from the rest of world to China GDP ratio.

Then we combine the log-linearized home economy clearing condition with the Euler equation and trade equations to derive the IS curve for home country. The detailed derivation is in Appendix 3.

$$y_t = E_t y_{t+1} - c \frac{1}{\sigma} \theta (R_t - E_t \pi_{t+1} - \bar{r}) - z_1 \theta E_t \Delta y_{t+1}^* - m_2 \theta E_t \Delta y_{t+1}^{ROW} - z_3 \theta \Delta q_{t+1} + \varepsilon_t^{IS} \quad (3.57)$$

where  $\varepsilon_t^{IS} = -c \frac{1}{\sigma} \theta (E_t \ln \varepsilon_{t+1} - \ln \varepsilon_t)$ .

From the Phillips curve (3.42)

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t - \frac{\alpha}{1 - 2\alpha} [\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1})]$$

Rewrite the linearized real marginal cost equation as following

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= w_t - p_t + (p_t - p_{H,t}) - a_t \end{aligned}$$

Then substitute equation (3.19), which represents the relationship between the real wage and leisure and consumption, into the previous condition

$$mc_t = \varphi n_t + \sigma c_t + (p_t - p_{H,t}) - a_t \quad (3.58)$$

From the equation (3.34) and (3.40), we can get

$$\begin{aligned} p_t - p_{H,t} &= \alpha (p_{F,t} - p_{H,t}) \\ &= \frac{\alpha}{1 - 2\alpha} q_t \end{aligned} \quad (3.59)$$

Then substituting (3.28), (3.59) and (A3.3) into (3.58) for  $n_t$ ,  $c_t$  and  $(p_t - p_{H,t})$ :

$$mc_t = \varphi (y_t - a_t) + \sigma c_t + (p_t - p_{H,t}) - a_t$$



$$\begin{aligned}
&= \varphi(y_t - a_t) + \sigma \left[ \frac{1}{c} (\Theta^{-1} y_t - z_1 y_t^* - m_2 y_t^{RoW} - z_3 q_t) \right] + \frac{\alpha}{1 - 2\alpha} q_t - a_t \\
&= \left( \sigma \frac{1}{c} \Theta^{-1} + \varphi \right) y_t - \frac{1}{c} \sigma z_1 y_t^* - \frac{1}{c} \sigma m_2 y_t^{RoW} + \left( \frac{\alpha}{1 - 2\alpha} - \frac{1}{c} \sigma z_3 \right) q_t - (1 + \varphi) a_t \quad (3.60)
\end{aligned}$$

The domestic potential level of output,  $y_t^p$ , is defined as the equilibrium output with the absence of price rigidity.  $y_t^p$  can be found by imposing the steady state value of  $m c_t$ , which is  $\tau$ , on equation (3.60). Then we have

$$\tau = \left( \sigma \frac{1}{c} \Theta^{-1} + \varphi \right) y_t^p - \frac{1}{c} \sigma z_1 y_t^* - \frac{1}{c} \sigma m_2 y_t^{RoW} + \left( \frac{\alpha}{1 - 2\alpha} - \frac{1}{c} \sigma z_3 \right) q_t - (1 + \varphi) a_t \quad (3.61)$$

We defined the output gap as  $x_t \equiv y_t - y_t^p$ , which is the deviation of log output,  $y_t$ , from its potential output,  $y_t^p$ .

Subtracting (3.60) from (3.61) gives

$$\widehat{m c}_t = \left( \sigma \frac{1}{c} \Theta^{-1} + \varphi \right) x_t \quad (3.62)$$

Substituting (3.62) into (3.42) we can have the Phillips curve in terms of output gap:

$$\begin{aligned}
\pi_t &= \beta E_t \pi_{t+1} + \lambda \left( \sigma \frac{1}{c} \Theta^{-1} + \varphi \right) x_t - \frac{\alpha}{1 - 2\alpha} [\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP} \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa_a x_t - \frac{\alpha}{1 - 2\alpha} [\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP} \quad (3.63)
\end{aligned}$$

where  $\kappa_a = \lambda \left( \sigma \frac{1}{c} \Theta^{-1} + \varphi \right)$  and  $\varepsilon_t^{PP}$  is the supply shock.

In particular, potential output is assumed to follow a random walk process with drift as Minford et al. (2021):

$$y_t^p - y_{t-1}^p = \Gamma^{yp} + \delta (y_{t-1}^p - y_{t-2}^p) + \varepsilon_t^{yp} \quad (3.64)$$

Equation (3.64) considers the permanent effect of the productivity shock,  $\varepsilon_t^{yp}$ .  $\Gamma^{yp}$  is the deterministic trend of the potential output, and  $\delta < 1$  ensures that process is trend stationary.

In order for the balance of payments to be maintained, there needs to be an equivalence between outflow of domestic money and inflow of foreign money in domestic currency (Minford et al., 2022).

$$\tilde{B}_{t+1}^f + IM_{F,t}^H + IM_{RoW,t}^H = (1 + R_t^*)\Phi_t \tilde{B}_t^f + \frac{IM_{H,t}^F + IM_{H,t}^{RoW}}{Q_t} \quad (3.65)$$

where the LHS represents the outflow of money to purchase foreign bonds and goods while the RHS denotes the bonds and interests receiving from foreign countries and the exports to the foreign countries.

Log-linearizing (3.65)

$$\begin{aligned} \frac{B^f}{Y} \tilde{b}_{t+1}^f &= \frac{B^f}{Y} (1 + R_t^* - R^*) + (1 + R^* + \phi_b) \frac{B^f}{Y} \tilde{b}_t^f + \frac{1}{Q} \frac{IM_{H,t}^F}{Y} (im_{H,t}^F - q_t) \\ &+ \frac{1}{Q} \frac{IM_{H,t}^{RoW}}{Y} (im_{H,t}^{RoW} - q_t) - \frac{IM_{F,t}^H}{Y} im_{F,t}^H - \frac{IM_{RoW,t}^H}{Y} im_{RoW,t}^H \end{aligned} \quad (3.66)$$

where  $\tilde{b}_{t+1}^f$  is the real foreign debt to GDP ratio at the end of time  $t$ ,  $R^*$  is the steady-state foreign interest rate.  $\frac{B^f}{Y}$ ,  $\frac{1}{Q}$ ,  $\frac{IM_{H,t}^F}{Y}$ ,  $\frac{IM_{RoW,t}^H}{Y}$ ,  $\frac{IM_{F,t}^H}{Y}$  and  $\frac{IM_{H,t}^{RoW}}{Y}$  are the steady-state ratios.

### 3.2.4 Foreign households and firms

Given that both domestic and foreign households, as well as firms, operate symmetrically, the foreign component of the model equations can be expressed as following<sup>1</sup>:

IS curve

$$\begin{aligned} y_t^* &= E_t y_{t+1}^* - c^* \frac{1}{\sigma^*} \Theta^* (R_t^* - E_t \pi_{t+1}^* - \bar{r}^*) - z_2 \Theta^* E_t \Delta y_{t+1} \\ &+ m_2 \Theta^* E_t \Delta y_{t+1}^{RoW} + z_3 \Theta^* E_t \Delta q_{t+1} + \varepsilon_t^{IS^*} \end{aligned} \quad (3.67)$$

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<sup>1</sup> See Appendix 4 for the detailed derivation.

Productivity

$$y_t^{p*} - y_{t-1}^{p*} = \Gamma^{yp*} + \delta^* (y_{t-1}^{p*} - y_{t-2}^{p*}) + \varepsilon_t^{yp*} \quad (3.68)$$

Phillips curve

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \kappa_a^* (y_t^* - y_t^{p*}) + \frac{\alpha}{1-2\alpha} [\beta^* E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP*} \quad (3.69)$$

Net export

$$nx_t^* = -nx_t \quad (3.70)$$

Foreign country import from Home country

$$im_{H,t}^F = \mu^* y_t^* + \psi^* q_t \quad (3.71)$$

Foreign country import from RoW

$$im_{RoW,t}^F = v^* y_t^* \quad (3.72)$$

### 3.2.5 Monetary policy

To close the dynamic world economy model, we assume that the central bank in home country adopts a Taylor rule as follows:

$$R_t = \rho R_{t-1} + (1 - \rho) [\phi_\pi \pi_t + \phi_y (y_t - y_t^p)] + \phi_q (q_t - q_{t-1}) + \varepsilon_t^R \quad (3.74)$$

where the interest rate smoothing parameter,  $\rho \in [0,1)$ , measures policy inertia. As  $\rho$  approaches 1, the last period policy has lasting effect on current policy.  $\phi_\pi$  and  $\phi_y$  are the responses to inflation and output, and  $\varepsilon_t^R$  is the policy error which can be interpreted as the monetary policy shock. The central bank of home country takes the exchange rate into the framework of monetary policy.  $\phi_q$  captures the responsiveness of monetary policy to the change of real exchange rates.

For the central bank in foreign country, a Taylor rule is assumed as follows:

$$R_t^* = \rho^* R_{t-1}^* + (1 - \rho^*) [\phi_\pi^* \pi_t^* + \phi_y^* (y_t^* - y_t^{p*})] - \phi_q^* (q_t - q_{t-1}) + \varepsilon_t^{R*} \quad (3.75)$$

where the coefficients have analogous interpretations, albeit the foreign central bank responds inversely to fluctuations in the real exchange rate.

Le et al. (2016a) and Le et al. (2018) have addressed the zero lower bound challenges arising after the US financial crisis within a closed economy model. Thus, to avoid the complexities of introducing a non-linear regime switch to the zero bound and implementing Quantitative Easing (QE), following Minford et al. (2021), we adopt the corporate bond rate as the relevant interest rate for the US in this model, which is Moody's seasoned AAA rated corporate bond yield. Since the interest rate in China does not hit the zero lower bound, the domestic interest rate employed is the treasury securities yields. Consequently, the monetary policy in the US can be implemented through various means such as QE, bank regulation, and direct adjustments in central bank lending or deposit rates. In this interpretation, the foreign Taylor rule is associated with the commercial credit rate. Importantly, this approach does not impact the international connections between the two countries.

### 3.2.6 Model list

The three-country model incorporates the following shocks:

$\varepsilon_t^{IS}, \varepsilon_t^{YP}, \varepsilon_t^{PP}, \varepsilon_t^R, \varepsilon_t^{IS*}, \varepsilon_t^{YP*}, \varepsilon_t^{PP*}, \varepsilon_t^{R*}$ . These represent, respectively, demand shock, productivity shock, supply shock, and monetary policy shock for the home country, and demand shock, productivity shock, supply shock, and monetary policy shock for the foreign country. All shocks, except productivity shocks, are assumed to follow an AR(1) process. Productivity shocks are non-stationary.

## Home country

IS curve

$$y_t = E_t y_{t+1} - c \frac{1}{\sigma} \theta (R_t - E_t \pi_{t+1} - \bar{r}) - z_1 \theta E_t \Delta y_{t+1}^* - m_2 \theta E_t \Delta y_{t+1}^{RoW} - z_3 \theta \Delta q_{t+1} + \varepsilon_t^{IS}$$

Productivity

$$y_t^p - y_{t-1}^p = \Gamma^{yp} + \delta (y_{t-1}^p - y_{t-2}^p) + \varepsilon_t^{yp}$$

Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_a (y_t - y_t^p) - \frac{\alpha}{1 - 2\alpha} [\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP}$$

Taylor rule

$$R_t = \rho R_{t-1} + (1 - \rho) [\phi_\pi \pi_t + \phi_y (y_t - y_t^p)] + \phi_q (q_t - q_{t-1}) + \varepsilon_t^R$$

Net export

$$xnx_t = m_1 im_{H,t}^F + m_2 ex_{RoW,t}^H - (n_1 im_{F,t}^H + n_2 im_{RoW,t}^H)$$

Home country import from foreign country

$$im_{F,t}^H = \mu y_t - \psi q_t$$

Home country import from the rest of world

$$im_{RoW,t}^H = \nu y_t$$

Balance of payments

$$\begin{aligned} \frac{B^f}{Y} \tilde{b}_{t+1}^f &= \frac{B^f}{Y} (1 + R_t^* - R^*) + (1 + R^* + \phi_b) \frac{B^f}{Y} \tilde{b}_t^f + \frac{1}{Q} m_1 (im_{H,t}^F - q_t) \\ &\quad + \frac{1}{Q} m_2 (im_{H,t}^{RoW} - q_t) - n_1 im_{F,t}^H - n_2 im_{RoW,t}^H \end{aligned}$$

## Foreign country

IS curve

$$y_t^* = E_t y_{t+1}^* - c^* \frac{1}{\sigma^*} \theta^* (R_t^* - E_t \pi_{t+1}^* - \bar{r}^*) - z_2 \theta^* E_t \Delta y_{t+1} \\ + m_2 \theta^* E_t \Delta y_{t+1}^{RoW} + z_3 \theta^* E_t \Delta q_{t+1} + \varepsilon_t^{IS^*}$$

Productivity

$$y_t^{p^*} - y_{t-1}^{p^*} = \Gamma^{yp^*} + \delta^* (y_{t-1}^{p^*} - y_{t-2}^{p^*}) + \varepsilon_t^{yp^*}$$

Phillips curve

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \kappa_a^* (y_t^* - y_t^{p^*}) + \frac{\alpha^*}{1 - 2\alpha^*} [\beta^* E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP^*}$$

Taylor rule

$$R_t^* = \rho^* R_{t-1}^* + (1 - \rho^*) [\phi_\pi^* \pi_t^* + \phi_y^* (y_t^* - y_t^{p^*})] - \phi_q^* (q_t - q_{t-1}) + \varepsilon_t^{R^*}$$

Net export

$$nx_t^* = -nx_t$$

Foreign country import from home country

$$im_{H,t}^F = \mu^* y_t^* + \psi^* q_t$$

Foreign country import from the rest of world

$$im_{RoW,t}^F = v^* y_t^*$$

## Rest of World

RoW current account balance, trade balance

$$\Xi im_{RoW,t}^H + (1 - \Xi) im_{RoW,t}^F = \Gamma ex_{RoW,t}^H + (1 - \Gamma) ex_{RoW,t}^F$$

Demand for RoW output

$$y_t^{RoW} = \Xi im_{RoW,t}^H + (1 - \Xi) im_{RoW,t}^F$$

The world's relative demand for China and US goods

$$ex_{RoW,t}^H = ex_{RoW,t}^F + \psi^{RoW} q_t$$

RUIP

$$E_t q_{t+1} - q_t = (R_t - E_t \pi_{t+1}) - (R_t^* - E_t \pi_{t+1}^*) + \phi_b \tilde{b}_t^f$$

Nominal exchange rate determination

$$s_t = q_t + p_t - p_t^*$$

## Shocks

Domestic demand shock

$$\varepsilon_t^{IS} = \rho_{IS} \varepsilon_{t-1}^{IS} + \epsilon_t^{IS}$$

Domestic productivity shock

$$\varepsilon_t^{yp} = \epsilon_t^{yp}$$

Domestic supply shock

$$\varepsilon_t^{PP} = \rho_{PP} \varepsilon_{t-1}^{PP} + \epsilon_t^{PP}$$

Domestic monetary policy shock

$$\varepsilon_t^R = \rho_R \varepsilon_{t-1}^R + \epsilon_t^R$$

Foreign demand shock

$$\varepsilon_t^{IS^*} = \rho_{IS^*} \varepsilon_{t-1}^{IS^*} + \epsilon_t^{IS^*}$$

Foreign productivity shock

$$\varepsilon_t^{yp^*} = \epsilon_t^{yp^*}$$

Foreign supply shock

$$\varepsilon_t^{PP^*} = \rho_{PP^*} \varepsilon_{t-1}^{PP^*} + \epsilon_t^{PP^*}$$

Foreign monetary policy shock

$$\varepsilon_t^{R^*} = \rho_{R^*} \varepsilon_{t-1}^{R^*} + \epsilon_t^{R^*}$$

## 3.3 Data and calibration

### 3.3.1 Data description

The data employed in this thesis covers the period between the third quarter of 2005 and the last quarter of 2021. The time horizon starts from China's transition from a dollar-peg to a managed floating exchange rate, determined by market supply and demand with reference to a basket of currencies, in July 2005. During this period, the maximum daily change in the RMB/USD exchange rate was initially set at 0.3 percent, which was later increased to 0.5 percent in May 2007. Subsequently, to better align with market dynamics, this limit on daily change was further expanded to 1.0 percent in April 2012, and later to 2.0 percent in March 2014. Additionally, in 2015, another reform was introduced, specifying that the daily fixing rate of the RMB against the US dollar would be determined based on the closing rate of the interbank foreign exchange market from the previous day, taking into account supply and demand conditions and price movements of major currencies.

To estimate the model, the data set obtains the observations for 22 series: output, consumption, inflation, nominal interest rate, labour force, imports, exports and foreign bond holding for both China and the US and the bilateral exchange rate of RMB to USD. Rest of world output is calculated by the demand equation. All variables are subject to natural logarithmic transformations, with the exception of variables in percentages such as interest rates and inflation. The data are seasonally adjusted and converted into real per capita terms unless indicated alternatively. The method used to remove the seasonal effect is the US Census Bureau's X12 which is implemented in EViews. A full description of the data used is given in the Appendix 5. The data is unfiltered as utilizing filtered data can potentially distort the model's dynamic properties in ways that might be challenging to identify, which is discussed later in next chapter.



### 3.3.2 Calibration

In this section the structure parameters are calibrated in accordance with both China and the US data and literature (Zheng and Guo, 2013; Minford et al., 2021; Minford et al., 2022). The parameter values and steady states are summarized in Table 3.1 and here gives some descriptions. The households discount factor for both countries are calibrated to 0.99, which indicates that the steady-state real interest rate is 1% quarterly. Foreign bond risk premium is calibrated to be 0.01 following Brzoza-Brzezina et al. (2022).

As this model assumes the same openness of both countries,  $\alpha$  is calibrated at 0.23 which is from a small open economy model of China (Zheng and Guo, 2013). For the home country, the degree of price stickiness  $\theta$  is set at 0.669. The inverse of consumption elasticity and inverse of labour elasticity are configured at 2.051 and 3.460, respectively. The income elasticity of home import from foreign country and the rest of world are initially set at unity. Following Minford et al. (2021), exchange rate elasticity is set to be 0.8. In the case of the foreign country, the price stickiness is set at 0.653, with the inverse of consumption elasticity and inverse of labour elasticity determined to be 3.550 and 2.658 respectively. As the domestic country setup, income elasticity of foreign import from home country and rest of world are set to be 1 while exchange rate elasticity of foreign import from home country is 0.8. The exchange rate elasticity of rest of world import from home country relative to foreign country is calibrated at 0.8.

In terms of the monetary policy rule, the domestic monetary policy inertia is calibrated at 0.561 following Minford et al. (2021). The coefficient of interest rate response to inflation is 2.6, response to output gap is 0.1 and response to exchange rate is 0.128. For the foreign country, the monetary policy inertia is calibrated at 0.217. The interest responses to inflation, output gap and exchange rate are 2.62, 0.59 and 0.143 respectively.

Table 3. 1 Steady-state coefficients

Parameter	Description	Value
Home country		
$\beta$	A quarterly discount factor	0.99
$\frac{C}{Y}$	Steady-state consumption to output ratio	0.3721
$\frac{X}{Y}$	Steady-state net export to output ratio	0.0320
$\frac{BF}{Y}$	Steady-state foreign bond holding to output ratio	0.4775
$m_1$	Steady-state China export to US to China output ratio	0.0406
$m_2$	Steady-state China export to rest of world to China output ratio	0.1866
$n_1$	Steady-state China import from US to China output ratio	0.0144
$n_2$	Steady-state China import from rest of world to China output ratio	0.1790
$\delta$	Mean reversion of productivity growth	0.99
$\phi_b$	Foreign bond risk premium	0.01
Foreign Country		
$\beta^*$	A quarterly discount factor	0.99
$\frac{C^*}{Y^*}$	Steady-state consumption to output ratio	0.6852
$\frac{X^*}{Y^*}$	Steady-state net export to output ratio	-0.0548
$m_3$	Steady-state US export to China to US output ratio	0.0065
$m_4$	Steady-state US export to rest of world to US output ratio	0.0834
$n_3$	Steady-state US import from China to US output ratio	0.0274
$n_4$	Steady-state US import from rest of world to US output ratio	0.1174

$\delta^*$	Mean reversion of productivity growth	0.99
<hr/>		
Rest of world		
<hr/>		
$\Xi$	Steady-state China import from rest of world to rest of world output ratio	0.1394
$\Gamma$	Steady-state China export to rest of world to rest of world output ratio	0.1454
<hr/>		

Table 3. 2 Calibrated coefficients

Parameter	Description	Calibration
Home		
$\alpha$	Degree of openness	0.23
$\theta$	Calvo-non-adjusting probability	0.669
$\sigma$	Inverse of consumption elasticity	2.051
$\varphi$	Inverse of labour elasticity	3.460
$\mu$	Income elasticity of home import from foreign	1
$\nu$	Income elasticity of home import from RoW	1
$\psi$	Exchange rate elasticity of home import from foreign	0.8
$\rho$	Monetary policy inertia	0.56
$\phi_\pi$	Monetary policy response to inflation	2.6
$\phi_y$	Monetary policy response to output gap	0.1
$\phi_q$	Monetary policy response to exchange rate	0.128
Foreign		
$\alpha^*$	Degree of openness	0.23
$\theta^*$	Calvo-non-adjusting probability	0.653
$\sigma^*$	Inverse of consumption elasticity	3.550
$\varphi^*$	Inverse of labour elasticity	2.658
$\mu^*$	Income elasticity of foreign import from home	1
$\nu^*$	Income elasticity of foreign import from RoW	1
$\psi^*$	Exchange rate elasticity of foreign import from home	0.8
$\rho^*$	Monetary policy inertia	0.217
$\phi_\pi^*$	Monetary policy response to inflation	2.62
$\phi_y^*$	Monetary policy response to output gap	0.59
$\phi_q^*$	Monetary policy response to exchange rate	0.143
$\psi^{RoW}$	Exchange rate elasticity of RoW import from China relative to US	0.8

# Chapter 4 Indirect Inference

## 4.1 Introduction

Indirect Inference is a statistical technique utilized for the estimation and testing of structural models, accommodating various sizes, complexities, and nonlinearities. The method involves generating simulated data from the model and comparing its behaviour with observed data, which is summarised by an auxiliary model. During estimation, parameters of the structural model are searched for closely matching its simulation with selected data. For testing, the model's simulation, using a specific set of parameter estimates, is compared with selected data, providing a joint test of both parameter values and model structure.

A series of studies have implemented indirect inference to estimate structural models. It is first proposed by Smith (1993) and then developed by Gregory and Smith (1991, 1993), Gouriéroux et al. (1993), Gouriéroux and Montfort (1995) and Canova (2007). Le et al. (2011, 2016) have explored its properties in small samples, where they find that indirect inference has greater test power and lower estimation bias than Maximum Likelihood.

The underlying idea of indirect inference is to find an auxiliary model to evaluate the model performance under the null hypothesis that the structural model depicts the truth of the economy. The comparison of performance of the auxiliary model estimated on simulated data with the performance of the auxiliary model estimated on actual data generates the evaluation criterion, which is the Wald test of the difference between the two sets of coefficients estimated from previous auxiliary model. The indirect inference estimator is the one that minimizes the distance between the two set sets of parameters in a suitable metric- i.e. that minimises the Wald statistic.

When employing indirect inference to assess a structural model, data can be simulated from the macroeconomic model, given the model's parameters and the error distributions. Structural parameters are selected in such a way that, when this model is simulated, it generates estimates of the auxiliary model whose results align closely with those derived from the actual data. Suppose a set of observed data  $y_t$  has a dimension of  $m$ , and  $t = 1, \dots, T$  and its probability density function is  $f(y_t|\beta)$ , where  $\beta$  is the vector of parameters of an assumed structural model. We can also specify another auxiliary model which possesses a tractable probability density function  $f(y_t|\theta)$ , where  $\theta$  represents the vector of parameters of this auxiliary model. The auxiliary model can be the reduced form of the unknown structural model, such as a VAR, or it can be other data descriptors such as moments or impulse responses.

We can generate simulated data from  $S$  independent draws from the structural macroeconomic model, then denote it as  $x_t(\beta)$  and we postulate the existence of a particular value of  $\beta$  represented by  $\beta_0$  such that  $\{x_t(\beta_0)\}_{s=1}^S$  and  $\{y_t\}_{t=1}^T$  share the same distribution.

By maximizing the likelihood function for auxiliary model defined for the actual data, the maximum likelihood estimates of parameters in the auxiliary model is

$$\hat{\theta} = \arg \max_{\theta} \sum_{t=1}^T \log f(y_t|\theta) \quad (4.1)$$

where  $\hat{\theta}$  captures specific characteristics of the observed data and it is a generally consistent estimator of  $\beta$ .

As we have simulated data  $\{x_t(\beta)\}_{s=1}^S$  which has a dimension of  $S \times T$ , the maximum likelihood estimates of parameters in the auxiliary model defined for the simulated data is

$$\tilde{\theta}(\beta) = \arg \max_{\theta} \sum_{s=1}^S \sum_{t=1}^T \log[f(x_t(\beta)|\theta)] \quad (4.2)$$

The indirect inference applies the method of simulated quasi-maximum likelihood (SQML) to make sure  $\tilde{\theta}(\beta)$  is the closest value to  $\hat{\theta}$ . The SQML finds the desired value by matching the representative actual data and simulated data as following:

$$b(\beta) = \arg \max_{\beta} \sum_{s=1}^S \sum_{t=1}^T \log[f(y_t|\tilde{\theta}(\beta))] \quad (4.3)$$

where the value of  $\beta$  gives a particular value of  $\theta$  that produces the maximum of the likelihood function given observed data. Suppose the existence of such a particular combination of parameters is true, then the simulated data and actual data are anticipated to satisfy the sufficient condition:

$$\theta = \text{plim } \hat{\theta} = \text{plim } \tilde{\theta}(\beta) \quad (4.4)$$

This implies that the set of parameterized auxiliary models must be sufficiently robust to encapsulate the fundamental characteristics of the data or to discern the variations in generative parameters.

## 4.2 Indirect inference test

Implementing the Indirect Inference test involves three primary steps, initially outlined by Minford et al. (2009), and subsequently expanded upon by Le et al. (2011) through the introduction of Monte Carlo experiments, and further by Le et al. (2016b) with the use of non-stationary data. A comprehensive explanation of the test can be found in those foundational papers. Below is a concise overview of the process for applying the Indirect Inference test to the DSGE model.

Step 1: Calculate shock processes

The first thing is to compute the structural residuals and innovations of the DSGE model, conditional on the observed data and the set of structural parameters. The number of independent residuals is supposed to be not larger than the number of endogenous variables. However, we do not restrict the residuals to be normally distributed. If the equations do not include expectation components, then the residuals can be straightforwardly derived from the equations and the actual data, whereas, when expectations are present, their estimation becomes necessary. In that context, we employ the robust instrumental variables methods introduced by McCallum (1976) and Wickens (1982), where the instruments are the lagged endogenous data, and therefore, the VAR model is the instrumental variables regression. Subsequently, the autoregressive coefficients, which stand for the shock persistence, and the innovation are estimated by considering the errors as autoregressive processes and employing the OLS method to account for the autoregressive behaviours.

#### Step 2: Derive the simulated data

According to Le et al. (2011), simulated data can be produced by bootstrapping the innovations that derived from the preceding step. The innovations are drawn in an overlapping way in accordance with the time vector to maintain any simultaneity among them, and subsequently add them into the model by integrating with the original shock processes. In the beginning, one vector of shocks is randomly drawn and the model is resolved by Dynare (Juilliard, 2001). Then the first period value becomes the vectors of lagged variables for the subsequent period. In period 2, the model is resolved by drawing the second vector of shocks, replacing it for this period, and this solution becomes the lagged variable vector for period 3, continuing in this manner until a full-size series of bootstrapped simulations is achieved. In this research, we set  $N$ , the number of bootstrapped simulations, to be 1000. As



each simulation is executed independently, it generates  $N$  sets of estimated parameters of the auxiliary model, which is denoted by  $\theta^s(\beta)$ .

Step 3: Compute the Wald statistic

The performance of the structural model is assessed by a Wald statistic. As mentioned previously, the null hypothesis is that the structural model with the given parameters depicts the true economy. Thus, we use a Wald statistic to decide whether to reject or not reject the null hypothesis. To construct a Wald statistic, the auxiliary model and both the observed data and pseudo data are needed. The OLS estimates can be applied to the auxiliary model to compute both the parameter vector from the actual data and the set of parameter vectors from bootstrapped samples, from which we can derive estimated coefficients  $\hat{\theta}$  and  $\theta^s(\beta)$  respectively.  $\overline{\theta(\beta)}$  is defined as the average value of the  $S$  sets of coefficients:

$$\overline{\theta(\beta)} = \frac{1}{S} \sum_{s=1}^S \theta^s(\beta) \quad (4.5)$$

The Wald statistic is calculated to measure the distance between the vector of auxiliary model coefficients from observed data and simulated data, which is expressed as

$$Wald = (\hat{\theta} - \overline{\theta(\beta)})' \Omega(\beta)^{-1} (\hat{\theta} - \overline{\theta(\beta)}) \quad (4.6)$$

where  $\Omega(\beta)$  denotes the variance and covariance matrix of  $(\theta^s(\beta))$ ;  $\Omega(\beta) = cov(\theta^i(\beta), \theta^j(\beta)) = \frac{1}{S} \sum_{s=1}^S ((\theta^s(\beta) - \overline{\theta(\beta)})(\theta^s(\beta) - \overline{\theta(\beta)})')$ .

Essentially, the Wald statistic judge the performance of the structural macroeconomic model by evaluating the divergence between the data descriptors such as a VAR, average impulse response functions and moments (Minford et al. 2016). If the structural model describes the economy well, the VAR estimates from simulated data should be not significantly

different those from actual data. If the Wald statistic falls below its critical value, the null hypothesis – denoted as  $\hat{\theta} = \overline{\theta(\beta)}$  and suggesting that the model not only fits the data well but also accurately represents real economic phenomena – is not rejected. However, the rejection implies that the model suffers a potential issue with the specification.

To ensure the model fits the data at the 95% confidence level, the Wald statistic for the actual data must be less than the 95th percentile of the Wald statistics derived from the simulated data. It is straightforward to represent the Wald result by a P-value, which is calculated as

$$\frac{100 - \text{the Wald percentile}}{100}$$

Or utilizing the transformed Mahalanobis distance to facilitate a more intuitive interpretation of the results:

$$MD = 1.645 \times \left( \frac{\sqrt{2W^\alpha} - \sqrt{2k - 1}}{\sqrt{2W^{0.95}} - \sqrt{2k - 1}} \right) \quad (4.7)$$

where  $W^\alpha$  is the Wald static from the actual data,  $W^{0.95}$  is the Wald static for the 95th percentile of the simulated data and  $k$  is the length of  $\hat{\theta}$ . The Wald statistic for real data is converted into a normalized t-statistic using the above formula established by Le et al. (2011). If the statistic is less than 1.645 then the null hypothesis is not rejected so that the model fits the data well. As it is normalized as a t-statistic, the resulting value is 1.645 at the 95% point of the distribution, and consequently, any value exceeding this would result in the rejection of the model.

### 4.3 Indirect inference estimation

As explained above, the aim of indirect inference estimation is to find the optimal parameters of the structural model which lead to a minimized distance between two sets of parameters from actual data and simulated data of one auxiliary model. Indirect inference can be employed to identify the parameters

that successfully pass the Wald test, while indirect inference estimation is utilized to determine the minimum Wald statistic from all Wald statistics that pass the test.

Following Le et al. (2011) and Le et al. (2013), we employ a robust Simulated Annealing (SA) algorithm to explore within the permissible parameter range of the model, aiming to identify the set that most closely aligns with the observed data behaviour. This algorithm in MATLAB is able to find a global minimum given factors like the initial temperature, the annealing schedule and the initial parameters. Thus, SA can find the minimum Wald statistic implied by the actual and simulated data.

Simulated Annealing (SA) commences its search utilizing initial parameter values, which are the calibrated coefficients, and calculates the initial Wald statistic. Subsequently, it initiates a loop, reiterating the three steps delineated in the preceding section. Within each loop, SA randomly selects a parameter set within specified bounds, with the new parameter set being influenced by the current optimal solution. Following this, a Wald statistic is calculated. If the new Wald is smaller than the preceding minimum statistic, the new Wald and its corresponding parameters are chosen for evaluating subsequent sequences. Ultimately, once certain criteria are satisfied, the loop ceases, and the minimum Wald from this loop sequence is assessed to determine whether it is smaller than the standardized t-statistic. Various stopping rules are available for the algorithm. A new loop is executed if the Wald statistic is not satisfied. If the Wald statistic still cannot be accepted after plenty of searching, it is suspected that the model has been mis-specified. Indirect inference tests the macroeconomic model unconditionally against observed data and re-estimate the model by the optimal fit.

In this study, the bounds are designated to be within 50% of the initial calibrated parameters, and the maximum number of iterations in every SA is

set to 300, as exceeding this number was found to cease achieving a smaller statistic.

## 4.4 Further discussion

### 4.4.1 Small sample properties

During the estimation process, the objective is to choose the auxiliary model and its attributes to minimize bias, while in testing, the goal is to set power as high as possible consistently with not over-rejecting tractable models, which is a goldilocks principle. Indirect inference has good properties in small samples, as noted above, with large samples it is like FIML in giving asymptotic efficiency.

Hall et al. (2012) employ the impulse response function as a specific data descriptor within the auxiliary model, exploring the properties of the indirect inference estimator in both small and large samples. Through Monte Carlo simulations, they demonstrate that indirect inference estimation possesses favourable small sample properties, considering both bias and efficiency of the estimator. These findings are subsequently validated by Guerron-Quintana et al. (2017).

Le et al. (2015) select VAR coefficients as the data descriptor, aligning with the choice made in this thesis. They find both direct inference, such as Bayesian Maximum Likelihood and conventional interval estimations, and indirect inference estimators, exhibit consistency and asymptotic normality in estimation. However, they note that indirect inference boosts significantly testing power in small samples compared to direct inference, a finding supported by Monte Carlo experiments.

Hence, employing the indirect inference procedure to both estimate the model and test its specification, especially with our available small samples, should consequently provide more dependable results from the estimation.

#### **4.4.2 Non-stationary data**

Typically, in the estimation of a DSGE model, the initial step involves filtering the data to ensure stationarity, which consequently results in stationary model residuals derived from the estimated model. However, argued by Meenagh et al. (2012), utilizing filtered data can potentially distort the model's dynamic properties in ways that might be challenging to identify. The widely used HP filter can distort the model's dynamic properties by modifying the lag dynamic structure and altering its forward-looking aspects, potentially creating misleading cycles.

Wickens (1982) highlights two primary implications of non-stationary data for modelling. Firstly, it allows distinguishing between temporary and permanent shocks, with the former having a transient impact in a stationary or trend stationary process and the latter having a permanent effect in a unit root process. Secondly, permanent shocks, interpreted as long-run growth path effects, cause a perpetual shift in the levels of endogenous variables sharing the same Balanced Growth Path, as opposed to temporary shocks, viewed as business cycle effects.

Given the significant interest in the model's expectations structure and impulse response functions and analysing business cycles, we prefer to use the original, unfiltered data to avoid such distortions. As explained above, the VECM model is employed so overcome these serious defects.

#### **4.5 The choice of the auxiliary model and non-stationary data**

As discussed in Le et al. (2011), the solution to a log-linearised DSGE model can be represented as a restricted vector autoregressive-moving-average (VARMA) model either in levels or in first differences in the presence of permanent shocks, and this can be approximately represented by a VAR.

Further details about employing a VAR to solve a DSGE model can be found by De Jong and Dave (2007), Del Negro et al. (2007), Del Negro and Schorfheide (2008), and Canova (2007). Based on the literature previously mentioned, a VAR model can inherently serve as an auxiliary model for evaluating a DSGE model, with the observed data being characterized by an unrestricted VAR model. When the model is identified with a restricted VAR, the structural restrictions of the DSGE model are reflected in the data generated from the model, ensuring consistency with the VAR. Consequently, the auxiliary model can be estimated without constraints on both the simulated and original data, which offers a distinct advantage.

If the shocks are stationary, a levels VAR may be employed; however, in the presence of non-stationary shocks, a VECM is necessary, as explored in the subsequent discussion (Meenagh et al., 2012; Le et al., 2016b). The DSGE model could generate non-stationary data either arising from the model structure in which state variables are functions of predetermined variables influenced by cumulative shocks, such as net foreign assets, or due to the inclusion of non-stationary variables such as a technology shock in the production function which is treated as an unobservable variable. The non-stationary exogenous variables will transmit non-stationary processes to the residuals in one or more structural model equations. Since the shocks are generated from the actual data, when these processes are treated as the observed variables, the number of the co-integrating vectors would be less than that of the endogenous variables. Hence, the VARX model where the non-stationary residuals present as the observed variables is allowed to represent the solution of the estimation model, and an unrestricted formation of this VARX model is performed as the auxiliary model.

Following Meenagh et al. (2012) and Le et al. (2016b), if the data are non-stationary, the VECM model is needed to create stationary errors. It approximates the reduced form of DSGE model and can be expressed as a

cointegrated VARX model. Suppose that a structural DSGE model is represented as a log-linearized form as the following:

$$A(L)y_t = B(L)E_t y_{t+1} + C(L)x_t + D(L)e_t \quad (4.8)$$

$x_t$  are exogenous variables and are assumed to be driven by

$$\Delta x_t = a(L)\Delta x_{t-1} + d + c(L)\epsilon_t \quad (4.9)$$

where  $y_t$  are  $p \times 1$  vector of endogenous variables,  $E_t$  is the rational expectation operator,  $x_t$  are  $q \times 1$  vector of exogenous variables which may contain both observable and unobservable variables, and they are non-stationary.  $e_t$  and  $\epsilon_t$  are vectors of i.i.d error process with zero means and covariance matrix  $\Sigma$ .  $L$  represents the lag operator as  $y_{t-s} = L^s y_t$ .  $A(L)$ ,  $B(L)$  and etc are matrix polynomial functions in the lag operator of order  $h$  whose roots of the determinantal polynomial lie outside the unit circle.  $y_t$  are non-stationary as they are linearly dependent on  $x_t$ .

The general solution of  $y_t$  is expressed as

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \quad (4.10)$$

where  $f$  is a vector of constant and polynomial functions in the lag operator. As  $y_t$  and  $x_t$  are both non-stationary variables, the solution of  $y_t$  above can be rewritten in terms of  $p$  cointegration relationships:

$$\begin{aligned} y_t &= [I - G(1)]^{-1}[H(1)x_t + f] \\ &= \Pi x_t + g \end{aligned} \quad (4.11)$$

where  $g$  is the vector of constants. The  $p \times p$  matrix  $\Pi$  has rank  $0 \leq r < p$ , where  $r$  measures the number of linearly independent cointegrating vectors.

In long run, the solution to the model is expressed as

$$\bar{y}_t = \Pi \bar{x}_t + g \quad (4.12)$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \quad (4.13)$$

$$\xi_t = \sum_{s=0}^{t-1} \epsilon_{t-s} \quad (4.14)$$

where  $\bar{y}_t$  and  $\bar{x}_t$  are the solution of  $y_t$  and  $x_t$  in the long run, respectively. The solution of  $\bar{x}_t$  can be decomposed into two components, one is a deterministic trend  $\bar{x}_t^d = [1 - a(1)]^{-1}dt$  and the other one is a stochastic trend  $\bar{x}_t^s = [1 - a(1)]^{-1}c(1)\xi_t$ . So  $\bar{x}_t = \bar{x}_t^d + \bar{x}_t^s$ . In this case, the VECM model is applied by subtracting  $y_{t-1}$  on both sides in equation (4.11):

$$\begin{aligned} \Delta y_t &= P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t - [I - G(1)](y_{t-1} - \Pi x_{t-1}) \\ &\quad P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t - [I - G(1)](y_{t-1} - \Pi x_{t-1}) \\ \omega_t &= M(L)e_t + N(L)\epsilon_t \end{aligned}$$

where  $\omega_t$  is a mixed moving average process. Furthermore, the previous VECM model can be approximately written in the form of a VARX model:

$$\Delta y_t = -K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \quad (4.15)$$

where  $\zeta_t$  represents a i.i.d process with zero mean. Given  $\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$  and  $\bar{y}_t = \Pi \bar{x}_t + g$ ,

The VARX model can also be written in the form of

$$\Delta y_t = K(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \quad (4.16)$$

where the time trend and the deterministic trend are embodied in  $\bar{x}_t$ .

According to the properties of the auxiliary model, either equation (4.15) or (4.16) can be chosen as the auxiliary model. Additionally, the equation (4.16) can be rewritten as

$$y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + t + q_t \quad (4.17)$$

where  $q_t$  is the vector of errors which comprises the lagged difference regressions and the deterministic time trend in  $\bar{x}_t$  that disturbs both the endogenous and exogenous variables.



By following Le et al. (2016b), equation (4.16) is chosen as the auxiliary model as it differentiates between the impact of the trend component of  $x_t$  and the temporary deviation of  $x_t$  from the trend. In our models, these two components exert different effects and, therefore, should be distinctly identified in the data to enable the tests to offer the most comprehensive discrimination. A benefit is that the parameters of the VARX can be estimated utilizing classical OLS methods. Meenagh et al. (2012) demonstrate that this procedure is extremely accurate through Monte Carlo experiments.

Le et al. (2011) propose two types of Wald statistic, the Full Wald, which utilizes the full joint distribution of VARX coefficients with the full covariance matrix thus all the endogenous variables are included in the auxiliary model, and the Directed Wald, which only comprises interested variables or key variables. Typically, the test's power increases with the inclusion of additional endogenous variables in the auxiliary model. Nonetheless, expanding the set of endogenous variables may result in uniform rejections (Le et al. 2015). Thus, the Direct Wald statistic is used in this thesis.

The open economy model derived in previous chapter illustrates that the domestic and foreign productivity shocks are the driving forces of the economy. In this case, the domestic and foreign output,  $y_t$  and  $y_t^*$  are chosen as endogenous variables, and domestic and foreign potential output,  $y_t^p$  and  $y_t^{p*}$  are selected as exogenous variables in the auxiliary model to evaluate the structural model. The VARX(1) model is described as follows

$$\begin{bmatrix} y_t \\ y_t^* \end{bmatrix} = B \begin{bmatrix} y_{t-1} \\ y_{t-1}^* \end{bmatrix} + C \begin{bmatrix} y_{t-1}^p \\ y_{t-1}^{p*} \\ t \end{bmatrix} + \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \quad (4.18)$$

where  $B = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$ ,  $C = \begin{bmatrix} \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{23} & \beta_{24} & \beta_{25} \end{bmatrix}$ .  $\xi_1$  and  $\xi_2$  are fitted stationary errors. In the Wald calculation, the parameters vector  $\beta$  comprises six coefficients of matrix  $B$  to describe the dynamic properties of the model and

data and includes the variances of the two errors to measure the size of variation:

$$\beta = [\beta_{13} \ \beta_{14} \ \beta_{15} \ \beta_{23} \ \beta_{24} \ \beta_{25} \ \text{var}(\xi_1) \ \text{var}(\xi_2)]'$$

The model will only pass the test if it can jointly replicate the data dynamics of domestic and foreign output levels, which means the model needs to jointly match the 8 coefficients in  $\beta$ .

# Chapter 5 Evaluate and Estimate the Benchmark Model

## 5.1 Introduction

In this chapter, the model introduced in Chapter 3 will be evaluated and estimated by Indirect Inference detailed in Chapter 4 using China and the US data during the period 2005Q3 and 2021Q4. In section 5.2, the test result of the model with its calibrated parameters is shown to ascertain its ability to fit the un-filtered non-stationary data. Then section 5.3 illustrates the results obtained from the Indirect Inference estimation of this model. Section 5.4 discusses the properties of the shocks with ADF and KPSS tests. The subsequent sections analyse the model's dynamics, including the impulse response functions, variance decomposition and historical decomposition. Finally, Section 5.8 concludes this chapter.

## 5.2 Indirect Inference test results based on calibration

Before proceeding with Indirect Inference (II) estimation, it is crucial to first ascertain whether the calibrated model passes the Wald test. Should the calibrated model fail to pass this test, we then proceed to II estimation. In this phase, the calibration values of the parameters are predetermined to serve as initial starting points. The testing process will then follow the methodology outlined in the preceding chapter. Table 5.1 reports the Transformed Wald statistic and P-value of the II test results where the domestic output and foreign output are selected as the key variables for testing this three-country model. As the table shows, the Trans Wald statistic is 68.14 which is much greater than the critical value of 1.65 and the P-value is smaller than 0.05, indicating rejection of the null hypothesis of fitness of the data. In conclusion, it is evident that the structural model, with its calibrated coefficients, does not adequately fit the data. This inadequacy could stem from two primary reasons.

Firstly, the chosen calibrated parameters might be unsuitable for the countries under study. Secondly, there could be defects within the structural model itself.

Table 5. 1 Calibration results

VARX(1)	Trans Wald	P-value
$y, y^*$	68.14	0.00

Consequently, we employ indirect inference estimation to determine whether the structural model can be conclusively rejected. If the structural model successfully passes this test, the corresponding estimated parameters that emerge are those that most appropriately align with the model.

### 5.3 Indirect Inference estimation results

Following the method discussed in previous chapter, the Indirect Inference is implemented to assess the model empirically. Table 5.2 reports the estimation results, the fourth column provides the estimated parameters while column 3 represents the calibrated parameters for comparison. It should be noted that discount factor for both countries and steady-state ratios are set to be fixed.

All of these coefficients have deviated from their initial calibration values which indicates that these calibrated coefficients are unsuitable. As it is assumed that home country and foreign country share the same degree of openness, the estimated degree of openness has increased significantly from 0.23 to 0.5916, implying that two countries have a greater degree of interconnectedness between the two countries within this open economy model. In the domestic country analysis, the estimated Calvo price rigidity parameter,  $\theta$ , is 0.7957, which is higher than its calibrated counterpart. This suggests that domestic prices demonstrate greater persistence in this data set. Meanwhile, the inverse of the consumption elasticity,  $\sigma$ , has fallen from 2.051 to 1.2341. This indicates that consumption is more responsive to interest rate fluctuations compared to the calibrated scenario. The inverse of labour

elasticity,  $\varphi$ , has decreased from 3.460 to 3.3654, implying that households are now more inclined to smooth out their working hours in response to changes in wage rates, as opposed to the behaviour suggested by the calibrated value. The income elasticity of home imports from both the foreign country and the rest of the world has declined from the calibrated benchmark of one to 0.7572 and 0.5001, respectively. This means that when the income of domestic households rises by 1%, their consumption of goods from the foreign country increases by approximately 0.75%, and their consumption of goods from the rest of the world increases by about 0.50%. The exchange rate elasticity of home import from the foreign country,  $\psi$ , also decreases from 0.8 to 0.6294. This suggests that domestic households are less responsive to exchange rate fluctuations than previously anticipated. Specifically, home imports from the foreign country are projected to decrease by 0.63% for every 1% depreciation in the real exchange rate. In terms of monetary policy, it is estimated to exhibit increased responsiveness to inflation, the output gap, and the exchange rate. The interest rate's sensitivity to inflation, represented by  $\phi_\pi$ , has significantly risen from 2.6 to 4.4477, indicating that the central bank is placing greater emphasis on inflation control. The monetary response to output gap,  $\phi_y$ , is 0.5032. Furthermore, the central bank's response to exchange rate changes has intensified, as evidenced by the increase of  $\phi_q$  from 0.128 to 0.2258, indicating a higher than calibrated response to exchange rate movements. Interest rate smoothing is estimated to be 0.2686 which is less than the calibrated value.

Turning to the foreign country, the degree of Calvo price stickiness,  $\theta^*$ , is found to be higher than both the home country's level and the calibrated value. The estimated inverse of consumption elasticity,  $\sigma^*$ , is 9.5379, exceeding both the calibration and home country's figures of 3.550 and 1.2341. This indicates that foreign households are less sensitive to fluctuations in interest rates compared to both expectations and their domestic counterparts. In contrast to

domestic households, foreign households show less responsiveness to wage changes, as reflected in the estimated value of  $\varphi^*$  at 9.4529. As the income elasticity of foreign import from home country is estimated to be 0.6158, the foreign country acts similar to home country when their income increases 1%, they would only spend less than 1% in the imports from counterpart country while foreign households will import 0.65% percent more goods from the rest of the world. Furthermore, the exchange rate elasticity of foreign import from home country,  $\psi^*$ , is estimated to be 0.9459, which is higher than the calibrated. This suggests that the foreign country will increase its imports from the home country by approximately 0.95% for every 1% depreciation in the real exchange rate. The inertia in the foreign monetary policy is observed to be larger than what was anticipated in the calibration, with the estimated parameter being 0.2859. This indicates a relatively higher smoothing in the foreign Taylor rule. Additionally, the monetary responses to inflation, the output gap, and the exchange rate in the foreign country are found to exceed both the calibrated values and those observed in the home country. This demonstrates a more aggressive stance in the foreign country's monetary policy.

For the rest of world, the exchange rate elasticity of import from home country relative the foreign country,  $\psi^{RoW}$ , is estimated to be 0.4286. This figure significantly bellows the calibrated value of 0.8, indicating that with a 1% depreciation in the real exchange rate, which corresponds to a higher value of  $q$ , the rest of the world tends to increase its imports from the home country by more than 0.43%, relative to imports from the foreign country.

Table 5. 2 Estimated coefficients

Parameter	Description	Calibration	II Estimation
Home			
$\alpha$	Degree of openness	0.23	0.5916
$\theta$	Calvo-non-adjusting probability	0.669	0.7957
$\sigma$	Inverse of consumption elasticity	2.051	1.2341
$\varphi$	Inverse of labour elasticity	3.460	3.3654
$\mu$	Income elasticity of home import from foreign	1	0.7572
$\nu$	Income elasticity of home import from RoW	1	0.5001
$\psi$	Exchange rate elasticity of home import from foreign	0.8	0.6294
$\rho$	Monetary policy inertia	0.56	0.2686
$\phi_\pi$	Monetary policy response to inflation	2.6	4.4477
$\phi_y$	Monetary policy response to output gap	0.1	0.5032
$\phi_q$	Monetary policy response to exchange rate	0.128	0.2258
Foreign			
$\alpha^*$	Degree of openness	0.23	0.5916
$\theta^*$	Calvo-non-adjusting probability	0.653	0.7014
$\sigma^*$	Inverse of consumption elasticity	3.550	9.5379
$\varphi^*$	Inverse of labour elasticity	2.658	9.4529
$\mu^*$	Income elasticity of foreign import from home	1	0.6158
$\nu^*$	Income elasticity of foreign import from RoW	1	0.6539
$\psi^*$	Exchange rate elasticity of foreign import from home	0.8	0.9459
$\rho^*$	Monetary policy inertia	0.217	0.2859
$\phi_\pi^*$	Monetary policy response to inflation	2.62	4.2006
$\phi_y^*$	Monetary policy response to output gap	0.59	0.8308
$\phi_q^*$	Monetary policy response to exchange rate	0.143	0.4492
$\psi^{RoW}$	Exchange rate elasticity of RoW import from China relative to US	0.8	0.4286

Table 5. 3 Estimation results

VARX(1)	Trans Wald	P-value
$y, y^*$	1.1798	0.0980

Following the application of Indirect Inference estimation, it is observed that the set of key variables, domestic output and foreign output is statistically significant and not rejected by the Indirect Inference test. With a critical value set at 1.645 for a 5% significance level, the Transformed Wald statistic from the II estimation stands at 1.1798, which is below the critical threshold. Additionally, the p-value associated with this statistic is 0.0980, exceeding the 0.05 significance level, thereby allowing the model to pass the test. In comparison, the calibration test previously yielded a result of 68.14 with a p-value of 0, this estimation has improved a lot. It demonstrates that the estimated model can explain the behaviour of data well.

## 5.4 Shock processes

Prior to conducting standard analyses such as impulse response functions (IRFs), variance analysis, and historical decomposition, I follow Le et al. (2014) to examine the properties of the errors. There are 8 shocks in the base model. Although these shocks are not observable, they can be backed out from the structural errors based on unfiltered data and estimated parameters. Table 5.4 shows the stationarity of each shock and also the estimated AR parameters. I conduct two different types of stationarity test for each shock process: the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test.

The null hypothesis of the ADF test is that the shock follows a unit root process against the alternative hypothesis that the shock is stationary. The results of the ADF test are presented in the second column of Table 5.4. These results demonstrate that the domestic and foreign supply shocks, as well as the monetary policy shocks, and foreign demand shock reject the null



hypothesis at the 1% significance level. The only exception is the domestic demand shock, which rejects the null hypothesis at the 5% significance level. However, the p-value of both domestic and foreign productivity shocks are close to 1 which indicates the non-rejection of the unit root process. One of the limitations of the ADF unit root test is its limited power in differentiating alternatives that closely resemble an  $I(1)$  process, as noted by Elliott et al. (1996). Essentially, the ADF test struggles to effectively distinguish between highly persistent stationary processes and non-stationary processes. Therefore, to reassess the structural error, I conduct the KPSS stationarity test.

The null hypothesis of the KPSS test is that the shock is stationary against the alternative hypothesis that the shock follows a unit root process. The outcomes of this test are detailed in the fourth column of Table 5.4. With the exception of productivity shocks, all other types of shocks do not reject the null hypothesis of stationarity. Therefore, it can be concluded that only domestic and foreign productivity shocks exhibit non-stationary characteristics. These two types of productivity shocks are presumed to follow an ARIMA (1,1,0) process, while the remaining shocks are modelled as AR (1) processes. Figure 5.1 and Figure 5.2 show the residuals calculated from the log-linearized model using estimated parameters.

Table 5. 4 Stationarity of shocks and estimated AR (1) parameters

Shocks	ADF p-value	KPSS statistic	Conclusion	AR(1) coefficient
Domestic Demand	0.0058*	0.4096	Stationary	0.7895
Domestic Supply	0.0000*	0.0599	Stationary	0.5058
Domestic Monetary Policy	0.0035*	0.3374	Stationary	0.7277
Domestic Productivity	0.9999	1.0150*	Non-stationary	0.9786
Foreign Demand	0.0003*	0.0892	Stationary	0.6936
Foreign Supply	0.0003*	0.0892	Stationary	0.3979
Foreign Monetary Policy	0.0004*	0.1469	Stationary	0.7901
Foreign Productivity	0.9999	1.0401*	Non-stationary	0.9845

Note:

- a. For the Augmented Dickey-Fuller (ADF) test, p-value with \*\*\*, \*\* and \* indicate a rejection of the unit root process at 10%, 5% and 1% significant level respectively.
- b. For the KPSS test, due to Kwiatkowski et al. (1992), statistic with \*\*\*, \*\* and \* indicate a rejection of the stationary process at 10%, 5% and 1% significant level respectively.

Figure 5. 1 Domestic shocks residuals

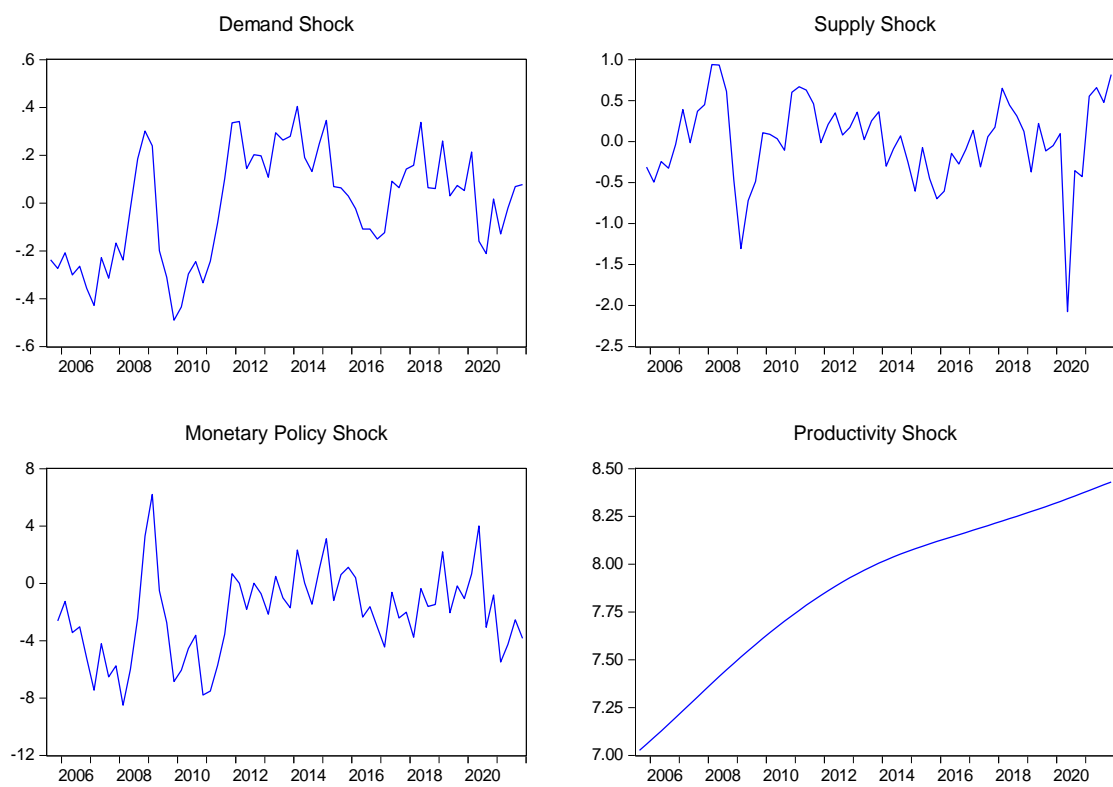
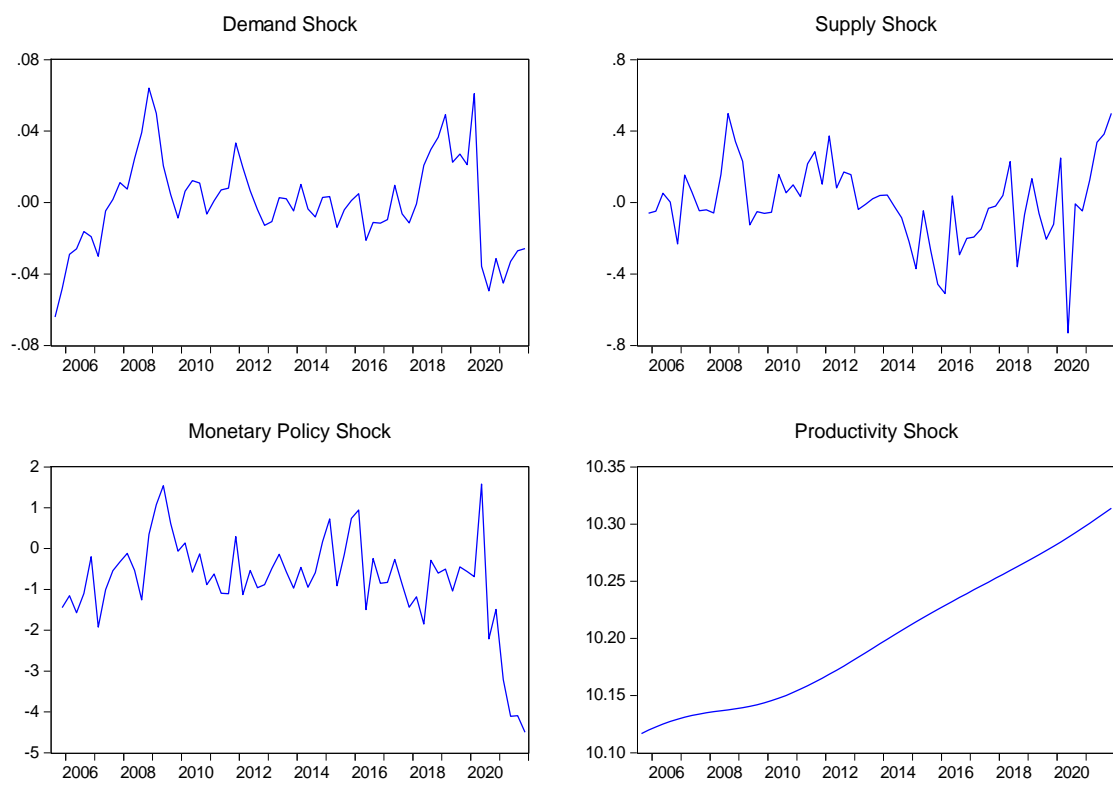


Figure 5. 2 Foreign shocks residuals



## 5.5 Impulse response functions

In this section, the impulse response functions to a couple of shocks are presented.

### 5.5.1 Demand shock

Figure 5.3 illustrates the response of macroeconomic variables to a 1% domestic demand shock. This positive shock increases domestic demand, shifting the domestic IS curve outward, which in turn increases domestic output and leads to a rise in the domestic inflation due to the Phillips curve trade-off. As the domestic output gap and inflation rise with this expansion, domestic central bank responds by raising interest rates to stabilize the domestic market. Consequently, the domestic country boosts its imports, particularly from the rest of the world. To finance these imports, domestic households sell foreign bonds, leading to a reduction in the risk premium.

This domestic shock leads to depreciation of real exchange rates and nominal exchange rates so that the imports from the foreign country become expensive then domestic country decrease the imports from the foreign country but imports from the rest of the world.

The spillover effect on foreign output is small compared to the response of domestic output. The foreign inflation raises because of the combination effect of increased imports of domestic produced goods and the raising prices of imported goods.

Conversely, Figure 5.4 presents the impacts of a 1% foreign demand shock. Given the symmetric nature of the three-country model, the effects are mirrored, with roles and responses of the domestic and foreign entities being reversed. The noticeable difference is the spillover effect on the domestic output, inflation and interest rates are much larger due to the steeper foreign Phillips curve.

Figure 5. 3 Impulse responses to a domestic demand shock

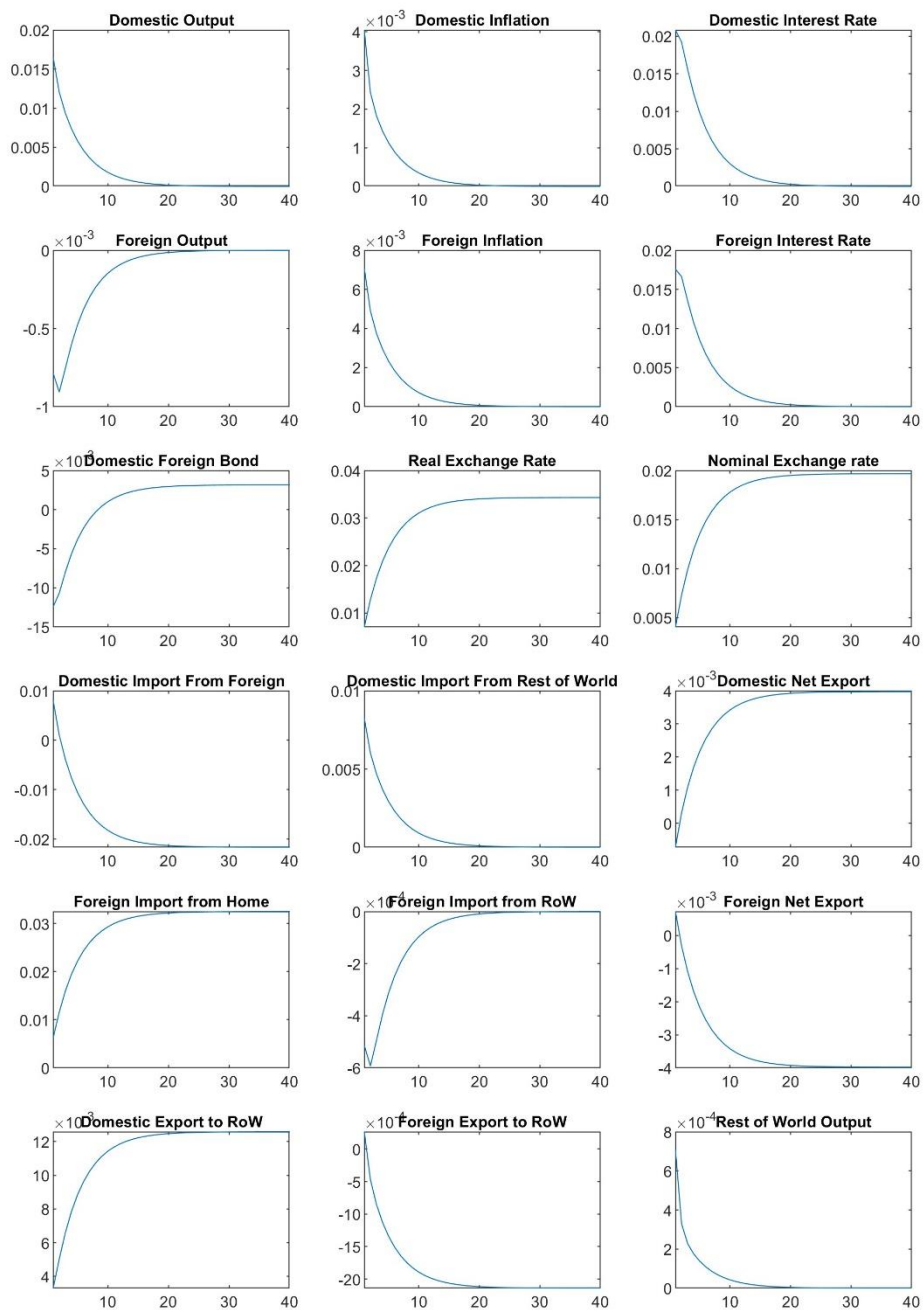
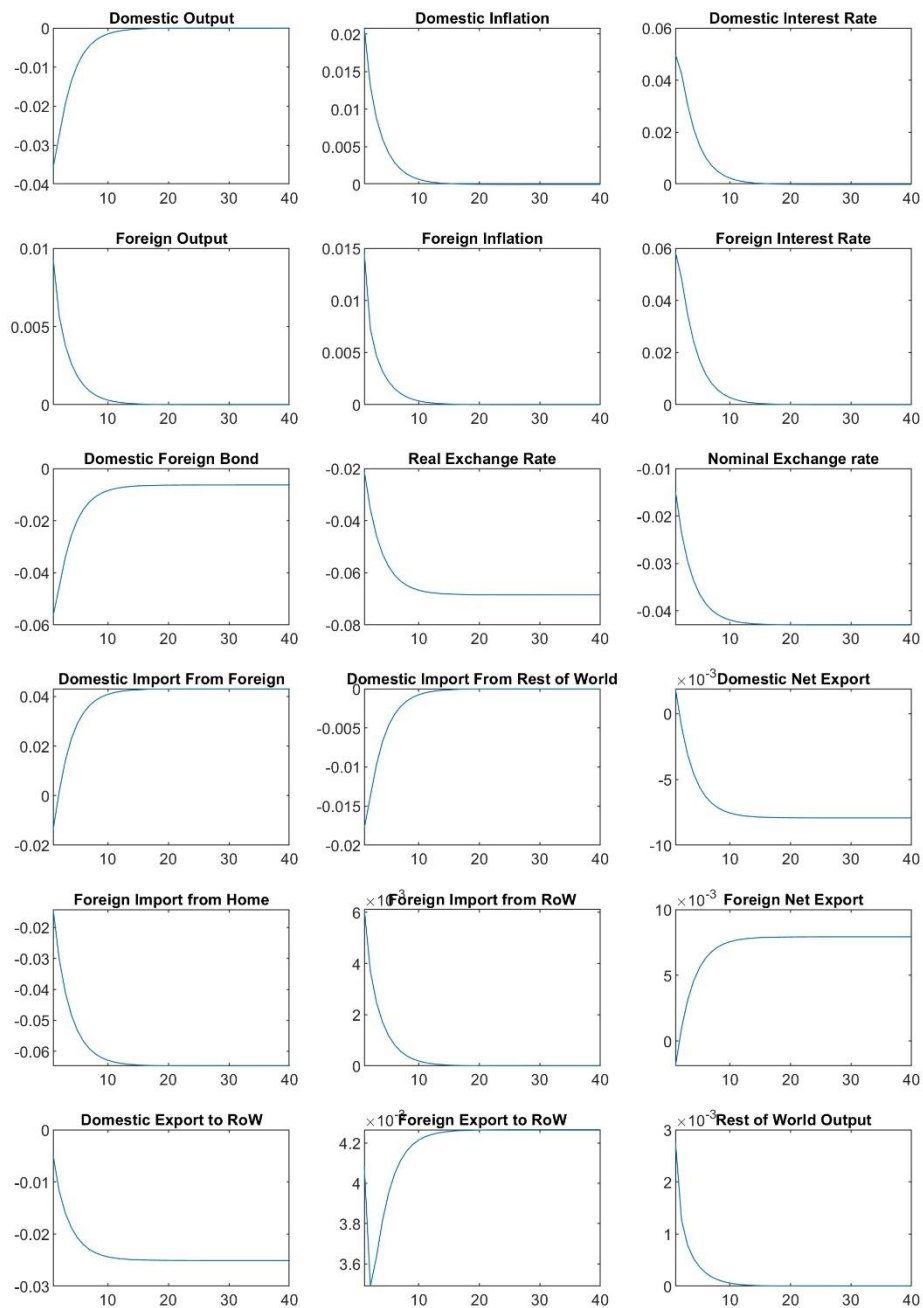


Figure 5. 4 Impulse responses to a foreign demand shock



### **5.5.2 Supply shock**

Figure 5.5 depicts the reaction of various macroeconomic variables to a negative domestic supply shock, also known as a cost-push shock which originates from exogenous cost factors. This type of shock results in increased production costs, which in turn shift the domestic Phillips curve upwards. This upward shift has a contractionary effect on aggregate demand, leading to a situation where inflation rises. Accompanying this increase in inflation is a corresponding rise in interest rates.

This shock also has spillover effects on the foreign country, leading to a similar pattern of declining output, along with increases in inflation and interest rates. As the domestic economy faces a more severe recession, it resorts to selling foreign bonds or borrowing more money from the foreign country to finance its economy. This depreciation of exchange rates makes the domestic country's exports more competitively priced, leading to an increase in net exports.

The effects of a foreign supply shock are analogous.

Figure 5. 5 Impulse responses to a domestic supply shock

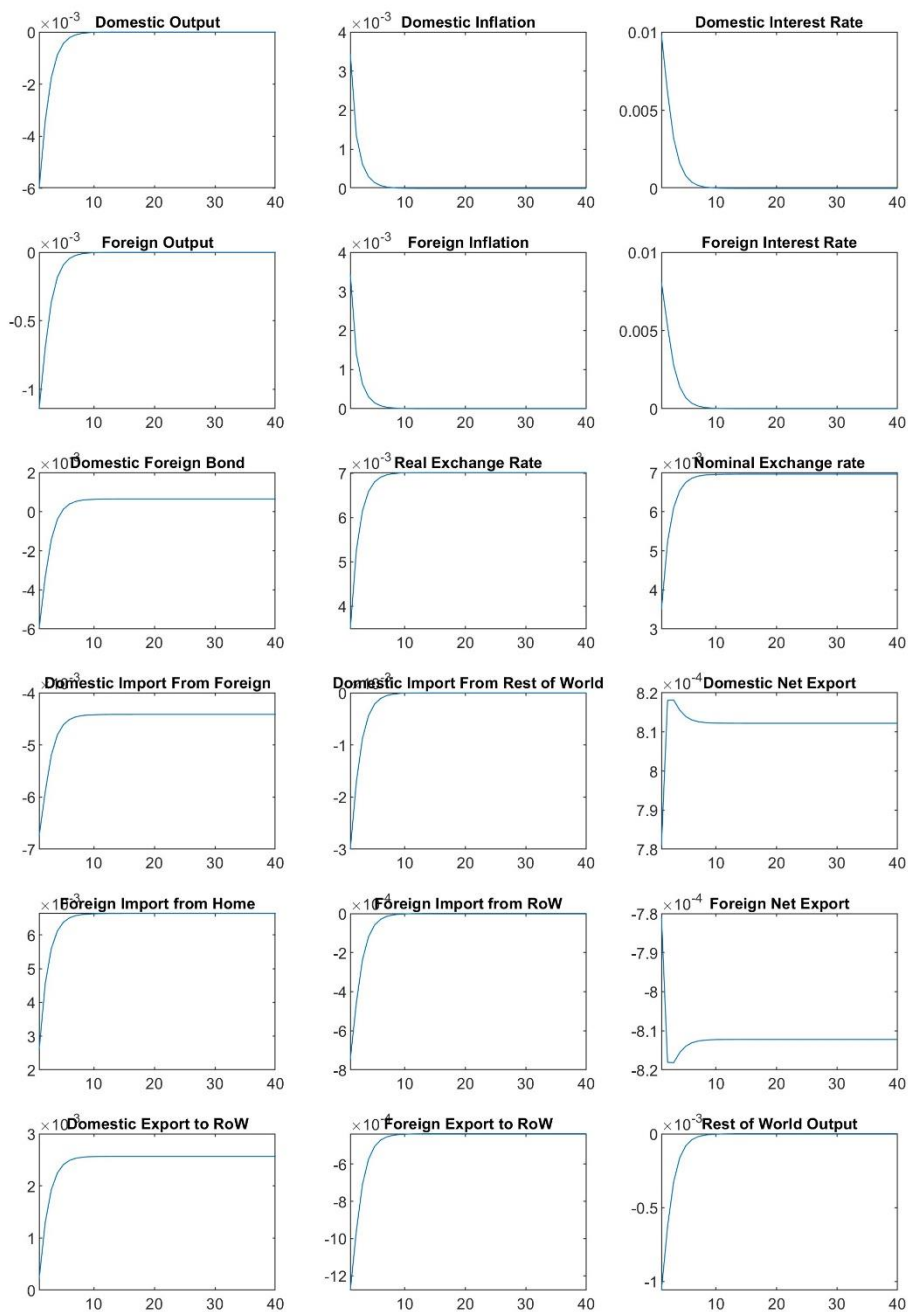
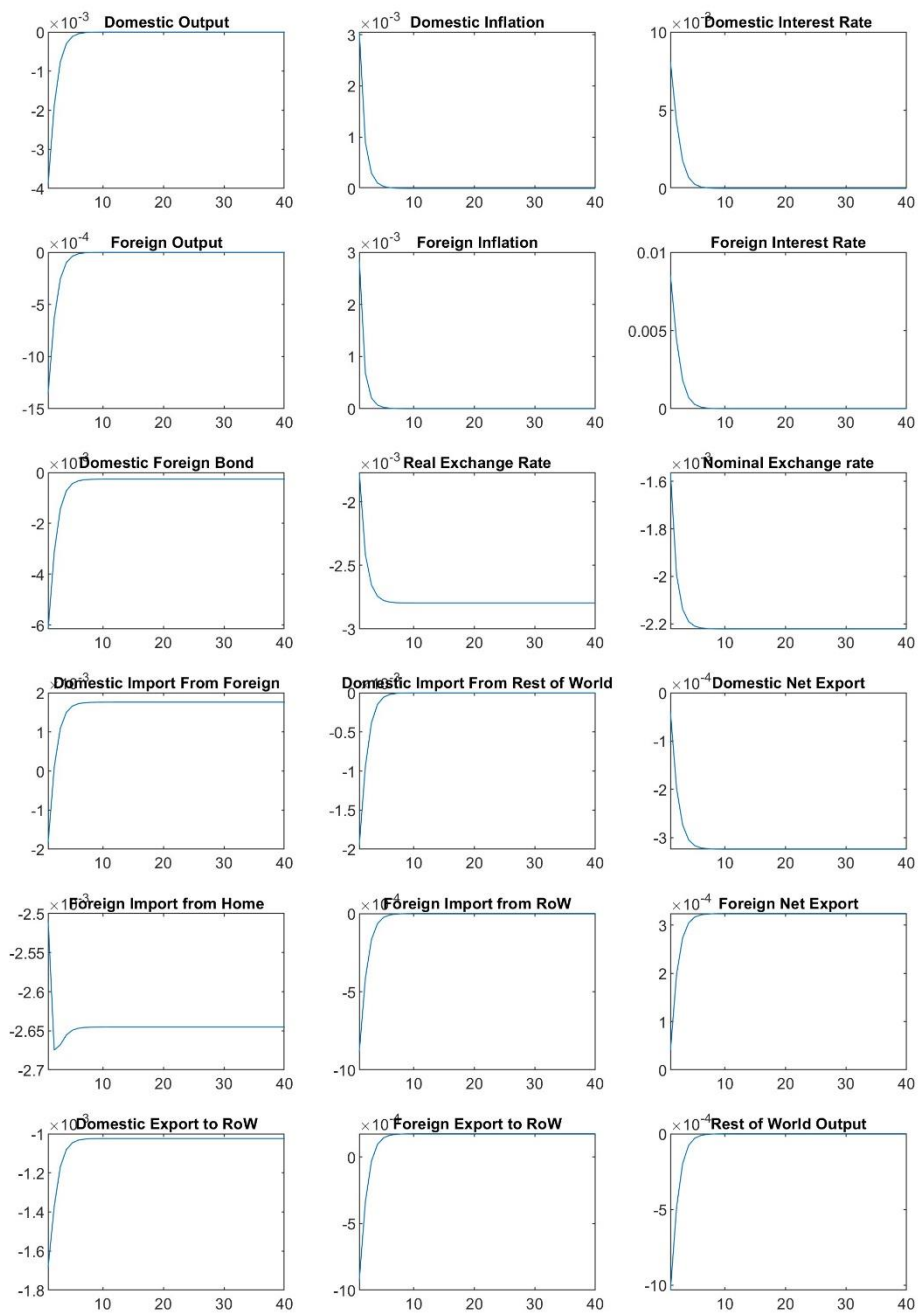




Figure 5. 6 Impulse responses to a foreign supply shock



### 5.5.3 Monetary policy shock

Figure 5.7 shows the IRFs of a domestic contractionary monetary policy shock. Despite being a contractionary shock, the domestic nominal interest rates decrease. This decrease in nominal interest rates, when combined with a larger reduction in domestic inflation, leads to an increase in real interest rates. The unexpected behaviour of nominal interest rates in the context of the Taylor rule can be attributed to the strong response of domestic central banks to the larger output gap and lower inflation. This contractionary monetary policy shock discourages demand.

The real exchange rates react positively to this contractionary shock while the nominal exchange rates respond negatively to the shock since the domestic real interest rates increase more than the foreign real interest while domestic nominal interest rates decrease when the foreign nominal interest rate increases.

In the foreign economy, output declines due to a combination of reduced domestic imports and an increase in domestic exports, the latter being more competitively priced as real exchange rate appreciates in terms of US dollars. The increase in foreign inflation prompts the foreign monetary authority to respond by raising interest rates.

The effects of a foreign monetary shock are analogous so both shocks lead to a decrease in the rest of world output.

Figure 5. 7 Impulse responses to a domestic monetary shock

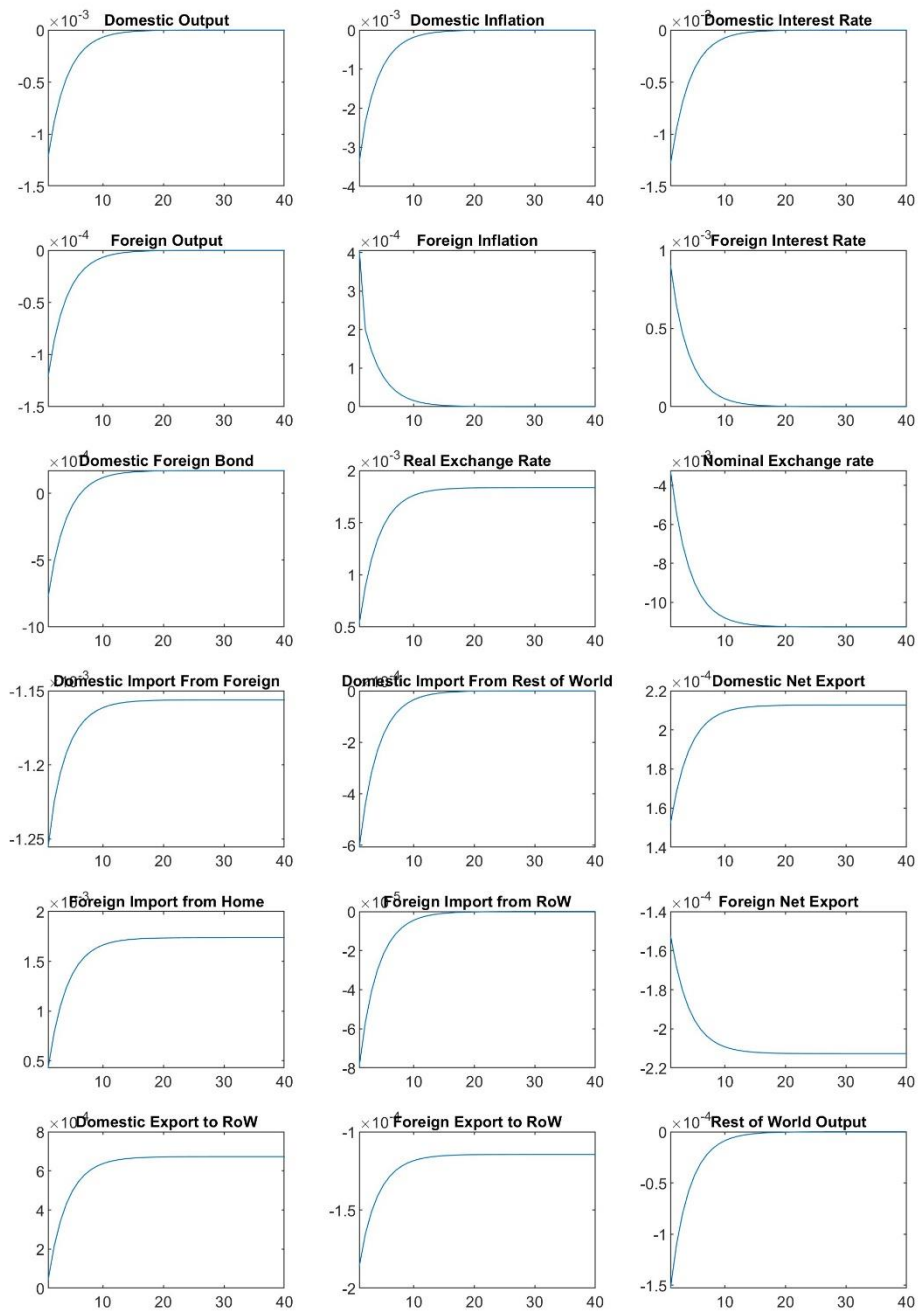
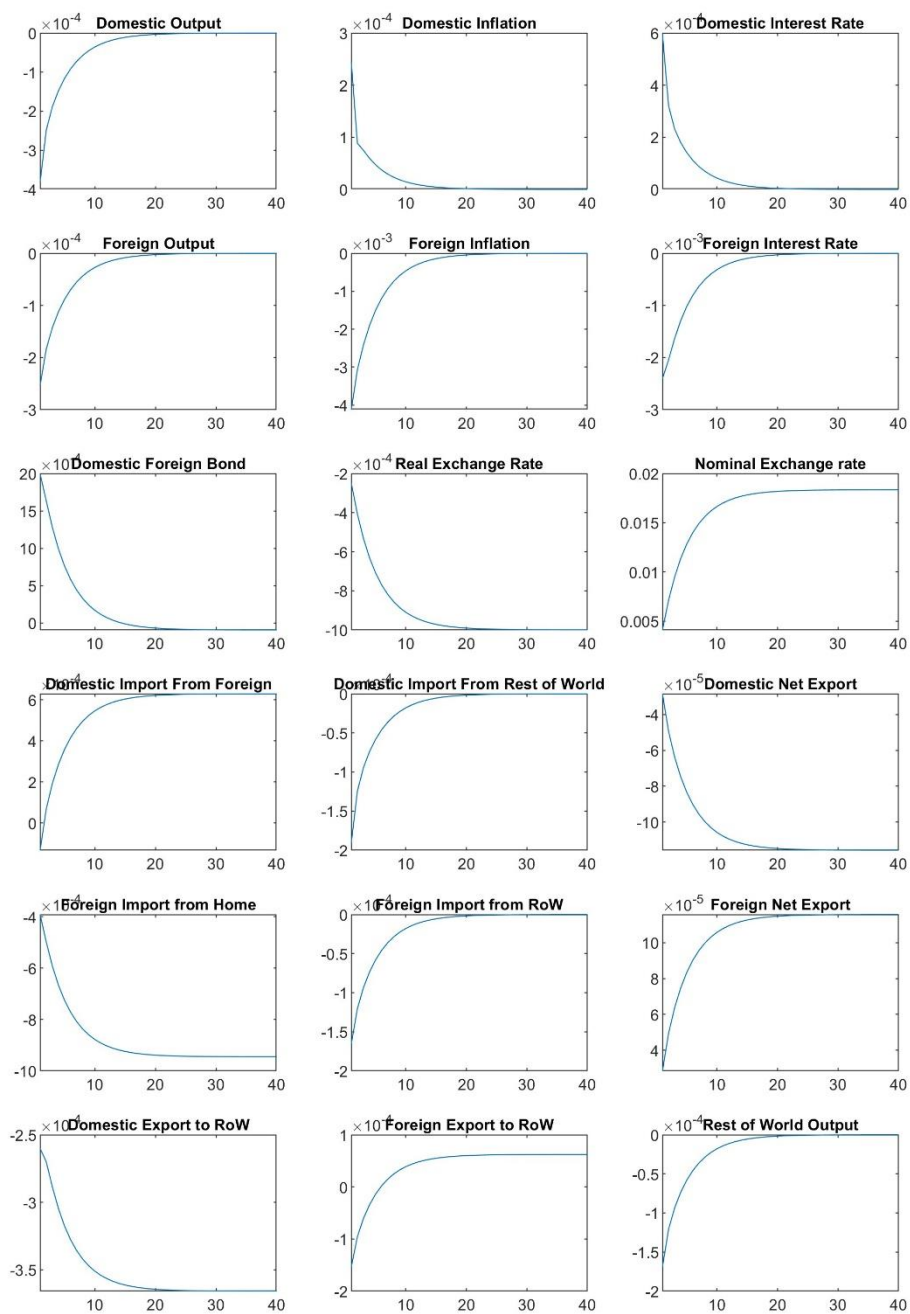


Figure 5. 8 Impulse responses to a foreign monetary shock



#### **5.5.4 Productivity shock**

Figure 5.9 demonstrates the effect of a permanent domestic productivity shock. This surge in productivity boosts output, thereby elevating the income of domestic households. Anticipating this higher income, households increase their consumption, which is initially over the contemporaneous supply at the start of the shock. This mismatch leads to a spike in inflation, which reaches a peak before gradually declining as supply catches up. The central bank reacts to this increasing inflation by raising nominal interest rates, which consequently also increases the real interest rate.

Foreign output rises slightly initially due to increased export to the domestic country. However, its output experiences a downturn later because it imports from domestic country as the goods become relatively cheaper.

The depreciation of nominal exchange rate and real exchange rate boost domestic exports which in turn boosts domestic output but suppress foreign output. Nam and Wang (2015) find the real exchange rate only appreciates slightly then raises persistently while Klein and Linnemann (2021) find the real exchange rate depreciates persistently after initial appreciation.

The reactions to a foreign productivity shock are similar, though in this case, domestic output does not exhibit an initial increase.

Figure 5. 9 Impulse responses to a domestic productivity shock

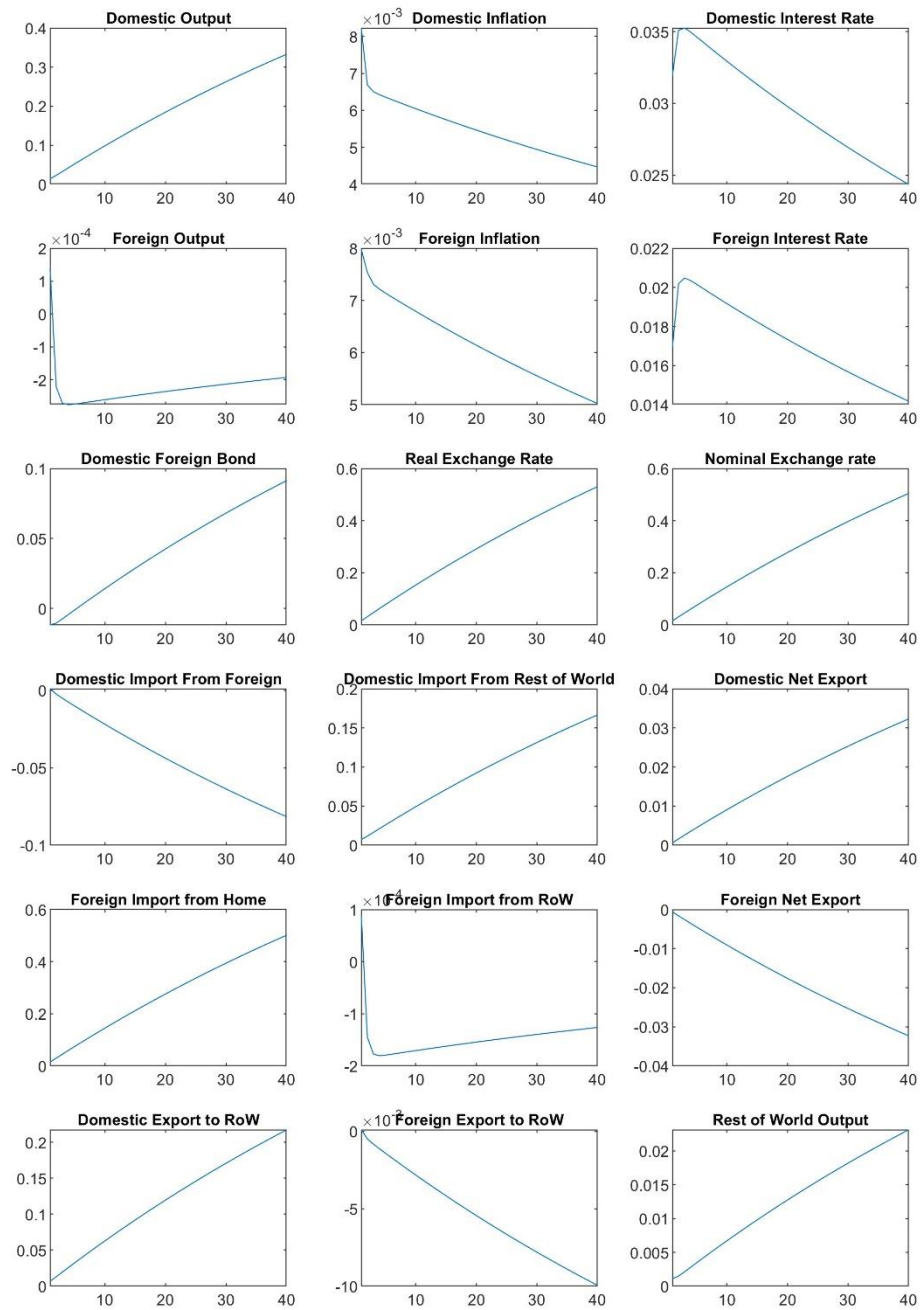
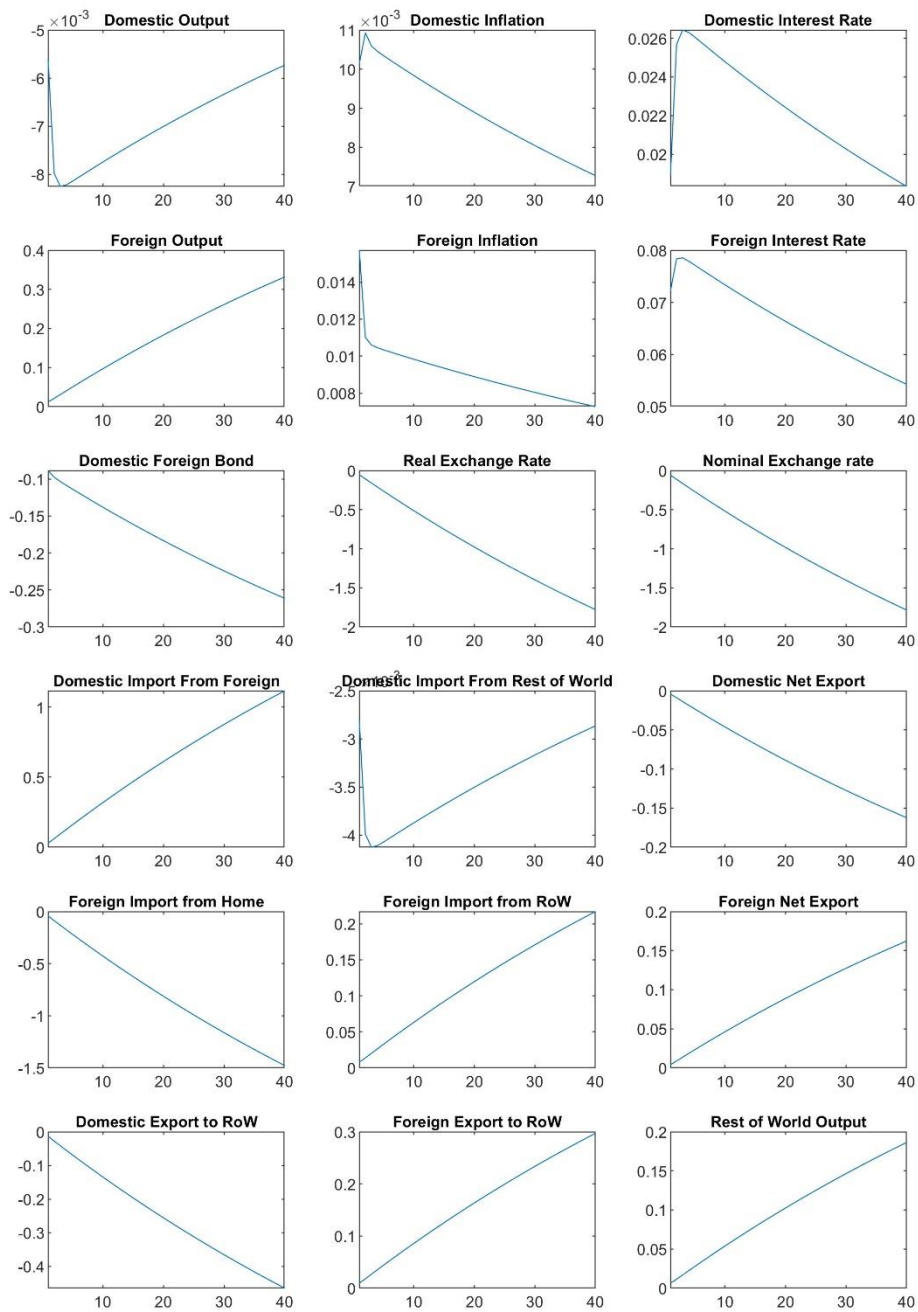


Figure 5. 10 Impulse responses to a foreign productivity shock



## 5.6 Variance decomposition

In this section, I attempt to answer the question of what the main drivers of domestic and foreign output are, inflation, nominal interest rate and real exchange rate based on the model estimation.

The purpose of variance decomposition is to assess the impact of individual shocks on specific variables of interest. Due to the non-stationary productivity shocks, the conventional method of computing variance decomposition as the sum of squared impulse functions is not applicable here. The way to decompose the variance is using the estimated parameters from the estimated model to compute the variation. The process involves the following steps: First, obtain residuals and innovations from the estimated model. Second, implementing bootstrapping for each shock, generating simulations of endogenous variables with other shocks treated as zero in each bootstrapping. Then compute the variances of the simulations, which provides a measure of each shock's contribution. Last, calculate the proportion, which represents the ratio of each shock's variance to the overall variance. The overall variance is the sum of the variances obtained from bootstrapping all structural shocks.

Table 5.5 reports the details of variance composition for short, medium and long-run time horizons. In the short term (one year ahead), domestic output is primarily influenced by domestic productivity shock, accounting for 41% of the variance. Domestic monetary policy shock can explain 26% of the variation. Domestic demand and supply shocks also play significant roles, contributing 16% and 15%, respectively. Foreign shocks, however, have a minimal impact on domestic output. Interestingly, foreign output is predominantly affected by domestic supply shocks, which account for 36% of its variation. This impact is even more pronounced than that on the home country's output. Domestic monetary shocks also influence foreign output in a similar manner. On the



other hand, foreign productivity shocks appear to have a limited effect on foreign output in the short term. Not surprisingly, due to the non-stationary productivity shock, both domestic output and foreign output are dominated by own productivity shock in longer periods. In 5 years later, domestic productivity shock accounts for 97% of domestic output and foreign productivity shock contributes 97% to foreign output as well. In the long run (10 years), domestic output is almost dominated by domestic productivity shock, and the same condition applies to foreign output, whose variance is explained by foreign productivity shock.

Another interesting point emerges from the variance decomposition analysis of inflation. Regardless of the time span considered, domestic inflation is overwhelmingly dominated by domestic monetary policy shocks, accounting for more than 92% of its variation. While a opposite trend is observed in the foreign country, where foreign monetary policy shocks explain 62% to 78% of inflation variation, it's important to note that domestic shocks also have spillover effects in this context.

The variance decompositions of the real and nominal exchange rates exhibit distinct patterns. In the short run, the nominal exchange rate is predominantly influenced by domestic monetary policy shocks (74%) and foreign monetary policy shocks (18%), with other shocks playing a negligible role. However, when examining the real exchange rate, domestic demand shock and domestic supply shock account for 14% and 18% of the variation in the short run, respectively, while the impact of domestic monetary shock is considerably reduced. Additionally, domestic productivity shock begins to exert an influence while foreign productivity shock does a little. In the medium and long term, monetary shocks continue to be the primary drivers of nominal exchange rate variations, with domestic productivity shocks having a more substantial impact. As for the real exchange rate, the effects of domestic monetary shocks

diminish significantly, while the contribution of domestic demand and supply shock significantly decreases, dropping from 22% and 26% to 7% and 2.5% and then to 2% and less than 1%. The productivity shock, which is a real shock, gains greater importance in explaining the variability of the real exchange rate over these longer periods, even though domestic shocks maintain a dominant role over foreign shocks.

Table 5. 5 Variance decomposition

1 year	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	15.87	0.39	20.27	4.26	4.88	21.78	13.64	0.46
Domestic supply	14.96	2.09	25.53	35.65	7.37	25.98	17.85	1.85
Domestic monetary	25.78	95.86	17.23	15.62	3.66	11.82	18.60	74.20
Domestic productivity	40.72	1.11	31.65	0.17	4.52	14.95	42.69	4.19
Foreign demand	1.09	0.15	1.43	3.73	0.22	2.79	1.41	0.06
Foreign supply	1.24	0.33	3.36	9.31	0.99	5.39	0.77	0.05
Foreign monetary	0.31	0.05	0.38	10.14	78.24	15.11	0.53	18.68
Foreign productivity	0.03	0.02	0.15	21.12	0.11	2.17	4.51	0.50
Total	100	100	100	100	100	100	100	100
5 years	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	0.93	0.40	15.49	0.26	7.18	20.73	6.90	0.57
Domestic supply	0.50	1.26	10.72	1.17	6.30	13.61	2.53	0.63
Domestic monetary	1.43	95.41	11.93	0.84	5.01	10.14	6.95	67.40
Domestic productivity	97.07	2.58	59.71	0.03	15.47	35.31	70.03	15.97
Foreign demand	0.03	0.07	0.50	0.10	0.15	1.21	0.24	0.02
Foreign supply	0.03	0.14	0.96	0.21	0.57	1.92	0.06	0.01
Foreign monetary	0.01	0.02	0.15	0.32	64.69	7.80	0.13	12.04
Foreign productivity	0.01	0.11	0.54	97.08	0.63	9.30	13.15	3.36
Total	100	100	100	100	100	100	100	100
10 years	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	0.12	0.39	9.40	0.05	5.19	14.38	2.38	0.40
Domestic supply	0.06	1.22	6.54	0.21	4.58	9.49	0.76	0.39
Domestic monetary	0.18	92.83	7.28	0.15	3.65	7.07	2.27	44.46
Domestic productivity	99.62	5.08	74.93	0.01	22.17	48.94	83.23	37.82
Foreign demand	0.00	0.08	0.32	0.02	0.12	0.89	0.08	0.02
Foreign supply	0.01	0.21	0.93	0.06	0.67	2.14	0.03	0.01
Foreign monetary	0.00	0.03	0.12	0.08	62.93	7.26	0.06	11.23
Foreign productivity	0.00	0.15	0.48	99.44	0.69	9.83	11.19	5.68
Total	100	100	100	100	100	100	100	100

## 5.7 Historical decomposition

In this section, I focus on the historical decompositions, which highlight the contributions of each shock to the volatility of real exchange rate for the period from 2005Q3 to 2021Q4.

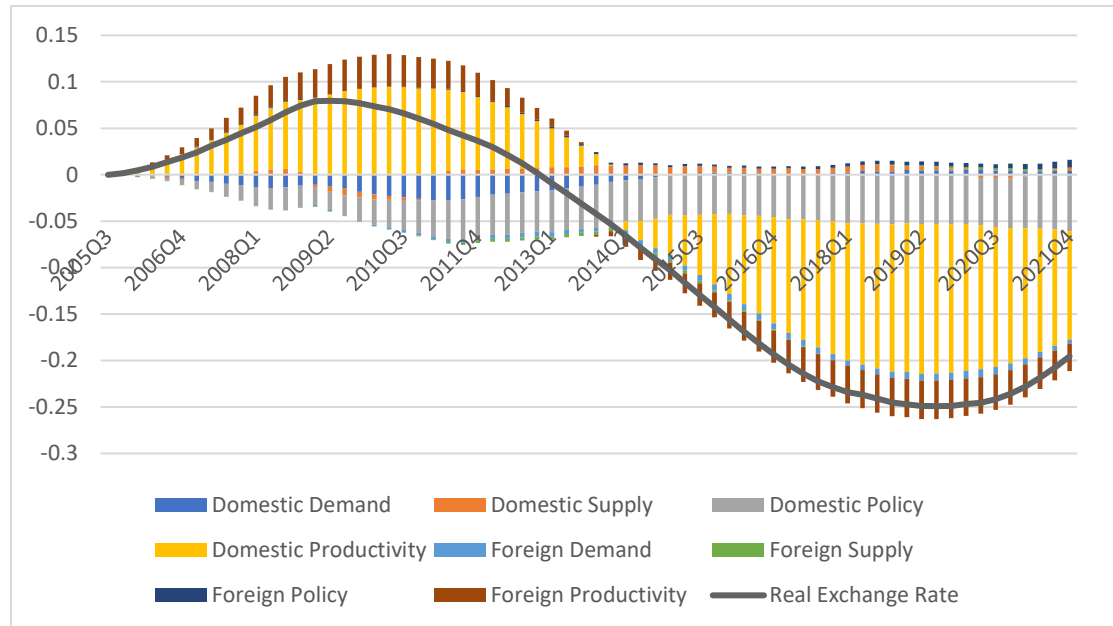
The method of calculating historical decomposition is similar to the way used in variance decomposition. I use the actual residuals and innovations from the estimated structural model to conduct the simulation. The distinction is using the complete sample of one shock while setting all other shocks to zero when assessing the contribution of that specific shock. The procedure is: first, obtain residuals and innovations from the estimated structural model. Second, initiate with the initial value of each variable and accumulate the variable by incorporating the actual shock at each period. This ensures that the value of the contribution at each time is based on the previous contribution plus the shock from the current period. Third, repeat the process outlined in step 2 for all shocks, and the cumulative effect is the sum of all contributions from all shocks at each period.

Figure 5.11 displays the impact of various shocks on the fluctuations of the real exchange rate, with the vertical axis indicating their respective contributions to the movement of the real exchange rate. Notably, domestic productivity shocks contributed to fluctuations in the real exchange rate most. Foreign productivity shocks exert a similar, albeit smaller, influence on the movements of the real exchange. The non-stationary productivity shocks predominantly drive the depreciation and appreciation of the real exchange rate.

Another interesting point is that China's monetary policy appears to consistently contribute to the appreciation of the real exchange rate throughout the observed period. The financial crisis of 2008, however,

temporarily slows down this trend of depreciation. During the Covid-19 pandemic, the appreciation of the real exchange rate is further moderated, as the impacts of productivity shocks dampen in this period.

Figure 5. 11 Historical decomposition of real exchange rate



## 5.8 Conclusion

This chapter initially evaluates the calibrated three-country model and concludes that it does not adequately fit the data behaviour when using the Indirect Inference method, as evidenced by the Transformed Wald statistic significantly exceeding the critical value. Subsequently, the model is re-estimated using the Indirect Inference method. The results of this test indicate that the model, with its newly estimated parameters, is not rejected for the sample period from 2005 to 2021 at the 5% significance level.

Further in the chapter, the impulse response functions are analysed to illustrate the transmission mechanisms within the model and how different shocks affect exchange rates and other variables. The analyses of variance decomposition and historical decomposition demonstrate that domestic productivity shock is the primary driver of fluctuations in the real exchange rate across short, medium, and long-term horizons. Monetary policy shocks exhibit greater importance than productivity shocks in explaining the variation in nominal exchange rate.

# Chapter 6 Model with News Shocks

## 6.1 Introduction

In the previous chapter, the benchmark model with eight domestic and foreign shocks is found to catch the data behaviour well using Indirect Inference method. In this chapter, news shocks are incorporated into the benchmark model to investigate whether news shocks could improve the model's ability to fit the data. A strand of the empirical literature has investigated the news effects of different shocks. Beaudry and Portier (2006), Jaimovich and Rebelo (2009) and Fujiwara et al. (2011) examine TFP news shocks; Schmitt-Grohe and Uribe (2012) test wage markups news shocks and government spending news shocks; Milani and Treadwell (2012) and Chen and Zhang (2015) investigate monetary policy news shocks. Thus, in this chapter, all of the eight shocks comprise both unanticipated contemporaneous shocks and anticipated news shocks.

This chapter is structured as follows: the introduction of news shocks is presented in section 6.2. The next section proposes the signal extraction process which is ignored in the literature and shows how it is incorporated into the model. Section 6.4 outlines the estimation results and compares it with the results in the base model. Then the following three sections show the impulse responses functions, variance decomposition and historical decomposition analysis. Finally, I conclude the news shock model in section 6.8.

## 6.2 News shocks

This three-country model is driven by eight exogenous forces: two non-stationary productivity shocks and six stationary shocks. These shocks are assumed to be subject to anticipated and unanticipated innovations in current setting. The anticipated innovations are formulated to be perceived four quarters ahead. The choice of a four-quarter anticipation period for specifying

the model is based on two key considerations: Firstly, a number of influential studies, including those by Fujiwara et al. (2011), Schmitt-Grohé and Uribe (2012), and Born et al. (2013), Nam and Wang (2015), Görtz and Tsoukalas (2017), have proposed or employed this duration. Secondly, following Le et al. (2020), the process of signal extraction is able to identify one news shock signal restriction clearly.

As proposed by Le et al. (2020), news shocks have a certain relationship with future shocks based on the signal extraction process. In the most literature, the structure of news is modelled in the following way where  $X$  is a series that contains both anticipated and unanticipated components:

$$X_t = \rho X_{t-1} + \epsilon_t + \epsilon_{t-4}^4 \quad (6.1)$$

where  $\epsilon_t$  is the unanticipated shock and  $\epsilon_{t-4}^4$  is the anticipated component, the news shock, which is known 4 periods ago and hits the economy at time period  $t$ , and  $\rho$  is the autocorrelation coefficient. The future shock is denoted by  $u_{t+1} = \epsilon_t + \epsilon_{t-4}^4$  so agents can learn information 4 quarters in advance. Anticipated shocks are observed in advance and expected prior to their materialization, while unanticipated shocks occur unexpected contemporaneously. Here we assume that shocks have mean 0, standard deviation  $\sigma_i^j$ ,  $Cov(\epsilon_t, \epsilon_{t-4}^4) = 0$ , and  $Cov(\epsilon_{m,t-4}^4, \epsilon_{n,t-4}^4) = 0, m \neq n$ . The previous assumptions indicate the anticipated and unexpected elements are uncorrelated across type, period, and time.

### 6.3 Signal extraction process

When a news shock is observed by agents, they would response to the shock with their knowledge to infer a statistical relationship. This rational behaviour enables agents to react to anticipated shocks even before they are realized. Minford and Peel (2019) call this procedure as signal extraction. In this process we assume that news shock has a direct connection to the future



resulting from a public shock only observed by agents but not directly apparent to econometrician<sup>2</sup>. The reaction of agents would be treated as observable error terms in the model by econometrician. To illustrate the signal extraction process, we use the productivity shock as the example. This method can be used to study other shocks from news or future.

Following Le et al. (2020), agents can exactly perceive what would happen in the future through their observations about R&D programmes or know the future with some random error. Thus, we assume that agents know what the shock will be in the forthcoming by having a statistical relationship between R&D and effects of the shock.

Before we move to the signal extraction regression, we need to clarify the elements in the signal. R&D spending is the experimental costs that firms spend on developing existing or new products and services, which is the source of future productivity,  $u_{t+1}$ . There is other unrelated experimental spending,  $v_{it}$ , that goes into R&D spending. With normalising coefficient of  $u_{t+1}$  to 1, we assume that the econometricians have the regression:

$$RD_t = u_{t+1} + v_{it} \quad (6.2)$$

For the agents who carrying out signal extraction, they establish a regression related to R&D and  $u_{t+1}$ :

$$u_{t+1} = \tau RD_t + \varepsilon_{t+1} \quad (6.3)$$

As the agents are impossible to have full information on the future, their rational expectation of  $u_{t+1}$ , is  $E_t u_{t+1}$ :

---

<sup>2</sup> Econometricians can identify shocks by collecting data on various factors, such as monetary announcements, technological developments, or financial events. However, in this context, we do not require econometricians to undertake such data collection. Instead, we rely solely on macroeconomic data for the estimation of shocks.

$$E_t u_{t+1} = \tau RD_t \quad (6.4)$$

Thus, the agents expect the next period productivity,  $u_{t+1}$ , would be  $\tau RD_t$ , where  $\tau$  is the parameter known by the agents.  $u_{t+1}$  will be the shock that comprises news shock.

However, the econometricians do not have data for R&D so that they are not able to model news shock directly. They can only use present and past values of the productivity shock that they observe to find out news shock. There are three points that the econometricians know. The first is that agents expect news shock with information of  $RD_t$ . The next is that they learn that  $RD_t = u_{t+1} + v_{it}$ . The last is they know that how the agents derive the expected future productivity by using optimal signal extraction process. The following is how the econometricians derive the process.

The econometricians believe that agents who use the signal extraction process have learnt the value of coefficient,  $\tau = \frac{cov(RD,u)}{var(RD)}$ . They can deduce a opposite relationship between  $RD_t$  and  $u_t$ , and also the coefficient of  $u_{t+1}$  being  $\frac{cov(RD,u)}{var(u)} = \tau \frac{var(RD)}{var(u)}$  and the regression error is  $w_t$ . Thus, they have

$$RD_t = \tau \frac{var(RD)}{var(u)} u_{t+1} + w_t \quad (6.5)$$

The previous equation could be estimated by agents who have data for R&D although it does not help agents understand the productivity better.

The econometricians postulate one period ahead news shock has a correlation with future productivity by this form:

$$\varepsilon_t^1 = \varpi u_{t+1} + \epsilon_t \quad (6.6)$$

The reason why the econometricians use this kind of form is that they know the two regressions agents have indicate a relationship between what agents expect to happen and the future productivity that would occur. Neither of these

two relationships are actually known to econometricians since they could not obtain R&D data, but they do understand how to derive them. In this way, they can determine the relationship between future productivity and the news shock. The following shows how this relationship is derived:

$$u_{t+1} = \varepsilon_{t+1} + \varepsilon_t^1 \quad (6.7)$$

Take expectation at time  $t + 1$ ,

$$E_t u_{t+1} = \varepsilon_t^1 = \tau RD_t = \tau^2 \frac{\text{var}(RD)}{\text{var}(u)} u_{t+1} + \tau w_t = \varpi u_{t+1} + \varepsilon_t \quad (6.8)$$

where  $\varpi = \tau^2 \frac{\text{var}(RD)}{\text{var}(u)}$  and  $\varepsilon_t = \tau w_t$ .

Thus, a one period ahead news shock is partly linked to future productivity, which is measured by  $\varpi$ , and partly unrelated to it as explained by error term,  $\varepsilon_t$ . We could simplify  $\varpi$  by

$$\varpi = \tau^2 \frac{\text{var}(RD)}{\text{var}(u)} = \left\{ \frac{\text{cov}(RD, u)}{\text{var}(RD)} \right\}^2 \frac{\text{var}(RD)}{\text{var}(u)} = \frac{\text{cov}(RD, u)^2}{\text{var}(RD)\text{var}(u)} \quad (6.9)$$

As  $\varpi$  indicates the correlation between future productivity and R&D and it is derived from the signal extraction process, we denote it as the signal extraction parameter. Since the econometrician do not have access to data on R&D or productivity shocks, they cannot regress the above equation to get the estimated parameters  $\varpi$  or  $\tau$ . They can, however, find out the size of the variances of these two elements based on the signal extraction parameter,  $\varpi$ . Under the condition that they know the signal extraction process are useful, they could determine the variances of two unknown stochastic variables,  $\varepsilon_t^1$  and  $\varepsilon_t$ , which are both crucial parts of the dynamic model. They would use indirect inference method to estimate  $\varpi$  indirectly.

To work out the variance sizes, the econometricians use equation (6.5) to calculate the variances related to  $RD_t$ . The explained variable of  $RD_t$  is productivity,  $u_t$ , so the explained variance is given by

$$\left[ \tau \frac{\text{var}(RD)}{\text{var}(u)} \right]^2 \text{var}(u) = \tau^2 \left[ \frac{\text{var}(RD)}{\text{var}(u)} \right] \text{var}(RD) \quad (6.10)$$

The unexplained part goes into the error term,  $w_t$ . Thus, the unexplained variance of  $RD_t$  is

$$\begin{aligned} \text{var}(w) &= \left\{ 1 - \tau^2 \left[ \frac{\text{var}(RD)}{\text{var}(u)} \right] \right\} \text{var}(RD) \\ &= (1 - \varpi) \left[ \frac{\text{var}(RD)}{\text{var}(u)} \right] \text{var}(u) \end{aligned} \quad (6.11)$$

Therefore, the variance of the unexplained part of  $\varepsilon_t^1, \epsilon_t$  is

$$\begin{aligned} \text{var}(\epsilon) &= \text{var}(\tau w) \\ &= \tau^2 \text{var}(w) \\ &= \tau^2 (1 - \varpi) \left[ \frac{\text{var}(RD)}{\text{var}(u)} \right] \text{var}(u) \\ &= \varpi (1 - \varpi) \text{var}(u) \end{aligned} \quad (6.12)$$

Hence the variance of the news shock,  $\varepsilon_t^1$  is given by

$$\begin{aligned} \text{var}(\varepsilon_t^1) &= \varpi^2 \text{var}(u) + \varpi (1 - \varpi) \text{var}(u) \\ &= \varpi \text{var}(u) \end{aligned} \quad (6.13)$$

It shows that the variance of the news shock is strictly correlated with the variance of productivity. When  $\varpi = \tau^2 \frac{\text{var}(RD)}{\text{var}(u)} = 0$ , this indicates that  $\text{var}(RD)$  have to be zero, which means there is no news for R&D spending, thus, there is no news and news shock has no variance. When  $\varpi = 1$  the variance of news shock is the same as the variance of future productivity, the news shock is equal to  $u_{t+1}$  and it does not have extra error term. Therefore, under rational expectation, we need to apply the restriction on the variance of the news shock. In general case, the value of  $\varpi$  should be between 0 and 1 as agents know only a part of full information of R&D. The news shock consists

of a future event,  $\varpi u_{t+1}$  and a random draw,  $\epsilon_t$ . The key to this derivation is that the variance of random error term,  $\epsilon_t$ , is strictly constrained by the variance of the future shock and the signal extraction parameter,  $\varpi$ . This restriction has been ignored in the rational expectations modelling literature.

We apply this process to other shocks. The structures for news shocks which is anticipated four periods ahead in the model are given by

Domestic demand shock

$$\epsilon_t^{IS} = \rho_{IS} \epsilon_{t-1}^{IS} + \epsilon_t^{IS} + \epsilon_{t-4}^{IS,4} \quad (6.14)$$

Domestic productivity shock

$$\epsilon_t^{yp} = \epsilon_t^{yp} + \epsilon_{t-4}^{yp,4} \quad (6.15)$$

Domestic supply shock

$$\epsilon_t^{PP} = \rho_{PP} \epsilon_{t-1}^{PP} + \epsilon_t^{PP} + \epsilon_{t-4}^{PP,4} \quad (6.16)$$

Domestic monetary policy shock

$$\epsilon_t^R = \rho_R \epsilon_{t-1}^R + \epsilon_t^R + \epsilon_{t-4}^{R,4} \quad (6.17)$$

Foreign demand shock

$$\epsilon_t^{IS^*} = \rho_{IS^*} \epsilon_{t-1}^{IS^*} + \epsilon_t^{IS^*} + \epsilon_{t-4}^{IS^*,4} \quad (6.18)$$

Foreign productivity shock

$$\epsilon_t^{yp^*} = \epsilon_{t-4}^{yp^*,4} \quad (6.19)$$

Foreign supply shock

$$\epsilon_t^{PP^*} = \rho_{PP^*} \epsilon_{t-1}^{PP^*} + \epsilon_t^{PP^*} + \epsilon_{t-4}^{PP^*,4} \quad (6.20)$$

Foreign monetary policy shock

$$\epsilon_t^{R^*} = \rho_{R^*} \epsilon_{t-1}^{R^*} + \epsilon_t^{R^*} + \epsilon_{t-4}^{R^*,4} \quad (6.21)$$

## 6.4 Indirect Inference estimation with news shocks

### 6.4.1 Estimation with perfect foresight

In chapter 5, we evaluate the performance of the base model without news and find it can fit the data well, as evidenced by a Trans Wald statistic of 1.1798. In this section, we present the estimation results of the three-country model incorporating news shocks. The data set is the same as the base model. Within this news shock model, the signal extraction parameter,  $\varpi$ , is set to be 1, which denotes the perfect foresight scenario. The final two rows of Table 6.1 display the Wald test results for the news model: one using coefficients estimated by the base model, and the other with coefficients that have been re-estimated specifically for the news model. If the news model is calibrated with the parameters estimated by the base model, then it cannot pass the Wald test as the Trans Wald statistic is 2.5394, which is larger than critical value, 1.645. However, when we re-estimate the news model, it can explain the movements of data well since the Trans Wald statistic is 1.3988 and p-value is 0.0650.

What's more, Table 6.1 illustrates two sets of estimated parameters of two models. The findings reveal notable deviations in these newly estimated values from the previous estimates. The estimated degree of openness,  $\alpha$ , is found to be lower than that in the base model. For the domestic country, firms have lower persistence in pricing as the value estimated is 0.7936. The inverse of consumption elasticity,  $\sigma$ , is estimated to be 1.8188 which is over the previous estimation, 1.2341. This indicates that domestic households are less sensitive to the changes in the real interest rate when they have more information. However, the inverse of labour elasticity,  $\varphi$ , has increased from 3.3654 to 5.8005, implying that the labour supply becomes less elastic.

In contrast to the import behaviour in the base model, the domestic country has both higher income elasticity of home import from the foreign country and higher income elasticity from the rest of world, as the values increasing from 0.7572 to 0.8833 and increasing from 0.5001 to 0.8156 respectively. The exchange rate elasticity of home import from foreign country,  $\psi$ , decreases from 0.6294 to 0.5919, showing less import if the real exchange rate appreciates.

Regarding monetary policy, the inertia,  $\rho$ , measured at 0.2127, is lower than the previously estimated value of 0.2686. The responsiveness of interest rates to inflation,  $\phi_\pi$ , decreases from 4.4477 to 3.5484. Additionally, the response of interest rates to the output gap,  $\phi_y$ , also shows a downward trend, decreasing from 0.5032 to 0.2377. However, there is an increase in the response of interest rates to exchange rate movements,  $\phi_q$ , with the value rising from 0.2258 to 0.3972.

On the foreign side, the probability of non-adjusting price is found to be 0.7980, which is higher than the previous estimation. Higher sensitivity of real interest rate movements and larger sensitivity of wage rate changes are revealed in the news model. Unlike the home country, the foreign country has lower income elasticity of foreign import from home and higher elasticity of foreign import from the rest of the world.

Turn to the foreign monetary policy, the estimated inertia shows a slight decline from 0.2859 to 0.2127. This figure aligns more closely with findings reported by Minford et al. (2021). The response of interest rates to inflation,  $\phi_\pi^*$ , is considerably lower than the previous estimate, with the current value being 3.5484. The response to the output gap exhibits a significant decrease, moving from 0.8308 to 0.3945. Similarly, the response of interest rates to exchange rate changes is estimated to be a smaller value of 0.2839. The response to output gap and to real exchange rate are higher than the

estimates in Minford et al. (2021) while the response to inflation is found to be higher.

Regarding the rest of the world, the exchange rate elasticity of its imports from China relative to the US is observed to be quite elastic, with a value of 1.3734, which is greater than 1. This implies that when the real exchange rate depreciates, goods made in China become cheaper relative to those made in the US. Consequently, the rest of the world tends to increase its imports by 1.37% from China when the depreciation rate is 1%.

Table 6. 1 Estimated coefficients and Wald test results

Parameter	Description	Calibration from base estimation	Estimated model
Home			
$\alpha$	Degree of openness	0.5916	0.5361
$\theta$	Calvo-non-adjusting probability	0.7957	0.7936
$\sigma$	Inverse of consumption elasticity	1.2341	1.8188
$\varphi$	Inverse of labour elasticity	3.3654	5.8005
$\mu$	Income elasticity of home import from foreign	0.7572	0.8833
$\nu$	Income elasticity of home import from RoW	0.5001	0.8156
$\psi$	Exchange rate elasticity of home import from foreign	0.6294	0.5919
$\rho$	Monetary policy inertia	0.2686	0.2127
$\phi_{\pi}$	Monetary policy response to inflation	4.4477	3.5484
$\phi_y$	Monetary policy response to	0.5032	0.2377



	output gap		
$\phi_q$	Monetary policy response to exchange rate	0.2258	0.3972
<hr/>			
Foreign			
<hr/>			
$\alpha^*$	Degree of openness	0.5916	0.5361
$\theta^*$	Calvo-non-adjusting probability	0.7014	0.7980
$\sigma^*$	Inverse of consumption elasticity	9.5379	5.3423
$\varphi^*$	Inverse of labour elasticity	9.4529	3.3161
$\mu^*$	Income elasticity of foreign import from home	0.6158	0.5936
$v^*$	Income elasticity of foreign import from RoW	0.6539	0.9102
$\psi^*$	Exchange rate elasticity of foreign import from home	0.9459	0.8217
$\rho^*$	Monetary policy inertia	0.2859	0.2127
$\phi_\pi^*$	Monetary policy response to inflation	4.2006	3.5484
$\phi_y^*$	Monetary policy response to output gap	0.8308	0.3945
$\phi_q^*$	Monetary policy response to exchange rate	0.4492	0.2839
<hr/>			
$\psi^{RoW}$	Exchange rate elasticity of RoW import from China relative to US	0.4286	1.3811
<hr/>			
Trans Wald		2.5394	1.3988
P value		0.0220	0.0650
<hr/>			

### 6.4.2 Tests with imperfect foresight

Up to this point, our emphasis has been on perfect foresight, with the signal extraction parameter,  $\varpi$ , set to 1. The previous finding does not indicate that news shocks enhance the model's capability to fit the data. Consequently, we investigate whether news shocks might exhibit improved performance or reverse under imperfect foresight. Following the approach of Le et al. (2020), we employ the Indirect Inference test to obtain Transformed Wald statistics with varying values of  $\varpi$  while maintaining the same set of parameters estimated by the perfect foresight condition. Table 6.2 shows the Transformed Wald statistic for different values of  $\varpi$ . Notably, all Transformed Wald statistics for signal extraction parameter values less than 1 are higher than the Transformed Wald statistic under perfect foresight. This suggests that news shocks deteriorate the model fit. Given that the fit of the model with perfect foresight is already inferior to the base model, news shocks adversely impact the fit of the three-country model, regardless of whether under perfect or imperfect foresight. The worst fit is at when  $\varpi$  is equal to 0.4 as the Transformed Wald statistic is the highest among these tests.

Since the model with perfect foresight exhibits the best fit among the news shock models, the subsequent analysis will be based on the perfect foresight model.

Table 6. 2 Transformed Wald statistic for different values of  $\varpi$

$\varpi$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Trans										
Wald	1.4693	1.5232	1.5054	1.6744	1.6312	1.4877	1.5328	1.5615	1.5271	1.3988

## 6.5 Impulse response functions

In this section, we discuss impulse responses of news shocks and compare them with impulse responses functions of the benchmark model to see whether news shocks change agent's reaction.

### 6.5.1 Demand news shock

Figure 6.1 illustrates the responses of various macroeconomic variables to a 1% standard deviation domestic demand news shock, which agents anticipate four quarters in advance. During the anticipation period, domestic output increases, until reach its peak at the moment the shock materializes. The anticipated increase in demand also leads to inflationary pressure and a subsequent rise in interest rates, as the central bank responds to the increasing inflation. As a result, this news shock induces a hump-shaped reaction, distinct from the responses to surprise shocks observed in the base model. This distinction is due to the demand shock is already known before its future realization, agents immediately respond to this information. The overall responses are larger in the news model.

Both real and nominal exchange rates begin to depreciate immediately upon anticipation of the demand news when the news on demand is anticipated. The responses to news shocks are larger than the responses in the base model.

The spillover effects of this anticipated domestic shock in this model become evident four periods ahead of the actual realization. These effects are more pronounced than the spillover observed in the base model because steeper IS curve and flatter Phillips curve.

Figure 6.2 displays the responses to a foreign demand news shock. While these reactions are symmetrical to those shown by a domestic demand news shock, as seen in the base model, the intensity of these responses differs

from the base model. Although foreign output generates larger increase when the news is materialized compared to the base, foreign inflation and foreign interest rate spike less than those in base model.

Real exchange rates and nominal exchange rates appreciate less than the appreciation in the base model. The spillover effects are dampened so the responses of domestic variables are smaller.

Figure 6. 1 Impulse responses to a domestic demand news shock (anticipated four quarters ahead)

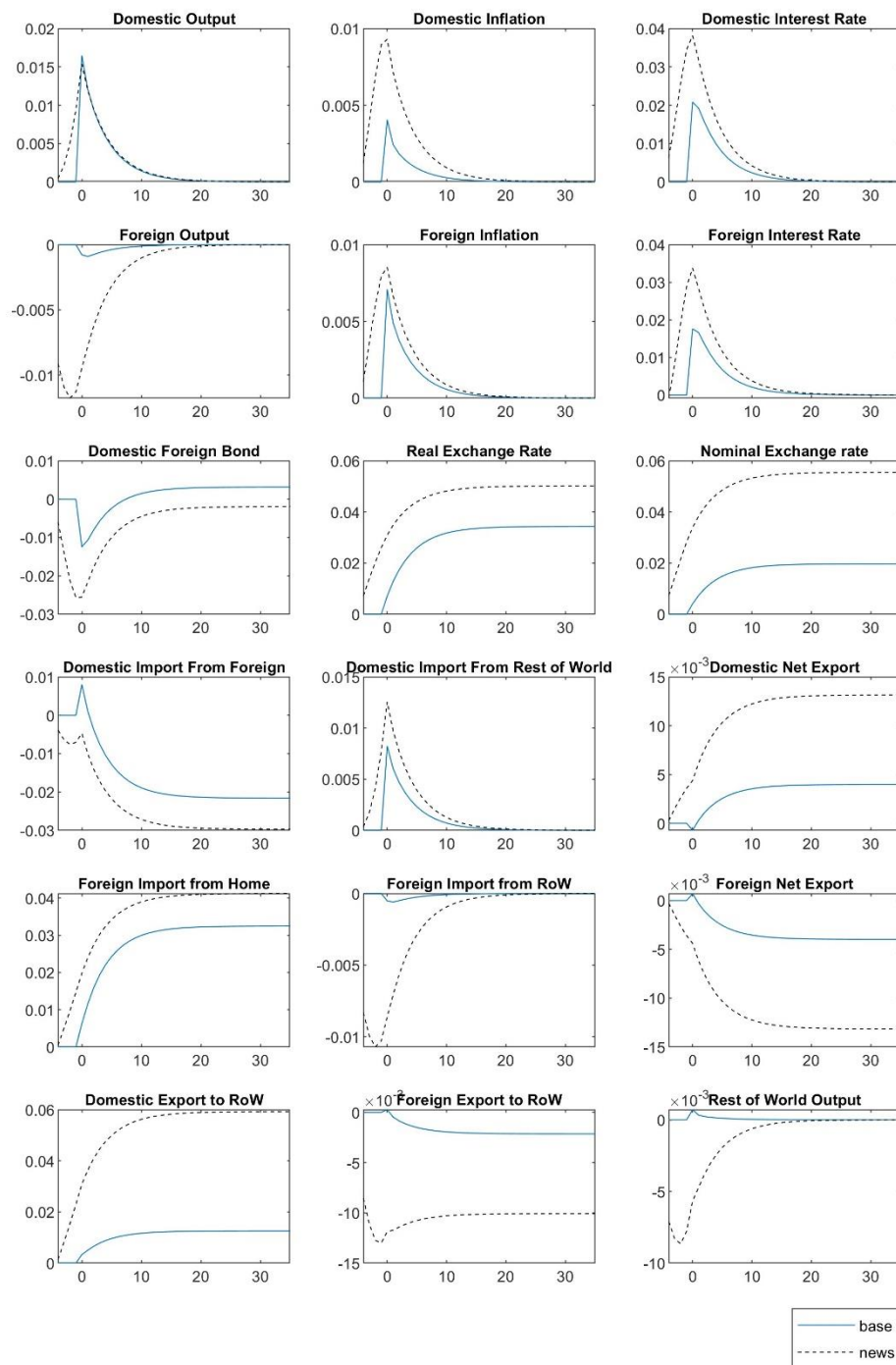
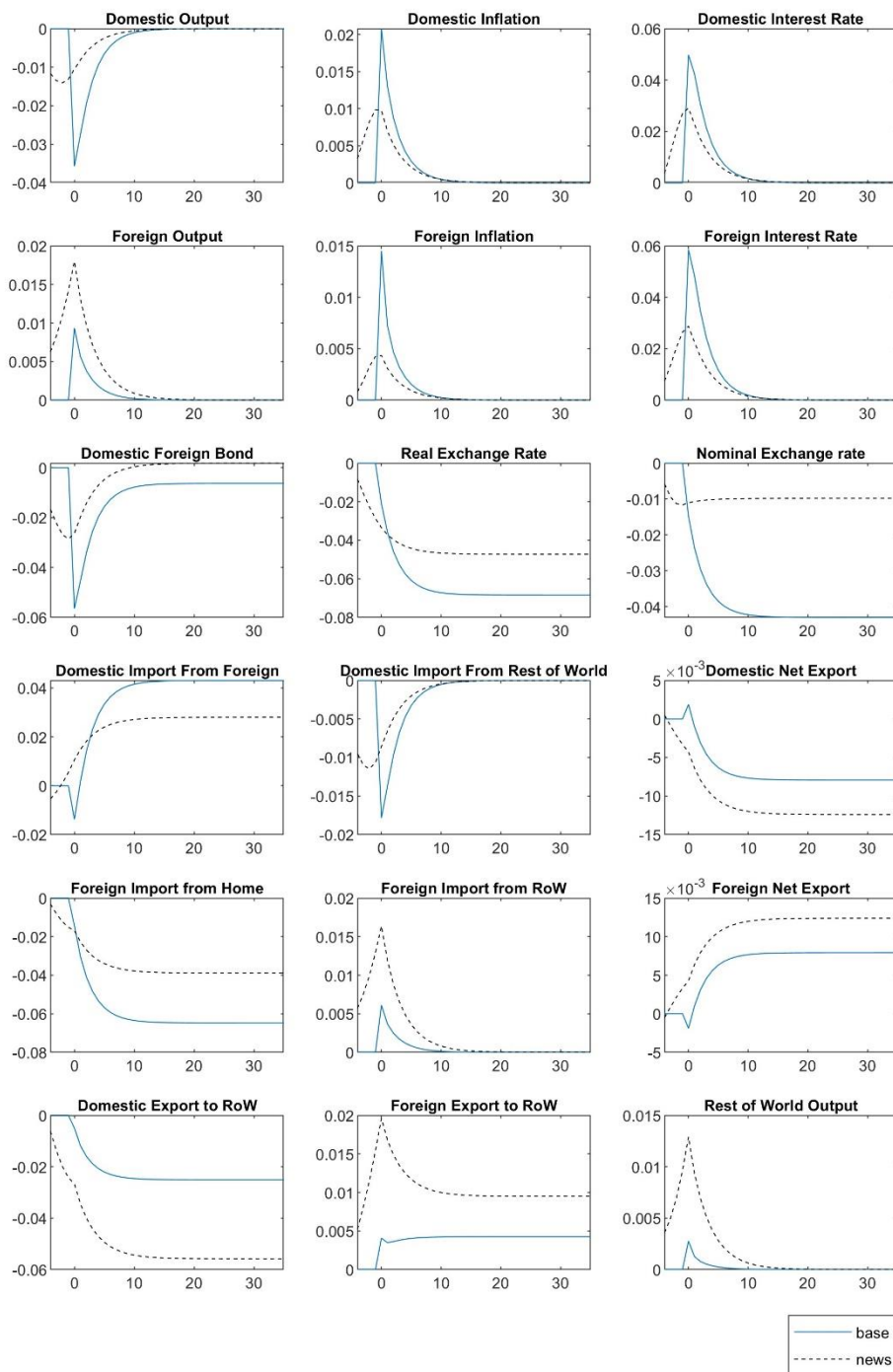


Figure 6. 2 Impulse responses to a foreign demand news shock (anticipated four quarters ahead)



### 6.5.2 Supply news shock

Figure 6.3 depicts how domestic and foreign variables respond to an anticipated supply shock. This expected shock initially triggers deflation, with the inflation rate peaking at the actual realization of the shock. In response, the domestic central bank adjusts interest rates to prevent from recession. Anticipating a forthcoming recession, households reduce their consumption until the shock materializes, then they start to consume more. The cautiousness leads to a smaller but longer recession as the decrease in output is less severe but more persistent compared to the decline observed in the base model.

This negative shock also leads to a depreciation of both the real and nominal exchange rates. The early response of the real exchange rate to the anticipated shock results in a roughly 0.2% smaller overall depreciation, while the nominal exchange rates experience a slightly greater depreciation than the appreciation seen in the base model.

The spillover effect of this supply shock also transmits inflationary pressures to the foreign country, leading to a rise in interest rates then a reduction in foreign output. However, this spillover impact on foreign output is more pronounced in the news model.

Figure 6.4 shows the responses to a foreign supply shock anticipated four periods in advance. The reactions of output, inflation, and nominal interest rates are similar to those seen with a domestic shock. The appreciation of exchange rates in this scenario is more significant than in the base model.

Figure 6. 3 Impulse responses to a domestic supply news shock (anticipated four quarters ahead)

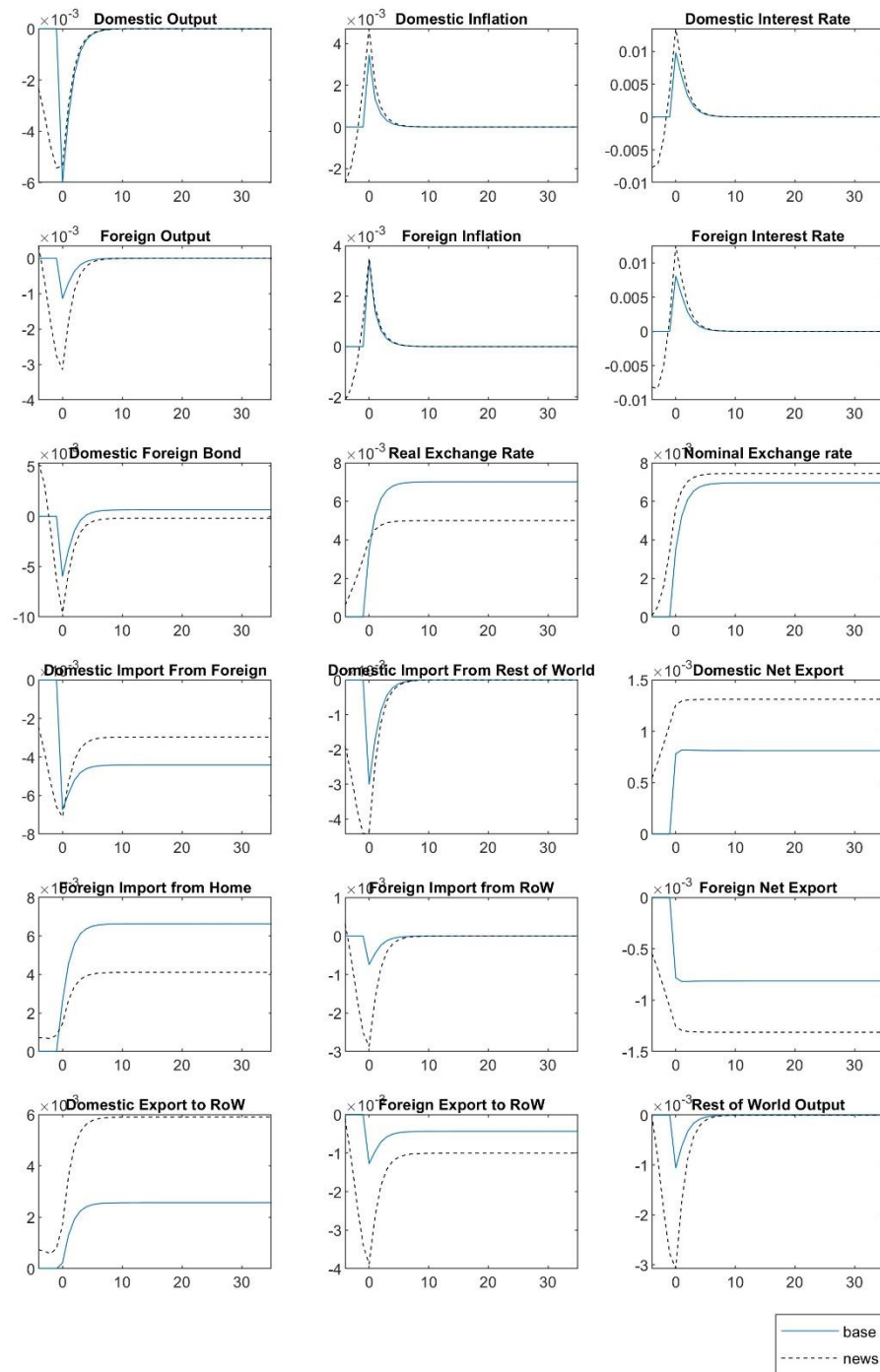
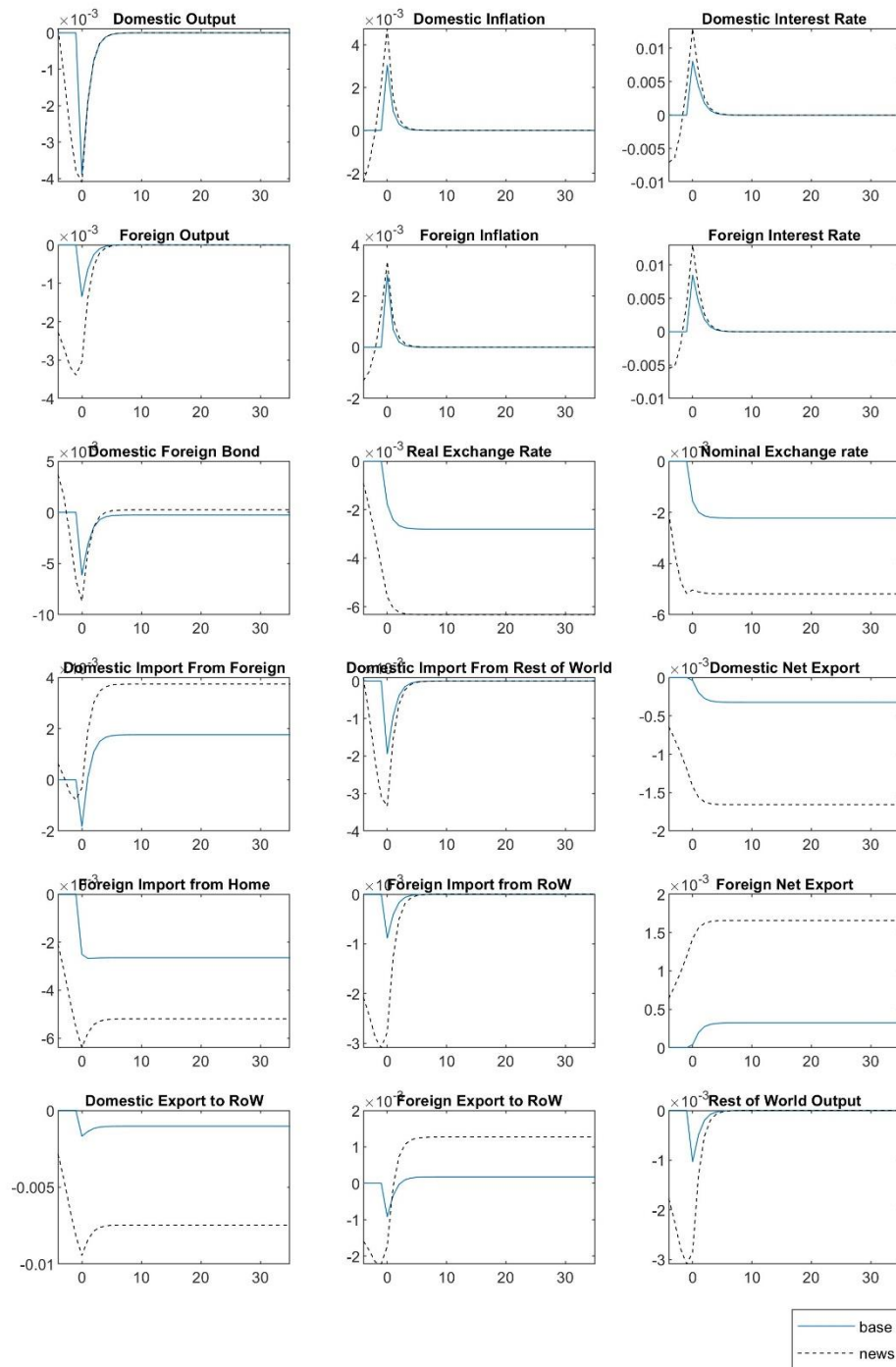




Figure 6. 4 Impulse responses to a foreign supply news shock (anticipated four quarters ahead)



### 6.5.3 Monetary news shock

Figure 6.5 presents the responses of various variables to an anticipated contractionary monetary policy shock. Inflation hits its lowest point when the contractionary shock actually takes place. As the news of this shock is anticipated four periods in advance, it potentially offers valuable insights into forthcoming interest rate decisions. Following the actual occurrence of the policy news shock, real interest rates increase. However, during the anticipation period, real interest rates actually experience a decrease, leading to an increase in domestic output.

The real exchange rate appreciates during the anticipated period and starts to depreciate once the shock materializes. The nominal exchange rate appreciates at the beginning of learning this shock. The eventual depreciation of the real exchange rate is smaller than what is seen in the base model, while the appreciation of the nominal exchange rate is more pronounced than in the base model.

In the foreign economy, there is an initial boost during the anticipation period in the news model. However, post-realization of the news, the foreign economy undergoes a recession that is more severe than what is observed in the base model.

A foreign anticipated contractionary monetary policy shock exhibits symmetrical responses as shown in Figure 6.6 except for a slight larger reaction of real exchange rate during the anticipation period.

Figure 6. 5 Impulse responses to a domestic monetary policy news shock (anticipated four quarters ahead)

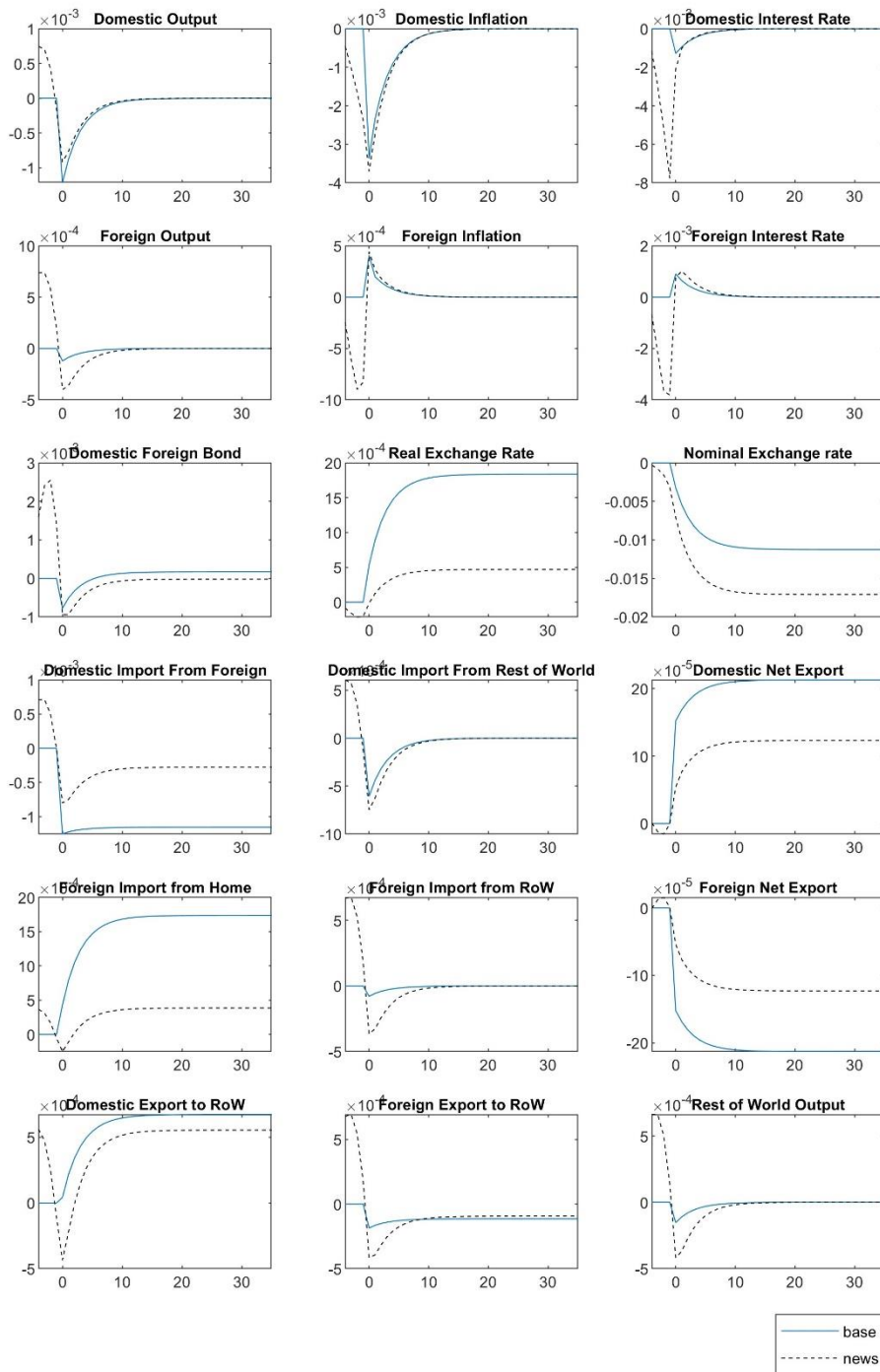
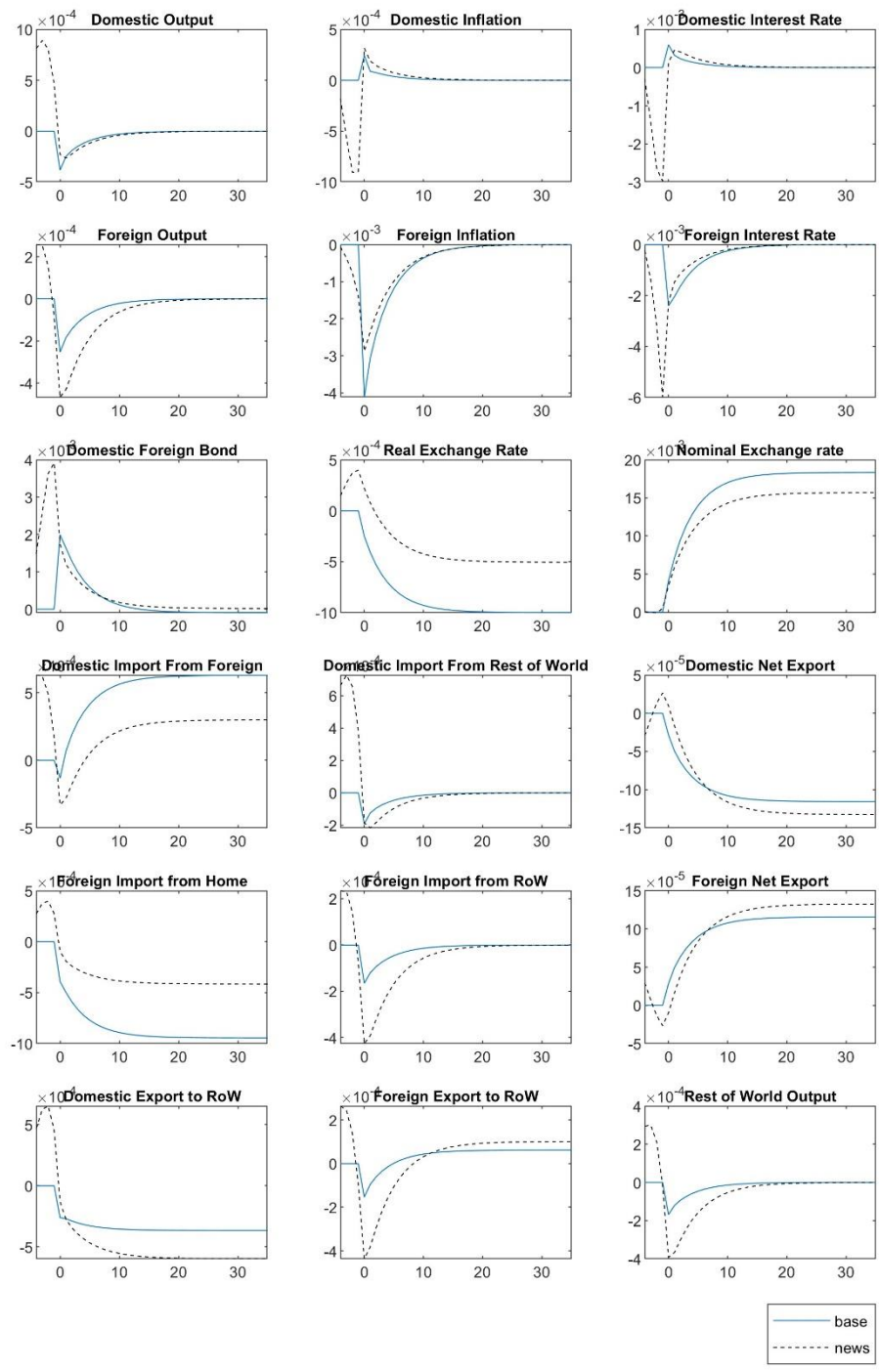


Figure 6. 6 Impulse responses to a foreign monetary policy news shock (anticipated four quarters ahead)



#### **6.5.4 Productivity news shock**

Figure 6.7 illustrates the responses to an expected domestic permanent productivity shock. The initial reaction of domestic output is slightly negative, then increases persistently with no delay, which is similar with the finding of Klein and Linnemann (2021). Domestic inflation and nominal interest rate arrive at the peak when the shock materializes. The responses of domestic interest rate are much stronger since the domestic inflation also reacts strongly.

In terms of exchange rates, both the real and nominal rates experience a significant depreciation immediately upon the shock's anticipation. The international spillover effect of the anticipated domestic productivity shock on foreign output is positive but relatively weak compared to the response observed in domestic output. This aligns with the findings of Klein and Linnemann (2021), which also indicate that other foreign variable responses to such a shock are small. However, compared with the base model, there is no apparent delay in the anticipation period.

Figure 6.8 displays the responses within the same framework to an anticipated permanent productivity shock originating from a foreign country. These reactions mirror those observed for a domestic shock, maintaining a symmetrical pattern. However, a notable difference is seen in the exchange rates, which react more strongly to the foreign productivity shock compared to the domestic one.

Figure 6. 7 Impulse responses to a domestic productivity news shock (anticipated four quarters ahead)

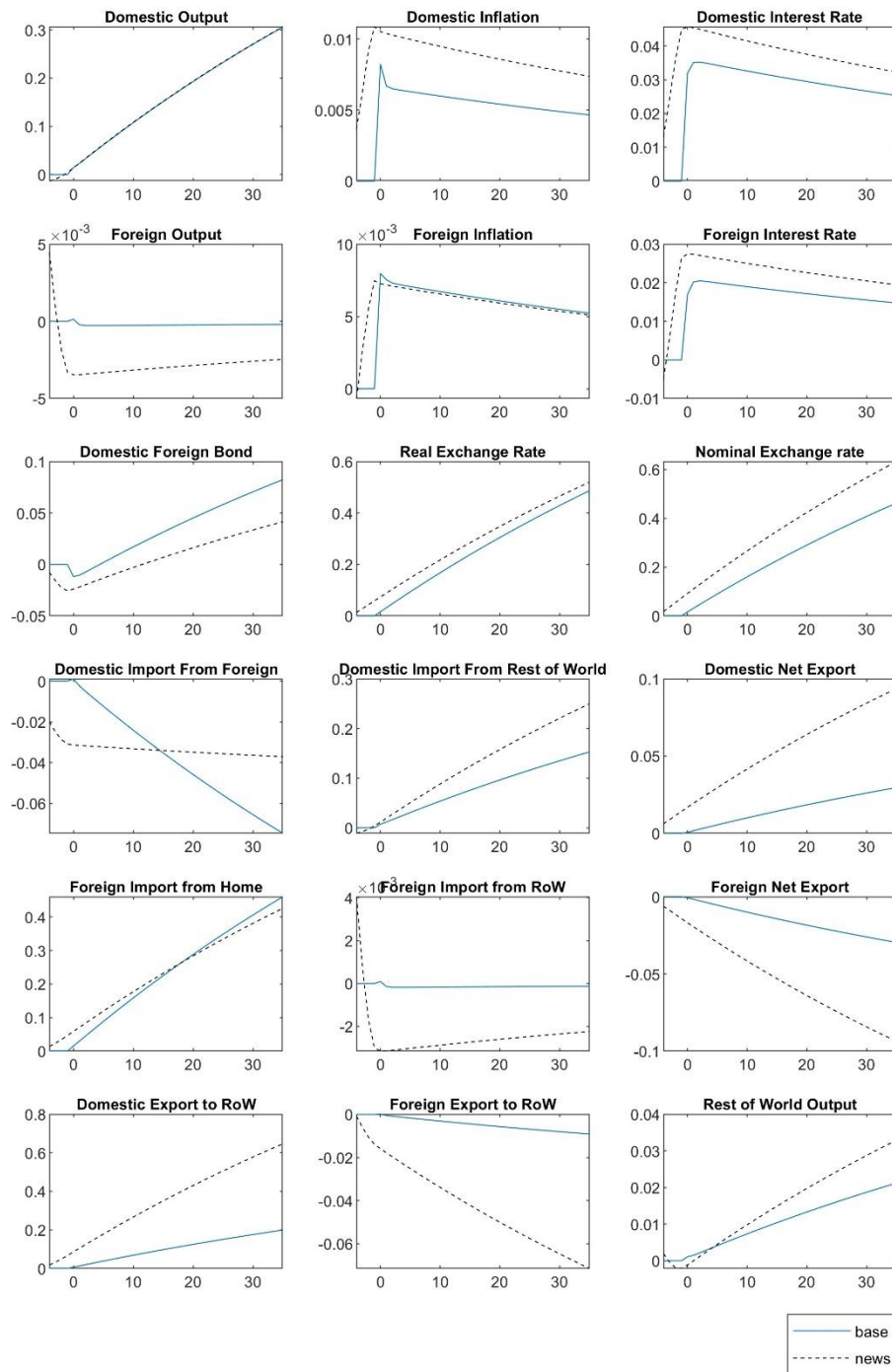
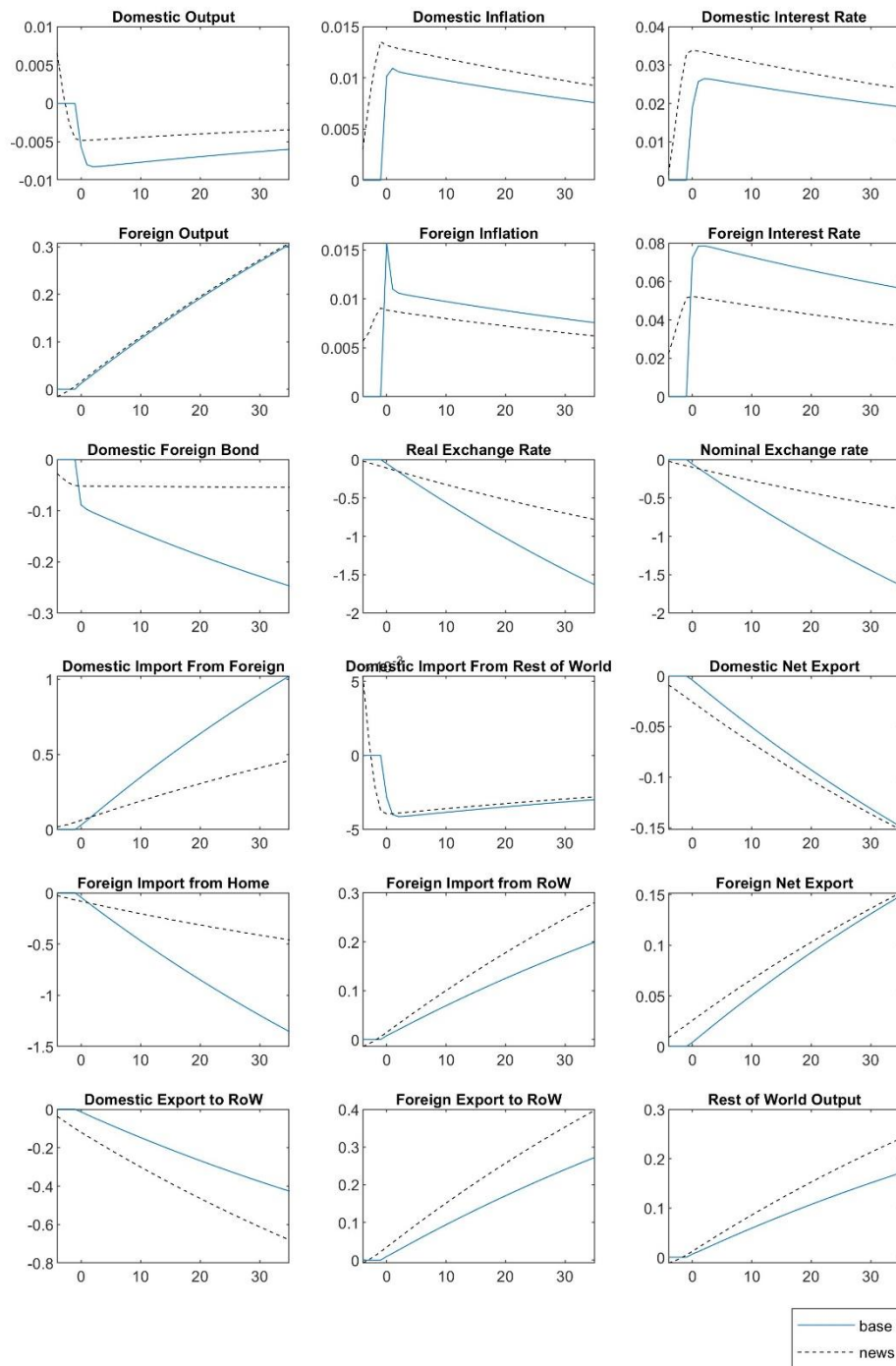


Figure 6. 8 Impulse responses to a foreign productivity news shock (anticipated four quarters ahead)



## 6.6 Variance decomposition

Table 6.3 presents the variance decomposition of different shocks under a scenario of perfect foresight. Since news shocks are anticipated four periods in advance, the variables begin to respond to these anticipated shocks prior to their actual materialization. Furthermore, since the news model is calibrated with parameters that fits the data, and these parameters differ from those used in the base model, the variation in the variables exhibits different behaviour.

In the short run, the domestic productivity shock explains 74% of variances of real exchange rates. The domestic demand shock has the second highest explanation power among these shocks. Domestic supply shock and foreign supply shock both account for 3% variation. However, productivity shock has less effects on the real exchange rate in the short term. Looking at the medium and long term, the dominance of domestic productivity shocks in explaining the variance becomes more pronounced, in contrast to the base model where their explanatory power is comparatively less.

In terms of nominal exchange rates, domestic monetary policy shocks and domestic productivity shocks are the two most important factors since they account for more than 85% variation. As the time passing, the foreign monetary policy shock contributes 10% of the movements of nominal exchange rates. Compared with the base model, foreign productivity shocks are less important.

For other variables, in the short run, the distinction is more obvious. Under the news model, domestic supply shock accounts for 43% of the variance in domestic output, a proportion that is higher than what is observed in the base model. However, these supply shocks exert less influence on other variables compared to the base model. Consistent with the base model, the variation in



domestic inflation is mainly driven by domestic monetary shocks. These shocks show even greater variation in the short term for both domestic and foreign variables in the news model. Foreign monetary shocks are still the most important shocks for the variation of foreign inflation in the medium and long run. Foreign demand shocks are the least important for the movements of foreign variables. Domestic shocks have larger spillover than those in the base model.

Table 6. 3 Variance decomposition with perfect foresight

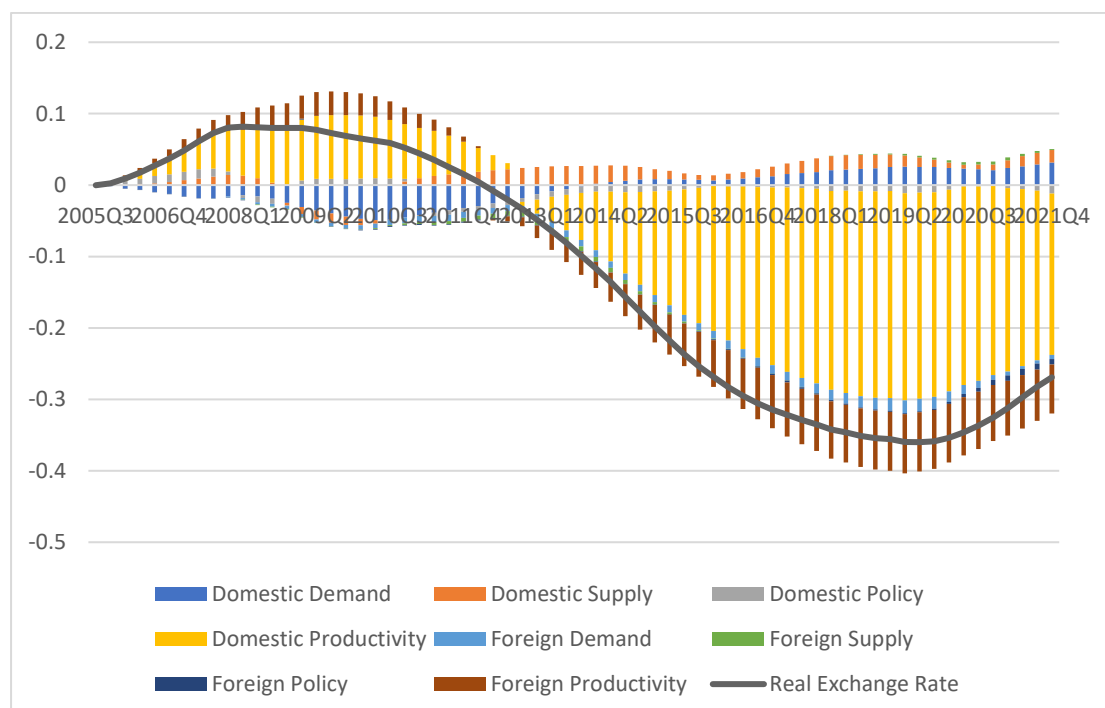
1 year	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	3.11	2.02	2.82	20.72	5.12	3.45	14.31	6.80
Domestic supply	42.58	4.09	3.73	11.26	8.24	9.02	3.77	1.45
Domestic monetary	23.11	82.97	81.51	48.35	56.45	57.25	1.14	39.66
Domestic productivity	10.44	3.94	5.92	1.94	4.83	3.30	73.89	48.54
Foreign demand	1.17	0.18	0.11	1.15	0.10	0.23	1.12	0.10
Foreign supply	5.66	1.48	1.28	14.89	1.74	1.79	3.20	2.68
Foreign monetary	13.91	5.28	4.60	1.50	23.43	24.80	1.07	0.20
Foreign productivity	0.02	0.05	0.03	0.20	0.10	0.16	1.52	0.57
Total	100	100	100	100	100	100	100	100
5 years	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	0.41	1.33	5.18	13.46	3.57	7.32	4.54	1.01
Domestic supply	1.34	3.02	6.72	10.80	5.86	12.83	1.22	0.46
Domestic monetary	1.26	85.73	52.89	25.96	17.10	36.33	0.21	61.55
Domestic productivity	96.43	7.14	29.74	4.49	10.69	20.72	91.01	24.07
Foreign demand	0.03	0.09	0.17	1.33	0.06	0.33	0.28	0.00
Foreign supply	0.22	1.12	2.24	8.36	1.72	3.97	0.82	0.11
Foreign monetary	0.30	1.48	2.91	3.86	60.85	17.80	0.06	12.57
Foreign productivity	0.00	0.10	0.15	31.73	0.16	0.70	1.85	0.23
Total	100	100	100	100	100	100	100	100
10 years	$y$	$\pi$	$R$	$y^*$	$\pi^*$	$R^*$	$q$	$s$
Domestic demand	0.05	1.26	4.20	3.82	3.30	6.31	1.73	0.70
Domestic supply	0.15	2.86	5.44	3.06	5.43	11.05	0.42	0.29
Domestic monetary	0.14	81.18	42.84	7.37	15.83	31.29	0.09	43.49
Domestic productivity	99.59	11.99	43.00	2.17	17.15	31.24	95.42	44.78
Foreign demand	0.00	0.08	0.13	0.38	0.05	0.28	0.10	0.00
Foreign supply	0.02	1.06	1.82	2.37	1.59	3.42	0.28	0.06
Foreign monetary	0.03	1.40	2.36	1.10	56.38	15.33	0.03	10.26
Foreign productivity	0.00	0.16	0.21	79.73	0.26	1.08	1.94	0.41
Total	100	100	100	100	100	100	100	100

## 6.7 Historical decomposition

In a manner akin to the base model which excludes news shocks, domestic productivity shocks emerge as the principal drivers of fluctuations in the real exchange rate. Foreign productivity shocks exhibit a similar pattern of influence, though their impact is comparatively smaller. Notably, in the context of the news model, monetary policy shocks from China do not exert a significant influence.

Given that news is anticipated four periods in advance in this model, the observable trends in response to these shocks are effectively advanced by the same duration. This anticipation leads to a scenario where the impact of the shocks on the variables becomes evident around four periods earlier than it would in the absence of such foresight.

Figure 6. 9 Historical decomposition of real exchange rate



## 6.8 Conclusion

This chapter explores the question of whether news shocks can enhance the model's capacity to fit data and explain exchange rate movements. The news model is evaluated by the powerful Indirect Inference test. Our findings indicate that while both the base and news models are capable of fitting the data, the news model exhibits relatively lower explanatory power, as reflected in its lower Wald statistic compared to that of the base model. What's more, the news model worsens the data fit further under the assumption of imperfect foresight.

However, when examining the impulse response functions, we observe that news shocks do influence agent's behaviour, though the impact is not markedly different from that of pure surprise shocks. In the analyses involving variance decomposition and historical decomposition, it becomes evident that news shocks exert only minor effects in the perfect foresight framework, particularly in the long run. This suggests that while news shocks are a relevant factor, their overall impact may be less significant than initially anticipated, which is consistent with the finding of Le et al. (2020).

# Chapter 7 Conclusion

The underlying motivation for this research stems from the internationalization of the renminbi (RMB). In 2005, China transitioned from a dollar-peg regime to a managed floating exchange rate system, which considers market supply and demand relative to a basket of currencies. Since this shift, China has progressively implemented reforms to further the internationalization of the RMB. This thesis explores two central questions concerning RMB exchange rate movements: firstly, identifying the sources of RMB exchange rate fluctuations within a three-country open economy model, and secondly, examining whether news shocks can provide higher explanation power of the behaviour of the exchange rate.

In Chapter 2, I review the literature on exchange rate determination through uncovered interest parity which is a common assumption in the open economy DSGE models. The chapter also reviews the effects of news shocks in both closed and open economies. However, it is observed that there is no unanimous consensus on the implications of news shocks, particularly regarding their influence on exchange rates, where clear evidence remains elusive.

Chapter 3 introduces the three-country open economy model featuring non-stationary productivity shocks and rest of world functioning as a transfer pot. Eight shocks in total – four from each economy – are incorporated to study their spillover effects on exchange rate movements.

Indirect Inference is introduced in Chapter 4 then it is applied to test and estimate the base model in Chapter 5. The estimation is conducted over the sample period from 2005Q3 to 2021Q4, corresponding to the period when China transitioned to a floating exchange rate regime. While the original calibration of the model fails to pass the test, the estimation successfully fits the data. Analysis from the estimated model highlights the reactions of real

exchange rates to various shocks, with domestic productivity shock emerging as the most significant. Monetary shocks have larger effects on the fluctuations of nominal exchange rates than productivity shocks.

Chapter 6 incorporates news shocks and signal extraction process into the base model which has been ignored by the literature. The findings suggest that the news model worsens the data fit under the assumption of perfect foresight, as indicated by a higher Wald statistic derived from the powerful Indirect Inference method. Moreover, the fit is further worsened under the assumption of imperfect foresight. When news shocks are perfectly anticipated, they do not substantially alter the conclusions drawn in the previous chapter. The responses to these shocks commence when the news is anticipated, and while the degrees of response vary, the domestic productivity shock remains the predominant driver of real exchange rate fluctuations.

This thesis makes two contributions. Firstly, it successfully employs a three-country open economy model to examine exchange rate movements between China and the US, demonstrating a good fit with the data. Secondly, the thesis incorporates news shocks, constrained by the signal extraction process, to explore their impact on exchange rate movements. It is found that the inclusion of news shocks does not markedly differ from the outcomes of the model without news shocks, providing an important perspective on the relative influence of anticipated information in exchange rate dynamics.

Despite the progress made in this thesis, there are further extensions which are worthy investigation. Firstly, incorporating a financial sector (Bernanke et al., 1999) into the existing framework could be beneficial, providing more insights into the fluctuations of exchange rates. While this substantial extension might improve the model's fit domestically, it is not expected to

significantly impact the international financial links which are central to our model.

Secondly, exploring the implications of the zero lower bound problem would be a valuable addition. This aspect has not been addressed in the current work because it does not arise in the model which features a Taylor rule that targets the corporate bond rate, preventing it from reaching zero. In the wake of the financial crisis, policies related to the effective lower bound have generated increasing attention. Le et al. (2016a) propose a conventional Taylor rule when quarterly nominal interest rate is above 0.25% and an unconventional but effective monetary policy by varying the supply of M0 in open market operations when the rate is at or lower than 0.25%. Investigating how these policies might impact exchange rate dynamics and monetary policy within this model could enrich future research. While incorporating the ZLB using T-bill rates might enhance the model's fit, it would introduce significant complexity without necessarily improving its ability to capture international linkages, considering the current model already demonstrates empirical fit with the data. While these extensions would be of interest, the fact that the model as specified and estimated is not rejected by the Indirect Inference test supports the specification used in this thesis.

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## Appendix 1 Price Dispersion

Let  $\hat{p}_t(i) \equiv p_t(i) - p_t$ .

A second order approximation of  $\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}$ :

$$\begin{aligned} \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} &= e^{(1-\epsilon)\log\left(\frac{P_t(i)}{P_t}\right)} \\ &= e^{(1-\epsilon)\hat{p}_t(i)} \\ &= 1 + (1-\epsilon)\hat{p}_t(i) + \frac{(1-\epsilon)^2}{2}\hat{p}_t(i)^2 \end{aligned}$$

As the definition of  $P_t$ ,  $1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di$ . A second-order approximation to this expression thus implies

$$E_i\{\hat{p}_t(i)\} = \frac{\epsilon-1}{2} E_i\{\hat{p}_t(i)^2\}$$

where  $E_i\{x(i)\}$  is the expectation or mean of  $x$ .  $\int_0^1 (1-\epsilon)\hat{p}_t(i) di = (1-\epsilon)E_i\{\hat{p}_t(i)\}$ ,  $\int_0^1 \frac{(1-\epsilon)^2}{2}\hat{p}_t(i)^2 di = \frac{(1-\epsilon)^2}{2} E_i\{\hat{p}_t(i)^2\}$ .

A second order approximation to  $\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}$  yields

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} = 1 - \frac{\epsilon}{1-\alpha}\hat{p}_t(i) + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \hat{p}_t(i)^2$$

Then we have

$$\begin{aligned} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= 1 - \frac{\epsilon}{1-\alpha} E_i\{\hat{p}_t(i)\} + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 E_i\{\hat{p}_t(i)^2\} \\ &= 1 - \frac{\epsilon}{1-\alpha} \frac{\epsilon-1}{2} E_i\{\hat{p}_t(i)^2\} + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 E_i\{\hat{p}_t(i)^2\} \\ &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right) \left(\frac{1-\alpha+\alpha\epsilon}{1-\alpha}\right) E_i\{\hat{p}_t(i)^2\} \\ &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right) \left(\frac{1-\alpha+\alpha\epsilon}{1-\alpha}\right) Var_i\{p_t(i)\} \end{aligned}$$

$$= 1 + \frac{1}{2} \left( \frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \text{Var}_i\{p_t(i)\}$$

where  $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ , and where the last equality follows from the observation that, up to second order

$$\begin{aligned} \int_0^1 (p_t(i) - p_t)^2 di &\cong \int_0^1 (p_t(i) - E_i\{p_t(i)\})^2 di \\ &\equiv \text{var}_i\{p_t(i)\} \end{aligned}$$

Finally, using the definition of  $d_t$  and up to a second order approximation

$$d_t \equiv (1-\alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di \cong \frac{\epsilon}{2\Theta} \text{Var}_i\{p_t(i)\}$$

## Appendix 2 Derivation of the New Keynesian Philips curve

$$\max_{\tilde{P}_{H,t}(h)} E_t \sum_{k=0}^{\infty} (\theta)^k \Delta_{t,t+k} \left\{ \left[ \left( \frac{\tilde{P}_{H,t}(h)}{P_{H,t+k}} \right)^{1-\gamma} - \left( \frac{\tilde{P}_{H,t}(h)}{P_{H,t+k}} \right)^{-\gamma} MC_{t+k} \right] Y_{t+k} \right\}$$

FOC for  $\tilde{P}_{H,t}(h)$

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right) \left\{ \left[ (1-\gamma) \frac{\tilde{P}_{H,t}(h)}{P_{H,t+k}} + \gamma MC_{t+k} \right] \frac{1}{\tilde{P}_{H,t}(h)} \left( \frac{\tilde{P}_{H,t}(h)}{P_{H,t+k}} \right)^{-\gamma} Y_{t+k} \right\} = 0$$

Dividing  $\left( \tilde{P}_{H,t}(h) \right)^{-\gamma-1}$ ,  $C_t^{-\sigma}$  and  $P_t$  on both sides to get

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k (C_{t+k})^{-\sigma} (P_{t+k})^{-1} Y_{t+k} P_{H,t+k}^{\gamma-1} \left[ (1-\gamma) \tilde{P}_{H,t}(h) + \gamma P_{H,t+k} MC_{t+k} \right] = 0$$

$$\tilde{P}_{H,t}(h) = \frac{\gamma}{\gamma-1} \frac{E_t \sum_{k=0}^{\infty} (\theta\beta)^k C_{t+k}^{-\sigma} P_{t+k}^{-1} P_{H,t+k}^{\gamma-1} Y_{t+k} P_{H,t+k} MC_{t+k}}{E_t \sum_{k=0}^{\infty} (\theta\beta)^k C_{t+k}^{-\sigma} P_{t+k}^{-1} Y_{t+k} P_{H,t+k}^{\gamma-1}} \quad (A2.1)$$

When  $\theta = 0$ , which means flexible prices, each firm is free to reset price every period. Thus, (A2.1) reduces

$$\tilde{P}_{H,t}^f(h) = \frac{\gamma}{\gamma-1} P_{H,t} MC_t = \tau P_{H,t} MC_t \quad (A2.2)$$

Under the condition of flexible prices, the optimal price  $\tilde{P}_{H,t}^f(h)$  set by firms is equal to a markup,  $\tau$ , over its nominal marginal cost  $P_{H,t} MC_t$ .

When  $\theta \neq 0$ , under stick prices condition, rewrite (A2.1)

$$(\gamma-1) \tilde{P}_{H,t}(h) E_t \sum_{k=0}^{\infty} (\theta\beta)^k C_{t+k}^{-\sigma} P_{t+k}^{-1} Y_{t+k} P_{H,t+k}^{\gamma-1} = \gamma E_t \sum_{k=0}^{\infty} (\theta\beta)^k C_{t+k}^{-\sigma} P_{t+k}^{-1} P_{H,t+k}^{\gamma} Y_{t+k} MC_{t+k} \quad (A2.3)$$

In the steady state

$$\frac{1}{1-\theta\beta} (\gamma-1) \tilde{P}_{H,t}(h) C^{-\sigma} P^{-1} Y P_H^{\gamma-1} = \frac{1}{1-\theta\beta} \gamma C^{-\sigma} P^{-1} Y P_H^{\gamma} MC$$

$$\frac{\tilde{P}_{H,t}(h)}{P_H} = \frac{\gamma}{\gamma - 1} MC = \tau MC$$

In the steady state, all firms which adjust prices will choose the same price as the domestic price, therefore

$$\frac{\tilde{P}_{H,t}(h)}{P_H} = \frac{\gamma}{\gamma - 1} MC = 1$$

The steady state value of real marginal cost is thus

$$MC = \frac{\gamma - 1}{\gamma} = \frac{1}{\tau}$$

Take the log-linearized form of LHS of (A2.3):

$$\begin{aligned} & (\gamma - 1)\tilde{P}_H(h) \left( 1 \right. \\ & \quad \left. + \tilde{p}_{H,t}(h) \right) E_t \sum_{k=0}^{\infty} (\theta\beta)^k C^{-\sigma} (1 - \sigma c_{t+k}) P^{-1} (1 + p_{t+k}) Y (1 \\ & \quad + y_{t+k}) P_H^{\gamma-1} (1 + (\gamma - 1)p_{H,t+k}) \\ & = (\gamma - 1)\tilde{P}_H(h) \left( 1 \right. \\ & \quad \left. + \tilde{p}_{H,t}(h) \right) E_t \sum_{k=0}^{\infty} (\theta\beta)^k C^{-\sigma} Y P^{-1} P_H^{\gamma-1} (1 - \sigma c_{t+k} + p_{t+k} + y_{t+k} \\ & \quad + (\gamma - 1)p_{H,t+k}) \\ & = (\gamma - 1)\tilde{P}_H(h) \left( 1 + \tilde{p}_{H,t}(h) \right) \left\{ \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^{\gamma-1} \right. \\ & \quad \left. + C^{-\sigma} Y P^{-1} P_H^{\gamma-1} E_t \sum_{k=0}^{\infty} [(\theta\beta)^k (-\sigma c_{t+k} + p_{t+k} + y_{t+k} \right. \\ & \quad \left. + (\gamma - 1)p_{H,t+k})] \right\} \end{aligned}$$

$$\begin{aligned}
&= (\gamma - 1)\tilde{P}_H(h) \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^{\gamma-1} \\
&\quad + (\gamma - 1)\tilde{P}_H(h) \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^{\gamma-1} \tilde{p}_{H,t}(h) \\
&\quad + (\gamma \\
&\quad - 1)\tilde{P}_H(h) C^{-\sigma} Y P^{-1} P_H^{\gamma-1} E_t \sum_{k=0}^{\infty} [(\theta\beta)^k (-\sigma c_{t+k} + p_{t+k} + y_{t+k} \\
&\quad + (\gamma - 1)p_{H,t+k})]
\end{aligned}$$

Take the log-linearized form of RHS of (A2.3):

$$\begin{aligned}
&\gamma E_t \sum_{k=0}^{\infty} (\theta\beta)^k C^{-\sigma} (1 - \sigma c_{t+k}) P^{-1} (1 + p_{t+k}) P_H^{\gamma} (1 + \gamma p_{H,t+k}) Y (1 + y_{t+k}) M C (1 \\
&\quad + \widehat{m}c_{t+k}) \\
&= \gamma \frac{1}{1 - \theta\beta} C^{-\sigma} P^{-1} P_H^{\gamma} Y M C \\
&\quad + \gamma C^{-\sigma} P^{-1} P_H^{\gamma} Y m c E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t+k} + p_{t+k} + \gamma p_{H,t+k} + y_{t+k} \\
&\quad + \widehat{m}c_{t+k}]
\end{aligned}$$

Let LHS=RHS:

$$\begin{aligned}
&(\gamma - 1)\tilde{P}_H(h) \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^{\gamma-1} + (\gamma - 1)\tilde{P}_H(h) \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^{\gamma-1} \tilde{p}_{H,t}(h) \\
&\quad + (\gamma \\
&\quad - 1)\tilde{P}_H(h) C^{-\sigma} Y P^{-1} P_H^{\gamma-1} E_t \sum_{k=0}^{\infty} [(\theta\beta)^k (-\sigma c_{t+k} + p_{t+k} + y_{t+k} \\
&\quad + (\gamma - 1)p_{H,t+k})] \\
&= \gamma \frac{1}{1 - \theta\beta} C^{-\sigma} P^{-1} P_H^{\gamma} Y m c \\
&\quad + \gamma C^{-\sigma} P^{-1} P_H^{\gamma} Y m c E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t+k} + p_{t+k} + \gamma p_{H,t+k} + y_{t+k} \\
&\quad + \widehat{m}c_{t+k}]
\end{aligned}$$

Using  $\frac{\tilde{p}_{H,t}(h)}{P_H} = \frac{\gamma}{\gamma-1} mc = 1$  and then cancelling terms that appear on both sides:

$$\begin{aligned}
& (\gamma - 1) \frac{1}{1 - \theta\beta} C^{-\sigma} Y P^{-1} P_H^\gamma \tilde{p}_{H,t}(h) \\
& \quad + (\gamma - 1) C^{-\sigma} Y P^{-1} P_H^\gamma E_t \sum_{k=0}^{\infty} [(\theta\beta)^k (-\sigma c_{t+k} + p_{t+k} + y_{t+k} \\
& \quad + (\gamma - 1) p_{H,t+k})] \\
& = (\gamma - 1) C^{-\sigma} P^{-1} P_H^\gamma Y E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t+k} + p_{t+k} + \gamma p_{H,t+k} + y_{t+k} \\
& \quad + \widehat{m}c_{t+k}]
\end{aligned}$$

Dividing  $(\gamma - 1) C^{-\sigma} Y P^{-1} P_H^\gamma$  on both sides:

$$\begin{aligned}
& \frac{1}{1 - \theta\beta} \tilde{p}_{H,t}(h) + E_t \sum_{k=0}^{\infty} [(\theta\beta)^k (-\sigma c_{t+k} + p_{t+k} + y_{t+k} + (\gamma - 1) p_{H,t+k})] \\
& = E_t \sum_{k=0}^{\infty} (\theta\beta)^k [-\sigma c_{t+k} + p_{t+k} + \gamma p_{H,t+k} + y_{t+k} + \widehat{m}c_{t+k}] \\
& \frac{1}{1 - \theta\beta} \tilde{p}_{H,t}(h) = E_t \sum_{k=0}^{\infty} [(\theta\beta)^k p_{H,t+k}] + E_t \sum_{k=0}^{\infty} [(\theta\beta)^k \widehat{m}c_{t+k}] \\
& \tilde{p}_{H,t}(h) = (1 - \theta\beta) \left[ E_t \sum_{k=0}^{\infty} (\theta\beta)^k (p_{H,t+k} + \widehat{m}c_{t+k}) \right] \tag{A2.4}
\end{aligned}$$

Equation (A2.4) indicates that the optimal nominal price is set to equal the expected discounted value of future nominal marginal costs.

Rewrite (A2.4) in the expanding form

$$\begin{aligned}
\tilde{p}_{H,t}(h) = & (1 - \theta\beta) [(p_{H,t} + \widehat{m}c_t) + \theta\beta(p_{H,t+1} + \widehat{m}c_{t+1}) + (\theta\beta)^2(p_{H,t+2} + \widehat{m}c_{t+2}) \\
& + \dots]
\end{aligned}$$

Write the previous one period forward:



$$\begin{aligned}
E_t \tilde{p}_{H,t+1}(h) &= (1 - \theta\beta) \left[ E_t \sum_{k=0}^{\infty} (\theta\beta)^k (p_{H,t+1+k} + \widehat{m}c_{t+1+k}) \right] \\
&= E_t \{ (1 - \theta\beta) [(p_{H,t+1} + \widehat{m}c_{t+1}) + \theta\beta(p_{H,t+2} + \widehat{m}c_{t+2}) + (\theta\beta)^2(p_{H,t+3} + \widehat{m}c_{t+3}) \\
&\quad + \dots ] \}
\end{aligned}$$

Multiplying  $\theta\beta$  by both sides,

$$\begin{aligned}
\theta\beta E_t \tilde{p}_{H,t+1}(h) &= E_t \{ (1 - \theta\beta) [\theta\beta(p_{H,t+1} + \widehat{m}c_{t+1}) + (\theta\beta)^2(p_{H,t+2} + \widehat{m}c_{t+2}) \\
&\quad + (\theta\beta)^3(p_{H,t+3} + \widehat{m}c_{t+3}) + \dots ] \}
\end{aligned}$$

Thus,

$$\tilde{p}_{H,t}(h) = (1 - \theta\beta)(p_{H,t} + \widehat{m}c_t) + \theta\beta E_t \tilde{p}_{H,t+1}(h) \quad (A2.5)$$

Following the assumption of price setting structure, the dynamics of the domestic price index satisfies:

$$P_{H,t} \equiv \left[ (1 - \theta)P_{H,t}(h)^{1-\gamma} + \theta P_{H,t-1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

Log-linearizing it around the zero-inflation steady state to yield:

$$p_{H,t} = (1 - \theta)p_{H,t}(h) + \theta p_{H,t-1} \quad (A2.6)$$

Rewrite (A2.6) further

$$\pi_{H,t} = (1 - \theta)(p_{H,t}(h) - p_{H,t-1}) \quad (A2.7)$$

Subtracting  $p_{H,t-1}$  from both sides of (A2.5) and substituting (A2.7) into it

$$\tilde{p}_{H,t}(h) - p_{H,t-1} = (1 - \theta\beta)(p_{H,t} + \widehat{m}c_t) - p_{H,t-1} + \theta\beta E_t \tilde{p}_{H,t+1}(h)$$

$$\frac{1}{1 - \theta} \pi_{H,t} = \pi_{H,t} + \frac{\theta\beta}{1 - \theta} E_t \pi_{H,t+1} + (1 - \theta\beta) \widehat{m}c_t$$

$$\frac{\theta}{1 - \theta} \pi_{H,t} = \frac{\theta\beta}{1 - \theta} E_t \pi_{H,t+1} + (1 - \theta\beta) \widehat{m}c_t$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \widehat{m}c_t$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda \widehat{m}c_t \quad (A2.8)$$

where  $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ . Equation (A2.8) is the New Keynesian Phillips curve for domestic inflation.

## Appendix 3 Derivation of IS Curve for Home Country

Solving for  $c_t$  with the market clearing condition:

Substituting (3.56) into (3.54) gives:

$$\begin{aligned} c_t &= \frac{1}{c} (y_t - x_n x_t) \\ &= \frac{1}{c} \{y_t - [m_1 i m_{H,t}^F + m_2 e x_{RoW,t}^H - (n_1 i m_{F,t}^H + n_2 i m_{RoW,t}^H)]\} \end{aligned} \quad (A3.1)$$

Combine (3.50) and (3.51)

$$\Gamma e x_{RoW,t}^H = y_t^{RoW} - (1 - \Gamma) e x_{RoW,t}^F$$

Using (3.52) to substitute  $e x_{RoW,t}^F$

$$\begin{aligned} \Gamma e x_{RoW,t}^H &= y_t^{RoW} - (1 - \Gamma)(e x_{RoW,t}^H - \psi^{RoW} q_t) \\ e x_{RoW,t}^H &= y_t^{RoW} + (1 - \Gamma)\psi^{RoW} q_t \end{aligned} \quad (A3.2)$$

Substituting (3.46), (3.47), (3.48) and (A3.2) into (A3.1) gives

$$\begin{aligned} c_t &= \frac{1}{c} \{y_t - [m_1(\mu^* y_t^* + \psi^* q_t) + m_2(y_t^{RoW} + (1 - \Gamma)\psi^{RoW} q_t) - n_1(\mu y_t - \psi q_t) \\ &\quad - n_2(v y_t)]\} \\ &= \frac{1}{c} \{[1 + (n_1 \mu + n_2 v)]y_t - (m_1 \mu^*)y_t^* - (m_2)y_t^{RoW} \\ &\quad - [m_1 \psi^* + m_2(1 - \Gamma)\psi^{RoW} + n_1 \psi]q_t\} \\ &= \frac{1}{c} (\Theta^{-1} y_t - z_1 y_t^* - m_2 y_t^{RoW} - z_3 q_t) \end{aligned} \quad (A3.3)$$

where  $z_1 = m_1 \mu^*$ ,  $z_2 = n_1 \mu + n_2 v$ ,  $z_3 = m_1 \psi^* + m_2(1 - \Gamma)\psi^{RoW} + n_1 \psi$  and  $\Theta^{-1} = 1 + z_2$ .

Substituting (A3.3) into the Euler equation (3.17) to get

$$\begin{aligned}
& \frac{1}{c}(\Theta^{-1}y_t - z_1y_t^* - m_2y_t^{RoW} - z_3q_t) \\
&= E_t \frac{1}{c}(\Theta^{-1}y_{t+1} - z_1y_{t+1}^* - m_2y_{t+1}^{RoW} - z_3q_{t+1}) \\
&\quad - \frac{1}{\sigma}(R_t - E_t\pi_{t+1} - \bar{r} + E_t\ln\epsilon_{t+1} - \ln\epsilon_t)
\end{aligned}$$

Rearrange it:

$$\begin{aligned}
& \Theta^{-1}y_t - z_1y_t^* - m_2y_t^{RoW} - z_3q_t \\
&= E_t(\Theta^{-1}y_{t+1} - z_1y_{t+1}^* - m_2y_{t+1}^{RoW} - z_3q_{t+1}) \\
&\quad - c \frac{1}{\sigma}(R_t - E_t\pi_{t+1} - \bar{r} + E_t\ln\epsilon_{t+1} - \ln\epsilon_t)
\end{aligned}$$

$$\begin{aligned}
\Theta^{-1}y_t &= E_t\Theta^{-1}y_{t+1} - E_tz_1(y_{t+1}^* - y_t^*) - E_tm_2(y_{t+1}^{RoW} - y_t^{RoW}) - E_tz_3(q_{t+1} - q_t) \\
&\quad - c \frac{1}{\sigma}(R_t - E_t\pi_{t+1} - \bar{r} + E_t\ln\epsilon_{t+1} - \ln\epsilon_t)
\end{aligned}$$

Multiplying both sides by  $\Theta$

$$\begin{aligned}
y_t &= E_t y_{t+1} - \Theta E_t z_1 (y_{t+1}^* - y_t^*) - \Theta E_t m_2 (y_{t+1}^{RoW} - y_t^{RoW}) - \Theta E_t z_3 (q_{t+1} - q_t) \\
&\quad - c \frac{1}{\sigma} \Theta (R_t - E_t \pi_{t+1} - \bar{r} + E_t \ln \epsilon_{t+1} - \ln \epsilon_t)
\end{aligned}$$

$$\begin{aligned}
y_t &= E_t y_{t+1} - z_1 \Theta E_t \Delta y_{t+1}^* - m_2 \Theta E_t \Delta y_{t+1}^{RoW} - z_3 \Theta \Delta q_{t+1} \\
&\quad - c \frac{1}{\sigma} \Theta (R_t - E_t \pi_{t+1} - \bar{r}) - c \frac{1}{\sigma} \Theta (E_t \ln \epsilon_{t+1} - \ln \epsilon_t)
\end{aligned}$$

$$y_t = E_t y_{t+1} - c \frac{1}{\sigma} \Theta (R_t - E_t \pi_{t+1} - \bar{r}) - z_1 \Theta E_t \Delta y_{t+1}^* - m_2 \Theta E_t \Delta y_{t+1}^{RoW} - z_3 \Theta \Delta q_{t+1} + \varepsilon_t^{IS}$$

where  $\varepsilon_t^{IS} = -c \frac{1}{\sigma} \Theta (E_t \ln \epsilon_{t+1} - \ln \epsilon_t)$ .

## Appendix 4 Derivation of Foreign Country Equations

The import of foreign country from home country is assumed to be the following expression

$$im_{H,t}^F = \mu^* y_t^* + \psi^* q_t$$

Foreign country import from RoW is denoted by below equation

$$im_{RoW,t}^F = v^* y_t^*$$

The net export of foreign country is the negative value of the net export of home country as the rest of world does not produce but only transfer products.

Thus,

$$\begin{aligned} nx_t^* &= -nx_t \\ &= -m_1 im_{H,t}^F - m_2 ex_{RoW,t}^H + (n_1 im_{F,t}^H + n_2 im_{RoW}^H) \end{aligned}$$

By using the identity equation

$$Y_t^* = C_t^* + NX_t^*$$

we have

$$\begin{aligned} c_t^* &= \frac{1}{c^*} (y_t^* - nx_t^*) \\ &= \frac{1}{c^*} \{y_t^* - [-m_1 im_{H,t}^F - m_2 ex_{RoW,t}^H + (n_1 im_{F,t}^H + n_2 im_{RoW}^H)]\} \\ &= \frac{1}{c^*} \{y_t^* - [-m_1 (\mu^* y_t^* + \psi^* q_t) - m_2 (y_t^{RoW} + (1 - \Gamma) \psi^{RoW} q_t) + n_1 (\mu y_t - \psi q_t) \\ &\quad + n_2 v y_t]\} \\ &= \frac{1}{c^*} \{(1 + m_1 \mu^*) y_t^* - (n_1 \mu + n_2 v) y_t + m_2 y_t^{RoW} \\ &\quad + [m_1 \psi^* + m_2 (1 - \Gamma) \psi^{RoW} + n_1 \psi] q_t \} \end{aligned}$$

As we have expressed that  $z_1 = m_1\mu^*$ ,  $z_2 = n_1\mu + n_2\nu$ ,  $z_3 = m_1\psi^* + m_2(1 - \Gamma)\psi^{RoW} + n_1\psi$  and we assume  $\Theta^{*-1} = 1 + z_1$ , the previous equation could be rewritten

$$\begin{aligned} c_t^* &= \frac{1}{c^*} \{(1 + z_1)y_t^* - z_2y_t + m_2y_t^{RoW} + z_3q_t\} \\ &= \frac{1}{c^*} \{\Theta^{*-1}y_t^* - z_2y_t + m_2y_t^{RoW} + z_3q_t\} \end{aligned} \quad (A4.1)$$

The Euler equation for US is

$$c_t^* = E_t c_{t+1}^* - \frac{1}{\sigma} (R_t^* - E_t \pi_{t+1}^* - \bar{r}^* + E_t \ln \epsilon_{t+1}^* - \ln \epsilon_t^*) \quad (A4.2)$$

Substitute (A4.1) into (A4.2) to get

$$\begin{aligned} y_t^* &= E_t y_{t+1}^* - c^* \frac{1}{\sigma^*} \Theta^* (R_t^* - E_t \pi_{t+1}^* - \bar{r}^*) - z_2 \Theta^* E_t \Delta y_{t+1} \\ &\quad + m_2 \Theta^* E_t \Delta y_{t+1}^{RoW} + z_3 \Theta^* E_t \Delta q_{t+1} + \varepsilon_t^{IS*} \end{aligned} \quad (A4.3)$$

Then substitute (3.37) and (3.38) into (3.39) we have

$$\begin{aligned} q_t &= s_t + [(1 - \alpha)p_{F,t}^* + \alpha p_{H,t}^*] - [(1 - \alpha)(s_t + p_{H,t}^*) + \alpha(s_t + p_{F,t}^*)] \\ &= (1 - 2\alpha)p_{F,t}^* + (2\alpha - 1)p_{H,t}^* \end{aligned}$$

Rewrite the previous equation

$$p_{H,t}^* = p_{F,t}^* + \frac{1}{2\alpha^* - 1} q_t$$

The CPI index of foreign country is  $P_t^* = \left[ (1 - \alpha^*)P_{F,t}^{*1-\eta^*} + \alpha^*P_{H,t}^{*1-\eta^*} \right]^{\frac{1}{1-\eta^*}}$ , thus the inflation for foreign country is

$$\begin{aligned} \pi_t^* &= (1 - \alpha^*)\pi_{F,t}^* + \alpha^*(p_{H,t}^* - p_{H,t-1}^*) \\ &= (1 - \alpha^*)\pi_{F,t}^* \\ &\quad + \alpha^* \left( \frac{1}{2\alpha^* - 1} q_t + p_{F,t}^* - \frac{1}{2\alpha^* - 1} q_{t-1} - p_{F,t-1}^* \right) \\ &= \pi_{F,t}^* + \frac{\alpha^*}{2\alpha^* - 1} (q_t - q_{t-1}) \end{aligned} \quad (A4.4)$$

The new Keynesian Phillips curve of foreign country is

$$\pi_{F,t}^* = \beta^* E_t \pi_{F,t+1}^* + \lambda^* \widehat{mc}_t^* \quad (A4.5)$$

where  $\lambda \equiv \frac{(1-\theta^*)(1-\theta^*\beta^*)}{\theta^*}$ .

$\widehat{mc}_t^*$  can be derived analogous to the home country counterpart

$$\widehat{mc}_t^* = \left( \sigma^* \frac{1}{c^*} \Theta^{*-1} + \varphi^* \right) x_t^* \quad (A4.6)$$

Substitute (A4.4) and (A4.6) into (A4.5) to get the new Keynesian Phillips curve in terms of output gap

$$\pi_t^* = \beta^* E_t \pi_{t+1}^* + \kappa_a^* (y_t^* - y_t^{p*}) + \frac{\alpha^*}{1 - 2\alpha^*} [\beta^* E_t (q_{t+1} - q_t) - (q_t - q_{t-1})] + \varepsilon_t^{PP*} \quad (A4.7)$$

# Appendix 5 Data Description

Table A5. 1 Data description

Symbol	Variable	Description	Source
$Y$	Domestic output	Nominal gross domestic product of China, CP <sup>3</sup>	FRED: CHNGDPNQDSMEI
$Y^P$	Domestic potential output	HP filter of domestic output	HP filter of domestic real output
$C$	Domestic Consumption	Nominal consumption of China, CP	DataStream
$P$	Domestic CPI level	GDP Price Deflator <sup>4</sup>	FRED: CHNCPIALLQINMEI
$\pi$	Domestic inflation rate	Percentage change in CPI	$\pi = (\ln P - \ln P_{-1}) * 100$
$R$	Domestic interest rate	3-month treasury securities yields for China / 4	FRED: IR3TTS01CNM156N
$N$	Domestic labour force	The number of working-age people who are either employed or looking for work.	DataStream
$EX_F^H$	Domestic export to foreign country	China's export to US, CP	US Census Bureau
$EX_{Row}^H$	Domestic export to the rest of world	China's total export minus its export to US, CP	OECD
$IM_F^H$	Domestic import from the foreign country	China's import from US, CP	US Census Bureau
$IM_{Row}^H$	Domestic import from the rest of world	China's total import minus its import from US, CP	OECD
$BF$	Home country holding of foreign bond	China's holding of US bond, CP	U.S. DEPARTMENT OF THE TREASURY
$Y^*$	Foreign output	Real gross domestic product of US, SA <sup>5</sup>	FRED: GDPC1

<sup>3</sup> CP stands for “current price”.

<sup>4</sup> Base period = 2012.

<sup>5</sup> SA stands for “seasonally adjusted”, other series are seasonally adjusted manually.



$Y^{p*}$	Foreign potential output	HP filter of foreign output	HP filter of foreign real output
$C^*$	Foreign consumption	US personal consumption, SA	FRED: PCE
$P^*$	Foreign price index level	GDP price deflator	FRED: GDPDEF
$\pi^*$	Foreign inflation rate	Percentage change in price deflator	$\pi^* = (\ln P^* - \ln P_{-1}^*) * 100$
$R^*$	Foreign interest rate	Moody's Seasoned Aaa Corporate Bond Yield / 4	FRED: DAAA
$N^*$	Foreign Labour force	Civilian Labor Force Level	FRED: CLF16OV
$EX_H^F$	Foreign export to home country	US's export to China, CP	US Census Bureau
$EX_{ROW}^F$	Foreign export to the rest of world	US's total export minus its export to China, CP	OECD
$IM_H^F$	Foreign import from home country	US's import from China, CP	US Census Bureau
$IM_{ROW}^F$	Foreign import from the rest of world	US's total import minus its import from China, CP	OECD
$BF^*$	Foreign country holding of home country bond	US holding of China's bond, CP	U.S. DEPARTMENT OF THE TREASURY
$\gamma^{ROW}$	Output of rest of world	Rest of world does not produce so its output equals to its export	$y_t^{ROW} = \Xi im_{ROW,t}^H + (1 - \Xi) im_{ROW,t}^F$
S	Nominal exchange rate	Chinese Yuan Renminbi to one U.S. Dollar	FRED: DEXCHUS
Q	Real exchange rate	Real units of Chinese Yuan Renminbi to one U.S. Dollar	$Q_t = \frac{S_t P_t^*}{P_t}$

Transformation of nominal variables to real variables:

$$Labour\ index = \frac{Labour\ force}{Labour\ force\ at\ base\ time}$$

$$Real\ variable = \ln \left( \frac{\frac{Nominal\ variable}{Deflator}}{Labour\ index} \right) * 100$$

Figure A5. 1 Real data series

