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Multi-Stage Robust Optimization of Real-Time Dynamic Dispatch with Fast-Acting Units in Resilient Power Systems

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Abstract—We propose a dynamic programming (DP) model for multi-stage stochastic-robust optimization (DMSR) to solve the real-time dispatch in power grids with fast-acting (FA) units for enhancing system resilience under extreme events. The proposed approach considers an offline solution and online dispatch. In the offline solution, the T -period real-time dispatch is formulated as a T -stage DMSR model, where its solution is based on an enhanced version of the fast robust dual dynamic programming (FRDDP) algorithm. In online dispatch, the mature cost-to-go functions coupling each stage are used as per-period policies to quickly direct the state adjustment of FA units and real-time dispatch decisions. In DMSR, the scenario-based technique is employed to model contingencies in the multi-stage framework, and the uncertainty set of wind power is constructed to reduce computing complexity. Case studies on the modified IEEE 14-Bus, 118-Bus and 300-Bus systems demonstrate the effectiveness of the proposed real-time dispatch method and solution methodology.

Index Terms—Fast-acting units, wind power, extreme events, multi-stage stochastic-robust optimization, enhanced fast robust dual dynamic programming

NOMENCLATURE

Indices and Sets

$s \in \mathcal{N}$	Index of contingency scenarios
$g \in \mathcal{G}$	Index of generation units
$e \in \mathcal{A}$	Index of BESS
$h \in \mathcal{B}$	Index of buses
$q \in \mathcal{G}_w$	Index of wind farms
$d \in \mathcal{D}$	Index of loads
$hk \in \mathcal{L}$	Index of lines between nodes h and k
$\mathcal{G}_{(N)FA}$	Set of (non-) fast acting units
Ξ	Uncertainty set of wind power
$\mathcal{T}$	Set of dispatch periods

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$\ell \in \Lambda_{t,s}$ Index of valid iterations and sample points set of upper bound problem at period t in scenario s

Parameters

$\mathcal{P}_s$	Probability of contingency scenario s
T	Total intra-day operation period
$\varsigma_{hk,t,s}$	Uncertain contingency parameters denoting whether line hk is interrupted at period t , scenario s
$(\cdot)^{\min/\max}$	Lower/upper output of equipment $(\cdot)$
η_e^c, η_e^d	Charging/discharging efficiency of BESS e at period t in scenario s
$c^{cur/she}$	Cost of wind curtailment and load shedding
$c_g^{fuel,k}$	Marginal fuel cost of unit g on segment k
c_g^u, c_g^d	Start-up and shut-down costs of unit g
C_e	Total capacity of BESS e
$U(D)R_g$	Ramp up/down rate limits of unit g
$\mathfrak{S}_{\ell \in \Lambda_{t,s}}$	Objective value of the problem $\bar{Q}_{t+1,s}$

Variables

$u_{g,t,s}$	Online indicator of unit g at period t , scenario s
$su_{g,t,s}$	Start-up indicator of unit g at period t , scenario s
$sd_{g,t,s}$	Shut-down indicator of unit g at period t , scenario s
$p_{g,t,s}^{(k)}$	Output of unit g at period t , scenario s and that on segment
$Soc_{e,t,s}$	SOC of BESS e at period t in scenario s
$Pw_{q,t,s}$	Output of wind farm q at period t , in scenario s
$Ps_{e,t,s}$	Charging/discharging power of BESS e at period t in scenario s
$F_{hk,t,s}$	Power flow of line hk at period t , scenario s
$\theta_{h,t,s}$	Phase angle of node h at period t , scenario s
$Pz_{d,t,s}$	Load shedding of node d at period t in scenario s
$Fw_{q,t,s}$	Forecast available wind power q at period t , scenario s
$\mathcal{E}_{q,t,s}$	Uncertain forecast error of available wind power from farm q at period t , scenario s
λ_ℓ	The coefficients of the convex combination of sample points in iteration ℓ

$\mathcal{A}_{t,s}^{+(-)}$ Slack variables for characterizing inner sample points in t -stage problem in scenario s

I. INTRODUCTION

EXTREME weather conditions, along with other high-impact and improbable events, continue to occur more frequently and with greater intensity [1], which pose significant challenges to the safe operation of power systems. In the meantime, wind energy has penetrated the grid to support zero-carbon transitions; however, because of unpredictable variability of wind, the real-time balancing operation of power systems would necessitate the commitment of costly reserve units or the provision of ancillary services, which could exacerbate the power system economic operations. The application of fast-acting (FA) units, which can provide sufficient flexibility to large-scale systems, is one of the critical balancing solutions in uncertain circumstances [2]. To find an effective control strategy for the operation of FA units, several works have studied the day-ahead unit commitment (UC) of FA units in power systems [3]-[5]. However, major investments in devising a resilient strategy for the real-time dispatch that can correspond with unexpected contingencies are lagging in this arena. The FA units often feature relatively short start-up and shut-down times, which will introduce numerous variables in large-scale systems and induce tractable obstacles in real-time computations.

The widespread real-time or intra-hour dispatch models are primarily based on the rolling horizon optimization, which is also referred to as the model predictive control (MPC) of power systems [6]. In addition, stochastic optimization (SO) and robust optimization (RO) methods are integrated into the MPC model to cope with corresponding uncertainties. In [7]-[9], the two-stage SO in combination with MPC is proposed to manage the real-time operation of power systems. However, the many scenarios embedded in the SO-based approach could hinder its fast computation for real-time applications. The two-stage RO is employed to handle this problem in the MPC model [10], which considers the worst-case scenario in the second stage. Although, the RO-based approach is considered over-conservative for relaxing the computational complexities. To mitigate the excessive conservatism of RO and the insufficient robustness of SO, the distributionally robust optimization (DRO) strategy is proposed [11]. DRO optimizes decisions based on the worst-case scenario distribution, offering a balance between RO's worst-case scenario focus and SO's reliance on a predetermined distribution [12]. Although various two-stage DRO methods have been implemented in day-ahead scheduling [12], [13], their complex computational requirements, due to the need for constructing intricate distribution fuzzy sets, limit their integration into MPC-based intra-day dispatch. The hybrid SO-RO method presents an alternative, addressing multiple uncertainties in optimization models more efficiently [14]. By appropriately allocating uncertainties to either SO or RO, this method strikes a balance between computational complexity and decision-making conservatism [15], showing promise for intra-day applications [16].

However, the use of SO, RO, and their improved or hybrid variants in the MPC model for real-time dispatch has the following drawbacks: 1) The myopic flaw in MPC framework,

which results in ineffective dispatch decisions since it lacks the complete information on entire day windows. 2) The two-stage scheme, applied to SO, RO, and their improvements, formulate uncertain wind scenarios in real-time operation at first, while the second stage problem makes economic dispatch decisions based on the realization of wind power generation [17]. The difficulty in such cases is known as the anticipativity defect of the two-stage optimization model [18].

To tackle the two problems, recently the dynamic programming (DP) formulated multi-stage scheme are presented in the hydropower dispatch [19], battery energy storage system (BESS) scheduling [20], and microgrid operations [21]. The DP framework fixes the myopic drawback of MPC as it uses the cost-to-go functions in each period to couple the future influences [22]. In addition, it enforces the non-anticipativity of dispatch decisions by extending the two-stage SO and RO model to a multi-stage process [18]. The DP approach can provide globally-optimal solutions to multi-stage SO and RO by sequential decomposition methodologies, which are referred to stochastic dual dynamic programming (SDDP) [23] and robust dual dynamic programming (RDDP) [24], respectively. In our previous research [25], a fast RDDP (FRDDP) method is developed to handle the DP based multi-stage RO model, which accelerates the upper approximation process of the conventional RDDP.

Nevertheless, the issue considered in this article cannot be addressed by previous studies. Here, the first obstacle is to model probable contingencies in the multi-stage framework. In the RO field, although the N-K criterion has been embedded in the multi-stage RO model in [26] to cope with cyclone's striking, the inter-temporal constraints introduced for uncertain variables could violate the stage-wise independent rule of RDDP and SDDP. This rule prohibits the uncertainty of one stage from being associated with the realization of other stages; otherwise, the whole DP formulated problem will be infeasible [24]. In the SO field, the integration of Markov chains into the multi-stage SO/SDDP model can represent the state transition of time-dependent uncertainties, which has been employed for modeling the generation of time-correlative wind turbines [21]. However, the introduction of Markov state lattices shifts the paradigm from a scenario-centric multi-stage model to a state-centric one, leading to a significant escalation in computational complexity [27].

An alternative strategy is to sample contingency scenarios from the statistically determined damage probability distribution for each line, which is transmuted from the Markov process [28]. This approach has found recent application in the day-ahead Unit Commitment (UC) [29] and for picking up critical loads [16] in a two-stage scheme; however, it has not studied in the multi-stage framework. Besides, the massive scenario trees of wind power generation will aggravate the computation burden of multi-stage SO, especially in large-scale systems. Another obstacle is the use of binary variables that indicate the state adjustment of FA units render the cost-to-go functions in each stage nonconvex, which will make the SDDP and RDDP intractable [24], [30]. Accordingly, an integer version of SDDP (SDDiP) is proposed in [31] to serve a tight solution for the DP based multi-stage SO model with integer recourse variables in each stage. The same motivation pertaining to SDDiP is employed in [26] to manage RDDP, but

the upper bound of which cannot be guaranteed to be global Lipschitz [32].

In response to the above problems, this paper focuses on the DP formulation of multi-stage hybrid SO-RO to attain a real-time dispatch for resilient power systems operated with variable wind generation and FA units. The contribution of our study is summarized below:

- 1) A DP model for multi-stage stochastic-robust optimization (DMSR) is presented to address the real-time dispatch of a resilient power system with FA units. DMSR extends the two-stage hybrid SO-RO model [14], [16] to the DP formulated multi-stage scheme for enforcing the non-anticipativity of dispatch decisions. Case studies show the combination of SO and RO can strike a compromise between the solution conservatism and computing complexity.
- 2) The practical application framework of the DMSR model for offline solution & online dispatch is presented. Compared to the MPC-based scheme, the DMSR model overcomes the myopic defect and greatly improves real-time power system computing efficiency by eliminating foresight horizon.
- 3) An enhanced FRDDP algorithm is developed to efficiently solve the DMSR model, where the relaxed inner approximation (RIA) method and Lagrangian hyperplanes are respectively employed in the upper and the lower approximations.

The rest of this paper is organized as follows. The detailed formulation of DMSR model is shown in Section II, and Section III presents the developed enhanced FRDDP methodology. In Section IV, the practical application procedure of proposed method for real-time dispatch is designed. Numerical results on the modified IEEE 14-bus, 118-bus, and 300-bus systems are demonstrated and discussed in Section V. Section VI concludes this paper.

II. MATHEMATICAL FORMULATION OF DMSR

A. Multi-stage SO-RO combination model

The compact form of DMSR model for the real-time dispatch of FA units in power systems is provided by (1). The formulation aims at finding the optimal dispatch policies for the periods between 1 to T according to the corresponding worst-case wind power generation realization. The proposed solution minimizes the expected cost of generation unit dispatch, wind curtailment, and load shedding for the N probable contingency scenarios. Each contingency scenario is sampled from the fragility curve [28] of transmission lines, which is described in [16] in detail.

$$\begin{aligned} \min & \sum_{s \in \mathcal{N}} \mathcal{P}_s \cdot \max_{\xi_s \in \Xi} \sum_{t \in \mathcal{T}} [c_t^\top x_{t,s} + q_t^\top p_{t,s} + d_t^\top (\xi_{t,s} + y_{t,s})] \\ \text{s.t.} & B_t[x_{t,s}(\xi_{t,s}) - x_{t-1,s}(\xi_{t-1,s})] + W_t[p_{t,s}(\xi_{t,s}) - p_{t-1,s}(\xi_{t-1,s})] \leq h_t \\ & D_t(\varsigma_{t,s})y_{t,s}(\xi_{t,s}) + H_t x_{t,s}(\xi_{t,s}) + F_t p_{t,s}(\xi_{t,s}) \leq g_t(\xi_{t,s}) \quad (1) \\ & x_{t,s} \in \mathbb{B}^m, p_{t,s} \in \mathbb{R}^r, y_{t,s} \in \mathbb{R}^n \\ & \forall t \in \mathcal{T}, \forall s \in \mathcal{N}, \forall \xi_s \in \Xi, \xi_s = \{\xi_{1,s}, \xi_{2,s}, \dots, \xi_{T,s}\} \end{aligned}$$

where:

$$\begin{aligned} x_{t,s} &= \{u_{g,t,s} \mid g \in \mathcal{G}_{FA}\}, p_{t,s} = \{(P_g, Soc_e)_{t,s} \mid g \in \mathcal{G}, e \in \mathcal{A}\} \\ y_{t,s} &= \{(Pw_q, Fw_q, Ps_e, Pz_d, \theta_h, F_{hk})_{t,s} \mid q \in \mathcal{G}_W, e \in \mathcal{A}, d \in \mathcal{D}\} \\ h(k) &\in \mathcal{B}, \xi_{t,s} = \{\xi_{q,t,s} \mid q \in \mathcal{G}_W\}, \varsigma_{t,s} = \{\varsigma_{hk,t,s} \mid hk \in \mathcal{L}\} \end{aligned}$$

In (1), for each contingency scenario s , $x_{t,s}$, $p_{t,s}$ and $y_{t,s}$ are the

decision variables at stage t , which depend on the realization of the wind power generation $\xi_{t,s}$. B_t , W_t , D_t , H_t and F_t are the coefficient matrices corresponding to the constraints described in Section B, where D_t contains the interruption parameters $\varsigma_{t,s}$ in scenario s . h_t represents a right-side constant matrix, and g_t is a function of $\xi_{t,s}$.

The stochastic DMSR model represents the randomness of system contingencies. This is attributed to the DMSR model leveraging multi-scenario-based stochastic optimization methodologies to depict intricate, time-dependent system contingency uncertainties. Nonetheless, considering that in every stage the extreme wind generation realization is screened out within each random contingency scenario, the stochastic of DMSR model can also refer to the randomness of the extreme scenarios.

The DMSR is reformulated in the DP form (2) using the L-shaped transformation, where the s -scenario objective function of stage t is simplified by $obj_{t,s}(\cdot)$.

$$\begin{aligned} Q_t &= \min \sum_{s \in \mathcal{N}} \mathcal{P}_s [obj_{t,s}(x_{t,s}, p_{t,s}, y_{t,s}) + Q_{t+1,s}(x_{t,s}, p_{t,s})] \\ \text{s.t.} & B_t x_{t,s} + W_t p_{t,s} \leq h_t \\ & D_t(\varsigma_{t,s})y_{t,s} + H_t x_{t,s} + F_t p_{t,s} \leq g_t(\xi_{t,s}) \\ & x_{t,s} \in \mathbb{B}^m, p_{t,s} \in \mathbb{R}^r, y_{t,s} \in \mathbb{R}^n \end{aligned} \quad (2)$$

In the contingency scenario s , the worst-case cost-to-go function $Q_{t,s}(x_{t-1,s}, p_{t-1,s})$ measures the total future cost for dispatch decisions $[x_{t-1,s}; p_{t-1,s}]$ in the worst-case wind power generation, which is calculated as:

$$Q_{t,s}(x_{t-1,s}, p_{t-1,s}) = \max \{Q_{t,s}(x_{t-1,s}, p_{t-1,s}; \xi_{t,s}) : \xi_{t,s} \in \Xi_t, \forall s\} \quad (3)$$

where, $Q_{t,s}(x_{t-1,s}, p_{t-1,s}; \xi_{t,s})$ is defined as:

$$\begin{aligned} \min & obj_{t,s}(x_{t,s}, p_{t,s}, y_{t,s}) + Q_{t+1,s}(x_{t,s}, p_{t,s}) \\ \text{s.t.} & B_t(x_{t,s} - x_{t-1,s}) + W_t(p_{t,s} - p_{t-1,s}) \leq h_t \\ & D_t(\varsigma_{t,s})y_{t,s} + H_t x_{t,s} + F_t p_{t,s} \leq g_t(\xi_{t,s}) \\ & x_{t,s} \in \mathbb{B}^m, p_{t,s} \in \mathbb{R}^r, y_{t,s} \in \mathbb{R}^n \end{aligned} \quad (4)$$

B. Specific real-time dispatch problem

Wind power generation is a time- and space-correlated stochastic process. In the traditional RO model [33], usually a boundary-prefixed polyhedral uncertainty set specifies the unknown parameter. However, this approach might be overly conservative because the uncertainty set could contain some low likelihood regions. The principal component analysis (PCA) methodology [34] is used in our study to create a data-driven polyhedral uncertainty set of wind power generation that characterizes the relative forecast error, as follows:

$$\Xi_t = \left\{ \xi_t \mid \begin{cases} \xi_t = \mu_t + D_t \delta \\ \|\delta\|_1 \leq \Gamma_1 \\ \|\delta\|_\infty \leq \Gamma_2 \end{cases} \right\} \quad (5)$$

where, μ_t and D_t represent the mean and covariance matrices, respectively, of the wind farm's historic relative prediction error at stage t . These two statistics are obtained using the PCA method. The normalized uncertain parameter is denoted by δ . Γ_1 and Γ_2 are the adjustable factor to strike a balance between

operating cost-effectiveness and robustness.

For a more profound understanding of the data-driven uncertainty set concept, Fig. 1 elucidates the formation of the data-driven wind uncertainty set. Within Fig. 1(a), parameters Γ_1 and Γ_2 define the foundational geometry of the uncertainty set, resulting in polyhedral boundaries. Upon incorporating the translation matrix μ_t and the rotation-stretching matrix D_t , the data-driven uncertainty set evolves to encompass all historical data. Fig. 1(b) demonstrates that, by leveraging the PCA method for extracting pertinent historical data to determine boundary placements and geometric dimensions, the data-driven uncertainty set is characterized by a reduced conservatism in comparison to its traditional box uncertainty set counterparts.

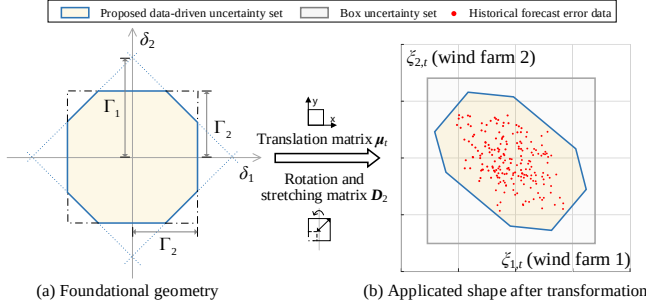


Fig. 1. Illustration of the formation for data-driven uncertainty set and comparison with the traditional box uncertainty set

Then, the real-time dispatch problem is described in the following contents in detail. For the brevity of statements, the cost-to-go functions are omitted and $t \in \mathcal{T}$ and $s \in \mathcal{N}$ are met for each variable. The t -stage objective of real-time dispatch in scenario s is demonstrated as

$$\min \sum_{g \in \mathcal{G}} \sum_{k \in K} c_g^{\text{fuel},k} P_{g,t,s}^k + \sum_{g \in \mathcal{G}_{FA}} (su_{g,t,s} + sd_{g,t,s}) + \sum_{q \in \mathcal{G}_W} c_q^{\text{cur}} (Fw_{q,t,s} + \xi_{q,t,s} - Pw_{q,t,s}) + \sum_{d \in \mathcal{D}} c_d^{\text{she}} Pz_{d,t,s} \quad (6)$$

The operations of FA, conventional units, and wind generators are limited by constraints (7)-(13) during the intra-day real-time dispatch. The start-up and shut-down states of conventional units are fixed by day-ahead parameters (7). FA unit states during real-time dispatch are adjustable variables whose start-up and shut-down costs are represented by (8)-(9). Constraints (10)-(12) would limit the generating and ramping capabilities of FA and conventional units, and (13) restricts the wind generator power output.

$$u_{g,t,s} = I_{g,t}, \forall g \in \mathcal{G}_{NFA} \quad (7)$$

$$su_{g,t,s} \geq c_g^u (u_{g,t,s} - u_{g,t-1,s}), su_{g,t,s} \geq 0, \forall g \in \mathcal{G}_{FA} \quad (8)$$

$$sd_{g,t,s} \geq c_g^d (u_{g,t-1,s} - u_{g,t,s}), sd_{g,t,s} \geq 0, \forall g \in \mathcal{G}_{FA} \quad (9)$$

$$P_{g,t,s} = \sum_{k \in K} P_{g,t,s}^k, P_{g,t,s}^{\min} u_{g,t,s} \leq P_{g,t,s} \leq P_{g,t,s}^{\max} u_{g,t,s}, \forall g \in \mathcal{G} \quad (10)$$

$$P_{g,t,s} - P_{g,t-1,s} \leq R_g^+ u_{g,t-1,s} + UR_g (1 - u_{g,t-1,s}), \forall g \in \mathcal{G} \quad (11)$$

$$P_{g,t-1,s} - P_{g,t,s} \leq R_g^- u_{g,t,s} + DR_g (1 - u_{g,t,s}), \forall g \in \mathcal{G} \quad (12)$$

$$0 \leq Pw_{q,t,s} \leq Fw_{q,t,s} + \xi_{q,t,s}, \forall q \in \mathcal{G}_W \quad (13)$$

The BESS operation is limited by (14)-(17). The SOC transition of BESS is modeled by (14)-(16), where the positive or negative values of $Ps_{e,t,s}$ corresponds to charging or discharging processes. Constraint (17) ensures the initial SOC state of BESS is recovered at the final stage of intra-day

dispatch, in which the penalty of auxiliary variables $\sigma_{e,s}^+$ and $\sigma_{e,s}^-$ is added to objective function of the T -stage problem.

$$Soc_{e,t-1,s} - Ps_{e,t,s} \eta_e^c / C_e \leq Soc_{e,t,s}^{\max}, \forall e \in \mathcal{A} \quad (14)$$

$$Soc_{e,t-1,s} - Ps_{e,t,s} / \eta_e^d C_e \geq Soc_{e,t,s}^{\min}, \forall e \in \mathcal{A} \quad (15)$$

$$Soc_{e,t,s} = Soc_{e,t-1,s} - Ps_{e,t,s} / C_e, \forall e \in \mathcal{A} \quad (16)$$

$$Soc_{e,T,s} + \sigma_{e,s}^+ - \sigma_{e,s}^- = Soc_{e,T,s}^{\text{ini}}, \forall e \in \mathcal{A} \quad (17)$$

Note that (16) estimates the SOC inaccurately at the end of this stage, since charging/discharging efficiency is neglected. To limit the impact of this approximation, the correction function (18) is configured at each adjacent stage of DMSR model. This linear BESS model subtly utilizes the multi-stage DP scheme to improve the computational efficiency of dispatch model. Meanwhile, our prior work [20], [25] provide the mathematical prove that the approach will not affect the convergence of decomposition method and manifests additional merits for practical application.

$$Soc_{e,t,s} = Soc_{e,t-1,s} - \max\{Ps_{e,t,s}, 0\} / (\eta_e^d C_e) - \eta_e^c \min\{Ps_{e,t,s}, 0\} / C_e, \forall e \in \mathcal{A} \quad (18)$$

The DC power flow during operation is restricted by (19)-(21), where transmission contingencies are considered and power balance constraint (22) is satisfied using the proposed solution.

$$(\theta_{h,t,s} - \theta_{k,t,s}) / x_{hk} = F_{hk,t,s}, \forall hk \in \mathcal{L}_f \cup \mathcal{L}_t \quad (19)$$

$$-F_{hk}^{\max} \varsigma_{hk,t,s} \leq F_{hk,t,s} \leq F_{hk}^{\max} \varsigma_{hk,t,s}, \forall hk \in \mathcal{L} \quad (20)$$

$$\theta^{\min} \leq \theta_{h,t} \leq \theta^{\max}, \forall h \in \mathcal{B} \quad (21)$$

$$\sum_{g \in \mathcal{G}_h} P_{g,t,s} + \sum_{e \in \mathcal{A}_h} Ps_{e,t,s} + \sum_{q \in \mathcal{G}_{Wh}} pw_{q,t,s} - \sum_{hk \in \mathcal{L}_f} pf_{hk,t,s} + \sum_{hk \in \mathcal{L}_t} pf_{hk,t,s} = \sum_{d \in \mathcal{D}_h} (L_{d,t} - Pz_{d,t,s}), \forall h \in \mathcal{B} \quad (22)$$

III. PROPOSED SOLUTION METHODOLOGY

A. Overall solution scheme of enhanced FRDDP

Different from SDDP, upper and lower bounds of cost-to-go functions are both introduced in the RDDP methodology, which are satisfied with $\underline{Q}_{t+1,s} \leq Q_{t+1,s} \leq \bar{Q}_{t+1,s}$. However, since the DMSR cost-to-go functions are nonconvex because of FA integer variables, the same decomposition method cannot be applied in FRDDP to generate the explicit boundaries above and below cost-to-go functions. Ref. [31] provides proof that any cost-to-go function of binary variables has a convex lower envelope, and the binary points coincide with the original cost-to-go function of FA binary variables. This issue validates the accuracy of cutting planes to update lower bounds in SDDiP. Accordingly, an enhanced FRDDP algorithm is developed in our study to demonstrate the effectiveness of RIA technique for refining the upper bounds. The first step in the enhanced FRDDP is to present the binary state variables in the proposed DMSR model. Since the state variable $x_{t,s}$ is already binary, we only provide the binary value of $p_{t,s}$ as $p_{t,s}^v$ [35].

To approximate the cost-to-go functions, the upper and lower bounds are refined by the corresponding upper and lower approximation processes. The upper bound and lower bound problems stated in (23) and (24), where $\Upsilon_t(\xi_{t,s}, \varsigma_{t,s})$ stands for the condensed feasible regions of dispatch variables, are solved

iteratively to accomplish the two stated processes.

$$\bar{Q}_{t,s} = \max_{\xi_{t,s} \in \Xi_t} \min \text{obj}_{t,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) + \bar{Q}_{t+1,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla) \quad (23)$$

$$\text{s.t. } (\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) \in \Upsilon_t(\xi_{t,s}, \varsigma_{t,s})$$

$$\mathbf{x}_{t,s} \in \mathbb{B}^m, \mathbf{p}_{t,s}^\nabla \in \mathbb{B}^r, \mathbf{y}_{t,s} \in \mathbb{R}^n$$

$$\underline{Q}_{t,s}(\xi_{t,s}) = \min \text{obj}_{t,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) + \underline{Q}_{t+1,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla) \quad (24)$$

$$\text{s.t. } (\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) \in \Upsilon_t(\xi_{t,s}, \varsigma_{t,s})$$

$$\mathbf{x}_{t,s} \in \mathbb{B}^m, \mathbf{p}_{t,s}^\nabla \in \mathbb{B}^r, \mathbf{y}_{t,s} \in \mathbb{R}^n$$

The overall execution of the enhanced FRDDP presented in Fig. 2 is divided into Forward pass and Backward pass procedures. In the Forward pass, extending from stage 1 to stage T , decision samples are obtained by solution of the lower bound problem. In the backward pass, extending from stage T to stage 1, inner sample points are added to refine the upper bound, and the inequalities are established to update the lower bound.

B. Hyperplanes-based lower approximation

The cutting-plane method can archive the lower approximation which is due to the characteristics of convex lower envelope in binary transformed cost-to-go functions. However, normal Benders cuts or its variants cannot provide a tight lower bound for DMSR model because the strong dual theorem does not hold for the corresponding mixed-integer variables [31]. In [31] and [35], certain Lagrangian cuts are proposed for formulating the lower DP bound in the proposed multi-stage SO model, where a zero-gap Lagrangian dual theorem validates its effectiveness. This strategy inspires our model to employ a similar technique for the lower approximation in the enhanced FRDDP. The following relaxation problem of $\underline{Q}_{t,s}$ is formulated to obtain the Lagrangian hyperplanes, based on the given state variables $\mathbf{x}_{t-1,s}$ and $\mathbf{p}_{t-1,s}^\nabla$, as shown in (25).

$$\mathcal{L}[\underline{Q}_{t,s}(\xi_{t,s})] = \max_{\pi_{t,s}^x, \pi_{t,s}^p} \Theta_{t,s} + \pi_{t,s}^x \top \mathbf{x}_{t-1,s} + \pi_{t,s}^p \top \mathbf{p}_{t-1,s}^\nabla$$

$$\text{where: } \Theta_{t,s} = \min \text{obj}_{t,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) + \underline{Q}_{t+1,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla) + \theta_{t,s} \quad (25)$$

$$\text{s.t. } \theta_{t,s} \geq -(\pi_{t,s}^x \top \mathbf{a}_{t,s} + \pi_{t,s}^p \top \boldsymbol{\beta}_{t,s})$$

$$\mathbf{a}_{t,s} = \mathbf{x}_{t-1,s}, \boldsymbol{\beta}_{t,s} = \mathbf{p}_{t-1,s}^\nabla$$

$$(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) \in \Upsilon_t(\xi_{t,s}, \varsigma_{t,s})$$

$$\mathbf{x}_{t,s} \in \mathbb{B}^m, \mathbf{p}_{t,s}^\nabla \in \mathbb{B}^r, \mathbf{y}_{t,s} \in \mathbb{R}^n, \mathbf{a}_{t,s}, \boldsymbol{\beta}_{t,s} \in \mathbb{R}^{m(r)}$$

In (25), $\mathbf{a}_{t,s}$ and $\boldsymbol{\beta}_{t,s}$ are auxiliary variables. Due to the non-smooth convex characteristic of the lower approximation problem (25), several methods such as the subgradient method [36] can be employed to effectively solve it. Although in Ref. [35], a level method is presented to address problem (25) within the framework of SDDiP, which shows a pronounced enhancement in computational efficiency compared to the classical subgradient approach when the sample paths (scenario trees) surpassing 3, this study leans towards the classical subgradient method to solve problem (25). The reason is the proposed enhanced FRDDP methodology screens the wind power output at each stage based on the worst-case scenario, which makes the sample path in it is only 1. As corroborated by

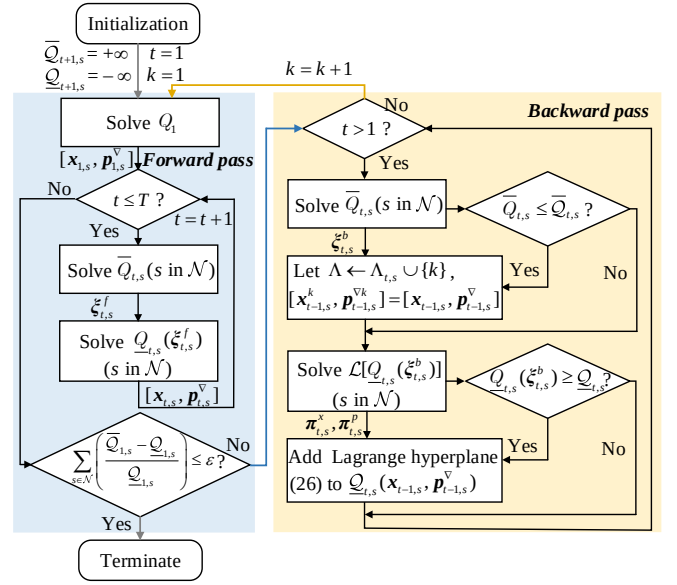


Fig. 2. Overall solution flowchart of enhanced FRDDP

numerical assessments in [35] and analyzes of [31], for a sample path of 1 or 2, the classical subgradient method's performance is on par with the level method. Additionally, prevalent toolkits in languages like Julia exist, facilitates the Lagrangian dual optimization using the subgradient strategy. Hence, driven by the efficient and programming friendly characteristics, this study employs the subgradient method to address problem (25).

After solving (25), the coefficient of Lagrangian hyperplanes $\pi_{t,s}^{x*}$ and $\pi_{t,s}^{p*}$ with the objective value $\Theta_{t,s}^*$ can be obtained. The Lagrangian hyperplanes are formulated as

$$\underline{Q}_{t,s}(\mathbf{x}_{t-1,s}, \mathbf{p}_{t-1,s}^\nabla) \geq \Theta_{t,s}^* + \pi_{t,s}^{x* \top} \mathbf{x}_{t-1,s} + \pi_{t,s}^{p* \top} \mathbf{p}_{t-1,s}^\nabla \quad (26)$$

C. RIA-based upper approximation

The RIA method was proposed for FRDDP in our prior research to improve the computational performance of upper approximation [25]. Compared to the internal approximation (IA) method used in conventional RDDP, RIA avoids the over-complicated extreme points enumeration via constructing a linear-based approximate convex hull as the upper bound of cost-to-go functions. The detailed comparison of IA and RIA can be referred to [25]. In the enhanced FRDDP, the RIA based upper approximation is employed to construct the upper bound of the binary transformed cost-to-go functions, where its effectiveness is demonstrated in this section. The RIA formulated upper problem is shown by (27).

$$\bar{Q}_{t,s} = \max_{\xi_{t,s} \in \Xi_t} \min \text{obj}_{t,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) + \bar{Q}_{t+1,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla)$$

$$\text{s.t. } \bar{Q}_{t+1,s}(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla) = \sum_{\ell \in \Lambda_{t,s}} \mathcal{G}_\ell \lambda_\ell + \mathbf{J}_{t,s}^\top (\mathbf{A}_{t,s}^+ + \mathbf{A}_{t,s}^-)$$

$$(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla, \mathbf{y}_{t,s}) \in \Upsilon_t(\xi_{t,s}, \varsigma_{t,s})$$

$$\sum_{\ell \in \Lambda_{t,s}} \lambda_\ell = 1, \lambda_\ell \geq 0, \forall \ell \in \Lambda_{t,s} \quad (27)$$

$$\sum_{\ell \in \Lambda_{t,s}} \lambda_\ell [\mathbf{x}_{t,s}^\ell; \mathbf{p}_{t,s}^{\nabla \ell}] + \mathbf{A}_{t,s}^+ - \mathbf{A}_{t,s}^- = [\mathbf{x}_{t,s}; \mathbf{p}_{t,s}^\nabla]$$

$$\mathbf{A}_{t,s}^+, \mathbf{A}_{t,s}^- \geq 0, \mathbf{A}_{t,s}^+ \in \mathbb{R}^{m+r}, \mathbf{A}_{t,s}^- \in \mathbb{R}^{m+r}$$

$$\mathbf{x}_{t,s} \in \mathbb{B}^m, \mathbf{p}_{t,s}^\nabla \in \mathbb{B}^r, \mathbf{y}_{t,s} \in \mathbb{R}^n, \lambda_\ell \in \mathbb{R}^1$$

In the RIA based approximate convex hull, when the iterative dispatch decisions are located inside the existing sample points, the upper bound of cost-to-go function is denoted by a convex combination of sample points with λ_ℓ as coefficients; otherwise, it is penalized by two large number boundary lines with adaptive slope $J_{t,s}$ [25]. A similar K -adapted method is employed in [26] to complete the upper approximation. However, it ignores the situation when the dispatch decision is located outside the existing sample points which leads the upper bound away from global Lipschitz [32]. To handle this problem, [32] presents a global Lipschitz dual inner approximation (DIA) method, which introduces an inner bilevel problem for the upper bound of SDDP. Besides, if the recourse variables are linear, DIA can be regarded as the dual counterpart of RIA [25]. However, since the introduction of integer variables expressing the FA unit states hinders the dual conversion of DIA, the inner bilevel problem of DIA results in the intractability of upper approximation.

In comparison, the inner problem of (27) is already a single layer optimization, making RIA more suitable for the DP based multi-stage problem with integer resource variables. The cost-to-go function is assumed to be convex in the aforementioned description. However, there is no discussion of the efficacy of RIA for binary transformed nonconvex cost-to-go functions. Fig. 3 demonstrates the applicability of enhanced FRDDP.

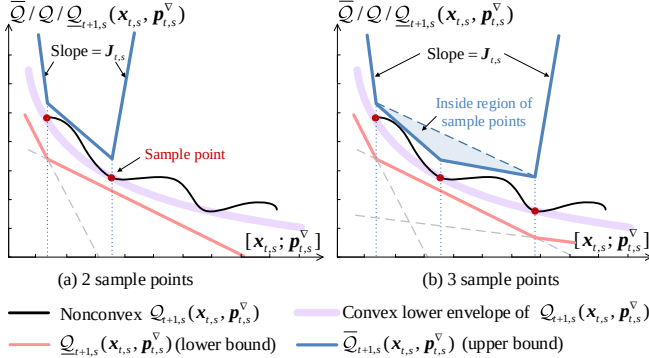


Fig. 3. Illustration of the effectiveness of enhanced FRDDP

In Fig. 3, since the convex lower envelope coincides with the cost-to-go functions at all sample points of dispatch decisions, the upper approximation to binary transformed cost-to-go functions can be equivalent to its lower envelope. Additionally, the RIA technique is proved to be tight and finite for the upper approximation of any convex cost-to-go function [25]. Therefore, although the binary transformed cost-to-go functions are nonconvex, the finite convergence of the enhanced FRDDP is guaranteed by the effectiveness of Lagrangian hyperplanes and the RIA method.

For the upper approximation max-min problem (27), when its inner-layer problem formulation is LP (meaning all recourse variables in the multi-stage problem are continuous), the problem (27) can be recast into a single-layer MILP problem leveraging either the dual theorem with Big-M method or the Karush-Kuhn-Tucker (KKT) optimality conditions. However, a more efficient vertex enumeration method for $\xi_{t,s} \in \Xi_t$ is recommended in Ref. [24]. This recommendation stems from the inherent decoupling of problems at each stage within the DP framework, which subsequently results in an exceedingly fewer number of extreme points for each Ξ_t . In the proposed DMSR

model, since the integer recourse variables are incorporated in each stage, the inner problem of (27) is characterized as MILP. Due to this, both the duality theorem and KKT conditions don't apply. Therefore, the only viable and efficient solution method is vertex enumeration.

D. Parallel computation of multiple scenarios

In the DMSR model, the state variables $x_{1,s}$ and $p_{1,s}$ for each scenario s , calculated by problem (2), are delivered to problem $Q_{t,s}$ ($t \geq 2$) in corresponding scenarios. Here, the N scenario problem can be handled in parallel since the recursive problems from stage 2 to T in each contingency scenario are independent. When the latest scenario's stage-2 problem is finished in the *backward pass*, the aggregated set of convex hulls and hyperplanes are delivered to refine the upper and lower bounds of first stage cost-to-go functions for $s \in N$. The schematic diagram illustrating the parallel computing framework within the enhanced FRDDP is depicted in Fig. 4.

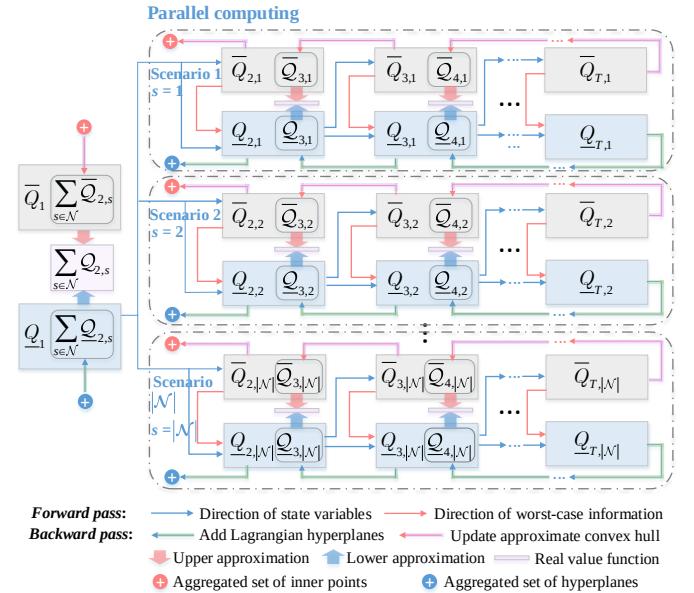


Fig. 4. Schematic diagram of the parallel computing framework

E. Discussion for FA units with maximum/minimum start and stop constraints

It is worth mentioning that when the maximum/minimum start/stop durations for FA units span beyond two dispatch intervals, there's a coupling of state variables across more than two stages, rendering the enhanced FRDDP approach inapplicable. An effective resolution for this challenge can be drawn from reference [35], specifically from formula (5a) - (5d) therein. In this method, the number of state variables representing the FA state in each stage is expanded to encompass the number of stages that overlap with the maximum/minimum start-stop durations. This modeling enhancement ensures that state variables are only interconnected in consecutive stages, even when the FA state constraints span beyond two stages.

IV. REAL-TIME APPLICATION OF THE PROPOSED PROCEDURE

A. Offline solution

The first step to use the proposed approach for the practical real-time dispatch of power systems is to obtain the mature

cost-to-go functions at each stage, which is completed by employing the enhanced FRDDP to solve the DMSR model. This procedure can be carried out offline or in the day-ahead of operating dispatch since it is relatively time-consuming.

We define the following t -stage expected cost-to-go function by the solution of DMSR model that represents the t -period dispatch policies considering the variable wind power generation and subsequent period contingencies.

$$\widehat{Q}_{t+1}(\mathbf{x}_t, \mathbf{p}_t^\nabla) = \sum_{s \in \mathcal{N}} \mathcal{P}_s \cdot \underline{Q}_{t+1,s}^*(\mathbf{x}_{t,s}, \mathbf{p}_{t,s}^\nabla) \quad (28)$$

where, $\underline{Q}_{t+1,s}^*$ stands for the lower bound of cost-to-go function in stage t and scenario s , which is close enough to the real cost-to-go function.

B. Real-time dispatch

The real-time dispatch decisions can be made swiftly after obtaining the dispatch policies from period 1 to T . For example, the following single-stage MILP problem is solved to yield the t -period state adjustment of FA units and other generation decisions:

$$\begin{aligned} \min \quad & \text{obj}_t(\mathbf{x}_t, \mathbf{p}_t^\nabla, \mathbf{y}_t) + \widehat{Q}_{t+1}(\mathbf{x}_t, \mathbf{p}_t^\nabla) \\ \text{s.t.} \quad & (\mathbf{x}_t, \mathbf{p}_t^\nabla, \mathbf{y}_t) \in \Upsilon_t(\xi_t^r, \varsigma_t^r) \\ & \mathbf{x}_t \in \mathbb{B}^m, \mathbf{p}_t^\nabla \in \mathbb{B}^r, \mathbf{y}_t \in \mathbb{R}^n \end{aligned} \quad (29)$$

where, ξ_t^r and ς_t^r are the real happened forecast errors of wind power generation and interruption events at period t .

The reverse binarization method [35] can restore the binary decision $\mathbf{p}_t^\nabla$ to the continues counterpart. Then, the intra-day real-time dispatch can be calculated by following the guidance of dispatch policies provided by the solution of T single-stage problems. The proposed procedure provides a more efficient and higher quality solution than that of the MPC approach, since it avoids the introduction of the extra look-ahead horizon and overcomes the short-sight defect.

Compared to the SO based cost-to-go functions applied as dispatch policies [19], [37], the combined SO-RO dispatch policy, shown in (28), omits massive wind power scenario trees and provides a deterministic and repetitive solution using the RDDP framework. Compared to the RO based counterpart [20], [25], the combined SO-RO dispatch policy utilizes the probabilities of future contingencies by aggregating multi-parallel scenarios, which enhance the resilience of dispatch decisions. The summarized processes for offline solution and online dispatch are provided in Fig. 5.

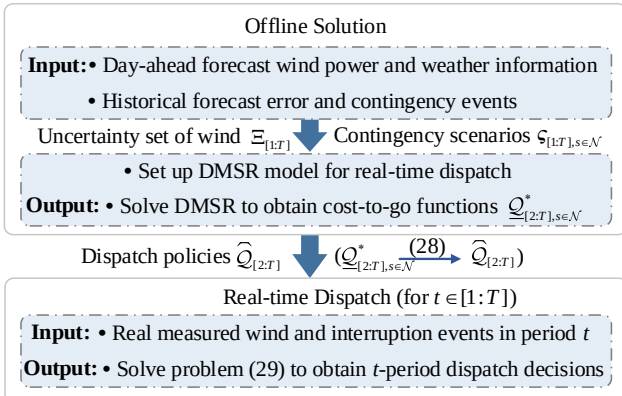


Fig. 5. Application framework of offline solution & online dispatch

V. CASE STUDY

A. Simulation setting

The modified IEEE 14-bus (MI14B), IEEE 118-bus (MI118B) and IEEE 300-bus (MI300B) systems, are intended to validate the performance of the proposed model and algorithm. The topology of MI14B system is drawn in Fig. 6, which consists of 2 wind farms (WF), 2 BESS, 2 conventional units and 2 FA units. The day-ahead forecast contingency, detailed equipment parameters, and the complete transmission information of the three test systems are available in [38]. Typhoon is considered as an extreme weather event in each case, whose influence will persist from 00:00 until 18:00, and the time interval for real-time dispatch is 15min. The test code is implemented on JuMP.jl toolkit of the Julia language, and the optimization problems are solved by Gurobi Optimizer 8.1.1 on a server with CPU Xeon E5-2678, 64 Gb RAM.

We consider the following cases which consider the other three real-time dispatch approaches for comparison:

Case I: MPC-based real-time dispatch, where the two-stage hybrid SO-RO scheme is utilized to model variable wind power generation and contingencies.

Case II: DP formulated multi-stage SO for real-time dispatch, which replaces the wind uncertainty set in DMSR with scenario trees [37] and is solved by SDDiP.

Case III: DP formulated multi-stage RO for real-time dispatch, where the DMSR model is solved in each contingency scenario to screen out the worst-case dispatch policies.

Case IV: The proposed DMSR model, which combines the SO and RO in the DP formulated multi-stage scheme.

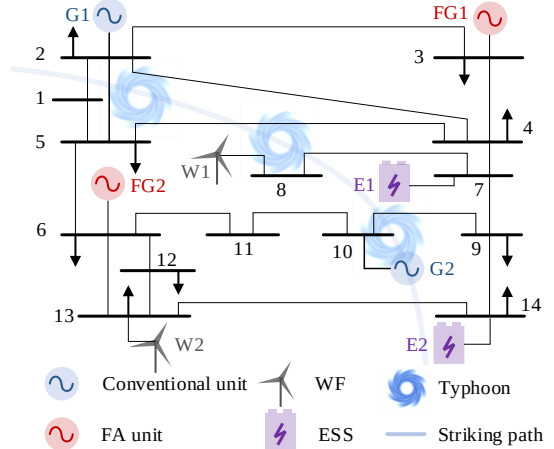


Fig. 6. Topology of the MI14B system

B. MI14B test system

For MI14B system, the factors of wind uncertainty set in the DMSR model are configured to $\Gamma_1 = 2$, $\Gamma_2 = 1.5$, which respectively represent the fluctuation range and spatial correlation of available wind power generation. Based on the historic data, the line outage probability is configured through the fragility curve, where the forecast route is shown in Fig. 6. Then 8 representative contingency scenarios with their probabilities are screened out via Monte Carlo sampling, which are shown in Fig. 7 for the DMSR model. In addition, the convergence criterion of enhanced FRDDP algorithm is set as $\varepsilon = 0.001$. The solution of dispatch policies for 96 periods are obtained in 475.53 seconds using the offline solution process.

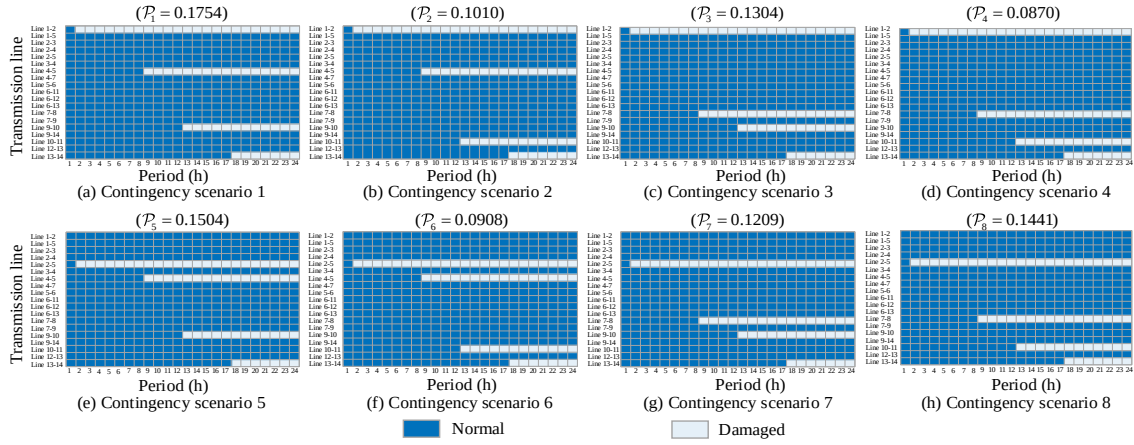


Fig. 7. Inputted contingency scenarios and the corresponding probabilities according to the striking route of typhoon

The in-sample results of DMSR model are analyzed to validate the effectiveness of dispatch decisions considering the worst-case wind generation. The typical scenario 7 in Fig. 7 is considered, where Buses 2-5, 7-8, 9-10 and 13-14 are on outage in periods 2:00, 9:00, 13:00 and 18:00. Fig. 8 displays the in-sample situation of wind fluctuation and consumption within the constructed PCA-based uncertainty set.

As shown in Fig. 8, when the wind power forecast is high, the worst-case realization of available wind power generation would be as high as possible to increase the large wind curtailment (WC). For example, the WF1 output is abandoned after 9:00 period once line 7-8 is on outage, which is otherwise at its upper bound of available power. Also, the variable wind power generation could reach its lower limit to cause critical load shedding when the generation is not abundant.

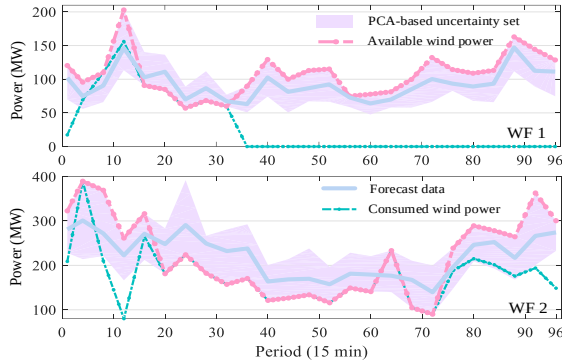
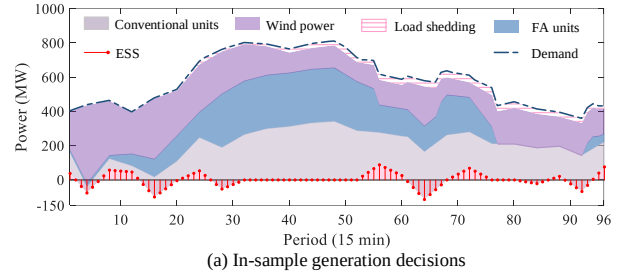
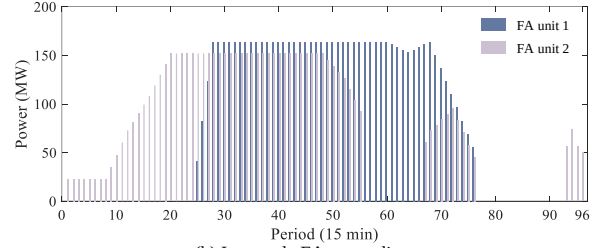


Fig. 8. In-sample fluctuation and consumption of wind power generation

To screen out the worst-case scenario, when the fluctuation range remains minimal, the available wind power often peaks at the upper bound during high generation periods. This saturation renders the system incapable of fully consuming the wind generation, leading to significant wind curtailment penalties. On the other hand, with a broader fluctuation range, the available wind power tends to oscillate between the upper and lower thresholds in successive stages. Such rapid fluctuations challenge both the generation units and the ESS, pushing them to their ramping speed limits, resulting in both wind curtailment and load shedding. These dynamics indicate that, even without budget constraint on the wind power uncertainty set, addressing the worst-case scenario is intricate. Such scenarios demand an optimized solution rather than a pre-emptive determination.



(a) In-sample generation decisions



(b) In-sample FA state adjustment

Fig. 9. In-sample dispatch decisions of DMSR model

Fig. 9(a) demonstrates the optimal in-sample generation decisions of the DMSR model and Fig. 9(b) provides the corresponding state adjustments of FA units. During the operation process, FA units would start up and shut down flexibly to accommodate the variable wind power generation, which would maintain the total load shedding (LS) at a low level. Furthermore, the DMSR model is compared with Cases I-III on the performances of application in the real-time dispatch. Specifically, 500 probably happened contingency scenarios, where each contains 10,000 possible wind power scenarios, are sampled by the Monte Carlo method for the out-of-sample tests. The statistical average results are summarized in Table I.

TABLE I
AVERAGE OUT-OF-SAMPLE RESULTS OF CASE I-IV ON MI14B

Case	Offline solution time (sec.)	Fuel cost ($10^3\$$)		WC+LS cost ($10^3\$$)	Total cost ($10^3\$$)
		FA units	Conventional units		
I	\	35.09	35.24	58.39	129.72
II	1,064.08	24.85	22.50	35.52	82.87
III	331.80	39.31	19.77	46.68	105.76
IV	475.53	30.28	21.63	39.47	91.38

Moreover, the out-of-sample studies are extended to cases where the wind power generation capacity is increased or decreased by 10%, and the average and the maximum values in

TABLE IV
OUT-OF-SAMPLE RESULTS OF DISPATCH DECISIONS ON MI118B AND MI300B

Test System	Case	FA generation cost ($10^3\$$)		LS cost ($10^3\$$)		Total cost ($10^3\$$)	
		Avg.	Max.	Avg.	Max.	Avg.	Max.
MI118B	I	299.51	497.36	392.03	842.67	1,610.21	2,349.38
	II	307.94	592.09	360.92	612.32	1,005.04	2,173.21
	III	381.80	438.80	453.57	493.14	1,430.16	1,664.55
	IV	354.07	501.24	394.65	521.01	1,132.18	1,842.29
MI300B	I	935.44	1,553.37	1,241.83	1,997.27	4,083.36	6,811.40
	II	977.90	1,744.60	632.46	1,729.24	3,194.80	6,062.79
	III	1,124.98	1,393.46	1,045.40	1,266.71	3,866.72	5,076.12
	IV	1,095.79	1,651.51	882.23	1,391.56	3,335.98	5,285.98

Cases I-IV are compared. The results are depicted in Fig. 10. In Table I and Fig. 10, there is a dispatch decision myopic because the MPC method applied in Case I has a limited rolling window and tends to track a predefined plan for flexible sources [39]. Comparatively, Cases II-IV would significantly improve the cost-effectiveness of dispatch decisions when they consider the whole-period uncertainties through the DP framework. Among Cases II-IV, Case II performs the best in reducing the average load shedding and wind curtailments; however, it increases the number of problems to be solved in scenario trees.

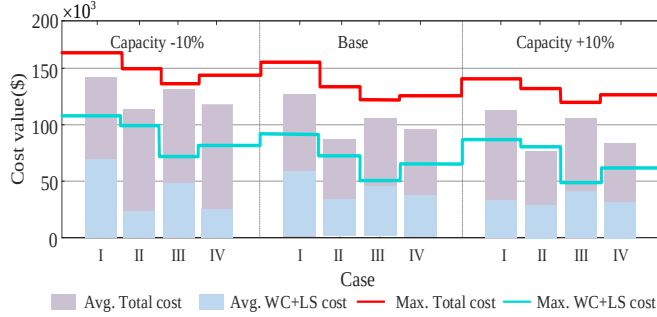


Fig. 10. Out-of-sample statistic outcomes at different wind penetration levels

Although Case III makes FA unit decisions directly in the worst contingency to behave slightly more economical in the maximum cost of LS and WC than Case IV, the solution in average level is overly conservative since ignoring the probability of contingency occurring. Benefiting from the combination of SO and RO, Case IV can achieve a solution that balances computational complexity and conservatism.

To evaluate the proposed data-driven uncertainty set (5), we compared it with three sizes of traditional Box uncertainty sets within the DMSR model. Table II displays the comparison of offline solution times and average out-of-sample performances. The data-driven set slightly prolongs offline solution times due to increased extreme points (as shown in Fig. 1 with two wind farms). However, its scheduling results are more cost-effective, showcasing that it can construct a more precise fluctuation boundary of wind power compared to the traditional Box set.

TABLE II
PERFORMANCES OF DMSR MODEL BASED ON DIFFERENT UNCERTAINTY SETS

Uncertainty set ([$-a$, a] denotes the error of ξ in Box set)	Offline solution time (sec.)	Fuel cost ($10^3\$$)	WC+LS cost ($10^3\$$)	Total cost ($10^3\$$)
Box [-15%, 15%]	367.01	50.74	49.30	100.04

Box [-20%, 20%]	369.22	60.11	37.91	98.02
Box [-25%, 25%]	363.45	67.86	35.85	103.71
Data-driven	475.53	51.91	39.47	91.38

C. MI118B and MI300B test systems

Moreover, large-scale systems MI118B and MI300B are introduced to further validate the effectiveness and scalability of proposed DMSR model, where the parameters configuration of DMSR model keep consistent with the MI14B system. Ref. [38] aggregates the day-ahead forecast information of demand and wind generation, detailed parameters and capacities of each equipment in MI118B and MI300B systems. Similar to MI14B, 200 probably happened contingency situations with 5,000 possible wind generation scenarios are sampled for the out-of-sample tests. It is worth mentioning that situations wherein $|G_W|$ wind farms connect to a single node versus those where $|G_W|$ wind farms attach to $|G_W|$ distinct nodes might yield divergent optimization outcomes due to transmission line constraints. However, such configurations do not amplify the computational intricacy of the overall model. Moreover, the structural validity of the model formulation and the efficacy of the enhanced FRDDP algorithm remain intact.

Since the MPC model in Case I doesn't need to obtain the dispatch policies offline, the time consumption of offline solution in Case II-IV and real-time computation process of Case I-IV are compared in Table III. Compared to Case III and IV, since the production of wind power's scenario trees in Case II exponentially increases the computational burden, the offline time consumption of it is higher. From the perspective of online computation, the calculation burden of Case II-IV have a notable decrease. The reason is the policy-guided online computation embeds all the future information into the mature cost-to-go functions, which avoids introducing the extra rolling horizon of MPC for coping with uncertainties. In Table IV above, we compare the average and maximum cost of FA generation, LS and total cost among the out-of-sample results in Case I-IV. Table IV has similar results with Table I, indicating that Case IV can balance the resilience of dispatch decisions in the extreme and normal conditions.

TABLE III
COMPUTATIONAL PERFORMANCES FOR DIFFERENT CASES

Case	MI118B		MI300B	
	Offline run time (sec.)	Online run time (sec./ period)	Offline run time (sec.)	Online run time (sec./ period)

I	\	73.39	\	102.44
II	5,639	5.80	23,787	8.16
III	1,972	3.91	4,258	4.88
IV	2,284	4.25	5,601	5.07

A non-flexible real-time dispatch (NF) model [25] is introduced to demonstrate the benefits of intra-day adjusting the state of FA units for enhancing the power system resilience in extreme conditions. In the NF model, we employ the same scenario-based technique of this study to simulate the contingency events, but decide the commitment of FA units during day-ahead, which denotes that the mature cost-to-go functions in NF model not contains the information guiding the state adjustment of FA units. Then, the same number of Monte Carlo sampling scenarios are used to compare the NF model and Case IV. The outcomes of numerical experiments are provided in Table V. In Table V, the FA unit commitment in real-time provides higher day-ahead economic and environmental benefits. Here, the NF model makes FA unit commitment for contingency and wind power generation forecasts, while the DMSR model adjusts its state after the current time-slot uncertainty is revealed.

TABLE V
COMPARISON FOR ADJUSTING FA UNIT STATES IN DIFFERENT TIMESCALES

Items (Avg.)	MI118B		MI300B	
	NF model	Proposed model	NF model	Proposed model
WC cost ($10^3\$$)	159.46	106.29	406.63	339.89
LS cost ($10^3\$$)	542.23	394.65	1,143.19	882.23
Load supplying ratio (%)	90.30	95.92	89.91	96.41
Total cost ($10^3\$$)	1,489.36	1,132.18	4,246.27	3,335.98

On the MI118B system, we assessed the DMSR model for different wind energy and ESS penetration levels. Fig. 11 illustrates average out-of-sample results for scenarios adjusting wind and ESS capacities by $\pm 10\%$ or $\pm 20\%$. As wind and ESS penetration grows, system load shedding reduces, signifying the enhancement of system resilience. However, the influence of the total dispatch cost is more multifaceted. At lower ESS penetration levels, the total dispatching cost exhibits an initial dip followed by a surge as the penetration level climbs. This is attributed to the fact that the adjustable capacity of the existing equipment of the system cannot accommodate excessive wind power, incurring sizable curtailment penalties. Yet, at elevated ESS levels, such as $+10\%$ and $+20\%$, ramping up wind penetration leads to a continued reduction in total dispatch cost. This underlines the role of ESS in improving the system's flexibility to handle wind power fluctuations.

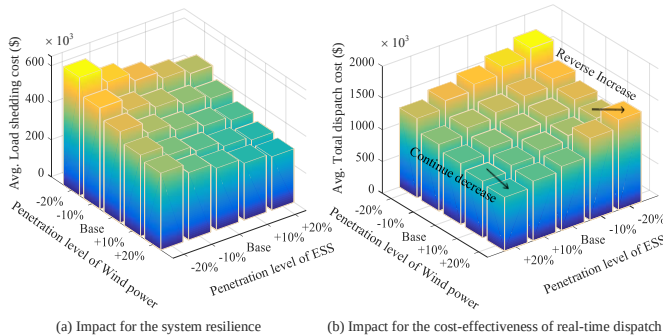


Fig. 11. Performance of DMSR model for different wind energy and ESS penetration levels on MI118B system

According to (28), the lower bound of the cost-to-go functions exhibits two properties that qualify it for dispatch policy applications. Firstly, once convergence is achieved through the enhanced FRDDP iterations, this lower bound closely aligns with the real cost-to-go functions, thereby seamlessly integrating anticipated future uncertainties into the present decision-making process. Furthermore, its piece-wise function formulation enhances the computational efficiency. Similarly, the RIA based upper bound embodies these attributes post the enhanced FRDDP convergence. To ascertain the efficacy of dispatch policies informed by both the converged lower and upper bounds, especially in the context of online computational agility for real-time dispatch scenarios, out-of-sample tests were performed. The encompassing solution quality and computational time consumption are comprehensively demonstrated in Table VI.

TABLE VI
REAL-TIME DISPATCH COMPARISON FOR DIFFERENT BOUND BASED DISPATCH POLICIES

Items (Avg.)	MI118B		MI300B	
	Lower bound	Upper bound	Lower bound	Upper bound
Online run time (sec. /period)	4.25	7.66	5.07	9.03
Total dispatch cost ($10^3\$$ /day)	1,132.18	1,132.15	3,335.98	3,336.22

In Table VI, it is evident that the optimality between the dispatch policies based on the lower bound and the upper bound is nearly identical, underscoring the efficient convergence of the enhanced FRDDP within a minimal gap tolerance. In terms of computational efficiency, the dispatch policies derived from the hyperplanes based lower bound showcase reduced complexity, making them more appropriate for deployment in large-scale systems.

D. Iterative performances of enhanced FRDDP

The iterative upper and lower bounds of the DMSR offline solution in the enhanced FRDDP are shown in Fig. 12. According to Fig. 12, the convergence of enhanced FRDDP is preserved although the problem size has grown. Moreover, in contrast to similar stochastic algorithms, like SDDiP, the enhanced FRDDP algorithm is a deterministic algorithm in which the sample points are obtained and the boundaries are adjusted in an orderly manner. This characteristic makes the dispatch policies repetitious and deterministic, which is the common practice in industrial applications.

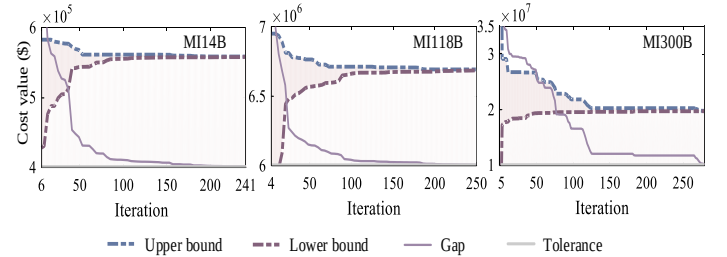


Fig. 12. Iterative process of enhanced FRDDP

Table VII shows the calculation performance of the enhanced FRDDP for six parametric configurations in the data-driven wind power uncertainty sets of DMSR model. Numerical

studies are tested for MI118B and MI300B systems. Table VII shows that the enhanced FRDDP algorithm in each range of uncertainty sets can successfully converge to a value below the tolerance. Moreover, although the iteration numbers vary, the solution time is only marginally impacted.

TABLE VII
CALCULATION PERFORMANCE OF ENHANCED THE FRDDP FOR DIFFERENT UNCERTAINTY SETS

(Γ_1, Γ_2)	MI118B			MI300B		
	Iteration num.	Gap (%)	Time (sec.)	Iteration num.	Gap (%)	Time (sec.)
(2.0, 1.5)	250	0.097	2,284.96	279	0.032	5,601.25
(2.0, 2.0)	240	0.075	2,149.98	272	0.094	5,517.91
(2.5, 1.5)	246	0.032	2,233.11	265	0.082	5,497.41
(2.5, 2.0)	263	0.089	2,324.14	258	0.038	5,349.26
(3.0, 2.0)	234	0.095	2,116.02	261	0.066	5,465.83
(3.0, 2.5)	259	0.077	2,308.12	280	0.052	5,692.20

VI. CONCLUSION

In this paper, the DMSR model is proposed for the resilient real-time dispatch of power systems with FA units in extreme events. In DMSR, the two-stage hybrid SO-RO model is extended to the DP formulated multi-stage scheme to enforce the non-anticipativity of dispatch decisions. Case studies show the combination of SO and RO achieves a solution that balances computational complexity with conservatism. The enhanced FRDDP is proposed to solve the DMSR model, which can handle the non-convex challenges arising from the introduction of integer variables denoting the FA unit states. The enhanced FRDDP shows good convergence and speed using the test systems, which reveals its potential applications to real-world power system operations.

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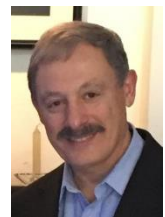
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