High-accuracy emulators for observables in $\Lambda$CDM, $N_{\text{eff}}, \Sigma m_{\nu}$, and $w$ cosmologies

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ABSTRACT

We use the emulation framework CosmoPower to construct and publicly release neural emulators of cosmological observables, including the cosmic microwave background (CMB) temperature and polarization power spectra, matter power spectrum, distance-redshift relation, baryon acoustic oscillation (BAO) and redshift-space distortion (RSD) observables, and derived parameters. We train our emulators on Einstein–Boltzmann calculations obtained with high-precision numerical convergence settings, for a wide range of cosmological models including $\Lambda$CDM, $w$CDM, $\Lambda$CDM + $N_{\text{eff}}$, and $\Lambda$CDM + $\Sigma m_{\nu}$. Our CMB emulators are accurate to better than 0.5 per cent out to $\ell = 10^3$, which is sufficient for Stage-IV data analysis, and our $P(k)$ emulators reach the same accuracy level out to $k = 50$ Mpc$^{-1}$, which is sufficient for Stage-III data analysis. We release the emulators via an online repository (CosmoPower Organisation), which will be continually updated with additional extended cosmological models. Our emulators accelerate cosmological data analysis by orders of magnitude, enabling cosmological parameter extraction analyses, using current survey data, to be performed on a laptop. We validate our emulators by comparing them to class and camb and by reproducing cosmological parameter constraints derived from Planck TT, TE, EE, and CMB lensing data, as well as from the Atacama Cosmology Telescope Data Release 4 CMB data, Dark Energy Survey Year-1 galaxy lensing and clustering data, and Baryon Oscillation Spectroscopic Survey Data Release 12 BAO and RSD data.

Key words: methods: data analysis – methods: statistical – cosmic background radiation – large-scale structure of Universe.

1 INTRODUCTION

Next-generation cosmic microwave background (CMB) surveys, such as the Simons Observatory (SO; Simons Observatory 2019), CMB-S4 (Abazajian et al. 2019; CMB-S4 2022), and CMB-HD (Sehgal et al. 2019), will provide high-signal-to-noise measurements of the CMB temperature and polarization anisotropy down to small scales, characterized by their power spectra. The final seasons of the Atacama Cosmology Telescope (ACT; Henderson et al. 2016) and of the South Pole Telescope (SPT; Benson et al. 2014) will also make progress in that direction in the forthcoming years. In addition, current and next-generation galaxy surveys, including the Dark Energy Survey (DES; Dark Energy Survey Collaboration 2016), the Kilo-Degree Survey (KiDS; Kuijken et al. 2015), the Hyper Suprime-Cam Survey (HSC; Aihara et al. 2018), the Vera C. Rubin Observatory (Ivezić et al. 2019), Euclid (Laureijs et al. 2011), Roman (Spiegel et al. 2015; Akeson et al. 2019), or SPHEREx (Doré et al. 2014), probe the matter power spectrum via weak gravitational lensing and galaxy clustering and spectroscopic galaxy surveys such as the Baryon Oscillation Spectroscopic Survey (BOSS, Dawson et al. 2013) and the Dark Energy Spectroscopic Instrument (DESI; DESI Collaboration et al. 2016) also probe the baryon acoustic oscillation (BAO) feature and cosmological velocities via redshift-space distortions (RSD).

Einstein–Boltzmann codes such as camb (Lewis, Challinor & Lasenby 2000) and class (Blas, Lesgourgues & Tram 2011; Lesgourgues 2011a) are routinely used to accurately compute linear-theory cosmological power spectra, as well as the background cosmic evolution (e.g. the distance-redshift relation). These codes also implement various prescriptions for the calculation of non-linear corrections to the matter power spectrum and quantities derived therefrom, e.g. Halofit (Smith et al. 2003) and HMCode (Mead et al. 2015, 2016, 2021). The computation of these cosmological quantities through a Boltzmann code represents a significant computational requirement in Markov Chain Monte Carlo (MCMC) methods, traditionally used to extract constraints on cosmological
parameters. MCMC methods require $\sim 10^2$–$10^6$ calls to a Boltzmann code as they achieve convergence through a similar number of likelihood evaluations. The problem is exacerbated if one requires high numerical accuracy in the computation of the cosmological quantities obtained from Boltzmann codes, as needed for upcoming surveys (e.g. McCarthy, Hill & Madhavacheril 2022).

Several groups have developed emulators of cosmological quantities in order to accelerate the MCMC calculations by bypassing the call to a Boltzmann code with a faster algorithm (e.g. Auld et al. 2007; Fendt & Wandelt 2007; Albers et al. 2019; Arich, Angulo & Zennaro 2021; Ginnther et al. 2022; Mootoovaloo et al. 2022; Bonici, Bianchini & Ruiz-Zapatero 2023). Recently, Spurio Mancini et al. (2022) developed CosmoPower, an emulation framework for cosmological quantities based on TensorFlow neural networks (Abadi et al. 2015) and with an architecture similar to Speculator (Alsing et al. 2020). The accuracy of the CosmoPower emulators presented in Spurio Mancini et al. (2022) was tested on CMB and large-scale structure (LSS) analysis of current and future data. More recently, Balkenhol et al. (2022) constructed CosmoPower emulators for the analysis of the SPT data, covering angular scales up to $\ell = 3000$, trained on high-precision camb spectra and covering LCDM, LCDM+$\Lambda$Neff, and LCDM + Alens models. These were subsequently used in the cand1 package (Balkenhol et al. 2024).

Here we use CosmoPower to provide high-accuracy emulators of CMB temperature (emulator acronym: TT), polarization (TE and EE), and lensing potential power spectra (PP), up to $\ell_{\text{max}} = 10^4$, as well as linear and non-linear matter power spectra (PKL and PKNL) up to $k = 50\ Mpc^{-1}$ in the redshift range $z \in [0, 5]$, the Hubble parameter (H), angular diameter distance (DA), $\sigma_8(z)$ (S8) for $z \in [0, 20]$, and various derived parameters (DEB; see Section 2 for a complete list).

With this suite of emulators, we are able to accurately reproduce marginalized posterior distributions from current CMB and LSS likelihood codes (based on CMB, matter power spectra, and BAO distances) within minutes on a laptop, whereas the original MCMC analyses took hours to days on large computing clusters. To accomplish this, we use the cobaya MCMC sampler and its implemented likelihoods (Torrado & Lewis 2021). Our CosmoPower emulators are wrapped into cobaya via a simple wrapper that will be available online. We reproduce parameter constraints from the ACT Data Release 4 (Aiola et al. 2020; Choi et al. 2020), Planck 2018 temperature, polarization, and lensing potential data (Planck Collaboration 2020a, b), DES-Y1 (Krause et al. 2017; DES Collaboration 2018; Troxel et al. 2018), and BAO distances from BOSS and other surveys (Beutler et al. 2011; Ross et al. 2015; Alam et al. 2017).

Our emulators are explicitly constructed with sufficient accuracy to remain valid for use in upcoming CMB analyses for the foreseeable future, including ACT, SPT, SO, CBMB-S4, and CBMB-HD, as well as ongoing galaxy surveys.

In addition to the emulators, we release the full pipeline to generate them, which can be used by the community to build emulators for other extensions to the standard cosmological model. The cosmological models covered here include $\Lambda$ Cold Dark Matter (CDM), wCDM, where $w$ is the dark energy equation of state; $\Lambda$CDM with varying $N_m$, where $N_m$ is the effective number of relativistic species at recombination; and $\Lambda$CDM with varying neutrino mass $\Sigma_m$. In $\Lambda$CDM, $\Lambda$CDM + $N_m$, and wCDM, we assume one massive and two massless neutrino states, as in the baseline Planck analyses (Planck Collaboration 2020a). In $\Lambda$CDM + $\Sigma_m$, we generate emulators for both the case of three species of neutrinos with degenerate mass, and the case of one massive and two massless neutrino states.

This paper is organized as follows. In Section 2, we outline our methodology, while in Section 3, we study and calibrate accuracy settings of Boltzmann solvers by comparing camb and class. This sets the accuracy target for our emulators, which we validate in Section 4. In Section 5, we illustrate the validity and applicability of our emulators on a variety of likelihoods mentioned above. We conclude in Section 6.

\section*{2 Method}

The general methodology used to construct training and testing sets for our neural network emulators follows that of Spurio Mancini et al. (2022). Here we provide a brief summary and refer the reader to that paper for further details. To compute all of the cosmological quantities on which our emulators are trained and tested we use class v2.9.4.\footnote{\texttt{https://github.com/lesgourg/class.public/issues/494}} We proceed in six steps, as follows.

(i) We specify the cosmological parameters upon which the emulators are built, i.e. the inputs for our neural networks. We choose the following six $\Lambda$CDM parameters, as defined in \texttt{class}:

(a) $\omega_b$, the physical baryon density;
(b) $\omega_cdm$, the physical dark matter density;
(c) $H_0$, the Hubble parameter today;
(d) $\tau$, the reionization optical depth;
(e) $n_s$, the scalar spectral index;
(f) $\ln10^{10}A_s$, where $A_s$ is the amplitude of primordial scalar perturbations.

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Min Value & Max Value \\
\hline
$\ln10^{10}A_s$ & 2.5 & 3.5 \\
$\Omega_{cdm}h^2$ & 0.08 & 0.20 \\
$\Omega_bh^2$ & 0.01933 & 0.02533 \\
$H_0$ [\text{km s}^{-1}\text{Mpc}^{-1}] & 39.99 & 100.01 \\
$\sigma_8$ & 0.8812(0.8) & 1.0692(1.2) \\
$\tau$ & 0.2 & 0.12 \\
$m_\nu$ (eV) & 0. & 2. \\
n & 2. & 0.33 \\
$N_{eff}$ & 1.5 & 5.5 \\
\hline
\end{tabular}
\caption{Parameter ranges used to generate the LHC of cosmological parameters used to compute the training data (see Section 2 for details).}
\end{table}

\footnote{\texttt{https://github.com/cosmopower/trained-models/SPT_high_accuracy}}
Figure 1. Comparison of lensed CMB power spectra computed with \texttt{class} (for default precision as the solid red line and ultra-high precision settings as the dotted yellow line labeled ‘class-pre’), \texttt{CosmoPower}, and \texttt{camb}. We show the dimensionless $\ell(\ell+1)C_{\ell}/2\pi$ for TT, TE, and EE, and $(\ell(\ell+1)C_{\ell}^{PP}/2\pi$ for PP (lensing convergence power spectrum) in a $\Lambda$CDM model with one set of cosmological parameters from the \textit{Planck} 2018 results (see text for details). The shaded areas indicate the forecast 1σ uncertainty for Advanced ACT, SO, and CMB-S4 (see Section 4 for details). This figure illustrates the emulated quantities, the relative difference between the spectra are shown in Fig. 4 (in per cent) and 5 (in units of CMB-S4 statistical error bars). On the EE plot, the thin black solid lines indicate the cosmic variance.

Figure 2. Same settings as Fig. 1 but for matter power spectra. We show the linear matter power spectra on the left and the non-linear spectra on the right (computed according to \texttt{IMCcode} with $\sigma_{8} = 3.13$ and $\Omega_{m} = 0.603$, within \texttt{class}). In both panels, we show the power at three redshifts, $z = 0$, 2.5, and 5 from top to bottom, spanning the range of the emulators. This figure illustrates the emulated quantities, the relative difference between these spectra are shown in percentages in Fig. 6.

In extensions, we also add $w$, $N_{\text{eff}}$, or $\Sigma m_{\nu}$, defined in the previous section.
Note that we opt for $H_0$ rather than for the angular size of the sound horizon at recombination, 1000, (which we obtain as a derived parameter), due to the slightly different definitions of the angular acoustic scale implemented in \texttt{camb} and \texttt{class}4. For MCMC analyses, if one needs to sample over 1000, rather than $H_0$, it is straightforward to do so, e.g. using a shooting method with the derived parameter emulators.5

1In \texttt{class}, $\theta_s$ is defined at the maximum of the visibility function, while in \texttt{camb} it is defined when the optical depth equals unity, and these times are slightly different. The $\theta_s$ of our emulator is the same as that in \texttt{class}.
2This adds an extra 20 ms to each MCMC step and would therefore increase slightly the convergence time of the chains. Note that the shooting method is also the standard way to sample over 1000, in \texttt{class}.

2We specify the range of the parameter values over which we train our emulators, reported in Table 1. These ranges should be conservative enough for all physically relevant analyses, assuming the use of data with similar or better constraining power than \textit{Planck} for the CMB, and DES or BOSS for galaxies. We emphasize that the emulators should not be used outside the ranges of Table 1 or for different cosmological models, as there is no guarantee on their accuracy in such cases. For the $\Lambda$CDM + $N_{\text{eff}}$ model, we let $N_{\text{eff}}$ vary between 1.5 and 5.5. In practice, we vary the parameter $N_{m}$ in \texttt{class} between 0.49 and 4.49 and obtain $N_{\text{eff}}$ as a derived parameter.6 In order to generate the data necessary for the matter power spectrum

$N_{m} = (T_{\text{conn}}/(411))^2$, where $T_{\text{conn}}$ is in units of $T_{\text{camb}}$ today. This is the convention used by \texttt{class}.

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at different redshifts, we add $z_{pk}$, the redshift at which we output the matter power spectra, as an extra varying input parameter to our grid.

(iii) We generate a Latin hypercube (LHC) of the parameter space. We set $N_{\gamma} = 128\,000$ as the number of samples in the LHC, which is the same as the number of spectra we aim at computing. To generate the LHC, we use the python library pyDOE.  

(iv) We use a computing cluster to calculate the $N_{\gamma}$ training samples in parallel. Note that despite the total number of training samples being similar to that needed for an MCMC, the task of generating training samples is embarrassingly parallel, since the computation for a given set of input parameters does not depend on any other – unlike an MCMC, which is a path-dependent calculation in parameter space. Given this, we find that computing resources are more optimally used by running each computation on a single thread (i.e. setting OMP_NUM_THREADS = 1 before running class). The total amount of disc space taken by the training and testing samples is $\approx 150$ GB. Note that for each sample, in addition to the cosmological observables that we want to emulate, we also save 14 derived parameters, such as $\sigma_8$ and 100$\theta_\star$ (see the end of this section for details). For each class of emulators (ACDM + extensions) the computation of the data requires $\approx 48$ h on a modern high-performance computing cluster. (The accuracy settings for the Boltzmann codes are given in Section 3.)

(v) We process the data so that the training samples can be used in the Cosmopower training pipeline. For all quantities (TT, TE, EE, PP, PKL, PKNL, DA, H, SS, and DBR), we select 80 per cent of the samples for training and leave the remaining ones for testing. The TT, EE, PP, PKL, PKNL, H, and DBR data consists of positive numbers with large dynamic ranges; to ease training, we take the logarithm.
of all these quantities before passing them to \cosmo. The TE power spectra are oscillatory, zero-crossing functions, and thus we cannot directly take the logarithm. In this case, we use Principal Component Analysis (PCA), which reduces the dimensionality of the data set and its dynamic range. We retain only 64 principal components for each spectrum. We verify that adding more does not lead to any significant improvement (Spurio Mancini et al. 2022).

(vi) We generate the emulators using the \cosmo functions and verify their accuracy on the test set. Generating each emulator involves training a neural network model. This operation takes $O(1 \text{ hr})$ for each emulator and is best performed on a GPU, to fully take advantage of the acceleration provided by the TensorFlow library, which \cosmo uses for its neural network implementations. We also stress that for a given cosmological model once the training is done and the emulator generated, this step does not need to be repeated. Note that we emulate TT, TE, EE, PP, PKL, PKNL, DA, H, S8, and DER separately. For all emulators, we use dense neural networks with four hidden layers, each with 512 nodes. When building the emulators for TT, TE, EE, PP, DA, H, S8, and DER, we do not include $\tau_{ph}$ in the mapping from parameters to data, since $\tau_{ph}$ has no effect on these quantities. Similarly, we also remove $\tau$ from the parameter sets for the PP, DA, H, S8, PKL, and PKNL emulators. The trained TT, TE, EE, PP, PKL, PKNL, DA, H, S8, and DER emulators are stored as pickle files. The size of each CMB power spectra emulator file is 25.9 MB, except the TE emulator which is lighter because of the PCA and is 6.3 MB. The PKL and PKNL emulators are 4.2 MB; the H, DA, and S8 emulator files are 13.5 MB; and the DER emulator is 3.2 MB. (The size of each emulator file is proportional to the number of data points that are saved: 11 000 multipoles for the CMB TT and EE spectra, the number of PCA weights for the TE spectra, 500 wavenumbers for PKL and PKNL, and 5000 redshifts for DA, H, and S8.)

For illustration, examples of CMB power spectra are shown in Fig. 1; examples of matter power spectra calculations are shown in Fig. 2; and examples of calculations of $H(z)$, $D_A(z)$, and $\sigma_8(z)$, which are used in the BAO and RSD calculations, are shown in Fig. 3.

2.1 Matter power spectrum

To construct the emulators for the linear power spectrum and its non-linear corrections, we add an extra input to the neural networks, namely, the redshift at which the power spectra are computed, $z_{ph}$. This represents an additional parameter, sampled between 0 and 5. For instance, the LHC used for our $\Lambda$CDM emulator has seven dimensions: the six $\Lambda$CDM parameters (cf. Table 1) augmented by $z_{ph}$. For each LHC sample we save the matter power spectra at $z_{ph}$ on a logarithmically spaced $k$-grid between $k_{min} = 10^{-4}$ Mpc$^{-1}$ and $k_{max} = 50$ Mpc$^{-1}$ with 500 points. (See Section 3 for our perturbation and non-linear settings.)

2.2 BAO and redshift-space distortions

Recent galaxy survey data allow us to constrain models based on various BAO distance measures, as well as using RSDs to constrain the parameter combination $f_s$, where $f$ is the growth rate of cosmic structures. To compute BAO distances and $f_s$, we need to save the angular diameter distance $D_A$, the Hubble parameter $H$, and $\sigma_8$ as a function of $z$, as well as the comoving sound horizon at baryon drag $r_s$, which is last in the list of derived parameters discussed above. We save $D_A(z)$, $H(z)$, and $\sigma_8(z)$ on a linearly spaced $z$-grid between $z_{min} = 0$ and $z_{max} = 20$ with 5000 points. The upper bound $z_{max} = 20$ is much higher than what is relevant to galaxy surveys, however we record the high-$z$ distances as they may be useful to other applications, such as studies of reionization or cosmic-dawn 21 cm measurements. With these quantities, we compute BAO distances and $f_s$, straightforwardly (see e.g. Alam et al. 2017). Note that $f_s = -1$
Figure 6. Relative difference of the linear (left) and non-linear (right) matter power spectra between two different settings of CLASS (high precision as the solid red line and ultra-high precision settings as the dotted-dashed yellow line labeled 'class-prec') and CosmoPower with respect to camb in percent.

Figure 7. Relative difference between CosmoPower and the 'true' high-precision CLASS prediction for CMB TT power spectra in per cent (top) and in units of CMB-S4 sensitivity (bottom). See end of Section 4 for details.
Figure 8. Relative difference between CosmoMC and the ‘true’ high-precision CLASS prediction for CMB EE power spectra in percent (top) and in units of CMB-S4 sensitivity (bottom). See end of Section 4 for details.

Figure 9. Relative difference between CosmoMC and the ‘true’ high-precision CLASS prediction for CMB TE power spectra in percent (top) and in units of CMB-S4 sensitivity (bottom). Note that the spikes are simply due to zero crossings of the TE power spectrum. See end of Section 4 for details.

\[ + \frac{z}{d\sigma_\tau(z)/dz} \] where the derivative can be evaluated numerically. This is how we compute \( f_\sigma \), as is done in CLASS.

2.3 Derived parameters

We use 14 commonly used derived parameters, namely:

(i) the angular size of the sound horizon at decoupling \( 100\theta_s \),
(ii) the amplitude of matter clustering \( \sigma_\tau \),
(iii) the primordial Helium fraction \( Y_P \),
(iv) the reionization redshift \( z_{\text{reion}} \),
(v) the number of effective relativistic degrees of freedom in the early Universe \( N_{\text{eff}} \),
(vi) the conformal time at which the visibility function reaches its maximum (i.e. the recombination time) \( t_{\text{rec}} \),
(vii) its associated redshift \( z_{\text{rec}} \),
(viii) the comoving sound horizon at recombination \( r_{\text{s,rec}} \).
(ix) the conformal angular diameter distance to recombination $r_{\text{a,rec.}}$.
(x) the conformal time at which the photon optical depth crosses unity $\tau_*$.
(xi) the redshift $z_*$ at which the photon optical depth crosses unity,
(xii) its associated comoving sound horizon $r_{\text{s,*}}$,
(xiii) and conformal angular diameter distance $r_*$,
(xiv) and the comoving sound horizon at baryon drag $r_d$.

These parameters are saved into a list for each sample in the LHC, with the ordering given above.

3 BOLTZMANN CODE ACCURACY AND SETTINGS

To ensure high precision in our calculations, we carefully study and calibrate the numerical precision parameters in camb and class.\footnote{See Lesgourgues (2011b) for calibration of the default numerical precision settings of class, which are determined to be sufficient for a Planck-like experiment, but require to be adjusted for our purpose, as explained in this section.} For the maximum multipole, we choose $\ell_{\text{max}} = 11,000$ in both codes.
Figure 12. Relative difference between CosmoPower and the 'true' high-precision CLASS prediction for the redshift evolution of the Hubble parameter (top row), angular diameter distance (middle row), and $\sigma_8$ (bottom row) in percent. See end of Section 4 for details.

Figure 13. Relative difference between CosmoPower emulators and the 'true' high-precision CLASS prediction for the derived parameters in percent. Each unit on the x-axis represents a derived parameter as indicated (see page 3 for the definitions of the fourteen parameters). Note that for $N_{\text{eff}}$ the error is null since this parameter is fixed, except in the $\Lambda$CDM + $N_{\text{eff}}$ model (third panel) where it is varied. In $\Lambda$CDM + $N_{\text{eff}}$, $N_{\text{eff}}$ is not an input parameter but a derived parameter (see Section 2) and hence there is a small error.

We chose $\ell_{\text{max}} = 11\,000$ as some forecasts and calculations, e.g. in the context of CMB-HD (Sehgal et al. 2019), require CMB power spectra at such high multipoles. For non-linear corrections to the matter power spectrum, we use HMcode (Mead et al. 2016) with fiducial values $\ell_{\text{min}} = 3.13$ and $n_s = 0.603$ in both codes. These values correspond to a dark matter-only scenario and are the same as in the baseline Planck 2018 analyses (Planck Collaboration 2020a), justifying our choice. For reionization modelling, we kept the default CLASS setting (i.e. the same as in camb).

We remark here that in future work it will be useful to train emulators in which these HMcode parameters are also varied, or for other non-linear regime prescriptions. Indeed, as shown by, e.g. McCarthy et al. (2022), future CMB power spectrum data will be sensitive to the impact of baryonic physics on the matter power spectrum (via CMB lensing); in addition, LSS statistics are sensitive to the non-linear and baryonic prescriptions at high values of $x$.

With emulators that include these parameters, we will be able to marginalize over this uncertainty.

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Figure 14. Comparison of 2D marginalized posterior probability distributions for ACT DR4 cosmological parameters in $\Lambda$CDM, between CosmoPower (empty red contours) and the full class calculation (solid blue contours, taken from Hill et al. 2022). With CosmoPower, the derived parameters $H_0$, $\sigma_8$, and $\Omega_m$ are added in post-processing of the chains. These results are obtained with the 'actpollite' ACT DR4 likelihood (see footnote 16) and Section 5.1 for details.

The TT, TE, EE, and PP spectra computed with camb\textsuperscript{10} are fully converged when using the following parameters (see e.g. Hill et al. 2022; McCarthy et al. 2022):

set_matter_power:

(i) $k_{\text{max}} = 10$
(ii) $k_{\text{per.logint}} = 130$
(iii) nonlinear = True

set_for_lmax:

(i) lens.potential.accuracy = 8
(ii) lens_margin = 2050

set_accuracy:

(i) AccuracyBoost = 2.0
(ii) lSampleBoost = 2.0
(iii) lAccuracyBoost = 2.0
(iv) DoLateRadTruncation = False

We use the camb spectra computed with these settings as a high-accuracy reference. This high-accuracy camb calculation takes $O(1 \text{ min})$ per sample on 16 threads.

\textsuperscript{10} We use camb v1.3.6.
It is possible to match class TT, TE, EE, TE, PKL, and PKNL spectra to camb to better than 0.1 per cent precision (for $\ell < 3000$ and $k < 10\text{Mpc}^{-1}$), by using the precision parameters in the cl\_ref\_pre file in the class GitHub repository (this file ensures convergence at the 0.01 percent level internally to class for TT and EE; Lesgourgues 2011b) and setting $\text{k\_max\_tau0\_over\_l\_max: 15}$ to ensure high accuracy at $\ell > 4000$ as discussed below). This is illustrated in Figs 4 and 6, where this calculation is shown as the dotted-dashed yellow lines, labeled 'class prec'. We refer to it as the ultra-high-precision class prediction. None the less, this computation takes $O(1h)$ which is prohibitively long to generate $O(10^7)$ spectra for training the emulators.

By optimizing the tradeoff between precision and computation time, we find that the minimal settings required in order to match the high-precision camb calculation with class v2.9.4 are obtained by setting the following three parameters:\footnote{Note that these parameters are optimized here, whereas McCarthy et al. (2022) used extremely high-precision class parameters without optimizing the tradeoff between precision and computation time.}

(i) $\text{accurate\_lensing: 1}$
(ii) $\text{k\_max\_tau0\_over\_l\_max: 15}$.
(iii) $\text{perturb\_sampling\_stepsize: 0.05}$
These are the precision parameters that we use in order to generate our emulator training data. The first parameter ensures that the lensed TT, TE, and EE spectra are converged for $\ell > 3000$. Without it, they have non-physical oscillatory features. The second parameter ensures convergence at high-$\ell$ for all the spectra, including the unlensed ones. The third parameter is critical to get high-accuracy spectra over the whole multipole range. It is particularly important to get a converged PP spectra for $\ell < l_{\text{switch limber}}$ (default value in class v2.9.4: $l_{\text{switch limber}} = 10$). Note that out of these three class parameters, only the second one has an impact on the linear and non-linear matter power spectra. This high-precision class calculation takes roughly the same amount of time as the camb one, i.e. $O(1\text{ min})$.

It is important to note that the first version of the emulators that we release in this paper are produced based on CMB and matter power spectra computed assuming RECFAST recombination model (Seager, Sasselov & Scott 1999; Scott & Moss 2009; Chluba, Fung & Switzer 2012), Big Bang Nucleosynthesis model from Parthenope (Conigracio et al. 2018) with a fiducial $N_{\text{eff}} = 3.046$ using v1.2 of the code for a neutron lifetime of 880.2 s identical to standard assumptions of Planck 2017 papers. Furthermore, for the non-linear modelling of matter transfer functions we use hmcodex from Mead et al. (2016). Another assumption that we make here is that the CMB monopole temperature is perfectly known and fixed to 2.7255 K, consistent with COBE/FIRAS measurements (Fixsen et al.)

Figure 16. Comparison of 2D marginalized posterior probability distributions between CosmoPower (blue contours) and the reference camb chains (red contours, dashed line) downloaded from the Planck legacy archive (footnote 20) for the Planck 2018 CMB lensing likelihood + DES-Y1 + BAO analysis. See Section 5.2 for details.
1996). We note that these settings, as well as the precision settings specified here, can affect the best-fitting models of future CMB and LSS experiments, and we advise they should be carefully quoted and specified.

4 EMULATORS

Our trained TT, TE, EE, PP, PKL, PKNL, H, DA, S8, and DER emulators in $\Lambda$CDM, $w$CDM, $\Lambda$CDM + $N_{\text{eff}}$, $\Lambda$CDM + $\Sigma m_n$, are made publicly available on the CosmoPower GitHub repository.\footnote{https://github.com/cosmopower-organization}

To predict the power spectra, they need to be imported in Python, via the CosmoPower package. They allow for rapid computation of power spectra and derived parameters, and hence of parameter posterior probability distributions in MCMC analyses (see Section 5). Loading the emulators takes $\approx 0.1$ s. Computing one set of TT, TE, EE, PKL, PKNL power spectra and derived parameters takes $\approx 60$ ms, compared to $O(1\text{ min})$ with class or camb with high-precision settings, i.e. we achieve a factor of 1000 speed-up. The GitHub repository also contains a set of notebooks showing how to use the emulators.

In Figs 1–6, we show the emulator’s predictions for one set of cosmological parameters, namely, the central values of the right column of table I in Planck Collaboration.
Figure 18. Comparison of 2D marginalized posterior probability distributions between CosmoLSS (blue contours) and the full calculation (red contours) for the Planck 2018 CMB likelihood BM_TTTEEE+lowl + lowE computed using camb in w-CDM and downloaded from the Planck legacy archive (footnote 20). See Section 5.4 for details. Note that our emulators cover the range $w > -2$ (which we used as a hard prior bound for our CosmoLSS chains), as this is the lower bound adopted in the DES Year 1 analysis (Krause et al. 2017), although the Planck chains allow for $w > -3$.

(2020a), with one massive neutrino with $m_\nu = 0.06$ eV and in $\Lambda$CDM.

In Fig. 1, we compare the power spectra from class, camb, and CosmoLSS explicitly. The class (i.e. the high-precision calculation discussed in Section 3), ‘class prec’ (i.e. the ultra-high-precision calculation discussed in Section 3), high-precision camb, and CosmoLSS power spectra are indistinguishable for $\ell < 10,000$. For $10,000 < \ell < 11,000$, some differences can be seen in the TT and EE power spectra: the class and CosmoLSS predictions tend to fall off compared to camb. Since this is less significant for the ‘class prec’ prediction, we attribute this to numerical precision settings, and do not investigate further as this is probably irrelevant for any CMB experiment in the foreseeable future. The shaded areas indicate the sensitivity of Advanced ACT (Henderson et al. 2016), SO (Simons Observatory 2019), and CMB-S4 (CMBS4 2022) forecast sensitivity. We refer to section 4 of Bolletti et al. (2022) for details on the TT and PP sensitivity curves and the links where they can be downloaded from. The TE sensitivity can be computed from TT and EE (see e.g. Spurio Mancini et al. 2022, for the formula we use).

13The CMB-S4 EE noise is from $54.190604d_2LAT_pol.default.noisecurves.deproj.SENS$.
Note that we use $f_{DE} = 0.3$ for Advanced ACT and $f_{SE} = 0.4$ for SO and CMR-S4. Also, note that all noise curves also assume the use of Planck data in combination with that from each ground-based experiment (see section 2 of Simons Observatory (2019) for details); the use of Planck is important in extending the forecast measurements to low multipoles, where otherwise atmospheric noise in the ground-based data would be very large.

Fig. 2 shows the same as Fig. 1 but for the linear and non-linear matter power spectra, computed at three redshifts ($z = 0$, 2.5, and 5). In these cases, the class, camb, and CosmoPower predictions are indistinguishable across the full $k$ range. The same is true for the H, DA, and S8 predictions, as can be seen in Fig. 3.

In Fig. 4, we show the relative difference between CMB angular power spectra (TT, TE, EE, PP) and camb, in per cent. The spikes in TE are where the spectra cross zero and are not problematic. As discussed before, the agreement between the ultra-high precision class prediction and camb is at the 0.1 per cent level until $\ell \sim 3000$, and degrades at higher $\ell$. None the less, the agreement...
between Cosmopower and both class predictions remains at the 0.1 per cent level across the whole range of multipoles, including $\ell > 3000$, except in two cases. First, in the low-$\ell$ part ($\ell < 10$) of the lensing convergence power spectrum, we see very small differences ($\leq 0.5$ per cent) between the ultra-high precision class prediction on one hand and the high-precision and Cosmopower prediction on the other hand (the agreement between the Cosmopower and high-precision class calculation remains better than 0.005 per cent). This difference can be reduced further by decreasing the parameter perturb_sampling_stepsize (see Section 3), at the cost of a longer runtime. Secondly, in the low-$\ell$ part ($\ell < 50$) of the EE power spectrum, we see differences between the Cosmopower and high-precision class prediction at the 0.6 per cent level. If needed, this could be solved by generating more training data.

To assess whether this level of difference is acceptable for CMB Stage-IV analyses, we compare the relative difference between the CMB spectra in terms of CMB-S4 sensitivity in Fig. 5. We see that the cosmapower power spectra agree with camb to better than 0.03$\sigma$ across all multipoles and with the high-precision class
prediction to better than 0.03σ. Therefore, such level of agreement is sufficient.

In Fig. 6, we show the relative difference between the linear and non-linear matter power spectra with respect to camb in percent, at three redshifts, z = 0, 2.5, and 5. In the left-hand panels, we see that the CosmoPower linear matter power spectra agree with the high-precision class predictions at the 0.05 percent level across the whole k range. The difference between these and the ultra-high precision class prediction is at the 0.3 percent level. Finally, the difference between the ultra-high precision class prediction and camb is at the 0.3 percent level up to k ≈ 20 Mpc⁻¹ and then becomes ≈ 0.8 per cent at higher k. In the right-hand panels, showing the non-linear matter power spectra, we see that differences between CosmoPower on one hand and class and camb on the other hand are more important for the non-linear matter power spectra, becoming larger in the non-linear regime, but remain at the 1–1.5 percent level, including at z = 5. The fall-off of the linear P(k) at small scales compared to camb is likely to be due to baryonic sound speed effect on non-linear scales that are treated differently in class and camb.

Since there is no direct measurement of the matter power spectrum, we cannot compare it with an instrumental sensitivity level. However, we can check the accuracy on observables based on the matter power spectrum. The lensing convergence power spectrum (PP, bottom right panel of Fig. 1) is one such example. Other examples include galaxy clustering and galaxy weak lensing observables and to a lesser extent, the lensing of CMB angular power spectra. We perform Stage III posterior inference analyses based on these observables and recover nearly identical constraints to those obtained running the full Boltzmann code calculations (see Section 5), giving us confidence that our emulators are sufficiently precise. We defer to future work a more detailed quantitative analysis, including the release of high-accuracy emulators for the matter power spectrum tested against (forecast) Stage-IV configurations.

The results shown in Figs 1–6, are for one set of cosmological parameters. However, it is important to quantify the precision of our emulators over the whole prior range of parameters (see Table 1), in all the cosmological models. We do so in Figs 7–13.14 These figures show the relative difference between the emulators, i.e. the CosmoPower prediction, and the exact high-precision class prediction on the testing data sets,15 containing ≈ 25,000 predictions. The results for TT are in Fig. 7, for EE in Fig. 8, for TE in Fig. 9, for PP in Fig. 10, for PKL and PKNL in Fig. 11, for H, DA, and S8 in Fig. 12, and for the derived parameters in Fig. 13. For TT, TE, EE, and PP, we show the difference in units of CMB-S4 sensitivity and in percentages, and for the other emulators, we show the difference in percentages (since there is no simple way to compare with instrumental sensitivity; see comments in the previous paragraph). In all the figures, the three shades of red correspond to 68 per cent, 95 per cent, and 99 per cent of the testing data set, from dark to light, respectively. For instance, for TT (Fig. 7), we find that our emulators predict the power spectra to better than 0.02σ at CMB-S4 precision level, 68 per cent of the time in all the four cosmological models considered here. The agreement for TE and EE is similar.

For PP the agreement is at the 0.02σ CMB-S4 precision level, 99 per cent of the time in all the four cosmological models considered here.

The quantity that is the least accurately emulated is the non-linear matter power spectrum, which is reproduced at the 1–1.5 per cent level 95 per cent of the time in all models (see Fig. 11). This level of agreement is satisfactory for application to current survey configurations, especially since non-linear modelling and baryonic uncertainty is at the ≈ 10 per cent level (see e.g. Mead et al. 2021) and even exceeds 30 per cent at k > 1 Mpc⁻¹ (see e.g. Amon & Efstathiou 2022).

The emulated redshift-dependent H(z), D_L(z), and σ_8(z) agree to better than 0.05 per cent between z = 0 and 20, for 99 per cent of the testing data set in all models (see Fig. 12). We find a similar agreement between emulated derived parameters and the testing data set (see Fig. 13).

5 ACCELERATED LIKELIHOOD ANALYSES

In this section, we run posterior inference analysis and use the emulators presented above in MCMC extractions of cosmological parameters. A full Stage-IV analysis will require optimization of likelihood codes alongside emulators which is beyond the scope of this paper and will be covered in future work. Therefore, here we demonstrate accelerated likelihood analyses for current (Stage II-III) data and reproduce marginalized constraints for ACT DR4 (Section 5.1). Planck lensing + DES + BAO (Section 5.2) and Planck TT,TE,EE in Λ CDM, Λ CDM + Neff and Λ CDM + S_8 (Section 5.4). We use cobaya (Torrado & Lewis 2021) for MCMC sampling. To analyse the resulting chains, we use GetDist (Lewis 2019).

We define a CosmoPower theory object within cobaya such that we are able to implement the CosmoPower call completely outside of the likelihoods. Our implementation of the CosmoPower theory object is available upon request. In a forthcoming paper, we plan to release an official CosmoPower wrapper for both cobaya and cosmofit (Zanin et al. 2015).

5.1 Accelerated ACT DR4 power spectrum analysis

We use the ACT Data Release 4 (DR4) 'acpol_lite' likelihood (Aiola et al. 2020; Choi et al. 2020) with CosmoPower to reproduce the original DR4 results. This is the foreground-marginalized likelihood publicly available in the pyactlike repository.16 For the CosmoPower runs, this likelihood involves the TT, TE, and EE emulators, relying on multipoles up to ℓ ≈ 4500.

In Fig. 14, we show the resulting 2D marginalized posterior probability distribution obtained in a few minutes on a laptop with CosmoPower as red empty contours, and compare it with the publicly available chains17 shown as the filled green contours. The reference class chains were obtained in Hill et al. (2022) using high-accuracy settings and took several days to converge. The overlap of the CosmoPower and reference contours is nearly perfect. We note that for the CosmoPower run we sample over H_0 and obtain θ_s as a derived parameter using our DER emulator in post-processing of the chains, whereas the reference class chains were computed with θ_s as an input parameter and H_0 was saved as a derived parameter.

14The 'mnu' results on this figure are for one massive and two massless neutrino states.
15The testing data are made of the exact computations which are not used for training the emulator neural networks. We use 20 per cent of our data set for testing.
16https://github.com/ACTCollaboration/pyactlike
17CLASS2p8_ACTPol_lite_DR41 leakfix_yip2_baseLCDM_iswip_kip_R0p01 downloaded from LAMBDA.
5.2 Accelerated Planck lensing + DES + BAO analysis

To further demonstrate the efficiency and wide range of our emulators, we reproduce cosmological parameters extracted from the Planck CMB lensing power spectrum + DES + BAO analysis. We compare with the publicly available Planck chains. The chains of interest are labeled DES; Lenspriors, lenseslensing_BAO.

We use the Python-native re-implementations of the Planck re-constructed lensing power spectrum likelihood, assuming their fiducial (conservative) multipole range. This likelihood is available in cobaya as planck_lensing_2018 (Planck Collaboration 2020b). We apply the priors used in the Planck lensing analysis: Gaussian priors on $n_s = 0.96 \pm 0.02$ and $\Omega_m h^2 = 0.222 \pm 0.005$, a flat prior $40 < H_0$/km/s/Mpc$^{-1} < 100$, and a fixed $\tau = 0.055$ (see Planck Collaboration 2020b, for details).

We use the DES-Y1 cosmic shear + galaxy auto- and cross-correlation (‘3 x 2-point’) likelihood, as implemented in des_y1 in cobaya, which is as described by Troxel et al. (2018), DES Collaboration (2018), and Krause et al. (2017).

Finally, we use the BAO likelihoods corresponding to BOSS DR12 (Aldam et al. 2017), the SDSS Main Galaxy Sample (Ross et al. 2015), and the 6dF survey (Beutler et al. 2011). These are implemented in cobaya as adss_dr12_consensus_bao, adss_dr7_mgsa, and sixdf_2011_bao.

For the CosmoPower runs, this likelihood combination involves the FNLK, H, DA, and FP emulators. Because this calculation requires the computation of the non-linear $P(k)$ at many redshifts, it is slightly more time-consuming. Indeed, we need to evaluate the matter power spectrum on a 2D grid in $(k, z)$, hence requiring us to call the emulator as many times as there are points in the z dimension. We set the $P(k,z)$ grid to cover redshifts between 0 and 15 with 15 linearly spaced points. With this choice, cobaya performs $\approx 16$ evaluations per second on an ARM64 MacBook Pro. We reach $R \approx 1 \approx 0.15$ after $\approx 30$ min on a laptop, with four chains taking a total of 56,000 accepted steps with acceptance rate of 0.2.

The results are shown in Fig. 16. The overlap between the CosmoPower and reference camb contours is perfect. As derived parameters, we show $S_\delta = \sigma_8(\Omega_m / 0.3)^0.5$ and $S_P^{25} \equiv 2\pi \sigma_8(\Omega_m / 0.3)^{0.25}$, which are most constrained by galaxy weak lensing and CMB lensing data, respectively. Our CosmoPower runs recover the reference constraints on $S_\delta$ and $S_P^{25}$ exactly. This validates the matter power spectrum emulators for current galaxy WL surveys.

5.3 RSD full shape analysis

To demonstrate that we can also compute $f_{\sigma_8}$ and use it in a likelihood analysis, using our S8 emulator (see Section 2), we perform a likehood analysis and use adss_dr12_consensus_fullshape with Planck lensing. We compare class and CosmoPower chains in Fig. 17.

5.4 Extensions

To include our emulators in extended cosmology, we run maximum-likelihood analyses in $\Lambda$CDM, $\Lambda$CDM + $N_{\text{eff}}$, and $\Lambda$CDM + $\Omega_m$. For the data and likelihood, we choose the official clik Planck 2018 priors16 kMTTTEEE+lowl + lowZ (Planck Collaboration 2020a) implementation, which we call through cobaya.

We show the $\Lambda$CDM, $\Lambda$CDM + $N_{\text{eff}}$, and $\Lambda$CDM + $\Omega_m$ constraints in Figs. 18-20, respectively. In each extension, we overlap the reference contours from camb chains (empty red contours) that we obtain online (see footnote 20). Our CosmoPower contours are in blue. In all cases the agreement with the reference Planck chains is excellent. To reach $R \approx 1 \approx 0.1$, CosmoPower needs approximately 25 min, while class or camb chains typically take $O(1)$ day to converge.

6 CONCLUSIONS

We have extended the work of Spario Mancini et al. (2022) by creating emulators for CMB temperature and polarization angular anisotropy power spectra, CMB lensing convergence power spectra, linear and non-linear matter power spectra, and the redshift evolution of $H(z)$, $D_A(z)$ and $\sigma_8(z)$, in $\Lambda$CDM and extensions, namely, $\Lambda$CDM, $\Lambda$CDM + $N_{\text{eff}}$ and $\Lambda$CDM + $\Omega_m$, with one or three massive states (see Section 2). All of these quantities are computed using high-precision class settings (see Section 4). They are sufficiently precise for Stage-IV CMB analyses (see Section 3) and current galaxy weak lensing and clustering surveys. We have tested all of our emulators in maximum-likelihood analyses, involving high-$\ell$ CMB, galaxy weak lensing, clustering, BAO, and RSD data (see Section 5).

The main outcome of this work is to open the door to fast parameter inference, via the widely used CMCMC method, with accelerated Boltzmann computations thanks to our CosmoPower emulators. For instance, as we demonstrate here, most of Stage-III parameter inference is now feasible on a laptop.

In a forthcoming paper, we plan release wrappers of our emulators so that they can be readily used within cobaya (Torrado & Lewis 2021) and cosmology (Zuntz et al. 2015).

We created an online repository to store our emulators and will continue updating it. For CMB polarization, we have made emulators for $E$ modes, which could be used to accelerate LiteBIRD (LiteBIRD Collaboration 2022) analyses dedicated on reionization history constraints (Zaldarriaga et al. 2008; Alis et al. 2022). We defer B-mode power spectra emulators to future work.

In addition to the applications considered here, these emulators can also be used to save time at the initial step of libraries for computations of cosmological LSS observables like class (Bolliet et al. 2018; Bolliet et al. 2022), ccl (Chisari et al. 2019), and other code which relies on the quantities emulated here. In fact, class now contains a fully-fledged wrapper for our emulators (see Bolliet et al. 2023) and a code repository17 and

16https://github.com/ACTCollaboration/plklike
17ACT DR4 Fortran likelihood, named scriptfull there.
18https://wiki.cosmos.esa.int/planck-legacy-archive
19https://github.com/CLASS-SZ/class_sz

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tutorial notebooks). The class.sz wrapper has recently been employed in a new likelihood code package for cluster number counts: cosmonec (Zubeldia & Bollett 2024).

Recent works (Cabass et al. 2022; Philcox & Ivanov 2022) have used the effective field theory of LSS (Baumann et al. 2012; Carrasco, Hertzberg & Senatore 2012) in order to derive constraints from spectroscopic surveys, based on the one-loop galaxy power spectrum and bispectra. These calculations are time-consuming (MCMC convergence takes 0(1)day using class.pt (Chudaykin et al. 2020). The bottleneck is the calculation of higher-order correlators.

The work presented here does not directly help to accelerate such analyses, but none the less the method can directly be applied in this context as well.

We note that class has been recently updated so it can compute high-ell CMB lensing faster and more accurately than what is possible with the version of the code used here. Therefore, it should be possible to generate class-based emulators with even more demanding high-accuracy settings. Other possible improvement include more refined recombination treatment, e.g. using CosmoRec (Chluba & Thomas 2011; Chluba & Thomas 2013) or HyRec (Ali-Haimoud & Hirata 2011; Lee & Ali-Haimoud 2020). As noted and discussed in (Gardiello et al. 2024; Jense et al. 2024), this will be useful for upcoming and future Stage-IV CMB analyses. Finally, the uncertainty in CMB temperature (which fixes the radiation density), even though it is small, will eventually become important to take into account as accuracy requirements increase. We leave this point of investigation to future work.

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**DATA AVAILABILITY**

The emulators are available online at CosmoPower Organisation, including tutorial notebooks. If you use the emulators, please cite our work as well as Spurio Mancini et al. (2022).

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