

# Detecting the heterodyning of gravitational waves

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## Abstract

Gravitational waves modulate the apparent frequencies of other periodic signals. Low-frequency gravitational waves could therefore be detected by observing frequency modulations in signals from higher-frequency sources, e.g. those from binary white dwarfs detected with the LISA gravitational-wave detector. We propose a concrete method to extract these modulations by coherently adding the cross-spectra of a large number of well-resolved quasi-monochromatic signals. We apply this method to the case of LISA, and find this method would enable the detection of background gravitational wave strain amplitudes of, e.g.  $A \simeq 10^{-10}$  at a frequency  $F \simeq 10^{-8}$  Hz, given current projections for the number and properties of Galactic binary white dwarfs and the sensitivity of the instrument. We also estimate (to within an order of magnitude) that this method could potentially compete with that of current pulsar timing arrays when using signals from binary neutron stars such as those expected to be observed with proposed detectors like DECIGO. Our results show that gravitational-wave detectors could be sensitive at frequencies outside of their designed bandwidth using the same infrastructure, which has the potential to

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open up unexplored and otherwise inaccessible parts of the gravitational wave spectrum.

Keywords: gravitational waves, gravitational wave detectors, super-massive black hole binaries, galactic binaries

## 1. Introduction

The field of gravitational-wave astronomy, as established with the first direct detection of gravitational waves (GWs) [1], is still in its infancy. So far, only GWs with frequencies between  $\sim 10 - 500$  Hz produced by the coalescence of black holes and neutron stars with masses  $\sim 1 - 100$  times the mass of our Sun have been detected [2]. New detectors and techniques are being developed to probe different regions of the GW frequency spectrum and to investigate numerous other potential GW sources; e.g. rotating neutron stars [3], binary white dwarfs (BWDs) [4], intermediate-mass and super-massive binary black holes (SMBHBs) [5], a background of primordial GWs [6], and dark matter [7, 8].

The sensitive bandwidth of laser interferometers (the only proven type of GW detector), is typically limited at low frequencies by spurious accelerations of the test masses, and at high frequencies by quantum uncertainty in the optical state and an intrinsically decreased response to GWs with wavelengths shorter than the interferometer's arms. Laser interferometers can be very sensitive at higher frequencies ( $\sim 1 - 100$  MHz), using cross-correlation and shorter arms [9, 10]. Increasing the sensitivity at lower frequencies is not straightforward, and even a space-based instrument such as LISA [11], subject to greatly reduced environmental noise, will not be sensitive to GWs below  $\lesssim 10^{-5}$  Hz. While marginal gains have been made in understanding and addressing the complex amalgam of low-frequency noise contributions encountered in laser interferometers (which include fundamental quantum limits) [12], it seems unlikely that their bandwidth will expand into lower frequencies by more than an order of magnitude in the coming decades.

Other detection techniques to probe new areas of the GW spectrum have been proposed and some have been tried. At high frequencies (kHz–GHz) these include techniques that exploit graviton-to-photon conversion (known as the inverse Gertsenshtein effect) [13, 14], optically levitated sensors, resonant mass detectors [15], and more [16].

At low frequencies, currently the only competitive method to search for GWs is using sets of time-resolved observations of pulsars, known as pulsar timing arrays (PTAs), which are sensitive in the nHz– $\mu$ Hz range [17–30]. GWs in this frequency range provide a probe of the most massive binary black holes ( $\gtrsim 10^9 M_{\odot}$ ) in the Universe, which are expected to form at the centre of merging galaxies [31]. Measuring the low-frequency GWs from individual SMBHBs or a stochastic background thereof allows investigation of their abundance and coevolution with their host galaxies. Several PTAs across the globe have recently found the first evidence for a low-frequency GW background [32–35], by monitoring the arrival times from dozens pulsars for more than a decade. GWs incident on a pulsar and/or the detector produce deviations of the apparent frequency or equivalently the arrival time of the radio pulses that are correlated between different pulsars. This detection technique thus exploits the interplay of electromagnetic pulses with GWs which results in a modulation of the pulse frequency.

Recently, work by Bustamante-Rosell *et al* [36] was published that proposed a new method to look for low-frequency gravitational waves<sup>4</sup>. This method for detecting (low-frequency) GWs uses the interactions between GWs of different frequencies. The basis of the method is the gravitational red- and blueshift induced by one GW onto the other. This mechanism can also be viewed as one GW perturbing the space-time along the direction of travel of the other GW, and thus modulating the arrival times of peaks and troughs of the other GW. Mathematically, the effect can be described as a multiplication or mixing of two GWs. From this description, it can be shown that the resulting GW signal contains Fourier components at the sum and difference of the frequencies of the two waves, with an amplitude proportional to the product of the amplitudes of the individual GWs. This elementary result of the mixing of two waves, also known as heterodyning, has been used in the processing of electromagnetic signals for over a century. Heterodyning effectively produces a frequency-shifted copy of one signal (known as a sideband) in the frequency range of a readily detectable second signal. As we show in this paper, this mechanism can be used in GW astronomy, where GW signals detectable with, e.g. laser interferometers can be used to detect low-frequency background GWs. This method of searching for low-frequency GWs is conceptually similar to the technique used by PTAs, with the crucial difference that instead of looking for disturbances in the periodic electromagnetic signal of pulsars, we look for disturbances in a periodic GW signal.

This method allows one to expand the sensitive bandwidth of GW detectors into low-frequency regimes using the detectors' existing infrastructures. Moreover, this method could enable a sensitivity to GWs in a bandwidth where no other detection methods exist, e.g. in the  $\mu\text{Hz}$  regime where the frequency ranges of space-based laser interferometers and PTAs leave a gap.

The work presented here is complementary to the work by Bustamante-Rosell *et al* [36]; we demonstrate a concrete cross-correlation search method for detector data and we simulate signals expected to be detected with LISA using an observationally-driven mock population of BWDs. In addition, as the heterodyning method is applicable to general periodic GW signals, we present order-of-magnitude estimates of the sensitivity that could be obtained with DECIGO [37], ET, and CE, which are expected to be able to observe large numbers of GW signals from BWDs and binary neutron stars (BNSs). The analysis and projections of the potential of the method in [36] for observations with LISA differ from ours; in section 6 we expand on the difference in findings.

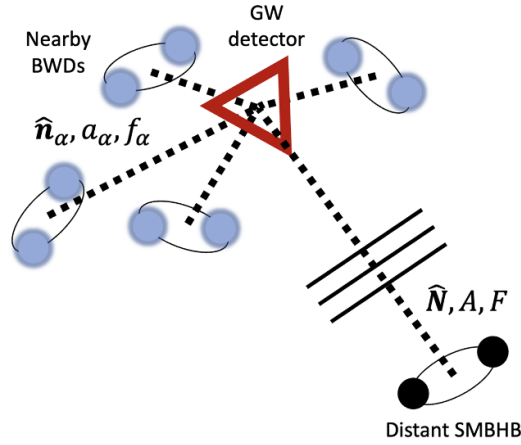
## 2. Theory

We consider a set of  $N \gg 1$  periodic GW sources which could be simultaneously observed for a long time (e.g. BWDs in our Galaxy that could be individually resolved by LISA [11]). We further assume that these sources emit quasi-monochromatic GWs, i.e. that their frequency does not significantly change within the observation time  $T$  (see section 6 for discussion of the implications of relaxing this assumption). In that case we can write the GW signal (in units of strain) from the  $\alpha$ th periodic source at distance  $d_\alpha$  as

$$h_\alpha(t) = a_\alpha \cos[2\pi f_\alpha t + \varphi_\alpha], \quad (\alpha = 1, 2, \dots, N), \quad (1)$$

with constant frequency  $f_\alpha$ , amplitude  $a_\alpha$ , and initial phase  $\varphi_\alpha$ . We refer to these GWs as carrier signals and to their sources as carrier sources.

<sup>4</sup> We note that our work was developed independently from Bustamante-Rosell *et al* [36], and we became aware of their work only after drafting this manuscript.



**Figure 1.** Sketch of the GW heterodyning method proposed in this work. The LISA GW detector aims to resolve GW signals from a large number of BWDs in the Milky Way [11]. If there is another GW, e.g. one emitted by a much more distant SMBHB, this could be indirectly detectable through the coherent modulations imparted on the measured GW frequency of the BWDs. Figure 1 adapted with permission from [36]. Copyrighted by the American Physical Society.

If there is an incident GW from a secondary, more distant source, this GW will perturb the spacetime at the location of the carrier sources and at the location of the observer (see figure 1). As a consequence, the frequency of the GW carrier signals are no longer constant but are modulated in time. For a background GW emitted by a distant point source in the direction  $\hat{N}$  this frequency modulation of the carrier signal is given by [38]

$$\frac{f_\alpha - f_\alpha(t)}{f_\alpha} = \frac{n_\alpha^i n_\alpha^j}{2(1 + \hat{N} \cdot \hat{n}_\alpha)} [h_{ij}^{\text{TT}}(t) - h_{ij}^{\text{TT}}(t_\alpha)], \quad (2)$$

where  $\hat{n}_\alpha$  and  $n_\alpha^i$  is the unit vector from the observer to the  $\alpha$ th carrier source and its components, respectively, and  $t_\alpha = t - d_\alpha(1 + \hat{N} \cdot \hat{n}_\alpha)/c$  is the retarded time coordinate that accounts for the propagation of the carrier wave. We will refer to the lower-frequency GW of interest that modulates the carrier signal as the background GW. Additionally,  $h_{ij}^{\text{TT}}(t)$  and  $h_{ij}^{\text{TT}}(t_\alpha)$  correspond to the metric perturbation due to the incident GW at the spacetime locations of the carrier source and the observer, respectively (in the terminology of PTAs [17, 18], the former is usually referred to as the ‘Earth term’ and the latter as the ‘pulsar term’).

It can be shown that the single-sided frequency spectrum of the modulated signal can then be written as [36]

$$\begin{aligned} \tilde{h}_\alpha(f) &\simeq a_\alpha e^{i\varphi_\alpha} \delta(f_\alpha - f) \\ &+ \frac{1}{2} a_\alpha A I_{\alpha,L} e^{i(\varphi_\alpha + \Phi_L)} \delta(f - f_\alpha + F_L) \\ &+ \frac{1}{2} a_\alpha A I_{\alpha,L} e^{-i(\varphi_\alpha + \Phi_L)} \delta(f - f_\alpha - F_L) \\ &+ \frac{1}{2} a_\alpha A I_{\alpha,D} e^{i(\varphi_\alpha + \Phi_{\alpha,D})} \delta(f - f_\alpha + F_{D,\alpha}) \\ &+ \frac{1}{2} a_\alpha A I_{\alpha,D} e^{-i(\varphi_\alpha + \Phi_{\alpha,D})} \delta(f - f_\alpha - F_{D,\alpha}), \end{aligned} \quad (3)$$

where  $I_{\alpha,L,D} = (f_{\alpha}/F_{L,D}) K(\hat{N}, \hat{n}_{\alpha}, h_{ij}^{\text{TT}}, d_{\alpha})$ , and  $K$  is a purely geometrical factor of order unity that accounts for the polarisation, propagation direction, and propagation distance of the background and carrier GWs. The first term in the spectrum given by equation (3) is the Fourier component corresponding to the carrier signal at the frequency  $f = f_{\alpha}$ . The modulation due to the background GW at the location of the observer manifests as two Fourier components with frequencies  $f = f_{\alpha} \pm F_L$  (second and third term in equation (3)), which we will refer to as the ‘local’ sideband terms. Similarly, the modulation of the carrier signal due to the background GW at the location of the carrier source produces sidebands with frequencies  $f = f_{\alpha} \pm F_{D,\alpha}$  (fourth and fifth term), which we will refer to as the ‘distant’ sideband terms. Note that the frequency  $F_L$  and phase offsets  $\Phi_L$  of the ‘local’ terms are independent of the carrier (they are equal to the frequency and phase of the modulating GW at the location of the observer), whereas the ‘distant’ terms have frequency and phase offsets  $F_{D,\alpha}$  and  $\Phi_{\alpha,D}$ , which depend on the location of the carrier source.

This mechanism, a sort of ‘GW heterodyning’ could allow the indirect detection of low-frequency GWs that may otherwise be undetectable when a GW detector is not sensitive to signals down to a frequency  $F$ , but is sensitive at much higher frequencies  $f_{\alpha} + F$ . Using this method, the upconverted background signal amplitude is  $A_{\text{sideband}} = A a_{\alpha} K f_{\alpha} / F$ . For example, if we take the carrier signal to be the GWs emitted by a typical BWD with amplitude  $a_{\alpha}$  (such as the BWDs that LISA aims to detect), and with frequency  $f_{\alpha} \sim 10^{-2}$  Hz, and we take the background signal to be GWs emitted by a SMBHB with amplitude  $A \sim 10^{-12}$  and frequency  $F_L \sim 10^{-8}$  Hz, the background sideband signal appears at an amplitude  $A_{\text{sideband}} \sim 10^{-6} a_{\alpha}$ .

This suppression relative to the carrier would mean the background signal amplitude is below the typical noise level of the detector. In the following section, we propose a method to amplify the signal which utilises the coherence of the modulation of multiple carrier signals. To this end, we construct and add  $N_p = N(N-1)/2 \gg 1$  different cross-spectra (one for each pair of carrier sources) such that the sideband terms sum up coherently to exceed the incoherent random noise.

### 3. Cross-correlation analysis

We propose a cross-correlation method for detecting a background gravitational wave signal that produces phase modulation of carrier GW signals. We will later use this method to make quantitative estimates of the expected signal-to-noise ratio that can be obtained for potential astrophysical GW sources using planned GW detectors.

We consider the time-domain output signal of the GW detector  $s(t)$  to be given by the sum of  $N$  carriers, all modulated by a single background GW signal with frequency  $F$  corresponding to either the ‘local’ ( $F = F_L$ ) or the ‘distant’ ( $F = F_D$ ) term, and noise  $n(t)$  characteristic of the detector:

$$s(t) = \sum_{\alpha=1}^N h_{\alpha}(t) + n(t). \quad (4)$$

For any carrier, we can apply a demodulation and phase-shift to the time-domain detector output and normalise it by the modulation index and the carrier amplitude:

$$s_{\alpha}(t) = \frac{\sqrt{2}}{a_{\alpha} I_{\alpha}} e^{-i(2\pi f_{\alpha} t + \varphi_{\alpha})} s(t). \quad (5)$$

This demodulation shifts the frequency of all Fourier components in the output by an amount  $f_{\alpha}$ , such that all sideband (heterodyne) signals are frequency shifted to the frequency  $\pm F$  of

the modulating background GW that produces them. Moreover, any heterodyne signals from background GWs will now appear with a Fourier amplitude equal to the background GW strain amplitude that produces them. In general, the demodulation frequency need not be constant in time, but could be adjusted over time to account for time-dependent changes in the carrier frequency. Specifically, the demodulation frequency and phase could be varied according to a predetermined carrier signal model, or they could be fit to the data post hoc (e.g. through maximising the demodulated carrier amplitude) when the frequency evolution is unknown *a priori*. After this frequency and phase shift, we can apply an appropriate low-pass filter to the data such that other terms, as long as they are well-separated from the carrier and modulation sideband, need not be considered<sup>5</sup>.

We consider the case where the time-domain detector output is discretised with a constant sampling frequency  $f_s$  for a total observation time  $T$ . Next, we take the single-sided discrete Fourier transform of the detector output, which yields a discrete complex amplitude spectrum  $S_\alpha^j$  for each carrier signal, which will have the form

$$S_\alpha^j = A e^{i\Phi_\alpha} \delta_{l(F)}^j + \frac{\sqrt{2}}{a_\alpha I_\alpha} \sqrt{\frac{\rho_\alpha^j}{T}} e^{i\eta_\alpha^j}, \quad (6)$$

where the index  $j = 1, 2, \dots, Tf_s/2$  runs over the frequency bins,  $l(F)$ <sup>6</sup> is the index of the bin that contains the background signal ( $\delta_l^j$  is the Kronecker delta),  $\rho_\alpha^j$  is the noise power spectral density of the detector, and  $\eta_\alpha^j$  are the random noise phases (where both noise parameters have undergone the frequency and phase shift described by equation (5)). The spectrum  $S_\alpha^j$  is unique for each carrier signal. As background GWs would modulate all carrier signals coherently (i.e. the sideband phase is deterministic), whereas the noise has a random phase, cross-correlating different carrier signals is advantageous. For each pair of carrier signals  $(\alpha\beta)$ , a cross-spectrum  $S_{\alpha\beta}^j = S_\alpha^j S_\beta^{j*}$  can be constructed which has the form

$$S_{\alpha\beta}^j = A^2 e^{i(\Phi_\alpha - \Phi_\beta)} \delta_{l(F)}^j + \frac{2}{a_\alpha a_\beta I_\alpha I_\beta} \frac{\sqrt{\rho_\alpha^j \rho_\beta^j}}{T} e^{i(\eta_\alpha^j - \eta_\beta^j)}, \quad (7)$$

where  $\Phi_\alpha - \Phi_\beta = \Phi_{\alpha\beta}$  is the phase difference of the modulating signal between the two carrier signals. From this expression it can be seen that  $\Phi_{\alpha\beta}$  is deterministic, and  $\eta_\alpha^j - \eta_\beta^j = \eta_{\alpha\beta}^j$  is random. Therefore, we can add up signal terms from different cross-spectra coherently, and the noise will average out. If we have  $N$  individually resolved carriers at our disposal we can construct  $N_p = N(N-1)/2$  different cross spectra and take a coherent weighted average of them

$$S^j = \frac{\sum_{(\alpha\beta)}^{N_p} \lambda_{\alpha\beta}^j S_{\alpha\beta}^j e^{-i\Phi_{\alpha\beta}}}{\sum_{(\alpha\beta)}^{N_p} \lambda_{\alpha\beta}^j}, \quad (8)$$

where  $\lambda_{\alpha\beta}^j$  are the weights of each cross-spectrum. Performing this coherent summation is possible as long as the relative modulation sideband phase  $\Phi_{\alpha\beta}$  can be determined for each

<sup>5</sup> The sidebands due to the local modulation can be considered well-separated in the frequency domain from the sidebands due to the distant modulation (pulsar/distant term) when  $|F_L - F_D| \gg 1/T$ . We also assume all carrier signals are well-separated from each other ( $|f_\alpha - f_\beta| \gg 1/T \forall \alpha, \beta$ ).

<sup>6</sup>  $l(F) = \lceil FT + \frac{1}{2} \rceil$ .

carrier pair  $(\alpha\beta)$ . For the modulation produced by the background GW at the detector ('local' term),  $\Phi_{\alpha\beta} = 0 \forall \alpha\beta$ . For the sideband due to the modulation produced at the source of the carrier GW signal ('distant' term),  $\Phi_{\alpha\beta}$  is a function of the relative positions of the background GW source and the carrier signal sources. In this case,  $\Phi_{ab}$  can be taken as free parameters that are fit to the data by maximising the total SNR for a particular sideband frequency, which would yield an upper estimate of the maximum background GW signal power at a certain frequency. Alternatively, a hypothetical background source position and frequency could be assumed, which prescribes a certain set of  $\Phi_{\alpha\beta}$  given the geometry of the source positions, which would then yield an upper limit of the estimated background GW strain at that frequency and sky position.

Note that the coherent average is constructed such that the expected real part of the signal bin is  $E[\text{Re}[S^{l(F)}]] = A^2$ . The squared signal-to-noise ratio can thus be defined for each bin

$$(\text{SNR}^j)^2 = \frac{(\text{Re}[S^j])^2}{\text{Var}(\text{Re}[S^j])}. \quad (9)$$

It can be shown that an optimal signal-to-noise ratio is found by taking the weights [39]

$$\lambda_{\alpha\beta}^j = \sum_{(\gamma\delta)}^{N_p} \left( [C^j]^{-1} \right)_{\alpha\beta,\gamma\delta} \simeq \left( \frac{1}{\sigma_{\alpha}^j \sigma_{\beta}^j} \right)^2 = \frac{(a_{\alpha} a_{\beta} I_{\alpha} I_{\beta})^2 T^2}{4 \rho_{\alpha}^j \rho_{\beta}^j}, \quad (10)$$

where  $C_{\alpha\beta,\delta\gamma}^j$  is the pair-wise cross-covariance matrix of the cross-spectra  $S_{\alpha\beta}^j, S_{\delta\gamma}^j$ , and  $\sigma_{\alpha,\beta}^j$  are the variances of frequency bin  $j$  in each carrier spectrum (equation (6)); the approximation holds in the weak-signal limit [39]. The SNR of a modulating background GW with frequency  $F$  and amplitude  $A$  can now be evaluated:

$$\left( \text{SNR}^{l(F)} \right)^2 \simeq \frac{A^4}{2} \sum_{(\alpha\beta)}^{N_p} \left( \frac{1}{\sigma_{\alpha}^{l(F)} \sigma_{\beta}^{l(F)}} \right)^2. \quad (11)$$

#### 4. Methods

The GW detector LISA is expected to observe a large number of continuous, periodic GW signals from BWDs in our Galaxy [4, 11, 40–45]. These BWDs could potentially serve as carrier sources that allow for the detection of low-frequency background GWs as described above.

The total number and properties of Galactic BWDs is subject to large uncertainty. To obtain a quantitative projection for the number, frequency, and amplitude of BWD GW signals that may be detected with LISA, we use an observationally driven parametric model of the Galactic white dwarf population, constructed by Korol *et al* [45]<sup>7</sup>. This model builds upon the spectroscopic samples of single white dwarfs and BWDs from the Sloan Digital Sky Survey (SDSS) and the Supernova Ia Progenitor survey (SPY) to produce a synthetic population of Galactic BWDs which are specified by their component masses, orbital frequencies, sky positions, and orientations. These source parameters are then used to calculate the GW signals of each BWD in the population. Part of the BWDs would emit GWs at low frequencies  $f \lesssim 3$  mHz and are

<sup>7</sup> <https://gitlab.in2p3.fr/korol/observationally-driven-population-of-galactic-binaries>, accessed 6 January, 2023.

**Table 1.** Input parameters used for generating synthetic populations of Galactic binary white dwarfs. The parameters  $\rho_{\text{WD},\odot}^{\text{Korol}}$ ,  $f_{\text{BWD},4\text{AU}}^{\text{Korol}}$ ,  $f_{\text{BWD},a_{\text{max}}}^{\text{Korol}}$ , and  $\alpha^{\text{Korol}}$  are used as input for the algorithm described by Korol *et al* [45] to model the sets of BWD carrier signals. These parameters represent the local WD density, the fraction of binaries with semi-major axes  $<4\text{AU}$ , the fraction of binaries with semi-major axes less than the maximum separation detectable with LISA ( $a_{\text{max}}$ ), and a power-law index specifying the BWD semi-major axis distribution, respectively (see Korol *et al* [45] for details). The values of these parameters were chosen to correspond to upper (*Optimistic*), median (*Moderate*), and lower (*Pessimistic*) observational limits. We chose observation times  $T$  between 1.0 and 10.0 yr.  $N$  indicates the resulting number of BWDs which are individually resolvable with LISA.

Model		Pessimistic	Moderate	Optimistic
$\rho_{\text{WD},\odot}^{\text{Korol}}$	$(10^{-3} \text{ pc}^{-3})$	4.11	4.49	4.87
$f_{\text{BWD},4\text{AU}}^{\text{Korol}}$		0.112	0.095	0.078
$f_{\text{BWD},a_{\text{max}}}^{\text{Korol}}$		0.008	0.009	0.010
$\alpha^{\text{Korol}}$		-1.18	-1.30	-1.45
$T$	(yr)	1.0	4.0	10.0
$N$		$7.0 \times 10^4$	$1.1 \times 10^5$	$1.9 \times 10^5$

predicted to be so numerous that they are not individually resolvable but constitute a confusion-limited foreground noise [42]. The rest, an estimated number of  $\sim \mathcal{O}(10^3 - 10^5)$  BWDs emit GWs at higher frequencies and are expected to be sufficiently loud that they are individually resolvable; these are the BWDs which can be used as carrier sources in our method.

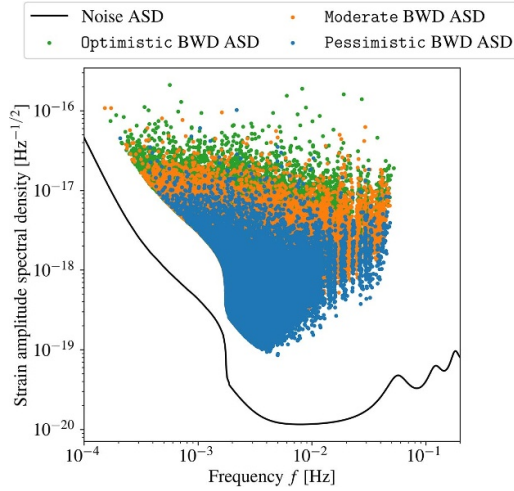
We consider three models with different carrier source and observation parameters, *Pessimistic*, *Moderate*, and *Optimistic*. For these models, we synthesised three BWD populations using different input parameters for the model of Korol *et al* [45]; specifically we vary the local WD density  $\rho_{\text{WD},\odot}^{\text{Korol}}$ , the WD binary fraction  $f_{\text{BWD}}^{\text{Korol}}$ , and the power-law index  $\alpha^{\text{Korol}}$ , which describes the BWD semi-major axis distribution (see Korol *et al* [45]). On the observation side we use three different values for the LISA mission lifetime  $T = 1.0, 4.0,$  and  $10.0\text{yr}$ , which sets the length of observation. To get an upper and lower limit for the resulting sensitivity to background GWs, we choose the model parameters such that *Pessimistic* and *Optimistic* models yield the lowest and highest number of BWDs within the current observational uncertainty while *Moderate* model corresponds to median values. The parameter values of the three different models are summarised in table 1.

In figure 2, we show the amplitude spectral density (ASD) of the BWD carriers for each model together with LISA's projected detector noise amplitude spectral density, as in [46], modified to account for the confusion noise due to unresolved BWDs derived by Korol *et al* [45]. Throughout this work we assume a BWD to be individually resolvable if  $a_{\alpha} \sqrt{T/\rho(f_{\alpha})} > 7$ , (where  $\rho(f_{\alpha})$  is the noise power spectral density evaluated at the BWD frequency), although the precise threshold does not affect the resulting sensitivity due to the dominant contribution of loud sources (see section 6).

## 5. Results

We estimate the sensitivity to background gravitational waves for the three models using our method, as in equation (11). Figure 3 shows the amplitude  $A$  versus frequency  $F$  of a background GW that could be detected with  $\text{SNR} = 2$ , corresponding to a  $\simeq 95\%$  detection probability. The differences between the *Pessimistic* and *Optimistic* models are less than one order of





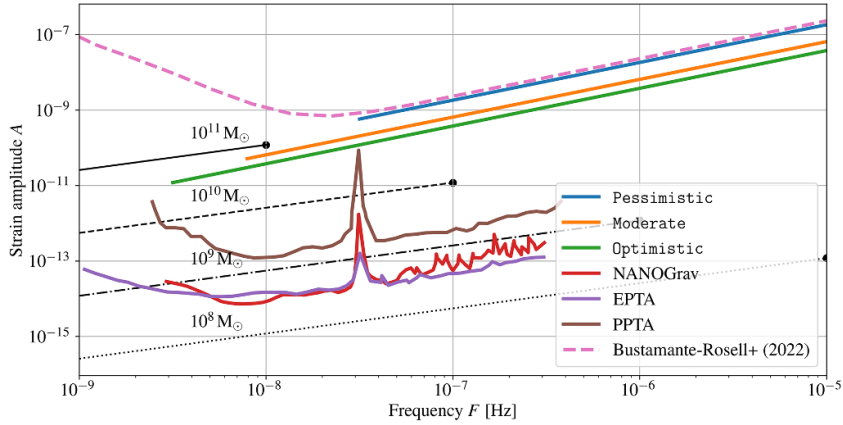
**Figure 2.** Amplitude spectral densities  $a_\alpha\sqrt{T}$  of gravitational wave signals from individually resolvable binary white dwarfs (BWDs) in three different models [45] as a function of their frequency  $f=f_\alpha$ . The solid line indicates the root of the projected noise power spectral density  $\sqrt{\rho}$  of LISA [45, 46]. BWDs are assumed to be individually resolvable if  $a_\alpha\sqrt{T/\rho(f_\alpha)} > 7$ .

magnitude in  $A$ . Our method is sensitive to GWs with frequencies as low as  $F \sim 10^{-8}$  Hz. GWs of these frequencies could be present in our Universe, e.g. as part of a (stochastic) background of GWs emitted by numerous individual sources [49]. At a frequency of  $F \simeq 10^{-8}$  Hz our method would be sensitive to amplitudes  $A \gtrsim 10^{-10}$ ; GWs of that amplitude at that frequency could, e.g. be emitted by a very massive SMBHB with a chirp mass of several  $\sim 10^{10} M_\odot$  at a distance  $D = 10$  Mpc, which is the scale of the Virgo cluster. The existence of such an object is unlikely, since observations from pulsar timing indicate that there is no SMBHB of that mass within a distance  $D \lesssim \mathcal{O}(1)$  Gpc [48]. Yet, analysing the GWs from BWDs could pose an independent method to probe the existence of these very massive SMBHBs.

We also consider the more general case of a number of carrier GW signals observed with an unspecified GW detector. For this case we assume that all  $N$  carrier signals have a similar frequency and are detected with the same SNR  $\sim a_\alpha\sqrt{T/\rho(f_\alpha)} = \text{const}$ . In figure 4, we show the correlated background GW amplitude that can be detected at an SNR of one, as a function of the number and individual SNR of the carrier signals.

We can apply this result to a proposed next-generation GW detector such as DECIGO [50, 52, 53], which operates in the dHz regime and is expected to observe GWs from a large number of compact binary stars. Assuming DECIGO observes GW signals from a population of  $N = 10^5$  binary neutron stars (BNSs) each observed with an SNR of  $\sim 10^4$  [50] at a typical frequency of  $f_\alpha = 0.1$  Hz, it would be possible to detect background GWs from SMBHBs with chirp masses of about  $\sim 10^9 M_\odot$  (at a fiducial distance  $D = 10$  Mpc and frequency  $F = 10^{-8}$  Hz). This would make the sensitivity of DECIGO to low-frequency GWs competitive with that of current PTAs (cf figure 3 and [54]).

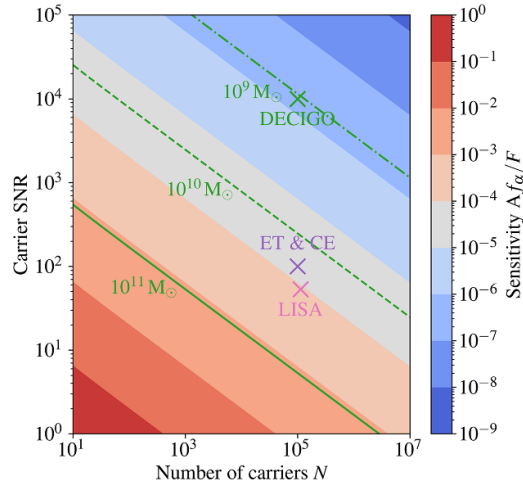
For reference, we also indicate in figure 4 the sensitivity that could be obtained using  $\sim 10^5$  carrier signals with an SNR  $\sim 10^2$  from compact binary coalescences, as expected to be detected using both Einstein Telescope (ET) and Cosmic Explorer (CE) [51]. These carrier signals would have frequencies between 10 and  $10^3$  Hz and could be observed for a duration



**Figure 3.** Sensitivity to low-frequency gravitational waves (GWs) that can be obtained by searching for correlated modulations in a set of well-resolved GW signals from binary white dwarfs (BWDs), as expected to be detected with LISA. For reference, we show the expected GW amplitudes of SMBHBs with chirp masses ranging from  $10^8$  to  $10^{11} M_{\odot}$  at a fiducial distance  $D = 10$  Mpc. These latter curves are cut off at the frequency of the Innermost Stable Circular Orbit  $f \lesssim 1$  kHz ( $M_{\odot}/M_c$ ) evaluated for equal-mass binaries [47], above which the SMBHBs quickly merge. We also show sensitivity curves from Bustamante-Rosell *et al* [36, figure 5], as well as the sensitivity curves of Pulsar Timing Arrays (PPTA [27]; EPTA [29]; NANOGrav [48]). The detection threshold (SNR = 2) is chosen to allow a consistent comparison to reported PTA sensitivities. In practice, we expect our method to show a reduction in sensitivity around  $F \simeq 1/\text{yr} \simeq 32$  nHz as seen for PTAs, where it would be difficult to distinguish a background GW from the Doppler modulation due the annual motion of LISA around the Sun. The sensitivity of our method is limited to frequencies  $F \gtrsim 1/T$  (e.g. 32 nHz in the *Pessimistic* model), below which the sensitivity is limited by the finite width of the frequency bins.

$T \lesssim 10^3$  s, which means the minimum detectable background GW frequency using our method is  $F \sim 10^{-3}$  Hz. Coherent background GW signals may be searched for using non-coincident carrier signals with a slight modification of the method described in section 3; a frequency-dependent phase correction ( $\phi_{\text{corr}} = 2\pi T_{\text{diff}}F$ ) must be applied to each carrier's demodulated spectrum (equation (6)), for a time difference between the signals  $T_{\text{diff}}$ . In case the background GW signal has a coherence time much shorter than the total observation time for all signals (i.e. the detector's lifetime), only coincident carrier signals can be cross-correlated to gain sensitivity.

The sensitivity of our method is fundamentally limited to frequencies  $F \gtrsim 1/T$ , as for lower frequencies the background signal cannot be distinguished from the carrier [36]. The same low-frequency limit due to observation time exists for PTAs. The high-frequency limit of our method is set by the Nyquist frequency of the detector output sampling,  $f_s/2$ , where for LISA



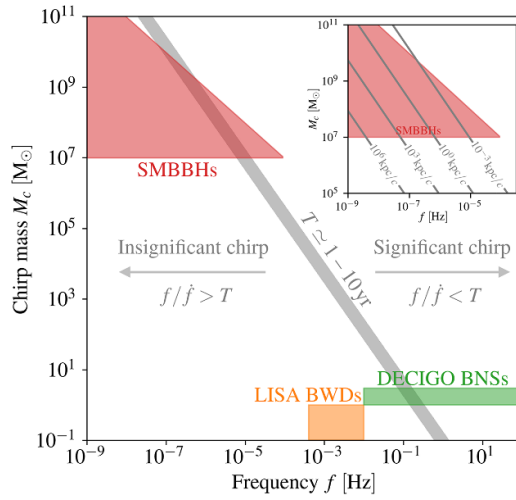
**Figure 4.** Order-of-magnitude estimate for the sensitivity to background gravitational waves (GWs) by cross-correlating a generic set of a number of GW signals  $N$  that are each detected with a certain SNR ('Carrier SNR'). The sensitivity (given by the colour scale) is expressed as the product of the background amplitude  $A$  times the typical frequency ratio of the background and carrier signals  $f_\alpha/F$ , where the detection threshold corresponds to an SNR equal to one. Furthermore, we indicate the sensitivity that could be obtained using a set of GW signals in the dHz regime from binary neutron stars as carriers, which could be done using data from DECIGO [50], and similarly the sensitivity using carrier signals detected using ET and CE [51]. We also show the sensitivity that could be obtained using the average SNR of binary white dwarf signals detected by LISA (in the Moderate model), as explicated in figure 3. For these detectors we assume typical carrier frequencies of  $f_\alpha \simeq 0.1$  Hz (DECIGO), 10 Hz (ET/CE), and  $10^{-3}$  Hz (LISA). For reference, we show contour lines that correspond to GW amplitudes from super-massive binary black holes with chirp masses ranging from  $10^9$  to  $10^{11} M_\odot$  at a fiducial distance  $D = 10$  Mpc, with a background frequency  $F = 10^{-8}$  Hz, and a carrier frequency  $f_\alpha = 0.1$  Hz.

$f_s \sim 1$  Hz [36]. PTAs have a much smaller sensitive bandwidth due to the low observation cadence of radio telescopes (once every several days or less).

## 6. Discussion

An important assumption used in deriving the sensitivity projections above is that the carrier signal frequency, amplitude, and phase are known without uncertainty. In practice, there will be non-negligible uncertainty in the determination of these parameters for the BWD signals detected with LISA, which means the normalisation and demodulation in equation (5) is subject to inaccuracy. The error in performing this operation on the data would propagate to an inaccuracy in the addition of carrier sidebands across different spectra (i.e. different sidebands would not line up in frequency exactly and their coherent summation would be imperfect). This uncertainty in the carrier GW parameters would thus reduce the signal-to-noise ratio of background GW signals, and needs to be accounted for in equation (11), which we leave for future work.

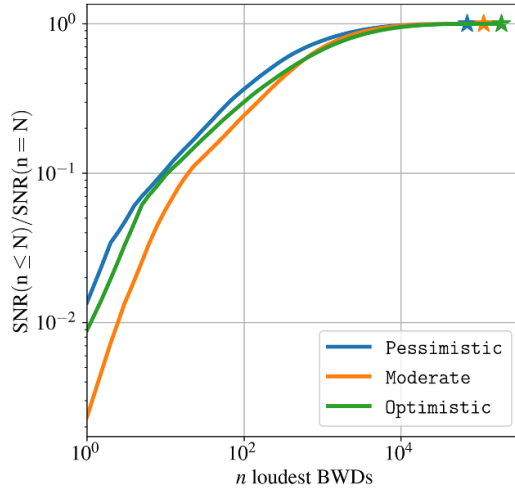
An effect of particular concern that manifests as uncertainty of the frequency of the carrier signal is phase noise imparted by the data acquisition system of the gravitational-wave detector.



**Figure 5.** Timescale  $f/\dot{f} = (5/96)(c^3/GM_c)^{5/3}(\pi f)^{-8/3}$  at which the frequency  $f$  of a compact binary with chirp mass  $M_c$  significantly increases due to energy loss through gravitational-wave emission. Coloured boxes indicate the parameter regions of background super-massive binary black holes (SMBBHs), LISA binary white dwarfs (BWDs), and DECIGO binary neutron stars (BNSs). This shows that LISA BWDs and most of the SMBBHs would not undergo significant frequency changes within the observation time  $T \approx 1-10$  yr, whereas most DECIGO BNSs would. The inset shows whether the SMBBHs would exhibit significant frequency changes within typical light travel times between a carrier source and the observer, i.e. whether ‘local’ and ‘distant’ sidebands overlap or not. For this figure we take the maximum GW frequency emitted by SMBBHs to correspond to the Innermost Stable Circular Orbit  $f \lesssim 1$  kHz ( $M_\odot/M_c$ ) evaluated for equal-mass binaries [47], which causes the diagonal cut-off.

As this noise would appear as random modulations of the carrier signal, it could obfuscate any background GWs that produce the same effect. Phase noise in the data acquisition system, due to, e.g. timing jitter of the sampling clocks, would produce irreducible correlated noise in the demodulated cross-spectra of different carriers. This effect might only be reduced by cross-correlating data obtained with different uncorrelated oscillators. Similarly, stochastic phase noise intrinsic to the carrier GW signal would reduce sensitivity to background GWs. In this case the effect on the sensitivity is limited as this noise will be uncorrelated between carriers and will be reduced in the average cross-spectrum (equation (8)).

In addition to these effective stochastic fluctuations of the carrier signal, there could be deterministic frequency changes of the carrier and background GWs. If the frequency of the background GWs changes significantly over the measurement time, i.e. if the GW background power spectral density is non-stationary, the coherent signal power would be spread over multiple frequency bins, leading to a lower SNR in each bin. An SMBHB background source might undergo a significant frequency evolution as its orbital period decays due to energy loss by GW emission. Figure 5 shows that this frequency change  $\dot{f}$  (‘chirp’) would not be significant for SMBBHs ( $M_c \gtrsim 10^9$ ) over the duration of observation  $T \approx 1-10$  yr. Figure 5 also shows the expected frequency changes of the LISA and DECIGO carrier signals. In particular, it shows that most DECIGO BNSs undergo significant frequency evolution over the duration of the detected signal. As discussed in section 3, these frequency changes could be compensated for at the demodulation stage.



**Figure 6.** Cumulative SNR of a background gravitational wave signal as a function of the  $n$  loudest binary white dwarfs (BWDs) in the set of carrier signals. Stars at the end of each line indicate the total number  $N$  of binaries in each model. In any model several  $10^2$  to  $10^3$  of the loudest BWDs are enough to achieve sensitivities similar to the entire sample.

Non-stationarity of the background GW PSD has another effect; the frequency change over a time equal to the typical light travel time between the carrier source and observer determines the frequency-space separation of the ‘local’ and ‘distant’ sideband terms, i.e.  $|F_L - F_D| \propto d_\alpha \dot{F}/c$ . If these terms are not separated in the spectrum, i.e. when  $|F_L - F_D| \lesssim 1/T$ , coherent summation of the ‘local’ terms of different cross-spectra is still possible but the ‘distant’ terms would add a small incoherent noise-like contribution to any signal bin. The inset of figure 5 shows that given typical light travel times between BWDs and the LISA detector of  $d_\alpha/c \simeq 10^{-1} - 10^1$  kpc/c [44], both separated and non-separated sidebands could be observed for background SMBHB GW sources. On the other hand, DECIGO will observe carrier signals from BNSs at much larger distances, e.g.  $d_\alpha \simeq 10^4$  kpc for a GW170817-like event [55], and therefore ‘local’ and ‘distant’ sidebands produced by a background SMBHB source ( $M_c \gtrsim 10^9 M_\odot$ ) would be well-separated in DECIGO data.

We note that for the sensitivity projections for LISA, the number  $N$  of individually resolvable BWDs in our models (see table 1) is larger by a factor up to  $\sim 10$  compared to previous estimates from Galaxy models combined with a binary population model [4, 42, 44, 56, 57], which reflects the large uncertainty of current predictions about the detectable BWD population. However, the exact total number of BWDs does not significantly affect the estimated sensitivity because the  $\sim \mathcal{O}(10^3)$  loudest BWDs signals provide the dominant contribution to the sensitivity. This is shown in figure 6; where we plot the normalised cumulative contribution of BWDs to the total SNR. It can be seen that several  $10^2$  to  $10^3$  BWDs are enough to achieve similar sensitivities to the total BWD population.

Importantly, we note that the estimates for detecting low-frequency gravitational waves using BWD carrier signals in LISA as obtained in Bustamante-Rosell *et al* [36] yield a sensitivity that is worse than the sensitivity we project for this scenario by roughly an order of

magnitude (for the *moderate* model, see figure 3). Bustamante-Rosell *et al* [36] obtain their main sensitivity estimate for the background GW amplitude by evaluating the Fisher information, which is equivalent to the maximum likelihood estimator in our equation (11). However, Bustamante-Rosell *et al* [36] make different assumptions about the carrier signals and their uncertainty. The lower sensitivity estimate found in their work is likely in part due to the inclusion of the uncertainty in the estimation of the carrier signal parameters, and the Fisher-information approach therefore presents a theoretical upper bound on the sensitivity that can be obtained with our method. Our estimate does not account for uncertainty in estimating the carrier signal parameters as explained above, whereby we potentially overestimate the sensitivity of our method. Another aspect that could account for some of the discrepancy between our sensitivity projections and those of Bustamante-Rosell *et al* [36] is the difference in BWD carrier signal populations used; the numbers of individually resolvable BWDs in the observationally-driven populations used in our work is greater than the number assumed in Bustamante-Rosell *et al* [36].

## 7. Conclusion

In this work, we have outlined a method to use a set of carrier gravitational wave sources to search for correlated frequency modulations caused by low-frequency background GWs. In this method demodulated cross-spectra of carrier sources are added coherently and with optimal weights such that any modulation common to the carrier sources is amplified with respect to random detector noise.

We considered the case of using our method to search for low-frequency GWs in data from LISA, which is expected to detect GWs from a large number of Galactic binary white dwarfs. The projected sensitivity that could thus be obtained (figure 3) ranges from strain amplitudes of  $A \sim 10^{-10}$  at  $F \sim 10^{-8}$  Hz to  $\sim 10^{-7}$  at  $\sim 10^{-5}$  Hz, and would cover a part of the GW spectrum where no other detection methods are currently available.

This sensitivity could potentially enable the detection of very massive SMBHBs with a chirp mass of several  $10^{10} M_{\odot}$  at a distance of  $D = 10$  Mpc, although the existence of such a system is all but ruled out by pulsar timing array observations [48].

Our results show that an even better sensitivity could be achieved using GW signals from compact binary stars detectable with next-generation GW detectors that operate in the dHz regime. In particular, using signals of binary neutron stars expected to be detected with DECIGO would yield a sensitivity competitive with that of current pulsar timing arrays.

Future detectors designed to detect GW signals in a higher frequency range could be used to indirectly probe GWs down to the frequencies given by the inverse instrument lifetime. Potentially, this allows one to probe GW frequencies in the nHz– $\mu$ Hz range, which are expected to be emitted by the most massive black holes in our Universe. Conveniently, this could be achieved without modification of the detector designs and with the same data outputs. This method could therefore prove a valuable tool in the exploration of the gravitational-wave spectrum and the development of gravitational-wave astronomy in general.

## Data availability statement

No new data were created or analysed in this study.

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