Eviction and Reception for Description Logic Concepts (Extended Abstract)

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Abstract

We study belief change for the case in which beliefs are expressed as concepts in description logic. We consider that the incoming information is in the format of a set of pointed interpretations and investigate eviction (removal of pointed models) and reception (addition of pointed models). We provide preliminary results in this setting, establishing whether $\mathcal{EL}_\perp$ and $\mathcal{ALC}$-concepts are eviction/reception compatible.

1. Introduction

In *Belief Change*[1, 2, 3], when confronted with a piece of information, an agent must modify its beliefs minimally: only beliefs in conflict with the incoming information can be removed. The principle of minimal change is conceptualised via sets of rationality postulates, whilst several classes of operators that abide by such postulates were proposed. The standard paradigms of belief change, such as the AGM paradigm[1] of *belief revision* and the KM paradigm[4] of *belief update*, assume the incoming information to be represented as formulae. In other areas, however, different forms of representing incoming information, such as sets of finite models, have been addressed. In the *learning from interpretations* setting [5], for example, the goal is to identify a concept that is satisfied by a given set of interpretations, classified as positive, while falsified by another given set of interpretations (classified as negative). Ideally, the concept should also follow the principle of minimal change, in the sense that the identified concept should be as close as possible to the set of positive interpretations.

Guimaraes et al. [6] have generalised the belief change paradigm to the setting where the incoming information is expressed as a set of interpretations, while the new corpus of beliefs should be finitely representable. Two main operations were proposed: eviction, which consists in removing the given set of interpretations; and reception, which consists in accommodating the given set of interpretations. Keeping the principle of minimal change and finite representation in this setting is challenging, and the authors have identified that eviction and reception are not definable in some logics. In this work, we deepen this investigation and consider concepts expressed in the $\mathcal{EL}_\perp$ and $\mathcal{ALC}$ description logics (DLs). We show that while eviction is definable in $\mathcal{EL}_\perp$, reception is not definable in $\mathcal{EL}_\perp$ nor $\mathcal{ALC}$.

2. Eviction and Reception

Following [7, 8], and [9], we use satisfaction systems to define logics. A *satisfaction system* is a triple $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$, where $\mathcal{L}$ is a language, $\mathfrak{M}$ is a set of models, and $\models$ is a satisfaction relation which contains all pairs $(M, \mathcal{B})$, where M is a model and $\mathcal{B}$ is a *base* (that is, a subset of $\mathcal{L}$), such that M satisfies $\mathcal{B}$ (i.e., $M \models \mathcal{B}$). We denote by $\mathsf{mod}_\Lambda(\mathcal{B})$ the set $\{M \in \mathfrak{M} \mid M \models \mathcal{B}\}$. We write simply $\mathsf{mod}(\mathcal{B})$ when the satisfaction system is clear from the context. Satisfaction systems allow us to be more flexible and precise regarding the precise scope of the operations and constructions we define.

This view also facilitates the generalisation of some results that do not depend on properties of the consequence relation of the logic.

The power set of a set A is denoted by $\mathcal{P}(A)$, while the set of all finite subsets of A is denoted by $\mathcal{P}_{\mathrm{f}}(A)$. We write $\mathcal{P}^*(A)$ to denote the non-empty subsets of A. An arbitrary set of models $\mathbb{M} \subseteq \mathfrak{M}$ within Λ is finitely representable iff there is $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ such that $\mathrm{mod}(\mathcal{B}) = \mathbb{M}$. $\mathrm{FR}(\Lambda)$ denotes the collection of all *finitely representable sets of models in* Λ, that is, the set $\{\mathbb{M} \subseteq \mathfrak{M} \mid \exists \mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L}) : \mathrm{mod}(\mathcal{B}) = \mathbb{M}\}$. Also, we say that a set of formulae $\mathcal{B} \subseteq \mathcal{L}$ is finitely representable iff there is a $\mathcal{B}' \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ with $\mathrm{mod}(\mathcal{B}) = \mathrm{mod}(\mathcal{B}')$.

Eviction turns the current belief state into a new one not satisfied by any of the input models. Reception turns the current belief state into a new one satisfied by all the input models.

Definition 1. *For all satisfaction systems* $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ *and* $\mathbb{M} \subseteq \mathfrak{M}$,

$$\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda) := \{\mathbb{M}' \in \mathrm{FR}(\Lambda) \mid \mathbb{M}' \subseteq \mathbb{M} \text{ and } \nexists \mathbb{M}'' \in \mathrm{FR}(\Lambda) \text{ with } \mathbb{M}' \subset \mathbb{M}'' \subseteq \mathbb{M}\}.$$

$$\mathrm{MinFRSups}(\mathbb{M}, \Lambda) := \{\mathbb{M}' \in \mathrm{FR}(\Lambda) \mid \mathbb{M} \subseteq \mathbb{M}' \text{ and } \nexists \mathbb{M}'' \in \mathrm{FR}(\Lambda) \text{ with } \mathbb{M} \subseteq \mathbb{M}'' \subset \mathbb{M}'\}.$$

Definition 2. *An* eviction operation *for a satisfaction system* $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ *is a function* $\mathrm{evc} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ *that satisfies the following postulates, for all* $\mathbb{M}, \mathbb{M}' \subseteq \mathfrak{M}$ *and for all* $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$:

(success) $\mathbb{M} \cap \mathrm{mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \emptyset$.

(inclusion) $\mathrm{mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subseteq \mathrm{mod}(\mathcal{B})$.

(finite retainment) *If* $\mathrm{mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) \subset \mathbb{M}' \subseteq \mathrm{mod}(\mathcal{B}) \setminus \mathbb{M}$ *then* $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$.

(uniformity) $\mathrm{MaxFRSubs}(\mathrm{mod}(\mathcal{B}) \setminus \mathbb{M}, \Lambda) = \mathrm{MaxFRSubs}(\mathrm{mod}(\mathcal{B}') \setminus \mathbb{M}', \Lambda)$ *implies* $\mathrm{mod}(\mathrm{evc}(\mathcal{B}, \mathbb{M})) = \mathrm{mod}(\mathrm{evc}(\mathcal{B}', \mathbb{M}'))$.

Definition 3. *A* reception operation *for a satisfaction system* $\Lambda = (\mathcal{L}, \mathfrak{M}, \models)$ *is a function* $\mathrm{rcp} : \mathcal{P}_{\mathrm{f}}(\mathcal{L}) \times \mathcal{P}(\mathfrak{M}) \to \mathcal{P}_{\mathrm{f}}(\mathcal{L})$ *that satisfies the following postulates, for all* $\mathbb{M}, \mathbb{M}' \subseteq \mathfrak{M}$ *and for all* $\mathcal{B} \in \mathcal{P}_{\mathrm{f}}(\mathcal{L})$:

(success) $\mathbb{M} \subseteq \mathrm{mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(persistence) $\mathrm{mod}(\mathcal{B}) \subseteq \mathrm{mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$.

(finite temperance) *If* $\mathrm{mod}(\mathcal{B}) \cup \mathbb{M} \subseteq \mathbb{M}' \subset \mathrm{mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M}))$ *then* $\mathbb{M}' \notin \mathrm{FR}(\Lambda)$.

(uniformity) $\mathrm{MinFRSups}(\mathrm{mod}(\mathcal{B}) \cup \mathbb{M}, \Lambda) = \mathrm{MinFRSups}(\mathrm{mod}(\mathcal{B}') \cup \mathbb{M}', \Lambda)$ *implies* $\mathrm{mod}(\mathrm{rcp}(\mathcal{B}, \mathbb{M})) = \mathrm{mod}(\mathrm{rcp}(\mathcal{B}', \mathbb{M}'))$.

For a discussion on the postulates presented in this section, see [6]. Neither reception nor eviction are definable in every logic. Eviction can only be defined in logics where $\mathrm{MaxFRSubs}(\mathbb{M}, \Lambda) \neq \emptyset$, for all sets of models $\mathbb{M}$, whilst reception can only be defined in logics where $\mathrm{MinFRSups}(\mathbb{M}, \Lambda) \neq \emptyset$, for all sets of models $\mathbb{M}$. Such logics are called respectively *eviction-compatible* and *reception-compatible*. Incompatibility is a problem inherent in belief change, and even when the incoming information is represented as formulae, some belief change operators, such as contraction[1], cannot be defined in some logics [10, 11, 12].

3. Description Logic Concepts: Eviction and Reception

In this section we establish preliminary results on eviction and reception compatibility for DL concepts. In particular, we show that $\mathcal{EL}_{\bot}$ is eviction-compatible (Theorem 4).

DL Concepts Let $\mathsf{N_C}$ and $\mathsf{N_R}$ be countably infinite and pairwise disjoint sets of concept names and role names, respectively. *$\mathcal{EL}$-concepts* are built according to the rule: $C, D ::= \top \mid A \mid (C \sqcap D) \mid (\exists r.C)$,

where $A \in \mathsf{N_C}$ and $r \in \mathsf{N_R}$. $\mathcal{EL}_\perp$-concepts extend $\mathcal{EL}$ by allowing $\perp$ (interpreted as the empty set). $\mathcal{ALC}$-concepts extend $\mathcal{EL}$-concepts with the rule $\neg C$ (recall that $C \sqcap \neg C$ is equivalent to $\perp$, so $\mathcal{ALC}$ extends $\mathcal{EL}_\perp$). We may omit parentheses if there is no risk of confusion. A *pointed interpretation* is a pair $(\mathcal{I}, d)$ where $\mathcal{I} = (\cdot^\mathcal{I}, \Delta^\mathcal{I})$ is an interpretation and $d \in \Delta^\mathcal{I}$. A pointed interpretation $(\mathcal{I}, d)$ *satisfies* a concept C iff $d \in C^\mathcal{I}$. The semantics of $\mathcal{EL}$, $\mathcal{EL}_\perp$, and $\mathcal{ALC}$ is defined using interpretations, as usual for DLs [13, 14].

Canonical Model Given a satisfiable $\mathcal{EL}_\perp$-concept D, we inductively define the tree-shaped interpretation $\mathcal{I}_D$ of D, with the root denoted d_D, as follows. When D is $\top$, we define $\mathcal{I}_\top$ as the interpretation with $\Delta^{\mathcal{I}_\top} := \{d_\top\}$ and all concept and role names interpreted as the empty set. For D a concept name $A \in \mathsf{N_C}$ we define $\mathcal{I}_A$ as the interpretation with $\Delta^{\mathcal{I}_A} := \{d_A\}$, $A^{\mathcal{I}_A} := \{d_A\}$, and all other concept and role names interpreted as the empty set. For $D = \exists r.C$, we define $\mathcal{I}_D$ as the interpretation with $\Delta^{\mathcal{I}_D} := \{d_D\} \cup \Delta^{\mathcal{I}_C}$. All concept and role name interpretations are as for $\mathcal{I}_C$ and we add (d_D, d_C) to $r^{\mathcal{I}_D}$, and assume d_D is fresh (i.e., it is not in $\Delta^{\mathcal{I}_C}$). Finally, for $D = D_1 \sqcap D_2$ we define $\Delta^{\mathcal{I}_D} := \Delta^{\mathcal{I}_{D_1}} \cup (\Delta^{\mathcal{I}_{D_2}} \setminus \{d_{D_2}\})$, assuming $\Delta^{\mathcal{I}_{D_1}}$ and $\Delta^{\mathcal{I}_{D_2}}$ are disjoint, and with all concept and role name interpretations as in $\mathcal{I}_{D_1}$ and $\mathcal{I}_{D_2}$, except that we connect d_{D_1} with the elements of $\Delta^{\mathcal{I}_{D_2}}$ in the same way as d_{D_2} is connected. In other words, we identify d_{D_1} with the root d_{D_2} of $\mathcal{I}_{D_2}$.

Homomorphism Given two pointed interpretations $(\mathcal{I}, d_0)$, $(\mathcal{J}, e_0)$, a *homomorphism* from $(\mathcal{I}, d_0)$ to $(\mathcal{J}, e_0)$ is a function $h : \Delta^\mathcal{I} \to \Delta^\mathcal{J}$ that satisfies: (i) $h(d_0) = e_0$; (ii) for all $A \in \mathsf{N_C}$, if $d \in A^\mathcal{I}$ then $h(d) \in A^\mathcal{J}$; and (iii) for all $r \in \mathsf{N_R}$, if $(d, d') \in r^\mathcal{I}$ then $(h(d), h(d')) \in r^\mathcal{J}$.

Our proof strategy for Theorem 4 is to invoke Theorem 1 (see [6]), which we recall here, together with the two technical lemmas below.

Theorem 1 (Theorem 16[6]). *A satisfaction system Λ is eviction-compatible iff for every $\mathbb{M} \subseteq \mathfrak{M}$ either (i) $\mathbb{M} \in \mathrm{FR}(\Lambda)$, (ii) $\mathbb{M}$ has an immediate predecessor in $(\mathrm{FR}(\Lambda) \cup \{\mathbb{M}\}, \subset)$, or (iii) there is no $\mathbb{M}' \in \mathrm{FR}(\Lambda)$ with $\mathbb{M} \subseteq \mathbb{M}'$.*

Lemma 2 (Adapted [15]). *For all satisfiable $\mathcal{EL}_\perp$-concepts C and D, we have that $\models C \sqsubseteq D$ iff there is a homomorphism from $(\mathcal{I}_D, d_D)$ to $(\mathcal{I}_C, d_C)$.*

Lemma 3. *For all satisfiable $\mathcal{EL}_\perp$-concepts C, if C is satisfiable then there is an $\mathcal{EL}_\perp$-concept C' such that $\models \perp \sqsubset C' \sqsubset C$.*

Let $\Lambda(\mathcal{EL}_\perp$-concepts$)$ be the satisfaction system for $\mathcal{EL}_\perp$-concepts.

Theorem 4. $\Lambda(\mathcal{EL}_\perp$-concepts$)$ *is eviction-compatible.*

Proof. We show that every *non-empty* $\mathbb{M} \subseteq \mathfrak{M}$ has an immediate predecessor in $(\mathrm{FR}(\Lambda(\mathcal{EL}_\perp$-concepts$))) \cup \{\mathbb{M}\}, \subset)$ (the case $\mathbb{M}$ is empty is easy as $\mathcal{EL}_\perp$ has the $\perp$ concept). Recall that an immediate predecessor of $\mathbb{M}$ in this case is a set of models $\mathbb{M}' \in \mathrm{FR}(\Lambda(\mathcal{EL}_\perp$-concepts$))$ such that $\mathbb{M}' \subset \mathbb{M}$ and there is no $\mathbb{M}''$ such that $\mathbb{M}' \subset \mathbb{M}'' \subset \mathbb{M}$. For this, we show that given a finitely representable set of models $\mathbb{M}$, there is no infinite chain $\mathbb{M}_1 \subset \mathbb{M}_2 \subset \ldots$ of sets of models in $(\mathrm{FR}(\Lambda(\mathcal{EL}_\perp$-concepts$)))$ such that $\mathbb{M}_i \subset \mathbb{M}$ for all $i \in \mathbb{N}$. Indeed, given $\mathbb{M}, \mathbb{M}' \in (\mathrm{FR}(\Lambda(\mathcal{EL}_\perp$-concepts$)))$, with $\mathbb{M}' \subset \mathbb{M}$, let C, D be $\mathcal{EL}_\perp$-concepts such that $\mathsf{mod}(C) = \mathbb{M}$ and $\mathsf{mod}(D) = \mathbb{M}'$. By the semantics of $\mathcal{EL}_\perp$, we have that $\models D \sqsubset C$. We want to show that there are finitely many $\mathcal{EL}_\perp$-concepts E such that $\models D \sqsubset E \sqsubset C$. Suppose there is no $\mathbb{M}' \in \mathrm{MaxFRSubs}(\Lambda(\mathcal{EL}_\perp$-concepts$))$ such that $\mathbb{M}' \subset \mathbb{M}$ with $\mathbb{M}' \neq \emptyset$. That is, $\perp$ is the only $\mathcal{EL}_\perp$-concept such that $\models \perp \sqsubseteq C$, or, in other words, there is no (satisfiable) $\mathcal{EL}_\perp$-concept C' such that $\models \perp \sqsubset C' \sqsubset C$. However, as $\mathbb{M}' \subset \mathbb{M}$, we have that $\mathbb{M} \neq \emptyset$, so C is satisfiable. This cannot be the case for $\mathcal{EL}_\perp$-concepts due to Lemma 3. By Lemma 3, if C is satisfiable then there is C' such that $\models \perp \sqsubset C' \sqsubset C$.

Then, there is $\mathbb{M}' \in (\mathrm{FR}(\Lambda(\mathcal{EL}_\perp$-concepts$)))$ such that $\mathbb{M}' \subset \mathbb{M}$ and $\mathbb{M}' \neq \emptyset$, so D is satisfiable. Let E be an $\mathcal{EL}_\perp$-concept such that $\models D \sqsubset E \sqsubset C$ (if there is no such concept then we are done). Since $\models D \sqsubset E$ we have that E is satisfiable. Then, by Lemma 2, there is a homomorphism from $(\mathcal{I}_E, d_E)$ to

$(\mathcal{I}_D, d_D)$. As D is finite, the number of concept and role names that occur in D is finite. Denote with $\mathsf{sig}(D)$ the set of concept and role names occurring D. Also, the existence of a homomorphism from $(\mathcal{I}_E, d_E)$ to $(\mathcal{I}_D, d_D)$ and the definition of $\mathcal{I}_E$ and $\mathcal{I}_D$ imply that the set of concept and role names occurring in E is a subset of $\mathsf{sig}(D)$. That is, there are finitely many concept and role names occurring in E and they are subset of $\mathsf{sig}(D)$. Moreover, by definition of $\mathcal{I}_E$ and $\mathcal{I}_D$, these interpretations correspond to tree-shaped labelled structures where the depth of (the tree corresponding to) $\mathcal{I}_E$ is less or equal to the depth n of $\mathcal{I}_D$. Since E was an arbitrary $\mathcal{EL}_\perp$-concept such that $\models D \sqsubset E$ this holds for all such concepts. As there are finitely many tree-shaped labelled structures with symbols in $\mathsf{sig}(D)$ and depth bounded by n, there are finitely many $\mathcal{EL}_\perp$-concepts E (up to logical equivalence) such that $\models D \sqsubset E$. Then there are finitely many $\mathcal{EL}_\perp$-concepts E (up to logical equivalence) such that $\models D \sqsubset E \sqsubseteq C$. This means that, by Theorem 16 in [6], $\Lambda(\mathcal{EL}_{\perp\mathrm{concepts}})$ is eviction-compatible. $\qquad\square$

The previous result does not hold for $\mathcal{EL}$ (without $\perp$) as it cannot express inconsistencies [6]. Let $\Lambda(\mathcal{ALC}\text{-concepts})$ be the satisfaction system for $\mathcal{ALC}$-concepts.

Theorem 5. $\Lambda(\mathcal{EL}_\perp\text{-concepts})$ *and* $\Lambda(\mathcal{ALC}\text{-concepts})$ *are not reception-compatible.*

The proof of Theorem 6 is based on [6].

Theorem 6. $\Lambda(\mathcal{ALC}\text{-concepts})$ *is not eviction-compatible.*

Acknowledgments

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