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**Raindrop size distribution (DSD) retrieval from polarimetric radar
observations using neural networks**

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12 **1 Introduction**

13 Rain is a major component of the water cycle, knowledge of its raindrop size distributions
14 (DSD) is critical for multiple Earth science applications, such as rainfall retrieval using remote
15 sensing (e.g., ground-based and spaceborne radar) (Dai and Han, 2014; Dai et al., 2014),
16 assessing soil loss due to raindrop splash (Fernandez-Raga et al., 2017; Dai et al., 2020),
17 determining aerosol-cloud-rainfall relationships, and the initialization and verification of cloud
18 microphysical models (May et al., 2011; Sarangi et al., 2017; Testik and Rahman, 2017). DSD
19 varies spatially and temporally both within specific storm types and across storm types and
20 climatic regimes (Bringi et al., 2003). Such variability is not easily reflected through point-
21 based DSD measurements such as disdrometer data. Since the introduction of polarimetric
22 weather radar, the estimation of DSD over large areas has been a long-standing goal in radar
23 meteorology (Bringi et al., 2003; Bringi et al., 2009; Kim et al., 2010; Anagnostou et al., 2013).
24 The precipitation information obtained from dual-polarized radar is now considered an
25 indispensable tool for studying the formation and evolution of precipitation (Brown et al., 2016;
26 Kumjian and Lombardo, 2017; Thompson et al., 2018; Kumjian et al., 2019).

27 Two classes of microphysics parameterization exist to describe DSD (bin
28 parameterization and bulk parameterization), which are distinguished by how they represent
29 the relationship between raindrop number concentration and its drop size. A disdrometer
30 parameterizes the DSD by discretizing the raindrop size distribution into a number of bins and
31 measuring the total number concentration per unit volume for each bin. In contrast, DSD
32 parameterization in dual-polarized radar assumes a prescribed analytic form for the size
33 distribution, such as the exponential, Gaussian, lognormal, or gamma function (Ulbrich, 1983;

34 Feingold and Levin, 1986; Torres et al., 1994). This is less computationally expensive and
35 easier; thus, it is more widely used (Feingold and Levin, 1986). In these models, DSD can be
36 characterized by only a few parameters related to characteristic diameter, concentration, and
37 shape distribution.

38 The advantage of dual-polarized radar is the accurate retrieval of DSD parameters by
39 determining the relationship between a group of radar observations and a group of DSD
40 parameters (Bringi et al., 2003; Le and Chandrasekar, 2011). For example, Bringi et al. (2003)
41 revealed that the differential reflectivity (Z_{DR}) is directly related to the drop median diameter
42 (D_0) defined in the exponential DSD function. A simple polynomial regression equation can be
43 established to estimate D_0 using Z_{DR} values measured by a polarimetric weather radar. Similar
44 explicit empirical relationships have been proposed for all DSD parameters and widely applied
45 to different types of polarimetric radar around the world. An enormous number of studies have
46 been presented in the past two decades (Zhang et al., 2001; Gorgucci et al., 2002; Brandes et
47 al., 2003; Brandes et al., 2004; Vivekanandan et al., 2004; Anagnostou et al., 2008; Gorgucci
48 et al., 2008; Bringi et al., 2009; Kim et al., 2010; Le and Chandrasekar, 2011; Anagnostou et
49 al., 2013; Yoshikawa et al., 2016; Brauer et al., 2020; Tokay et al., 2020; Wen et al., 2020;
50 Abhijeet et al., 2023).

51 However, there are two major concerns regarding the traditional DSD retrieval algorithms.
52 First, the regression relationship between the dual polarization parameters and the DSD
53 parameters has application scenario limitations. A simple polynomial regression may not fully
54 exploit the in-depth relationship using available observations (Bringi and Chandrasekar, 2001;
55 Tokay et al., 2013). For example, as reported by Anagnostou et al. (2008), the D_0 - Z_{DR}

relationship is only applicable in the case of $0.3 \leq Z_{DR} \leq 2.5$ dB (Kim et al., 2010). The fitted relationship between the shape parameter μ and the radar variables (i.e. Z_H and Z_{DR}) is quite weak when using the regression method. Furthermore, to obtain large-scale DSD parameters using dual polarization radar, the DSD retrieval equation established using historical observations at a specific station is applied to a large area for future conditions (Dai and Han, 2014; Dai et al., 2014; Dai et al., 2019). In other words, the spatial and temporal prediction ability of the radar-DSD relationship must be considered in the DSD retrieval algorithm. Therefore, our current understanding of radar-DSD relationship extrapolation as well as its prediction in space and time is inadequate.

Machine learning algorithms such as neural networks are considered to be more accurate than polynomial regression at predicting outcomes without being explicitly programmed (Hagan et al., 1997). A recurrent neural network (RNN) is a unique class of neural networks that allows previous outputs to be used as inputs in the hidden states, but it is available in limited time periods (Rumelhart et al., 1986). Based on conventional RNN, long short-term memory (LSTM) network is an extension system to deal with the vanishing gradient problem and extent memory in sequence modeling, which can effectively help RNNs to remember inputs for a long time according to the importance it assigns to the information (Hochreiter and Schmidhuber, 1997). Due to LSTM's ability to handle time series data with significant time lags between important events, it is regarded as a stable and effective approach in general sequence modeling (Yang et al., 2020). Therefore, for our study, LSTM can be employed to establish a radar-based DSD retrieval model using extensive historical disdrometer data.

Therefore, this study proposes a novel DSD retrieval model using dual-polarization radar

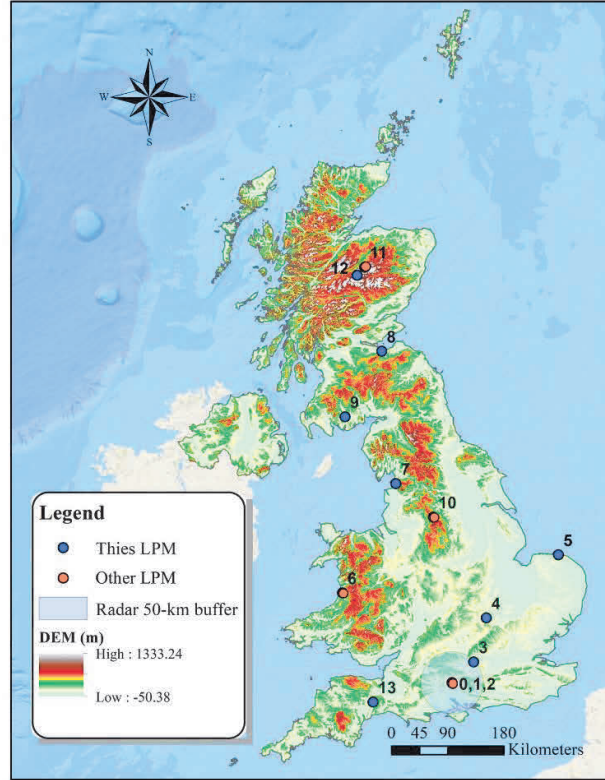
78 observations based on LSTM techniques, taking an X-band dual-polarization radar as a case
79 study. The fitting performance and spatial and temporal prediction abilities of the model are
80 compared with traditional methods. It is expected that the new retrieval model will be more
81 representative and applicable on large spatial and temporal scales. This study contributes to
82 improving quantitative precipitation estimates using radar polarimetry and facilitates a better
83 understanding of precipitation microphysics.

84 The remainder of this paper is organized as follows. Section 2 illustrates the study data
85 sources. Section 3 describes the methods used, including a detailed introduction of the
86 traditional regression method and the proposed scheme. Section 4 presents the results of DSD
87 and compares the temporal prediction abilities of the two methods. Finally, the conclusion
88 summarizes the key findings, limitations, and future work.

89 **2 Data sources**

90 Two types of disdrometers were chosen in this study (Fig. 1). Information related to
91 location, type, and coverage period of the disdrometers is shown in Table 1. The records
92 available in this study are excluded from abnormal measured particles by the threshold of
93 velocity–diameter relationship (Ji et al., 2019), rain rate $> 0.1 \text{ mm h}^{-1}$. Disdrometer Verification
94 Network (Pickering et al., 2018) consists of 14 Thies laser precipitation monitors (LPMs)
95 around UK from 2017 to 2019. Due to the 14th disdrometer in DiVeN located at Coverhead has
96 no DSD data, we used 13 LPMs to investigate in this study. The LPM subdivides particles into
97 appropriate classes. The scale of this classification is smaller for small and slow particles than
98 for large and fast particles. The measured particles are divided into diameter (the volume
99 equivalent diameter) and speed (particle speed) classes in a two-dimensional field, comprising

100 20 different diameter (0.125–8.000 mm) and 22 speed (0–20 m s⁻¹) classes, resulting in a total
 101 of $20 \times 22 = 440$ classes. This kind of disdrometer measures the fall velocity by monitoring the
 102 duration of precipitation in its laser beam. The sample area of LPM is 45.6 cm². The data output
 103 is integrated in 1-min intervals.



104
 105 Fig. 1. Location of disdrometers including four focused analysis Thies laser precipitation
 106 monitors (LPMs, in orange dots), other nine LPM stations (in blue dots), and a Joss–Waldvogel
 107 disdrometer (JWD, station ID is sets to 0) at Chilbolton Observatory in UK.

108 Table 1. Information of JWD and LPM disdrometers.

Type	Stiation ID	Name	Latitude	Longitude	Period	Records
JWD	0	Chilbolton	51° 08' N	01° 26' W	Apr. 2004 – Sep. 2017	291523
	1	Chilbolton	51° 08' N	01° 26' W	Feb. 2017 – Sep. 2019	79749
LPM	2	Chilbolton2	51° 08' N	01° 26' W	Feb. 2017 – Mar. 2018	34102
	3	RUAO	51° 26' N	00° 38' W	Feb. 2017 – Sep. 2019	69096

4	Cranfield	52° 04' N	00° 38' W	Feb. 2017 – Aug. 2019	63422
5	Weybourne	52° 57' N	01° 17' E	Feb. 2017 – Sep. 2019	63570
6	Aberystwyth	54° 25' N	04° 00' W	Feb. 2017 – Sep. 2019	118443
7	Lancaster	54° 01' N	02° 46' W	Feb. 2017 – Sep. 2019	80661
8	Edinburgh	55° 55' N	03° 10' W	Feb. 2017 – Sep. 2019	65313
9	Laurieston	54° 58' N	04° 04' W	Apr. 2017 – Sep. 2019	120703
10	Holme Moss	53° 32' N	01° 51' W	Mar. 2017 – Sep. 2019	89336
11	Cairngorm	57° 08' N	03° 40' W	Jun. 2017 – Sep. 2019	135820
12	Feshie	57° 00' N	03° 51' W	Jun. 2017 – Sep. 2019	30511
13	Dunkeswell	54° 12' N	01° 59' W	Nov. 2017 – Sep. 2019	63540

An impact-type RD-69 JWD disdrometer is located in Chilbolton Observatory in the south of the UK. Its observational data are available from April 2004 to September 2017. However, there are several months of missing data from this JWD (from mid-August 2004 to mid-December 2004 and from July 2005 to May 2006) because the disdrometer instrument was returned to the manufacturer for maintenance. All data are accessible from the British Atmospheric data Centre at <https://www.chilbolton.stfc.ac.uk>. The sampling period and collection area of the disdrometer are 10 s and 50 cm², respectively, and the 10 s measurement data averaged over a 1-min period to filter time variation. Its drop sizes ranging from 0.3–5.0 mm were measured in 127 increments. These long-term DSD data were used to estimate the temporal predictability of the proposed model. The data from the DSD datasets of JWD and LPM utilized in this study was subjected to a quality control procedure based on our previous studies (Dai et al., 2020; Zhu et al., 2021, 2023), which included the removal of outliers and non-rainfall information.

To further analyze the performance of the proposed model on radar observations, an X-band dual-polarization Doppler weather radar (NXPol) was used in this study. It was deployed at the Chilbolton Facility (51.14° N, 1.44° W) for Atmospheric and Radio Research (CFARR), UK, spanning from November 2016 to June 2018. Operating within the framework of the

NERC National Centre for Atmospheric Science (NCAS) Atmospheric Measuring Facility, this radar constitutes a fundamental element and functions in alignment with the continuous and comprehensive observations overseen by the NERC National Centre for Atmospheric Science (Neely III et al., 2018). NXPOL operates at a frequency of 9.375 GHz with a 1° beamwidth and a 2.4 m antenna diameter, featuring an antenna gain of 44 dB and peak power exceeding 55 kW. Its average power usage is approximately 80 W, enabling a typical operating range of 100~150 km. The radar's scanning capabilities include azimuth range of 0° to 360° and elevation range of -1° to 181°, with azimuth/elevation speed ranging from 1° to 36° per second.

3 Methodology

3.1 Raindrop size distribution estimation

Bin parameterization discretizes the raindrop diameter size into a number of bins. The concentration of raindrops in bin i within a unit volume, which is measured by a disdrometer at a discrete time t , is derived from the following equation (Ulbrich, 1983; Islam et al., 2012):

$$N_m(D_i, t) = \frac{n_i(t)}{AV_i \Delta t \Delta D_i} \quad (1)$$

where m is a measured quantity; D_i is the central drop diameter of bin i (mm); $n(t)$ is the counted number of drops in bin i at time t ; A is the disdrometer sensor area ($4.56 \times 10^{-3} \text{ m}^2$ for LPM and $5.00 \times 10^{-3} \text{ m}^2$ for JWD); Δt is the sample interval (60 s); ΔD_i is the i^{th} bin width (mm); and V_i is the terminal fall speed of raindrops (m s^{-1}). With a small bin size, the bin microphysics parameterization can effectively reflect the real DSD. However, when there are a large number of bins, and thus a large number of processed equations, the bin parameterization becomes computationally expensive and therefore less applicable for theoretical analysis than bulk microphysical parameterization.

Bulk parameterization assumes a prescribed analytic form with only a couple of DSD parameters. The normalized gamma distribution (Eq. 2) is generally used because it enables simple comparisons of the DSD and reduces DSD uncertainty owing to a lack of restrictions on the shape of raindrop spectra (Ulbrich, 1983; Islam et al., 2012; Song et al., 2017). The generic form of the distribution is:

$$N(D) = N_w f(\mu) \left(\frac{D}{D_0}\right)^\mu \exp\left[-(3.67 + \mu) \frac{D}{D_0}\right] \quad (2)$$

where $N(D)$ is the number of raindrops per unit volume. There are three key parameters in this equation; i.e., the shape parameter μ , the generalized intercept parameter N_w , and the drop median volume diameter D_0 . $f(\mu)$ is a function of the shape parameter as described in Montopoli et al. (2008).

Occasionally, the mass-weighted mean drop diameter (D_m) can be used instead of D_0 to describe the DSD (Ulbrich, 1983):

$$\frac{D_0}{D_m} = \frac{3.67 + \mu}{4 + \mu} \quad (3)$$

Based on this definition, these parameters can be formulated as the ratio of different moments of the DSD, as follows (Montopoli et al., 2008):

$$D_m = \frac{\int D^4 N(D) dD}{\int D^3 N(D) dD} \quad (4)$$

$$N_w = \frac{256}{6} \frac{\int D^3 N(D) d(D^5)}{\int D^4 N(D) d(D^4)} \quad (5)$$

$$\mu = \frac{(7 - 11\eta) - \sqrt{(7 - 11\eta)^2 - 4(\eta - 1)(30\eta - 12)}}{2(\eta - 1)} \quad (6)$$

where dD denotes the infinitesimal increment of D used for integration, as do $d(D^5)$ and $d(D^4)$ in Equation (5), η depends on the second, fourth, and sixth moments of the DSD, which is

written as:

$$\eta = \frac{\int D^4 N(D) d(D^2)}{\int D^2 N(D) dD \int D^6 N(D) dD} \quad (7)$$

3.2 Radar polarized characteristic observations

The most important characteristic observations of conventional single-polarized Doppler radar are the radar reflectivity (Z) and the radial velocity (RV), whereas dual-polarized radar employs the reflectivity (Z_H), the differential reflectivity (Z_{DR}), and the specific differential phase (K_{DP}) to infer precipitation types and amounts (Gorgucci et al., 1992; Scarchilli et al., 1996). The dual-polarized radar technique typically achieves significantly higher accuracies (Hall et al., 2017). It alternately transmits vertically and horizontally polarized electromagnetic waves and receives polarized backscattered signals. Three dual-polarization parameters, including Z_H (in $\text{mm}^6 \cdot \text{m}^{-3}$), Z_{DR} (in dB), and K_{DP} (in $\text{deg} \cdot \text{km}^{-1}$), can be calculated from the disdrometer data using the T-matrix scattering technique (Barber and Yeh, 1975; Yang et al., 2022):

$$Z_{H,V} = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int |f_{H,V}(D)|^2 N(D) dD \quad (8)$$

$$Z_{DR} = 10 \log_{10} \frac{Z_H}{Z_V} \quad (9)$$

$$K_{DP} = \frac{180\lambda}{\pi} \int \text{Re}[f_{HH}(D) - f_{VV}(D)] N(D) dD \quad (10)$$

where f_H and $f_V(D_i)$ represent the backscatter amplitude of a raindrop for horizontal and vertical polarizations, while $f_H(0, D_i)$ and $f_V(0, D_i)$ represent the standard forward scattering amplitudes. K_w is the dielectric factor of water, and λ signifies the radar wavelength.

3.3 Traditional regression method (TRM)

In the Rayleigh scattering, Z is a function of the sixth moment of the DSD. According to Eq. (8), Z_H is a function of $N(D)$. If all drops have equilibrium shapes whose axis ratio is a function of D , then both Z_{DR} and K_{DP} become functions of $N(D)$, albeit to different extents (Bringi and Chandrasekar, 2001). Thus, the microphysical origin of radar characteristic observations is discussed in relation to the drop size distribution. This microphysical link between radar observations and the characteristic DSD diameter is important and has been extensively studied (Bringi et al., 2009).

The traditional DSD retrieval algorithm generally establishes an empirical relationship between DSD parameters and radar observations using the polynomial regression method, which we term the traditional regression method (TRM) in this study. This work follows the algorithm of Bringi et al. (2003), Brandes et al. (2004), and Kim et al. (2010) to derive DSD parameters. First, the D_0 is derived from Z_{DR} using a four-parameter regression equation:

$$D_0 = a + bZ_{DR} + cZ_{DR}^2 + dZ_{DR}^3 \quad (11)$$

Then, N_w can be easily estimated using one of the moments of the DSD such as Z_H or Z_{DR} . In this study, the liquid water content (LWC) is first estimated from Z_H and Z_{DR} , then N_w is obtained using Eq. (12) and (13):

$$LWC = aZ_H 10^{(bZ_{DR} + cZ_{DR}^2 + dZ_{DR}^3 + eZ_{DR}^4)} \quad (12)$$

$$N_w = \frac{3.67^4}{\pi \rho_w} \frac{LWC}{D_0^4} \quad (13)$$

Lastly, as μ is closely related to the slope of the exponential DSD ($N(D) = N_0 D^\mu e^{-\lambda D}$), λ , μ are estimated by:

$$\mu = a + b\lambda + c\lambda^2 \quad (14)$$

$$\lambda = mZ_{DR}^n \quad (15)$$

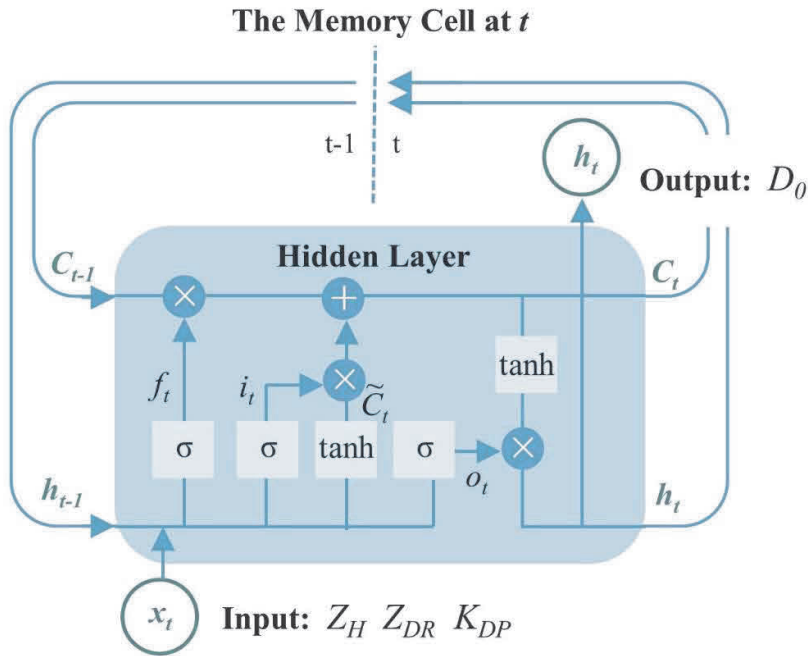
To minimize the error due to sampling effects when estimating μ , a data-filtering method is adopted that filters $R > 5 \text{ mm h}^{-1}$, which was proposed by Zhang et al. (2001) and has been applied in many studies (Gorgucci et al., 2008; Bringi et al., 2009).

3.4 Regression method using LSTM

LSTMs can automatically learn the complex long-term dependencies in multiple hidden neurons, overcoming the limitations of traditional RNNs in time series prediction. A multi-layer LSTM architecture is used in this study to derive DSD parameters from radar observations. Three schemes with different inputs and outputs are designed. To compare with the traditional method, the radar measurements chosen to derive DSD parameters are exactly the same with the traditional method. The first scheme (LSTM- D_0) uses Z_{DR} as the input to derive D_0 . The second scheme (LSTM- N_w) uses Z_H and Z_{DR} as inputs and N_w as the output. In the third scheme (LSTM- μ), Z_{DR} is used as input to derive μ . The possible improvement of LSTM schemes with different combinations of measurements that differ from the traditional method is possible. A detailed discussion of this issue is given in the Section 5.

The LSTM scheme used in this study is configured with one input layer (Z_H , Z_{DR} , and K_{DP} at each time step), one hidden layer, and one output layer (one of the three DSD parameters at each time step). The hidden layer is comprised of a number of memory cells, which can transfer the data flow and unit state in the neuron chain. Each memory cell has three gates named forget gate (f_t), input gate (i_t), and output gate (o_t), using for data updating or removing, and controlling the cell state adjustment when some transformation occurs (Le et al., 2019). As

230 shown in Fig. 2, the memory cell of the LSTM- D_0 scheme is schematized at the moment t . The
 231 scheme is similar to that of LSTM- D_0 , and the difference between LSTM- N_w , and LSTM- μ is
 232 that the outputs are N_w and μ , respectively. There are two kinds of layers (σ layer and tanh layer)
 233 are contained in a memory cell to update the new cell state. Specifically, the sigmoid layer
 234 determines whether the new information should be accepted or abandoned, then the tanh
 235 function weights the past values, setting their importance level (Le et al., 2019).



236
 237 Fig. 2. Architecture of the LSTM- D_0 . Note that x_t , h_t , and C_t are the input, output, and cell state
 238 at time step t , and σ and tanh are the sigmoid function and Tanh function respectively.
 239 Additionally, f_t , i_t , and o_t represent the forget gate, input gate, and output gate, respectively, and
 240 $\tilde{C}_t$ denotes the candidate update.

241 First, f_t decides which information from the previous cell state (C_{t-1}) should be preserved.
 242 The decision is made by a σ layer to squash values between 0 and 1, which calculates a
 243 forgetting factor for each element of the cell state:

$$f_t = \sigma(W_{xf}x_t + W_{hf}h_{t-1} + b_f) \quad (16)$$

where b_f represents the bias parameter, W_{xf} and W_{hf} are the weights applied to the current input x_t and the previous hidden state h_{t-1} of f_t , respectively.

The subsequent phase is to determine what information from x_t should be stored in the cell state. Initially, i_t employs a σ function, identifies the values for modification. Then, a tanh layer generates a candidate update ($\tilde{C}_t$) for augmenting the state.

$$i_t = \sigma(W_{xi}x_t + W_{hi}h_{t-1} + b_i) \quad (17)$$

$$\tilde{C}_t = \tanh(W_{xc}x_t + W_{hc}h_{t-1} + b_c) \quad (18)$$

where b_i and b_c represents the bias terms for i_t and $\tilde{C}_t$, W_{xi} , W_{hi} , W_{xc} and W_{hc} are the weight parameters applied to x_t and h_{t-1} of i_t and $\tilde{C}_t$. After that, combine f_t , i_t , and $\tilde{C}_t$ to compute the new cell state C_t :

$$C_t = f_t C_{t-1} + i_t \tilde{C}_t \quad (19)$$

Finally, the output values h_t are decided by o_t and C_t together. After using the σ layer to decide which parts of the cell state to output, the cell state is passed through the tanh layer and multiplied by the output of the σ gate:

$$o_t = \sigma(W_{xo}x_t + W_{ho}h_{t-1} + b_o) \quad (20)$$

$$h_t = o_t \tanh(C_t) \quad (21)$$

where b_o represents the bias terms for o_t , W_{xo} and W_{ho} are the weight parameters applied to x_t and h_{t-1} of o_t .

3.5 Evaluation methods

To quantify the performance difference between the two TRM and LSTM, three indicators

are investigated in more detail; i.e., the Nash-Sutcliffe efficiency coefficient (NSE, used to assess the predictability of models), the root mean squared error (RMSE), and the mean bias error (MBE). The definitions of these indicators are shown in Eqs. (22)–(24):

$$NSE = 1 - \frac{\sum (y_t - \hat{y}_t)^2}{\sum (y_t - \bar{y})^2} \quad (22)$$

$$RMSE = \sqrt{\frac{1}{N} \sum (y_t - \hat{y}_t)^2} \quad (23)$$

$$MBE = \frac{1}{N} \sum (y_t - \hat{y}_t) \quad (24)$$

where y_t and $\hat{y}_t$ represent the observed value and the predicted value at time t respectively, and $\bar{y}$ indicates the average observed data.

4 Results

4.1 DSD parameters retrieval from disdrometers

Aiming to illustrate the instruments performance, four disdrometers (1-Chilbolton, 6-Aberystwyth, 10-Holme Moss, 11-Cairngorm) in DiVeN was chosen as visualization examples through section 4.1–4.3. These stations were chosen based on their geographic diversity, with two located in England, one in Scotland, and one in Wales. The three parameters (D_0 , N_w and μ) are firstly derived using Eqs. (3) to (7). To explore the general DSD characteristics at the four different locations, the probability density distributions of these parameters are shown in Fig. 3. The general D_0 distribution patterns at Aberystwyth and Holme Moss are similar, while μ distribution curves at Chilbolton and Holme Moss almost overlap. At the Cairngorm site, the peak values of the key parameters other than N_w are very similar to those at the other sites, but the peak density of D_0 is slightly higher and the peak density of μ is much lower. The probability density function (PDF) of the three parameters can reveal the local DSD

characteristics and the PDF diversity can be used to examine the DSD retrieval algorithm under different situations.

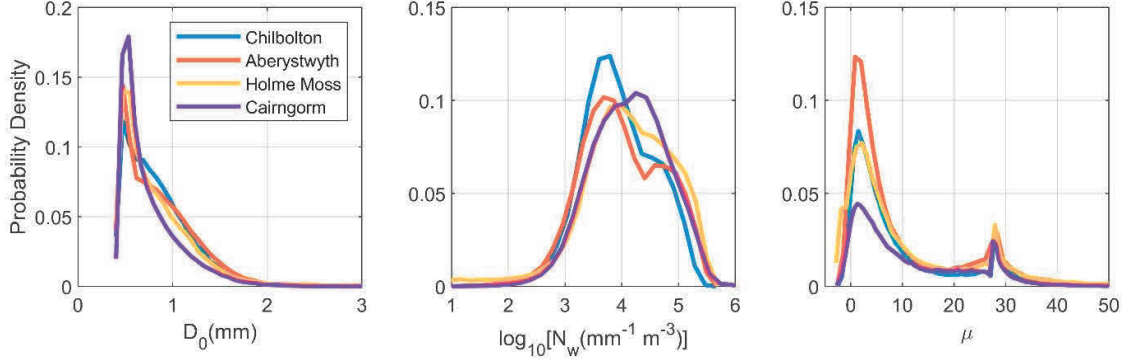


Fig. 3. PDF of DSD parameters for disdrometers.

The empirical relationship between rainfall and radar observations is the foundation of radar rainfall estimation, which depends on the local raindrop size distribution. The Z_H - R and R - K_{DP} relationships are estimated using long-term disdrometer data sets for radar rainfall estimation, as shown in Fig. 4. DSDs from hail, snow (judged by the present weather code of disdrometers) and extreme light rain ($R < 0.1 \text{ mm h}^{-1}$) have been removed. The fitted coefficients of fitted curves are different among the four disdrometer sites, proving the spatial limitations of the constant empirical equation. In the subsequent TRM and LSTM models, the DSD parameters (D_0 , N_w and μ) obtained from the disdrometer are utilized as the actual target values for the models. The calculated dual-polarization radar parameters (Z_H , Z_{DR} and K_{DP}) derived from these DSD parameters serve as the input values for the models.

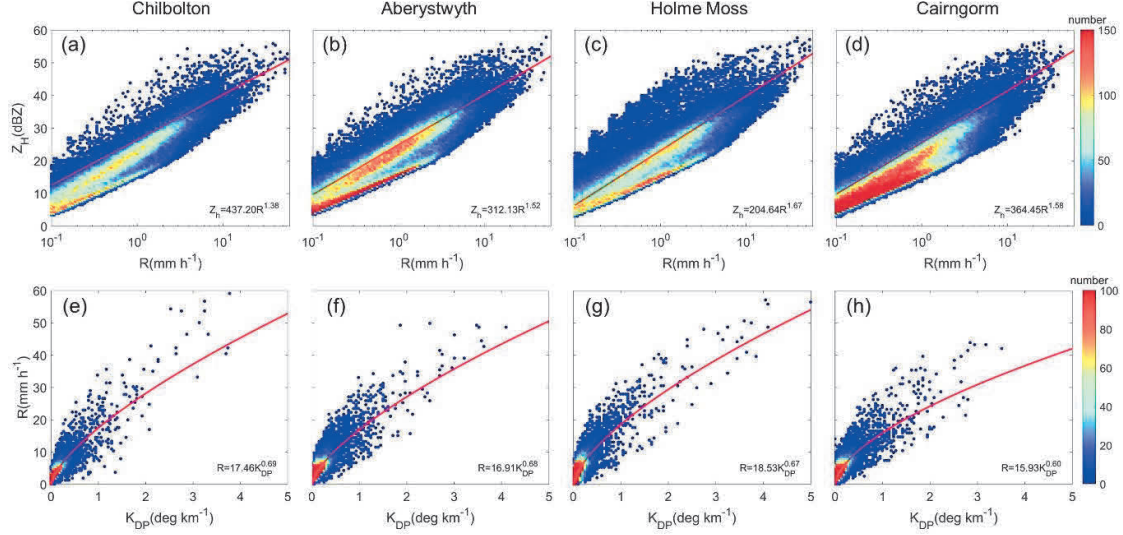


Fig. 4. Number of minutes per x-axis unit and y-axis unit of relationships between Z_H – R and R – K_{DP} for disdrometer observations.

4.2 Radar DSD retrieval using TRM

With long-term observations from four disdrometers, the simulated D_0 – Z_{DR} relationships are estimated using Eq. (11) and displayed in Fig. 5. According to Fig. 5, D_0 is estimated fairly accurately over a wide range. Simulations can also be utilized to evaluate the performance of D_0 estimation. Fig. 6(a-d) show scatterplots of TRM- D_0 versus true D_0 for different locations. The break point of D_0 is observed at which the fitted effect becomes unreliable, which occurs at approximately 2.0, 1.9, 2.3, and 2.0 mm for the four locations, indicating that the traditional regression method for D_0 estimation is more suitable for small rain drops. At each LPM station, 80% of data are chosen to establish the relationship model, and the remaining data are used to calculate validation indicators including RMSE, NSE, and MBE of fitted parameter, as shown in Table 2 for D_0 . The table shows that except Holme Moss station has little poor estimation in D_0 , this size parameter can be estimated to an efficiency coefficient no less than 0.90 and RMSE

no more than 0.12 mm for all locations.

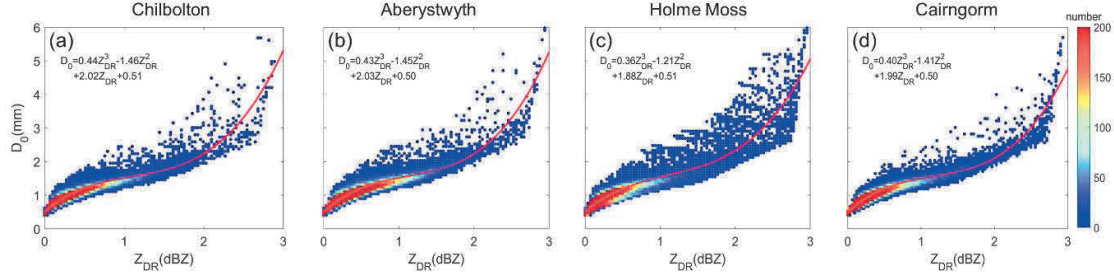


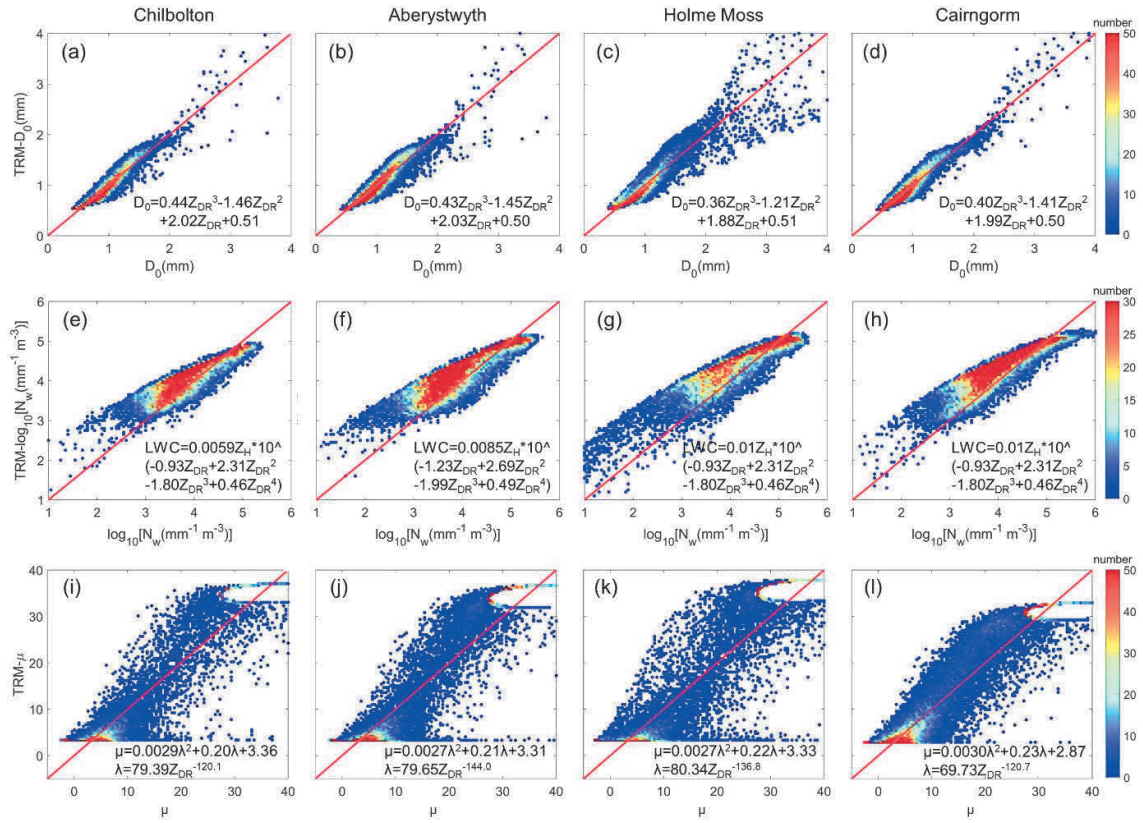
Fig. 5. Number of minutes per x-axis unit and y-axis unit of relationships between $Z_{DR} - D_0$ for disdrometer observations.

Table 2. Comparison of estimator fitted skill for D_0 .

Station ID	TRM			LSTM		
	RMSE	NSE	MBE	RMSE	NSE	MBE
1	0.09	0.93	0.01	0.08	0.95	0.00
2	0.10	0.93	0.02	0.07	0.95	0.00
3	0.10	0.92	0.01	0.09	0.94	-0.01
4	0.11	0.92	0.00	0.09	0.93	0.00
5	0.12	0.93	0.01	0.10	0.94	0.00
6	0.10	0.93	0.00	0.09	0.95	0.00
7	0.09	0.92	0.00	0.09	0.95	-0.01
8	0.09	0.92	0.01	0.08	0.94	0.00
9	0.11	0.91	-0.02	0.08	0.95	0.00
10	0.17	0.89	0.01	0.10	0.95	0.00
11	0.08	0.94	0.01	0.07	0.96	0.00
12	0.11	0.93	-0.02	0.09	0.94	0.01
13	0.10	0.93	-0.01	0.09	0.94	-0.01

Fig. 6(e-h) give information about TRM- $\log_{10}N_w$ results for the four locations. Due to the wide variability of N_w , $\log_{10}N_w$ is used for the comparison. Despite some fluctuations, the overall trend of estimated TRM- $\log_{10}N_w$ matches that of observed $\log_{10}N_w$ relatively closely. Table 3 compares fitting abilities of TRM- $\log_{10}N_w$ for the whole DiVeN sites. The RMSE values of TRM- $\log_{10}N_w$ range from a minimum value of 0.32 (1-Chilbolton) to a maximum value of 0.44 (10-Holme Moss), and only Holme Moss site has a positive MBE, verifying that most

326 TRM underestimate N_w .



327
328 Fig. 6. Number of minutes per x-axis unit and y-axis unit of D_0 (a-d), $\log_{10}N_w$ (e-h) and μ (i-l)
329 between TRM-derived and disdrometer-observed values.

330 Table 3. Comparison of estimator fitted skill for $\log_{10}N_w$.

Station ID	TRM			LSTM		
	RMSE	NSE	MBE	RMSE	NSE	MBE
1	0.32	0.70	-0.16	0.17	0.93	0.01
2	0.36	0.73	-0.19	0.18	0.92	-0.02
3	0.33	0.71	-0.16	0.19	0.90	0.00
4	0.35	0.69	-0.19	0.19	0.89	0.01
5	0.36	0.75	-0.18	0.20	0.90	0.01
6	0.40	0.64	-0.25	0.18	0.94	0.00
7	0.39	0.56	-0.26	0.22	0.89	0.01
8	0.38	0.52	-0.26	0.19	0.92	0.01
9	0.41	0.64	-0.23	0.17	0.94	0.00
10	0.44	0.74	0.24	0.19	0.94	0.00
11	0.41	0.56	-0.29	0.14	0.95	0.01
12	0.41	0.56	-0.29	0.20	0.91	0.00

13	0.33	0.72	-0.16	0.19	0.91	0.01
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Fig. 6(i-l) show four scatterplots of TRM- μ versus μ in the μ estimate. The estimated error in TRM is high for all μ values in all locations (Fig. 6i-l and Table 4). The RMSE of TRM- μ is even reach 10 at Holme Moss (Table 4). As λ was first estimated using Z_{DR} , the results of μ estimated under such conditions exhibit a greater error than the other parameters. For this reason, it is very difficult to accurately estimate μ using the traditional regression method, especially in the presence of measurement errors (Gorgucci et al., 2002).

Table 4. Comparison of estimator fitted skill for μ .

Station ID	TRM			LSTM		
	RMSE	NSE	MBE	RMSE	NSE	MBE
1	6.26	0.77	-0.36	3.83	0.87	-0.20
2	7.08	0.77	-0.72	4.05	0.85	0.28
3	6.09	0.75	-0.60	4.14	0.81	0.07
4	4.93	0.77	0.42	3.42	0.83	0.03
5	7.68	0.68	0.58	3.14	0.82	0.02
6	5.96	0.78	-0.53	3.03	0.91	0.23
7	4.74	0.78	-0.43	3.67	0.85	-0.12
8	5.31	0.75	-0.26	4.14	0.82	-0.03
9	5.77	0.76	0.27	3.14	0.90	-0.04
10	10.00	0.67	0.41	4.00	0.87	0.29
11	4.86	0.81	-0.76	3.43	0.90	-0.33
12	5.25	0.77	0.40	3.98	0.86	0.17
13	5.72	0.78	-0.05	3.81	0.83	0.02

4.3 Radar DSD retrieval using LSTM network

Our meticulously designed LSTM model is tailored for the inversion of DSD parameters using dual-polarization radar observation inputs. The LSTM models in this study were developed based on the deep learning frameworks TensorFlow (Abadi et al., 2016) and Keras (Chollet, 2015). Comprising multiple LSTM layers, the architecture adeptly captures intricate temporal dependencies inherent in sequential data. Moreover, the dropout technique between

LSTM layers has been introduced to counteract overfitting (Srivastava et al., 2014), which selectively deactivated neurons during training, promoting the acquisition of more robust and generalized patterns. Regarding optimization, we've opted for the Adam optimizer coupled with the Mean Squared Error (MSE) loss function. The batch size, representing the number of training samples used in each iteration, is set at 500 to strike a balance between computational efficiency and model performance. The influence of data on model performance is a pivotal factor. To enhance precision, all input and output data undergo preprocessing and normalization before training. This normalization ensures data consistency in terms of scales, preventing the dominance of any single feature in the learning process.

The lengths of the time series for the four LPM locations are 79,749, 118,443, 89,336, and 135,820, respectively for Chilbolton, Aberystwyth, Holme Moss, and Cairngorm sites. A scatterplot of DSD parameters estimated using LSTM models of each site versus values observed by disdrometers is depicted in Fig. 7. In each dataset, 60%, 20%, and 20% of data are used to train the model, test the model, and evaluate the calibration performance respectively. The validation effects are given in Section 4.4, where the temporal and spatial predictability of these models is investigated. The corresponding indicators of the estimates are also indicated in Table 2-4. A close agreement is observed between simulated and observed D_0 values. Compared with the traditional regression method, the proposed method exhibits a higher efficiency coefficient and lower deviations. For example, LSTM- D_0 and observed D_0 samples are almost symmetrically distributed on the diagonal completely (Fig. 7a-d). For large D_0 values, especially larger than 2 mm, there is no significant drop in performance; thus, estimates using the proposed method are more robust than TRM estimates.

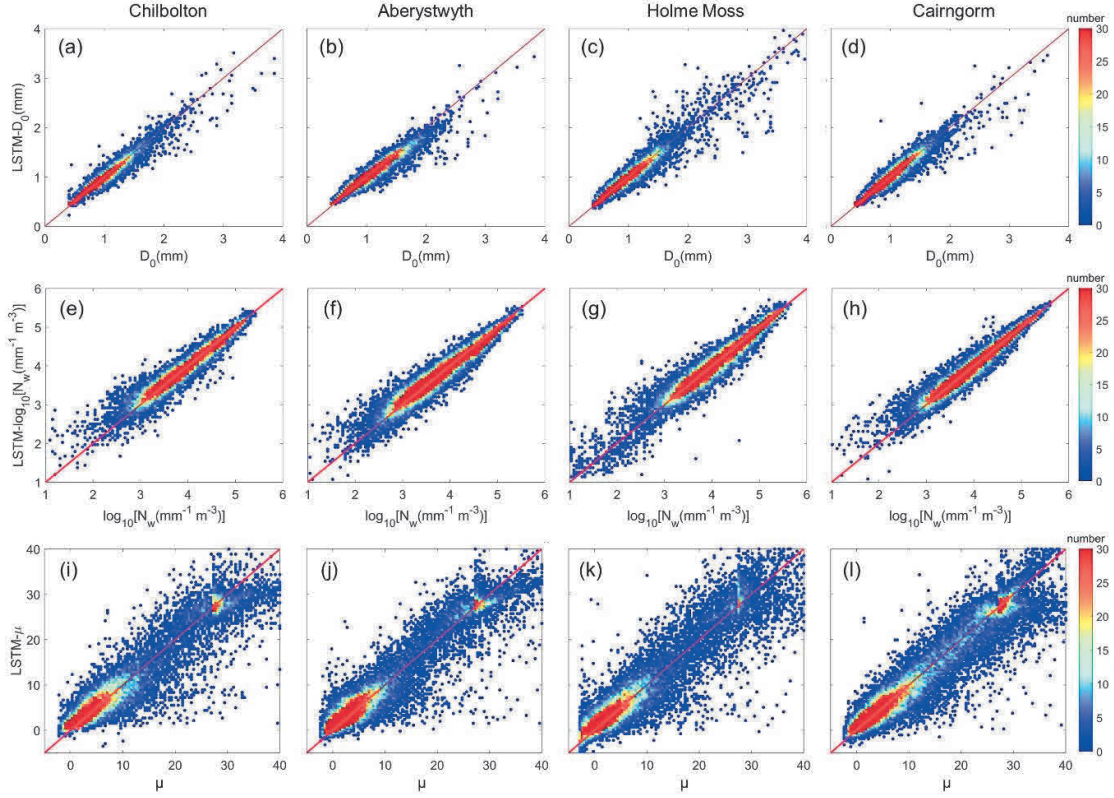


Fig. 7. Number of minutes per x-axis unit and y-axis unit of D_0 (a-d), $\log_{10}N_w$ (e-h) and μ (i-l) between LSTM-derived and disdrometer-observed values.

In terms of N_w , the $\log_{10}N_w$ is estimated fairly well with negligible bias over a wide range for all locations (Fig. 7e-h), indicating a remarkable improvement in performance due to exploiting the strength of the LSTM technique. Notably, the estimated error decreases with increasing $\log_{10}N_w$. In Fig. 7(i-l), The fitted effects are relatively poor between the modeled and observed μ . However, compared with the poor performance using the TRM scheme, LSTM can achieve significantly better outcomes when estimating μ .

According to Table 3 and Table 4, the performance of all LSTM schemes is better than that of the TRM schemes. Taking Chilbolton as an example, the NSE of fitted $\log_{10}N_w$ increases from 0.70 using TRM to 0.93 using LSTM, and the RMSE decreases from 0.32 to 0.17. The

MBE errors of D_0 and $\log_{10}N_w$ decrease to zero for 69.2% and 38.5% sites, respectively. The percentage increase in NSE values for D_0 , $\log_{10}N_w$ and μ is 2.2%, 32.9% and 13.0% at Chilbolton, and 6.7%, 27.0% and 29.9% at Holme Moss.

There is also a huge improvement in the fitting performance when estimating μ using the LSTM scheme. The NSE of fitted μ increases from 0.77 to 0.87 for Chilbolton, and from 0.67 to 0.87 for Holme Moss. The LSTM performance is still a little poor compared to D_0 and N_w retrieval, indicating the complexity of estimating μ using dual-polarization data. However, the improvement increases the practical applicability of the estimated μ parameters. A similar conclusion is also found for the other disdrometers.

4.4 Temporal and spatial predictability of DSD retrieval methods

4.4.1 Temporal predictability of DSD retrieval methods

The performance of DSD estimation methods is related not only to the fitted effects at the disdrometer location, but also to its spatial and temporal predictability. Due to the lack of disdrometers, the estimator should exhibit good performance for various radars over a long-term period, or at least for all radar pixels of a single radar. For this reason, we investigate and compare the temporal predictability of the two estimation methods using long-term historical observations.

Temporal prediction ability relates to the model established using historical data being applicable to future situations. Five-fold cross-validation is used to comprehensively assess the predictability performance of the model. Fig. 8 shows 50 ensemble five-fold cross-validation analyses for the three DSD parameters at Chilbolton JWD station. In each ensemble, four-fifth data are chosen randomly to establish the estimator, and the remaining data are used for

validation. The largest, smallest, and mean performance of the two methods for different parameters are shown in Fig. 8. The LSTM exhibits less variations in performance than TRM, indicating that the LSTM estimator has a higher capacity for temporal extrapolation.

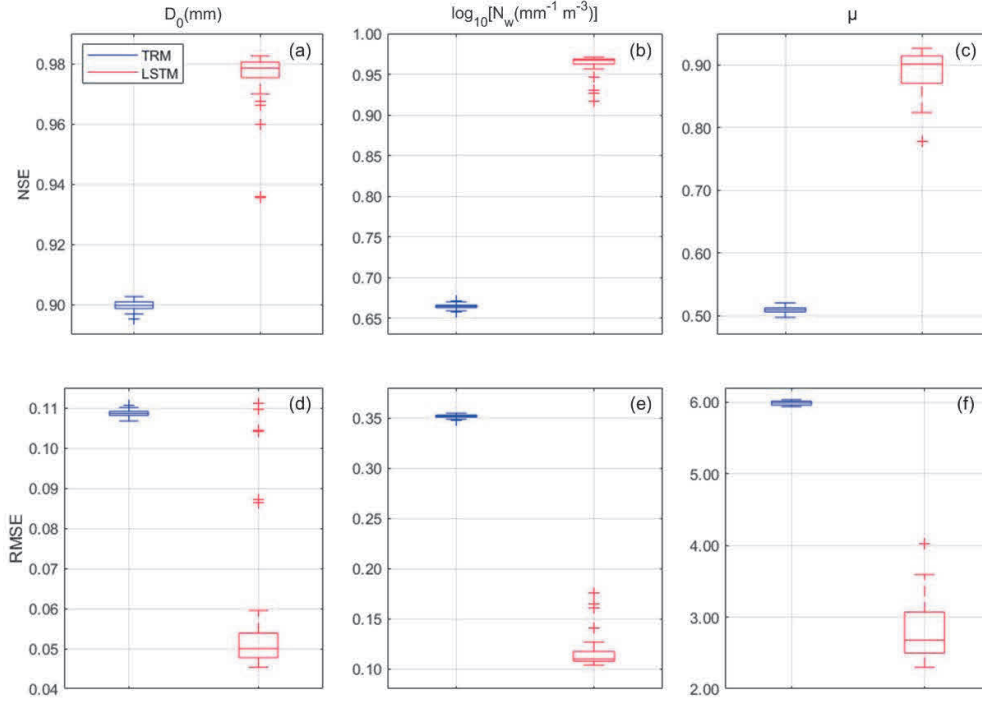


Fig. 8. Performance of temporal predictability for ensemble five-fold cross-validation analysis using long-term JWD data at Chilbolton station.

The average values for all ensembles, termed the mean temporal predictability are 0.98, 0.97, and 0.90 for the three key parameters using LSTM. On the contrary, TRM schemes exhibit poor and unstable performance, as shown by the red boxes in Fig. 8. The mean temporal prediction abilities of the TRM schemes are only 0.90, 0.66, and 0.51. The minimum correlation for the three LSTM schemes is 0.94, 0.91, and 0.78, which is still much higher than that of the TRM schemes. In terms of the predicted RMSE errors, LSTM schemes are also more robust.

To investigate the effect of the training period on performance, the estimators are configured to run with 1 to 13 years of data covering the period from 2003 to 2016 at Chilbolton. The observations from 2017 are used for the evaluation. The relationship between NSE, RMSE, and training years is shown in Fig. 9. For the TRM estimator, the performance increases gradually with training duration for D_0 . In contrast, the performance of the LSTM estimator remains stable. In terms of μ , the performance increase is substantial within four years of calibration for both TRM and LSTM estimators. The increase in performance is less significant after this point. Therefore, it is concluded that at least four years of historical data are required to obtain reliable μ estimates at Chilbolton.

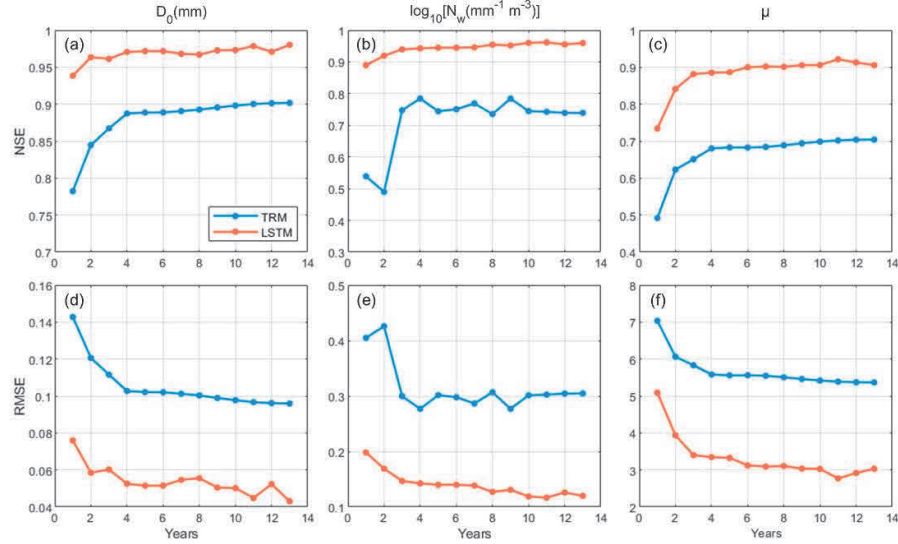


Fig. 9. Relationships between training periods and modeled performance for D_0 (a), $\log_{10}N_w$ (b) and μ (c) using long-term JWD data at Chilbolton station.

4.4.2 Spatial predictability of DSD retrieval methods

The spatial predictability indicates whether the estimator established using data at a given location can be extended to other locations. To investigate the spatial predictability, the

estimator established using data from one disdrometer is evaluated using the observations of other disdrometers. In this study, thirteen LPMs in DiVeN are used for evaluating spatial extrapolation of raindrop characteristics using TRM and LSTM.

The comparisons between mean NSE and mean RMSE indicators for model spatial extrapolated sites are shown in Fig. 10. The blue plusses and circles indicate the TRM and LSTM schemes, respectively. Except the μ parameters, the increased NSE and decreased RMSE with increasing distance indicates that estimator performance is spatially dependent. However, the improvement is not linear; i.e., some NSE values at long distance are better than those at medium distance. This is acceptable because the performance of the estimators is not fully dominated by the effect of distance. As can be seen in Fig. 10, all the LSTM prediction performances are better than TRM except for the Cairngorm site of D_0 . All NSEs of D_0 predicted by different models exceed 0.90, indicating that although the overall prediction ability of TRM for D_0 is worse than that of LSTM, it still has a good spatial extrapolation effect. For the most difficult μ parameters to predict, the RMSEs of TRM are often more than twice that of LSTM, and the same site Cairngorm is still the worst predicted by LSTM. However, the mean NSE value of LSTM at Cairngorm is only 0.0026 different than the TRM, and the RMSE is almost the same as the TRM. In addition, comparing the low RMSE and high NSE of each parameter at Cairngorm in Table 2-4, results show that the LSTM scheme for this site may have little poor spatial extrapolation performance.

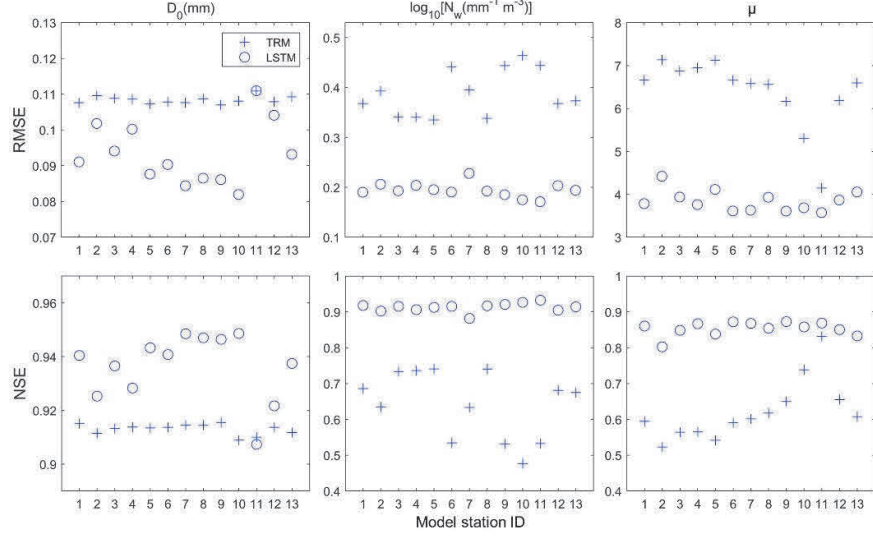


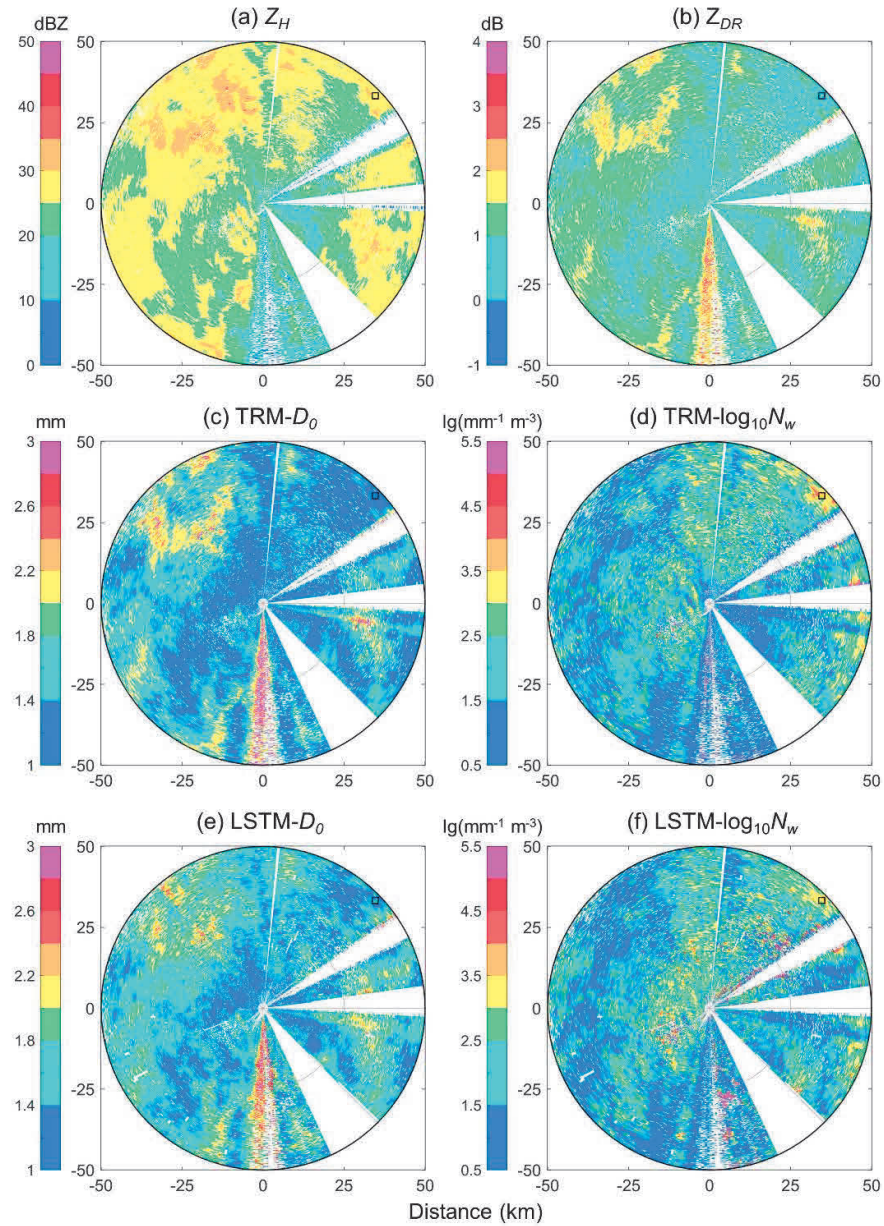
Fig. 10 Averaged RMSE and NSE for spatial extrapolated sites of TRM and LSTM models based on DiVeN data for D_0 , $\log_{10}N_w$ and μ .

The average NSE in other stations is defined as the spatial predictability. Almost all the spatial predictabilities of the LSTM method are better than that of the TRM. When estimating D_0 and N_w with a medium separation distance, the predictability of the LSTM schemes can reach 0.95 and 0.93, whereas the maximum prediction ability of TRM schemes is less than 0.92 and 0.74, respectively, indicating poor spatial extrapolation for N_w by the TRM scheme. The mean spatial predictability using LSTM for LPM sites is 0.94, 0.91 and 0.85 for the three key parameters.

4.4.3 DSD retrieval using real radar observation

Furthermore, the performance of the DSD inversion model on real radar data is analyzed using the NXPOL observations. The radar was located at the Chilbolton Observatory. Its center coincides with the station 1 and 2 locations and is 47.47 km from the RUAO station. It was therefore used to make comparisons with the disdrometer at the RUAO station. After the attenuation correction (Bringi et al., 1990; Zhang et al., 2021) and nonweather clutter removal

463 with radar correlation coefficient less than 0.98, the Z_H and Z_{DR} sequences of the target stations
 464 were extracted. Fig. 11 shows the radar image of Z_H , Z_{DR} and retrieved D_0 , $\log_{10}N_w$ from
 465 13:58:08 to 14:04:47 on May 17, 2017 for a 1.5° elevation angle, and the black square in the
 466 figure indicates the location of the RUAO station.



467

468 Fig. 11 A 1.5° elevation PPI of Z_H (a), Z_{DR} (b) and retrieved D_0 (c, e) and $\log_{10}N_w$ (d, f) from
 469 13:58:08 to 14:04:47 on May 17, 2017, and the black square in each subplot indicates the

location of the RUAO station.

Fig. 12 illustrates the contrast between the DSD parameters observed from the disdrometer and those retrieved from NXPOL radar for the rainfall event that transpired at the RUAO station between 10:00 and 14:30 on May 17, 2017. In the diagram, the blue line signifies the observed DSD sequence obtained from the disdrometer. The orange crosses portray the DSD sequence acquired through LSTM, while the yellow asterisks correspond to the DSD series computed using TRM.

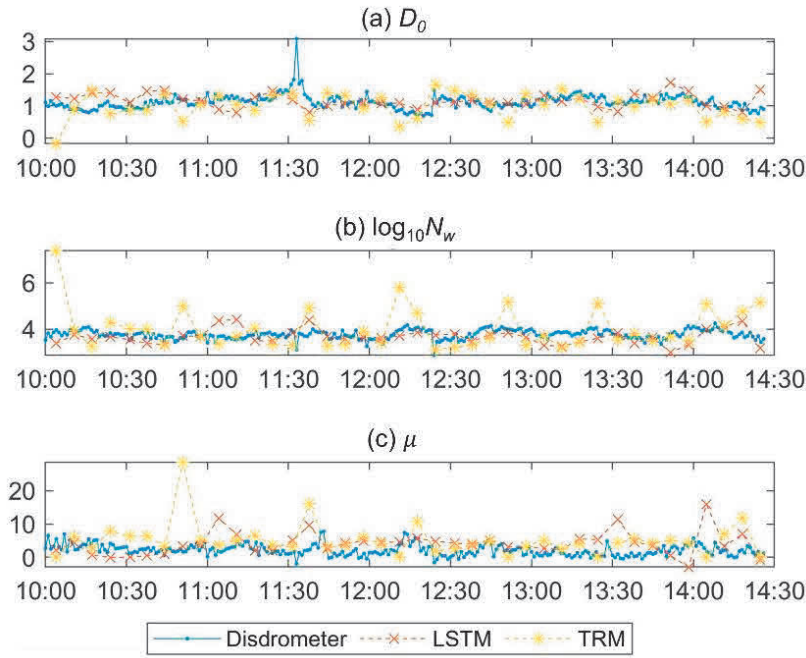


Fig. 12 D_0 , $\log_{10} N_w$ and μ values observed from the disdrometer and retrieved from the NXPOL observations using TRM and LSTM models of RUAO station at 10:00 to 14:30 on May 17, 2017.

It can be observed that the radar DSD values acquired through LSTM and TRM are in close agreement with the observed values, indicating the capability of the refined X-band radar data for DSD retrieval. The RMSE of LSTM results is notably lower than that of TRM results,

suggesting that the X-band radar-DSD retrieval method proposed in this study exhibits reduced uncertainty and greater stability compared to conventional methods. Tables 5 and 6 present the comparison of RMSE between the DSD values derived from radar using TRM and LSTM against the observed DSD values from the disdrometer. Notably, Table 6 includes not only the LSTM model from the RUAO station but also LSTM models from other DiVeN sites, assessing the spatial extrapolation performance of the models when applied to real radar data. The results indicate that, regarding the inversion of D_0 , only the LSTM model generated by the station 2 yields a relatively larger RMSE, exceeding TRM- D_0 . Moreover, for most stations, LSTM- D_0 exhibits RMSE values below 0.30. The performance of LSTM- $\log_{10}N_w$ and LSTM- μ in terms of RMSE also surpasses the TRM outcomes reported in Table 5, particularly demonstrating substantial improvement for LSTM- μ . Clearly, LSTM holds a significant advantage in DSD retrieval when employed with actual radar data.

Table 5. RMSE between the disdrometer-derived DSD and radar-retrieved DSD using TRM.

TRM- D_0	TRM- $\log_{10}N_w$	TRM- μ
0.37	0.88	6.29

Table 6. RMSE between the disdrometer-derived DSD and radar-retrieved DSD using LSTM at 13 stations.

Station ID	LSTM- D_0	LSTM- $\log_{10}N_w$	LSTM- μ
1	0.30	0.57	2.18
2	0.56	0.70	5.99
3	0.26	0.40	2.70
4	0.27	0.41	2.72
5	0.50	0.95	2.55
6	0.30	0.51	2.04
7	0.34	0.56	2.26
8	0.26	0.45	2.25
9	0.33	0.50	3.01
10	0.37	0.51	3.14

11	0.26	0.66	2.11
12	0.26	0.57	2.04
13	0.28	0.49	2.86

5 Discussion and conclusion

The estimation of DSD over large spatial and temporal scales is a long-standing goal in radar meteorology. Instead of the traditional regression method, this study proposes a new type of DSD retrieval model using dual-polarization radar observations based on LSTM techniques. Multi-layer long short-term memory network are used to derive DSD parameters from radar observations. Three schemes (LSTM- D_0 , LSTM- N_w , and LSTM- μ) are designed with different combinations of Z_{DR} and Z_H as inputs. The temporal and spatial predictabilities of the schemes are then estimated to investigate estimator performance over a large area and a long-term period. Based on a series of experiments using fourteen disdrometers, we make the following conclusions.

(1) The LSTM scheme is a highly effective way to improve the accuracy of DSD parameter retrieval, which is expected to replace traditional polynomial regression methods. Based on a series of experiments using 13 LPMs with same coverage periods, we conclude that the performance of all LSTM estimators is better than that of the traditional method. For example, at the Holme Moss site, the NSE values for D_0 , $\log_{10}N_w$ and μ estimates are 6.7%, 27.0% and 29.9% higher.

(2) The poor estimation performance of μ using the traditional regression estimator complicates its use in practical applications. The average RMSE performance improvement of 40% using the LSTM model makes the estimated parameters more practically applicable. However, the radar-based μ estimation still suffers from large uncertainty.

(3) Estimator performance differs between disdrometers at different locations. The key factor affecting the performance is the estimator method itself.

(4) Temporal and spatial predictability is a key concern for DSD estimators. The mean temporal predictability increases by 0.08, 0.31, and 0.39 for the three key parameters when using LSTM whereas the mean spatial predictability increases by 0.03, 0.19, and 0.23.

However, some limitations are noted that may have affected the outcome of this work. The first concern relates to the measurement errors of the disdrometers and dual-polarization radar. The proposed scheme estimates the radar measurements Z_H and Z_{DR} using disdrometer data and assumes that the original data are error free. However, accounting for radar measurement uncertainty in DSD parameter retrieval is complex. Currently, the radar product is obtained through a series of quality control procedures. In terms of disdrometer error, however, uncertainty may exist in the sampling, for example. Therefore, all disdrometers in DiVeN were introduced to reduce the possible influence of measurement errors.

Secondly, the temporal and spatial predictability could be analyzed more thoroughly by employing a denser disdrometer network and longer data collection periods. Although this study demonstrates that the proposed scheme exhibits better prediction performance in both time and space, estimator prediction performance could be revealed more clearly using different locations and times.

Thirdly, although there are differences in spatial and temporal resolution between radar and raindrop spectrometer observations, these variations can be minimized through the use of conversion methods (Yin et al., 2007). For example, Dai et al. (2023) calculated global-scale rainfall erosivity using IMERG and DPR data as replacements for disdrometers, with a spatial

resolution of $0.1^\circ \times 0.1^\circ$ and a time interval of 30 minutes.

Finally, to compare with the traditional methods, the parameters chosen to derive DSD parameters are exactly the same as the traditional methods, which restricts the performance of the proposed schemes. A different combination of measurements may produce a better LSTM scheme. For example, we find the estimated D_0 can be further slightly improved if using both Z_H and Z_{DR} as inputs. More importantly, LSTM is a popular machine learning method in long-term analysis; other methods such as the Random Forest deep learning method may also exhibit better performance. In summary, this study presents new insights to retrieve DSD parameters from radar-simulated measurements using machine learning algorithms, and highlights the importance of radar polarimetry and precipitation microphysical processes. It is expected that the novel retrieval model proposed in this study can be applied on large temporal and spatial scales, making it a more representative method for estimating raindrop size distributions.

Acknowledgments

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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