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Multi-fidelity deep neural network with Monte Carlo dropout technique for uncertainty-aware risk recognition of backward erosion piping in dikes

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Abstract: Backward erosion piping (BEP) is an increasingly critical failure mechanism in dike systems, often triggered by floods resulting from extreme rainfall events, which are exacerbated by the ongoing shifts in climate patterns. This study embarks on a nuanced examination of BEP, a phenomenon that substantially undermines flood control and emergency management endeavors. Addressing the complexities of BEP incidents and the consequential variable impact on dike infrastructure, we have incorporated Monte Carlo dropout techniques within a multi-fidelity deep learning framework to enhance predictive accuracy and provide an uncertainty-aware assessment of BEP risk. Drawing on 16 sets of physical model experiments emulating dike structures from the lower Yangtze River basin in China, we conducted a detailed study of the BEP evolution mechanism under various hydrological scenarios intensified by climate change. These experiments allowed us to observe the initiation and progression of BEP in different conditions. The development of a structural equation model quantifies the effects of several critical factors—including those exacerbated by climatic variability—on BEP dynamics. Seven key factors were identified as influential to BEP risk levels, integrating distinct BEP traits, hydrological attributes, and dike engineering conditions. A Monte Carlo dropout-enhanced multi-fidelity deep neural network (MFDNN) was crafted, synthesizing low-fidelity experimental data with high-fidelity field case studies, to construct an advanced model for BEP risk level identification. Compared against four sophisticated machine learning models, our MFDNN demonstrated superior performance, effectively blending experimental insights with real-world occurrences. The proposed model emerges as a scientifically robust and pragmatic tool for delineating BEP risk levels in dike systems, providing vital guidance for categorizing BEP incidents and forging targeted, climate-informed emergency response strategies.

Keywords: Backward erosion piping (BEP); Risk level; Criticality; Multi-fidelity deep neural network (MFDNN); Data science; Dike

1. Introduction

In the realm of geotechnical engineering, piping emerges as a critical phenomenon, implicated in nearly half of all dam failures [1]. A comprehensive review of the existing literature on piping entailed a thorough investigation of case studies to identify the primary patterns of piping failure. The failure mechanism most commonly observed was determined to be concentrated leak erosion, which is typically linked with conduits or other forms of structural damage in dams [2]. Remarkably, backward erosion piping (BEP) accounts for up to one-third of piping failures, manifesting through

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erosion processes in embankments and beneath impermeable earthen dams or levees [3]. BEP is characterized by the incremental erosion of the finer soil fractions from the foundations of flood protection systems (FPS)[4]. Initiation of this process occurs on the downstream side of the FPS, often precipitated by localized damage that exposes permeable soils, thereby establishing elevated local hydraulic gradients. This incipient damage propagates retrograde towards the upstream side of the water-retention structure, culminating in forming a fully developed conduit (refer to Fig. 1) that undermines the structural integrity of the entire system. BEP is acknowledged as a complex interplay of sediment transport, subterranean water flow, hydrodynamics, and soil mechanics. Historical documentation reveals a multitude of dike failures globally, including significant incidents in the United States, China, and the Netherlands, attributable to BEP [5]. A notable instance occurred during the 1998 Yangtze River basin flood, where 51.2% of the 1,702 dikes assessed sustained damage as a result of BEP. These extensive experiences in flood mitigation underscore the criticality and intricacy of BEP as a hazard in the management of dike systems, highlighting the necessity for continued research and development in this area.

Contemporary investigations into BEP predominantly evaluate the probability of its occurrence within FPS structures. Traditional methods for design assessment include Bligh's approach, Terzaghi's technique, and the concept of the blanket method [6][7]. A streamlined analytical framework offers a method to differentiate critical from non-critical states in the event of localized impairments on the leeward side of flood protection structures, commonly known as "sand boils" [8][9]. An evidence-based approach for evaluating the erosion potential within these structures was developed, leveraging data from a comprehensive analysis of over 1,462 dam embankments [10]. This approach has significantly enriched our understanding of erosion potential. Further investigations into the interactions between soil particles beneath dike foundations have led to the development of improved equations. These equations are instrumental in identifying the pivotal gradient, substantially advancing our knowledge of Bulkhead Erosion Potential dynamics [11][12][13].

In view of the complexity of the phenomenon, several experimental works have been carried out to quantify the dependence of BEP on the relevant, measurable soil, geometry, and load parameters. Investigations have been conducted into various aspects, including the erosion mechanism at the tip of the BEP channel [14], critical gradients, formation of pipes, erosion mechanisms [15], and the impact of soil particle size, distribution, shape, specific gravity, and flow at the BEP outlet [16][17][18]. Regarding external factors influencing BEP progression, studies have examined the effects of particle characteristics, soil structure of the dike foundation, and the geometry of the BEP channel [19][20][21][22][23]. Concerning hydraulic conditions, the impact of suddenly applied critical and supercritical hydraulic loads on BEP was scrutinized [24]. Variations in the erosion mechanism of BEP were noted when differing upstream water head rise rates were applied [25][26]. Innovative testing methods have been developed for more accurate data acquisition during BEP development [27], examining the regularity of BEP evolution across different test specimen sizes. Large-scale backward erosion BEP experiments were undertaken to explore the relationship between upstream gradient and progression rate, revealing significant pipe volumes relative to previous laboratory tests and an approximate linear relationship between pipe length and volume [28]. Field testing along the Mississippi and Waal Rivers measured head loss in sand boils, comparing the results with predictions from a basic theoretical model. This demonstrated that head loss is not a constant but varies with grain size, sand boil throat diameter, and flow rate

[29].

The capability of predicting the evolution of BEP in FPSs is highly desirable due to the transient nature of the phenomenon. BEP events are inherently dynamic, influenced by a multitude of factors that can vary significantly over time and space, making real-time prediction and monitoring essential for effective flood risk management. Although the experiments clarify the relevance of various quantities with respect to the evolution of the erosion process, in situ conditions are often very different from the laboratory environment because of the time and spatial variability of the driving factors (hydraulic loads, soil properties, seasonality of the water table, presence of localized defects, and damage). These discrepancies highlight the challenges in extrapolating laboratory findings directly to real-world scenarios without incorporating adaptive and robust predictive models. To this end, numerical predictions offer a valuable tool to assess the likelihood of BEP progression, helping designers and decision makers quantify the health of the systems and plan maintenance accordingly. Such predictive capabilities are crucial for proactive management, allowing for timely interventions that can prevent catastrophic failures and optimize resource allocation. The scientific community has developed a trio of key computational methodologies to address the complexities of predicting BEP. First, some methods focus on modeling the progression of erosion by improving the representation of soil conductivity [30][31][32][19]. These methods typically involve detailed characterizations of soil permeability and hydraulic conductivity, enabling more accurate simulations of water flow and soil particle migration. Second, others have merged the discrete element method (DEM) with simulations of fluid dynamics [33][34]. This hybrid approach allows for a granular analysis of soil particle interactions and fluid movements, providing insights into the microscale processes that drive BEP development. Third, there are sophisticated multiphase models that simulate changes in particle density due to erosion, based on well-defined constitutive laws and their influence on fluid flow [35][36][37][38]. These models integrate the complex interactions between solid and fluid phases, capturing the feedback mechanisms that govern the initiation and evolution of BEP. By combining these advanced computational techniques, researchers can achieve a more comprehensive understanding of BEP dynamics, facilitating the development of more effective mitigation strategies and decision-support tools.

Predicting the evolution of BEP hazards in dikes and discerning their level of danger are critical yet challenging tasks for frontline flood control personnel [39]. This complexity primarily arises from the multitude of factors influencing BEP incidents in dikes, making it difficult to judge their trends based on a single factor. Extensive physical model experiments and numerical simulations have shown that BEP development, characterized by the migration and reformation of soil particles in the embankment, does not inevitably lead to dike failure. Dikes often exhibit multiple BEP occurrences simultaneously during floods, potentially numbering in the hundreds. Implementing emergency measures on all these instances simultaneously can significantly strain rescue efforts, with the risk of overlooking critical BEP leading to dike breaches. Prioritizing high-risk BEP incidents through risk level identification and classification can markedly enhance the efficiency and scientific approach of emergency interventions. The risk level of BEP can be defined based on factors such as the rate of soil particle migration, the size and location of the affected area, the saturation level of the dike, and the historical performance of the dike under similar conditions. High-risk BEP incidents are those that exhibit rapid progression, affect critical structural points, or occur in dikes with a history of previous failures under similar conditions. Medium-risk incidents might show moderate progression or affect less critical areas, while low-risk incidents are those with

slow progression and occur in non-critical zones. However, the complexity of BEP, combined with limitations in model scale and boundary conditions in physical experiments and numerical simulations, often impedes precise identification, leading to scant research on classifying BEP risk levels. Recently, the application of artificial intelligence has introduced a new paradigm for addressing complex engineering challenges. Data-driven methods have made some progress in BEP prediction [3]. This approach leverages multilayer machine learning models for real-time predictions at the system level and has demonstrated its potential in assessing BEP risks in flood control infrastructure through a case study. However, research in this area remains limited, particularly due to the scarcity of case-specific data on BEP incidents. Moreover, the results of data-driven predictive methods based on machine learning are often questioned due to their inherent uncertainties, which can undermine their credibility. Therefore, quantifying uncertainty and establishing uncertainty-aware data-driven methods are crucial for enhancing the reliability and acceptance of these predictive models.

In this study, a methodology for multi-fidelity identification of Bank Erosion Problem (BEP) risk levels is proposed, incorporating a diverse range of factors influencing BEP incidents and the availability of dike hazard information. This methodology is further enhanced by the integration of the Monte Carlo dropout technique to quantify the uncertainty of predictions. Initially, a series of physical model experiments were conducted based on the current conditions of the dikes in the lower Yangtze River basin, China, to reveal the evolutionary mechanisms of BEP and quantitatively analyze the impact of various factors. A database was then established, consisting of abundant but less precise low-fidelity data from these experiments, and scarce but more accurate high-fidelity data from engineering cases in the Yangtze River basin. By leveraging the strengths of data with varying fidelities, a multi-fidelity BEP risk level identification model was trained, incorporating the Monte Carlo dropout method to evaluate prediction uncertainty. This significantly enhances the model's robustness and accuracy in real-world applications.

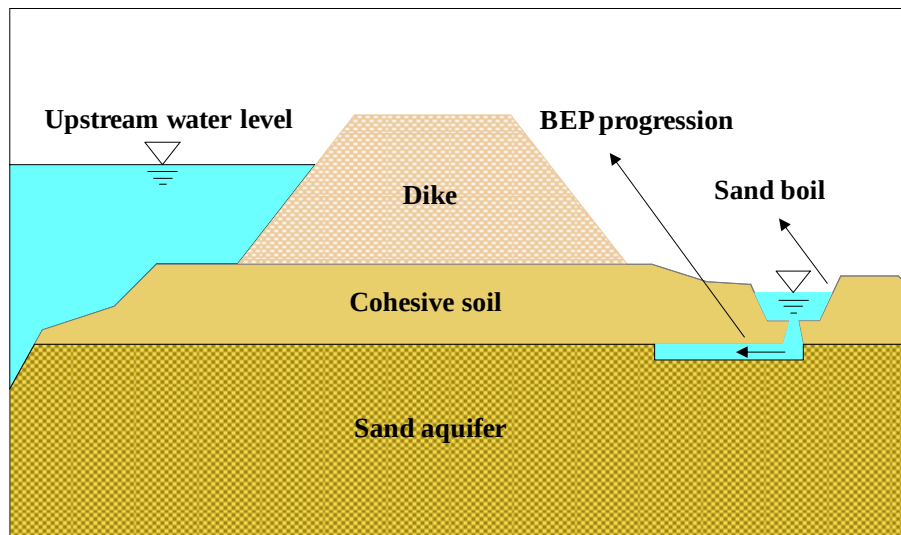


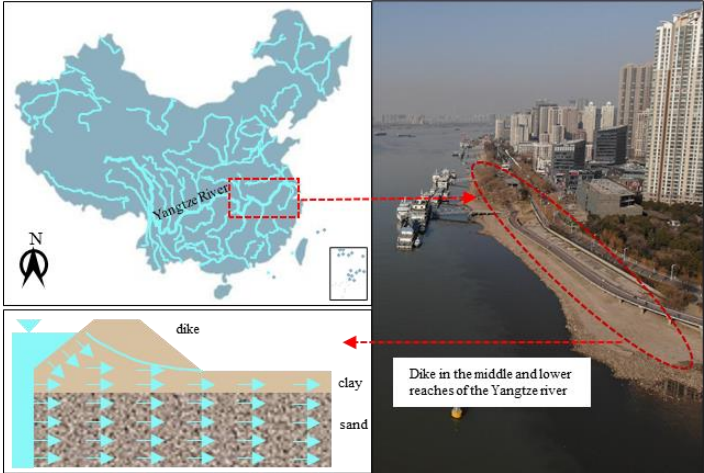
Fig. 1. Schematic description of the backward erosion piping (BEP).

2. Methodology

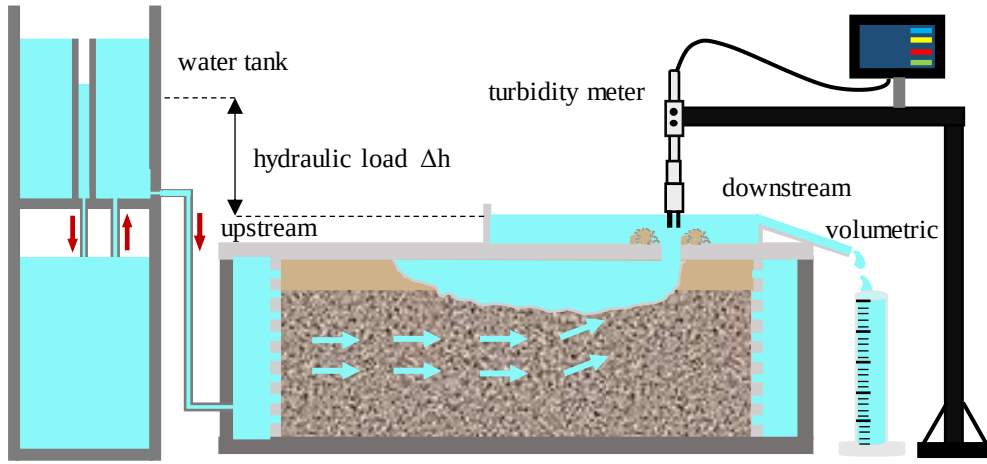
2.1. Design of experimental model

Fig. 2 presents the experimental setup conducted on the Yangtze River dike, the subject of our

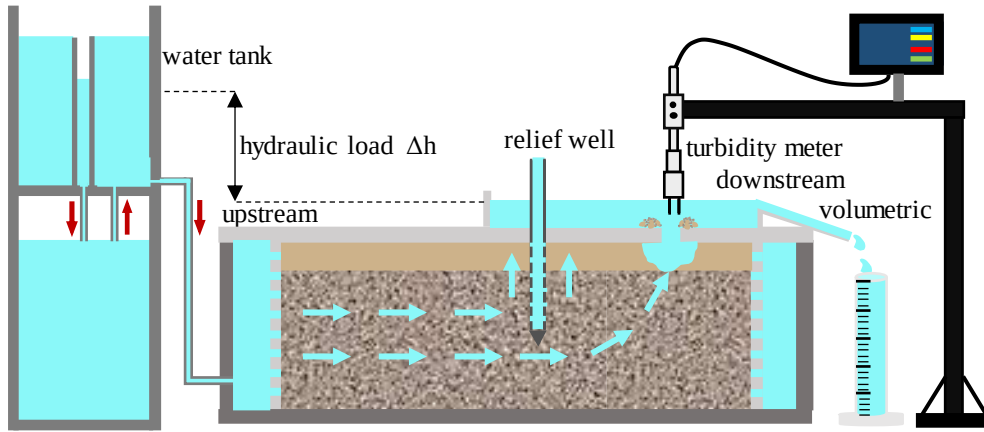
158 investigation. The dike's substructure is characterized by a dual-layer composition: an upper stratum
 159 of sedimentary clay and a lower stratum of permeable fine sand. The experimental methodology is
 160 executed in a sandbox model, detailed schematically in Fig. 3. Employing a mesoscale model
 161 measuring 60 cm in length, 30 cm in width, and 30 cm in height, the setup is enclosed with a clear
 162 cover plate, which is strategically perforated to accommodate the installation of apparatus for
 163 backward erosion BEP and relief well simulations. Two distinct configurations are prepared: one
 164 for a conventional backward erosion BEP test, as depicted in Fig. 3a, and another incorporating
 165 relief well interventions, as shown in Fig. 3b. The exact dimensions of these configurations are
 166 delineated in Fig. 3c.



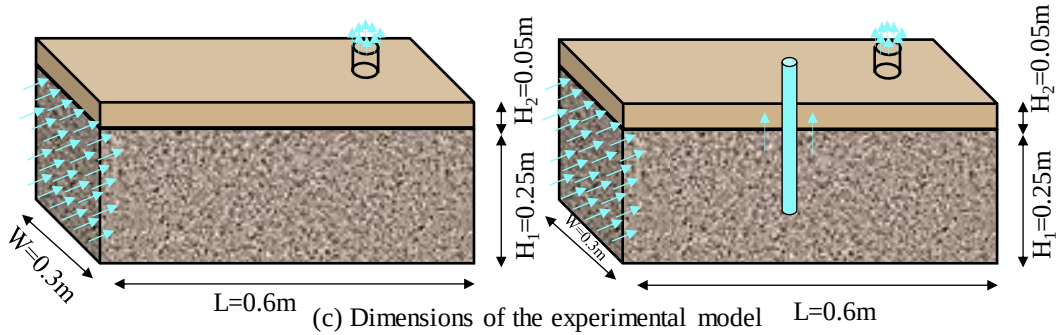
167
 168 **Fig. 2. Schematic figure of dike foundation structure.**



(a) Sink model for piping simulation



(b) Sink model with pressure-reducing wells for piping simulation



(c) Dimensions of the experimental model

Fig. 3. Schematic figure of test model.

During the experiments, a PPR pipe with a 20 mm diameter outlet, wrapped in a filtration net and a permeable sponge, simulates the relief well, maintaining a counter-filtration rate consistent with typical engineering standards at 18%. As demonstrated in Fig. 4, Piezometric tubes are embedded within the model to track pore water pressure variations during erosion. A turbidity recorder positioned at the outlet continuously measures turbidity fluctuations, thereby capturing real-time data on sediment concentration throughout the experiment.

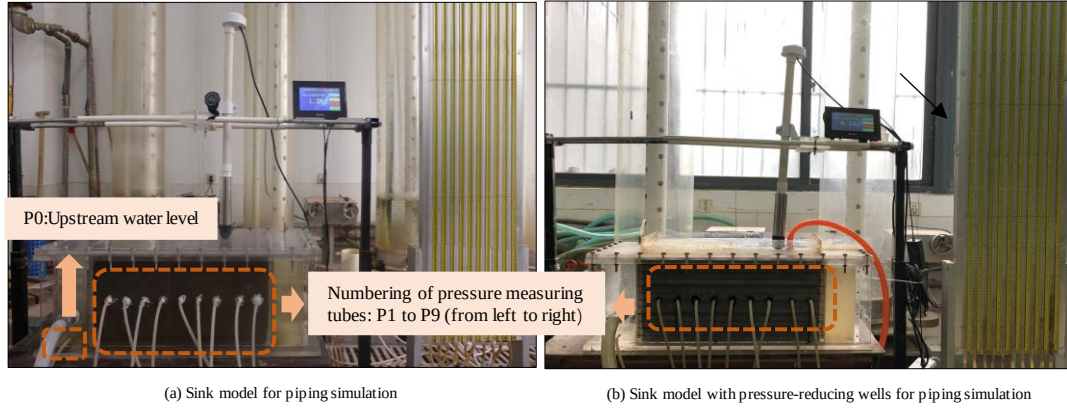


Fig. 4. Physical figure of test model.

2.2. Experimental materials and plans

The permeability of surface sedimentary clay is considerably lower than that of the bottom sand layer. In an effort to bolster the precision of the test outcomes, the clay employed in these experiments is meticulously sourced from the Yangtze River dike site. This clay, once collected onsite, undergoes a sequence of drying, crushing, and screening processes. Particles that measure less than 5mm are designated as test materials, and the samples are subsequently prepared with a 20% water content. For the purpose of these experiments, we use sand originating from the Yangtze River Basin, which has been comprehensively cleaned. Prior to initiating the sandbox model tests, both particle analysis and seepage deformation tests are conducted to ascertain the distinctive characteristics of the test material. Test results indicate that the test material's median particle size (d_{50}) is 0.14 mm, while the control particle size (d_{60}) is 0.19 mm. Furthermore, the test material exhibits a non-uniformity coefficient (C_u) of 1.86, a curvature coefficient (C_c) of 1.01, and a permeability coefficient (k) of $8.7 \times 10^{-3} \text{ cm} \cdot \text{s}^{-1}$.

Through a systematic analysis of existing literature, we have identified that the density of the embankment foundation cover layer, thickness of the cover layer, diameter of the BEP, and distance of the BEP all exert a certain level of influence on BEP. However, there has been limited research addressing the interface characteristics between the foundation cover layer and the sand layer. In traditional binary embankment foundations, a mixed transitional layer of soil and sand has gradually formed at the interface due to years of geological sedimentation and animal activity. Nonetheless, in some sections of the embankment that have been subject to human intervention, excavation has disrupted these transitional layers, and subsequent backfilling operations have resulted in distinct interfaces between the cover and sand layers. Consequently, this paper considers the differences in geological conditions of the embankment foundation and traditional influencing factors. Given that a significant portion of the embankments in the Yangtze River basin have been fortified against BEP through the installation of relief wells, our experimental design also incorporates tests involving BEP with relief wells. To quantify the impact of various external factors on the risk of collapse due to BEP under consistent experimental conditions, we have devised a systematic experimental study drawing upon previous research and engineering characteristics, ultimately conducting 16 sets of mesoscale physical model experiments as shown in Table 1.

Table 1 Experimental plan for BEP.

Number	Thickness of soil	Density of soil layer	Thickness of soil-sand	Diameter of BEP	Distance of	Depth of relief	Location of relief well
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	layer(cm)	(g/cm ³)	contact layer(cm)	(cm)	BEP(cm)	well(cm)	
E1	2.5	1.45	0	3	40	\	\
E2	5	1.45	0	3	40	\	\
E3	10	1.45	0	3	40	\	\
E4	5	1.4	0	3	40	\	\
E5	5	1.5	0	3	40	\	\
E6	5	1.45	2.5	3	40	\	\
E7	5	1.45	5	3	40	\	\
E8	5	1.45	0	2	40	\	\
E9	5	1.45	0	5	40	\	\
E10	5	1.45	0	3	30	\	\
E11	5	1.45	0	3	50	\	\
E12	5	1.45	0	3	40	10	Ahead 10 cm (away from the dike)
E13	5	1.45	0	3	40	15	Ahead 10 cm (away from the dike)
E14	5	1.45	0	3	40	30	Ahead 10 cm (away from the dike)
E15	5	1.45	0	3	40	30	Behind 10 cm (closer to the dike)
E16	5	1.45	0	3	40	30	Beside 10 cm

2.3. Theoretical background of Multi-fidelity deep neural network surrogate model (MFDNN)

This subsection details the MFDNN model, which crafts precise responses through the integration of information from multiple fidelity sources. A critical aspect of multi-fidelity modeling is the effective utilization of the relationship between low-fidelity and high-fidelity data. In this approach, auto-regressive method[40] is utilized, as expressed in Equation (1)

$$y_H = \rho(x)y_L + \delta(x) \quad (1)$$

where y_L and y_H denote the low-fidelity and high-fidelity data, respectively; $\rho(x)$ is a scaling factor that quantifies the correlation between, y_H , y_L and δ is the corresponding noise. This approach is capable of yielding satisfactory outcomes when managing the linear correlation present between data of low and high fidelity. To effectively capture the nonlinear correlation between low- and high-fidelity data, an enhanced autoregressive scheme is employed, as described in Equation (2): [41]

$$y_H = \alpha F_l(x_H, y_L) + (1 - \alpha)F_{nl}(x_H, y_L), \alpha \in [0,1] \quad (2)$$

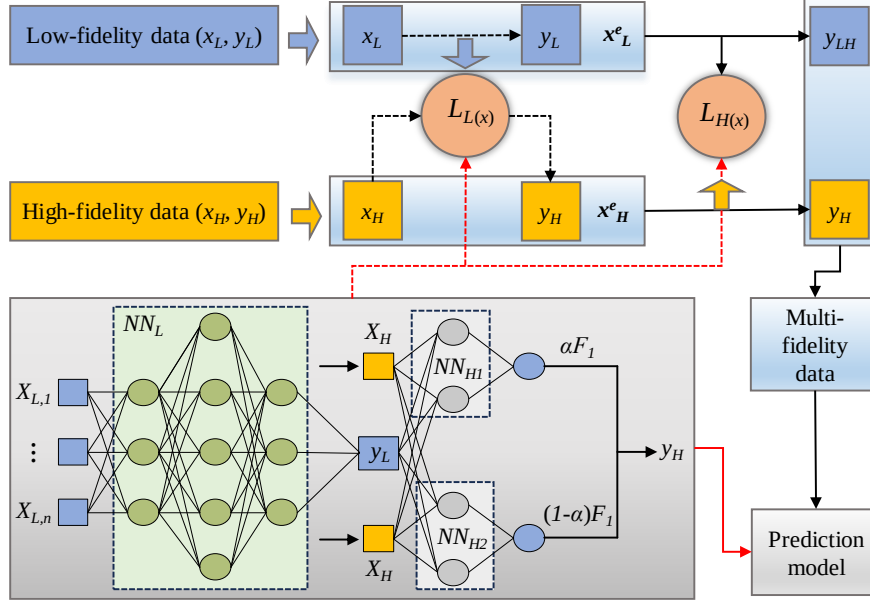


Fig. 5. Architecture of the MFDNN.

Fig. 5 illustrates the architecture of a Multi-Fidelity Deep Neural Network (MFDNN). This innovative network architecture capitalizes on data from two distinct fidelity levels: low-fidelity (NN_L) and high-fidelity (NN_H) [42]. The development of this multi-fidelity neural network draws inspiration from Bayesian methods, optimizing the integration of both low- and high-fidelity data to enhance model performance. Initially, the network trains a preliminary prediction model $L_L(x)$ with a dataset of low-fidelity (x_L, y_L) . Following this, it processes high-fidelity input data (x_H) through a dedicated branch (NN_{H1}). In parallel, another branch (NN_{H2}) synthesizes the output y_L from the low-fidelity model with the high-fidelity input, facilitating a sophisticated fusion of data. This branching mechanism enables the seamless integration of predictions from the low-fidelity model with direct high-fidelity inputs, significantly enhancing prediction accuracy. Moreover, the architecture innovatively combines high-fidelity input data x_H with the low-fidelity model's output y_{HL} to create an extended vector x_{eH} . This vector serves as the foundation for training a subsequent high-fidelity prediction model $L_H(x_e)$. A learned fusion parameter α merges the outputs from the two high-fidelity models, yielding the ultimate high-fidelity output y_H . This fusion strategy not only uncovers the inherent relationships between low- and high-fidelity data but also empowers the model to master data interactions through adaptive learning, moving beyond the limitations of fixed, static equations. Additionally, the model incorporates a unique expansion vector x_{eL} to derive more precise predictions y_{LH} for low-fidelity data points x_L . By enhancing the low-fidelity data and integrating it with high-fidelity data for further training, the network achieves remarkable accuracy in predicting combined multi-fidelity data. The integration of Bayesian methods allows for the adaptive adjustment of model parameters by updating probability distributions based on observed data, ensuring robust predictions despite data fidelity variability. The auto-regressive scheme initially uses ordinary least squares (OLS) to estimate baseline parameters, followed by iterative optimization techniques such as gradient descent or the Levenberg-Marquardt algorithm to refine scaling factors and noise parameters, minimizing prediction errors. The enhanced auto-regressive scheme incorporates non-linearities through polynomial regression or kernel methods, further optimized using backpropagation to update neural network weights and biases. Cross-validation techniques are employed throughout the training process to prevent overfitting and ensure

generalization to unseen data, with final model parameters selected based on validation dataset performance.

This advanced multi-fidelity modeling approach significantly boosts the model's performance and reliability, enabling optimal utilization of available data resources while ensuring high prediction accuracy [43][44]. The loss function typically involves a term for the discrepancy between the predicted high-fidelity output y_H and the true high-fidelity data, often using coefficient of determination (R^2), mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE), and the mean absolute percentage error (MAPE) is calculated as shown in Equations (3)~(7).

$$R^2 = 1 - \frac{\sum_{i=1}^N (y_{H,i} - y_{H,i})^2}{\sum_{i=1}^N (y_{H,i} - \bar{y}_{H,i})^2} \quad (3)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_{H,i} - y_{H,i}| \quad (4)$$

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_{H,i} - y_{H,i})^2 \quad (5)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_{H,i} - y_{H,i})^2} \quad (6)$$

$$MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{y_{H,i} - y_{H,i}}{y_{H,i}} \right| \quad (7)$$

Where N is the number of data points; $y_{H,i}$ is the true high-fidelity output for the i^{th} data point and $\hat{y}_{H,i}$ is the predicted high-fidelity output from the model, $\bar{y}_{H,i}$ is the average of the true values.

2.4. Monte Carlo dropout for uncertainty estimation in deep learning model

Monte Carlo Dropout (MCD) is a sophisticated technique that quantifies the uncertainty in neural network predictions, drawing on Bayesian inference principles [45]. It leverages dropout not merely as a regulariser but also as a strategic means to approximate the posterior distribution of the network weights. During the training phase, dropout is employed to selectively and randomly deactivate subsets of neurons, thereby crafting a progressively refined network at each iteration. This introduction of randomness helps to mitigate the risk of overfitting by preventing the co-adaptation of neurons [46]. For prediction and uncertainty quantification, MCD uniquely applies dropout during the inference phase, allowing the network to engage in multiple stochastic forward passes. This approach deviates from the typical practice of disabling dropout post-training. As a result of the stochastic neuron exclusion, each forward pass culminates in a distinct prediction set, tantamount to sampling from the approximate posterior distribution. Collating these varied predictions yields not a single deterministic output but a robust probabilistic distribution. The variance within this distribution is indicative of the prediction's uncertainty; a higher variance signals increased uncertainty and, conversely, decreased confidence in the model's outputs. Thus, MCD enhances the predictive power of neural networks by providing a nuanced, probabilistic framework for understanding and articulating forecast uncertainty. By integrating MCD, our model becomes uncertainty-aware, allowing for more reliable and informative predictions that account for the inherent uncertainties in the data. This uncertainty-aware approach is crucial for improving the robustness and credibility of our predictive models, particularly in the context of risk assessment

and emergency response in flood control systems.

The MCD technique starts by employing dropout during the training phase, which involves randomly deactivating neurons in the network to prevent overfitting and ensure that the model generalizes well to unseen data. During the inference phase, unlike typical dropout applications where dropout is turned off, MCD keeps dropout active to perform multiple stochastic forward passes. Each pass generates a different subset of active neurons, producing a distinct output. By aggregating these outputs, we obtain a probabilistic distribution rather than a single deterministic prediction. This probabilistic approach measures uncertainty by calculating the variance among the predictions; higher variance indicates higher uncertainty, thus providing insights into the confidence level of the model's predictions. By incorporating MCD, our model is enhanced to handle uncertainty more effectively, making it particularly valuable for applications in risk assessment where understanding the confidence and reliability of predictions is essential.

2.5. Implementation process of uncertainty-aware multi-fidelity deep learning method

This research details the key steps involved in implementing the multi-fidelity deep neural network (MFDNN) combined with Monte Carlo (MC) to achieve uncertainty-aware multi-fidelity deep learning.

First, in the data preprocessing phase, we collected a large amount of experimental data and high-quality on-site BEP case data, serving as low-fidelity and high-fidelity data, respectively. The low-fidelity data consists of abundant laboratory data, which, although of lower accuracy, is plentiful in quantity. The high-fidelity data, on the other hand, is collected from on-site case studies, which is highly accurate despite being limited in quantity. We utilized the Scikit-learn library to standardize these data, ensuring consistency in input data.

In the model architecture design, we used the TensorFlow framework to construct the MFDNN model. The low-fidelity network (LL) is used for initial predictions and comprises an input layer and two fully connected layers (with 64 and 32 neurons, respectively). After each fully connected layer, a 50% Dropout layer is added to prevent overfitting. The high-fidelity networks (NNH1 and NNH2) are more complex, handling high-fidelity data and fused data, respectively. Each network includes an input layer and two fully connected layers (with 128 and 64 neurons, respectively), followed by a 50% Dropout layer after each fully connected layer to improve prediction accuracy. The low-fidelity data, being abundant but less accurate, allows the model to use fewer neurons to avoid overfitting, making computation more efficient. Conversely, high-fidelity data, being scarce but highly accurate, requires more neurons to capture subtle differences and complex features in the data, thus enhancing prediction accuracy. The high-fidelity networks need a more complex structure to fully utilize the high-precision data, thereby improving performance and reliability in the overall fused model.

During the training process, we first trained the low-fidelity model using the Adam optimizer and mean squared error (MSE) loss function. Then, we combined the output of the low-fidelity model with the high-fidelity input data to form an extended vector, which was used to train the high-fidelity model. In the inference phase, we kept the Dropout layers active and performed multiple stochastic forward passes to generate a series of prediction results. We assessed the model's uncertainty by calculating the mean and variance of these predictions.

To quantify uncertainty and achieve uncertainty-aware functionality, we customized the Dropout layer to remain active during inference, performing multiple stochastic forward passes to

generate a series of prediction results. This approach forms a probability distribution used to quantify uncertainty. Finally, we evaluated the model's uncertainty by calculating the mean and variance of the prediction results and assessed model performance using metrics such as R^2 , MAE, MSE, RMSE, and MAPE.

2.6. Research framework

The research commences by amalgamating high-fidelity data from on-site case monitoring and contrasting this with low-fidelity observations derived from physical experimental models. This combined dataset informs the development and refinement of the Multi-Fidelity Deep Neural Network (MFDNN), which assimilates risk assessments leveraging both sets of data, bolstered by strategic enrichment of low-fidelity samples. To encapsulate the uncertainty of predictions and facilitate the digital management of dike disasters, the MFDNN is further augmented with a Monte Carlo Dropout (MCD) layer. This layer acts during both the training and predictive phases, enabling the network to estimate the uncertainty associated with its predictions through stochastic forward passes. By incorporating MCD, the MFDNN becomes uncertainty-aware, providing a quantitative measure of predictive uncertainty and enhancing the digitalization of risk management with a probabilistic assessment of BEP risks. The MFDNN, empowered by Bayesian optimization, undergoes iterative enhancements until its forecasts correlate with the high-fidelity data, leading to improved convergence and elevated predictive precision. This advancement marks a significant leap in the accurate identification of BEP risks in dikes, contributing to the advancement of digital disaster management strategies. The enhanced research framework, inclusive of the MCD component for uncertainty quantification, is depicted in Fig 6.

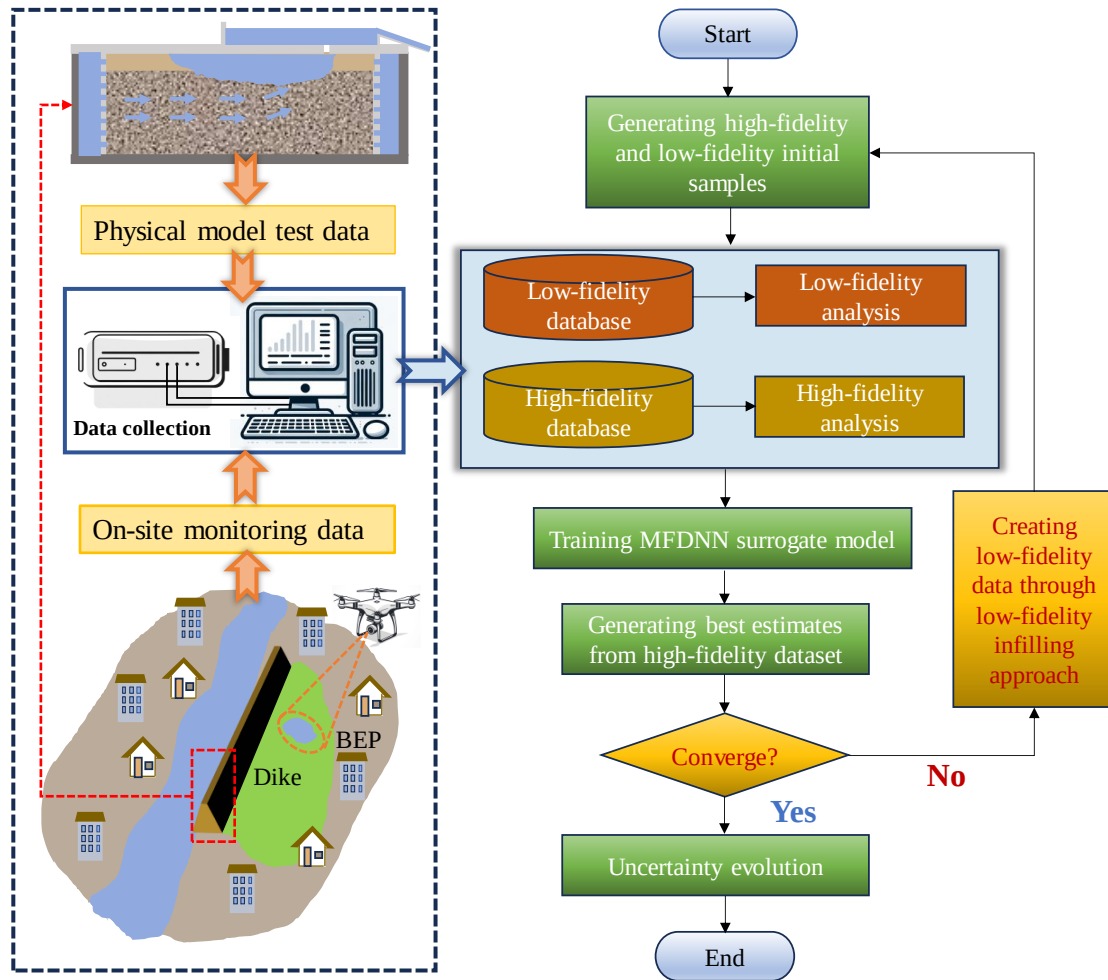


Fig. 6. Framework of the research.

3. Results

3.1. Analysis of experimental data for BEP

Fig. 7~9 encapsulates the intricate interactions between hydraulic gradients, flow dynamics, and turbidity during BEP experiments. As the density of the overlying layer increases, the critical and erosive hydraulic gradients trend upwards, signifying that denser materials offer better resistance to the water pressures that incite BEP. Moreover, the hydraulic gradient is affected by the layer's thickness, indicating that a thicker overlay bolsters the defense against the sand stratum's erosion. The interface's characteristics between the overlay and the sand layer crucially influence these gradients. Smooth interfaces result in lower gradients, which may lead to quicker onset and progression of BEP, whereas interfaces with a soil-sand mix heighten the gradients, thereby strengthening the resistance to BEP. The dimensions and spatial extent of the BEP have considerable implications. Larger diameters and elongated channels tend to reduce the hydraulic gradients, potentially expediting BEP development.

The turbidity observations align with the hydraulic gradient and flow rate findings. In conditions leading to lower hydraulic gradients and flow rates, such as with larger BEP channels or more distant BEP locations, the turbidity at the outlet typically increased, signaling the approach of critical and destructive phases of BEP. Turbidity levels spiked, reflecting the severity of the erosion

and suggesting a more imminent risk of dike failure. According to the data, relief wells have a significant impact on the development of BEP. Deeper wells correlate with lower gradients and thus play a role in impeding the formation and growth of BEP. BEP that occur outside the relief wells (positioned away from the dike) exhibit a higher gradient, resulting in a greater resistance to erosion. Conversely, BEP observed within the relief wells (located closer to the dike) demonstrate a lower gradient, which corresponds to a higher risk factor. Combining the insights from hydraulic gradients, flow rates, and turbidity patterns reveals a comprehensive picture of the conditions conducive to BEP. These findings underscore the necessity of considering the overlying layer's density and thickness, the interface quality, the relief wells' positioning, and the physical characteristics of the piping itself when assessing the risk of BEP.

After conducting a detailed analysis of 16 sets of backward erosion BEP physical model experiments, this research observed some regular and similar phenomena during the development of BEP. Taking the phenomena of one set of experiments (E1) as shown in Fig. 10 as an example, our analysis is as follows: In the initial stages of BEP, its development is relatively slow, with clear water flow and only a small amount of sediment being flushed out, as shown in Fig. 10 (A). As the water level gradually rises, the BEP enters a rapid development phase, where the sediment content at the BEP outlet significantly increases, and the flow rate also increases accordingly, as revealed in Fig. 10 (B) and (C). When the BEP is about to penetrate the dike, the flow rate and sediment content at the outlet sharply rise, making the BEP phenomena extremely intense, as demonstrated in Fig. 10 (D). The condition of the BEP channels after destruction is illustrated in Fig. 10 (E) and (F). Based on these observations, we can classify the risk levels of BEP into three stages: slow development, rapid development, and imminent destruction. In the slow development stage, BEP is relatively mild with weak destructiveness and lower risk; during the rapid development stage, the intensity of water and sand ejection increases, enhancing its destructiveness; in the stage nearing destruction, BEP become extremely severe, posing the highest risk of causing dike failure.

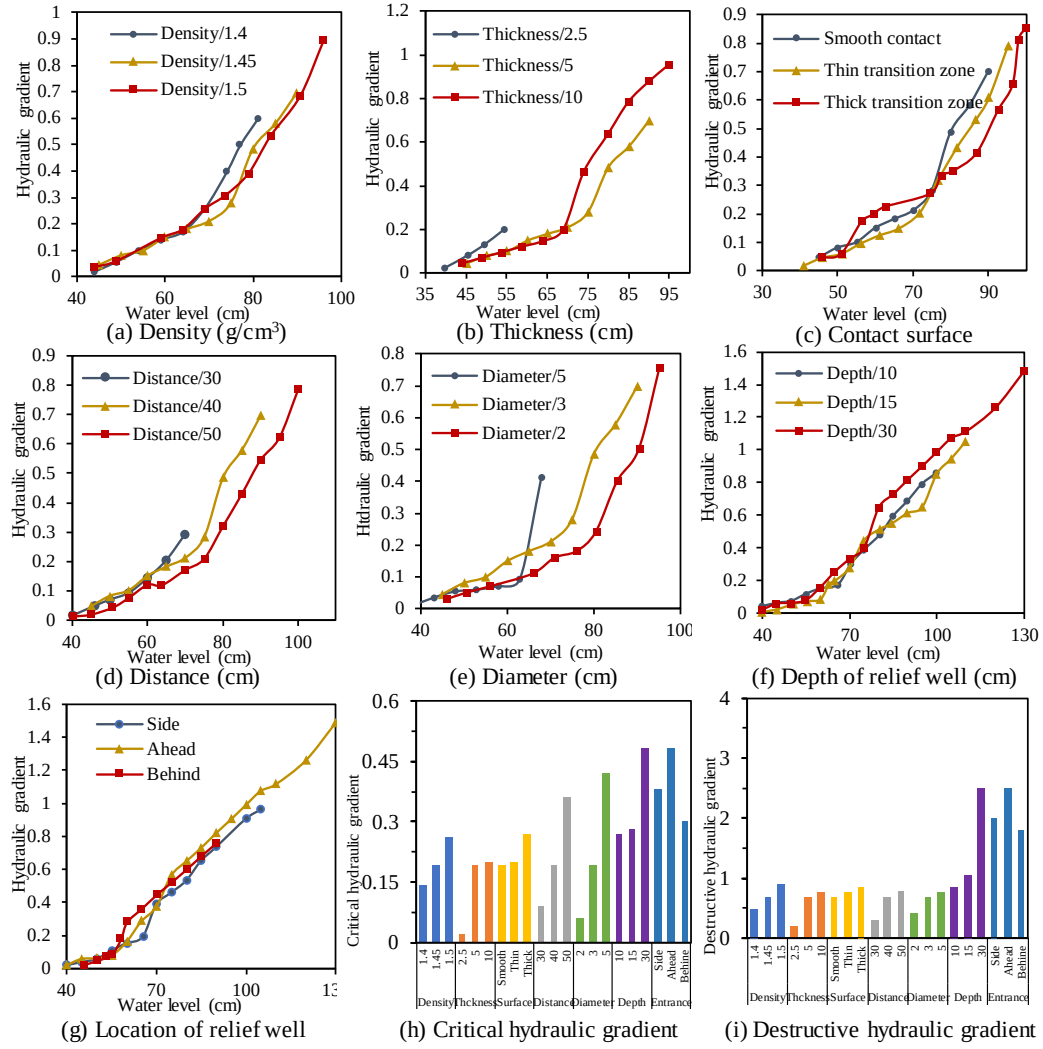


Fig. 7. Hydraulic gradient variation law in BEP experiments.

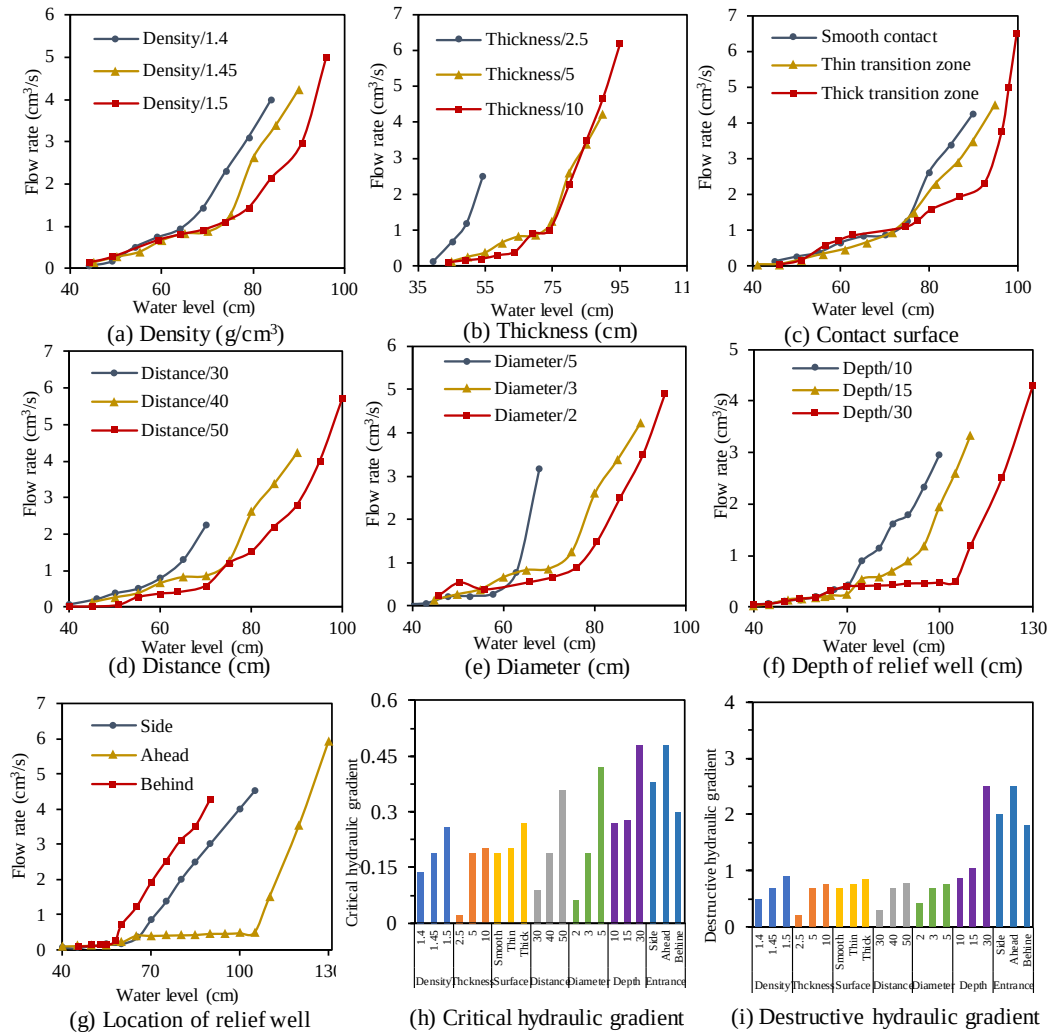


Fig. 8. Flow rate variation law in BEP experiments.

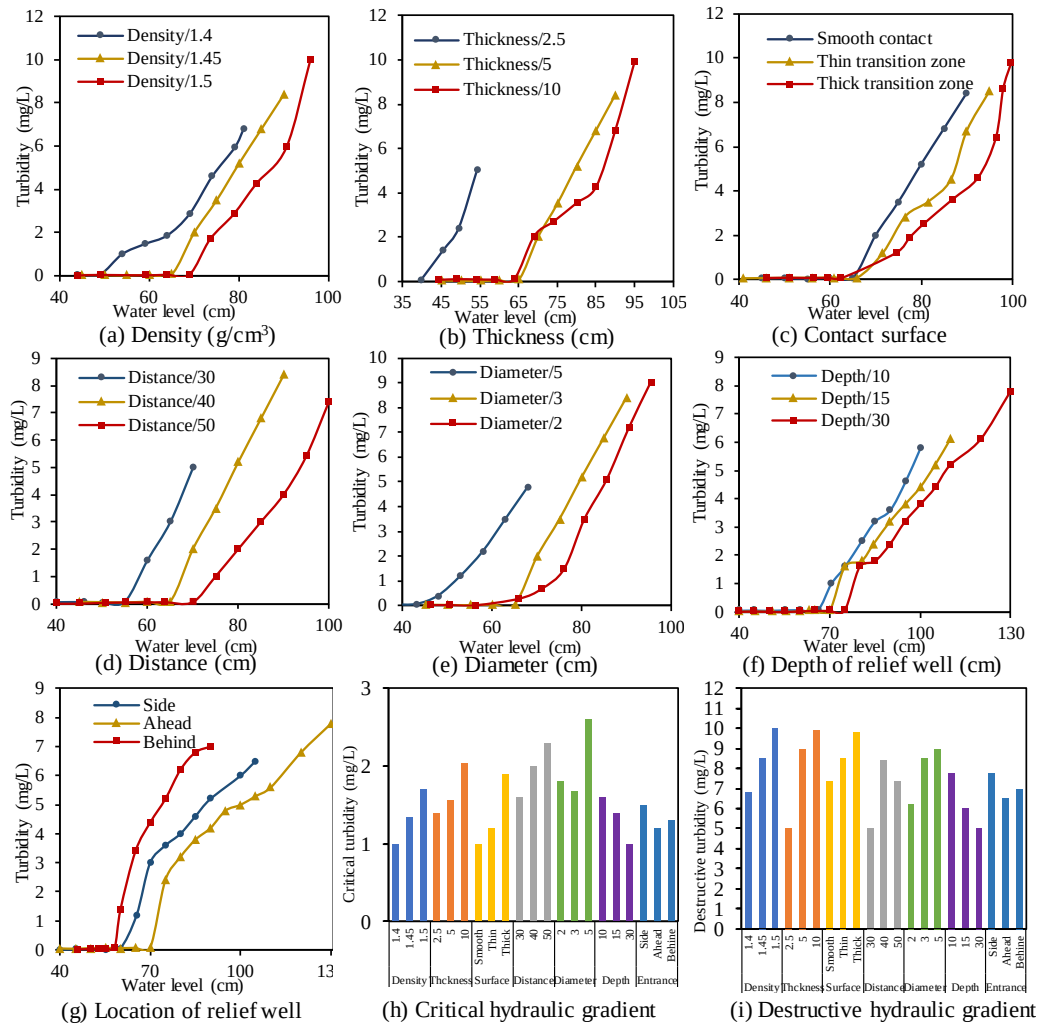


Fig. 9. Turbidity variation law in BEP experiments.

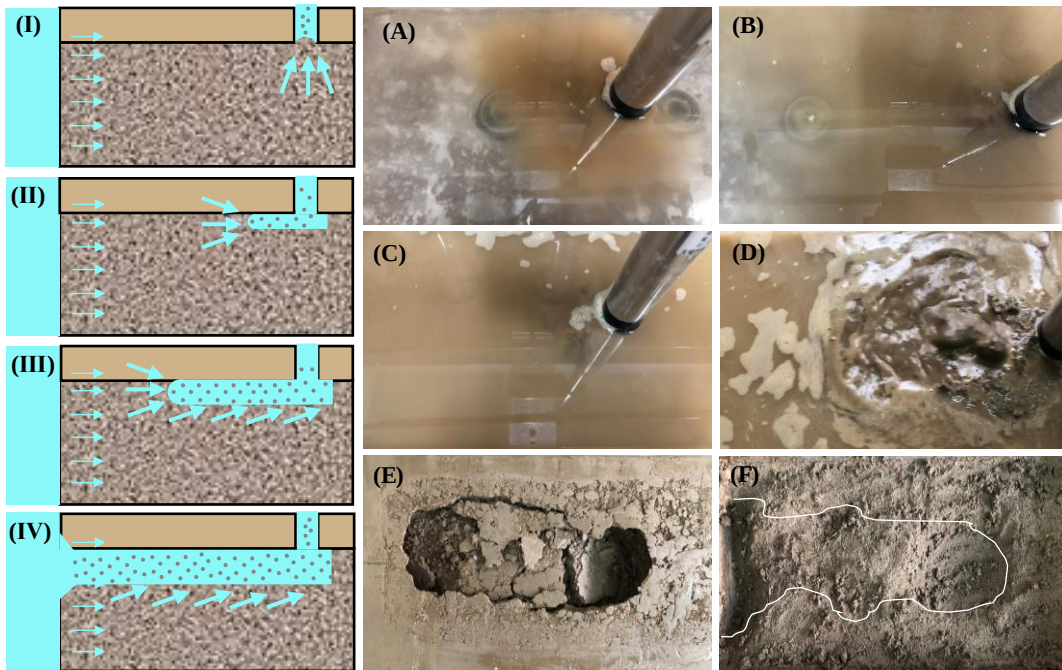


Fig. 10. The phenomenon of BEP experiments (E1).

3.2. Correlation analysis of factors affecting BEP

With numerous factors influencing BEP risk, identifying those with high correlation is crucial. The structural equation model excels in pinpointing factors with significant correlation, essential for determining the various elements impacting BEP risk. Based on experimental data, this study developed a structural equation model to explore the relationships between upstream water level, BEP outlet flow rate, turbidity at the BEP outlet, thickness and density of the covering soil layer, characteristics of the soil-sand contact surface, BEP diameter, distance of BEP occurrence from upstream, depth of relief wells, and their location, and the risk levels of BEP. As depicted in Fig. 11, the structural equation model shows that the BEP risk level is influenced by multiple environmental and engineering parameters. The upstream water level significantly positively affects the flow rate (0.727), which in turn significantly impacts turbidity (0.817). These factors notably influence the BEP risk level, with upstream water level and BEP outlet turbidity positively correlated with the BEP risk level, while BEP outlet flow rate is negatively correlated. The thickness of the soil-sand contact surface, density, and thickness of the covering soil layer are negatively correlated with BEP risk level, with notable impacts from the soil-sand contact surface thickness (-0.152) and covering soil layer density (-0.272). The distance of BEP occurrence from the dike base negatively correlates with BEP risk level and is significantly impactful, while the diameter of BEP (Diameter of BEP) has a lower correlation (0.187) but remains an important factor. The positive correlation (0.232) between the depth of the relief well and risk level, and the slight negative impact (-0.006) of the relief well location on risk level, indicate that the configuration of relief wells affects BEP risk level, with depth having a more pronounced impact than location.

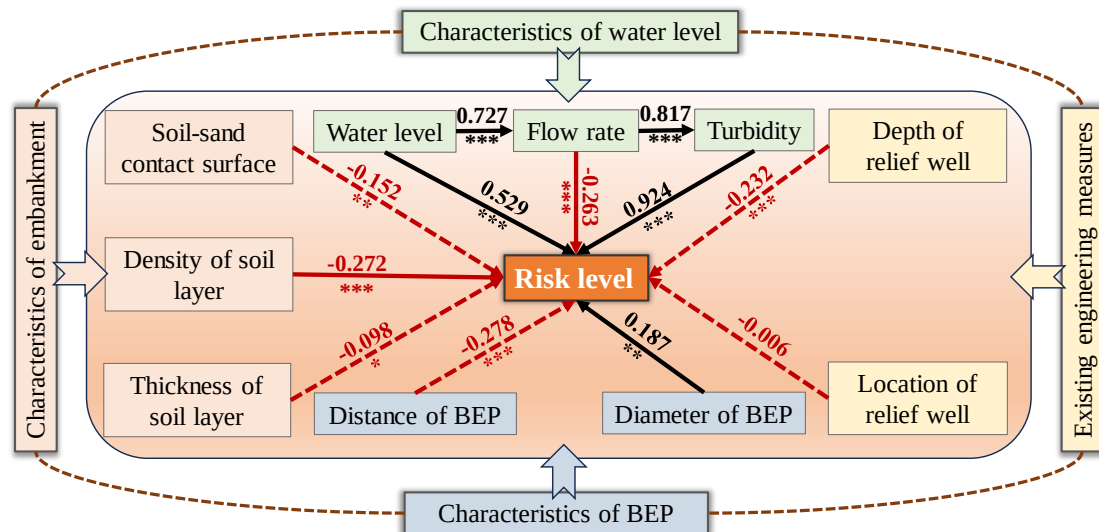


Fig.11 The structural equation model visualizes the relationship of factors affecting BEP. The dotted and solid lines are insignificant and significant effects. The level of significance is denoted by * $p < 0.05$, ** $p < 0.01$, and * $p < 0.001$. Numbers above the arrow line indicate correlation. The black and red lines represent positive and negative correlations.**

Given the challenges in measuring and recording certain factors in actual embankment engineering projects, this study strives to harness the full potential of the BEP case dataset to establish a viable BEP risk level identification model. The research involved a critical analysis of the factors influencing BEP risk levels, streamlining them based on their significance to formulate a pragmatic index system. This system encompasses BEP hazard characteristics, water level features,

and the intrinsic conditions of embankment projects.

Specifically, BEP hazard characteristics include variables such as the proximity of the BEP incident point to the bank base, the diameter of the BEP, and the turbidity at the BEP outlet. Water level characteristics pertain to the levels in front of the embankment, while the embankment conditions cover aspects like the geological state of the embankment foundation, the caliber of foundational construction, and existing engineering measures, including the implementation of pressure relief wells. Although turbidity at the BEP outlet can be quantified in experimental setups, corresponding monitored values in case data are often absent. Hence, turbidity has been categorized into four evaluative criteria: clear, light sand, heavy sand, and turbulent sand discharge, based on a synthesis of experimental observations and turbidity readings. While experimental data indicate that the depth of relief wells influences the BEP risk level, the precise parameters of these wells are typically not logged in the engineering datasets. Consequently, the model accounts for the presence of relief wells as part of the embankment's inherent conditions. Moreover, the thickness and density of the experimentally established covering layer serve as proxies for construction quality. These factors are integrated and stratified into five levels that align with the dike grade classifications documented in the BEP cases. Lastly, the thickness of the soil-sand mixture contact layer in the embankment foundation is adopted as an evaluative metric for the geological conditions of the foundation, which is also recorded in the actual BEP case repository.

3.3. Multi-Fidelity model parameter settings

In this study, a MFDNN model was designed and implemented, focusing on accurately identifying and assessing the risk levels of BEP in dikes. Uniquely, this model integrates two input layers, one for low-fidelity (LF) indoor experimental data and the other for high-fidelity (HF) field case data. Each input layer consists of 7 nodes that precisely map key characteristics like BEP diameter, distance of BEP, upstream water level, geological conditions, dike quality, and the presence of mitigation measures. The output of the model is the risk level of BEP, categorized into three levels based on regular conclusions obtained from experiments and records of on-site BEP cases: low risk, medium risk, and high risk. These risk levels are then labeled within the dataset.

The architecture of the model further includes three hidden layers, with the first two layers each equipped with 64 neurons, and the third layer fitted with 32 neurons. All these layers employ the ReLU activation function as shown in Equation (8), which enhances the model's non-linear learning capability and helps prevent the issue of gradient vanishing, enabling the network to capture more complex feature patterns.

$$f(x) = \max(0, x) \quad (8)$$

The core fusion layer F employs a weighted average strategy, as shown in Equation (9), to integrate information from the two different fidelity data sources. In the equation, α is a predefined hyperparameter, whose value is adjusted based on the reliability and relevance of the data sources. This adjustment is made to balance the contributions of low-fidelity and high-fidelity data to the final prediction. The output layer consists of three neurons and utilizes the softmax function as shown in Equation (10) to output the probability distribution of risk levels. This enables effective classification into three risk categories: low risk, medium risk, and high risk.

$$F = \alpha \cdot NN_{LF}(X_{LF}) + (1 - \alpha) \cdot NN_{HF}(X_{HF}) \quad (9)$$

$$\sigma(z)_j = \frac{e^z j}{\sum_{k=1}^K e^z k} \quad (10)$$

3.4. Application and evaluation of MFDNN in BEP risk level recognition

To further validate the efficacy of this model in identifying embankment erosion problems (BEP), this study collected 241 sets of BEP risk data from actual Yangtze River embankment sections. Among these, 161 data sets were chosen as training samples, with the remaining 80 designated for validation. The research followed these steps: firstly, seven key influencing factors were selected as model inputs, with BEP risk level designated as the model output. Risk levels were categorized into three tiers: level 1 for low risk, level 2 for medium risk, and level 3 for high risk. The model was trained using the training samples to establish non-linear relationships between input factors and risk level outputs. Subsequently, the model made predictions on both training and validation samples. Comparison between predicted and actual risk levels, as illustrated in Fig. 12, demonstrated superior performance of the model on the training set compared to the validation set. This disparity is primarily attributed to the model being finely tuned to the training data during its learning process, rendering it more adept at handling such data. On the validation set, the model exhibited a coefficient of determination (R^2) value of 0.89, Mean Absolute Error (MAE) of 0.15, Mean Squared Error (MSE) of 0.08, and Root Mean Squared Error (RMSE) of 0.28, indicative of its high predictive accuracy.

Fig. 13(a)(b) illustrates the results obtained through the application of the Monte Carlo Dropout technique to evaluate uncertainty distribution across various risk levels within both the training and test datasets. In the figure, we use standard deviation to represent the uncertainty score. The uncertainty score quantifies the level of uncertainty or variability in the data, with a higher standard deviation indicating greater uncertainty. In the training dataset, there is a consistent uncertainty distribution among low, medium, and high risk levels, characterised by tightly clustered medians and moderate interquartile ranges indicating a similar spread of uncertainty scores within each risk category. However, in the test dataset, a notable trend emerges. The median uncertainty scores for low and medium-risk levels are significantly lower compared to those for high-risk levels. This trend is emphasized by the wider dispersion of uncertainty scores for high-risk levels, as indicated by their extended range and interquartile width. These findings suggest a heightened difficulty in accurately predicting high-risk events compared to low-risk events, likely due to the complexity of high-risk scenarios or limited data availability. By employing an uncertainty-aware approach through Monte Carlo Dropout, our model can better account for the variability and unpredictability inherent in the data, providing more reliable and informative predictions. This uncertainty-aware methodology highlights the challenges in predicting high-risk scenarios and underscores the importance of incorporating uncertainty quantification in risk assessment models.

Fig. 14(c)(d) depicts a scatter plot illustrating the relationship between the maximum predicted probability and uncertainty score. As the maximum prediction probability increases, signaling heightened confidence in a specific prediction, the corresponding uncertainty score gradually diminishes. This observed trend holds true across both datasets, underscoring the efficacy of the Monte Carlo dropout technique in delivering predictive certainty. Predictions characterized by high model confidence display minimal uncertainty, whereas those with lower forecast probabilities exhibit higher levels of uncertainty. In Fig. 14(a), the prediction outcomes from the training set

exhibit a conspicuous inverse correlation between the maximum prediction probability and uncertainty score. Meanwhile, Fig. 14(b) portrays darker points denoting outliers in the test set, suggesting that the uncertainty within the test set's prediction outcomes surpasses that of the training set, aligning with anticipated outcomes. Analysis of prediction uncertainty underscores potential areas necessitating additional training data or further refinement of the model.

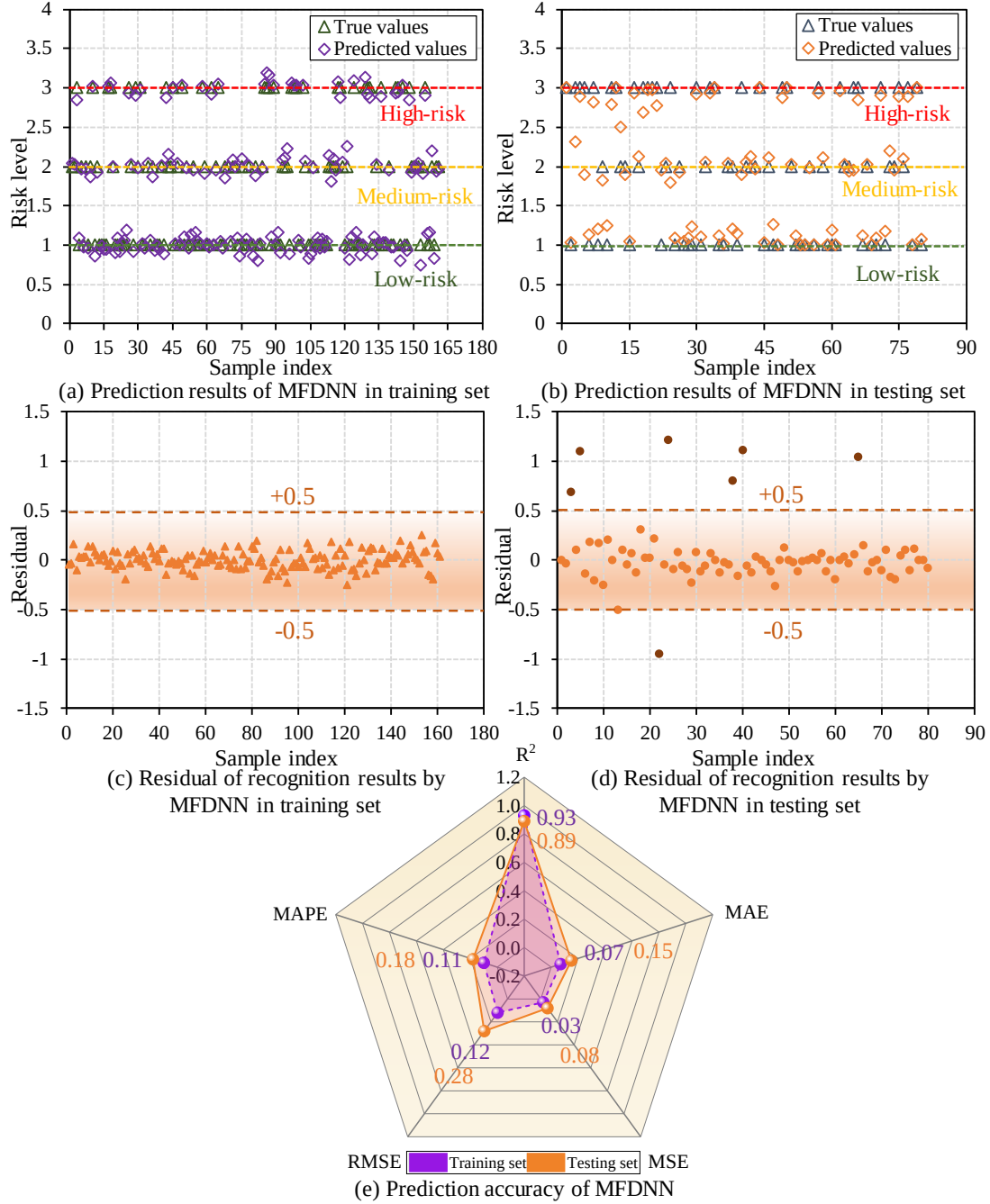


Fig. 12. Prediction results by MFDNN.

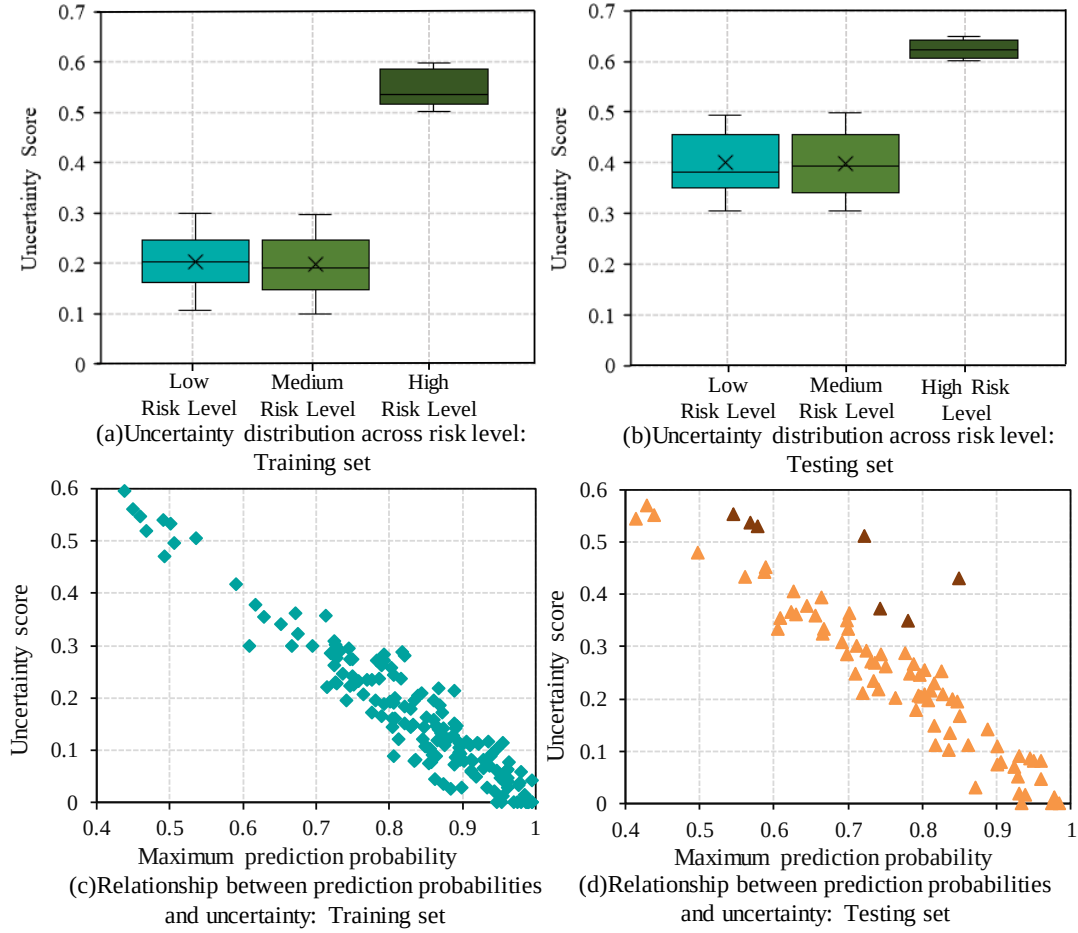


Fig. 13. Results of uncertainty-aware evaluation.

4. Discussion

4.1. Influence of different model

The study conducted a comparative analysis of the performance of MFDNN against other prominent machine learning techniques in terms of predictive accuracy. Each model was trained 15 times, and the results were subjected to statistical analysis using standard errors to reflect the reliability and statistical significance of the computed results. As depicted in Table 2, we conducted a comprehensive assessment of various models using key statistical metrics such as R^2 , MAE, MSE, RMSE, and MAPE on both the training and test sets. The results presented are the averages of 15 repeated experiments. The MFDNN demonstrated superior performance across all evaluation metrics on the test set. It achieved an R^2 value of 0.88, surpassing the artificial neural network (ANN) with an R^2 of 0.85, the support vector machine (SVM) with an R^2 of 0.86, the random forest (RF) with an R^2 of 0.78, and the K nearest neighbor (KNN) method with an R^2 of 0.84. The MFDNN also recorded the lowest mean absolute error (MAE) of 0.15, indicating a higher accuracy in predictions compared to ANN (0.21), SVM (0.23), RF (0.19), and KNN (0.26). Furthermore, the MFDNN achieved the lowest mean squared error (MSE) at 0.09, significantly lower than ANN (0.14), SVM (0.15), RF (0.12), and KNN (0.17). Its root mean square error (RMSE) was 0.27, and the mean absolute percentage error (MAPE) was 0.19, both of which were the lowest among all models tested.

These results underscore MFDNN's robust generalization capability and its effectiveness in minimizing prediction errors when applied to unseen data.

Fig. 14 and 15 provide a visual comparison of the performance of various models on the training and test sets, respectively. Both figures display key statistical metrics including R^2 , MAE, MSE, RMSE, and MAPE, with T-shaped error bars indicating the variability in each metric. In Fig. 14, which illustrates the training set performance, the MFDNN shows the highest R^2 value at 0.92, significantly outperforming other models such as ANN (0.87), SVM (0.89), RF (0.85), and KNN (0.89). The MFDNN also achieves the lowest MAE (0.07), MSE (0.03), RMSE (0.13), and MAPE (0.11), highlighting its superior accuracy and consistency during training. Fig. 15 depicts the performance of the test set. The MFDNN continues to lead with an R^2 value of 0.88, outperforming ANN (0.85), SVM (0.86), RF (0.78), and KNN (0.84). It also records the lowest MAE (0.15), MSE (0.09), RMSE (0.27), and MAPE (0.19), demonstrating its robust generalization capability and effectiveness in minimizing prediction errors on unseen data.

Fig. 16 and 17 provide a detailed box plot comparison of the performance metrics for various models on the training and test sets, respectively. Each box plot illustrates the distribution of critical statistical metrics, including R^2 , MAE, MSE, RMSE, and MAPE, with dual Y-axes to accommodate the different scales of these metrics. In Fig. 16, which represents the training set performance, the MFDNN displays the highest median R^2 value, close to 0.92, with relatively narrow interquartile ranges for MAE, MSE, RMSE, and MAPE, indicating consistent performance across multiple runs. The MFDNN's MAE, MSE, RMSE, and MAPE values are all lower compared to other models such as ANN, SVM, RF, and KNN, underscoring its accuracy and reliability during training. Fig. 17 shows the performance metrics on the test set. Here, the MFDNN also stands out with a median R^2 value of around 0.88, higher than ANN, SVM, RF, and KNN. The box plots for MFDNN reveal lower and more compact distributions for MAE, MSE, RMSE, and MAPE, highlighting its robustness and minimal prediction errors on unseen data. The other models exhibit more comprehensive interquartile ranges and higher median values for these error metrics, suggesting more variability and less accuracy in their predictions compared to MFDNN.

Tabel 2 Comparison of recognition accuracy in different model.

Model	Training set					Testing set				
	R^2	MAE	MSE	RMSE	MAPE	R^2	MAE	MSE	RMSE	MAPE
MFDNN	0.92	0.07	0.03	0.13	0.11	0.88	0.15	0.09	0.27	0.19
ANN	0.87	0.09	0.05	0.21	0.16	0.85	0.21	0.14	0.36	0.25
SVM	0.89	0.09	0.07	0.27	0.24	0.86	0.23	0.15	0.36	0.28
RF	0.85	0.12	0.08	0.25	0.18	0.78	0.19	0.12	0.34	0.25
KNN	0.89	0.11	0.07	0.34	0.21	0.84	0.26	0.17	0.42	0.26

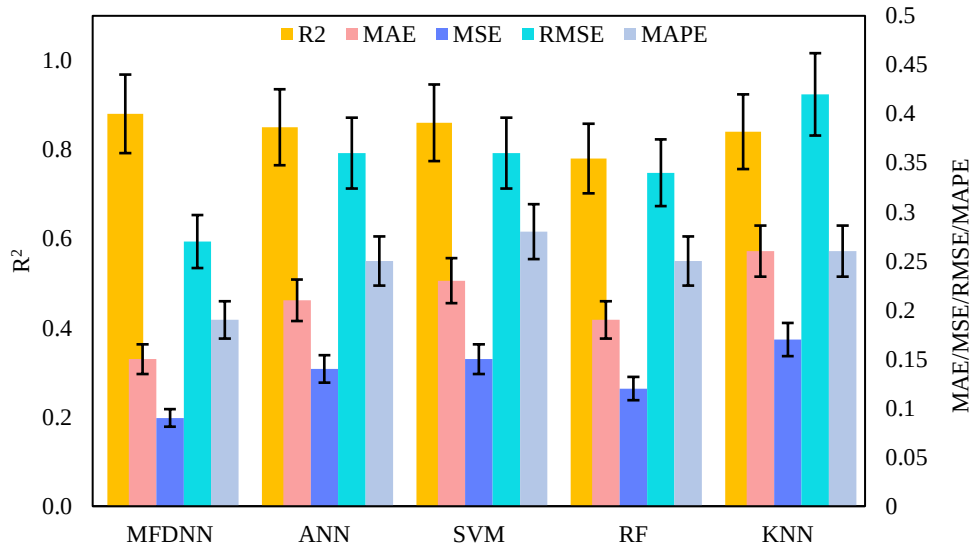


Fig. 14. Training set performance metrics for various models.

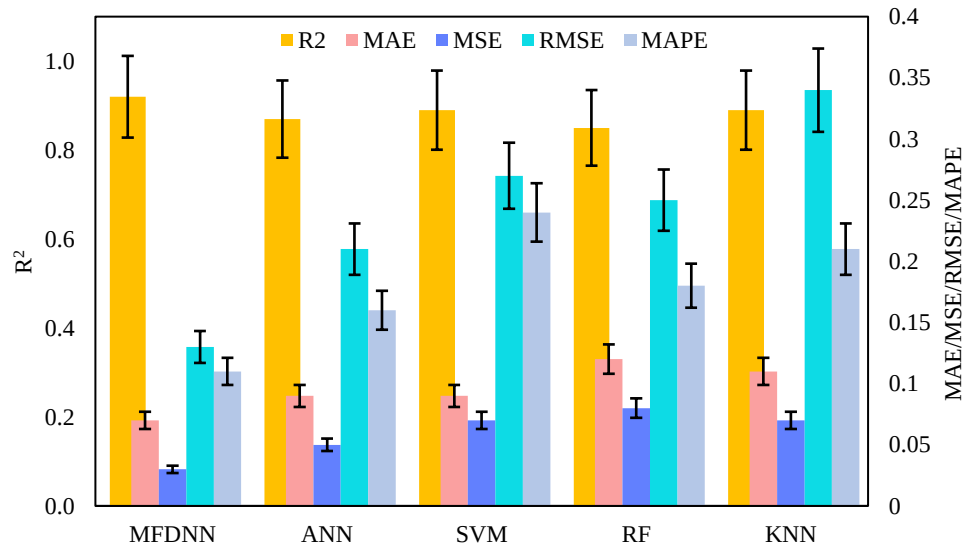


Fig. 15. Testing set performance metrics for various models.

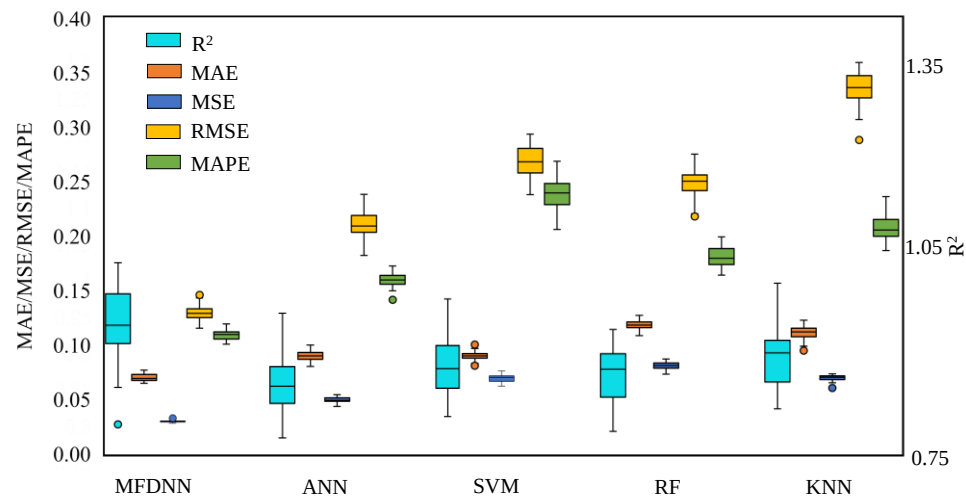


Fig. 16. Training set performance metrics box plot for various models.

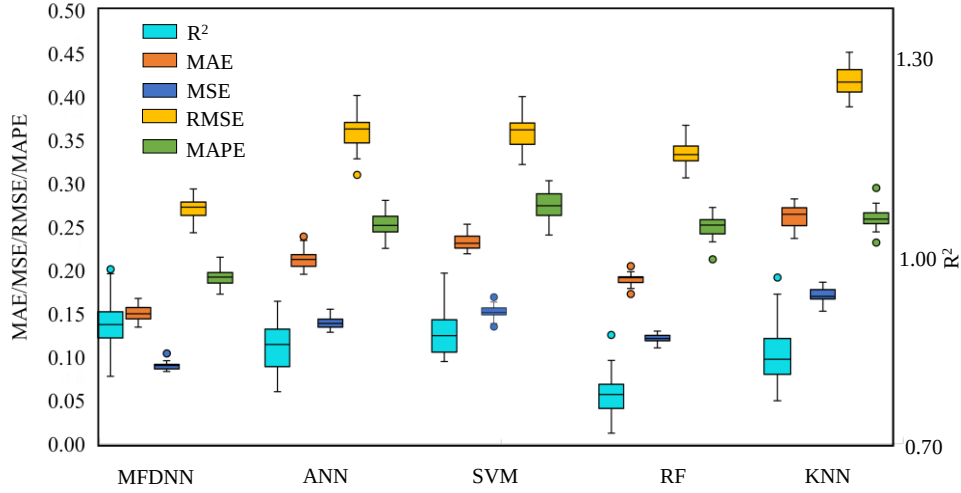


Fig. 17. Testing set performance metrics box plot for various models.

4.2. Influence of high-fidelity data's percentage

In this study, we delve into the efficacy of MFDNN models augmented by high-fidelity data across various scales. Each model was trained 15 times, and the results were then analyzed statistically, with standard errors calculated to reflect the reliability and statistical significance of the findings. Referencing Table 3, we scrutinize the cognitive accuracy of the model on both the training and test sets, employing pivotal statistical metrics including R^2 , MAE, MSE, RMSE, and MAPE. The results presented are the averages of 15 repeated experiments. Our findings reveal a notable enhancement in model performance as the proportion of high-fidelity data increases within the training set. Particularly noteworthy is the substantial leap in R^2 value observed when the high-fidelity data proportion reaches 30%, soaring from 0.85 with solely low-fidelity data (0% high-fidelity data) to 0.91. This trend persists on the test set, as evidenced by the escalation in MFDNN's R^2 value from 0.79 to 0.89 with the augmentation of high-fidelity data, underscoring the pivotal role of high-fidelity data in bolstering the model's generalization prowess. The MAE, MSE, RMSE, and MAPE metrics also show consistent improvements with increasing high-fidelity data, reinforcing the effectiveness of high-fidelity data in enhancing overall model accuracy and reliability.

Fig. 18 and 19 provide a comprehensive analysis of the model's performance across different proportions of high-fidelity data in both training and test sets. T-shaped error bars represent key statistical metrics, including R^2 , MAE, MSE, RMSE, and MAPE. In Fig. 18, which displays the training set metrics, the MFDNN demonstrates the highest R^2 value at 0.92. The model with 30% high-fidelity data shows a marked improvement, achieving an R^2 of 0.91, an MAE of 0.09, an MSE of 0.07, an RMSE of 0.13, and a MAPE of 0.13. As the proportion of high-fidelity data increases from 0% to 30%, there is an apparent enhancement in performance, indicated by the decreasing error metrics and increasing R^2 values. Fig. 19 illustrates the performance on the test set, where the MFDNN again outperforms other models across all metrics. With 0% high-fidelity data, the R^2 value is 0.79, which rises significantly to 0.89 when the high-fidelity data proportion reaches 30%. Correspondingly, the error metrics show a consistent decline, with the model achieving an MAE of 0.18, an MSE of 0.12, an RMSE of 0.30, and a MAPE of 0.21 at 30% high-fidelity data, showcasing its superior performance in minimizing prediction errors.

Fig. 20 and 21 provide a detailed box plot comparison of the model's performance metrics

across different proportions of high-fidelity data. In Fig. 20, representing the training set, the MFDNN shows consistently high performance with the highest median R^2 value close to 0.92. The box plots reveal a clear trend: as the proportion of high-fidelity data increases, the error metrics generally decrease, indicating improved model accuracy and stability. Fig. 21 depicts the test set performance, showing that the inclusion of high-fidelity data enhances model performance. The MFNN with 30% high-fidelity data achieves the highest median R^2 value of 0.89, along with the lowest error metrics. The model's performance improves markedly from an R^2 of 0.79 with 0% high-fidelity data, underscoring the importance of high-fidelity data in enhancing the model's generalization capabilities and reducing prediction errors on unseen data.

Table 3 Comparison of recognition accuracy in different high-fidelity data's percentage.

Model	Training set					Testing set				
	R^2	MAE	MSE	RMSE	MAPE	R^2	MAE	MSE	RMSE	MAPE
MFDNN	0.92	0.07	0.03	0.13	0.11	0.88	0.15	0.09	0.27	0.19
0	0.85	0.11	0.12	0.15	0.16	0.79	0.27	0.19	0.34	0.35
10%	0.89	0.12	0.08	0.17	0.18	0.84	0.25	0.16	0.33	0.25
20%	0.88	0.10	0.09	0.15	0.16	0.87	0.22	0.14	0.35	0.26
30%	0.91	0.09	0.07	0.13	0.13	0.89	0.18	0.12	0.30	0.21

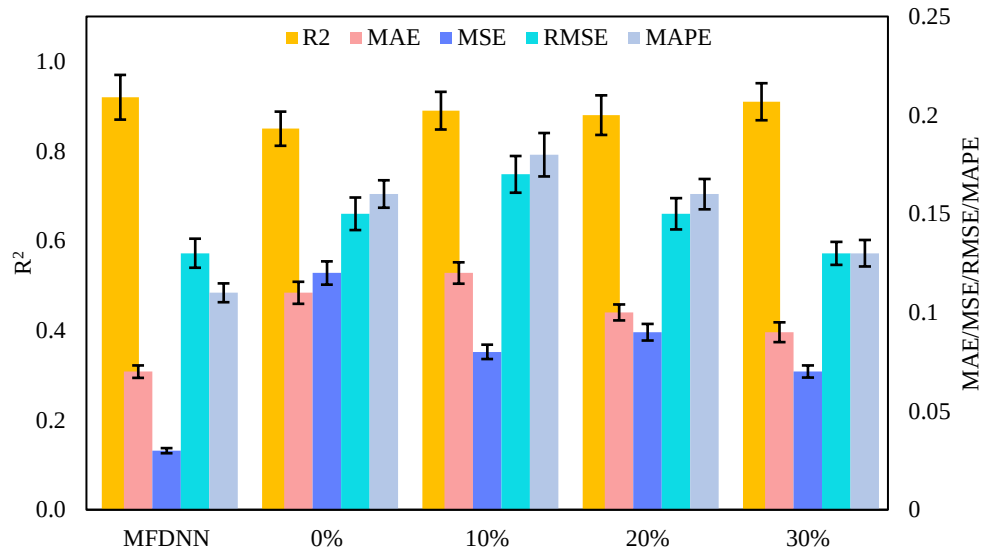


Fig. 18. Training set performance metrics for different proportions of high-fidelity data.

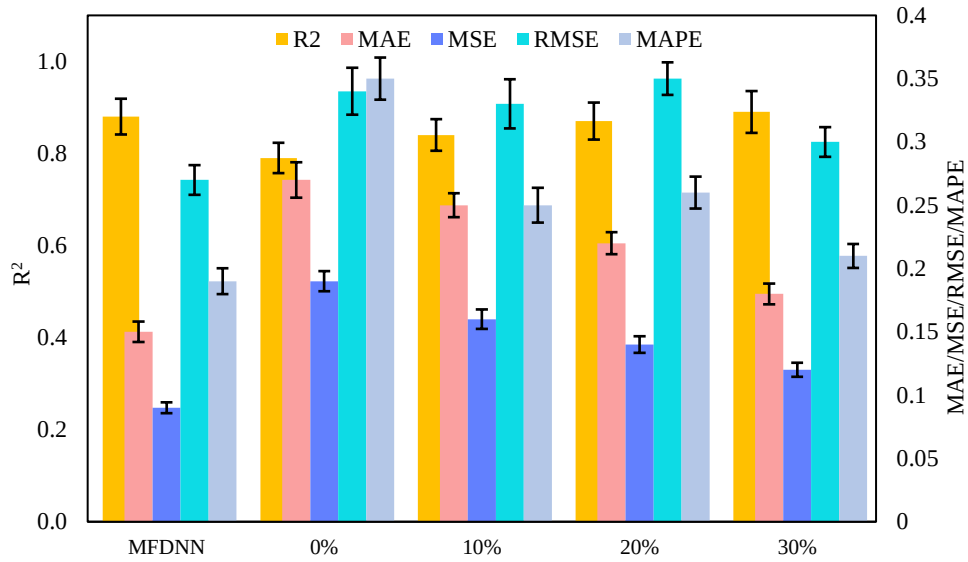


Fig. 19. Testing set performance metrics for different proportions of high-fidelity data.

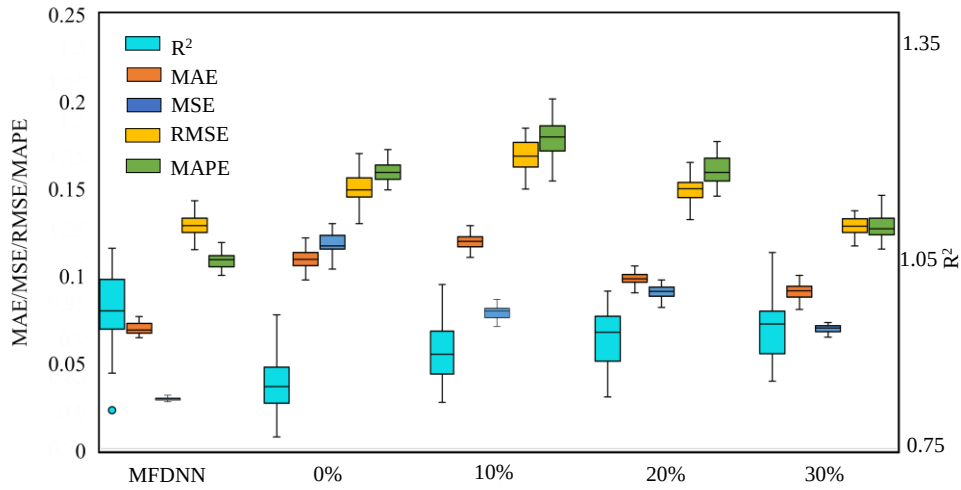


Fig. 20. Box plot of training set performance metrics for different proportions of high-fidelity data.

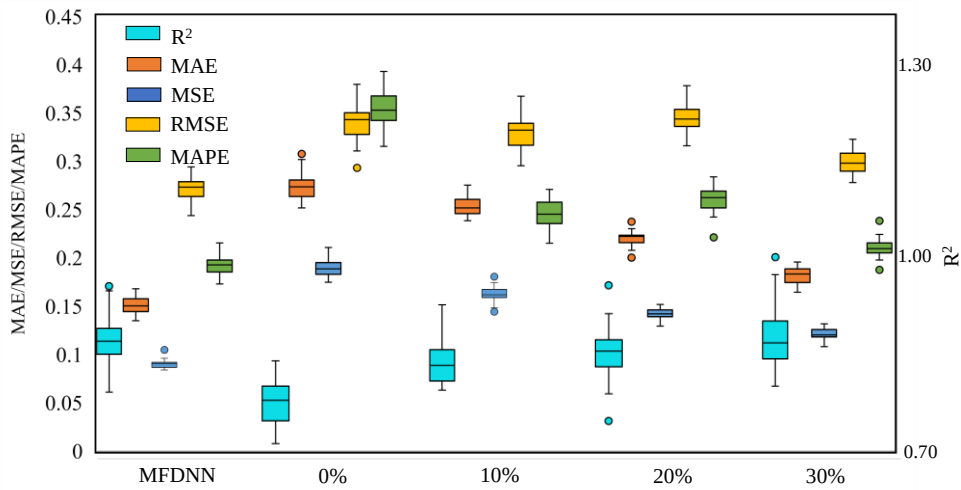


Fig. 21. Box plot of testing set performance metrics for different proportions of high-fidelity data.

data.

5. Conclusions

In this study, confronted with the challenge of limited availability of high-fidelity data, we propose a multi-fidelity approach utilizing deep learning algorithms to develop an accurate data-driven prediction model for assessing piping risk levels. By utilizing a combination of extensive experimental data and a smaller set of high-quality field BEP case data, we introduce an innovative MFDNN model tailored for evaluating BEP risk levels. The model's effectiveness is validated against a test set, demonstrating its capability to accurately identify BEP risk levels influenced by various factors.

To further validate the feasibility of our approach, we comprehensively compare the performance of our multi-fidelity model with models relying solely on extensive or limited high-fidelity data, as well as models lacking enhanced multi-fidelity data. Results indicate a significant improvement in prediction accuracy with our method compared to traditional models trained on multi-fidelity data. Additionally, our study delves into the key factors influencing model performance, including machine learning algorithm selection and the proportion of high-fidelity data. Notably, the proportion of high-fidelity data is found to have a substantial impact on model performance, while the choice of machine learning algorithm plays a lesser role.

Overall, the multi-fidelity deep learning model proposed in this study performs excellently in predicting piping risk levels, particularly in scenarios with limited high-fidelity data. Beyond enhancing prediction accuracy, this model provides a robust tool for identifying high-risk scenarios, which is vital for engineering safety management and the development of emergency response plans. This research not only contributes to the theoretical understanding of multi-fidelity modelling but also offers practical solutions for real-world engineering challenges. By integrating advanced computational techniques with practical applications in dike safety and flood management, our study aligns with the journal's objectives of advancing knowledge in the field of flood control and engineering safety. Future research will focus on refining the model structure, extending its application in real-world engineering contexts, and exploring automated hyperparameter tuning methods to achieve more efficient and accurate risk assessment.

CRedit authorship contribution statement

Hongchen Liu: Conceptualization, Methodology, Software, Investigation, Formal analysis, Writing – original draft, Writing – review & editing. **Huaizhi Su:** Supervision, Resources, Software, Funding acquisition. **Jiaquan Yang:** Conceptualization, Methodology, Visualization. **Haijiang Li:** Software, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

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