Defining the Ideal Criteria for Stable Skeletonisation in Object Point Clouds

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***Abstract*— In recent years, there has been notable interest in skeletonization methods for 3D object models, driven by their broad applicability in fields such as computer graphics and robotics. However, existing studies have lacked a clear quantitative standard for evaluating skeletonization quality. This paper extends prior research on point cloud skeletonization to examine the intrinsic properties of the process across diverse object shapes, aiming to provide intuitive insights into the quality of resulting skeletons. Additionally, we propose a novel concept of stable convergence of contraction, leveraging distributions of geometric curvature and vectorial normal changes.**

I. INTRODUCTION

The exploration of skeletonization methods for 3D object models has surged in recent years, driven by their versatile applications across computer graphics and robotics [1]–[4]. However, despite the extensive research conducted in this domain, a notable gap persists in the lack of established quantitative benchmarks for assessing the quality of resulting skeletons [2], [5]. This work addresses this deficiency by delving into the inherent characteristics of skeletonization processes across objects of varying shapes. Through a comprehensive analysis, we aim to offer intuitive insights into the quality of generated skeletons, paving the way for more informed decision-making in practical applications.

Furthermore, we introduce a novel concept termed stable convergence of contraction, which leverages distributions of geometric curvature and vectorial normal changes. This addition to the evaluation metrics not only enhances the understanding of skeletonization methods but also opens new avenues for improving their performance. By presenting our methodology for evaluating metrics and defining high-quality skeletons, followed by the presentation of experimental results and observations, we contribute to advancing the field of skeletonization research. Our findings hold promise for refining existing techniques and informing future developments, ultimately enhancing the effectiveness of skeletonization methods in real-world applications.

II. METHODOLOGY

In this section, we look into the meaningful geometrical properties of stable contraction and skeletonisation, addressing the challenge of evaluating the skeletonisation results, after the brief introduction of Laplacian-based skeletonisation (LBC).

A. Overview of Laplacian-based skeletonisation

The Laplacian-based skeletonization pipeline (Fig. 1) extracts skeleton vertices and connections from contracted point clouds using Laplacian weights. It starts with the k-nearest neighbors (KNN) algorithm to compute neighbor points for each point in the input cloud. These neighbor points are processed using Delaunay triangulation and Principal Component Analysis (PCA) to derive neighbor rings. A Laplacian matrix is then constructed from these rings, and an iterative contraction process shrinks the point cloud by balancing contraction and attraction weights until a termination condition is met. The resulting contracted points form an approximation of the medial surface, known as the surface skeleton. This surface skeleton is abstracted into skeleton vertices using farthest point sampling, with connections established based on neighbor ring information. Finally, the skeleton undergoes necessary refinements to achieve the desired structure.

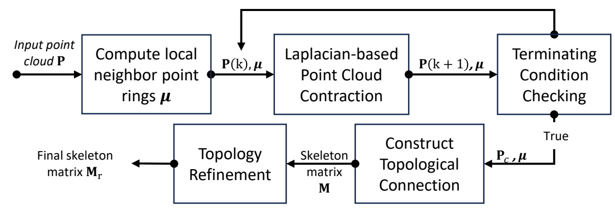

Fig. 1: General structure of Laplacian-based contraction of point clouds [4], [6].

B. Characteristics of the stable contraction & skeletonisation

The evaluation of Laplacian-based contraction, crucial for generating surface and curve skeletons, faces significant challenges. We analyze changes in surface normal vectors and curvatures to assess contraction quality, utilizing cosine similarity to measure directional insights. Additionally, we examine curvature differences between original and contracted point clouds.

To gauge stability, we propose assessing boundedness and stable convergence through symmetry in normal vector and curvature differences, respectively. Boundedness ensures contraction results remain within expected boundaries, while distributions in normal vector and curvature differences converging to a symmetric and unimodal pattern indicate stable convergence. The normal vector difference is given by

$$D_{\theta,k,i} = \frac{\arccos(S_C(n_{k,i}, n_{0,i}))}{\pi}, \tag{1}$$

where

$$S_C(\mathbf{n}_{k,i}, \mathbf{n}_{0,i}) = \frac{\mathbf{n}_{k,i} \cdot \mathbf{n}_{0,i}}{\|\mathbf{n}_{k,i}\| \|\mathbf{n}_{0,i}\|}. \tag{2}$$

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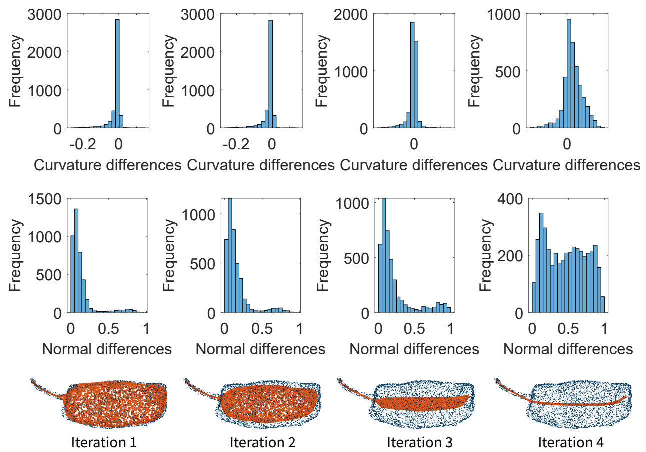

Fig. 2: Changes of normal vector and curvature differences within contraction process.The selected point cloud object is chilli.

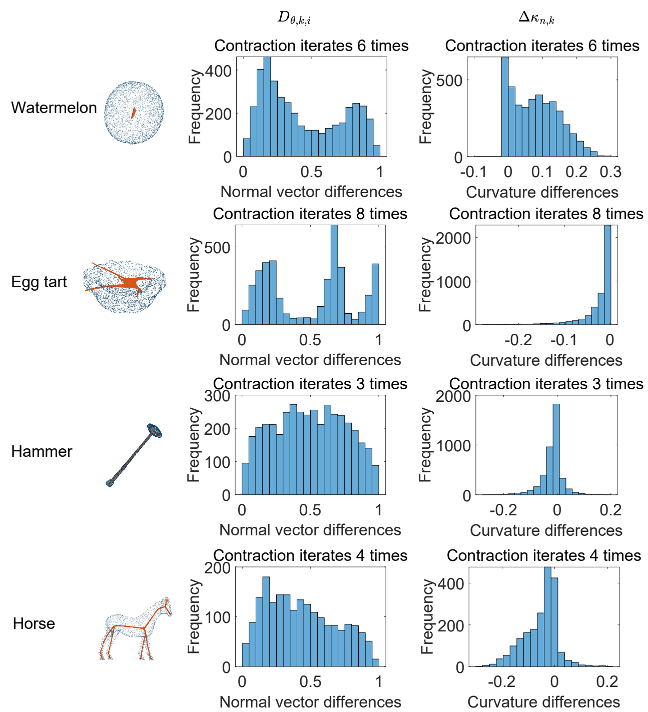

Fig. 3: Curvature vector and normal differences between the input and point cloud after contraction by Laplacian-based skeletonisation. The obtained contracted point sets (red) are expected to be bounded by the original point clouds (grey).

$\mathbf{n}_{k,i}$ and $\mathbf{n}_{0,i}$ are the surface normal vectors of i-th point in the point cloud contracted k times and the original point cloud respectively. And the curvature differences are defined by

$$\Delta\kappa_{n,k} = \kappa_{n,k} - \kappa_{n,0}, \tag{3}$$

where $\kappa_{n,k}$ and $\kappa_{n,0}$ are the normal curvature of i-th point in the point cloud contracted k times and the original point cloud respectively. Understanding these metrics aids in evaluating surface skeleton quality and contributes to generating superior curve skeletons for various applications.

III. Results and Discussion

In this section, we present the experimental results of our analysis on Laplacian-based skeletonisation conducted on real-scanned point cloud data. Using point cloud models from the OmniObject3D dataset [7], we analysed the classic Laplacian-based skeletonisation method [6].

As illustrated in Fig. 2, the distributions of normal vector differences and curvature differences evolve during the contraction process. The curvature difference (3) remains minimal and maintains a consistent shape throughout, while the normal vector difference (1) distribution gradually stabilises over iterations. This stabilisation aligns with the criteria defined in Section II-B, indicating the stable convergence of the chilli point cloud contraction. Fig. 3 further contrasts the distribution patterns of well-contracted point clouds with those of undesirable contraction results, clearly showing the stable and unstable patterns, respectively.

These findings highlight the importance of curvature and surface normal vector differences in indicating stable convergence during contraction and Laplacian-based skeletonization. The method continuously corrects distribution patterns to ensure stability throughout the process.

IV. Conclusions

This study explores how contraction methods, using Laplacian-based contraction as an example, achieve stable convergence in skeletonization. We identify stability indicators through changes in curvature and normal vectors. Future work will focus on scoring the quality of skeletons.

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