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Citation for final published version:

Shi, Haojie, Xiong, Houbo, Gan, Wei, Lin, Yumian and Guo, Chuangxin 2025. Fully distributed planning method for coordinated distribution and urban transportation networks considering three-phase unbalance mitigation. *Applied Energy* 377 (Part A) , 124449. 10.1016/j.apenergy.2024.124449

Publishers page: <http://dx.doi.org/10.1016/j.apenergy.2024.124449>

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Fully Distributed Planning Method for Coordinated Distribution and Urban Transportation Networks Considering Three-phase Unbalance Mitigation

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HIGHLIGHTS

- The temporal-space relationship between EV traffic flow and three-phase charging load are characterized and considered in the proposed planning model.
- DNR and charging flow optimization are coordinated and optimized to manage voltage imbalances effectively.
- A distributed framework utilizing an improved ADMM algorithm is developed to solve the proposed planning problem.
- The effectiveness of the proposed model and distributed framework is validated using real-world distribution and transportation networks of Zhejiang province of China.

ARTICLE INFO

Article history:

Received 00 December 00

Received in revised form 00 January 00

Accepted 00 February 00

Keywords:

Electric vehicles

Three-phase unbalanced distribution network

Nodal voltage imbalance

Urban transportation network

Fully distributed framework

ABSTRACT

Electric vehicles (EVs) are regarded as one of the silver bullets for addressing the global climate warming issue due to their zero-carbon emission. However, the proliferation of EVs increases the demand on distribution network (DN), which already face challenges due to three-phase asymmetrical loads and impedance mismatches. The combination of charging demand and traditional three-phase loads aggravated the nodal voltage imbalance within DN. To tackle these issues, this paper presents a coordinated planning model for a three-phase unbalanced distribution network (TUDN) and urban transportation network (UTN). The model optimizes the placement of fast charging stations (FCS), line expansions, and road enhancements, using a distribution network reconfiguration (DNR) strategy to improve charging traffic flow (CTF) and reduce voltage imbalances. To ensure privacy encryption, a fully distributed framework using the alternating direction method of multipliers (ADMM) is designed to solve this problem, where a two-layer iterative process (TIP) is further developed to improve the convergence of ADMM, taking into account the integer variables in the model. Numerical simulations on a modified IEEE 34-bus system and a real-world system in China demonstrate the model's effectiveness, achieving a 43.84% reduction in maximum voltage imbalance and outperforming other algorithms in iteration count and computation time.

1. Introduction

Electric vehicles (EVs) are recognized as a promising solution to the global climate crisis due to their zero carbon emissions. According to the China National Development and Reform Commission, new energy

vehicles are projected to account for about 20% of total vehicles sales by 2025 [1]. In response to this rapid growth in demand, the number of charging infrastructures in China is being significantly expanded, resulting in a tight interconnection between distribution network (DN) and urban transportation network (UTN) [2–5].



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Nomenclature

Abbreviations

EVs	Electric vehicles.
DN	Distribution network.
UTN	Urban transportation network.
FCS	Fast charge station.
CTF	Charging traffic flow.
TAP	Traffic assignment problem.
DNR	Distribution network reconfiguration.
TUPF	Three-phase unbalanced power flow.
TUDN	Three-phase unbalanced distribution network.
TCL	Three-phase charging load.
ADMM	Alternating direction method of multipliers.
TIP	Two-layer iterative process.

Indices and Sets

$q, \mathcal{Q}$	Index and set of nodes in UTN.
$a, \mathcal{A}$	Index and set of roads in UTN.
$t, \mathcal{T}$	Index and set of time periods.
$\mathcal{G}_{od}^{uEV}, \mathcal{G}_{od}^{EV}$	Sets of paths of O-D pair.
$od, \mathcal{O}\mathcal{D}$	Index and set of O-D pairs. y^{road}
$\varphi(p)$	Set of three-phase $\{a, b, c\}$
$\mathcal{P}(d), \mathcal{P}(g)$	Sets of DG nodes, substation nodes,
$\mathcal{P}(tr), \mathcal{P}$	transfer nodes, electric power nodes in TUDN.
$\mathcal{M}(j)$	Set of roads connected to electric node j .
$\mathcal{B}$	Set of branches.

Parameters

β	Proportion of electric vehicle fast charging users.
M_{ex}	Upper bound of expansion number of road.
$t_{a,0}$	Noncongested travel time on road a .
$C_{a,0} (\Delta C_a)$	Traffic flow capacity of existing road (expansion road).
$\lambda^{road}, \lambda^{line}$	Annual investment cost coefficient of road, electric.
λ^{fcs}	power lines and FCS.
$\kappa^{road}, \kappa^{line}$	Discount factor, which leverages long-term investment.
κ^{fcs}	cost and short-term operation cost.
r^{in}	Annual interest rate.
y^{line}, y^{fcs}	Expected work life of road, electric power lines and FCS.
c^{time}	Coefficient of travel time cost.
$P^{fcs, ch}$	Power capacity of each FCS.
$\lambda_{n,0}^{fcs}$	Number of existing FCS of road n .
κ^{gen}	3×1 cost vectors of active/reactive power outputs of substations.
$\bar{C}^{un}$	The penalty coefficient of voltage imbalance.
$\bar{U}, \underline{U}$	Upper/lower bound of voltage magnitude.
P_b^{min}	3×1 vectors of active power limits of DG at substation node f .

M	Big constant used in the big-M method.
$S^{gen, max}$	3×1 vectors of apparent power limits at substation node.
$e_{od, r, a}$	Correlation coefficient of alternative paths and roads, if alternative path r of O-D pair od passes road a , the value is 1, otherwise is 0.
$q_{od, t}$	Traffic demand of O-D pair od .
$\mathcal{N}^n$	Total number of electric power node.
U_0	three-phase reference voltage.
S_{ij}^{max}	3×1 vectors of apparent power limits of branch (i, j) .
P_t^{EV}	Charging rate of traffic flow.
S_{ij}^{max}	Maximum number of switching times.
Z_{ij}	3×3 complex matrix of the three-phase impedance of expanded branch (i, j) .
Z_{ij}^{exp}	3×3 complex matrix of the three-phase impedance of candidate branch (i, j) .
Z_{ij}^{exi}	3×3 complex matrix of the three-phase impedance of existing branch (i, j) .

Variables

$f_{od, r, t}^{EV} (f_{od, r, t}^{uEV})$	Traffic flow of VCU (VNCU) through candidate path r of O-D pair od .
$x_{a, t}^{EV}, x_{a, t}^{uEV}$	Traffic flow of EVCU (VNCU) on transportation road a .
$Z_{a, b}^{EV}, \mu_{a, t}^{EV}, \varphi_{od, t}^{EV}, \mu_{a, t}^{EV}, \lambda_{a, t}^{con}$	Binary variables for the number of transportation roads.
$\mu_{a, t}^{EV}, \varphi_{od, t}^{EV}, \lambda_{a, t}^{con}$	Dual variables of (2)–(6) in the traffic assignment model formulated as (1)–(7).
$f_{n, t}^{EV}$	Charging traffic flow assigned to road n .
$P_{n, d, t}^{ch}$	Charging power of phase d at electric power n .
δ_{ij}	Binary planning variable of branch (i, j) .
x_n^{fcs}	Planned number of FCS on electric power node n .
$P_{p, t}^{pre}$	3×1 vectors of forecast available power outputs of DG at node p .
$P_{p, t}^{open}$	3×1 vectors of actual power outputs of DG at node p .
$\chi_{ij, t}^{open}$	Binary variable that depicts open a switch of branch (i, j) , if open the branch, the value is 1, otherwise is 0.
$\chi_{ij, t}^{close}$	Binary variable that depicts close a switch of branch (i, j) , if close the branch, the value is 1, otherwise is 0.
$\mathcal{V}^{un}$	Voltage three-phase imbalance.
$\varepsilon_{a, t}, \mathcal{V}_{j, d, t}^{au}$	Auxiliary variables for the linearization of nonlinear term of $\lambda_{a, t}^{con} Z_{a, b}, \mathcal{V}^{un} U_{j, d, t}, \delta_{ij} P_{ij, t}, \delta_{ij} Q_{ij, t}$.
$U_{j, d, t}$	Squared voltage magnitude of phase d at electric power j .
$P_{ij, t} (Q_{ij, t})$	3×1 vectors of active (reactive) power flow that leaves node i toward node j .
$P_{p, t} (Q_{p, t})$	3×1 vectors of active (reactive) power calculated at node p .
$P_{g, t} (Q_{g, t})$	3×1 vectors of active (reactive) power outputs of substations.
$P_{j, t}^L (Q_{j, t}^L)$	3×1 vectors of active (reactive) regular loads at node j .
$P_{j, t}^{EV}$	3×1 vectors of charging loads at node j .
$U_{j, t}$	3×1 vectors of squared voltage magnitude at node j .

In response to the proliferation of EVs, many previous works have been devoted to the charging station planning. Refs. [6–7] proposed joint planning for EVs charging stations and distributed energy source. Ref. [8] improved the charging efficiency of stations in medium voltage DN. However, these studies [6–8] overlook traffic flow models and focus on planning charging stations at single sites. Some research publications have explored coordinated planning methods for DN and UTN, considering power flows in DN and traffic flows in UTN to optimize the location and size of EV charging stations [9–17]. Refs. [9–14] proposed coordinated planning methods between the two systems, and determined optimal expansion plans for power lines, sites of distributed generation (DG) and

fast charge stations (FCS). Ref. [15] considered the tight coupling of UTN, DN and natural gas distribution networks, establishing a coordinated planning model for urban energy infrastructures. Refs. [16–17] proposed planning strategies for coordinated distribution and urban transportation networks in a market-based environment. However, these studies [9–17] assumed DN as a three-phase balanced system, using a single-phase equivalent model to simplify computations. In fact, DN is usually unbalanced due to numerous three-phase asymmetric loads and impedance mismatches [18–20]. Therefore, with the increasing number of EVs, the combination of charging demand and traditional three-phase loads

aggregates the nodal voltage imbalances within DN, leading to system losses, feeder trips, and other grid security issues [21].

Past works [22–26] have proposed various approaches to address the three-phase imbalance problem caused by the development of EVs. In [22], a voltage control scheme for DN is proposed to realize energy loss minimization, peak load reduction and load unbalance management. In [23], vehicle-to-grid technology is applied to optimize the charging behaviors of EVs. In [24,25], EVs are incentivized to participate in solving nodal voltage imbalances issue through price incentives. In [26], a peer-to-peer trading strategy among FCS clusters is proposed. Refs. [22–25] focused on EVs in low-voltage areas or regional parking lots connected to DN through slow charging piles, although the optimal charging strategies proposed effectively mitigated the nodal voltage imbalances, they did not consider the coupling between DN and UTN, treating the charging load distribution of EVs as a predetermined boundary condition. Additionally, while ref. [26] introduced a transaction strategy considering the coupling between DN and UTN, it focused on cooperative scheduling rather than cooperative planning between the two systems.

In fact, reasonable planning for the placement of FCS, line expansions, and road enhancements can guide EV users to travel more efficiently, optimizing the charging traffic flow (CTF) by solving the traffic assigned problem (TAP), thereby mitigating the three-phase imbalance problem within DN. Refs. [3,6–17,27–29] proved that collaborative planning of coupled DN and UTN systems can enhance the operational safety of DN. Furthermore, several studies have demonstrated that distribution network reconfiguration (DNR) strategy can effectively address nodal voltage imbalance issue using a three-phase unbalanced power flow model (TUPF) [22,30–32].

In addition, DN and UTN are two independent systems, traditional centralized solution algorithms require both systems to share parameter information, which violates the privacy protection requirements. In this regard, ref. [9] utilized the Lagrangian relaxation (LR) method to protect the privacy of DN and UTN. LR is the most representative method where coupling constraints are relaxed, and Lagrangian multipliers are iteratively updated [33]. The alternating direction method of multipliers (ADMM) is an improved form of LR. Refs. [34,35] have demonstrated the superiority of ADMM for fast convergence in distributed optimization by introducing quadratic penalty terms into the traditional Lagrangian function. Ref. [36] established a distributed planning framework based on ADMM to obtain

optimal planning results for an electricity-gas coupled system with multi-energy hubs. However, the effectiveness of an ADMM-based distributed solving framework in coordinated planning model for distribution and urban transportation networks have been rarely discussed.

To address these challenges, this paper proposes a coordinated planning model for a three-phase unbalanced distribution network (TUDN) and UTN coupled system using a fully distributed planning method based on ADMM algorithm. The primary contributions of this work, as compared with existing studies (summarize in Table 1), are as follows:

- (1) Unlike current power and transportation network planning methods that overlook the nodal voltage three-phase imbalance issue in DNs, this paper proposes a coordinated planning model for a TUDN and UTN. The model employs TUPF to characterize the temporal-space relationship between traffic flow distribution and three-phase charging load (TCL) on electric power nodes. The model integrates the charging demand of EVs across each phase of the electric power nodes, converting the cumulative charging demand of EVs into TCL.
- (2) The proposed planning model simultaneously address the allocation of charging piles and the expansion strategy of three-phase power lines, as well as the expansion strategy of roads within UTN. It aims to guide EV users to travel efficiently and improve the asymmetrical distribution of TCL. Based on the traffic flow distribution among EV users, DNR and CTF optimization are coordinated and optimized to manage voltage imbalances effectively.
- (3) A fully distributed framework based on improved ADMM is established, using TCL in TUDN and CTF in UTN as decoupling information. This approach decomposes the coordinated planning model into two subproblems, enabling TUDN and UTN to solve regional issues independently. Furthermore, a two-layer iterative process (TIP) ensures convergence, taking into account the integer variables in the model. This method fixes the integer variables, representing expansion results in the regional subproblems during each ADMM iteration, and then iteratively performs improved ADMM loops.
- (4) The proposed coordinated planning method and distributed solving framework are validated using a real-world case in Zhejiang Province, China, confirming the effectiveness and reliability of the proposed planning approach.

Table 1 Comparative features of previous researches for coordinated distribution and urban transportation networks.

References	Coordinated planning model of TN and UTN	Considering TAP	Utilizing model	TUPF	Implement strategy	DNR	Considering privacy protection of DN and UTN
[2], [3]	✓	✓	✗		✗		✗
[9], [11]–[17]	✓	✓	✗		✗		✗
[10]	✓	✓	✗		✗		✓
[6]–[8], [27]–[29]	✓	✗	✗		✗		✗
[22], [26]	✗	✓	✓		✓		✗
[23]	✗	✗	✓		✗		✓
[24], [25]	✗	✗	✓		✗		✗
This work	✓	✓	✓		✓		✓

The remainder of this paper is organized as follows. Section 2 illustrates the mathematical models of the UTN and TUDN, and linearizes each model into a mixed-integer linear program (MILP). Section 3 describes the distributed framework and solution methodology. Section 4 and 5 focus on

the validation of the proposed model and solution methodology. This validation is conducted initially on a modified IEEE 34-node distribution network, then applied to a real-world system in Zhejiang Province, China. Finally, section 6 presents the conclusions of this paper.

2. Mathematical model of UTN and TUDN

In section 2.1 and 2.2, the models for UTN and TUDN are described, respectively. Section 2.3 introduces the transformation using primal-dual optimality conditions, and explains the application of the Big-M method and McCormick's envelope method. These techniques are utilized to linearize the nonlinear terms in the models.

Fig. 1 illustrates the coupled system for UTN and TUDN as described in this paper. The traffic flow is determined under the TAP operational constraints, after which the CTF is allocated to different roads based on the traffic flow distribution, supplied by the corresponding electric power nodes. Then, the cumulative charging demand of EVs on each road is converted into TCL of the corresponding power node. For instance, considering the geographical connection between a specific road a and an electric power node j , the node j supplies electricity to road a through installed FCS. Thus, the charging demand of EVs arriving at road a is equivalent to the TCL of node j .

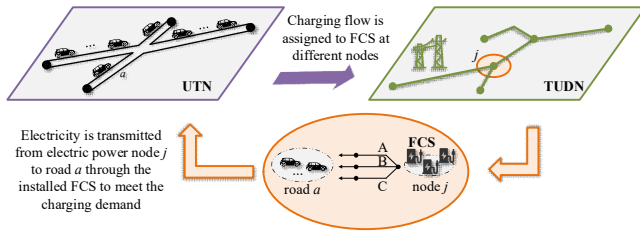


Fig. 1 The representation of the proposed coupled system of UTN and TUDN.

2.1. Urban transportation network model 公式节 1

The UTN is composed of sets of nodes and roads, denoted as $\mathcal{N}$ and $\mathcal{A}$, respectively. Assume at time t , vehicles leave from various origins and travel to their respective destinations, described as O-D pairs. The number of EVs in each O-D pair is represented as $q_{od,t}$. For a specific O-D pair $od \in \mathcal{OD}$, each traveler can choose a different path from a set of available paths to reach their destination, which is denoted by $\mathcal{G}_{od}^{uEV}$ ($\mathcal{G}_{od}^{EV}$). Each path is composed of several roads, each with corresponding parameters such as a capacity limit $C_{a,0}$ and a congested travel time $t_{a,0}$. The coupled constraints about the relationship between traffic flow on paths and roads are as follows. Constraints (1a)–(1b) limit the traffic flow balance for vehicle fast-charging users (VFU) and vehicle non-fast charging users (VNCU), respectively. Constraints (1c)–(1d) ensure that the total VCU and total VNCU across all candidate paths are equal to the CTF and non-CTF of O-D pair od , respectively. This model is a static traffic flow model, focusing primarily on the spatial coupling between traffic flow, routes flow and roads flow, without considering the coupling of traffic flow over time periods. This model has been widely used in planning problems [9, 10] and long-term operation problem [26]. Although semi-dynamic and dynamic traffic flow models have been proposed, they are more suitable for medium-term or long-term planning problems [37, 38] or short-term operational scheduling problems [39–41], with dynamic models typically taking the form of complex integrals. Therefore, considering the applicable time scale, the type of problem, and the complexity of solving complex, this paper adopts the static traffic model.

$$x_{a,t}^{EV} = \sum_{od \in \mathcal{OD}} \sum_{r \in \mathcal{G}_{od}^{EV}} e_{od,r,a} f_{od,r,t}^{EV}, \quad \forall a \in \mathcal{A} \quad (1a)$$

$$x_{a,t}^{uEV} = \sum_{od \in \mathcal{OD}} \sum_{r \in \mathcal{G}_{od}^{uEV}} e_{od,r,a} f_{od,r,t}^{uEV}, \quad \forall a \in \mathcal{A} \quad (1b)$$

$$\sum_{r \in \mathcal{G}_{od}^{EV}} f_{od,r,t}^{EV} = \beta q_{od,t}, \quad \forall od \in \mathcal{OD} \quad (1c)$$

$$\sum_{r \in \mathcal{G}_{od}^{uEV}} f_{od,r,t}^{uEV} = (1-\beta) q_{od,t}, \quad \forall od \in \mathcal{OD} \quad (1d)$$

$$x_{a,t}^{EV} \geq 0, x_{a,t}^{uEV} \geq 0, f_{od,r,t}^{EV} \geq 0, f_{od,r,t}^{uEV} \geq 0 \quad (1e)$$

The road expansion plan increases the traffic flow capacity of each road. Therefore, traffic flow is constrained not to exceed the expanded road capacity, which is formulated as: 公式节 2

$$C_a = C_{a,0} + \Delta C_a S_a, \quad \forall a \in \mathcal{A} \quad (2a)$$

$$x_{a,t}^{EV} + x_{a,t}^{uEV} \leq C_a, \quad \forall a \in \mathcal{A} \quad (2b)$$

$$S_a = \sum_{b=1}^{\lfloor \log_2 M_{\max} \rfloor + 1} 2^{b-1} Z_{a,b}, \quad \forall a \in \mathcal{A} \quad (2c)$$

Constraint (2a) describes the expanded road capacity, which is the sum of the existing and newly built lanes capacity, where S_a is the number of newly built lanes on road a . Constraint (2b) restricts the traffic flow on road a , ensuring it does not exceed the road's capacity. Constraint (2c) defines the computational formula for calculating the number of newly built lanes, where $\lfloor \cdot \rfloor$ represents the rounded-down calculation. For example, if the maximum expansion number of road a is 3, then the $\lfloor \log_2 M_{\max} \rfloor$ is equal to 2, thus the upper bound of b is 3.

In traffic theory, two common concepts are social optimal (SO) and user equilibrium (UE). The planning model of UTN based on SO aims to minimize the total extra travel time resulting from road congestion, which is formulated as: 公式节 3

$$\begin{cases} \min F_{so} = \sum_{t \in T} \sum_{a \in \mathcal{A}} t_{a,0} (x_{a,t}^{EV} + x_{a,t}^{uEV}) - \sum_{t \in T} \sum_{od \in \mathcal{OD}} t_{od,0} q_{od,t} \\ \text{s.t. (1a)–(1e), (2a)–(2c)} \end{cases} \quad (3)$$

In the SO model, the UTN requires a central management authority, to ensure that all users comply with central instructions, thereby achieving the goal of no road congestion and minimizing total social travel time. The UE model builds on the SO model by introducing a road congestion time variable, which acts as the dual multiplier for the capacity constraint (Eq.(2b)) in the original SO model. This extension modifies the UTN's objective function to reflect the difference between actual travel time (considering road congestion) and ideal commuting time. As a result, for certain OD pairs, the travel time for EV users on different paths becomes equivalent and no greater than the travel time required for other candidate paths. Interested readers can refer to Ref. [2], which provides a complete and detailed proof of this theory. Therefore, the UE principle [42] is applied in the proposed model in this paper, stated as:

$$\min C_{TN} = F_{TN}^{inv} + \frac{8760}{T} F_{TN}^{tra} \quad \text{公式节 4} \quad (4a)$$

$$F_{TN}^{inv} = \lambda^{\text{road}} \kappa^{\text{road}} \sum_{a \in \mathcal{A}} S_a, \quad \lambda^{\text{road}} = \frac{r^{\text{in}} (1 + r^{\text{in}})^{y^{\text{road}}}}{(1 + r^{\text{in}})^{y^{\text{road}}} - 1} \quad \text{公式节 5} \quad (5a)$$

$$F_{TN}^{tra} = c^{\text{time}} \left[\sum_{t \in T} \sum_{a \in \mathcal{A}} (t_{a,0} + \lambda_{a,t}^{\text{con}}) (x_{a,t}^{EV} + x_{a,t}^{uEV}) - \sum_{t \in T} \sum_{od \in \mathcal{OD}} t_{od,0} q_{od,t} \right] \quad (5b)$$

The objective function (4a) aims to minimize the total investment C_{TN} of UTN. The investment cost F_{TN}^{inv} (5a) represents the cost of road expansion. In constraint (5b), the first summation term represents the practical travel time, while the second summation term represents the ideal minimum travel time. The difference between these two terms represents the total extra travel time, which is then converted into operation cost F_{TN}^{tra} through a coefficient c^{time} representing the travel time cost.

The UTN planning model must first satisfy constraints (1a)–(1e), (2a)–(2c), which are considered the primal problem. Additionally, constraints (6a)–(6f) are a set of dual constraints realizing the transition from SO to UE, which constraint (6e) representing the strong duality condition. 公式节 6

$$\mu_{a,t}^{\text{EV}} - \lambda_{a,t}^{\text{con}} = t_{a,0}, \forall a \in \mathcal{A} \quad (6a)$$

$$\mu_{a,t}^{\text{uEV}} - \lambda_{a,t}^{\text{con}} = t_{a,0}, \forall a \in \mathcal{A} \quad (6b)$$

$$\varphi_{od,t}^{\text{EV}} - \sum_{a \in \mathcal{A}} e_{od,r,a} \mu_{a,t}^{\text{EV}} \leq 0, \forall od \in \mathcal{OD} \quad (6c)$$

$$\varphi_{od,t}^{\text{uEV}} - \sum_{a \in \mathcal{A}} e_{od,r,a} \mu_{a,t}^{\text{uEV}} \leq 0, \forall od \in \mathcal{OD} \quad (6d)$$

$$\beta \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} q_{od,t}^{\text{EV}} + (1-\beta) \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} q_{od,t}^{\text{uEV}} - \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} C_a \lambda_{a,t}^{\text{con}} = \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} t_{a,0} \left(x_{a,t}^{\text{EV}} + x_{a,t}^{\text{uEV}} \right) - \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} t_{od,0} q_{od,t} \quad (6e)$$

$$\lambda_{a,t}^{\text{con}} \geq 0, \forall a \in \mathcal{A} \quad (6f)$$

Where $\lambda_{a,t}^{\text{con}}$ is the dual variable of constraint (2b), $\mu_{a,t}^{\text{EV}}$, $\mu_{a,t}^{\text{uEV}}$, $\varphi_{od,t}^{\text{EV}}$, and $\varphi_{od,t}^{\text{uEV}}$ represent the dual variables of constraints (1a), (1b), (1c) and (1d), respectively.

The travel plans of EVs influence the traffic flow on different paths within UTN. Subsequently, the total charging demand is allocated to different roads along each path based on the determined traffic flow. –

$$f_{n,t}^{\text{EV}} \leq \sum_{od \in \mathcal{OD}} \sum_{r \in \mathcal{G}_{od}^{\text{EV}}} e_{od,r,n} f_{n,t}^{\text{EV}}, \forall n \in \mathcal{A} \quad (7a)$$

$$f_{od,r,t}^{\text{EV}} \leq \sum_{n \in \mathcal{A}} e_{od,r,n} f_{n,t}^{\text{EV}}, \forall od \in \mathcal{OD}, \forall r \in \mathcal{G}_{od}^{\text{EV}} \quad (7b)$$

$$\sum_{n \in \mathcal{A}} f_{n,t}^{\text{EV}} = \beta \sum_{od \in \mathcal{OD}} q_{od,t} \quad (7c)$$

Constraints (7a)–(7b) describe the temporal-space relationship between CTF assigned to a road and EV traffic flow passing through the path r that intersects the road. By combining constraint (7a) and constraint (7d), the total CTF allocated to each FCS is equal to the total charging demand of EVs. Constraint (7c) ensures that the sum of CTF on each road is equal to the total number of VCU. 公式节 8

The CTF assigned to each road is constrained by the charging capacity of the existing FCS and the newly built FCS, which is formulated as:

$$P_{n,d,t}^{\text{ch}} = \frac{1}{3} P_t^{\text{EV}} f_{n,t}^{\text{EV}}, \forall n \in \mathcal{A}, \forall d \in \varphi(p) \quad (8a)$$

$$\sum_{d \in \varphi(p)} P_{n,d,t}^{\text{ch}} \leq P_{n,0}^{\text{fcs, ch}} \left(x_{n,0}^{\text{fcs}} + x_n^{\text{fcs}} \right), \forall n \in \mathcal{A} \quad (8b)$$

Constraint (8a) establishes the relationship between the three-phase power output of FCS on each road and the CTF assigned to the corresponding road. Constraint (8b) restricts the total power output of FCS on each road, ensuring it does not exceed the sum of charging capacity of existing FCS and newly built FCS.

2.2. Three-phase unbalanced distribution network model

The planning model for TUDN aims to determine the optimal placement of FCS, and expansion strategy of power lines to accommodate the rapid growth of EV users' charging demand. Additionally, based on the charging demand distribution among EV users, DNR is employed to further alleviate the nodal voltage imbalances issue. Therefore, the objective function of TUDN is to minimize the total cost C_{DN} , which comprises the sum of investment cost, operation cost, and implementation cost of DG, which is formulated as:–

$$\min C_{\text{DN}} = F_{\text{DN}}^{\text{inv}} + F_{\text{DN}}^{\text{op}} + F_{\text{DN}}^{\text{dnr}} \quad \text{公式节 9} \quad (9)$$

$$\begin{cases} F_{\text{DN}}^{\text{inv}} = \lambda^{\text{line}} \kappa^{\text{line}} \sum_{(i,j) \in \mathcal{B}} \delta_{ij} + \lambda^{\text{fcs}} \kappa^{\text{fcs}} \sum_{n \in \mathcal{A}} x_n^{\text{fcs}} \\ \lambda^{\text{line}} = \frac{r^{\text{in}} (1 + r^{\text{in}})^{y^{\text{line}}}}{(1 + r^{\text{in}})^{y^{\text{line}}} - 1}, \lambda^{\text{fcs}} = \frac{r^{\text{in}} (1 + r^{\text{in}})^{y^{\text{fcs}}}}{(1 + r^{\text{in}})^{y^{\text{fcs}}} - 1} \end{cases} \quad \text{公式节 10} \quad (10a)$$

$$F_{\text{DN}}^{\text{op}} = \frac{8760}{T} \sum_{w \in \mathcal{P}(\mathcal{g})} \sum_{t \in \mathcal{T}} \left(\kappa^{\text{gen}} \right)^T \left(P_{w,t} + Q_{w,t} \right) + \sum_{p \in \mathcal{P}(\mathcal{d})} \sum_{t \in \mathcal{T}} C_{\text{cut}} \left(E_{3 \times 1} \right)^T \left(P_{p,t}^{\text{pre}} - P_{p,t} \right) \quad (10b)$$

$$F_{\text{DN}}^{\text{dnr}} = \frac{8760}{T} \kappa^{\text{act}} \sum_{t \in \mathcal{T}} \left(\chi_{ij,t}^{\text{open}} + \chi_{ij,t}^{\text{close}} \right) + C^{\text{un}} \nu^{\text{un}} \quad (10c)$$

The investment cost (10a) encompasses the total expenditure on expanding power lines and built new FCS. The operation cost (10b) includes the combined cost of energy supplied by substations and the penalty cost for curtaining renewable energy sources (RES), where $E_{3 \times 1}$ is a 3×1 vector with each element being 1. The implementation cost of DNR (10c) accounts for the action cost of power lines switch and the penalty cost of nodal voltage three-phase imbalances.

TUPF model is used to simulate the operation of the DN, which preserves the coupling relationship between each phase voltage through matrix H , with the explicit formulation provided as follows: 公式节 11

$$\sum_{(i,j) \in \mathcal{B}} P_{ij,t} - \sum_{(j,i) \in \mathcal{B}} P_{ji,t} + \sum_{p=j, p \in \mathcal{P}(\mathcal{d})} P_{p,t} + \sum_{g=j, p \in \mathcal{P}(\mathcal{g})} P_{g,t} = P_{j,t}^{\text{L}} + P_{j,t}^{\text{EV}}, \forall i \in \mathcal{M}(j) \quad (11a)$$

$$\sum_{(i,j) \in \mathcal{B}} Q_{ij,t} - \sum_{(j,i) \in \mathcal{B}} Q_{ji,t} + \sum_{p=j, p \in \mathcal{P}(\mathcal{d})} Q_{p,t} + \sum_{g=j, p \in \mathcal{P}(\mathcal{g})} Q_{g,t} = Q_{j,t}^{\text{L}}, \forall i \in \mathcal{M}(j) \quad (11b)$$

$$\left(P_{ij,t} \circ P_{ij,t} \right) + \left(Q_{ij,t} \circ Q_{ij,t} \right) \leq x_{ij,t} \left(S_{ij}^{\text{max}} \circ S_{ij}^{\text{max}} \right), \forall (i,j) \in \mathcal{B}, \forall (j,i) \in \mathcal{B} \quad (11c)$$

$$P_p^{\text{min}} \leq P_{p,t} \leq P_{p,t}^{\text{pre}}, \forall p \in \mathcal{P}(\mathcal{d}) \quad (11d)$$

$$\left(P_{p,t} \circ P_{p,t} \right) \oslash \left(P_{p,t} \circ P_{p,t} + Q_{p,t} \circ Q_{p,t} \right) \geq \left(\lambda^{\text{min}} \right)^2 E_{3 \times 1}, \forall p \in \mathcal{P}(\mathcal{d}) \quad (11e)$$

$$\left(P_{g,t} \circ P_{g,t} \right) + \left(Q_{g,t} \circ Q_{g,t} \right) \leq \left(S^{\text{gen,max}} \circ S^{\text{gen,max}} \right), \forall g \in \mathcal{P}(\mathcal{g}) \quad (11f)$$

$$U_{j,t} \leq U_{i,t} - \tilde{Z}_{ij} S_{ij,t}^* - \tilde{Z}_{ij}^* S_{ij,t} + M(1 - x_{ij,t}) E_{3 \times 1}, \forall (i,j) \in \mathcal{B}, \forall (j,i) \in \mathcal{B}, \forall i \in \mathcal{M}(j) \quad (11g)$$

$$U_{j,t} \geq U_{i,t} - \tilde{Z}_{ij} S_{ij,t}^* - \tilde{Z}_{ij}^* S_{ij,t} - M(1 - x_{ij,t}) E_{3 \times 1}, \forall (i,j) \in \mathcal{B}, \forall (j,i) \in \mathcal{B}, \forall i \in \mathcal{M}(j) \quad (11h)$$

$$U_{i,t} = U_0 \circ U_0, \forall i \in \mathcal{P}(\mathcal{g}) \quad (11i)$$

$$(U_{\text{min}} \circ U_{\text{min}}) \leq U_{i,t} \leq (U_{\text{max}} \circ U_{\text{max}}), \forall i \in \mathcal{P} / \mathcal{P}(\mathcal{g}) \quad (11j)$$

$$Z_{ij} = Z_{ij}^{\text{exi}} + \delta_{ij} (Z_{ij}^{\text{exp}} - Z_{ij}^{\text{exi}}), \forall (i,j) \in \mathcal{B}, \forall (j,i) \in \mathcal{B} \quad (11k)$$

$$H = \begin{bmatrix} 1 & e^{-j2\pi/3} & e^{j2\pi/3} \\ e^{j2\pi/3} & 1 & e^{-j2\pi/3} \\ e^{-j2\pi/3} & e^{j2\pi/3} & 1 \end{bmatrix} \quad (11l)$$

$$-\mathcal{V}_{\text{un}} \leq \frac{U_{j,d,t} - U_{j,t}^{\text{base}}}{U_{j,t}^{\text{base}}} \leq \mathcal{V}_{\text{un}}, \forall d \in \varphi(p), \forall j \in \mathcal{P} \quad (11m)$$

$$U_{j,t}^{\text{base}} = \frac{1}{3} \sum_{d \in \varphi_p} U_{j,d,t}, \forall j \in \mathcal{P} \quad (11n)$$

Constraints (11a)–(11b) describe the TUPF balance conditions, where $P_{j,t}^{\text{EV}} = [P_{j,a,t}^{\text{EV}}, P_{j,b,t}^{\text{EV}}, P_{j,c,t}^{\text{EV}}]^T$. Constraint (11c) ensures that the apparent

power flow on branch (i, j) does not exceed the allowed limit, with $\circ$ indicating the Hadamard product calculation. Constraint (11d) ensures that the available power outputs of DG do not exceed the forecasted values. Constraint (11e) defines the relationship between active and reactive power

outputs of DG through a fix power factor, expressed as $\lambda^{\min}$. Constraint (11f) sets the power output limit of substation. Constraints (11g)–(11h) describe the voltage drop for branch (i, j) using the big-M method, where

$\tilde{Z}_{ij}$ is the Hadamard product of matrices H and Z_{ij} , and $(\cdot)^*$ is the complex conjugation. Constraint (11i) sets the substation voltage as the reference value, while constraint (11j) limits the boundaries of nodal voltage magnitude. Constraint (11k) defines the three-phase impedance of new power lines after expansion, if δ_{ij} is 1, the impedance of the new

power line is $Z_{ij}^{\exp}$, otherwise, it equals to the impedance of the existing power line. The constant matrix H is specified in (11l). According to Ref. [18], it is assumed that the three-phase voltages at each node are nearly balanced $\frac{U_j^a}{U_j^b} \approx \frac{U_j^b}{U_j^c} \approx \frac{U_j^c}{U_j^a} \approx e^{j2\pi/3}$, resulting in matrix H . Constraints (11m)–(11n) limit the voltage three-phase imbalance, with the maximum imbalance $\mathcal{V}_{in}$ set as 6% [44, 45].

DN typically maintain a radial topology to reduce system losses and avoid feeder trip phenomenon [46]. Thus, the distribution system reconfiguration plan must satisfy sufficient constraints to ensure a radial topology in TUDN operation, which are formulated as:...

$$\sum_{(i,j) \in \mathcal{B}} x_{ij,t} = \mathcal{N}^n - 1 - \sum_{k \in \mathcal{P}(\text{tr})} (1 - u_{k,t}) \quad (12a)$$

$$x_{ij,t} + x_{ji,t} \leq 1, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B} \quad (12b)$$

$$x_{ij,t} \leq u_{j,t}, x_{ji,t} \leq u_{j,t}, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B}, \forall i \in \mathcal{M}(j) \quad (12c)$$

$$\sum_{(i,j) \in \mathcal{B}} x_{ij,t} + \sum_{(j,i) \in \mathcal{B}} x_{ji,t} \geq 2u_{j,t} \quad (12d)$$

$$\sum_{(i,j) \in \mathcal{B}} r_{ij,t} + \sum_{(j,i) \in \mathcal{B}} r_{ji,t} + g_{j,t} = \frac{1}{\mathcal{N}^n}, \forall j \in \mathcal{P} / \mathcal{P}(\text{tr}) \quad (12e)$$

$$\sum_{(i,j) \in \mathcal{B}} r_{ij,t} + \sum_{(j,i) \in \mathcal{B}} r_{ji,t} + g_{j,t} = \frac{1}{\mathcal{N}^n} u_{j,t}, \forall j \in \mathcal{P}(\text{tr}) \quad (12f)$$

$$0 \leq g_{j,t} \leq 1, \forall j \in \mathcal{P}(\text{g}) \quad (12g)$$

$$g_{j,t} = 0, j \in \mathcal{P} / \mathcal{P}(\text{g}) \quad (12h)$$

$$0 \leq r_{ij,t} \leq x_{ij,t}, \forall (i, j) \in \mathcal{B} \quad (12i)$$

$$(x_{ij,t} + x_{ji,t}) - (x_{ij,t-1} + x_{ji,t-1}) \leq \chi_{ij,t}^{\text{close}}, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B} \quad (12j)$$

$$(x_{ij,t-1} + x_{ji,t-1}) - (x_{ij,t} + x_{ji,t}) \leq \chi_{ij,t}^{\text{open}}, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B} \quad (12k)$$

$$\chi_{ij,t}^{\text{close}} + \chi_{ij,t}^{\text{open}} \leq 1, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B} \quad (12l)$$

$$\sum_{t \in \mathcal{T}} (\chi_{ij,t}^{\text{open}} + \chi_{ij,t}^{\text{close}}) \leq 2S_{ij}^{\text{max}}, \forall (i, j) \in \mathcal{B}, \forall (j, i) \in \mathcal{B} \quad (12m)$$

Constraint (12a) ensures a radial topology by restricting that the total number of nodes in operation must equal to the total number of nodes in operation, where $u_{k,t}$ is a binary variable representing the operational state of a transfer node, if the transfer node k is used, the value is 1, otherwise, it is 0. It is noted that a transfer node is defined as a node with no DG or load, and the related proof is provided in [46]. Constraint (12b) ensures there is only one power flow direction on each branch. Constraints (12c)–(12d) specify the state of power lines connected to a corresponding transfer node, if a transfer node is not used, all adjusted power lines are disconnected, otherwise at least two adjusted power lines must be in operation. Constraints (12e)–(12i) introduce a fictitious flow to ensure a radial topology. Constraints (12e)–(12f) enforce network connectivity, where $r_{ij,t}$ is the fictitious flow associated with branch (i, j) , also referred to as a commodity [46,48]. Constraints (12g)–(12h) limit the commodity to be generated only at the substation node, so the $g_{j,t}$ is 0 if the node j has no

substation. Constraint (12i) illustrates the correlation between power flow and fictitious power flow. Constraints (12j)–(12m) limit the total maximum number of switch operations within the entire period.

2.3. Linearizing the nonlinear term

1) Linearization of UTN

Objective function (5b) includes a nonlinear term representing the product of the dual variable and traffic flow $\lambda_{a,t}^{\text{con}} (x_{a,t}^{\text{EV}} + x_{a,t}^{\text{uEV}})$. This term is transformed into $\lambda_{a,t}^{\text{con}} C_a$ using the Karush-Kuhn-Tucker (KKT) criteria combined with constraint (2b). According to constraints (2a) and (2c), there is a bilinear term $\lambda_{a,t}^{\text{con}} Z_{a,b}$ in the expanded expression of $\lambda_{a,t}^{\text{con}} C_a$ which is also in (6e). The nonlinear term $\lambda_{a,t}^{\text{con}} Z_{a,b}$ can be linearized into $\varepsilon_{a,b,t}$ using the Big-M method. Thus, the objective function (5b) and constraint (6e) can be substituted by expression (13a) and constraint (13b), respectively, where $\varepsilon_{a,b,t}$ is limited by a set of linear inequalities as shown in (13c)...

$$F_{\text{TN}}^{\text{tra}} = c^{\text{time}} \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} \left[t_{a,0} (x_{a,t}^{\text{EV}} + x_{a,t}^{\text{uEV}}) + C_{a,0} \lambda_{a,t}^{\text{con}} \right] + \Delta C_a \sum_{b=1}^{\lfloor \log_2 M_{\text{ex}} \rfloor + 1} 2^{b-1} \varepsilon_{a,b,t} \quad (13a)$$

$$c^{\text{time}} \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} t_{od,0} q_{od,t} + \beta \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} q_{od,t} \phi_{od,t}^{\text{EV}} + (1 - \beta) \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} q_{od,t} \phi_{od,t}^{\text{uEV}} - \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} C_{a,0} \lambda_{a,t}^{\text{con}} - \Delta C_a \sum_{b=1}^{\lfloor \log_2 M_{\text{ex}} \rfloor + 1} 2^{b-1} \varepsilon_{a,b,t} = \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}} t_{a,0} (x_{a,t}^{\text{EV}} + x_{a,t}^{\text{uEV}}) - \sum_{t \in \mathcal{T}} \sum_{od \in \mathcal{OD}} t_{od,0} q_{od,t} \quad (13b)$$

$$|\varepsilon_{a,b,t} - \lambda_{a,t}^{\text{con}}| \leq M(1 - Z_{a,b}), |\varepsilon_{a,b,t}| \leq M \cdot Z_{a,b} \quad (13c)$$

2) Linearization of TUDN

Eq. (11c) and (11f) are circular constraints, which can be linearized using the quadratic constraint linearization method illustrated in Fig. A1. In this paper, (11c) and (11f) are substituted by two squared constraints (13d) and (13f), respectively.

$$\begin{cases} -S_{ij}^{\text{max}} \leq P_{ij,t} \leq S_{ij}^{\text{max}}, -S_{ij}^{\text{max}} \leq Q_{ij,t} \leq S_{ij}^{\text{max}} \\ -\sqrt{2}x_{ij,t}S_{ij}^{\text{max}} \leq P_{ij,t} + Q_{ij,t} \leq \sqrt{2}x_{ij,t}S_{ij}^{\text{max}} \\ -\sqrt{2}x_{ij,t}S_{ij}^{\text{max}} \leq P_{ij,t} - Q_{ij,t} \leq \sqrt{2}x_{ij,t}S_{ij}^{\text{max}} \\ \forall (i, j) \in \mathcal{B} \end{cases} \quad (13d)$$

$$\begin{cases} -S^{\text{gen,max}} \leq P_{g,t} \leq S^{\text{gen,max}}, -S^{\text{gen,max}} \leq Q_{g,t} \leq S^{\text{gen,max}} \\ -\sqrt{2}S^{\text{gen,max}} \leq P_{g,t} + Q_{g,t} \leq \sqrt{2}S^{\text{gen,max}} \\ -\sqrt{2}S^{\text{gen,max}} \leq P_{g,t} - Q_{g,t} \leq \sqrt{2}S^{\text{gen,max}} \\ \forall (i, j) \in \mathcal{B}, g \in \mathcal{P}(\text{g}) \end{cases} \quad (13e)$$

Constraints (11g)–(11h) involve complex product calculations and result in nonlinear terms when combined with constraint (11k). The Big-M method is applied to linearize these constraints, as shown in Eq. (A1)–Eq. (A8) in Appendix A.

Constraint (11e) is a nonlinear term representing the division between coupled variables, can be linearized into constraint (13f) using the inequality transformation method.

$$-\frac{\sqrt{1-(\lambda^{\min})^2}}{\lambda^{\min}} P_{p,t} \leq Q_{p,t} \leq \frac{\sqrt{1-(\lambda^{\min})^2}}{\lambda^{\min}} P_{p,t}, \forall p \in \mathcal{P}(d) \quad (13f)$$

Constraint (11m) is a nonlinear term similar to constraint (11e), which can be transformed into constraint (13g) after combining with constraint (11n). However, it includes a nonlinear term representing the production of two continuous variables $\mathcal{V}^{\text{un}} U_{j,d,t}$, where $\mathcal{V}^{\text{un}} \in [0, 0.06]$, $U_{j,d,t} \in [0.95, 1.05] U_0$. For the product of two continuous variables limited by upper and lower boundaries, the term $\mathcal{V}^{\text{un}} U_{j,d,t}$ can be substituted by a set of linear inequality (13h) through introducing an auxiliary variable $\mathcal{V}_{j,d,t}^{\text{au}}$ based on McCormick's envelope method [49]. Therefore, constraint (11m) can be substituted by (13i).

$$-\frac{1}{3} \mathcal{V}^{\text{un}} \sum_{d \in \phi_p} U_{j,d,t} \leq U_{j,d,t} - U_{j,t}^{\text{base}} \leq \frac{1}{3} \mathcal{V}^{\text{un}} \sum_{d \in \phi_p} U_{j,d,t}, \forall j \in \mathcal{P} \quad (13g)$$

$$\begin{cases} \mathcal{V}_{j,d,t}^{\text{au}} - \mathcal{V}^{\min} U_{j,d,t} - U^{\min} \mathcal{V}^{\text{un}} + U^{\min} \mathcal{V}^{\min} \geq 0 \\ -\mathcal{V}_{j,d,t}^{\text{au}} + \mathcal{V}^{\max} U_{j,d,t} - U^{\min} \mathcal{V}^{\max} + U^{\min} \mathcal{V}^{\text{un}} \geq 0 \\ -\mathcal{V}_{j,d,t}^{\text{au}} + U^{\max} \mathcal{V}^{\text{un}} - U^{\max} \mathcal{V}^{\min} + \mathcal{V}^{\min} U_{j,d,t} \geq 0 \\ \mathcal{V}_{j,d,t}^{\text{au}} + U^{\max} \mathcal{V}^{\max} - U^{\max} \mathcal{V}^{\min} - \mathcal{V}^{\max} U_{j,d,t} \geq 0 \end{cases} \quad (13h)$$

$$-\frac{1}{3} \sum_{d \in \phi_p} \mathcal{V}_{j,d,t}^{\text{au}} \leq U_{j,d,t} - U_{j,t}^{\text{base}} \leq \frac{1}{3} \sum_{d \in \phi_p} \mathcal{V}_{j,d,t}^{\text{au}} \quad (13i)$$

Where $\mathcal{V}^{\min}$ and $\mathcal{V}^{\max}$ are equal to 0 and 0.06, respectively. $U^{\min}$ and $U^{\max}$ are equal to $0.95U_0$ and $1.05U_0$, respectively.

Therefore, the proposed coordinated planning model has been transformed into a MILP problem. Due to space limitation and to highlight key points, this paper does not consider uncertainty. However, in long-term planning problems, uncertainty can be addressed in two ways: one that improving the prediction accuracy of uncertain variables or through clustering algorithms. Both methods can obtain highly accurate predictable value as boundary conditions for planning. One that improving the model itself. The proposed coordinated planning model is essentially a MILP problem, where the integer variables include line expansion results, the road expansion number, the new installed FCS number, the switching states of tie-switches, and the traffic flow of each road. The continuous variables include the CTF on each road, line power flow, et al. If uncertainty is considered, this model can be reconstructed as a two-stage robust optimization problem or a stochastic optimization problem. In the first stage, decisions to determine integer variables representing planning results are made. In the second stage, integer variables are fixed to determine the optimal power flow and CTF at the operational level. In robust optimization, the second stage aims to find the worst-case scenario considering uncertainty to make optimal decisions. In stochastic optimization, the second stage aims to optimize the mathematical expectation of multiple scenarios.

3. Solution methodology

In this section, we illustrate the solution methodology for autonomously solving each subproblem. Firstly, the consensus constraint is presented in section 3.1, explaining the coupling variables between TUDN and UTN. Then, a fully distributed framework based on an improved ADMM is established in section 3.2. To enhance the convergence property of ADMM, the TIP is formulated in section 3.3. Finally, the solution procedures are detailed in section 3.4.

3.1. Consensus constraint between TUDN and UTN

Power nodes in TUDN transmit electricity to the corresponding roads by installed FCS. Thus, the power demand of EVs from each power node in TUDN is equal to the power output of the FCS on the corresponding road, which is determined by the CTF assigned to that road. The mathematical model is formulated as: 公式节 14

$$P_{j,d,t}^{\text{EV}} = P_{n,d,t}^{\text{ch}}, \forall d \in \phi(p), \forall n \in \mathcal{J}(j) \quad (14a)$$

Where $\mathcal{J}(j)$ represents set of roads supplied by power node j .

3.2. Fully distributed framework base on improved ADMM

To provide a clearer understanding of the distributed framework, we first establish the centralized solution algorithm as shown in Eq. (14b). This model is a MILP, which requires TUDN and UTN to share their respective information.

$$\begin{cases} \min C = C_{\text{TN}} + C_{\text{DN}} \\ \text{s.t. (1a)–(1e), (2a)–(2c), (6a)–(6d), (6f), (7a)–(7c), (8a)–(8b) \\ \quad (11a)–(11b), (11d), (11i)–(11l), (11n), (12a)–(12m) \\ \quad (13b)–(13e), (13h)–(13i), (A5)–(A8), (14a) \end{cases} \quad (14b)$$

To solve the proposed coordinated planning model in a decentralized way, the fully distributed framework based on an improved ADMM is established. This framework relaxes the consensus constraint (14a) via augmented Lagrangian functions. Hence, the planning model is decomposed into TUDN and UTN subproblems. After relaxation, TUDN and UTN solve their planning models locally, then exchange the coupling variables and start next solution iteration. The optimal planning results are obtained when the differences between the coupling variables are small enough. The optimization problems for TUDN and UTN at the m th iteration are formulated as Eq. (14c) and (14d), respectively.

$$\begin{cases} \min C = C_{\text{UTN}} + \sum_{t \in \mathcal{T}} \sum_{d \in \phi(p)} \sum_{n \in \mathcal{A}} \left[\lambda_{\text{TN},n,d,t}^m \left(P_{n,d,t}^{\text{ch}} - \overline{P_{n,d,t}^{\text{co}}}^m \right) + 0.5 \rho^m \left(P_{n,d,t}^{\text{ch}} - \overline{P_{n,d,t}^{\text{co}}}^m \right)^2 \right] \\ \text{s.t. (1a)–(1e), (2a)–(2c), (6a)–(6d), (6f), (7a)–(7c) \\ \quad (8a)–(8b), (13b)–(13c) \end{cases} \quad (14c)$$

$$\begin{cases} \min C = C_{\text{TPUD}} + \sum_{t \in \mathcal{T}} \sum_{d \in \phi(p)} \sum_{j \in \mathcal{P}} \left[\lambda_{\text{DN},j,d,t}^m \left(P_{j,d,t}^{\text{EV}} - \overline{P_{j,d,t}^{\text{co}}}^m \right) + 0.5 \rho^m \left(P_{j,d,t}^{\text{EV}} - \overline{P_{j,d,t}^{\text{co}}}^m \right)^2 \right], \forall n \in \mathcal{J}(j) \\ \text{s.t. (11a)–(11b), (11d), (11i)–(11l), (11n), (12a)–(12m) \\ \quad (13d)–(13e), (13h)–(13i), (A5)–(A8) \end{cases} \quad (14d)$$

Where $\lambda_{\text{TN},n,d,t}^m$ and $\lambda_{\text{DN},j,d,t}^m$ are the Lagrangian multipliers. ρ^m is the penalty factor in Lagrangian function. $\overline{P_{n,d,t}^{\text{co}}}^m$ is average values of coupling variables based on the results obtained after the $m-1$ th iteration, calculated as (14e).

$$\overline{P_{n,d,t}^{\text{co}}}^m = 0.5(P_{j,d,t}^{\text{EV},m} + P_{n,d,t}^{\text{ch},m}), \forall j \in \mathcal{P}, \forall d \in \phi(p), \forall n \in \mathcal{J}(j) \quad (14e)$$

Additionally, the coordinated results of TUDN and UTN is realized by updating the Lagrangian multipliers using the latest coupling variables values which is given as (14f).

$$\begin{cases} \lambda_{TN,n,d,t}^{m+1} = \lambda_{TN,n,d,t}^m + \rho^{m+1} (P_{n,d,t}^{ch,m} - \overline{P_{n,d,t}^{co}}^{m+1}) \\ \lambda_{DN,j,d,t}^{m+1} = \lambda_{DN,j,d,t}^m + \rho^{m+1} (P_{j,d,t}^{EV,m} - \overline{P_{n,d,t}^{co}}^{m+1}) \end{cases}, \forall j \in \mathcal{P}, \forall n \in \mathcal{J}(j) \quad (14f)$$

The iteration process continues until primal residual ($\sigma_{P,TN}$ and $\sigma_{P,DN}$) and dual residual (σ_D) are within tolerance, as in (14g).

$$\begin{cases} \sigma_{P,TN} = \left\| P_{n,d,t}^{ch,m} - \overline{P_{n,d,t}^{co}}^{m+1} \right\|_2^2, \sigma_{P,DN} = \left\| P_{j,d,t}^{EV,m} - \overline{P_{n,d,t}^{co}}^{m+1} \right\|_2^2 \\ \sigma_D = \left\| \rho^{m+1} \left(\overline{P_{n,d,t}^{co}}^m - \overline{P_{n,d,t}^{co}}^{m+1} \right) \right\|_2^2, \\ \sigma_{P,TN} \leq \varepsilon_P, \sigma_{P,DN} \leq \varepsilon_P, \sigma_D \leq \varepsilon_D, \forall n \in \mathcal{J}(j) \end{cases} \quad (14g)$$

Where $\sigma_{P,TN}$ and $\sigma_{P,DN}$ are sets of primal residuals, ensuring that coupling variables are close enough. σ_D is the dual residual, ensuring the stability of the results. ε_P and ε_D are primal residual and dual residual, respectively.

For standard ADMM, the penalty factor is fixed as a constant, which slows down the convergence process. Refs. [50, 51] set the penalty factor as 15000 and 0.01, respectively, to achieve convergence. However, determining the suitable order of magnitude in advance is challenging. Therefore, a dynamic updating strategy for penalty factor [52,53] is utilized to improve the convergence, which is shown as:

$$\rho^{m+1} = \begin{cases} \mu^{up} \rho^m & \frac{\sigma_{P,TN}}{\varepsilon_P} > \eta \frac{\sigma_D}{\varepsilon_D} \\ \rho^m / \mu^{down} & \frac{\sigma_D}{\varepsilon_D} > \eta \frac{\sigma_{P,TN}}{\varepsilon_P} \\ \rho^m & \text{otherwise} \end{cases} \quad (14h)$$

Where η , μ^{up} , and μ^{down} are all parameters greater than 1. η is the threshold for increasing or decreasing the penalty multiplier, μ^{up} and μ^{down} are the expansion multiplier and reduction multiplier, respectively.

3.3. Two-Layer Iterative Process

Although the distributed framework described in section 3.2 can solve the TUDN and UTN in a distributed way, its convergence cannot be guaranteed due to the presence of integer variables representing the planning results in this model. To ensure computational tractability, a two-layer iterative process (TIP) is applied in this section. During this process, the inner layer executes the improved ADMM to solve TUDN and UTN, which are linear problems in this layer because the integer variables are fixed. The outer layer determines the planning results with the coupling variables fixed. When the planning decisions obtained from the two successive outer layer iterations remain the same, the following inner ADMM loops will solve the same problem, and TIP stops. Additionally, the convergence of TIP can be proven as Eq. (B1)-(B4) in Appendix B.

3.4. Solution procedures

The flowchart of the proposed distributed solution technique based on ADMM and TIP is illustrated in Fig. 2.

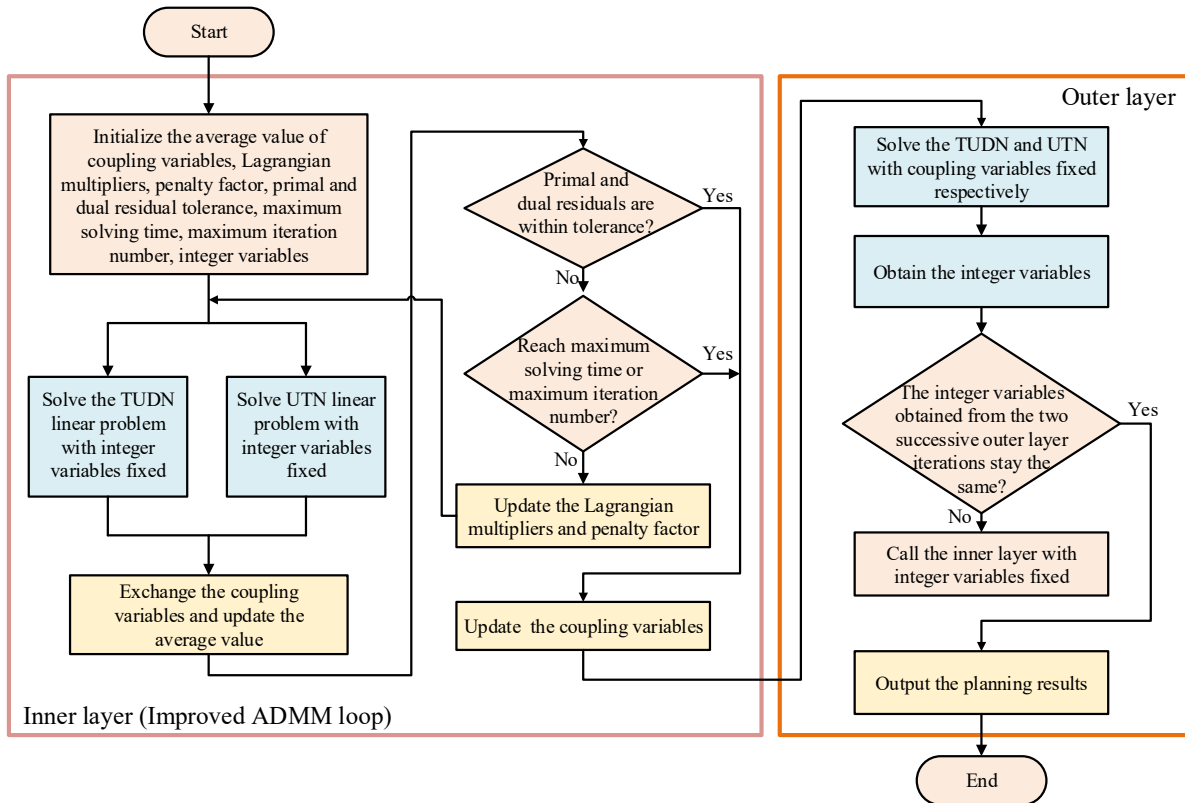


Fig. 2 Flowchart of the proposed solution methodology.

4. Case study I: coupled modified IEEE 34-bus distribution and 12-node transportation system

4.1. Simulation settings

In this section, the proposed method is tested on a coupled system comprising a modified IEEE 34-bus distribution test system [57] and a transportation test system with 20-road [10]. Fig. C1 displays the topological information of the revised distribution system with 5 initially opened tie-switches, along with the transportation system. Single-phase photovoltaic units (PV) are installed in electric nodes 3 and 13 while single-phase wind turbines (WT) are installed in electric nodes 19 and 22. The output curve of PV and WT are illustrated in Fig. 3. The TUDN needs handle both traditional three-phase loads and charging loads from EV uses. The parameters for the traditional loads include the power sequence reference value and the relative values of each nodal three-phase load, as shown in Fig. 4. The parameters for the charging loads include the total charging load and average charging load at each time, as shown in Fig. C2. Table 2 specifies the position of each system parameter within the relevant

tables and figures. The optimization problem is solved using Gurobi Optimizer 10.0.0 on a server with an Intel Core i5 CPU and 16 GB of RAM.

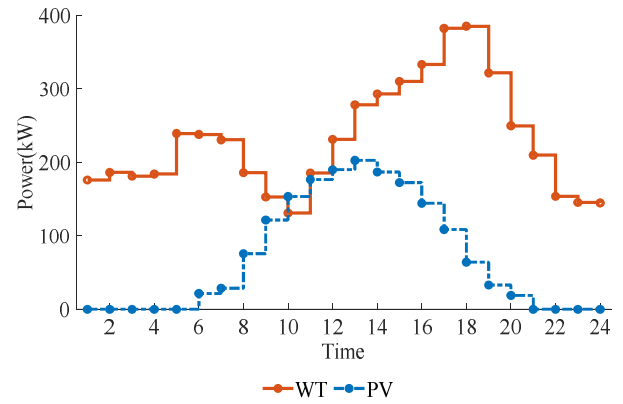


Fig. 3 DG power output curves

Table 2 Simulation system in Case study I

Indices	Parameters	Position
Road parameters	topological information	Fig. C1
	power capacity of each FCS, investment cost of FCS	Table C1
	noncongested travel time, initial FCS distribution, initial road capacity, expansion road capacity, investment cost of road expansion	Ref. [58]
EV users parameters	travel plans	Ref. [58]
	average charging power at each time	Fig. C2
	coefficient of travel time cost, proportion of electric vehicle fast charging users	Table C2
Power lines parameters	topological information	Fig. C1
	original three-phase impedance, candidate three-phase impedance, expansion cost	Ref. [58]
Operation parameters within TPUD	DG power curve	Fig. 3
	traditional power load	Fig. 4
	coefficient of substation power output, penalty coefficient of RES curtailment, penalty coefficient of voltage imbalance, tie-switches act cost, voltage grade	Table C3
	initial penalty multiplier, multiplier expansion or contraction factor, threshold for increasing or decreasing the penalty multiplier	Table C4

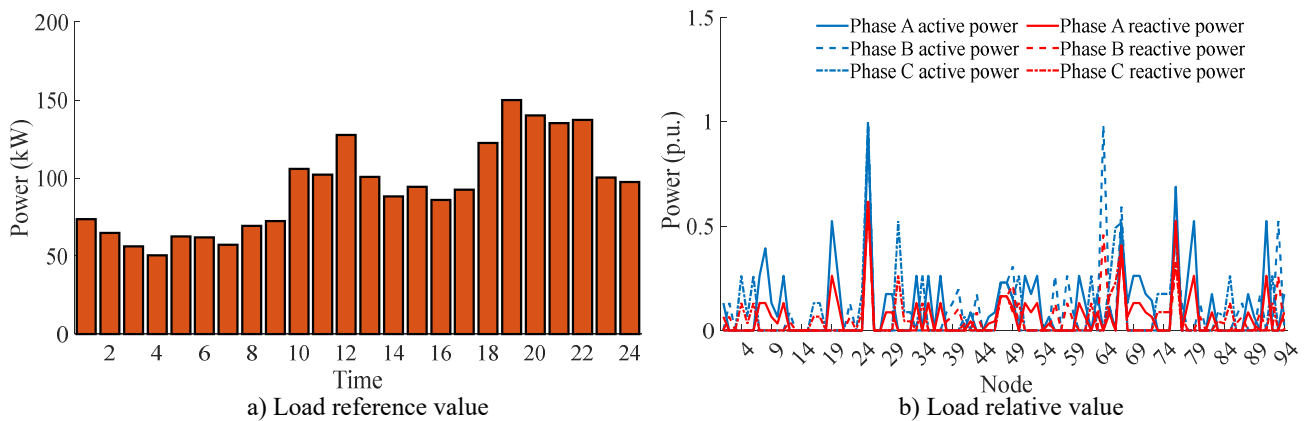


Fig. 4 Curves of load reference and three-phase load relative value in Case study I

Five cases are designed to validate the proposed coordinated planning model and distributed solving framework. The specific verification goals

and the characteristics of different cases are summarized in Table 3 and Table 4.

Table 3 Verification goals of different cases

Verification goal	Case I	Case II	Case III	Case IV	Case V
CTF distribution optimization	✓	✓			
Nodal voltage imbalance optimization	✓	✓	✓		
Effectiveness and reliability of the proposed distributed framework	✓			✓	✓

Table 4 Characteristics of different cases

Case	Improved ADMM	Standard ADMM	Centralized solution algorithm	Coordinated planning	Separately planning	DNR
Case I	✓			✓		✓
Case II			✓		✓	✓
Case III	✓			✓		
Case IV		✓		✓		✓
Case V			✓	✓		✓

4.2. Coordinated planning results analysis

The number of newly built lanes and FCS number on each road can be found in Table C5. Fig. 5 illustrates the overall distribution of FCS within UTN, along with the expansion results of power lines for TUDN. As shown in Table C5, the newly built lanes are primarily concentrated on roads 2, 3, 7 and 10 due to their large initial capacity. The strategic planning decision aims to guide vehicle users towards these roads to reduce congestion time. However, as shown in Fig. 5, FCS installations are primarily centralized on roads 3, 9, 18, 19 and 20, indicating that CTF is not strictly positively correlated with traffic flow distribution. This phenomenon arise from the further optimization of CTF based on traffic flow distribution.

WT have been installed at power nodes 3 and 22, corresponding to roads 3 and 9. The downstream node of 22 is equipped with PV, corresponding to roads 18 and 20. These four roads concentrate the most CTF, thereby alleviating the power supply pressure of the TUDN by utilizing nearby RES. Additionally, the DNR strategy at time 18 is highlighted in Fig. 5. It can be found that the tie-switch of branch 29 (starting from power node 28, ending at power node 29) is opened, transforming nodes 19 and 20, which were originally terminal nodes, into upstream nodes of 29, 30 and 31. As shown in Fig. 5, road 19 is also installed with additional FCS, and the power demand of its corresponding power node is supplied not only by the substation but also by upstream node 19, which is equipped with PV. This results also alleviates the power supply pressure by absorbing the PV outputs.

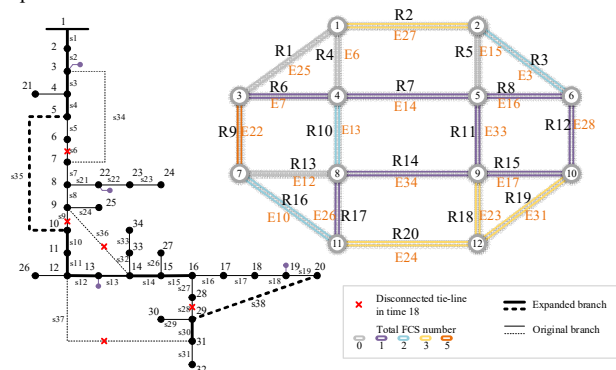


Fig. 5 Planning results and the DNR strategy at time 18

We next turn to the expansion results of power lines, as depicted in Fig. 5, the expanded power lines are primarily distributed at the beginning of the network, thereby enhancing the transmission capacity of the substation. Additionally, downstream branches of node 13 and 19 have also been expanded, reflecting the concentration of CTF. This expansion of FCS and power lines has contributed to the reasonable optimization of CTF distribution, consequently improving the safety of TUDN.

4.3. CTF distribution optimization analysis

To validate whether the CTF is further optimized based on the proposed coordinated planning method, this section compares the planning results between Case I (coordinated planning method) with Case II (UTN planning method). Case I is finished firstly, then the TUDN is planned taking the CTF assigned to each road as the boundary, as illustrated in Fig. 6.

In Case II, CTF is primarily focused on roads 2, 7, 9 and 16. This means the UTN planner decided to construct FCS exclusively on roads with substantial capacity, without considering the operational constraints of TUDN. As a result, the CTF distribution in Case II is more dispersed compared to the results in Case I, driven by the goal of minimizing investment. This strategy aims to reduce the cost of FCS installation by directing EV users to charge at existing FCS locations scattered throughout their original positions.

As shown in Fig. 6(a), the CTF is further optimized by considering the TUDN safety operation requirements during UTN planning method under the coordinated solving process. The maximum CTF is found on road 3, with its corresponding power node located at the beginning of the network, which has access to a WT. Therefore, the charging demand on road 3 can be supplied by the nearby WT, alleviating power supply pressure. Additionally, the CTF is concentrated on roads 9 and 20, where a WT is installed at the corresponding power node of road 9. Moreover, road 20 has a large capacity, and its corresponding power node is located downstream of the DG node, reducing travel time and relieving power supply pressure on the substation.

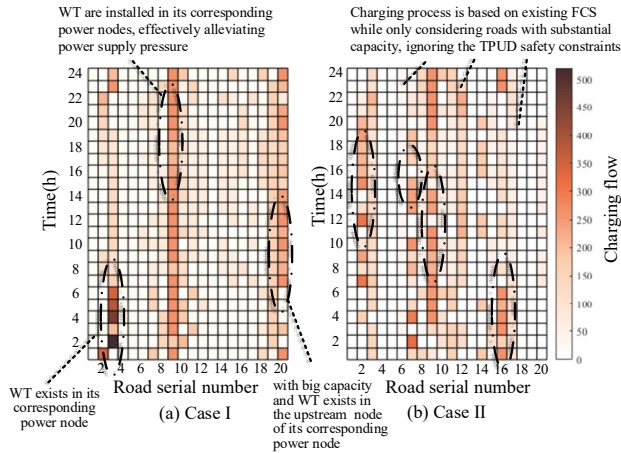


Fig. 6 CTF in Case I and Case II

4.4. Voltage imbalance optimization analysis

We further analyze the effect of mitigating the nodal voltage imbalance issue by introducing a new case, case III. The DNR strategy is applied in both Case I and Case II. However, in Case III, the initial topology of TUDN is utilized without additional tie-switches. The nodal voltage curve at time 18 and the degree of three-phase unbalance at all times are shown in Fig. 7.

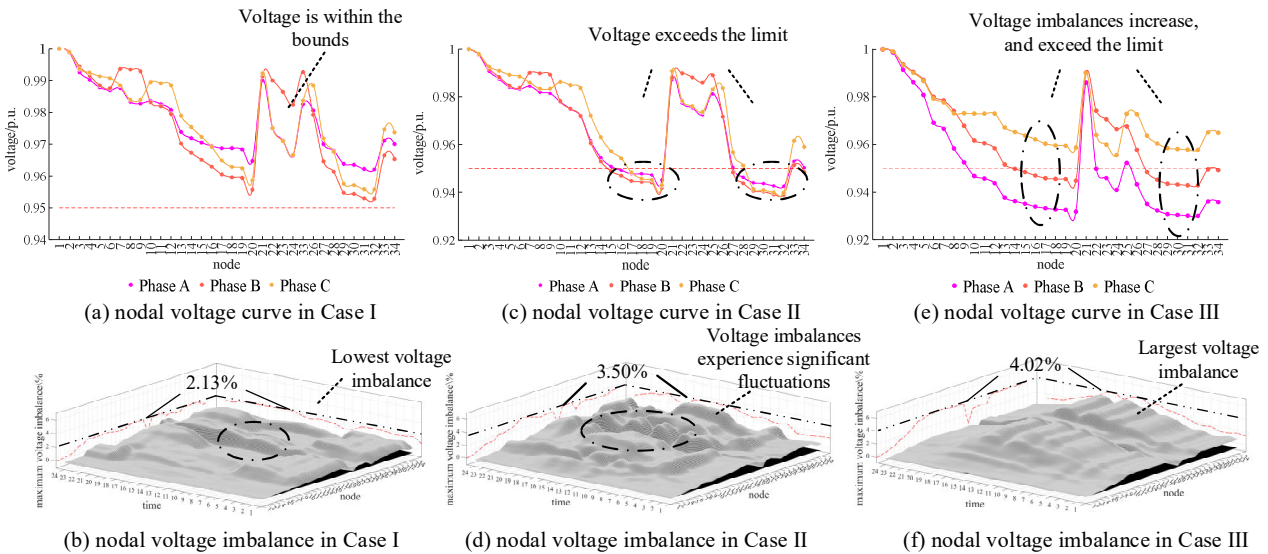


Fig. 7 Nodal voltage curve at time 18 and nodal voltage imbalances at all time

4.5. Effectiveness and reliability of the proposed distributed framework analysis

To validate the effectiveness of the dynamic updating strategy of the penalty factor, the solution process of the improved ADMM (Case I) and standard ADMM (Case IV) are compared in Fig. 8. In this comparison, the improved ADMM dynamically updates the penalty multipliers, whereas the standard ADMM uses fixed penalty multipliers. The maximum residual refers to the larger of primal and dual residuals at each iteration.

As can be seen in Fig. 7(e) and Fig. 7(f), when the TUDN operates within the initial topology without DNR strategy, the voltage imbalance is the largest due to the inflexibility of tie-switches in adjusting load distribution, leading to a significant voltage drop at the terminal node. Conversely, in Fig. 7(c) and Fig. 7(d), the nodal voltage imbalance is effectively reduced due to the introduction of the DNR strategy. However, in Case II, where the UTN is optimized first with the goal of minimizing investment, followed by the TUDN planning, the nodal voltage at the terminal node still exceeds the limit. Moreover, although the voltage imbalance issue has been effectively mitigated by the DNR strategy, its imbalance value experiences significant fluctuations. This fluctuation is attributed to the dispersed distribution of CTF, as shown in the color diagram in Fig. 7(b).

Fig. 7(a) and Fig. 7(b) show the results of the proposed coordinated planning. A comparison of the nodal voltage imbalance between Case I and Case II reveals a significant reduction in nodal voltage imbalance fluctuation with the collaborative planning method. Combined with the color diagram in Fig. 7(a), this decline in voltage imbalance results from optimizing the CTF distribution, making it more concentrated while taking into account the safety requirement of the TUDN. Comparing the nodal voltage imbalance in Case I with Case III, it can be found that the maximum voltage unbalance degree is effectively reduced due to implementation of the DNR strategy. Furthermore, during the evening peak time, the TUDN optimizes the network topology, ensuring that the voltage at the terminal node remains within the allowed bounds, as depicted in Fig. 7(a).

For Case I, the acceleration/deceleration factors $\mu^{\text{up}} / \mu^{\text{down}}$ are both picked as 2. The initial penalty factor is set as 0.1, and the gap tolerance for ADMM is 0.1kW. TIP converges after 3 outer iterations and 37 inner iterations when the planning decisions obtained from the two successive outer iterations remain unchanged. At the first iteration, UTN and TUDN solve the MILP to determine the planning results. Then, with the integer variables fixed, the solution procedure converges to a local optimum after 11 inner iterations. A huge jump shows up in the 12th iteration when the second outer iteration process begins. However, the TIP does not converge after the second inner solution period finished, so the third outer iteration

process begins, resulting in another jump in the 25th iteration. After another 13 inner iterations, the whole solution procedure converges, with a total parallel calculation time of 4596.15 seconds.

Then we turn to analyze the solution process in Case IV. For the standard ADMM, using the same initial penalty factor and residual tolerance, the penalty factor remains fixed without dynamic updating. At the third outer iteration, the maximum residual keeps oscillating, indicating that this method is not converging after 3 outer iterations and 117 inner iterations, with a total time cost of 9463 seconds already.

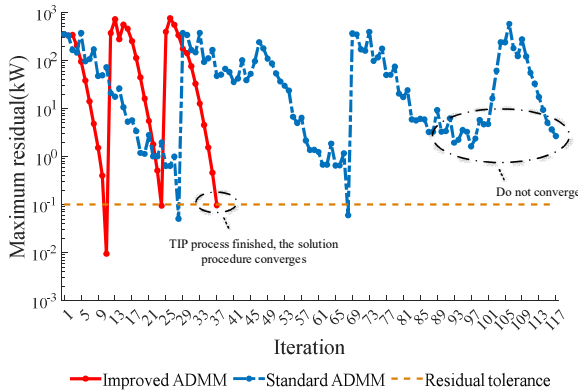


Fig. 8 Evolution of max-residual in improved ADMM and standard ADMM.

The planning results obtained from the distributed solving algorithm may be locally optimal. To evaluate the quality of the solution, a comparison of investments for Case I and Case V is presented. Case V employs a centralized solving algorithm, where all agents share parameters and solve the MILP problem established by Eq. (15), ensuring a globally optimal solution. Case I uses the improved ADMM algorithm to solve the collaborative planning model in a distributed way. Therefore, comparing the solutions from these two cases can verify the reliability of the planning results obtained by the proposed distributed solution framework.

For the FCS sites, line expansions and road enhancements plans, the solutions obtained from the centralized solution method are consistent, except for the DNR strategy. As shown in Table 5 and Table 6, under the allowed maximum switching actions, the centralized solution involves more frequent switching actions, with 6 tie-switches operating three times, reaching the maximum operation number. In contrast, the distributed solution involves fewer tie-switches actions. This indicates that the centralized solution enables the TPUD to perform DNR more effectively, optimizing power flow and resulting in an annual power output cost of \$1869.435 thousand. Under the distributed solution framework, the annual power output cost of the substation is \$1941.354 thousand. The error between the two solution methods is defined as (15). The calculation shows that the distributed solution results in a 3.8% economic loss in the substation's output cost. 公式节 15

$$\text{loss} = \frac{C(\mathcal{C}) - C(\mathcal{D})}{C(\mathcal{C})} \times 100\% \quad (15)$$

Where $C(\mathcal{C})$ and $C(\mathcal{D})$ are the cost obtained by the centralized solution method and distributed solution method, respectively.

Table 5 DNR strategy of centralized planning

time/h	Opened tie-switches
0-1	s6,s28,s35,s36,s37
1-2	s6,s9,s28,s36,s37
2-7	s9,s28,s34,s36,s37
7-10	s6,s9,s28,s36,s37
10-11	s6,s28,s35,s36,s37
11-20	s6,s9,s28,s36,s37
20-24	s9,s28,s34,s36,s37

Table 6 DNR strategy of distributed planning

time/h	Opened tie-switches
0-1	s9,s28,s34,s36,s37
1-20	s6,s9,s28,s36,s37
20-24	s9,s28,s34,s36,s37

Next, we analyze the error in the CTF obtained by the two solution methods. Define the error $= f(\mathcal{C}) - f(\mathcal{D})$, as the difference between the CTF obtained by the centralized solution and that obtained by the distributed solution. The error between the two solutions at peak charging power time is shown in Fig. 9. As can be seen, the CTF distributions obtained by the two solution methods are almost identical, with a maximum error of 4.91 vehicles. This indicates that the operating results of the UTN obtained by the distributed solution remain reliable.

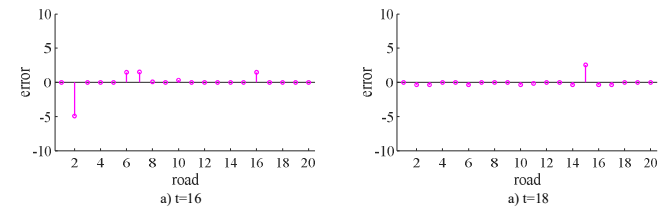


Fig. 9 Error of CTF between centralized solution and distributed solution at peak charging power time

Then we analyze the total costs between the two solutions as shown in Table 7. There is a significant disparity in investment for TUDN. However, the investment for the UTN is Case I remains nearly the same as in Case V. From the perspective of total investment, it can be observed that the results obtained by the two models are very close, and the maximum error is only 1%, which is acceptable in practice. Although the planning results obtained by the proposed method lead to higher investment, they still demonstrate a certain level of reliability. Moreover, the distributed solving framework effectively avoids information disclosure between the UTN and TUDN, ensuring privacy protection.

Table 7 Comparison of the cost between different cases

Case	Total cost/10 ⁴ \$	TUDN/10 ⁴ \$	UTN/10 ⁴ \$
Case I	706.89	489.93	216.96
Case V	699.53	484.57	215.96
Error(%)	1.05	1.11	0.46

5. Case study II: real-world system in China

5.1. Simulation setting

To further validate the effectiveness and reliability of the proposed coordinated planning model and distributed solving framework, a real-world system in Zhejiang Province, China is applied. The geographical relationship between power lines and roads is depicted in Fig. 10. For simplicity, only the main sections of overhead lines are shown in Fig. 10, while the complete topology of this area is presented in Fig. C3, comprising 91 power nodes in DN and 35 nodes in UTN. Power lines impedance, road information can be found in [59]. Traditional power load distribution is shown in Fig. 11. Total loads are 8512kW and 4630kvar on phase A, 5611.8kW and 3140kvar on phase B, 6708kW and 3654kvar on phase C. The travel plans, which have time-variant characteristics, are given in Table C6. Other basic parameters remain the same as in Case study I. The planning results are provided in Table C7.

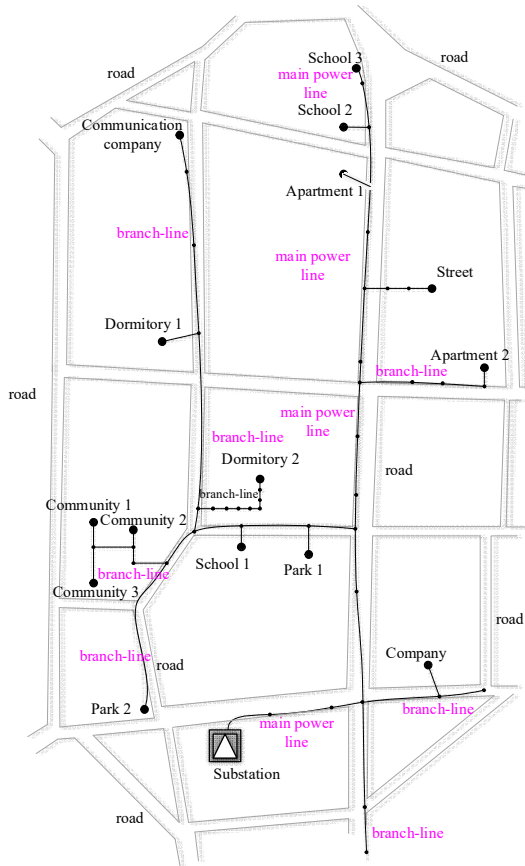


Fig. 10 A abstract map of a real-world system in China, Zhejiang province

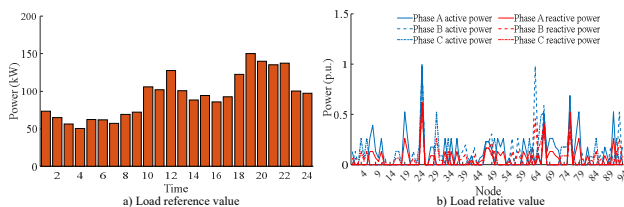


Fig. 11 Curves of load reference and three-phase load relative value in Case study II

5.2. Voltage imbalance optimization analysis

To analyze the results of voltage imbalance optimization, three cases with the variables outlined in Table 4 are adopted. Table 8 shows the maximum voltage unbalance degree in different cases. In Case I, the proposed coordinated planning model is utilized, which integrates the UTN and TUDN, and introduces the DNR strategy. As can be seen, the proposed planning method effectively optimizes the voltage imbalance, resulting in the lowest voltage unbalance degree compared to the other cases.

Table 8 Maximum voltage unbalance degree in different cases

Case	Maximum voltage unbalance degree
Case I	3.29%
Case II	4.61%
Case III	5.86%

5.3. Solving efficiency analysis

Like other dual decomposition methods, penalty factors of ADMM have an impact on algorithm performance. The parametric sensitivity issue for Case study I is investigated in Table 9. As the table shows, without the dynamic updating strategy for penalty factor, the convergence property is not satisfactory when the penalty factors are predetermined to be too small or too large. After implementing the dynamic updating strategy (parameters are explained in Table C4), as shown in Table 9 and Table 10, the varying penalty strategy results in fewer iterations in both test systems. Therefore, it is a preferable strategy to improve the computational efficiency of ADMM.

Table 9 Comparison of different penalty factors

Penalty Factors	Constant ρ		Varying ρ	
	Iteration number	Solving time (s)	Iteration numbers	Solving time (s)
$\rho^1 = 0.01$	Cannot converge within 200 iterations		78	6868.29
$\rho^1 = 0.1$	48	5269.57	37	4596.15
$\rho^1 = 2$	73	6504.53	62	6139.83
$\rho^1 = 10$	Cannot converge within 200 iterations		91	7162.13

Table 10 Solution performance in different test system

Test system	Constant ρ		Varying ρ	
	Iteration number	Solving time (s)	Iteration numbers	Solving time (s)
Case study I	48	5269.57	37	4596.15
Case study II	72	7719.11	57	6999.82

To analyze the impact of integer variables introduced by the TUPF model and DNR strategy on solving efficiency, a comparative analysis is conducted. This analysis includes four models. Model I employs the proposed coordinated planning model using the TUPF model and DNR strategy, solved in a centralized way. Models II-IV solve different established models, as detailed in Table 11, using the proposed distributed framework. The solving results for each case are shown in Table 12.

Table 11 Differences between the four models

Model	TUPF	DNR	Single-phase equivalent model	Distributed Framework
Model I	✓	✓		
Model II	✓	✓		✓
Model III	✓			✓
Model IV			✓	✓

Based on the proposed coordinated planning model and distributed framework, the computation time is 4596.15 seconds for the small test system and 6999.82 seconds for the large test system. Compared to model III, which uses TUPF model but without DNR strategy, and model IV, which uses the single-phase equivalent model, the proposed model and distributed framework increases solution complexity. However, the

proposed model enhances the operational model of the DN to a three-phase level and reduces the maximum three-phase imbalance by adopting the DNR strategy. Considering the long-term effectiveness of the planning results, the maximum additional computation time of 6624 seconds is acceptable for engineering requirements. Therefore, the proposed planning model and distributed framework hold significant practical value in real-world applications.

Table 13 shows the computational performance of the proposed distributed solution algorithm compared to other solution methods. The proposed distributed framework, based on an improved ADMM algorithm, demonstrates superiority in both iteration number and calculation time, making it highly suitable for practical applications.

Table 12 Solving efficiency in different cases

Model	Case study I			Case study II		
	Solving time/s	Iteration number	Maximum voltage unbalance degree	Solving time/s	Iteration number	Maximum voltage unbalance degree
Model I	1205.45	0	2.13%	1911.38	0	3.26%
Model II	4596.15	37	2.13%	6999.82	57	3.29%
Model III	1308.4	29	4.02%	2805.56	53	5.86%
Model IV	211.28	11	/	376.94	35	/

Table 13 Comparison of different solution method

Method	Case study I			Case study II		
	Iteration numbers	Solving time/s	Cost/10 ⁴ \$	Iteration numbers	Solving time /s	Cost/10 ⁴ \$
Centralized way	0	1205.45	699.53	0	1911.38	1031.20
LR	Cannot converge within 200 iterations			Cannot converge within 200 iterations		
Enhanced Benders	58	6416.59	705.38	69	9846.5	1034.80
Proposed solution algorithm	37	4596.15	706.89	57	6999.82	1034.45

6. Conclusion

In this paper, a coordinated planning model is proposed for TUDN and UTN, extending the traditional single-phase equivalent model of DN to a TUPF model with asymmetrical impedance on power lines and load mismatches. Additionally, a distributed framework utilizing an improved ADMM algorithm is developed to solve the proposed planning problem, enabling the UTN and TUDN to solve their regional issues independently. The specific findings are drawn as follows: 1) The temporal-space relationship between traffic flow and TPL is effectively characterized and considered in the proposed planning model. The optimal placement of FCS, line expansions, and road enhancements can guide EV users to travel

efficiently, thereby improving the distribution of CTF and addressing the asymmetrical distribution of TCL. 2) Based on the traffic flow distribution among EV users, implementing the DNR strategy can optimize the operational topology of DN by modifying the state of tie-switches, efficiently reducing voltage imbalances. 3) The proposed fully distributed framework can decompose the coordinated planning model into two subproblems, protecting each system's privacy. The dynamic updating strategy for the penalty factor used in the model can effectively improve the solving efficiency. 4) Numerical simulations on a modified IEEE 34-bus system and a real-world system in China demonstrate the significant practical value of the proposed planning model in real-word applications. It achieves a 43.84% reduction in maximum voltage imbalance and outperforms other algorithms in iteration count and computation time.

Acknowledgements

This work is supported by National Natural Science Foundation of China (U22B2098).

Appendix A 公式节 1

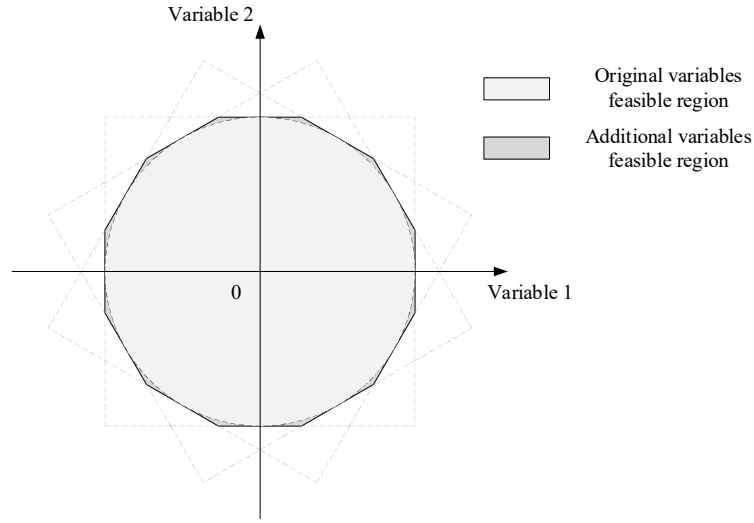


Fig. A1 Circular constraint linearization method

The constraints (11g)–(11h) involve complex product calculations. Firstly, we expand them into (A1) and (A2), respectively, where R'_{ij} and X'_{ij} are the real and imaginary parts of the complex matrix $\tilde{Z}_{ij}$, respectively. According to the illustration of $\tilde{Z}_{ij}$ and combined with Eq. (11k), the two terms R'_{ij} and X'_{ij} are expressed as (A3) and (A4), respectively. 公式章 1 节 1

$$U_{j,t} \leq U_{i,t} - 2R'_{ij}P_{j,t} - 2X'_{ij}Q_{j,t} + M(1 - x_{ij,t})E_{3 \times 1}, \forall (i, j) \in \mathcal{B} \quad (A1)$$

$$U_{j,t} \geq U_{i,t} - 2R'_{ij}P_{j,t} - 2X'_{ij}Q_{j,t} - M(1 - x_{ij,t})E_{3 \times 1}, \forall (i, j) \in \mathcal{B} \quad (A2)$$

$$R'_{ij} = R^{\text{exi}'}_{ij} + \delta_{ij} \left(R^{\text{exp}'}_{ij} - R^{\text{exi}'}_{ij} \right), \forall (i, j) \in \mathcal{B} \quad (A3)$$

$$X'_{ij} = X^{\text{exi}'}_{ij} + \delta_{ij} \left(X^{\text{exp}'}_{ij} - X^{\text{exi}'}_{ij} \right), \forall (i, j) \in \mathcal{B} \quad (A4)$$

Where $R^{\text{exi}'}_{ij}$ and $X^{\text{exi}'}_{ij}$ are the real and imaginary parts of the complex matrix $H \circ Z^{\text{exi}}_{ij}$, $R^{\text{exp}'}_{ij}$ and $X^{\text{exp}'}_{ij}$ are the real and imaginary parts of the complex matrix $H \circ Z^{\text{exp}}_{ij}$.

There are bilinear terms $\delta_{ij}P_{j,t}$ and $\delta_{ij}Q_{j,t}$ after expanding Eq. (A1)–(A2) combined with (A3)–(A4). These bilinear terms can be substituted by $v_{ij,t}$ and $\tau_{ij,t}$ through the Big-M method, respectively. Therefore, Eq. (A1)–(A2) can be replaced by expressions (A5) and (A6), respectively, where $v_{ij,t}$ and $\tau_{ij,t}$ are constrained by Eq. (A7) and Eq. (A8), respectively.

$$U_{j,t} \leq U_{i,t} - 2 \left[R^{\text{exi}'}_{ij}P_{j,t} + \left(R^{\text{exp}'}_{ij} - R^{\text{exi}'}_{ij} \right)v_{ij,t} \right] - 2 \left[X^{\text{exi}'}_{ij}Q_{j,t} + \left(X^{\text{exp}'}_{ij} - X^{\text{exi}'}_{ij} \right)\tau_{ij,t} \right] + M(1 - x_{ij,t})E_{3 \times 1}, \forall (i, j) \in \mathcal{B} \quad (A5)$$

$$U_{j,t} \geq U_{i,t} - 2 \left[R^{\text{exi}'}_{ij}P_{j,t} + \left(R^{\text{exp}'}_{ij} - R^{\text{exi}'}_{ij} \right)v_{ij,t} \right] - 2 \left[X^{\text{exi}'}_{ij}Q_{j,t} + \left(X^{\text{exp}'}_{ij} - X^{\text{exi}'}_{ij} \right)\tau_{ij,t} \right] - M(1 - x_{ij,t})E_{3 \times 1}, \forall (i, j) \in \mathcal{B} \quad (A6)$$

$$|v_{ij,t} - P_{j,t}| \leq M(1 - \delta_{ij}), |v_{ij,t}| \leq M\delta_{ij}, \forall (i, j) \in \mathcal{B} \quad (A7)$$

$$|\tau_{ij,t} - Q_{j,t}| \leq M(1 - \delta_{ij}), |\tau_{ij,t}| \leq M\delta_{ij}, \forall (i, j) \in \mathcal{B} \quad (A8)$$

Appendix B 公式节 2

The convergence of TIP can be proven as follows.

Rewrite the proposed coordinated planning model as: 公式章 2 节 1

$$\min_{x, Z} \{ f(x, Z) : \text{s.t. } x \in \mathcal{I}_x, Z \in \mathcal{I}_Z \} \quad (B1)$$

Where x and Z are the continuous and discrete variables, respectively. $\mathcal{I}_x$ and $\mathcal{I}_Z$ are the corresponding feasible regions, respectively.

The TIP can be decomposed into two steps as follows:

a) Solve each subproblem with planning results fixed:

$$x^{m+1} = \arg \min_x f(x, Z^m) \quad (B2)$$

b) Solve each subproblem with coupling variables fixed:

$$Z^{m+1} = \arg \min_Z f(x^{m+1}, Z) \quad (B3)$$

A non-increasing sequence can be obtained through (B2)-(B3):

$$f(x^{m+1}, Z^{m+1}) \leq f(x^{m+1}, Z^m) \leq f(x^m, Z^m) \quad (B4)$$

Since (x, Z) is always a feasible solution, the values of $f(x, Z)$ are bounded below by the optimal value of (B1). Although the non-increasing sequence described in (B4) is not a strictly decreasing sequence, simulation results in [54-56], which utilize similar algorithms, show that the below-bounded non-increasing sequence will converge to a local optimum within finite iterations.

Appendix C 公式节 3

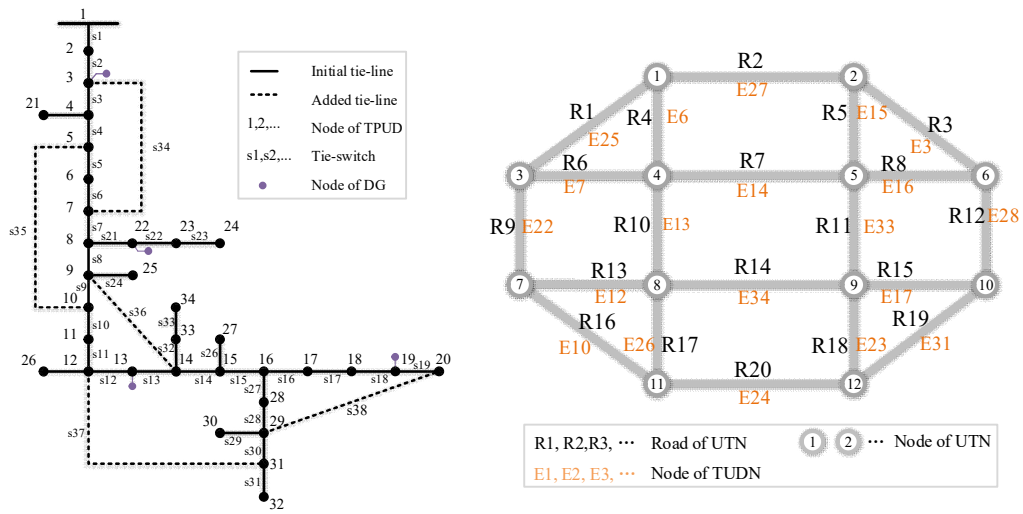


Fig. C1 Topology of coupled modified IEEE 34-bus distribution and 12-node transportation system

Table C1 Parameters of FCS	
Parameters	Value
power capacity of each FCS $P^{fcs, ch}$ (kW)	100
investment cost of FCS κ^{fcs} /(\$/kW)	40

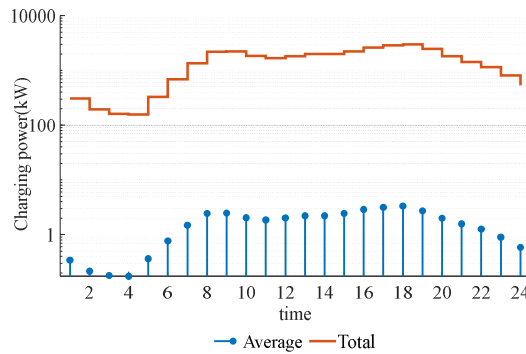


Fig. C2 Total charging power and average charging power at each time

Table C2 Parameters of travel process	
Parameters	Value
coefficient of travel time cost c^{time} (\$/h)	6

proportion of electric vehicle fast charging users β 0.2

Table C3 Parameters of distribution network operation

Parameters	Value
coefficient of substation power output κ^{gen} (\$/kW)	0.08
penalty coefficient of RES curtailment C_{cut} (\$/kW)	0.64
penalty coefficient of voltage imbalance C^{un}	10^4
tie-switches act cost κ^{act} (\$)	1.00
voltage grade (kV)	12.6

Table C4 Parameters of distribution network operation

Parameters	Value
initial penalty multiplier ρ^1	0.1
multiplier expansion factor μ^{up}	2
multiplier contraction factor μ^{down}	2
threshold for increasing or decreasing the penalty multiplier η	10
residual tolerance $\varepsilon_P / \varepsilon_D$ (kW)	0.1

Table C5 Planning results of UTN in Case study I

Road serial number	Newly lanes number	Newly FCS number	Congestion time/min	Road serial number	Newly lanes number	Newly FCS number	Congestion time/min
1	0	0	0	11	0	0	0
2	2	1	7.5	12	0	1	0
3	3	0	9.5	13	0	0	0
4	0	0	0	14	0	0	0
5	0	0	0	15	0	0	0
6	0	0	0	16	0	0	0
7	3	0	0	17	0	0	12.5
8	2	0	7	18	0	3	6
9	0	3	0	19	0	3	0
10	3	1	10	20	0	2	0

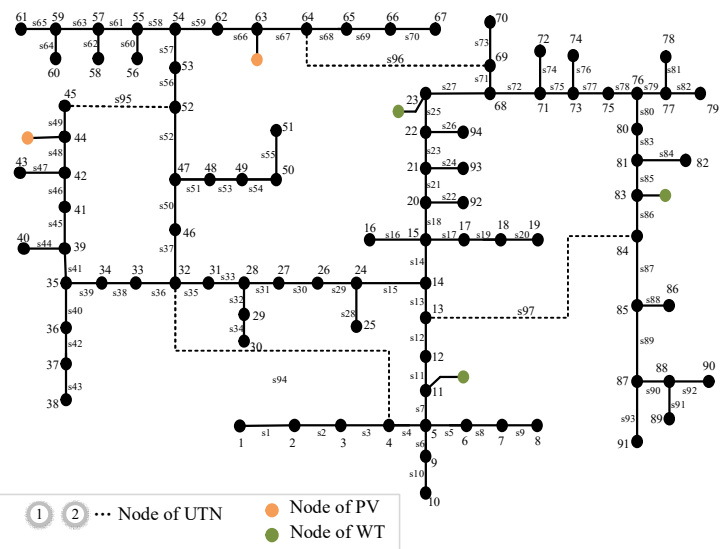
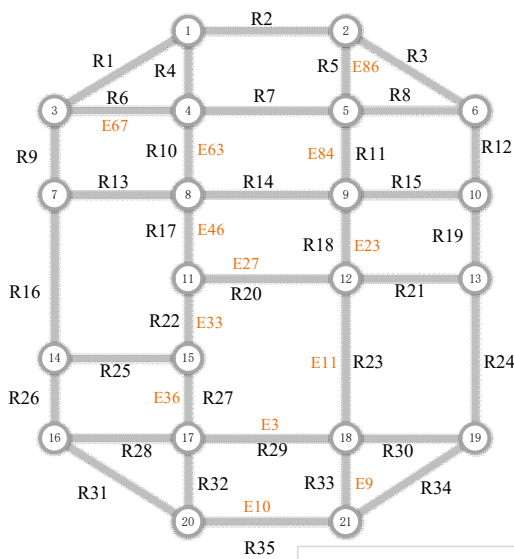


Fig. C3 Topology of coupled modified 91-bus distribution and 21-node transportation system of a real-word system in China, Zhejiang province

Table C6 Trip plans of vehicle drivers and trip rates in Case study II

time/h	O-D Pair											
	3-6	3-13	4-13	4-21	7-13	7-19	8-19	8-21	14-19	14-21	11-21	16-19
1	230	510	627	503	667	368	695	406	406	610	493	561
2	328	465	606	508	636	300	747	571	330	698	571	674
3	377	472	801	442	594	353	768	545	280	695	502	683
4	370	263	640	411	669	416	820	539	320	624	547	551
5	379	517	657	488	575	383	730	479	314	584	556	742
6	491	465	647	363	681	340	732	439	267	616	462	681
7	486	550	795	673	751	363	907	631	498	690	603	655
8	424	579	747	613	752	477	824	575	432	666	588	701
9	515	507	715	611	727	410	754	547	439	807	553	660
10	443	476	764	599	754	279	802	665	471	638	528	615
11	463	586	744	589	676	497	817	542	416	772	619	731
12	422	509	820	563	757	366	797	692	370	567	602	720
13	346	475	750	603	704	432	725	616	381	642	492	716
14	345	486	724	608	731	452	758	605	460	674	647	653
15	389	481	868	605	758	432	826	584	354	759	712	778
16	313	506	752	551	743	460	768	652	466	575	611	612
17	438	513	764	616	824	422	696	618	417	576	543	722
18	500	600	200	550	300	500	350	450	500	600	430	420
19	498	604	808	709	887	518	927	700	470	830	710	865
20	382	416	676	566	736	374	841	575	293	579	394	717
21	392	464	726	480	682	318	709	519	275	670	514	622
22	284	503	674	521	688	382	636	548	365	540	520	524
23	375	398	671	534	585	319	778	555	377	557	567	572
24	387	475	644	555	607	263	593	556	376	559	461	642

Tab C7 Planning results of Case study II

Planning results	
Expanded line / (Start node-end node)	1-2、2-3、3-4、4-5、5-11、13-14、15-20、 20-21、22-23、32-33、35-39、41-42、42-44、 44-45、52-53、64-65、75-76、76-80
Extended lanes/ Road serial number (Extended lanes number)	7(2)、8(3)、18(2)、19(1)、21(2)、26(3)、
Newly built FCS/ Road serial number (Newly built FCS number)	6(2)、18(1)、20(1)、 23(1)、35(2)

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