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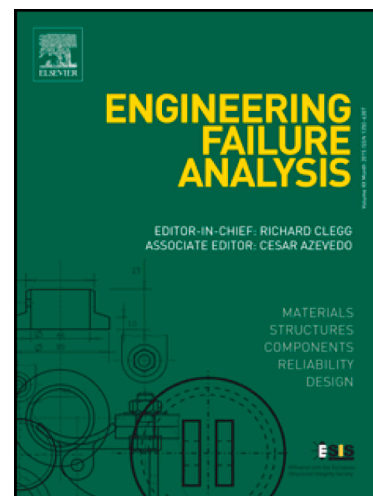
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TVMS calculation model for gear tooth tip chipping failure with EHL

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Hights

- (1) Considering the transmission of meshing forces by the oil film at the chipping defect under elastohydrodynamic lubrication and introducing elastohydrodynamic torsional stiffness k_t , this study investigates the variation of time-varying mesh stiffness using this computational method.
- (2) Considering the changes in surface roughness with varying defect sizes and establishing an elastohydrodynamic lubrication stiffness calculation model, this study investigates the impact of surface roughness on the friction coefficient.
- (3) Investigating the influence of oil film stiffness and roughness on time-varying mesh stiffness under chipping conditions, this study finds that as chipping faults occur, the roughness at the chipping defect increases, altering the meshing contact area. The occurrence of chipping leads to system load imbalance, causing significant fluctuations in oil film stiffness.

Abstract: This paper proposes a calculation method for Time-Varying Mesh Stiffness (TVMS) in the presence of tooth tip chipping defects under elastohydrodynamic lubrication (EHL). A contact stiffness model for tooth tip chipping defects under EHL is established, and an analytical equation for the TVMS of chipping defects under EHL is derived. The torsional stiffness at the chipping defect under EHL is introduced, considering the influence of roughness at the chipping defect on the characteristics of the oil film and its stiffness. The effects of chipping defect size, torsional stiffness of the chipping defect under EHL, roughness, and oil film stiffness on TVMS are analyzed, providing methodological support for improving the calculation accuracy of gear TVMS.

Keywords: tooth tip chipping; time-varying mesh stiffness (TVMS); elastohydrodynamic lubrication (EHL); surface roughness

Introduction

Gears have the advantages of high transmission efficiency, high precision, and strong bearing capacity, and are widely used in aerospace, engineering machinery, marine engineering, and other fields. However, gears are often subjected to poor load conditions, heavy lubrication, and high-intensity cyclic stress during operation. This can lead to issues such as pitting, root cracks, tooth fractures, and other faults, which can seriously affect the normal operation of machinery and equipment, and even result in significant economic losses or safety risks. Therefore, studying the generation mechanisms and evolution of gear failure, as well as identifying early failure characteristics, is crucial to ensuring the operational safety of gear transmission systems.

Numerous scholars, both domestically and internationally, have extensively researched the fault mechanisms, dynamic characteristics, and fault diagnosis of gear systems. Vibration-based signal and neural network diagnostics have become effective tools for detecting gear damage and fatigue [1]. The study investigates crack propagation and fracture surfaces and establishes an empirical model.[2]. Establishing dynamic models and the simulation of faults can help explain vibration characteristics, failure mechanisms, and life prediction, among other factors. Local damage such as pitting, cracking, and spalling increases the risk of fault propagation and further damage. As a result, the TVMS is reduced. TVMS, being a core parameter of the gear transmission system, significantly influences the vibration, noise, and efficiency of gear transmissions[3][4][5]. In recent years, studies on TVMS have commonly employed the finite element analysis method and potential energy method, However, the finite element method's calculation process is complex, and its cycle time is too long, making it primarily useful for verifying the accuracy of the potential energy method. The results of the two methods are generally in good agreement. Additionally, the solution model for time-varying meshing stiffness can be directly derived using the potential energy method, combined with geometrical and material parameters, Efficient failure simulation can also be achieved by utilizing the dynamic model of gears[6]. Therefore, simulation technology based on analytical models has become a research hotspot for studying gear failure mechanisms, condition monitoring, and fault diagnosis.

Yang et al. [7] were the first to model the gear tooth as a variable section cantilever beam model and applied cantilever beam theory to calculate meshing stiffness. Chen et al. [8] used the finite element method (FEM) to calculate the mesh stiffness of a gear pair and examined the impact of tooth cracks on mesh stiffness. Tian et al. [9] enhanced the calculation theory by incorporating shear energy and including shear stiffness in the calculation formula. The expression for TVMS was derived based on the potential energy method to estimate the meshing stiffness of cracked gear faults [10][11][12][13][14]. Liang et al. [15] derived equations for the TVMS of gear teeth at different crack levels, established a stiffness model for pitting in gears and investigated the effect of the severity of pitting faults on the meshing stiffness. Ma et al. [16] proposed an improved meshing stiffness model that considered the transition curve between the root and base circles, a TVMS model with different crack types, and further refined the model by incorporating the extended tooth contact effect [17], modifying the base stiffness of the fillet [18], and accounting for tooth shape correction [19]. Luo et al. [20] proposed a method for calculating meshing stiffness for gear spalling failures that does not depend on specific geometry.

Current research mainly focuses on the meshing stiffness of gears under failures such as pitting, cracking, and spalling, with less attention given to top chipping failures. Alireza Behvar et al. [21] investigated fatigue cracks within the internal gears of a differential, proposing that cracks are more likely to form at various heights within the tooth rather than solely at the tooth surface or near the surface. This challenges the traditional view that cracks primarily originate at or near the surface of the tooth. Wang et al. [22] simulated chipping failures in gears use a periodic impulse function and investigated their vibration response. chaari et al. [23] considered the dynamic response of gears in spalled and failed states, analyzing the tooth fracture curve under friction by simplifying it to a

straight line. They assumed that only the tooth width is affected by failure defects, neglecting the changes in cross-sectional area and moment of inertia. To better match the actual working conditions, Li et al. [24] developed a TVMS calculation method for tooth top chipping faults that accounts for variations in tooth width, area and moment of inertia, also investigated the characteristic response of composite planetary gear sets under tooth top chipping faults. However, in their model, the tooth fracture line is still simplified to a straight line, which may lead to estimation errors in TVMS calculations for tooth top chipping faults. Meanwhile, Liu et al. [25] proposed an improved potential energy method to derive a bending fracture line equation, which produced theoretical results more consistent with finite element analysis. Yang et al. [26] used a modified potential energy method that considers the stress distribution in the chipping region caused by chipping defects.

In the above literature, contact deformation stiffness is primarily calculated using Hertzian contact theory [27][28][29] and finite element nonlinear algorithms [30][31]. However, when considering the effects of EHL, the mechanical behavior of the contact body may change significantly due to its interaction with the lubricant. Lu et al. [32] examined load distribution and transmission error in spur gears, accounting for extended tooth contact and EHL. Similarly, Huang et al. [33][34] included EHL effects in their analytic-slice coupling model, calculated TVMS, and analyzed the impacts of gear modifications. Cheng et al. [35][36] calculated TVMS for the elastic-fluid contact taking into account a three-dimensional rough surface. Wang and Zhu [37] proposed a helical gear TVMS model that incorporates lubrication and friction, where the friction coefficient is obtained using an elastoplastic model under different surface roughness conditions. According to the literature [38], TVMS is typically obtained through the mean slope method. However, stiffness models for EHL contact and rough surface contact primarily use the local slope method. Due to the coordination of deformations across multiple contact points in gear meshing design, the deformation of the lubrication film cannot be directly equated to contact deformation.

Given the aforementioned limitations, the primary objective of this study is to propose a computational method for the TVMS of gear tip chipping defects under EHL. This method primarily considers the transmission of meshing forces through the oil film at the site of the chipping conditions. As a result, a TVMS model for gear tip chipping faults under EHL is established. This model aims to elucidate the influence of different tooth surface morphologies on TVMS, reveal the intrinsic relationship between internal fault mechanisms and external manifestations, and holds significant value for identifying early weak fault features. The main innovations of this method are as follows:

- (4) Considering the impact of the oil film on meshing force transmission at tooth tip chipping defects under EHL, taking into account the lubrication insufficiency as the defect size increases, and introducing elastohydrodynamic torsional stiffness k_t' , this study investigates variations in TVMS using the proposed computational method.
- (5) Accounting for changes in surface roughness with varying defect sizes and establishing an EHL stiffness calculation model, this study examines the impact of surface roughness on the friction coefficient.
- (6) This study explores the influence of oil film stiffness and surface roughness on TVMS under chipping conditions. It finds that as chipping faults occur, the roughness at the defect site increases, altering the meshing contact area. The presence of chipping leads to an imbalance in system load, resulting in significant fluctuations in oil film stiffness.

1 Model of TVMS for gear tooth tip chipping failure

1.1 Analysis of force distribution on gear tooth tip chipping failure

Tooth tip chipping fault is a form of failure mode where part of the material breaks away near the tooth tip[25]. As shown in Fig. 1, the tooth top chipping defect starts from the tooth tip, where w_d is the width of the defect, l_d is the height of the defect, t_d is the depth of the defect, L is the width of the tooth, A_x and is the area of the tooth cross-section along the width of the tooth at any position on the involute line. The main focus of this paper is the early failure of gears, the tooth tip chipping defect is of relatively small size.

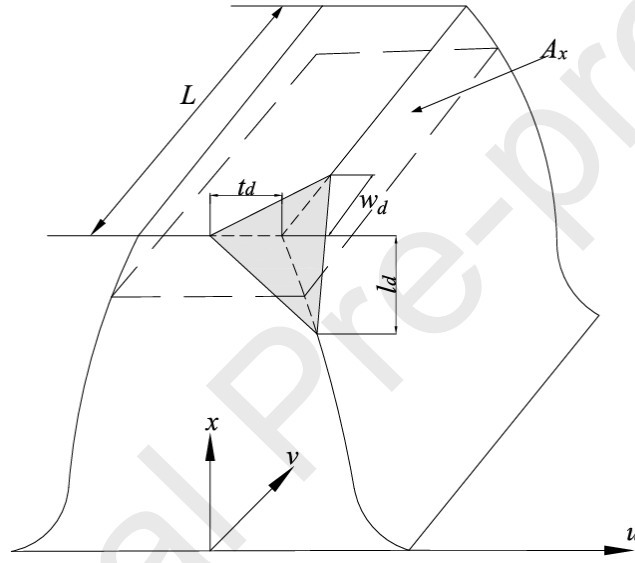


Fig. 1. Model of the tooth tip chipping

In applying the potential energy method to calculate the gear meshing stiffness, the gear tooth is considered as a variable cross-section cantilever beam, and the case where the root circle is not aligned with the base circle is taken into account, as shown in Fig. 2. The tooth profile includes transition curve AB, involute curve BE and top curve EF, of which A and B are the starting point and end point of the transition curve and their shapes are determined by the fillet angle of the top end of the tooth, G is the intersection point of the involute curve and the base circle, R_r and R_d are the radius of the root circle and the base circle, respectively, and R_t is the radius of the transition circle passing through the B point, d indicates the distance from the action point to the root circle, h indicates the distance from the action point to the tooth center axis, x indicates the distance from any point on the involute line to the root circle.

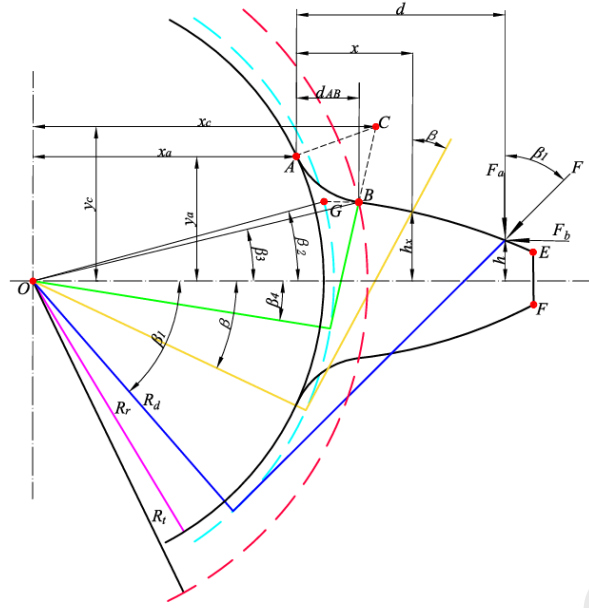
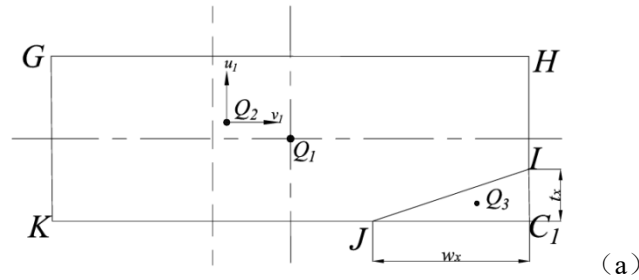


Fig. 2. Schematic diagram of a healthy gear

Fig. 3(a), and (b) depict the tooth apex cross-section, the tooth apex chipping failure model, and the tooth apex chipping cantilever beam structure, respectively. The cross-section axes in the tip region are specifically shown in Fig. 3(a), which can depict the changes in the defect geometric parameters caused by tooth apex chipping failures, and the rectangular shape GHC_1K is the tooth apex cross-section of the healthy gear when the center of mass is Q_1 . Polygon $GHIJK$ for the top of the tooth chipping cross-section, at this time the center of mass for the Q_2 , obviously chipping defects so that the tooth width, the load force area is reduced, Triangle IC_1J is the cross-sectional area of chipping defects with the center of mass Q_3 , where t_x denotes the depth of the defect and w_x denotes the width of the defect. In Fig. 3(b), the cantilever beam structure of the chipping defect at the top of the tooth is represented, D is the starting point of the chipping defective portion, d_{AD} represents the distance from the starting point of the chipping defective portion D to the root of the tooth along the direction of the tooth height, and L_{max} represents the distance from the root of the tooth to the tip of the tooth, which can be represented by the equation:

$$L_{max} = R_d [(\alpha_a + \beta_2) \sin \alpha_a + \cos \alpha_a] - x_a \quad (1)$$

$$\text{Where, } \alpha_a = \sqrt{\frac{R_a^2}{R_d^2} - 1} - \beta_2 \quad (2)$$



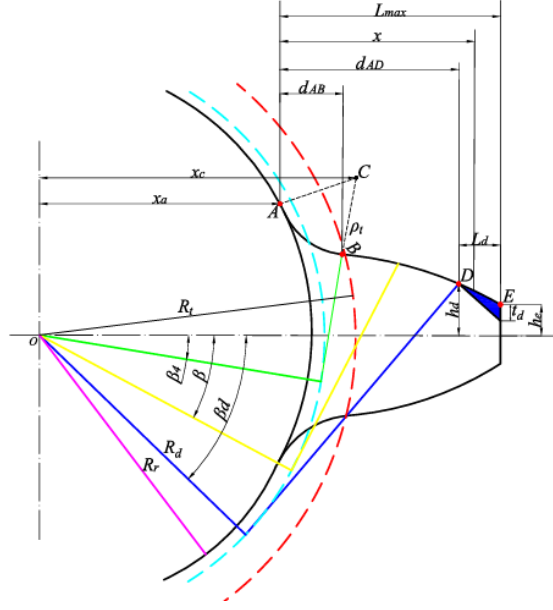


Fig. 3. Model of gear tooth tip chipping: (a) Tooth tip Cross-section Diagram; (b) Cantilever Beam Model

$$\beta_2 = \frac{\pi}{2Z} + \tan \alpha_0 - \alpha_0 \quad (3)$$

where α_0 is the standard pressure angle and Z is the number of gear teeth.

$$d_{AD} = L_{max} - L_d \quad (4)$$

According to the cantilever beam model, it is possible to calculate the depth of the defect t_x and the width of the defect w_x of the arm beam at position x .

$$t_x = \frac{h_d + t_d - h_e}{L_d} (x - L_{max}) + h_x + t_d - h_e, d_{AD} \leq x \leq L_{max} \quad (5)$$

$$h_e = R_d [(\beta_1 + \beta_2) \cos \beta_1 - \sin \beta_1] \quad (6)$$

$$h_x = \begin{cases} y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2}, & 0 \leq x \leq d_{AB} \\ R_d [(\beta_2 - \beta) \cos \beta + \sin \beta], & d_{AB} \leq x \leq d \end{cases} \quad (7)$$

$$w_x = \frac{w_d}{t_d} t_x, d_{AD} \leq x \leq L_{max} \quad (8)$$

where h_d , h_x , h_e are half of the thickness of the chord tooth at position D, position x , and the chipped tip at the top of the tooth, respectively.

The equations for the derivation of the tooth width and cross-sectional area at the top chipped tip of the tooth are:

$$L_{chip} = L - w_x, x > d_{AD} \quad (9)$$

$$A_{chip} = 2h_x L - \frac{1}{2} w_x t_x, x > d_{AD} \quad (10)$$

As shown in Fig. 3(a), due to the occurrence of tooth apex chipping failure, at this time the area of meshing force action is changed and the center of mass will be shifted, according to the parallel axis theorem the rotational inertia $J_{V,r}$ of the whole cross-section and the rotational inertia $J_{V,t}$ at the chipping defect can be expressed as respectively:

$$J_{V,r} = \frac{2}{3} h_x^3 L + 2h_x L \Delta u_r^2 \quad (11)$$

$$J_{V,t} = \frac{1}{36} w_x t_x^3 + \frac{1}{2} w_x t_x \Delta u_t^2 \quad (12)$$

where Δu_r denotes the perpendicular distance from the center of mass Q_1 to V_1 , and Δu_t denotes the perpendicular distance from the center of mass Q_3 to the V_1 -axis, which is expressed by the equation:

$$\Delta u_r = \left[\frac{1}{2} w_x t_x \left(h_x - \frac{1}{3} t_x \right) \right] / \left[2h_x L - \frac{1}{2} w_x t_x \right] \quad (13)$$

$$\Delta u_t = \Delta u_r + h_x - \frac{1}{3} t_x \quad (14)$$

Therefore, the moment of inertia of the cross-section about the shaft with a toothed-top chipping failure can be inferred as:

$$J_V = J_{V,r} - J_{V,t} \quad (15)$$

Based on the inference and calculation of the above equations, the bending stiffness k_b , axial compression stiffness k_a , shear stiffness k_s , Hertzian contact stiffness k_h and rounded base stiffness k_f of the tooth apex chipping defect faults can be calculated, and the equations can be inferred as:

$$\begin{aligned} \frac{1}{k_b} = & \int_0^{d_{AB}} \frac{3[R_d - x_a \cos \beta_1 - x \cos \beta_1]^2}{2EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]^3} dx \\ & + \int_{-\beta_d}^{-\beta_a} \frac{3\{1 + \cos \beta_1[(\beta_2 - \beta) \sin \beta - \cos \beta]\}^2 (\beta_2 - \beta) \cos \beta}{2EL [\sin \beta + (\beta_2 - \beta) \cos \beta]^3} d\beta \\ & + \int_{-\beta_d}^{-\beta_a} \frac{R_d^3 \{1 + \cos \beta_1[(\beta_2 - \beta) \sin \beta - \cos \beta]\}^2 (\beta_2 - \beta) \cos \beta}{EJ_V} d\beta \end{aligned} \quad (16)$$

where E is Young's modulus.

$$\begin{aligned} \frac{1}{k_a} = & \int_0^{d_{AB}} \frac{\sin^2 \beta_1}{2EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]} dx \\ & + \int_{-\beta_d}^{-\beta_1} \frac{(\beta_2 - \beta) \cos \beta \sin^2 \beta_1}{2EL [\sin \beta + (\beta_2 - \beta) \cos \beta]} d\beta \\ & + \int_{-\beta_1}^{-\beta_d} \frac{R_d (\beta_2 - \beta) \cos \beta \sin^2 \beta_1}{EA_{chip}} d\beta \end{aligned} \quad (17)$$

where ρ_t is the radius of the transition curve, which can be defined as:

$$\rho_t = c^* m / (1 - \sin \alpha_0) \quad (18)$$

In the above equation, c^* is the tip clearance coefficient, m modulus, and m is pressure angle.

$$\begin{aligned} \frac{1}{k_s} = & \int_0^{d_{AB}} \frac{1.2(1+\nu) \cos^2 \beta_1}{EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]} dx \\ & + \int_{-\beta_d}^{-\beta_1} \frac{1.2(1+\nu)(\beta_2 - \beta) \cos \beta \cos^2 \beta_1}{EL [\sin \beta + (\beta_2 - \beta) \cos \beta]} d\beta \\ & + \int_{-\beta_1}^{-\beta_d} \frac{2.4(1+\nu) R_d (\beta_2 - \beta) \cos \beta \cos^2 \beta_1}{EA_{chip}} d\beta \end{aligned} \quad (19)$$

Based nonlinear contact forces[39], Based on nonlinear contact forces, the nonlinear Hertzian contact stiffness [40] can be expressed as

$$k_h = \frac{E^{0.9} L_{chip}^{0.8} F_i^{0.1}}{1.37}, F_i = F \times LSR_i \quad (20)$$

where F_i is the gear mesh force for the i st tooth team in contact and LSR_i is the load distribution ratio for the i st tooth pair.

The gear fillet-foundation deflection should also be considered in the total deflection. Corresponding fillet-foundation stiffness can be calculated according to Sainsot et al.[41].

$$\frac{1}{k_f} = \frac{\cos^2 \beta_1}{EL_{chip}} \left\{ L^* \left(\frac{u_f}{S_f} \right)^2 + M^* \left(\frac{u_f}{S_f} \right) + P^* (1 + Q^* \tan^2 \beta_1) \right\} \quad (21)$$

where β_1 is the pressure angle of the action point, and the parameters of L^* , M^* , P^* , Q^* , u_f , and S_f are given in Table 1.

Table 1

Values of coefficient

	A_i	B_i	C_i	D_i	E_i	F_i
$L^*(h_f, \theta_f)$	-5.574×10^{-5}	-1.9986×10^{-4}	-2.3015×10^{-4}	4.7702×10^{-3}	0.0271	6.8045
$M^*(h_f, \theta_f)$	60.111×10^{-5}	28.100×10^{-3}	-83.431×10^{-4}	-9.9256×10^{-3}	0.1624	0.9086
$P^*(h_f, \theta_f)$	-50.592×10^{-5}	185.50×10^{-3}	0.0538×10^{-4}	53.3×10^{-3}	0.2895	0.9236
$Q^*(h_f, \theta_f)$	-6.2042×10^{-5}	9.0889×10^{-3}	-4.0964×10^{-4}	7.8297×10^{-3}	-0.1472	0.6904

1.2 Calculation of TVMS for tooth tip chipping failure

1.2.1 Calculation of TVMS for tooth tip chipping failure without EHL

In the calculation of TVMS for chipped gear pair, the simplified computational model, as shown in Fig. 4, shows the meshing process of the gear pair at the defect during meshing, and the occurrence of chipping faults at the top of the teeth causes changes in the cross-sectional moments of inertia, cross-sectional area, and length of the meshing line.

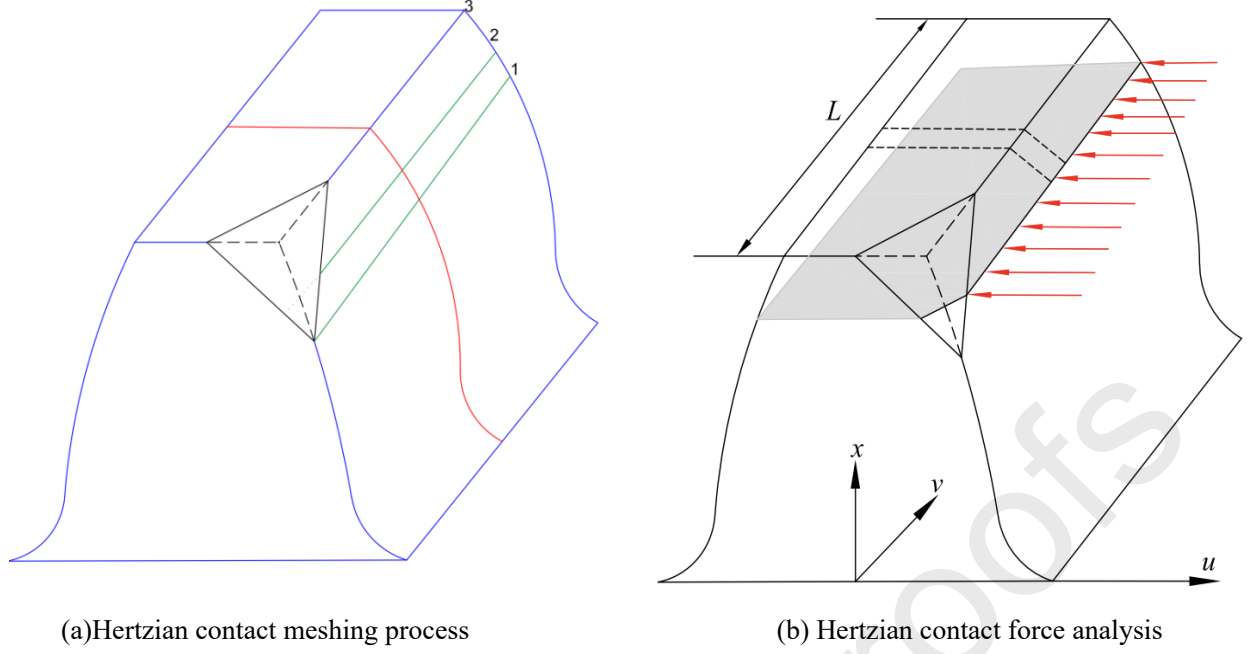


Fig. 4. Contact Force Diagram During Gear Meshing

If the elastic flow lubrication is not considered, the chipping defect EF has no contact during meshing, as shown in Fig. 4, at this time, the cross-sectional load-acting area is reduced, and the force is unbalanced, but the gears work in a complex environment, often in heavy oil environments, so it is very necessary to consider the elastic flow lubrication.

Under rigid conditions, the point of action of the total load is changed due to the torsional effect, which, together with the chipping tip notch at the top of the tooth, leads to a change in the center of mass. The position of the gear load force F_r is deflected from the vertical center axis u_1 by Δv_F . Therefore, the synthetic load will torsionally twist the toothed top chipped tooth with a torque whose torsional energy, according to the cantilever beam theory, is expressed as P_τ :

$$P_\tau = \frac{F^2}{2k_\tau} = \int_0^d \frac{T_e^2}{2GJ_D} dx \quad (22)$$

$$G \text{ being the shear modulus } G = \frac{E}{2(1+\nu)}.$$

J_D is the polar moment of inertia of polygon GHIJK, J_v is the rotational moment of inertia of the chipping tip defect region around the v_1 -axis, and J_u is the rotational moment of inertia of the chipping tip defect region around the u_1 -axis, which can be expressed as Eq:

$$J_D = J_v + J_u \quad (23)$$

Calculate the rotational moment of inertia $J_{u,r}$ of the healthy cross-section around u_1 and the rotational

moment of inertia $J_{u,t}$ of the chipped tip defect region around u_1 , respectively.

$$J_{u,r} = \frac{1}{6} h_x L^3 + 2 h_x L \Delta V_r^2 \quad (24)$$

$$J_{u,t} = \frac{1}{36} t_x w_x^3 + \frac{1}{2} w_x t_x \Delta V_t^2 \quad (25)$$

where ΔV_r denotes the horizontal distance from the center of mass Q_1 to the u_1 -axis and ΔV_t denotes the horizontal distance from the center of mass Q_3 to the u_1 -axis, which can be expressed by the equation.

$$\Delta V_r = \frac{\frac{1}{2} w_x t_x \left(\frac{l}{2} - \frac{1}{3} w_x \right)}{2 h_x L - \frac{1}{2} w_x t_x} \quad (26)$$

$$\Delta V_t = \Delta V_r + \frac{1}{2} L - \frac{1}{3} w_x \quad (27)$$

Then the rotational moment of inertia of the chipping defect region around the u_1 -axis after the occurrence of tooth apex chipping can be expressed by Eq.28.

$$J_u = J_{u,r} - J_{u,t}, x > d_{AD} \quad (28)$$

From there, the equivalent torque can be calculated:

$$T_e = F_r \Delta V_F \quad (29)$$

$$F_r = F \cos \beta_1 \quad (30)$$

$$\Delta V_F = \frac{w_x}{2} - \Delta V_r, x > d_{AD} \quad (31)$$

The torsional stiffness k_τ can be calculated as:

$$\frac{1}{k_\tau} = \begin{cases} 0, x < d_{AD} \\ \int_{-\beta_1}^{-\beta_d} \frac{2(1+\nu) \cos^2 \beta_1 \Delta V_F^2 R_b (\beta_2 - \beta) \cos \beta}{E J_D} d\beta, x > d_{AD} \end{cases} \quad (32)$$

Therefore, when a toothed top chipping failure occurs on gear 1, the meshing stiffness of the chipped single

tooth pair can be expressed as So when a toothed top chipping failure occurs on one of the gear toothed tops, the meshing stiffness of the single tooth pair of the chipped gear pair k_t is:

$$k_t = \frac{1}{\frac{1}{k_h} + \frac{1}{k_{b1}} + \frac{1}{k_{a1}} + \frac{1}{k_{s1}} + \frac{1}{k_{r1}} + \frac{1}{k_{f1}} + \frac{1}{k_{b2}} + \frac{1}{k_{a2}} + \frac{1}{k_{s2}} + \frac{1}{k_{f2}}} \quad (33)$$

To achieve a smooth transition between single-tooth and double-tooth meshing regions, a continuous function can be used to describe the variation of stiffness with respect to time or the gear rotation angle. For the single-tooth meshing region, the stiffness can be expressed as k_{single} , for the double-tooth meshing region, the stiffness can be expressed as k_{double} .

This continuous function would ensure that the stiffness transitions smoothly between the single-tooth and double-tooth meshing zones, maintaining stability in the gear system's dynamic behavior.

$$k(t) = k_{single}(1 - f(t)) + k_{double} \cdot f(t) \quad (34)$$

$$f(t) = \frac{1}{2}(1 - \cos(\pi \cdot \frac{t - t_0}{T})) \quad (35)$$

Here, $f(t)$ is a continuously varying interpolation function, typically taking values in the range [0,1], representing the smooth transition from single-tooth to double-tooth meshing.

When multiple pairs of single teeth have meshed simultaneously, the total TVMS k_t' for the multiple pairs of gears meshing in contact shall be determined as:

$$k_t' = \sum_{i=1}^N k_t^i \quad (36)$$

The related parameters of the gear pair are shown in Table 2.

Table 2

Basic Parameters of Gear Pair

Parameter	Value	Parameter	Value
Tooth number, Z_1/Z_2	25/53	Pressure angle($^\circ$),	20
Modules, m	3	Poisson's ratio, ν	0.3
Young's modulus(Pa)	2.068×10^{11}	Tooth width (mm)	38

1.2.2 Calculation of TVMS with EHL under gear tip chipping failure.

To better calculate the effect of chipping defect size on the TVMS under elastic flow lubrication, as well as to consider the case where the oil film can transmit the gear meshing force at the first instant of meshing, the model is shown in Fig. 5.

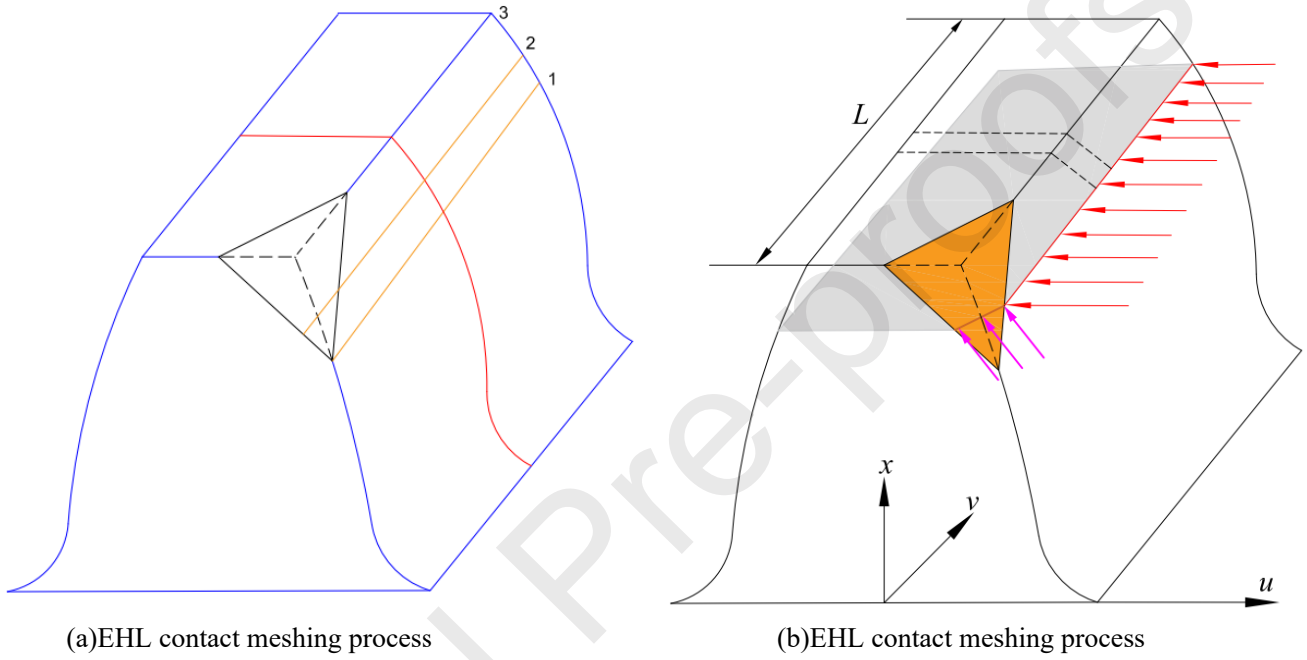


Fig. 5. Contact State at Gear Meshing with EHL

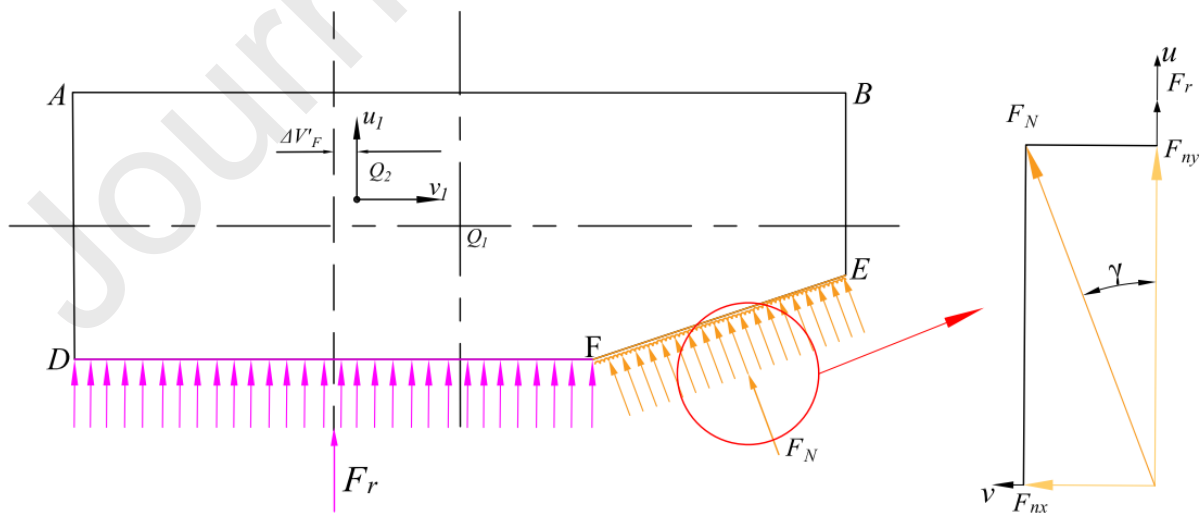


Fig. 6. Analysis of Torsional Effects and Force Distribution on Defective Gears under EHL

Lubrication conditions, as shown in Fig.6, when the chipping fault gear meshing, due to the presence of

lubricant between the gears, under the action of the meshing force, the chipping defects in the oil film can transfer the meshing force, according to the law of parallelogram force analysis, this time, the lubricant transfer of the meshing force F_N will produce a transverse force on the gears F_{nx} , the resulting chipping region of the asymmetric distribution of the force may cause torsional effect of gears, the torsional effect. The lubrication torsional stiffness is introduced to differentiate from the torsional effect under rigid conditions.

$$T_e' = (F_r + F_{ny})\Delta V_F' \quad (37)$$

$$\Delta V_F' = \frac{w_x}{2} - \Delta V_r', x > d_{AD} \quad (38)$$

$$k_r' = \begin{cases} 0, x < d_{AD} \\ \int_{-\beta_1}^{-\beta_d} \frac{2(1+\nu)\cos^2\beta_1\Delta V_F'^2 R_b(\beta_2 - \beta)\cos\beta}{EJ_D} d\beta, x > d_{AD} \end{cases} \quad (39)$$

Because of the relatively small size of the chipping defect, γ is close to zero, F_{nx} is very small, and the meshing force F_N transmitted by the oil film is almost equal to the gear meshing force, disregarding the moment F_{nx} along the y axis.

2 EHL model considering surface roughness.

The contact mode of the gear is line contact, the line contact model of the gear pair is shown in Fig. Along with the occurrence of chipping failure, the roughness at the defect gradually increases, where the line contact model of the Newtonian fluid considering the roughness, the Reynolds equation can be expressed as[42]:

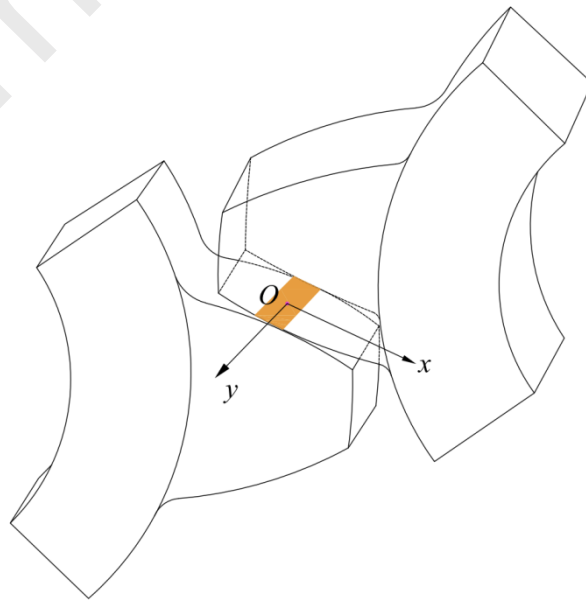


Fig. 7. Gear Line Contact Model

$$\frac{\partial}{\partial x} \left(\phi_x \frac{\rho h^3}{12\eta} \frac{\partial p_h}{\partial x} \right) + \frac{\partial}{\partial y} \left(\phi_y \frac{\rho h^3}{12\eta} \frac{\partial p_h}{\partial y} \right) = \left(\frac{u_1 + u_2}{2} \right) \frac{\partial(\rho h_T)}{\partial x} + \left(\frac{u_1 - u_2}{2} \right) \frac{\partial(\rho \phi_s)}{\partial x} \quad (40)$$

where P_h is the fluid dynamic water pressure, h is the oil film thickness, h_T is the average gap between the two surfaces, η and ρ are the viscosity and density of the lubricant, respectively, and ϕ_x and ϕ_y denote the pressure flow coefficients in the x and y directions, respectively[43], μ_1 and μ_2 represent the speeds of the master and slave gears, respectively.

Considering the Hertzian geometry and elastic deformation under elastic-fluid lubrication contact, the lubrication film thickness h can be expressed as:

$$h(x, y) = h_0(t) + \frac{x^2}{2R_x} + v(x, y, t) + \delta_1(x, y, t) + \delta_2(x, y, t) \quad (41)$$

where $h_0(t)$ denotes the initial film thickness, R_x denotes the local radius of curvature at the contact width, $\delta_1(x, y, t)$ and $\delta_2(x, y, t)$ denotes the surface roughness of the contact surface, respectively, and $v(x, y, t)$ is the elastic deformation with the following expression:

$$v(x, y, t) = \frac{2}{\pi E} \iint_{\Omega} \frac{p(\xi, \zeta) + p_c(\xi, \zeta)}{\sqrt{(x - \xi)^2 + (y - \zeta)^2}} d\xi d\zeta \quad (42)$$

where p is the fluid dynamic pressure, p_c is the rough contact pressure, Ω denotes the contact area of the gear pair, ξ and ζ are variables defining the contact area, and E is the effective modulus of elasticity.

In the Reynolds equation, the function of dynamic water pressure can be expressed in terms of viscosity and density:

$$\eta = \eta_0 \exp \left\{ (\ln \eta_0 + 9.67) \left[\left(1 + 5.1 \times 10^{-9} p_h \right)^{Z_0} - 1 \right] \right\} \quad (43)$$

$$\rho = \rho_0 \left[1 + \frac{0.6 \times 10^{-9} p_h}{1 + 1.7 \times 10^{-9} p_h} \right] \quad (44)$$

where η_0 is the initial viscosity, ρ_0 is the initial lubricating density, Z_0 is the coefficient of viscous pressure, and p_h is the fluid dynamic water pressure.

According to the literature[44][45][46][47], in the proposed elastic-plastic model, which incorporates elastic deformation, elastic-plastic deformation, and fully plastic deformation, the pressure exerted on the micro convex body can be expressed by Eq:

$$\begin{aligned}
P_a = & \frac{2}{3} E' n \beta^{0.5} \sigma^{1.5} \left(\frac{\sigma}{\sigma_s} \right) \frac{1}{\sqrt{2\pi}} \int_{h^* - y_s^*}^{h^* - y_s^* + w_1^*} w^* e^{-0.5(\sigma/\sigma_s \cdot z^*)^2} dz^* \\
& + 2\pi \cdot h_d \cdot n \beta \sigma \frac{1}{\sqrt{2\pi}} \left(\frac{\sigma}{\sigma_s} \right) \int_{h^* - y_s^* + w_2^*}^{\infty} w^* e^{-0.5(\sigma/\sigma_s \cdot z^*)^2} dz^* + \pi \cdot h_d \cdot n \beta \sigma \frac{1}{\sqrt{2\pi}} \left(\frac{\sigma}{\sigma_s} \right) \int_{h^* - y_s^* + w_1^*}^{h^* - y_s^* + w_2^*} w^* e^{-0.5(\sigma/\sigma_s \cdot z^*)^2} \\
& \times \left[1 - 0.6 \frac{\ln w_2^* - \ln w_1^*}{\ln w_2^* - \ln w_1^*} \right] \times \left[1 - 2 \left(\frac{w^* - w_1^*}{w_2^* - w_1^*} \right)^3 + 3 \left(\frac{w^* - w_1^*}{w_2^* - w_1^*} \right)^3 \right] dz^*
\end{aligned} \quad (45)$$

where $w^* = z^* h^* + y_s^*$, normalized by σ , h denotes the distance between two rough surfaces marked by a mean line, β , n denote the radius and density of the rough surface, respectively, and h_d denotes the Vickers hardness. w_1 , w_2 denote the critical interferences for initial yielding and full plastic flow, respectively, $w_2 = 54w_1$. σ_s and y_s denote the labeling deviation of the vertices and the distance of the mean line of the surface height from the surface vertices, respectively, and the following equations can be written according to McCool[48]:

$$\begin{cases} \sigma_s = \sqrt{1 - \frac{3.7169 \times 10^{-4}}{(n\beta\sigma)^2}} \sigma \\ \sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \\ y_s = \frac{0.0459}{n\beta\sigma} \sigma \end{cases} \quad (46)$$

The product of $n\beta\sigma$ can be viewed as a constant, which, according to relevant studies, can be set at 0.05[49].

By integrating the fluid dynamic pressure and contact pressure in the calculation area, the fluid dynamic pressure bearing capacity and contact bearing capacity are obtained respectively, and the total bearing capacity can be obtained by adding the two, which can be expressed as:

$$F_T = \iint_{A_h} p_h dx dy + \iint_{A_c} p_c dx dy \quad (47)$$

From the rheological properties of the lubricant, it can be seen that when the shear stress τ of the fluid is relatively small, when $\tau < \tau_0$, τ varies linearly with the shear rate and behaves as a Newtonian fluid; when $\tau > \tau_0$, τ varies nonlinearly with the shear rate and behaves a non-Newtonian fluid. In this paper, we address elastic fluid dynamics calculations, where the fluid shear stress is relatively large, and the shear stress can be derived from the Eyring model.

$$\tau_h = \tau_0 \arcsin h \left(\frac{\eta \dot{\gamma}}{\tau_0} \right) \quad (48)$$

where τ_0 is the Eyring shear stress, so the effect of surface roughness on the coefficient of friction under elastofluid lubrication can be expressed as:

$$\mu = \frac{F_h + F_a}{F_T} = \frac{\iint_{A_h} \tau_h(x, y) dx dy + \iint_{A_a} \mu_a p_a(x, y) dx dy}{\iint_{A_h} p_h dx dy + \iint_{A_a} p_c dx dy} \quad (49)$$

where μ_a is the boundary friction coefficient and the dynamic pressure zone area $A_h = A_{nom} - A_a$, where A_{nom} is the nominal area and A_a is the actual contact area.

At steady state, the oil film pressure on the contact zone should be equal to the applied load so that the equilibrium equation can be established:

$$\iint_{\Omega} p(x, y, t) dx = F(t) \quad (50)$$

where $F(t)$ denotes the meshing force of one tooth pair, and then the numerical solution of the above elastic-fluid equations can be computed based on the Thomas model[50] to obtain the oil film pressure p and the oil film thickness, and the normal oil film is computed by using the additional force ΔF in the contact force:

$$K_{film} = \frac{\Delta F}{\Delta h} = \frac{A \sum_{m=1}^M \sum_{k=1}^K \Delta p_{m,k}(t)}{\frac{1}{MK} \sum_{m=1}^M \sum_{k=1}^K \Delta h_{m,k}(t)} \quad (51)$$

where Δh denotes the additional deformation induced by ΔF , M and K denote the discrete increments of the contact film in the x and y directions, respectively. A refers to the contact film area. The relevant parameters are shown in Table 3.

Table 2

Parameters in the simulations

Parameter	Value	Parameter	Value
Lubricant ambient density (kg/m ³), ρ_0	890	Ree-Erying shear stress (Pa), τ_0	0.5×10^7
Lubricant ambient viscosity (Pa s), η_0	0.05	Lubricant ambient temperature (°C)	25

3 TVMS model considering surface roughness

The gear meshing process can be divided into meshing in and out, and friction in the different meshing processes to do the role of different directions, with the occurrence of chipping failure, the top of the tooth chipping defects at the surface roughness gradually increased, the friction coefficient is also increasing. The bending stiffness k_{ib} , axial compression stiffness k_{ia} , shear stiffness k_{is} , Hertzian contact stiffness k_{ih} , and fillet base stiffness k_{if} , for a tooth apex chipping defect fault considering surface roughness, can be expressed as:

$$\begin{aligned}
\frac{1}{k_{tb}} &= \int_0^{d_{AB}} \frac{3[R_d - \mu(x_a + x) \cos \beta_1]^2}{2EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]^3} dx \\
&+ \int_{-\beta_d}^{-\beta_1} \frac{3\{1 + \mu \cos \beta_1[(\beta_2 - \beta) \sin \beta - \cos \beta]\}^2 (\beta_2 - \beta) \cos \beta}{2EL [\sin \beta + (\beta_2 - \beta) \cos \beta]^3} d\beta \\
&+ \int_{-\beta_1}^{-\beta_d} \frac{R_d^3 \{1 + \mu \cos \beta_1[(\beta_2 - \beta) \sin \beta - \cos \beta]\}^2 (\beta_2 - \beta) \cos \beta}{EJ_V} d\beta
\end{aligned} \tag{52}$$

$$\begin{aligned}
\frac{1}{k_{ta}} &= \int_0^{d_{AB}} \frac{\mu^2 \sin^2 \beta_1}{2EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]} dx \\
&+ \int_{-\beta_d}^{-\beta_1} \frac{\mu^2 (\beta_2 - \beta) \cos \beta \sin^2 \beta_1}{2EL [\sin \beta + (\beta_2 - \beta) \cos \beta]} d\beta \\
&+ \int_{-\beta_1}^{-\beta_d} \frac{R_d (\beta_2 - \beta) \cos \beta \mu^2 \sin^2 \beta_1}{EA_{chip}} d\beta
\end{aligned} \tag{53}$$

$$\begin{aligned}
\frac{1}{k_{ts}} &= \int_0^{d_{AB}} \frac{1.2\mu^2(1+\nu)\cos^2 \beta_1}{EL \left[y_c - \sqrt{\rho_t^2 - (x_c - x_a - x)^2} \right]} dx \\
&+ \int_{-\beta_d}^{-\beta_1} \frac{1.2\mu^2(1+\nu)(\beta_2 - \beta) \cos \beta \cos^2 \beta_1}{EL [\sin \beta + (\beta_2 - \beta) \cos \beta]} d\beta \\
&+ \int_{-\beta_1}^{-\beta_d} \frac{2.4\mu^2(1+\nu)R_d(\beta_2 - \beta) \cos \beta \cos^2 \beta_1}{EA_{chip}} d\beta
\end{aligned} \tag{54}$$

$$k_h = \frac{E^{0.9} L_{chip}^{0.8} F_i^{0.1}}{1.275}, F_i = F \times LSR_i \tag{55}$$

where F_i is the gear meshing force for the i th tooth team in contact and LSR_i is the load distribution ratio for the i th tooth pair.

$$\frac{1}{k_{ff}} = \frac{\cos^2 \beta_1}{EL_{chip}} \left\{ L^* \left(\frac{uf}{Sf} \right)^2 + M^* \left(\frac{uf}{Sf} \right) + P^* (1 + Q^* \tan^2 \beta_1) \right\} \tag{56}$$

Parameters uf and Sf are gear parameters, while L^* 、 M^* 、 P^* and Q^* can be calculated using the following equations:

$$X_i^*(h_{fi}, \theta_f) = A_i / \theta_f^2 + B_i h_{fi}^2 + C_i h_{fi} / \theta_f + D_i / \theta_f + E_i h_{fi} + F_i \tag{57}$$

As shown in Fig.8, the illustrations depict Hertzian contact under smooth surfaces in Fig.8(a), smooth EHL contact in Fig.8(b), and rough surface EHL contact in Fig.8(c). Due to differences in surface morphology and lubrication, elastohydrodynamic lubrication, and surface roughness are indispensable components in the calculation of time-varying mesh stiffness.

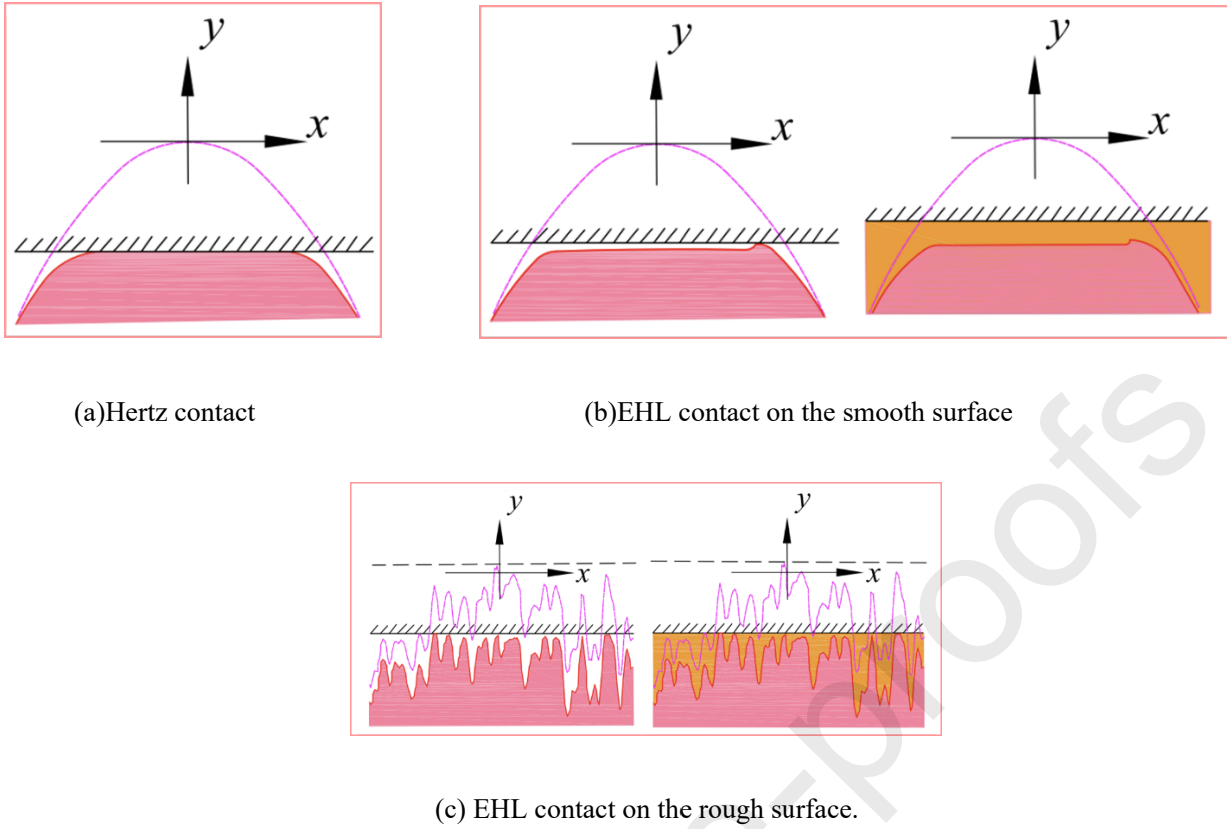


Fig. 8. Elastohydrodynamic Lubrication Stiffness Model

4 Results and Discussion

4.1 The impact of chipping size on TVMS

Time-varying meshing stiffness is the gear in the meshing process, due to the elastic deformation produced by the force of the gear teeth to resist the ability, and with the rotation of the gear and producing cyclical changes, the increase in the size of the chipping tip will increase the gear meshing process of the shock and vibration, aggravate the wear of the tooth surface, make the gear transmission performance decline, but also will make the time-varying mesh stiffness size and change law more complex and unstable. To investigate the effect of chipping defect size on the time-varying meshing stiffness, the height l_d , width w_d , and depth t_d of the chipping defects are investigated separately. The chipping dimensions are varied in three directions as shown in Fig. 9.

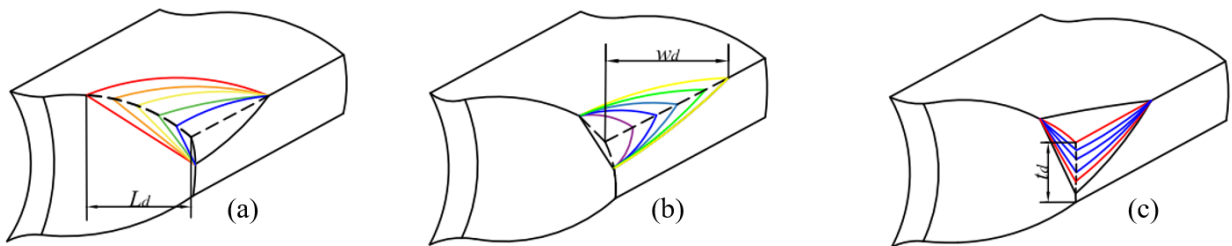


Fig. 9. Different locations of gear tooth tip chipping failure: (a) Chipping defect height; (b) Chipping defect width;

As shown in Fig.10, with a fixed defect width and depth, the impact of varying defect height on TVMS is analyzed. When the height of the edge chipping defect increases, the effect of the defect on TVMS becomes more pronounced. Additionally, as the defect height increases, the reduction in TVMS becomes significantly larger. Therefore, variations in the height of the tip chipping defect can influence both the magnitude of TVMS and the duration of the defect's impact.

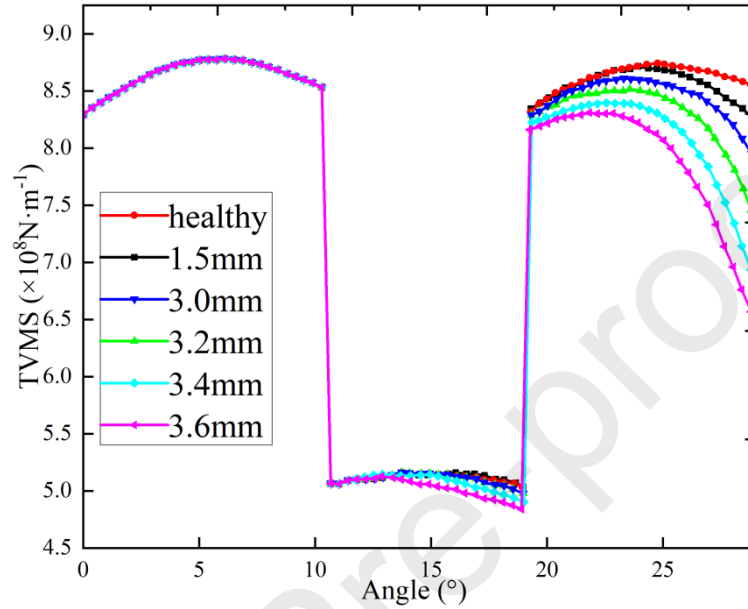


Fig. 10. The effect of defect height on TVMS

As shown in Fig.11, the impact of defect width on TVMS is analyzed, with defect height and depth held constant while varying the defect width. It is evident that as the tip chipping defect occurs, the TVMS curve decreases further with the increase in defect width. This indicates that defect width only affects the magnitude of TVMS, but does not influence the duration of the defect's impact.

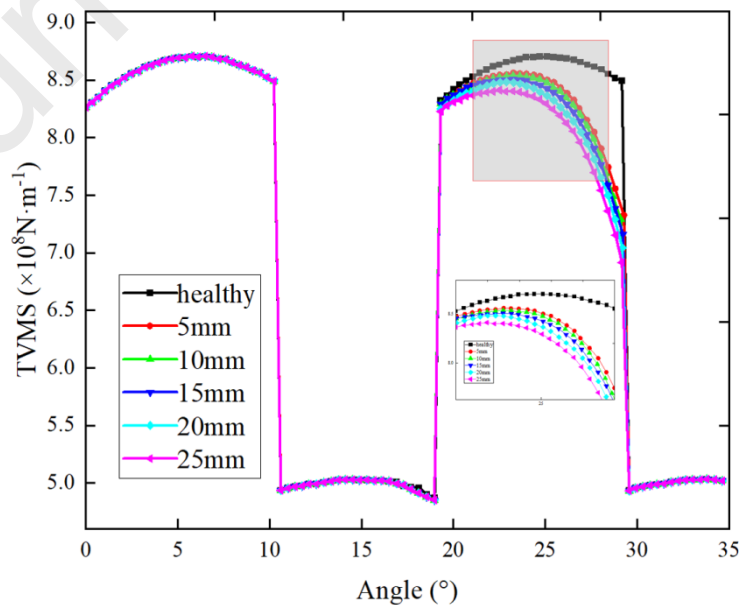


Fig 11 The influence of defect width on TVMS

When the defect height and defect width are certain, the effect of defect depth on the time-varying engagement stiffness is investigated, as shown in Fig. 12, which exhibits the variation of the defect depth on the time-varying engagement stiffness curve. Unlike the changes brought about by defect height and defect depth, the change in defect depth does not affect the tooth width and the extent of the defect, but it affects the cross-sectional area, which leads to a change in the rotational inertia, and therefore the effect of defect depth on the time-varying meshing stiffness is very limited.

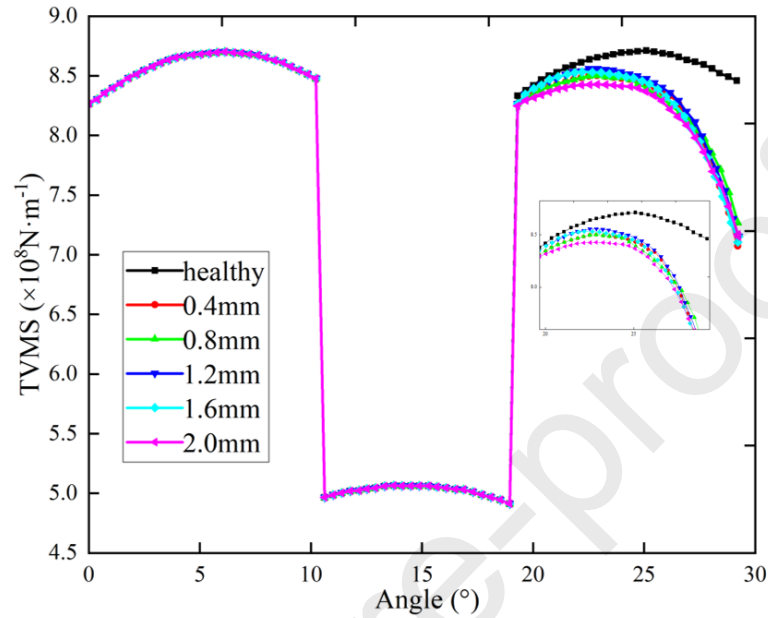


Fig. 12. The effect of defect depth on TVMS

4.2 The influence of torsional effects

Torsional effects and their influence on asymmetric forces are investigated using TVMS modeling of chipping tip failures under elastic-fluid lubrication. To better characterize the influence of torsional effects, a torsional stiffness can be introduced. Torsional stiffness, as an own property of gear stiffness, is the ability of a gear to exhibit resistance to deformation when subjected to a torque, which is added to the total TVMS to assess the impact and significance of torsional stiffness. The results were studied and analyzed through three chipping tip defect sizes, as shown in Table 4, and whether torsional stiffness was introduced as a variable, for which three assumptions needed to be made: It is assumed that chipping defects and lubricants do not introduce friction; Assume that the temperature is always constant; Assuming that the gear pair is adequately lubricated, the oil film at the defect transmits the meshing force.

Table 4

Three Types of Defect Sizes

A slight chip case			A medium chip case			A serious chip case		
l_d	w_d	t_d	l_d	w_d	t_d	l_d	w_d	t_d

As shown in Fig.13-Fig.15, the corresponding results for minor tip chipping defects, moderate tip chipping defects, and severe tip chipping defects are respectively presented. In the case of minor tip chipping defects, because the chipping size is relatively small, the oil film can quickly fill the defect and transmit the meshing force under the push of the meshing force. The load contact area remains essentially unchanged, reducing the asymmetry in load distribution. EHL can mitigate the torsional effect caused by tip chipping failure; under rigid conditions, the torsional effect is influenced by the small load contact area, affecting the rotational inertia. However, at this time, the tooth tip chipping defect is minor, and the asymmetry in load distribution is not significant, so the torsional effect has a very limited impact on the stiffness of the tooth surface.

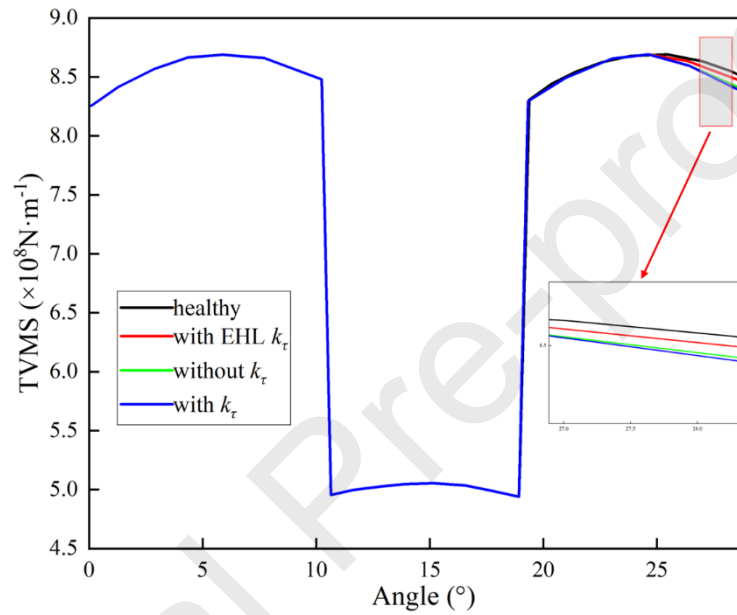


Fig.13. The influence of small defect torsional stiffness on TVMS

In the case of moderate tip chipping defects, as shown in Fig. 14, as the size of the tip chipping increases, the differences become more pronounced, although the overall differences are not obvious from the figures. However, due to the increased size of the tip chipping defects at this time, the oil film cannot quickly fill the defects under the action of the meshing force, resulting in less smooth transmission of the meshing force. At this time, the tip chipping failure under EHL has a more significant impact on the TVMS.

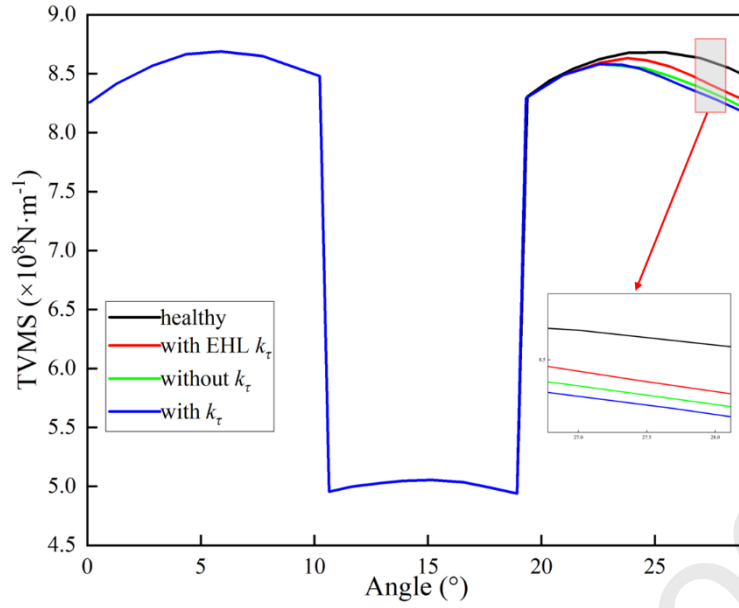


Fig.14. The influence of moderate defect torsional stiffness on TVMS

In the case of severe tooth chipping defects, as shown in Fig. 15, the load transmission capacity of the lubricating oil during the gear meshing process is significantly weakened with the increase in chipping defect size. This leads to a reduced ability to cushion and distribute the meshing forces effectively, and it also increases the friction coefficient in the contact areas of the gear surfaces.

From the perspective of time-varying mesh stiffness, the combined effect of tooth chipping and insufficient lubrication results in noticeable fluctuations in gear mesh stiffness. Firstly, the tooth profile changes caused by the tooth chipping reduce the contact area between the meshing teeth, which weakens the load-bearing capacity at the contact points. The center of mass shifts, and k_t' gradually approaches the torsional stiffness under rigid conditions, manifesting as periodic decreases in time-varying mesh stiffness. Secondly, as the effectiveness of the lubricant diminishes, the friction between the gears increases, making the dynamic characteristics of the mesh stiffness more unstable

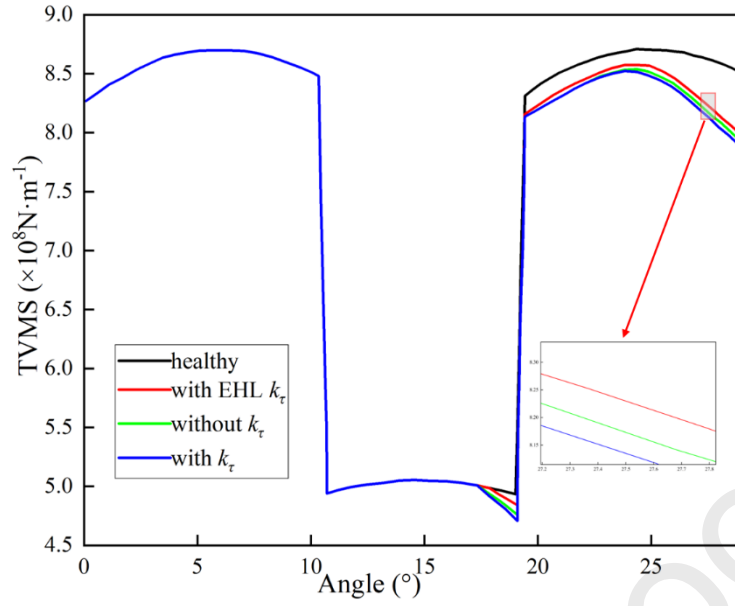


Fig.15. The influence of severe defect torsional stiffness on TVMS

Overall, the influence of torsional stiffness on the total TVMS under rigid conditions is not particularly significant. The primary reasons are as follows: The total TVMS of a gear pair is determined by a combination of factors including the Hertzian contact stiffness, single tooth stiffness, and base circle stiffness. Torsional stiffness is merely one component of single-tooth stiffness, which also encompasses bending stiffness, axial compression stiffness, and shear stiffness. Consequently, the impact of each component on the total TVMS is limited. However, under EHL, the lubricant's ability to transmit meshing forces balances the load, thereby reducing the effect of tip-chipping defects on TVMS. This demonstrates that torsional stiffness is a crucial parameter for the calculation of TVMS in the presence of tip-chipping defects.

4.3 The effect of surface roughness on friction coefficient

Under different tooth surface roughness conditions, the distribution of oil film thickness is depicted in Fig.16, where all parameters except surface roughness are constants. As surface roughness increases, the oil film thickness also increases, a relationship influenced by the flow factor in the modified Reynolds equation. Furthermore, with increasing roughness, the minimum film thickness location approaches the contact area's center. Additionally, as surface roughness increases, central and minimum film thickness values tend to converge.

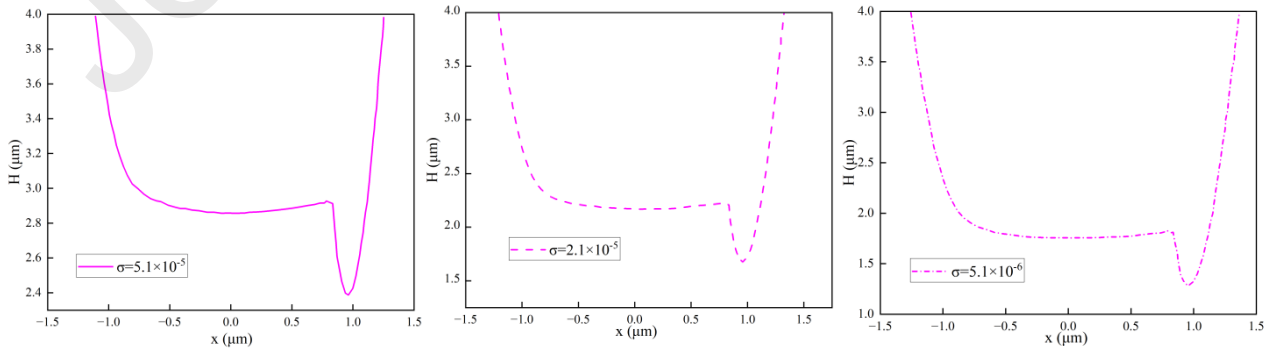


Fig.16. The effect of surface roughness on film thickness

According to the above formula, the influence of surface roughness on friction coefficient under EHL conditions can be calculated. As depicted in Fig.17, during the meshing process of gears, the friction coefficient increases with surface roughness. This increase occurs due to the formation of more tiny contact points during mutual contact, leading to an increase in contact area. As the contact points change with gear rotation and contact status, the friction coefficient reaches its maximum at the tooth root or crest and gradually decreases thereafter. Particularly in the area of tip chipping, where the cross-section is irregularly protruding and the surface is rough, fluid lubrication effectiveness is compromised, resulting in increased frictional resistance.

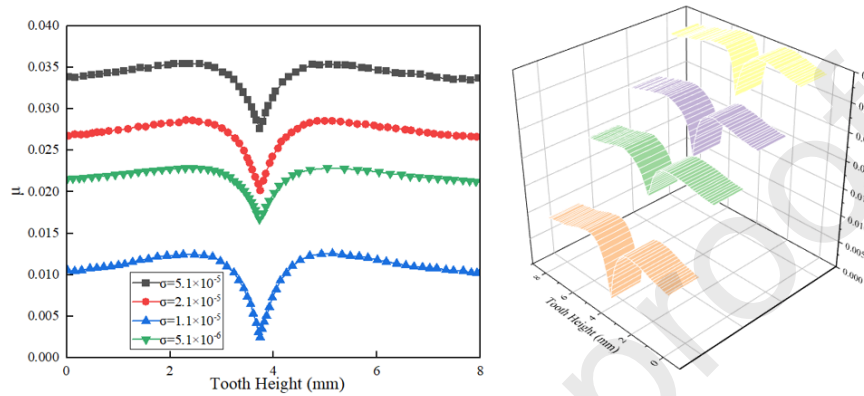


Fig.17. The influence of surface roughness on tooth surface friction coefficient

As shown in Fig.18, with the occurrence of tip chipping failure, the number and height of asperities increase accordingly. As surface roughness gradually increases, these asperities come into contact more frequently during the gear meshing process, leading to increased local pressure at the contact points. The presence of these asperities prevents the complete formation and maintenance of the lubricating oil film at these contact points, thereby reducing the load-bearing capacity of the oil film and further exacerbating the load pressure on the asperities.

Increasing roughness causes the distribution of asperities on the gear surface to become more uneven. This uneven distribution results in uneven pressure distribution on the gear contact surface, leading to localized high-pressure and low-pressure areas. As roughness increases, the height of pressure peaks decreases, and with high roughness, these peaks can nearly disappear.

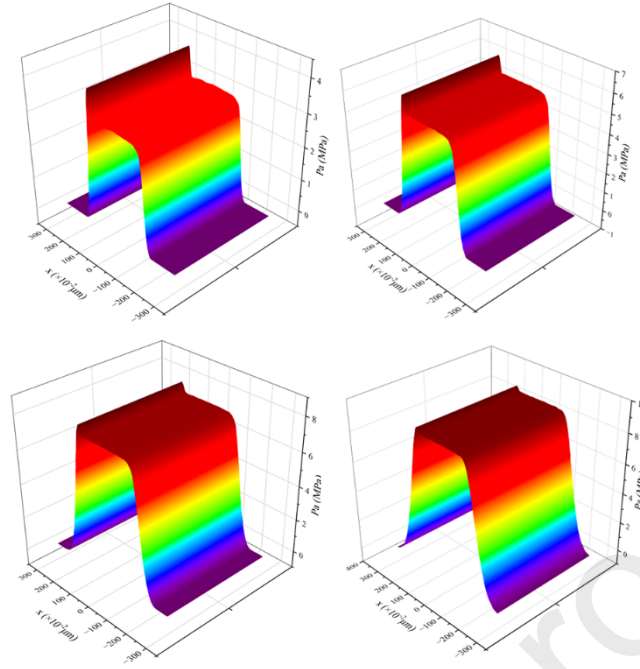


Fig.18. The effect of surface roughness on the distribution of contact pressure on rough surfaces

4.4 The effect of surface roughness on TVMS

As shown in Figure 19, when oil film lubrication is not considered, the TVMS curve begins to exhibit a certain degree of fluctuation as surface roughness gradually increases. Moreover, the degree of fluctuation becomes more pronounced with greater surface roughness. This is because surface roughness leads to microscopic irregularities on the gear meshing contact surfaces, causing localized variations in contact stiffness. When the roughness is high, the actual contact area is smaller, resulting in greater fluctuations in localized contact stiffness.

The surface roughness changes also cause dynamic load distribution during the gear meshing process. When rough surfaces lead to uneven load distribution on the tooth surfaces, it results in fluctuations in TVMS. Additionally, rough surfaces increase friction and wear during the gear meshing process. The increase in friction force leads to changes in contact stiffness, while wear alters the tooth surface morphology, further affecting TVMS.

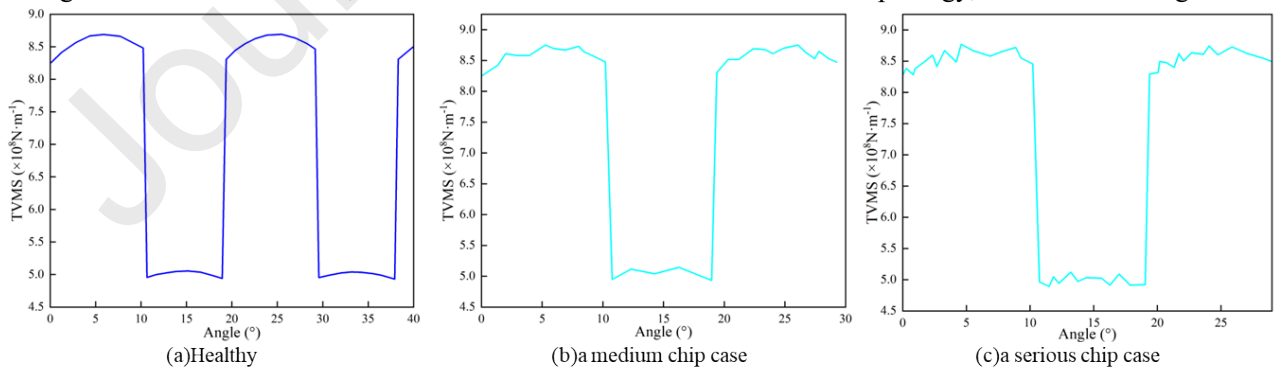


Fig.19. Time-varying mesh stiffness of gear pairs under different roughness levels

As shown in Fig.20, the calculation of oil film stiffness is illustrated for healthy and gear tooth tip chipping failure conditions. The study reveals that both healthy and chipped gear pairs exhibit significant fluctuations in oil

film stiffness during alternate meshing cycles of gear pairs. These fluctuations are attributed to the oscillations in gear tooth loads. Notably, gear tooth tip chipping intensifies the oscillations in gear load variations. Compared to the oil film stiffness in healthy conditions, tip chipping failure results in an increase in oil film stiffness during the period of chipped tooth engagement, particularly at the moment of chipping, where a sudden mutation occurs.

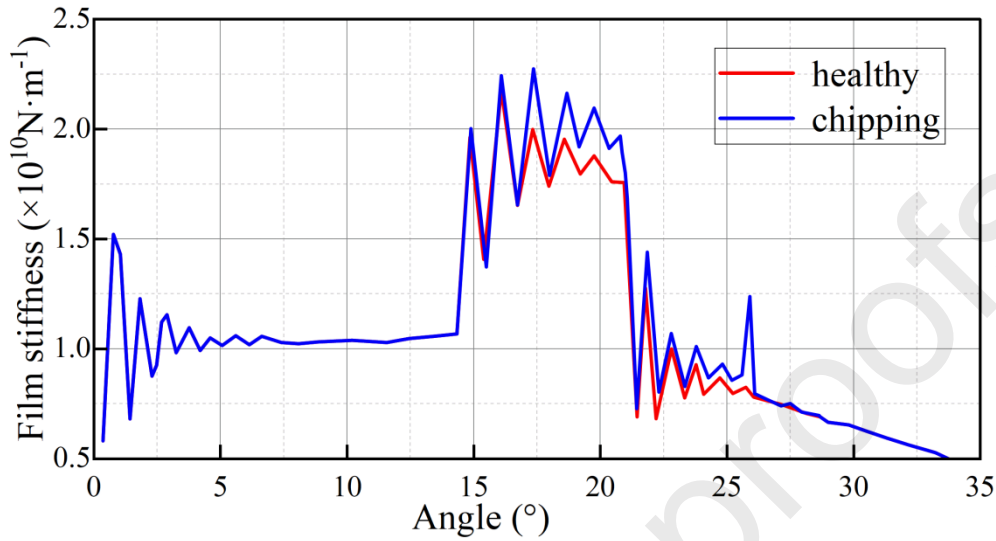


Fig.20. EHL Stiffness Considering Surface Roughness

In the context of tip chipping, oil film stiffness refers to the resistance generated by the oil film against relative motion between the meshing surfaces of gears. Under chipped conditions, an increase in surface roughness of the gear teeth makes it difficult for the oil film to form effectively at the tooth tips, thereby affecting the oil film stiffness. As depicted in Fig.21, with an increase in the size of gear tooth tip chipping defects, there is a significant alteration in the geometric shape of the meshing area, leading to a reduction in the effective contact area. This reduction impairs the formation and maintenance of the oil film, thereby influencing the TVMS.

As the tooth chipping enlarges, the thickness of the oil film decreases, leading to time-varying characteristics of the oil film stiffness, which gradually diminishes. For smaller tooth chipping sizes, the change in oil film stiffness is relatively minor; however, with larger tooth chipping sizes, the reduction in oil film stiffness becomes more pronounced. In the early stages of tooth chipping formation, there is an increase in pressure concentration within the contact area. As the chipping size continues to grow, the failure of the oil film and the irregularities of the contact surface cause the mesh stiffness to start decreasing.

When the tooth chipping size is small, the oil film can still maintain a certain thickness and stiffness, resulting in a relatively uniform pressure distribution in the gear contact area, and the time-varying characteristics of mesh stiffness remain stable. However, as the chipping size increases, local pressure rises, and the oil film stiffness gradually decreases, leading to a possible initial increase in mesh stiffness, followed by a decrease as the oil film thickness reduces. When the tooth chipping size becomes large, the oil film can hardly sustain sufficient thickness, and the contact surface may exhibit direct metal-to-metal contact, causing a significant drop in mesh stiffness.

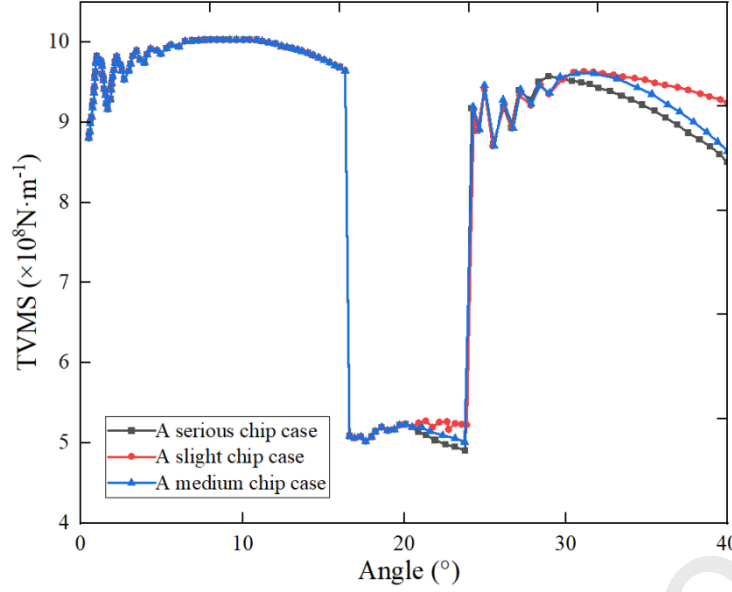


Fig.21. TVMS with Oil Film Stiffness for Different tooth tip Chipping Sizes

4.5 Verification of Stiffness Calculation

To verify the accuracy of the time-varying mesh stiffness (TVMS) calculations under different defects, a comparison is made with the dynamic analysis of a straight cylindrical gear rotor system with tooth tip chipping, based on an improved TVMS model proposed in reference [26]. This model accounts for the calculation of TVMS under the torsional effects of tooth tip chipping in frigid conditions. The method proposed in this paper is compared with the results from the reference, as shown in Figure 22. The variation trends of TVMS under different defects calculated by the proposed method are largely consistent with those from the reference. Furthermore, the TVMS calculations under EHL considered in this paper also align with the general trend. The results demonstrate that this method not only accurately computes TVMS under EHL but also achieves a high level of precision in the calculations.

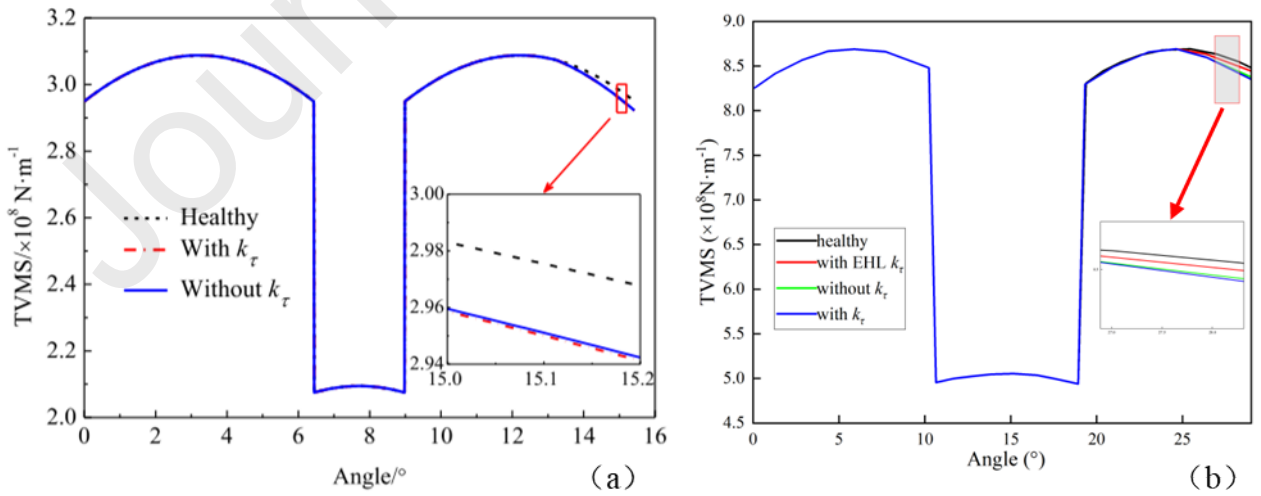


Fig.22. Comparisons of (b) the proposed method and (a) results in Ref. [26]

5 Conclusion

This paper proposes a time-varying mesh stiffness calculation method for gear tooth tip chipping failure considering EHL. Based on this method, the influence of tip chipping failure under EHL on TVMS is studied. The effects of surface roughness and oil film stiffness under chipped conditions on TVMS are also considered. The conclusions drawn from this study are as follows:

1. Under rigid conditions, the size of tip chipping defects decreases the TVMS, and as the size of the chipping increases, the reduction in TVMS also increases. Under EHL, the oil film transmits meshing forces at the tooth tip chipping defect, balancing the load and mitigating the impact of tooth tip chipping defects on TVMS. Therefore, torsional stiffness is an important indicator.

2. As the surface roughness increases, the oil film thickness between contacting tooth surfaces increases while dynamic pressure decreases. Consequently, to maintain total pressure amplitude, surface roughness significantly increases. The larger the surface roughness, the higher the friction coefficient, which is attributable to the larger friction coefficient caused by greater surface roughness. Under various EHL conditions with different surface roughnesses, the friction coefficient sharply decreases when the sliding-to-rolling ratio at nodes is zero, indicating pure rolling friction.

3. As the size of gear tooth tip chipping defects increases, it significantly alters the geometric shape of the meshing area, reducing the effective contact area. This decrease in contact area diminishes the ability to form and maintain the oil film, thereby affecting the time-varying mesh stiffness.

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Declaration of interests

- ☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
- ☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

Fig. 1. Model of the tooth tip chipping

Fig. 2. Schematic diagram of a healthy gear

Fig. 3. Model of gear tooth tip chipping: (a) Tooth tip Cross-section Diagram; (b) Cantilever Beam Model

Fig. 4. Contact Force Diagram During Gear Meshing

Fig. 5. Contact State at Gear Meshing with EHL

Fig. 6. Analysis of Torsional Effects and Force Distribution on Defective Gears under EHL

Fig. 7. Gear Line Contact Model

Fig. 8. Elastohydrodynamic Lubrication Stiffness Model

Fig. 9. Different locations of gear tooth tip chipping failure: (a) Chipping defect height; (b) Chipping defect width; (c) Chipping defect depth

Fig.10. The effect of defect height on TVMS

Fig. 11. The influence of defect width on TVMS

Fig. 12. The effect of defect depth on TVMS

Fig.13. The influence of small defect torsional stiffness on TVMS

Fig.14. The influence of moderate defect torsional stiffness on TVMS

Fig.15. The influence of severe defect torsional stiffness on TVMS

Fig.16. The effect of surface roughness on film thickness

Fig.17. The influence of surface roughness on tooth surface friction coefficient

Fig.18. The effect of surface roughness on the distribution of contact pressure on rough surfaces

Fig.19. Time-varying mesh stiffness of gear pairs under different roughness levels

Fig.20. EHL Stiffness Considering Surface Roughness

Fig.21. TVMS with Oil Film Stiffness for Different tooth tip Chipping Sizes

Fig.22. Comparisons of (b) the proposed method and (a) results in Ref. [26]

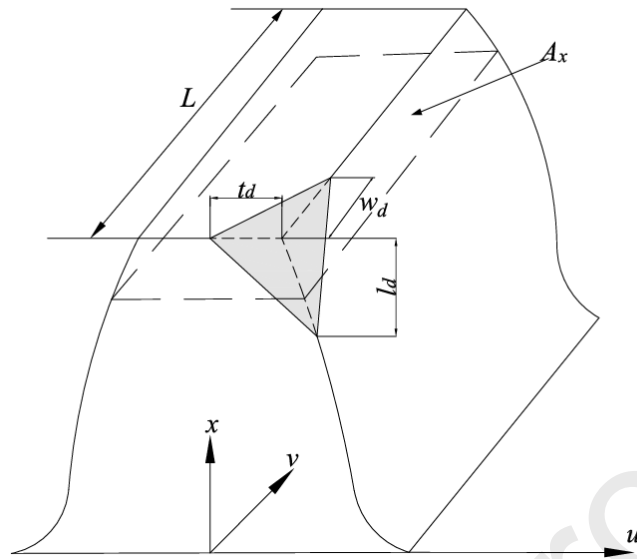


Fig. 23. Model of the tooth tip chipping

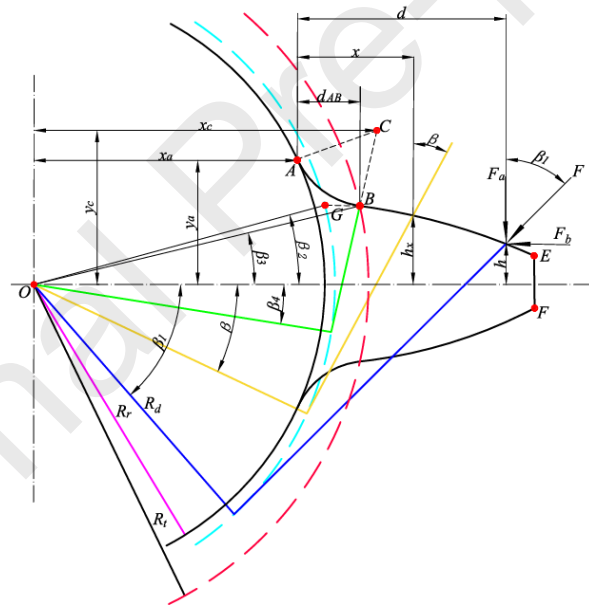
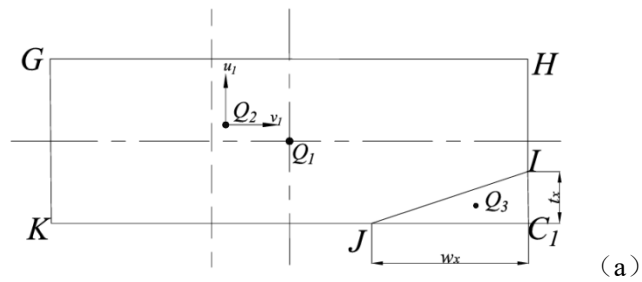
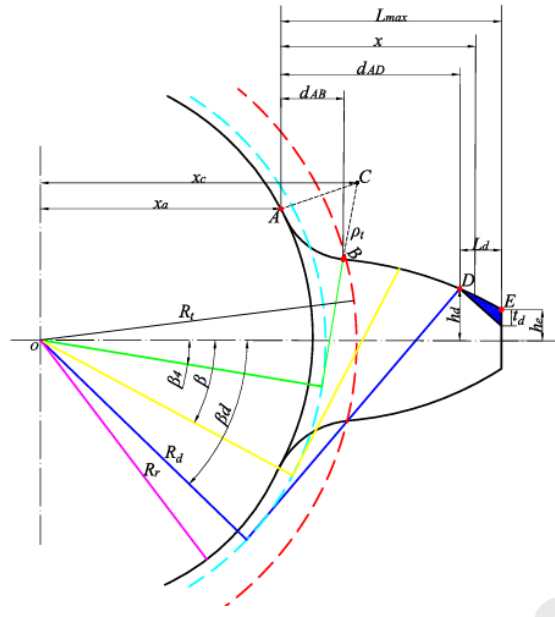


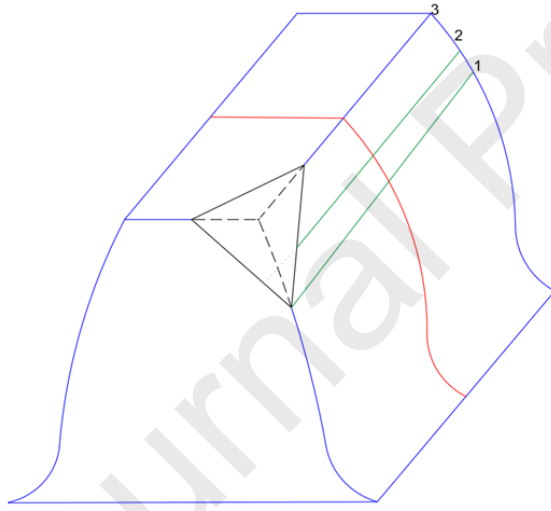
Fig. 24. Schematic diagram of a healthy gear



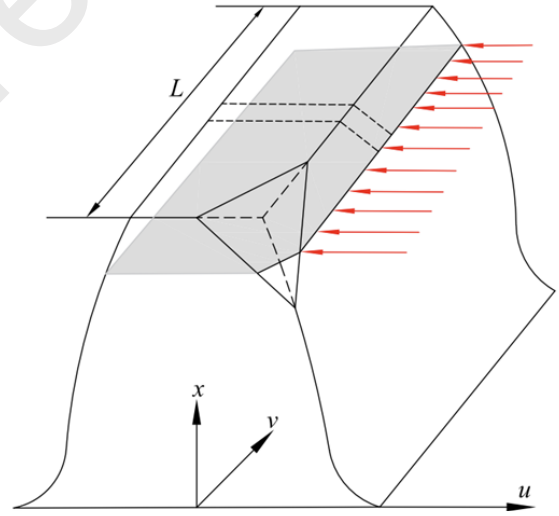


(b)

Fig. 25. Model of gear tooth tip chipping: (a) Tooth tip Cross-section Diagram; (b) Cantilever Beam Model



(a)Hertzian contact meshing process



(b) Hertzian contact force analysis

Fig. 26. Contact Force Diagram During Gear Meshing

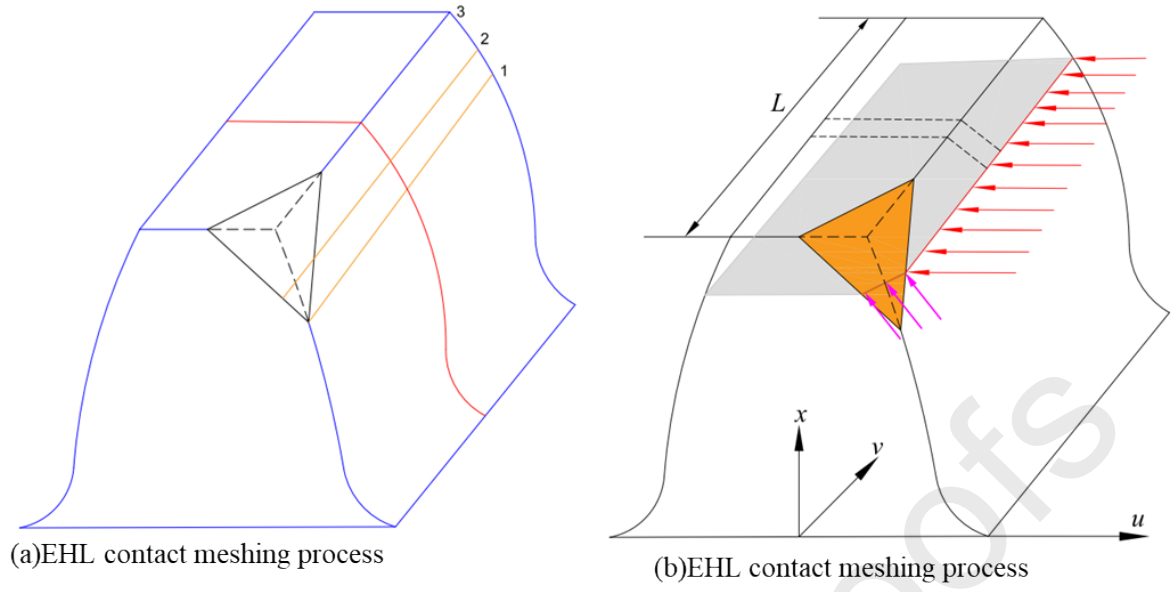


Fig. 27. Contact State at Gear Meshing with EHL

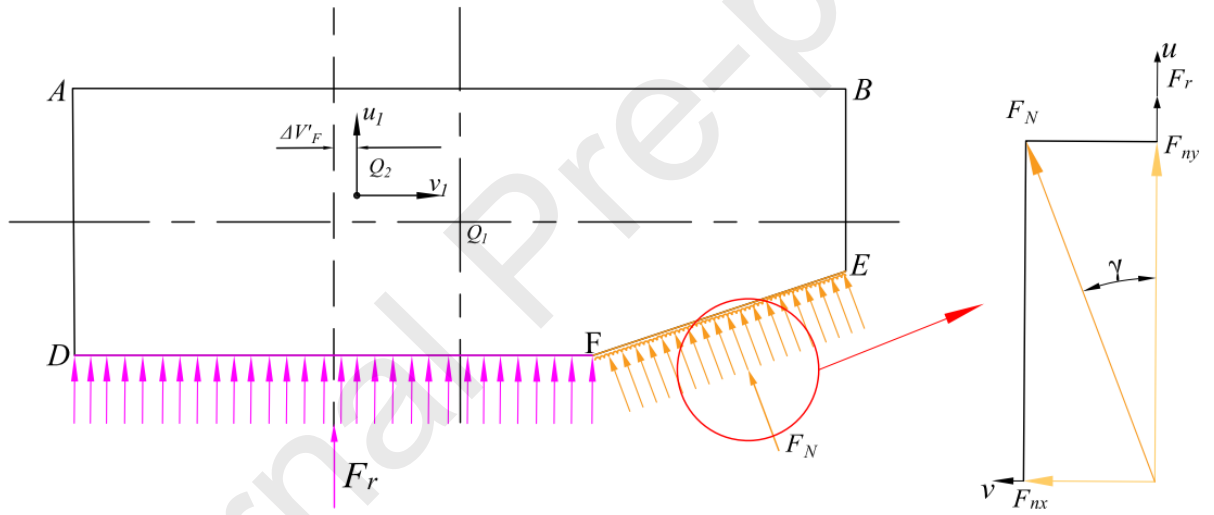


Fig. 28. Analysis of Torsional Effects and Force Distribution on Defective Gears under EHL

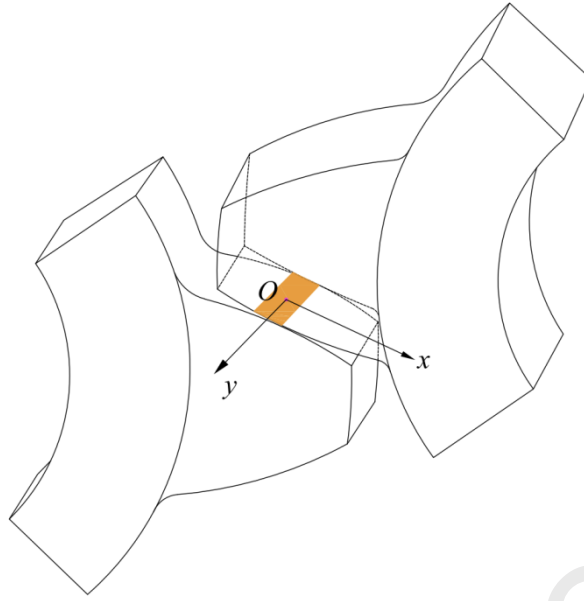
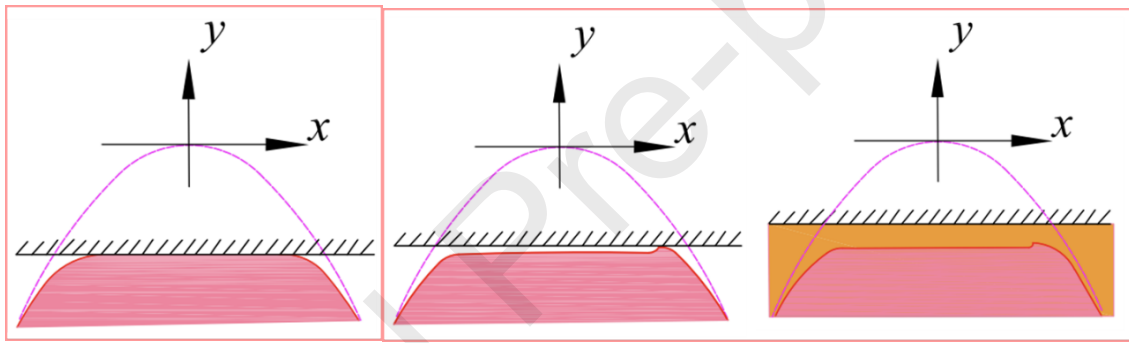
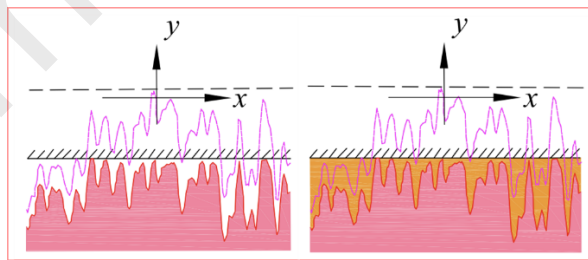


Fig. 29. Gear Line Contact Model



(a) Hertz contact

(b) EHL contact on the smooth surface



(c) EHL contact on the rough surface.

Fig. 30. Elastohydrodynamic Lubrication Stiffness Model

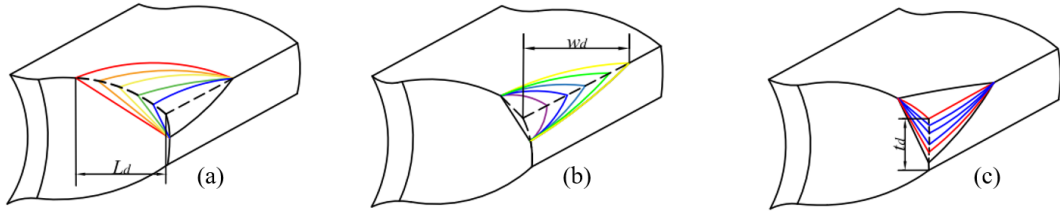


Fig. 31. Different locations of gear tooth tip chipping failure: (a) Chipping defect height; (b) Chipping defect width; (c) Chipping defect depth

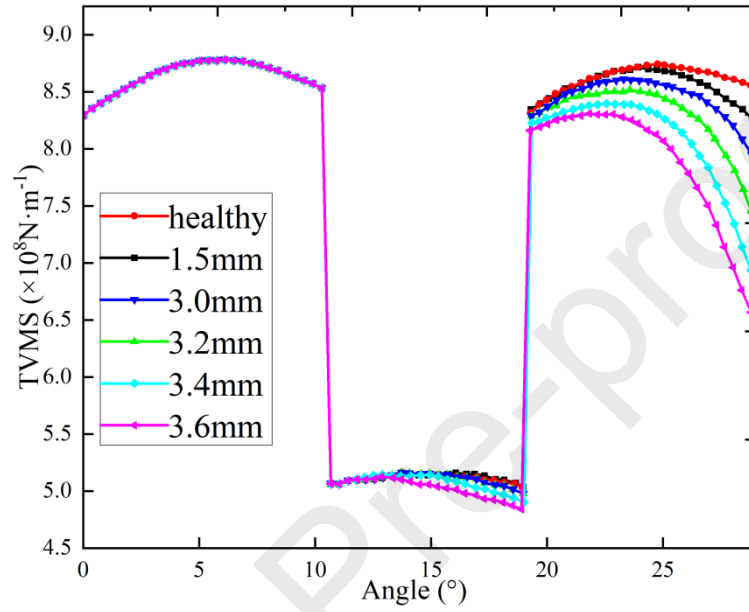


Fig.32. The effect of defect height on TVMS

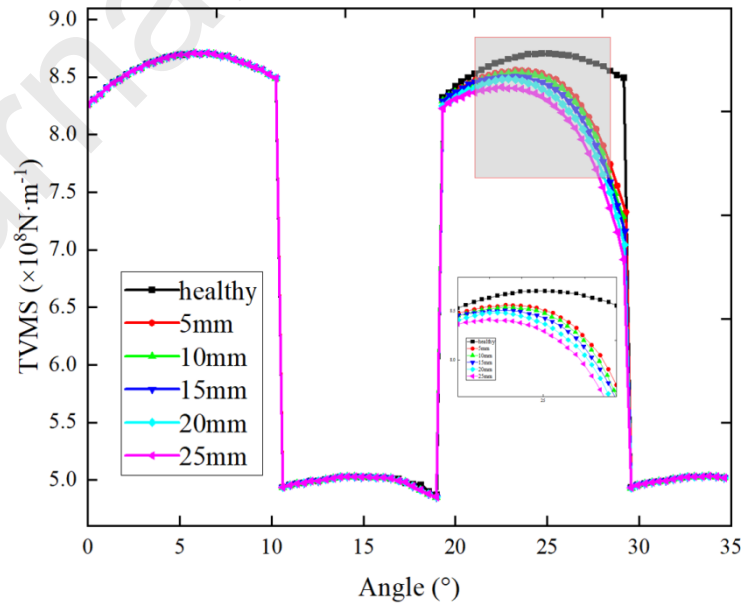


Fig. 33. The influence of defect width on TVMS

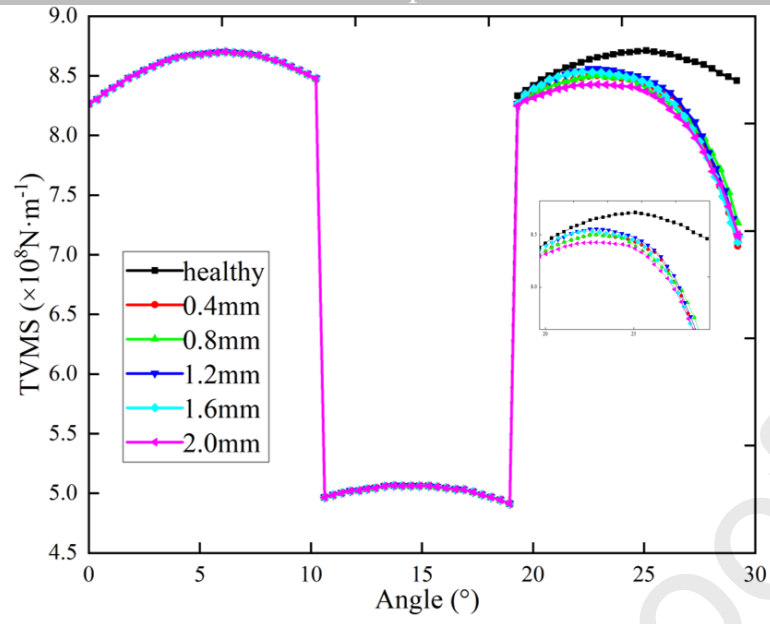


Fig. 34. The effect of defect depth on TVMS

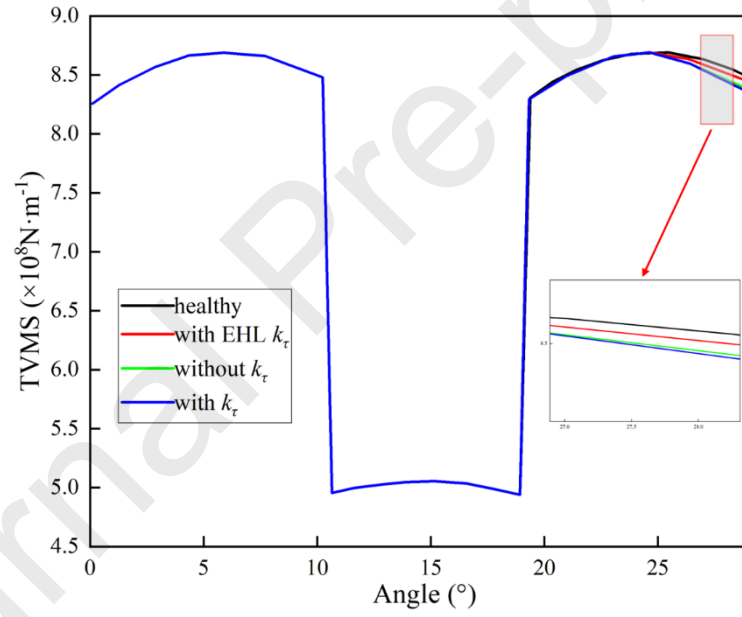


Fig.35. The influence of small defect torsional stiffness on TVMS

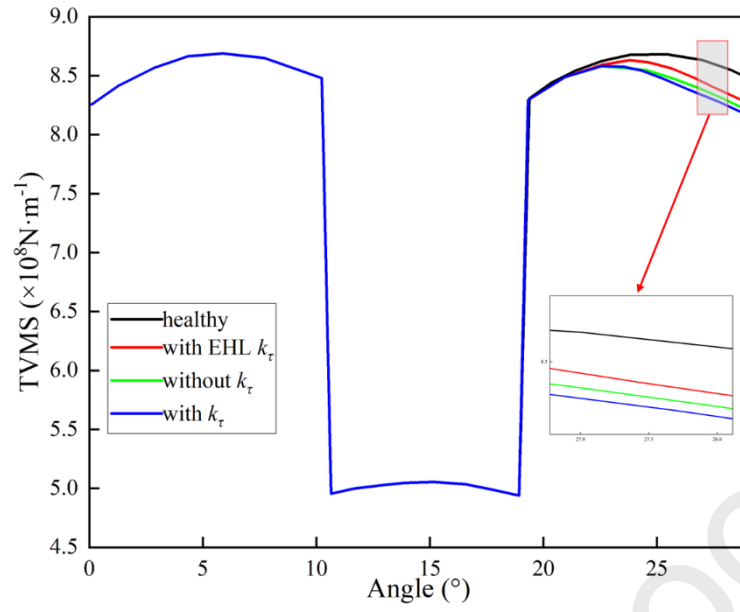


Fig.36. The influence of moderate defect torsional stiffness on TVMS

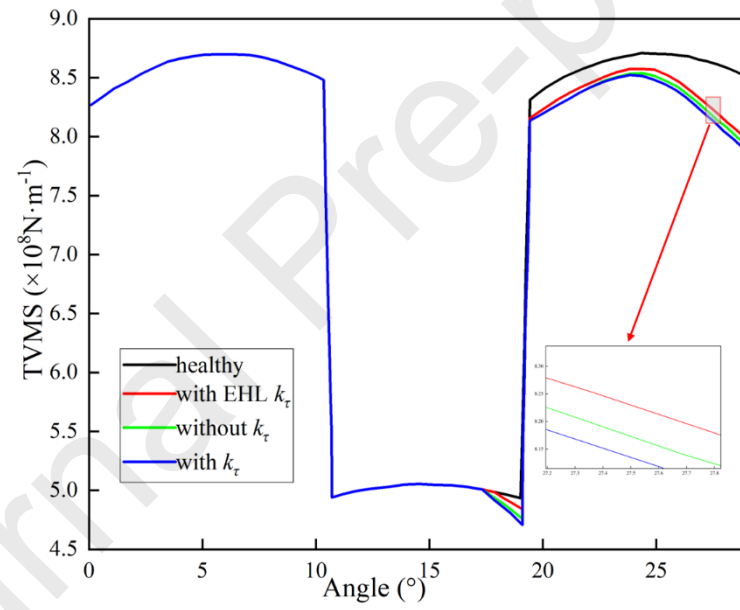


Fig.37. The influence of severe defect torsional stiffness on TVMS

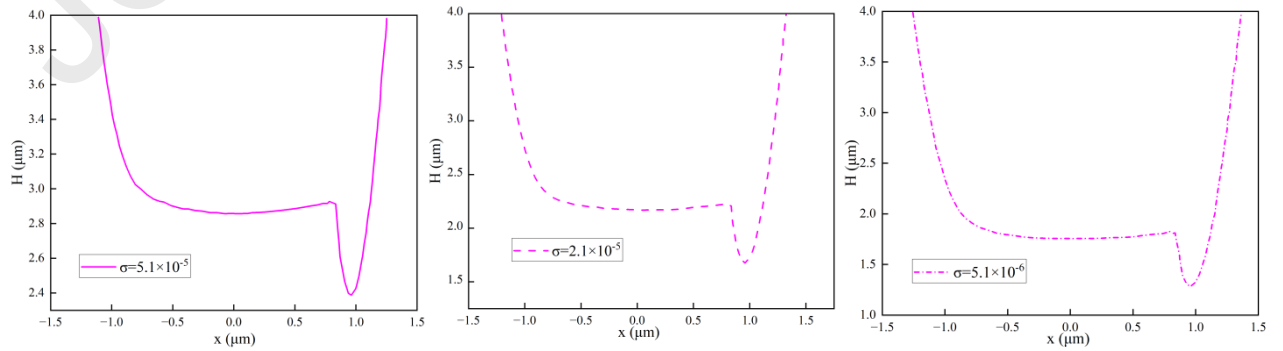


Fig.38. The effect of surface roughness on film thickness

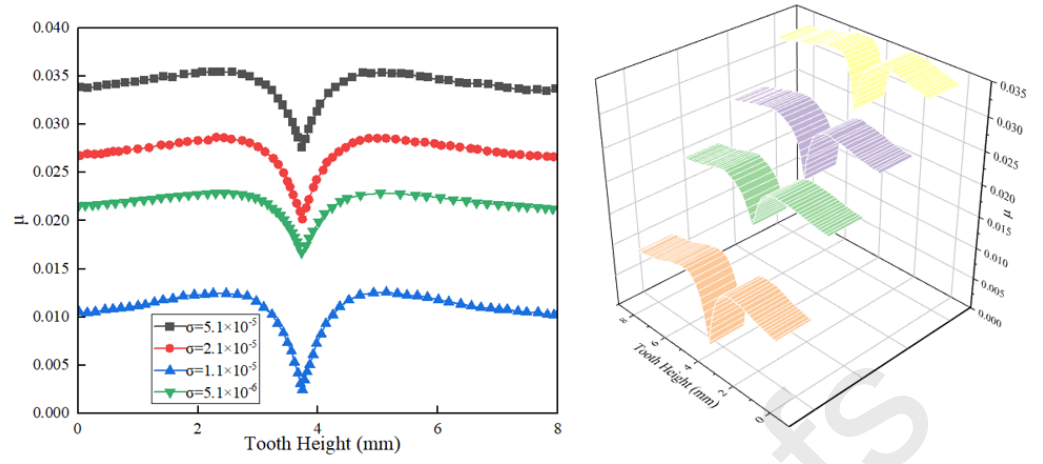


Fig.39. The influence of surface roughness on tooth surface friction coefficient

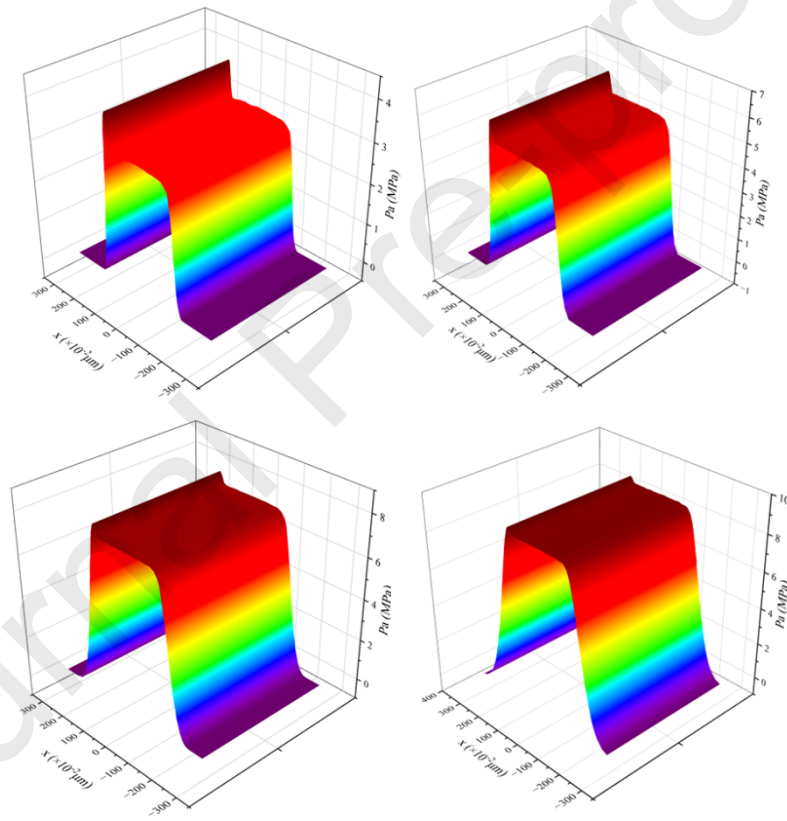


Fig.40. The effect of surface roughness on the distribution of contact pressure on rough surfaces

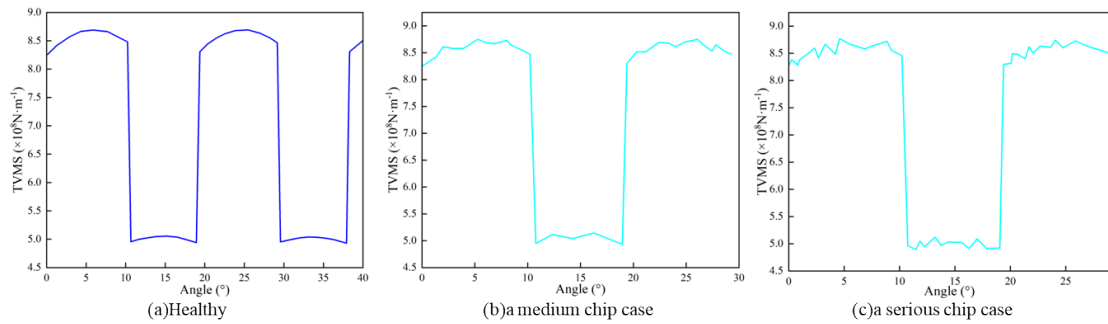
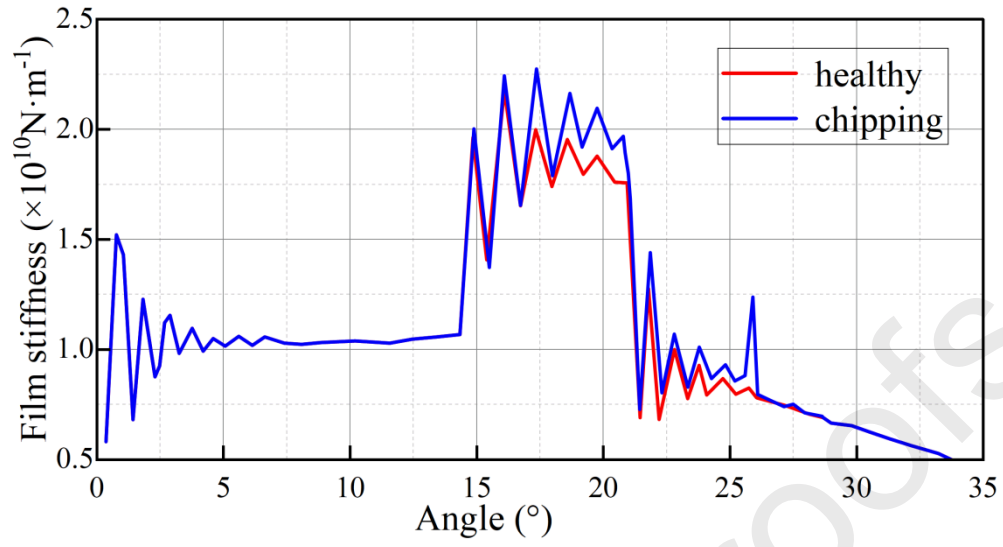
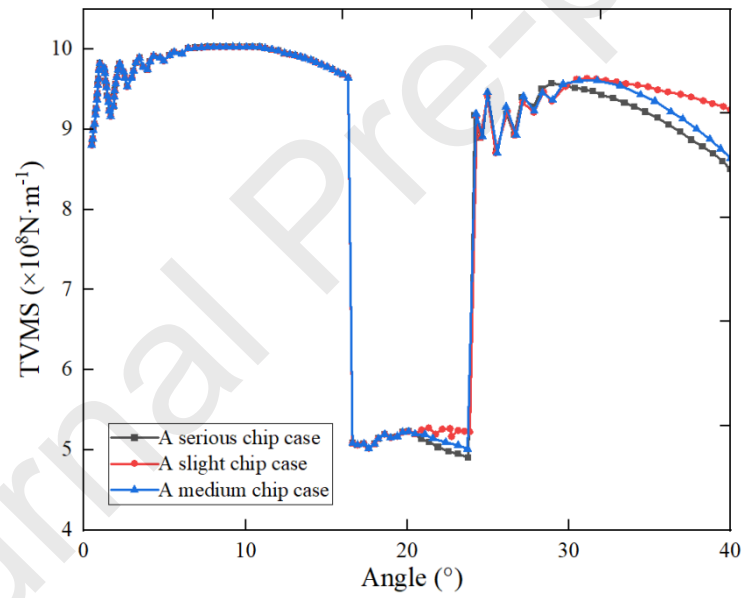


Fig.41. Time-varying mesh stiffness of gear pairs under different roughness levels**Fig.42.** EHL Stiffness Considering Surface Roughness**Fig.43.** TVMS with Oil Film Stiffness for Different tooth tip Chipping Sizes

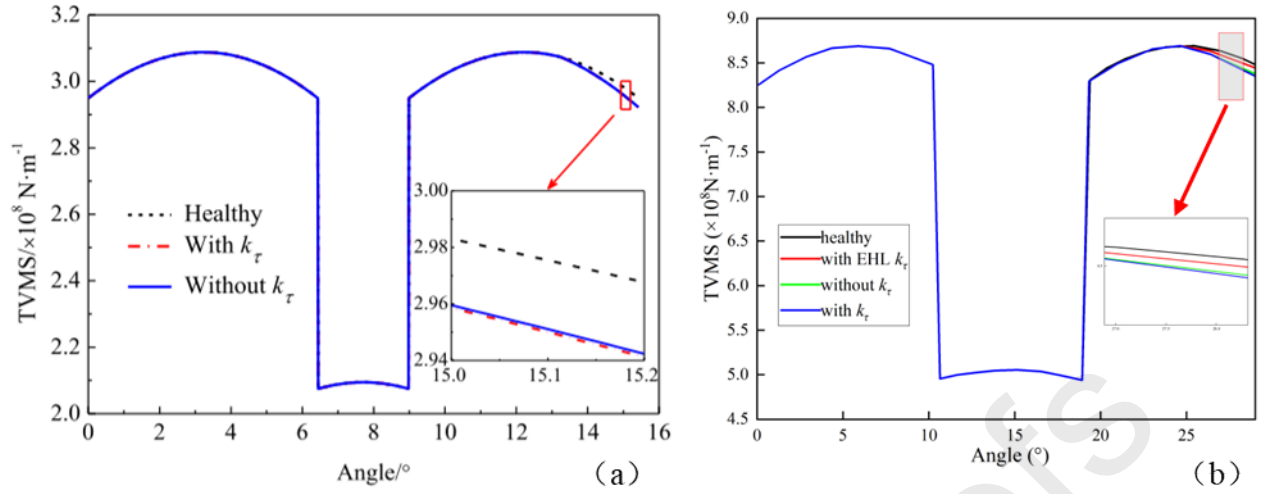


Fig.44. Comparisons of (b) the proposed method and (a) results in Ref. [26]

Table 1 Values of coefficient

Table 2 Basic Parameters of Gear Pair

Table 3 Parameters in the simulations

Table 4 Three Types of Defect Sizes

Table 1

Values of coefficient

	A_i	B_i	C_i	D_i	E_i	F_i
$L^*(h_f, \theta_f)$	-5.574×10^{-5}	$-1.9986 \times 10^{-}$	$-2.3015 \times 10^{-}$	4.7702×10^{-3}	0.0271	6.8045
$M^*(h_f, \theta_f)$	60.111×10^{-5}	28.100×10^{-3}	$-83.431 \times 10^{-}$	$-9.9256 \times 10^{-}$	0.1624	0.9086

$P^*(h_f, \theta_f)$	-50.592×10^{-4}	185.50×10^{-3}	0.0538×10^{-4}	53.3×10^{-3}	0.2895	0.9236
$Q^*(h_f, \theta_f)$	-6.2042×10^{-4}	9.0889×10^{-3}	-4.0964×10^{-4}	7.8297×10^{-3}	-0.1472	0.6904

Table 2**Basic Parameters of Gear Pair**

Parameter	Value	Parameter	Value
Tooth number, Z_1/Z_2	25/53	Pressure angle($^\circ$),	20
Modules, m	3	Poisson's ratio, ν	0.3
Young's modulus(Pa)	2.068×10^{11}	Tooth width (mm)	38
Tip clearance coefficient c^*	0.25	Addendum coefficient	1

Table 3

Parameters in the simulations

Parameter	Value	Parameter	Value
Lubricant ambient density (kg/m ³), ρ_0	890	Ree-Eyring shear stress (Pa), τ_0	0.5×10^7
Lubricant ambient viscosity (Pa s), η_0	0.05	Lubricant ambient temperature ($^\circ\text{C}$)	25

Table 4

Three Types of Defect Sizes

A slight chip case			A medium chip case			A serious chip case		
l_d	w_d	t_d	l_d	w_d	t_d	l_d	w_d	t_d
0.1m m	0.5m m	0.1m m	2.0m m	1.0m m	0.8m m	3.2m m	1.5m m	2.0m m