

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/173269/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Pan, Diankun, Liang, Yong, Zhang, Zhimin and Wu, Zhangming 2025. Design and dynamics of a cantilevered bistable buckled piezoelectric beam for vibrational energy harvesting. Mechanical Systems and Signal Processing 224 , 112013. 10.1016/j.ymssp.2024.112013

Publishers page: <http://dx.doi.org/10.1016/j.ymssp.2024.112013>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



Design and dynamics of a cantilevered bistable buckled piezoelectric beam for vibrational energy harvesting

Diankun Pan^{1*}, Yong Liang¹, Zhimin Zhang¹, Zhangming Wu^{1,2}

¹ Key Laboratory of Impact and Safety Engineering, Ministry Education of China, Ningbo University, Ningbo, 315211, China

² School of Engineering, Cardiff University, Cardiff, CF24 3AA, Wales

Abstract: In this paper, a novel family of low-cost, magnet-free bistable piezoelectric energy harvesters with a simple structure is designed, fabricated, and tested to evaluate their nonlinear dynamics and performance for harvesting energy from broadband vibrations. A laser-machined bistable structure, consisting of a buckled beam and two supporting beams, is employed as the host structure for constructing this energy harvester with piezoelectric transducers. The integration of buckled beams and constraints provided by supporting beams allows for the configuration of this bi-stable buckled piezoelectric beam under cantilevered boundary conditions without requiring external operation. The proposed harvester's static mechanical properties and dynamic responses are predicted using a finite element model, while its basic dynamics are understood through a simple analytical model. The frequency-sweep results demonstrate that the proposed harvester exhibits a broadband characteristic compared to the linear piezoelectric beam with a similar configuration, and various vibration modes and their corresponding performance of energy harvesting are analyzed and characterized. The potential of this proposed harvester is explored by adjusting the geometry parameters, such as the width of the supporting beam and thickness, to alter its dynamics and energy harvesting performance. Finally, a nonlinear energy harvesting array consisting of four proposed harvesters with adjacent broadbands is fabricated to enhance overall performance, achieving a broadband width of 13.7 Hz at an acceleration of 0.75 g. The proposed method introduces a novel design philosophy for nonlinear vibrational energy harvesters.

Keywords: Bistable; Cantileverd boundary; Piezoelectric; Energy harvesting; Buckled

1. Introduction

Piezoelectric energy harvesting, which is a very convenient mechanism for capturing mechanical vibrations and converting them into useful electric power, has attracted considerable attention for the replacement of millions of batteries in portable and wearable electronics and wireless sensor nodes in the past decades [1-5]. Typically, a linear piezoelectric energy harvester uses a resonant-based configuration, such as a cantilever with piezoelectric materials, which is widely explored and originally adopted for harvesting vibrational energy due to their relatively simple structures and well-established theory, analytical models, experiments, and optimization methods [6-8]. The energy output of linear energy harvesters is generally satisfactory for applications in which the excitation frequency remains constant and closely matches the natural frequency. However, achieving this is not always feasible for ubiquitous environmental vibrations characterized by a wide spectrum of excitation frequencies. [9, 10].

To enhance the frequency bandwidth of energy harvesting, various strategies have been employed, including the utilization of multiple resonators [11-14], frequency tuning [15-17], internal resonance [18-21], multi-modal resonance [22-24], and nonlinear resonances [25-28]. While these mechanisms offer advantages in certain aspects, they also encounter challenges and limitations. For instance, active frequency tuning requires external power and has a limited tuning range. The utilization of piezoelectric arrays with multiple cantilever beams increases both the size and cost, potentially necessitating a more complex processing electric circuit. Additionally, internal or multi-modal

resonance demands a more intricate structure while the coupling effect reduces the response of independent modes. The nonlinear resonances have been extensively investigated in the past few decades. A common approach involves incorporating bi-stable or multi-stable structures with softening and hardening nonlinear behaviors into the design of piezoelectric vibrational energy harvesters. This design strategy offers several advantages, including the ability to operate across a wide range of excitation frequencies and enhance output power [27]. The methods for inducing bistability in piezoelectric energy harvesting systems can generally be categorized into two approaches [10, 27]. The first approach involves introducing the nonlinearity through the prestress induced by the mechanical boundary condition. A typical design of this method is based on a buckled piezoelectric beam that is subjected to an axial preload or precompression under specific boundary conditions, such as two clamped ends or two simply supported ends. The depth of the potential well is determined directly by the magnitudes of the axial preload or precompression. Cottone *et al.* [29] revealed that the power generation of the buckled piezoelectric beam demonstrated superior performance when subjected to wideband random vibrations, as compared to its unbuckled state. Friswell *et al.* [30] vertically mounted the tip mass to the cantilever piezoelectric beam and then excited it in the transverse direction at its base, and the system exhibited high nonlinearity with two potential wells. Masana and Daqaq [31] investigated the comparative performance of a clamped-clamped piezoelectric beam subjected to axial loads with different magnitudes, revealing that a bistable harvester can outperform its monostable counterpart when the inter-well oscillation is triggered. Xu *et al.* [32] developed the piezoelectric bulked beam with a simply supported boundary and the height of the potential barrier and the distance between the potential well was adjusted by the axial compressive displacement. Andò *et al.* [33] introduced a vibrational energy harvester that utilized a bistable clamped-clamped beam and two piezoelectric transducers, which undergo beam impacts whenever the beam transitions between its two stable states. Mariello *et al.* [34] proposed a novel method, known as Package-Induced Preloading (PIP), for inducing out-of-plane buckling in flexible piezoelectric transducers and subsequently achieving bistability. The exploration of designs with more intricate mechanical boundary conditions was undertaken to introduce nonlinearities. Wu *et al.* [35] presented a novel bistable energy harvester comprising a host piezoelectric beam with a sinusoidal pre-deformed shape, along with two cantilever beams assembled onto the beam. Zou *et al.* [36] proposed a vibration energy harvester consisting of a piezoelectric cantilever beam and a cylindrical cam-roller-spring structure that generates negative stiffness and piecewise nonlinearity. Yang and Cao [37] utilized elastic steel slices and springs to develop a novel multi-stable energy harvester that facilitates the transition between potential wells. The implementation of this approach necessitates additional components to achieve the desired boundary conditions, with most designs opting for restrictions at both ends of the beam. Consequently, while enhancing the performance of the harvester, it also introduces increased complexity to the system. This is unfavorable when aiming to reduce size and enhance reliability. The utilization of thermal prestress-induced bistable composite laminates is an alternative approach to achieve nonlinearities in host structures [38]. Arrieta *et al.* [39] introduced cantilevered piezoelectric bi-stable composites for broadband energy harvesting without external devices.. The laminates exhibit a large size due to their bifurcation characteristic of bi-stability, and the excitation level required to induce inter-well motion is considerably higher compared to other methods [40].

The second approach to achieving bistability involves utilizing magnetic repulsion or attraction load. One classical example is a cantilever piezoelectric beam with a tip magnet, which establishes bistability through the magnetic force between the tip and external magnets. Stanton *et al.* [41] validated this example numerically and experimentally, and the results showed that the nonlinear harvester coexists with high energy attractors over a large frequency and amplitude range. De Paula *et al.* [42] addressed the influence of nonlinearities in this bistable energy harvester subjected to random vibrations. The ferromagnetic cantilever with two symmetrically positioned permanent magnets near its end is another typical example. Erturk and Inman [43] investigated this type of bistable energy harvester by experiments and a theoretical model. The harvester can achieve more stable positions, which is beneficial for

expanding the working bandwidth and reducing the excitation requirement, depending on factors such as magnet configuration, including magnetization and number. Zhou *et al.* [44] proposed the theoretical model and experimental investigation of a broadband piezoelectric vibrational energy harvester with a triple-well potential induced by a magnetic field. Zhou *et al.* [45, 46] developed a novel quad-stable energy harvester to improve the harvesting efficiency. Deng *et al.* [47] introduced a poly-stable vibrational energy harvester, which is realized through a synergetic poly-stable beam array consisting of multiple cantilever beams equipped with mutually repelling magnets at their tips. Lai *et al.* [48] designed a multi-stable array consisting of piezo-magnetoelastic beams, which effectively scavenge vibrations characterized by low amplitude and low frequency. Sun *et al.* [49] presented a novel penta-stable harvester design that utilized interaction forces among a rectangular magnet and two annular magnets. Mei *et al.* [50] introduced the magnetic-linkage effect to create time-varying potential wells in the nonlinear piezoelectric energy harvester capable of effectively operating over a wide frequency bandwidth under low-level base excitation. Fan *et al.* [51] combined a cantilever beam with a magnetic tip, and a clamped-clamped beam with central magnet mass, to obtain a wideband piezoelectric energy harvester with both bistability and parametric resonance characteristics. The multi-stability system with magnets offers advantages in terms of expanding the effective bandwidth and reducing excitation levels compared to the bistable system. However, the increasing complexity associated with designing and analyzing a larger number of magnets and piezo beams poses challenges, making it unfavorable for size reduction and stability enhancement. The magnetic field may also cause interference with other electronic devices, and the interaction between magnets and other components can easily affect the desired potential wells.

The aforementioned approaches, in summary, require additional components such as magnets, springs, and supports. Moreover, the design of the nonlinear system is intricate and its anti-risk capability is relatively weak. The present work proposes a novel, simple, cost-effective, magnet-free, bistable piezoelectric vibrational energy harvester to tackle this issue. The previous study introduced a series of bistable structures that were developed using the laser manufacturing technique [52]. The typical case comprises a wide main beam and two narrow supporting beams, which are cut by laser from a thin stainless steel plate in a single processing step. As the heat-affected zones vary in distance, the resulting deformation discrepancy between the main beam and supporting beams generates precompression that acts upon the main beam. Consequently, the main beam undergoes pre-compression and exhibits bi-stability due to constraints by the supporting beams. Therefore, laser manufacturing serves as both the fabrication method and pre-stressing technique for this family of bistable structures.

The nonlinearity of the broadband piezoelectric energy harvester is primarily attributed to the host structure, thus making it a promising option for achieving broad frequency range energy harvesting. In this study, we leverage this bistable structure as the host framework to devise a novel series of piezoelectric vibrational energy harvesters. The remaining sections of the paper are structured as follows: Section 2 provides a comprehensive overview of the fundamental conceptual design, followed by Section 3 which presents the experiments and numerical methodology. This is then followed by Section 4, where an analytical model is introduced. The findings and subsequent discussion are presented in Section 5. Finally, conclusions are summarized in Section 6.

2. Conceptual design

A previously proposed family of laser-machined bistable structures is utilized as the host structure for the construction of a novel type of bistable vibrational energy harvester. Initially, a planar prototype of the bistable host structure, comprising three rectangular loops, is meticulously designed. The outer loop determines the overall shape of the structure, while the two symmetrical inner loops create two slits that divide the entire structure into five distinct parts: a wider main beam, two narrower supporting beams, and two ends, as illustrated in Fig. 1. The prototype is laser-cut from a thin stainless steel plate, and upon completion of the fabrication process, the main beam exhibits buckling and bistability under the constraint of two supporting beams, as depicted in Fig. 1. The bistable feature of this structure is attributed to the mechanisms whereby two heat effect zones at the cut edge exert a more pronounced influence on

the narrow supporting beam in comparison to the wider main beam, resulting in differential deformation between them. Specifically, the supporting beam experiences greater shortening than the main beam. As a result, precompression occurs in the main beam due to this difference in deformation, leading to the bistability of the entire structure. According to this mechanism, the laser processing parameters and the width of the supporting beam are crucial variables to control the bistable characteristics, which have been previously confirmed in Ref. [52]. Additionally, an investigation was also conducted in [52] to explore the snap-through properties of this bistable structure under cantilevered boundary conditions, which serves as the fundamental basis of this study.

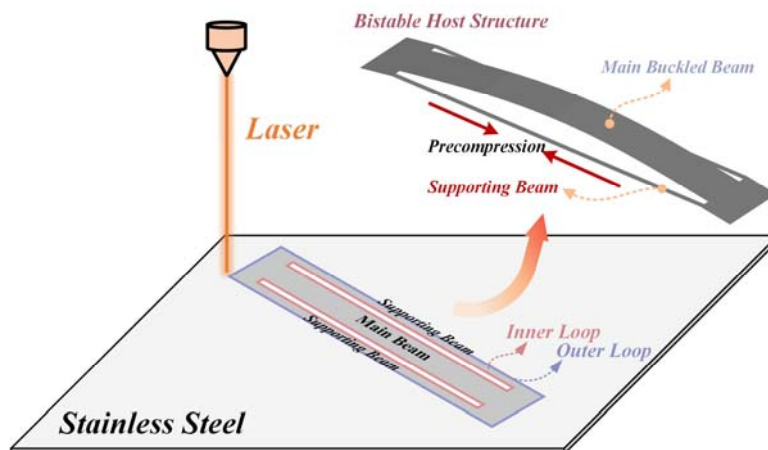


Fig. 1 Bistable host structure manufactured by laser technique.

The laser-machined bistable structure mentioned above is utilized as the host structure for this bistable piezoelectric vibrational energy harvester (BPVEH). As depicted in Fig. 2, two pieces of piezoelectric ceramics (PZT-5H) are employed as the energy transducers and bonded to both surfaces of the main buckled beam. One end of the host structure is fixed to establish a typical cantilevered boundary condition, while the other end serves as a placement for the proof mass. The bistable piezoelectric energy harvester can be integrated with a vibration source to capture vibrational energy. The potential advantages of this bistable energy harvester are as follows: Firstly, the bistable host structure based on laser manufacturing has a simple design without the need for additional components. Secondly, due to the utilization of a bistable beam and internal integrated constraint conditions, BPVEH can function like a cantilever beam and is easily connected to other components, with a relatively low fundamental frequency compared to double-end constraint situations. Thirdly, by bonding piezoelectric material to the main buckled beam, it can achieve large and uniform deformation during stable state transitions, thereby fully exploiting its energy conversion capabilities. In this study, the dynamic behaviors of this novel BPVEH are thoroughly investigated and then its performance in vibrational energy harvesting is also evaluated.

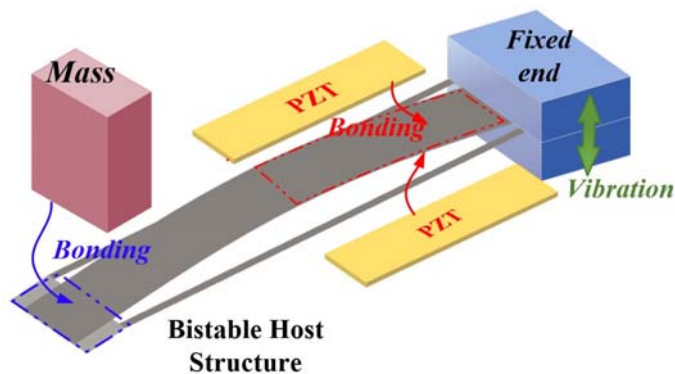


Fig. 2 Conception of the proposed bistable piezoelectric vibrational energy harvester.

3. Experiments and finite element analysis

3.1 Fabrication

The bistable host structure is initially designed and fabricated based on the previous investigation. The schematic diagram of the bistable host structure's basic geometry is presented in Fig. 3(a). The overall length of the structure measures 50mm, while its width is 8 mm. The main beam has a width of 5 mm, which is ten times that of the supporting beams. Both types of beams have an equal length of 42 mm. To prevent stress concentration, a radius of 0.2 mm is applied to the corners where slits intersect with the main beam and supporting beams, as depicted in Fig. 3(a). The two ends of the entire structure possess different lengths; one serves as a fixed region while the other acts as a connection region with proof mass.

The samples were fabricated from a stainless steel plate with a thickness of 0.15 mm using a laser system. The laser parameters were carefully selected as follows: the average power was set at 1000 W and the scan speed was maintained at 30 mm/s. Nitrogen assisting gas, co-axially aligned with the laser beam, emerged from the conical nozzle during the process. Initially, two inner loops representing slits were cut, followed by completion of the outer loop encompassing the entire shape. After undergoing laser machining without any additional operations, upon removal from the substrate, the experimental sample exhibited two stable states as depicted in Fig. 3(b). As anticipated, the main beam exhibited buckling as a result of the axial compression constraint imposed by two supporting beams. Subsequently, the bistable host structure was obtained and meticulously affixed with cyanoacrylate adhesives to its surfaces were two pieces of piezoelectric ceramics (PZT-5H) measuring 0.24 mm in thickness. To decrease the fundamental frequency and enhance inertia, several magnetic cubes with the desired mass were securely attached to the free end of the structure. Finally, under cantilevered boundary conditions, as depicted in Fig. 3(c), the experimental BPVEH was mounted onto the shaker using a clumper, showcasing its two stable states.

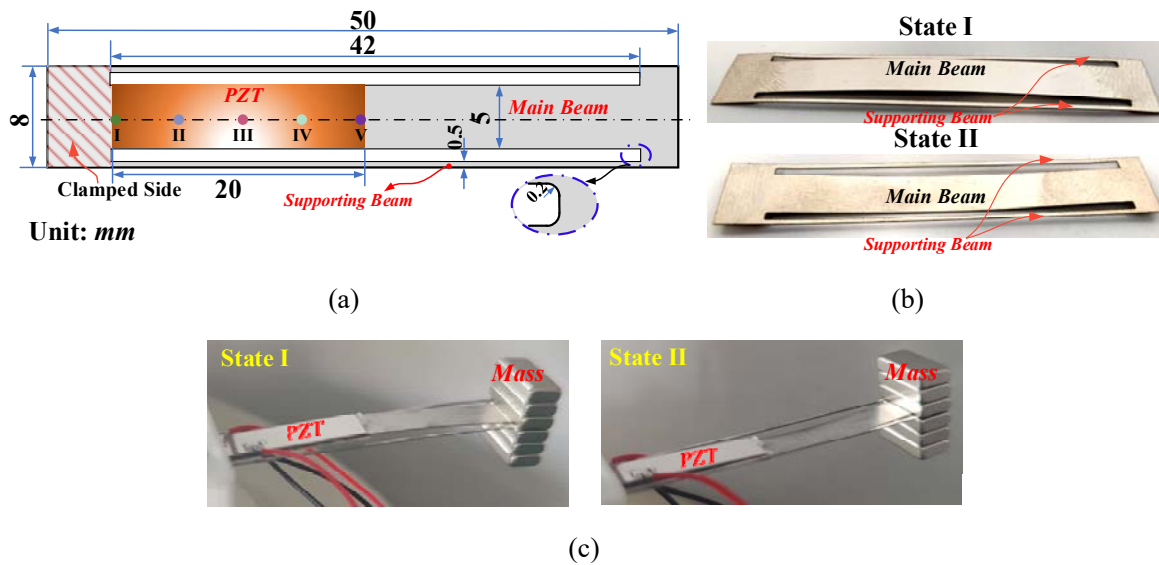


Fig. 3 (a) Geometry of bistable host structure; (b) Two states of bistable host structure; (c) Two stable states of experimental BPVEH with cantilevered boundary condition.

3.2 Quasi-static experiment

Before the vibrational testing, a quasi-static experiment was carried out first to obtain the load-displacement responses of BPVEH during snap-through between two stable states. As illustrated in Fig. 4(a), an experimental setup was established, which consists of a load cell, a screw system, and a movable stage. The sample was fixed to a support truss to form the cantilevered boundary condition, and its free end was restricted to the load cell by two nuts. Before conducting the vibrational testing, a preliminary quasi-static experiment was conducted to obtain the load-displacement responses of BPVEH during the snap-through process between two stable states. As illustrated in Fig.

4(a), an experimental setup was established, comprising a load cell, a screw system, and a movable stage. The sample was fixed to a support truss to create a cantilevered boundary condition, with its free end restricted to the load cell using two nuts. The load cell was installed on a movable stage driven by the screw system, allowing for forward and backward movement. The load cell has an accuracy of 3 mN and a measurement range of -3~3 N. To trigger snap-through and snap-back responses in one test, the load cell first moves forward with a given displacement and then returns to the starting point. To ensure a quasi-static process, the load cell moves at a speed of 4 mm/min during measurement. The initial measurement point for the load is adjusted to zero, ensuring that the sample is in one of its stable states. Throughout testing, both vertically applied load and displacement are simultaneously captured and recorded, resulting in load-displacement curves corresponding to snap-through responses. The experimental results will be later discussed alongside simulation results.

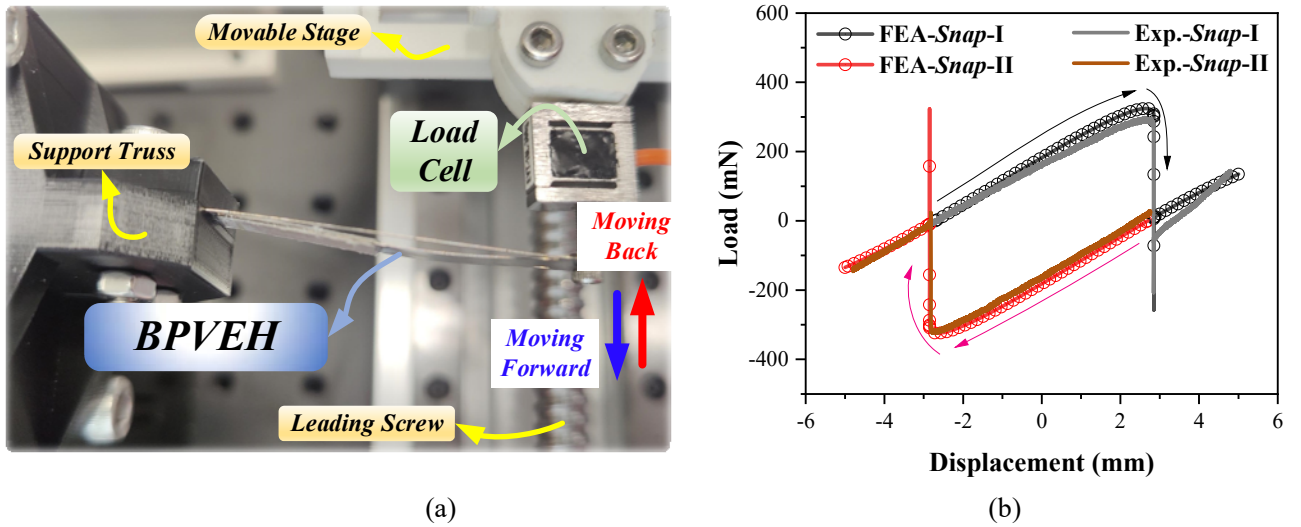


Fig. 4 (a) Experimental set-up for measurement of load-displacement response; (b) Load-displacement curves of snap-through behavior of BPVEH.

3.3 Finite element analysis

3.3.1 Static analysis

A finite element analysis (FEA) is conducted using the commercial software ABAQUS to gain insights into the fundamental characteristics of this bistable piezoelectric energy harvester. The shell element S4R (doubly curved shell elements with 4 nodes per element) and piezoelectric element C3D20E (quadratic piezoelectric element with 20 nodes per element) are employed for the bistable host structure and PZT transducers, respectively. The PZTs are bonded to the surfaces of the main beam using a 'tie' constraint during assembly. The electrical potential on the bonding surfaces of PZTs is set to zero throughout the entire analysis, and uniform potential is imposed on free surfaces through equation constraints. Geometrically nonlinear algorithms are enabled by activating the 'Nlgeom' switch for the entire duration of the analysis. Material properties can be found in Tab. 1. Convergence study and efficiency analysis have been performed to determine an optimal mesh size of 0.5mm.

A preliminary static finite element analysis is conducted to determine the fundamental characteristics of BPVEH, such as stable states and snap-through responses. The primary focus of this simulation is on the final state of the bistable structure after fabrication, thus disregarding residual stress development and transient physical effects during the laser machining process. In order to simulate precompression resulting from deformation discrepancies between the main beam and supporting beams, a negative equivalent temperature gradient ΔT is applied to two supporting beams. However, it is challenging to predict the pre-compression accurately applied to the main beam in experimental samples; hence, the temperature gradient ΔT remains undetermined.

The bistability of BPVEH can be characterized by its snap-through properties, which have been experimentally

measured to obtain the load-displacement responses. Therefore, these experimental results can be utilized to determine the temperature gradient ΔT in FEA. Consequently, the static FEA aims at achieving two objectives: 1) determining the equivalent temperature gradient ΔT applied to the supporting beams; and 2) comprehending the characteristics of snap-through responses. The degrees of freedom at the clamped end are constrained in accordance with the experimental setup of the cantilever boundary, while the other end remains unconstrained during the initial analysis step. There are three ‘static, general’ steps. In the first step, the temperature gradient is ΔT applied to two supporting beams, and a nominal lateral displacement as a disturbance is applied to the free end. Next, this disturbance is deactivated in the second step, which results in the first stable state related to the disturbance being obtained. In the third step, a lateral displacement in the opposite direction of the disturbance is applied at the mid-point of the free-end edge to trigger the snap-through behavior from the current stable state to another one. The reaction load can be extracted against the displacement in this analysis step, and then compared to the experimental results.

Tab. 1 Basic material properties

Material	Properties
Steel	Density $\rho=7900 \text{ kg/m}^3$
	Elastic modulus $E=187.5 \text{ GPa}$
	Poisson's ratio $\nu=0.25$
	The coefficient of thermal expansion $\alpha=14.4 \times 10^{-6}/^\circ\text{C}$
PZT	Density $\rho=7600 \text{ kg/m}^3$
	Modulus $E=56 \text{ GPa}$
	Poisson's ratio $\nu=0.289$
	Piezoelectric constants $d_{31}=-280 \times 10^{-12} \text{ m/V}$, $d_{33}=650 \times 10^{-12} \text{ m/V}$
	Permittivity $\varepsilon=3200\varepsilon_0$, $\varepsilon_0=8.85 \times 10^{-12} \text{ F/m}$
Adhesive	Density $\rho=1140 \text{ kg/m}^3$
	Elastic modulus $E=17.2 \text{ MPa}$
	Poisson's ratio $\nu=0.42$

The load-displacement curves predicted by FEA exhibit a favorable agreement with experimental results, as shown in Fig. 4(b) when the equivalent temperature gradient ΔT is set to 137°C . Hence, this temperature gradient can reasonably simulate the snap-through response, and the presented finite element model effectively captures the bistability characteristics of this BPVEH. During the Snap-I process from stable state I to II, the load increases almost linearly with displacement firstly then drops suddenly, and the snapping point with a maximum load, which is termed snapping load, is very close to the snap-down point where the load decreases vertically. After the snap-down point, a new equilibrium branch emerges. A similar trend is observed during the Snap-II process from stable state II back to I. Consequently, the load-displacement curves exhibit a distinct hysteresis loop, with two branches discernible within a specific displacement range that can be identified by two snap-down points.

The PZTs influence the stable states of the bistable host structure. The longitudinal curvature and out-of-plane displacement of the central line of the main beam at the two stable states are extracted from FEA results to evaluate this effect, as depicted in Fig. 5 (a). The shapes of stable states exhibit a purely bending mode, with an out-of-plane displacement w greater than 2 mm at the free end. Due to local reinforcement, the longitudinal curvature shows piecewise distribution characteristics. The curvature at the position where PZT is bonded is significantly lower than in other parts. Nevertheless, this low curvature indicates that there exists initial deformation in the piezoelectric layer during stable states. The initial deformation results in the prestress of PZT at stable states, which is indicated by an initial voltage represented by the EPOT value obtained from FEA. As illustrated in Fig. 5(b), the voltage within the piezoelectric layer is correlated with the displacement of the free end during the snap-through processes. The magnitude of the initial voltage is 4.5 V, denoted as dash lines in Fig. 5(b). Similar to the load-displacement responses,

initially, the voltage increases and then decreases after reaching a peak point. If snap-through behavior occurs, it will surpass this maximum point, serving as a criterion for determining its occurrence.

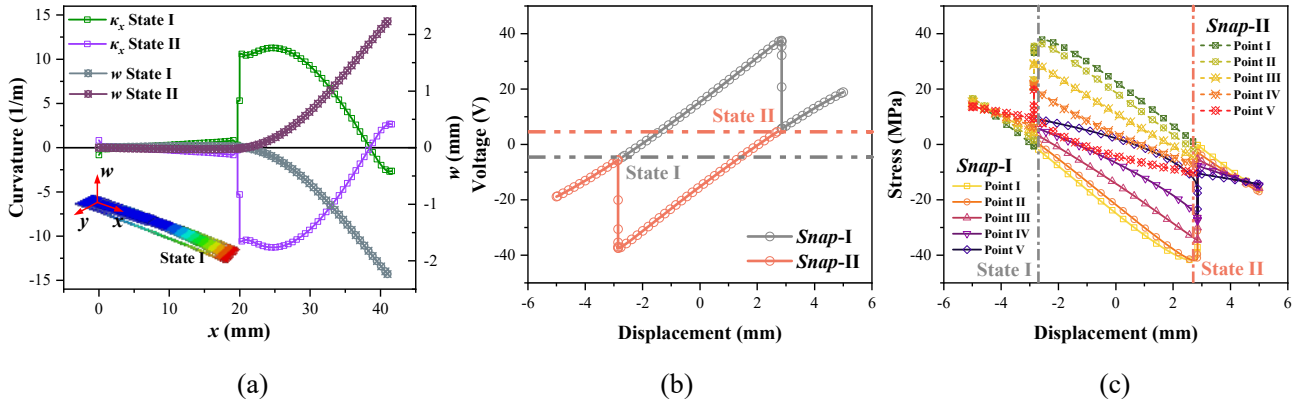


Fig. 5 (a) The curvature and out-of-plane displacement of central lines of the main beam; (a) The voltage-displacement curves predicted by FEA during the snap-through processes; (c) The longitudinal stress-displacement curves of five monitoring points predicted by FEA during the snap-through processes.

The PZT material is fragile and prone to breakage, therefore it is imperative to conduct a thorough examination of its safety and reliability. In this regard, the longitudinal stresses at five designated monitoring points are extracted from FEA results depicting the snap-through behavior, as illustrated in Fig. 3(a) and shown in Fig. 5(c). It should be noted that the selected PZT is positioned on the upper surface of the main beam, resulting in an initial stress state where all values on the free surface are tensile during stable state I. With the Snap-I process, the stress undergoes a transition from tensile to compressive status, reaching a maximum value. For the Snap-II process, it is crucial to consider the maximum value of tensile stress since the compressive failure stress of PZT is significantly higher than its tensile failure stress. The maximum values of tensile stress at five monitoring points gradually decrease, and Point I experiences a maximum tensile stress of 37.8 MPa, which remains below the ultimate tensile strength of PZT (76 MPa). Therefore, during transitions between two stable states, the PZTs are deemed safe. Additionally, it should be noted that there is minimal difference in stress statuses between Points I and II, indicating non-linear distribution along the longitudinal axis.

3.3.2 Dynamic analysis

After conducting static finite element analysis, a dynamic analysis is performed on BPVEH. Initially, a linear perturbation analysis step is appended after the BPVEH takes one of the stable shapes. In this step, the Lanczos solver, embedded in the finite element solver, has been utilized to calculate the first-order natural frequency and corresponding elastic displacement mode shape of BPVEH. The majority of ambient vibrations typically fall within the low-frequency range of 0~30 Hz. [53, 54], and six magnetic cubes with a total mass of 0.44 g are employed to lower the fundamental frequency to fall into this range. As shown in Fig. 6(a), the first-order mode of BPVEH is a bending shape and the corresponding frequency is 20.2 Hz, which is within the range of 10 to 30 Hz. Moreover, the situation without bistability, when the temperature gradient is removed, is also considered, as shown in Fig. 6(b). In this situation, the harvester has a flat state, and the first-order natural frequency is 14.3 Hz, which is much lower than that of the BPVEH. If the two silts are removed further, a typical linear piezoelectric beam (LPB) is obtained, and its first-order natural frequency is 16.7 Hz shown in Fig. 6(c), which is also lower than that of BPVEH.

To further investigate the dynamic response of BPVEH, a 'dynamic, implicit' step is conducted after two 'static, general' steps to achieve one stable state. Taking advantage of the simplified structure of BPVEH, two types of dynamic simulations can be established: frequency-sweep response under a specific acceleration level and single-frequency vibration response. To simulate sinusoidal frequency-sweep excitation with a designated acceleration, the excitation signal is generated by

$$Amp = A \sin(2\pi \int (f_0 + kt) dt) \quad (1)$$

where A is the amplitude of acceleration, k is the frequency sweep rate, and t represents the time that an implicit kinetic analysis step takes. The above-obtained excitation acceleration signal applied to the clamped end is input into the “Tabular Amp” function in ABAQUS to simulate the frequency-sweep response. The voltage responses of the piezoelectric layer associated with the linearly varying frequency are obtained and compared with the experimental results. Additionally, both forward sweeping and reverse sweeping are performed at identical acceleration levels, respectively. To simulate the time-voltage responses of BPVEH under a specific frequency and acceleration level for studying typical vibration modes, a harmonic displacement excitation is applied to the clamped end as a boundary condition in another ‘dynamic, implicit’ step. The predicted results of frequency-voltage and time-voltage responses will be discussed later alongside the experimental findings.

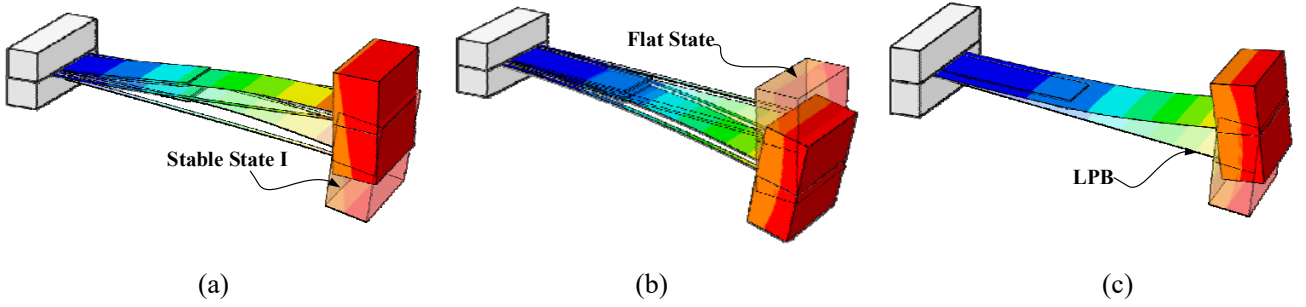


Fig. 6 First-order mode shapes predicted by FEA: (a) BPVEH; (b) Flat harvester without temperature gradient; (c) Linear piezoelectric beam (LPB).

3.4 Dynamic experiments

To acquire the nonlinear vibration responses of BPVEH, encompassing frequency-sweep and single-frequency responses, a dynamic experimental system was established as depicted in Fig. 7, which was procured from a commercial vendor. This system comprised a shaker, a power amplifier, and a controller that integrated the function of signal generation. Furthermore, this system can be effectively managed by a software program to realize different operational modes. The experimental sample was securely mounted on the shaker driven by the power amplifier and controller to generate the vibration. An IEPE (Integrated Electronics Piezo-Electric) accelerometer was attached to the shaker to measure and provide feedback on the current acceleration level to the controller. Subsequently, a closed-loop system was established to achieve the desired excitation conditions, such as frequency-sweep vibration at a specific acceleration. The open-circuit voltage of the piezoelectric elements in parallel connection was measured using an oscilloscope. Frequency-sweep experiments were conducted by inputting a frequency range of 10 to 40 Hz and three acceleration levels (0.5g, 1.0g, and 1.5g, where $g=9.85 \text{ m/s}^2$) into the vibration system through control software. The sweep speed was set at 30Hz/min. Finally, the frequency-voltage responses of BPVEH were obtained by the experiments. Moreover, the single-frequency operation mode was implemented to generate harmonic vibrations at a specific frequency, enabling the acquisition of time-voltage signals corresponding to distinct vibration modes of BPVEH.

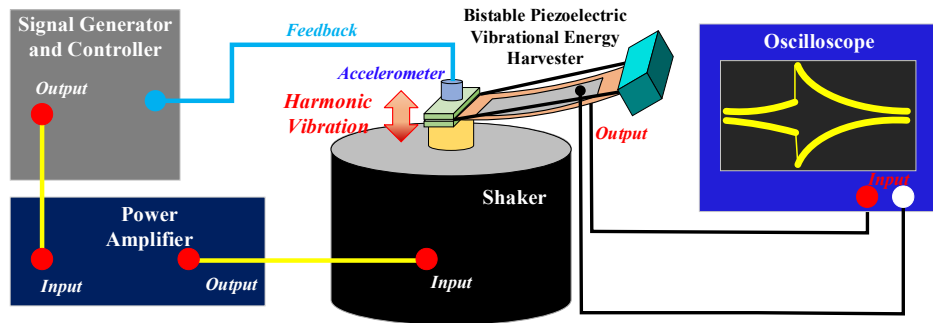


Fig. 7 Experimental system for vibration testing.

4. Analytical Model

Although the finite element model can give fairly reliable results for dynamic analysis, the simulation process is generally time-consuming due to the adoption of the ‘dynamic, implicit’ step, especially for the simulation of frequency-sweep with a long time. Therefore, it is necessary to exploit a simple analytical model to predict the dynamic response efficiently. This cantilever piezoelectric energy harvester could be described by a lumped parameter model based on the Duffing equation, as shown in Fig. 8(a). The governing electromechanical equations are

$$\begin{cases} m\ddot{x}(t) + c\dot{x}(t) + F_r - \theta v(t) = F(t) \\ C_p \dot{v}(t) + v(t)/R + \theta \dot{x}(t) = 0 \end{cases} \quad (2)$$

where m and c are the equivalent mass and damping in the fundamental mode, respectively; θ is the equivalent electrometrical coupling coefficient; C_p is the equivalent capacitance of the piezoelectric materials; R is the load resistance; $v(t)$ is the voltage across the electrical load resistance; $x(t)$ is the relative displacement of the mass to the base; F_r is the equivalent nonlinear restoring load. $F(t)$ is the external harmonic mechanical load as an excitation term, which is given by,

$$F(t) = -mA \sin(\omega t) \quad (3)$$

The nonlinear characteristic is reflected by the restoring load F_r that depends on the dynamics. Here, the load-displacement responses predicted by FEA are employed to describe the restoring load F_r , as shown in Fig. 8(b). Two load-displacement responses depend on the direction of the snap-through, and the following rule is illustrated in Fig. 8(b). When the initial equilibrium position is State I represented by Point B , the mass moves on the path of F_{r1} involving the branches AB and BD meanwhile the snap-through phenomenon does not occur. If the mass moves across Point C which is the snapping point and reaches snap-down Point D , the mass will plummet down to Point E on the path of F_{r2} and then reach the second equilibrium position, namely Point B' , which means the snap-through behavior is completed. Next, the mass will move on the path of F_{r1} involving branches $A'B'$ and $B'D'$ until the mass breaks the current equilibrium status and realizes the changes in the path when the snap-down point is D' . Therefore, two branches are possible for a displacement range between D and D' , which could be described from X_{10} to X_{20} as shown in Fig. 8(b).

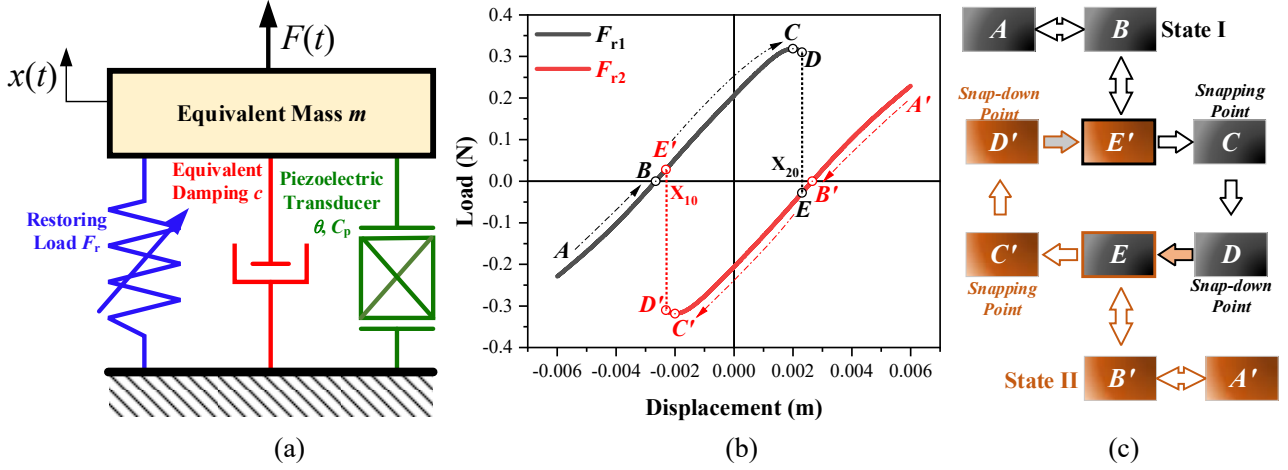


Fig. 8 (a) The equivalent analytical model; (b) The restoring load-displacement curves; (c) The corresponding rule of the load path.

Here, to describe the restoring load F_r , an eighth-order polynomial is employed,

$$F_r = \begin{cases} F_{r1} = k_0 + k_1x + k_2x^2 + k_3x^3 + k_4x^4 + k_5x^5 + k_6x^6 + k_7x^7 + k_8x^8 \\ F_{r2} = -k_0 + k_1x - k_2x^2 + k_3x^3 - k_4x^4 + k_5x^5 - k_6x^6 + k_7x^7 - k_8x^8 \end{cases} \quad (4)$$

The coefficients of the restoring load could be obtained by fitting to the results of FEA, as illustrated in Tab. 2. The bifurcation of branches X_{10} is -0.00231m . As previous analysis, the restoring load depends not only on the current value of displacement $x(t)$ but also on the number of snaps in the history of motion, when the displacement is in the range from X_{10} to X_{20} . If the starting stable position is State I, the restoring load is F_{r1} when the number of snaps is even or zero and the restoring load is F_{r2} when the number of snaps is odd in the range between X_{10} to X_{20} . The path of restoring load should be determined by the number of snaps and the displacement $x(t)$. The electromechanically coupled equations can be used in an ordinary differential equation solver for numerical simulations (the ‘NDSolve’ command of Mathematica is used here. The remaining parameters of this model identified experimentally are listed in Tab. 2.

Tab. 2 Parameters of the analytical model

Equivalent mass	$m=0.00487\text{ kg}$
Equivalent damping	$c=0.036\text{ Ns/m}$
Equivalent capacitance	$C_p=1.4 \times 10^{-8}\text{ F}$
Equivalent electromechanical coupling coefficient	$\theta=9.0 \times 10^{-5}\text{ N/V}$
Fitted polynomial coefficients of the restoring load	$k_0=0.2058\text{ N}; k_1=72.7309\text{ N/m}; k_2=-1.160.2 \times 10^3\text{ N/m}^2;$ $k_3=5.0606 \times 10^4\text{ N/m}^3; k_4=-6.5876 \times 10^8\text{ N/m}^4; k_5=-3.8277 \times 10^{11}\text{ N/m}^5;$ $k_6=-7.7419 \times 10^{13}\text{ N/m}^6; k_7=-6.9118 \times 10^{15}\text{ N/m}^7; k_8=-2.3387 \times 10^{17}\text{ N/m}^8$

5. Results and discussion

5.1 Frequency sweep

The linear frequency sweeps are conducted in two directions across the frequency range of 10~40 Hz, based on its first-order fundamental frequency. The open-circuit voltage outputs corresponding to different frequencies under three acceleration levels (0.5 g, 1.0 g, and 1.5 g) are obtained through three approaches: experiments, finite element analysis (FEA), and analytical modeling, as illustrated in Fig. 9. Here, to simulate the open-circuit condition, the load resistance R is set to $1\text{ M}\Omega$ in the analytical model. Overall, the results from FEA and analytical modeling exhibit a good agreement with the experimental findings.

When the excitation level is 0.5 g, the softening-type response is captured in the forward frequency sweep, which means the response peak bends to the lower frequency. For the experimental results, the frequency of the peak is 19 Hz, which is very close to 19.04 Hz and 19.05 Hz predicted by FEA and the model, respectively. It illustrates that the FEA and the analytical model are adequate for the prediction of the basic dynamics of the harvester. It should be noted that the starting voltage of FEA corresponding to the stable state is not zero as in the previous static analysis. When the excitation level is 0.5 g, the forward frequency sweep captures a softening-type response, characterized by a downward bending of the response peak towards lower frequencies. In terms of experimental results, the peak frequency measures 19 Hz, which closely aligns with the predicted values of 19.04 Hz and 19.05 Hz obtained from FEA and the analytical model respectively. This demonstrates that both FEA and the analytical model are sufficiently accurate in predicting the fundamental dynamics of the harvester. It should be noted that, unlike previous static analysis where the starting voltage was assumed to be zero, FEA indicates a non-zero starting voltage corresponding to the stable state. The softening feature in the reverse sweep demonstrates a more pronounced trend, accompanied by snap-through behavior observed in both experimental and FEA results. However, the harvester's response in FEA analysis stops at the second stable state with a negative voltage after the reverse sweep, whereas the model underestimates the amplitude of the response peak compared to experimental results.

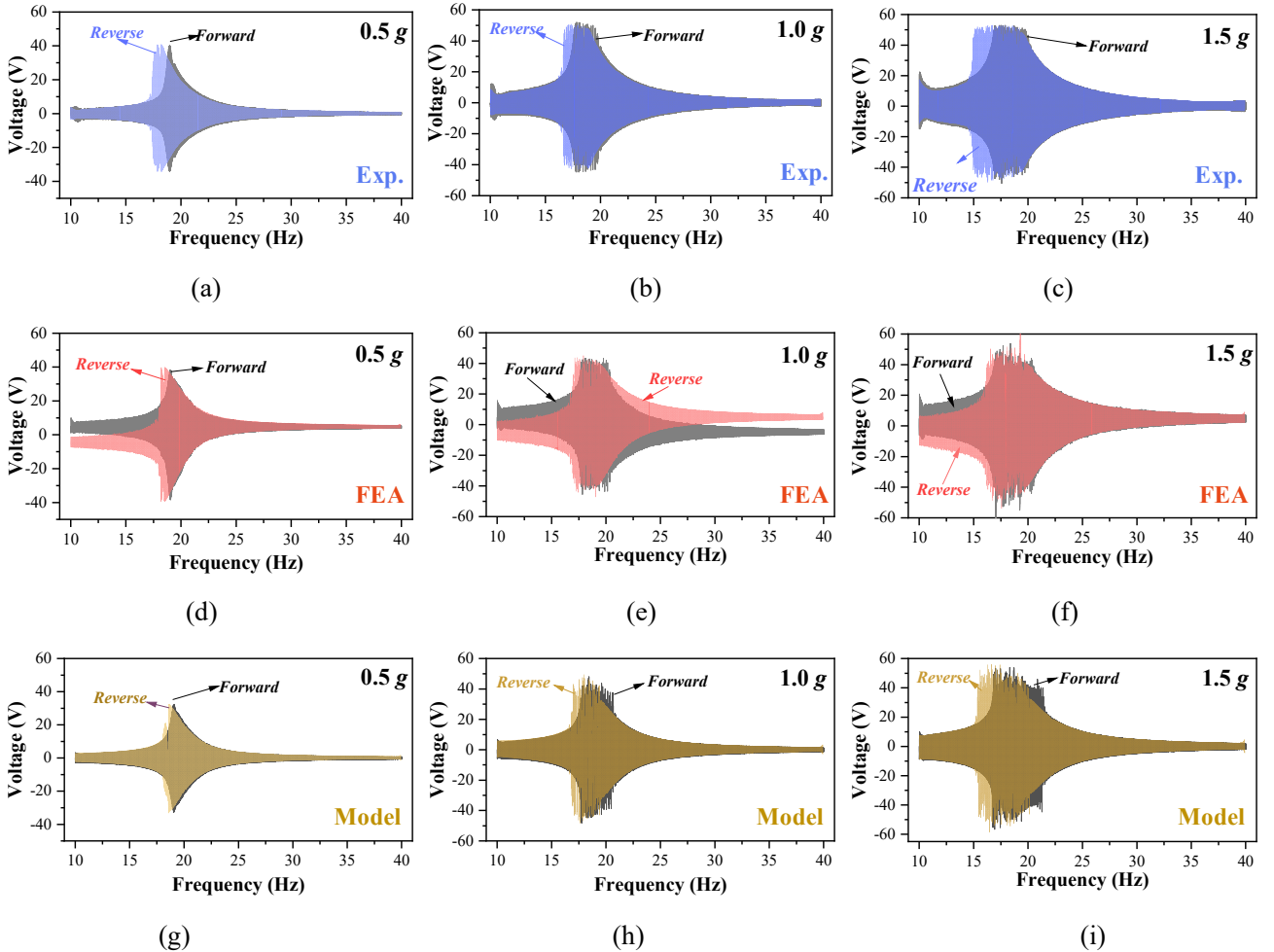


Fig. 9 The results of frequency-sweep of BPVEH under three acceleration levels given by experiments ((a), (b), and (c)), finite element model ((d), (e), and (f)), and analytical model ((g), (h), and (i)).

When the acceleration reaches 1.0 g, the cross-well vibration with snap-through behavior is triggered, resulting in a broadband response peak regardless of the sweep direction. The experimental results show that the broadband response covers a frequency range from 16.64 Hz to 18.94 Hz during forward sweeps and from 17.61 Hz to 19.71 Hz during reverse sweeps. In comparison, both FEA and model predictions indicate frequency ranges of 17.15 Hz to

19.23 Hz and 17.09 Hz to 19.45 Hz respectively for reverse sweeps, while for forward sweeps, they demonstrate broadband ranges of 17.73 Hz to 20.14 Hz and 17.88 Hz to 20.43 Hz respectively. This demonstrates the reliability of both finite element analysis (FEA) and analytical models in accurately predicting the fundamental characteristics of the proposed harvester. With the increase in excitation level to 1.5g, the broadband response peak exhibits a wider bandwidth; however, there is no significant change in its amplitude during the experiments. In comparison to the forward sweep, the reverse sweep captures a broader frequency range of 14.98 Hz to 19.37 Hz, and these experimental results demonstrate better agreement with the model predictions than those obtained from FEA.

Moreover, a linear piezoelectric beam (LPB) is utilized as a reference sample to demonstrate the nonlinear characteristics of BPVEH. The geometric details and experimental setup of the linear piezoelectric beam are depicted in Fig. 10(a) and (b), respectively. To ensure comparability, the same outer outline, PZT configuration, and proof mass as BPVEH are adopted while maintaining identical laser processing parameters during fabrication. It should be noted that the extraction wires are relocated to the center of the beam to simplify assembly operations and this modification has undergone experimental validation, confirming its minimal impact on output results. As illustrated in Fig. 10(c), the experimental frequency-sweep results align well with those obtained from FEA. Under the acceleration of 1g, the open-circuit voltage response of the linear harvester exhibits a single prominent resonance peak at approximately 17 Hz. As depicted in Fig. 10(d) and (e), both the experimental and FEA predicted time-voltage curves corresponding to its resonance mode display a typical sinusoidal waveform resembling the excitation signal, with only one peak observed in the Fast Fourier Transform (FFT) plot at the excitation frequency. The resonance behavior of this linear piezoelectric beam yields a higher voltage amplitude reaching nearly 50 V, slightly surpassing that of BPVEH. However, BPVEH demonstrates superior performance by maintaining high voltage amplitudes across a wider range of frequencies compared to this linear piezoelectric beam.

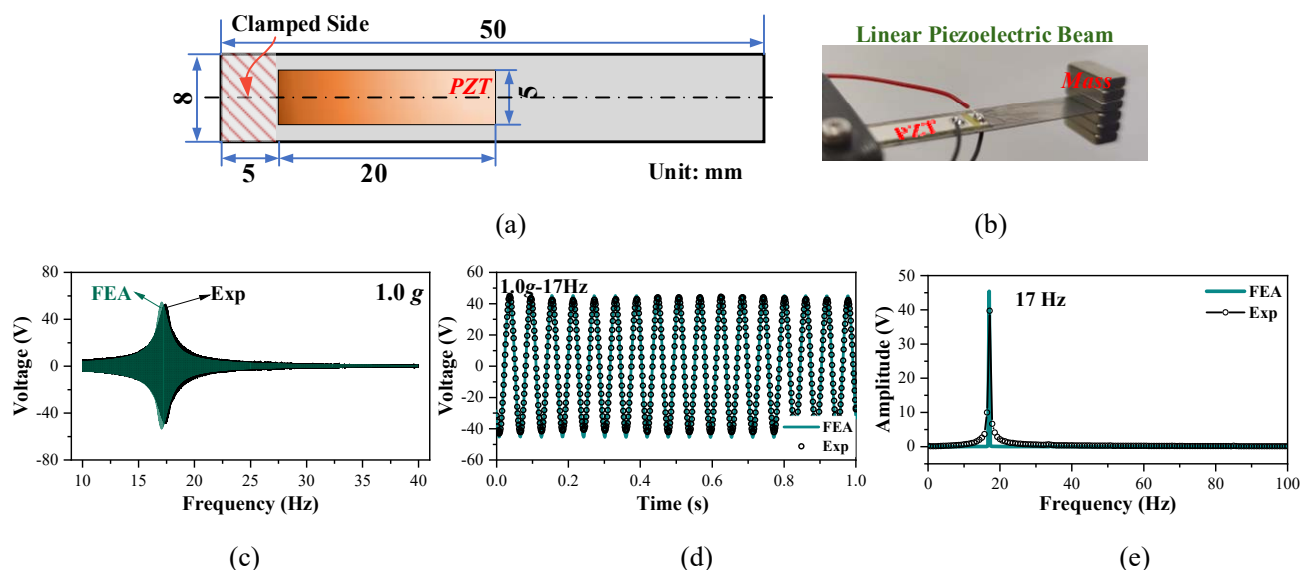


Fig. 10 (a) The basic geometry of the linear piezoelectric beam; (b) The sample of the linear piezoelectric beam; (c) The experimental and FEA results of frequency-sweep of the linear piezoelectric beam; (d) Experimental and FEA time-voltage curves of the linear piezoelectric beam at the excitation condition of 1g and 17Hz; (e) FFT plots of the linear piezoelectric beam.

In addition, the resonance response of a linear piezoelectric beam leads to a substantial increase in voltage output. However, when the excitation level is further increased to 1.5 g, the dynamic stress experienced by piezoelectric transducers exceeds their ultimate strength, resulting in their failure during experimental testing. Nevertheless, BPVEH can withstand this higher excitation level while ensuring the integrity of the piezoelectric transducer remains intact. Therefore, bistability serves as an inherent protective mechanism for piezoelectric transducers under relatively high excitation levels.

5.2 Vibration modes

5.2.1 Single-well vibration

When the snap-through behavior is not triggered, the mass will follow a single restoring load path, as illustrated in Fig. 8(b). This mode is referred to as single-well vibration, which is commonly observed in frequency sweep results. At the frequency range away from the main peak, the vibration modes are all single-well. For instance, two frequencies 15 Hz and 25 Hz at the acceleration of 1 g are utilized to demonstrate the characteristics of this vibration mode. The finite element mode and the analytical model are utilized to predict the time-voltage signals in comparison with the experimental results. As depicted in Fig. 11(a) and (b), a good agreement is observed among the three sets of time-voltage curves, which exhibit harmonic waveforms resembling those of linear harvester. However, FFT plots in Fig. 11(d) and (e) reveal the presence of a weak superharmonic component at order 2 alongside the primary excitation harmonic frequency, indicating a characteristic nonlinearity.

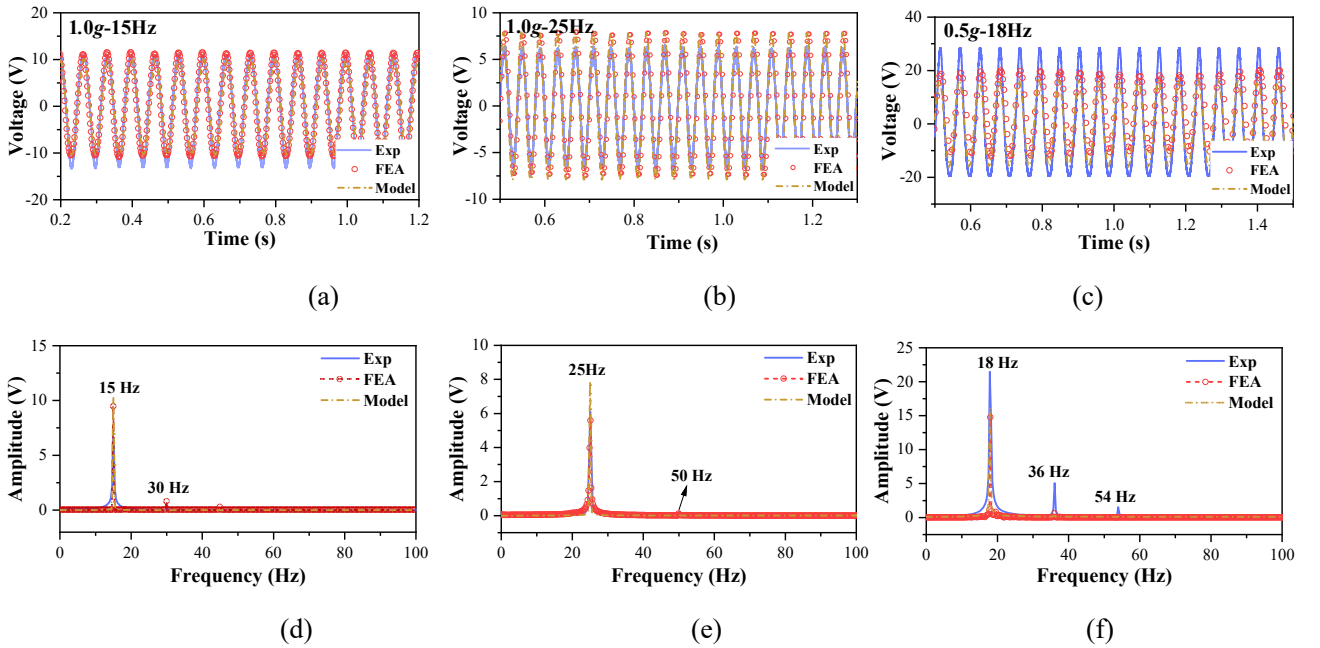


Fig. 11 Typical time-voltage curves of single-well vibration mode: (a) 1.0 g and 15 Hz, (b) 1.0 g and 25 Hz, and (c) 0.5 g and 18 Hz; and the corresponding FFT plots: (d) .0 g and 15 Hz, (e) 1.0 g and 25 Hz, and (f) 0.5 g and 18 Hz.

Besides the two typical single-well vibration modes observed on either side of the main broadband peak, another characteristic single-well vibration is present in the vicinity of the peak when the excitation level is relatively low. For example, when the acceleration is 0.5g, the vibration response at 18 Hz is the single-well mode, and the corresponding time-voltage responses are depicted in Fig. 11(c). The experimental waveform deviates slightly from a pure sinusoid, and the voltage magnitudes corresponding to the valleys and peaks are not equal, but both FEA predictions and the model maintain harmonic characteristics. Therefore, the experimental amplitude of the time-voltage response is observed to be higher than the predicted values by both analytical methods. The experimental FFT plots as shown in Fig. 11(f) exhibit three distinct peaks, namely the primary response corresponding to the excitation frequency, as well as second-order and third-order superharmonic responses. It should be noted that the finite element model and analytical model fail to accurately predict these latter two superharmonic responses. This discrepancy can be attributed to the fact that these responses represent a critical state between single-well mode and cross-well mode in experiments, which proves challenging for both analytical approaches to capture. Additionally, the amplitudes observed in FFT plots do not exhibit consistency with those of the corresponding time-voltage waveforms, which can be attributed to the inherent asymmetry in voltage magnitudes within time-voltage waveforms.

5.2.2 Cross-well vibration

The vibration mode involving snap-through is referred to as cross-well vibration, which is a highly complex phenomenon. Experimental observations have identified two types of cross-well vibration based on the regularity of snap-through behavior. The first type exhibits periodic snap-through behavior under excitation conditions of 1.5 g and 18 Hz. As depicted in Fig. 12(a) and (b), the time-voltage wave predicted by FEA shows better agreement with experimental results. The voltage response of this particular type of cross-well vibration demonstrates a certain regularity; however, the amplitude of the voltage wave is not stable, and intermittent high-frequency oscillations occur, indicating the occurrence of snap-through behavior. However, the analytical model is too simple to give this complicated response but the periodic snap-through behavior is captured, as shown in Fig. 12(c). Therefore, in the case of this type of cross-well vibration, snap-through behavior occurs regularly; however, the frequency of snap-through is lower than the excitation level, as confirmed by the FFT plots presented in Fig. 12(d). The FFT plots indicate the simultaneous existence of superharmonic components (4/3 order: 24Hz, 5/3 order: 30Hz) and subharmonic components (1/3 order: 6Hz, 2/3 order: 12Hz). For the experimental results, a greater number of high-order superharmonic components were observed below 150 Hz. The snap-through frequency of 6 Hz predicted by the two methods aligns with the experimental findings. However, there is a discrepancy in terms of the occurrence pattern: while FEA and experimental results show adjacent snap-through behaviors, the model's outcomes exhibit intermittent occurrences that are separated by uniform single-well motions.

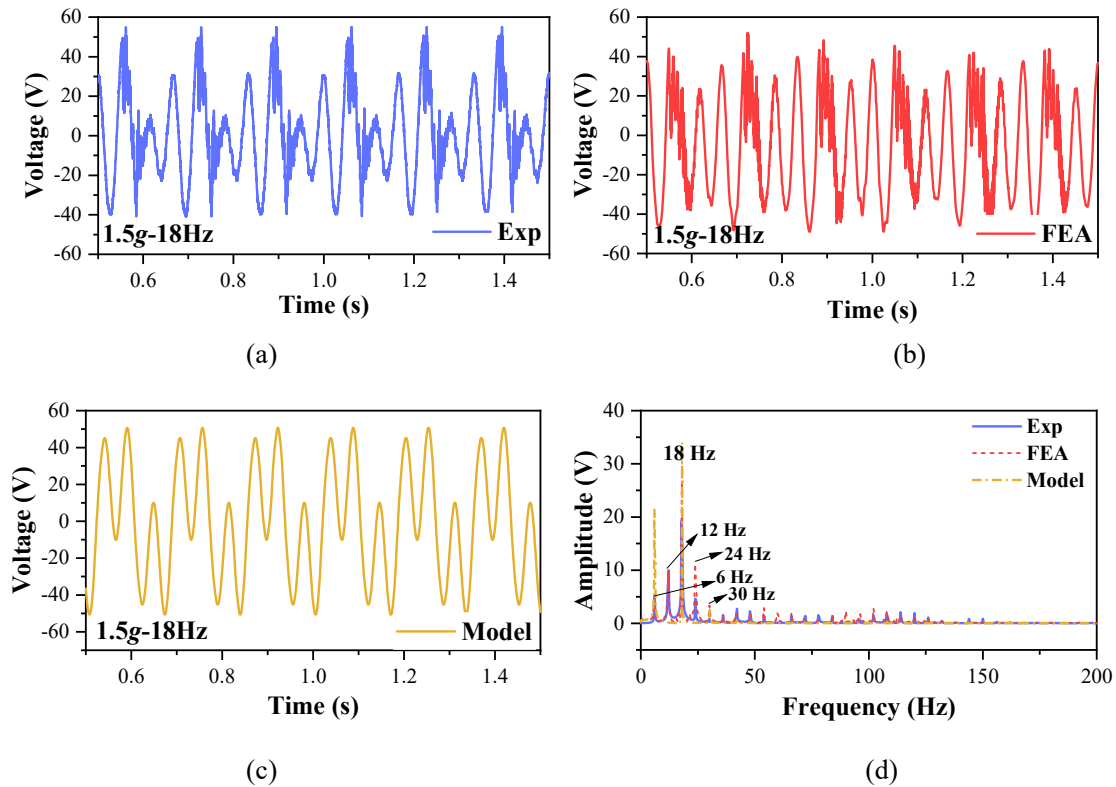


Fig. 12 Time-voltage curves of periodic cross-well vibration at 1.5 g and 18 Hz given by: (a) experiments, (b) FEA, and (c) analytical model; and the corresponding FFT plots (d).

The other type of cross-well vibration mode is observed through experiments and finite element analysis (FEA) at an excitation condition of 1 g and 18 Hz, as depicted in Fig. 13(a) and (b). The irregular time-voltage waveform indicates that this response mode exhibits chaotic cross-well vibration. The FFT plots in Fig. 13(d) reveal that the dominant response corresponds to the excitation frequency, while the continuous band below 40 Hz confirms the chaotic nature of the response. However, the analytical model with only one degree of freedom also can produce a chaotic response and predict a subdominant response at 3.5 Hz, which is not observed in experiments and FEA simulations.

According to experimental observations, it is worth noting that triggering cross-well vibrations with periodic snap-through behaviors is more challenging compared to chaotic cross-well vibrations. Therefore, the emergence of periodic cross-well vibrations requires a higher excitation level and occurs at relatively low frequencies.

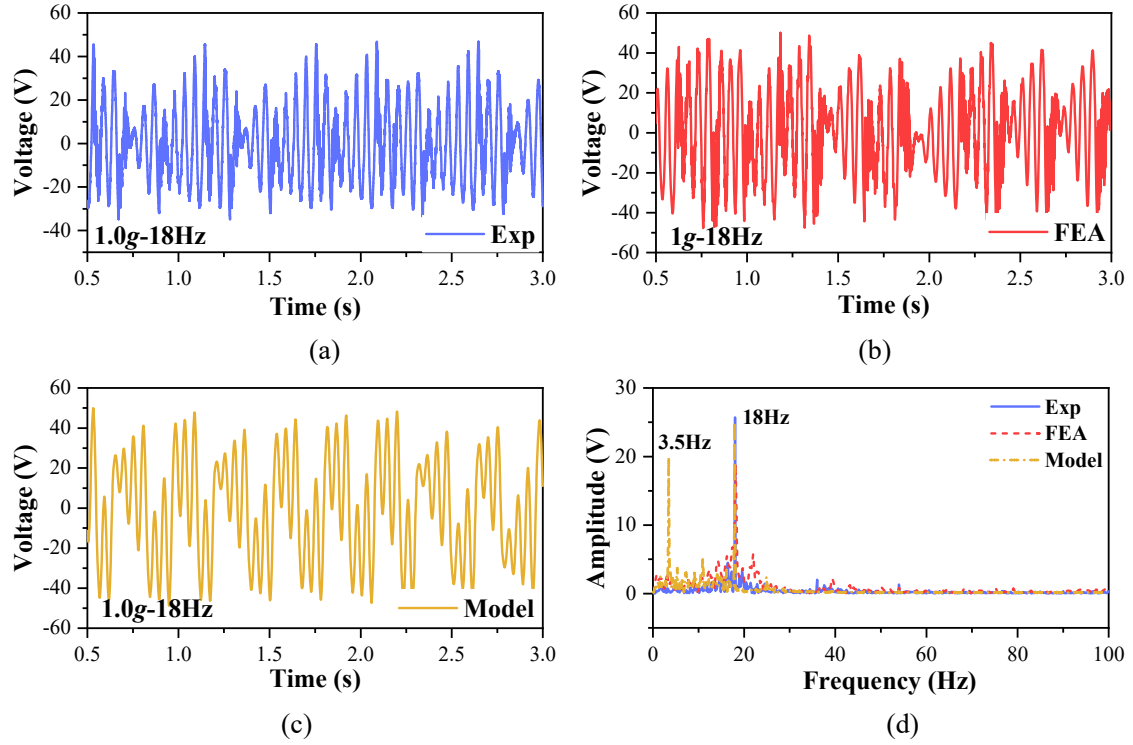


Fig. 13 Time-voltage curves of chaotic cross-well vibration at 1.0 g and 18 Hz given by: (a) experiments, (b) FEA, and (b) analytical model; and the corresponding FFT plots (d).

5.3 Output Power

To demonstrate the energy harvesting performance of BPVEH, two characterization methods are employed. Firstly, a capacitance of 47 μF is charged using a standard AC-DC circuit, and the voltage across the capacitor terminals is measured utilizing an oscilloscope. The recorded voltages over time under different excitation conditions are depicted in Fig. 14(a). The voltage exhibits a gradual increase and then reaches a stable plateau value. The five excitation scenarios corresponding to different vibration modes discussed earlier have been tested. In the case of single-well vibrations at 15 Hz and 25 Hz with an acceleration of 1.0 g, the voltage plateau value is relatively low compared to the main broadband response peak. However, for a single-well vibration at 0.5 g and 18 Hz, the plateau voltage is significantly higher due to a similar resonant response. In comparison to single-well vibrations, the two cross-well vibration scenarios at 18 Hz with accelerations of 1.0 g and 1.5 g exhibit higher plateau voltage values.

Secondly, in the experiments, the output power of the BPVEH was measured under different vibration modes using a standard AC-DC circuit. This circuit comprised a bridge rectifier, a filtering capacitor, and a tunable resistor as illustrated in Fig. 15. The output voltage generated by the parallel connection of PZTs is rectified and stored in a capacitor before being discharged through a variable resistive load R . The steady-state voltages V_R were recorded for various load resistances R , and the average steady-stage powers P vs. load resistance R can be calculated by

$$P = V_R^2 / R \quad (4)$$

The measured powers of BPVEH vs. load resistance are shown in Fig. 14(b), and the maximum powers corresponding to optimal load resistance are listed in Tab. 3. When subjected to a base acceleration of 1.0 g, BPVEH operates under single-well vibration mode at frequencies of 15 Hz and 25 Hz, as previously analyzed, resulting in relatively low maximum output power. However, the single-well vibration at 0.5 g and 18 Hz exhibits a significantly higher

maximum output power compared to the previous two scenarios, owing to its higher output capability. When the vibration mode is switched to cross-well, there is a significant increase in the maximum power output. At the same frequency of 18 Hz, the maximum power output corresponding to cross-well vibration under an acceleration of 1.0 g is 2.5 times higher than that under single-well vibration mode. With an increase in acceleration to 1.5 g, the maximum power output shows an increment of 8.3% compared to the situation at a 1.0 g acceleration level. The periodic cross-well vibration exhibits slightly superior performance compared to the chaotic cross-well vibration, albeit necessitating a higher excitation level. Furthermore, the normalized power density considering both the size of the piezoelectric transducer and the acceleration is calculated and also presented in Tab. 3. Notably, the power density of chaotic cross-well vibration is twice that of periodic cross-well vibration. This observation underscores that the frequency associated with snap-through behavior does not significantly impact output power, which is contingent upon output voltage amplitude.

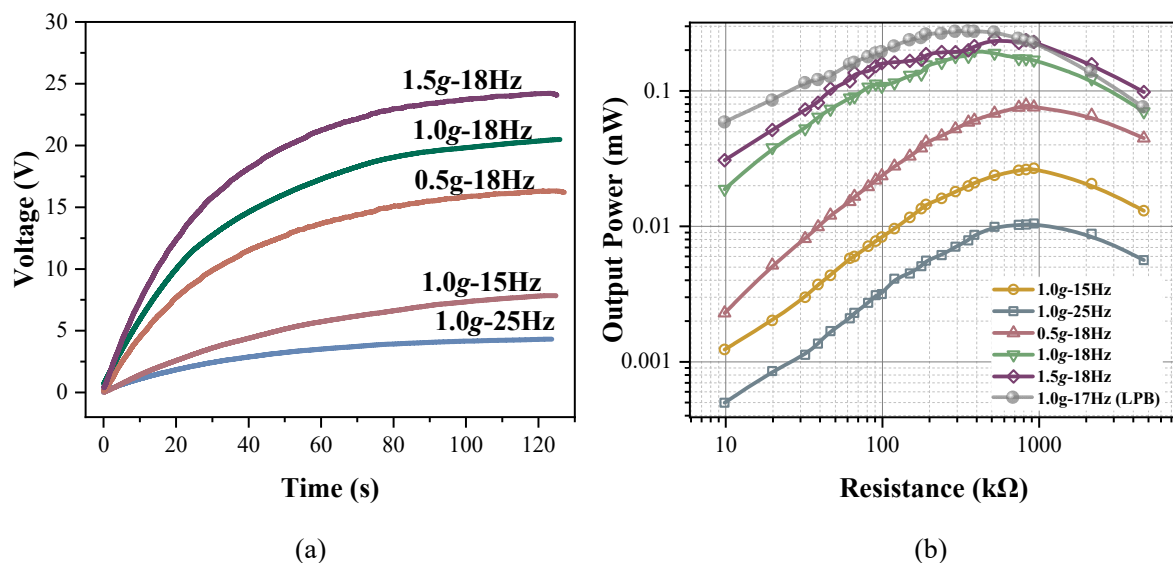


Fig. 14 (a) Time-voltage curves of charging capacity; (b) The output power with resistance curves.

The output power of the linear piezoelectric beam was also measured, as depicted in Fig. 14(b) and Tab. 3. Its maximum output power, corresponding to its resonance mode, reaches 0.277 mW, surpassing that of BPVEH. However, the linear harvester's maximum output power experiences a rapid decline when the excitation frequency deviates from its natural frequency. In contrast, for this BPVEH, the output power can be maintained within the frequency range associated with cross-well vibration mode. Additionally, it should be noted that PZT material used in the linear harvester is prone to breakage at resonant frequencies under relatively high excitation levels; whereas BPVEH exhibits a higher tolerance towards such excitations.

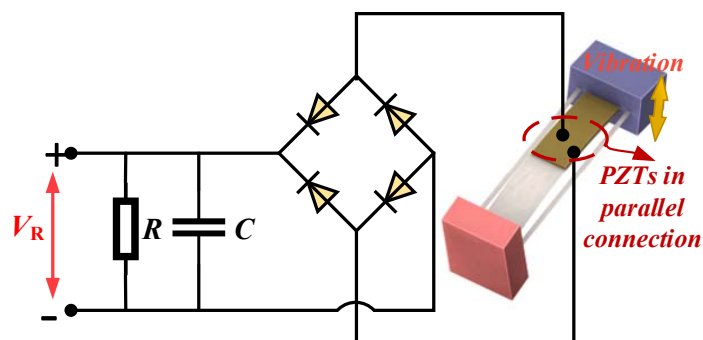


Fig. 15 AC-DC measurement circuit for measurement of output power.

Tab. 3. Summary of output powers

Excitation Condition		Vibration Mode	Maximum Power (mW)	Optimal Resistance (kΩ)	Normalized Power Density (mW/cm ³ /g ²)
Acceleration (g)	Frequency (Hz)				
0.50	18	Single-well	0.078	830	1.95
1.00	15	Single-well	0.027	930	0.68
1.00	25	Single-well	0.011	930	0.28
1.00	18	Chaotic cross-well	0.199	390	4.98
1.50	18	Periodic cross-well	0.241	520	2.68
1.00	15	Cross-well with continuous snap- through (Sample II)	0.381	390	9.53
0.75	20	Cross-well with continuous snap- through (Sample III)	0.204	290	9.07
1.00	17	Resonant Mode (LPB)	0.277	350	6.93

5.4 The method to adjust the performance of BPVEH

The performance of BPVEH relies on the laser-machined bistable host structure. Previous studies have shown that both the processing parameters of the laser and the geometry parameters can be utilized to adjust the bistability of the host structure. However, controlling bistability using laser processing parameters is considerably more complex and challenging. Therefore, we focus on utilizing geometry parameters to demonstrate the designability and controllability of our proposed BPVEH device. Among these parameters, adjusting the width of the supporting beam becomes a primary choice for tailoring BPVEH performance since it directly influences deformation discrepancies between main beams and supporting beams, which are crucial in determining bistability. As shown in Fig. 16(a), the geometry of BPVEH is maintained except for the width of supporting beam W_2 . The width of the supporting beam is increased to 0.8 mm and 1.0 mm, which is termed Sample I and Sample II. The thickness t is also a crucial geometric parameter that can be adjusted to induce bistability. The thickness t is increased to 0.2 mm, and the width of the supporting beam is adjusted to 0.8 mm, resulting in the acquisition of Sample III. All three samples were fabricated using the same processing parameter as before, and the experimental Sample II is shown in Fig. 16(a).

The load-displacement responses of these samples were measured, as shown in Fig. 16(b). To compare the precompression of these samples, the finite element model is employed to find the equivalent temperature gradient. As shown in Fig. 16(b), after adjusting the FEA results to align with the experimental findings, the corresponding temperature gradients for these three samples are successively 50°C, 29°C, and 26 °C. The mechanical characteristics of the snap-through properties for these three samples are presented and compared with the initial sample in Tab. 4, aiming to demonstrate the tailoring potentials of the proposed harvester. As expected, Samples I and II exhibit a decreased snapping load compared to the initial sample due to the augmented width of the supporting beam. With the decline of the snapping load, the displacement corresponding to the stable states is lowered too. Moreover, the distance between two snap-down points is decreased, and the area of the hysteresis loop is reduced simultaneously. This demonstrates that altering the width of the supporting beam proves to be an effective means of modifying bistability. By increasing the thickness to 0.2 mm based on Sample I, Sample III is obtained, which exhibits the lowest snapping load among all samples and closely positioned snap-down points. Consequently, it can be inferred

that bistability is further weakened in Sample III.

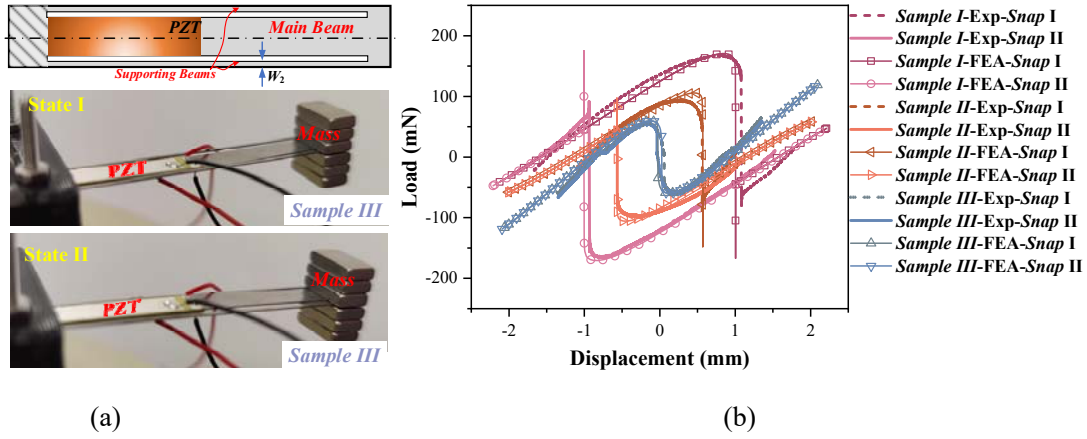


Fig. 16 (a) Two stable states of experimental sample III; (b) The load-displacement curves of three samples during two snap-through processes.

Tab. 4 Typical mechanical characters of the snap-through properties

	Width of supporting beam W_2 (mm)	Thickness t (mm)	Stable Position (mm)		Snapping load (mN)		Position of snapping load (mm)		Position of snap-down point (mm)	
			Exp	FEA	Exp	FEA	Exp	FEA	Exp	FEA
Initial sample	0.5	0.15	-2.68/2.42	2.69	293.8/-320.1	324.5	2.78/-2.81	2.64	2.84/-2.82	2.85
Sample I	0.8	0.15	-1.49/1.45	1.57	167.5/-165.3	171.7	0.81/-0.80	0.83	1.08/-0.93	1.00
Sample II	1.0	0.15	-0.94/0.95	1.14	94.3/-98.9	105.7	0.31/-0.29	0.45	0.56/-0.56	0.57
Sample III	0.8	0.20	0.88/-0.86	0.91	58.5/-60.6	59.7	0.20/-0.16	0.12	0.06/-0.03	0.02

The experimental frequency-sweep results of these three samples are illustrated in Fig. 17. For Sample I, at an acceleration of 0.5 g, the occurrence of cross-well vibration is indicated by a peak with a flat head. As the acceleration increases, the frequency range associated with cross-well vibration expands. The occurrence of a continuous snap-through behavior in the form of cross-well vibration mode, characterized by a sharp peak, is observed at approximately 13.5 Hz during reverse frequency sweep when the acceleration reaches 1.5 g. This new mode of cross-well vibration has not been previously observed. For Sample II, at an acceleration of 0.5 g, the frequency range of cross-well vibration in the reverse sweep direction (14.8~21.6Hz) is even wider than that of Sample I at an acceleration of 1 g. When the acceleration increases to 0.75 g, this frequency range becomes broader and a hardening-type response (14.5~17.6 Hz) emerges in the reverse sweep direction. The mode corresponding to the hardening peak exhibits cross-well vibration with continuous snap-through behavior, where voltage amplitude increases with frequency along this branch. Furthermore, when the acceleration level reaches 1.0 g, this cross-well vibration mode also appears in the forward sweep direction. It should be noted that there is some intermittent snap-through and low voltage amplitude cross-well vibration response following behind the hardening branch. For Sample III, the cross-well vibration with continuous snap-through behavior can be triggered at an acceleration of 0.5g. In the results of reverse frequency-sweep, an evident hardening response appears in the frequency range of 14.2~20.1 Hz, while the cross-well vibration response with intermittent snap-through follows and persists in a frequency range of 20.1~26

Hz. However, no hardening response is observed in the forward frequency sweep direction. When the acceleration increases to 0.75g, a relatively narrow but distinct frequency range of 20.2~22.8 Hz corresponding to the hardening response emerges, along with a wider range of 12~21.9 Hz captured during reverse sweep direction testing. The weak cross-well vibration response with intermittent snap-through extends over broadband up to 31.5 Hz and 27.5 Hz in two directions of frequency sweep respectively.

When comparing the frequency-sweep results in two directions, it is observed that the discrepancy between these two results under the same excitation level becomes increasingly apparent with the excitation level, especially when the response of cross-well vibration with continuous snap-through behaviors exists. This discrepancy is attributed to the hysteretic feature resulting from the inherent nonlinearities in the system, leading to a staggered arrangement of two response peaks around the first mode in both sweep directions. Under the same acceleration level, the cross-well vibration with continuous snap-through behaviors is easier to induce in the reverse sweep. It is explained that the entrance into this vibration with the large amplitude feature during reverse sweeps may be caused by the coalescence of vibrational energy carried by the cross-well motion and the first mode at their linear fundamental frequency. Once the reverse sweep passes through the first mode, there is a drop-off in amplitude until returning to the low-amplitude region with the single-well vibration. However, it does not exist in the forward sweep due to the system being low-energy orbit when approaching the first mode. When the excitation level increases to some extent, the system may enter the high-energy orbit and then trigger the cross-well vibration with continuous snap-through behavior in the forward sweep, just like sample III under the acceleration level of 0.75 g.

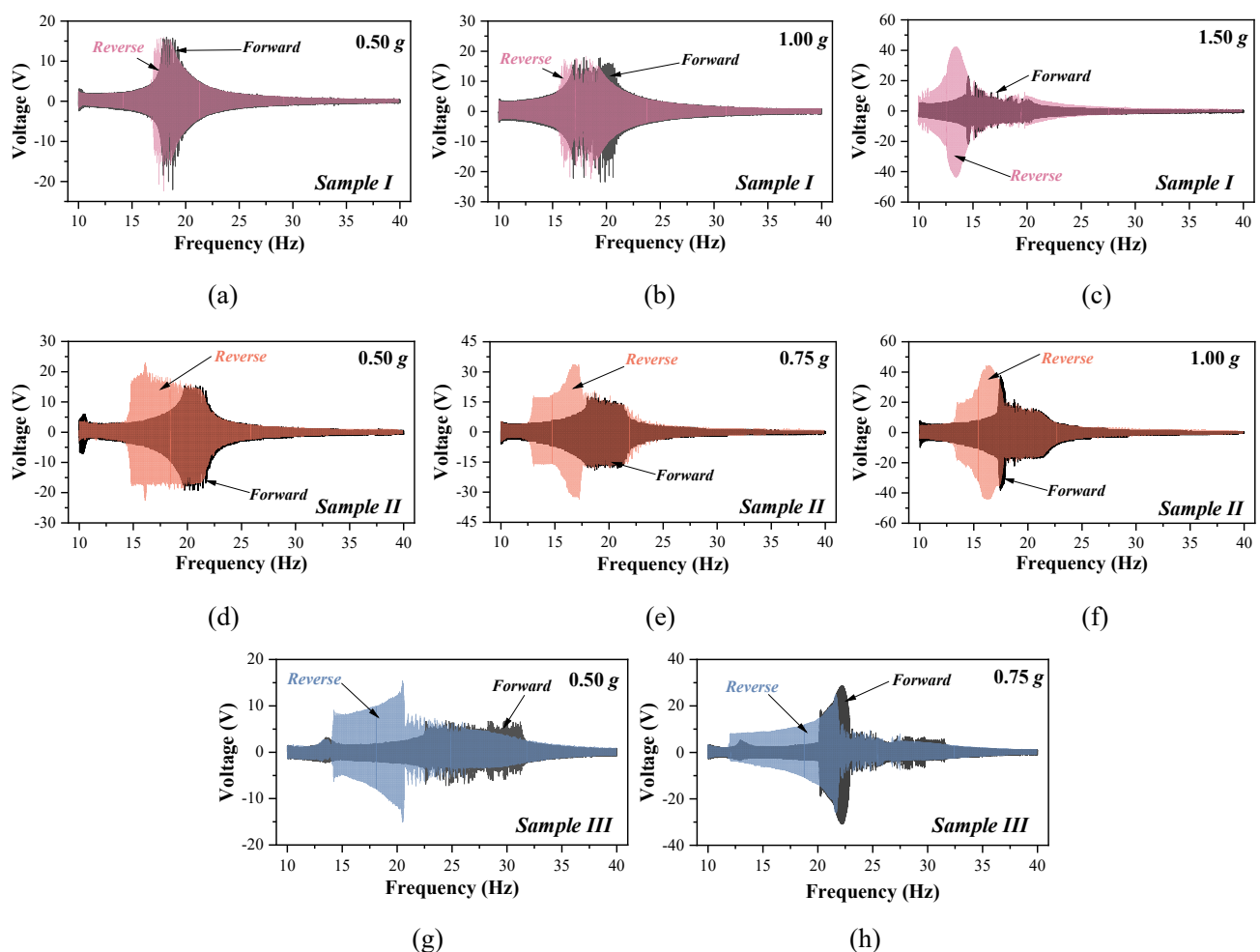


Fig. 17 The experimental frequency-sweep results of three samples at the different acceleration levels.

The cross-well vibration with continuous snap-through behavior has not been previously analyzed. In this study, two typical voltage-time signals of this vibration mode were captured through experiments and FEA from Sample II under

the excitation condition of 1.0 g and 13 Hz, as well as Sample III under the excitation condition of 0.75 g and 20 Hz, as illustrated in Fig. 18(a) and (b). The voltage signal exhibits primarily harmonic characteristics, with some fluctuations distributed along the main curve. From the FFT plots shown in Fig. 18(c) and (d), it can be observed that the snap-through frequency of the main response is equal to the excitation frequency, which represents a dominant feature of this vibration mode. Furthermore, these fluctuations correspond to subharmonic responses occurring at integer multiples of the excitation frequency. Comparing both responses, it can be noted that Sample II exhibits a higher voltage amplitude than Sample III.

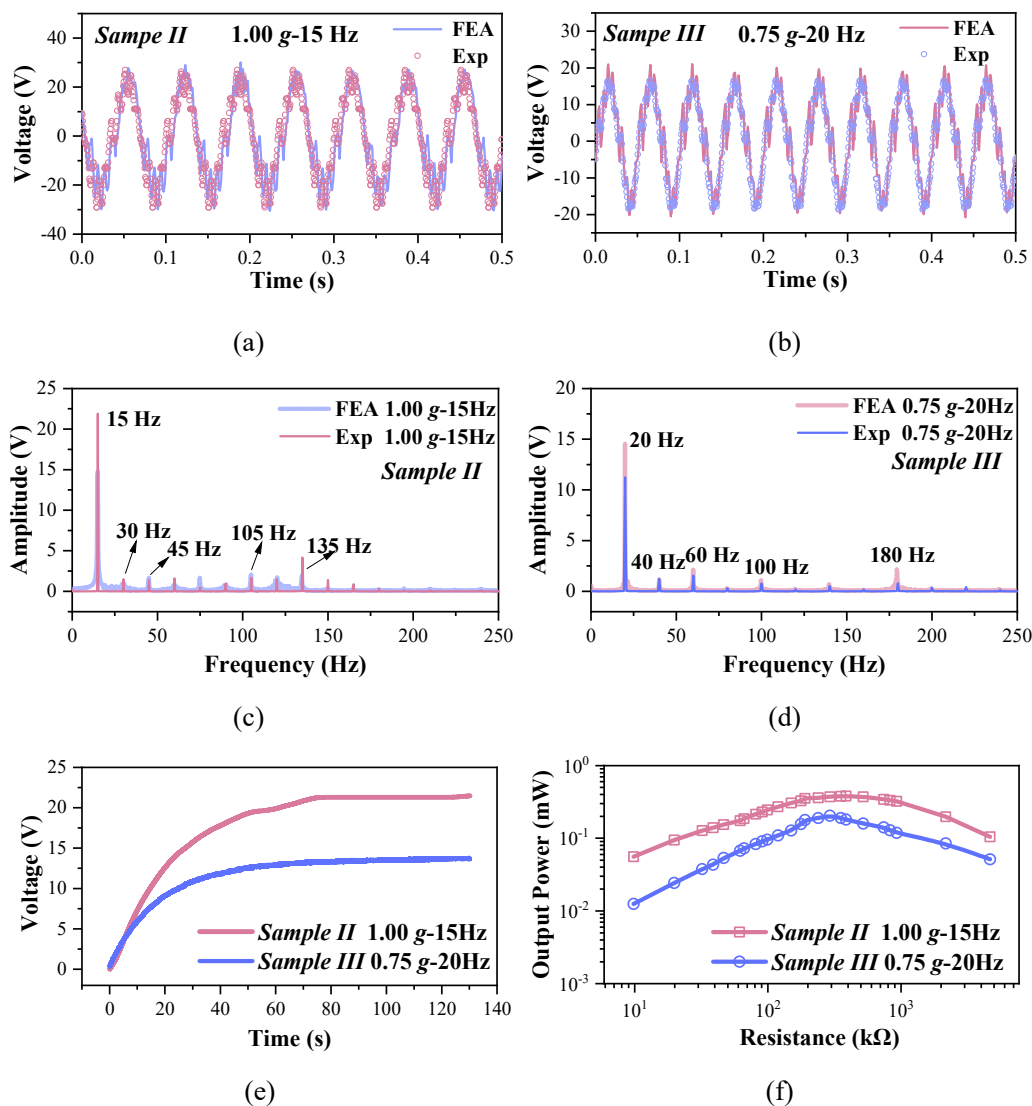


Fig. 18 (a) The cross-well oscillation time-voltage responses with continuous snap-through of Sample II at the excitation condition of 1.0g and 15Hz; (b) The cross-well oscillation response with continuous snap-through of Sample III at the excitation condition of 0.75g and 20Hz; (c) FFT plots of Sample II at the excitation condition of 1.0g and 15Hz; (d) FFT plots of Sample III at the excitation condition of 0.75g and 20Hz; (e) The time-voltage curve of charging capacitor; (f) The time-voltage curves of charging the capacitor under two types of cross-well oscillation with snap-through; (f) The resistance load vs output power under two types of cross-well oscillation with continuous snap-through.

Under this vibration mode, the performances of the charging capacitor ($47 \mu\text{F}$) and the output powers are also characterized as previously demonstrated in Fig. 18(e) and (f). In comparison to previous results, it is evident that the time required to reach the plateau voltage is significantly reduced; however, the values of plateau voltage remain relatively low at approximately 21 V and 13 V for these two cases respectively. However, Sample II and III exhibit maximum output powers of 0.381 mW and 0.204 mW correspondingly. The measured results of output power and

power density are summarized in Tab. 3. Under an acceleration of 1 g, Sample II surpasses the linear harvester in terms of maximum power while also exhibiting the highest normalized power density among all samples tested thus far. As for Sample III, its normalized power density exceeds that of both the linear harvester and the initial sample. The results demonstrate that the dominant mode for energy harvesting is the cross-well vibration with continuous snap-through behavior. Therefore, the primary effective bandwidth of Samples II and III corresponds to the branch of hardening response. The output of the intermittent cross-well vibration bandwidth is relatively low and can be considered as the secondary effective bandwidth.

Compared to the four samples, the initial sample demonstrates the highest snapping load, indicating a greater difficulty in triggering cross-well vibration. However, it also exhibits the highest voltage amplitude of the flattened response peak with intermittent cross-well vibration corresponding to the primary effective bandwidth, wherein the output power remains relatively stable regardless of snap-through behavior frequency. When the bistability, which includes the snapping load and displacement amplitude corresponding to the stable states, is weakened by adjusting geometric parameters such as the width of the supporting beam and thickness, it results in a reduction in the required excitation level to trigger cross-well vibration and leads to the emergence of cross-well vibration with continuous snap-through behavior. At this time, the primary effective bandwidth lies within the hardening branch characterized by a non-linear increase in output with frequency. Following this, there exists a secondary effective bandwidth that covers a higher frequency range but exhibits lower output levels. Moreover, the discrepancy between results in two sweeping directions could be mitigated when the bistability is weakened; however, the effective bandwidth in the reverse direction generally surpasses that in the forward direction at the same excitation level.

5.5 The nonlinear piezo-beam array

The single BPVEH exhibits relatively low output power and a limited effective bandwidth. However, the proposed BPVEH features a simple structure and flexible boundary conditions, enabling the accumulation of multiple BPVEHs within a compact size. An effective approach is to construct a multistable array comprising several BPVEHs. To enhance the overall effective bandwidth of the array, each BPEH can possess a slightly different but neighboring effective bandwidth. In this regard, Sample III is selected as the constructing member due to its wider effective bandwidth, lower excitation level requirement, and ample safety margin for piezoelectric material. Moreover, Sample III exhibits a relatively high natural frequency, thereby providing a broader range for bandwidth adjustment.

Four samples were individually fabricated, and four metal blocks were affixed to their tips to adjust their fundamental frequency. The energy harvesting array was assembled by mounting four BPVEHs on the same support, as depicted in Fig. 19(a). The metal mass blocks were produced using the SLM (Selective Laser Melting) technique, featuring identical bonding areas with the samples but varying heights to control the mass, as illustrated in Fig. 19(b). The array was subjected to an excitation level of 0.75g acceleration, and the oscilloscope monitored the voltage responses of four BPVEHs individually. To investigate the broadband effect of the array, a resistance load was connected to each BPVEH, and the corresponding transient output powers were correlated with frequency, as depicted in Fig. 19(c) and (d). Here, an optimal resistance of 290 k Ω for sample III shown in Tab. 3 was employed, but it should be noted that the optimal resistance may not be suitable for all excitation conditions due to its dependence on vibration modes, as illustrated in Tab. 3. Here, the load resistance primarily serves to demonstrate the effective bandwidth. The output power of the secondary effective bandwidth corresponding to the intermittent cross-well vibration is too low, so the primary effective bandwidth is considered first.

In the forward sweep direction, all four BPEHs exhibit a hardening-type primary response characterized by cross-well oscillation with continuous snap-through vibration mode. The increasing tip mass results in adjacent and overlapping effective frequency bandwidths for the four BPEHs, resulting in a total array effective bandwidth of 9.4 Hz, covering the range from 18.8 Hz to 28.2 Hz. In the reverse sweep direction, this effective bandwidth expands to 13.7 Hz, extending from 13.2 Hz to 26.9 Hz. Due to the hardening-type response, the output power exhibits a steep

increase within the effective bandwidth, with a maximum transient power of 4.2 mW observed during the forward sweep of No. 4 BPEH. However, at the starting frequency, this power is below 0.5 mW. It is worth noting that there are overlapping sections in the output power of all four BPEHs' main responses. Therefore, it becomes intriguing to accumulate these powers from all four BPEHs to evaluate the overall array performance, as depicted in Fig. 19(e). The average transient powers of the forward and reverse sweep in the effective bandwidth are 1.1 mW and 0.6 mW, respectively. It should be noted that the beams are independent and their outputs are not integrated really. Here, this rough estimation, however, serves as a means to illustrate the broadband possibility of this array. It is possible to design a more complicated energy harvesting management circuit that effectively extracts the power from the array in the future. Also, more parametric works will be done in the following studies, and then the array can be designed by utilizing BPVEHs with varying parameters, resulting in improved average power, expanded effective bandwidth with a better distribution of the resonance frequencies, and more compact size.

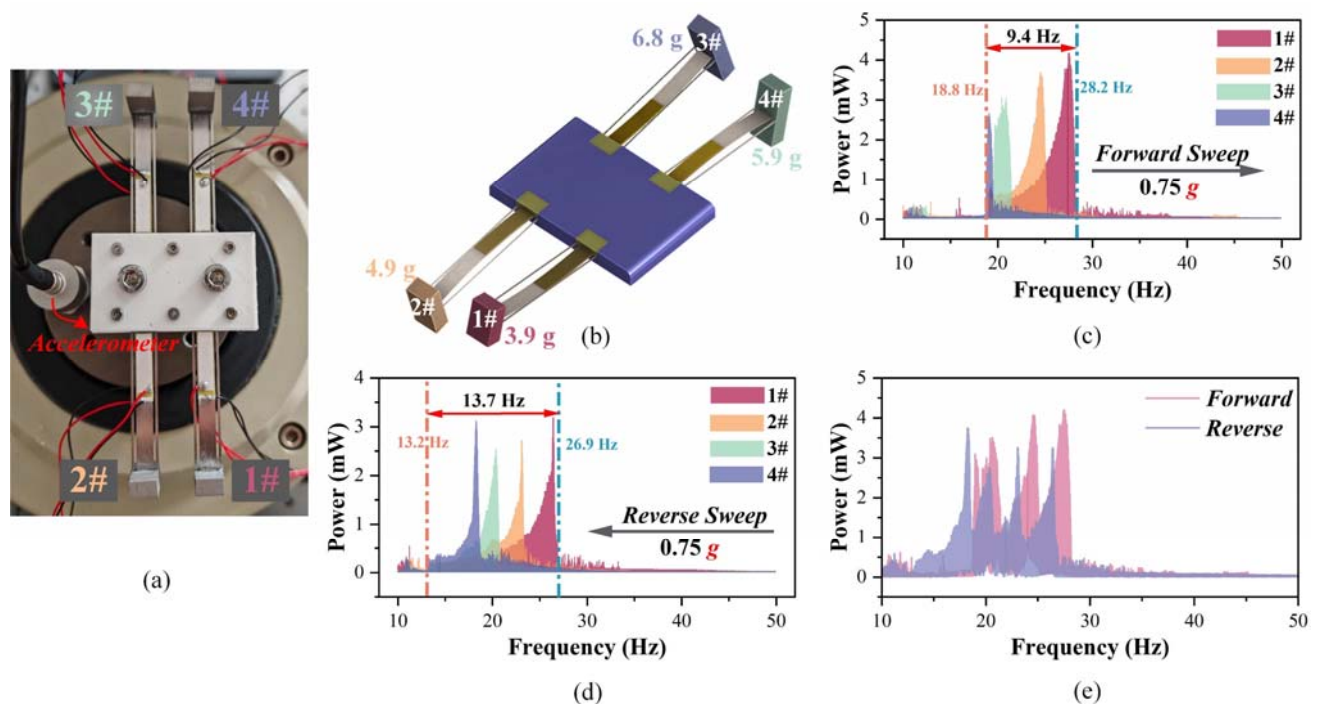


Fig. 19 (a) The experimental sample of the energy harvesting array; (b) The diagram of the energy harvesting array; (c) The frequency-power responses in the forward frequency sweep; (d) The frequency-power response in the reverse frequency sweep; (e) The sum of power in two frequency-sweep directions.

6. Conclusions

The present study presents a novel, cost-effective, magnet-free, bistable piezoelectric vibrational energy harvester that has been designed, prototyped, and tested to evaluate its performance in harvesting energy from wideband vibrations. A finite element model is established to analyze the static properties and dynamic behaviors of the proposed harvester, while a simple analytical model is also developed for effectively predicting its dynamic response. Several key findings of the investigation are outlined as follows:

- (1) By utilizing laser machining as the fabrication and pre-stressing method simultaneously, the integration of the buckled beam with constraints provided by supporting beams allows for the functionality of a cantilever beam in BPVEH. Its static snap-through behaviors are characterized by two load-displacement paths between two snapping points;
- (2) With the increase in excitation level, the softening response peak of BPVEH transforms into a broader and flatter peak, while the single-well vibration transitions into cross-well vibration with intermittent snap-through occurrences. If the excitation level continues to rise, a hardening response peak characterized by continuous snap-through

phenomena may emerge at a lower frequency range if the bistability is relatively weak.

(3) Compared to the linear harvester at its resonant frequency, the BPVEH exhibits a comparable high level of output power under the intermittent cross-well vibration mode and demonstrates enhanced tolerance towards higher excitation levels due to its strong bistability. The generation of higher power is observed in cases where continuous snap-through occurs during cross-well vibration for BPVEH with weaker bistability.

(4) The bistability can be mitigated by adjusting the thickness of the substrate and the width of the supporting beam, thereby inducing a continuous snap-through phenomenon in cross-well vibration at a relatively low excitation level and achieving a peak response with hardening characteristics. Simultaneously, as bistability decreases, the effective bandwidth of energy harvesting shifts from the flattened peak to the hardening peak.

(5) The nonlinear piezo-beam array consisting of four identical BPVEHs with varying proof masses demonstrates good broadband characteristics and achieves the widest effective bandwidth of 13.7 Hz. Therefore, the proposed energy harvester presents a promising approach to achieving broadband features without relying on magnetic components, thereby opening up numerous possibilities for environmental energy harvesting.

The present study demonstrates the fundamental feasibility of the proposed energy harvester for broadband vibration. However, future improvements should be made to enhance its performance, such as investigating a more sophisticated nonlinear circuit capable of accommodating the nonlinear characteristics of the energy harvesting array and optimizing the geometric parameters to achieve a better distribution of resonance frequencies across the array.

7. Acknowledgment

This work was supported by the National Natural Science Foundation of China [grant No. 12202215].

References

- [1] Wei C, Jing X. A comprehensive review on vibration energy harvesting: Modelling and realization. *Renewable and sustainable energy reviews*. 2017;74:1-18.
- [2] Wu Y, Fang J, Wu C, Li C, Sun G, Li Q. Additively manufactured materials and structures: A state-of-the-art review on their mechanical characteristics and energy absorption. *International Journal of Mechanical Sciences*. 2023;246:108102.
- [3] Sanislav T, Mois GD, Zeadally S, Folea SC. Energy Harvesting Techniques for Internet of Things (IoT). *IEEE Access*. 2021;9:39530-49.
- [4] Sezer N, Koç M. A comprehensive review on the state-of-the-art of piezoelectric energy harvesting. *Nano Energy*. 2021;80:105567.
- [5] Chen G, Zhu Y, Huang D, Zhou S. Self-powered and self-sensing devices based on piezoelectric energy harvesting. *Science China Technological Sciences*. 2024;67:1631-67.
- [6] Anton SR, Sodano HA. A review of power harvesting using piezoelectric materials (2003–2006). *Smart Materials and Structures*. 2007;16:R1.
- [7] Sodano HA, Inman DJ, Park G. A review of power harvesting from vibration using piezoelectric materials. *Shock and Vibration Digest*. 2004;36:197-206.
- [8] Safaei M, Sodano HA, Anton SR. A review of energy harvesting using piezoelectric materials: state-of-the-art a decade later (2008–2018). *Smart Materials and Structures*. 2019;28:113001.
- [9] Pellegrini SP, Tolou N, Schenk M, Herder JL. Bistable vibration energy harvesters: A review. *Journal of Intelligent Material Systems and Structures*. 2012;1045389X12444940.
- [10] Tran N, Ghayesh MH, Arjomandi M. Ambient vibration energy harvesters: A review on nonlinear techniques for performance enhancement. *International Journal of Engineering Science*. 2018;127:162-85.
- [11] Ferrari M, Ferrari V, Guizzetti M, Marioli D, Taroni A. Piezoelectric multifrequency energy converter for power harvesting in autonomous microsystems. *Sensors and Actuators A: Physical*. 2008;142:329-35.
- [12] Xue H, Hu Y, Wang Q. Broadband piezoelectric energy harvesting devices using multiple bimorphs with different operating frequencies. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*. 2008;55:2104-8.

- [13] Castagnetti D. Experimental modal analysis of fractal-inspired multi-frequency structures for piezoelectric energy converters. *Smart Materials and Structures*. 2012;21:094009.
- [14] Liu H, Qian Y, Lee C. A multi-frequency vibration-based MEMS electromagnetic energy harvesting device. *Sensors and Actuators A: Physical*. 2013;204:37-43.
- [15] Hu Y, Xue H, Hu H. A piezoelectric power harvester with adjustable frequency through axial preloads. *Smart Materials and Structures*. 2007;16:1961.
- [16] Vinod RC, Prasad MG, Yong S, Frank TF. A vibration energy harvesting device with bidirectional resonance frequency tunability. *Smart Materials and Structures*. 2008;17:015035.
- [17] Eichhorn C, Tchagsim R, Wilhelm N, Woias P. A smart and self-sufficient frequency tunable vibration energy harvester. *Journal of micromechanics and microengineering*. 2011;21:104003.
- [18] Cao DX, Leadenham S, Erturk A. Internal resonance for nonlinear vibration energy harvesting. *The European Physical Journal Special Topics*. 2015;224:2867-80.
- [19] Nie X, Tan T, Yan Z, Yan Z, Hajj MR. Broadband and high-efficient L-shaped piezoelectric energy harvester based on internal resonance. *International Journal of Mechanical Sciences*. 2019;159:287-305.
- [20] Chen L, Jiang W. A piezoelectric energy harvester based on internal resonance. *Acta Mechanica Sinica*. 2015:1-6.
- [21] Sun R, Zhou S, Li Z, Cheng L. Dual electromagnetic mechanisms with internal resonance for ultra-low frequency vibration energy harvesting. *Applied Energy*. 2024;369:123528.
- [22] Wu H, Tang L, Yang Y, Soh CK. A novel two-degrees-of-freedom piezoelectric energy harvester. *Journal of Intelligent Material Systems and Structures*. 2013;24:357-68.
- [23] Sun R, Li Q, Yao J, Scarpa F, Rossiter J. Tunable, multi-modal, and multi-directional vibration energy harvester based on three-dimensional architected metastructures. *Applied Energy*. 2020;264:114615.
- [24] Tao K, Tang L, Wu J, Lye SW, Chang H, Miao J. Investigation of Multimodal Electret-Based MEMS Energy Harvester With Impact-Induced Nonlinearity. *Journal of Microelectromechanical Systems*. 2018;27:276-88.
- [25] Daqaq MF, Masana R, Erturk A, Quinn DD. On the role of nonlinearities in vibratory energy harvesting: a critical review and discussion. *Applied Mechanics Reviews*. 2014;66:040801.
- [26] Neiss S, Goldschmidtboeing F, Kroener M, Woias P. Analytical model for nonlinear piezoelectric energy harvesting devices. *Smart Materials and Structures*. 2014;23:105031.
- [27] Fang S, Zhou S, Yurchenko D, Yang T, Liao W-H. Multistability phenomenon in signal processing, energy harvesting, composite structures, and metamaterials: A review. *Mechanical Systems and Signal Processing*. 2022;166:108419.
- [28] Liang H, Hao G, Olszewski OZ. A review on vibration-based piezoelectric energy harvesting from the aspect of compliant mechanisms. *Sensors and Actuators A: Physical*. 2021;331:112743.
- [29] Cottone F, Gammaitoni L, Vocca H, Ferrari M, Ferrari V. Piezoelectric buckled beams for random vibration energy harvesting. *Smart Materials and Structures*. 2012;21:035021.
- [30] Friswell MI, Ali SF, Bilgen O, Adhikari S, Lees AW, Litak G. Non-linear piezoelectric vibration energy harvesting from a vertical cantilever beam with tip mass. *Journal of Intelligent Material Systems and Structures*. 2012;23:1505-21.
- [31] Masana R, Daqaq MF. Relative performance of a vibratory energy harvester in mono-and bi-stable potentials. *Journal of Sound and Vibration*. 2011;330:6036-52.
- [32] Xu C, Liang Z, Ren B, Di W, Luo H, Wang D, et al. Bi-stable energy harvesting based on a simply supported piezoelectric buckled beam. *Journal of Applied Physics*. 2013;114:114507.
- [33] Andò B, Baglio S, Bulsara AR, Marletta V. A bistable buckled beam based approach for vibrational energy harvesting. *Sensors and Actuators A: Physical*. 2014;211:153-61.
- [34] Mariello M, Blad TWA, Mastronardi VM, Madaro F, Guido F, Staufer U, et al. Flexible piezoelectric AlN transducers buckled through package-induced preloading for mechanical energy harvesting. *Nano Energy*. 2021;85:105986.
- [35] Nan W, Yuncheng H, Jiyang F. Bistable energy harvester using easy snap-through performance to increase output

power. *Energy*. 2021;226:120414.

- [36] Zou D, Liu G, Rao Z, Zi Y, Liao W-H. Design of a broadband piezoelectric energy harvester with piecewise nonlinearity. *Smart Materials and Structures*. 2021;30:085040.
- [37] Yang T, Cao Q. Novel multi-stable energy harvester by exploring the benefits of geometric nonlinearity. *Journal of Statistical Mechanics: Theory and Experiment*. 2019;2019:033405.
- [38] Emam SA, Inman DJ. A review on bistable composite laminates for morphing and energy harvesting. *Applied Mechanics Reviews*. 2015;67:060803.
- [39] Arrieta A, Delpero T, Bergamini A, Ermanni P. Broadband vibration energy harvesting based on cantilevered piezoelectric bi-stable composites. *Applied Physics Letters*. 2013;102:173904.
- [40] Betts DN, Bowen CR, Kim HA, Guyer RA, Le Bas P-Y, Inman DJ. Modelling the dynamic response of bistable composite plates for piezoelectric energy harvesting. *Proceedings of "AIAA SciTech Conference"*, Maryland, USA2014.
- [41] Stanton SC, McGehee CC, Mann BP. Nonlinear dynamics for broadband energy harvesting: Investigation of a bistable piezoelectric inertial generator. *Physica D: Nonlinear Phenomena*. 2010;239:640-53.
- [42] De Paula AS, Inman DJ, Savi MA. Energy harvesting in a nonlinear piezomagnetoelastic beam subjected to random excitation. *Mechanical Systems and Signal Processing*. 2015;54:405-16.
- [43] Erturk A, Inman D. Broadband piezoelectric power generation on high-energy orbits of the bistable Duffing oscillator with electromechanical coupling. *Journal of Sound and Vibration*. 2011;330:2339-53.
- [44] Zhou S, Cao J, Inman DJ, Lin J, Liu S, Wang Z. Broadband tristable energy harvester: Modeling and experiment verification. *Applied Energy*. 2014;133:33-9.
- [45] Zhou Z, Qin W, Zhu P. Improve efficiency of harvesting random energy by snap-through in a quad-stable harvester. *Sensors and Actuators A: Physical*. 2016;243:151-8.
- [46] Zhou Z-y, Qin W-y, Zhu P. Energy harvesting in a quad-stable harvester subjected to random excitation. *AIP Advances*. 2016;6:025022.
- [47] Deng H, Du Y, Wang Z, Ye J, Zhang J, Ma M, et al. Poly-stable energy harvesting based on synergetic multistable vibration. *Communications Physics*. 2019;2:21.
- [48] Lai S-K, Wang C, Zhang L-H. A nonlinear multi-stable piezomagnetoelastic harvester array for low-intensity, low-frequency, and broadband vibrations. *Mechanical Systems and Signal Processing*. 2019;122:87-102.
- [49] Sun S, Su X, Chen X, Xu J, Leng Y, Bao H, et al. Design and performance of a novel magnetically induced penta-stable piezoelectric energy harvester. *Smart Materials and Structures*. 2024;33:025016.
- [50] Mei X, Nan H, Dong R, Zhou R, Jin J, Sun F, et al. Magnetic-linkage nonlinear piezoelectric energy harvester with time-varying potential wells: Theoretical and experimental investigations. *Mechanical Systems and Signal Processing*. 2024;208:110998.
- [51] Fan Y, Ghayesh MH, Lu T-F. A broadband magnetically coupled bistable energy harvester via parametric excitation. *Energy Conversion and Management*. 2021;244:114505.
- [52] Pan D, Wu Z, Dai F, Tolou N. A novel design and manufacturing method for compliant bistable structure with dissipated energy feature. *Materials & Design*. 2020;196:109081.
- [53] Kulah H, Najafi K. Energy scavenging from low-frequency vibrations by using frequency up-conversion for wireless sensor applications. *IEEE Sensors Journal*. 2008;8:261-8.
- [54] Karami MA, Inman DJ. Electromechanical Modeling of the Low-Frequency Zigzag Micro-Energy Harvester. *Journal of Intelligent Material Systems and Structures*. 2011;22:271-82.