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Research paper

Optimizing residential flexibility for sustainable energy management in distribution networks

Manoharan Premkumar [a,*], Sowmya Ravichandran [b], Ahmad O. Hourani [c], Thamer A.H. Alghamdi [d,e,**]

[a] Department of Electrical and Electronics Engineering, Dayananda Sagar College of Engineering, Affiliated to Visvesvaraya Technological University, Bengaluru, Karnataka 560078, India
[b] Department of Electrical and Electronics Engineering, Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal, Karnataka 576104, India
[c] Hourani Center for Applied Scientific Research, Al-Ahliyya Amman University, Jordan
[d] Wolfson Centre for Magnetics, School of Engineering, Cardiff University, Cardiff CF24 3AA, UK
[e] Electrical Engineering Department, Faculty of Engineering, Al-Baha University, Al-Baha 65779, Saudi Arabia

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ABSTRACT

In search of a sustainable and green economy, many initiatives have been undertaken to promote clean energy and enhance local flexibility. Residential flexibility, achieved through home appliances capable of adjusting their consumption profiles, offers a feasible solution for operators to address challenges such as congestion and balancing in distribution systems. This paper considered an improved approach for aggregators to provide flexibility in distribution systems. By leveraging load flexibility resources, the model facilitates the rescheduling of real-time and shifting appliances to meet the demands of Balance Responsible Parties (BRPs) or Distribution System Operators (DSOs). This study uses a number of approaches to solve the recommended model effectively despite the problem's inherent complexity. An extensive test case with twenty residential houses equipped with seven types of appliances each is run in order to confirm and compare the optimization algorithms' performance. The results show that by rescheduling home appliance loads across 24 hours, the aggregator may effectively accommodate flexibility requests from DSOs/BRPs while optimizing the expenses associated with user compensation. To further improve the optimization process, this study uses a new Reinforced Learning Quantum Inspired Grey Wolf Optimization (RLQIGWO). Through the integration of reinforcement learning and quantum mechanics principles into the original grey wolf optimizer, RLQIGWO achieves better performance in load balancing, resource utilization, and execution of tasks. The findings demonstrate that the proposed RLQIGWO improves the efficacy and competence of flexibility options in distribution networks, paving the way to a more adaptable and strong energy management strategy.

1. Introduction

The environmental crisis has increased the European Union's concern about the significant impact of the energy sector on the environment. The EU has launched a number of initiatives targeted at promoting energy sustainability in order to address issues (Mata et al., 2020). These strategies centre on encouraging clean energy sources, including renewable energy, and making use of local flexibility to shift the economy toward one that is more environmentally friendly, low-carbon, and sustainable. The development of a smart grid, which promises major advantages, including effective energy management and the seamless integration of renewable energy sources, is key to this shift (Haider et al., 2016; de Lavoreille et al., 2022). But these developments also present new difficulties for grid management and operation, especially in distribution networks. The complexity of controlling energy distribution and consumption in smart grids calls for advanced approaches. Deploying enhanced communication capabilities is essential for Demand-Side Management (DSM) and Demand Response (DR) plans to be implemented successfully (Luo et al., 2022; Paterakis et al., 2015; Palensky and Dietrich, 2011). These solutions take advantage of home

* Corresponding author.
** Corresponding author at: Wolfson Centre for Magnetics, School of Engineering, Cardiff University, Cardiff CF24 3AA, UK.
E-mail addresses: mprem.me@gmail.com (M. Premkumar), alghamdit1@cardiff.ac.uk (T.A.H. Alghamdi).

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appliances' flexibility to allow end users to actively assist in fixing distribution network issues. Energy flexibility is a key component of the energy market strategy and is described as the ability to change load profiles by shifting, lowering, or reducing consumption over different periods (Azizi et al., 2021; Lu et al., 2022).

Future power systems are going to develop into much more flexible systems that can constantly adjust their operating points to match variations in generation and demand in real-time. In order to manage the rapidly changing nature of future power systems, it is necessary to install a variety of technical ancillary services, also known as flexibility services, in response to this increased requirement for flexibility (Golmohamadi et al., 2021; Brunner et al., 2020). Typically, flexibility services are purchased to meet the needs for flexibility at the local and systemic levels. Locally, flexibility services help distribution system operators (DSOs) make their electrical distribution networks more flexible (Chinde et al., 2021). DSOs can more effectively control local grid efficiency and stability thanks to these services. These flexibility services for DSOs and TSOs are sourced from flexible energy resources (FERs) (Mugnini et al., 2019; Mekonnen et al., 2012; Kilkki et al., 2018). At the moment, the majority of FERs utilized to provide flexibility services across the system are conventional fuel-based generators. It is the responsibility of these generators to provide a steady supply of power and balance the system. In a similar vein, DSOs handle local flexibility requirements by using conventional controllers and devices. However, relying just on these traditional methods is not enough for power systems in the future, particularly considering the significant penetration of renewable energy sources that are predicted (Luo et al., 2023; Anvari-Moghaddam et al., 2015; Chen et al., 2018).

Renewable energy generation is intermittent, which adds another layer of complexity and calls for more sophisticated and adaptable flexibility solutions. New forms of FERs must be incorporated by both DSOs and TSOs in order to meet future flexibility demands (Al Zahr et al., 2022; Ghazvini et al., 2024). Distribution networks' smart and engaged home consumers offer a very attractive source of flexibility. Flexible devices that can be regulated in accordance with system operators' needs are a feature of these smart houses. Enhancing the flexibility of the appliance allows home consumers to sell their flexibility to DSOs and TSOs, thereby earning money in exchange for their flexible capacities. System operators can optimize the flexibility of active residential consumers connected to distribution systems through this dynamic interaction (Finck et al., 2020). Including residential users in flexible markets and integrating smart home technologies can make power systems more resilient and capable of responding to fluctuations in generation and demand in real-time. By increasing grid stability and efficiency and enabling consumers to participate in the energy ecosystem actively, this approach helps to create a more adaptable and sustainable power system in the future (Asadi and Golmohamadi, 2022; Bahmani et al., 2022).

A clearly defined set of procedures and active participation from all parties are necessary for the effective running of these local energy communities. Aggregators guarantee that these procedures are specified precisely and followed, facilitating smooth transactive energy exchanges (Sousa et al., 2018; Kiljander et al., 2019). This is the process of purchasing and selling energy inside the local network in order to maximize the use of available resources and take care of certain local limitations, such as distributing the load more evenly or reducing congestion in the distribution network. Aggregators are utilized in practice to enable real-time monitoring and management of energy consumption by utilizing smart grid infrastructure and advanced communication technologies. They can then react to real-time signals from DSOs and TSOs by dynamically adjusting the aggregated load (Lezama et al., 2020; Elghitani and Zhuang, 2018). When there is a large penetration of renewable sources, this capability becomes especially important because the fluctuation in generation calls for a flexible and responsive demand side. In addition, aggregators are better equipped to handle home appliance rescheduling when complex optimization algorithms are integrated into the system. Exact control over flexible resources is made possible by these algorithms, which guarantee that demands for flexibility are fulfilled effectively and that participating consumers profit economically to the fullest extent possible (Elghitani and Zhuang, 2018; Lezama et al., 2017; Lipari et al., 2018). Aggregators harness distributed energy resources' full potential by enabling small customers to contribute to system stability and efficiency actively and it fosters a low-carbon, sustainable energy future in addition to supporting the integration of renewable energy sources by offering a manageable buffer. Aggregators enable a decentralized approach that improves the power system's resilience and adaptability by removing the need for conventional centralized control techniques and establishing a more stable framework for energy management (Lampropoulos et al., 2018; Bezmaslov et al., 2022; Khajeh et al., 2022).

This work builds upon the paradigm first presented by Sousa et al. (2018) by adding an aggregator similar to Lezama et al. (2020), that oversees the community of Home Energy Management Systems (HEMS). The aggregator's job is to reschedule household appliances so that the grid is more flexible. With the help of this improved model, aggregators can provide DSOs and BRPs with customizable services that maximize resource use in exchange for financial remuneration. This work utilizes a sophisticated Grey Wolf Optimizer (GWO) called the Reinforced Learning Quantum Inspired Grey Wolf Optimization (RLQIGWO) algorithm due to the complex nature of the fundamental mathematical formulation. The purpose of this algorithm is to quickly identify the best solution for shifting home appliances around so that the next day is flexible. The model's ability to manage the complex and dynamic aspects of resource management and energy demand is improved by the RLQIGWO algorithm, which offers a reliable solution for next-generation energy systems. The main objective is to guarantee that these requests are fulfilled effectively and to keep the expenses of paying end users for their involvement to a minimum. The RLQIGWO algorithm, which incorporates quantum computing concepts (Lampropoulos et al., 2018; Bezmaslov et al., 2022) and reinforcement learning (Yin et al., 2023; Hu and Yu, 2023; Sundar Ganesh et al., 2024) into the GWO framework (Mirjalili et al., 2014; Premkumar et al., 2024), is presented in this paper due to the complexity of the problem. By combining the advantages of adaptive learning from reinforcement learning with the exploration powers of quantum computing, this hybrid technique seeks to improve the algorithm's performance. A thorough case study with twenty residential homes furnished with seven varieties of household appliances is carried out to evaluate and contrast the effectiveness of the evolutionary algorithms. The findings reveal that the aggregator can successfully fulfil flexibility requests from DSOs/BRPs by rescheduling home appliance loads over 24 hours while also optimizing the costs connected with user compensation. Moreover, the incorporation of RLQIGWO demonstrates superior performance in task execution, resource utilization, and load balancing compared to existing algorithms. The effective utilization of energy flexibility involves engaging residential consumers to adjust their energy usage patterns in response to signals from grid operators. By leveraging the flexibility of home appliances, consumers can contribute to alleviating congestion, balancing loads, and enhancing the overall resilience of distribution networks. This approach not only helps in managing peak demand and integrating renewable energy sources more effectively but also offers economic benefits to consumers who participate in demand response programs. The major contributions of this study are as follows.

- The model considered in this study integrates the flexibility of both devices to increase/reduction of energy consumption by home appliances within the DSO/aggregator framework.
- The model accounts for comprehensive consumption profiles of appliances, capturing the specifics of both real-time and shifting reduction/increase capabilities.
- Each appliance's baseline profile includes precise details on duration and start times, enhancing the accuracy of the flexibility provision.

- The study employed the RLQIGWO algorithm to handle scalable optimization challenges, extending its applicability to many appliances and devices. The performance comparison is also made considering other state-of-the-art algorithms.
- The study also introduced a compensation mechanism for shifting devices based on an activation structure rather than the energy managed, ensuring fair and efficient remuneration for flexibility provision.

The paper is organized as follows: Section 2 provides a review of existing energy management systems that utilize aggregators, situating the proposed work within the current methods. Section 3 details the problem description and presents the mathematical modelling that supports the model considered in this study. Section 4 presents the RLQIGWO algorithm as an effective tool for solving the considered problem. Section 5 outlines the case study, which is based on real load profiles of various home appliances and also presents and analyzes the results of different case studies. Section 6 concludes with a summary of the findings and suggestions for future research directions.

2. Related works

Numerous researches have examined how residential consumers and intelligent homes can take part in the delivery of flexibility services in the framework of residential flexibility. Many research investigations have focused on using active residential consumers to provide local flexibility. For instance, one study proposed a centralized control system for smart home appliances aimed at delivering local flexibility services, where the DSO sets dynamic tariffs and daily network tariffs to manage distribution network congestion. However, this study did not address the facility of the wider flexibility services (Fotouhi Ghazvini et al., 2019). Additionally, the authors of (Lipari et al., 2018) examined the provision of local services through aggregated commercial customers, focusing primarily on the aggregation method and the interaction between the DSO and the aggregator rather than on the scheduling of appliances and the flexible capacity potential of the customers. In response to DSO demands for flexibility, an additional study presented a real-time rescheduling approach for shiftable appliances. Evolutionary algorithms were used to tackle the scheduling problem. However, the operational limitations of many appliances were not taken into consideration in this study, including household thermal comfort needs and how these affect appliance operations (Lezama et al., 2020). In their work (Olivella-Rosell et al., 2018), the authors outlined a market framework that enables households to engage in local flexibility services. While they provided a thorough model, they needed precise mathematical representations of household equipment and their variable functions. In addition, the authors of Monteiro et al. (2020) suggested using flexible energy supplies to solve DSOs' operating problems. These studies, however, frequently need to pay more attention to the thorough modelling of such adaptable energy sources.

Diverse aggregator representations have been explored in the literature to aggregate flexibility and energy for various objectives. Optimization techniques are commonly employed in these studies. For instance, the authors of Olivella-Rosell et al. (2018) and Roos et al. (2014) introduced a load aggregator for energy consumer portfolios. In order to reduce overall consumption costs, this aggregator gathers flexibility from loads and storage using an optimization model. Although the model depicts each customer's physical system, it does not take into consideration moving loads of equipment or their complicated profiles. The study suggests a potential 4 % reduction in portfolio energy costs, although the high costs and inefficiency of batteries remain a challenge. For aggregators to increase their involvement in local and wholesale energy markets, especially the day-ahead wholesale energy market, a number of studies suggest the best bidding strategies. The authors of Di Somma et al. (2019) and Wang et al. (2020) developed an optimization model for aggregator participation in the day-ahead market, considering demand flexibility. This model, based on a stochastic mixed-integer linear programming (MILP) approach, accounts for uncertainties in intermittent generation and market prices to determine optimal bidding policies that maximize expected profits. Using an ising spin-based model, the authors of Prieto-Castrillo et al. (2018) investigated consumer demand flexibility in a regional power trading scenario. More relationships at lower levels help all parties engaged, according to this large-scale investigation that assessed the cumulative advantages for all involved individuals and the overall aggregation profit, which rises with the number of aggregators. The authors of Carreiro et al. (2015) presented the Energy Box Aggregator (EBAg), a mediator between end-users and system operators (SO). EBAg coordinates large-scale flexibility with installed devices, gathering demand-side flexibility from clusters of end-users to meet system service requirements. By adjusting power demand within a planning horizon, EBAg helps balance load and supply, mitigate peaks, and manage the intermittent behaviour of renewable sources. In order to reduce the difference in load flexibility offered by each cluster and to maximize aggregator profits, the model takes a multi-objective optimization approach, taking into account payments to end-user groups and income from the SO. This method provides an option to invest in reserve and peak generating capacity while improving overall system efficiency (Essiet et al., 2019; Almeida et al., 2024).

Several studies have examined the capability of smart homes to provide flexibility using detailed mathematical representations of appliances. For instance, the authors (Gasca et al., 2022) projected two approaches for controlling Thermostatically Controllable Loads (TCLs) and computing their available flexibility. While this research utilized mathematical models for TCL appliances, it did not assess the flexibility potential of other types of flexible appliances. The authors of Jacobsen et al. (2015) developed an ICT-based architecture for managing, forecasting, aggregating, and scheduling loads for numerous end-users. This scalable infrastructure, centred around an aggregator, facilitates residential participation in DR programs. After validating the findings with 200,000 homes in a large-scale simulation, the study concluded that end-user compensation plans that are suitable might result in a more environmentally friendly and safe energy supply. The authors of Firoozi et al. (2020) proposed the creation of local energy communities by nearby residential customers in order to offer frequency restoration reserves as an across-the-system service, but this study focused solely on electric vehicles and battery energy storage as shared flexible energy resources. In another study, the authors of Patnam and Pindoriya (2021) offered a comprehensive analysis of how smart houses can participate in DR programs, either individually or in aggregate. However, the study did not include individual mathematical models of household appliances. Additionally, the authors of Hui et al. (2017) proposed the aggregation of household air conditioners to play active roles in providing operating reserves. Further, the authors of Paridari and Nordström (2020) introduced a new method for forecasting the flexibility and scheduling of their water heaters to deliver frequency suppression reserves for TSOs. The authors of Abapour et al. (2020) proposed a competitive framework where DR aggregators offer their services to network operators. The programme use Nash equilibrium to address the challenge, resulting in a 7 % rise in network operator revenues. It is based on price elasticity and consumer benefit functions.

For EV flexibility aggregation for auxiliary services, the authors of Wu et al. (2016) established a stochastic optimization model. A bi-level problem is transformed into a MILP problem that can be solved using programming methods with equilibrium constraints by this model, which also includes conditional value at risk to quantify uncertainties in the EV bidding process. The authors of Spinola et al. (2018) introduced an approach using clustering techniques combined with metaheuristics to aggregate distributed resources. This ensures the necessary resources and energy amounts for every cluster for day-ahead operations. The authors of Henriquez et al. (2018) presented an approach to find the optimal process of a demand response aggregator in wholesale electricity markets. The model includes load curtailment and flexible loads while accounting for market price uncertainties and balancing requirements impacting aggregator activities. A market-based method for aggregators was established by the authors of Olivella-Rosell et al. (2018), who described the relationships between operational methods and local market parties. The study suggests that such a local market could delay grid upgrades, reduce energy costs, and increase grid capacity.

This paper considered an aggregator positioned between the BRP/DSO and customers, as similar to (Lezama et al., 2020). The role of this is to collect flexibility from residential households to meet the DSO's flexibility needs with day-ahead planning. The following sections elaborate on the specific details and assumptions underlying this approach.

Problem Formulation

This section outlines the framework and key assumptions of the proposed model. It also offers the mathematical structure of an optimization model with the objective of reducing the aggregator's expenses related to flexibility provision.

2.1. Aggregator flexibility

The usage of current Home Energy Management Systems (HEMS) at the end-user level to control appliances with shifting abilities in response to aggregator demands for flexibility has been reported in (Sousa et al., 2018). The HEMS optimized the shifting times of various appliances and rescheduled them to exploit financial returns for the flexibility provided. The model considered in this study is illustrated in Fig. 1, and the selected model is described as follows (Basit et al., 2017; Lezama et al., 2019):

- The focus shifts to the aggregator's role in managing HEMS with different DR-capable devices.
- Two types of DR devices are considered: those whose consumption can be shifted to different periods and those with real-time control capabilities.
- The aggregator is equipped to respond to flexibility requests from a DSO or a BRP, which provide monetary compensation for each power unit of flexibility provisioned.
- The aggregator uses a flexibility management system to reschedule appliances to align closely with the flexibility curve required by the DSO. End-users can register their devices for flexibility provision and configure preferences such as allowed shiftable times, expected remuneration for flexibility activation, and prioritization of devices for activation.
- The necessary infrastructure, including smart metering systems, communication lines, and HEMS, is assumed to be in place.
- Both the DSO/BRP and the aggregator have access to baseline power consumption forecasts provided by a third party, representing normal consumption if no DR is activated.

2.2. Formulation of optimization model

By compensating households that take part in the DR programme according to their preferences and modifications to their baseline consumption profiles, the aggregator seeks to match a flexibility request from the DSO or BRP. The problem considered in this study is formulated as a Mixed-Integer Non-Linear Programming (MINLP) problem, as discussed in (Lezama et al., 2020).

Let denote the following parameters. **Set A:** $A = \{1, \dots, N_I\}$ as the collection of all appliances with the ability to shift their operating times and **Set B:** $B = \{1, \dots, N_J\}$ as the collection of all appliances capable of reducing their power consumption, recorded in the aggregator's Energy Management System (EMS). The appliance's characteristics are discussed as follows. (i) Shifting Capabilities: Each appliance i with shifting capabilities is defined by the tuple: $A_i = [t_{start}(i), O(i), p_A(i,k)] \in A$, where $t_{start}(i)$ is the baseline start time for the device i, $O(i)$ represents the duration of the operation for appliance i, and $p_A(i,k)$ denotes the power profile of appliance i during the interval $k = \{1, \dots, O(i)\}$; (ii) Real-time Control Capabilities: Each appliance j with real-time control capabilities is defined by the tuple: $[B_j = [p_B(j), Int_{start}(j,t)] \in B]$, where $p_B(j)$ is the maximum power consumption of appliance j and $Int_{start}(j,t)$ is the baseline intensity of appliance j at time t.

It is possible for users to customize their settings using the HEMS interface. The user-defined tuple for appliances that have shifting capabilities is as follows:

$$PrefA(i) = [t_{allow}(i), D_{allow}(i), C_A(i)] \tag{1}$$

where $t_{allow}(i)$ is the earliest permissible start time for device i, $D_{allow}(i)$ is the duration for which the aggregator can control the device i, and $C_A(i)$ is the expected remuneration in EUR for shifting the operation of appliance i within the permissible window. Given such parameters, the constraints for shifting appliances of Set A are defined as follows:

$$t_{allow}(i) \le t_{new}(i) \le t_{allow}(i) + D_{allow}(i) \tag{2}$$

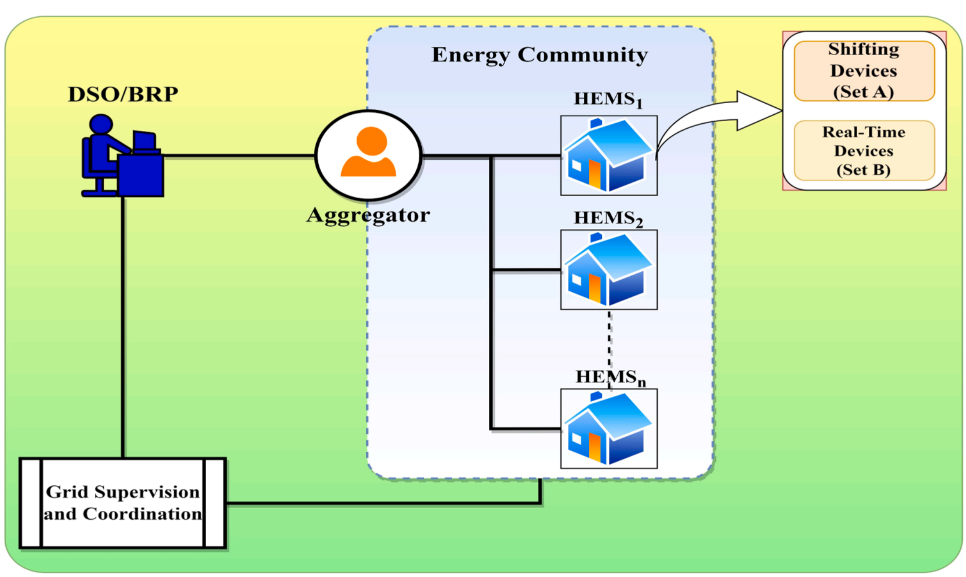


Fig. 1. Model considered in this study.

where $t_{new}(i)$ is the new starting time for appliance i. For appliances with reduction capabilities, the user-defined tuple is:

$$PrefB(j) = [t_{allow}(j), D_{allow}(j), I_{\min}(j), I_{\max}(j), C_B(j)] \tag{3}$$

where: $t_{allow}(j)$ and $D_{allow}(j)$ define the allowed periods for intensity modifications, $I_{\min}(j)$ and $I_{\max}(j)$ are the minimum and maximum permissible intensity adjustments for appliance j, and $C_B(j)$ is the remuneration expected in EUR/kWh for the power increase or reduction within the allowed window. The constraints for modifying the power profiles of appliances of Set B are as follows.

$$I_{\min}(j) \leq Int_{new}(j) \leq I_{\max}(j) \tag{4}$$

$$\text{Int}_{\text{new}(j,t)} = \begin{cases} \text{Int}_{\text{new}(j,t)} & \text{if} t_{\text{allow}(j)} \leq t \leq t_{\text{allow}(j)} + D_{\text{allow}(j)} \\ \text{Int}_{\text{start}(j,t)} & \text{otherwise} \end{cases} \tag{5}$$

where $\text{Int}_{\text{new}(j,t)}$ is a variable within the range $[0,1]$ for each $t \in NT$, representing the percentage modification of the baseline profile. The flexibility provided by the aggregator, $F_{agg}(t)$ is defined as the difference between the baseline consumption profile and the new scheduled profile:

$$F_{agg}(t) = P_{base}(t) - P_{flex}(t) \tag{6}$$

where: $P_{base}(t)$ is the baseline consumption profile and $P_{flex}(t)$ is the resulting consumption profile after rescheduling appliances. Remember that an impartial third party should establish the baseline profile and should indicate the anticipated power usage in the event that there are no changes or rescheduling. In order for the aggregator to calculate the flexibility provided, it is assumed that it has access to each household's baseline consumption data. The following equations capture the baseline profile of all devices.

$$P_{base}(t) = \sum_{i=1}^{N_I} A_{base}(i,t) + \sum_{j=1}^{N_J} B_{base}(j,t) \tag{7}$$

Eq. (7) represents the total power consumption at any given time t, combining the power usage of all devices with shifting abilities (set A) and reduction abilities (set B).

$$A_{\text{base}(i,t)} = \begin{cases} p_{A(i,t-t_{\text{start}(i)}+1)} & \text{if} t_{\text{start}(i)} \leq t \leq t_{\text{start}(i)} + O_{t(i)} - 1 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

Eq. (8) defines the power consumption profile $A_{\text{base}(i,t)}$ of an appliance i with shifting capabilities at time t. The function $t_{\text{start}(i)}$ indicates the baseline start time of the appliance and $O_{t(i)}$ represents the duration of its operation. For example, if a washing machine starts its baseline process at $t_{start}(i) = 5$ (1:00 am) and runs for $O(i) = 7$ periods (105 minutes), it consumes power $p_A(i,t)$ during this interval.

$$B_{base}(j,t) = p_B(j) \cdot Int_{start}(j,t) \tag{9}$$

Eq. (9) defines the power consumption profile $B_{base}(j,t)$ of an appliance j with reduction capabilities at time t. The term $p_B(j)$ is the maximum power of device j, and $Int_{start}(j,t)$ is the baseline power of the device at time t, which ranges between 0 and 1, presenting the consumption in percentage.

The aggregator is responsible for determining the new starting periods $t_{new}(i)$ for shifting devices and new powers $Int_{new}(j,t)$ for reduction devices. The new consumption profile is defined as follows:

$$P_{flex}(t) = \sum_{i=1}^{N_I} A_{flex}(i,t) + \sum_{j=1}^{N_J} B_{flex}(j,t) \tag{10}$$

Eq. (10) represents the total power consumption at any given time t, after the aggregator has optimized the starting times and intensities of the appliances.

$$A_{\text{flex}(i,t)} = \begin{cases} p_{A(i,t-t_{\text{new}(i)}+1)} & \text{if} t_{\text{new}(i)} \leq t \leq t_{\text{new}(i)} + O_{t(i)} - 1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Eq. (11) describes the power consumption profile $A_{\text{flex}(i,t)}$ of an appliance i with shifting capabilities at time t after rescheduling. Here, $t_{\text{new}(i)}$ is the new starting time optimized by the aggregator.

$$B_{flex}(j,t) = p_B(j) \cdot Int_{new}(j,t) \tag{12}$$

Eq. (12) describes the power consumption profile $B_{flex}(j,t)$ of an appliance j with reduction capabilities at time t after the aggregator has determined the new intensity $Int_{new}(j,t)$. Therefore, the input parameters and decision variables are summarized as follows. (i) Input Parameters: $t_{start}(i), O(i), p_A(i,t), p_B(j)$, and $Int_{start}(j,t)$; (ii) Decision Variables: $t_{new}(i)$ and $Int_{new}(j,t)$.

The objective function is the minimization of the total compensation provided to families plus a penalty for any discrepancy in the flexibility obtained by the DSO/BRP in order to maximize the aggregator's profitability. The objective function is represented as follows:

$$\text{Minimize} f = \left(\sum_{i=1}^{N_I} \text{Rem}_A(i) + \sum_{j=1}^{N_J} \text{Rem}_B(j) \right) + C_{\text{DSO}} \cdot F_{\text{match}} \tag{13}$$

The remuneration paid to households for shifting the operation of appliance i is defined as:

$$\text{Rem}_{A(i)} = \begin{cases} C_{A(i)} & \text{if} t_{\text{start}(i)} \neq t_{\text{new}(i)} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

where: $C_{A(i)}$ is a flat payment in EUR given to the household for shifting the operation of appliance i. This payment is made regardless of the number of periods the appliance is shifted. The remuneration for modifying the baseline profile of appliance j with reduction capabilities is given by:

$$\text{Rem}_B(j) = C_B(j) \cdot \sum_{t=1}^{T} |B_{\text{base}}(j,t) - B_{\text{flex}}(j,t)| \tag{15}$$

where: $C_B(j)$ is the compensation in EUR per kWh, $B_{\text{base}}(j,t)$ denotes the baseline power consumption and $B_{\text{flex}}(j,t)$ is the adjusted power consumption at time t. The penalty for the mismatch between the flexibility provided by the aggregator $(F_{\text{agg}}(t))$ and the flexibility procured by the DSO $(F_{\text{DSO}}(t))$ is calculated as follows.

$$F_{\text{match}} = \sum_{t=1}^{T} |F_{\text{agg}}(t) - F_{\text{DSO}}(t)| \tag{16}$$

where: C_{DSO} is the penalty rate in EUR per kWh for any discrepancy in each period t. By minimizing the total compensation paid to homes and the penalties incurred because of flexibility mismatches, the objective function makes sure that the aggregator maximizes overall revenues. The selected model in this study takes into account all fines and incentives required for properly running the DR program.

3. Reinforced learning quantum inspired grey wolf optimization algorithm

Inspired by the social structure and hunting habits of grey wolves, the GWO is a well-known metaheuristic algorithm (Mirjalili et al., 2014). Despite its effectiveness, GWO occasionally experiences early convergence and a lack of diversity in the search field. In order to overcome the drawbacks of GWO, the RLQIGWO method combines quantum-inspired techniques to improve exploration capabilities with reinforced learning to update search agents' placements adaptively.

3.1. Grey Wolf Optimizer (GWO)

The GWO algorithm mimics the leadership hierarchy and hunting strategy of grey wolves in nature (Mirjalili et al., 2014). The hierarchy is divided into four levels: (i) Alpha (α) wolves are the leaders whose solution is represented as the best solution; (ii) Beta (β) wolves are the second-best solution, assisting the alpha wolves; (iii) Delta (δ) wolves are the third-best solutions, subordinate to alpha and beta wolves; (iv) Omega (ω) wolves are the remaining wolves, following the others.

Wolves encircle prey using the following equations:

$$D = \left|C \cdot X_p - X\right| \tag{17}$$

$$X(t+1) = X_p - A \cdot D \tag{18}$$

where $\boldsymbol{X_p}$ is the position vector of the prey, X is the position vector of a grey wolf, and $\boldsymbol{A}$ and $\boldsymbol{C}$ are coefficient vectors.

The hunting behaviour is guided by the alpha, beta, and delta wolves, which are considered to have better knowledge about the potential location of the prey. The positions are updated as follows:

$$X_1 = X_\alpha - A_1 \cdot |C_1 \cdot X_\alpha - X| \tag{19}$$

$$X_2 = X_\beta - A_2 \cdot \left|C_2 \cdot X_\beta - X\right| \tag{20}$$

$$X_3 = X_\delta - A_3 \cdot |C_3 \cdot X_\delta - X| \tag{21}$$

$$X(t+1) = \frac{X_1 + X_2 + X_3}{3} \tag{22}$$

3.2. Reinforced-learning-based position update

The purpose of Reinforcement Learning (RL) in the proposed algorithm is to help in adapting the search agent's positions based on past experiences and the received feedback (fitness values) (Hu and Yu, 2023; Mazyavkina et al., 2021). In Q-learning, the agent maintains a Q-table that stores Q-values for state-action pairs. In the RLQIGWO, the state is represented by the current position of the wolf, and the action is the movement to a new position. The Q-value update rule is given by:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left[r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right] \tag{23}$$

where: $Q(s,a)$ is the Q-value for state s and action a, α is the learning rate (how much new information overrides old information), r is the reward received after taking action a from state s (in RLQIGWO, the fitness value), γ is the discount factor (importance of future rewards), $\max_{a'} Q(s', a')$ is the maximum Q-value for the next state s'. In the RLQIGWO algorithm, the positions are updated based on the Q-table as follows:

$Q = 0$ (Q table initialization) (24)

The Q table values are updated as follows.

$$Q(i,j) \leftarrow Q(i,j) + \alpha(f(i) - Q(i,j)) \tag{25}$$

where: $Q(i,j)$ is the Q-value for the j^{th} dimension of the i^{th} search agent, and $f(i)$ is the fitness value of the i^{th} search agent. Finally, the wolf position is updated as follows.

$$X(i,j) \leftarrow X(i,j) + Q(i,j) \tag{26}$$

where: $X(i,j)$ is the position of the j^{th} dimension of the i^{th} search agent. The pseudocode for the reinforced-learning-based position update is provided in Algorithm 1.

Algorithm 1. : Pseudocode for the Reinforced-Learning-Based Position Update

Initialize Q-table Q with zeros
Set learning rate α
For each search agent i:
  For each dimension j:
    Evaluate fitness $f(i)$
    Update Q-value:
      $Q(i,j) \leftarrow Q(i,j) + \alpha * (f(i) - Q(i,j))$
    Update position based on Q-value:
      $X(i,j) \leftarrow X(i,j) + Q(i,j)$
    Boundary checking:
      If $X(i,j) > lb(j$, then $X(i,j) \leftarrow ub(j$
      If $X(i,j) < lb(j$, then $X(i,j) \leftarrow lb(j$

3.3. Quantum-inspired based position update

The quantum-inspired update enhances the GWO by addressing some of the limitations of the classical GWO, such as premature convergence and insufficient exploration of the search space (Gu and Zhou, 2019; Nie et al., 2017; Wang et al., 2012). The quantum superposition technique and the addition of random angles guarantee that the search agents cover a larger portion of the search space. By doing so, it becomes easier to find superior global solutions and avoid local optima. The best solutions (alpha, beta, and delta) have varying effects on each search agent's position through the use of quantum-inspired updates. When these factors are combined using the sine and cosine functions, the population becomes more diverse and variable, allowing for a deeper examination. Quantum tunnelling, simulated through the quantum-inspired position updates, allows search agents to escape local optima. The property of quantum tunnelling is crucial in ensuring that the algorithm does not get stuck in suboptimal solutions and can continue to search for the global optimum. The quantum-inspired mechanism balances exploration and exploitation by adjusting positions based on the best wolves and a random quantum component.

Quantum mechanics is a fundamental theory in physics that describes the physical properties of nature at the scale of atoms and subatomic particles. Quantum mechanics introduces several key concepts that differ significantly from classical mechanics. The principles of quantum mechanics relevant to optimization algorithms include superposition, entanglement, and quantum state representation. The principle of superposition states that a quantum system can exist in multiple states simultaneously. In contrast to classical bits, which can be either 0 or 1, quantum bits (qubits) can be in a state that is a linear combination of both 0 and 1:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{27}$$

where $|\alpha|^2 + |\beta|^2 = 1$. A quantum state can be represented as a probability amplitude, and measurements collapse the state into one of the basis states with a certain probability. Entanglement is a phenomenon where the quantum states of two or more objects are interconnected, such that the state of one object cannot be described independently of the state of the other objects, even when a large distance separates the

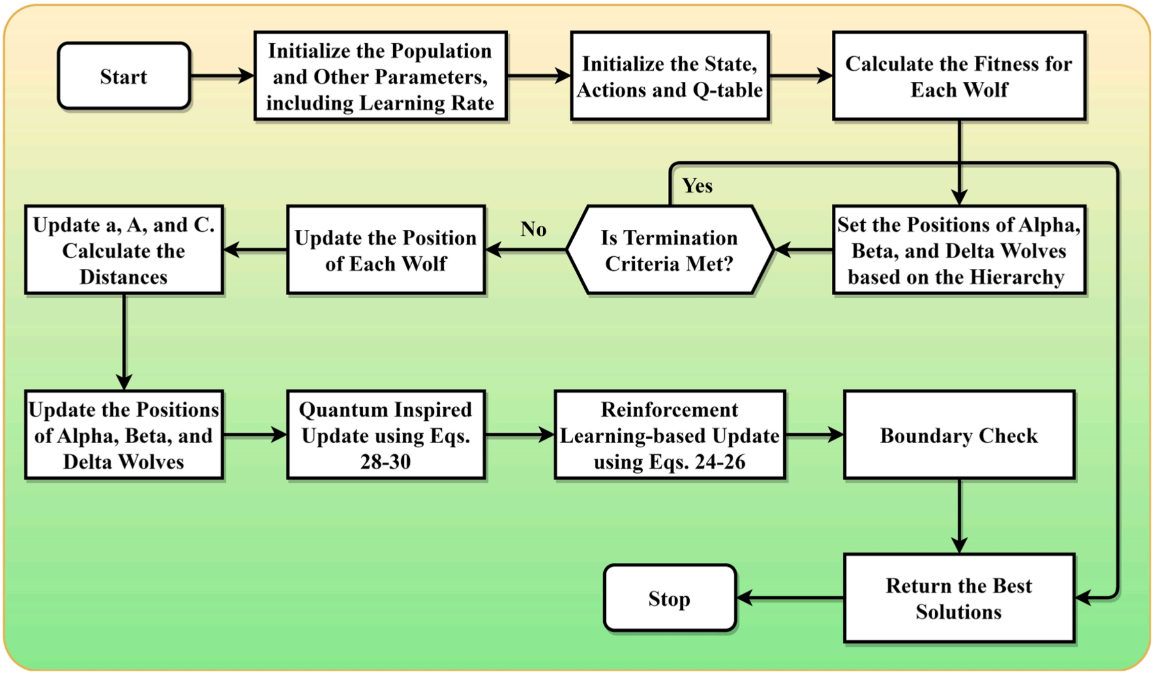


Fig. 2. Flowchart of the proposed RLQIGWO.

objects. Quantum tunnelling allows particles to pass through potential barriers that they would not be able to cross, according to classical mechanics. This property helps in escaping local minima in optimization problems.

The quantum-inspired update leverages the principle of superposition to enhance the diversity and exploration capabilities of the search process in optimization. The new position is influenced by the positions of the best solutions (alpha, beta, and delta wolves in GWO) and a random component representing the quantum superposition. Each search agent's position is considered a quantum state influenced by the alpha, beta, and delta positions. A random angle θ is introduced to simulate quantum superposition:

$$\theta = 2\pi \cdot \text{rand} \tag{28}$$

where: rand generates a random number in the range [0, 1]. The new position is computed as a linear combination of the influences of the alpha, beta, and delta wolves, weighted by sine and cosine functions to introduce diversity:

$$r = |\sin(\theta) \cdot X_1 + \cos(\theta) \cdot X_2 + X_3| \tag{29}$$

where: X_1, X_2, X_3 are the positions influenced by the alpha, beta, and delta wolves, respectively. The calculated position is scaled to fit within the problem's boundaries:

$$X'(i,j) = r \cdot (ub(j) - lb(j)) + lb(j) \tag{30}$$

where: $ub(j)$ and $lb(j)$ are the upper and lower bounds of the j^{th} dimension, respectively, and $X'(i,j)$ is the new position of the j^{th} dimension of the i^{th} search agent. The pseudocode for the reinforced-learning-based position update is provided in Algorithm 2.

Algorithm 2. : Pseudocode for the Quantum-Inspired Position Update

```
For each search agent i:
  For each dimension j:
    Generate a random angle θ in the range [0, 2π]
    Calculate the quantum-inspired position:
      r ← |sin(θ) * X1(j) + cos(θ) * X2(j) + X3(j)|
    Scale the position to the problem boundaries:
      X(i, j) ← r * (ub(j) − lb(j)) + lb(j)
    Boundary checking:
      If X(i, j) > lb(j), then X(i, j) ← ub(j)
      If X(i, j) < lb(j), then X(i, j) ← lb(j)
```

The proposed RLQIGWO algorithm is a new optimization algorithm that combines the social hierarchy and hunting behaviour of grey wolves (GWO) with RL and quantum-inspired mechanisms. This combination enhances the exploration and exploitation capabilities of the algorithm, making it more efficient at solving complex optimization problems. In the RLQIGWO algorithm, both RL-based position updates and quantum-inspired position updates are combined to enhance the search process. The steps are as follows.

- Calculate the fitness of each search agent.
- Update the positions of the alpha, beta, and delta wolves based on their fitness.

- Calculate random angle θ, compute new position r using superposition and scale the new position to problem boundaries.
- Update Q-values using the fitness values and adjust the positions based on Q-values.
- Ensure the positions remain within the specified boundaries.
- Return the best fitness and position.

The pseudocode of the proposed RLQIGWO algorithm is provided in Algorithm 3. The flowchart of the proposed RLQIGWO algorithm is shown in Fig. 2.

Algorithm 3. : Pseudocode for the Proposed RLQIGWO Algorithm

Initialize parameters: population size (N), maximum iterations (T), and learning rate (α)
Initialize random positions of search agents within bounds and their solutions
Initialize Q-table Q with zeros
For each iteration from 1 to T:
  Evaluate fitness for each search agent $X(i)$
  Update alpha, beta, and delta wolves based on fitness
  For each search agent i:
    For each dimension j:
      Calculate a, A, and C and calculate distances
      Update positions $X1, X2, X3$ using alpha, beta, and delta wolves
      Quantum-Inspired Update:
        $\theta \leftarrow 2\pi \times rand$
        $r \leftarrow |\, sin(\theta) \times X1 + cos(\theta) \times X2 + X3 \,|$
        $X(i,j) \leftarrow r\,.(ub(j) - lb(j)) + lb(j)$
      Reinforced Learning Update:
        $Q(i,j) \leftarrow Q(i,j) + \alpha \times (f(i) - Q(i,j))$
        $X(i,j) \leftarrow X(i,j) + Q(i,j)$
      Boundary checking:
        If $X(i,j) > lb(j)$, then $X(i,j) \leftarrow ub(j)$
        If $X(i,j) < lb(j)$, then $X(i,j) \leftarrow lb(j)$
Return: Best score and best position.

The RLQIGWO algorithm effectively combines the strengths of GWO, reinforcement learning, and quantum-inspired mechanisms. This new approach enhances the exploration and exploitation capabilities of the optimizer, making it suitable for solving complex and high-dimensional optimization problems. The adaptive learning and quantum-inspired updates help in avoiding local optima and ensure a diverse search space exploration, leading to more robust and efficient optimization results.

3.4. Complexity

The time complexity of the RLQIGWO algorithm can be determined by analyzing the operations performed during initialization and each iteration of the main loop. The initialization phase has a time complexity of $O(\mathrm{N} \times \mathrm{dim})$ (dim denotes the problem dimension), the position update for all search agents and dimensions is $O(N \times \mathrm{dim})$, and combining all these steps, the time complexity for each iteration is $[O(N \times T_f + N \times \mathrm{dim})$. Given that T_f is the dominant term, the overall time complexity for T iterations is $O(\mathrm{T} \times N \times T_f + \mathrm{T} \times N \times \mathrm{dim})]$. The storage requirements of the algorithm determine the space complexity. The space complexity for the position storage is $O(N \times \mathrm{dim})$, leader position storage is $O(\mathrm{dim})$, Q-table storage is $O(N \times \mathrm{dim})$, and the fitness value storage is $O(N)$. The overall space complexity is $O(N \times \mathrm{dim} + \mathrm{dim} + N + \mathrm{T})$.

4. Results and discussions

This section details the case study considered in this study, along with the control parameter settings of the state-of-the-art algorithms, and discusses the results obtained for different case studies, including the benchmark problem analysis.

4.1. Results of benchmark problems

To validate the performance of the proposed algorithm, 5 traditional CEC benchmark problems, i.e., F1-F5, are considered with the different problem dimensions to check the scalability and adaptability of the proposed algorithm. The functions F1-F5 are unimodal test functions with variable problem dimensions. These functions are selected to validate the scalability and adaptability of the proposed algorithm, and mostly, these functions are used to assess the exploration ability of any algorithms. The performance is compared with five other algorithms, such as Quantum-Inspired Particle Swarm Optimization (QPSO) (Agrawal et al., 2021), Oppositional Based Learning-Grey Wolf Optimizer (OBLGWO) (Yu et al., 2021), Successful History-Based Adaptive DE Variants with Linear Population Size Reduction (LSHADE) (Mohamed et al., 2019), Reinforcement Learning-Artificial Bee Colony (RLABC) algorithm (Cui et al., 2022), and the GWO. The parameter settings of all algorithms for the benchmark problems are selected based on the original version. The population size is selected as 30, and the maximum iterations are selected as 500 for all algorithms.

The statistical results, such as the minimum (MIN), mean (MEAN), maximum (MAX), standard deviation (STD)of the objective function values, and Runtime (RT) are included in Table 1 for the 30, 100, and 500 problem dimensions. The outcomes recorded in Table 1 show that for most of the benchmark functions, including different dimensions, the proposed RLQIGWO is performing better. This statement is proved by analyzing the MIN, MEAN, and STD values. All these values are computed after 30 individual executions due to the stochastic nature of all algorithms for a fair comparison. After careful observations, it is noticed that the RLABC algorithm is also performing better for larger dimensions. The proposed algorithm performs equally, and other algorithms struggle to handle the large dimension problems. The

Table 1
Statistical analysis of benchmark problems for different problem dimensions.

Functions	Dim	Algorithm	MIN	MAX	MEAN	STD	RT
F1	30	RLQIGWO	5.17E−123	2.20E−115	4.40E−116	9.85E−116	0.088
		GWO	7.87E−29	3.13E−27	9.07E−28	1.31E−27	0.069
		RLABC	8.95E−107	2.39E−97	4.77E−98	1.07E−97	0.109
		OBLGWO	4.73E−82	8.92E−80	3.27E−80	3.85E−80	2.384
		QPSO	5.88E+02	2.42E+03	1.43E+03	8.67E+02	0.156
		LSHADE	3.39E−42	1.41E−40	3.62E−41	5.89E−41	0.416
F2		RLQIGWO	2.62E−63	1.23E−60	2.71E−61	5.35E−61	0.072
		GWO	4.40E−17	2.50E−16	1.32E−16	7.70E−17	0.091
		RLABC	1.67E−56	3.75E−52	8.82E−53	1.61E−52	0.081
		OBLGWO	1.46E−57	1.66E−50	3.39E−51	7.40E−51	2.478
		QPSO	1.96E+01	4.65E+01	2.99E+01	1.04E+01	0.172
		LSHADE	1.08E−24	1.04E−23	3.46E−24	3.91E−24	0.325
F3		RLQIGWO	2.90E−107	3.22E−102	6.45E−103	1.44E−102	0.234
		GWO	1.55E−07	1.75E−05	5.11E−06	7.15E−06	0.209
		RLABC	2.87E−97	1.95E−80	3.89E−81	8.70E−81	0.291
		OBLGWO	9.04E+03	5.66E+04	3.91E+04	2.01E+04	2.491
		QPSO	1.39E+04	5.84E+04	3.58E+04	1.87E+04	0.194
		LSHADE	2.01E−13	3.86E−10	9.21E−11	1.66E−10	0.359
F4		RLQIGWO	8.18E−58	4.96E−52	9.92E−53	2.22E−52	0.078
		GWO	1.49E−07	7.24E−07	3.97E−07	2.65E−07	0.069
		RLABC	9.28E−55	1.44E−47	2.87E−48	6.43E−48	0.088
		OBLGWO	6.44E+00	6.99E+01	4.28E+01	2.65E+01	2.491
		QPSO	4.57E+01	7.42E+01	6.48E+01	1.12E+01	0.178
		LSHADE	1.16E−11	4.58E−10	1.96E−10	2.05E−10	0.425
F5		RLQIGWO	2.17E+01	2.31E+01	2.26E+01	6.56E−01	0.091
		GWO	2.62E+01	2.72E+01	2.68E+01	5.28E−01	0.094
		RLABC	1.22E−03	3.09E−02	1.61E−02	1.23E−02	0.144
		OBLGWO	2.71E+01	2.81E+01	2.76E+01	4.18E−01	2.509
		QPSO	1.03E+05	5.02E+06	1.35E+06	2.06E+06	0.147
		LSHADE	2.51E+01	2.55E+01	2.53E+01	1.38E−01	0.394
F1	100	RLQIGWO	3.32E−119	2.15E−108	4.30E−109	9.60E−109	0.153
		GWO	3.45E−13	3.96E−12	1.87E−12	1.69E−12	0.153
		RLABC	3.70E−109	3.46E−94	6.95E−95	1.55E−94	0.193
		OBLGWO	1.94E−80	3.52E−75	8.05E−76	1.53E−75	2.294
		QPSO	1.07E+05	1.28E+05	1.16E+05	8.97E+03	0.269
		LSHADE	4.18E−30	2.19E−28	5.13E−29	9.36E−29	0.463
F2		RLQIGWO	1.11E−62	8.69E−59	2.47E−59	3.60E−59	0.156
		GWO	3.63E−08	7.10E−08	5.63E−08	1.26E−08	0.147
		RLABC	4.78E−57	8.63E−47	1.73E−47	3.86E−47	0.194
		OBLGWO	9.23E−56	1.97E−50	5.67E−51	8.20E−51	2.588
		QPSO	2.91E+02	3.53E+02	3.27E+02	2.28E+01	0.313
		LSHADE	7.41E−18	1.55E−17	1.11E−17	3.09E−18	0.397
F3		RLQIGWO	3.72E−95	2.77E−90	7.83E−91	1.20E−90	0.528
		GWO	9.26E+01	4.59E+02	2.21E+02	1.39E+02	0.503
		RLABC	6.13E−85	1.41E−59	3.48E−60	6.11E−60	0.716
		OBLGWO	6.18E+05	1.11E+06	9.00E+05	2.17E+05	2.809
		QPSO	2.10E+05	4.18E+05	2.67E+05	8.76E+04	0.663
		LSHADE	1.32E−01	1.42E+02	3.48E+01	6.14E+01	0.563
F4		RLQIGWO	5.87E−53	3.83E−50	9.23E−51	1.64E−50	0.166
		GWO	3.51E−01	1.73E+00	8.46E−01	5.22E−01	0.147
		RLABC	8.51E−55	3.32E−47	6.65E−48	1.49E−47	0.250
		OBLGWO	8.43E+00	9.68E+01	7.78E+01	3.88E+01	2.559
		QPSO	8.80E+01	9.53E+01	9.26E+01	2.78E+00	0.316
		LSHADE	3.26E−05	1.87E−02	3.91E−03	8.26E−03	0.350
F5		RLQIGWO	9.52E+01	9.81E+01	9.62E+01	1.18E+00	0.169
		GWO	9.66E+01	9.78E+01	9.74E+01	5.31E−01	0.169
		RLABC	2.61E−03	5.74E−02	3.34E−02	2.37E−02	0.184
		OBLGWO	9.80E+01	9.84E+01	9.82E+01	1.52E−01	2.584
		QPSO	1.58E+08	2.91E+08	2.54E+08	5.42E+07	0.288
		LSHADE	9.57E+01	9.79E+01	9.64E+01	9.55E−01	0.394
F1	500	RLQIGWO	5.86E−115	1.47E−106	5.28E−107	5.76E−107	0.544
		GWO	1.40E−03	1.71E−03	1.56E−03	1.20E−04	0.519
		RLABC	3.43E−112	5.58E−89	1.12E−89	2.50E−89	0.791
		OBLGWO	7.19E−78	1.10E−70	2.43E−71	4.79E−71	2.788
		QPSO	1.13E+06	1.26E+06	1.22E+06	5.20E+04	1.100
		LSHADE	4.40E−23	3.34E−22	1.69E−22	1.21E−22	1.025
F2		RLQIGWO	2.72E−58	3.09E−55	1.08E−55	1.49E−55	0.575
		GWO	1.05E−02	1.59E−02	1.34E−02	1.97E−03	0.503
		RLABC	4.23E−54	2.36E−52	5.77E−53	1.00E−52	0.744
		OBLGWO	1.32E−54	6.60E−48	1.42E−48	2.90E−48	2.828
		QPSO	2.60E+137	2.37E+156	4.76E+155	6.55E+04	1.194
		LSHADE	3.43E−14	9.98E−14	6.87E−14	2.68E−14	1.047
F3		RLQIGWO	9.53E−87	2.16E−77	4.37E−78	9.62E−78	2.369
		GWO	1.94E+05	3.03E+05	2.44E+05	4.38E+04	2.319

(continued on next page)

Table 1 (*continued*)

Functions	Dim	Algorithm	MIN	MAX	MEAN	STD	RT
		RLABC	2.87E−74	4.65E−43	9.30E−44	2.08E−43	4.575
		OBLGWO	1.40E+07	3.49E+07	2.73E+07	9.14E+06	4.691
		QPSO	4.73E+06	6.09E+06	5.28E+06	4.96E+05	3.013
		LSHADE	1.41E+03	1.65E+05	6.30E+04	6.41E+04	2.769
F4		RLQIGWO	4.73E−49	6.15E−40	1.23E−40	2.75E−40	0.503
		GWO	6.08E+01	7.18E+01	6.65E+01	4.74E+00	0.478
		RLABC	2.68E−54	4.31E−51	1.18E−51	1.86E−51	0.938
		OBLGWO	5.60E+01	9.61E+01	7.95E+01	1.64E+01	2.816
		QPSO	9.84E+01	9.91E+01	9.88E+01	2.76E−01	1.103
		LSHADE	4.67E+01	9.77E+01	7.62E+01	2.56E+01	0.931
F5		RLQIGWO	4.94E+02	4.96E+02	4.95E+02	9.14E−01	0.606
		GWO	4.98E+02	4.98E+02	4.98E+02	2.40E−01	0.531
		RLABC	2.31E−03	6.00E−01	2.18E−01	2.44E−01	0.978
		OBLGWO	4.96E+02	4.97E+02	4.96E+02	3.64E−01	2.794
		QPSO	5.08E+09	5.44E+09	5.27E+09	1.38E+08	1.094
		LSHADE	4.97E+02	4.98E+02	4.98E+02	1.24E−01	1.016

performance is also verified based on the computational time. For the same, the RT values are recorded in Table 1. Based on the RT values, it is obvious that the original GWO has less RT values. The RT values of the proposed RLQIGWO are slightly greater than the original GWO but lesser than other selected algorithms. Therefore, based on the statistical metrics and RT values, it is clear that the proposed algorithm is superior to the other selected algorithms. The superior performance of the proposed algorithm is due to the fact that the proper balancing between the exploration and exploitation using the reinforced learning and the quantum principles. The results also proved that the proposed algorithm is more adaptable and able to solve large-dimension problems due to more scalability.

In order to visualize the convergence behaviour of all algorithms for different problem dimensions, the convergence curves are plotted and shown in Fig. 3. After careful observation of Fig. 3; it is noticed that the proposed algorithm is performing better with faster convergence than the other algorithms. Similarly, the convergence behaviour of RLABC is also equally better than the proposed algorithm, especially when the problem dimensions are high. Another variant of GWO, i.e., OBLGWO, is performing well for low-dimension problems but has a poorer performance than the original GWO when the problem dimensions are increased. Excluding Function F5, the proposed algorithm shows better performance for all functions of all dimensions.

The sensitivity of the proposed algorithm is validated by adjusting the different learning rates. As per the earlier discussions, the proposed algorithm has the learning rate as the algorithmic-specific parameters. Therefore, the sensitivity analysis is made by adjusting the learning rate. The learning rates are selected as 0.05, 0.1, 0.15, 0.2, and 0.25 by keeping the maximum number of iterations as 500 and the population size as 30. Three tests functions, such as F1, F2, and F3, are considered with the problem dimensions of 30. The convergence graphs of all functions with different learning rates are shown in Fig. 5. In addition, the fitness values of all test functions with different learning rates are recorded in Table 2.

By observing Table 2 and after careful observations, it is decided to go with the learning rate of 0.05. There is less impact of the learning rate on the convergence rate. The convergence speeds of different learning rates are almost similar. There is a minor difference with respect to fitness values. For real-world problems, it is often recommended to use a lower learning rate to ensure stable convergence, even though it may require slightly higher iterations and slightly increase the computational burden.

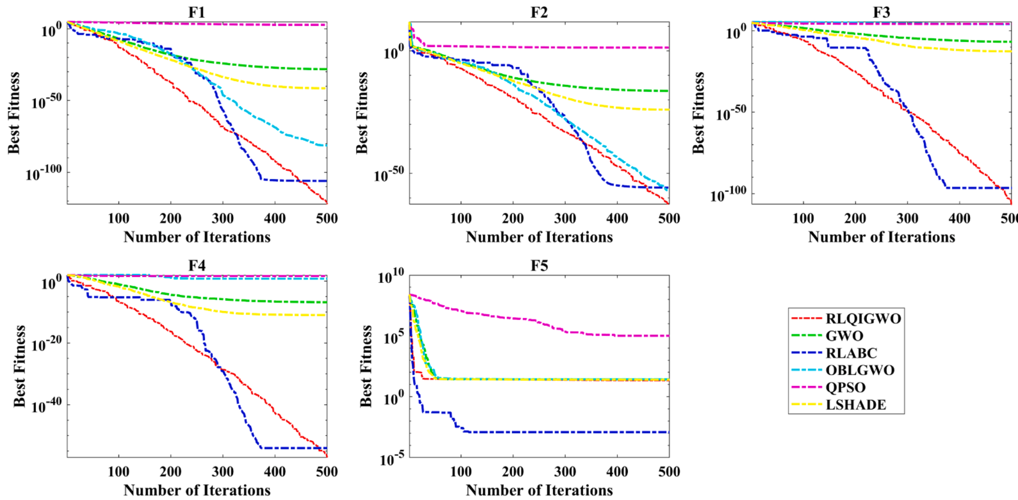
4.2. Descriptions of the data and the case study

In this analysis, this paper suggests that the aggregator exercises direct control over certain household devices registered by users through voluntary flexibility contracts. These contracts allow users to set preferences via their HEMS. Unlike the study by Sousa et al. (2018), the study in this paper includes both shifting and real-time devices. The key distinctions between these two types of appliances are highlighted as follows (Basit et al., 2017): (i) Shifting appliances have a set consumption schedule that can be shifted to a different time, either earlier or later, typically incentivized by the aggregator and are highly suitable for providing flexibility in energy management. For instance, dishwashers, tumble dryers, and washing machines; (ii) Real-time appliances allow for minor adjustments in their energy consumption during operation, either increasing or decreasing their load and offer limited flexibility, as changes depend on the immediate needs and priorities of users. For instance, lighting, air conditioners, desktop computers, and televisions.

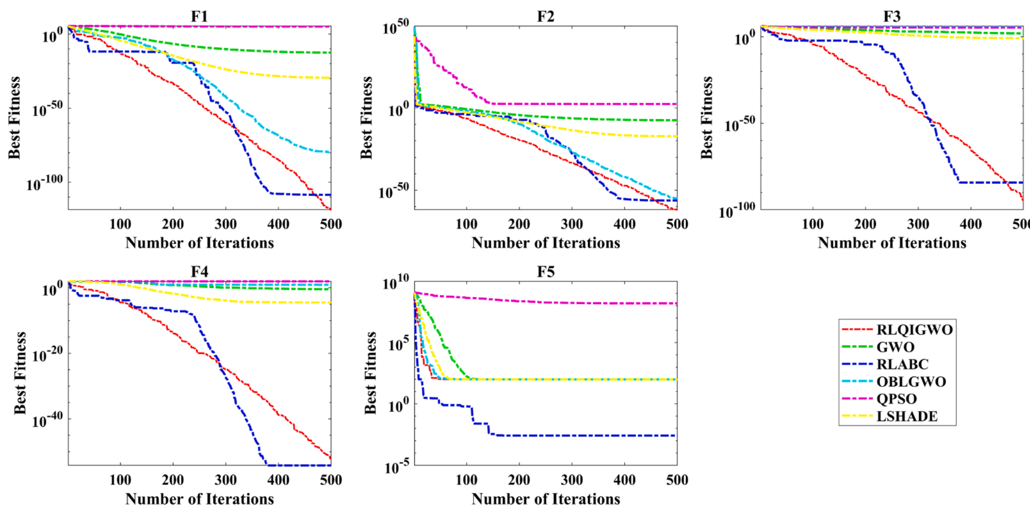
This study examines households equipped with either shifting or real-time devices. Specifically, the households include (i) Shifting devices represented in Fig. 5, including washing machines, tumble dryers, and dishwashers, and (ii) Real-time device consumption patterns depicted in Fig. 6, including lighting, air conditioners, desktop computers, and televisions. Characteristic consumption profiles and descriptions of these devices are summarized by Stamminger (2009). The authors of Curtis (2017) conducted a study on DR aggregators, revealing that a minimum of 200 kW of DR is required for financial viability. Based on the consumption profiles in this study, it is essential to aggregate 20 households with like characteristics to meet this threshold. To simulate this, this study has generated 20 profiles for every appliance type by means of a stochastic function with a uniform distribution, varying by 5 % around the typical profiles shown in Figs. 5 and 6. Table 3 outlines the consumption patterns and types of devices considered.

With 20 similar households, a total maximum consumption of about 217 kW is attained. The baseline profiles (i.e., the starting time $t_{start}(i)$) of devices A_i and B_j were created by distributing the appliances as follows: 10 % from periods 1–40 (00:00–10:00), 30 % from periods 41–56 (10:00–14:00), 10 % from periods 57–76 (14:00–19:00), and 50 % from periods 77–88 (19:00–22:00). This distribution strategy aims to replicate a typical daily energy consumption pattern. Fig. 7 displays the resulting baseline pattern, represented by the vector $P_{base}(t)$. This vector includes 96 power consumption values corresponding to the aggregated energy use of all appliances over 24 hours in 15-minute intervals. From Fig. 7, it is observed that the peak hours of consumption are recognized between 10:00–14:00 and 19:00–23:00 hours.

To model user preferences for shifting devices, this study utilized a randomized uniform function. This function generated permissible operation windows ranging from 0 to 64 intervals (16 hours). The expected compensation was set to fluctuate by ±30 %, around 0.2 EUR, simulating various levels of user participation. For real-time devices, another stochastic uniform function was applied to determine allowable power modifications. These ranged from 0 to 0.4, translating to up to



(a)



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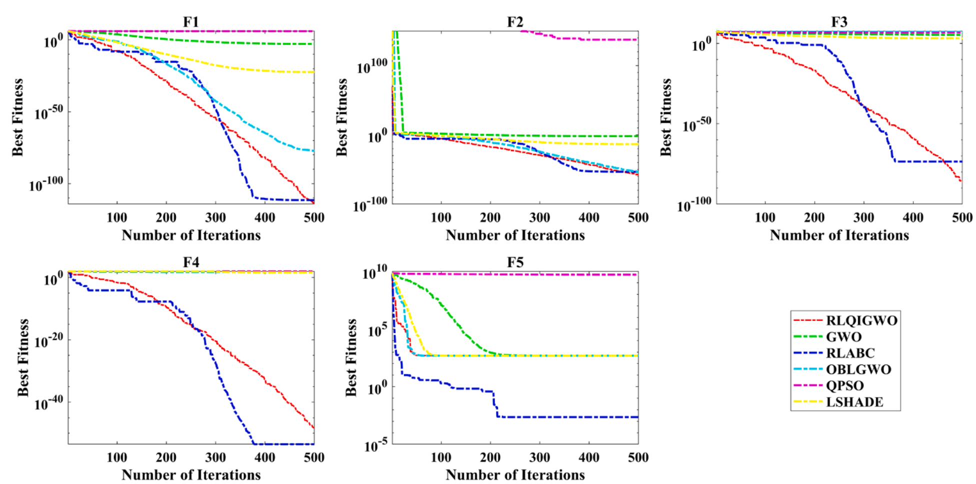

(c)

Fig. 3. Convergence curves for the problems with dimensions: (a) 30 dimensions, (b) 100 dimensions, (c) 500 dimensions.

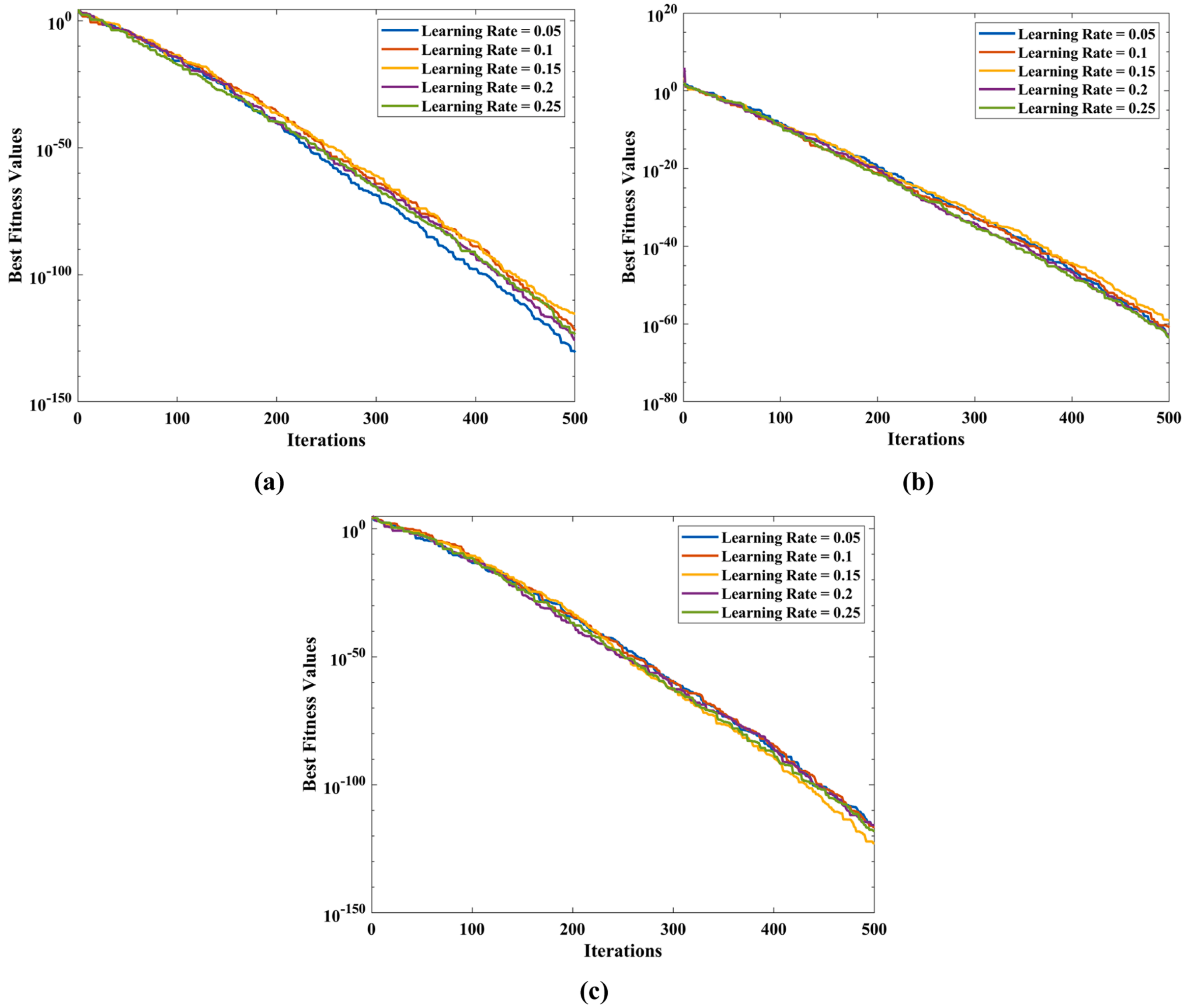

Fig. 4. Convergence curves for the different learning rates: (a) Function F1, (b) Function F2, (c) Function F3.

40 % of the power of the devices. The probable compensation for these modifications varied by ±30 %, around 0.09 EUR per kWh adjusted. The foundation of the studied model is the idea that a DSO or BRP would seek to purchase flexibility from an aggregator. This concept is supported by frameworks like the universal smart energy framework, established by the Backers et al. (2014), which formalizes the contractual relationships between aggregators and DSOs for trading flexibility. The case study considered a scenario where the DSO requests flexibility for the following day and the request is depicted in Fig. 8 and is characterized as an input vector $F_{\mathrm{DSO}}(t)$, which includes 96 intervals.

In Fig. 8, it is observed that the positive data indicate a rise in consumption, and the negative data indicate a decrease in consumption. These values span 24 hours, divided into 15-minute intervals, and indicate the power requirements for up-regulation or down-regulation. The case study integrates various assumptions to create a realistic baseline and flexibility requests from the DSO. The study presents different prices and consumption levels at different time intervals of the day to reflect the average daily consumption profile and user behaviour. The proposed model seeks to offer a thorough scenario for examining the interactions among DSOs, aggregators, and customers inside a DR framework. It does this by taking into account both shifting and real-time device preferences. Through the integration of diverse user behaviours and market conditions, the model provides a significant understanding of the pragmatic elements of flexibility trading within the energy industry.

4.3. Parameter settings

This section discusses the parameter settings for the proposed flexibility management model of the following algorithms: QPSO (Agrawal et al., 2021), OBLGWO (Yu et al., 2021), LSHADE (Mohamed et al., 2019), RLABC algorithm (Cui et al., 2022), and the GWO. To guarantee optimal performance, the settings of the comparing algorithms and the proposed RLQIGWO are usually adjusted according to the problem's complexity. To improve exploration and exploitation, the OBLGWO incorporates oppositional-based learning into the GWO. The parameters for OBLGWO are opposition-based learning frequency at every 50 iterations, the parameter a decreases linearly from 2 to 0 throughout iterations, while A and C are vectors with components in the range $[0, 2]$ and are recalculated at each iteration. The parameter settings for LSHADE are scale factor and crossover rate, and these are adaptively controlled based on success-history mechanisms and the initial values are often set

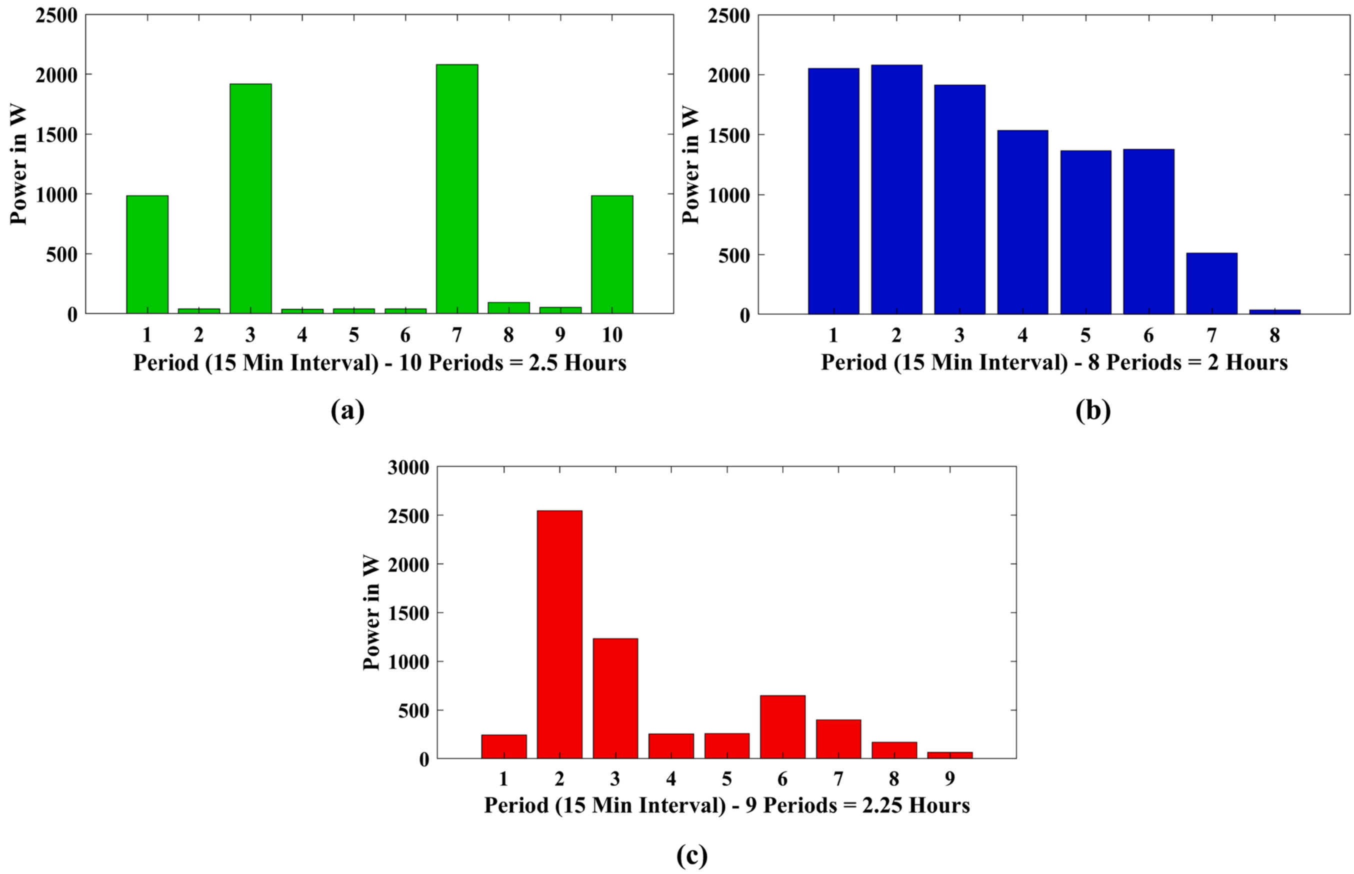

Fig. 5. Consumption patterns (15 min interval) of devices with shifting abilities: (a) Dishwasher, (b) Tumble dryer, (c) Washing machine.

Table 2
Fitness values for different learning rates of various test functions.

Function	Learning Rate	Best Fitness
F1	0.05	3.50E−131
	0.1	1.11E−122
	0.15	6.21E−116
	0.2	2.26E−126
	0.25	3.72E−124
F2	0.05	7.00E−64
	0.1	1.24E−61
	0.15	8.97E−60
	0.2	2.00E−63
	0.25	3.62E−64
F3	0.05	3.99E−117
	0.1	2.16E−117
	0.15	1.59E−123
	0.2	1.76E−116
	0.25	6.36E−119

to 0.5. QPSO integrates quantum mechanics principles into PSO to enhance the search capabilities. The parameter settings for QPSO are a contraction-expansion coefficient between 0.5 and 1.0, inertia weight is 0.9, and social and cognitive factors are 1.5. The RLABC algorithm combines reinforcement learning techniques with the ABC algorithm to improve the foraging behaviour of bees. The parameter settings for RLABC are half of the population size, the number of scout bees is 5 % of the population size, learning rate, discount factor, and exploration-exploitation balance are set as 0.05, 0.6, and 0.5, respectively. The parameter settings for GWO and RLQIGWO are similar to OBLGWO. Additionally, the learning rate for RLQIGWO is set as 0.05. The population size for all the algorithms is set as 30, and the maximum number of iterations is set as 10000. The algorithms are implemented in the MATLAB software platform and executed in a laptop with an Intel i5 processor with a clock frequency of 4.44 GHz and 16 GB memory. Each algorithm is executed 30 times individually for a fair comparison.

4.4. Performance comparison

This study conducted a comprehensive performance comparison of several algorithms, including RLQIGWO, to evaluate their convergence abilities and cost-optimization effectiveness for the aggregator. Table 4 details the statistics for the fitness values obtained by the algorithms. These statistics include the MIN, MEAN, MAX, and STD of the objective function values. Additionally, Table 4 lists the mean compensation compensated by the aggregator to users (Rem_A and Rem_B), penalties compensated to the DSO due to flexibility mismatches ($DSO_{mismatch}$), and the mean time for computation by each algorithm to find a solution. The results indicate that algorithms such as RLQIGWO, RLABC, and LSHADE achieve satisfactory solutions within similar optimization times, except for LSHADE, which takes twice as long as RLQIGWO. Other algorithms, including QPSO, GWO, and OBLGWO, exhibit similar poor average fitness values. This variability can be attributed to the parameter sensitivity inherent to the algorithms. Overall, the proposed RLQIGWO achieved the best MIN and MEAN fitness values, QPSO showed the lowest standard deviation but had the overall poorest performance, and GWO and OBLGWO had the worst average fitness values, followed by QPSO, which, although better than GWO and OBLGWO, was still poorer to RLQIGWO, RLABC, and LSHADE.

To further validate the performance and robustness of the algorithms, this study analyzed their convergence behaviour. Fig. 9 illustrates the mean convergence curves for the selected algorithms. The following observations were made: (i) QPSO, GWO, and OBLGWO exhibited similar poor convergence curves; (ii) RLQIGWO, RLABC, and

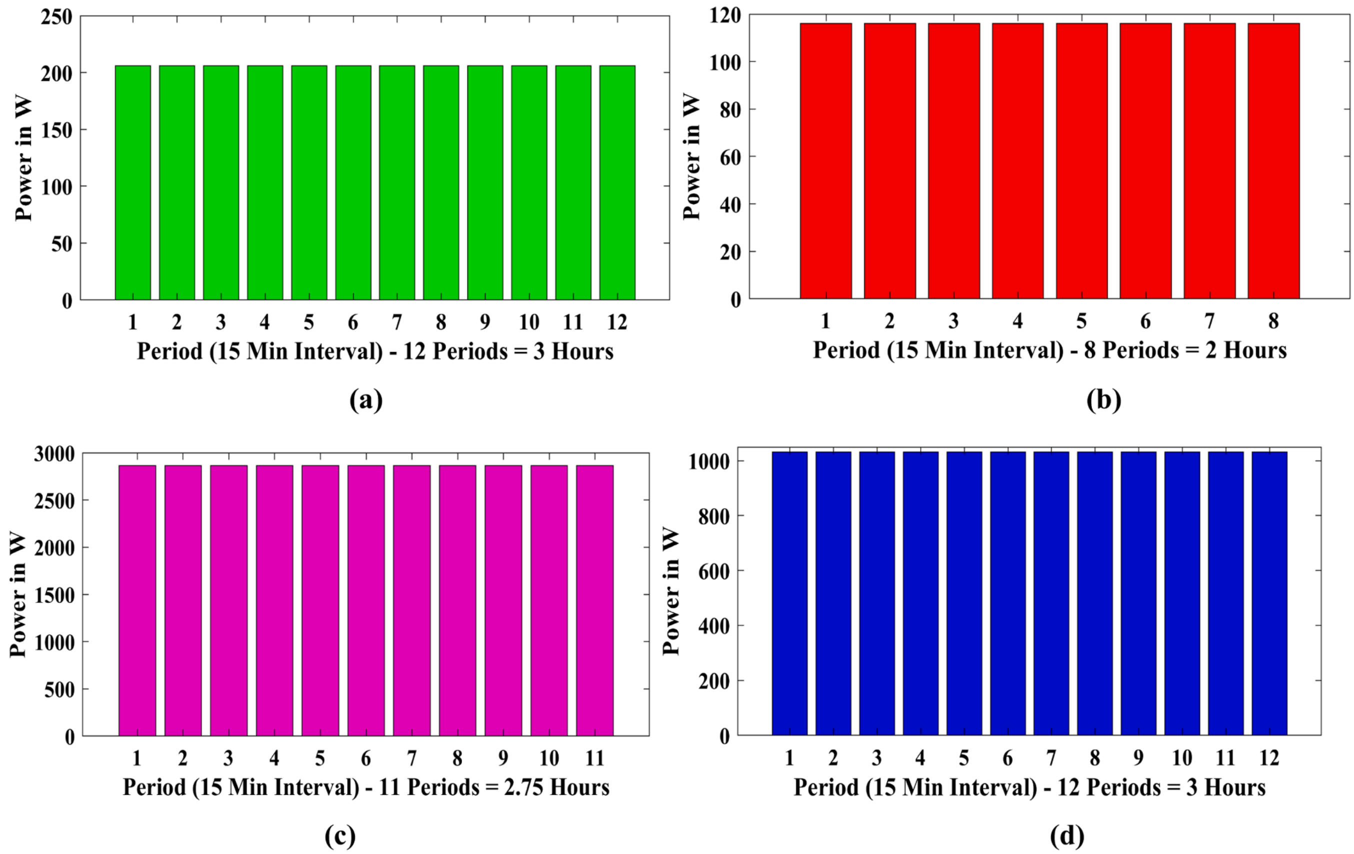

Fig. 6. Consumption patterns (15 min intervals) of appliances with real-time capabilities: (a) Television, (b) Desktop, (c) Air conditioner, (d) Lighting system.

Table 3
Power and energy consumption of houses with different appliances (Lezama et al., 2020).

Appliances	Peak power (kW)	Energy (kWh)	Remuneration (EUR)
Shifting Devices			
Dishwasher	2	5.34	±30 % of 0.2
Tumble dryer	2	10.84	
Washing machine	2.5	5.77	
Real-Time Devices			
Air conditioner	3	22.20	±30 % of 0.09
Lighting system	1	12.00	
Desktop	0.12	0.96	
Television	0.2	2.40	
Aggregated (single house)	10.82	59.51	
Aggregated (20 homes)	216.4	1190.20	

LSHADE displayed similar convergence curves but RLQIGWO showed the best average fitness value with superior convergence characteristics; (iii) the proposed RLQIGWO achieved a final average fitness comparable to the best-performing algorithms despite having a similar convergence curve. To provide a deeper understanding of the resulting profiles, remuneration, and flexibility matching, this study focused on the best solution found by RLQIGWO, one of the top performers. The detailed analysis highlights: (i) RLQIGWO's superior fitness values indicate its effectiveness in minimizing the aggregator's costs; (ii) The proposed algorithm successfully balances remuneration to users and penalties paid to the DSO, optimizing overall performance; (iii) The convergence analysis confirms RLQIGWO's robustness in finding optimal solutions efficiently.

The RLQIGWO algorithm outperforms other algorithms in terms of convergence capabilities and cost minimization for several key reasons, such as adaptive learning, dynamic decision-making, enhanced exploration, and diverse solution generation. The RL component of RLQIGWO enables effective exploration and it utilizes the search space by enabling it to learn and refine its solution over time adaptively. Using RL, the algorithm can dynamically modify its parameters to guide the proposed algorithm to provide diverse solutions. RLQIGWO has a quantum mechanics concept that offers improved exploration possibilities. The proposed RLQIGWO algorithm investigates several possible solutions because of the laws of quantum physics, which lowers the possibility of becoming stuck in local optima. Finding global optima depends on the quantum mechanics framework, which also facilitates the generation of diverse solutions and ensures a wider search space. The proposed RLQIGWO balances exploration and exploitation by adaptively adjusting parameters depending on the state. The proposed RLQIGWO solves the flexibility management problem effectively due to its combination with RL, quantum-inspired processes, and GWO. Among the evaluated algorithms, RLQIGWO is the most efficient due to its exceptional convergence skills and ability to minimize aggregator costs. This detailed evaluation highlights the importance of parameter tuning and adaptive mechanisms in enhancing algorithm performance.

In addition to the above mentioned algorithms, the performance also compared with the traditional PSO algorithm (Eberhart and Kennedy, 1995), and two other new algorithms, called Global-Best guided Firefly Algorithm (GBFA) (Zare et al., 2023), and Fata morgana algorithm (FATA) (Qi et al., 2024). The algorithmic parameters are selected based on the original version. The results obtained by the PSO and FATA is not satisfied and it is less than RLQIGWO, QPSO, LSHADE, OBLGWO, RLABC, GBFA, and GWO due to their inability to handle the constrained real-world problems. The results obtained by the GBFA is comparable but still worse than the proposed algorithm. Table 4 also recorded the

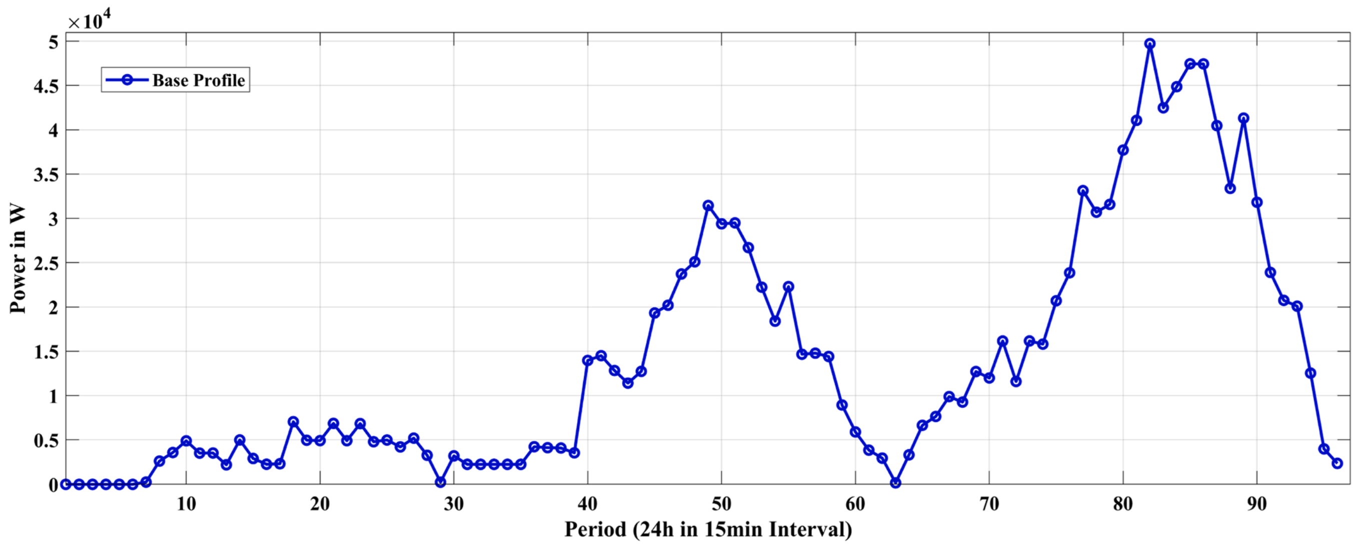

Fig. 7. Baseline consumption profile.

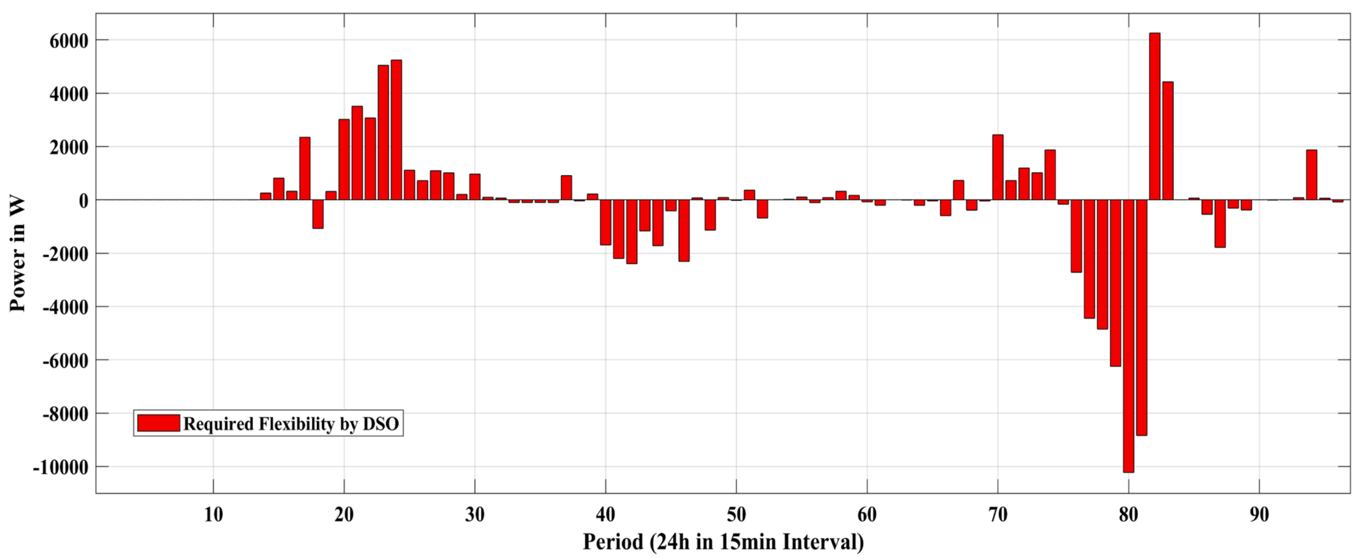

Fig. 8. DSO's request for flexibility for the next 24 hours (15 min intervals).

Table 4
Statistical results of all algorithms, including penalties, remuneration, and computational time.

Algorithms	Objective Function Values (EUR)				Aggregator Cost (EUR)		Computation Time (Sec.)	Mean FRT
	MIN	MAX	MEAN	STD	Rem_A and Rem_B	$DSO_{mismatch}$		
RLQIGWO	**13.9910**	**15.3209**	**14.7772**	0.3855	**9.7858**	**4.9914**	4.2584	**1.0**
QPSO	21.0081	21.9244	21.5843	**0.2784**	12.1974	9.3869	6.7265	4.0
LSHADE	14.5450	18.6878	16.6344	1.1486	10.4862	6.1482	8.0141	3.0
OBLGWO	25.4454	27.1225	26.3051	0.6186	12.8867	13.4184	4.4363	6.5
RLABC	14.3255	16.4719	15.4762	0.5669	10.1506	5.3256	4.5793	2.0
GWO	25.4454	28.1526	26.5793	0.8066	12.7452	13.8341	**4.2569**	6.5
PSO	25.8915	29.4715	27.3941	0.9783	12.9741	14.4200	4.2743	9.0
GBFA	22.8285	24.9769	23.1453	0.4934	12.1478	10.9975	7.4189	5.0
FATA	25.5332	27.2458	26.1197	0.5983	12.8543	13.4408	6.7937	8.0

mean Friedman's Ranking Test (FRT) values. Based on the mean FRT values, the proposed algorithm stands first among all algorithms, followed by RLABC, LSHADE, QPSO, GBFA, OBLGWO, GWO, FATA, and PSO.

4.5. Rescheduling analysis for flexibility provision

As per the objective function, the proposed model aims to find the optimal rescheduling of household appliances to adapt the baseline consumption pattern, as illustrated in Fig. 7 and align it with the flexibility demanded by the operator, as shown in Fig. 8. The comparison

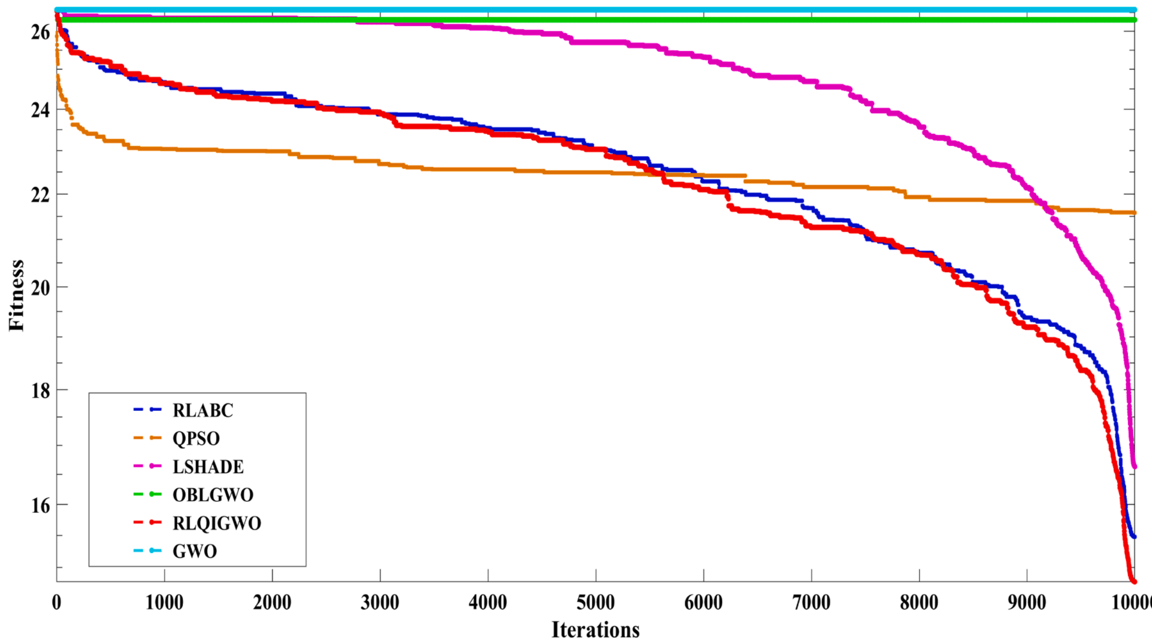

Fig. 9. Average fitness curve obtained by all selected algorithms.

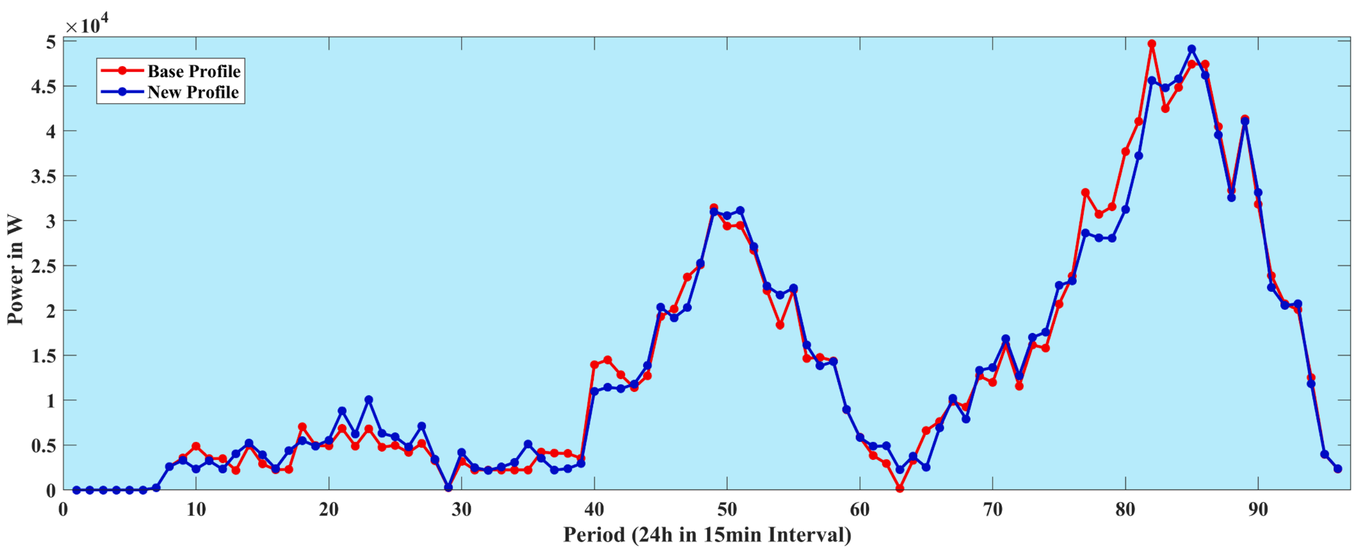

Fig. 10. Baseline and new profile after rescheduling to adopt the flexibility request.

between the baseline profile and the adapted profile after employing the optimization framework is illustrated in Fig. 10. Consumption during peak hours is significantly reduced, adhering to the flexibility demand from the operator. There is a noticeable increase in power consumption during non-peak hours, indicating a shift to earlier hours of the day. It is important to note that these modifications can only be implemented if the DSO activates the procured flexibility. The aggregator and the DSO must have a contract in place specifying compensation and settlement guidelines for this process to run effectively. To evaluate the effectiveness of the flexibility matching, Fig. 11 shows the flexibility demanded by the operator alongside the flexibility provisioned by the aggregator after device rescheduling. The aggregator is able to reschedule devices in a manner that approximates the flexibility demanded by the operator to a considerable degree. From the aggregator's perspective, modifying the baseline schedule involves deviating from the best scheduling of devices. The aggregator's ability to handle energy resources optimally is predicated on its technological capabilities. To reduce the compensation needed for offering flexibility, the aggregator seeks to accomplish the necessary changes with the least expensive devices. The fitness function is designed to allow the aggregator to accept some penalty for flexibility mismatches if it results in a lower overall cost. For example, if the cost of shifting or adjusting consumption is high, it might be more cost-effective to incur a penalty. The model's approach to rescheduling optimizes the balance between reducing peak-hour consumption and increasing non-peak-hour usage, aligning closely with the DSO's flexibility requests. Through effective contractual agreements and strategic cost management, the aggregator can provide the required flexibility while minimizing costs and penalties, ultimately enhancing the efficiency and reliability of the energy system.

Additionally, this study investigates the impact of flexibility provided by different types of devices, specifically focusing on real-time and shiftable devices. The distinction in their flexibility contributions is pivotal in determining the compensation mechanisms within the DR program. Fig. 12 presents the rescheduling and baseline profiles for

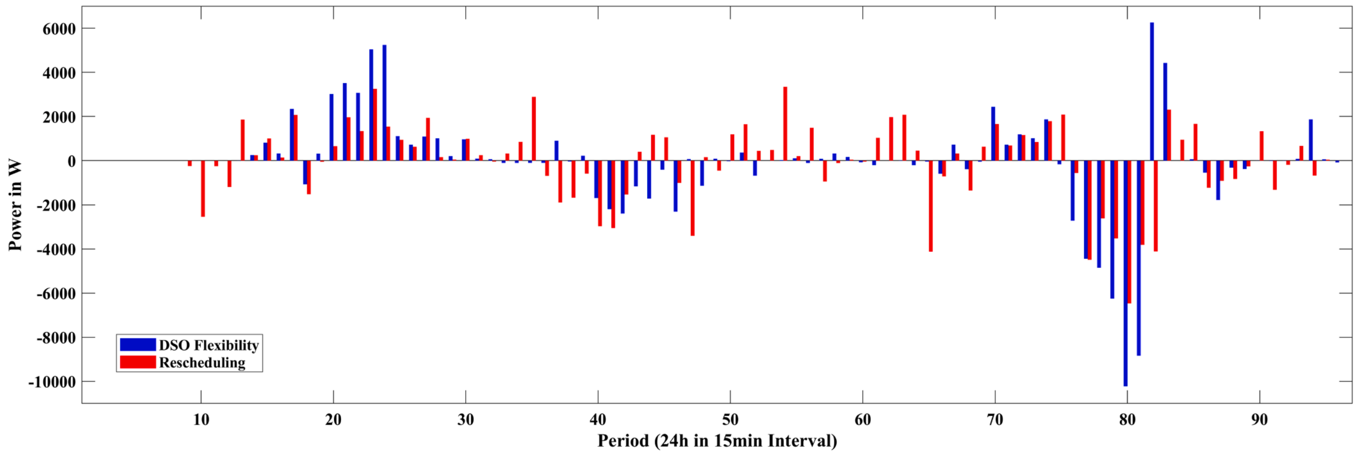

Fig. 11. DSO Flexibility and rescheduled flexibility provision.

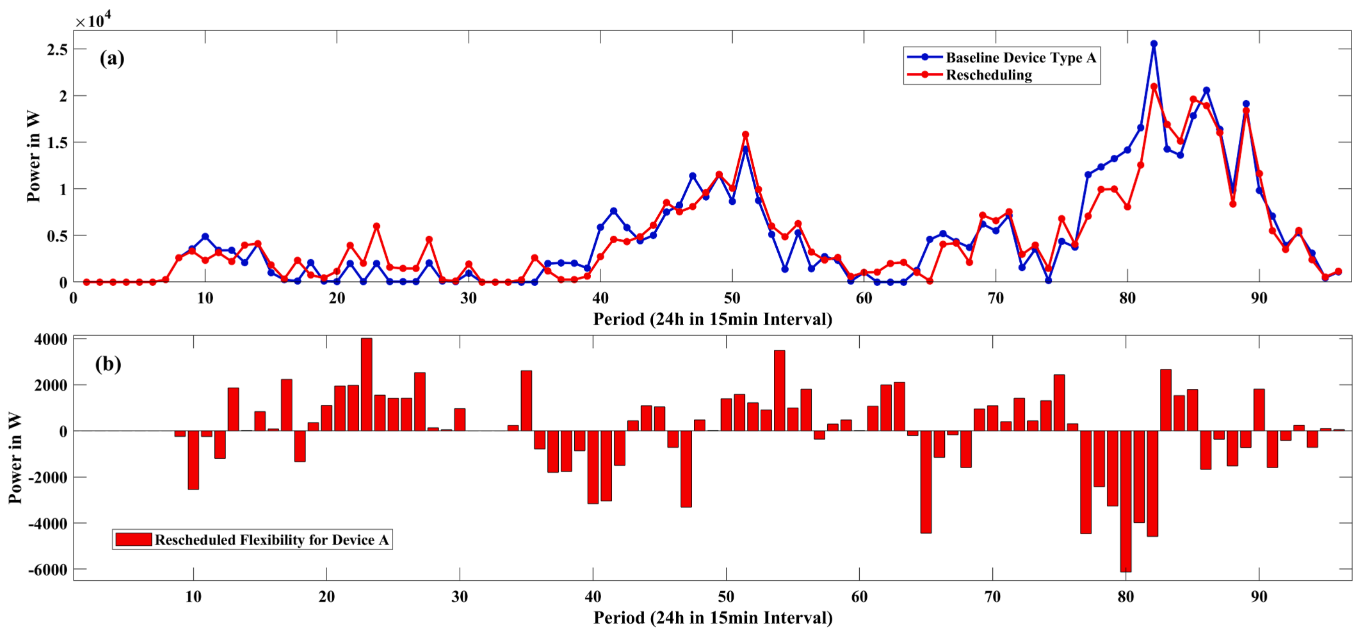

Fig. 12. Profiles of Set A devices; (a) Baseline and rescheduling, (b) Flexibility provision.

shifting devices (Fig. 12a), and their flexibility provision (Fig. 12b). Shifting devices demonstrate a substantial contribution to flexibility, particularly during peak times. At 20 hours, i.e., 80th 15 Min interval, the maximum flexibility given by shifting devices for reduction achieves 6 kW, as shown in Fig. 12(b). Fig. 13 provides similar data for real-time devices. Real-time devices contribute significantly less to flexibility compared to shifting devices. The maximum flexibility offered by real-time devices is nearer to 900 W at 19 hours, i.e., 76th 15 Min interval, as indicated in Fig. 13(b). The observed differences in flexibility contributions between shifting and real-time appliances can be attributed to the following primary factors: (i) By decreasing the load during the initially planned period and raising the load during the shifted period, shifting devices have two effects on the baseline profile; (ii) The aggregator compensates users by paying ±30 % of 0.2 EUR for shifting the pattern to another allowed interval, which is often more cost-effective than compensating in EUR/kWh, as required for real-time appliances; (iii) Since their load may be accurately adjusted during a specified period without affecting future scheduled events, real-time appliances provide superior control; (iv) Due to their precise control capabilities, real-time devices are valuable for avoiding penalty mismatches by making targeted adjustments.

Fig. 12(a) shows the baseline and rescheduled profiles after optimization. The rescheduling involves shifting consumption from peak to non-peak periods. Fig. 12(b) illustrates the flexibility provision by shifting devices. Peaks in flexibility are evident around certain hours, indicating substantial shifts in load.

Fig. 13(a) displays the baseline and adjusted profiles for real-time devices. Adjustments are smaller and more frequent, reflecting the limited but precise flexibility of real-time devices. Fig. 13(b) shows the flexibility provided by real-time devices. The flexibility contribution is lower compared to shifting devices but offers fine-tuned control. This analysis highlights the crucial role of shifting and real-time devices in providing flexibility within a DR program. Shifting devices offer significant flexibility through dual impacts on the baseline profile and cost-effective compensation mechanisms. In contrast, real-time devices, despite their lower flexibility contribution, provide precise control to avoid penalties, ensuring balanced and efficient energy management. In order to evaluate the effect of the DSO's penalty on the mismatch in the flexibility provided by the aggregator, this research additionally carried out an extra experiment. In order to do this, the penalty cost is adjusted between 0.05 and 1 EUR, and the suggested RLQIGWO algorithm is used to carry out the optimization process. This results in the lowest fitness

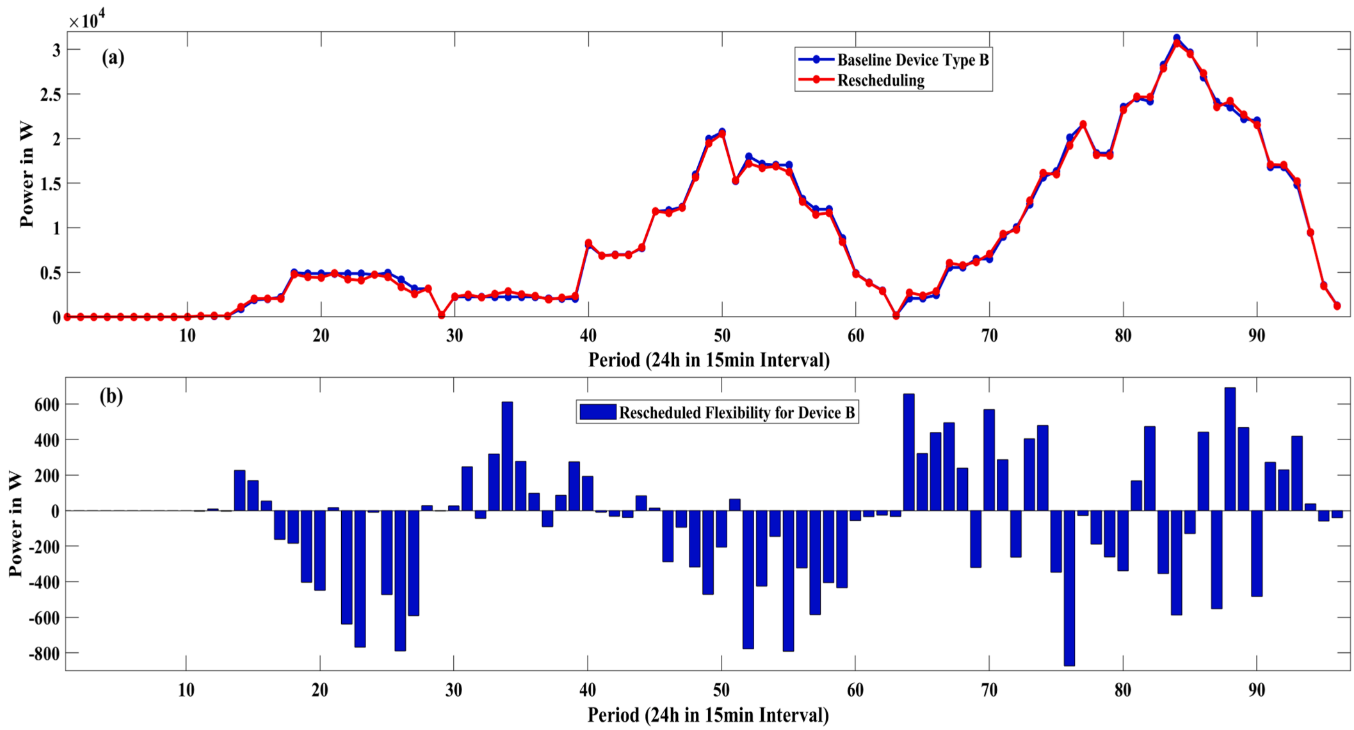

Fig. 13. Profiles of Set B devices; (a) Baseline and rescheduling, (b) Flexibility provision.

Table 5
Flexibility and aggregator cost during DSO penalty variations.

C_{DSO} in EUR/kWh	Flexibility in kW			Aggregator Price in EUR			
	$Flex_A$	$Flex_B$	Total	Rem_A	Rem_B	Penalty	Total
0.05	23.4542	0.0000	23.4542	0.1554	0.0000	1.3366	1.4921
0.1	49.0200	23.6668	72.6868	0.7013	0.6213	1.3307	2.6533
0.3	113.8280	29.2873	143.1153	5.4821	1.2252	3.5941	10.3015
0.5	122.0860	39.7571	161.8431	9.5114	1.2981	2.8117	13.6211
0.7	128.5000	47.0885	175.5885	10.4823	1.4700	4.4357	16.3880
0.9	117.7060	39.7674	157.4734	9.6026	1.3939	7.4644	18.4609
1	128.2580	47.1918	175.4498	9.4315	1.4774	8.0578	18.9666

value, as will be covered in the evaluation of performance section. This study also intended to assess the degree of flexibility offered and the compensation that various device types received. Table 5 provides a detailed summary of the flexibility offered by the aggregator, divided down by the types of appliances that are being utilized, as well as the associated compensation and fines that the aggregator has to pay. When the DSO levies a small fine, the aggregator chooses to pay the fine instead of turning on flexibility.

For example, the flexibility of Set B appliances is reduced to zero by the aggregator at a penalty of 0.05 EUR. It makes sense as the real-time device activation price (Set B) is larger than the penalty, at ±30 % of 0.09 EUR/kWh. As the penalty costs increase above the activation cost of Set B devices (e.g., 0.1 EUR and above), the aggregator starts utilizing the flexibility provided by these devices. Sometimes, the aggregator collects more flexibility overall than what the DSO requests. Figs. 13(b) and 12(b) illustrate that during hours 6–8, the aggregator increases consumption with some Set A devices while simultaneously decreasing consumption with Set B devices. One potential solution to improve flexibility matching and reduce penalties is the usage of market-based policies. These strategies eliminate the transitional between users and the operator, allowing for more direct and efficient transactions. However, implementing such a decentralized optimization approach poses significant challenges. The additional experiment demonstrates that the penalty cost levied by the operator significantly influences the aggregator's strategy for providing flexibility. At lower penalty costs, the aggregator finds it more economical to pay the penalty rather than activate costly flexibility. Conversely, higher penalty costs incentivize the aggregator to utilize available flexibility to avoid penalties. Balancing flexibility provision between increasing and decreasing consumption across different devices is crucial for achieving optimal results.

4.6. Limitations and challenges

The present system is predicated on flexible compensation and penalty rates assuming fixed monetary rates for the flexibility, which makes cost computation and engagement by users easy. But in practice, these rates may vary over time because of factors such as market structure, demand for energy, and even the users themselves. In addition, dynamic pricing is tied to user behaviour and user provision of flexibility. For instance, users can be willing to give more flexibility at peak periods given high energy prices, whilst there could be less participation in opting for cheaper or no incentives in off-peak periods. Such variability implies that the flat rates model in its current with no changes to user charges may not be able to take into account all the interrelations as they would be expected to in the current energy market. As such, this should be the focus of subsequent studies in order to imitate how the fluctuations in the incentives offered to the exploitation of flexibility sources change on its usage. In the proposed model, the relationships among DSOs, aggregation, and users are treated in a simplistic way and with a clear-cut agreement on the roles and terms of the parties involved. However, it can be seen that these relationships are not that simplistic. Other stakeholders, such as regulatory bodies, energy suppliers, and others, are often involved in a complex network of relationships. Each of these parties may have distinct goals and compliance, e.g. regional restrictions and motivation structures, which might make the aggregator's job of flexibility management more complex. For instance, differing priorities among stakeholders could influence how flexibility is provisioned, remunerated, and utilized. These complicated contractual dynamics are likely to affect the overall efficiency and cost-effectiveness of the flexibility provision, and further research is needed to model these complexities more accurately.

Another limitation of the current model has to do with the understanding of the users as if they are the same with regard to participation in the demand response programs. However, this is not true as, based on previous studies, user behaviour is multi-faceted. It can change due to a variety of reasons, including individual motivation, economic status, environmental issues, as well as the perceived incentive to join a demand response program. For instance, users who consume more electricity may also adjust their consumption patterns but are also less responsive to financial incentives. This behavioural diversity brings in unpredictabilities in forecasting how much flexibility that would be expected is going to be available. In addition, technological barriers such as the use of old machines that cannot carry out the tasks required or poor network connectivity might also affect responsiveness. It is important to acknowledge this variability in user engagement so as to enhance the reliability and strength of the provision of flexibility when applied in practice. The proposed model has been shown to be effective within a small and bounded system, but extending it to a large distribution network is problematic. The more connected devices, users, and stakeholders, the higher the challenge of managing and enhancing flexibility in real time. For example, the need to process more extensive data sets and the need for coordination to achieve flexibility across thousands of devices may be unsustainable in areas with limited technology or resources. In large-scale implementations, optimization algorithms not only have to process more data, but they also have to do it quickly and adapt to changes in supply and demand. It is possible that the model can become less efficient and effective at this scale, and some new computational resources and optimization methods, for example, decentralized or cloud-based, should be included.

The model in this study is currently built with some assumptions on the structure of the energy system, including the regulatory framework and market structures. However, the energy systems that exist in different regions are very different and may work under different circumstances. For instance, some may have heavily regulated markets for energy, while others may have relatively more open markets for the energy business. These differences can greatly affect the application of the proposed model. While in some systems, technical infrastructure like smart meters or advanced grid management systems can be present in many places, in other systems, the lack of infrastructure can hinder the aggregator from managing flexibility effectively. It is also important to note that the model's flexibility to different regional circumstances, market regulations, and energy systems will be key to its applicability. Future research should examine the model's applicability to other energy systems and take into account local legislation, market conditions, and available facilities. The considered model has good results in ideal conditions, but when it is applied to real-life large-scale energy systems, several things could be improved. Real energy systems are much more complicated than the simplified models used in simulation and may contain many interdependent parts, external connections and possible sources of disruption. There may be variable user activity, regional infrastructural factors, and some technical factors which may not be envisaged at the time of modelling but can be a challenge when the model is being applied. For instance, in a large-scale energy system, outages, weather conditions, or variations in energy demand could alter flexibility requirements, and it would not be possible to satisfy the DSO's demand with the procured flexibility. The extent to which the model can be scaled up and be accurate and efficient in such unpredictable settings will be informed by more work on flexible, scalable and robust optimization techniques.

While the considered model demonstrates strong potential in controlled environments, the practical challenges of dynamic pricing, complex contractual relationships, variable user engagement, scaling to larger systems, and diverse energy infrastructures must be addressed to ensure its effectiveness in real-world applications. Future work should focus on refining the model to account for these real-world complexities, such as incorporating dynamic pricing models, expanding user behaviour modelling, and improving scalability. Additionally, decentralized optimization approaches and advanced computational resources may be necessary to handle the increased complexity and variability of real-world energy systems, ensuring that the model remains both efficient and adaptable in diverse, large-scale applications.

5. Conclusions and future directions

This study introduced a robust mathematical framework designed to aid aggregators in optimizing the management of household devices for flexibility facilities. Through the use of the proposed methodology, aggregators can minimize the expenses related to rescheduling home appliances and simultaneously react to DSOs' requests for flexibility. Two device types were included in our analysis: Set A, which had shifting capabilities, and Set B, which had real-time energy adjustment capabilities. The problem's inherent large-scale, non-linear character was addressed by using state-of-the-art algorithms as the main optimization tools to derive close to optimal solutions, including RLQIGWO, RLABC, QPSO, GWO, OBLGWO, PSO, FATA, GBFA, and LSHADE. According to the results, aggregators may more effectively match the flexibility they offer with the demands of the DSO and lower the fees they have to pay users for their contributions to flexibility by using the suggested model. RLQIGWO was found to be the most efficient algorithm out of all the selected algorithms. Because of the way it integrated a quantum-inspired method for a good diverse solution, reinforcement learning for adaptive improvement, and the GWO's inherent capabilities for effective search procedures, the proposed RLQIGWO algorithm performed better than other approaches. The findings of this study have significant suggestions for the broader field of smart grid management and DR strategies. The flexibility management model, with the RLQIGWO algorithm, provides a viable path for enhancing the optimization of DR strategies and flexibility management. By improving the efficiency and cost-effectiveness of flexibility provision, this work contributes to the development of more adaptive and responsive energy management systems. These flexibility models are critical for the future of sustainable and resilient power grids. The integration of RLQIGWO could lead to smarter, more dynamic interactions between aggregators and DSOs. These models ultimately support the transition to a more flexible and efficient energy market.

The contractual connections between DSOs, aggregators, and users, including fixed monetary rates, indicate areas that guarantee further research. By doing away with middlemen and promoting direct transactions, investigating decentralized market-based techniques may also improve flexibility provision. Enhancing the model to learn the requirement to balance rising and falling loads at the same time also be essential. To further enhance the model's efficiency, real-time data and strong forecasting techniques can be used to predict user behaviour and energy demand. Finally, the application of self-tuning and adaptive techniques strengthens optimization robustness in different circumstances.

Ethical Approval

This study is based on simulation. The authors have verified that it does not necessitate ethical approval.

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CRediT authorship contribution statement

Manoharan Premkumar: Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Sowmya Ravichandran:** Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Conceptualization. **Ahmad O. Hourani:** Writing – review & editing, Visualization, Resources, Formal analysis. **Thamer A. H. Alghamdi:** Writing – review & editing, Visualization, Funding acquisition, Formal analysis.

Declaration of Competing Interest

We affirm that the authors do not possess any conflicting interests as delineated by the publisher, nor any other interests that could potentially be viewed as affecting the findings and/or discourse presented in this manuscript.

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Data availability

No data was used for the research described in the article.

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