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Distributed Optimization Strategy for Networked Microgrids Based on Network Partitioning

Jingjing Wang^a, Liangzhong Yao^{a*}, Jun Liang^b, Jun Wang^a, Fan Cheng^a

Abstract: The integration of a large number of distributed resources into an active distribution network presents significant challenges, including high control dimensionality, strong output uncertainty, and low utilization of renewable energy. This paper introduces a distributed optimization strategy for networked microgrids based on network partitioning to alleviate the computational burden, reduce operating costs, and enhance the utilization of renewable energy. The active distribution network is partitioned into networked microgrids, and a two-layer distributed optimization model is developed for their management. The first layer focuses on intra-day distributed optimal dispatch, balancing power and load by managing various flexible resources and the exchange power between virtual microgrids. The second layer, real-time distributed power tracking optimization, coordinates flexible resources within virtual microgrids to mitigate photovoltaic power fluctuations and track intra-day dispatch instructions. Simulation results demonstrate that the proposed network partitioning method reduces dispatch costs by 5.3% and increases the utilization of distributed PV by 3%, compared to the NP method that only considering modularity. Moreover, calculation times for intra-day dispatch and real-time power tracking are reduced by approximately 26% and 50%, respectively, compared to centralized control.

Keywords: High penetration photovoltaic integration, Network partitioning, Networked microgrids, Distributed optimization

1. Introduction

1.1. Background and Motivation

In recent years, there has been a significant surge in the integration of distributed renewable energy, such as distributed photovoltaic (PV) systems, into distributed networks. The combination of characteristics such as low grid-integration voltage levels, high penetration rates, and power output uncertainty presents significant challenges to the operation and control of active distribution networks and the utilization rate of distributed PV^[1]. Despite the flexible resources, including micro gas turbines (MGT), energy storage systems (ESS), and flexible loads (FL), such as air conditioners and electric vehicles, offer substantial flexible regulation resources, their large quantity and widespread distribution characteristics present significant challenges for traditional operation and control methods, leading to communication challenges, high costs, low reliability, and complex decision-making processes^{[2][3]}.

Networked microgrids (NMGs) are clusters of virtual microgrids (VMs) that are physically connected and functionally interoperable^{[4]-[5]}. NMGs can be formed by dividing an active distribution network and clustering multi-type flexible resources within it^[6]. By employing a thoughtful partitioning approach, the flexible adjustment capabilities of each VM can be enhanced, ensuring a balance between power supply and demand within VM. Additionally, transforming from the traditional centralized control mode of the active distribution network to a model of coordination and autonomous operation both between and within VM offers several benefits, such as reduces the computational burden, improves the interaction effect between VM, enhances the utilization rate of renewable energy, and lowers the dispatch costs of the active distribution network.¹

1.2. Literature Review

Several network partitioning (NP) methods and clustering algorithms have been investigated and employed in various operational scenarios. In [7]-[8], the active distribution network was partitioned into interconnected virtual microgrids with self-sufficient capacity, aiming to minimize the unbalanced power within each microgrid and reduce the interactive power between virtual microgrids. In [9], a clustering algorithm was employed to group closely distributed PV and ESS into the same area, taking into account electric distances between nodes and participating in peak shaving. In [10], a real-time distributed cluster algorithm was introduced to classify distributed ESS into two types of virtual power plants, addressing high-frequency and low-frequency power requirements, respectively. A consensus

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clustering approach for bi-objective power NP was introduced in [11] to explore the Pareto frontier of the bi-objective NP. [12]-[15] utilized various clustering or intelligent algorithms to partition the active distribution network, and focused on voltage or reactive local optimization to minimize the power losses between areas. In [16], a network cluster method for an active distribution network based on measurement information was proposed, dividing the active distribution network into multiple microgrids in the event of a power failure to enhance reliability. A planning method was used to establish distributed generator clusters within the active distribution network, aiming to minimize net-load power while considering the uncertainties of network topology and the isolated operating probabilities of distributed generator clusters[17]. However, existing researches primarily focus on a single metric and are based on typical scenarios or utilizes typical operational data. Given that the partitioned clusters may lack reasonable and adequate flexibility based on a single metric or typical operation data, multiple metrics and the uncertainty of renewable energy should be taken into account when conducting NP.

Energy management is of prime importance in achieving stable and economic operations of multi-microgrids. [4][5] provided an overview of existing energy management systems, which include centralized, decentralized, and distributed control. Centralized control governs the energy and power balance of the entire system and offers optimal results. However, it imposes a heavy computational burden and communication requirements and is suitable for systems with low variability^[18-19]. Decentralized control, on the other hand, grants autonomy to each microgrid as an independent agent^[20-22], resulting in reduced computational burden and low privacy requirements. Nonetheless, the lack of communication between adjacent microgrids limits optimization outcomes. In contrast, distributed control fosters communication between adjacent MGs, thereby enhancing optimization results by incorporating additional information compared to decentralized control^[23-26]. This approach is well-suited for large-scale multi-microgrid systems.

Several approaches have been proposed in the research of distributed control. A double-layer distributed optimal control approach for multi-microgrids to compensate for differences between upper and lower layers was introduced in [23]. [24] proposed a distributed online dynamic dispatch strategy for NMGs, operating without forecast data to achieve autonomous operation of microgrids. [25] presented a transactive energy framework with distributionally robust optimization for coordinated energy management of NMGs, facilitating power exchange between NMGs and DSOs, as well as between NMGs. [26] introduced a distributed leader-follower based adaptive consensus control approach for NMGs, enabling coordination among multiple distributed resources for frequency control. Despite the advantages of distributed control, such as lower computational complexity and plug-and-play capabilities compared to other control methods, coordinating information and energy across multiple microgrids, hierarchical levels, and time scales effectively remains a challenging task according to existing researches.

Tab 1. Researches on network partitioning and distribution control of NMGs

Refs.	Network partitioning	Distributed control of NMGs	Scenarios	Mathematical model
[9]	✓	×	Cluster distributed PV and ESS to participate peak shaving	Cluster algorithm
[10]	✓	×	Classify ESS into two types to addressing high-frequency and low-frequency power requirements	Cluster algorithm
[12-15]	✓	×	Partition ADN into several areas to realize local optimize of voltage or reactive power	Intelligent algorithm
[23]	×	✓	Compensate differences between upper and lower layers	Double-layer distributed optimization
[24]	×	✓	Autonomous operation or emergency dispatch	Distributed online Lyapunov-based optimization
[25]	×	✓	Facilitating power exchange between NMGs and DSOs, as well as between NMGs	Robust optimization
This paper	✓	✓	Realize autonomous operation of VM and distributed optimization between VMs	Intelligent algorithm; Double-layer optimization

1.3. Main focuses and contributions

Tab. 1 compares the recent research works in the NMGs subject area. To tackle the shortage of research on NP and the challenges of distributed control mentioned above, this paper introduces a distributed optimization strategy for NMGs based on NP, as illustrated in Fig.1. The major contributions are as follows:

1. A robust flexibility metric for VM which considers the balance between supply and demand under the renewable energy uncertainty. A NP model, incorporating both modularity and robust flexibility, is introduced to partition the active distribution network into NMGs. This approach enhances the ability to manage the uncertainty of renewable energy in VMs and achieve balance between power supply and demand.

2. Distributed optimization models are established for NMGs to verify the reasonableness of network partitioning, and to demonstrate the advantages of enhancing renewable energy utilization and reducing operation costs. The intra-day optimization model coordinates and dispatches flexible resources to achieve a balance between power supply and demand when NMGs operate independently. When NMGs operate interactively, the intra-day optimization model can achieve interactions between VMs as well as between VM and the upper grid based on Synchronous Alternating Direction Method of Multipliers (SADMM) algorithm. The goal is to dispatch flexible resources within VMs to reduce operating costs and increase the utilization of renewable energy. The real-time power tracking model aims to coordinate flexible resources within VMs to mitigate PV power fluctuations and follow intra-day dispatch instructions.

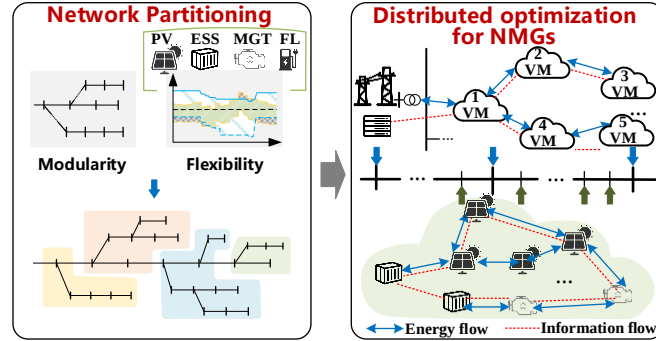


Fig.1 The framework for distributed optimization of NMGs based on NP

The remainder of this paper is organized as follows. The NP method for the active distribution network is introduced in Section 2. The two-layer distributed optimal model for NMGs is introduced in Section 3. Section 4 gives the relative simulation results and analysis, and Section 5 presents the main conclusions.

2. NP method of active distribution network

The integration of high-penetration distributed renewable energy, such as photovoltaics, into active distribution network increases the control complexity and decision dimensions. Dividing the active distribution network into NMGs enables coordinated and autonomous operation, thereby reducing operational complexity. This chapter introduces a robust flexibility metric to measure the supply and demand balance capacity of VMs and divide the active distribution network into NMGs. By focusing on modularity and robust flexibility, this approach aims to enhance the capacity to address PV uncertainty.

2.1. Uncertainty modelling of PV

The truncated versatile distribution (TVD) model is employed to characterize the uncertainty of PV [27]. TVD model has three flexible shape parameters, allowing it to more accurately capture the shape of the distribution. Moreover, the TVD model has a limited interval, and both its cumulative density function and inverse cumulative density function of TVD have analytical forms. This makes the TVD model particularly effective for accurately representing PV. The probability density function (PDF) of TVD is defined as:

$$f(x) = \begin{cases} \frac{1}{s} \cdot \frac{\alpha \beta e^{-\alpha(x-\gamma)}}{(1 + e^{-\alpha(x-\gamma)})^{\beta+1}} & m \leq x \leq n \\ 0 & x \leq m, x \geq n \end{cases} \quad (1)$$

$$s = (1 + e^{-\alpha(n-\gamma)})^{-\beta} - (1 + e^{-\alpha(m-\gamma)})^{-\beta} \quad (2)$$

Where x is a random variable. α , β and γ are the shape parameters of the TVD model, respectively. m and n represent the upper and lower limits of standardized interval, respectively. When it is applied to describe the power of renewable energy, $m=0$, $n=1$. s represents the standardized coefficients.

At a given confidence level c , the inverse cumulative density function is:

$$F^{-1}(c) = \gamma - \frac{1}{\alpha} \ln \left[\left(s \cdot c + (1 + e^{-\alpha(m-\gamma)})^{-\beta} \right)^{-1/\beta} - 1 \right] \quad (3)$$

The historical forecast power data and actual power data of PV are divided into distinct boxes based on the forecast data range. In each box, the TVD model is used to match the actual power, and the probability distribution model of the

PV actual power in each box is obtained.

2.2. Metrics of NP

2.2.1. Modularity metric

The modularity metric is applied to assess the structural strength of a partition [28]. It represents the sum of the differences between the actual and expected weights of connections between two nodes. Higher modularity indicates that the actual weights are significantly greater the expected weight, resulting in more concentrated nodes within each partition and tighter connections between them. The modularity metric is defined as:

$$\rho = \frac{1}{2m} \sum_i \sum_j (e_{ij} - \frac{k_i k_j}{2m}) \delta(i, j) \quad (4)$$

Where e_{ij} represents the weight between node i and j , which is represented as the electrical distance in this paper. $m = (\sum_i \sum_j e_{ij})/2$ represents the total line weights in VM; $k_i = \sum_j e_{ij}$ represent the total line weights connected to node i ; $\delta(i, j) = 1$ if node i and j are in the same VM, otherwise $\delta(i, j) = 0$.

2.2.2. Robust flexibility metric

VM requires flexible adjustable capabilities to address the uncertainties of renewable energy resources [29]. Robust flexibility assesses the supply and demand balance capacity while accounting for the robust interval of PV output.

The power demand within the VM is net-load power, and the power is provided by ESS, FL and MGT.

(1) Power demand

The day-ahead PV forecast power are divided, and the PDF of PV actual power in each box using TVD are obtained. Subsequently, the confidence intervals of the PV actual power are determined based on the quantile theory, as shown in Eq. (5).

$$F_{i,t}^{nd} = [F_{i,t}^{nd}, \bar{F}_{i,t}^{nd}] \quad (5-a)$$

$$F_{i,t}^{nd} = \sum_{r \in N_i} PV_{r,t}^{pre,dn} + \sum_{a \in N_i} P_{a,t}^{ESS} + \sum_{b \in N_i} P_{b,t}^L + \sum_{c \in N_i} P_{c,t}^{MGT} \quad (5-b)$$

$$\bar{F}_{i,t}^{nd} = \sum_{r \in N_i} PV_{r,t}^{pre,un} + \sum_{a \in N_i} P_{a,t}^{ESS} + \sum_{b \in N_i} P_{b,t}^L + \sum_{c \in N_i} P_{c,t}^{MGT} \quad (5-c)$$

Where N_i represents the node set of the i th VM. $PV_{r,t}^{pre,dn}$ and $PV_{r,t}^{pre,un}$ represent the upper and lower quantiles of the PV actual power PDF. $P_{a,t}^{ESS}$, $P_{b,t}^L$, and $P_{c,t}^{MGT}$ represent the operating power of ESS, FL, and MGT, respectively.

(2) Flexible power providers

(a) Energy Storage System

Energy storage system offers upper and lower regulation capabilities through discharge and charge. However, constraints on maximum charge and discharge power, as well as capacity, must be taken into account.

$$\bar{P}_{a,t}^{ESS,d} = \min\{(E_a - \underline{E}_a) / \Delta t, P_a^{ESS,N}\} \quad (6-a)$$

$$\bar{P}_{a,t}^{ESS,c} = \min\{(\bar{E}_a - E_{a,t}) / \Delta t, P_a^{ESS,N}\} \quad (6-b)$$

Where E_a represents the capacity of the a th ESS. $P_a^{ESS,N}$, $\bar{E}_a$, and $\underline{E}_a$ represent the rated charge and discharge power, and the maximum and minimum capacity of a th ESS, respectively.

(b) Flexible Load

Flexible load serves as a crucial demand response resource of the power system. The user response willingness model is applied to measure the response willingness under different power price scenarios [30]:

$$\Delta \bar{P}_{b,t}^L = \mu_t^b \times P_{b,t}^L \quad (7-1)$$

$$\mu_t^b = \varphi \left(e^{P_t^{real} / P_{max} - 1} - e^{P_{base} / P_{max} - 1} \right) \quad (7-2)$$

Where $P_{b,t}^L$ represents the operational power of b th FL at time t , μ_t^b represents the response willingness of b th FL at time t . φ represents the subjective response coefficient of FL. P_{base} represents the expected electricity price, P_t^{real} represents the Time-of-use (TOU) electricity price, and P_{max} represents the maximum TOU price throughout the day. The FL response ratio is mainly influenced by the TOU price.

(c) Micro Gas Turbine

Micro Gas turbine plays an important role as a flexible resource in VM. Constraints include operational constraints and ramp constraints:

$$\Delta \bar{P}_{c,t}^{\text{MGT}} = \min\{(\bar{P}_c^{\text{MGT}} - P_{c,t}^{\text{MGT}}), \Delta P^{\text{MGT}}\} \quad (8-a)$$

$$\Delta P_{c,t}^{\text{MGT}} = \min\{P_{c,t}^{\text{MGT}} - \underline{P}_c^{\text{MGT}}, \Delta P^{\text{MGT}}\} \quad (8-b)$$

Where $\bar{P}_c^{\text{MGT}}$ and $\underline{P}_c^{\text{MGT}}$ represent the maximum and minimum output power of c th MGT, respectively. ΔP^{MGT} represents the maximum ramp power of the MGT.

The regulation capacity of multi-type flexible resources is aggregated using *Minkowski Sum*.

$$F_{i,t}^{\text{pr}} = P_1 \oplus P_2 \oplus \dots \oplus P_j = P_{a,t}^{\text{ESS}} \oplus \Delta P_{b,t}^{\text{L}} \oplus \Delta P_{c,t}^{\text{MGT}} \quad (9)$$

The aggregated regulation capacity at different times can be obtained and shown as:

$$R_i = \{[E_{i,1}^{\text{pr}}, \bar{F}_{i,1}^{\text{pr}}], [E_{i,2}^{\text{pr}}, \bar{F}_{i,2}^{\text{pr}}], \dots, [E_{i,T}^{\text{pr}}, \bar{F}_{i,T}^{\text{pr}}]\} \quad (10)$$

(3) Robust flexibility metric

The interval form is used to describe the robust flexibility supply-demand balance capability of the VM, as shown in Fig. 2.

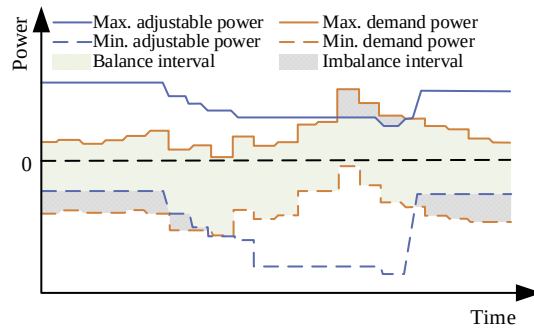


Fig.2 Intervals of power demand and power supply

The robust flexibility of VM can be calculated as follows.

$$F_{i,t}^{\text{bl}} = \begin{cases} 1, E_{i,t}^{\text{pr}} \leq E_{i,t}^{\text{nd}} \leq \bar{F}_{i,t}^{\text{nd}} \leq \bar{F}_{i,t}^{\text{pr}} \\ 0, \bar{F}_{i,t}^{\text{pr}} \leq E_{i,t}^{\text{nd}} \parallel E_{i,t}^{\text{pr}} \geq \bar{F}_{i,t}^{\text{nd}} \\ \min\{\bar{F}_{i,t}^{\text{pr}} / \bar{F}_{i,t}^{\text{nd}}, E_{i,t}^{\text{pr}} / E_{i,t}^{\text{nd}}\}, \text{otherwise} \end{cases} \quad (11)$$

The robust flexibility of all NMGs in the active distribution network is shown as:

$$F = \left(\sum_t \sum_i F_{i,t}^{\text{bl}} \right) / I \quad (12)$$

Where I represent the total number of NMGs after NP.

2.2.3. NP method

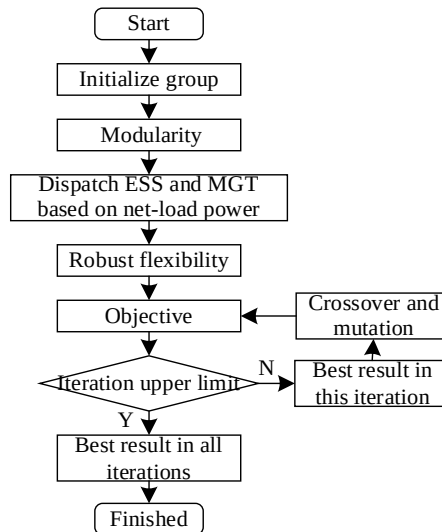


Fig. 3 NP process

The NP objective function, as shown in Eq. (13), is established to maximize the robust flexibility of each VM while ensuring the modularity of the NMGs.

$$f = \max(\rho + F) \quad (13)$$

Where ρ represents the modularity of NMGs, as indicated in Eq. (4), while F represents the robust flexibility of NMGs, as shown in Eq. (12).

The method described in [31] is adopted to conduct NP on the active distribution network using the genetic algorithm. The NP process is shown in Fig. 3.

In each iteration process, the NP form with the best objective is retained. In the subsequent iteration, new NP forms are generated through a continuous process of crossover and mutation, based on the previous best result. The NP form with the best objective across all iterations is selected as the final NP result, which determines the topology and number of VMs.

3. Two-layer distributed optimal model for NMGs

To address the heavy computational burden and meet the time scale required for dispatch, this paper proposes a two-layer distributed framework. Each VM can operate independently or interactively after partitioning. This approach transforms the global optimization of the active distribution network into coordinated optimization among NMGs the power balancing within VM. By allowing each VM to make independent decisions, the operational complexity of the active distribution network is reduced, and solution efficiency is enhanced.

The upper-layer distributed optimization model is based on day-ahead forecast data, with a dispatch period set to 4 hours and a 15-minute time scale for rolling optimization of the NMGs. This time scale aligns with the intra-day dispatch period of the power grid. The lower-layer focuses on real-time power balancing within the VM. Given the real-time fluctuations in PV output, which may cause discrepancies between the power of the VM and the dispatch instructions, real-time smoothing of PV power is necessary to track the intra-day dispatch instructions. Fig. 4 illustrates the temporal relationship between these two layers.

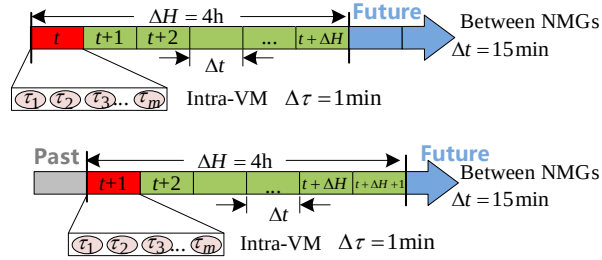


Fig. 4. Time relationship between different layer

3.1. Intra-day distributed optimal dispatch among NMGs

3.1.1. NMGs intra-day distributed optimal dispatch model

Whether the NMGs operate independently or interactively, the goal of the optimal dispatch is to minimize the operating costs within each VM. Given the uncertainty of PV power, the optimal dispatch model is presented as follows.

(1) Objective function

Each VM aims to minimize the operation cost, shown as follows:

$$\min \sum_{t \in T} (C_{i,t}^{PV} + C_{i,t}^{ESS} + C_{i,t}^L + C_{i,t}^{MGT} + C_{i,t}^{EX} + C_{i,t}^{GD} + C_{i,t}^P) \quad (14-a)$$

$$C_{i,t}^{PV} = \sum_{r \in N_i} \left(c_{r,t}^{PV,b} \int_0^{P_r^{PV}(t)} (P_r^{PV} - P_r^{PV,A}) f(P_r^{PV,A}) dP_r^{PV,A} - c_{r,t}^{PV,s} \int_{P_r^{PV}}^{P_r^{PV,max}} (P_r^{PV,A} - P_r^{PV}) f(P_r^{PV,A}) dP_r^{PV,A} \right) \quad (14-b)$$

$$C_{i,t}^{ESS} = \sum_{a \in N_i} c_{a,t}^{ESS} (P_{a,t}^{ESS,d} + P_{a,t}^{ESS,c}) \quad (14-c)$$

$$C_{i,t}^L = \sum_{b \in N_i} c_{b,t}^L |\Delta P_{b,t}^L| \quad (14-d)$$

$$C_{i,t}^{MGT} = \sum_{c \in N_i} c_{c,t}^{MGT} P_{c,t}^{MGT} \quad (14-d)$$

Where P_r^{PV} represents the dispatch power of the r th PV. $P_r^{PV,A}$ represents the actual power of the r th PV. $f(P_r^{PV,A})$ represents the PDF of the actual power of the r th PV at time t . $P_r^{PV,max}$ is the capacity of the r th PV. The stochastic cost of PV $C_{i,t}^{PV}$ includes both the power purchase cost resulting from overestimation, occurring when the actual power is lower than the dispatched power, and the PV curtailment penalty resulting from underestimation when the actual power

exceeds the dispatched power. $P_{a,t}^{\text{ESS,d}}$ and $P_{a,t}^{\text{ESS,c}}$ represent the discharge and charge power of a th ESS at time t , respectively. $\Delta P_{b,t}^{\text{L}}$ represents the adjustment power of the b th FL at time t . $P_{c,t}^{\text{MGT}}$ represents the power of the c th MGT at time t . $c_{r,t}^{\text{PV,b}}$, $c_{r,t}^{\text{PV,s}}$, $c_{a,t}^{\text{ESS}}$, $c_{b,t}^{\text{L}}$, and $c_{c,t}^{\text{MGT}}$ represents the cost coefficient of multi-type flexible resources at time t .

The VM which is connected to the upper grid has interactive power with the grid, the cost is shown as follows.

$$C_{i,t}^{\text{GD}} = c_{i,t}^{\text{GD}} P_{i,t}^{\text{GD}} \quad (14\text{-e})$$

Where $P_{i,t}^{\text{GD}}$ represents the interactive power with the upper grid of the i th VM. $c_{i,t}^{\text{GD}}$ represents the cost coefficients of interactive power with the upper grid by the i th VM. To reduce or avoid the influence on the upper grid of the reverse power flow, the active distribution network is punished when transporting power to the upper grid.

There is interactive power among NMGs, the cost is shown as follows.

$$C_{i,t}^{\text{EX}} = \left(\sum_{j \in N, i \neq j} c_{ij,t}^{\text{EX}} P_{ij,t}^{\text{EX}} \Delta t \right) \quad (14\text{-f})$$

Where $c_{ij,t}^{\text{EX}}$ represent the cost coefficient of interactive power between NMGs. $P_{ij,t}^{\text{EX}}$ represents the interactive power between the i th VM and j th VM. When VM operates independently, this cost does not exist within the objective.

(2) Constraints

(a) Power flow constraints based on *Distflow*:

$$P_i^{\text{GD}} + P_i^{\text{EX}} + \sum_{r \in N_i} M_{i,r}^{\text{PV}} P_{r,t}^{\text{PV}} + \sum_{a \in N_i} M_{i,a}^{\text{ESS}} (P_{a,t}^{\text{ESS,d}} - P_{a,t}^{\text{ESS,c}}) + \sum_{c \in N_i} M_{i,c}^{\text{MGT}} P_{c,t}^{\text{MGT}} - \left(P_{b,t}^{\text{L}} - \sum_{b \in N_i} M_{i,b}^{\text{L}} P_{b,t}^{\text{FL}} \right) = \sum_{j,k \in N_i} P_{j,k,t}^{\text{LN}} - \sum_{i,j \in N_i} (P_{i,j,t}^{\text{LN}} - P_{i,j,t}^{\text{LS}}) \quad (15\text{-a})$$

$$Q_i^{\text{GD}} + Q_i^{\text{EX}} + \sum_{r \in N_i} M_{i,r}^{\text{PV}} Q_{r,t}^{\text{PV}} + \sum_{a \in N_i} M_{i,a}^{\text{ESS}} Q_{a,t}^{\text{ESS}} + \sum_{c \in N_i} M_{i,c}^{\text{MGT}} Q_{c,t}^{\text{MGT}} - \left(Q_{b,t}^{\text{L}} - \sum_{b \in N_i} M_{i,b}^{\text{L}} Q_{b,t}^{\text{FL}} \right) = \sum_{j,k \in N_i} Q_{j,k,t}^{\text{LN}} - \sum_{i,j \in N_i} (Q_{i,j,t}^{\text{LN}} - Q_{i,j,t}^{\text{LS}}) \quad (15\text{-b})$$

$$V_{i,t}^2 - V_{j,t}^2 = 2(R_{i,j} P_{i,j,t}^{\text{LN}} + X_{i,j} Q_{i,j,t}^{\text{LN}}) - (R_{i,j}^2 + X_{i,j}^2) ((P_{i,j,t}^{\text{LN},2} + Q_{i,j,t}^{\text{LN},2}) / V_{i,t}^2) \quad (15\text{-c})$$

$$V^{\min} \leq V_{i,t} \leq V^{\max} \quad (15\text{-d})$$

$$P_{i,j,t}^{\text{LS}} = R_{i,j} (P_{i,j,t}^{\text{LN},2} + Q_{i,j,t}^{\text{LN},2}) / V_{i,t}^2 \quad (15\text{-e})$$

$$Q_{i,j,t}^{\text{LS}} = X_{i,j} (P_{i,j,t}^{\text{LN},2} + Q_{i,j,t}^{\text{LN},2}) / V_{i,t}^2 \quad (15\text{-f})$$

Where $M_{i,r}^{\text{PV}}$, $M_{i,a}^{\text{ESS}}$, $M_{i,b}^{\text{FL}}$ and $M_{i,c}^{\text{MGT}}$ are incidence matrices of PV, ESS, FL, and MGT, respectively, e.g., $M_{i,r}^{\text{PV}}$ is 1 if PV r is located in node i and otherwise, it is 0. $V_{i,t}$ is the node voltage. $P_{i,j,t}^{\text{LN}} + jQ_{i,j,t}^{\text{LN}}$ is the power on line (i, j) with the impedance $R_{i,j} + jX_{i,j}$, $P_{i,j,t}^{\text{LS}} + jQ_{i,j,t}^{\text{LS}}$ is the power loss.

Eqs. (15-a) and (15-b) represent the active and reactive power balance at nodes, where the active/reactive power at a node equals the difference between power flowing into and flowing out the node. Eq. (15-c) describes the voltage drop between nodes, Eq.(15-d) represents the voltage constraints of the distribution network, while Eq.(15-e) and (15-f) represent the losses, which are a nonlinear term in the model. Typically, these losses are linearized using second-order cone relaxation to simplify the calculations.

This paper does not optimize the reactive power. In the power flow, the related reactive power is handled according to the following rules: The reactive power regulation capacity of PV and ESS meets the converter requirements^{[33]-[34]}. The reactive power regulation capacity of VMT meets the operation constraints. Loads are considered to operate with a constant power factor, where $\cos \varphi = 0.95$.

(b) Linearize the stochastic cost of PV

This paper employs the piecewise linearization method to address the integral part of the stochastic cost of PV. Firstly, 1000 points are evenly selected within the range of $[0, P_r^{\text{PV,max}}]$. Then the corresponding stochastic cost $C_{r,t}^{\text{RN,rd}}$ of each point is calculated using numerical integration. The stochastic cost function of different segments can be achieved through piecewise linear fitting, as shown in the following.

$$C_{r,t}^{\text{RN,rd}} = C_{r,t}^{\text{RN,rd,pl(start)}} + \sum_l K_{l,r,t}^{\text{RN}} \delta_{l,r,t}^{\text{RN}}, \forall t \quad (16\text{-a})$$

$$P_{r,t}^{PV} = P_{r,t}^{PV,pl(start)} + \sum_l \delta_{l,r,t}^{RN}, \forall t \quad (16-b)$$

$$I_{l,r,t}^{RN} - I_{l-1,r,t}^{RN} \leq 0, \forall t, l \quad (16-c)$$

$$0 \leq \delta_{l,r,t}^{RN} \leq \delta_{l,r,t}^{RN,max} I_{l,r,t}^{RN}, \forall t, l \quad (16-d)$$

$$\delta_{l-1,r,t}^{RN} \geq \delta_{l-1,r,t}^{RN,max} I_{l-1,r,t}^{RN}, \forall t, l \quad (16-e)$$

Where $I_{l,r,t}^{RN}$ is the auxiliary 0-1 variable of segment l , and $I_{l',r,t}^{RN} = 1 (l' = 1, \dots, l)$ when $P_r^{PV}(t)$ locates in the range of $[P_r^{PV,pl(l)}(t), P_r^{PV,pl(l+1)}(t)]$. $\delta_{l,r,t}^{RN,max}$ is the length of different segments, which satisfying $\delta_{l,r,t}^{RN,max} = P_r^{PV,pl(l+1)} - P_r^{PV,pl(l)}$. $(P_r^{PV,pl(start)}, C_{r,t}^{RN,rd,pl(start)})$ is the starting point of segment $l=1$. $K_{l,r,t}^{RN}$ is the slope of segment l , which satisfying $K_{l,r,t}^{RN} = (C_{r,t}^{RN,rd,pl(l+1)} - C_{r,t}^{RN,rd,pl(l)}) / (P_r^{PV,pl(l+1)} - P_r^{PV,pl(l)})$.

(c) Operation constraints of flexible resources

$$\bar{P}_{a,t}^{ESS,d} \leq P_{a,t}^{ESS} \leq \bar{P}_{a,t}^{ESS,c} \quad (17-a)$$

$$-\Delta \bar{P}_{b,t}^L \leq \Delta P_{b,t}^L \leq \Delta \bar{P}_{b,t}^L \quad (17-b)$$

$$P_{c,t}^{MGT} \leq P_{c,t}^{MGT} \leq \bar{P}_{c,t}^{MGT} \quad (17-c)$$

(d) Constraints on interactive power between grid and VM

$$-\bar{P}_{i,t}^{GD} \delta_{i,t}^{GD} \leq P_{i,t}^{GD} \leq \bar{P}_{i,t}^{GD} \delta_{i,t}^{GD} \quad (18)$$

Where $\bar{P}_i^{GD}$ and $\underline{P}_i^{GD}$ represent the maximum and minimum power that i th VM can interact with the upper grid, respectively, which are influenced by the tie-line capacity and maximum regulation capacity of the VM. $\delta_{i,t}^{GD}$ represents the connection state of the i th VM with the upper grid at time t .

(e) Constraints on interactive power between NMGs

$$\zeta_{ij,t} P_{ij,t}^{EX} \leq P_{ij,t}^{EX} \leq \zeta_{ij,t} \bar{P}_{ij,t}^{EX}, \forall i \in I, \forall t \in H \quad (19)$$

Where $\zeta_{ij,t} \in \{0,1\}$ represent the connection state between the i th MG and j th MG at time t ; $\underline{P}_{ij}^{EX}$ and $\bar{P}_{ij}^{EX}$ represent the maximum and minimum interactive power between NMGs, which are influenced by the tie-line capacity and maximum regulation capacity of the MG. When MG operates independently, there is no interactive power among NMGs.

3.1.2. SADMM algorithm for NMGs

NMGs can directly solve the optimal model when operates independently. However, when NMGs interact, each VM focuses on its own optimization, but VMs are interconnected, and the power flow on the tie-line impacts the optimization results of each VM. To deal with the influence of power flow on the tie-line between the two interacting VMs, decoupling the NMGs is necessary to address the electric connections among them, as illustrated in Fig. 5.

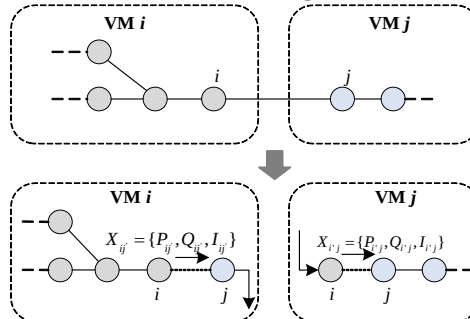


Fig. 5 Decoupling among NMGs

The boundary node i in the i th VM is mirrored as i' in j th VM, and likewise, node j in the j th VM is mirrored as j' in the i th VM. Given the power flow direction, the mirrored node i' in the j th VM is defined as the source node, while the mirrored node j' in the i th VM is defined as the load node. Each VM contains the tie-line ij and the corresponding power flow, allowing each VM to solve the optimization independently and comprehensively.

Due to the independent solutions of each VM, the power flow at same tie-line in different VMs may achieve different results. During decision-making processes within VMs, the value $X_{ij} = \{P_{ij}, Q_{ij}, I_{ij}\}$ of the virtual tie-line ij'

can be calculated through power flow within the i th VM, as well as $X_{i'j} = \{P_{i'j}, Q_{i'j}, I_{i'j}\}$ of the virtual tie-line $i'j$ in the j th VM. The iterative solution ensures that the state variables of the tie - line on both sides converge to the same value.

For the i th VM, the reference tie-line variables at the $m+1$ th iteration are defined as the average tie-line variables between the i th VM and its adjacent j th VM at the m th iteration.

$$X_{ij}^{m+1,ref} = X_{ij}^{m+1,ref} = \frac{X_{ij}^m + X_{ij'}^m}{2} \quad (20)$$

Where $X_{ij}^{m+1,ref}$ and $X_{ij'}^{m+1,ref}$ represent the reference virtual tie-line variables of the i th VM and the j th VM at the $m+1$ th iterations, respectively. X_{ij}^m and $X_{ij'}^m$ represent the virtual tie-line variables of the i th VM and the j th VM at the m th iteration, respectively.

The Lagrangian Multiplier incorporates the difference between the virtual tie-line variables and their references in each iteration. This difference can be interpreted as a penalty coefficient and is used in penalty cost formulation. The penalty cost is added into the objective formulation in Eq.(14-a) to influence the optimization result. The *lagrangian multiplier* of each VM can be updated by:

$$\Omega_{ij}^{m+1} = \Omega_{ij}^m + \left\| \left(X_{ij}^m - X_{ij}^{m,ref} \right) \right\|_2^2 \quad (21-a)$$

$$\Omega_{ij'}^{m+1} = \Omega_{ij'}^m + \left\| \left(X_{ij'}^m - X_{ij'}^{m,ref} \right) \right\|_2^2 \quad (21-b)$$

Where Ω_{ij}^m and $\Omega_{ij'}^m$ represent the set of *lagrangian multipliers* of the virtual tie-line ij' and $i'j$ at the m th iteration, respectively. where $\Omega_{ij}^m = \{\lambda_{ij}^P, \lambda_{ij}^Q, \lambda_{ij}^I\}$, $\Omega_{ij'}^m = \{\lambda_{ij'}^P, \lambda_{ij'}^Q, \lambda_{ij'}^I\}$.

The virtual tie-line consistency penalty cost between the i th VM and the j th VM is introduced to ensure consistency in mirrored virtual tie-line, which is described as follows:

$$C_{i,t}^P = \sum_{j \in N_i, j \neq i} \sum_{\lambda_{ij} \in \Omega_{ij}} \left(\lambda_{ij}^m \left(X_{ij,t}^m - X_{ij,t}^{m,ref} \right) + \frac{\rho}{2} \left\| X_{ij,t}^m - X_{ij,t}^{m,ref} \right\|_2^2 \right) \quad (22)$$

Where ρ denotes the iteration step size.

The convergence criterion includes raw residual, representing the difference in tie-line variables calculated between both side VMs, and the dual residual, representing the difference in reference tie-line variables across adjacent iterations. The two convergence criteria are defined as follows:

$$r_{ij}^m = r_{ij}^m = \left\| X_{ij}^m - X_{ij}^m \right\|_2^2 \leq \varepsilon \quad (23-a)$$

$$s_{ij'}^m = s_{ij'}^m = \left\| X_{ij'}^{m,ref} - X_{ij'}^{m-1,ref} \right\|_2^2 \leq \varepsilon \quad (23-b)$$

Where r_{ij}^m , r_{ij}^m , $s_{ij'}^m$, and $s_{ij'}^m$ represent the raw residual and dual residual of the i th VM and the j th VM at the m th iteration, respectively. ε is the tolerance level.

All nonlinear components in this paper are converted into linear ones, ensuring that the optimization model remains convex and guaranteeing convergence.

3.2. Real-time power tracking within VM

During the real-time stage, each VM should track the intra-day dispatch instructions and achieve real-time power balance within the VM, while accounting for the real-time PV power fluctuation among Δt . In this stage, the real-time adjustment of FL and MGT is a challenge to achieve, therefore, their real-time adjustment capability is not taken into consideration. Additionally, ESS is considered adjustable within constraints.

The objective of the i th VM in Δt is to minimize the unbalanced power when PV power fluctuations.

$$\min \Delta P_\tau = \sum_{r \in N_i} \left(P_{r,\tau}^{PV,real} + \Delta P_{r,\tau}^{PV} \right) + \sum_{a \in N_i} \left(P_{a,\tau}^{ESS} + \Delta P_{a,\tau}^{ESS} \right) + \sum_{b \in N_i} P_{r,\tau}^L + \sum_{c \in N_i} P_{r,\tau}^{MGT} + \sum_{i \in I} \left(P_{i,t}^{GD} + P_{i,t}^{EX} \right), \forall \tau \in \Delta t \quad (24)$$

Where $P_{r,\tau}^{PV,real}$ represents the actual power of PV, while τ is the time within Δt .

Constraints include operational constraints of multi-type flexible resources, shown as follows:

$$0 \leq \Delta P_{r,\tau}^{PV} \leq P_{r,\tau}^{PV,real} \quad (25-a)$$

$$\bar{P}_{a,\tau}^{ESS,d} \leq P_{a,\tau}^{ESS} + \Delta P_{a,\tau}^{ESS} \leq \bar{P}_{a,\tau}^{ESS,c} \quad (25-b)$$

$$P_{b,\tau}^L = P_{b,t}^L, P_{c,\tau}^{\text{MGT}} = P_{c,t}^{\text{MGT}}, P_{i,\tau}^{\text{EX}} = P_{i,t}^{\text{EX}}, P_{i,\tau}^{\text{GD}} = P_{i,t}^{\text{GD}}, \forall \tau \in \Delta t \quad (25\text{-c})$$

Where $P_{r,\tau}^{\text{PV,real}}$ represent the MPPT power of PV.

This paper adopts the consensus algorithm as described in [35]. The consensus algorithm achieves global consistency through limited information exchange among distributed resources. The communication topology is established based on the graph theory. Taking $\sigma_* = c_*(\Delta P_* / \Delta P_*^{\text{max}})$ as the consensus index, PV and ESS within the VM are iterated according to the Eq. (26).

$$\sigma_{k+1,m}^{\text{PV}} = \sigma_{k,m}^{\text{PV}} + \sum_{n \in N_i} d_{m,n}^{\text{PV}} (\sigma_{k,n}^{\text{PV}} - \sigma_{k,m}^{\text{PV}}) + \varepsilon \Delta P_k, \{m, n \in N_i\} \quad (26\text{-a})$$

$$\sigma_{k+1,m}^{\text{ESS}} = \sigma_{k,m}^{\text{ESS}} + \sum_{n \in N_i} d_{m,n}^{\text{ESS}} (\sigma_{k,n}^{\text{ESS}} - \sigma_{k,m}^{\text{ESS}}) + \varepsilon \Delta P_k, \{m, n \in N_i\} \quad (27\text{-b})$$

$$\Delta P_{k+1} = \Delta P_\tau - \sum_{r \in N_i} \Delta P_{k+1,r}^{\text{PV}} - \sum_{a \in N_i} \Delta P_{k+1,a}^{\text{ESS}} \quad (27)$$

Where ΔP_k represents the unbalanced power in the k th iteration. ΔP_{k+1}^* represents the adjustment of multi-type flexible resources. $d_{m,n}^*$ represent the communication adjacent matrix of different types of flexible resources. ΔP_τ represent the unbalanced power resulting from PV fluctuation at time τ .

The solution process of the double-layer distributed optimization is shown as:

1	for t=1: T _{max}
	upper-layer
2	for iter=1: Iter _{max}
3	for i=1: I
4	each VM solves the intra-day dispatch model independently and saves the virtual tie-line variables. Eqs. (15)-(19).
5	end
6	if qualify formula (23); break ;
7	else update Eq. (20) and Eq. (21);
8	end
9	end
	lower-layer
10	Reserve dispatch power of flexible resources, and interactive power between VMs and outer among ΔH .
11	for $\tau=1$: τ_{max} (among Δt)
12	for k=1: K
13	Calculate the consensus index σ_* of each flexible resource.
14	Update $k+1$ th consensus index of each flexible resource using Eqs. (25).
15	if the difference of Eq. (27) meets the convergence criterion; break;
16	end
17	end
18	Update the operation state of flexible resource, $t+1=\tau_{\text{max}}$
19	end

4. Case studies

The effectiveness of the proposed method is investigated based on the 141-bus active distribution network [36]. The voltage of the active distribution network is 10kV. The total load is 10.6MW, with half of the load nodes containing flexible loads. The configuration of PV, ESS, and MGT are shown in Tab.2.

Tab.2 Configuration of PV, ESS, and MGT

Type	Node	Capacity
PV	[4,17,27,34,39,48,56,57,65-68,77,96,98,99,106-107,113,114,116,126,132,133-134]	0.3MW
	[8,12,20,21,32,51,52,64,66,78,81,87,95,100,119,123,129,130,139,141]	0.4MW
ESS	[24,48,54,119]	0.5MW/1MWh
	[96,126]	0.4MW/1MWh

The Time-of-use (TOU) electricity price is determined according to electricity prices for general industrial and commercial sales in Beijing [37]. The TOU electricity price curve and response willingness are shown in Fig. 6(a). Combined with equations (7), the response willingness closely follows the shape of the TOU price curve. When the TOU price is high, the response willingness of FL increase accordingly, conversely, the willingness decreases when the TOU price is low.

Day-ahead forecast data, intra-day forecast data, and actual power data of PV are presented in Fig.6(b). The time scale for day-ahead and intraday forecast data is 15 minutes, while real-time data is recorded at 1-minute intervals. In this paper, the prediction error of the load is not considered, the actual load power is assumed to match the predicted value. Simulations are conducted using MATLAB 2020a environment with the GUROBI solver.

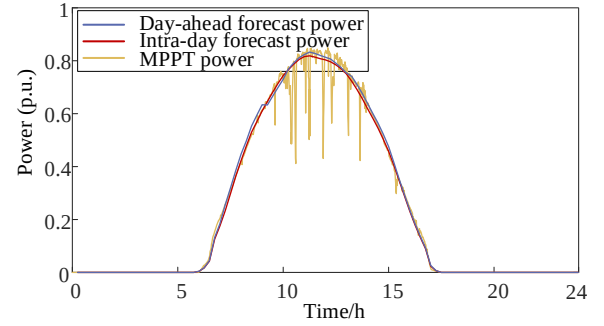
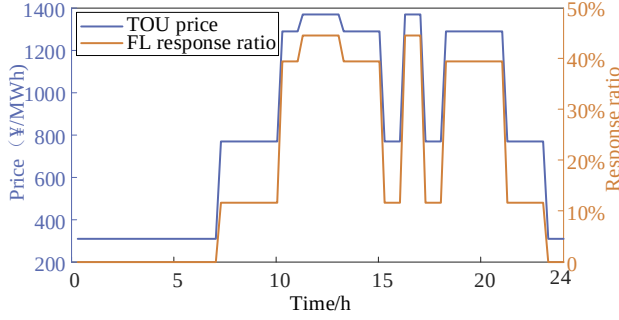


Fig. 6(a) TOU electricity price and response willingness of FL

Fig. 6(b) PV forecast and output power

4.1. Results and analysis of NP

The active distribution network is divided into NMGs based on the NP method introduced in section 2. The genetic algorithm is employed to solve the NP model. The population size is set to 50, with a maximum of 200 iterations, and an elite reservation method is applied to ensure algorithm convergence.

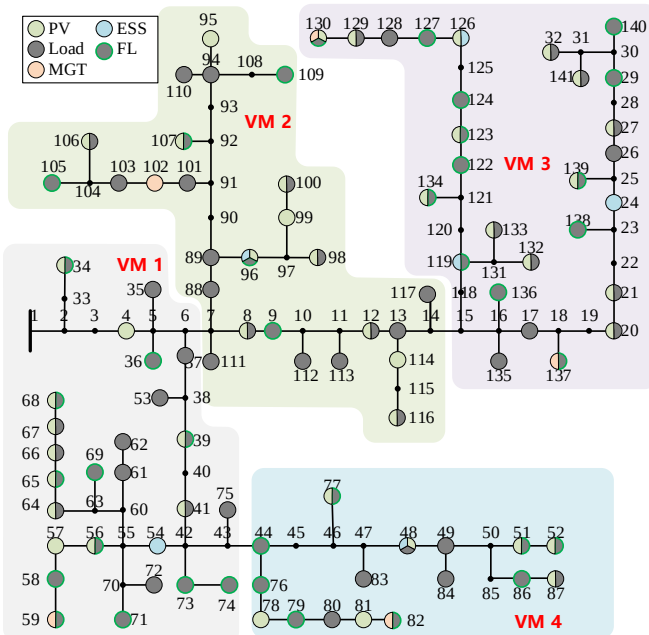
Three NP methods are employed to compare and validate the effectiveness of the proposed method in this paper. Schemes 2 and 3 employs the same solution method. However, the objective of Scheme 2 only considers modularity. The topology and number of VMs after partitioning are determined by the NP method.

Scheme 1: no NP.

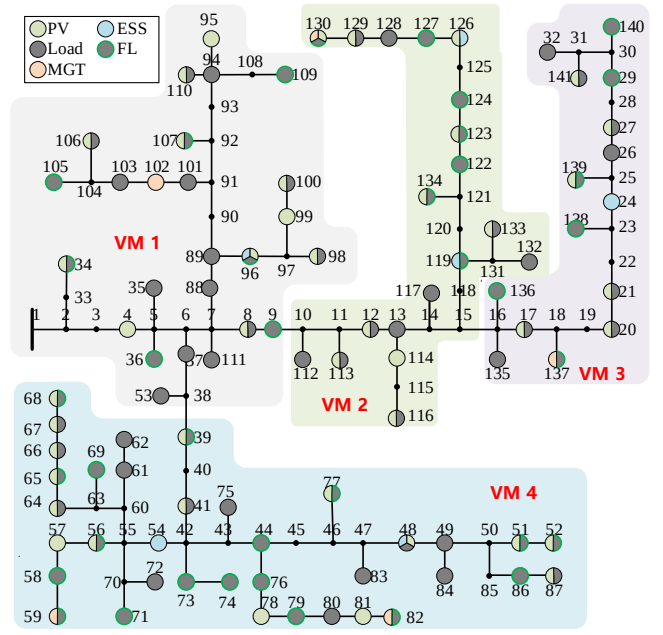
Scheme 2: NP only considers modularity.

Scheme 3: proposed NP method.

The NP results for schemes 2 and 3 are illustrated in Fig. 7. Modularity and robust flexibility metrics under three schemes are presented in Tab. 3. Fig. 8 depicts the robust flexibility intervals for schemes 2 and 3.



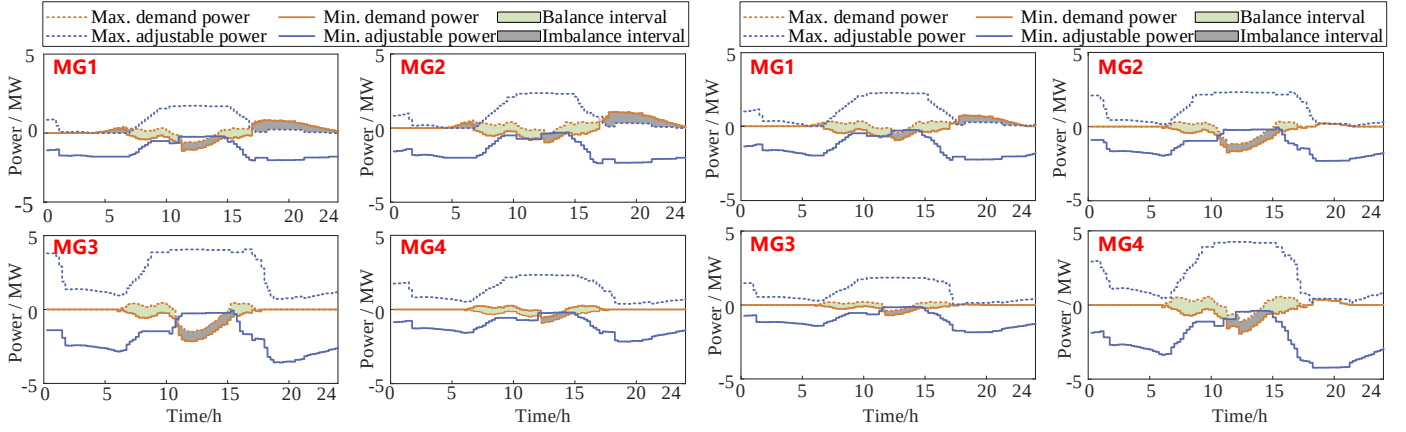
(a) NP result for Scheme 2



(b) NP result for Scheme 3

Fig. 7 NP results for different Schemes
Tab. 3 Modularity and robust flexibility metrics

	Modularity	Robust flexibility				
		VM1	VM2	VM3	VM4	Average
Scheme 1	/			0.82		
Scheme 2	0.74	0.43	0.74	0.43	0.74	0.43
Scheme 3	0.67	0.66	0.67	0.66	0.67	0.66



(a) Robust flexibility intervals for Scheme 2

(b) Robust flexibility intervals for Scheme 3

Fig. 8 Robust flexibility intervals

Tab. 3 reveals that Scheme 2 exhibits better structural performance, because Scheme 2 only focus on modularity. However, the robust flexibility of individual VMs varies significantly. As illustrated in Fig.8(a), VM1 and VM2 experience power unbalances at most time, whereas other VMs maintain surplus adjustable capacity at the same time. This discrepancy indicates a mismatch between the adjustment capacity of each VM and its power demand.

In Scheme 3, the robust flexibility metric for each VM is more evenly distributed. As depicted in Fig. 8(b), the unbalanced intervals of VMs are notably reduced. Scheme 3 ensures a more equitable distribution of adjustable and demand power among NMGs while maintaining satisfactory modularity. Consequently, the proposed NP method achieves a balanced allocation of flexible regulation ability and demand, along with satisfactory modularity.

4.2. Results and analysis of VM independent operation

Using the NP method described in Schemes 2 and 3, independent operation simulations were conducted for each VM to assess their robustness in island scenarios. During the simulations, VM1 remains connected to the upper grid, while the tie-lines between VM2, VM3, and VM4 are all disconnected. The cost coefficient of the ESS is set at 150¥/MWh, while the compensation cost coefficient for FL is set at 100¥/MWh lower than the TOU price. The cost coefficient of MGT is set at 690¥/MWh. The purchase and sale cost coefficient between VMs and between VMs and the upper grid are all set equal to the TOU price. Additionally, to enhance the utilization of distributed PV, the purchase coefficient of PV is set to TOU price, while the PV curtailment penalty coefficient is higher than TOU price.

The optimal results of Schemes 2 and 3 are illustrated in Fig. 9, with the total energy for both schemes shown in Tab. 4, and the operating costs presented in Tab.5.

Tab. 4 Total load in two schemes

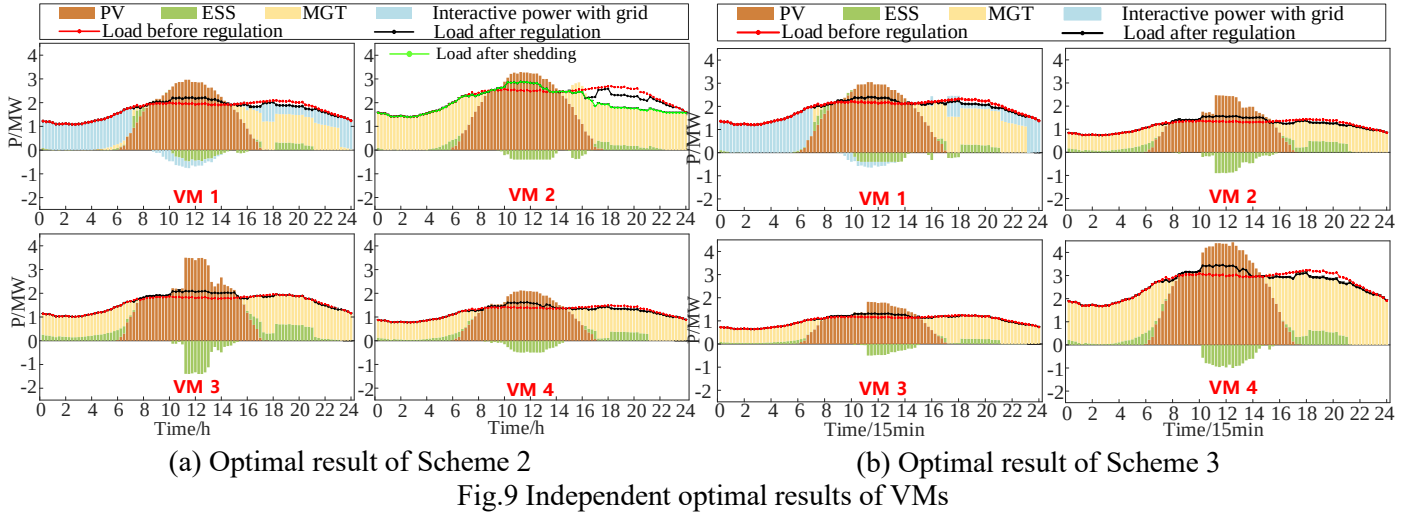
	Load before regulation/MWh	Flexible load regulation/MWh	Shed load/MWh
Scheme2	162.47	7.93	2.45
Scheme3		7.61	0

Tab. 5 Independent operate costs of VMs

	Scheme2				Scheme3			
	VM1	VM2	VM3	VM4	VM1	VM2	VM3	VM4
PV /¥	1557.4	1918.1	7765.7	1230.5	1566.7	4437.3	2675.2	3039.8
ESS /¥	432.8	517.3	1198.4	605.7	533.8	822.6	402.3	1048.3
FL /¥	2240.3	3189.2	1778.5	1582.1	1811.2	2108.9	886.1	3322.1
MGT /¥	6377.6	17953	11565	10062	7981.1	8650.9	8773.0	22766.0

Grid and VM /¥	7852.2	0	0	0	6707.6	0	0	0
Total /¥		77825.8				77532.9		

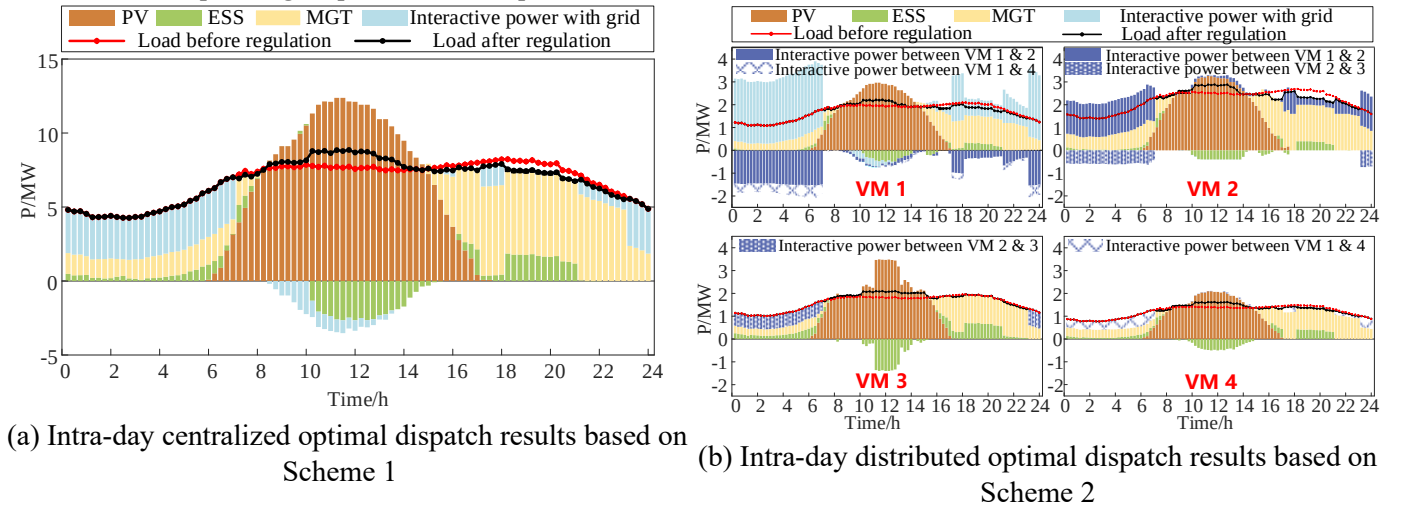
As shown in Tab.5, the total cost difference between the two schemes is minimal. However, in Scheme 2, a source-load unbalance and load shedding occur after 5pm due to the decrease in PV power generation and the increase in load for VM2. By comparing the total load before and after regulation, as well as the load after shedding, it becomes clear that 7.93 MWh of the load participates in demand response, while 2.45 MWh must be shed due to insufficient power supply. According to Fig. 8(a), the robust flexibility of VM2 during this period is nearly zero, indicating that VM2 lacks the capacity for flexible adjustment. In contrast, in Scheme 3, 7.61 MWh of the load participates in demand response, and no load shedding occurs, as each VM is able to meet load demands and maintain a power-load balance. This demonstrates that the proposed NP method exhibits strong robustness in extreme scenarios, ensuring that each VM can operate safely in islanded conditions.

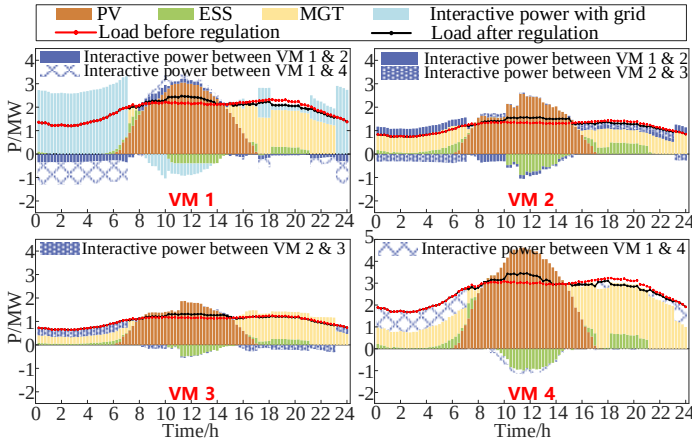


4.3. Results and analysis of Intra-day distributed optimal dispatch for NMGs

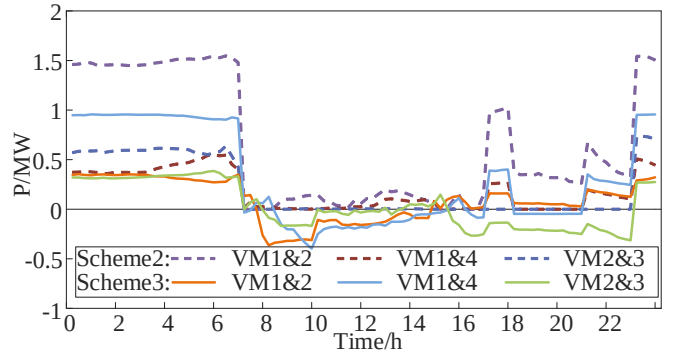
After NP, NMGs can interact with each other. A centralized optimal simulation is conducted based on Scheme 1, while distributed interactive optimal operation simulations of NMGs are conducted based on Schemes 2 and 3. During the simulation, the cost coefficient between VMs and between VMs and the upper grid is set to the TOU price, while other cost coefficients remain the same as in independent operation.

The intra-day distributed optimal dispatch results of NMGs obtained based on three schemes are illustrated in Fig. 10, and the corresponding dispatch costs are presented in Tab. 6.





(c) Intra-day distributed optimal dispatch results based on Scheme 3



(d) Interactive power between VM

Fig. 10 Intra-day dispatch results

At noon, the PV power exceeds the load in each VM in both Schemes 2 and 3. In Scheme 2, while the excess power from VM1 is transmitted to the upper grid, it is also routed to other connected VMs, such as VM2 and VM4. However, due to variations in flexibility among these VMs, PV power curtailment occurs as a result of the surplus power from other VMs. Conversely, in Scheme 3, VM1 exhibits better flexibility, allowing excess power from other VMs to be transmitted to VM1 and then to the upper grid, thereby reducing total PV power curtailment. During other periods, each VM can dispatch flexibility resources within its own and coordinate interactive power between VMs and between VM and the upper grid based on the regulation cost and purchase/sale cost. Comparing the interactive power between VMs in Fig 10(d), the interactive power of Scheme 3 is noticeably less than that in Scheme 2, indicating that NMGs in Scheme 3 possess higher flexibility and rely less on external sources.

As the control of the active distribution network shifts from centralized global optimization to each VM's individual optimum after NP, the total operation cost of the active distribution network may not be superior to centralized control. Comparing the total cost among the three schemes in Tab.6, the difference between the cost of Scheme 1 and Scheme 3 is approximately 3.1%, while the difference with Scheme 2 is about 6.4%. The distributed optimal dispatch cost of Scheme 3 is 3.3% less than that in Scheme 2.

Comparing the independent operation costs in Tab.5 with the interactive operation costs in Tab.6 for Schemes 3, a notable reduction of 5.3% in total costs is observed when NMGs operate interactively. This reduction is attributed to the inherent flexibility of each VM in addressing power imbalances by comparing adjustment costs within and outside the VM during interactive operation, thereby reducing the total operating cost of the active distribution network. Thus, NMGs can operate interactively under normal conditions to reduce the costs, while operate independently during emergencies or faults to ensure the safety of operations within the VM.

4.4. Results and analysis of Real-time power tracking within VM

Actual PV output data with a time scale of 1 minute is utilized to reflect real-time power fluctuations. Real-time distributed optimization simulations within VMs are carried out based on the three schemes, allowing for a comparison of the real-time power adjustment capacity of each scheme.

Fig. 11 focuses on Scheme 3 to observe the operation of each flexible resource within the VM under real-time power fluctuations.

From Fig. 11, it is evident that when the PV is underestimated, the PV tracks the intra-day dispatch instruction. When PV is overestimated, the VM compensates for the PV power shortage by coordinating the output of ESSs and MGTs to meet dispatch instructions, thereby minimizing tie-line power difference. Considering the potential damages due to frequent changes in charging and discharging cycles of electrochemical energy storage caused by the frequent PV fluctuation, hybrid energy storage, which consists of energy-type and power-type energy storage, can be employed to mitigate PV power fluctuations ^[38].

Fig.12 illustrates the MPPT power and actual power of NMGs under Schemes 2 and 3 in independent and interactive operation. When NMGs operate independently, the utilization rate of the PV in Scheme 2 is 89.8%, while in Scheme 3 it reaches 91.2%. However, when NMGs operate interactively, the utilization rate of PV in Scheme 2 increases slightly to 90%, whereas in Scheme 3, it significantly rises to 94.2%. Scheme 3 exhibits a marked increase of 4.2% in PV utilization rate when operate interactively compared to independent operation. Interactive operation based on the NMGs partitioned by the proposed method can significantly enhance the PV utilization rate compared to independent operation.

Tab. 6 Intra-day distributed optimal dispatch costs

	Scheme 1	Scheme2				Scheme3			
		VM1	VM2	VM3	VM4	VM1	VM2	VM3	VM4
PV /¥	6711.3	1500.6	1992.2	7744.8	1241.9	1535.60	3443.20	2345.30	2357.30
ESS /¥	2905.4	451.4	557.4	1210.5	612.7	479.90	835.80	431.60	1054.00
FL /¥	8327.1	1970.2	3442.7	1778.6	1727.7	2217.00	1952.50	1033.20	3050.70
MGT /¥	37325	8816.3	13131	8289.3	7245.9	8166.00	6976.20	7868.30	17195.00
Grid and VM /¥	16165	14331	0	0	0	12712.00	0.00	0.00	0.00
Between NMGs /¥	/	-8264	5318	1268.1	1677.9	-1423.1	1323.5	-1147.8	1247.4
Total /¥	71433.8	76044.2				73653.6			

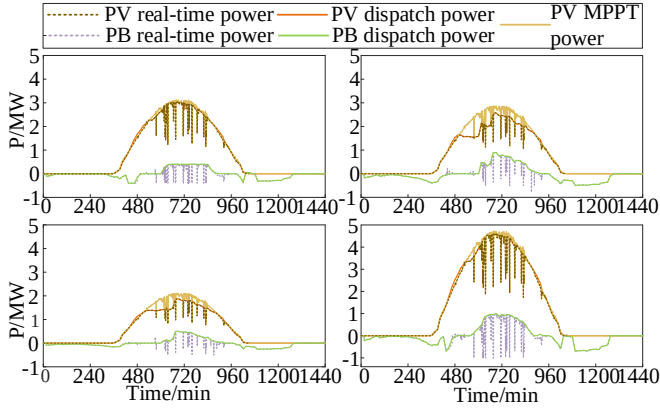


Fig. 11 Real-time distributed optimization results for Scheme3

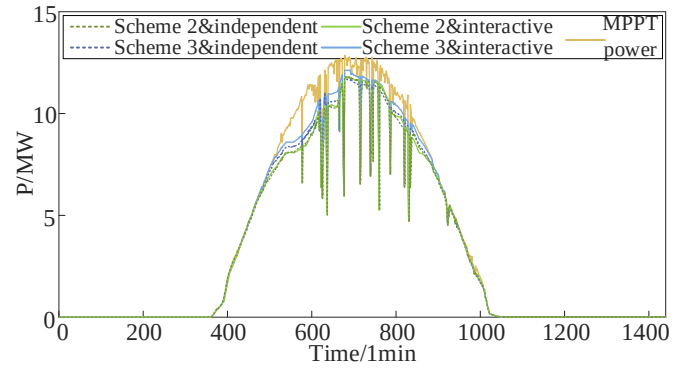


Fig.12 Total PV power under different schemes and modes

PV energy curtailment still occurs. One reason is that the energy storage system has reached its charging limit and can no longer absorb additional energy. Another reason is that ESS prefer to reserve capacity during periods with a low curtailment penalty coefficient to minimize curtailment during the periods when the curtailment penalty coefficient is higher, thereby reducing the overall curtailment cost.

PV power fluctuation may still lead to an imbalance between source and load within the VM due to the limited adjustment ability within VM. Tab. 7 presents the corresponding unbalanced energy for the three schemes.

Tab.7 Unbalance energy

		Unbalance energy/MWh	Total/MWh
Scheme 2	VM1	0.34	1.09
	VM2	0.23	
	VM3	0.19	
	VM4	0.33	
Scheme 3	VM1	0.12	0.34
	VM2	0.13	
	VM3	0.02	
	VM4	0.07	

It can be seen from Tab. 7 that the unbalanced energy of Scheme 3 is much smaller than that of Scheme 2 when the PV output fluctuates. The NMGs divided by Scheme 3 have stronger real-time adjustment capacity.

4.5. Analysis of computation efficiency

The main purpose of the distributed optimization operation is to reduce the computational burden of the NMGs. Therefore, the convergence speed and solution speed of the algorithm are crucial. Fig.13 gives the convergence curves of the intra-day distributed optimal dispatch model.

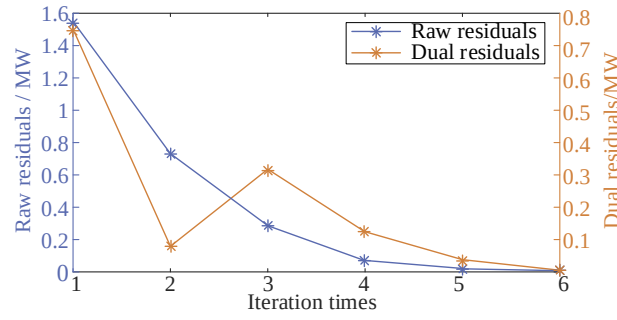


Fig. 13 Converge curve of Raw residual and dual residual

This study uses the Parallel Computing Toolbox in MATLAB, with eight simulation workers. Each VM solves independently. As depicted in Fig. 13, the SADMM algorithm tends to converge after six iterations.

Tab.8 provides the solution times of the intra-day dispatch and real-time power tracking in Schemes 2 and 3.

Fig.8 Solution times in different Schemes

	Intra-day($t=50$)/s	Real-time ($\tau=980$)/s
Scheme1	65	0.016
Scheme3	48	0.0051

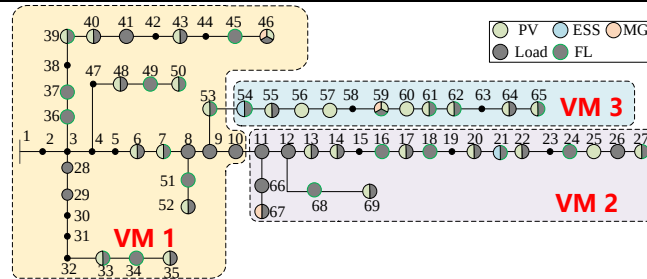
Tab. 8 illustrates that in Scheme 1, where there is no NP, the active distribution network contains numerous decision variables, resulting in slow iterative convergence. Conversely, in Scheme 3, the dimension of the decision variables within each VM is reduced, leading to significantly accelerated convergence. NP substantially reduces optimization computation time, which can meet the requirements for intra-day dispatch and real-time power tracking.

4.6 Application in IEEE-69 bus Distribution Systems

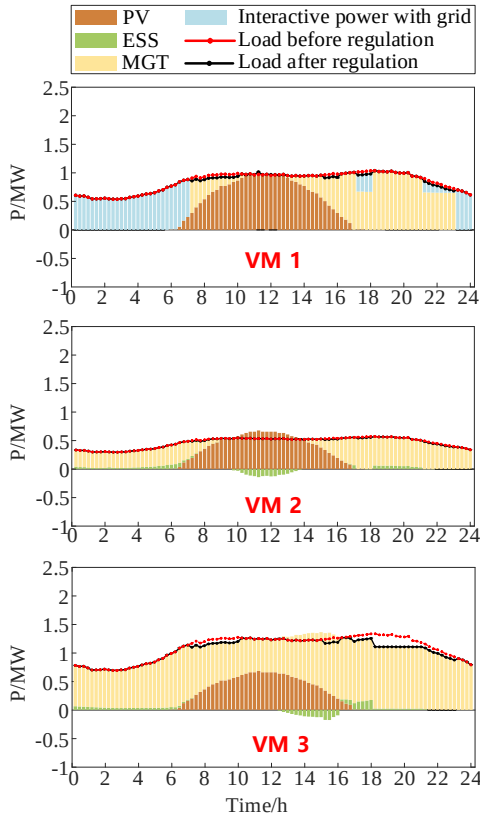
To verify the robustness and applicability of the proposed method across different distribution network, the IEEE-69 bus distribution network is utilized for network partitioning and distribution optimal dispatch. The network partitioning result and the independent and interactive dispatch result within and between VMs are illustrated in Fig. 14.

Tab.9 Operation costs of IEEE-69 bus distribution system

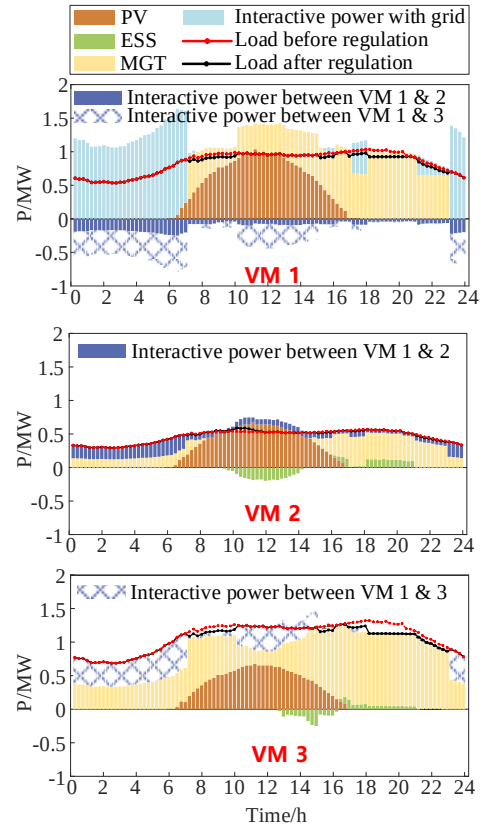
	Independent optimal			Interactive optimal		
	VM1	VM2	VM3	VM1	VM2	VM3
PV /¥	495.08	328.40	329.05	492.11	329.96	328.18
ESS /¥	0	122.61	93.12	0	165.27	61.29
FL /¥	247.75	48.19	516.15	225.50	132.77	408.74
MGT /¥	5267.00	4468.40	14241.00	7190.60	2872.90	10819.00
Grid and VM /¥	1969.40	0	0	3804.80	0.00	0.00
Between NMGS /¥	/	/	/	-4890.60	1562.70	3327.90
Total/¥	28126.15			26831.12		



(a) Network partitioning result of IEEE-69 bus distribution system



(b) Independent optimal results of IEEE-69 bus distribution system



(c) Interactive optimal results of IEEE-69 bus distribution system

Fig.14 Results of IEEE-69 bus distribution system

Fig.14(a) demonstrates that the divided virtual microgrids can maintain power-load balance throughout the day when operating independently. Tab.9 indicates a significant 4.6% reduction in total costs when NMGs operate interactively. The proposed strategy exhibits strong robustness and applicability under diverse conditions.

5. Conclusion

To address the challenges posed by significant uncertainty, low utilization of renewable energy, and control complexities in active distribution network with high penetration of renewable energy, this paper proposes a distributed optimal operation strategy for NMGs based on NP. Considering the robustness interval of renewable output power, a NP model incorporating modularity and robust flexibility is established to partition the active distribution network into NMGs, thereby enhancing the balance capacity between power supply and demand within VM. Furthermore, intra-day dispatch and real-time power tracking models are developed and solved using distributed methods. The flexible resources are dispatched and the interactive power among NMGs is coordinated to reduce operating costs and improve the utilization of renewable energy. Additionally, the flexible resources within VM are coordinated to mitigate power fluctuation and track intra-day dispatch instructions in a real-time power tracking model. Through simulation and comparative analysis, the results demonstrate:

1. The proposed NP method, which incorporates modularity and robust flexibility, effectively partitions the active distribution network into NMGs and ensures a robust power balance capacity within each VM. Simulation results demonstrate that NMGs partitioned by this NP method can achieve power balance between supply and demand, thereby avoiding load shedding.
2. The proposed distributed optimal method based on NP effectively dispatches flexible resources and coordinates interactions between VMs and between VM and the upper grid, resulting in reduced operating costs and increased utilization of renewable energy. Simulation results in this paper indicate that when NMGs operate interactively, the proposed NP method reduces dispatch costs by 3.3% and enhances distributed PV utilization by 4.2% compared to the NP method considering modularity alone.
3. The partitioned NMGs can operate in different modes. During emergencies or faults, NMGs can operate independently to ensure a reliable power supply, while under normal conditions, they can operate interactively to reduce costs and increase the utilization of renewable energy. The simulation results presented in this paper verify the

effectiveness of independent operation, and demonstrate a 5.3% reduction in operating costs and a 3% increase in renewable energy utilization during interactive operation compared to independent operation.

4. The proposed two-layer distributed optimal decomposes the global optimization model into sub-models and solves them in parallel locally. This approach reduces decision-making dimensions and accelerates solution speeds. Simulation results in this paper demonstrate that applying NP to the active distribution network reduces calculation times for intra-day dispatch and real-time power tracking by approximately 26% and 50%, respectively, compared to centralized control.

This research is applicable to active distribution networks with high penetration of distributed renewable energy, energy storage, and flexible loads, aiming to fully utilize the regulation capabilities of various types of flexible resources. Based on this research, future work will focus on evaluating the regulation capabilities of microgrids and exploring their interaction mechanisms with the higher-level grid and neighboring microgrids. Additionally, the network partitioning method proposed in this paper is valuable for site planning of Soft Open Point (SOP), suggesting that tie-lines of VMs could be developed into SOPs, thereby enhancing the flexibility of active distribution networks.

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