

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/175087/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Liu, Jinlong, Zou, Zhiqiang, Gao, Kang, Yang, Jie, He, Siyuan and Wu, Zhangming 2025. A novel digital unit cell library generation framework for topology optimization of multi-morphology lattice structures. *Composite Structures* 354 , 118824. [10.1016/j.compstruct.2024.118824](https://doi.org/10.1016/j.compstruct.2024.118824)

Publishers page: <https://doi.org/10.1016/j.compstruct.2024.118824>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



A novel digital unit cell library generation framework for topology optimization of multi-morphology lattice structures

Jinlong Liu ^a, Zhiqiang Zou ^a, Kang Gao ^{a,b*}, Jie Yang ^c, Siyuan He ^d, Zhangming Wu ^{b**}

^a Key Laboratory of Concrete and Prestressed Concrete Structures of the Ministry of Education, School of Civil Engineering, Southeast University, Nanjing, China

^b School of Engineering, Cardiff University, The Parade, Cardiff CF24 3AA, UK

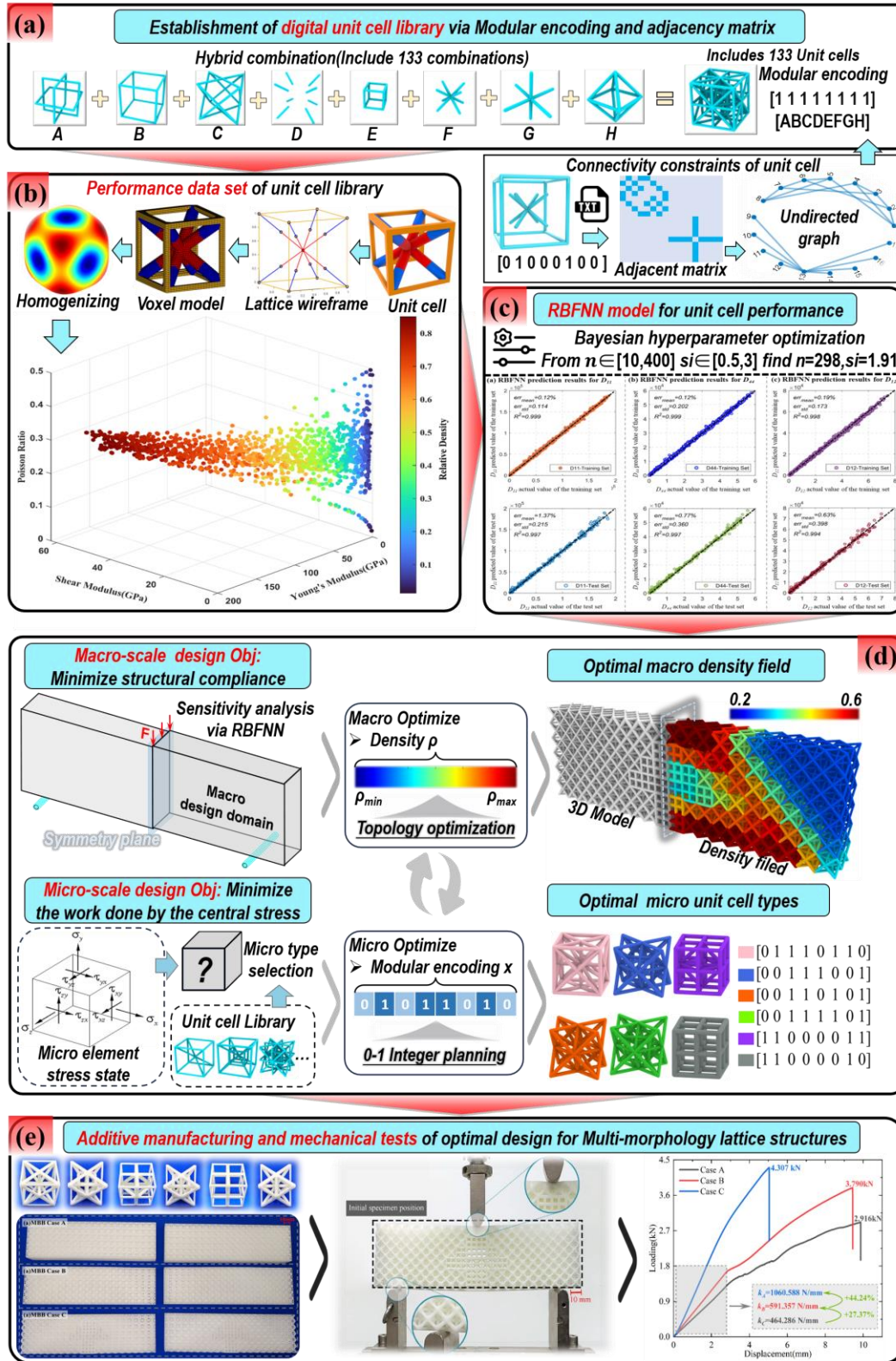
^c School of Engineering, RMIT University, PO Box 71, Bundoora, VIC 3083 Australia

^d State Key Laboratory of Bioelectronics, School of Biological Science and Medical Engineering, Southeast University, Nanjing, 210096, China

Abstract

Although single-unit cell lattice structures are commonly used in engineering, multi-morphology composite lattice structures offer enhanced mechanical properties and diverse functionalities by tailoring their microstructures. This study presents a novel framework for generating a digital unit cell library to optimize the design of multi-morphology lattice structures. The framework involves creating the library using modular encoding and an adjacency matrix while addressing connectivity constraints. A voxel model is employed to streamline the homogenization process of the various unit cells in the library. This homogenization dataset trains a Radial Basis Function Neural Network (RBFNN) to evaluate the elasticity tensor of the unit cells. The compliance of the multi-morphology lattice structure is minimized by identifying optimal unit cell volume fractions and types through topology optimization and 0-1 integer programming. The former utilizes sensitivity analysis via RBFNN to determine the optimal volume fractions, while the latter focuses on minimizing the element strain energy and considers constraints that satisfy the optimal volume fractions of unit cells. The effectiveness and feasibility of this method are demonstrated through three benchmark numerical examples. Experimental results from additive manufacturing samples show that the proposed multi-morphology lattice structures achieve a 44.24% increase in initial stiffness and a 47.70% increase in ultimate strength compared to single-unit cell lattice structures.

Keywords: lattice structure, unit cell library, radial basis function neural network, topology optimization, additive manufacturing



Acronyms

AM	additive manufacturing
DLP	digital light processing
LCD	liquid crystal display
FDM	fused deposition method
BCC	body-centered cubic
FCC	face-centered cubic
SNBCC	spherical node-body-centered-cubic
GBCC	graded-strut body-centered cubic
TPMS	triply periodic minimal surface
TO	topology optimization
RBFNN	radial basis function neural network

1. Introduction

Lattice structures outperform solid materials(such as steel and plastic) by offering lightweight and enhancing multifunctional properties, like energy absorption[1,2], heat dissipation[3], sound absorption[4], electromagnetic wave absorption[5], and biocompatibility performance[6]. Compared with the stochastic cellular structure, lattice structures provide superior strength along with remarkable programmability advantages by controlling micro-features[7–9]. For instance, by integrating diverse micro-unit cells in specific regions of lattice structure, it is possible to create multi-morphology lattice structures that not only exhibit enhanced stiffness and bearing capacity but also possess immense potential for multifunctional properties improvements[10–13]. Additionally, advances in additive manufacturing (AM) technologies like Digital Light Processing (DLP) and Liquid Crystal Display (LCD),etc. have overcome accuracy and defect challenges associated with traditional fused deposition method(FDM)[14–17], which have paved the way for the fabrication of sophisticated multi-morphology lattice structures. However, most existing research remains largely confined to a limited range of simplistic or single micro unit cell, which often struggle to satisfy the developmental demands for advanced structures that achieve higher performance with reduced material usage.

Investigation of diverse unit cells to improve the performance of lattice structure has, therefore, attracted increasing attention in the past few years[18,19]. Typically, the generation of unit cells can be dichotomized into two methodologies: manual generation and mathematical generation[20]. Traditional manually generated unit cells, encompassing structures such as Kagome, octet truss, Kelvin, body-centered cubic (BCC), face-centered cubic (FCC), and cubic configurations, are known for their merits in stiffness, stability, and energy absorption capacities[21–24]. Besides, Ren et al.[25] introduced a lattice structure termed spherical node-body-centered-cubic(SNBCC), showcasing superior stiffness and diminished stress concentration. Similarly, Bai et al.[26] unveiled a lattice configuration named graded-strut body-centered cubic(GBCC), outperforming BCC lattices in terms of stiffness. In contrast, mathematical generation techniques leverage parameterized geometry for the facile definition of diverse unit cells, offering a more efficient alternative to manual methods, notably in the creation of unit cell libraries. Liu et al.[27] delineated a methodology to effectively generate a truss unit cell library by defining 15 key nodes and a transformation matrix, addressing constraints from manufacturability. Li et al.[28] introduced a graph-based deep learning generative model, creating a comprehensive latent space for a wide range of truss unit cells; however, many of the unit cells fail to connect effectively with each other

62 to form lattice structures. Hamza Sulayman Abdullahi et al.[29] utilize a hierarchical approach to generate
63 candidate unit cells and establish a unit cell library after screening, but only 0.196% of these cells meet
64 the connectivity constraints.

65 Based on the above research concerning the microstructure of unit cells, many design methods have
66 been proposed for lattice structures, which are primarily categorized into two types: uniform and
67 functionally graded. Uniform lattice structures are designed through the arrangement of identical micro-
68 unit cells, which can be optimized for either high strength, negative Poisson's ratio or enhanced heat
69 transfer by parameter optimization or topology optimization (TO)[30–33]. Building upon this foundation,
70 and to ensure that the optimized lattice structures meet the smoothness requirements for applications,
71 Montemurro et al.[34–36] and Bertolino[37] separately developed multiscale topology optimization
72 design framework for multi-material lattice structures and anisotropic cellular materials based on NURBS
73 hyper-surfaces. In addition, achieving improvements in various performances through variations in
74 cellular porosity to create functionally graded lattice structures is simpler and more feasible[38–40]. Under
75 the assumption of scale separation, functionally graded lattice structures as well as other multi-scale
76 structures, can be designed with greater efficiency and accuracy through homogenization-based topology
77 optimization.[41–43] Zhang et al.[44], Chen et al.[45] and Simsek et al.[46] independently utilized
78 homogenization theory to formulate regression equations for the elastic modulus of plate, truss, and triply
79 periodic minimal surface (TPMS) unit cells, considering changes in micro volume. Integrating these
80 equations with the variable density method facilitated homogenization-based topology optimization,
81 reducing the compliance. Furthermore, to overcome the challenge in accurately evaluating the
82 homogeneous mechanical properties of unit cells with multiple design variables using regression
83 equations, Wu et al.[47]and Wang et al.[48] utilized neural network as surrogate models for calculating
84 the micro unit cell's homogeneous elastic tensor in the optimization design of lattice structures.

85 Although initial improvements in the mechanical properties of lattice structures were driven by the
86 functional gradient of single unit cells [50,51], incorporating multi-morphology unit cells can enhance the
87 stress adaptability and functionality of the lattice structure in different local regions[52,53]. To take
88 advantage of this, many studies utilize multiple design variables to characterize the artificial hybrid unit
89 cell, enabling conversion to various microstructures by adjusting parameter values and driving the
90 homogenization-based topology optimization design of multi-morphology lattice structure[49–52].
91 Additionally, multi-material interpolation models[53], Kriging metamodels and sigmoid functions[54,55]

are employed to define multi-morphology lattices for topology optimization. Contrary to the limited diversity of artificial hybrid unit cells featuring two or three morphologies, adopting the unit cell library for multi-morphology lattice structures increases available microstructural configurations and broadens their physical properties[56–58]. Alzahrani et al.[59] emphasize the significance of lattice unit cells selection from the library, advocating for assigning unit cells with superior mechanical properties to regions of higher relative density. On this basis, Liu et al.[27] additionally took into account the ratio of mechanical properties to density proposed weighted selection functions. However, these selection functions exclusively consider the Young's modulus and volume fraction of unit cells, overlooking the variance in global elasticity tensor between different unit cells and stress states in different regions of macrostructure. Finally, the oversight results in suboptimal unit cell configurations in regions predominantly subjected to shear or coupling forces, thereby limiting the potential for performance improvement in multi-morphology lattices.

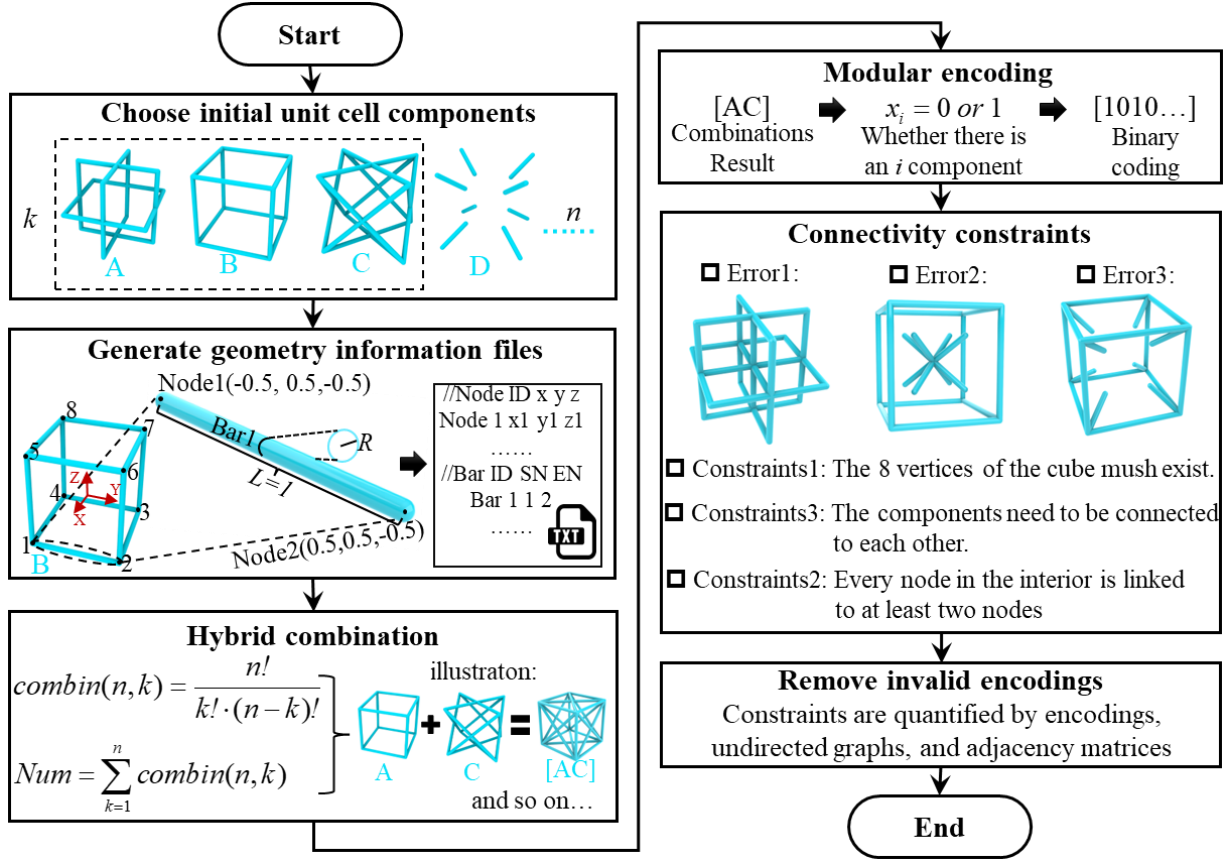
This study aims to develop a rapid method for generating a digital unit cell library for optimizing the design of multi-morphology lattice structures. The research is organized as follows: Section 2 delves into a digital generation technique for the unit cell library based on combinatorial mathematics and graph theory. Additionally, a radial basis function neural network (RBFNN) surrogate model is established to quickly evaluate the mechanical properties of the unit cells. Section 3 introduces an optimization design method for multi-morphology lattice structures based on the RBFNN model. Section 4 validates the effectiveness of the proposed method in designing high-performance, lightweight lattice structures through three numerical examples. Finally, Section 5 showcases the optimized design results through Digital Light Processing (DLP) technology for additive manufacturing and mechanical testing.

2. Construction and performance evaluation of unit cell library

In this section, taking the truss lattice as an example, a simple and efficient method of unit cell library generation is proposed by defining the basic component with modular encoding, which can effectively ensure the diversity and connectivity of the lattice. Meanwhile, the voxel model is derived through the modular encoding of each unit cell, and then homogenized to obtain the corresponding elasticity tensor. Finally, based on the established unit cell library, hyperparameter optimization and training of the RBFNN are conducted to develop a surrogate computational model that maps the design variables to the elasticity tensor.

121 2.1 Unit cell construction method

122 The process of establishing a unit cell library is summarized in the flowchart shown in Fig. 1 and
 123 involves the following steps. By modifying the base components, this method can be extended to construct
 124 a unit cell library for arbitrary truss lattices, ensuring connectivity between them.



126 **Fig. 1** Unit cell library creation process

127 *Step 1.* The process begins by determining the geometries of the n base components, represented by
 128 letter elements, and records their node coordinates and the node numbers at the ends of the rods into a .txt
 129 file. For details, see section 2.1.1.

130 *Step 2.* The hybridization and combination of k base components into varying unit cells is analogous
 131 to a combinatorial problem with letter elements. As k ranges from 1 to n , the methodology exhaustively
 132 identifies all feasible unit cell hybrids. For details, see section 2.1.2.

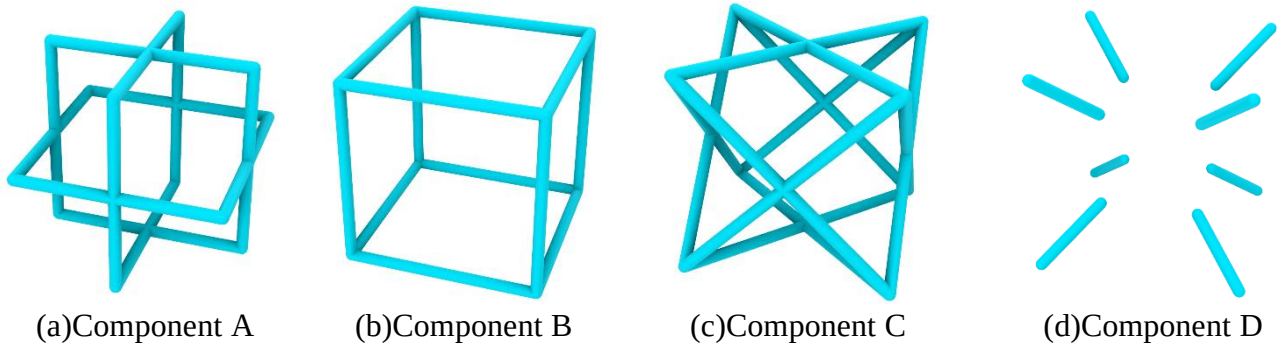
133 *Step 3.* The introduction of 0-1 Boolean value enables the quantification of the existence of each base
134 component within a combination, leading to the generation of n -bit binary encodings representing each
135 possible unit cell combination. For details, see section 2.1.2.

136 *Step 4.* The adjacency matrix of an undirected graph serves to eliminate modular encodings that fail
137 to meet connectivity constraints, thus ensuring that the unit cells contribute to the cohesiveness of the
138 lattice structure. For details, see section 2.1.3.

139 The initially established .txt file is crucial in translating the geometric model into the graph structure,
140 allowing for the identification of connectable components in the hybrid lattice. The specific details of the
141 process unfold as follows.

142 2.1.1 Base components of unit cell

143 The choice of the basic components is crucial for determining the performance range of the library.
144 Eight selected base components, shown in Fig. 2, are derived from proven truss unit cells with excellent
145 mechanical properties, including BCC, RBCC, FCC, octet, and tesseract[9,20]. These components have
146 been widely used in the optimization design of lattice structures[27,29,53–55,60], exhibiting symmetry
147 across three orthogonal central planes as well as cubic symmetry. The nodes and rods of each unit cell
148 component are coded in the order of counterclockwise rotation from the bottom left corner to the top, the
149 reference frame of the node coordinates is the spatial body-centered coordinate system, and the two
150 shortest points along the length of the rod are defined as the nodes at rods' ends, and all the information
151 is written in a .txt file for the subsequent hybridized combination and homogenization analysis.

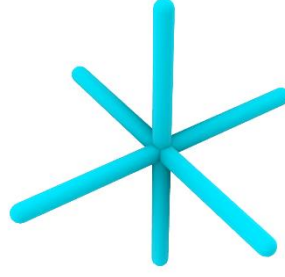




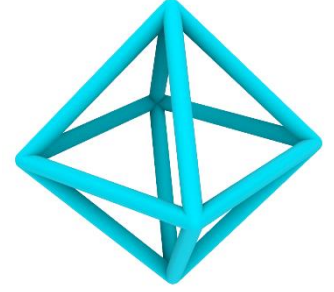
(e)Component E



(f)Component F



(g)Component G



(h)Component H

Fig. 2 The selected base component

2.1.2 Hybridized combinations and modular encoding

The process of manually selecting and generating a new unit cell from the eight unit cell components in Fig. 2 is essentially a classical combinatorial mathematical problem, which can be conveniently solved by exhausting all possible combinations using Eq.(1), which allows for the initial identification of candidate unit cells.

$$Num = \sum_{k=1}^n \frac{n!}{k! \cdot (n-k)!} \quad (1)$$

where $Num=255$ indicates the total number of possible combinations; $n=8$ is the number of base components; and k is the number of base components contained in the unit cell produced by the hybridization combination.

Fig. 3 provides examples of unit cells derived from hybridized combinations with different values of k and the corresponding modular encoding, respectively. The modular encoding characterizes the combination type contained in the current unit cell, using a 0-1 Boolean value to create a binary encoding of length n , with components represented by A~H. For example, if components A and B are not included with $k=6$, the corresponding modular encoding $\mathbf{x}_c$ is [0 0 1 1 1 1 1 1]. This approach is straightforward and can quickly parameterize all candidate unit cells.

k	Hybrid combination								Result	Modular encoding							
1	+		+	+	+	+	+	+	= 	$[01000000]$							
2	+	+		+	+	+	+		+	= 	$[00100010]$						
3	+		+	+		+	+		+	+	= 	$[01010100]$					
4	+		+	+		+		+		+	+	= 	$[01011100]$				
5	+		+	+		+	+		+		+		= 	$[01010111]$			
6	+	+		+		+		+		+		+		= 	$[00111111]$		
7	+		+		+		+		+		+		+		= 	$[01111111]$	
8		+		+		+		+		+		+		+		= 	$[11111111]$
	A	B	C	D	E	F	G	H									

Fig. 3 Hybrid combination of lattice unit cells

2.1.3 Connectivity constraints of unit cell

To ensure that the created unit cell library exhibits superior manufacturability and mechanical properties, it is imperative to meticulously screen all obtained candidates. From both the macro lattice structure and micro unit cell perspectives, we can derive three connectivity constraints for the truss unit cell, along with corresponding implementation methods. It is important to note that the theoretical constraint methods presented are universally applicable for generating an interconnected digital unit cell library using any basic truss components; the only requirement is to adjust the corresponding constraint parameters accordingly.

(1) Nodes must contain the 8 vertices of the cube: unit cells in different regions of the macroscopic lattice structure are typically connected to each other through their own vertices or face-centered points, resulting in overall connectivity. The more intersections between different unit cells, the more stable the

lattice structure becomes, so it is necessary to ensure that there are at least 8 vertices shared between each unit cell and the surrounding unit cells. To satisfy this constraint, the sum of the second to fourth elements in modular encoding must be greater than or equal to 1, as shown in Eq.(2). This is because only component B, C, or D possess the 8 vertices of the cube.

$$\sum_{i=2}^4 x_i - 1 \geq 0 \quad (2)$$

(2) No dangling components: Each component of the unit cell must be connected to form an effective load transfer path and ensure manufacturability. Since all the nodes and connections of the unit cell components are defined in the corresponding .txt files mentioned above, the unit cell generated by the combination can easily obtain all the nodes and connections through modular encoding, which is exactly the same definition of the undirected graph in Eq.(3).

$$\begin{aligned} G &= (V, E), V \neq \emptyset \\ V(G) &= \{v_1, v_2, v_3, \dots, v_n\} \\ E(G) &= \{(v_1, v_2), (v_1, v_3), \dots, (v_i, v_j)\} \end{aligned} \quad (3)$$

where G denotes an undirected graph; V is the set of vertices; E is the set of edges, an unordered binary geometry consisting of elements in V ; for an undirected graph G , the unordered pairs (v_i, v_j) and (v_j, v_i) denote the same edge.

Considering that connectivity graphs in graph theory require paths between any two nodes in G , transforming the geometric information of the unit cell into an undirected graph G can reveal hanging components that do not satisfy the requirement. And the basic property of the connectivity graph is that it consists of only one connected component, meaning the graph is fully connected, so the connectivity analysis can be carried out on the undirected graph G of the unit cell through the adjacency matrix $\mathbf{A}_m$, so as to filter out all invalid encodings in candidate unit cells with more than one connectivity component.

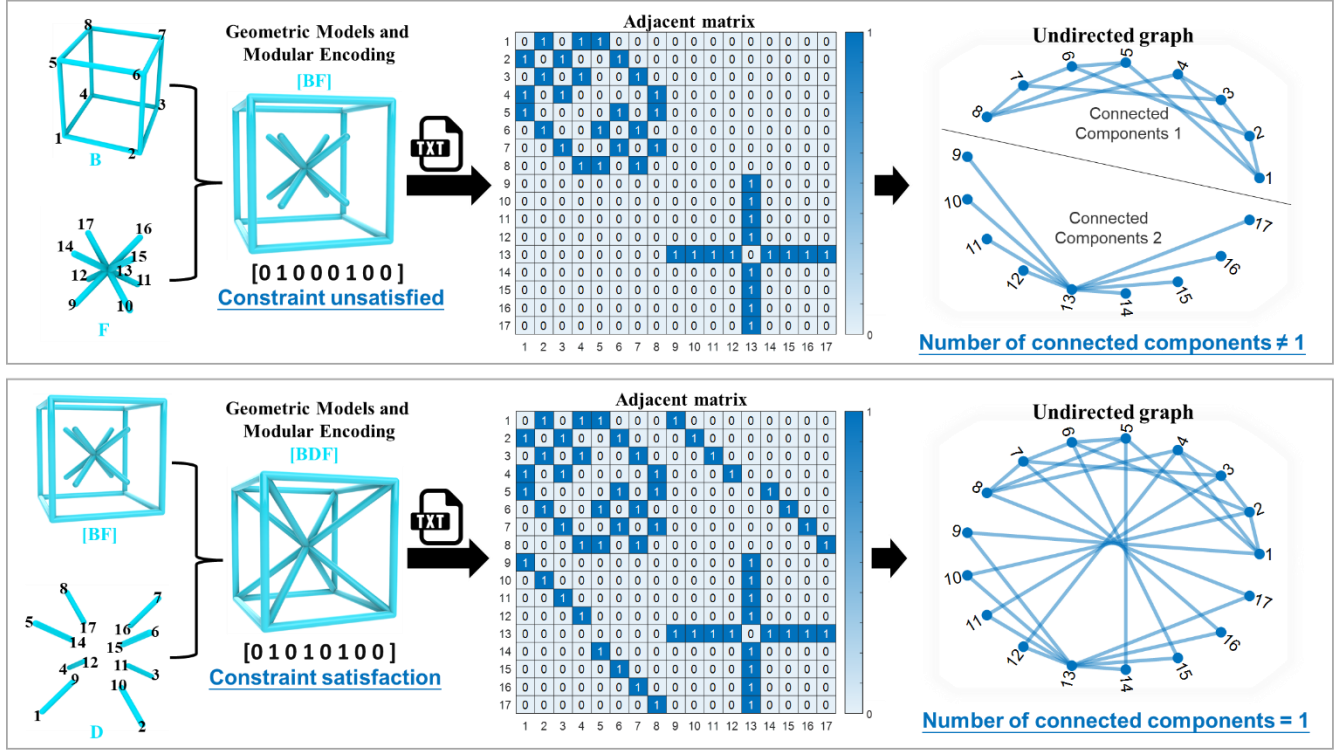


Fig. 4 Screening method based on adjacency matrix of undirected graph

Fig. 4 shows two examples of determining whether there is a hanging combination in the geometric model by the adjacency matrix A_m of the undirected graph G .

The process is as follows:

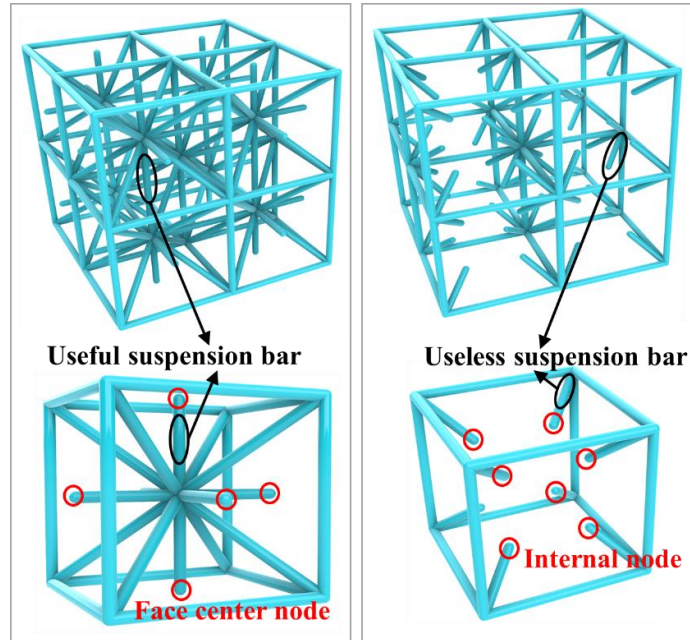
- **Read the .txt file:** Obtain the node and connection information of the unit cell according to the modular encoding;
- **Generate the adjacency matrix A_m :** According to Eq.(4), the matrix A_m is constructed. In this matrix, the element a_{ij} represents the connection weight of the i -th node and the j -th nodes in the undirected graph G . If there is a connection between two nodes, the weight is 1; otherwise, it is 0.
- **Determine the number of connectivity components:** This is done using Laplace matrix L_m , The matrix L_m is calculated as shown in Eq.(5), and the eigenvalue decomposition is performed on L_m . The number of eigenvalues equal to 0 indicates the number of connectivity components.

$$a_{ij} = \begin{cases} 1 & \text{if } v_i v_j \in E \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

$$L_m = D_m - A_m \quad (5)$$

216 where a_{ij} is the element of the i -th row and j -th column of the adjacency matrix $\mathbf{A}_m$; $\mathbf{L}_m$ is the Laplace
 217 matrix corresponding to the undirected graph G_m ; $\mathbf{D}_m = \text{diag}(d(v_1), d(v_2), \dots, d(v_n))$ denotes the degree matrix,
 218 and $d(v_n)$ denotes the degree of the first node v_n , i.e., it is the number of v_n 's neighbors, which is defined
 219 as the following set of nodes $\{v_j \in V | v_n v_j \in E\}$.

220 **(3) All internal nodes must be connected to at least two nodes:** to prevent the presence of suspended,
 221 non-functional rods within the unit cell. These suspended rods neither transfer loads effectively within the
 222 unit cell nor contribute to the connectivity between unit cells in the lattice structures. As illustrated in Fig.
 223 5, the reason this constraint is not simply defined as disallowing suspended rods is that Component G (Fig.
 224 2) can create a situation where rods are suspended. However, this type of suspension is acceptable if it
 225 aids in connecting different locations within the lattice structure, particularly when the suspension occurs
 226 at the center of a face rather than within the cell. To enforce this constraint, we examine the sum of each
 227 row or column in the adjacency matrix $\mathbf{A}_m$, which indicates the number of neighbors for each node. If any
 228 element in the vector obtained by summing the rows of the adjacency matrix equals 1, and the fourth digit
 229 of the modular encoding is 1, the constraint is not satisfied. This scenario indicates that the unit cell
 230 contains Component D (Fig. 2), which does not meet the requirement.



232 **Fig. 5** Useful and useless suspension bars for improving connectivity

2.2 Homogenization method of unit cells

The overall stiffness matrix of the lattice structure depends on its own microstructure. Thus, estimating the elasticity tensor of each microscopic unit cell is a crucial step for optimizing the lattice structure in this study. Notably, the homogenization framework based on the 3D voxel model, as established by Dong et al.[61] and building on the method developed by Andreassen et al.[62] can greatly simplify the homogenization process for unit cell libraries. This framework uses a voxel model, which is a hexahedral mesh defined by a 3D logical matrix. It can achieve the required accuracy and solve the problem efficiently when the voxel resolution is set to 50, while also effectively accounting for the volume of overlapping nodes compared to using truss or beam elements[61]. For example, considering the BCC unit cell shown in Fig.6, the volume V_{vol} is calculated as 0.1635 from Eq.(6). This value is only 0.426% different from the geometric model volume of 0.1642, which was calculated using Rhino7 software for a unit cell side length 1.

$$V_{vol} = \frac{\sum_{i=1}^{n_x} \sum_{j=1}^{n_y} \sum_{k=1}^{n_z} \mathbf{VLC}_{ijk}}{n_x n_y n_z} \quad (6)$$

where $\mathbf{VLC}$ is the 3D logical matrix of combined voxel model. n_x, n_y, n_z are the number of voxels in the x, y, z direction, which is equal to the size of the resolution of the voxel model.

Fig. 6 shows the homogenization process after the synthesis model is obtained in this paper by taking BCC with modular code [0 1 0 1 0 1 0 0] as an example. In other words, the lattice wireframe is generated by reading the txt files of the base component of the current unit cell according to the modular encoding. When the voxel resolution is 50, the voxel number along the x, y and z axes is 50. Then, the 3D logical matrix of each base component is determined by the rod radius size, and the 3D logical matrix of the combined voxel model is obtained by the fetch and union operation. The element type is set as an eight-node hexahedral element, the global stiffness matrix and load vector are established, and the corresponding element displacement is obtained by applying periodic element strain conditions. Then the element displacement and global displacement field are used to calculate the elastic tensor matrix $\mathbf{D}$, where the elements can be calculated according to Eq.(7).

$$D_{ij} = \frac{1}{|\Omega|} \sum_{(e)} \int_{\Omega^{(e)}} \left(\chi_{(e)}^{0(i)} - \chi_{(e)}^{(i)} \right)^T \mathbf{K}_e \left(\chi_{(e)}^{0(j)} - \chi_{(e)}^{(j)} \right) d\Omega^{(e)} \quad (7)$$

259 where $\chi_{(e)}^{0(i)}$ is the element displacements corresponding to the prescribed unit test strain field[61]. $\mathbf{K}_e$ is
 260 the element stiffness matrix. Ω is the total volume of the unit cell of the lattice structure. $\chi_{(e)}^{(i)}$ is the
 261 displacement field obtained from the global stiffness equation.

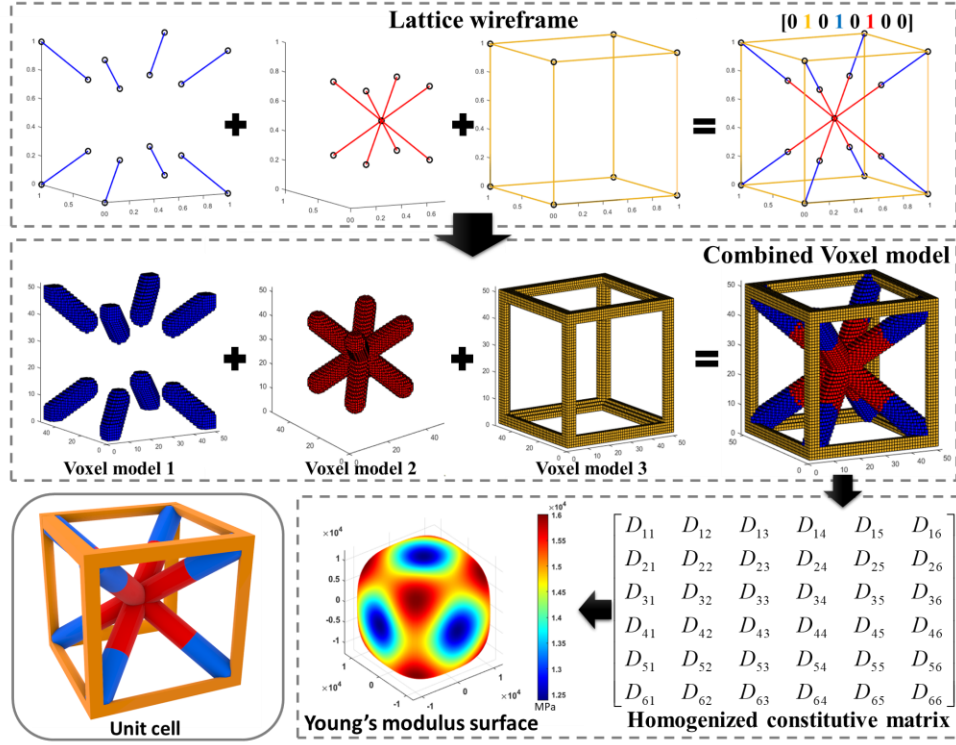


Fig. 6 Homogenization process of unit cell

2.3 Unit cell performance evaluation model

2.3.1 Performance data set

Based on the above modular encoding unit cell generation, screening and homogenization methods, the design variable in the data set include the modular encoding and volume fraction of unit cell. The physical characteristics of the data set include the elastic tensor of the unit cell under different geometric parameters, the relative elastic modulus, Poisson's ratio, and shear modulus calculated according to Eq.(8), in addition, the ratio of the unit cell to the mechanical characteristics of the material E/E_0 , ν/ν_0 , G/G_0 , and so on. Here $E_0= 210\text{GPa}$, $\nu_0=0.3$, $G_0= 808\text{GPa}$.

272

$$\mathbf{S} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & -\nu_{13}/E_1 & 0 & 0 & 0 \\ -\nu_{21}/E_2 & 1/E_2 & -\nu_{23}/E_2 & 0 & 0 & 0 \\ -\nu_{31}/E_3 & -\nu_{32}/E_3 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{23} \end{bmatrix} \quad (8)$$

273

274

275

where $\mathbf{S}$ is the inverse of $\mathbf{D}$. Since the unit cell composed of hybrids exhibits cubic symmetry, the relative elastic modulus and Poisson's ratio in all directions are the same as in the shear model, that is $E_1=E_2=E_3$, $G_1=G_2=G_3$, $\nu_{12}=\nu_{13}=\nu_{21}=\nu_{23}=\nu_{31}=\nu_{32}$.

276

277

278

279

280

281

282

283

284

285

286

287

288

289

290

291

292

293

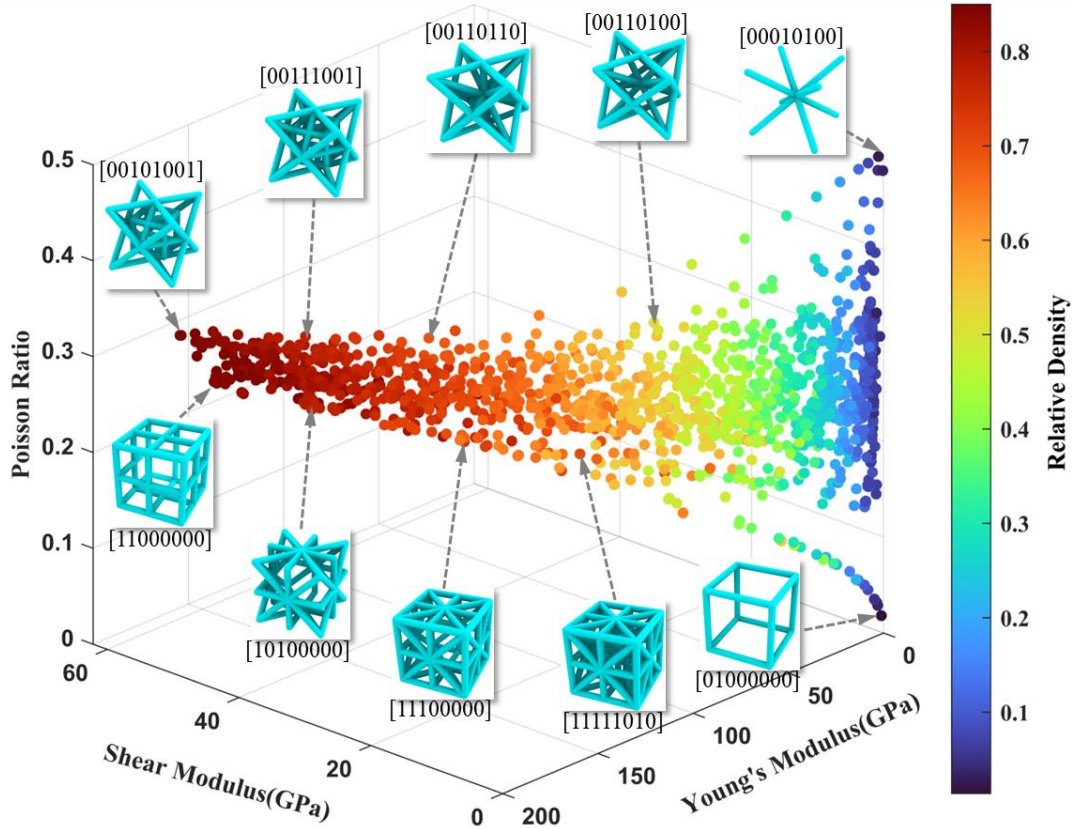
294

295

The high quality of the unit cell library is primarily reflected in three aspects: a wide distribution of mechanical properties, a rich variety of unit cell types, and the absence of invalid rods with effective connectivity between all unit cells. Based on the eight base components selected in this paper, 133 effective unit cells can be screened from the 255 unit cells generated using the construction method outlined. This represents a proportion of 52.2%, which is significantly higher than the 0.196% reported by Hamza Sulayman Abdullahi et al.[29]. This indicates that a substantial number of invalid configurations can be avoided by carefully selecting and configuring the base components. The resulting library includes not only commonly used unit cells such as cross, FCC, and BCC, but also many new configurations. The high quality of the unit cell library is primarily reflected in three aspects: a wide distribution of mechanical properties, a rich variety of unit cell types, and the absence of invalid rods, ensuring effective connectivity between all unit cells. Based on the eight base components selected in this study, 133 effective unit cells were identified from the 255 unit cells generated using the outlined construction method. This represents a proportion of 52.2%, indicating that a substantial number of invalid configurations can be avoided through careful selection and configuration of the base components. The resulting library includes not only commonly used unit cells such as cross, FCC, and BCC, but also many novel configurations. Using these effective unit cells, different rod radii were assigned to generate voxel models for homogenization, with the relative density calculated using Eq.(6) to determine V_{vol} . The range of rod radii for each unit cell was set between 0.03 and 0.43 times the cube edge length. Different radius values within this range were selected within this range to ensure that each unit cell contains at least one sample point in the V_{vol} ranges of 0-10%, 10-20%, 20-30%, ..., 70-80%, and 80-85%. Additionally,

296 samples with V_{vol} greater than 85% were removed, as high-density lattice structures approach solid
 297 materials. 1399 samples satisfying the above requirements were generated.

298 Fig. 7 illustrates the range of mechanical properties for 1,399 samples in the unit cell library dataset.
 299 The ranges for equivalent Young's modulus, shear modulus, Poisson's ratio, and relative density are
 300 [0.021, 151.69] MPa, [0.0029, 58.51] MPa, [0.019, 0.495], and [1.37, 84.99] %, respectively. Although
 301 research by Panetta et al. and Liu et al.[27]'s generates more unit cell types by varying rod radius and key
 302 point positions, the Young's modulus and shear modulus ranges in the current work are broader within the
 303 given density range [1.37, 84.99]%. Furthermore, the connectivity between different types of unit cells is
 304 more advantageous compared to the libraries constructed by Liu et al. and Li et al. [27,28]. The framework
 305 presented here is adaptable to any number of base components, and as the number of base components
 306 increases, the variety of unit cell configurations obtained through hybridization will become more diverse.



307

308 **Fig. 7** Relative Young's modulus, shear modulus, and Poisson ratio for the unit cell library

2.3.2 Radial Basis Function Neural Network

To drive subsequent optimization designs of multi-morphology lattice structures based on proposed unit cell library, it is imperative to establish a computational model for the elastic tensor $\mathbf{D}$ that accounts for the unit cell's relative density ρ_e and modular encoding $\mathbf{x}_c$. Given that the RBFNN exhibits superior local approximation capabilities, essential for accurately capturing the microscopic performance characteristics of various unit cells, we have chosen to employ RBFNN for hyperparameter optimization and training using the dataset established in Section 2.3.1. This approach ensures an accurate assessment of the performance of different unit cells. In this configuration, the input layer consists of a discrete modular encoding $\mathbf{x}_c=[x_1, x_2, \dots, x_8]$ and a continuous relative density value ρ_e , while the hidden layer is composed of n radial basis functions, as represented in Eq.(9) below.

$$\phi_i(\mathbf{x}) = \exp\left(-\frac{1}{2s_i^2} \|\mathbf{x} - \mathbf{c}_i\|^2\right) \quad (9)$$

where $\mathbf{x}=[\mathbf{x}_c, \rho_e]$ is the input vector. $\mathbf{c}_i=[c_{i1}, \dots, c_{i9}]$ is the center vector of the i -th radial basis function, determined during the training of the RBFNN. s_i is the spread parameter of the radial basis function.

The output vector $\mathbf{y}=[D_{11}, D_{44}, D_{12}]$ is calculated by Eq.(10). For the elasticity tensor of unit cell in unit cell library, $D_{11}=D_{22}=D_{33}$, $D_{44}=D_{55}=D_{66}$ and $D_{12}=D_{13}=D_{21}=D_{23}=D_{31}=D_{32}$.

$$y_j = \sum_{i=1}^n w_{ij} \phi_i(\mathbf{x}) + b_j \quad j = 1, 2, 3 \quad (10)$$

where w_{ij} is the weight from the i hidden layer neuron to the j output layer neuron. b_j is the bias of the j output layer neuron.

The choice of RBFNN hyperparameters will directly influence the accuracy of the computation of the unit cell elasticity tensor D_{ij} . Compared to other hyperparameter optimization methods, such as grid and random search, the Bayesian optimizer offer several advantages. It automates and streamlines the optimization process by leveraging historical information, considering uncertainty, and adapting to continuous hyperparameter spaces. Therefore, to ensure that the developed RBFNN is accurate and without overfitting, a Bayesian optimizer was utilized to optimize the hyperparameters of a RBFNN, including the spread parameter s_i of the RBF function in Eq.(9) and the maximum number of hidden layer neurons n in Eq.(10).

Fig. 8 illustrates the hyperparameter search scope of RBFNN and the chosen objective function was the mean of squared errors obtained through five-fold cross-validation, which enhanced the robustness and reliability of the evaluation process by minimizing the risk of overfitting and accounting for different data configurations. The convergence criterion of the hyperparametric optimization is set to reach the maximum number of iterations 100 or then the rate of change of the objective function is less than 0.001.

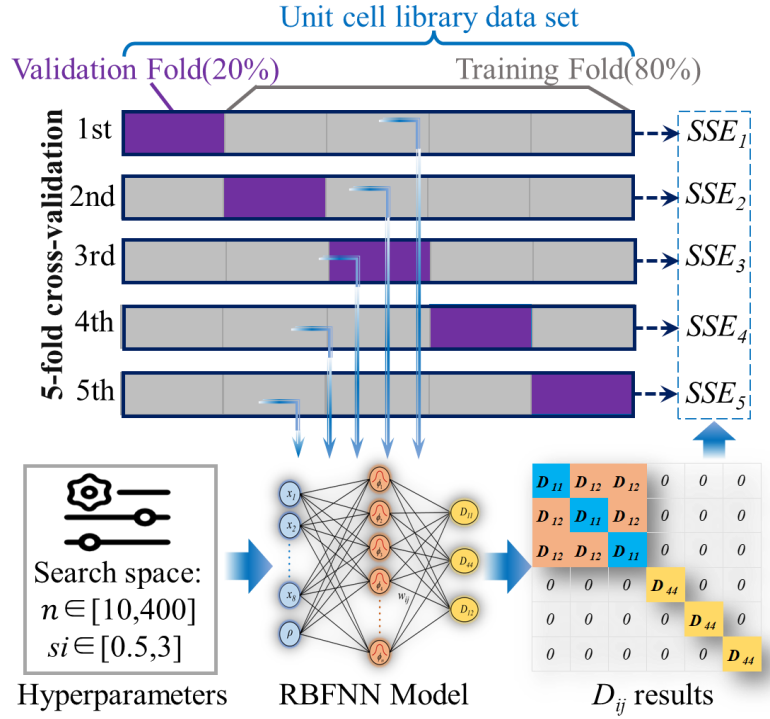


Fig. 8 The process of five-fold cross-testing in hyperparametric optimization.

Through hyperparameter optimization, the optimal hyperparameters of RBFNN are determined as $n=298$ and $s_i=1.91$. Fig. 9 depicts the performance evaluation result of the RBFNN model, optimized through hyperparameter tuning. This evaluation reveals that the model achieves remarkable precision in predicting the unit cell's elasticity tensor values within the library database. Specifically, the coefficient of determination (R^2) for both the training and test datasets consistently lies between 0.994 and 0.999. Furthermore, the model demonstrates an exceptionally low average relative error rate (err_{mean}) of merely 1.3%, coupled with a standard deviation of the error rate (err_{std}) below 0.4%. These metrics underscore the RBFNN model's high accuracy and robustness against overfitting, underscoring its substantial reliability for calculating the elasticity tensor of unit cells.

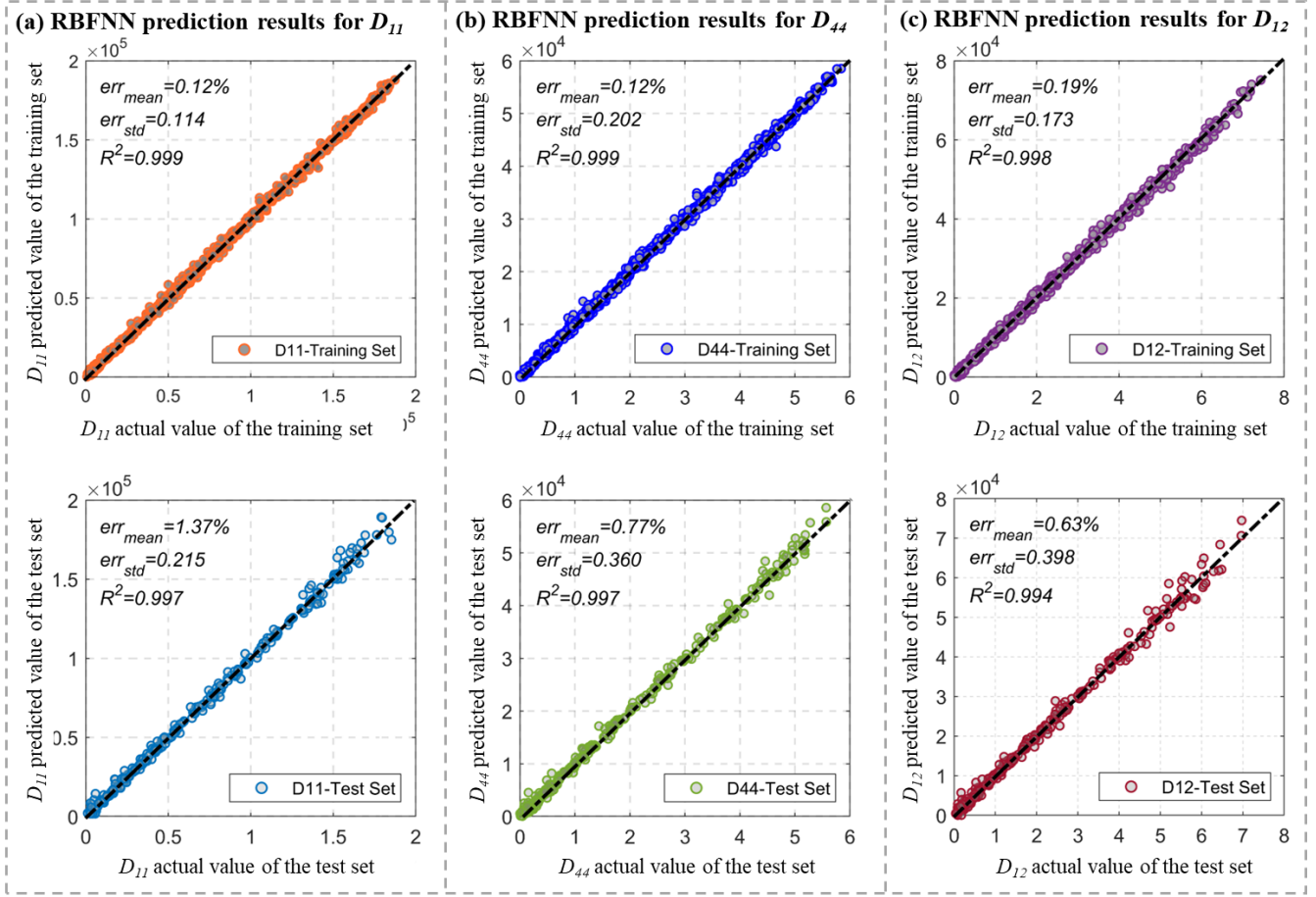


Fig. 9 Performance obtained by training RBFNN with optimal hyperparameters.

3 Design of multi-morphology structure using digital unit cell library

In this section, we propose an optimization design method for multi-morphology lattice structures using unit cell library, as depicted in Fig. 10. As mechanical characteristics of the macro lattice structure depend on the type and volume fraction of the micro unit cell, the topology optimization design of the lattice structure is first carried out to obtain the ideal volume fraction of the microscopic unit cell, and determine the stress vector of each unit cell. Then, a 0-1 integer programming model is developed with the objective of minimizing the element strain energy, taking into account the limitations of the volume fraction of the microscopic cells. The model will traverse the various modular encodings x in the unit cell library to determine the optimal type of each microscopic unit cell. After updating the unit cell type, secondary topology optimization is performed for the macroscopic lattice structure. Throughout the process, the updating of the stiffness matrix and sensitivity information relies on a Radial Basis Function Neural Network (RBFNN). Specifically, the elastic tensor for each element during the optimization

process can be computed using Eq.(10), which is then used to assemble the stiffness matrix. For the detailed derivation of sensitivity calculations using RBFNN, please refer to Eq.(13) in Section 3.1. The detailed process of calculating the radius of unit cell in the post-processing in Fig. 10 is shown in Appendix A.

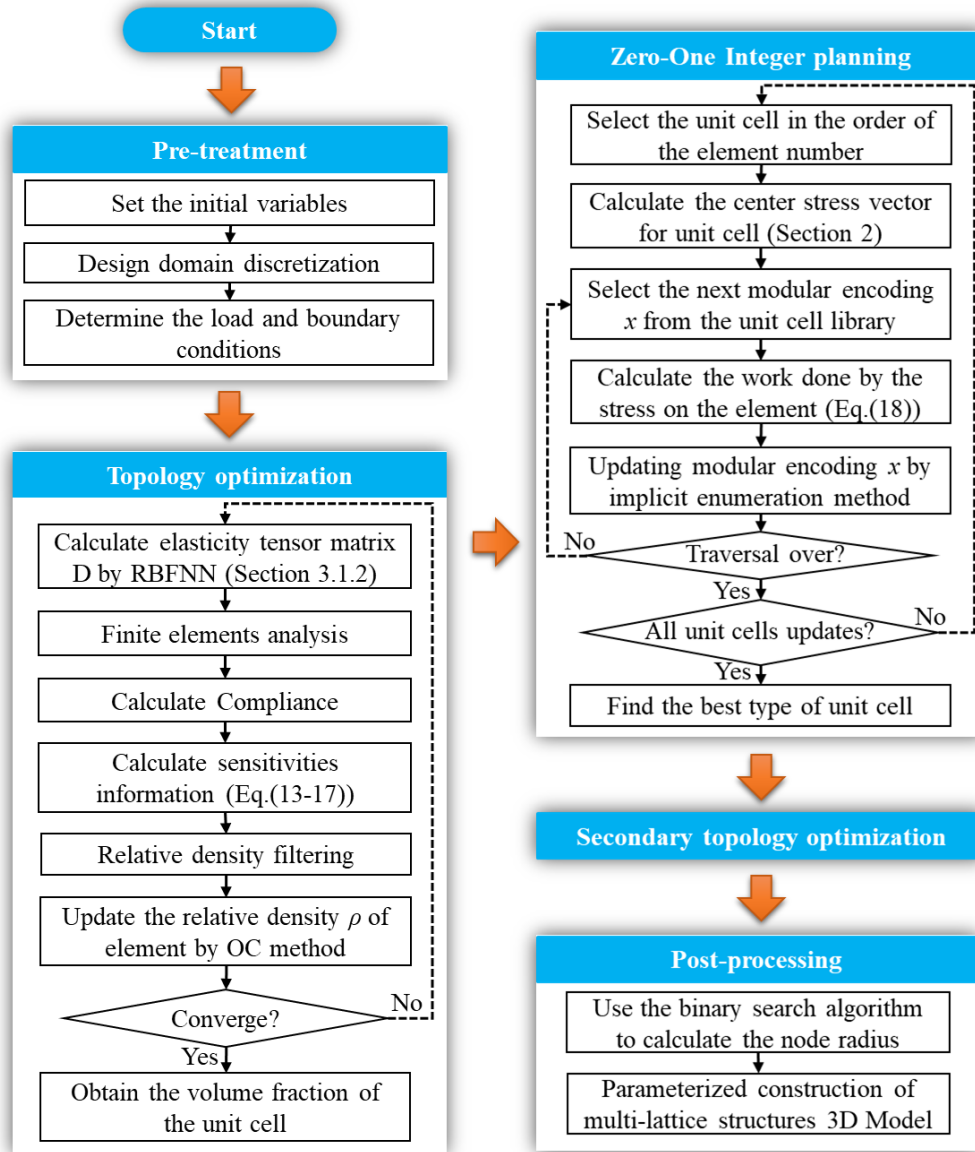


Fig. 10 Flowchart of the proposed optimization method

3.1 Density field design of multi-morphology structures

The unit cell through homogenization can be regarded as a set of solid elements with different volume fractions that make up the macro lattice structure. The optimization variable of the topology optimization is the density value ρ , which is equivalent to the volume fraction V_v of the micro unit cell, and the obtained optimal density field guides the selection of the volume rate of the unit cell inside the lattice structure. The formula for density field updating of multi-lattice structures is given in Eq.(11):

$$\begin{aligned}
 &\text{Find: } \rho = \{\rho_e, \dots, \rho_N\} \\
 &\text{Minimize: } C(\rho) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^N \mathbf{u}_e^T \mathbf{K}_e \mathbf{u}_e \\
 &\text{Subject to: } \sum_{e=1}^N \rho_e V_e - \eta V_{sum} \leq 0 \\
 &\quad : \mathbf{K} \mathbf{U} = \mathbf{F} \\
 &\quad : 0 < \rho_{min} \leq \rho \leq \rho_{max} \quad e = 1, \dots, N
 \end{aligned} \tag{11}$$

where $\mathbf{K} = \sum_e \mathbf{K}_e = \sum_e \int_{\Omega_i} \mathbf{B}^T \mathbf{D}_e \mathbf{B} d\Omega_i$ is the global stiffness matrix of lattice structures, and $\mathbf{D}_e$ uses the RBFNN of Eq.(10) for calculation. C is the compliance of the lattice structure. $\mathbf{U}$ and $\mathbf{F}$ are the global displacement and load vectors of lattice structure, respectively. $\mathbf{K}$ is the global stiffness matrix of the lattice structure, which is determined by the type and volume rate of the unit cell. ρ_{min} and ρ_{max} are the minimum and maximum volume rates of the unit cell, respectively. η is the volume rate of the lattice structure. V_e and V_m are the volumes of hexahedral units and lattice structures internally filled with unit cells, respectively. N is the number of all unit cells in the lattice structure. $\mathbf{u}_e$ and $\mathbf{k}_e$ are the displacement vector and stiffness matrix of the unit cell, respectively.

The density field of lattice structure is updated by the Optimality Criterion rule[63] and density filtering is introduced to ensure the existence of Eq.(11) solution[64]. In addition, sensitivity analysis is needed to drive topology optimization. Consider that elastic tensor matrix $\mathbf{D}$ of the unit cell depends on its own volume rate and the type of structure, so the expression for the sensitivity of the objective function is as follows Eq.(12).

$$\frac{\partial C_e}{\partial \rho_e} = -\mathbf{u}_e^T \frac{\partial \mathbf{k}_e}{\partial \rho_e} \mathbf{u}_e = -\mathbf{u}_e^T \left(\int_{\Omega_e} \mathbf{B}^T \frac{\partial \mathbf{D}_e(\rho_e, \mathbf{x})}{\partial \rho_e} \mathbf{B} d\Omega_e \right) \mathbf{u}_e \tag{12}$$

where C_e is the compliance of i -th unit cell at the macro scale. $\mathbf{B}$ is a geometric matrix of 8-node hexahedral elements. Ω_e is the elemental domain. By combining Eq.(9) with Eq.(10) and applying the chain rule of calculus, we can derive the partial derivative of the RBFNN output vector $\mathbf{y}=[D_{11}, D_{44}, D_{12}]$ with respect to the density ρ , as shown in Eq.(13).

$$\begin{aligned} \frac{\partial y_j}{\partial \rho_e} &= \sum_{i=1}^n w_{ij} \frac{\partial \phi_i}{\partial \rho_e} = \sum_{i=1}^n w_{ij} \frac{\partial \phi_i}{\partial \|\mathbf{x} - \mathbf{c}_i\|^2} \frac{\partial \|\mathbf{x} - \mathbf{c}_i\|^2}{\partial \rho_e} \\ &= -\sum_{i=1}^n w_{ij} \frac{\phi_i}{s_i^2} (\rho_e - c_{i9}) \end{aligned} \quad (13)$$

where c_{i9} is the eighth component of the center vector of the i -th hidden layer node. Through Eq.(13), the $\partial \mathbf{D}_e(\rho_e, \mathbf{x}) / \partial \rho_e$ in Eq.(12) can be calculated according to Eq.(14).

$$\frac{\partial \mathbf{D}_e(\rho_e, \mathbf{x})}{\partial \rho_e} = \begin{bmatrix} \partial D_{11} / \partial \rho_e & & & & & & \\ \partial D_{12} / \partial \rho_e & \partial D_{11} / \partial \rho_e & & & & & \\ \partial D_{12} / \partial \rho_e & \partial D_{12} / \partial \rho_e & \partial D_{11} / \partial \rho_e & & & & \\ 0 & 0 & 0 & \partial D_{44} / \partial \rho_e & & & \\ 0 & 0 & 0 & 0 & \partial D_{44} / \partial \rho_e & & \\ 0 & 0 & 0 & 0 & 0 & \partial D_{44} / \partial \rho_e & \end{bmatrix} \quad (14)$$

3.2 Selection unit cell types for multi-morphology structures

Once the density field is determined by the topology optimization, the selection of unit cells can be analyzed from the following two points. Stress and Load Considerations: The stress experienced by each unit cell within the macro lattice structure serves as an internal force for the lattice and an external load for the microscopic unit cell. Thus, the stress vector $\boldsymbol{\sigma}_e$ for all elements must be computed under the determined density field. This stress vector $\boldsymbol{\sigma}_e$ is then used as the external load for the micro unit cell. Unit Cell Response: Different unit cells, characterized by distinct elastic tensor matrices $\mathbf{D}$, will exhibit varying strains under specific volume fractions and applied stresses. For each unit cell type, the strain is computed in response to the applied stress $\boldsymbol{\sigma}_e$, leading to the determination of the strain energy U_e of the microscopic unit cell.

410 In summary, to select the most suitable unit cell, the following procedure is used: calculate the stress
 411 vector σ_e for all elements based on the density field, apply this stress as an external load to each unit cell,
 412 and then determine the corresponding strains to calculate the strain energy U_e .

413 Obviously, under the same external load and volume rate, the better the mechanical properties of the
 414 unit cell to resist deformation, the corresponding strain energy is smaller. The process is a 0-1 integer
 415 programming problem as shown in Eq.(15) below. The search space of the optimization variable x of Eq.
 416 (15) is the 133 modular encodings in the unit cell library, and the optimal solution can be obtained by
 417 using the implicit enumeration method, which determines the best selection of each unit cell in the lattice
 418 structure.

$$\begin{aligned}
 &\text{Find: } x = \{x_1, \dots, x_8\} \\
 &\text{Minimize: } U_e(x) = \frac{1}{2} \sigma_e^T \epsilon_e V_e \\
 &\text{Subject to: } \mathbf{D}_e(x) \epsilon_e = \sigma_e \\
 &\quad : V_{vol} = \rho_e \\
 &\quad : x_i = 0 \text{ or } 1 \quad i = 1, \dots, 8
 \end{aligned} \tag{15}$$

420 where x and V_e is the modular encoding and the volume of unit cell, respectively. $\sigma_e = \mathbf{D}_e \mathbf{B}_e \mathbf{u}_e$ is the unit
 421 cell center stress vector $[\sigma_x^e, \sigma_y^e, \sigma_z^e, \sigma_{xy}^e, \sigma_{yz}^e, \sigma_{zx}^e]$. $\epsilon_e = \mathbf{D}_e^{-1} \sigma_e$ is the strain vector corresponding to σ_e . $\mathbf{D}_e$ is
 422 calculated using RBFNN

423 4. Numerical examples

424 In this section, we demonstrate the advantages and robustness of the proposed method through three
 425 numerical examples. All examples were computed on a standard desktop PC equipped with an Intel(R)
 426 Core(TM) i7-13700F processor (16 cores, 2.1 GHz) and 32 GB of RAM. The material properties in the
 427 examples are set with an elastic modulus E_0 of 210GPa, a Poisson's ratio ν_0 of 0.3. The cubic unit cells
 428 composing the lattice structure have an edge length of $L=10$ mm. The minimum density ρ_{min} and the
 429 maximum density ρ_{max} are set separately in each numerical example, while the optimized lattice structure's
 430 volume fraction is chosen to be $\eta = (\rho_{min} + \rho_{max})/2$. The modular encoding for base unit cell is set as [1 1 1
 431 1 1 1 1 1], and the initial density of all unit cells is equal to the lattice structure's volume fraction η .
 432 Convergence criteria for topology optimization is defined as the Euclidean norm between the density from
 433 the previous optimization iteration and the current density being less than 1×10^{-6} [65], or upon reaching

the maximum number of iterations, set at 1000. The finite element analysis used Ferrari et al.[65]'s MATLAB based framework.

4.1 Cantilever beam

The first example concerns the optimization design for a cantilever beam, with the minimum and maximum density values (ρ_{min}, ρ_{max}) and the efficiency parameter(η) set to 0.1, 0.6, and 0.35, respectively. Fig. 11 illustrates the geometric parameters and boundary conditions of this case, where L denotes the side length of 10mm for the element. The domain's left region is fixed, while a concentrated force F of 1000N is applied at the nodes in the middle of the right side, acting in the negative Z -direction. In terms of computational efficiency, the optimization for a design domain comprising 3,000 elements with a total of 14,229 degrees of freedom took a reasonable time of 34.06 minutes. The time spent on the topology optimization of the macroscopic density field was 848.7394 seconds, with an average iteration time of 1.9923 seconds, while the selection of the microscopic unit cell required 1194.8398 seconds.

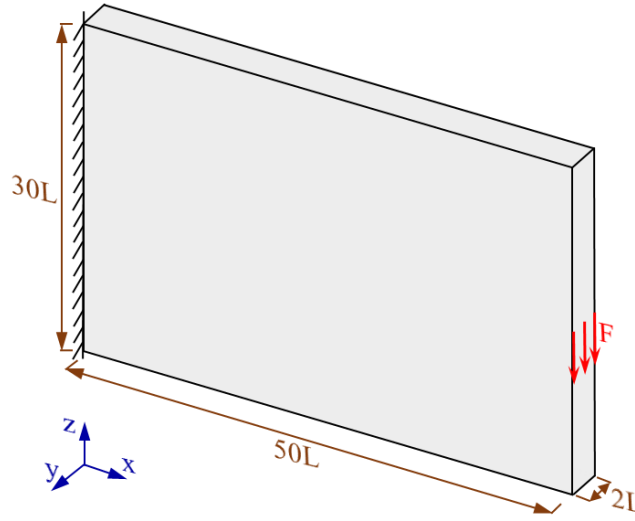


Fig. 11 Boundary conditions and design of the 3D Cantilever beam

Fig. 12 plot the iterative history of global compliance during the optimization process at a volume fraction of 35%, encompassing three distinct stages: topology optimization of the base lattice structure, unit cell selection, and secondary topology optimization. It can be observed that the compliance smoothly decreases throughout the optimization, with the volume fraction consistently maintained, validating the numerical stability of the methodology proposed in this study. Notably, the compliance of the base lattice structure was reduced from 520.977 mJ to 280.818 mJ through density field design. Subsequent 0-1 integer

programming for unit cell selection led to a further 6.468% decrease in compliance. Finally, the secondary topology optimization resulted in an additional reduction of 2.024 mJ in compliance. These findings demonstrate the effectiveness of the optimization design framework presented in this paper.

Fig. 13 illustrates the geometric model of the final optimized structure, in which more stable spherical reconstructions are employed at the nodes of different unit cells. This has been further highlighted by magnified local views. The optimization yielded six distinct unit cells, each with unique morphological and mechanical characteristics, strategically populated within the most appropriate stress-bearing regions. For instance, on the upper and lower sides of the fixed end, where the axial tensile and compressive stresses reach their maximum values, there exists a distribution of unit cells predominantly consisting of cubic rods. In other regions, the introduction of crossed diagonal struts enables the unit cells to better accommodate shear forces and non-axial loading conditions.

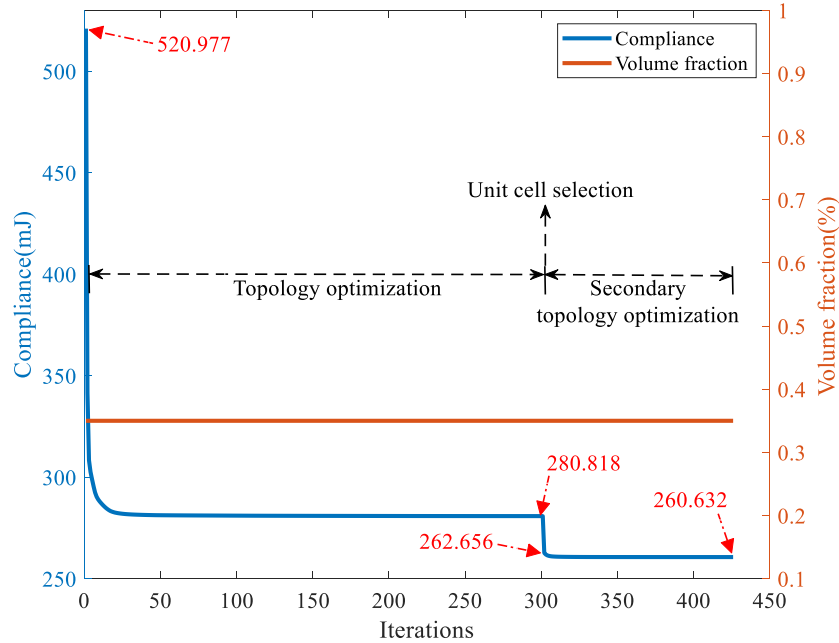


Fig. 12 Iteration histories of objective and constraint function for 3D Cantilever beam

To better investigate the mechanical characteristics of the selected unit cell, Fig. 14(a-c) presents the distribution of mechanical properties within the final optimized cantilever beam's unit cell. Fig.17(a-c) depict the relative elastic modulus, relative shear modulus, and relative Poisson's ratio, as well as their respective ranges of density distribution for the various types of unit cells. It is observed that at the same density level, the gray and purple unit cells exhibit the highest elastic moduli, while possessing the smallest shear moduli and Poisson's ratios, which are more favorable for resisting uniaxial stress. Meanwhile, the

orange, blue, and green unit cells, which fill the high-density core stress regions, demonstrate larger elastic moduli and Poisson's ratios, and their elastic moduli are also situated in an optimal range, allowing them to better accommodate the combined effects of shear and tensile compressive loads.

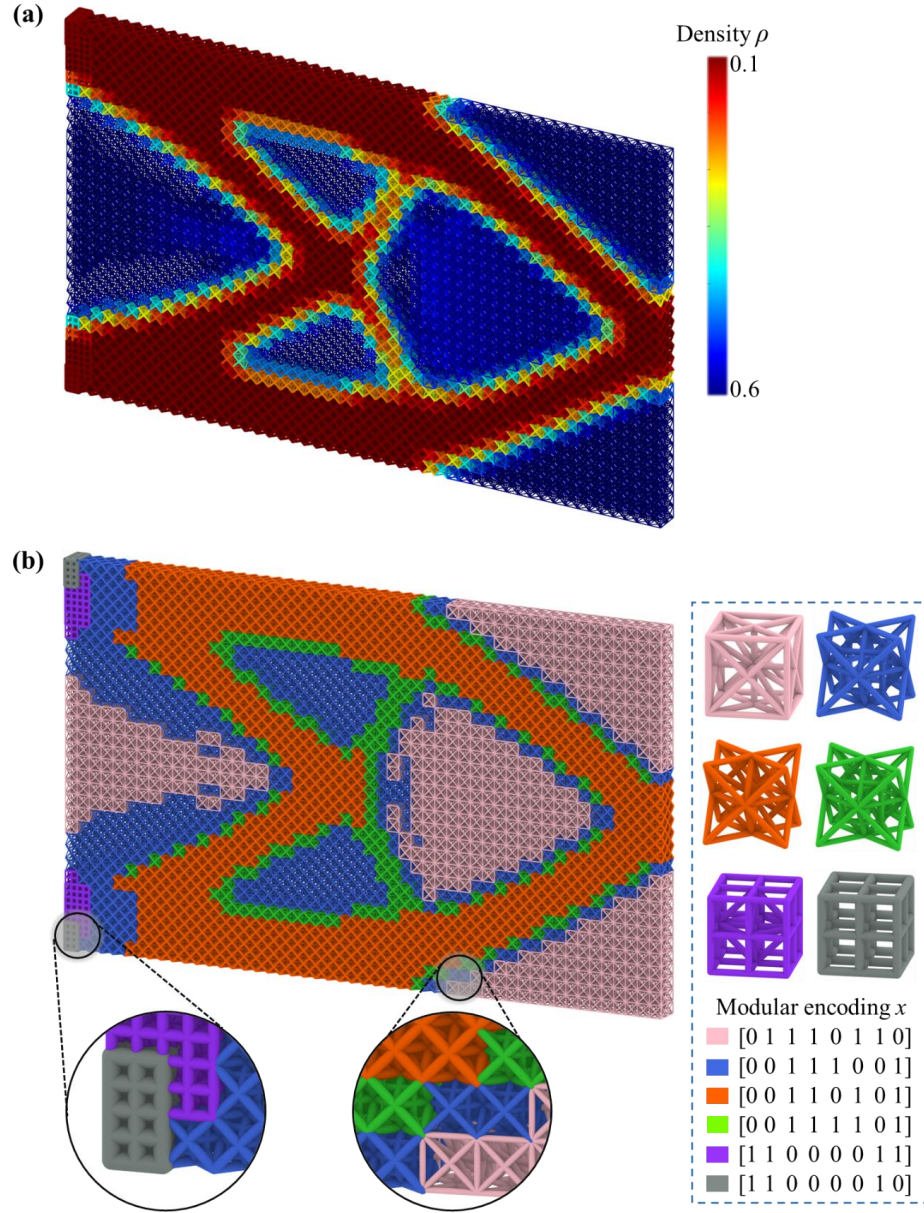


Fig. 13 The geometric model of the final optimization result of cantilever beam

In addition, the distributions of the von Mises principal stress for the initial structure (1st Iteration), topology optimization result (301th Iteration), and the final optimized result (426th Iteration) are depicted in Fig. 14(d). It is observed that the preliminary topology optimization leads to a 56.38% reduction in the mean von Mises principal stress, with the stress distribution tending more towards the lower stress regions,

482 suggesting a material distribution that is more aligned with the trajectories of stress. However, an increase
 483 of 15.42% in the maximum von Mises principal stress indicates a certain degree of stress concentration.
 484 Fortunately, subsequent to the selection of the unit cell, the final optimized result not only further reduces
 485 the average principal stress by 5.97%, but also limits the increase in maximum von Mises principal stress
 486 to only 4.86% compared to the original structure, thus mitigating local stress concentrations.

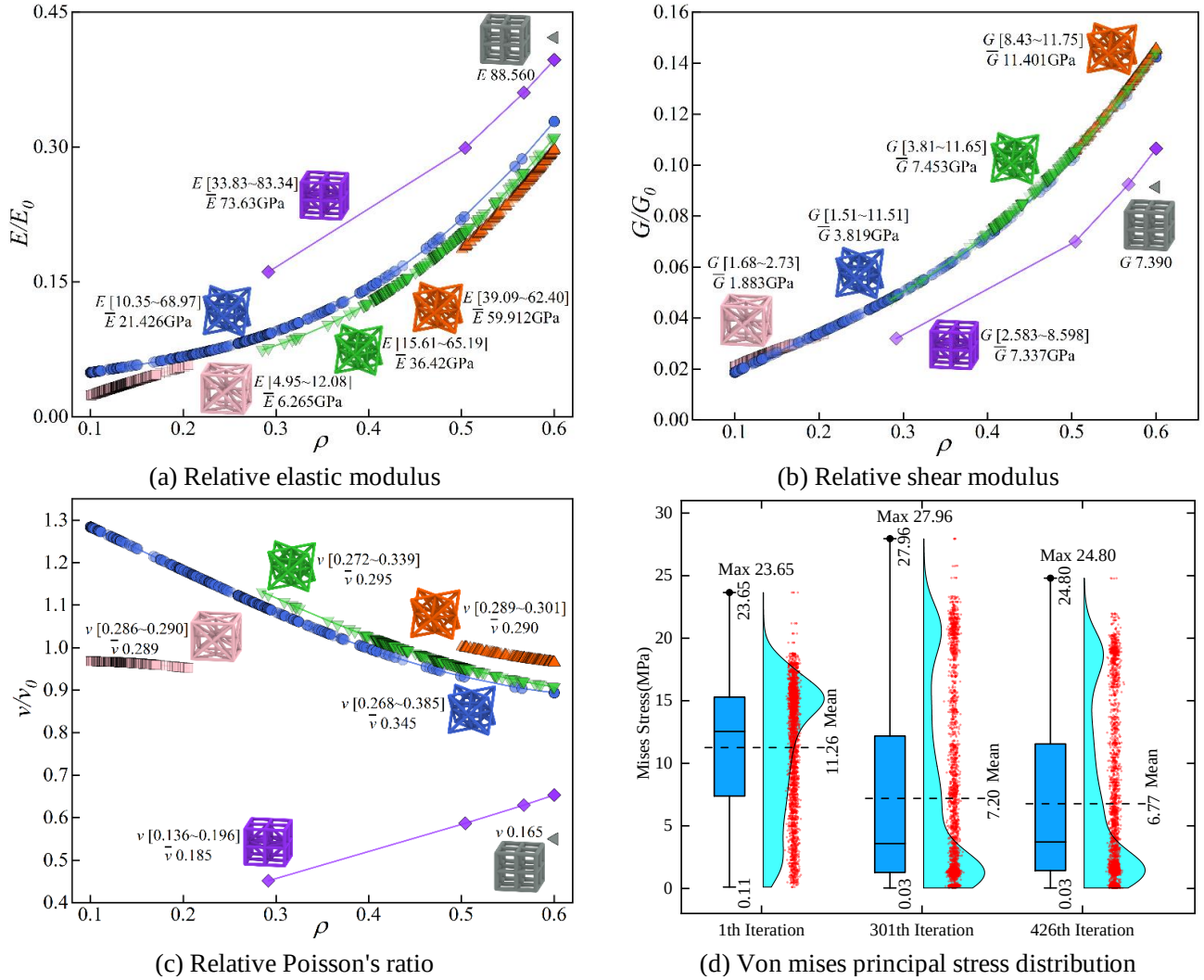
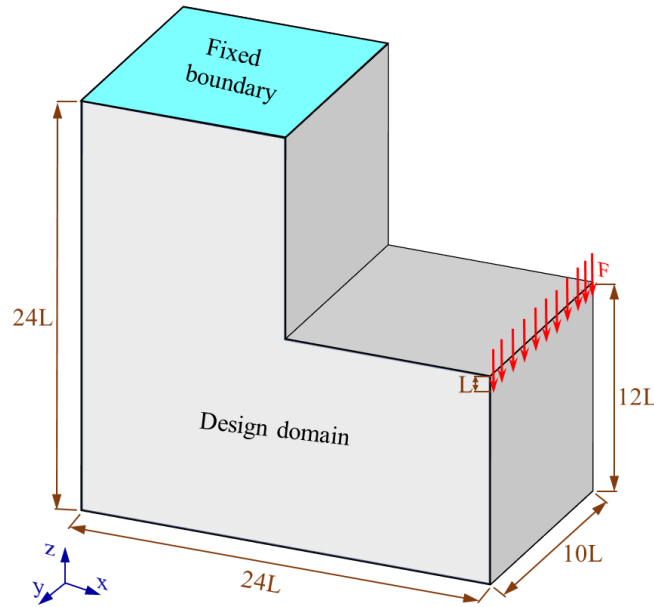


Fig. 14 Mechanical properties of unit cells in final optimized cantilever beam

4.2 L-shaped beam

489 The second case study concerns a optimization design for an L-shaped beam, with density constraints
 490 and volume fractions identical to those in the first example. The upper surface of the design domain is
 491 fixed, and the 500N node force is applied to element nodes on the right surface 10mm from the top edge.
 492 Specific geometric parameters and boundary conditions are illustrated in Fig. 15. The design domain

493 consists of 4,320 elements with a total of 15,873 degrees of freedom. The optimization process took a
 494 total of 43.46 minutes, with the macroscopic topology optimization consuming 1121.9699 seconds,
 495 averaging 6.3032 seconds per iteration, while the microscopic unit cell selection required 1485.5947
 496 seconds. Fig. 16 displays the global compliance update history throughout the entire optimization process,
 497 during which the volume fraction of the structure remained stable. The compliance of the base lattice
 498 structure was initially reduced from 1114.83 mJ to 426.46 mJ through topology optimization.
 499 Subsequently, the introduction of new unit cell types through unit cell selection led to a further reduction
 500 in compliance by -5.754%. Finally, the compliance converged to 400.929 mJ through secondary topology
 501 optimization, demonstrating the scalability of the proposed method for irregular design domains.



502
 503 **Fig. 15** Boundary conditions and design of the L-shaped beam

504 Fig. 17 showcases the geometric model reconstruction of the optimized L-shaped beam result. The
 505 distribution of mechanical properties and densities of the two predominant unit cell types, purple and gray,
 506 is more extensive in Fig. 18 (a-c) due to an increased number of fixed-end elements. All six unit cells
 507 encompass the initial eight base components (Section 2.1). The gray and purple unit cells, primarily
 508 composed of cubes (containing Components A and B), are located in constrained areas, offering a more
 509 direct material distribution for uniaxial stress-bearing. In contrast, the remaining unit cells exhibit FCC
 510 and BCC-like structural features (containing Components C, D, and F), enabling them to better adapt to
 511 complex stress states not dominated by a single stress type. Additionally, some unit cells possess
 512 hypercubic structures (containing Component E), such as the blue and green unit cells, which are

distributed between the cubic types and the BCC, FCC types, aiding in the stress transition within the lattice structure. In conclusion, the unit cell selection methodology presented in this study is reliable and effective. It successfully identifies the optimal unit cells from the library to match stress states in different regions of the lattice structures.

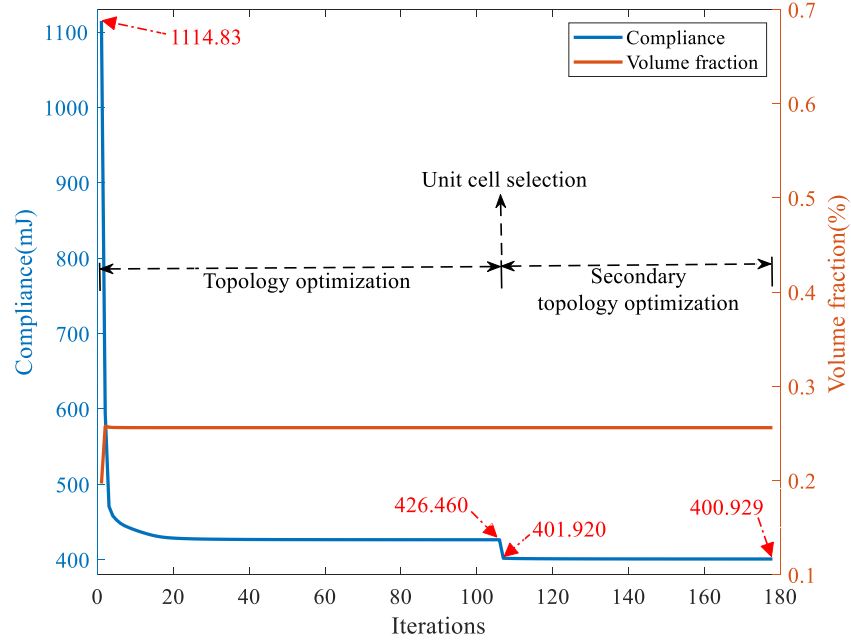


Fig. 16 Iteration histories of objective and constraint function for L-shaped beam

Moreover, the variation in the Von Mises principal stress distribution also demonstrates the validity of the proposed optimization design method and the chosen unit cell results. As indicated in Fig. 18(d), the final optimization result after the introduction of multiple unit cells (178th Iteration) is superior to the initial topology optimization with a single unit cell (106th Iteration), with a reduction of 6.42% in the maximum Mises principal stress and a decrease of 33.77% in the average Mises principal stress compared to the original structure.

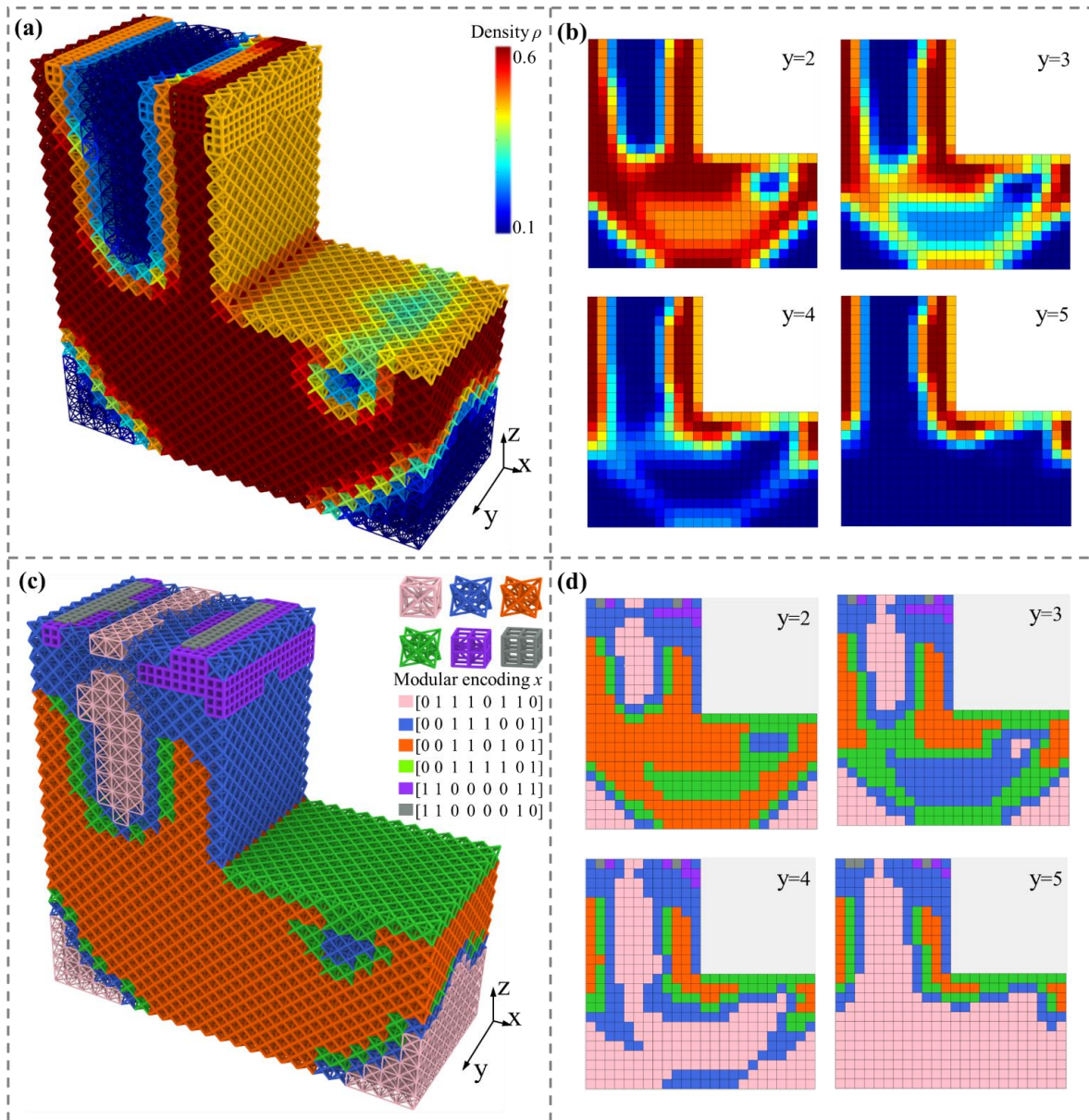


Fig. 17 The geometric model of the final optimization result of optimized L-shaped beam

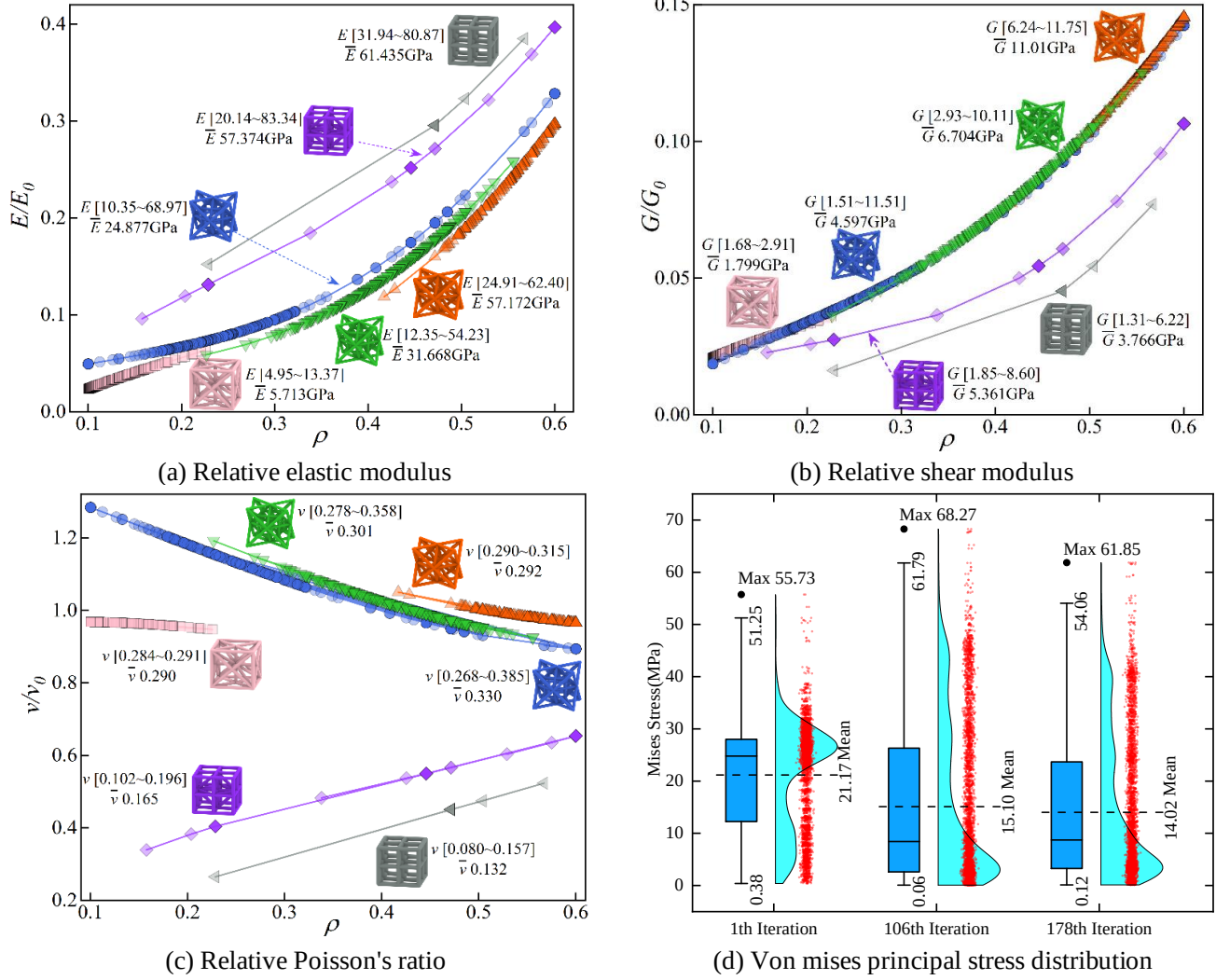


Fig. 18 Mechanical properties of unit cells in final optimized L-shaped beam

4.3 MBB beam

The third numerical example is MBB beam, with the parameters ρ_{min} , ρ_{max} , and η set to 0.2, 0.6, and 0.4, respectively. Fig. 19 presents the specific geometric dimensions and boundary conditions, by introducing symmetry boundary conditions, the design domain is reduced to half of the original structure, and $F=1000\text{N}$ applied to the element nodes. The global compliance variation throughout the optimization process is depicted in Fig. 20. As expected, the global compliance initially converges from 277.659 mJ to 173.869 mJ after initial topology optimization, and then further decreases by 8.76% due to the choice of unit cell, ultimately converging to 157.131 mJ through Secondary topology optimization. The entire optimization process takes only 34.2318 seconds.

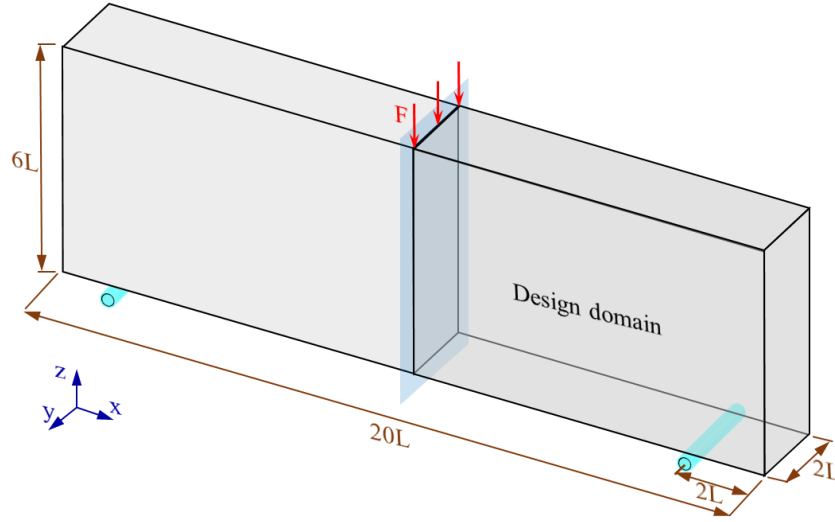


Fig. 19 Boundary conditions and design of the MBB beam

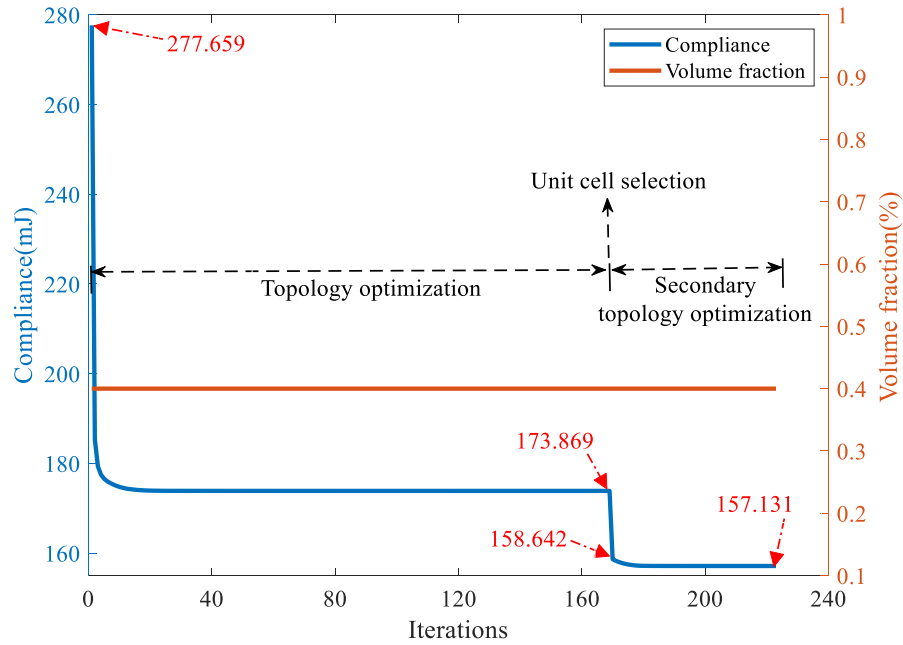


Fig. 20 Iteration histories of objective and constraint function for 3D MBB beam

Fig. 21 displays the geometric reconstruction models of the original structure, the initial topology optimization result, and the final optimization outcome. In the final optimization result, apart from the cubic components retained at the symmetric boundary for the tension and compression sides, the remaining unit cells concentrate material at the face-centered and body-centered cross points, which aligns more closely with the stress conditions of the MBB beam structure. It is noteworthy that due to the increased minimum density, the pink unit cells appearing in the first two examples are absent in this case,

547 indicating that within the present unit cell library, the mechanical performance of this unit cell is not
 548 outstanding at volume fractions above 0.2.

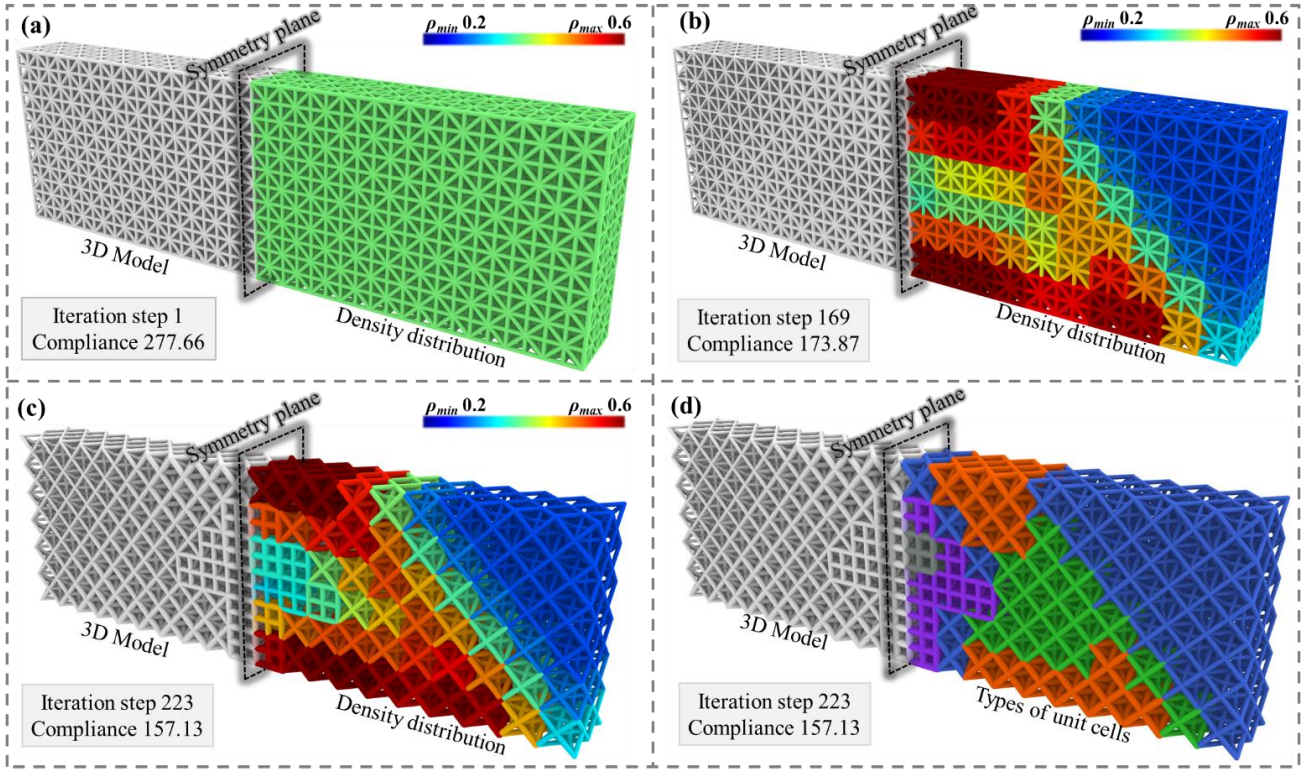


Fig. 21 Topology optimization process for 3D MBB beam

5 Experimental investigations

To further investigate the effectiveness of the proposed method, the additive manufacturing and mechanical testing of the numerical example results is presented in this section. We selected a DLP 3D printer (SimpNeed P1000) and its standard white resin (SimpNeed UV Ultra) for the fabrication of the lattice structures in Appendix B. The detailed physical properties of the printing material are listed in Appendix C below. Mechanical testing was conducted at the Mechanics Experimental Center at Southeast University, using an INSTRON 3367 testing machine to perform three-point bending tests on the MBB beams, with a displacement loading rate of 2 mm/min.

Fig. 22(a-c) illustrates the deformation behavior of three sets of specimens prior to failure. The observation are as follows: The specimen in Case A failed due to localized damage, characterized by micro-rods breaking and shearing off at the supports and under the loading head. This indicates that the design was unable to effectively manage concentrated stresses, leading to premature failure at specific

points. In Case B, where the optimal material distribution was achieved through topology optimization, no significant local fractures were observed. However, the microstructural unit cells experienced significant buckling at the supports. This buckling is attributed to the uniformity of the unit cell type, which may have resulted in inadequate support under compressive loads. Case C demonstrated an effective solution to the issues of local buckling and fracture by incorporating a variety of unit cells. In this case, no significant deformation was observed in the unit cells at the supports and under the loading head prior to failure.

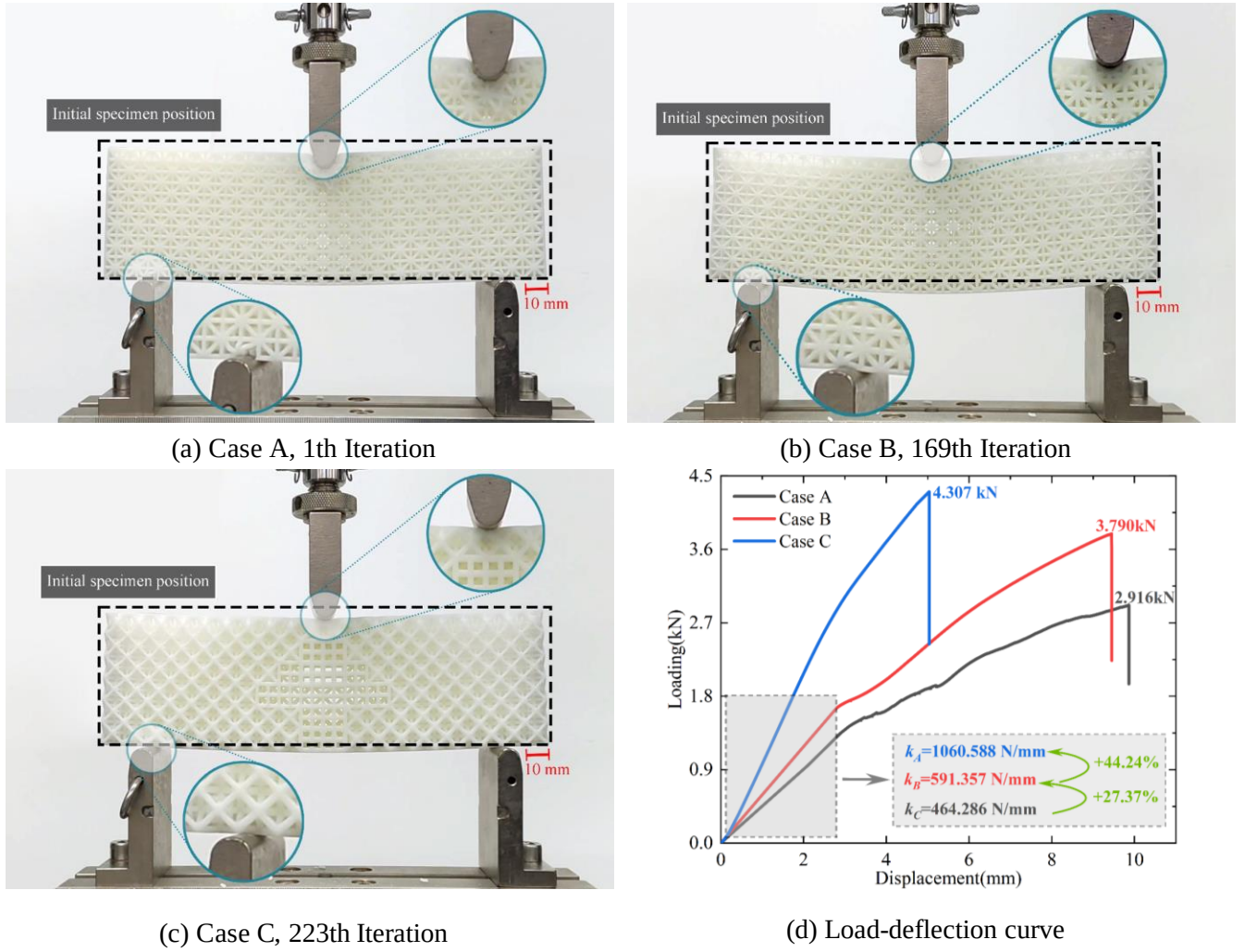


Fig. 22 Test result of MBB beam during three-point bending

Fig. 22(d) records the full load-deflection curves at midspan during testing. It is evident that the final optimization result incorporating unit cell selection (Case C) outperforms the other cases in terms of stiffness and load-bearing capacity, confirming the numerical results presented in Section 4.3. For the initial elastic section, which reflects the stiffness of the overall lattice structure, Case C demonstrated a

575 notable improvement of 44.24% over Case B. Case B itself showed a 27.37% increase in stiffness
576 compared to Case A. Regarding load-bearing capacity, Case C exhibited increases of 13.64% and 47.70%
577 over Cases B and A, respectively. In summary, the optimization design methodology proposed in this
578 study effectively enhances both the initial stiffness and ultimate strength of lattice structures, achieving
579 these improvements without increasing material usage.

580 **6 Limitations and future work**

581 In the realm of numerical computation methods, the voxel-based approach is indeed straightforward;
582 however, it often fails to accurately capture complex three-dimensional stress fields, particularly at the
583 intersections of rods. Future research should focus on developing precise geometric models by
584 parameterizing unit cells using modular encoding and relative density, which could subsequently be
585 applied for homogenization[66].

586 Furthermore, the definition of compliance as presented in Eq.(11) is only applicable under zero
587 Dirichlet boundary conditions. A more comprehensive definition of compliance, based on the total
588 potential energy of the structure, should be considered for optimization designs that involve non-zero
589 Dirichlet boundary conditions or non-homogeneous Neumann-Dirichlet boundary conditions[67,68].
590 Additionally, both existing studies and the current work rarely incorporate the scale separation condition,
591 which is emphasized in the literature[37], into the problem framework and numerical examples to enhance
592 optimization efficiency. Specifically, the characteristic length scales across different sizes often do not
593 vary by 2 or 3 orders of magnitude. This aspect warrants greater attention in future research efforts.

594 On the other hand, while the proposed method exhibits good computational efficiency in benchmark
595 examples, the implicit enumeration method utilized for optimizing the unit cell types results in a linear
596 increase in computational cost as the number of grids increases. Therefore, future work could benefit from
597 the introduction of dynamic clustering algorithms for the optimization design of complex or large-scale
598 structures in practical engineering applications. By categorizing elements into different clusters based on
599 stress vectors derived from density optimization results and performing unit cell type optimization only
600 once for each cluster centered on its core, design efficiency could be significantly enhanced. Moreover,
601 the generation and selection process of the unit cell library in this study did not take into account additional
602 constraints imposed by specific materials and fabrication processes. For example, considerations related

603 to overhang angles and wall thickness when using PLA material for 3D printing should be integrated into
604 the future expansion of this research.

605 **7 Conclusion**

606 In this study, we introduced an innovative approach for creating a digital unit cell library aimed at
607 optimizing the design of multi-morphology lattice structures. This method's primary advantage lies in its
608 ability to quickly generate a vast array of unit cells that not only adhere to connectivity constraints but
609 also exhibit a wide range of mechanical properties. During optimization, an RBFNN surrogate model is
610 utilized to select 133 candidate unit cells, allowing for better adaptation to structural regions with varying
611 stress conditions, thus significantly enhancing mechanical performance without increasing material use.

612 Specifically, the use of modular encoding and an adjacency matrix ensures that both unit cells and
613 the resulting multi-morphology lattice structures comply with connectivity requirements. The integration
614 of modular encoding with the voxel model significantly simplifies the homogenization process for the unit
615 cell library. Instead of relying on regression equations to evaluate the elasticity tensor of individual unit
616 cells, a data-driven RBFNN was employed as a surrogate model. This approach allows for the efficient
617 evaluation of the elasticity tensors for various unit cells. The explicit formulation of the RBFNN also
618 facilitates the rigorous derivation of sensitivity for topology optimization, enabling the precise
619 determination of the optimal volume fraction for each unit cell within the multi-morphology lattice
620 structures.

621 To identify the optimal unit cell for different stress regions within the multi-morphology lattice
622 structure, a 0-1 integer programming model was employed with the objective of minimizing element strain
623 energy. The effectiveness of the framework was demonstrated through three numerical examples, which
624 achieved significant reductions in compliance and efficient calculations. Mechanical tests further
625 validated the framework, showing notable improvements in both stiffness and load-bearing capacity of
626 the optimized multi-morphology lattice structures. The proposed framework is expected to be a valuable
627 computational tool for designing lattice structures in various fields, including civil engineering, aerospace,
628 biomedical, and mechanical engineering.

629 **Appendix A**

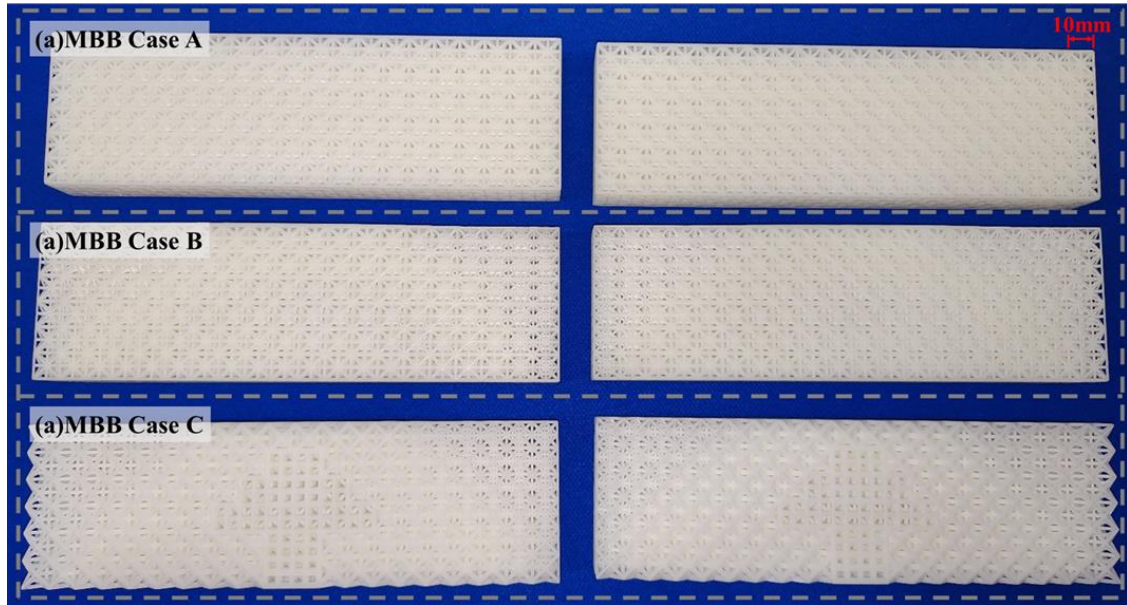
630 The radius calculation process for each unit cell in the multi-morphology lattice structures is as
631 follows:

632 (1) Predefine variables: Set the search range for rod radius to be 0.03 to 0.43 times the unit cell side
633 length L ; generate the lattice wireframe based on the modular encoding optimization results (as shown in
634 Fig. 6).

635 (2) Bisection search: Each update regenerates the voxel model based on the radius and lattice
636 wireframe; use Eq.(6) to compute the unit volume fraction (density) from the voxel model's logical matrix;
637 repeat the above process until the error between the computed volume fraction and the optimized cell
638 density is less than 1%.

639 Appendix B

640 Fig. 23 shows the three sets of MBB beams corresponding to the initial lattice structure (Case A), the
641 density-optimized lattice structure (Case B), and the final optimized result (Case C). To account for
642 potential random errors in mechanical testing, each set of MBB beams included two identical specimens.
643 Each unit cell has a side length of 10 mm, with the thinnest internal micro-rods having a diameter of only
644 0.595 mm. No significant deformation or fracture was observed in the micro-rods of any specimens. The
645 Digital Light Processing (DLP) process was carried out with a scanning speed of 7500 mm/s and a layer
646 thickness of 0.03 mm.



647
648 **Fig. 23** DLP manufacturing results of the MBB beam

649 **Appendix C**

650 **Table 1** The physical characteristics of SPR6000B epoxy resin

Property	Test standard	Magnitude
tensile strength	ASTM D638M	45.1 MPa
compression strength	ASTM D638M	85.3 MPa
elongation at break	ASTM D638	9.3~10.2 %
bending strength	ASTM D790	65.9 MPa
bending modulus	ASTM D790	2178~2210 MPa
Distortion temperature	ASTM D680 045MPa	46.8°C
Poisson's ratio		0.41
Density		1.15 g/cm ³

651 **Declaration of competing interest**

652 The authors declare that they have no known competing financial interests or personal relationships
653 that could have appeared to influence the work reported in this paper.

654 **Data availability**

655 Data will be made available on request.

656 **Acknowledgement**

657 The work described in the present paper is fully funded by a research grant from the National Natural
658 Science Foundation of China (52208151), Natural Science Foundation of Jiangsu Province
659 (BK20210254), 2021 High-level Personnel Project Funding of Jiangsu Province (JSSCBS20210069) as
660 well as the funding from UKRI-EP SRC (EP/Y023455/1). The authors are grateful for the financial
661 support. The damage dataset supporting the conclusions of this article will be made available by the
662 authors, without reservation.

663 **References**

664 [1] Schaedler TA, Ro CJ, Sorensen AE, Eckel Z, Yang SS, Carter WB, et al. Designing Metallic
665 Microlattices for Energy Absorber Applications. Adv Eng Mater 2014;16:276–83.
666 <https://doi.org/10.1002/adem.201300206>.

667 [2] Zhang W, Yin H, Wu Y, Jin Q, Wu L, Wen G, et al. A novel auxetic 3D lattice structure for enhancing
668 energy absorption. *Compos Struct* 2023;326:117620. <https://doi.org/10.1016/j.compstruct.2023.117620>.

669 [3] Lotfi B, Sunden BA. A novel trussed fin-and-elliptical tube heat exchanger with periodic cellular
670 lattice structures. *Int J Numer Methods Heat Fluid Flow* 2022;33:1076–115. [https://doi.org/10.1108/HFF-](https://doi.org/10.1108/HFF-04-2022-0206)
671 04-2022-0206.

672 [4] Zhang P, Li Z, Liu B, Zhou Y, Zhao M, Sun G, et al. Sound absorption performance of micro-
673 perforated plate sandwich structure based on triply periodic minimal surface. *Journal of Materials*
674 *Research and Technology* 2023;27:386–400. <https://doi.org/10.1016/j.jmrt.2023.09.237>.

675 [5] An Q, Li D, Liao W, Liu T, Joralmon D, Li X, et al. A Novel Ultra-Wideband Electromagnetic-
676 Wave-Absorbing Metastructure Inspired by Bionic Gyroid Structures. *Adv Mater* 2023;35:2300659.
677 <https://doi.org/10.1002/adma.202300659>.

678 [6] Puppi D, Mota C, Gazzarri M, Dinucci D, Gloria A, Myrzabekova M, et al. Additive manufacturing
679 of wet-spun polymeric scaffolds for bone tissue engineering. *Biomed Microdevices* 2012;14:1115–27.
680 <https://doi.org/10.1007/s10544-012-9677-0>.

681 [7] Liu C, Pham M. Spatially Programmable Architected Materials Inspired by the Metallurgical Phase
682 Engineering. *Adv Mater* 2023:2305846. <https://doi.org/10.1002/adma.202305846>.

683 [8] Wagner MA, Lumpe TS, Chen T, Shea K. Programmable, active lattice structures: Unifying stretch-
684 dominated and bending-dominated topologies. *Extreme Mech Lett* 2019;29:100461.
685 <https://doi.org/10.1016/j.eml.2019.100461>.

686 [9] Benedetti M, Du Plessis A, Ritchie RO, Dallago M, Razavi N, Berto F. Architected cellular
687 materials: A review on their mechanical properties towards fatigue-tolerant design and fabrication.
688 *Materials Science and Engineering: R: Reports* 2021;144:100606.
689 <https://doi.org/10.1016/j.mser.2021.100606>.

690 [10] Xu S, Shen J, Zhou S, Huang X, Xie YM. Design of lattice structures with controlled anisotropy.
691 *Mater Design* 2016;93:443–7. <https://doi.org/10.1016/j.matdes.2016.01.007>.

692 [11] Alberdi R, Dingreville R, Robbins J, Walsh T, White BC, Jared B, et al. Multi-morphology lattices
693 lead to improved plastic energy absorption. *Mater Design* 2020;194:108883.
694 <https://doi.org/10.1016/j.matdes.2020.108883>.

695 [12] Wang B, Cheng GD, Jiang L. Design of multi-tubular heat exchangers for optimum efficiency of
696 heat dissipation. *Eng Optim* 2008;40:767–88. <https://doi.org/10.1080/03052150802054027>.

- 697 [13] Mahmoudi M, Ghannadpour SAM, Nedjad KH. Functionally graded multi-morphology lattice
698 structures as an optimized sandwich core via digital light processing additive manufacturing. *Mater*
699 *Design* 2024;238:112710. <https://doi.org/10.1016/j.matdes.2024.112710>.
- 700 [14] Peng X, Kuang X, Roach DJ, Wang Y, Hamel CM, Lu C, et al. Integrating digital light processing
701 with direct ink writing for hybrid 3D printing of functional structures and devices. *Addit Manuf*
702 2021;40:101911. <https://doi.org/10.1016/j.addma.2021.101911>.
- 703 [15] Roohani I, Entezari A, Zreiqat H. Liquid crystal display technique (LCD) for high resolution 3D
704 printing of triply periodic minimal surface lattices bioceramics. *Addit Manuf* 2023;74:103720.
705 <https://doi.org/10.1016/j.addma.2023.103720>.
- 706 [16] Egan PF, Khatri NR, Parab MA, Arefin AME. Mechanics of 3D-Printed Polymer Lattices with
707 Varied Design and Processing Strategies. *Polymers* 2022;14:5515.
708 <https://doi.org/10.3390/polym14245515>.
- 709 [17] Karamooz Ravari MR, Kadkhodaei M, Badrossamay M, Rezaei R. Numerical investigation on
710 mechanical properties of cellular lattice structures fabricated by fused deposition modeling. *Int J Mech*
711 *Sci* 2014;88:154–61. <https://doi.org/10.1016/j.ijmecsci.2014.08.009>.
- 712 [18] Veloso F, Gomes-Fonseca J, Morais P, Correia-Pinto J, Pinho ACM, Vilaça JL. Overview of
713 Methods and Software for the Design of Functionally Graded Lattice Structures. *Adv Eng Mater*
714 2022;24:2200483. <https://doi.org/10.1002/adem.202200483>.
- 715 [19] Park K-M, Min K-S, Roh Y-S. Design Optimization of Lattice Structures under Compression: Study
716 of Unit Cell Types and Cell Arrangements. *Materials* 2021;15:97. <https://doi.org/10.3390/ma15010097>.
- 717 [20] Helou M, Kara S. Design, analysis and manufacturing of lattice structures: an overview. *Int J*
718 *Computer Integr Manuf* 2018;31:243–61. <https://doi.org/10.1080/0951192X.2017.1407456>.
- 719 [21] Effective properties of the octet-truss lattice material. *J Mech Phys Solids* 2001;49:1747–69.
720 [https://doi.org/10.1016/S0022-5096\(01\)00010-2](https://doi.org/10.1016/S0022-5096(01)00010-2).
- 721 [22] Lumpe TS, Stankovic T. Exploring the property space of periodic cellular structures based on crystal
722 networks. *Proc Natl Acad Sci* 2021;118:e2003504118. <https://doi.org/10.1073/pnas.2003504118>.
- 723 [23] Jin N, Wang F, Wang Y, Zhang B, Cheng H, Zhang H. Failure and energy absorption characteristics
724 of four lattice structures under dynamic loading. *Mater Design* 2019;169:107655.
725 <https://doi.org/10.1016/j.matdes.2019.107655>.
- 726 [24] Deshpande VS, Ashby MF, Fleck NA. Foam topology: bending versus stretching dominated
727 architectures. *Acta Mater* 2001;49:1035–40. [https://doi.org/10.1016/S1359-6454\(00\)00379-7](https://doi.org/10.1016/S1359-6454(00)00379-7).

[25] Ren X, Xiao L, Hao Z. Multi-property cellular material design approach based on the mechanical
 behaviour analysis of the reinforced lattice structure. *Mater Design* 2019;174:107785.
<https://doi.org/10.1016/j.matdes.2019.107785>.

[26] Bai L, Zhang J, Xiong Y, Chen X, Sun Y, Gong C, et al. Influence of unit cell pose on the mechanical
 properties of Ti6Al4V lattice structures manufactured by selective laser melting. *Addit Manuf*
 2020;34:101222. <https://doi.org/10.1016/j.addma.2020.101222>.

[27] Liu Y, Zhuo S, Xiao Y, Zheng G, Dong G, Zhao YF. Rapid Modeling and Design Optimization of
 Multi-Topology Lattice Structure Based on Unit-Cell Library. *J Mech Des* 2020;142:091705.
<https://doi.org/10.1115/1.4046812>.

[28] Zheng L, Karapiperis K, Kumar S, Kochmann DM. Unifying the design space and optimizing linear
 and nonlinear truss metamaterials by generative modeling. *Nat Commun* 2023;14:7563.
<https://doi.org/10.1038/s41467-023-42068-x>.

[29] Abdullahi HS, Zhuo S, Fu D, Gao S. Design of multi-topology lattice structures based on a high-
 quality unit cell library. *Mechanics Based Design of Structures and Machines* 2024;1–27.
<https://doi.org/10.1080/15397734.2024.2353323>.

[30] Jeong S, Yoo HH. Shape optimization of bowtie-shaped auxetic structures using beam theory.
Compos Struct 2019;224:111020. <https://doi.org/10.1016/j.compstruct.2019.111020>.

[31] Xue H, Luo Z. Design of Auxetic Coronary Stents by Topology Optimization. In: Miller K, Wittek
 A, Joldes G, Nash MP, Nielsen PMF, editors. *Computational Biomechanics for Medicine*, Cham: Springer
 International Publishing; 2020, p. 17–31. https://doi.org/10.1007/978-3-030-42428-2_3.

[32] Gao J, Xue H, Gao L, Luo Z. Topology optimization for auxetic metamaterials based on isogeometric
 analysis. *Comput Methods Appl Mech Eng* 2019;352:211–36. <https://doi.org/10.1016/j.cma.2019.04.021>.

[33] Montemurro M, Refai K, Catapano A. Thermal design of graded architected cellular materials
 through a CAD-compatible topology optimisation method. *Composite Structures* 2022;280:114862.
<https://doi.org/10.1016/j.compstruct.2021.114862>.

[34] Montemurro M, Bertolino G, Panettieri E. Topology optimisation of architected cellular materials
 from additive manufacturing: Analysis, design, and experiments. *Structures* 2023;47:2220–39.
<https://doi.org/10.1016/j.istruc.2022.12.032>.

[35] Montemurro M, Roiné T, Pailhès J. Multi-scale design of multi-material lattice structures through a
 CAD-compatible topology optimisation algorithm. *Engineering Structures* 2022;273:115009.
<https://doi.org/10.1016/j.engstruct.2022.115009>.

759 [36] Montemurro M, Bertolino G, Roiné T. A general multi-scale topology optimisation method for
 760 lightweight lattice structures obtained through additive manufacturing technology. *Composite Structures*
 761 2021;258:113360. <https://doi.org/10.1016/j.compstruct.2020.113360>.

762 [37] Bertolino G, Montemurro M. Two-scale topology optimisation of cellular materials under mixed
 763 boundary conditions. *International Journal of Mechanical Sciences* 2022;216:106961.
 764 <https://doi.org/10.1016/j.ijmecsci.2021.106961>.

765 [38] Chen D, Gao K, Yang J, Zhang L. Functionally graded porous structures: Analyses, performances,
 766 and applications – A Review. *Thin-Walled Structures* 2023;191:111046.
 767 <https://doi.org/10.1016/j.tws.2023.111046>.

768 [39] Chen D, Gao K, Yang J, Kitipornchai S. An introduction to functionally graded porous materials and
 769 composite structures. *Machine Learning Aided Analysis, Design, and Additive Manufacturing of*
 770 *Functionally Graded Porous Composite Structures*, Elsevier; 2024, p. 3–15. [https://doi.org/10.1016/B978-](https://doi.org/10.1016/B978-0-443-15425-6.00001-8)
 771 [0-443-15425-6.00001-8](https://doi.org/10.1016/B978-0-443-15425-6.00001-8).

772 [40] Zou Z, Liu J, Gao K, Chen D, Yang J, Wu Z. Inverse design of functionally graded porous structures
 773 with target dynamic responses. *International Journal of Mechanical Sciences* 2024;280:109530.
 774 <https://doi.org/10.1016/j.ijmecsci.2024.109530>.

775 [41] Seoane B, Castellanos S, Dikhtiarenko A, Kapteijn F, Gascon J. Multi-scale crystal engineering of
 776 metal organic frameworks. *Coord Chem Rev* 2016;307:147–87.
 777 <https://doi.org/10.1016/j.ccr.2015.06.008>.

778 [42] Provatas N, Dantzig JA, Athreya B, Chan P, Stefanovic P, Goldenfeld N, et al. Using the phase-field
 779 crystal method in the multi-scale modeling of microstructure evolution. *JOM* 2007;59:83–90.
 780 <https://doi.org/10.1007/s11837-007-0095-3>.

781 [43] Wu J, Sigmund O, Groen JP. Topology optimization of multi-scale structures: a review. *Struct*
 782 *Multidiscip Optim* 2021;63:1455–80. <https://doi.org/10.1007/s00158-021-02881-8>.

783 [44] Zhang J, An Q. Topology optimization of fibre reinforced polymer lattice structures for additive
 784 manufacturing. *Compos Sci Technol* 2023;242:110144.
 785 <https://doi.org/10.1016/j.compscitech.2023.110144>.

786 [45] Chen L, Che J, Liang S, Wang Y. Multiscale topology optimization of gradient lattice structure based
 787 on volume parametric modeling. *Compos Struct* 2024;328:117746.
 788 <https://doi.org/10.1016/j.compstruct.2023.117746>.

789 [46] Simsek U, Gayir CE, Kiziltas G, Sendur P. An integrated homogenization-based topology
790 optimization via RBF mapping strategies for additively manufactured FGLS and its application to bandgap
791 structures. *Int J Adv Manuf Technol* 2020;111:1361–74. <https://doi.org/10.1007/s00170-020-06207-8>.

792 [47] Wu C, Luo J, Zhong J, Xu Y, Wan B, Huang W, et al. Topology Optimisation for Design and
793 Additive Manufacturing of Functionally Graded Lattice Structures using Derivative-Aware Machine
794 Learning Algorithms. *Addit Manuf* 2023;103833. <https://doi.org/10.1016/j.addma.2023.103833>.

795 [48] Wang S, Zhang J, Zhou H, Zhang L, Cao K. Multi-scale Topology Optimization for Manufacturable
796 Graded Lattice Structure via Neural Network Prediction Model. In Review; 2023.
797 <https://doi.org/10.21203/rs.3.rs-2903295/v1>.

798 [49] Wang C, Zhu JH, Zhang WH, Li SY, Kong J. Concurrent topology optimization design of structures
799 and non-uniform parameterized lattice microstructures. *Struct Multidiscip Optim* 2018;58:35–50.
800 <https://doi.org/10.1007/s00158-018-2009-0>.

801 [50] Zhang C, Liu J, Yuan Z, Xu S, Zou B, Li L, et al. A novel lattice structure topology optimization
802 method with extreme anisotropic lattice properties. *J Comput Des Eng* 2021;8:1367–90.
803 <https://doi.org/10.1093/jcde/qwab051>.

804 [51] Zhang C, Wu T, Xu S, Liu J. Multiscale topology optimization for solid-lattice-void hybrid
805 structures through an ordered multi-phase interpolation. *Comput-Aided Des* 2023;154:103424.
806 <https://doi.org/10.1016/j.cad.2022.103424>.

807 [52] Zhao M, Li X, Zhang DZ, Zhai W. TPMS-based interpenetrating lattice structures: Design,
808 mechanical properties and multiscale optimization. *Int J Mech Sci* 2023;244:108092.
809 <https://doi.org/10.1016/j.ijmecsci.2022.108092>.

810 [53] Costa MR, Sohouli A, Suleman A. Multi-scale and multi-material topology optimization of gradient
811 lattice structures using surrogate models. *Compos Struct* 2022;289:115402.
812 <https://doi.org/10.1016/j.compstruct.2022.115402>.

813 [54] Liu X, Gao L, Xiao M. Multiscale concurrent topology optimization of hierarchal multi-morphology
814 lattice structures. *Comput Methods Appl Mech Eng* 2023;415:116209.
815 <https://doi.org/10.1016/j.cma.2023.116209>.

816 [55] Liu X, Gao L, Xiao M. Multiscale topology optimization framework for natural frequency
817 maximization of multi-morphology lattice structures. *Compos Struct* 2024;328:117720.
818 <https://doi.org/10.1016/j.compstruct.2023.117720>.

819 [56] Zok FW, Latture RM, Begley MR. Periodic truss structures. *J Mech Phys Solids* 2016;96:184–203.
820 <https://doi.org/10.1016/j.jmps.2016.07.007>.

821 [57] Martínez Cendrero A, Franco Martínez F, Solórzano Requejo WG, Díaz Lantada A. Open-source
822 library of tissue engineering scaffolds. *Mater Design* 2022;223:111154.
823 <https://doi.org/10.1016/j.matdes.2022.111154>.

824 [58] Lee D, Chen WW, Wang L, Chan Y-C, Chen W. Data-Driven Design for Metamaterials and
825 Multiscale Systems: A Review. *Adv Mater* 2024;36:2305254. <https://doi.org/10.1002/adma.202305254>.

826 [59] Alzahrani M, Choi S-K, Rosen DW. Design of truss-like cellular structures using relative density
827 mapping method. *Mater Design* 2015;85:349–60. <https://doi.org/10.1016/j.matdes.2015.06.180>.

828 [60] Zhou H, Zhang DZ, He N, Zhao M. Topology optimization of multi-morphology composite lattice
829 structure with anisotropy properties. *Compos Struct* 2023;321:117294.
830 <https://doi.org/10.1016/j.compstruct.2023.117294>.

831 [61] Dong G, Tang Y, Zhao YF. A 149 Line Homogenization Code for Three-Dimensional Cellular
832 Materials Written in matlab. *J Eng Mater Technol* 2019;141:011005. <https://doi.org/10.1115/1.4040555>.

833 [62] Andreassen E, Andreassen CS. How to determine composite material properties using numerical
834 homogenization. *Comput Mater Sci* 2014;83:488–95. <https://doi.org/10.1016/j.commatsci.2013.09.006>.

835 [63] Sigmund O. A 99 line topology optimization code written in Matlab. *Struct Multidiscip Optim*
836 2001;21:120–7. <https://doi.org/10.1007/s001580050176>.

837 [64] Sigmund O, Petersson J. Numerical instabilities in topology optimization: A survey on procedures
838 dealing with checkerboards, mesh-dependencies and local minima. *Struct Optimization* 1998;16:68–75.
839 <https://doi.org/10.1007/BF01214002>.

840 [65] Ferrari F, Sigmund O. A new generation 99 line Matlab code for compliance topology optimization
841 and its extension to 3D. *Struct Multidisc Optim* 2020;62:2211–28. [https://doi.org/10.1007/s00158-020-](https://doi.org/10.1007/s00158-020-02629-w)
842 [02629-w](https://doi.org/10.1007/s00158-020-02629-w).

843 [66] Refai K, Montemurro M, Brugger C, Saintier N. Determination of the effective elastic properties of
844 titanium lattice structures. *Mechanics of Advanced Materials and Structures* 2020;27:1966–82.
845 <https://doi.org/10.1080/15376494.2018.1536816>.

846 [67] Montemurro M. On the structural stiffness maximisation of anisotropic continua under
847 inhomogeneous Neumann–Dirichlet boundary conditions. *Composite Structures* 2022;287:115289.
848 <https://doi.org/10.1016/j.compstruct.2022.115289>.

849 [68] Montemurro M, Rodriguez T, Pailhès J, Le Texier P. On multi-material topology optimisation
850 problems under inhomogeneous Neumann–Dirichlet boundary conditions. *Finite Elements in Analysis*
851 *and Design* 2023;214:103867. <https://doi.org/10.1016/j.finel.2022.103867>.
852