International Commodity-Tax Competition and Asymmetric Producer Prices*

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Abstract: In this paper, I extend the seminal commodity-tax competition model of Kanbur and Keen (1993) letting asymmetric producer prices between countries of different sizes. Unlike Kanbur and Keen, I show that there are multiple equilibria, and at some equilibria, small-country consumers cross-shop from the large. Besides, tax harmonization can benefit the small country, and a minimum tax rate can decrease a country's tax revenue.

Keywords: Commodity-tax competition, cross-border shopping, asymmetric prices, tax harmonization, minimum tax rate.

JEL Classification Numbers: H20, H77

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1 Introduction

The free movement of goods allows consumers to engage in cross-border shopping in many countries, including the US, Canada, and the European Union. The ease of international travel between some countries, such as Bulgaria and Turkey further extends shopping across borders. This has been putting pressure on the commodity-tax rates of national governments for a long time by moving their tax bases. As a result, various discussions have been arising among exposed countries on coordination of tax policies.[1]

Motivated by these facts, a large literature in economics has developed models to analyze the effects of competition and coordination between countries on commodity taxes. These models mainly depart from the seminal model of Kanbur and Keen (1993), called as *KK* in the following, and assume that a commodity's producer price is the same in all countries. However, producer prices can differ among countries due to different input costs or technology levels. Additionally, governments can impose different taxes on production, such as environmental taxes as discussed in Cremer and Gahvari (2006), which can differ producer prices. The exchange rate can also cause a difference in prices as in Asplund et al. (2007). Di Matteo and Di Matteo (1996) state that this is an important determinant of cross-border shopping between the US and Canada, and DHA (2022) reports that depreciating Turkish Lira increased cross-border sales from Turkey to Bulgaria substantially in recent years. Thus, the equal producer prices assumption restricts the analysis in the literature. To overcome this restriction, I extend KK's model allowing different producer prices between countries.

In my model, there are two countries with different population sizes. There is a single commodity and the governments impose a unit tax on it to maximize their tax revenues. So, for a consumer, the price of the commodity is the sum of its producer price and the tax. Different from KK, producer prices can be asymmetric between the countries. If the border is closed, people can buy the commodity only from their own country. So, countries choose their tax rates independently. However, if the border is open, people can also shop in the foreign country by paying a travel cost. In this case, countries determine their tax rates considering each other's rates. This defines a simultaneous-move game and I analyse its pure-strategy Nash equilibria. I compare closed- and open-border taxes, and evaluate the effects of tax-coordination policies. As in KK, I investigate tax harmonization and minimum tax rate policies. Under harmonization, countries choose the same tax rate, and under the minimum tax rate, they set their taxes above a pre-determined minimum.

Different from KK, depending on the parameter values, we can have a unique equilibrium, multiple equilibria, or no equilibrium in the model. At some equilibria, the small country chooses a higher tax rate yet its consumers cross-shop from the large. Besides, tax harmonization can benefit the small country and a minimum tax rate can reduce tax revenues.

My paper mainly contributes to the literature on international commodity-tax competition. The first paper on this topic was Mintz and Tulkens (1986). Because their model was very general it did

[1] See, for example, Cnossen (2003) for a survey and Šemeta (2011) for a speech.

not allow precise predictions. KK developed a more specific model giving sharper results. Their main focus was on competition between two countries with different population sizes. Later models mainly departed from KK and analyzed commodity-tax competition in different environments. For example, Trandel (1994) developed a model to investigate the effects of asymmetric population distribution between countries on taxes. Haufler (1996) built a model in which people get utility from a public good as well as a private one. Ohsawa (1999) analyzed competition among more than two countries. Nielsen (2001) developed a model in which the countries differ in their geographical sizes and Agrawal (2012) allowed multiple regions in a country. Lastly, Devereux et al. (2007) built a model allowing price-elastic demands. However, all the papers in the literature assumed that producer prices are the same in different countries. I contribute to this literature by analyzing commodity-tax competition under asymmetric producer prices.

Behrens et al. (2009) developed a commodity-tax-competition model in which producer prices can differ between countries but in a limited manner. Specifically, in their model, for the consumers of one country, the other country's producer prices are always higher because of a trade tax. Besides, different from this paper they analyzed international trade, not cross-border shopping. Karakosta (2018) also has a similar model in which producer prices can differ due to an emission tax but again without cross-border shopping. My model differs from theirs by letting shopping between borders and any level of difference in producer prices.[2]

The paper proceeds as follows. Section 2 presents the model, and Section 3 analyses the equilibria, Section 4 investigates tax-coordination policies, and finally Section 5 concludes. The proofs of all lemmas and propositions are given in the appendix.

2 Model

As in KK, I employ a partial-equilibrium model with two countries and a single-taxed good. The countries are located on the real line. The "large" country extends from -1 to 0 and the "small" from 0 to 1. I call them small and large because of their populations which are distributed uniformly with density h in the small country and H in the large such that $H \geq h > 0$. So, the large country's population is more than the small.

The producer price of the good is given as p in the small country and as P in the large, where $p, P > 0$. Different from KK, I allow $p \neq P$. Producer prices can be different in the two countries due to different marginal costs of production. This may stem from differences in input costs (wages, rents, etc.), production technologies, or government policies (taxes or subsidies on production). The small country imposes a unit tax $t \geq 0$ on the goods sold in its borders and the large country a unit

[2]There is also a large literature on capital-tax competition. Some recent papers in this literature are Cai and Treisman (2005), Kempf and Rota-Graziosi (2010), Ogawa (2013), Keen and Konrad (2013),Haufler and Lulfesmann (2015), Hindriks and Nishimura (2015), Pi and Chen (2017), Eichner and Pethig (2015). This literature lets asymmetries between countries in various dimensions including production technologies which can lead to different producer prices.

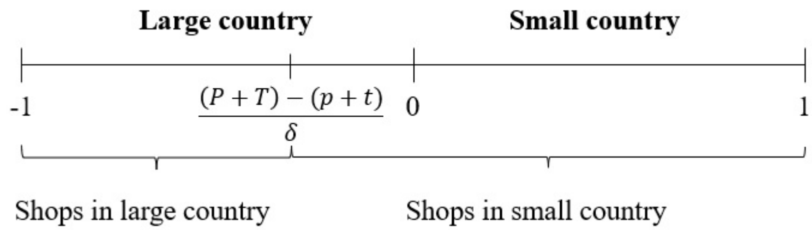


Figure 1: Cross-border shopping when the good is cheaper in the small country—that is, $p+t < P+T$.

tax $T \geq 0$. The value of the good is v for the small-country consumers and V for the large country, where $v, V > 0$. Note that I do not make any assumption about producer prices or consumer valuations in the two countries other than that they are positive. They can be different or equal in the two countries, and if they are different, they can be higher in any country. So, "small" and "large" refer only to the population of the countries. Each consumer buys a unit of the good if its consumer price, which is equal to the sum of producer price and unit tax, is less than or equal to its value. To be more precise, the consumer price is $p + t$ in the small country and $P + T$ in the large. I call the consumer price of the good simply as *price* from now on. If the border between the countries is closed, consumers can buy the goods only from their country producers located at each point in the country. If the border is open, consumers can buy the good either from their own country or from the other at the border. If they buy the good from their own country, they pay no transportation cost. However, if they buy it from the other country, they pay a transportation cost of $\delta > 0$ for each unit of distance they travel, including the return cost.

A small-country consumer living at a distance s from the border buys the good from the large if and only if its benefit is non-negative and higher than buying it from the small:

$$0 \leq v - (P + T) - \delta s \tag{1}$$

and

$$v - (p + t) \leq v - (P + T) - \delta s. \tag{2}$$

Equation 2 implies that

$$s \leq \frac{(p + t) - (P + T)}{\delta};$$

the analogous condition for the large country is $s \leq \frac{(P+T)-(p+t)}{\delta}$. Figure 1 illustrates cross-border shopping when the good is cheaper in the small country.

As in KK, both country governments aim to maximize tax revenue.[3] I assume that the value of

[3] In the literature of commodity tax competition while some papers, including KK, Ohsawa (1999), Nielsen (2001), Devereux et al. (2007), assume that the governments aim to maximize total tax revenue, some other papers, such as Haufler (1996) and Agrawal (2012), assume that governments aim to maximize social welfare. The former approach

the good is high enough for the consumers so that they buy the good at imposed tax rates. Under this assumption when the border is closed each country chooses the maximum tax rate under the constraint that the consumers buy the good. So, the small country imposes a tax of $v - p$ and the large $V - P$. The small country's tax revenue is $(v - p)h$ and the large's $(V - P)H$. When the border is open the two countries choose their tax rates simultaneously considering each other's actions, which transforms the model into a game. I investigate the pure-strategy Nash equilibria. I assumed that taxes are non-negative. But even if they can be negative, which we can interpret as subsidies, my results will be the same since governments aim to maximize revenue therefore they will never choose a negative rate.

3 Nash Equilibria

To analyze the Nash equilibria of the model, I first derive both countries' best-response correspondences. When the border is open, the small country's revenue function is

$$r(t,T) = \begin{cases} \max\left\{0, th\left(1 - \frac{(p+t)-(P+T)}{\delta}\right)\right\} & \text{if } P+T \leq p+t, \\ \min\left\{th + tH\frac{(P+T)-(p+t)}{\delta}, t(h+H)\right\} & \text{if } p+t \leq P+T. \end{cases}$$

If the good's price is higher in the small country, some of the small-country consumers shop in the large. So, the small country collects tax only from its remaining consumers. If the price in the small country is too high, all of its consumers shop in the large. Then, the small country can not collect any tax. If the price in the small country is lower, the opposite occurs. A similar interpretation also applies to the large country's revenue function:

$$R(t,T) = \begin{cases} \max\left\{0, TH\left(1 - \frac{(P+T)-(p+t)}{\delta}\right)\right\} & \text{if } p+t \leq P+T, \\ \min\left\{TH + Th\frac{(p+t)-(P+T)}{\delta}, T(H+h)\right\} & \text{if } P+T \leq p+t. \end{cases}$$

Using the revenue functions, we can derive the best-response correspondences as given in Lemma 1. Note that $\theta = \frac{h}{H}$ denotes the ratio of population densities.

follows public-choice literature motivated by the idea that governments are Leviathans. I also take this approach since it can better describe the behavior of governments. Additionally, as discussed by KK, if we assume that governments use tax revenues to maximize social welfare by providing a public good with a high enough value, the two approaches coincide.

Lemma 1. *The small country's best-response correspondence, $t(T)$, is*

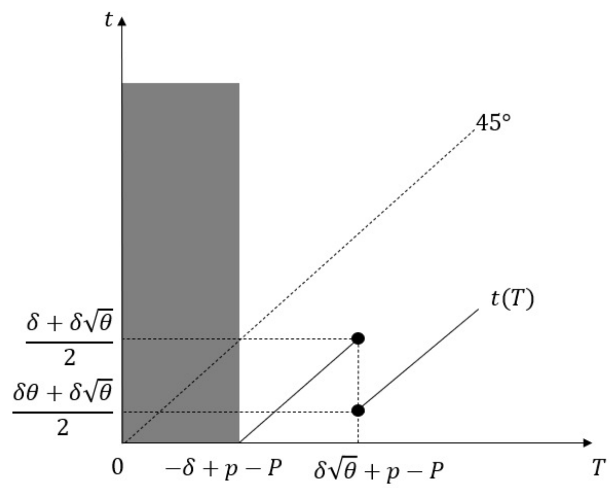
$$t(T) = \begin{cases} [0, \infty) & \textit{if } T \leq -\delta + p - P, \\ \frac{\delta + P + T - p}{2} & \textit{if } -\delta + p - P < T < \delta\sqrt{\theta} + p - P, \\ \left\{ \frac{\delta + P + T - p}{2}, \frac{\delta\theta + P + T - p}{2} \right\} & \textit{if } T = \delta\sqrt{\theta} + p - P, \\ \frac{\delta\theta + P + T - p}{2} & \textit{if } \delta\sqrt{\theta} + p - P < T, \end{cases} \tag{3}$$

and the large country's, $T(t)$, is

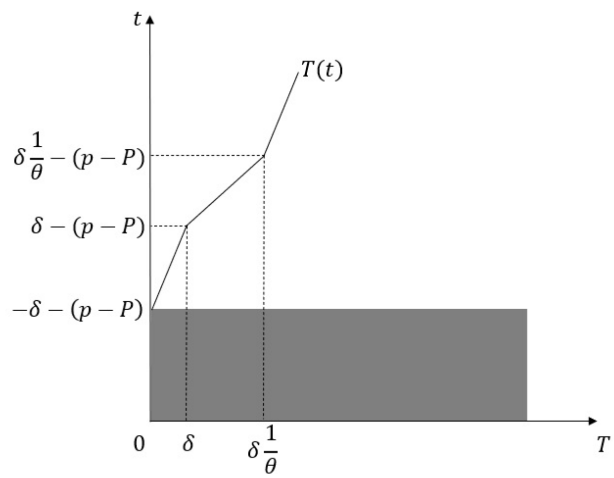
$$T(t) = \begin{cases} [0, \infty) & \textit{if } t \leq -\delta - (p - P), \\ \frac{\delta + p + t - P}{2} & \textit{if } -\delta - (p - P) < t \leq \delta - (p - P), \\ p + t - P & \textit{if } \delta - (p - P) < t \leq \delta\frac{1}{\theta} - (p - P), \\ \frac{\delta\frac{1}{\theta} + p + t - P}{2} & \textit{if } \delta\frac{1}{\theta} - (p - P) < t. \end{cases} \tag{4}$$

Figure 2 illustrates the best-response correspondences presented in Lemma 1. As seen in the figure, the two countries' best-response correspondence takes different shapes. This is because of their sizes. In Figure 2a, if the large country's tax rate is low enough, any rate is optimal for the small. Because, whatever the small's tax rate, all consumers shop from the large since the price in the large is much lower. If the large country's tax rate is moderate, small's best action is choosing a high enough tax rate so that it collects the maximum revenue from its consumers that shop domestically. In this region, if the large country chooses a higher tax rate, so does the small. However, if the large country's tax rate is high enough, it is more profitable for the small to choose a low enough tax rate to attract large-country consumers. This maximizes its tax revenue by benefiting from the high population of the large. This policy shift creates a discontinuity in the small's best-response correspondence. In Figure 2b, if the small country's tax rate is low enough, any rate is optimal for the large. As the small country's tax rate increases, so does the large country's. In this case, we do not observe any discontinuity in the large country's best response. Because for the large there is no opportunity to benefit from the high population of the small.

Before moving to the equilibria, I discuss how the relative size of the countries, θ, and their price difference, $p - P$, affect best-response correspondences. As I presented in Lemma 1, both countries' best-response correspondences are piecewise. Within each piece, relative size and the price difference of the countries have a similar effect on the best responses. Specifically, in each piece of the small country's best response, while θ has a non-decreasing effect, $p - P$ has a non-increasing effect. For example, when $\delta\sqrt{\theta} + p - P < T$, $t(T)$ increases with θ and decreases with $p - P$. For the large country, in each piece, while θ has a non-increasing effect, $p - P$ has a non-decreasing effect: For example, when $\delta\frac{1}{\theta} - (p - P) < t$, $T(t)$ decreases with θ and increases with $p - P$. However, on the whole domain of the best responses relative size and price difference have different effects: They



(a) The small country's best-response correspondence when its producer price is high enough ($\delta < p - P$).



(b) The large country's best-response correspondence when its producer price is high enough ($p - P < -\delta$).

Figure 2: Best-response correspondences of the small and large countries.

affect the occurrence of different pieces. Specifically, if we did not include θ in the model by assuming that it is equal to one, we would lose the discontinuity in the small's best response. This can be seen in Figure 2a clearly. If $\theta = 1$, the discontinuity points overlap. Similarly, if $\theta = 1$, the piece of the large country's best response defined over $t \in \left[\delta - (p - P), \delta\frac{1}{\theta} - (p - P)\right]$ does not exist. On the other hand, the price difference generates the multi-valued pieces of the correspondences defined over the low rates of the other country—the shaded areas in Figure 2. If prices were equal—that is, $p - P = 0$—we would lose these pieces since the domain that they are defined over would disappear. Therefore, including both θ and $p - P$ in the model is necessary to understand different effects. Below, I also discuss how they affect Nash equilibria.

To continue, I call a pure-strategy Nash equilibrium simply as an *equilibrium* and denote the small country's equilibrium tax rate with t_N and the large country's with T_N.

Proposition 1. *The equilibria of the model are given as follows.*

i) If the small country's producer price is lower enough than the large country's—that is, $p - P \leq -2\delta - \delta\theta$*—there exists a continuum of equilibria with* $T_N \in [0, -2\delta - \delta\theta - (p - P)]$ *and* $t_N = \frac{\delta\theta + P + T_N - p}{2}$.

ii) If the small country's producer price is close to the large country's—that is, $-2\delta - \delta\theta < p - P \leq \frac{2\delta + \delta\theta - 3\delta\sqrt{\theta}}{2}$*—there exists a unique equilibrium with* $t_N = \frac{2\delta\theta + \delta - (p-P)}{3}$ *and* $T_N = \frac{2\delta + \delta\theta + p - P}{3}$.

iii) If the small country's producer price is higher than the large country's by a small amount—that is, $\frac{2\delta + \delta\theta - 3\delta\sqrt{\theta}}{2} < p - P < \max\left\{\delta, \frac{2\delta\frac{1}{\theta} + \delta - 3\delta\sqrt{\theta}}{2}\right\}$*—there is no equilibrium.*

iv) If the small country's producer price is higher than the large country's by a moderate amount—that is, $\max\left\{\delta, \frac{2\delta\frac{1}{\theta} + \delta - 3\delta\sqrt{\theta}}{2}\right\} \leq p - P \leq \delta\frac{1}{\theta} + 2\delta$*—there exists a unique equilibrium with* $t_N = \frac{2\delta + \delta\frac{1}{\theta} - (p-P)}{3}$ *and* $T_N = \frac{2\delta\frac{1}{\theta} + \delta + p - P}{3}$.

v) If the small country's producer price is much higher than the large country—that is, $\delta\frac{1}{\theta} + 2\delta < p - P$*—there exists a continuum of equilibria with* $t_N \in \left[0, -2\delta - \delta\frac{1}{\theta} + p - P\right]$ *and* $T_N = \frac{\delta\frac{1}{\theta} + p + t_N - P}{2}$.

As I state in the first part of Proposition 1 if the small country's producer price is low enough, we have a continuum of equilibria. This is because, whatever the large country's tax rate, all consumers shop in the small. Thus, any tax rate is optimal for the large. Similarly, a continuum of equilibria arises when the small country's producer price is much higher. At these equilibria, all small-country consumers cross-shop in the large. If the small's producer price is close to the large's or moderately higher than it, we have a unique equilibrium. Finally, if the small's producer price is slightly higher, then no equilibrium exists. KK assume that the producer prices are the same in the two countries, that is, $p - P = 0$. Therefore, in their model, there is a unique equilibrium.

How does the price difference, $p - P$, lead to different types of equilibria? As we see in Lemma 1, both countries' best-response correspondences are piecewise. Price difference determines the boundaries of the intervals that each piece is defined over. So, change in it shifts the domain of each piece and this creates different types of equilibria. In Figure 3, I illustrate the equilibria in Proposition 1 for all values of $p - P$. In the graphs the intersection points of $t(T)$ and $T(t)$ denote the equilibrium points. For Parts (ii) and (iv) there are multiple graphs in the figure, but the equilibrium taxes are the same under each case as given in Proposition 1. Part (iii) also has two cases—both without any equilibrium. In the figure, we can follow how price difference, $p - P$, shifts best responses. As the price difference increases from Part (i) to (v), the large country's best response, $T(t)$, shifts down. We observe a similar shift towards left in the small country's best response as the price difference decreases from Part (v) to (i). These shifts lead to the different types of equilibria or non-existence of equilibrium given in Proposition 1.

In the following, I call the unique equilibrium in the second part of Proposition 1 *Type A equilibrium*, the unique equilibrium in the fourth part *Type B*, the multiple equilibria in the first part *Type C*, and the multiple equilibria in the fifth part *Type D*.

How do relative size, θ, and producer price difference, $p - P$, affect the equilibria? Firstly, even if we assume that the countries have the same sizes—that is, $\theta = 1$—we will still have all types of equilibria in Proposition 1 including the non-existence result. It is the asymmetric producer prices that create different types of equilibria in the model. If the countries had the same producer prices, we would only have Type A equilibrium. However, both θ and $p - P$ affect equilibrium tax rates. Their effects can be in the same or opposite directions. For example, at Type A equilibrium, while θ increases the tax rate of the small country, $p - P$ decreases it. At Type B equilibrium, they both decrease it. For the large country, they both increase tax rate at Type A equilibrium. But at Type B equilibrium, while θ decreases the tax rate, $p - P$ increases it. Therefore, it is important to include both parameters in the model.

Proposition 2. *Under Type A equilibrium, there are cross-border sales from the small country to the large. Additionally, the small country's tax rate is lower than the large if and only if its producer price is not too much lower than the large—that is,* $t_N < T_N$ *if and only if* $-\frac{1}{2}\delta(1 - \theta) < p - P$.

Under the Type A equilibrium, the price of the good in the small country is lower than in the large. Thus, large-country consumers close to the border shop in the small. At this equilibrium, to benefit from the large country's high population, the small country chooses a tax rate to attract some large-country consumers. This result is in line with one of the main conclusions of the preceding literature, including KK and Trandel (1994).

At this equilibrium, cross-border sales from the small country to the large do not necessarily imply that the small country's tax rate is lower than the large. The small country's tax rate is lower than the large if and only if its producer price is not too low. If its producer price is lower enough than the large, small country imposes a higher tax rate than the large and still attracts

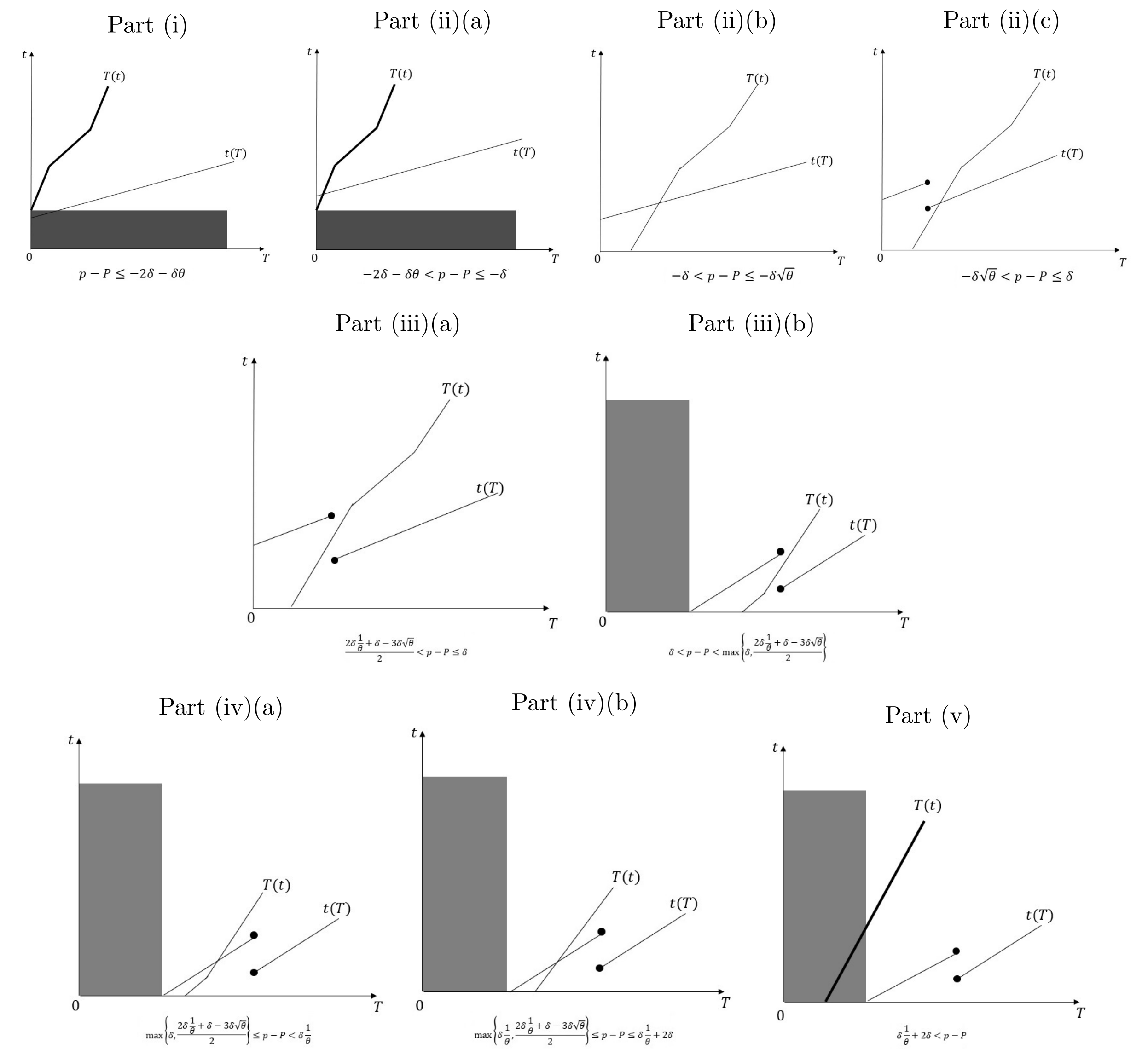


Figure 3: Equilibria in each part of Proposition 1. The intersection points of the correspondences, $t(T)$ and $T(t)$, denote the equilibrium points. Price difference increases from Part (i) to (v).

some large-country consumers. The antecedent literature, like KK and Trandel (1994), assumes that producer prices are the same in both countries. Thus, they find that the small country chooses a lower tax rate than the large. My result shows that this conclusion does not always hold.[4] As I state below, Type B equilibrium changes the results substantially.

Proposition 3. *Under the Type B equilibrium, there are cross-border sales from the large country to the small. Additionally, the large country's tax rate is higher than the small—that is,* $T_N > t_N$*.*

As I state in Proposition 3, under Type B equilibrium, some small-country consumers cross-shop from the large. This is because we have Type B equilibrium if the large country's producer price is lower than the small. This enables the large country to attract small-country consumers. This result is the opposite of Type A equilibrium. It is also contrary to the main result of antecedent literature, including KK and Trandel (1994). Nielsen (2002) obtains a similar result but the governments aim to maximize social welfare in his model. Nielsen develops his model to explain examples of small country consumers shopping from the large, like Canadians shopping from the US (Ferris, 2000), and Finns and Norwegians shopping from Sweden (Asplund et al., 2007). My result proposes another explanation for cross-border shopping in large countries.

Under Type B equilibrium, although small-country consumers shop in the large country, the large country's tax rate is higher than the small. It imposes a higher tax rate using its lower producer price. This result is the same as Type A equilibrium when the producer price of the large country is low enough. The higher tax rate of the large county supports the results in the preceding literature, such as KK and Trandel (1994). However, contrary to the previous literature, cross-border sales are from the large country in my result.

How about Type C and D equilibria? We can show that under Type C equilibria, all consumers shop from the small country and the small country's tax rate is higher than the large. This is because the small country's producer price is much lower than the large. A Type C equilibrium is like a stronger version of Type A. Under Type D equilibria, all consumers shop from the large country and the large country's tax rate is higher than the small. These equilibria represent a more extreme version of Type B equilibrium.[5]

4 Tax Coordination

In this section, I investigate the effects of tax coordination policies. With a coordination policy countries keep their borders open but choose their tax rates following the policy. I compare tax revenues under competition and coordination. As in KK, two policies are analyzed: Tax harmonization and minimum tax rate.

[4]The fact that the small country chooses a higher tax rate when its producer price is low resembles the result that more productive countries can choose a higher capital-tax rate. See, for example, Kempf and Rota-Graziosi (2010), Ogawa (2013), Hindriks and Nishimura (2015), and Eichner and Pethig (2015) for capital-tax models.

[5]The formal statements of the results under Type C and D equilibria are available from the author upon request.

Tax Harmonization. Under tax harmonization, both countries set a common tax rate, τ, which is between the two countries' tax rates under tax competition. If under tax competition $T_N < t_N$, then $\tau \in [T_N, t_N]$, or vice versa. In the following, to avoid non-generic cases I assume that the countries have different sizes.

Proposition 4. *Assume that the small country is strictly smaller than the large—that is, $\theta < 1$. At Type A and B equilibria, tax harmonization affects the countries' tax revenues as follows.*

i) Tax revenue of the country that has the lower tax rate under tax competition decreases.

ii) The other country's tax revenue increases if and only if the harmonization rate, τ, is high enough. Specifically, there exits $\bar{\tau} \in [\min\{t_N, T_N\}, \max\{t_N, T_N\}]$ such that the other country's tax revenue increases if and only if $\bar{\tau} < \tau$.

As I state in Proposition 4, the tax revenue of the country that has the lower tax rate under tax competition decreases with harmonization. Because, if the harmonization rate is equal to the higher of the two competitive tax rates, the revenue of the country decreases as it is not able to choose its best response. If its revenue is decreasing even at the highest harmonization rate, then it will also decrease at other rates. Since, under harmonization, the revenue of a country is an increasing function of the harmonization rate. The other country's revenue increases with tax harmonization if and only if the harmonization rate is high enough. This is because, similar to Part (i), if the harmonization rate is equal to the lower of two competitive tax rates, the country's tax revenue decreases since it cannot choose its best response. However, if the harmonization rate is equal to the higher of the two rates, the country's tax revenue increases as its customer base enlarges. Considering that tax revenues increase with the harmonization rate, tax harmonization increases the revenue of the country with the higher competitive tax rate if and only if the harmonization rate is high enough. In KK's model, the small country always chooses a lower tax rate. Thus, KK find that tax harmonization harms the small country and benefits the large country if and only if the harmonization rate is high enough. From one perspective, my result generalizes KK's to a more general setting by showing that tax harmonization hurts one of the countries and benefits the other if and only if the harmonization rate is high enough. However, I show that the effect of tax harmonization on a country's tax revenue does not depend on the country's size; rather, it depends on the country's tax rate under competition. From another perspective, my result reverses KK's by showing that tax harmonization can hurt the large country.

How does tax harmonization work under Type C and D equilibria? At these equilibria, under competition, all consumers shop from the country that imposes the higher tax rate. So clearly harmonization does not affect the revenue of the country with the lower competitive tax rate, but decreases the revenue of the other country.[6]

[6] The formal statement of tax-harmonization results under Type C and D equilibria are available from the author upon request.

Minimum Tax Rate. Under this policy both countries choose their taxes above a minimum rate. The minimum rate is determined between the two countries' tax rates under competition. To understand the effects of the minimum tax rate on revenues, first, we need to analyze how this policy affects best-response correspondences.

In Figure 4, I give an illustration of the effect of a minimum tax rate, μ, on best-response correspondences. The large country's best-response correspondence disappears on its domain below that minimum tax rate—that is, on the interval $[0, \mu]$. This part of the large country's best response is denoted with a dashed line. On the rest of its domain, the minimum tax rate is non-binding for the large country's best-response correspondence. Similarly, the small country's best-response correspondence disappears on its domain for the values below the minimum tax rate, i.e. on the interval $[0, \mu]$. For the large country's tax rate higher than the minimum rate but lower than a certain level, the small country's best response does not change since the minimum rate is not binding. As the large country's tax rate gets larger, the minimum rate binds the small country's best response. This part of the small country's best response is shown with a dashed line. In this situation, for a while, the small country chooses a tax rate larger than the large country's as it has a continuous revenue function. When the large country's tax rate is large enough, the small country chooses a tax rate smaller than the large country. Because the smallest tax rate that it can choose is the minimum rate, the large country chooses the minimum rate for a while. This choice is shown with a horizontal line in the figure. As the large country's tax rate gets larger, the minimum tax rate becomes non-binding for the small country. So, the small country chooses the same response it gives under competition.

The minimum tax rate affects revenues via equilibrium taxes and cross-border shopping.[7] Clearly, it increases both countries' equilibrium tax rates. Its effect on cross-border shopping depends on the relative increase in tax rates. If cross-border shopping decreases, the revenue of the country in which consumers cross-shop increases. Because both its tax rate and customer base increase. The other country's revenue is affected in opposite directions: While the higher tax rate increases its revenue, reduced cross-border shopping decreases it. The net effect depends on the relative size of these two effects. If the minimum tax rate increases cross-border shopping, now the revenue of the country in which consumers cross-shop is affected in opposite directions. It can be shown that, as long as cross-border shopping does not stop, this country's revenue will be a quadratic and concave function of the minimum rate as given in Figure 5. So, its revenue increases for moderate levels of minimum rate, but decreases at other levels. After this intuition, I present my formal result in the following proposition.

Proposition 5. *Assume that the small country is strictly smaller than the large—that is, $\theta < 1$. For Type A and B equilibria, we have the following results.*

[7] Under minimum-tax rate, because of the discontinuity in the small country's best-response correspondence, the existence of equilibrium is not guaranteed. However, following KK, I skip the existence problem and focus on the effects of minimum-tax rate on tax revenues.

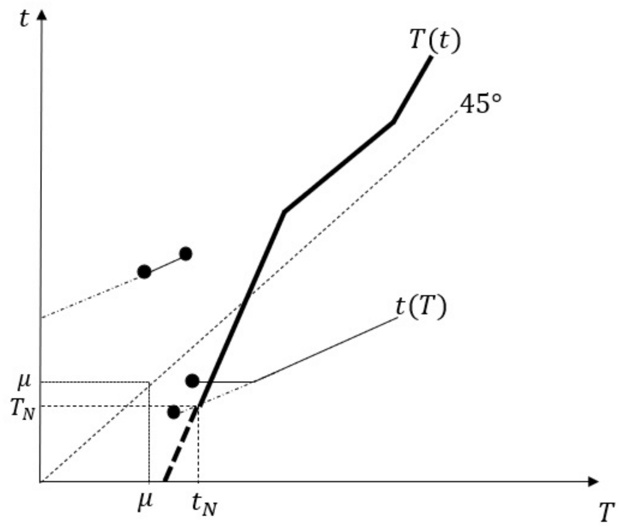


Figure 4: Best-response correspondences under the minimum-tax rate, μ, when producer prices are close enough to each other in the two countries ($-\delta < p - P < \delta$).

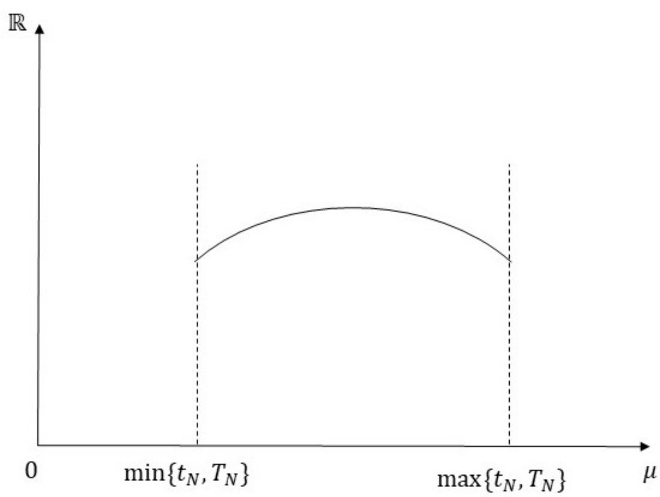


Figure 5: Tax revenue of the country which is affected in two opposite directions by the minimum tax rate.

i) If the small country's producer price is higher than the large by a small amount, a minimum-tax-rate policy with a high enough minimum rate increases both countries' tax revenue if and only if the two countries' producer-price difference is in a certain interval. Specifically, if $\frac{1}{4}\delta(1-\theta) < p - P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$ *and* $\mu \in [\delta - (p-P), T_N]$*, there exists* $(p-P)_m$ *and* $(p-P)_M$ *such that minimum tax rate increases both countries' tax revenue if and only if* $(p-P)_m < p - P < (p-P)_M$*.*

ii) Otherwise, the minimum tax rate increases both countries' tax revenue if and only if the minimum rate is low enough. Specifically, there exists $\bar{\mu} \in [\min\{t_N, T_N\}, \max\{t_N, T_N\}]$ *such that the minimum tax rate increases both countries' tax revenue if and only if* $\mu \leq \bar{\mu}$*.*

As I state in the second part of Proposition 5, in most cases, the minimum tax rate increases both countries' revenue if and only if the minimum rate is low enough. This is because of the two-way effect of minimum tax on revenues that I explained above. However, as stated in the first part of the proposition, if the small country's producer price is higher than the large by a small amount and the minimum tax rate is high enough, the minimum tax rate increases both countries' tax revenue if and only if their producer price difference is in a certain interval. Indeed, the large country's revenue always increases. However, change in the small country's revenue depends on producer-price difference between the countries. Because, in this case, the minimum tax rate halts cross-border shopping. So, producer prices do not affect tax revenues. Yet, in the same situation, under competition, the small country's revenue is a quadratic and convex function of producer-price difference. I plot them in Figure 6. Then, as it is clear in the figure, the minimum rate increases the small country's tax revenue if and only if the producer price difference is in an interval around the minimum of the small country's competitive tax revenue. This shows that the minimum tax rate can decrease a country's tax revenue contrary to KK.

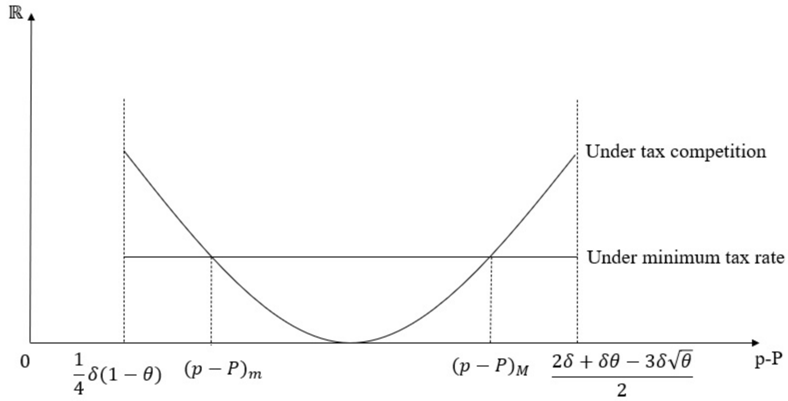


Figure 6: Small country's tax revenue under competition and minimum tax rate when its producer price is higher than the large by a small amount $\left(\frac{1}{4}\delta(1-\theta) < p - P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}\right)$ and the minimum tax rate is high enough $(\delta - (p-P) \leq \mu \leq T_N)$.

As a final note because Type C and D have multiple equilibria they do not let us study the minimum tax rate. We are not able to compare revenues under competition and coordination in a clear way.

5 Conclusion

I analyzed commodity-tax competition between two countries with different population densities and producer prices. Contrary to the previous literature, I showed that the small country can choose a higher tax rate and its consumers cross-shop from the large country. Additionally, tax harmonization can benefit the small country, and the minimum tax rate can decrease a country's tax rate. In my model, countries choose their tax rates simultaneously. An interesting future-research project can be letting countries have different producer prices when they choose their tax rates sequentially as in Wang (1999).

A Appendix: Proof of Lemmas and Propositions

A.1 Proof of Lemma 1

Small country's best-response function.

Assume that $T \leq p - P - \delta$. Then, whatever t is all small-country consumers shop from the large country. So, we have $t(T) \in [0, \infty)$.

Assume that $p - P - \delta < T \leq p - P$. There is no tax rate such that the large-country consumers shop from the small country. Thus, the small country's problem is $\max_t th\left(1 - \frac{(t+p)-(T+P)}{\delta}\right)$ s.t. $0 \leq t$. Solving the problem, we have $t(T) = \frac{\delta+(T+P)-p}{2}$.

Assume that $p - P < T$. If the small country maximizes its revenue by choosing $t \leq (T+P) - p$, its problem is given as $\max_t th + tH\frac{(T+P)-(t+p)}{\delta}$ s.t. $0 \leq t \leq (T+P) - p$. Solving the problem, we have

$$t = \begin{cases} (T+P) - p & \text{if } T < \delta\theta + p - P, \\ \frac{\delta\theta+(T+P)-p}{2} & \text{if } \delta\theta + p - P \leq T. \end{cases} \tag{5}$$

If the small country maximizes its revenue by choosing $t \geq T + P - p$, its problem is given as $\max_t th\left(1 - \frac{(t+p)-(T+P)}{\delta}\right)$ s.t. $T + P - p \leq t$. Solving the problem, we have

$$t = \begin{cases} \frac{\delta+(T+P)-p}{2} & \text{if } T \leq \delta + p - P, \\ (T+P) - p & \text{if } \delta + p - P < T. \end{cases} \tag{6}$$

I analyze which way of maximizing its revenue is optimal for the small country under different values of θ. First, assume that $\theta < 1$. Take $T \in (p-P, \delta\theta+p-P)$. If the small country maximizes its revenue by choosing $t \leq (T+P)-p$, $t = (T+P)-p$ by Equation 5, and its revenue is $[(T+P)-p]h$. If it maximizes its revenue by choosing $t \geq (T+P)-p$, $t = \frac{\delta+(T+P)-p}{2}$ by Equation 6, and its revenue is $h\frac{(\delta+(T+P)-p)^2}{4\delta}$. We have $h\frac{(\delta+(T+P)-p)^2}{4\delta} > [(T+P)-p]h$ if and only if $(\delta - [(T+P)-p])^2 > 0$ which always holds as we have $\delta\theta < \delta$ and $T < \delta\theta + p - P$. So, we have $t(T) = \frac{\delta+(T+P)-p}{2}$. Take $T \in [\delta\theta + p - P, \delta + p - P]$. If the small country maximizes its revenue by choosing $t \geq (T+P)-p$, $t = \frac{\delta+(T+P)-p}{2}$ by Equation 5, and its revenue is $h\frac{(\delta+(T+P)-p)^2}{4\delta}$. If it maximizes its revenue by choosing $t \leq (T+P)-p$, $t = \frac{\delta\theta+(T+P)-p}{2}$ by Equation 6, and its revenue is $h\frac{(\delta\theta+(T+P)-p)^2}{4\delta\theta}$. We have $h\frac{(\delta+(T+P)-p)^2}{4\delta} > h\frac{(\delta\theta+(T+P)-p)^2}{4\delta\theta}$ if and only if $\delta\sqrt{\theta} > (T+P-p)$. So, we have

$$t(T) = \begin{cases} \frac{\delta+(T+P)-p}{2} & \text{if } T < \delta\sqrt{\theta} + p - P, \\ \left\{\frac{\delta+(T+P)-p}{2}, \frac{\delta\theta+(T+P)-p}{2}\right\} & \text{if } T = \delta\sqrt{\theta} + p - P. \end{cases} \tag{7}$$

Take $T > \delta + p - P$. If the small country maximizes its revenue by choosing $t \leq (T+P)-p$, $t = \frac{\delta\theta+(T+P)-p}{2}$ by Equation 5, and its revenue is $h\frac{(\delta\theta+(T+P)-p)^2}{4\delta\theta}$. If it maximizes its revenue by choosing $t \geq (T+P)-p$, $t = (T+P)-p$ by Equation 6, and its revenue is $[(T+P)-p]h$. It is easy to show that the latter revenue is higher. Thus, $t(T) = \frac{\delta\theta+(T+P)-p}{2}$. Collecting the results, if $\theta < 1$, we have

$$t(T) = \begin{cases} [0.\infty) & \text{if } T \leq p - P - \delta \\ \frac{\delta+(T+P)-p}{2} & \text{if } p - P - \delta < T < \delta\sqrt{\theta} + p - P, \\ \left\{\frac{\delta+(T+P)-p}{2}, \frac{\delta\theta+(T+P)-p}{2}\right\} & \text{if } T = \delta\sqrt{\theta} + p - P, \\ \frac{\delta\theta+(T+P)-p}{2} & \text{if } \delta\sqrt{\theta} + p - P < T. \end{cases} \tag{8}$$

Second, assume that $\theta = 1$. Take $T \in (p - P, \delta + p - P)$. If the small country maximizes its revenue by choosing $t \leq (T + P) - p$, $t = (T + P) - p$ by Equation 5, and its revenue is $[(T + P) - p]h$. If it maximizes its revenue by choosing $t \geq (T + P) - p$, $t = \frac{\delta+(T+P)-p}{2}$ by Equation 6, and its revenue is $h\frac{(\delta+(T+P)-p)^2}{4\delta}$. We have $h\frac{(\delta+(T+P)-p)^2}{4\delta} > [(T + P) - p]h$ if and only if $(\delta - [(T + P) - p])^2 > 0$ which always holds as we have $\theta = 1$ and $T < \delta\theta + p - P$. So, we have $t(T) = \frac{\delta+(T+P)-p}{2}$. Take $T = \delta\theta + p - P = \delta + p - P$. If the small country maximizes its revenue by choosing $t \leq (T + P) - p$, $t = \frac{\delta\theta+(T+P)-p}{2}$ by Equation 5, and if it maximizes its revenue by choosing $t \geq (T + P) - p$, $t = \frac{\delta+(T+P)-p}{2}$ by Equation 6. Because, both tax rates are equal, as $\theta = 1$, we have $t(T) = \frac{\delta+(T+P)-p}{2}$. Take $T > \delta + p - P$. If the small country maximizes its revenue by choosing $t \leq (T + P) - p$, $t = \frac{\delta\theta+(T+P)-p}{2}$ by Equation 5, and its revenue is $h\frac{(\delta\theta+(T+P)-p)^2}{4\delta\theta}$. If it maximizes its revenue by choosing $t \geq (T + P) - p$, $t = (T + P) - p$ by Equation 6, and its revenue is $[(T + P) - p]h$. It is easy to show that the former revenue is higher. Thus, we have $t(T) = \frac{\delta+(T+P)-p}{2}$. Collecting the results, if $\theta = 1$, we have

$$t(T) = \begin{cases} [0.\infty) & \text{if } T \leq p - P - \delta \\ \frac{\delta+(T+P)-p}{2} & \text{otherwise.} \end{cases} \tag{9}$$

The whole results give us the small country's best-response function:

$$t(T) = \begin{cases} [0, \infty) & \text{if } T \leq p - P - \delta, \\ \frac{\delta+P+T-p}{2} & \text{if } p - P - \delta < T < \delta\sqrt{\theta} + p - P, \\ \left\{\frac{\delta+P+T-p}{2}, \frac{\delta\theta+P+T-p}{2}\right\} & \text{if } T = \delta\sqrt{\theta} + p - P, \\ \frac{\delta\theta+P+T-p}{2} & \text{if } \delta\sqrt{\theta} + p - P < T. \end{cases}$$

Large country's best-response function.

Assume that $t \leq -(p - P) - \delta$. Then, whatever T is all large-country consumers shop from the small country. So, we have $T(t) \in [0, \infty)$.

Assume that $-(p - P) - \delta < t \leq -(p - P)$. There is no tax rate such that the small-country consumers shop from the large country. Thus, the large country's problem is $\max_T TH\left(1 - \frac{(T+P)-(t+p)}{\delta}\right)$

s.t. $0 \leq T$. Solving the problem, we have $T(t) = \frac{\delta + (t+p) - P}{2}$.

Assume that $-(p-P) < t$. If the large country maximizes its revenue by choosing $T \leq (t+p) - P$, its problem is given as $\max_T TH + Th\frac{(t+p)-(T+P)}{\delta}$ s.t. $0 \leq T \leq (t+p) - P$. Solving the problem, we have

$$T = \begin{cases} (t+p) - P & \text{if } t < \delta\frac{1}{\theta} - (p-P), \\ \frac{\delta\frac{1}{\theta} + (t+p) - P}{2} & \text{if } \delta\frac{1}{\theta} - (p-P) \leq t. \end{cases} \tag{10}$$

If the large country maximizes its revenue by choosing $T \geq t + p - P$, its problem is given as $\max_T TH\left(1 - \frac{(T+P)-(t+p)}{\delta}\right)$ s.t. $t + p - P \leq T$. Solving the problem, we have

$$T = \begin{cases} \frac{\delta + (t+p) - P}{2} & \text{if } t \leq \delta - (p-P), \\ (t+p) - P & \text{if } \delta - (p-P) < t. \end{cases} \tag{11}$$

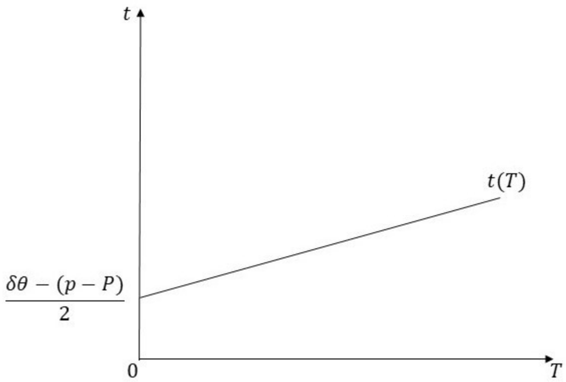
I analyze which way of maximizing its revenue is optimal for the small country under different values of θ.

First, assume that $\theta = 1$. Then, clearly, the best-response function of the large country is analogous to the small country. So, we have

$$T(t) = \begin{cases} [0.\infty) & \text{if } t \leq -(p-P) - \delta \\ \frac{\delta + (t+p) - P}{2} & \text{otherwise.} \end{cases} \tag{12}$$

Second, assume that $1 < \theta$. Take $t \in (-(p-P), \delta - (p-P))$. If the large country maximizes its revenue by choosing $T \leq (t+p) - P$, $T = (t+p) - P$ by Equation 10, and its revenue is $[(t+p) - P]H$. If it maximizes its revenue by choosing $T \geq (t+p) - P$, $T = \frac{\delta + (t+p) - P}{2}$ by Equation 11, and its revenue is $H\frac{(\delta + (t+p) - P)^2}{4\delta}$. It is easy to show that the former revenue is higher. Thus, we have $T(t) = \frac{\delta + (t+p) - P}{2}$. Take $t \in [\delta - (p-P), \delta\frac{1}{\theta} - (p-P)]$. Whether the large country maximizes its revenue by choosing $T \leq (t+p) - P$ or $T \geq (t+p) - P$, it is optimal to choose $T = (t+p) - P$ by Equations 10 and 11. So, we have $T(t) = (t+p) - P$. Take $t > \delta\frac{1}{\theta} - (p-P)$. If the large country maximizes its revenue by choosing $T \leq (t+p) - P$, $T = \frac{\delta\frac{1}{\theta} + (t+p) - P}{2}$ by Equation 10, and its revenue is $H\frac{(\delta\frac{1}{\theta} + (t+p) - P)^2}{4\delta\frac{1}{\theta}}$. If it maximizes its revenue by choosing $T \geq (t+p) - P$, $T = (t+p) - P$ by Equation 11, and its revenue is $[(t+p) - P]H$. It is easy to show that the latter revenue is higher if and only if $t \leq \delta\frac{1}{\theta} - (p-P)$. Collecting the results, we have

$$T(t) = \begin{cases} [0, \infty) & \text{if } t \leq -(p-P) - \delta, \\ \frac{\delta + p + t - P}{2} & \text{if } -(p-P) - \delta < t \leq \delta - (p-P), \\ p + t - P & \text{if } \delta - (p-P) < t \leq \delta\frac{1}{\theta} - (p-P), \\ \frac{\delta\frac{1}{\theta} + p + t - P}{2} & \text{if } \delta\frac{1}{\theta} - (p-P) < t. \end{cases} \tag{13}$$



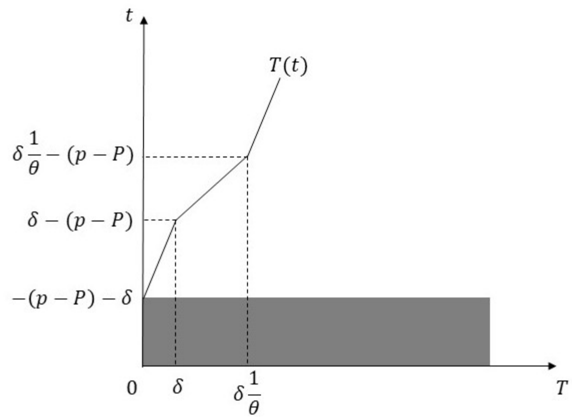


Figure 7: Best-response correspondences in the Proof of Proposition 1 Part (i) and (ii) when $-2\delta - \delta\theta < p - P \leq -\delta$.

□

A.2 Proof of Proposition 1

In the following, I assume that $\theta \leq 1$. So, we have

$$t(T) = \begin{cases} [0, \infty) & \text{if } T \leq p - P - \delta, \\ \frac{\delta+P+T-p}{2} & \text{if } p - P - \delta < T < \delta\sqrt{\theta} + p - P, \\ \left\{\frac{\delta+P+T-p}{2}, \frac{\delta\theta+P+T-p}{2}\right\} & \text{if } T = \delta\sqrt{\theta} + p - P, \\ \frac{\delta\theta+P+T-p}{2} & \text{if } \delta\sqrt{\theta} + p - P < T, \end{cases}$$

and

$$T(t) = \begin{cases} [0, \infty) & \text{if } t \leq -(p - P) - \delta, \\ \frac{\delta+(t+p)-P}{2} & \text{if } -(p - P) - \delta < t \leq \delta - (p - P), \\ (t + p) - P & \text{if } \delta - (p - P) < t \leq \delta\frac{1}{\theta} - (p - P), \\ \frac{\delta\frac{1}{\theta}+(t+p)-P}{2} & \text{if } \delta\frac{1}{\theta} - (p - P) < t. \end{cases} \tag{14}$$

To prove the proposition, I present both countries' best-response correspondences and find their intersection points for each part of the proposition. Additionally, Part (iii) of the proposition is implied by Parts (ii) and (iv). Thus, I give the proof of Part (iii) after the proof of Part (iv).

Part i: Assume that $(p - P) \leq -2\delta - \delta\theta$. Then, the best-response correspondences of the small and large countries are as given in Figure 7. Under the parameter restrictions of Part (i), it is easy to show that the best-response correspondences intersect as in Figure 8. Solving for the intersection points, we get a continuum of equilibria such that $T_N \in [0, -2\delta - \delta\theta - (p - P)]$ and

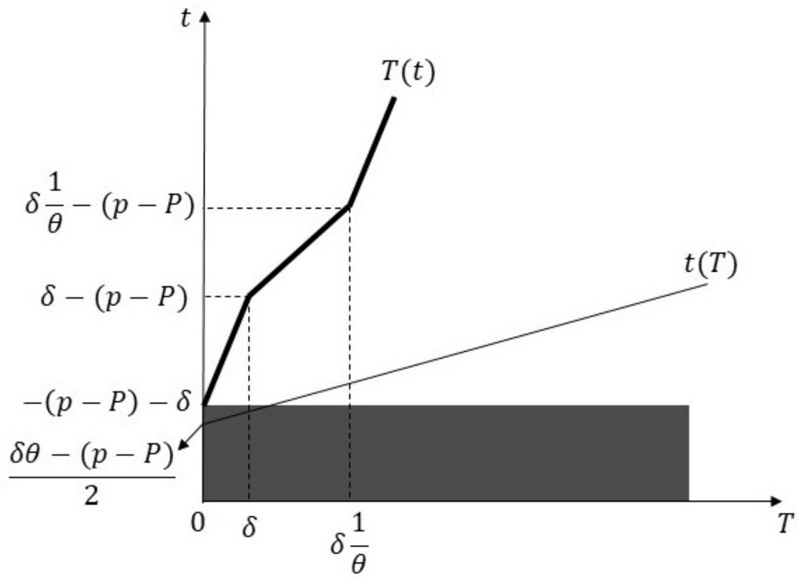


Figure 8: Equilibria in the Proof of Proposition 1 Part (i).

$t_N = \frac{\delta\theta + T_N - (p-P)}{2}$.

Part ii: I prove Part (ii) of the proposition under three cases, as the shape of the best-response correspondences differs. First, assume that $-2\delta - \delta\theta < p - P \leq -\delta$. Then, the best-response correspondences of the small and large countries are given in Figure 7. Under the parameter of our restrictions, it is easy to show that the best-response correspondences intersect as in Figure 9. Solving for the intersection points, we get the unique equilibrium $(t_N, T_N) = \left(\frac{2\delta\theta+\delta-(p-P)}{3}, \frac{2\delta+\delta\theta+(p-P)}{3}\right)$.

Second, assume that $-\delta < p - P \leq -\delta\sqrt{\theta}$. Then, the best-response correspondences of the small and large countries are given in Figure 10. It is easy to show that the best-response correspondences intersect only if $t \leq \delta - (p-P)$. For the other values of t, if solve for an intersection point, given our assumption on the parameters, the values of t or T fall out of the interval that we assume for it. If we solve for an intersection point when $t \leq \delta - (p-P)$, we get the unique point $(t_N, T_N) = \left(\frac{2\delta\theta+\delta-(p-P)}{3}, \frac{2\delta+\delta\theta+(p-P)}{3}\right)$. We can show that, given our assumption on the parameters, $t_N \leq \delta - (p-P)$ and $T_N \geq 0$. Thus, this intersection point is the unique equilibrium.

Third, assume that $-\delta\sqrt{\theta}. < (p-P) \leq \delta$. Then, the best-response correspondences of the small and large countries are given in Figure 11. It is easy to show that the best-response correspondences intersect only if $t \leq \delta - (p-P)$ and $\delta\sqrt{\theta} + p - P \leq T$. In other cases, if we solve for an intersection point, given our assumption on the parameters, the values of t or T fall out of the interval that we assume for it. If we solve for an intersection point when $t \leq \delta - (p-P)$ and $\delta\sqrt{\theta} + p - P \leq T$, we get the unique point $(t_N, T_N) = \left(\frac{2\delta\theta+\delta-(p-P)}{3}, \frac{2\delta+\delta\theta+(p-P)}{3}\right)$. We need $t_N \leq \delta - (p-P)$ and $\delta\sqrt{\theta} + p - P \leq T_N$ which require $p - P \leq \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$.

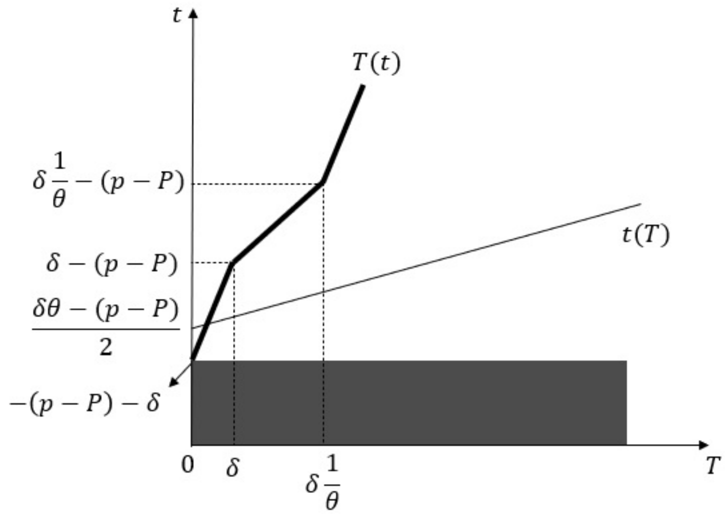


Figure 9: Equilibrium in the Proof of Proposition 1 Part (ii) when $-2\delta - \delta\theta < p - P \leq -\delta$.

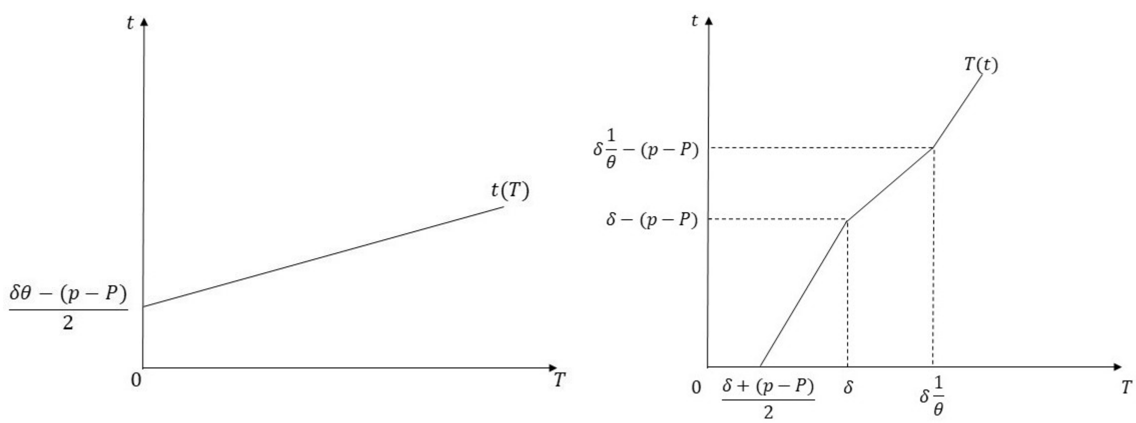


Figure 10: Best-response correspondences in the Proof of Proposition 1 Part (ii) when $-\delta < p - P \leq -\delta\sqrt{\theta}$.

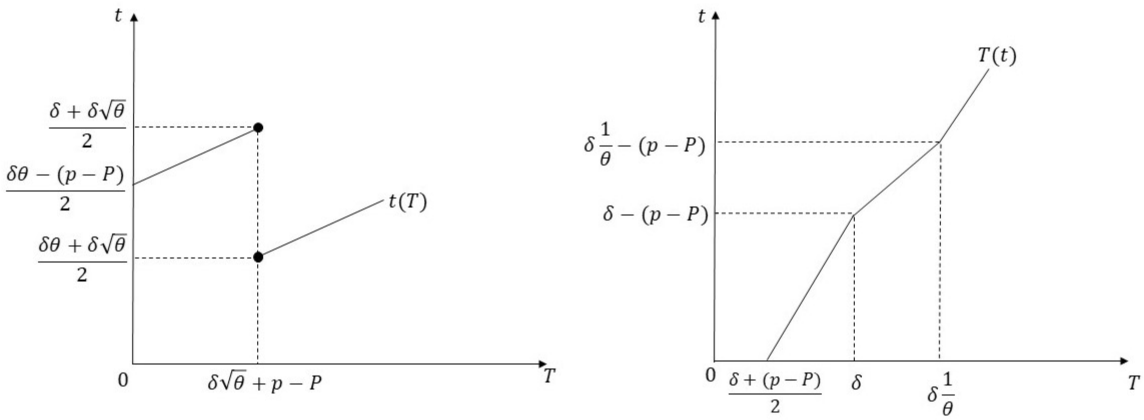


Figure 11: Best-response correspondences in the Proof of Proposition 1 Part (ii) when $-\delta\sqrt{\theta} \leq (p-P) \leq \delta$.

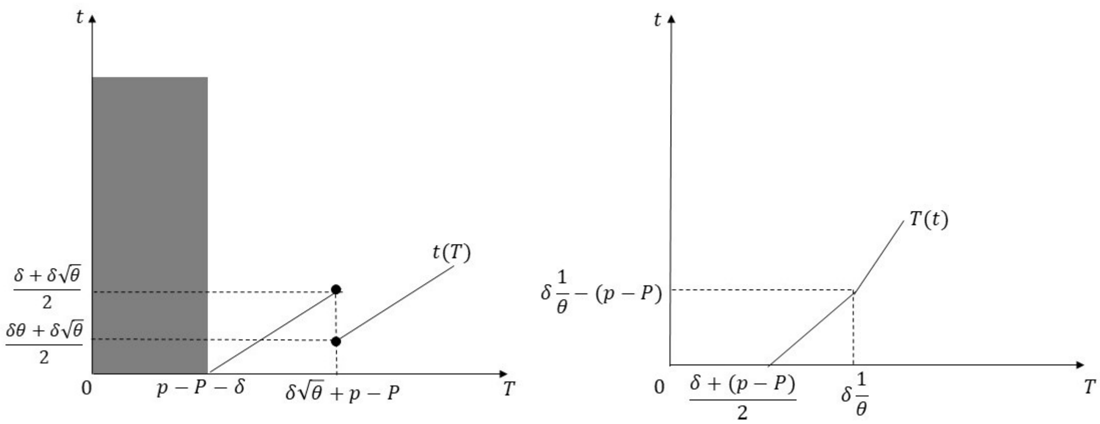


Figure 12: Best-response correspondences in the Proof of Proposition 1 Part (iv) when $\delta < p-P < \delta\frac{1}{\theta}$.

So, collecting the results, if $-\delta < p-P \leq \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$, we have a unique equilibrium $(t_N, T_N) \left(\frac{2\delta\theta+\delta-(p-P)}{3}, \frac{2\delta+\delta\theta+}{3}\right.$

Part iii. Proof Part (iii) is given after proof of Part (iv) as it is implied by the proofs of Part (ii) and (iv).

Part iv: I prove Part (iv) of the proposition under two cases, as the shape of the best-response correspondences differs under the two. First, assume that $\delta \leq p-P < \delta\frac{1}{\theta}$. Then, the best-response correspondences of the small and large countries are given in Figure 12. It is easy to show that the best-response correspondences intersect only if $\delta\frac{1}{\theta} - (p-P) \leq t$ and $p-P-\delta \leq T \leq \delta\sqrt{\theta}+p-P$. In other cases, if solve for an intersection point, given our assumption on the parameters, the values of t or T fall out of the interval that we assume for it. If we solve for an intersection point when $\delta\frac{1}{\theta} - (p-P) \leq t$ and $p-P-\delta \leq T \leq \delta\sqrt{\theta}+p-P$, we get the unique point $(t_N, T_N) = \left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}, \frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)$. We need $\delta\frac{1}{\theta}-(p-P) \leq t_N$ and $p-P-\delta \leq T_N \leq \delta\sqrt{\theta}+p-P$

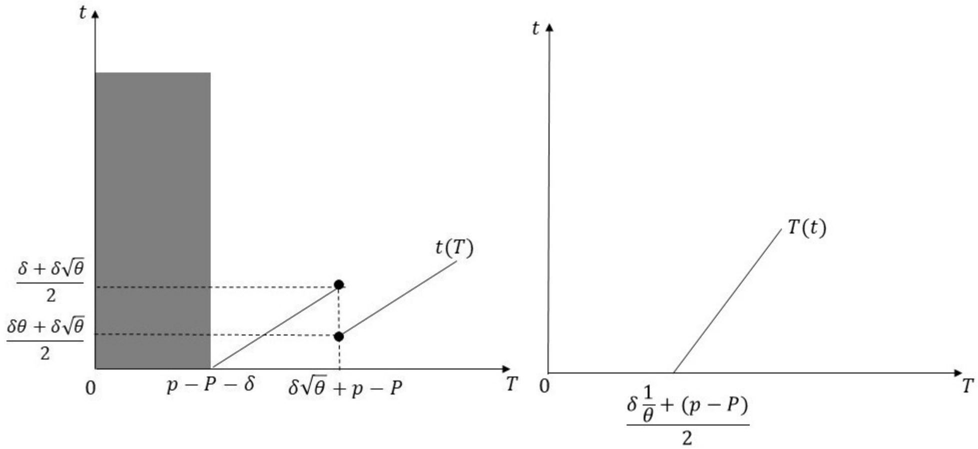


Figure 13: Best-response correspondences in the Proof of Proposition 1 Part (iv), when $\delta\frac{1}{\theta} \leq (p-P) \leq \delta\frac{1}{\theta} + 2\delta$, and in Part (v).

which require $\frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2} \leq p - P$.

Second, assume that $\delta\frac{1}{\theta} \leq (p-P) \leq \delta\frac{1}{\theta} + 2\delta$. Then, the best-response correspondences of the small and large countries are given in Figure 13. It is easy to show that the best-response correspondences intersect only if $p-P-\delta \leq T \leq \delta\sqrt{\theta}+p-P$. For the other ranges of T, if solve for an intersection point, given our assumption on the parameters, the values of t or T fall out of the interval that we assume for it. If we solve for an intersection point when $p-P-\delta \leq T \leq \delta\sqrt{\theta}+p-P$, we get the unique point $(t_N, T_N) = \left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}, \frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)$. We need $p - P - \delta \leq T_N \leq \delta\sqrt{\theta} + p - P$ which requires $\frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2} \leq p - P$.

Collecting the results in both parts of the proof, if $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} \leq p - P \leq \delta\frac{1}{\theta} + 2\delta$, we have a unique equilibrium $(t_N, T_N) = \left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}, \frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)$.

Part iii. By the second part of the proof of Part (ii), there is no equilibrium if $\frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2} < p - P < \delta$. By the proof of Part (iv), there is no equilibrium if $\delta < p - P < \max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\}$. Combining the two results, there is no equilibrium if $\frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2} < p - P < \max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\}$.

Part v: Assume that $\delta\frac{1}{\theta} + 2\delta < p - P$. Then, the best-response correspondences of the small and large countries are as given in Figure 13. Then, under the assumptions of Part (i), it is easy to show that the best-response correspondences intersect as in Figure 14. Solving for the intersection points, we get a continuum of equilibria such that $t_N \in \left[0, p - P - 2\delta - \delta\frac{1}{\theta}\right]$ and $T_N = \frac{\delta\frac{1}{\theta}+p-P+t_N}{2}$.

□

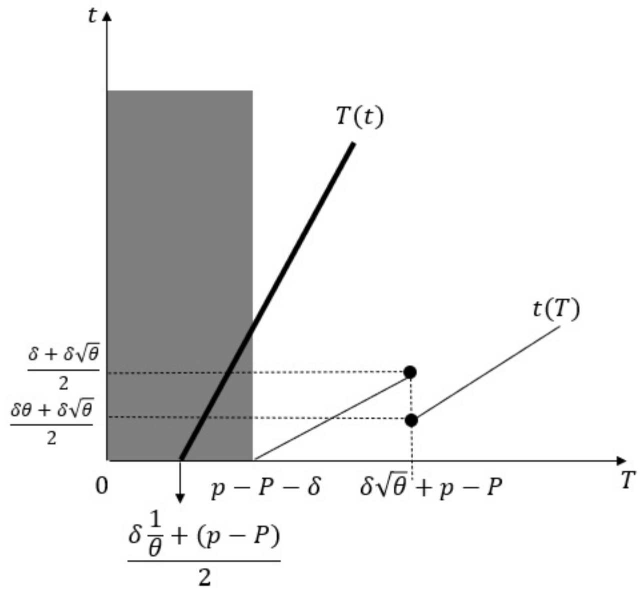


Figure 14: Equilibria in the Proof of Proposition 1 Part (v).

A.3 Proof of Proposition 2

We have cross-border sales from small to the large country if and only if $t_N + p < T_N + P$. Rewriting the condition under Type A equilibrium, by Proposition 1 Part (ii), we get $p - P < \delta(1-\theta)$. By the same proposition, we have Type A equilibrium if and only if $-\delta < p - P \leq \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$ and $\frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2} < \delta(1-\theta)$, the condition for cross-border sales from small to large country always holds.

By Proposition 1 Part (ii), we have $t_N < T_N$ under Type A equilibrium if and only if $\frac{2\delta\theta+\delta-(p-P)}{3} < \frac{2\delta+\delta\theta+(p-P)}{3}$, that is $-\frac{1}{2}\delta(1-\theta) < p - P$.

□

A.4 Proof of Proposition 3

We have cross-border sales from the large country to the small country if and only if $T_N + P < t_N + p$. Rewriting the condition using Proposition 1 Part (iv), we have $\delta\left(\frac{1}{\theta} - 1\right) < p - P$. We have Type B equilibrium if and only if $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} \leq p - P \leq \delta\frac{1}{\theta} + 2\delta$. If $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} = \delta$, we have $\delta\left(\frac{1}{\theta} - 1\right) < \delta \leq p - P$ and so the condition for cross-border sales from the large country is satisfied. If $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} = \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}$, we have $\delta\left(\frac{1}{\theta} - 1\right) < \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2} \leq p - P$ and so the condition for cross-border sales from the large country is satisfied. Thus, there are always cross-border sales from the large to the small country under a Type B equilibrium.

By Proposition 1 Part (iv), we have $t_N < T_N$ if and only if $-\frac{1}{2}\delta\left(\frac{1}{\theta}-1\right) < p-P$ which always holds as under a Type B equilibrium we have $0 \leq p-P$ by Proposition 1 Part (iv).

□

A.5 Proof of Proposition 4

I prove the proposition under four cases depending on the parameter values.

Case 1: Assume that $-2\delta-\delta\theta < p-P < -\frac{1}{2}\delta(1-\theta)$. We have cross-border sales in the small country and $T_N < t_N$ by Proposition 2. Additionally, the tax revenues are given as $R(t_N,T_N) = T_N H\left(1-\frac{(P+T_N)-(p+t_N)}{\delta}\right)$ and $r(t_N,T_N) = t_N h + t_N H\frac{(P+T_N)-(p+t_N)}{\delta}$ by Proposition 1 Part (ii) and Proposition 2. Under tax harmonization, take $\tau \in [T_N,t_N]$. Then, it is easy to show that we have cross-border sales in the small country, and the tax revenues are given as $R(\tau) = \tau H\left(1-\frac{P-p}{\delta}\right)$ and $r(\tau) = \tau h + \tau H\frac{P-p}{\delta}$.

Part i: We have $R(t_N) = R(t_N,t_N) < R(t_N,T_N)$ as $\frac{(P+T_N)-(p+t_N)}{\delta} < \frac{P-p}{\delta}$. Thus, considering that $R(\tau)$ is increasing, we have $R(\tau) < R(t_N,T_N)$ for all $\tau \in [T_N,t_N]$.

Part ii: We have $r(t_N,T_N) < r(t_N)$ as $\frac{(P+T_N)-(p+t_N)}{\delta} < \frac{P-p}{\delta}$. We also have $r(T_N) = r(T_N,T_N) < r(t_N,T_N)$. Thus, considering that $r(\tau)$ is increasing, there exits $\bar{\tau}$ such that $r(t_N,T_N) < r(\tau)$ if and only if $\bar{\tau} < \tau$.

Case 2: Assume that $-\frac{1}{2}\delta(1-\theta) < p-P < 0$. We have cross-border sales in the small country and $t_N < T_N$ by Proposition 2. Additionally, the tax revenues are given as $R(t_N,T_N) = T_N H\left(1-\frac{(P+T_N)-(p+t_N)}{\delta}\right)$ and $r(t_N,T_N) = t_N h + t_N H\frac{(P+T_N)-(p+t_N)}{\delta}$ by Proposition 1 Part (ii) and Proposition 2. Under tax harmonization, take $\tau \in [t_N,T_N]$. Then, it is easy to show that we have cross-border sales in small the country, and the tax revenues are given as $R(\tau) = \tau H\left(1-\frac{P-p}{\delta}\right)$ and $r(\tau) = \tau h + \tau H\frac{P-p}{\delta}$.

Part i: We have $r(T_N) = r(T_N,T_N) < r(t_N,T_N)$ as $\frac{P-p}{\delta} < \frac{(P+T_N)-(p+t_N)}{\delta}$. We also have $r(T_N) = r(T_N,T_N) < r(t_N,T_N)$. Thus, considering that r is increasing, $r(\tau) < r(t_N,T_N)$ for all $\tau \in [t_N,T_N]$.

Part ii: We have $R(T_N) > R(t_N,T_N)$ as $\frac{P-p}{\delta} < \frac{(P+T_N)-(p+t_N)}{\delta}$. We also have $R(t_N) = R(t_N,t_N) < R(t_N,T_N)$. Thus, considering that R is increasing, there exists $\bar{\tau} \in [t_N,T_N]$ such that $R(t_N,T_N) < R(\tau)$ if and only if $\bar{\tau} < \tau$.

Case 3: Assume that $0 < p-P \leq \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$. We have cross-border sales in the small country and $t_N < T_N$ by Propositions 2. Additionally, the tax revenues are given as $R(t_N,T_N) = T_N H\left(1-\frac{(P+T_N)-(p+t_N)}{\delta}\right)$ and $r(t_N,T_N) = t_N h + t_N H\frac{(P+T_N)-(p+t_N)}{\delta}$ by Proposition 1 Part (ii) and Proposition 2. Under tax harmonization, take $\tau \in [t_N,T_N]$. Then, it is easy to show that we have cross-border sales in the large country, and the tax revenues are given as $R(\tau) = \tau H + \tau h\frac{p-P}{\delta}$ and $r(\tau) = \tau h\left(1-\frac{p-P}{\delta}\right)$.

Part i: We have $r(T_N) = r(T_N,T_N) < r(t_N,T_N)$. Thus, considering that r is increasing, $r(\tau) < r(t_N,T_N)$ for all $\tau \in [t_N,T_N]$.

Part ii: We have $R(T_N) > R(t_N, T_N)$ as $\frac{p-P}{\delta} > \frac{p+t_N-(P+T_N)}{\delta}$. We also have $R(t_N) = R(t_N, t_N) < R(t_N, T_N)$. Thus, considering that R is increasing, there exists $\bar{\tau} \in [t_N, T_N]$ such that $R(t_N, T_N) < R(\tau)$ if and only if $\bar{\tau} < \tau$.

Case 4: Assume that $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} \le p - P \le \delta\frac{1}{\theta} + 2\delta$. We have cross-border sales in the large country and $t_N < T_N$ by Propositions 3. We have $r(T_N) = r(T_N, T_N) < r(t_N, T_N)$. Additionally, the tax revenues are given as $R(t_N, T_N) = T_N H + T_N h \frac{p+t_N-(P+T_N)}{\delta}$ and $r(t_N, T_N) = t_N h \left(1 - \frac{p+t_N-(P+T_N)}{\delta}\right)$ by Proposition 1 Part (iv) and Proposition 3. Under tax harmonization, take $\tau \in [t_N, T_N]$. Then, it is easy to show that we have cross-border sales in the large country, and $R(\tau) = \tau H + \tau h \frac{p-P}{\delta}$ and $r(\tau) = \tau h \left(1 - \frac{p-P}{\delta}\right)$.

Part i: We have $r(T_N) = r(T_N, T_N) < r(t_N, T_N)$. Thus, considering that r is increasing, $r(\tau) < r(t_N, T_N)$ for all $\tau \in [t_N, T_N]$.

Part ii: We have $R(T_N) > R(t_N, T_N)$ as $\frac{p-P}{\delta} > \frac{p+t_N-(P+T_N)}{\delta}$. We also have $R(t_N) = R(t_N, t_N) < R(t_N, T_N)$. Thus, considering that R is increasing, there exists $\bar{\tau} \in [t_N, T_N]$ such that $R(t_N, T_N) < R(\tau)$ if and only if $\bar{\tau} < \tau$.

□

A.6 Proof of Proposition 5

Under a minimum-tax rate, I show the equilibrium tax rates by t_m and T_m for the small and large countries, respectively.

Part i: Assume that $-\delta\sqrt{\theta} < p - P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$. Then, we have $t_N = \frac{2\delta\theta+\delta-(p-P)}{3} < \frac{2\delta+\delta\theta+(p-P)}{3} = T_N$ by Proposition 1 Part (ii), and cross-shopping in the small country by Proposition 2. These imply that we have $R(t_N, T_N) = H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$. Assume further that $\frac{1}{4}\delta(1-\theta) < p-P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$. so that we have $\delta-(p-P) < T_N < \delta\frac{1}{\theta}-(p-P)$. Then, the equilibrium is as given in Figure 15.

Take the minimum tax rate $\mu \in [\delta - (p-P), T_N]$. Then, we have $t_m = \mu < \mu + p - P = T_m$. There are cross-border sales if and only if $\mu + t_m \neq T_m + P$, that is, $\mu \neq \mu$ which never holds. So, there is no cross-border sale under the minimum tax rate.

We have $R(\mu) = (\mu + p - P)H$. Then $R(\mu) > R(t_N, T_N)$ if and only if $(\mu + p - P)H > TH\left(1 - \frac{T+P-(t+p)}{\delta}\right)$, that is, $\delta(\mu + p - P) > \left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$. For the minimum value of μ, the left-hand side of this inequality is δ^2. Additionally, we have $\delta^2 > \left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$ if and only if $p - P < \delta(1-\theta)$ which always holds as $\frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{3} < \delta(1-\theta)$.

We have $r(\mu) = \mu h$. Then, $r(\mu) > r(t_N, T_N)$ if and only if $\mu h > H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$, that is, $\mu\delta\theta > \left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$. It is easy to show that the graph of both sides of this inequality is as given in Figure 16. So, there exists $(p-P)_m$ and $(p-P)_M$ such that a minimum tax rate increases the small country's tax revenue if and only if $(p-P)_m < p - P < (p-P)_M$.

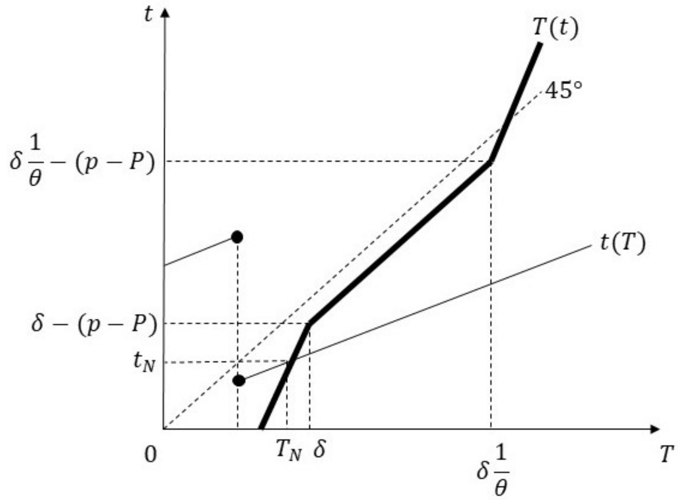


Figure 15: Equilibrium in the proof of Proposition 5 Part (i).

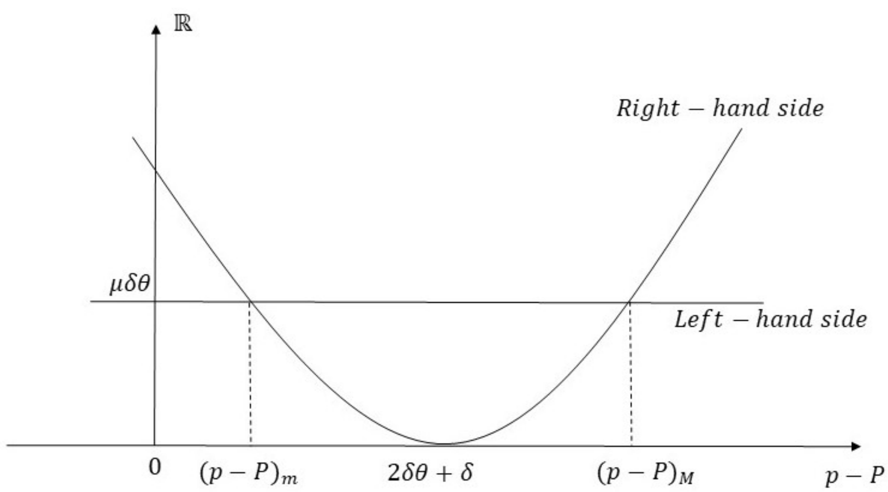


Figure 16: Graph of the inequality in the proof of Proposition 5 Part (i).

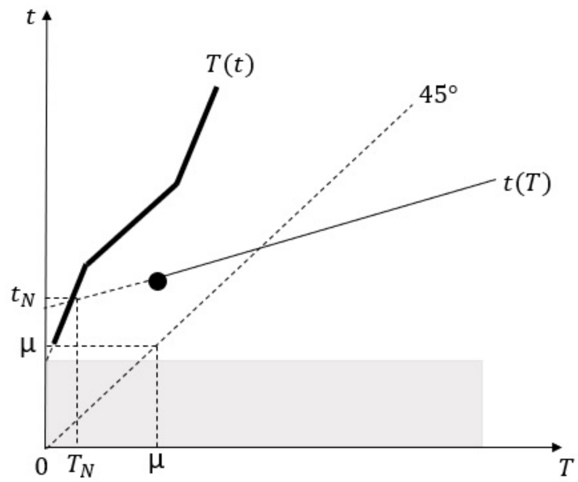


Figure 17: Best-response correspondences in Case 1 of proof of Proposition 5 Part (ii).

Part ii: I prove Part (ii) of the proposition by considering different cases for the value of $p-P$, as the figure of the best-response correspondences and the range of the feasible values of the minimum-tax rate differ under each case.

Case 1: Assume that $-2\delta - \delta\theta < p - P \leq \delta$. Then, under a minimum tax rate $\mu \in [T_n, t_n]$, the best-response correspondences are given in Figure 17. As shown by the dashed lines and the lightly shadowed area, both countries' best-response correspondences are irrelevant for the other country's tax rate below μ. Then, as seen in the graph, the best-response correspondences do not intersect and we do not have an equilibrium. So, I continue to the proof, by focusing on the remaining cases.

Case 2: Assume that $-\delta < p - P < \min\left\{-\delta\sqrt{\theta}, -\frac{1}{2}\delta(1-\theta)\right\}$. Then, we have $T_N = \frac{2\delta+\delta\theta+(p-P)}{3} < \frac{2\delta\theta+\delta-(p-P)}{3} = t_N$ by Proposition 1 Part (ii), and cross-shopping in the small country by Proposition 2. These imply that we have $R(t_N, T_N) = H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$ and the equilibrium is as given in Figure 18.

Following the discussion in Section 4, under a minimum-tax rate $\mu \in [T_N, t_N]$, we can ignore $T(t)$ for $t < \mu$. Additionally, as T can not be less than μ, for values of $t > \mu$, we have $T = \mu$ until the value of t is such that $T(t) = \mu$. And $t(T)$ will get longer before the discontinuity point and will be equal to μ before the value of T is such that $t(T) = \mu$. Then, we have $T_m = \mu < \frac{\delta\theta+\mu-(p-P)}{2} = t_m$. Under the minimum tax rate, there are cross-border sales in the small country if and only if $t_m + p < T_m + P$, that is, $\delta\theta + (p-P) < \mu$ which always holds as $\mu \geq T_N$.

We have $R(\mu) = T_m H\left(1 - \frac{T_m+P-(t_m+p)}{\delta}\right) = \mu H\frac{2\delta-\mu+(p-P)+\delta\theta}{2\delta}$. Notice that $R(\mu) > R(t_N, T_N)$

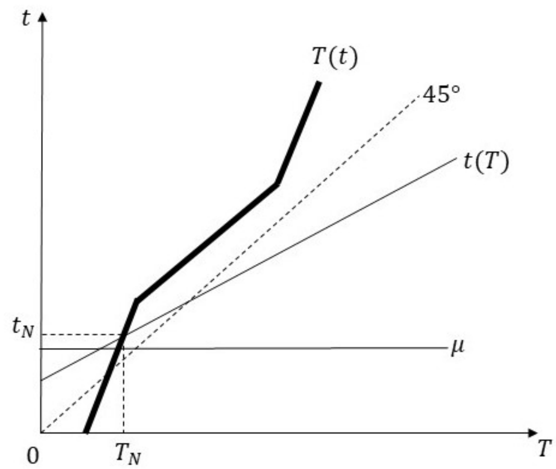


Figure 18: Equilibrium in Case 2 of proof of Proposition 5 Part (ii).

if and only if $\frac{[\delta\theta+2\delta+(p-P)]\mu-\mu^2}{2} > \frac{[\delta\theta+2\delta+p-P]^2}{9}$. It is easy to show that the graph of the left-hand side of this inequality is as given in Figure 19. Then, the minimum tax rate increases the large country's tax revenue if and only if $\frac{\delta\theta+2\delta+(p-P)}{3} < \mu < \frac{\delta\theta+2\delta+(p-P)}{2} + \frac{\delta\theta+2\delta+(p-P)}{2} - \frac{\delta\theta+2\delta+(p-P)}{3}$, that is, $\frac{\delta\theta+2\delta+(p-P)}{3} < \mu < \frac{2[\delta\theta+2\delta+(p-P)]}{3}$ which always holds as $T_N < \mu < t_N < \frac{2[\delta\theta+2\delta+(p-P)]}{3}$.

We have $r(\mu) = t_m h + t_m H \frac{T_m+P-(t_m+p)}{\delta} = \frac{1}{\delta} H \left(\frac{\delta\theta+\mu-(p-P)}{2}\right)^2$. Then, $r(\mu) > r(t_N, T_N)$ if and only if $\mu > \frac{\delta\theta+2\delta+(p-P)}{3}$ which always holds as $\mu > T_N$.

So, for Case 2, we have $\bar{\mu} = t_N$.

Case 3: Assume that $-\frac{1}{2}\delta(1-\theta) < p - P < -\delta\sqrt{\theta}$. Then, we have $t_N = \frac{2\delta\theta+\delta-(p-P)}{3} < \frac{2\delta+\delta\theta+(p-P)}{3} = T_N$ by Proposition 1 Part (ii), and cross-shopping in the small country by Proposition 2. These imply that we have $R(t_N, T_N) = H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$ and the equilibrium is as given in Figure 20.

Following the discussion in Section 4, under a minimum-tax rate $\mu \in [t_N, T_N]$, we can ignore $T(t)$ for $t < \mu$. And $t(T)$ will get longer before the discontinuity point and will be equal to μ before the value of T such that $t(T) = \mu$. Then, we have $t_m = \mu < \frac{\delta+\mu+(p-P)}{2} = T_m$. There are cross-border sales in the small country if and only if $t_m + p < T_m + P$, that is, $p - P < \delta - \mu$. Notice that we have $\delta - T_N < \delta - \mu < \delta - t_N$, that is, $\delta - \frac{2\delta+\delta\theta+(p-P)}{3} < \delta - \mu < \delta - \frac{2\delta\theta+\delta+(p-P)}{3}$. By the left-hand side of this inequality, we have $\frac{\delta-\delta\theta-(p-P)}{3} < \delta - \mu$. Assume that we have $\frac{\delta-\delta\theta-(p-P)}{3} < p - P$. This implies that $\frac{1}{4}\delta(1-\theta) < p - P$ which contradicts our assumption for Case 3. So, there are always cross-border sales in the small country.

We have $R(\mu) = T_m H\left(1 - \frac{T_m+P-(t_m+p)}{\delta}\right) = H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2$. Notice that $R(\mu) > R(t_N, T_N)$ if and only if $H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2 > H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$, that is, $\mu > \frac{2\delta\theta+\delta-(p-P)}{3} = t_N$ which always holds as $\mu > t_N$.

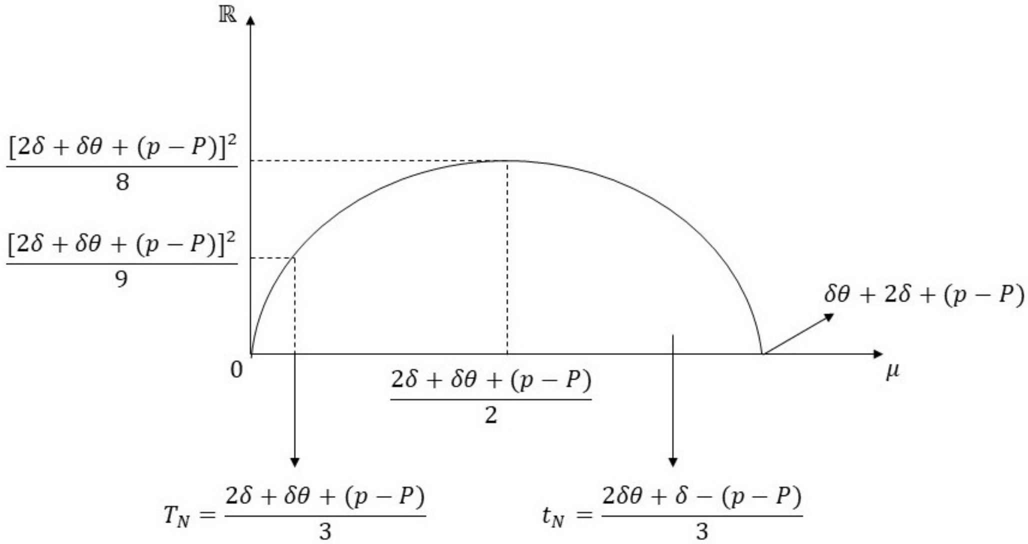


Figure 19: Graph of the inequality in Case 1 of proof of Proposition 5 Part (ii).

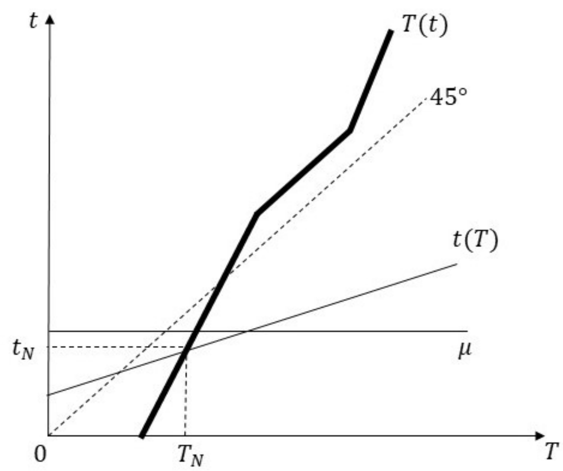


Figure 20: Equilibrium Case 3 of proof of Proposition 5 Part (ii).

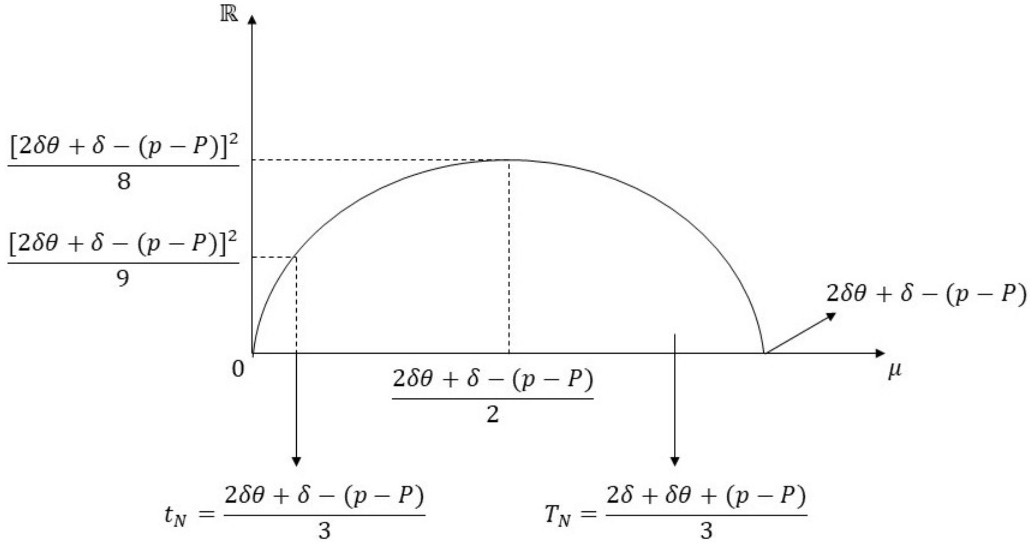


Figure 21: Graph of the inequality in Case 3 of proof of Proposition 5 Part (ii).

We have $r(\mu) = t_m h + t_m H \frac{T_m+P-(t_m+p)}{\delta} = H\frac{1}{\delta}\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2}$. Then, $r(\mu) > r(t_N, T_N)$ if and only if $\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2} > \left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$. It is easy to show that the graph of the left-hand side of the above inequality is given in Figure 21. Then, the minimum tax rate increases the small country's tax revenue if and only if $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2\delta\theta+\delta-(p-P)}{2} + \frac{2\delta\theta+\delta-(p-P)}{2} - \frac{2\delta\theta+\delta-(p-P)}{3}$ that is $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2[2\delta\theta+\delta-(p-P)]}{3}$ which always holds as $t_N < \mu < T_N < \frac{2[2\delta\theta+\delta-(p-P)]}{3}$.

So, for Case 3, we have $\bar{\mu} = T_N$.

Case 4: Assume that $-\delta\sqrt{\theta} < p - P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$. Then, we have $t_N = \frac{2\delta\theta+\delta-(p-P)}{3} < \frac{2\delta+\delta\theta+(p-P)}{3} = T_N$ by Proposition 1 Part (ii), and cross-shopping in the small country by Proposition 2. These imply that we have $R(t_N, T_N) = H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$, and the equilibrium given in Figure 22.

Case 4.i: Assume that $-\delta\sqrt{\theta} < p - P < \frac{1}{4}\delta(1-\theta)$ so that we have $T_N < \delta - (p-P)$. Take the minimum tax rate $\mu \in [t_N, T_N]$. Then, we have $t_m = \mu < \frac{\delta+\mu+(p-P)}{2} = T_m$. We have cross-border sales in the small country if and only if $t_m + p < T_m + P$, that is, $\mu < \delta - (p-P)$ which always holds as $\mu < T_N < \delta - (p-P)$. So, there are always cross-border sales in the small country.

We have $R(\mu) = T_m H\left(1 - \frac{T_m+P-(t_m+p)}{\delta}\right) = H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2$. Notice that $R(\mu) > R(t_N, T_N)$ if and only if $H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2 > H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$, that is, $\mu > \frac{2\delta\theta+\delta-(p-P)}{3} = t_N$ which always holds as $\mu > t_N$.

We have $r(\mu) = t_m h + t_m H\frac{T_m+P-(t_m+p)}{\delta} = H\frac{1}{\delta}\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2}$. Then $r(\mu) > r(t_N, T_N)$ if and only if $H\frac{1}{\delta}\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2} > H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$, that is, $\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2} > \left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$. It is easy to show that the graph of the left-hand side of this inequality is given in Figure 23. Then,

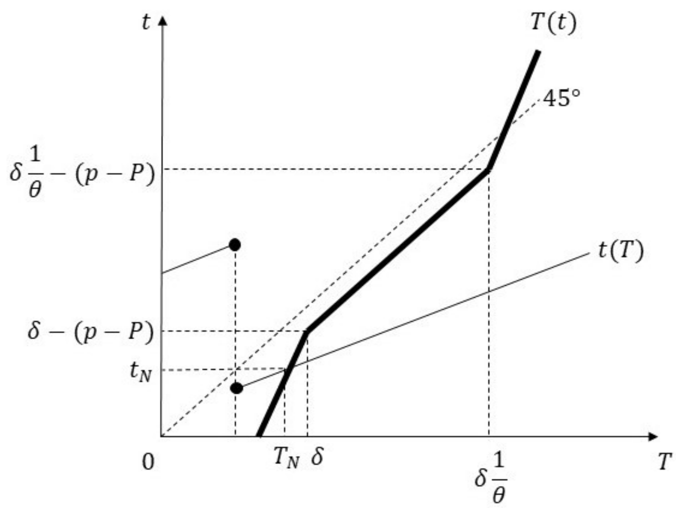


Figure 22: Equilibrium in Case 3 of proof of Proposition 5 Part (ii).

the minimum tax rate increases the small country's tax revenue if and only if $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2\delta\theta+\delta-(p-P)}{2} + \frac{2\delta\theta+\delta-(p-P)}{2} - \frac{2\delta\theta+\delta-(p-P)}{3}$, that is, $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2[2\delta\theta+\delta-(p-P)]}{3}$.

So, for Case 4.i, we have $\bar{\mu} = \frac{2[2\delta\theta+\delta-(p-P)]}{3}$.

Case 4.ii: Assume that $\frac{1}{4}\delta(1-\theta) < p-P < \frac{2\delta+\delta\theta-3\delta\sqrt{\theta}}{2}$. so that we have $\delta-(p-P) < T_N < \delta\frac{1}{\theta}-(p-P)$. Take the minimum tax rate $\mu \in [t_N, \delta-(p-P)]$. Then, we have $t_m = \mu < \frac{\delta+\mu+(p-P)}{2} = T_m$. There is still a cross-border sale in the small country if and only if $t_m + p < T_m + P$, that is, $p-P < \delta-\mu$. Notice that by our assumption about the parameters, in this case, we have $\delta-(\delta-(p-P)) < \delta-\mu < \delta-\frac{2\delta\theta+\delta+(p-P)}{3}$, that is, $p-P < \delta-\mu < \frac{2\delta-2\delta\theta+p-P}{3}$. So, we always have a cross-border sale in the small country.

We have $R(\mu) = T_m H\left(1-\frac{T_m+P-(t_m+p)}{\delta}\right) = H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2$. Notice that $R(\mu) > R(t_N, T_N)$ if and only if $H\frac{1}{\delta}\left(\frac{\mu+\delta+(p-P)}{2}\right)^2 > H\frac{1}{\delta}\left(\frac{\delta\theta+2\delta+p-P}{3}\right)^2$, that is, $\mu > \frac{2\delta\theta+\delta-(p-P)}{3} = t_N$ which always holds as $\mu > t_N$.

We have $r(\mu) = t_m h + t_m H\frac{T_m+P-(t_m+p)}{\delta} = H\frac{1}{\delta}\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2}$. Then, $r(\mu) > r(t_N, T_N)$ if and only if $H\frac{1}{\delta}\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2} > H\frac{1}{\delta}\left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$, that is, $\frac{-\mu^2+[2\delta\theta+\delta-(p-P]\mu}{2} > \left(\frac{2\delta\theta+\delta-(p-P)}{3}\right)^2$. It is easy to show that the graph of the left-hand side of this inequality is given in Figure 24. Then, the minimum tax rate increases the small country's tax revenue if and only if $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2\delta\theta+\delta-(p-P)}{2} + \frac{2\delta\theta+\delta-(p-P)}{2} - \frac{2\delta\theta+\delta-(p-P)}{3}$, that is, $\frac{2\delta\theta+\delta-(p-P)}{3} < \mu < \frac{2[2\delta\theta+\delta-(p-P)]}{3}$.

So, for Case 4.ii, we have $\bar{\mu} = \frac{2[2\delta\theta+\delta-(p-P)]}{3}$.

Case 5: Assume that $\max\left\{\delta, \frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2}\right\} < p-P < \delta\frac{1}{\theta}$. Then, we have $t_N = \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < T_N = \frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}$ by Proposition 1 Part (iv), and cross-shopping in the large country by Proposition 2. These imply that we have $R(t_N, T_N) = h\frac{1}{\delta}\left(\frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = h\frac{1}{\delta}\left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$

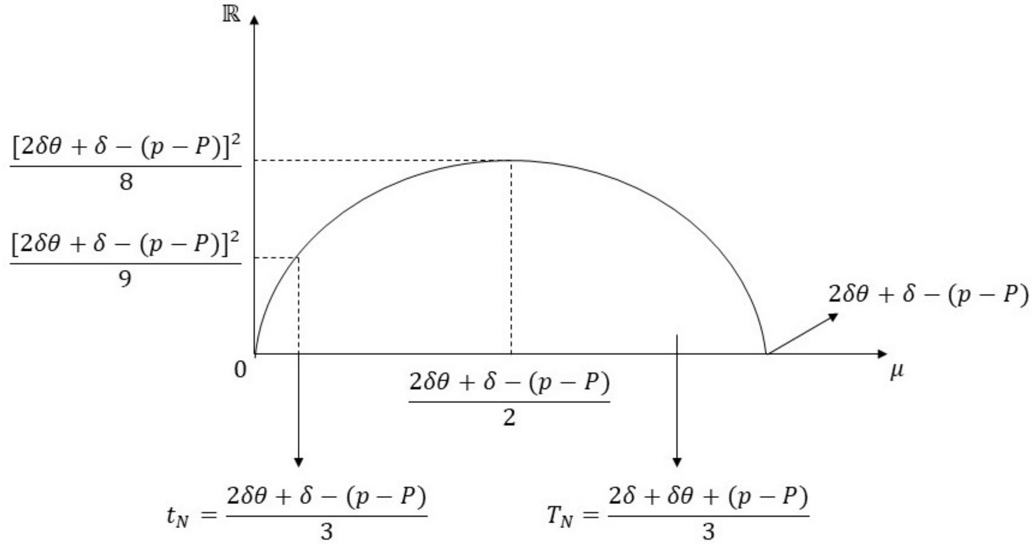


Figure 23: Graph of the inequality in Case 4(i) of the proof of Proposition 5 Part (ii).

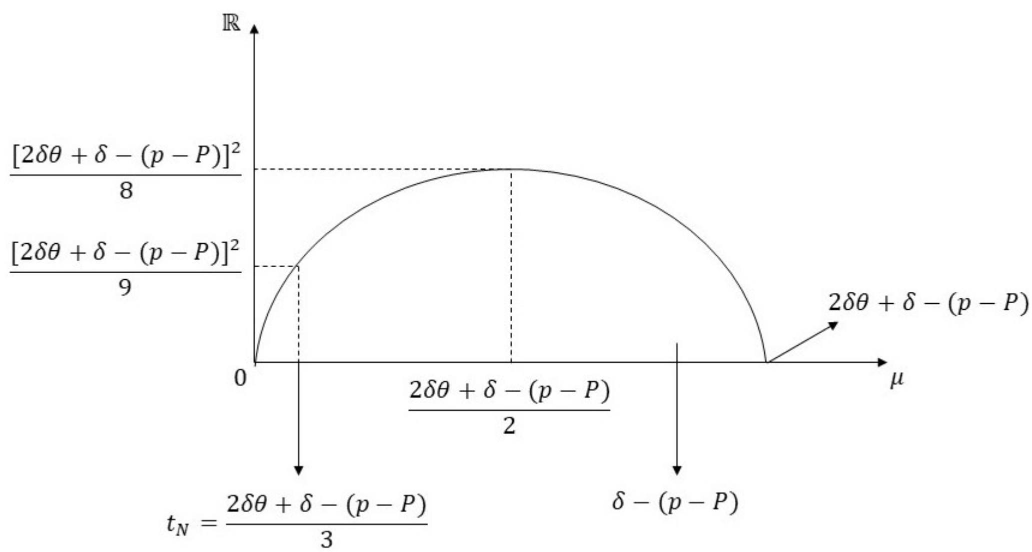


Figure 24: Graph of the inequality in Case 4(ii) of proof of Proposition 5 Part (ii).

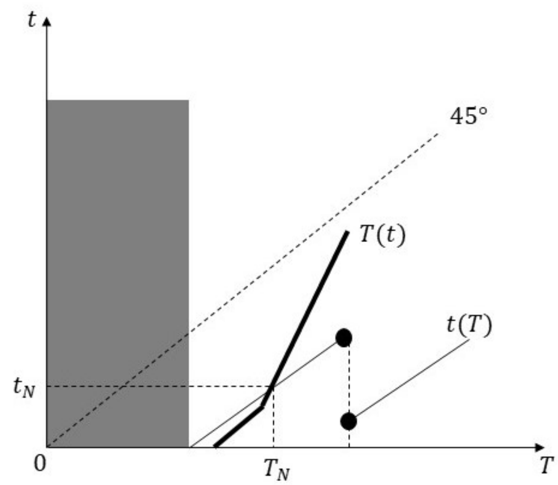


Figure 25: Equilibrium in Case 5 of proof of Proposition 5 Part (ii).

and the equilibrium given in Figure 25. Following the discussion in Section 4, under a minimum-tax rate $\mu \in [t_N, T_N]$, we can ignore $T(t)$ for $t < \mu$. And $t(T)$ will be equal to μ before the μ-line cuts $t(T)$ in its middle part, get longer before the discontinuity point, and will be equal to μ before the value of T is such that $t(T) = \mu$. Then, we have $t_m = \mu < \frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2} = T_m$. There is a cross-border sale in the large country if and only if $T_m + P < t_m + p$, that is, $\delta\frac{1}{\theta} - (p-P) < \mu$. Assume that $\mu < \delta\frac{1}{\theta} - (p-P)$ so that there is a cross-border sale in the small country. For this to hold, we need $t_N < \delta\frac{1}{\theta} - (p-P)$ which implies that $p - P < \delta\frac{1}{\theta} - \delta$. For this to hold, we need $\frac{2\delta\frac{1}{\theta}+\delta-3\delta\sqrt{\theta}}{2} < \delta\frac{1}{\theta} - \delta$, that is, $1 < \sqrt{\theta}$ which never holds. So, there is always cross-border sale in the large country.

We have $R(\mu) = T_m H + T_m h\frac{t_m+p-(T_m+P)}{\delta} = h\frac{1}{\delta}\left(\frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2}\right)^2$. Then, $R(\mu) \geq R(t_N, T_N)$ if and only if $h\frac{1}{\delta}\left(\frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2}\right)^2 \geq h\frac{1}{\delta}\left(\frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)^2$, that is, $\mu \geq \frac{\delta\frac{1}{\theta}+2\delta-(p-P)}{3} = T_N$ which always holds as $\mu > T_N$.

We have $r(\mu) = t_m h\left(1 - \frac{t_m+p-(T_m+P)}{\delta}\right) = h\frac{1}{\delta}\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2}$. Then, $r(\mu) \geq r(t_N, T)N)$ if and only if $h\frac{1}{\delta}\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2} > h\frac{1}{\delta}\left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$, that is, $\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2} > \left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$. It is easy to show that the graph of the left-hand side of this inequality is given in Figure 26. Then, the minimum tax rate increases the small country's tax revenue if and only if $\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < \mu < \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{2} + \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{2} - \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}$, that is, $\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < \mu < \frac{2[2\delta+\delta\frac{1}{\theta}-(p-P)]}{3}$.

So, for Case 5, we have $\bar{\mu} = \frac{2[2\delta+\delta\frac{1}{\theta}-(p-P)]}{3}$.

Case 6: Assume that $\delta\frac{1}{\theta} < p-P < \delta\frac{1}{\theta}+2\delta$. Then, we have $t_N = \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < T_N = \frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}$ by Proposition 1 Part (iv), and cross-shopping in the large country by Proposition 2. These imply that we have $R(t_N, T_N) = h\frac{1}{\delta}\left(\frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)^2$ and $r(t_N, T_N) = h\frac{1}{\delta}\left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$ and the equilibrium is as given in Figure 27.

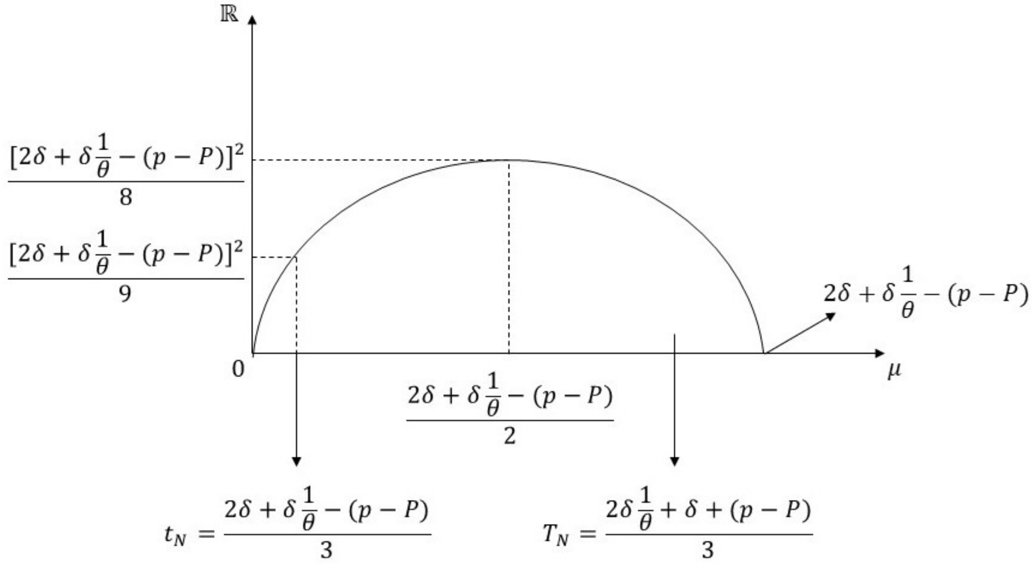


Figure 26: Graph of the inequality in the proof of Case 5 of proof of Proposition 5 Part (ii).

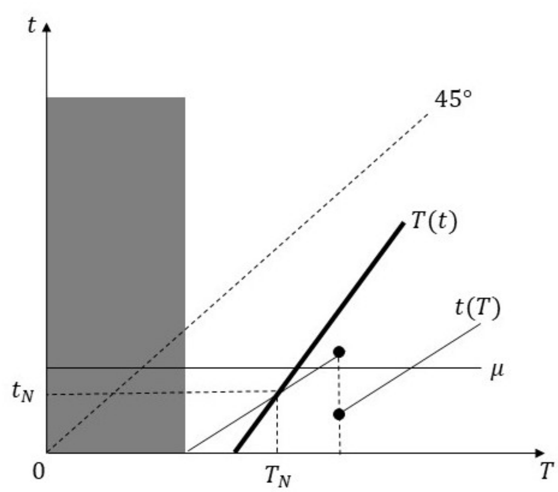


Figure 27: Equilibrium in Case 6 of the proof of Proposition 5 Part (ii).

Following the discussion in Section 4, under a minimum-tax rate $\mu \in [t_N, T_N]$, we can ignore $T(t)$ for $t < \mu$. And $t(T)$ will be equal to μ before μ-line cuts $t(T)$ in its middle part, get longer before the discontinuity point, and will be equal to μ before the value of T is such that $t(T) = \mu$. Then we have $t_m = \mu < \frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2} = T_m$. There is a cross-border sale in the large country if and only if $T_m P < t_n + p$, that is, $\delta\frac{1}{\theta} - (p-P) < \mu$ which always holds as $\delta\frac{1}{\theta} - (p-P) < 0 < t_N < \mu$.

We have $R(\mu) = T_m H + T_m h \frac{t_m+p-(T_m+P)}{\delta} = h\frac{1}{\delta}\left(\frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2}\right)^2$. Then, $R(\mu) \geq R(t_N, T_N)$ if and only if $h\frac{1}{\delta}\left(\frac{\delta\frac{1}{\theta}+\mu+(p-P)}{2}\right)^2 \geq h\frac{1}{\delta}\left(\frac{2\delta\frac{1}{\theta}+\delta+p-P}{3}\right)^2$, that is, $\mu \geq \frac{\delta\frac{1}{\theta}+2\delta-(p-P)}{3} = t_N$ which always holds as $\mu > t_N$.

We have $r(\mu) = t_m h\left(1 - \frac{t_m+p-(T_m+P)}{\delta}\right) = h\frac{1}{\delta}\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2}$. Then, $r(\mu) \geq r(t_N, T_N)$ if and only if $h\frac{1}{\delta}\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2} > h\frac{1}{\delta}\left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$, that is, $\mu\frac{2\delta-\mu+\delta\frac{1}{\theta}-(p-P)}{2} > \left(\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}\right)^2$. It is easy to show that the graph of the left-hand side of the above inequality is as given in Figure 28. Then, the minimum tax rate increases the small country's tax revenue if and only if $\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < \mu < \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{2} + \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{2} - \frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3}$, that is, $\frac{2\delta+\delta\frac{1}{\theta}-(p-P)}{3} < \mu < \frac{2[2\delta+\delta\frac{1}{\theta}-(p-P)]}{3}$.

So, for Case 6, we have $\bar{\mu} = \frac{2[2\delta+\delta\frac{1}{\theta}-(p-P)]}{3}$.

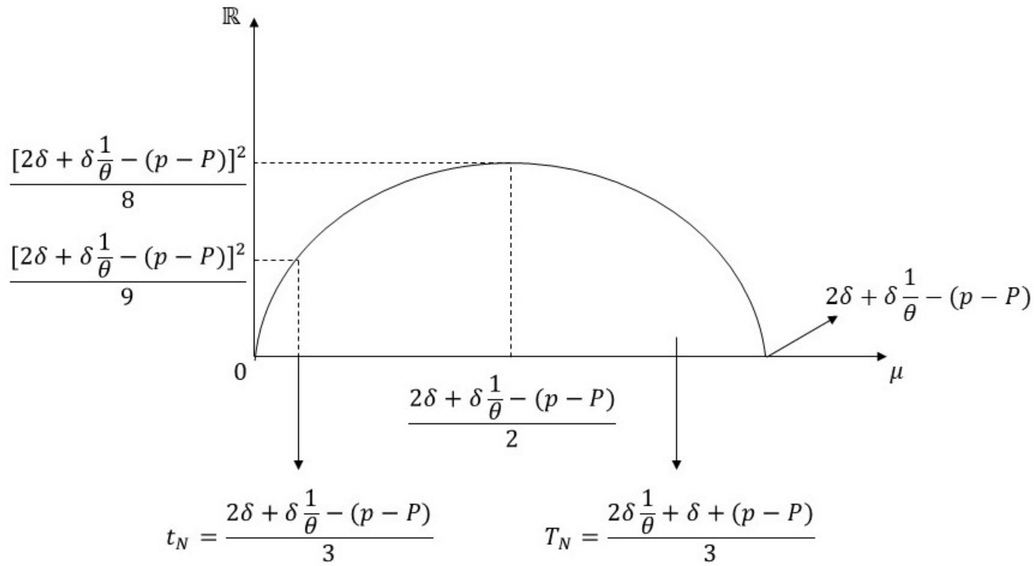


Figure 28: Graph of the inequality in Case 6 of proof of Proposition 5 Part (ii).

□

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