Wealth Inequality, Entrepreneurship and Aggregate Output: A Tale of Two Centuries in the UK*

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Abstract: This paper investigates the long-run nexus between wealth inequality and aggregate output using a DSGE model in which wealth inequality endogenously affects individual entrepreneurship incentives, thereby influencing aggregate output. Our model passes the indirect inference test against the UK data from 1870 to 2015. We find that shocks to aggregate TFP, entrepreneurial barriers, government grant support and general government spending played significant roles in shaping historical inequality dynamics in the UK. Directly removing entrepreneurial barriers or indirectly providing government grant support to the private sector such as through inclusive loan subsidies are effective means of reducing inequality and stimulating output growth.

Keywords: Wealth Inequality, Aggregate Output, Entrepreneurship, Indirect Inference

JEL: E10, C63, O40, D31, N30

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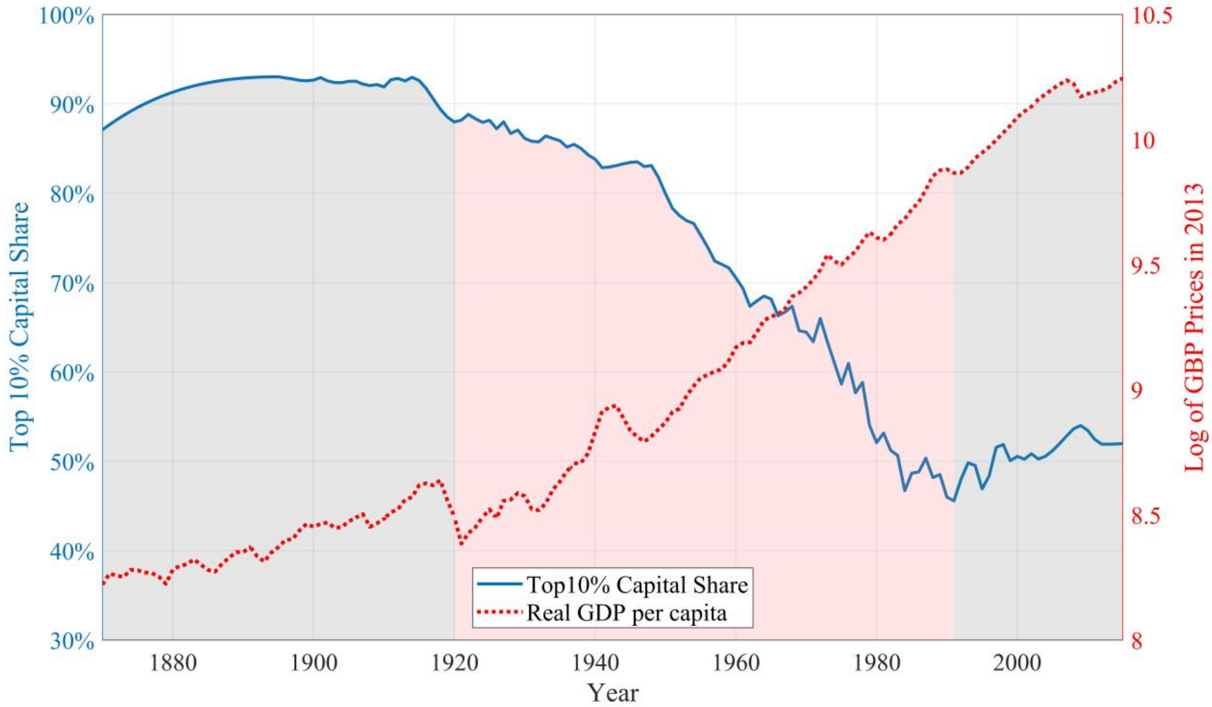
1. Introduction

The nexus between inequality and output attracts enduring research interest of economists since Kuznets (1955). Mainstream theoretical economists often utilize deterministic models to explore how initial wealth distributions shape the balanced growth path of the macroeconomy (Bertola, 1993; Alesina & Rodrik, 1994; Galor & Moav, 2004; Garbinti et al., 2020), whereas empirical econometricians frequently rely on reduced-form panel regressions to investigate this nexus over short time intervals (Barro, 2000; Banerjee & Duflo, 2003; Bagchi & Svejnar, 2015; Ericsson & Molinder, 2020). However, the relationship between wealth inequality and aggregate output is complex and varies over time (Fig. 1): they evolved in the same direction before World War I and from the 1990s onward (grey-shaded areas), whereas they exhibited an inverse co-movement from 1920 to 1990 (pink-shaded area). These patterns indicate that both the static theoretical models and the short-sample empirical estimates are inadequate to fully capture the dynamic interaction between inequality and output. To address this limitation, we re-examine the long-term relationship between wealth inequality and aggregate output using a dynamic structural model.

To achieve this, we develop a multi-agent dynamic stochastic general equilibrium (DSGE) model with heterogeneity in individual wealth levels. At the core of our model is the idea that endogenous wealth levels determine individual entrepreneurial barriers, and individuals choose to undertake entrepreneurial activities when the expected marginal utility of future returns exceeds the utility of current consumption sacrificed for entrepreneurial entry costs. Government grant support can mitigate the strengthened relationship between entrepreneurship and inequality by alleviating entrepreneurial barriers for the less wealthy. To empirically validate our model, we utilize historical data from the Bank of England (BOE) spanning from 1870 to 2015 and use indirect inference (II) to estimate the structural parameters, followed by a series of shock analyses and counterfactual simulations. Our results suggest that rising aggregate TFP and government grant support, and reductions in entrepreneurial barriers are key drivers of both output growth and the reduction of wealth inequality. Our counterfactual simulations find that if the entrepreneurial barriers rate had been reduced by 5% cumulatively in the late 1980s, it would have resulted in an additional 55% cumulative output growth and a 4% reduction in inequality over the subsequent three decades. Moreover, early and continuous government support to the private sector may yield significant long-term benefits.

Fig. 1 Wealth Inequality and Aggregate Output in the UK from 1870 to 2015



This paper makes several contributions. Theoretically, we construct a dynamic model that captures the evolving relationship between wealth inequality and aggregate output over time and across different economic states. In terms of mechanisms, only a limited number of studies have explored the interconnected roles of entrepreneurship and inequality in shaping economic outcomes. For instance, Cagetti & De Nardi (2006) emphasize how financial constraints, by raising collateral requirements, force entrepreneurs—particularly wealthier ones—to operate smaller-scale businesses, ultimately reducing inequality. In contrast, our model highlights the impact of wealth-driven disparities in entrepreneurial barriers, demonstrating that reducing these barriers for less wealthy individuals can enhance productivity while mitigating inequality. This approach bridges the gap between the “inequality-entrepreneurship” and “entrepreneurship-output” channels, emphasizing how targeted policies, such as government grant support, can mitigate inequality while fostering aggregate economic growth. Empirically, we quantitatively explain the effects of various shocks on inequality across different historical periods and provide an analysis of the potential effects of hypothetical policy adjustments. The analytical framework developed in this paper provides a flexible approach that can be extended to empirical studies beyond the context of the UK.

The next section conducts a comprehensive literature review. Section 3 sets out our DSGE model, followed by a detailed data description in Section 4. Section 5 describes the indirect inference method for model estimation and testing. The empirical results are presented in

Section 6 before conclusions (Section 7) are drawn.

2. Related Literature

Theoretical research on the nexus between wealth inequality and aggregate output has been continuously enriched since the 1990s. The prevailing theory is that wealth endowments drive individual investment decisions in physical and human capital, and thus aggregate output (Aghion et al., 1999). The seminal literature on this theory can be traced to Galor & Zeira (1993), who develop an OLG model in which imperfections in credit markets and the indivisibility of human capital investment diversify individual decisions and their labor skills given heterogeneous wealth endowments. The broader idea behind this theory is the effect of wealth-induced unfairness of opportunity on the drivers of aggregate output. Economists are accustomed to interpreting opportunity disparities in the context of the inequality-output nexus in terms of heterogeneity in labor skills (Galor & Zeira, 1993; Alesina & Rodrik, 1994; Persson &Tabellini, 1994), education levels (Maoz & Moav, 1999; Yang & Zhou, 2022), savings preferences (Fishman & Simhon, 2002; Galor & Moav, 2004; Garbinti et al., 2020), consumption propensity (Foellmi & Zweimüller, 2006; Carroll et al., 2017; Bental & Kragl, 2021), risk-taking ability (Foellmi & Oechslin, 2008; Ghiglino & Venditti, 2011) and health status (Martin & Baten, 2022). However, another crucial aspect of opportunity is overlooked—entrepreneurship.

The importance of entrepreneurship to economic development has been well established, either through the knowledge spillover (Acs et al., 2009; Akcigit & Kerr, 2018) or through the improvements in organizational management (Tether & Tajar, 2008; Hervas-Oliver et al., 2016), both of which boil down to the entrepreneurial forms of innovation. Furthermore, the fact that entrepreneurship is dependent on wealth holdings (Blanchflower & Oswald, 1998; Levine & Rubinstein, 2017; Aghion, et al., 2019) and credit constraints (Evans & Jovanovic, 1989; Cagetti & De Nardi, 2006; Coibion et al., 2020), is also well established. However, these two critical relations have not been effectively integrated in previous literature investigating the inequality-output nexus. This paper addresses this gap by providing a comprehensive examination of the interconnections between wealth inequality, entrepreneurship, and aggregate output. Specifically, unlike Cagetti & De Nardi (2006), who assume entrepreneurship as an indivisible choice and model increasing default costs for wealthier entrepreneurs, we consider entrepreneurship as a divisible activity, where individuals can allocate time between entrepreneurial and labor activities. This approach reflects the reality that even entrepreneurs contribute labor to manage and operate their businesses (Cagetti & De Nardi, 2006). Moreover, our setup, which links entrepreneurial barriers to relative wealth levels, more intuitively captures the fact that wealthier individuals are more able to afford high entrepreneurial cost and can more easily engage in entrepreneurship. This setting also facilitates the integration of the entrepreneurial mechanism into a dynamic framework, rather than limiting it to static analysis.

Empirically, the primary approach employed to examine the inequality-output nexus has predominantly been the use of panel data analysis, with samples covering numerous countries and spanning several years or decades (e.g., Deininger & Squire, 1998; Castelló & Domenech, 2002; Bagchi & Svejnar, 2015; Ericsson & Molinder, 2020). While these studies have yielded insights that are generally less controversial than theoretical models, the identification of causal factors remains challenging. The inherent difficulty lies in identifying causation amid the multifaceted nature of the economic output, where numerous accompanying factors may contribute to the observed effects. Reduced-form regressions often struggle to disentangle causes driving growth from those driven by it. Moreover, both the choice of estimation method and the composition of the sample can have a substantial impact on the estimated nexus (Halter et al., 2014). In this context, our study takes a distinctive approach by investigating the long-term inequality-output nexus within the framework of a DSGE model. The dynamic framework allows for a more nuanced examination of the causal mechanisms at play, providing valuable insights into how wealth inequality affects economic output over time. Noteworthy among few prior dynamic studies is the work of Álvarez-Peláez & Díaz (2005) who set up a Ramsey model with the "minimum consumption" constraint for households. Our research is related to this paper in the dynamic approach, but with a different mechanism—entrepreneurship.

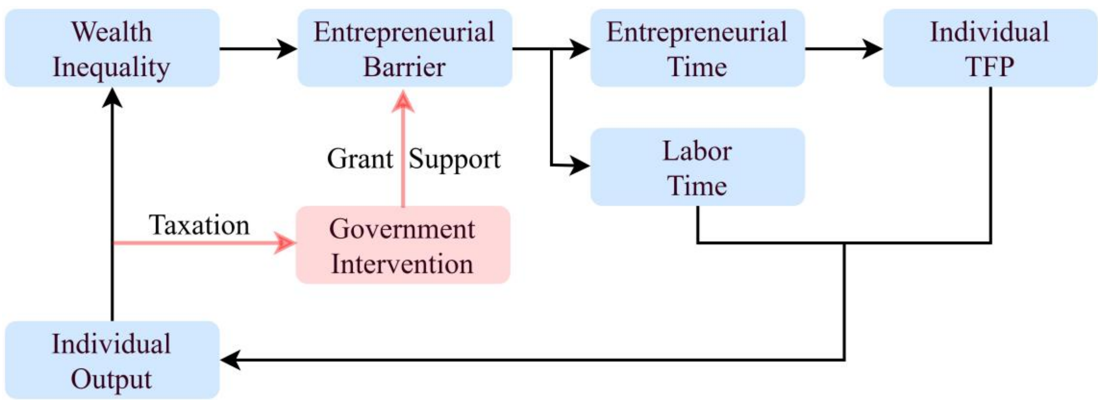
3. The DSGE Model

In this section, we expand on our rigorously-based DSGE model, which features an economy comprised of two distinct groups—the wealthy and the less wealthy—each with constant population weights denoted by $\mu_i;\ i = 1, 2$. The assumption of constant population weights has been applied in numerous macroeconomic studies due to data availability (Shi, 1999; Foellmi & Zweimüller, 2006; Yang et al., 2024). Although this approach inevitably overlooks long-term social mobility across wealth classes, Clark & Cummins (2015) provide a realistic basis for this assumption, showing that wealth inheritance across five generations in the UK remained persistently consistent, suggesting that the top wealth group in the UK has been relatively stable over a century.

We model each wealth group as a representative agent who can allocate divisible time between entrepreneurial activities and working for their own business (Quadrini, 2000). This setting diverges from the approaches of Cagetti & De Nardi (2006) and Bayer et al. (2024), who assume that individuals must choose between two mutually exclusive, indivisible states: entrepreneurship or labor. Although Cagetti & De Nardi (2006) argue that entrepreneurs in their model run their "own business" and "use their own labor"—acting both as business owners and workers—the indivisible time allocation serves as a useful simplification for models involving a large number of heterogeneous individuals. In contrast, our model allows each representative agent to allocate divisible time between entrepreneurial activities and regular labor, thereby acknowledging that entrepreneurs must also contribute labor to their ventures. This setting can be understood by considering that each representative agent is essentially an aggregation of numerous individuals within the same wealth group. While each individual may face a binary decision, the aggregation of many individuals leads to a representative agent whose time allocation between entrepreneurship and labor is effectively continuous.

Both entrepreneurial time and regular labor in our model serve as inputs into their production process. Notably, entrepreneurial time affects the individual's total factor productivity (TFP), thereby influencing their output. At the same time, entrepreneurship also entails sunk costs, which are typically reflected in administrative taxes and fees in preparation and operation, and other non-wage employment expenses. Given that wealthier individuals are either less financially constrained or better able to absorb sunk costs, we naturally link individuals' entrepreneurial costs to their relative wealth levels, following Hurst & Lusardi (2004). Consequently, initial wealth inequality implies that individuals face different entrepreneurial barriers. Each agent determines their time allocation by balancing the marginal returns from entrepreneurship against the marginal returns from regular labor, ultimately impacting individual output through changes in TFP and labor inputs, leading to new wealth effects. This loop can potentially lead to a continuous exacerbation of wealth inequality. Government interventions, such as direct tax redistribution and inclusive financial support to lower entrepreneurial barriers for the less affluent, effectively mitigate this adverse dynamic. The interactions between these mechanisms (as depicted in Fig. 2) helps explain the long-term inequality-output dynamics observed in the historical data.

Fig. 2 Flow chart of the model mechanisms



3.1 Individual Behavior

We first consider individual decision-making process, which involves maximizing the expected utility as represented by equation (1). $C_{i,t}$ represents consumption, $N_{i,t}$ and $Z_{i,t}$ denote labor time and entrepreneurship time for agent i, respectively. $u_{i,t}$ and $v_{i,t}$ are idiosyncratic consumption preference shocks and leisure preference shocks.

$$\mathrm{U}\left(C_{i,t}, N_{i,t}, Z_{i,t}\right) = \Phi u_{i,t} \frac{\left(C_{i,t}\right)^{1-\psi_1}}{1-\psi_1} - (1-\Phi) v_{i,t} \frac{\left(N_{i,t}+Z_{i,t}\right)^{1+\psi_2}}{1+\psi_2} \tag{1}$$

Under the assumption of perfectly competitive labor and capital markets, agents face a budget constraint as represented by equation (2). Specifically, after producing output $Y_{i,t}$, agent i pays a proportion τ_t of this output as income tax to the government and earns income from holding government bonds, amounting to $r_{t-1}b_{i,t}$, along with a lump-sum transfer T_t. Each agent must then decide how to allocate resources among consumption, investment in their own production $I_{i,t}$, (which incurs capital adjustment costs, represented by $AJ_{i,t}$), changing bonds purchasing $b_{i,t+1} - b_{i,t}$, and covering the sunk costs associated with entrepreneurship $\varphi_{i,t}\pi_t Z_{i,t}$. The sunk costs are composed of the entrepreneurial time allocation $Z_{i,t}$, the unit cost of entrepreneurial time π_t, and a wealth-group-specific markup $\varphi_{i,t}$. These costs represent necessary expenditures such as administrative fees, non-wage labor costs, and other operational expenses incurred during the establishment and management of a business.

$$(1-\tau_t)Y_{i,t} + r_{t-1}b_{i,t} + T_t = C_{i,t} + \left(b_{i,t+1} - b_{i,t}\right) + \left(I_{i,t} + AJ_{i,t}\right) + \varphi_{i,t}\pi_t Z_{i,t} \tag{2}$$

$I_{i,t}$ is calculated as $K_{i,t} - (1-\delta)K_{i,t-1}$, where $K_{i,t-1}$ represents the capital stock at the beginning of period t. $AJ_{i,t}$ is given by $\Gamma\left(I_{i,t}/K_{i,t-1}\right)K_{i,t-1}$ is the capital adjustment cost. Since our focus is not on the micro-level details of firms' investment costs, we use a quadratic adjustment cost, known as the "Hayashi" form (Hayashi, 1982), which is sufficient to capture aggregate investment behavior (Thomas, 2002).

$$AJ_{i,t} = \frac{\xi}{2}\left(\frac{I_{i,t}}{K_{i,t-1}} - \delta\right)^2 K_{i,t-1} = \frac{\xi}{2}\frac{\left(K_{i,t} - K_{i,t-1}\right)^2}{K_{i,t-1}}$$

Given that the economy is divided into two wealth groups with relatively stable membership within each group, we simply assume that each group's representative agent employs their own labor and capital for closed production, following the form of equation (3). In this paper, entrepreneurship serves as a form of innovation, either through knowledge spillovers (Akcigit & Kerr, 2018) or through improvements in organizational management (Hervas-Oliver et al., 2016), and its return is reflected in enhanced TFP. Therefore, we endogenize individual TFP $A_{i,t}$ as a function of entrepreneurial time $Z_{i,t}$ and the TFP growth shock $v_{A,t}$, as described in equation (4). $v_{A,t}$ with a mean of 1, represents either the success rate of entrepreneurship or the inherent risk involved, which is homogeneous across all individuals.

$$Y_{i,t} = A_{i,t}\left(K_{i,t-1}\right)^{\alpha}\left(N_{i,t}\right)^{1-\alpha} \tag{3}$$

$$\frac{A_{i,t+1}}{A_{i,t}} = \theta_1 + \theta_2 Z_{i,t} v_{A,t} \tag{4}$$

As the model is deliberately simple, one can easily obtain the following optimal decisions from first order conditions. Following Yang et al. (2024), we approximate the optimal decision rule for $Z_{i,t}$ full of complexity to equation (5) where $\theta_3 = \theta_2\beta/\left[\left(1+gY_{i,t}\right)^{\psi_1-1} - \beta\right]$. See Appendix A for Lagrange optimization and the full model listing.

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} + \varphi_{i,t}\pi_t = v_{A,t}\frac{A_{i,t}}{A_{i,t+1}}(1-\tau_t)Y_{i,t}\theta_3 \tag{5}$$

Given a perfectly competitive labor market, $(1-\alpha)Y_{i,t}/N_{i,t}$ represents the implicit real wage rate $w_{i,t}$ for agent i. The term $\varphi_{i,t}\pi_t$ represents the entrepreneurial cost that must be borne when giving up one unit of regular labor time to engage in entrepreneurial activities. Therefore, we can interpret $\varphi_{i,t}\pi_t/w_{i,t}$ as the relative opportunity cost of one unit of entrepreneurial time. We define the variable $\pi_t' = \pi_t/\overline{w}_t$ as the entrepreneurial barriers rate, and the new markup $\varphi_{i,t}'$ captures the difference between $w_{i,t}$ and the average wage rate $\overline{w}_t$. Consequently, equation (5) can be rewritten as equation (6) and further linearized to obtain equation (7). $\ln\pi_t'$ follows an *AR*(1) process.

$$\frac{A_{i,t+1}}{A_{i,t}} = \frac{(1-\tau_t)\theta_3}{\left(1-\tau_t+\varphi_{i,t}'\pi_t'\right)(1-\alpha)}N_{i,t}v_{A,t} \tag{6}$$

$$\ln A_{i,t+1} - \ln A_{i,t} = \phi_{1,i} - \phi_{2,i}\left(\ln\pi_t' + \ln\varphi_{i,t}' + \frac{\tau_t}{1-\tau}\right) + \ln N_{i,t} + \varepsilon_{A,t} \tag{7}$$

$$\ln\pi_t' = \gamma_\pi \ln\pi_{t-1}' + \varepsilon_{\pi,t} \tag{8}$$

3.2 Markup of Barriers Rate and Government Intervention

Agent i's markup $\varphi_{i,t}'$ measures the additional entrepreneurial barriers faced relative to the societal average. It is well-documented that wealthy individuals tend to face fewer entrepreneurial barriers due to advantages such as lower financing costs, stronger bargaining power, and extensive social networks (Levine & Rubinstein, 2017; Coibion et al., 2020). Therefore, we standardize the markup for the wealthy to one, i.e., $\varphi'_{1,t} = 1$ and specify $\varphi'_{2,t}$ for the less wealthy as being negatively related to their relative wealth, $K_{2,t-1}/K_{1,t-1}$.

In the absence of government intervention, credit market preferences for wealthier entrepreneurs would ultimately lead to an uncontrolled exacerbation of inequality. However, numerous studies have shown that government intervention such as guarantees and bailouts effectively prevents this undesirable outcome, and these fiscal windfalls and increased subventions have played a crucial role in protecting smaller and younger enterprises (Moro et al., 2020; Bi et al., 2024; Garmaise & Natividad, 2024). Based on this solid body of evidence, we further incorporate the government grant support ratio to the private sector (GS_t) into the markup for the less wealthy, $\varphi'_{2,t}$, which results in the final form represented by equation (9). The linearized version of this relationship is given in equation (10).

$$\varphi'_{2,t} = \left(K_{1,t-1}/K_{2,t-1}\right)^{\rho_1} {GS_t}^{-\rho_2} \tag{9}$$

$$\ln\varphi'_{2,t} = \rho_1\left(\ln K_{1,t-1} - \ln K_{2,t-1}\right) - \rho_2 \ln GS_t \tag{10}$$

Government spending is divided into two categories: general spending G_t, which follows an exogenous AR(1) process as described by equation (12), and spending on private grant support, $G_t\left(GS_t - \varepsilon_{CR,t}\right)$, where $\varepsilon_{CR,t}$ represents the shock to the support rate GS_t. Both categories of spending are funded through income tax revenues collected from residents. This balance of payments for the government is represented by equation (11) where Y_t is the aggregate output.

$$G_t + G_t\left(GS_t - \varepsilon_{CR,t}\right) = \tau_t Y_t \tag{11}$$

$$\ln G_t = \gamma_G \ln G_{t-1} + \varepsilon_{G,t} \tag{12}$$

3.3 Aggregate Economy

To close the model, we aggregate the budget constraints of the two representative agents according to their population weights, and assume that the bond market clears via Walras' Law, yielding the initial market clearing condition: $(1-\tau_t)Y_t - \pi_t \sum \mu_i \varphi_{i,t} Z_{i,t} + T_t = C_t + K_t - (1-\delta)K_{t-1} + \sum \mu_i AJ_{i,t}$. For simplicity, we assume that the government's lump-sum transfer payments, T_t, are funded by the total entrepreneurial costs paid by individuals, and that the aggregate capital adjustment cost constitutes an aggregate error term $\epsilon_{x,t}$. Finally, we derive the ultimate market clearing conditions for the economy, represented by the aggregate variables (equations (13)-(15)) and expressed in the final form as equation (16). See Appendix A for the full model listing.

$$Y_t = \mu_1 Y_{1,t} + \mu_2 Y_{2,t} \tag{13}$$

$$K_t = \mu_1 K_{1,t} + \mu_2 K_{2,t} \tag{14}$$

$$C_t = \mu_1 C_{1,t} + \mu_2 C_{2,t} \tag{15}$$

$$(1-\tau_t)Y_t = C_t + K_t - (1-\delta)K_{t-1} + \epsilon_{x,t} \tag{16}$$

4. UK Data over Two Centuries

In this section, we present a comprehensive overview of our data, encompassing meticulous deliberation on sampling, sources, measurements, and treatments to ensure accuracy and relevance for our research objectives. The mid-19th century marked a pivotal point in the development trajectory of industrialized nations, transitioning from the Malthusian epoch to the mature phase of the industrialization process (Galor, 2018). In parallel, the formation of a complete labor market and labor organizations were gradually established, evidenced by the emergence of numerous "new model" unions in Britain and the US during the 1860s and the enactment of the *Trade Unions Act* in Britain in 1871 (Mathias & Postan, 1978). To align our research with these historical milestones, we select the sample period from 1870 to 2015. Our data is drawn from authoritative sources, primarily the Bank of England (BOE) database, "A Millennium of Macroeconomic Data," and the World Inequality Database (WID), both of which are widely recognized and utilized in academic research.

4.1 Data Without Special Preprocessing

The following data require no specialized treatment and are directly sourced from the aforementioned databases. Real GDP and real household consumption, both measured in chained volume at 2013 market prices, are used to represent aggregate output and consumption, respectively.[1] The real interest rate is calculated by taking the annual average of the three-month rate, adjusted for one-year-ahead inflation expectations at the beginning of each year. Aggregate labor is measured by multiplying the total number of employees by their weekly working hours, then dividing by the total weekly hours of the entire population. This straightforward methodology effectively accounts for both the relative labor participation rate and the impact of population growth on the labor force.

We also consider a time-varying capital share (α), which is calculated as one minus the labor share collected from the BOE database. This approach allows us to capture the true variation in output over the long term. Additionally, aggregate productivity is measured by the Solow residual from the aggregate Cobb-Douglas production function, with capital services and aggregate labor as inputs.

4.2 Data With Preprocessing

A conventional method for estimating domestic capital stock involves using the capital stock formation formula $K_t = I_t + (1 - \delta)K_{t-1}$ where K_t stands for the capital stock at the beginning of the next period $(t + 1)$. However, this approach poses challenges, such as accounting for variations in the depreciation rate over extended periods and ensuring consistency with the model's capital adjustment costs (Piketty, 2014). To address these issues, we adopt the national wealth minus net foreign assets approach, following Piketty & Zucman (2014), which provides a more consistent estimation.

Entrepreneurial barriers could occur due to a number of reasons, but constructing an indicator that captures all of the factors that influence entrepreneurship over such a long period of time is almost impossible. To estimate the aggregate entrepreneurial barriers rate (π_t'), we draw on well-documented findings by Djankov et al. (2010) and Kanniainen & Leppämäki (2009) that the increases in tax burdens and in trade union power are significantly detrimental to entrepreneurial activities. The rate is calculated as the average of the ratio of trade union members (TUM) to total employees and the ratio of indirect taxes to GDP.[2] This composite indicator simply takes into account the impact of both output-side (e.g., the impact of various indirect

[1] Break-adjusted data are used to account for the secession of Southern Ireland from the UK (Great Britain and Northern Ireland) in 1922. All data are presented in their original form without stationarization, as such transformation could alter the implied shocks. Non-stationarity does not pose an obstacle to the indirect inference estimation method employed in this paper.

[2] Indirect taxes in the UK, including taxes on production and productivity such as VAT, excise duties, and customs duties, among others, when combined with the trade union membership (TUM) ratio, can serve as an effective measure of the overall entry barrier associated with entrepreneurial activities. This measure is distinct from the income tax τ_t. To maintain additivity, both the TUM ratio and the indirect tax revenue over GDP are normalized by dividing each by its respective average.

taxes on market dynamism) and input-side (e.g., the impact of trade unions on labor costs) factors on entrepreneurial barriers. Income tax is charged for both groups of individuals in our model and thus we measure this average income tax rate by the ratio of direct taxes on income and wealth to GDP. Government support (GS_t), representing the proportion of government credit support extended to the private sector, is calculated as the ratio of grants to personal sector to total government expenditure, based on data from the BOE.

Individual income and individual capital are estimated using a combination of group shares and aggregate values. Leveraging data from WID, we gather information on the top 10% and bottom 90% shares of income and capital, supplementing any missing data through spline interpolation (see Fig. 1 for inequality trends). Individual consumption is determined by individual budget constraints once borrowing dynamics reach equilibrium. The labor force for individuals is approximated using aggregate value, while individual productivity is measured using Solow residuals derived from individual production functions.

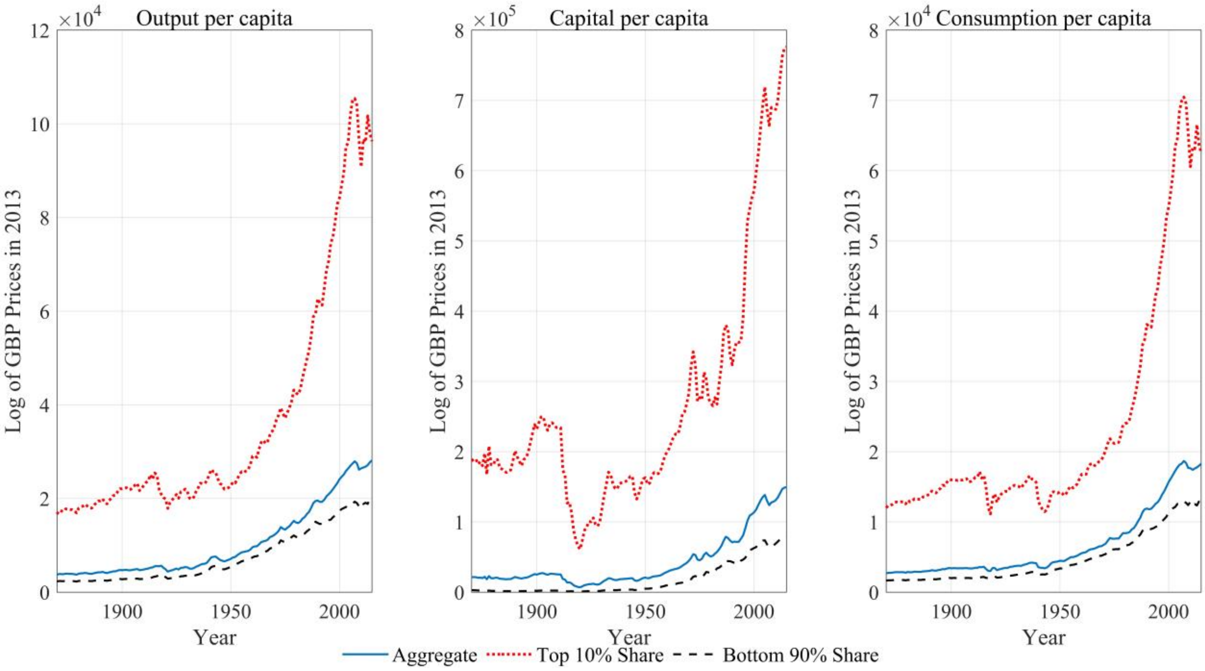
While "net personal wealth" data from the WID forms the basis of our analysis, it is important to clarify that this measure encompasses a broader range of assets than productive capital. Regarding the transformation of wealth into productive capital, we understand this process as an aggregation of various asset types, each contributing to production through distinct mechanisms. For example, housing wealth can generate rental income through leasing functions, while equity investments yield profits through traditional production processes. These different streams are consolidated into a single production function for each wealth group, allowing us to capture the aggregate productive capacity of wealth without explicitly modeling each asset type. While this approach abstracts from the details of wealth composition, it aligns with our model's focus on macroeconomic dynamics over long historical periods. In addition, we adopt a conservative two-agent framework, where the less wealthy group aggregates a diverse population, including individuals with little or no business wealth and even those with negative net wealth. This approach mitigates discrepancies between the model and empirical data, smoothing outliers in the wealth distribution's left tail.

4.3 Presentation of Historical Data

Fig. 3 illustrates the historical trends of real output per capita, real capital per capita, and real consumption per capita in the UK from 1870 to 2015. At an aggregate level, all three indicators exhibit an overall upward trend over the long term. However, significant downturns are evident during key historical events, including both World Wars, the post-war recession of the 1970s, the early 1990s recession, and the 2007 financial crisis. Among these three metrics, capital stock experienced the most pronounced negative impact during these periods of economic distress. From a distributional perspective, the top 10% share of output, capital, and consumption exhibited more substantial fluctuations, particularly in the case of capital stock. This heightened volatility led to the "flat Z" pattern in wealth/capital inequality depicted in Fig. 1.

Fig. 4 presents the historical trends of other variables and time-varying parameters in the UK. Among the variables depicted, real government spending per capita stands out, showing a generally steady increase over time. However, during both World Wars, government spending surged significantly, reflecting heightened fiscal needs. Consistent with this, the proportional income tax rate (τ) also increased dramatically during the two wars, indicating an intensified fiscal effort to support wartime expenditures. In contrast to overall government spending, government support to the private sector experienced sharp declines during both World Wars, followed by rapid increases during the post-war recovery phases. This pattern suggests the crucial role of government grant support to the private sector in fostering economic recovery, which aligns with the conceptual foundation of our model.

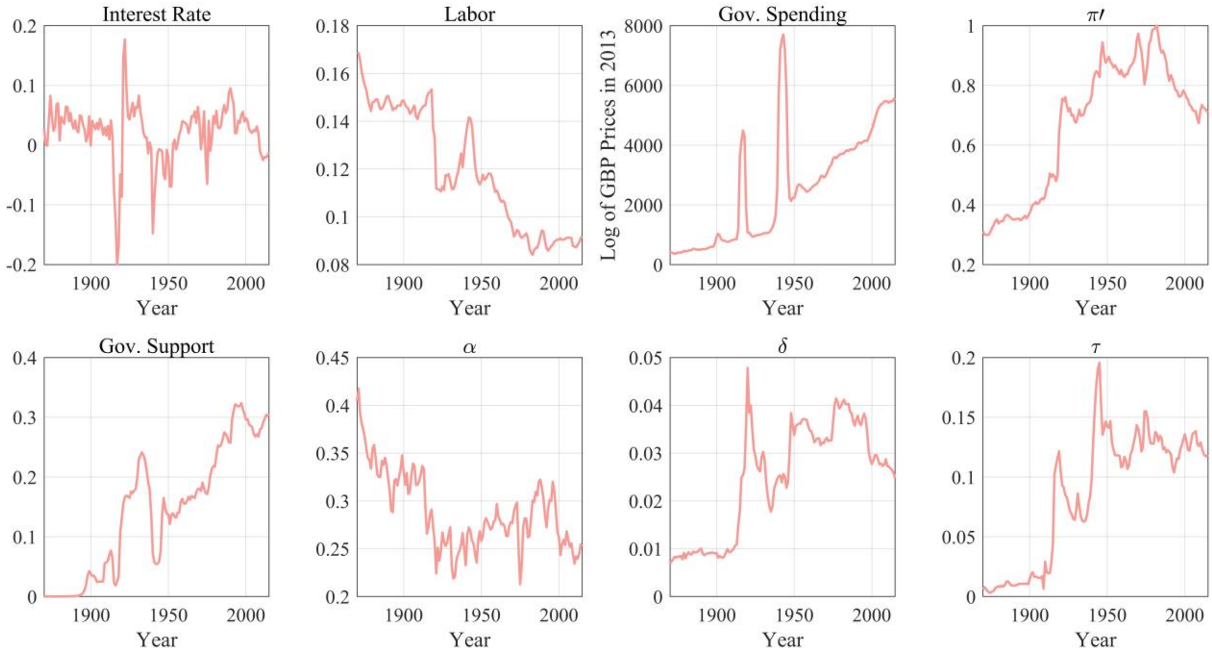
Fig. 3 Real output, capital and consumption in the UK from 1870 to 2015



The share of capital in the production function (α) and the capital depreciation rate (δ) also exhibited notable changes over the long term. Particularly, δ, defined as the ratio of the composite value of capital consumption to domestic capital stock, showed a substantial rise during World War I, indicating significant capital destruction. As noted by Piketty (2014) and Karabarbounis & Neiman (2014), a rising depreciation rate is associated with an increase in the capital-output or wealth-income ratio, contributing to the decline in the labor share of output and worsening inequality after the 1970s and vice versa at the beginning of the 20th century.

Therefore, treating both α and δ as time-varying parameters is crucial for capturing these historical dynamics effectively.

Fig. 4 Time series plots for other variables and time-varying parameters



5. Parameter Estimation Methodology

Our model involves a substantial number of structural parameters. For common parameters that can be inferred from observed data, we calibrate directly based on sample data and relevant literature. Meanwhile, we employ indirect inference (II), a structural estimation method, to estimate the values of other parameters. This section provides the principles and application of the II estimation method. Structural estimation offers inherent advantages over reduced-form regressions in identifying a model's internal mechanisms and conducting counterfactual analysis. It involves generating simulated data from the structural model by incorporating stochastic shocks and comparing its properties to those of the actual data. If both the model specification and parameter values are correct, the properties of the actual data should align with those of the simulated data distribution at a critical minimum probability. Recent applications of the II method are found in Le et al. (2017), Fu & Gregory (2019), Dong et al. (2019, 2023), Gala et al. (2020), Aronsson et al. (2022) and Yang et al. (2024).

To explain how II works, let $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$ denote the structural model parameters and auxiliary model parameters, respectively. We test the null hypothesis $\boldsymbol{\theta} = \boldsymbol{\theta_0}$ for a given set of structural parameter values $\boldsymbol{\theta_0}$. First, we use the actual data to estimate the auxiliary model parameters, $\widehat{\boldsymbol{\beta}}$. Next, we simulate S samples using the structural model and the actual data with S sets of stochastic shocks, then estimate the auxiliary model parameters for each simulated sample to obtain the estimators $\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})$, where $s = 1, \cdots, S$. Under the null hypothesis, $\widehat{\boldsymbol{\beta}}$ and the average of $\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})$ (i.e., $\overline{\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})} = \sum \widetilde{\boldsymbol{\beta}_{\boldsymbol{s}}}(\boldsymbol{\theta_0}) \,/S$) are convergent. The following Wald statistic, $Wald_a$, which asymptotically follows a χ^2 distribution with degrees of freedom equal to the number of elements in $\boldsymbol{\beta}$, is used as the criterion:

$$Wald_a = \left[\widehat{\boldsymbol{\beta}} - \overline{\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})}\right]' \Omega^{-1} \left[\widehat{\boldsymbol{\beta}} - \overline{\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})}\right]$$

where $\boldsymbol{\Omega}$ is the variance-covariance matrix of $\widehat{\boldsymbol{\beta}} - \widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})$.

Next, we replace $\widehat{\boldsymbol{\beta}}$ in $Wald_a$ by $\widetilde{\boldsymbol{\beta}}_{\boldsymbol{s}}(\boldsymbol{\theta_0})$ for each $s = 1, \cdots, S$ to compute an alternative Wald statistic, $Wald_s$. If the null hypothesis is true, $Wald_a$ and $\{Wald_s\}$ should come from the same χ^2 distribution. Therefore, we conclude that if $Wald_a$ is less than the one-tailed critical value of the χ^2 distribution based on the simulated $\{Wald_s\}$ for a given confidence interval, the null hypothesis cannot be rejected, indicating that the estimated model is true.

5.1 Auxiliary model

The II method differs from another prevailing structural estimation method—Simulated Moment Method (SMM)—in terms of the auxiliary model and the stochastic process. SMM typically sets moments (mean, variance or covariance) as the auxiliary model. In contrast, the auxiliary model in II can take various forms, such as vector autoregression model (VAR), vector error correction model (VECM), impulse response function, or moments. Given the significant changes in long-run historical data, moments are unsuitable as an auxiliary model in this study. A VARX(1) (i.e., VAR(1) with non-stationary variables **X** that drive the model solution) has been proved to be a well approximation of a DSGE model with non-stationary exogenous variables (Le et al. (2017).[3] Therefore, after thoroughly examining the Granger causality relationships among the endogenous variables (the selected variables and their causal estimates are presented in Table 1), we choose the following form for the VARX(1):

$$\begin{bmatrix} \ln Y_t \\ \ln K_t \\ \ln K_{1,t} \end{bmatrix} = \boldsymbol{\beta_{VAR}} \cdot \begin{bmatrix} \ln Y_{t-1} \\ \ln K_{t-1} \\ \ln K_{1,t-1} \end{bmatrix} + \boldsymbol{\alpha_{VAR}} \cdot \boldsymbol{X_t} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}; \quad \boldsymbol{\beta_{VAR}} = \begin{bmatrix} \beta_{VAR,1} & \beta_{VAR,2} & \beta_{VAR,3} \\ \beta_{VAR,4} & \beta_{VAR,5} & \beta_{VAR,6} \\ \beta_{VAR,7} & \beta_{VAR,8} & \beta_{VAR,9} \end{bmatrix}$$

where $\boldsymbol{\beta_{VAR}}$ and $\boldsymbol{\alpha_{VAR}}$ are coefficient matrices, and $\boldsymbol{X_t}$ is a vector of the exogenous non-stationary variables, including $\ln A_{1,t-1}$, $\ln A_{2,t-1}$ and a time trend. The auxiliary parameter vector used for computing the Wald statistic includes all the 9 elements in $\boldsymbol{\beta_{VAR}}$ and the 3 variances of the

[3] Le et al. (2017) find that a VARX with one-period lag and 3 or more endogenous variables typically has a great rejection power, while raising the order or the number of variables further would boost the rejection power so much that any hope of seeking a tractable model not to be rejected would be hard.

VARX residuals. The χ^2 distribution that $Wald_a$ asymptotically follows has 12 degrees of freedom.

Table 1. Granger Test on the Auxiliary Model

Equation	Null hypothesis	χ^2 statistic	df	P-value
$\ln Y$	$\ln K$ & $\ln K_1$ cannot Granger-causes $\ln Y$ jointly	9.3393	2	0.94%
$\ln K$	$\ln Y$ & $\ln K_1$ cannot Granger-causes $\ln K$ jointly	6.2782	2	4.33%
$\ln K_1$	$\ln Y$ & $\ln K$ cannot Granger-causes $\ln K_1$ jointly	6.2857	2	4.32%

Note: The test results for the above three endogenous variables indicate causal relationships among them, confirming that they are suitable for forming the VAR. Furthermore, identification deteriorates when any of the endogenous variables is replaced by another, supporting the appropriateness of the specified auxiliary model.

5.2 Simulation

Our approach to running simulations includes the following steps:

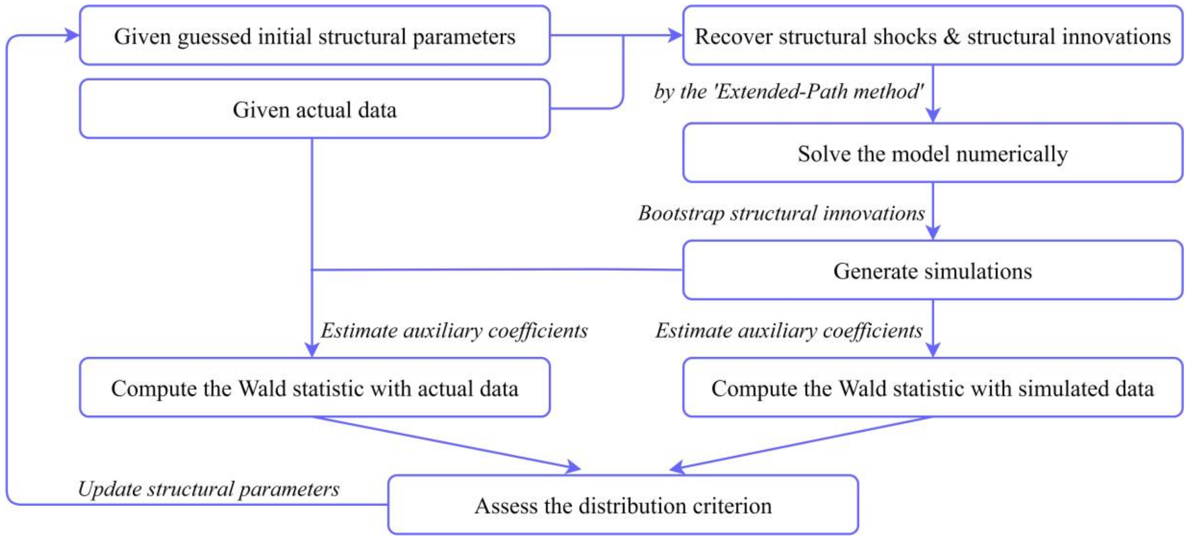
- *Step1*: Given the null hypothesis and the actual data, we first recover the structural shocks from the structural model, which is different from many SMMs that employ artificial shocks. For each structural error, we estimate its stochastic process and collect the regression residuals called the "structural innovations".

- *Step2*: In consideration of non-stationary productivity evolutions, we solve the model numerically to achieve the path of endogenous variables without structural innovations.

- *Step3*: We bootstrap the structural innovations and combine the solved paths of endogenous variables to generate simulations.

The procedure of our II estimation is summarized in Fig. 5.

Fig. 5 The Flowchart of II Estimation Procedure



6. Baseline Empirical Results

This section presents the baseline results for the estimated model. First, we provide the calibrated and II-estimated structural parameter values and evaluate the efficiency of the estimation. We then analyze the model's responses to temporary shocks and historical shocks, identifying the primary contributors to both output and inequality over the span of two centuries.

6.1 Parameters and II Estimation Efficiency

Our model listing includes 13 parameters that can be directly calibrated using sample averages and relevant literature. Among these, 10 parameters are constants, while the remaining 3 are time-varying, as detailed in Section 4. Table 2 summarizes the economic interpretation and values assigned to these parameters. We employ the II method to estimate the other 8 structural parameters in the model, whose interpretations and estimated values are presented in Table 3. Notably, the estimated marginal effect of barriers rate on individual TFP for the less wealthy is significantly higher than for the wealthy ($\phi_{2,1} < \phi_{2,2}$), indicating that the less wealthy are more sensitive to entrepreneurial barriers. This finding underscores the importance of government grant support.

Table 2. Calibrated Parameters

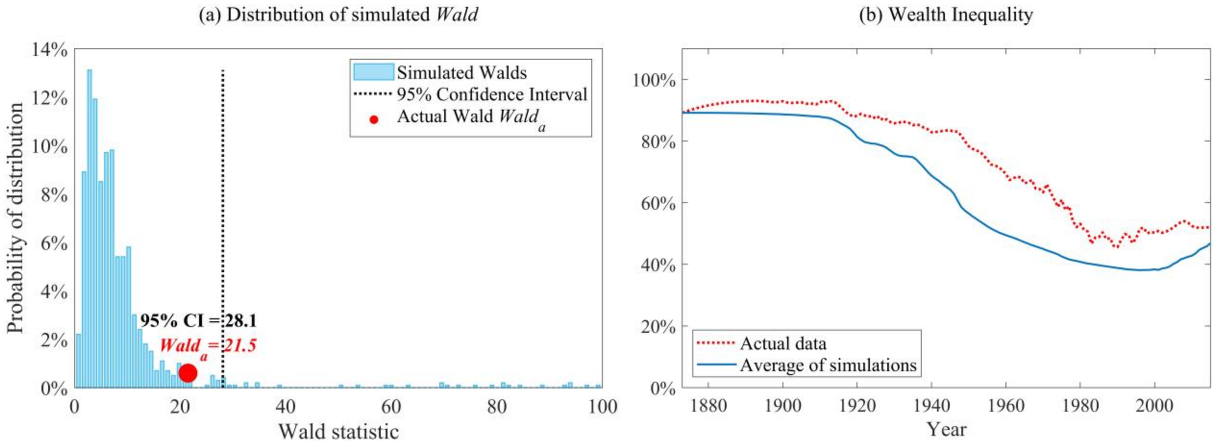
Economic Interpretation	Symbol	Value
Constant Parameters		
Consumption weight in individual utility	Φ	0.500
Utility discount rate	β	0.970
Population weight of the wealthy	μ_1	0.100
Population weight of the less wealthy	μ_2	0.900
Steady-state aggregate output/consumption ratio	Y/C	1.561
Steady-state aggregate capital/consumption ratio	K/C	5.915
Steady-state real interest rate	r	0.020
Drift in linearized productivity equation for group *i*	$\phi_{1,i}$	0.016
Coefficient of lagged term in Gov. spending AR(1)	γ_G	1.002
Coefficient of lagged term in barriers rate AR(1)	γ_π	0.987
Time-varying Parameters		
Share of capital in Cobb-Douglas production	α	0.21~0.42 (mean 0.29)
Capital depreciation rate	δ	0.01~0.05 (mean 0.02)
Proportional income tax rate	τ	0~0.2 (mean 0.09)

Table 3. Estimated Parameters

Economic Interpretation	Symbol	Value
Elasticity of consumption in individual utility	ψ_1	0.840
Elasticity of leisure in individual utility	ψ_2	0.980
Factor of capital adjustment cost	ξ	0.930
Individual labor's response factor to productivity growth	η_3	0.820
Marginal effect of barriers rate on individual TFP for the wealthy	$\phi_{2,1}$	0.050
Marginal effect of barriers rate on individual TFP for the less wealthy	$\phi_{2,2}$	0.400
Marginal effect of relative capital gap on barriers rate for the less wealthy	ρ_1	0.070
Marginal effect of Gov. support on barriers rate for the less wealthy	ρ_2	0.063

Given the structural estimates, the critical value for the 95% confidence interval of the χ^2 distribution from 1,000 simulated samples is 28.1, which is greater than the actual $Wald_a$ statistic of 21.5, with a p-value of 6.5%. This indicates that the null hypothesis—both the model specification and the structural estimators are true—cannot be rejected (Fig. 6(a)). Additionally, the final model-generated simulations, averaging across 1000 samples, closely replicate the "flat Z" pattern of wealth inequality observed in the UK over the past two centuries (Fig. 6(b)). This suggests that our model effectively captures key factors driving the long-term evolution of wealth inequality in the UK.

Fig. 6 II Estimation Efficiency



6.2 Temporary Impulse Response and Historical Shock Decomposition

Using the estimated model, we can clearly observe the dynamic impact of different shocks on the macroeconomy. In line with the model specification, we identify eight distinct structural errors, as introduced in Table 4. To analyze the evolution of each error, we rigorously perform stationarity tests, which provide insights into the temporal behavior of individual structural errors. Subsequently, the residuals from these tests are collected as true historical shocks.

Table 4. Structural Errors

Economic Interpretation	Symbol	AR coefficient
Aggregate TFP error	$\varepsilon_{A,t}$	0.459
Consumption preference error for the wealthy	$\varepsilon_{C1,t}$	0.713
Consumption preference error for the less wealthy	$\varepsilon_{C2,t}$	0.712
Labor supply error for the wealthy	$\varepsilon_{N1,t}$	0.538
Labor supply error for the less wealthy	$\varepsilon_{N2,t}$	0.660
Government spending error	$\varepsilon_{G,t}$	0.528
Credit expansion rate	$\varepsilon_{GS,t}$	0.823
Aggregate entrepreneurial barriers rate	$\varepsilon_{\pi,t}$	0.349

In a complex economy, even very brief shocks can result in widespread effects that persist far beyond the duration of the shock itself. To capture these effects, we generate eight types of temporary shocks (defined as one standard deviation of each type of historical shock) and independently impose them on the stable economy depicted by our model. We then observe the responses of key macroeconomic indicators—aggregate output, capital, consumption, and wealth inequality—over subsequent periods, in what is known as an impulse response analysis.

We start by examining four idiosyncratic shocks. Given the asymmetry of these shocks between the two groups, it is intuitive to expect an inevitable impact on inequality. Aggregate temporary shocks undoubtedly affect macroeconomic indicators, but they also have distributional consequences (Fig. 8). Our results first confirm the positive role of inclusive technological progress in both economic growth and redistribution (Fig. 8(a)). The redistributive effect can be understood as sustained productivity gains driven by accelerated capital accumulation among the less wealthy, which spurs entrepreneurial activities. Temporary increases in government grant support also sustainably enhance output and reduce inequality, although the output effect is relatively modest (Fig. 8(c)). However, a temporary reduction in entrepreneurial barriers (a negative shock) yields the most favorable outcomes in terms of both output and redistribution (Fig. 8(d)). Conversely, a temporary surge in general government spending crowds out grant support to the private sector, resulting in adverse effects on both output and redistribution (Fig. 8(b)). Thus, when balancing output growth with inequality reduction, a potential "free lunch" policy recommendation is to reduce overall entrepreneurial barriers or increase government grant support to the private sector. These efforts appear more feasible than striving for substantial inclusive TFP growth.

Fig. 7 confirms this intuition, but each idiosyncratic shock also has prolonged effects on aggregate economic outcomes. Specifically, a temporary surge in consumption demand by the wealthy (Aggregate temporary shocks undoubtedly affect macroeconomic indicators, but they also have distributional consequences (Fig. 8). Our results first confirm the positive role of inclusive technological progress in both economic growth and redistribution (Fig. 8(a)). The redistributive effect can be understood as sustained productivity gains driven by accelerated capital accumulation among the less wealthy, which spurs entrepreneurial activities. Temporary increases in government grant support also sustainably enhance output and reduce inequality, although the output effect is relatively modest (Fig. 8(c)). However, a temporary reduction in entrepreneurial barriers (a negative shock) yields the most favorable outcomes in terms of both output and redistribution (Fig. 8(d)). Conversely, a temporary surge in general government spending crowds out grant support to the private sector, resulting in adverse effects on both output and redistribution (Fig. 8(b)). Thus, when balancing output growth with inequality reduction, a potential "free lunch" policy recommendation is to reduce overall entrepreneurial barriers or increase government grant support to the private sector. These efforts appear more feasible than striving for substantial inclusive TFP growth.

Fig. 7(a)) significantly boosts aggregate consumption but also reduces overall capital accumulation, ultimately lowering aggregate output. As a result, even though this shock initially reduces inequality, it comes at the cost of a longer-term decline in total output, which eventually affects other groups and leads to a resurgence of inequality. Similarly, a temporary increase in consumption demand by the less wealthy exacerbates inequality while also presenting similar aggregate economic challenges. Aggregate temporary shocks undoubtedly affect macroeconomic indicators, but they also have distributional consequences (Fig. 8). Our results first confirm the positive role of inclusive technological progress in both economic growth and redistribution (Fig. 8(a)). The redistributive effect can be understood as sustained productivity gains driven by accelerated capital accumulation among the less wealthy, which spurs entrepreneurial activities. Temporary increases in government grant support also sustainably enhance output and reduce inequality, although the output effect is relatively modest (Fig. 8(c)). However, a temporary reduction in entrepreneurial barriers (a negative shock) yields the most favorable outcomes in terms of both output and redistribution (Fig. 8(d)). Conversely, a temporary surge in general government spending crowds out grant support to the private sector, resulting in adverse effects on both output and redistribution (Fig. 8(b)). Thus, when balancing output growth with inequality reduction, a potential "free lunch" policy recommendation is to reduce overall entrepreneurial barriers or increase government grant support to the private sector. These efforts appear more feasible than striving for substantial inclusive TFP growth.

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Fig. 7(d) show that a temporary increase in labor supply preference—regardless of the group—not only enhances aggregate economic performance over a longer period but also results in a slight reduction in inequality, though via different paths. The inequality effect in Aggregate temporary shocks undoubtedly affect macroeconomic indicators, but they also have distributional consequences (Fig. 8). Our results first confirm the positive role of inclusive technological progress in both economic growth and redistribution (Fig. 8(a)). The redistributive effect can be understood as sustained productivity gains driven by accelerated capital accumulation among the less wealthy, which spurs entrepreneurial activities. Temporary increases in government grant support also sustainably enhance output and reduce inequality, although the output effect is relatively modest (Fig. 8(c)). However, a temporary reduction in entrepreneurial barriers (a negative shock) yields the most favorable outcomes in terms of both output and redistribution (Fig. 8(d)). Conversely, a temporary surge in general government spending crowds out grant support to the private sector, resulting in adverse effects on both output and redistribution (Fig. 8(b)). Thus, when balancing output growth with inequality reduction, a potential “free lunch” policy recommendation is to reduce overall entrepreneurial barriers or increase government grant support to the private sector. These efforts appear more feasible than striving for substantial inclusive TFP growth.

Fig. 7(c) is not surprising, as a temporary increase in labor preference by the wealthy translates into higher income, which boosts individual consumption. This crowds out consumption by the less wealthy, accelerating their capital accumulation and eventually reducing inequality. These findings suggest that economies characterized by individual diligence are expected to grow faster than those dominated by personal hedonism.

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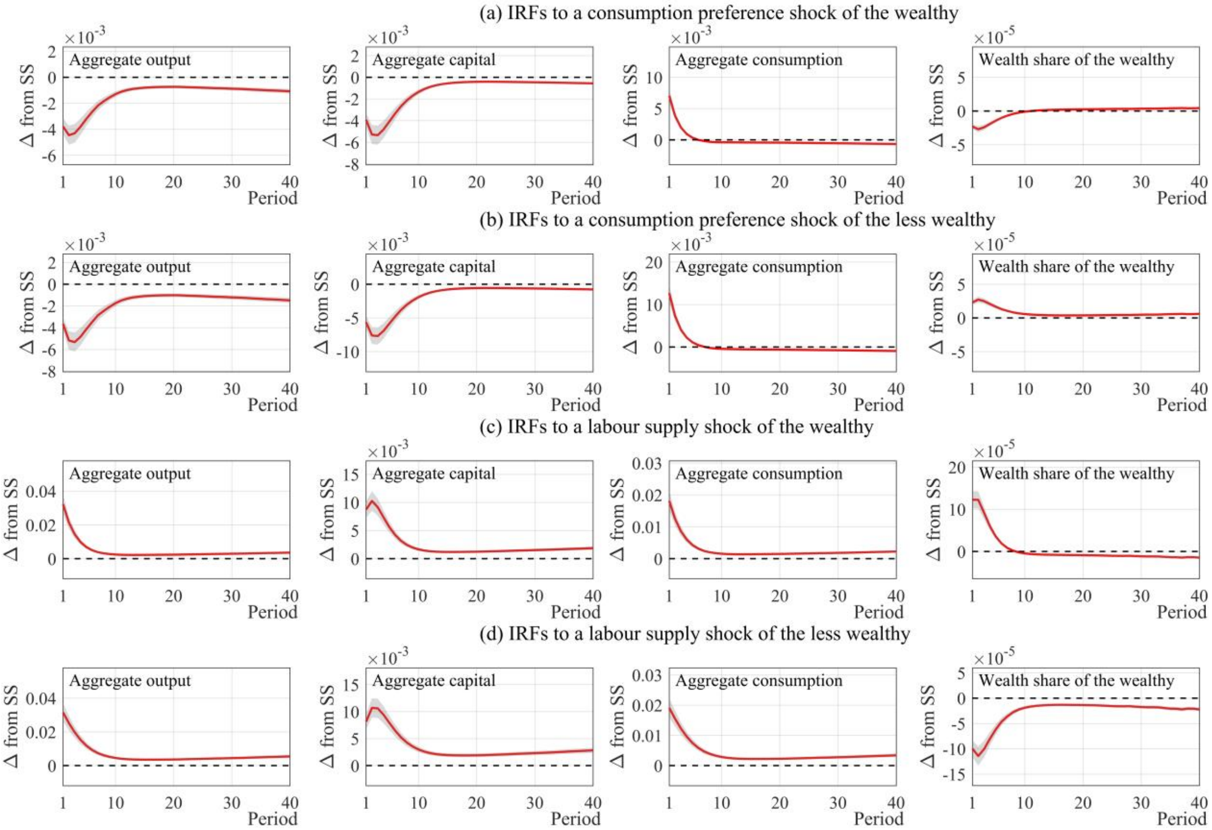
Fig. 7 Responses to One SD Idiosyncratic Shocks ±2SE

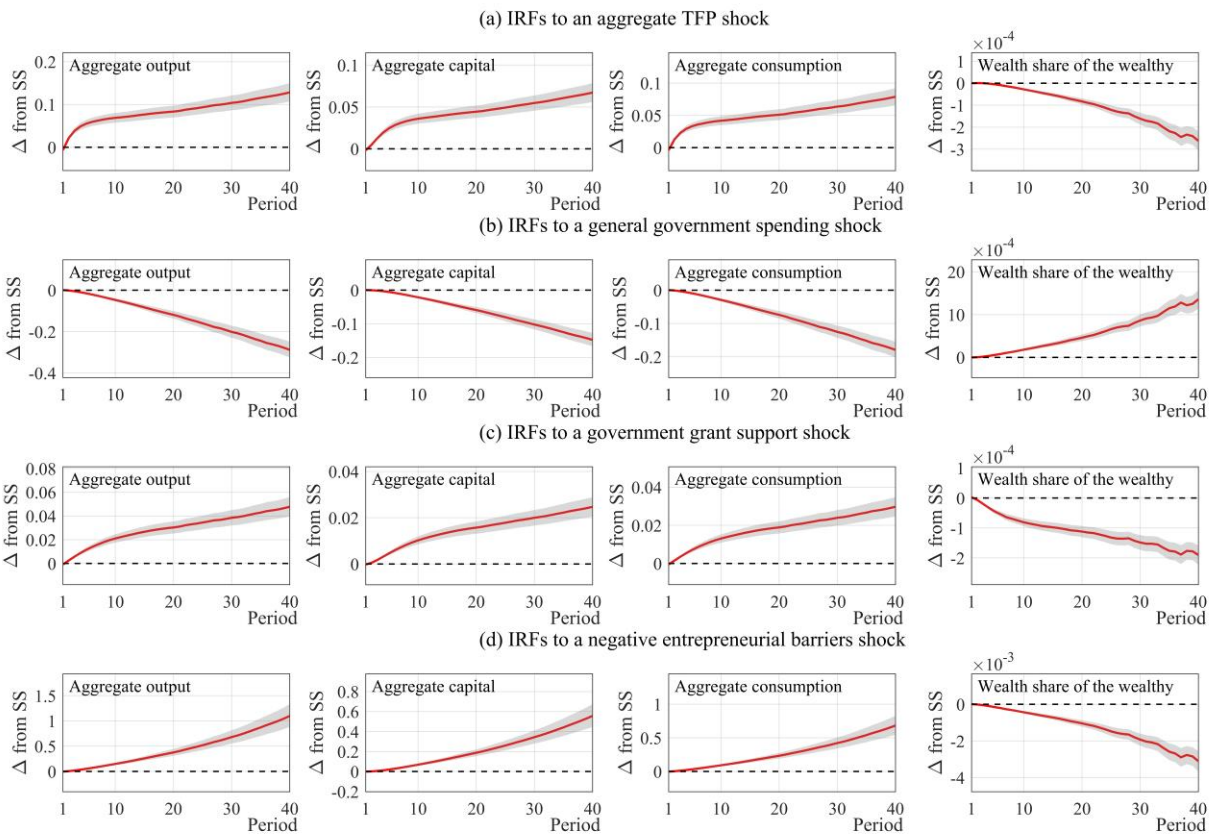


Note: The solid lines represent the impulse responses to idiosyncratic shocks, respectively, while the shaded bands indicate the impulse responses when the shock varies by ±2 standard errors (SE).

The analysis of temporary shocks provides valuable insights into the economic dynamics depicted by our model. However, we are also interested in identifying the key shocks that have shaped the impressive "flat Z" pattern of wealth inequality over the past century and a half through the historical shock decomposition. To achieve this, we conduct counterfactual simulations by applying each type of historical shock independently to observe its unique contribution.

Fig. 9 presents the decomposition, showing that all types of shocks contributed to a gradual increase in inequality (relative to the 1870s level) in the years leading up to the 1920s. However, subtle changes occurred around World War I. Some shocks, particularly the shocks to entrepreneurial barriers, began to have an active effect on declining inequality. In the following decades, all shocks contributed to the decline in inequality until the late 1940s. Over the next three decades, the shocks to aggregate TFP, entrepreneurial barriers, government grant support, and idiosyncratic shocks all played a significant role in reducing inequality, with aggregate TFP shocks contributing the most. However, general government spending shocks hindered the decline in inequality during this period, and their adverse impact grew stronger over time. Additionally, entrepreneurial barriers shocks contributed less and less to reducing inequality, and eventually became one of the forces driving inequality back up by the late 1970s. A more detailed observation shows that in the few years after the early-1980s, the contribution of the entrepreneurship barriers shock to the rebound in inequality declined temporarily. This may be related to a series of market liberalization policies of the Thatcher government. Since the late 1980s, as the contribution of general government spending shocks and entrepreneurial barriers shocks to the rebound in inequality became stronger, and the role of aggregate TFP shocks in suppressing inequality weakened, inequality in the UK ended its 70-year decline.

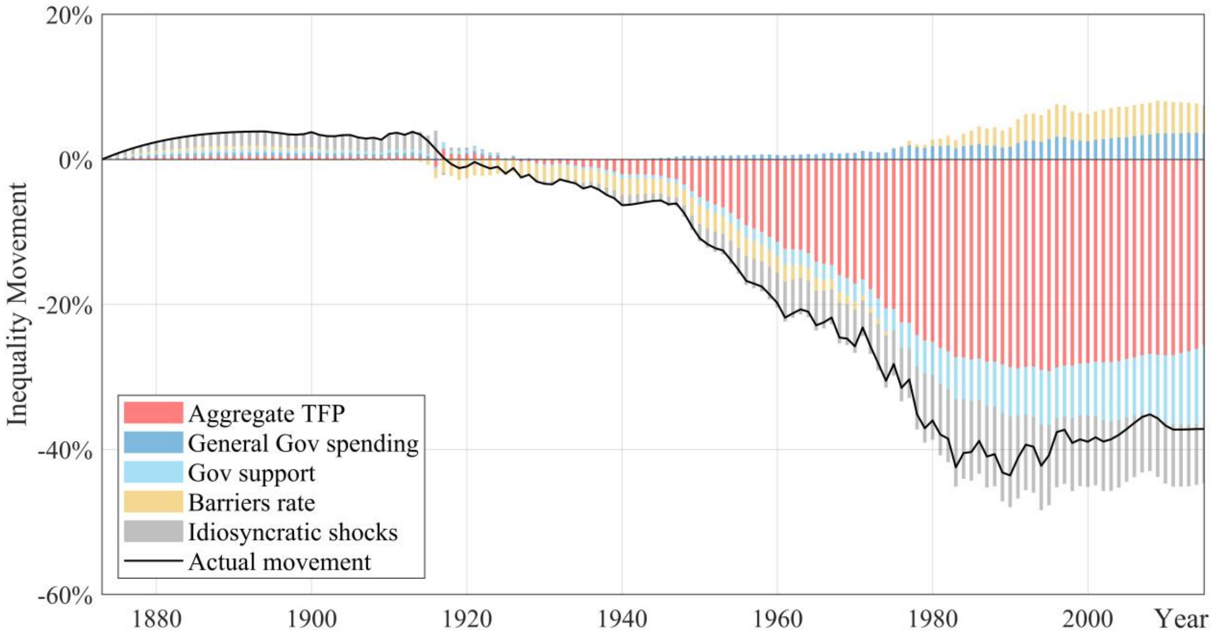
Fig. 8 Responses to One SD Aggregate Shocks ±2SE



Note: The solid lines represent the impulse responses to aggregate shocks, respectively, while the shaded bands indicate the impulse responses when the shock varies by ±2 standard errors (SE).

Overall, in addition to the positive role played by aggregate TFP shocks and idiosyncratic shocks (possibly related to changes in individual preferences) in curbing inequality after World War II, it is noteworthy that government support shocks have played an increasingly significant role in combating inequality. This underscores the importance of active government intervention in entrepreneurship and financing markets. The decomposition analysis also suggests that the UK government could potentially do better in curbing inequality by directly and persistently lowering entrepreneurial barriers and more reasonably arranging government spending.

Fig. 9 Historical Shock Decomposition of Inequality



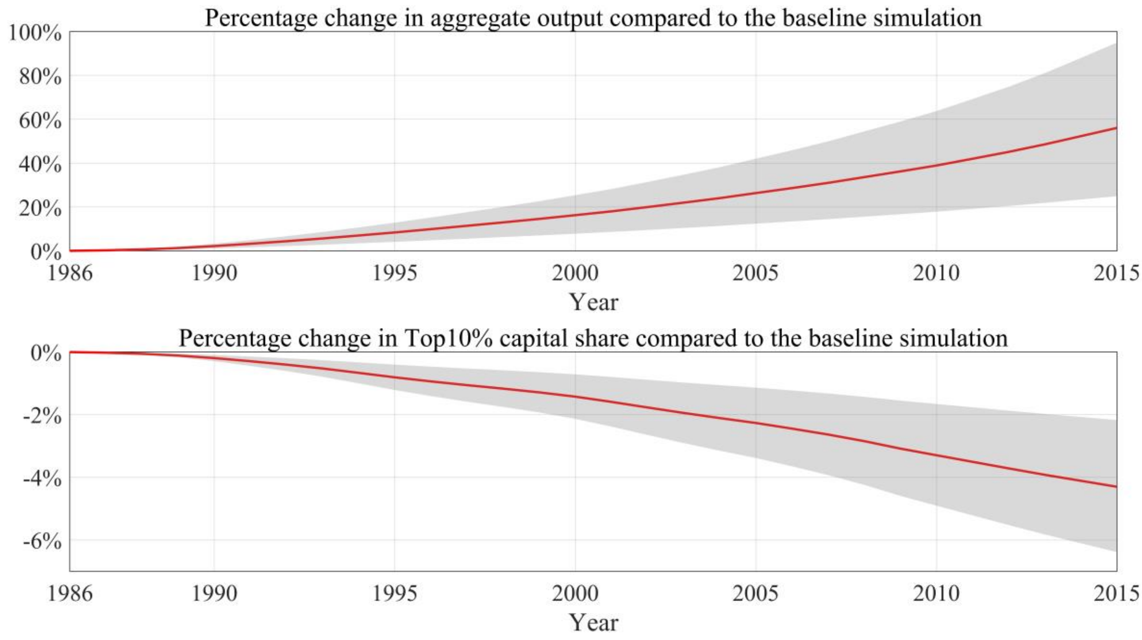
7. Policy Discussions

In the previous section, the impulse response analysis indicated that the shocks to aggregate TFP, entrepreneurial barriers, and government grant support had the most pronounced effects on reducing inequality, while also having significant impacts on output. In particular, these shocks were crucial during historical periods marked by notable shifts in inequality trends. The exogenous shock of TFP is uncontrollable, while its endogenous progress is jointly determined by the aggregate entrepreneurial barriers and the strength of government grant support. Therefore, we turn our attention to two areas where government intervention may be particularly effective: reducing entrepreneurial barriers and increasing government grant support to the private sector. This section presents counterfactual simulations and discusses the implications of these two key interventions.

7.1 Entrepreneurial Barriers

The entrepreneurial barriers rate in Britain began to significantly contribute to the rebound in inequality in the late 1980s, but such a trend was not observed before that (Fig. 9). This is likely related to the series of neoliberal policies implemented by the Thatcher government in the early to mid-1980s and the weakening of the liberal trend in the late 1980s.[4] Although there has been ongoing debate regarding whether Thatcher's neoliberal policies exacerbated inequality in the UK (Machin, 1997), the historical shock decomposition presented earlier suggests that the worsening inequality since the late 1980s should not be blamed on the Thatcher government. Instead, the shock to the entrepreneurial barriers rate caused by the cooling of the neoliberal trend can explain a large part of the rebound in inequality.

Fig. 10 Policy Effect by A Continuous Decline in Entrepreneurial Barriers Rate



Note: The red lines represent the responses of aggregate output and the top 10% capital share to a 5% cumulative reduction in entrepreneurial barriers from 1986 to 1990. The gray area indicates the impact after the policy intensity is adjusted up and down by 50%.

We wonder what the impact would have been on the UK economy if the reduction in entrepreneurial barriers during the early to mid-1980s had continued for a longer period. To explore this, we use a counterfactual simulation that adheres to the Lucas critique, employing the "true" structural model tested II and incorporating the shocks implied by these policies. This approach involves multi-period sequential shocks, rather than the single temporary shocks used in impulse response analysis. Specifically, we introduce a shock to entrepreneurial barriers that lasts from 1986 to 1990, resulting in a cumulative 5% reduction over five years, and observe the changes in aggregate output and inequality over the following decades. Fig. 10 depicts the policy effect. We observe that this policy indeed creates a "free lunch"—sustained reduction in

[4] The combined effect of these liberal policies is reflected in our barriers rate index as changes on both the outputs side (e.g., the impact of changes in various indirect taxes on market vitality) and the inputs side (e.g., the effect of labor policies on employment costs, particularly the weakening of trade unions).

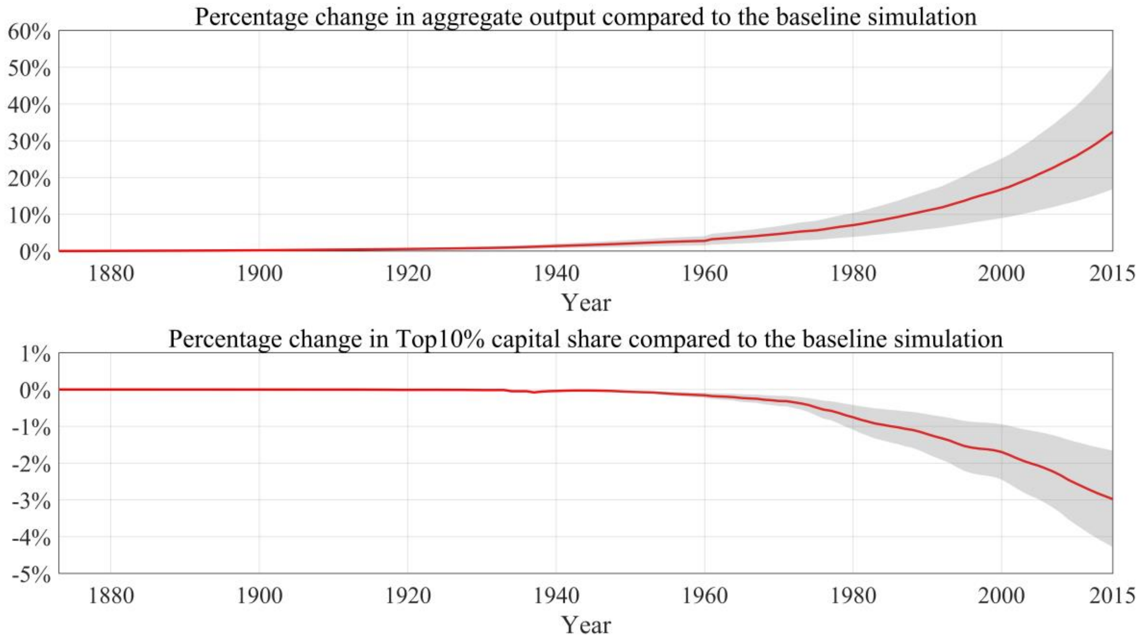
entrepreneurial barriers over five years would lead to a continuous output growth (approximately 55% cumulative growth over 30 years) and a continuous reduction in inequality (approximately 4% cumulative decline over 30 years). Moreover, this policy shows rapid effectiveness. This finding supports the Thatcher neoliberal policies to some extent.

7.2 Government Grant Support

The historical shock analysis shows that in the three decades after the 1940s, the shocks to government grant support played a significant role in reducing inequality. Consistent with this finding, the late 1940s marked the beginning of the British welfare system, where the less wealthy benefited more from policies that generally improved the living conditions of citizens. In contrast, during the classical gold standard period between the 1870s and World War I, money issuance and credit market development were severely constrained, which was particularly unfavorable for the entrepreneurship of the less wealthy. This period coincided with the lead-up to the first turning point in the “flat Z” shape of British inequality.

In this experiment, we assume that Britain started government grant support in the early 1870s to help the less wealthy finance their entrepreneurship. Specifically, we introduce continuous government support shocks into the model, increasing the government support ratio by a cumulative 5% over the five years from 1873 to 1878, and observe the long-term changes in the simulation results. Fig. 11 shows that if the UK government had started increasing government grant support to the private sector in the early 1870s, even with a policy duration as short as one government term, the policy effect would still exhibit a long-term “free lunch”—gradual output growth and a steady decrease in inequality. Although the impact of this policy shift may not be immediately apparent (since it indirectly affects TFP, rather than having a direct impact like entrepreneurial barriers), the cumulative effect over time could lead to a qualitative transformation (it takes about eighty years of accumulation in our experiment) and ultimately yield significant benefits (more than a century later, cumulative output increased by approximately 35%, and inequality decreased by about 3%). Given that increasing government grant support would only require modest adjustments to the structure of government spending without excessive market intervention, this finding has important policy implications: governments should provide financial support to the private sector early and continuously, as the long-term benefits may far outweigh the immediate results.

Fig. 11 Policy Effect by A Continuous Increase in Government Grant Support



Note: The red lines represent the responses of aggregate output and the top 10% capital share to a 5% cumulative increase in government support ratio from 1873 to 1878. The gray area indicates the impact after the policy intensity is adjusted up and down by 50%.

8. Concluding Remarks

This paper investigates the long-run relationship between wealth inequality and economic growth. We develop a DSGE model featuring two agents with heterogeneous wealth levels, where wealth status influences individual entrepreneurial costs, thereby affecting both output and inequality. Using a comprehensive UK dataset spanning from 1870 to 2015, we employ indirect inference to estimate the structural parameters and validate the predictions of our model.

In the empirical section, we examine the model's responses to various temporary shocks and find that the co-movement between output and inequality varies greatly with idiosyncratic shocks, leading to a complex inequality-output nexus for regression-based analyses. Three specific temporary shocks—positive shocks to aggregate TFP and government grant support, and negative shocks to the entrepreneurial barriers rate—are found to simultaneously reduce inequality while stimulating output growth. Historical shock decomposition analysis suggests that, in addition to the positive role of aggregate TFP shocks and idiosyncratic shocks in reducing inequality after World War II, government support shocks have become increasingly crucial in combating inequality. This highlights the growing importance of active government intervention in entrepreneurship and financing markets. In policy experiments, we find that a sustained reduction in entrepreneurial barriers over a short period in the late 1980s would have fostered continuous output growth and inequality reduction, supporting the efficacy of early Thatcher-era neoliberal policies. We also find the importance of early and sustained government support to the private sector, as the long-term benefits of such policies can substantially outweigh the immediate effects.

This paper is only the first step in a broader research agenda. The explanatory effect of the theoretical model in other countries needs to be further investigated. The redistributive effect of taxation is deliberately omitted in this paper because it has been thoroughly studied, but it can be endogenized in a more comprehensive model to study optimal taxation. In addition, it has been noticed that physical capital may have a greater impact on inequality in the early stage of economic development, while the influence of human capital has greater importance in modern and contemporary times (Galor & Moav, 2004; Foreman-Peck & Zhou, 2021). One promising extension is to refine the asset types in the growth-inequality nexus.

Competing Interests: The author(s) declare none.

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Appendix A. Lagrange optimization and Model Listing

Based on the model settings (1)-(4), we can write the agent i's optimal problem as the following Lagrangian and solve the first-order conditions for the control variables $C_{i,t}$, $b_{i,t+1}$, $N_{i,t}$, $Z_{i,t}$ and $K_{i,t+1}$ respectively, thus obtaining the optimal rules of equations (A1)-(A5).

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \Bigg\{ \beta^s U_{i,t+s} + \lambda_{t+s} \Bigg[(1-\tau_{t+s})Y_{i,t+s} + (1+r_{t-1+s})b_{i,t+s} - \varphi_{i,t+s}\pi_{t+s}Z_{i,t+s} + T_{t+s} - C_{i,t+s} - b_{i,t+1+s} - K_{i,t+s} + (1-\delta)K_{i,t-1+s} - \frac{\xi}{2}\frac{\left(K_{i,t+s}-K_{i,t-1+s}\right)^2}{K_{i,t-1+s}} \Bigg] \Bigg\}$$

$$\lambda_t = \Phi u_{i,t}\left(C_{i,t}\right)^{-\psi_1} \tag{A1}$$

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1} \tag{A2}$$

$$(1-\Phi)v_{i,t}\left(N_{i,t}+Z_{i,t}\right)^{\psi_2} = \lambda_t(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} \tag{A3}$$

$$(1-\Phi)v_{i,t}\left(N_{i,t}+Z_{i,t}\right)^{\psi_2} + \lambda_t\varphi_{i,t}\pi_t = v_{A,t}\theta_2\frac{A_{i,t}}{A_{i,t+1}}E_t\left[\sum_{s=0}^{\infty}\beta^s\frac{\Phi u_{i,t+1}(1-\tau_{t+s})Y_{i,t+s}}{\left(C_{i,t+s}\right)^{\psi_1}}\right] \tag{A4}$$

$$\lambda_t\left(1+\xi\frac{K_{i,t}-K_{i,t-1}}{K_{i,t-1}}\right) = E_t\left\{\lambda_{t+1}\left[(1-\tau_{t+1})\alpha\frac{Y_{i,t+1}}{K_{i,t}} + \frac{\xi}{2}\frac{\left(K_{i,t+1}\right)^2}{\left(K_{i,t}\right)^2} + 1 - \delta - \frac{\xi}{2}\right]\right\} \tag{A5}$$

Now we consider the approximation of (A4). First, use $E_t\left[(1+r_{t-1+s})\beta u_{i,t+s}(C_{i,t+s})^{-\Psi_1}\right] = E_t\left[u_{i,t-1+s}(C_{i,t-1+s})^{-\Psi_1}\right]$ to derive the following relation,

$E_t\left[\sum_{s=1}^{\infty}\beta^s\frac{\Phi u_{i,t+s}(1-\tau_{t+s})Y_{i,t+s}}{\left(C_{i,t+s}\right)^{\psi_1}}\right] = E_t\left\{\sum_{s=1}^{\infty}\frac{\Phi u_{i,t}(1-\tau_{t+s})Y_{i,t+s}}{\left[\prod_{k=0}^{s-1}(1+r_{t+k})\right]\left(C_{i,t}\right)^{\psi_1}}\right\}$. Then,

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} + \varphi_{i,t}\pi_t = v_{A,t}\theta_2\frac{A_{i,t}}{A_{i,t+1}}E_t\left[\sum_{s=1}^{\infty}\frac{(1-\tau_{t+s})Y_{i,t+s}}{\prod_{k=0}^{s-1}(1+r_{t+k})}\right] \tag{A6}$$

$Y_{i,t+s}$ $(s>1)$can be predicted by $Y_{i,t}(1+g_{Y,i})^s$; $\tau_{t+s}(s>1)$can be predicted by τ_t; r_{t+s} $(s>1)$ can be predicted by r_t, then

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} + \varphi_{i,t}\pi_t = v_{A,t}\theta_2\frac{A_{i,t}}{A_{i,t+1}}(1-\tau_t)Y_{i,t}\sum_{s=1}^{\infty}\frac{(1+g_{Y,i})^s}{(1+r_t)^s} \tag{A7}$$

Since the steady-state of the Euler equation (A2) implies that $(1+g_{C,i})^{\Psi_1} = \beta(1+r_t)$. $g_{Y,i} = g_{C,i}$ on the BGP. (A7) can be written as

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}}+\varphi_{i,t}\pi_t=v_{A,t}\theta_2\frac{A_{i,t}}{A_{i,t+1}}(1-\tau_t)Y_{i,t}\sum_{s=1}^{\infty}\left[\frac{\beta(1+g_{Y,i})}{(1+g_{Y,i})^{\Psi_1}}\right]^s$$

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}}+\varphi_{i,t}\pi_t=v_{A,t}\frac{A_{i,t}}{A_{i,t+1}}(1-\tau_t)Y_{i,t}\frac{\theta_2\beta(1+g_{Y,i})}{(1+g_{Y,i})^{\Psi_1}-\beta(1+g_{Y,i})}$$

By defining $\theta_3=\frac{\theta_2\beta(1+g_{Y,i})}{(1-\alpha)(1+g_{Y,i})^{\Psi_1}-\beta(1+g_{Y,i})}$, we yield (A8), i.e., equation (5) in Section 4.

$$(1-\tau_t)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}}+\varphi_{i,t}\pi_t=v_{A,t}\frac{A_{i,t}}{A_{i,t+1}}(1-\tau_t)Y_{i,t}\theta_3 \tag{A8}$$

The term $\varphi_{i,t}\pi_t$ represents the entrepreneurial cost that must be borne when giving up one unit of regular labor time to engage in entrepreneurial activities. Therefore, we can interpret $\varphi_{i,t}\pi_t/w_{i,t}$ as the relative opportunity cost of one unit of entrepreneurial time. We define the variable $\pi_t'=\pi_t/\overline{w}_t$ as the entrepreneurial barriers rate, and the new markup $\varphi_{i,t}'$ captures the difference between $w_{i,t}$ and the average wage rate $\overline{w}_t$. Since $(1-\alpha)Y_{i,t}/N_{i,t}=w_{i,t}$, defining $\pi_{i,t}'=\pi_t/w_{i,t}$, equation (5) can be rewritten as equation (A9), i.e., equation (6) in Section 4.

$$\frac{A_{i,t+1}}{A_{i,t}}=\frac{(1-\tau_t)\theta_3}{(1-\tau_t+\varphi_{i,t}'\pi_t')(1-\alpha)}N_{i,t}v_{A,t} \tag{A9}$$

The full model listing is shown below where (A10) and (A18) are obtained from individual Euler equations. Nonlinear relations are retained in (A11) and (A12) to guarantee that the aggregate economy always equals the sum of individual behavior. (A13) is the linearized market clearing condition. (A14) and (A15) are linearized individual production functions, followed by the linearized individual capital equations (A16) and (A17).[5] Equation (A19) transforms from the consumption aggregation equation. Approximating $N_{i,t}/(N_{i,t}+Z_{i,t})$ to 1 and ignoring constant terms yield the linearized equations (A20) and (A21) for individual labor.[6] The rest of the list has been shown in the model section. Individual bonds are removed from the list because they take small share over individual capital resource which we are not interested in.

$$r_t=\psi_1\left(E_t\ln C_{2,t+1}-\ln C_{2,t}\right)-\ln\beta+\varepsilon_{c2,t} \tag{A10}$$

$$\ln Y_t=\ln\left[\mu_1\exp(\ln Y_{1,t})+\mu_2\exp(\ln Y_{2,t})\right] \tag{A11}$$

$$\ln K_t=\ln\left[\mu_1\exp(\ln K_{1,t})+\mu_2\exp(\ln K_{2,t})\right] \tag{A12}$$

$$\ln C_t=(1-\tau)\frac{Y}{C}[\ln Y_t+\ln(1-\tau_t)]-\frac{K}{C}[\ln K_t-(1-\delta)\ln K_{t-1}]+\varepsilon_{M,t} \tag{A13}$$

[5] $\eta_1=(r+\delta)/[(2+r)\xi+r+\delta]$ and $\eta_2=(1+r)\xi/[(2+r)\xi+r+\delta]$. The approximation $E_t(Y_{i,t+1}/C_{i,t+1})\approx Y_{i,t}/C_{i,t}$ regardless of some constant terms is adopted in this linearization.

[6] As observations on $Z_{i,t}$ are unavailable, and θ_2 and $(N_{i,t}+Z_{i,t})$ are not identified separately, we define parameter $\eta_3=1/[\theta_2(N_{i,t}+Z_{i,t})]$ and estimate it instead of θ_2.

$$\ln Y_{1,t} = \alpha \ln K_{1,t-1} + (1-\alpha)\ln N_{1,t} + \ln A_{1,t} \quad \text{(A14)}$$

$$\ln Y_{2,t} = \alpha \ln K_{2,t-1} + (1-\alpha)\ln N_{2,t} + \ln A_{2,t} \quad \text{(A15)}$$

$$\ln K_{1,t} = \eta_1 \ln Y_{1,t} + \eta_2 \ln K_{1,t-1} + (1-\eta_1-\eta_2)E_t \ln K_{1,t+1} + \eta_1\left(\frac{1}{\psi_1} - \frac{1}{r+\delta}\right) r_t - \frac{\eta_1}{1-\tau}\tau_{t+1} \quad \text{(A16)}$$

$$\ln K_{2,t} = \eta_1 \ln Y_{2,t} + \eta_2 \ln K_{2,t-1} + (1-\eta_1-\eta_2)E_t \ln K_{2,t+1} + \eta_1\left(\frac{1}{\psi_1} - \frac{1}{r+\delta}\right) r_t - \frac{\eta_1}{1-\tau}\tau_{t+1} \quad \text{(A17)}$$

$$\ln C_{1,t} = E_t \ln C_{1,t+1} - \frac{1}{\psi_1}(r_t + \ln\beta) + \varepsilon_{c1,t} \quad \text{(A18)}$$

$$\ln C_{2,t} = \ln\left[\exp(\ln C_t) - \mu_1 \exp(\ln C_{1,t})\right] - \ln\mu_2 \quad \text{(A19)}$$

$$\ln N_{1,t} = \frac{1}{1+\psi_2}\left\{\ln Y_{1,t} - \psi_1 \ln C_{1,t} - \psi_2\eta_3\left(\ln A_{1,t+1} - \ln A_{1,t}\right) + \ln[(1-\tau_t)(1-\alpha)]\right\} + \varepsilon_{N1,t} \quad \text{(A20)}$$

$$\ln N_{2,t} = \frac{1}{1+\psi_2}\left\{\ln Y_{2,t} - \psi_1 \ln C_{2,t} - \psi_2\eta_3\left(\ln A_{2,t+1} - \ln A_{2,t}\right) + \ln[(1-\tau_t)(1-\alpha)]\right\} + \varepsilon_{N2,t} \quad \text{(A21)}$$

$$\ln A_{1,t+1} = \ln A_{1,t} + \phi_{1,1} - \phi_{2,1}\left(\ln\pi'_t + \frac{\tau_t}{1-\tau}\right) + \ln N_{1,t} + \varepsilon_{A,t} \quad \text{(A22)}$$

$$\ln A_{2,t+1} = \ln A_{2,t} + \phi_{1,2} - \phi_{2,2}\left[\ln\pi'_t + \frac{\tau_t}{1-\tau} + \rho_1\left(\ln K_{1,t-1} - \ln K_{2,t-1}\right) - \rho_2 GS_t\right] + \ln N_{2,t} + \varepsilon_{A,t} \quad \text{(A23)}$$

$$\ln G_t = \gamma_G \ln G_{t-1} + \varepsilon_{G,t} \quad \text{(A24)}$$

$$GS_t = \ln Y_t + \ln\tau_t - \ln G_t + \varepsilon_{GS,t} \quad \text{(A25)}$$

$$\ln\pi'_t = \gamma_\pi \ln\pi'_{t-1} + \varepsilon_{\pi,t} \quad \text{(A26)}$$

Appendix B. Data Description

The data used in this study are sourced from WID and the database "A Millennium of Macroeconomic Data" issued by BOE.

Variable	Definition	Source
R	Real interest rate	Annual average of 3-month rates minus 1-year head expectation at start of year
Y	Real GDP per capita	GDP at 2013 market price over POP, geographically-consistent estimate based on post-1922 borders
C	Real consumption per	Household consumption at 2013 market price over POP, geographically-

	capita	consistent estimate based on post-1922 borders
K	Real capital per capita	Domestic capital stock (=NW-NFA) at 2013 market price over POP, geographically-consistent estimate based on post-1922 borders
NW	National wealth	Market-value national wealth (WID)
NFA	Net foreign assets	Market-value net foreign assets (WID)
G	General government spending per capita	General government spending at 2013 market price over POP, geographically-consistent estimate based on post-1922 borders
POP	Total population	Total population of GB and Northern Ireland, geographically-consistent estimate based on post-1922 borders
N	Labor supply	Estimate by EM over POP multiplied by AWH over (24 by 7)
EM	Employment	
AWH	Average weekly hours	
TUM	Trade union members	
IDT	Indirect taxes over GDP	Nominal indirect tax revenues over nominal GDP
τ	Direct taxes over GDP	Nominal direct tax revenues over nominal GDP
$1-\alpha$	Labor share of GDP	
δ	Depreciation rate of capital stock	Depreciation of capital consumption over domestic capital stock
GS	Government grant support ratio	Government support to private sector over the general government spending
Y_1	Real GDP per capita for the rich	Real GDP per capita multiplied by top10% pre-tax income share (WID)
K_1	Real capital per capita for the rich	Real capital per capita multiplied by top10% wealth share (WID)
C_1	Real consumption per capita for the rich	Real consumption per capita multiplied by top10% pre-tax income share (WID)