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A model-informed approach to robust calibration of experimental guided wave modes with cyberphysical systems

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The multilayered, and anisotropic nature of fiber-reinforced composite structures presents challenges in monitoring the initiation and progression of damage. Non-intrusive inspection methods for such structures in safety-critical aerospace applications are vital to the continuous monitoring regime and facilitate the transition to a predictive maintenance regime. Acousto-ultrasonic techniques have shown great promise with regards to non-intrusive monitoring of progressive structural damage and relies on the efficacy of the onboard monitoring system to acquire signals and extract essential features on the edge. These signal features, when correlated with parameterized damage metrics, enable the characterization of damages, and help to realize an automated framework for assessment of structural integrity. This study presents a physics-informed and experimental data driven approach for isolation and reconstruction of fundamental guided wave modes in a multilayered carbon fiber reinforced composite panel. A daisy chained edge device along with a sparse transducer array captured structural-acoustic responses to a frequency-sweeps of harmonic excitations. A combined data-driven and model-informed reconstruction algorithm integrated the acquired experimental data, the semi-analytical composite waveguide dispersion characteristics and the point-to-point system transfer function at each transducer location, to reconstruct the ultrasonic guided wave modes. Studies on deterministic wave mode identification approach, based on the reconstruction errors between experimental and reconstructed signal Hilbert envelopes, generated high-fidelity reconstructions, validated by strong concordance with empirical data. A probabilistic approach based on Bayesian inference has been proposed to quantify uncertainties associated with both experimental measurements and model predictions. The joint parameter distribution results demonstrated the model's efficacy in evaluating a statistical dependence between the posterior predictive estimates that closely resembles the underlying physics governing the model. The results showed high concordance with the experimental acousto-ultrasonic signals, highlighting the ability to capture the true signal characteristics along with the associated confidence estimates. Future work will focus on investigating changes in the modal properties of the guided waves as a function of parameterized structural degradation. The estimated damage signatures will help to quantify the uncertainties of the experimental measurements and model predictions. This will facilitate the development of a physics-informed, robust and non-intrusive structural health monitoring system with real-time decision making capabilities.

I. Nomenclature

SHM	=	Structural health monitoring
UGW	=	Ultrasonic guided wave
SAFE	=	Semi-analytical finite element
CFRC	=	Carbon fiber reinforced composite
FFT	=	Fast Fourier transform
STFT	=	Short time Fourier transform
AIC	=	Akaike information criterion

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TF	= Time-frequency representation
SWT	= Synchrosqueezed wavelet transform
HHT	= Hilbert Huang transform
S_0	= Symmetric fundamental ultrasonic guided wave mode.
A_0	= Antisymmetric fundamental ultrasonic guided wave mode.
x	= Distance between actuating transducer and sensing transducer (m)
t	= time (s)
ω	= Circular frequency (rad/s)
ω_0	= Central radial harmonic frequency.
n_s	= Number of sensing transducers bonded to the test structure.
$X(t)$	= User defined actuation signal.
T_A	= Actuating transducer transfer function.
X_A	= Transfer function transformed actuation pulse given by $T_A X(t)$.
$H(\omega)$	= Dispersion characteristics of the composite waveguide under consideration.
Y_S	= Transfer function transformed structural acoustic response given by $H(\omega)X_A$
T_S	= Sensing transducer transfer function.
$Y(t)$	= Acousto-ultrasonic signal recorded at the sensors.
n_ω	= Number of harmonic components of signal within a chosen frequency window.
$g(\omega)$	= Complex frequency response of the actuation signal.
$U_m(x, t)$	= Lamb wave propagating in a plate like structure as a function of x and t
U_{S_0}	= Symmetrical S_0 guided wave mode amplitude
U_{A_0}	= Asymmetrical A_0 guided wave mode amplitude
k_S	= S_0 mode wave number (m^{-1})
k_A	= A_0 mode wave number (m^{-1})
α_M	= Set of parameters denoting the elastic and geometric properties of the waveguide.
V_{PS}	= Phase velocity of S_0 mode, given by $\frac{\omega}{k_S}$ (m/sec)
V_{PA}	= Phase velocity of A_0 mode, given by $\frac{\omega}{k_A}$ (m/sec)
V_{gS}	= Group velocity of S_0 mode, given by $\frac{d\omega}{dk_S}$ (m/sec)
V_{gA}	= Group velocity of A_0 mode, given by $\frac{d\omega}{dk_A}$ (m/sec)
$H(x)(t)$	= Hilbert envelope of the experimental structural-acoustic response.
$H(x^M)(t)$	= Hilbert envelope of the reconstructed signal. Also given by $H(x_{S_0}^M + x_{A_0}^M)$.
$\pi(\theta \mathcal{D})$	= Probability density function on random parameter θ conditional upon the observations in $\mathcal{D}$
$\pi(U_{S_0}, U_{A_0}, \alpha_M)$	= Prior distribution over U_{S_0} , U_{A_0} and α_M

II. Introduction

ENGINEERING structures perform under a multitude of operational loads including tensile forces, compression, torsion, impact and fatigue, all under varied environmental and operational conditions throughout their service life. Ensuring their structural integrity is fundamentally important and the evolving paradigm of continuous structural monitoring regimes within a data-rich environment is rapidly gaining in importance. Advanced aerospace and automotive engineering designs employ complex, high-performance multilayered composite materials, which exhibit intricate failure mechanisms such as delaminations [1], porosity [2], fibre-breakage [3], and matrix-fibre debonding [4] amongst others. Once a damage is initiated, its progression to eventual failure can occur rapidly, often with minimal warning, particularly in CFRCs which are widely employed in safety critical engineering applications [5, 6]. These complexities create substantial challenges in detecting the onset of structural damage, tracking its progression, and assessing the risks associated with continued operation of such structures.

Therefore, it is crucial to develop and validate an onboard SHM system that can describe the condition of a structure in real time. Such systems must continuously monitor and assess the current state of the structure, evaluate the operational risks associated with any detected damage and predict optimal points for maintenance intervention based on ongoing degradation patterns [7–11]. Existing literature underscores the significant potential of incorporating cyberphysical systems into SHM to develop a predictive maintenance regime for these structures; however, there remains a scarcity of studies that have successfully implemented these systems in practical applications [12, 13]. To enable

practical predictive maintenance, it is essential to establish concrete damage signatures. The interaction of ultrasonic guided wave modes with the composite waveguide and the alterations in their characteristics upon encountering damage are well documented [14, 15]. Therefore, the identification and isolation of fundamental ultrasonic guided wave modes are amongst the most crucial feature extraction operations that can be performed on the acquired structural acousto-ultrasonic responses.

A multitude of signal processing methods have been developed to isolate these ultrasonic guided wave modes. 2D-FFT is considered to be an effective method to separate wave modes where the acquired amplitude-time series signal is converted to the frequency-wavenumber domain. The resulting frequency-wavenumber spectrum was used to extract wave modes based on the amplitude peaks. Employing a digital filter to the frequency-wavenumber representations was reported to enhance the process of wave mode separation. A frequency dependent bandpass filter was applied to a frequency-wavenumber representation of data to separate the A_0 mode for delamination inspection [16]. Other works on using a filtering technique in conjunction with 2D-FFT can be found in [17–19]. However, the requirement of extensive spatially sampled data that restricts the practicality of 2D-FFT to operational components. Under the TF regime, the STFT has been a prominent tool to produce TF scalograms with an aim to identify wave modes but failed to provide a good resolution at high frequency points [20, 21]. Another TF representation is the wavelet transform which decomposes the signal into a set of basic vector components based on a scaling and a translation coefficient [22]. The peak magnitude of the wavelet transform was used to extract the wave modes [23, 24]. The SWT is another tool that was applied to multimodal ultrasonic signals to generate their TF counterparts. This method optimised the TF representation by reassigning the scale variable of the wavelet transform to the frequency variable. Image processing techniques such as watershed algorithm and region building were applied to the TF representations to isolate ultrasonic guided wave modes before performing the inverse SWT to reconstruct the temporal representation of the isolated modes [25]. The STFT is more computationally efficient and simpler to implement, making it suitable for real-time applications. However, STFT has fixed window sizes, leading to resolution issues, especially at higher frequencies, and may not provide as precise a time-frequency representation as wavelet-based methods. Therefore, a balance needs to be struck between computational efficiency and resolution, depending on the specific requirements of the application.

Analysis of non-stationary data has also been performed with the HHT, which involves empirical mode decomposition and the Hilbert transform [26]. HHT was used for detecting wave modes from laser-generated signals [27], overlapped lamb waves in metal plates [28], vibrations of a typical 3 degree of freedom structural model and a 15-storeyed building amongst others [29]. However, the method was not effective in the frequency ranges that contained multiple modes. Consequently, this issue was addressed by developing a hybrid methodology involving the combination of the HHT with the wavelet transform to analyse highly overlapped signals which yielded better results [30]. Recently, there have been innovative applications and major improvement to HHT where the conventional HHT was combined with other intelligence algorithms, like artificial neural network and convolutional neural network amongst others to form more powerful signal processing tools for analyzing structural health monitoring information [31]. Although a significant progress was made towards the development of signal processing techniques to isolate wave modes, most of the established techniques fail when the wave modes overlap in the time and time-frequency domains. Additionally, the available techniques largely focused on isolation than reconstruction of wave modes with minimal consideration of real-time health monitoring applicability.

In this paper, we present an experimental data driven and semi-analytical model informed methodology to individually reconstruct the fundamental ultrasonic guided wave modes which can then be superimposed to produce signal reconstructions which exhibit high concordance with their experimental counterparts. Moreover, the proposed reconstruction model is a crucial building block of the digital layer of the envisioned autonomous cyberphysical SHM framework. A lightweight edge computing device with a sparse array of transducers was utilized to acquire structural acoustic responses of a carbon fiber reinforced epoxy panel subjected to harmonic excitations over a range of actuation frequencies. A semi-analytical finite element formulation was employed to model the composite waveguide, which describes the dispersion behavior of the propagating guided waves with phase and group velocity values of the fundamental ultrasonic guided wave modes. The dispersion characteristics derived from the semi-analytical model combined with the amplitude and phase data of the experimental actuation signals, were used to generate accurate semi-analytical model informed and experimental data-driven reconstructions of the structural acoustic responses at the transducer positions. A space-time signal decomposition technique was employed to reconstruct the spatial resolution of signals and a robust Bayesian approach was used to quantify the lack of a dense network of sensors over the physical span of the structure while isolating the fundamental ultrasonic guided wave modes.

This paper is structured as follows. section III begins by stating the properties of the composite panel used in this study, describes the signal generation and reception framework, the experimental setup and delineates the reconstruction

algorithm along with the important constituents. section IV delineates the results obtained from both the deterministic and probabilistic reconstruction methodologies and compares them against the acquired experimental signals. Finally, section V summarizes the conclusions and highlights future directions.

III. Materials, methods and experiments

As the name suggests, the cyberphysical framework is composed to two main components - the physical and the digital components. The physical layer of the CyberSHM framework makes use of smart edge computing nodes constituted by a sparse array of transducers and daisy-chained edge devices for signal acquisition and processing. This setup enables the edge nodes to

- 1) interrogate any thin-walled structure with a user-defined actuation signal sweeps over frequency bandwidths,
- 2) acquire acousto-ultrasonic structural responses and
- 3) deploy scripts on the edge for a range of tasks including file manipulation, signal feature extraction, classification, and transmitting essential data to the digital layer for further analysis.

These capabilities facilitate threshold-free ultrasonic guided wave based active inspection on thin-walled plate-like structures in real-time. The conceptualization, development and experimental validation of the data acquisition and in-line classification functionalities of the CyberSHM physical layer has been performed and can be found in our previous study [32]. The same physical layer was used for data acquisition in this study. The digital layer hosts a SAFE model to analyse elastic wave dispersion in a laminated composite waveguide. This model captures essential characteristics of the guided wave behaviour and simulates the dispersion phenomena within composite materials. Additionally, a Bayesian model for inverse identification of damage location and characteristics from the acousto-ultrasonic measurements is hosted in the digital layer. The inverse identification algorithm would be scalable to exploit the multicore architecture of cloud computing platforms. The experimental setup consisted of a symmetric 12-layered CFRP panel of dimension $500 \times 500 \times 3$ mm, with a layup configuration of $[+45_3/-45_3]_S$. The panel was equipped with a sparse array of NANO30 transducers, affixed using ProCure 60 Cyanoacrylate adhesive. Transducers S_1 , S_2 , and S_3 were arranged in a linear array, positioned at different distances from the actuator A in the 0° direction, as depicted in figure 1. Two

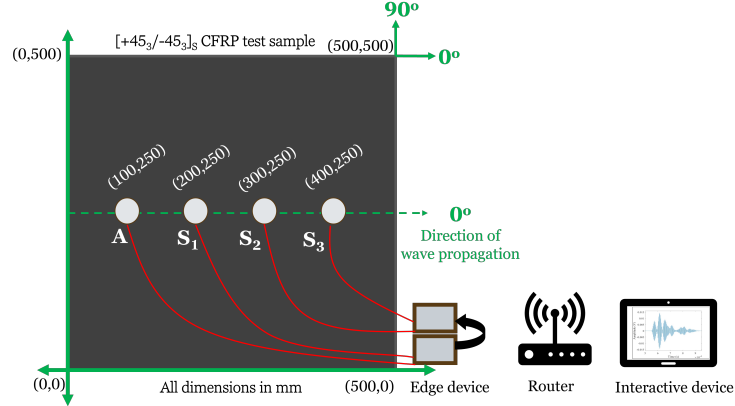


Fig. 1 Linear array of actuator A and three transducers S_1 – S_3 used to acquire acousto-ultrasonic responses for signal reconstruction (all dimensions in mm)

edge devices were connected in a daisy chain configuration, forming a four-channel signal generation and reception system. These edge devices were configured to apply a $2V_{pp}$ 10-cycle toneburst sinusoidal pulse to the panel, capture acousto-ultrasonic time-history responses, and conduct in-line classification to differentiate essential acoustic event representative signals from non-essential ones. The signals classified as essential acoustic event representative were used in signal reconstruction. The reconstruction algorithm developed in this study is experimental data-driven and semi-analytical model-informed. Consider n_s number of sensing transducers bonded to the CFRP panel at varying linear distances from the actuating transducer. The model is developed under the assumption that the first and the second arriving ultrasonic guided wave modes in the recorded signal are the symmetric S_0 and the asymmetric A_0 modes respectively. To reconstruct the ultrasonic guided wave modes accurately, it is crucial to consider the transfer function in the ultrasonic guided wave trajectory (figure 2). An actuating transducer generates a user defined actuation

signal $X(t)$, which is then transformed by the actuating transducer transfer function T_A into X_A , the signal that enters the sample. As X_A propagates through the sample, it is modified by the sample's dispersion characteristics, represented by $H(\omega)$, resulting in the signal Y_S . This output signal from the sample is then picked up by a sensing transducer, which transforms it through its transfer function T_S into the final recorded signal $Y(t)$. The overall relationship between the input and the output signals can be expressed as $Y(t) = H(\omega)T_S T_A X(t)$ highlighting the cumulative effect of the transfer functions on the transformation of the input signal to the output signal. To reliably reconstruct the ultrasonic guided

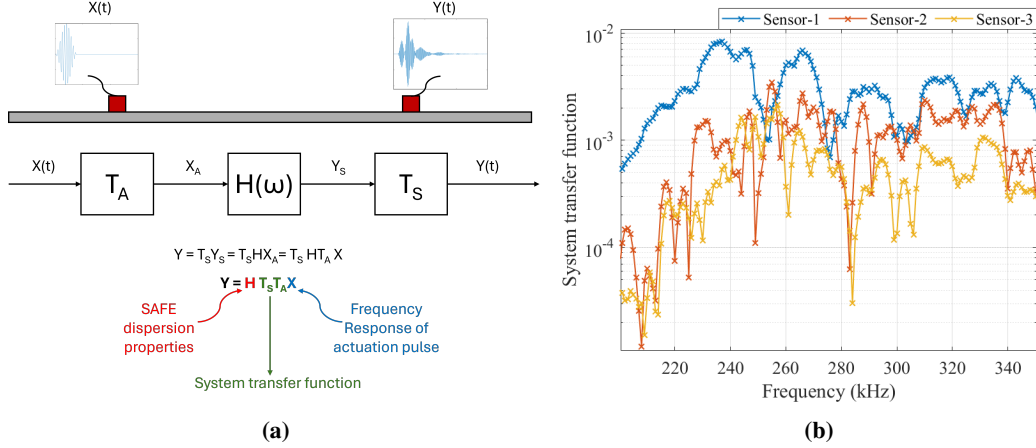


Fig. 2 (a) Schematic diagram of the point-to-point system transfer function employed in the signal reconstruction algorithm. (b) System transfer function plot across the frequency points defined in the actuation signal frequency response curve shown in figure 4b

wave modes, it is essential for the developed model to closely approximate the behavior of the actual structural system. This was accomplished by creating a SAFE model of the composite waveguide, which accounts for the fiber orientation, modulus of elasticity, modulus of rigidity, thickness and direction of guided wave propagation. The semi-analytical method is a well-established technique for determining wavenumbers, phase velocities, and group velocities along the propagation direction at specific frequencies [33]. A key advantage of this method is its ability to discretize the waveguide's cross-section while analytically solving an eigenvalue problem in the direction of wave propagation by incorporating a harmonic exponential term. This approach enables the extraction of the phase and group velocities for the S_0 and A_0 modes, as shown in figure 3. The phase and group velocity data, along with harmonic frequency components of the actuation pulse, $g(\omega)$ in figure 4b and the system transfer function illustrated in figure 2 were incorporated to generate unit amplitude S_0 and A_0 mode reconstructions as explained below. An ultrasonic guided wave signal $U_m(x, t)$ propagating in a plate like structure can be modelled as a function of displacement x and time t using,

$$U_m(x, t) = U_{S_0} e^{j(k_{S_0} x - \omega t)} + U_{A_0} e^{j(k_{A_0} x - \omega t)} + \varepsilon(x, t) \quad (1)$$

In equation 1, x is the distance from the actuating transducer, ω is the actuation frequency, t is the time vector, U_{S_0} and U_{A_0} are the amplitudes and k_{S_0} and k_{A_0} are the wavenumbers of the S_0 and A_0 modes respectively extracted from the semi-analytical model. Here ε denotes the error function to account for measurement and prediction discrepancies. The two fundamental ultrasonic guided wave modes were modelled using the phase velocity values obtained from a semi-analytical model. The amplitude coefficients U_{S_0} and U_{A_0} were then evaluated using the experimentally recorded signal as reference thereby making the model physics informed and experimental data driven. It is important to note that although the CFRC panel is excited by a toneburst sinusoidal ultrasonic signal at a specific monotonic frequency $\omega = \omega_0$, the signal in fact injected into the sample will be composed of a range of frequencies between a window $\omega + \Delta\omega$ and $\omega - \Delta\omega$ because of the interactions of the ultrasonic guided wave modes with the multilayered composite material. Therefore, the resultant signal would be a summation of equation 1 across all the frequency points in the window $\omega - \Delta\omega$ to $\omega + \Delta\omega$.

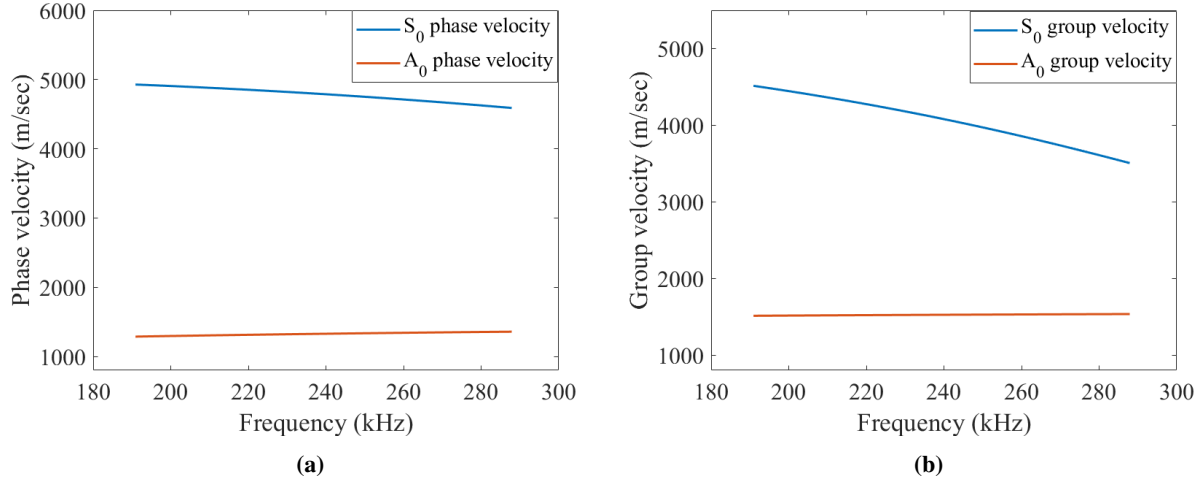


Fig. 3 (a) Phase velocity values of the S_0 and A_0 modes across the frequency points defined in the actuation signal frequency response curve shown in figure 4b. (b) Group velocity values of the S_0 and A_0 modes across the frequency points defined in the actuation signal frequency response curve shown in figure 4b

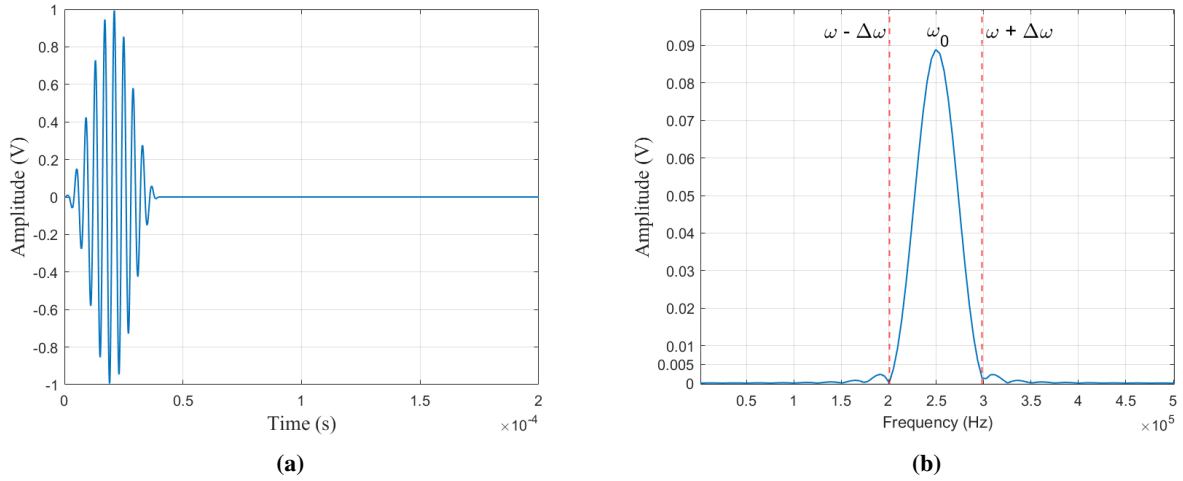


Fig. 4 (a) $2V_{pp}$, 10 period, Hanning-windowed toneburst actuation signal (b) Frequency response of the actuation signal

$$\begin{aligned}
 U_m(x, t) &= \int_{\omega-\Delta\omega}^{\omega+\Delta\omega} U_{S_0} e^{j(k_S(\omega)x - \omega t)} g(\omega) d\omega + \int_{\omega-\Delta\omega}^{\omega+\Delta\omega} U_{A_0} e^{j(k_A(\omega)x - \omega t)} g(\omega) d\omega \\
 &= \frac{1}{2\Delta\omega} \left[\int_{\omega-\Delta\omega}^{\omega+\Delta\omega} U_{S_0} e^{jk_S(\omega)x} g(\omega) e^{-j\omega t} + \int_{\omega-\Delta\omega}^{\omega+\Delta\omega} U_{A_0} e^{jk_A(\omega)x} g(\omega) e^{-j\omega t} \right] d\omega
 \end{aligned} \quad (2)$$

Noting that k_S and k_A are functions of ω , the wave numbers for the individual wave modes, in the following equation specifically the S_0 mode, can be expressed as a first order approximation about $\omega = \omega_0$ as

$$k_S(\omega)x - \omega t = k_S(\omega_0)x - \omega_0 t + \left. \frac{\partial(k_S(\omega)x - \omega t)}{\partial\omega} \right|_{\omega_0} \Delta\omega + \mathcal{O}(\Delta\omega)^2 \quad (3)$$

Noting the wavenumber-group velocity relationship $\frac{\partial k_S}{\partial\omega} = \frac{1}{V_{gs}}$ and $\Delta\omega = n\delta\omega$ as the number of discrete frequency steps

away from ω_0 , we have

$$k_s(\omega)x - \omega t = (k_0x - \omega_0t) + n\delta\omega \left(\frac{x}{V_{gs}} - t \right) \quad (4)$$

Evaluating the integrals in equation 2, as a discrete summation over small frequency step size $\delta\omega_i$, we have

$$\bar{U}_{S_0}(x, t) = \frac{U_{S_0}}{2\Delta\omega} \sum_i g(\omega_i) e^{jk_0x - \omega_0t} e^{jn\delta\omega_i \left(\frac{x}{V_{gs}} - t \right)} \delta\omega_i \quad (5)$$

Noting that $\Delta\omega = n_\omega\delta\omega$, we get the expressions for the reconstructions of the individual modes after some algebraic manipulation, as

$$\bar{U}_{S_0}(x, t) = \frac{U_{S_0}}{2n_\omega} e^{j\omega_0 \left(\frac{x}{V_{ps}} - t \right)} \left[\sum_{n=0}^{n_\omega} g(\omega + n\delta\omega) e^{jn\delta\omega \left(\frac{x}{V_{gs}} - t \right)} \right] \quad (6)$$

Using similar formulation for the A_0 term,

$$\bar{U}_{A_0}(x, t) = \frac{U_{A_0}}{2n_\omega} e^{j\omega_0 \left(\frac{x}{V_{pA}} - t \right)} \left[\sum_{n=0}^{n_\omega} g(\omega + n\delta\omega) e^{jn\delta\omega \left(\frac{x}{V_{gA}} - t \right)} \right] \quad (7)$$

The reconstructed ultrasonic guided wave symmetric S_0 and assymetric A_0 modes were then superimposed to obtain the reconstructed signal $U_m(x, t)$.

$$U_m(x, t) = \bar{U}_{S_0}(x, t) + \bar{U}_{A_0}(x, t) \quad (8)$$

It is important to note that the reconstructed signal obtained in equation 8 is the superimposition of unit amplitude reconstructions of the S_0 and A_0 guided wave modes, implying that the amplitude of the final full-length reconstructed signal is yet to be determined. To achieve this, the Hilbert transform wave envelopes [34] were calculated for both the recorded signals and their reconstructed counterparts were calculated. Subsequently, an objective function was established based on the Euclidean norm of the difference between the Hilbert envelopes of the recorded and reconstructed signals as expressed in equation 9.

$$\varepsilon(x, t) = \sqrt{\sum_{k=1}^N \|H(x^e)(t) - H(x^M)(t)\|^2} \quad (9)$$

Here, $H(x^e)(t)$ represents the Hilbert analytic signal of the experimental data $x^e(t)$, and $H(x^M)(t)$ represents the Hilbert analytic signal of the reconstructed data $x^M(t)$. The objective function was minimized to estimate the S_0 and A_0 mode amplitudes U_{S_0} and U_{A_0} and generate highly accurate full-length reconstructions of the structural-acoustic responses at each sensor location. The subsequent subsections section IV.A and section IV.B delineate the deterministic and probabilistic regimes employed in this study to estimate the modal amplitudes.

IV. Results and discussions

The parameters in consideration include U_{S_0} , U_{A_0} , V_{PS} , V_{PA} and AIC [35] based signal onset correction factor δn . The objective function established in equation 9 was utilized to optimize these parameters through two distinct approaches: deterministic approach and probabilistic approach. The deterministic approach provided reconstructions that closely aligned with the experimental data, demonstrating the efficacy of integrating the semi-analytical model with empirical measurements. The Bayesian inference based probabilistic joint parameter estimation methodology quantified and combined the uncertainties associated with both the model predictions and the experimental measurements in the signal reconstructions. The subsequent subsections elaborate on the results obtained from both the optimization regimes, highlighting the potential of utilizing this framework to establish reliable damage signatures.

A. Data driven and semi-analytical model informed deterministic approach to modal amplitude and model parameter estimation

Initially, the objective function defined in equation 9 was minimized at each sensor location individually to calibrate U_{S_0} , U_{A_0} , and δn . Since the optimization was performed separately for each sensor, the algorithm required nine initial guess values for these parameters, three for each sensor. After the optimization, the reconstruction algorithm

was re-executed using the calibrated S_0 and A_0 mode amplitudes and the AIC correction to generate full-length reconstructions of the signals at each sensor location. The final reconstructions exhibit high concordance with the experimental signals as illustrated in figure 5. Although this optimization regime produced reconstructions that closely

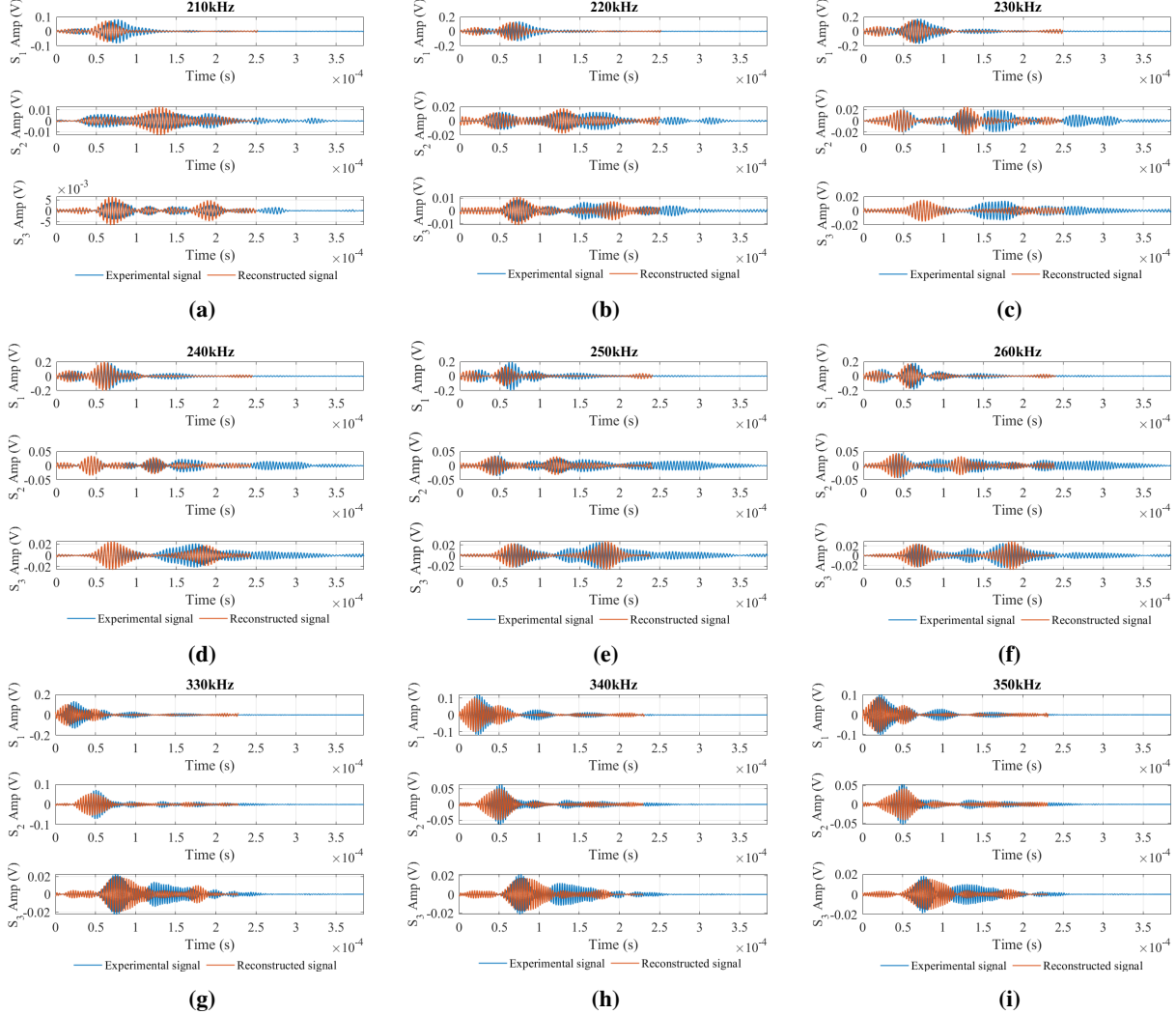


Fig. 5 Semi-analytical model informed and experimental data driven reconstruction of the S_0 and A_0 ultrasonic guided wave modes derived from deterministic point optimization performed independently at each sensor location.

matched the experimental counterparts, it is important to note that this regime would be computationally prohibitive with an increased number of transducers, given that the process involved separate optimization for each sensor location. To address this, a weighted-cumulative error function was formulated, accounting for contributions from all the three transducers simultaneously as expressed in equation 10.

$$\varepsilon_c(x, t) = \sum_{i=1}^{n_s} w_i \cdot \varepsilon_i(x, t) \quad (10)$$

Because the optimization is not repeated independently for each sensor, the parameter space could be expanded to include V_{PS} and V_{PA} at the central actuation frequency, ω_0 , thereby enhancing the robustness of the model without adding additional computational load. In this approach, optimization was only executed once, jointly calibrating the reconstructed U_{S_0} and U_{A_0} and δn along with V_{PS} and V_{PA} . This regime required only five initial guess values in total

unlike the previous regime, where the amplitudes were independently optimized at each sensor location. This method computes the optimized amplitudes by propagating the optimized estimates across all the three transducers through the system transfer function. The transfer function established at each sensor was multiplied with the optimized S_0 and A_0 amplitude estimates to generate location-specific modal amplitude values. Finally, the S_0 and A_0 modes with the calibrated amplitude values were superimposed to generate full-length signal reconstructions that exhibited high concordance with their experimental counterparts as shown in figure 6. The evolution of the reconstructed S_0 and A_0

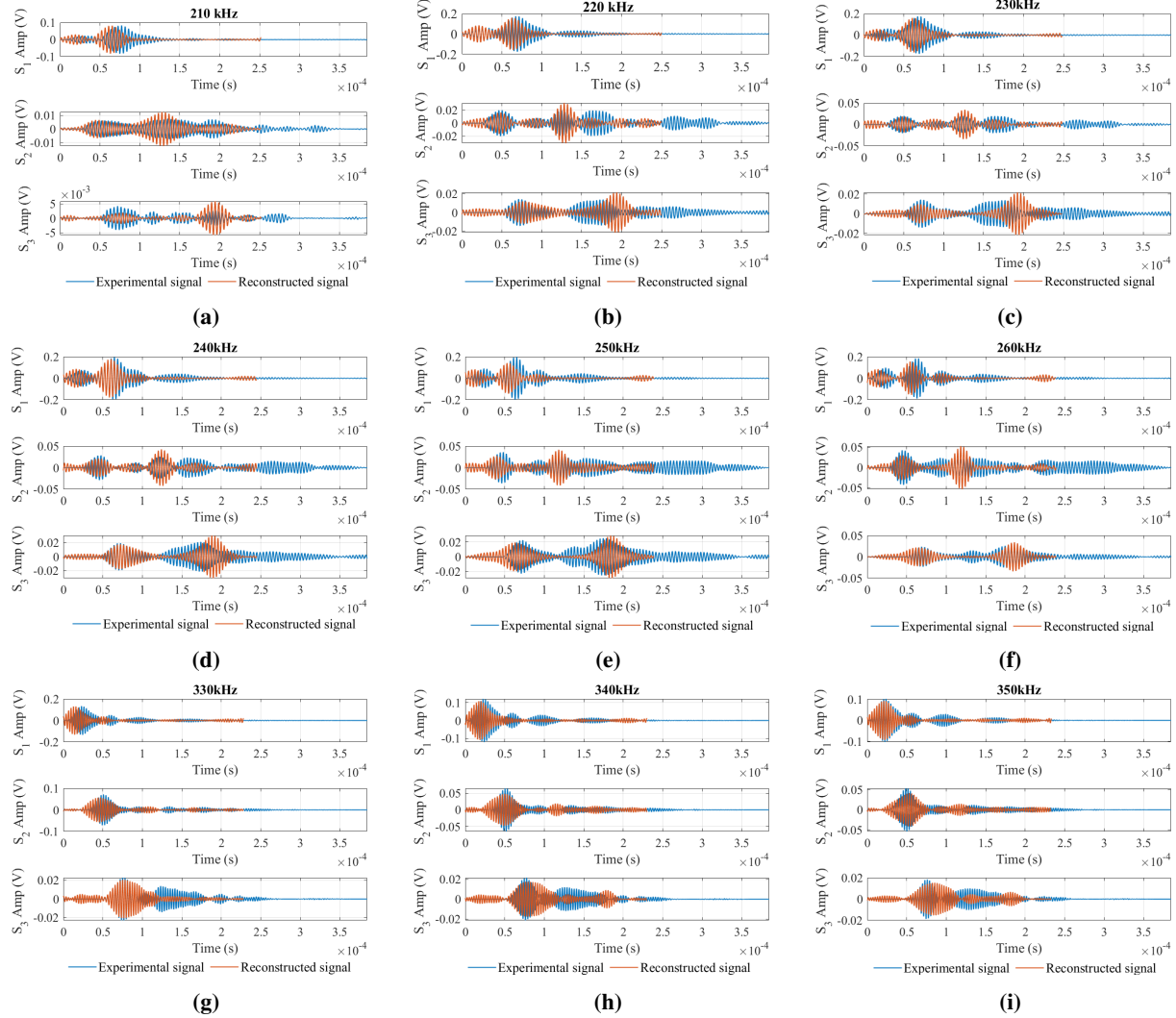


Fig. 6 Semi-analytical model informed and experimental data driven reconstruction of the S_0 and A_0 ultrasonic guided wave modes derived from deterministic point optimization using a weighted-cumulative error function across the three transducer locations.

mode amplitudes across the actuation frequency range of 200kHz–350kHz was investigated as a function of actuation frequency, as depicted in figure 7. The propagation characteristics of the S_0 and the A_0 modes differ significantly, with the S_0 mode primarily involving in-plane compression and tension along the direction of propagation as opposed to the A_0 mode which is characterized by out-of-plane flexural or bending waves. Due to its longitudinal stress-wave propagation regime, the S_0 mode exhibits more efficient transmission of energy compared to the A_0 mode. This is because of its reduced interaction with the plate’s boundaries, which minimizes scattering and attenuation effects while concentrating energy along its path. In contrast, the flexural motion of the A_0 mode is significantly influenced by the plate thickness and boundaries, resulting in greater dispersion and energy loss. As the actuation frequency increases, the phase and group velocities of the S_0 mode converge, resulting in reduced dispersion and allowing the mode to propagate

with minimal energy loss. But, the A_0 mode exhibits increased dispersion at higher frequencies, resulting in greater energy loss and reduced amplitude [33, 36]. The reconstructed S_0 and A_0 modes reflect these characteristics, as depicted in figure 7. Additionally, the amplitude response of the full-length reconstructions of the ultrasonic guided wave signals was analyzed at each monotonic frequency point within the specified range. Both experimental and reconstructed responses demonstrated a high degree of agreement, indicating that energy content of the reconstructed signals at a given frequency closely resembles that of their experimental counterparts at the same frequency.

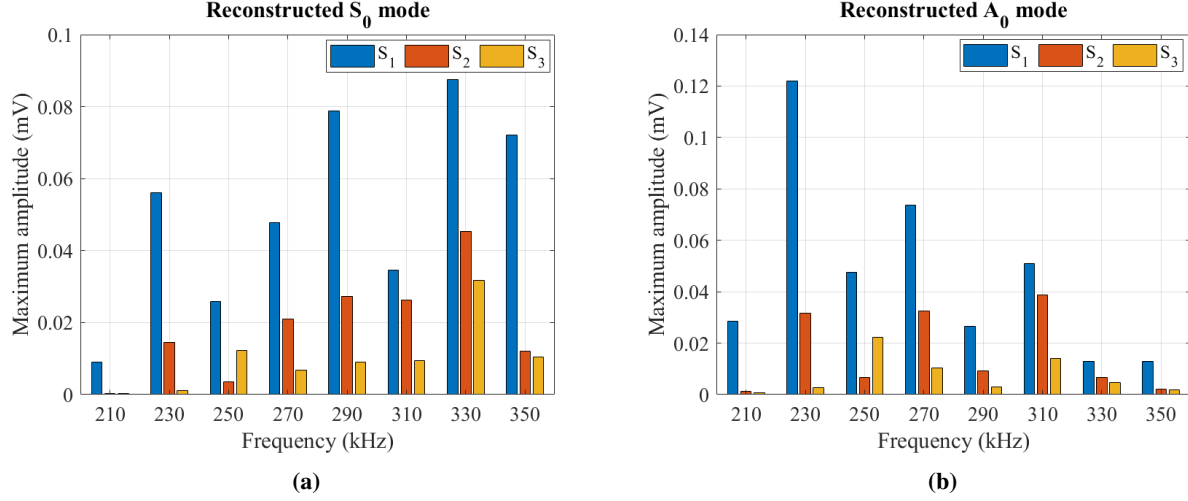


Fig. 7 (a) Evolution of the reconstructed S_0 mode amplitude in the actuation frequency range 200kHz–350kHz. (b) Evolution of the reconstructed A_0 mode amplitude in the actuation frequency range 200kHz–350kHz.

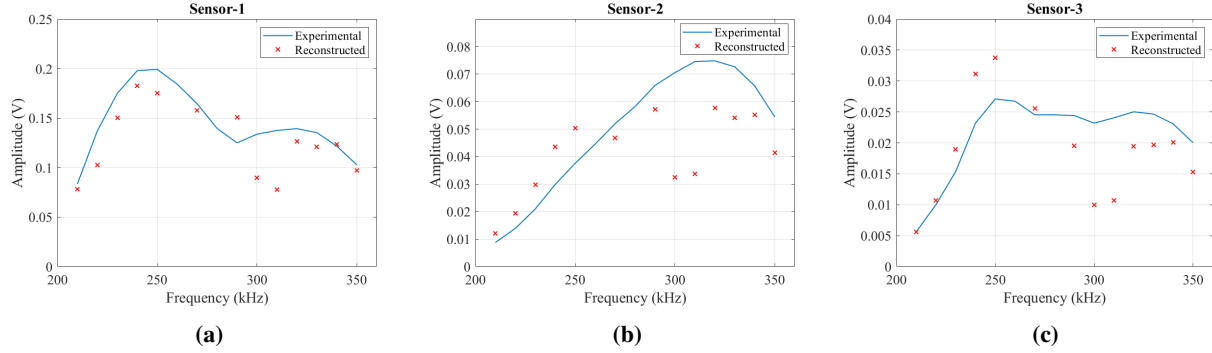


Fig. 8 High concordance between the amplitude response curves of the full-length reconstructed and acquired experimental signals in the frequency range 200kHz-350kHz at (a) transducer-1, (b) transducer-2 and (c) transducer-3

To summarize, the deterministic optimization approach combined the data from the semi-analytical model, which is informed by the underlying physics of guided wave interactions in the multilayered anisotropic composite waveguide with the acquired experimental data. This method enabled accurate calibration of S_0 and A_0 mode amplitudes, as well as phase velocities producing reconstructions that closely matched their experimental counterparts. The subsequent step is to enhance the robustness and reliability of the modal amplitude and model parameter estimations by employing a probabilistic joint parameter estimation framework. This approach accounts for both the aleatoric uncertainty inherent in the acquired experimental data and the epistemic uncertainty associated with the semi-analytical model predictions, thereby ensuring that these uncertainties are quantified and reflected in the final reconstructions.

B. Data driven and semi-analytical model informed probabilistic approach to modal amplitude and model parameter estimation

This algorithm implements an adaptive metropolis-Hastings MCMC method to perform probabilistic joint parameter estimation. This methodology offers two primary advantages: it determines the amplitudes of the individual wave modes and allows for effective damage assessment by analyzing the parameter α_M . This parameter encompasses the elastic and geometric properties of the waveguide, enabling an evaluation of whether the system has deviated from its pristine condition or a previously established baseline. The algorithm is based on Bayesian inference, where the posterior distribution of the parameters is updated based on the likelihood of observing the experimental data, given the proposed model parameters. A process flow diagram of this algorithm is illustrated in figure 9.

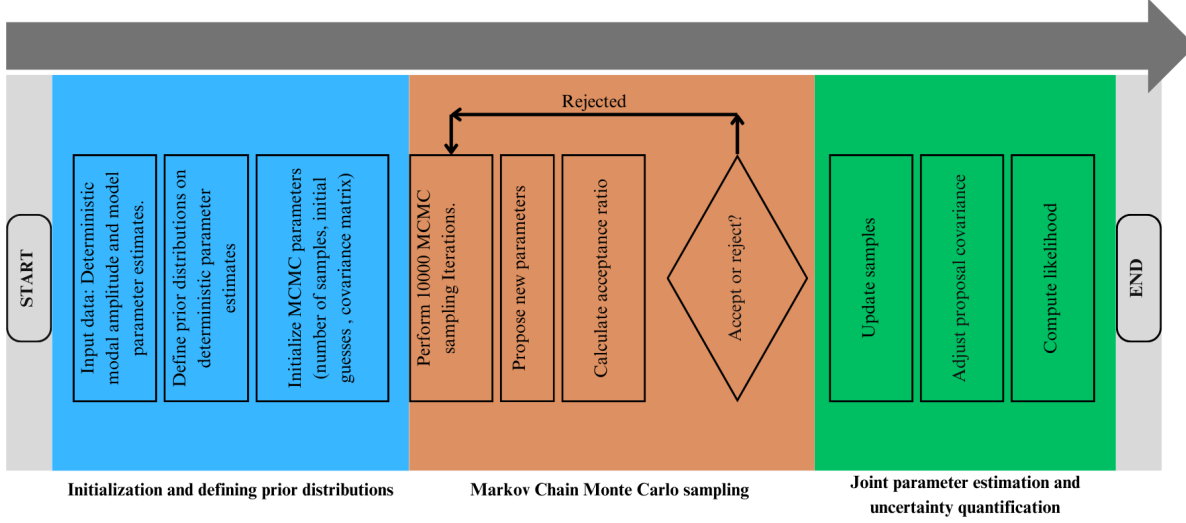


Fig. 9 Process flow diagram of the adaptive Metropolis-Hastings approach employed in this study for joint probabilistic estimates of modal amplitude and dispersion parameters.

$$\pi(U_{S_0}, U_{A_0}, \alpha_M \mid k_{S_0}, k_{A_0}, H(x)(t)) \propto e^{(-\frac{1}{2} \varepsilon^T \Sigma^{-1} \varepsilon)} \pi(U_{S_0}, U_{A_0}, \alpha_M) \quad (11)$$

$$\text{where } \varepsilon = H \left(x_{S_0}^M + x_{A_0}^M \right) (t) - H(x^e)(t)$$

This Bayesian approach derives the posterior distribution of the individual wave characteristics U_{S_0} , U_{A_0} and the model parameters V_{PS} and V_{PA} , conditional on the predictions of the wavenumbers k_{S_0} and k_{A_0} which are functions of α_M . The priors on U_{S_0} , U_{A_0} , V_{PS} and V_{PA} were defined using the standard normal distribution; δn was defined using a standard uniform distribution. The mean values for the priors were extracted from the deterministic parameter estimations. An affine transformation was applied wherein each parameter's prior range was defined by a mean and a variation parameter. The range was computed by adjusting the mean by a fraction of its own value, thus allowing the prior to account for possible fluctuations around the optimized values. In each iteration, the Metropolis-Hastings algorithm proposes a new set of parameters based on a multivariate normal distribution. This proposal is then accepted or rejected based on the ratio of posterior probabilities of the proposed samples to the current samples, known as the acceptance ratio. The adaptive feature dynamically adjusts the proposal covariance matrix based on the acceptance rate. The likelihood computes the Euclidean norm of the difference between the experimental and reconstructed Hilbert transform signal envelopes and returns a probability density function based on this norm, which guides the MCMC sampling process. A lower norm indicates a better fit, leading to a higher likelihood value and a higher norm suggests a poor fit resulting in a lower likelihood value. The posterior distribution is proportional to the likelihood function multiplied by the prior distribution assumed for these wave amplitudes and model parameters. The error function is directly proportional to a Gaussian likelihood function, with the ε vectors representing the differences between the Hilbert envelopes of the reconstructed and experimental signals at each sensor. Minimizing these differences across all available sensors allows for the joint identification of modal amplitudes and model parameters. By sampling from the posterior distribution, the algorithm captures both aleatoric and epistemic uncertainties arising from the inherent

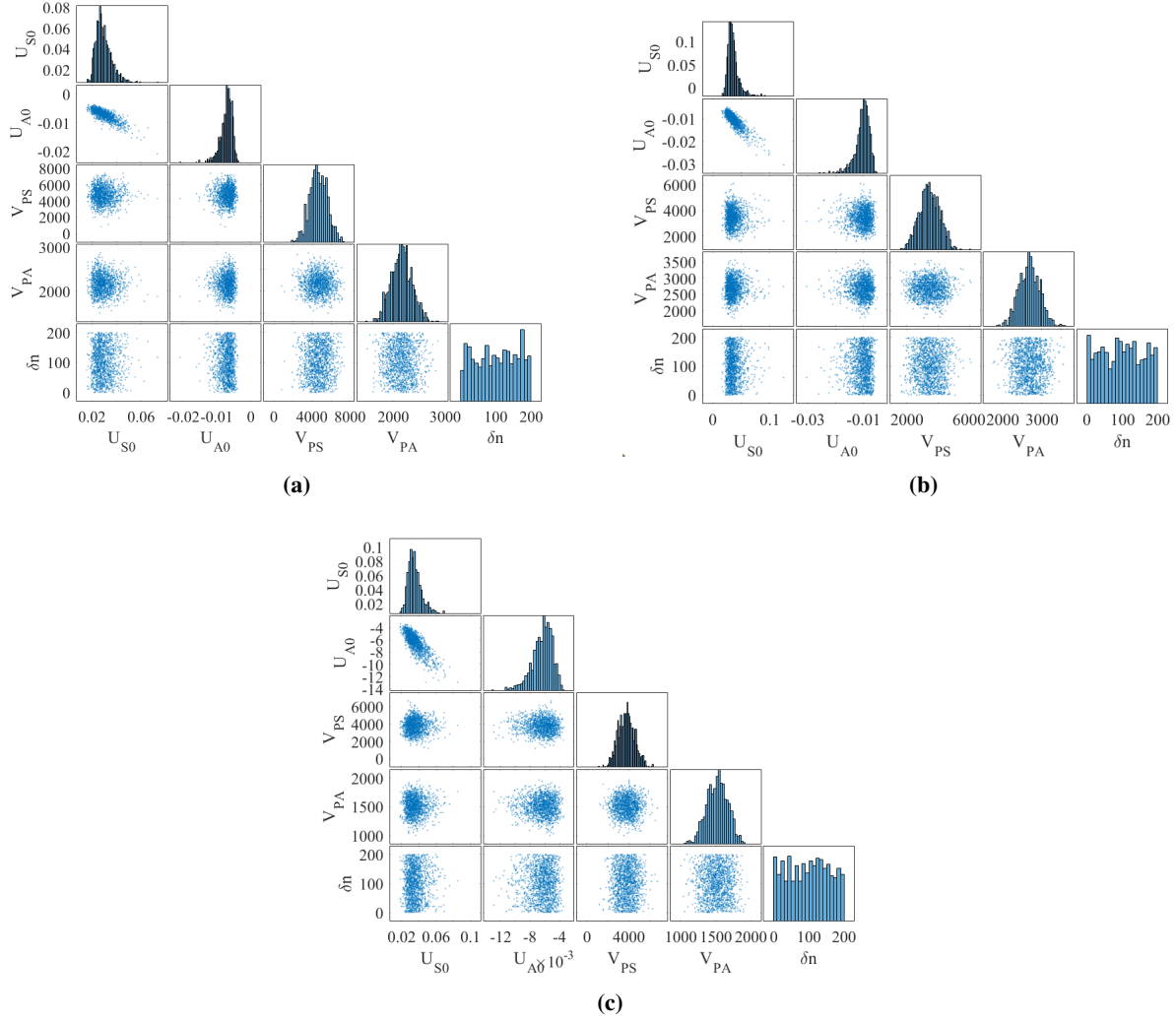


Fig. 10 Probabilistic modal amplitude and model parameter distribution plots at (a) 330kHz, (b) 340kHz and (c) 350kHz

randomness in the experimental data and from the lack of knowledge in the semi-analytical model, respectively. Unlike the deterministic point optimization regime, output from the Bayesian joint parameter estimation is not just a single best-fit set of parameters but rather a distribution of plausible values for the modal amplitudes and model parameters as evidenced in the diagonal elements of the joint distribution plots in figure 10. An interesting observation from these joint parameter distribution plots is the negative slope in the U_{S_0} vs. U_{A_0} subplot which is indicative of a negative correlation between the two parameters. This alignment with the fundamental S_0 and A_0 mode characteristics shown in figure 7 implies that the model's physics-informed formulation is capturing the distinct propagation behaviours of the S_0 and A_0 modes. This relationship that is reflected in the Bayesian modal amplitudes underscores the model's accuracy in representing the underlying physics by assigning a statistical dependence that closely mirrors the characteristics of these modes. The 10000 samples for each parameter, fitting the posterior distribution defined in equation 11 were utilized to generate the posterior predictive Hilbert envelopes at each sensor location. The mean posterior predictive envelope along with 75% and 95% confidence intervals is shown overlaid with the recorded experimental signals in figure 11. The high concordance between the Bayesian posterior predictive hilbert envelope and the experimental signal demonstrates the model's efficacy in capturing the true signal characteristics within a reasonable range of uncertainty. This robust fundamental wave mode calibration approach not only proposes a range of plausible parameter values but also generates highly accurate reconstructions of the acquired acousto-ultrasonic signals.

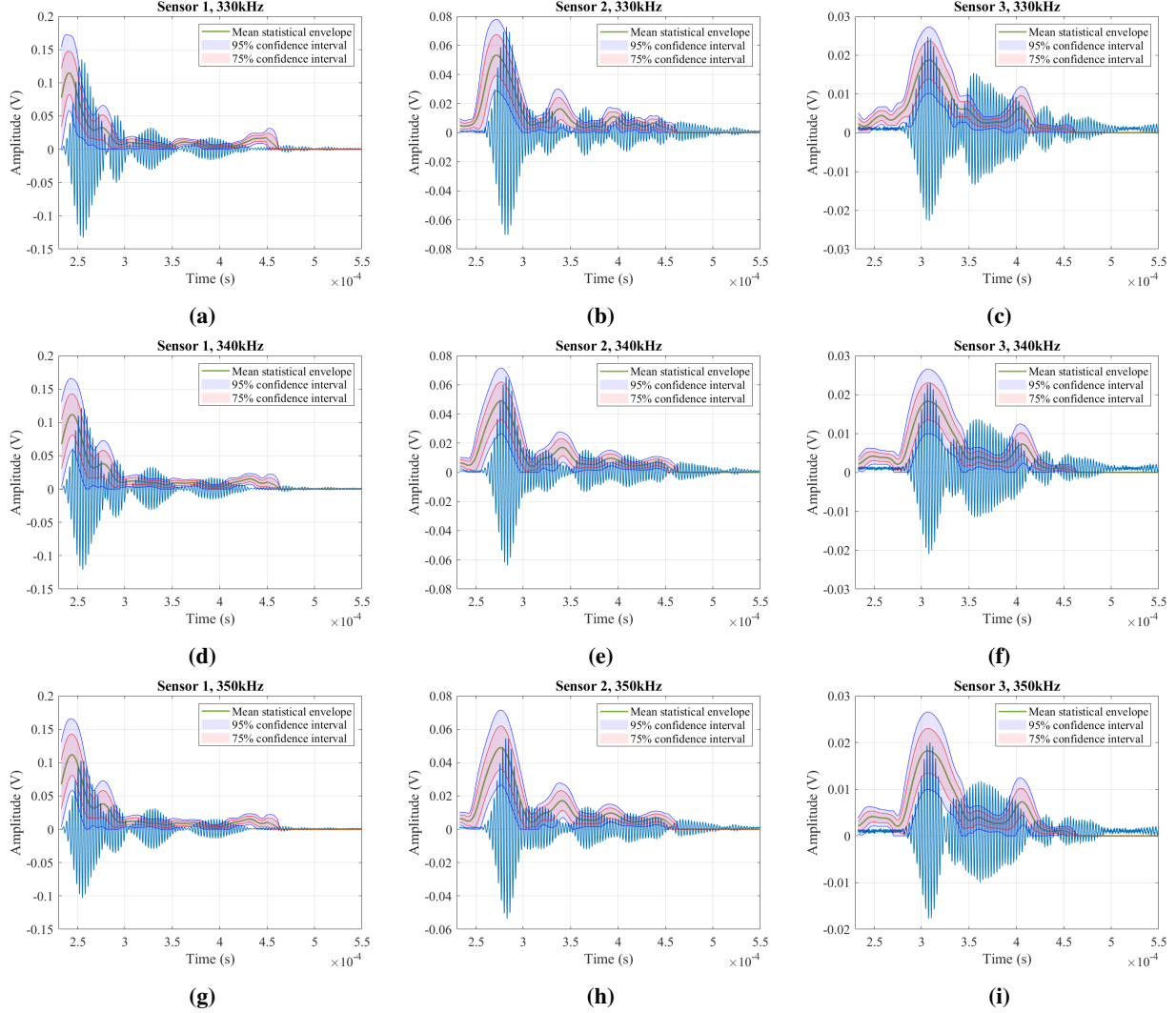


Fig. 11 Comparison of the experimental signal with the mean statistical Hilbert envelope, overlaid with 75% and 95% confidence intervals.

V. Conclusion

This study presents a semi-analytical model-informed and experimental data-driven approach, for effective isolation and reconstruction of fundamental ultrasonic guided wave modes. A threshold-free guided wave detection and classification framework in CyberSHM enabled precise interrogation of a 12-layered CFRC panel, using a harmonic excitation pulse sweeping over a defined frequency band. A combined model-informed and data driven reconstruction algorithm leveraged the amplitude and phase information of the actuation pulse, the dispersion characteristics of the fundamental S_0 and A_0 guided wave modes from the semi-analytical model and the point-to-point system transfer function between the transducer and the CFRC to reconstruct the fundamental ultrasonic guided wave modes which were subsequently superimposed to generate full-length acousto-ultrasonic signal reconstructions.

The Hilbert envelopes of the experimental and reconstructed signals were calculated and an objective function based on the Euclidean norm of their difference was established. Two deterministic point optimization regimes computed the analytical modal amplitudes and model parameters to produce high-fidelity reconstructions of the experimental signals. To enhance the reliability and robustness of the results, a Bayesian inference-based probabilistic joint parameter estimation framework was employed where the uncertainties associated with both experimental measurements and model predictions were quantified by presenting a distribution of plausible values for the modal amplitudes and model parameters rather than deterministic single point estimates. The relationship reflected in the Bayesian fundamental mode

amplitudes underscored the model's accuracy in representing the underlying physics by assigning a statistical dependence that closely mirrors the characteristics of these modes. The high concordance between the Bayesian posterior predictive Hilbert envelope and the experimental signal demonstrates the models efficacy in capturing the true signal characteristics within a reasonable range of uncertainty. Accurately reconstructing the fundamental ultrasonic guided wave modes and quantifying the uncertainty in the model predictions and experimental data is an important step in enhancing the reliability of these wave mode reconstructions for damage detection. The cyberphysical nature of this framework will be fully realized when both the physical layer, featuring autonomous threshold-free active signal detection and classification, and the digital layer, encompassing the semi-analytical waveguide model and reconstruction algorithm, are deployed on a daisy-chained edge device to enable continuous, periodic, or on-demand structural interrogation.

Future studies will extend this framework to panels with controlled damage, analyzing changes in guided wave mode content as a function of introduced damage. This will facilitate the establishment of reliable damage signatures, which can be incorporated into a comprehensive structural health metric, advancing predictive and condition-based maintenance capabilities for composite structures. Studies on two-dimensional waveguides with recalibration of waveguide properties throughout its loading phases will be undertaken with a sparse array of distributed sensors to demonstrate the validity of the proposed approach to operational thin-walled aerostructures. Ultimately, the presence of distinct damage signatures in the fundamental guided wave modes will provide insight into the interaction of damage with propagating wave modes and will help in classifying the damage types and their level of severity.

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