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Region-inspired distributed optimal dispatch of flexibility providers in coordinated transmission-distribution framework

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Abstract: To exploit the flexibility of generator-load resources for improving the whole system economy and to protect the information privacy at the transmission and distribution system operator (TSO-DSO) interface, the coordinated optimal dispatch of transmission and distribution networks should be determined in a distributed manner. This paper proposes a two-stage distributed TSO-DSO dispatch method with boundary power feasible flexibility region (FFR) pre-estimated and inspired. In the first stage, given the self-government characteristics of TSO and DSOs, the concept of TSO-DSO FFR consists of DSO provision FFR and TSO desirability FFR, which respectively define the set of security-constrained boundary power operating points from distribution and transmission perspective. The convex optimal power flow based approaches for TSO and DSO FFRs are correspondingly deployed to identify the power flexibility ranges at the TSO-DSO interconnection bus. In the second stage, an FFR-driven distributed decision-making framework is presented to determine the dispatch solutions of flexibility resources in TSO and DSOs, which are solved using a hierarchical loop iteration algorithm. Case studies on the T24-D3 and T118-D30 systems validate superior dispatch performance and high computation efficiency.

Keywords: Optimal dispatch, distributed optimization, flexibility providers, transmission and distribution networks, region inspiration

Nomenclature

Sets

Ω_B/Ω_L	Set of boundary/existing lines
Ω_G (Ω_G^i)	Set of generators (at bus i)
Ω_l	Set of buses
T	Set of time slots

Parameters

$P_g^{G,min}/P_g^{G,max}$	Minimum/maximum active power generated by generator g
$Q_g^{G,min}/Q_g^{G,max}$	Minimum/maximum reactive power generated by generator g
$P_{ij}^{min}/P_{ij}^{max}$	Minimum/maximum active power of boundary line ij
$Q_{ij}^{min}/Q_{ij}^{max}$	Minimum/maximum reactive power of boundary line ij
$P_{ij,t}^{T,*}/Q_{ij,t}^{T,*}$	Optimal active/reactive power flows from transmission bus i to distribution bus j at time slot t
$P_{ij,t}^{D,*}/Q_{ij,t}^{D,*}$	Optimal active/reactive power flows from distribution bus i to transmission bus j at time slot t
$P_{j,t}^{REF}$	Predicted nominal value of renewable energy maximum power output for bus j at time slot t
$P_{i,t}^{ES,min}/P_{i,t}^{ES,max}$	Minimum/Maximum power boundary of BESS at bus i of slot t

$$P_{j,t}^{LF}/Q_{j,t}^{LF}$$

Predicted nominal value of active/reactive load demand for bus j at time slot t

$$G_{ij}/B_{ij}/R_{ij}/X_{ij}$$

Conductance/susceptance/resistance /reactance of line ij

$$G_j/B_j$$

Shunt conductance/susceptance at bus j

$$E_{i,t}^{ES,min}/E_{i,t}^{ES,max}$$

Minimum/maximum energy boundary of BESS at bus i of slot t

$$I_{ij}^{max}/S_{ij}^{max}$$

Maximum current magnitude/apparent power of line ij

$$\varphi_{i,t}$$

Power factor angle of load demand for bus i at time slot t

$$N_t$$

Number of time slots

$$V_i^{min}/V_i^{max}$$

Minimum/maximum voltage magnitude of bus i

$$\Delta t$$

Duration time between two adjacent hours

Variables

$$P_{g,t}^G/Q_{g,t}^G$$

Active/reactive power generated by generator g at time slot t

$$P_{j,t}^{REC}$$

Power curtailment of renewable energy for bus j at time slot t

$$P_{j,t}^{LC}/Q_{j,t}^{LC}$$

Active/reactive load shedding power for bus j at time slot t

$$P_{j,t}^{ES}$$

Power of BESS for bus j at time slot t

$$P_{ij,t}/Q_{ij,t}$$

Active/reactive power flow of line ij at time slot t

$P_{ij,t}^{D/T}/Q_{ij,t}^{D/T}$	Active/reactive power injections from the distribution/transmission bus i to transmission/distribution bus j at time slot t
$L_{ij,t}$	Square of current magnitude of line ij at time slot t
$U_{i,t}$	Square of voltage magnitude of bus i at time slot t
$a_{ij,t}/b_{ij,t}$	Artificial variables introduced to relax the transmission power flow model

1. Introduction

Proliferation of distributed energy resources (DERs) in distribution networks has reformed the traditional supply pattern where power flows from the transmission network unidirectionally to the distribution network [1], [2]. When power surplus from DERs occurs after supplying the local load in a distribution network, the surplus could be fed back to the transmission network through interconnection bus, which could make the power through the interconnection bus bi-directional and time-varying, given the uncertainty of renewable energy outputs and load fluctuations [3]. Therefore, on the one hand, uncertainty on the distribution side will be propagated to the transmission side causing operational challenges, but on the other hand, with the coordination of the transmission system operator (TSO) and the distribution system operator (DSO), the flexibility of resources within the distribution system, such as DERs and demand response, can be explored to mitigate the uncertainty [4]. Considering DERs and demand response indeed interact with the transmission side through the TSO-DSO interface, i.e., the interconnection bus, the determination of the exchanging power will affect the economic and secure operation of both the transmission and distribution sides.

In the transmission network, flexibility providers could be combined cycle units, hydropower plants, and aggregated loads. By contrast, in a distribution network, flexibility providers are usually distributed generators (DGs), battery energy storage systems (BESSs), and demand response [5]. Unlike the transmission system, not only TSO but also DSO might have very limited and even no knowledge about the locations, capacities, and operational characteristics of these flexibility providers connected in the distribution network [6], which results in compromised power output forecasting and deterioration of the economic operation. As the transmission and the distribution network are interconnected through the interconnection bus [7], limited power exchanging information at the TSO-DSO interface would be available. With the barrier of information privacy and communication limitation of the flexible resources in distribution networks, the coordinated TSO-DSO dispatch should be conducted in a distributed manner.

Therefore, coordination of the transmission and distribution sides would rely on distributed optimization algorithms. Some methods of this type, such as Lagrangian relaxation methods [1], [2], [8], optimality condition decomposition [9]-[11], and heterogeneous decomposition [6], [12], [13], have been adopted to resolve the problem of TSO-DSO coordinated optimal dispatch. The optimality condition based decomposition and heterogeneous decomposition methods rely on the Karush-Kuhn-Tucker condition and strong duality [14]. For the former, the

exchanged information between the TSO and DSO is homogeneous, while being heterogeneous for the latter. When the problem formulation of the TSO-DSO coordinated optimal dispatch becomes complicated, the strong duality may not hold, and the derivation of the dual form becomes invalid. By contrast, the Lagrangian relaxation based methods present superior scalability by relaxing consistency constraints of interconnection buses to formulate two subproblems respectively for the TSO and DSO. Then, the Lagrangian multipliers are iteratively updated to coordinate the two subproblems [15] until certain convergence conditions are met. To enhance the convergence ability, augmented Lagrangian relaxation methods, e.g., auxiliary problem principle (APP) [16], alternating direction method of multipliers (ADMM) [17], and analytical target cascading (ATC) [18], are proposed with adding quadratic penalty terms into the originally linear Lagrangian function. However, since these methods start with randomly selecting the initial value and need to tentatively tune the boundary exchanging information to finally achieve consistency in each scenario, their computational performances are usually unsatisfactory.

Feasible flexibility region (FFR) [19], [20] that is usually generated by flexibility margin estimation and can depict allowable power exchanged through the TSO-DSO interface has received increasing research interests [4], [21]. However, FFR is usually identified from the DSO perspective and commonly considered as the aggregated security-constrained flexibility of the DERs and loads in the distribution network at the TSO-DSO interface to the transmission network. Since the FFR model is based on a spatiotemporal coupling problem and presents high computational complexity, reference [22] provides an inner approximation method for identifying the FFR focusing on efficiently solving the FFR model. This method guarantees feasibility of the FFR but sacrifices the coverability to a certain extent as the inner approximation is intrinsically conservative. Hence, to fully unlock the flexibility of TSO-DSO coordination with the FFR, the outer approximation method that can guarantee problem feasibility while avoiding over-optimism is more appropriate for depicting the FFR.

Most of the existing works studied the feasibility of power exchange at the TSO-DSO interface merely from the perspective of the DSO. However, given that it depends not only on the power injection capability of the DSO, but also on the power accommodation ability of the TSO, reference [23] proposes a more precise FFR model jointly considering the DSO power injection capability and the TSO power desirability. Reference [24] reviewed main FFR generation algorithms, and classified them into three categories: i) Monte Carlo sampling based methods [20], ii) elimination methods [22] and iii) optimization-based methods [4], [23]-[27]. With the Monte Carlo method, a large number of operating points of the exchanging power are randomly sampled and then fed to a sequence of network-constrained optimal power flow (OPF) problems to directly identify the margin. Compared with the Monte Carlo and the elimination methods, the optimization-based methods present superior computational efficiency and accuracy, especially when the numbers of flexibility providers and interconnected distribution networks are large.

TSO-DSO FFR can be utilized to inspire the distributed coordination of the TSO and the DSO. Since the power flexibility is estimated by active and reactive powers injected

through the TSO-DSO interface, TSO-DSO FFR is indeed a region on the P-Q domain. With the TSO-DSO FFR being pre-estimated, an initial setpoint to the exchanging power can be selected within the feasible region on the P-Q domain, while an optimal operating point can be finally achieved through searching this P-Q domain. TSO-DSO FFR indeed provides distributionally interacting manner that enables the TSO and the DSO to optimize their dispatch and explore their flexibility providers. With the FFR inspired distributed coordination framework, the feasible operating points to the exchanging powers can be pre-estimated and then be interacted. This, indeed, upgrades the iterative procedure of the existing Lagrangian relaxation methods by shrinking the searching space and reducing the communication burden and the number of iterations. Thereby, both the computational accuracy and the efficiency of distributed TSO-DSO coordination can be consequentially improved.

This paper proposes a TSO-DSO FFR inspired distributed TSO-DSO coordination framework to explore the potential of flexibility providers simultaneously within the transmission and the distribution sides. To overcome the barriers of communication burden and information privacy, FFRs are generated respectively from the perspectives of the transmission and the distribution sides. The two types of FFRs are referred as DSO provision FFR and TSO desirability FFR, which are essentially the sets of feasible operating points of active and reactive exchanging powers that are obtained by solving a series of network-constrained OPF problems. With the DSO provision FFR and TSO desirability FFR, the TSO-DSO coordinated optimal dispatch problem can be decomposed into two subproblems with the Lagrangian relaxation framework. These two phases consist of the TSO-DSO FFR inspired distributed TSO-DSO coordination framework, which is illustrated in Fig. 1.

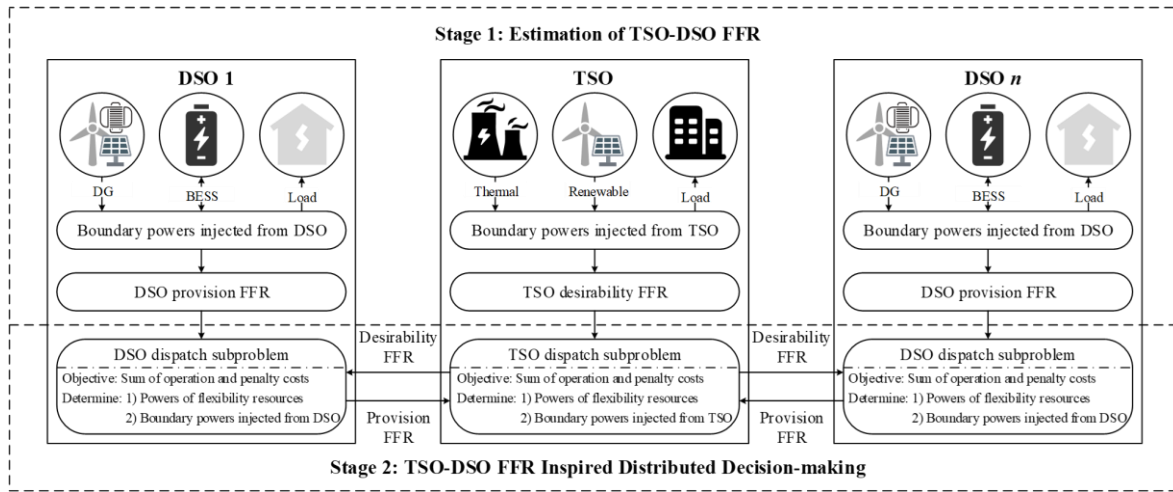


Fig. 1. FFR inspired distributed TSO-DSO coordination framework.

The contributions of this paper are threefold:

- 1) With the novel concept of FFR, aggregating estimation models of the DSO provision FFR and TSO desirability FFR at the TSO-DSO interfaces, respectively from distribution and transmission perspectives, are proposed in a second-order cone programming (SOCP) relaxed manner.
- 2) Focusing on TSO-DSO coordinated optimal dispatch, a TSO-DSO FFR inspired distributed framework that leverages the DSO provision FFR and TSO desirability FFR is proposed in which the pre-estimated FFR indicates the feasible region of the corresponding dispatch subproblem.
- 3) A computationally friendly hierarchical loop iterative algorithm is developed to solve the TSO-DSO coordinated dispatch problem under the proposed framework. Different from existing distributed algorithms, the optimal dispatch solution can be directly achieved by traversing the vertices of FFRs.

The rest of the paper is organized as follows: Section 2 provides the concept division, optimization model and generation approach of TSO-DSO FFR. The FFR-inspired distributed optimal dispatch problem as well as its solving method for transmission and distribution networks are presented in Section 3. Section 4 conducts the case studies and shows the simulation discussions, followed by the conclusions.

2. Estimation of TSO-DSO FFR

This section introduces the two types of TSO-DSO FFRs at the TSO-DSO interfaces that are respectively depicted from the views of the transmission and the distribution sides. Two optimization problems that help estimate TSO-DSO FFRs are firstly formulated. To preserve the convexity of the two models, DistFlow power flow model is adopted. Thereafter, an adaptive optimization-based algorithm is deployed to identify the two types of TSO-DSO FFRs.

Respecting to the security constraints, the DSO regulates its available flexible resources, such as DGs, BESSs, and distributed loads, to finally guarantee an acceptable power injection/withdrawal to/from the transmission side. Similarly, the TSO dispatches the flexibility providers, e.g., renewable energy stations, thermal generators, and aggregating loads, within the transmission system referring to the power injections/withdrawals from connected distribution systems.

Fig. 2 illustrates the relationship of the DSO provision FFR, the TSO desirability FFR, and the synthesized TSO-DSO FFR. As shown in Fig. 2, 1) as the outputs of renewable energy resources and load demands are time-varying, the TSO-DSO FFR is time-varying as well; 2) the DSO provision FFR is determined by the flexibility providers within the distribution side and the TSO desirability FFR is determined by the flexibility providers within the transmission side; and 3) the synthesized TSO-DSO FFR, as the name suggests, is

produced by intersecting the DSO provision FFR and the TSO desirability FFR, representing jointly allowed exchanging powers between the TSO and the DSO.

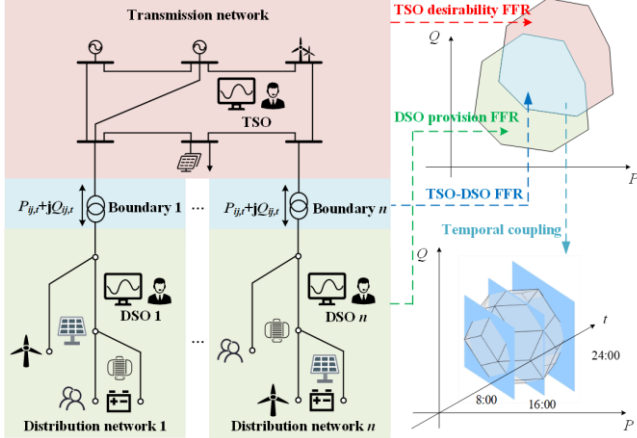


Fig. 2. The two types of TSO-DSO FFR.

2.1. DSO provision FFR

The DSO provision FFR is defined as a set of operating points to the exchanging powers, $P_{ij,t}^D$ and $Q_{ij,t}^D$, that satisfy the operational constraints (1)-(16) of the distribution side. With the DistFlow power flow model and SOCP relaxed line flow constraints [28], the DSO provision FFR can be described with the following constraints.

1) Distribution Power Balance Constraints:

$$\sum_{g \in \Omega_G^i} P_{g,t}^G + \sum_{ij \in \Omega_L} (P_{ij,t} - L_{ij,t} R_{ij}) + P_{ij,t}^D + P_{j,t}^{ES} + P_{j,t}^{REF} - P_{j,t}^{REC} = P_{j,t}^{LF} - P_{j,t}^{LC}, \forall ij \in \Omega_B, \forall j \in \Omega_I, \forall t \in T \quad (1)$$

$$\sum_{g \in \Omega_G^i} Q_{g,t}^G + \sum_{ij \in \Omega_L} (Q_{ij,t} - L_{ij,t} X_{ij}) + Q_{ij,t}^D = Q_{j,t}^{LF} - Q_{j,t}^{LC}, \forall ij \in \Omega_B, \forall j \in \Omega_I, \forall t \in T \quad (2)$$

Constraints (1) and (2) respectively enforce the active and reactive power balance at the distribution buses.

2) Distribution Power Flow Constraints:

$$\|2P_{ij,t} \quad 2Q_{ij,t} \quad L_{ij,t} - U_{i,t}\|_2^2 \leq L_{ij,t} + U_{i,t}, \forall ij \in \Omega_L, \forall i \in \Omega_I, \forall t \in T \quad (3)$$

$$U_{i,t} - U_{j,t} = 2(R_{ij}P_{ij,t} + X_{ij}Q_{ij,t}) - (R_{ij}^2 + X_{ij}^2)L_{ij,t}, \forall ij \in \Omega_L, \forall t \in T \quad (4)$$

Constraint (3) is the SOCP relaxation form of the Distflow model and constraint (4) calculates the voltage magnitudes of distribution buses.

3) Distribution Network Security Constraints:

$$L_{ij,t} \leq (I_{ij}^{max})^2, \forall ij \in \Omega_L, \forall t \in T \quad (5)$$

$$(V_i^{min})^2 \leq U_{i,t} \leq (V_i^{max})^2, \forall i \in \Omega_I, \forall t \in T \quad (6)$$

Constraints (5) and (6) respectively enforce the limits of line capacity and bus voltage of distribution networks.

4) DG Operational Constraints:

$$P_g^{G,min} \leq P_{g,t}^G \leq P_g^{G,max}, \forall g \in \Omega_G, \forall t \in T \quad (7)$$

$$Q_g^{G,min} \leq Q_{g,t}^G \leq Q_g^{G,max}, \forall g \in \Omega_G, \forall t \in T \quad (8)$$

$$0 \leq P_{i,t}^{REC} \leq P_{i,t}^{REF}, \forall i \in \Omega_I, \forall t \in T \quad (9)$$

Constraints (7) and (8) indicate the bounds of active and reactive power outputs from diesel generators, and constraint (9) enforces the power curtailment limit of distributed renewable energy.

5) BESS Operational Constraints:

$$P_{i,t}^{ES,min} \leq P_{i,t}^{ES} \leq P_{i,t}^{ES,max}, \forall i \in \Omega_I, \forall t \in T \quad (10)$$

$$E_{i,t}^{ES,min} \leq \sum_{\tau=1}^t P_{i,\tau}^{ES} \Delta t \leq E_{i,t}^{ES,max}, \forall i \in \Omega_I, \forall t \in T \quad (11)$$

Constraints (10) and (11) enforce the power and accumulated energy consumption of each BESS referring to the discrete-time feasible region model in [29]. It can be seen that the power variables of different time slots are temporal coupling. However, with the method proposed in [29], the boundary parameters $P_{i,t}^{ES,min}$, $P_{i,t}^{ES,max}$, $E_{i,t}^{ES,min}$ and $E_{i,t}^{ES,max}$ can be calculated, thereby deriving the following power flexibility model of BESSs:

$$\phi_J^{ES,min} \leq \sum_{t \in T} P_t^{ES} \leq \phi_J^{ES,max}, \forall J \subseteq T$$

where $\phi_J^{ES,min}$ and $\phi_J^{ES,max}$ are parameters calculated from Δt , $P_{i,t}^{ES,min}$, $P_{i,t}^{ES,max}$, $E_{i,t}^{ES,min}$ and $E_{i,t}^{ES,max}$.

6) Distribution Load Response Constraints:

$$0 \leq P_{i,t}^{LC} \leq P_{i,t}^{LF}, \forall i \in \Omega_I, \forall t \in T \quad (12)$$

$$0 \leq Q_{i,t}^{LC} \leq Q_{i,t}^{LF}, \forall i \in \Omega_I, \forall t \in T \quad (13)$$

$$Q_{i,t}^{LC} = P_{i,t}^{LC} \tan \phi_{i,t}, \forall i \in \Omega_I, \forall t \in T \quad (14)$$

Constraints (12) and (13) are the load shedding constraints. Constraint (14) enforces the relationship between active and reactive powers.

7) Distribution Power Injection Constraints:

$$P_{ij}^{min} \leq P_{ij,t}^D \leq P_{ij}^{max}, \forall ij \in \Omega_B, \forall t \in T \quad (15)$$

$$Q_{ij}^{min} \leq Q_{ij,t}^D \leq Q_{ij}^{max}, \forall ij \in \Omega_B, \forall t \in T \quad (16)$$

Constraints (15) and (16) limit the active and the reactive powers injected at the interconnection bus from the DSO.

2.2. TSO desirability FFR

Similarly, the TSO desirability FFR is a set of operating points to the exchanging power, $P_{ij,t}^T$ and $Q_{ij,t}^T$, that satisfy the operational constraints of the transmission side. Different from the DSO provision FFR, the TSO desirability FFR, as in (17)-(31), is described based on an AC power flow model with SOCP relaxed line power constraints [30].

1) Transmission Power Balance Constraints:

$$\sum_{g \in \Omega_G^i} P_{g,t}^G + \sum_{ij \in \Omega_L} P_{ij,t} + P_{ij,t}^T + P_{j,t}^{REF} - P_{j,t}^{REC} - G_j U_{j,t} = P_{j,t}^{LF} - P_{j,t}^{LC}, \forall ij \in \Omega_B, \forall j \in \Omega_I, \forall t \in T \quad (17)$$

$$\sum_{g \in \Omega_G^i} Q_{g,t}^G + \sum_{ij \in \Omega_L} Q_{ij,t} + Q_{ij,t}^T + B_j U_{j,t} = Q_{j,t}^{LF} - Q_{j,t}^{LC}, \forall ij \in \Omega_B, \forall j \in \Omega_I, \forall t \in T \quad (18)$$

Constraints (17) and (18) respectively enforce the active and reactive power balance at the transmission buses.

2) Transmission Power Flow Constraints:

$$P_{ij,t} = -G_{ij} U_{i,t} + G_{ij} a_{ij,t} - B_{ij} b_{ij,t}, \forall ij \in \Omega_L, \forall t \in T \quad (19)$$

$$Q_{ij,t} = B_{ij} U_{i,t} - G_{ij} b_{ij,t} - B_{ij} a_{ij,t}, \forall ij \in \Omega_L, \forall t \in T \quad (20)$$

$$a_{ij,t}^2 + b_{ij,t}^2 \leq U_{i,t} U_{j,t}, \forall ij \in \Omega_L, \forall t \in T \quad (21)$$

Constraints (19) and (20) calculate the active and reactive power flows in transmission lines, and (21) represents the SOCP relaxed bus voltage constraints. All these constitute a strict convex relaxation for transmission power flow, which has been investigated in [30] in the sacrifice of little quality.

3) Transmission Network Security Constraints:

$$P_{ij,t}^2 + Q_{ij,t}^2 \leq (S_{ij}^{max})^2, \forall ij \in \Omega_L, \forall t \in T \quad (22)$$

$$(V_i^{min})^2 \leq U_{i,t} \leq (V_i^{max})^2, \forall i \in \Omega_I, \forall t \in T \quad (23)$$

Constraints (22) and (23) enforce the line capacity and bus voltage of the transmission network.

4) *Generator Operational Constraints:*

$$P_g^{G,min} \leq P_{g,t}^G \leq P_g^{G,max}, \forall g \in \Omega_G, \forall t \in T \quad (24)$$

$$Q_g^{G,min} \leq Q_{g,t}^G \leq Q_g^{G,max}, \forall g \in \Omega_G, \forall t \in T \quad (25)$$

$$0 \leq P_{i,t}^{REC} \leq P_{i,t}^{REF}, \forall i \in \Omega_I, \forall t \in T \quad (26)$$

Constraints (24) and (25) enforce the upper and lower bounds of active and reactive powers of thermal generators, and (26) enforces the power curtailment limit of renewable energy stations.

5) *Transmission Load Response Constraints:*

$$0 \leq P_{i,t}^{LC} \leq P_{i,t}^{LF}, \forall i \in \Omega_I, \forall t \in T \quad (27)$$

$$0 \leq Q_{i,t}^{LC} \leq Q_{i,t}^{LF}, \forall i \in \Omega_I, \forall t \in T \quad (28)$$

$$Q_{i,t}^{LC} = P_{i,t}^{LC} \tan \phi_{i,t}, \forall i \in \Omega_I, \forall t \in T \quad (29)$$

Constraints (27) and (28) are the active and reactive power shedding constraints of aggregating loads.

6) *Transmission Power Injection Constraints:*

$$P_{ij}^{min} \leq P_{ij,t}^T \leq P_{ij}^{max}, \forall ij \in \Omega_B, \forall t \in T \quad (30)$$

$$Q_{ij}^{min} \leq Q_{ij,t}^T \leq Q_{ij}^{max}, \forall ij \in \Omega_B, \forall t \in T \quad (31)$$

Constraints (30) and (31) limit the active and reactive powers injected at the interconnection buses from the transmission side.

2.3. FFR construction algorithm

As the DSO provision FFR and the TSO desirability FFR are described with linear and SOCP constraints (1)-(16) and (17)-(31), the two FFR models in Sections 2.1 and 2.2 are both convex hulls. Although discrete, when the boundary values that satisfy constraints (1)-(16) and (17)-(31) are located and dense enough, the regions formed by connecting those discrete boundary values could approximately depict the complete DSO provision FFR and the TSO desirability FFR with high accuracy. With a number of boundary values, a piece-wise linear approximated region can be generated to represent the original convex hull. To this end, with the idea of iteratively generating new boundary values with existing ones and gradually refining the boundary of the convex hull, a fast FFR construction algorithm is proposed.

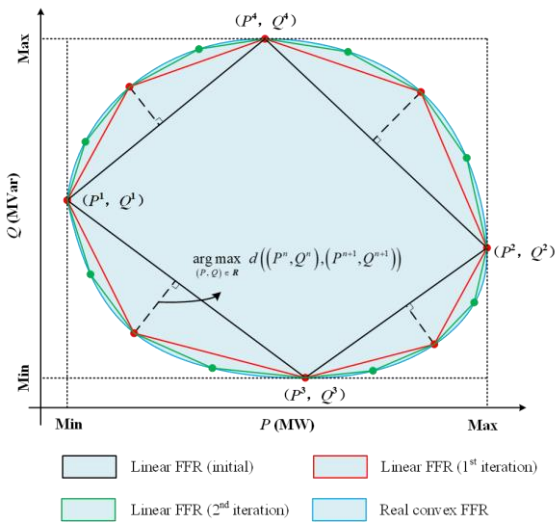


Fig. 3. Illustration of the fast FFR construction algorithm.

The steps of the proposed algorithm are as follows and illustrated in Fig. 3. This algorithm is applicable in constructing both DSO provision FFR and TSO desirability

FFR. The DSO provision FFR is taken as the example in introducing the algorithm.

Step 1: Initialize the set of boundary values as $R_0 = \emptyset$ and set the convergence threshold ε_0 .

Step 2: Obtain the initial four boundary values by solving (32) with $\alpha = \pm 1$, $\beta = 0$ and $\alpha = 0$, $\beta = \pm 1$. Correspondingly, the leftmost, rightmost, downmost and upmost operating points can be found when the values of (α, β) are set to $(1,0)$, $(-1,0)$, $(0,1)$ and $(0,-1)$. Problem (32) can be solved using the primal dual interior point method [31]. The initial four boundary values are shown in Fig. 3 and are connected with black lines to show the polygon prototype.

$$\begin{aligned} \min \quad & \alpha P_{ij,t}^D + \beta Q_{ij,t}^D, \quad ij \in \Omega_B \\ \text{s. t.} \quad & (1) - (16) \end{aligned} \quad (32)$$

In (32), α and β are two coefficients that jointly determine a searching direction [4]. With different α and β , namely different searching directions, the objective can capture the corresponding boundaries of the DSO provision FFR. In general, the DSOs will solve a series of model (32) with different α and β , to depict the boundary. However, exploring excessive boundary values to over-refine the FFR, which is computationally unfriendly and unnecessary, has been improved in our proposed method.

Step 3: Search in the clockwise direction, for any two positionally adjacent boundary values, generate a new boundary operating point by maximizing the Euclidean distance to the line connecting the two existing boundary values. The newly found boundary values are added to the set R .

Step 4: Evaluate the area of surrounded polygon after every time finding a new boundary operating point and calculate the increased area. If the increased area is larger than the threshold, repeat Step 3; Otherwise, terminate and output R .

Remark 1: The new boundary operating point generated by this algorithm always locates on the boundary of the convex hull. Different from the existing FFR construction methods, the proposed method requires no preset number of the initial boundary values but directly takes the accuracy as the final convergence threshold.

Remark 2: The counterpart of problem (32) for the TSO desirability FFR can be formulated as problem (33).

$$\begin{aligned} \min \quad & \alpha P_{ij,t}^T + \beta Q_{ij,t}^T, \quad ij \in \Omega_B \\ \text{s. t.} \quad & (17) - (31) \end{aligned} \quad (33)$$

Considering the transmission network generally interconnects with multiple distribution networks and thus the TSO indeed interact with multiple DSOs simultaneously, the flexibility ranges at the TSO-DSO interfaces are trade-off when solving this model. For a certain TSO-DSO interface, the TSO desirability FFR can be depicted in the following two steps: 1) Derive the power flexibility at the interconnection bus with the power flexibilities of the rest of transmission buses, and 2) intersect the aggregated power flexibility with the local power flexibility at the interface. In this way, the TSO desirability FFR at each interconnection bus can be separately estimated.

Finally, with the DSO provision FFR and TSO desirability FFR estimated at one TSO-DSO interface respectively, the power flexibility at this boundary should be the intersection of them considering the power injection consistency. It is worthwhile to mention that, for the DSO provision FFR, the decision variables of this optimization problem include active and reactive power outputs from DGs, BESSs and flexible

loads within the distribution network while, for the TSO desirability FFR, the decision variables are the active and reactive outputs from flexible generation-load resources in the transmission network.

3. FFR-inspired distributed decision-making solution

This section completes the TSO-DSO FFR inspired distributed TSO-DSO coordination framework. With the constructed DSO provision FFR and TSO desirability FFR in advance, the distributed TSO-DSO coordinated dispatch problem is decomposed into two subproblems respectively for the TSO and the DSO and a hierarchical algorithm is proposed to iteratively solve the two subproblems to achieve an optimal solution to the original optimal dispatch problem.

3.1. DSO optimal dispatch subproblem

The operational constraints of a distribution system have been formulated as in (1)-(16). In addition to them, the power injection consistency constraint [32], i.e., the power injected to the DSO is equal to the withdrawal from the TSO through the interconnection bus, is included. Since constraints (1)-(16) constitute the DSO provision FFR, the feasible operating points of power injections representing the distribution system flexibility can be formulated as $(P_{ij,t}^D, Q_{ij,t}^D) \in \mathbf{R}^D$. The DSO dispatch subproblem minimizes the total operation costs the flexibility resources, which comprise the following three terms:

1) DG Operation Cost:

$$Cost_{DG} = \sum_{t \in T} \left[\sum_{g \in \Omega_G} C_g^G(P_{g,t}^G) + \sum_{i \in \Omega_I} C_i^{REC}(P_{i,t}^{REC}) \right] \quad (34)$$

In (34), C_g^G and C_i^{REC} are, respectively, the production and curtailment cost functions for distributed diesel generator g and renewable energy at bus i . The production cost functions of diesel generators are considered as piece-wise linear [33].

2) BESS Operation Cost:

$$Cost_{BESS} = \sum_{t \in T} \left[\sum_{i \in \Omega_I} C_i^{ES}(P_{i,t}^{ES}) \right] \quad (35)$$

In (35), C_i^{ES} represents the cost function of the BESS connected at bus i .

3) Distributed Load Shedding Cost:

$$Cost_{DL} = \sum_{t \in T} \left[\sum_{i \in \Omega_I} C_i^{LC}(P_{i,t}^{LC}) \right] \quad (36)$$

In (36), C_i^{LC} is the cost function for shedding load at bus i .

Thus, the DSO optimal dispatch subproblem can be summarized as follows.

$$\begin{aligned} \min_{P_{g,t}^G, Q_{g,t}^G, P_{i,t}^{REC}, P_{i,t}^{ES}, P_{i,t}^{LC}, Q_{i,t}^{LC}} & Cost_{DG} + Cost_{BESS} + Cost_{DL} \\ \text{s.t.} & \begin{cases} (P_{ij,t}^D, Q_{ij,t}^D) \in \mathbf{R}^D \\ P_{ij,t}^D = P_{ij,t}^{T,*}, Q_{ij,t}^D = Q_{ij,t}^{T,*} \end{cases} \end{aligned} \quad (37)$$

3.2. TSO optimal dispatch subproblem

The operational constraints of the transmission system are formulated as in (17)-(31). Similarly, power injection consistency constraint is included. The TSO dispatch subproblem minimizes the total cost of dispatching the flexibility resources. The total cost consists of the following terms:

1) Thermal Generator Production Cost:

$$Cost_{Gen} = \sum_{t \in T} \left[\sum_{g \in \Omega_G} C_g^G(P_{g,t}^G) \right] \quad (38)$$

where, like the distributed diesel generator production cost, the quadratic generation cost function is also deployed and piece-wise linearized.

2) Renewable Energy Station Curtailment Cost:

$$Cost_{Re} = \sum_{t \in T} \left[\sum_{i \in \Omega_I} C_i^{REC}(P_{i,t}^{REC}) \right] \quad (39)$$

3) Aggregating Load Shedding Cost:

$$Cost_{AL} = \sum_{t \in T} \left[\sum_{i \in \Omega_I} C_i^{LC}(P_{i,t}^{LC}) \right] \quad (40)$$

Similarly, the powers injected at the interface from the transmission side should be equal to those from the distribution side and can be estimated in the TSO desirability FFR. Thus, the power injection consistency constraints are also added to the TSO optimal dispatch subproblem, which can be organized as in (41).

$$\begin{aligned} \min_{P_{g,t}^G, Q_{g,t}^G, P_{i,t}^{REC}, P_{i,t}^{LC}, Q_{i,t}^{LC}} & Cost_{Gen} + Cost_{Re} + Cost_{AL} \\ \text{s.t.} & \begin{cases} (P_{ij,t}^T, Q_{ij,t}^T) \in \mathbf{R}^T \\ P_{ij,t}^T = P_{ij,t}^{D,*}, Q_{ij,t}^T = Q_{ij,t}^{D,*} \end{cases} \end{aligned} \quad (41)$$

3.3. The iterative solving procedure

The power injection consistency constraints (37) and (41), are reformulated into the objective functions with augmented Lagrangian penalty coefficients as in (42) and (43).

$$\begin{aligned} Cost_{DP} &= \sum_{t \in T} \left\{ v_{ij,t,1} (P_{ij,t}^{T,*} - P_{ij,t}^D) + [w_{ij,t,1} (P_{ij,t}^{T,*} - P_{ij,t}^D)]^2 \right. \\ &\quad \left. + v_{ij,t,2} (Q_{ij,t}^{T,*} - Q_{ij,t}^D) + [w_{ij,t,2} (Q_{ij,t}^{T,*} - Q_{ij,t}^D)]^2 \right\} \end{aligned} \quad (42)$$

$$\begin{aligned} Cost_{TP} &= \sum_{ij \in \Omega_B} \sum_{t \in T} \left\{ v_{ij,t,1} (P_{ij,t}^T - P_{ij,t}^{D,*}) + [w_{ij,t,1} (P_{ij,t}^T - P_{ij,t}^{D,*})]^2 \right. \\ &\quad \left. + v_{ij,t,2} (Q_{ij,t}^T - Q_{ij,t}^{D,*}) + [w_{ij,t,2} (Q_{ij,t}^T - Q_{ij,t}^{D,*})]^2 \right\} \end{aligned} \quad (43)$$

In (42)-(43), $v_{ij,t}$ and $w_{ij,t}$ represent the linear and quadratic term multipliers of the penalty functions for power inconsistencies between TSO-DSO interconnection buses i and j at time slot t , and the subscripts 1 and 2 respectively indicate the penalty coefficients for active and reactive power inconsistencies. The quadratic penalty terms can be piece-wise linearized as in reference [1].

Finally, the distributed TSO-DSO coordinated dispatch problem can be formulated as in (44).

$$\begin{cases} \text{DSO} \begin{cases} \min_{P_{g,t}^G, Q_{g,t}^G, P_{i,t}^{REC}, P_{i,t}^{ES}, P_{i,t}^{LC}, Q_{i,t}^{LC}} Cost_{DG} + Cost_{BESS} + Cost_{DL} + Cost_{DP} \\ (P_{ij,t}^D, Q_{ij,t}^D) \in \mathbf{R}^D \\ (P_{ij,t}^{D,*}, Q_{ij,t}^{D,*}) \downarrow \uparrow (P_{ij,t}^{T,*}, Q_{ij,t}^{T,*}) \end{cases} \\ \text{TSO} \begin{cases} \min_{P_{g,t}^G, Q_{g,t}^G, P_{i,t}^{REC}, P_{i,t}^{LC}, Q_{i,t}^{LC}} Cost_{Gen} + Cost_{Re} + Cost_{AL} + Cost_{TP} \\ (P_{ij,t}^T, Q_{ij,t}^T) \in \mathbf{R}^T \end{cases} \end{cases} \quad (44)$$

In (44), $P_{ij,t}^D$ and $Q_{ij,t}^D$ are determined in the DSO dispatch subproblem given the optimal exchanging power $(P_{ij,t}^{T,*}, Q_{ij,t}^{T,*})$ from the TSO desirability FFR. Similarly, $P_{ij,t}^T$ and $Q_{ij,t}^T$ are

optimized in the TSO dispatch subproblem in the presence of the optimal exchanging power $(P_{ij,t}^{D,*}, Q_{ij,t}^{D,*})$ from the DSO provision FFR.

To solve the distributed TSO-DSO coordinated dispatch problem, a TSO-DSO FFR inspired hierarchical iterative algorithm is proposed. Given the DSO provision FFR $\mathbf{R}^D$ and TSO desirability FFR $\mathbf{R}^T$ for all time slots, the proposed algorithm consists of an inner loop and an outer loop as shown in Fig. 4. In the inner loop, the TSO and DSO dispatch subproblems are iteratively solved with invariant penalty multipliers, while in the outer loop, the penalty multipliers are dynamically updated with respect to the results from the inner loop. The steps of TSO-DSO FFR inspired hierarchical iterative algorithm are as follows.

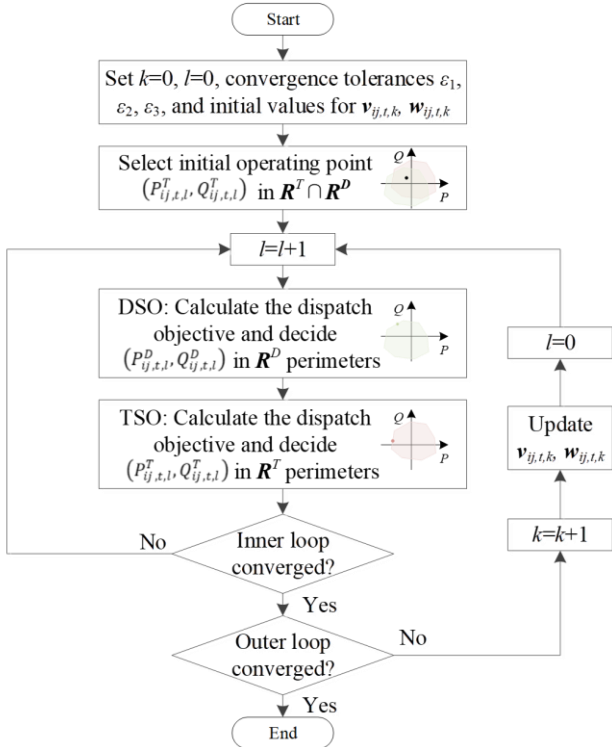


Fig. 4. Solution procedure for FFR inspired distributed TSO-DSO dispatch.

Step 1: Initialize the iteration indexes of inner and outer loops as $l=0$ and $k=0$ and set the convergence tolerance of the inner loop as ε_1 and that of the outer loop as ε_2 and ε_3 . Select the initial values of penalty multipliers $\mathbf{v}_{ij,t,k}$, $\mathbf{w}_{ij,t}$ and exchanging power $(P_{ij,t,l}^T, Q_{ij,t,l}^D)$ from the synthesized TSO-DSO FFR, i.e., $\mathbf{R}^T \cap \mathbf{R}^D$.

Step 2: Set $l=l+1$. Calculate the objective of DSO dispatch subproblem with $(P_{ij,t,l-1}^T, Q_{ij,t,l-1}^D)$ as parameters and $(P_{ij,t,l}^D, Q_{ij,t,l}^D)$ in the perimeters of DSO provision FFR $\mathbf{R}^D$ as decision variables.

Step 3: Calculate the objective of TSO dispatch subproblem with $(P_{ij,t,l}^D, Q_{ij,t,l}^D)$ as parameters and $(P_{ij,t,l}^T, Q_{ij,t,l}^T)$ in the perimeters of TSO desirability FFR $\mathbf{R}^T$ as decision variables.

Step 4: If the inner loop convergence condition (44) is satisfied, go to Step 5; otherwise, go to Step 2.

$$\left| \frac{Cost_{ij,l} - Cost_{ij,l-1}}{Cost_{ij,l}} \right| \leq \varepsilon_1 \quad (45)$$

In (45), $Cost_{ij,l}$ represents the objective function of TSO i or DSO j in iteration l .

Step 5: If the outer loop convergence condition (46) is satisfied, terminate and output the solution; otherwise, set $k=k+1$ and update the penalty multipliers $\mathbf{v}_{ij,t,k}$ and $\mathbf{w}_{ij,t,k}$ as in (47).

$$\|\mathbf{c}_{ij,t,k}\| \leq \varepsilon_2, \|\mathbf{c}_{ij,t,k} - \mathbf{c}_{ij,t,k-1}\| \leq \varepsilon_3 \quad (46)$$

$$\mathbf{v}_{ij,t,k+1} = \mathbf{v}_{ij,t,k} + 2\mathbf{w}_{ij,t,k} \circ \mathbf{w}_{ij,t,k} \circ \mathbf{c}_{ij,t,k},$$

$$\mathbf{w}_{ij,t,k+1} = \lambda \mathbf{w}_{ij,t,k} \quad (47)$$

In (47), $\mathbf{c}_{ij,t,k} = (P_{ij,t,k}^T - P_{ij,t,k}^D, Q_{ij,t,k}^T - Q_{ij,t,k}^D)$; symbol $\circ$ means the Hadamard product; and λ is the scaling coefficient. The method of updating Lagrangian penalty multipliers in the outer loop and returning to the inner loop has been proven to converge to the optimal solution [34].

Step 6: Set $(P_{ij,t,0}^T, Q_{ij,t,0}^D) = (P_{ij,t,l}^T, Q_{ij,t,l}^D)$, $l=0$, and go back to Step 2.

It is worthwhile to mention that with the generated piecewise linearized FFRs, the dispatch subproblems for TSO and DSOs are linear programming problems that can be efficiently solved with commercial solvers. In fact, since the DSO provision FFR $\mathbf{R}^D$ and the TSO desirability FFR $\mathbf{R}^T$ are indeed polygons, the optimal dispatch solutions to the TSO and the DSO should locate on the boundary of $\mathbf{R}^D \cap \mathbf{R}^T$. In the presence of the boundary polygon, the dispatch subproblems can be converted as unconstrained by simply traversing them to identify the optimal dispatch solutions respectively for TSO and DSOs.

4. Case studies

Two test systems, i.e., T24-D3 and T118-D30, are designed to validate the performance of the proposed method. All numerical simulations are conducted using a 64-bit laptop with 1.80 GHz CPU and 8 GB RAM and optimization problems are solved with CPLEX 12.6.3.

4.1. Simulation description

The T24-D3 system consists of an IEEE 24-bus transmission network and three IEEE 33-bus distribution networks. Three wind farms are allocated at transmission buses 3, 10, and 19 with the identical capacity of 500 MW. The characteristics of distributed wind generators, solar generators, and loads in various distribution networks are different, as shown in Table 1, and the hourly predicted powers of distributed wind generators, solar generators and loads in three DSOs are shown in Fig. 5. The remaining parameters are set referring to reference [35].

The T118-D30 system consists of an IEEE 118-bus transmission network connected with 30 IEEE 33-bus distribution networks. DSO1s are added to transmission buses 7, 18, 19, 20, 27, 28, 29, 33, 34 and 35, DSO2s are installed at transmission buses 39, 53, 57, 58, 70, 74, 75, 76, 77 and 84, and DSO3s are added to transmission buses 90, 102, 104, 105, 106, 107, 108, 109, 112 and 118. In the transmission system, ten wind farms with the installed capacity of 250 MW are connected at buses 6, 18, 32, 42, 56, 70, 77, 99, 113, and 116. The parameters of thermal generators are given in [36]. The convergence tolerances of FFR construction and hierarchical loop iteration algorithms are set as 0.001, 0.01, 0.001 and 0.001.

Table 1 Parameters of renewable generators and loads of three DSOs in T24-D3

DSO No.	Wind/MW	Solar/MW	Peak Load/MW
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1	87	87	150
2	40	100	105
3	172	86	233

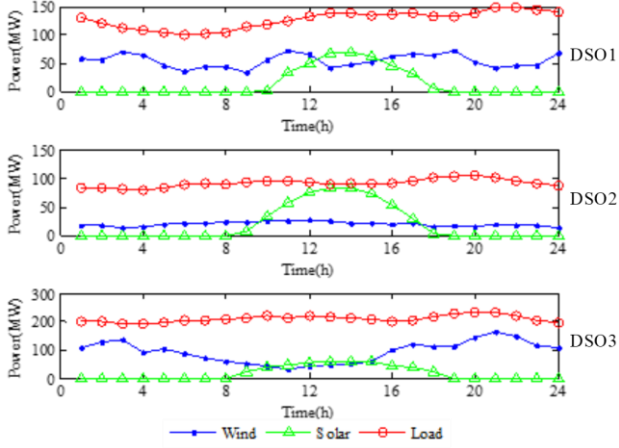


Fig. 5. Predicted powers of renewable energy and load demand in T24-D3 distribution levels.

4.2. Solution of FFR inspired distributed TSO-DSO dispatch

In order to verify the exactness of the proposed method, we compare the cost results of the conventional centralized and FFR inspired dispatch methods in Table 2. Note that, the centralized method cannot be deployed in practice due to information exchanging limitation between the TSO and the DSOs, but its solution can serve as the benchmark of the global optimum. As shown in Table 2, for both cases, the gaps of the results with the two methods are relatively small. The proposed method can yield a solution with a gap less than 0.2% to the optimal solution from the TSO, the DSO, and the whole system perspectives. This demonstrates the exactness of the FFR inspired dispatch method.

Table 2 Cost results of centralized and proposed methods

Case	Method	TSO Cost/\$	DSO Cost/\$	Total Cost/\$
T24-D3	Centralized	544512	1542	546054
	Proposed	543912	1542	545454
T118-D30	Centralized	1043996	3416	1047412
	Proposed	1042766	3414	1046180

Fig. 6 shows the hourly dispatch results for the TSO and the DSOs in T24-D3 system. Although the TSO injects powers to DSOs at most hours of the typical day, it also purchases powers from DSOs in the middle of the day, i.e., hours 11-12 and 14-17 for DSO1, and hours 11-16 for DSO2. Since the maximum capacity of distributed solar generation in DSO3 is smaller than that in DSO1 and DSO2, DSO3 always purchases powers from the TSO during the day. Within the transmission system, as shown in Fig. 6(a), powers generated by the cheap thermal generators 2 and 4 regulate with uncertainties related to wind farm output and aggregating load variation. At hour 9, the aggregated load is shed when the output of wind farms becomes small, and the power generated by thermal generators becomes insufficient. In DSO1, as shown in Fig. 6(b), since the power generation from DGs and BESSs is insufficient, the distributed loads are shed at hours 16-18. Moreover, BESS is charged at hours 1, 5, 20, 21 and 24, and discharged at peak hours 11 and 14-18. In DSO2, as shown in Fig. 6(c), the dispatchable DGs are OFF at hours 13 and 14 because of the redundant solar energy at noon. The distributed loads are shed at peak hours 18-21 to

balance powers of DGs and BESSs. Besides, BESSs transfer renewable energy by charging at hours 1-7 and 23-24 and discharging at hours 9-10 and 16-22. In DSO3, as shown in Fig. 6(d), although BESSs discharge at hours 19-21, the distributed loads are shed due to the limited powers generated by DGs.

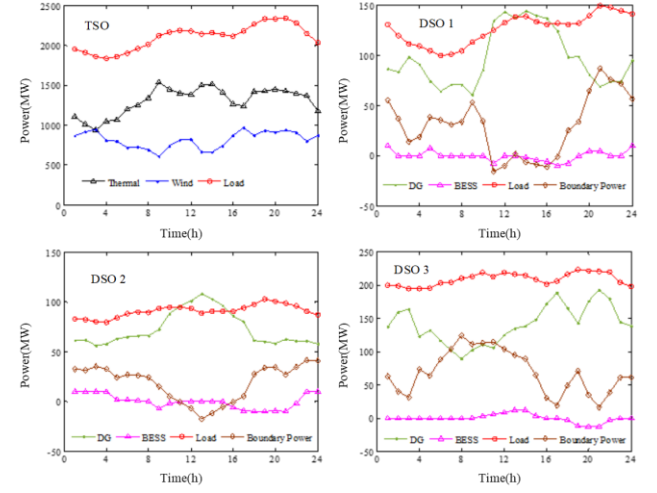


Fig. 6. Results of the proposed dispatch for T24-D3.

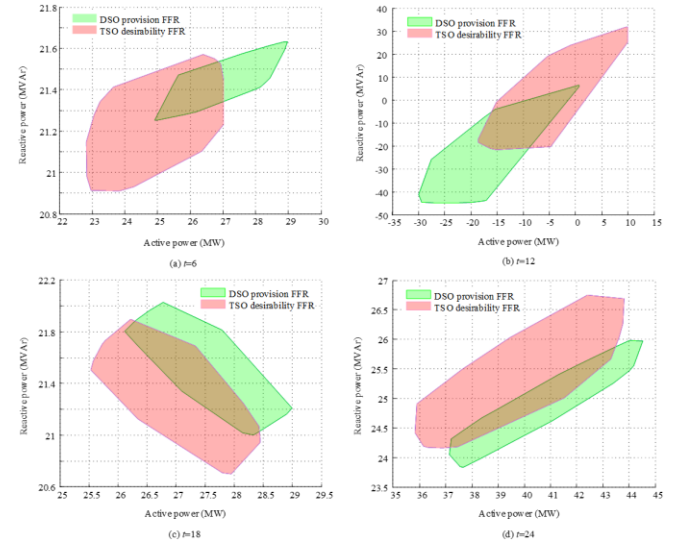


Fig. 7. FFRs at the TSO and DSO2 interface at hours (a) 6, (b) 12, (c) 18, and (d) 24.

Fig. 7 illustrates the active and reactive power FFRs at the interface of TSO and DSO2 at different hours. The FFRs estimated are depicted from the perspective of the TSO and thus positive values indicate that the TSO injects power to the DSO. Similarly, negative values mean that the DSO injects power to the TSO. When the power through the interconnection bus increases, the net loads of the DSO and the TSO respectively increase and decrease. As shown in Fig. 7, FFRs at the interface of TSO and DSO2 are time-variant, contributed by uncertainties of renewable energy and load fluctuation. TSO desirability FFR and DSO2 provision FFR have a common region at each hour, and boundary powers in this region can be negative at hour 12 since numbers of distributed solar generators are added to DSO2 and outputs of them are redundant at noon. In fact, this common region provides the feasible area for determining the consistent powers injected at the TSO-DSO interconnection bus. As the TSO desirability FFR and DSO provision FFR are depicted

by SOCP relaxed problems, they are naturally convex and compact.

4.3. Comparison of different dispatch methods

The results with isolated dispatching and the proposed dispatch method are compared in Table 3 to validate the performance of the FFR inspired distributed TSO-DSO coordination. With the isolated dispatching, TSO and DSO are not aware of the flexibility resources of each other, and it is difficult for them to accurately estimate the boundary mutually injected powers. Thus, flexibility resources in both sides are not likely to be appropriately dispatched. In Table 3, S1 represents the typical wind-solar-load scenarios and S2 indicates the critical ones. As seen from this table, the solutions with the proposed dispatch method are more economical than the ones with the isolated dispatching in both cases. For the case with T118-D30, the total cost of the proposed method in scenario S2 is \$2474 and \$4649 less than

that of the isolated method in scenarios S1 and S2, respectively. By coordinating the flexibility resources in both TSO and DSOs, the proposed method refrains from overusing or underutilizing the local generators, thus cost-effectiveness performs better. For both cases in scenario S1, the DSO cost of the proposed method is higher than that of the isolated method. This is because, with TSO-DSO interaction, more DGs are accommodated to achieve local power balance in DSOs. By considering differential source-load scenarios, the proposed method outperforms the isolated one, with both the maximum and average load shedding significantly reduced. Taking Case T24-D3 as an example, the average load shedding of the proposed method is 81.15% less than that of the isolated method in scenario S2. Besides, the maximum load shedding is higher in scenario S2 because the net load is heavy in scenario S2, which causes more congestion in either transmission or distribution network.

Table 3 Dispatch results of isolated and proposed methods

Case	Method		Max Load Shedding/MW	Ave Load Shedding/MW	TSO Cost/\$	DSO Cost/\$	Total Cost/\$
T24-D3	Isolated	S1	126.96	14.06	553554	529	554083
		S2	289.99	39.41	591689	3029	594718
	Proposed	S1	12.65	0.96	543912	1542	545454
		S2	27.84	7.43	546761	1628	548389
T118-D30	Isolated	S1	200.14	22.24	1050579	440	1051019
		S2	380.17	30.77	1046426	6768	1053194
	Proposed	S1	2.52	0.18	1042766	3414	1046180
		S2	25.63	2.07	1045157	3388	1048545

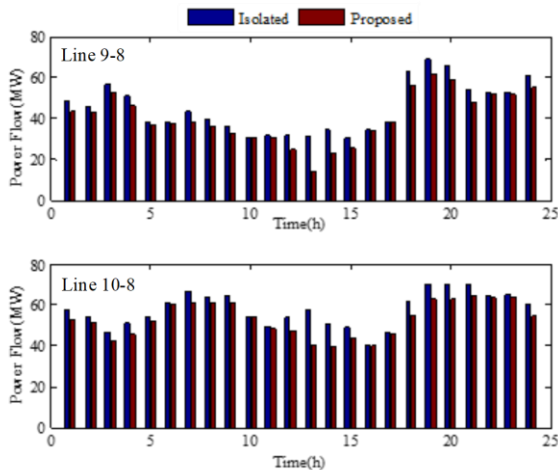


Fig. 8. Comparison of line flow between isolated and proposed methods.

By integrating DGs and BESSs for supplying distributed loads, with the proposed method, DSOs can contribute to relieving the transmission line congestion in contrast with the isolated method. Fig. 8 shows the line flows on lines 9-8 and 10-8 in Case T24-D3. It can be shown that the power flows obtained by the proposed method are significantly less than those of the isolated method at hours 12-15 when the load is relatively heavy. More distribution loads are supplied by the local DGs at these time periods due to the high transmission locational marginal price at the TSO-DSO interconnection bus, thus boundary powers interacted between transmission and distribution networks are reduced. The maximum power flow deviations of lines 9-8 and 10-8 achieved by the proposed method are respectively 55% and 30% less than those of the isolated method, thus the line congestion is mitigated.

4.4. Effect of identifying TSO-DSO FFR

To further investigate the performance of the proposed FFR-inspired dispatch method, the iteration number, computation time and optimization cost of different distributed optimization methods, such as APP, ADMM and ATC, are compared in Table 4. Besides, the optimization gap of these methods is calculated to evaluate their computation accuracy.

Table 4 Comparison of different iteration solution methods

Case	Comparison Item	Method			
		APP	ADMM	ATC	Proposed
T24-D3	Iteration $l/(k,l)$	30	23	(4,11)	(2,6)
	Computation time/s	194.75	96.23	94.71	19.83
	Total cost/\$	547625	547244	547187	545454
	Optimization gap	0.29%	0.22%	0.21%	0.11%
T118-D30	Iteration $l/(k,l)$	45	28	(6,32)	(4,9)
	Computation time/s	1098.88	769.34	579.66	89.34
	Total cost/\$	1050286	1050275	1049577	1046180
	Optimization gap	0.27%	0.27%	0.21%	0.12%

As observed from Table 4, for Case T24-D3, the proposed method converges after 2 outer loops and 6 inner loops when the boundary power inconsistencies between two outer loops are relatively small, and the total computation time is 19.83 seconds. While for the ATC using the same penalty coefficients as the proposed method, this method converges in 94.71 seconds after 4 outer loops and 11 inner loops. In contrast, the proposed method achieves a much better convergence property. This is because the proposed method utilizes the estimated FFR in the first stage to provide the

feasible boundary powers for each inner loop iteration. Compared with the APP and ADMM based methods, the proposed method also shows its superiority in terms of computational efficiency. Furthermore, from the accuracy perspective, the proposed method obtains the minimum optimization gap among other methods. For Case T118-D30, the proposed method converges after 4 outer loops and 9 inner loops, and the total computation time is 89.34 seconds, which performs best in computation time compared with other methods. Moreover, the proposed method yields solution with gap less than 0.2%, thus good enough to be utilized.

5. Conclusion

This paper presents a two-stage distributed optimal dispatch method with which the first stage estimates the FFR at the TSO-DSO interface and the second stage determines the FFR-inspired dispatch solution of flexibility resources in TSO and DSOs, with the boundary powers in the pre-estimated FFRs being selected as the shared variables. Case studies show the computation advantages and high dispatch decision-making performance of the proposed method. The FFR-inspired distributed dispatch method offers acceptable model-solving time for day-ahead dispatch independent of case scale, which improves the convergence ability of existing distributed optimization algorithms.

Future research will focus on the FFR-driven real-time control of TSO and DSOs to improve the whole system flexibility. Moreover, network flexibility, such as topology optimization and structure reconfiguration, can be considered into the distributed decision-making framework.

Acknowledgments

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