

The total accommodation capability curve for a distribution network considering N-1 criterion

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ABSTRACT

The total accommodation capability curve (TAC curve) can completely describe the DG accommodation capability of a distribution network. The urban distribution network generally adopts the security constraints under N-1 criterion, but the formulation of the TAC curve in the existing studies only considers the network constraints under normal operation conditions considering N-0 criterion. To fill this gap, this paper develops a TAC curve model considering security constraints under N-1 criterion in distribution networks. The model is based on alternating current (AC) power flow because the problem of voltage override after DG integration cannot be ignored. Then, the solution and plotting method of the TAC curve are proposed. The method is based on the DC power flow model combined with voltage calibration correction, which is easy to solve and accurate. Finally, test systems are used to verify the proposed model and method. The rules between the N-1 TAC curve and the N-0 TAC curve and the main factors affecting the TAC curve are analyzed. The application of the TAC curve to planning is also provided.

1. Introduction

The future distribution network will face great challenges from low carbon requirements. The electrification of traffic and heat is expected to happen in the distribution network, which will greatly increase the peak power demand. Apart from the power supply from the main grid, low carbon distributed generation (DG) such as wind turbines and photovoltaic panels are increasingly integrated into the distribution network. However, the high penetration of DG will bring great challenges to the secure operation of distribution networks. In particular, it may cause reverse power flows [1] and the node voltage violations [2] in distribution networks. Due to the density of buildings in urban distribution networks, DG often needs to be installed in a high density in a limited space, resulting in the impact of a high penetration of DG integration on urban distribution network being particularly obvious. Hence, the research on the complete and accurate evaluation of the maximum allowable power output of DG in a distribution network has attracted attention [1].

Existing studies generally use "accommodation capability" to characterize the ability of a distribution network to integrate DG, which is

defined as the maximum allowed DG power output under certain security constraints of the distribution network [3]. There are two main methods to calculate the accommodation capability of a distribution network: the optimization-based method and the power flow-based method. The optimization-based method obtains the accommodation capability of DG through establishing the optimization models [4,5]. However, due to the complexity of the methods, the optimization-based methods are difficult to formulate a unified accommodation capability model under different DG integration scenarios. The power flow-based method obtains the DG accommodation capability by gradually increasing the DG output and examining the security constraints via power flow analysis. Following the process, the maximum DG output that does not violate the security constraints of the network is finally obtained as the system accommodation capability [6,7]. However, these studies did not consider the impact of the connected location of the load/DG on the results. The optimization-based method and power flow-based method are accurate, but their results normally reflect the accommodation capability of DG under limited selected scenarios, which cannot describe the overall picture of the accommodation capability of the distribution network.

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The total supply capability curve (TSC curve) based on the distribution system security region (DSSR) in [8] provides an alternative to address this problem. DSSR is a collection of all operating states in a distribution network that meet security constraints under certain criterion such as N-0 or N-1 criterion [9]. All operating states within the boundaries of the DSSR are secure, while the operating states outside the boundaries are insecure. The TSC curve consists of the load values at all security boundary points of the DSSR. A security boundary point is a critical security point of a distribution network on a certain load distribution [8]. Therefore, the TSC curve fully considers all load distribution scenarios. Literature [10] further implemented the security region methodology to active distribution networks with high DG penetration. On this basis, literature [11] proposes the total accommodation capability curve (TAC curve), which is composed of the sum of DG output of all DSSR security boundary points. A security boundary point is the critical security point of a distribution network on a certain DG and load distribution [11]. Therefore, the TAC curve completely describes all DG and load distribution scenarios. However, the existing TAC curve only considers the operation constraints under normal conditions (i.e. security constraints under N-0 criterion) and does not consider the security constraints under N-1 criterion.

Constraints under N-1 criterion are considered in distribution networks, particularly for urban distribution networks with high reliability requirements. The N-1 criterion ensures that the distribution network can still operate normally when one component of a distribution network fails [12]. Consequently, it is important to ensure the reliable power supply to the consumers. However, the consideration of N-1 criterion will significantly increase the amount of computation. To our knowledge, only literature [13] and [14] have studied the accommodation capability of distribution network to DG under N-1 criterion. Literature [13] proposes a model for a novel concept of maximum capability boundary (MCB), which establishes a new version of AC power flow constraints by reconstructing the admittance matrix considering the N-1 criterion. However, this model is complex since the calculation of this model is based on AC power flow constraints and cannot consider all scenarios of DG integration. Literature [14] considers the N-1 criterion and adopts a hybrid algorithm to calculate the accommodation capability of the system. The algorithm continuously reduces the solution space through the mixed Benders decomposition and dichotomy, and then alternately iterates until the optimal solution is obtained. However, this algorithm is computationally complex, easy to fall into local optimal solution, and cannot fully describe the accommodation capability considering N-1 criterion.

In summary, the existing TAC curve does not consider N-1 criterion, whereas the conventional accommodation capability model considering N-1 criterion cannot fully describe the accommodation capability. In order to solve these problems, this paper proposes a TAC curve model that considers N-1 criterion. The main contributions are as follows:

- 1) A security region-based TAC curve considering N-1 criterion is innovatively developed. It can describe the whole picture of the capability of distribution networks for DG accommodation. The proposed TAC curve can be used in DG/load access planning, distribution network wire replacement planning and operation of distribution networks, which can improve the capability of distribution network to accommodate DG.
- 2) A method for solving the TAC curve is established. For high computational efficiency, the method firstly uses the DC power flow model to obtain the DG security boundary, which is formed by the critical values of DG output that just breach the boundary. AC power flow simulation is then used to verify and correct the boundary points for the formulation of the TAC curve.
- 3) Two rules of TAC curve are found. Firstly, it shows that N-1 TAC curve is lower than the N-0 TAC curve or remains unchanged. Secondly, it is found that the accommodation capability reflected in the

TAC curve is positively correlated with the downstream load of the bottleneck feeder segment.

The rest of this paper is organized as follows. Section II introduces the concepts and models of security regions, and security boundaries of distribution network which refers to the boundary formed by operating states with critical security. Section III establishes the TAC curve model considering N-1 criterion. Section IV further provides the method of solving the TAC curve model. Section V validates the methods through case studies and discusses the role of the TAC curve in the planning and operation of distribution networks. Section VI concludes this paper.

2. Preliminaries

The section first provides the security criteria in distribution networks, whereas the concepts of security regions and security boundaries, which are obtained based on these security criteria, are introduced subsequently. These concepts are important for modelling and plotting the TAC curve in Section III and IV.

2.1. Security and security constraints for distribution network

The TAC curve considering N-1 security criterion needs to meet both the N-0 and N-1 security criteria. Before going through the N-0 and N-1 criteria in distribution networks, the definitions of the operating state and the state space for distribution networks are first provided for clarity.

2.1.1. Operating state and state space

The operating state is a set of independent variables that can uniquely determine the system state in the security analysis. The operating state \mathbf{W} is then expressed as:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_L \\ \mathbf{W}_{DG} \end{bmatrix} = \begin{bmatrix} [S_{L1}, S_{L2}, \dots, S_{Lx}, \dots, S_{Ln}]^T \\ [S_{DG1}, S_{DG2}, \dots, S_{DGy}, \dots, S_{DGm}]^T \end{bmatrix} \quad (1)$$

where \mathbf{W} can be divided into two parts: \mathbf{W}_L , which summarizes the power load connected at different nodes, and \mathbf{W}_{DG} , which represents all the DG injections. For conciseness, the nodes connected with power load are defined as load nodes, while the nodes with DG generation are defined as DG nodes. In (1), S_{Lx} is the outflow power of the x th load node, and n is the number of load nodes; S_{DGy} is the injected power of the y th DG node, and m is the number of DG nodes. The power at the load or DG node is all based on the apparent power amplitude, which stipulates that the power flowing from the grid to the load is positive, and the negative sign only indicates the opposite direction. Appendix A presents a list of symbols for this paper.

The operating state of the distribution network varies with different nodal power load/generation. The state space Θ is defined as the set of all possible operating states, which is expressed as:

$$\Theta = \left\{ \mathbf{W} \mid \begin{array}{l} 0 \leq S_{Lx} \leq S_{Lx}^{\max}, \forall 1 \leq x \leq n \\ -S_{DGy}^{\max} \leq S_{DGy} \leq 0, \forall 1 \leq y \leq m \end{array} \right\} \quad (2)$$

where the superscript max represents the maximum value.

2.1.2. N-0 criterion and constraints under N-0 criterion

N-0 criterion requires that under the normal operation of the distribution network, the power flows through the power lines and the substation transformers do not exceed their capacities, and the voltage is sustained within a reasonable range.

The N-0 criterion of the operating state \mathbf{W} during normal operation of the distribution network is:

$$\begin{cases} \Delta \mathbf{V} \in \mathbf{C}_v \\ [\mathbf{S}_B, \mathbf{S}_T] \in \mathbf{C}_c \end{cases} \quad (3)$$

where C_v is the node voltage variation constraints; $\Delta V = [\Delta V_1, \Delta V_2, \Delta V_r, \dots, \Delta V_{n+m}]$ is the vector of the node voltage offset; C_c is the component capacity constraints which consider the bi-directional power flows; $S_B = [S_{B,1}, S_{B,2}, \dots, S_{B,j}, \dots, S_{B,l}]$ is the vector of the magnitude of the line flows; $S_T = [S_{T,1}, S_{T,2}, \dots, S_{T,i}, \dots, S_{T,p}]$ is the vector of the power flows through the substation transformers.

2.1.3. N-1 criterion and constraints under N-1 criterion

N-1 criterion refers to when a single component fails, the system isolates the fault through switching operation, makes the faulty component out of operation, and transfers the non-fault segment to meet the capacity and voltage constraints of the component after the transfer.

This paper only considers the faults on power lines. Since the research target of is to calculate the accommodation capability of the distribution network, the fault component does not include DG.

The N-1 criterion of distribution network operating state \mathbf{W} is:

$$\begin{cases} \Delta V(k) \in C_v^{N-1} \\ [S_B(k), S_T(k)] \in C_c^{N-1} \end{cases} \quad (4)$$

The components that quit operation due to their faults or system faults is represented as ψ_k ; where $\Delta V(k)$ represents the vector of the node voltage offset after ψ_k quitting operation; C_v^{N-1} is the node voltage variation constraints considering N-1 criterion; $S_B(k)$ represents the vector of the line flow after ψ_k quitting operation; $S_T(k)$ represents the vector of the power flow through the substation transformer after ψ_k quitting operation; C_c^{N-1} is the component capacity constraints considering N-1 criterion.

2.2. Security region and security boundary for distribution network

The accommodation capability curve is plotted by the strict security boundary points of the distribution system security region, so it is necessary to introduce the security region and security boundary of the distribution network.

2.2.1. Distribution network security region

The distribution system security region is the set of all operating states in the state space that meet the security constraints [15]. After DG is integrated into the distribution network, if the load of the distribution network is approaching its maximum carrying capacity, N-1 criterion sometimes might be looser than N-0 criterion. Therefore, the N-1 security region model needs to consider both N-0 and N-1 criterion [10].

Currently, the distribution network security region is mainly applied to urban distribution networks. In the urban distribution network, capacity violation will lead to serious faults. Compared with AC power flow, DC power flow can highlight the main contradiction in the security of capacity constraints [9], so the DC power flow model is adopted. The mathematical model of the distribution network security region is shown in Eq. (5).

$c_{B,j}$ is the capacity of the power lines, \mathbf{B} is the set of all power lines; r is the network loss coefficient; $\Omega_{B,j}^L$ and $\Omega_{B,j}^{DG}$ respectively represent the set of all load nodes and DG nodes downstream of line j in normal operation; $S_{B,j}$ is the vector of the power flow on the j th line in normal operation; Ω_{DSSRN1}^{Bj} is the security sub-region after the j th feeder segment fails; $S_{B,j}(k)$ represents the vector of the power flow on the j th line after ψ_k quitting operation; Ψ is contingency set, $\Psi = \{\psi_1, \dots, \psi_k, \dots, \psi_{n_f}\}$; $\Omega_{B,j(k)}^L$ and $\Omega_{B,j(k)}^{DG}$ respectively represent the set of downstream load nodes and DG nodes of line j after ψ_k quitting operation.

Ω_{DSSR} indicates the distribution system security region considering N-1 criterion, which is constructed by taking the intersection of the security region considering N-0 criterion Ω_{DSSRN0} and security region only considering N-1 criterion Ω_{DSSRN1} ; Ω_{DSSRN1} is constructed by taking the intersection of the security sub-region Ω_{DSSRN1}^{Bj} .

Ω_{DSSRN1} considers the following operations after a fault occurs: After isolating the faulted feeder segment, the service to the upstream of a faulted feeder segment will be restored by opening the first switchable component upstream to the faulted segment, and the service to the downstream will be restored through back feed via closing a normally open tie switch. $-c_{B,j} \leq S_{B,j}(k) = (1+r) \left(\sum_{x \in \Omega_{B,j(k)}^L} S_{Lx} + \sum_{y \in \Omega_{B,j(k)}^{DG}} S_{DGy}(k) \right) \leq c_{B,j}$ in Eq. (5) indicates that after the transfer, the downstream load and DG of the feeder segment must meet the feeder capacity constraints, which reflects the operation state after the operation of the feeder fault is completed.

For the convenience of calculation, given the matrix form of Eq. (5), as shown in Eq. (6):

$$\Omega_{DSSR} = \{ \mathbf{W} | -\mathbf{c}^{N-1} \leq (1+r) [\mathbf{A}_L^{N-1}, \mathbf{A}_{DG}^{N-1}] \times \mathbf{W} \leq \mathbf{c}^{N-1} \} \quad (6)$$

where \mathbf{A}_L^{N-1} is a load coefficient matrix considering both N-0 and N-1 security constraints; \mathbf{A}_{DG}^{N-1} is a DG coefficient matrix considering both N-0 and N-1 security constraints. \mathbf{c}^{N-1} is a constant vector formed by the component capacity value considering both N-0 constraints and N-1 security constraints.

2.2.2. Distribution network security boundary

The security boundary consists of boundary points with critical security [16]. Critical security, referred to as criticality, means that for a secure operating state, there is at least one load increase, the new operating state will be insecure.

Depending on whether strict criticality is met, security boundaries can be divided into strict and non-strict security boundaries [16]. The accommodation capability curve points are composed of strict security boundary points [17].

Strict criticality in the distribution network refers to the condition where, for a certain secure operating state, any load increase alone causes the system insecure [18]. The mathematical description of strict criticality is:

$$\Omega_{DSSR} = \left\{ \begin{array}{l} \Omega_{DSSRN0} \cap \Omega_{DSSRN1} \\ \Omega_{DSSRN0} = \left\{ -c_{B,j} \leq S_{B,j} = (1+r) \left(\sum_{x \in \Omega_{B,j}^L} S_{Lx} + \sum_{y \in \Omega_{B,j}^{DG}} S_{DGy} \right) \leq c_{B,j}, \forall j \in \mathbf{B} \right\} \\ \Omega_{DSSRN1} = \Omega_{DSSRN1}^{B1} \cap \Omega_{DSSRN1}^{B2} \cap \dots \cap \Omega_{DSSRN1}^{Bj} \cap \dots \cap \Omega_{DSSRN1}^{Bl} \\ \Omega_{DSSRN1}^{Bj} = \left\{ -c_{B,j} \leq S_{B,j}(k) = (1+r) \left(\sum_{x \in \Omega_{B,j(k)}^L} S_{Lx} + \sum_{y \in \Omega_{B,j(k)}^{DG}} S_{DGy}(k) \right) \leq c_{B,j} \right\} \\ \forall j \in \mathbf{B} - \psi_k, \forall \psi_k \in \Psi \end{array} \right\} \quad (5)$$

An operating state $\mathbf{W} = [S_1, S_2, \dots, S_n]^T$ is located on the security boundary Ω_{DSSR} , i.e. $\mathbf{W} \in \Omega_{DSSR}$. The j load is increased to form a new point $W = [S_1, \dots, S_j + \varepsilon, \dots, S_n]^T$, where ε is an arbitrarily small positive number. If for $\forall \varepsilon > 0$ and $j \in \{1, 2, \dots, n\}$, there is $W \notin \Omega_{DSSR}$, then the operating state \mathbf{W} is strictly critical.

The strict security boundary (referred to as the security boundary in the rest of the paper) is defined as the set of all operating states with strict criticality [16]. Similar to TSC [19], TAC (which describes the accommodation capability of the distribution network) should select a strict boundary where any increase or reduction in DG output would cause system insecurity.

2.2.3. DG security boundary

In the security boundaries, the boundaries formed by the critical value of DG output crossing the boundaries are termed as DG security boundaries [11]. DG security boundary can be further classified into upper DG boundary and lower DG boundary.

The upper DG boundary consists of all operating states with DG growth criticality [20]. DG growth criticality means that any DG growth in output will cause the system to violate security constraints. Its mathematical definition is:

$$\begin{cases} \mathbf{W} \in \Omega_{DSSR} \\ \mathbf{W}' = [\mathbf{W}_L, S_{DG1}, \dots, S_{DGy} + \varepsilon, \dots, S_{DGm}] \notin \Omega_{DSSR} \\ \forall y \in \{1, 2, \dots, m\}, \lim_{\varepsilon \rightarrow 0^+} \end{cases} \quad (7)$$

where m is the number of DG nodes, \mathbf{W}_L is the load part of \mathbf{W} , and S_{DGy} is the injected power at the y DG node. Given a secure operating state \mathbf{W} with the security region, \mathbf{W}' represents a new operating state formed by adding a small value ε to one DG power injection in \mathbf{W} (e.g. S_{DGy} in (7)). This formula shows that if any new operating state \mathbf{W}' generated in this way lies outside the security region, \mathbf{W} has strict DG growth criticality.

In the same way, the lower DG boundary comprises all operating states with DG reduction criticality [20]. This content will not be repeated here.

3. The TAC curve model

The TAC curve describes the capability of a distribution network to accommodate DG, that is, the maximum allowable DG output. In previous study [11], the TAC curve under N-0 criterion was proposed. Referring to the formation of the TAC curve under N-0 criterion in [11], this section formulates the TAC curve considering N-1 criterion. Specifically, the points on the TAC curve are formed by DG security boundary points. By using the serial number regarding the DG security boundary points as the horizontal coordinate and the total DG output at the boundary point as the vertical coordinate, the total accommodation capability curve is formed in order from small to large, which is termed as the TAC curve for brevity.

As stated in the last section, the DG security boundary can be classified into the upper DG boundary and the lower DG boundary. Based on their boundary points, the upper TAC curve and the lower TAC curve can be modeled respectively. In this study, the TAC curve models are obtained based on AC power flow, which are detailed as follows.

3.1. The upper TAC curve model

The serial number of the upper DG boundary points is taken as the horizontal coordinate, and the total DG output of the boundary points is taken as the vertical coordinate. The curves are plotted in order from small to large. The mathematical model of the upper TAC curve considering the N-1 criterion is shown in Eq. (8).

$$C_{TAC^+}^{N-1} = \left\{ \begin{array}{l} |\text{Val}(\mathbf{W}_{DG\alpha})| \leq |\text{Val}(\mathbf{W}_{DG(\alpha+1)})| \\ -\mathbf{c}_e^{N-1} = (1+r)[\mathbf{A}_{Le}^{N-1}, \mathbf{A}_{DGe}^{N-1}] \times \mathbf{W}_a \\ -\mathbf{c}_{ne}^{N-1} < (1+r)[\mathbf{A}_{Lne}^{N-1}, \mathbf{A}_{DGne}^{N-1}] \times \mathbf{W}_a \leq \mathbf{c}_{ne}^{N-1} \\ \|\mathbf{A}_{DGe}^{N-1}\| < 0 \\ \mathbf{S}_{L,\min} \leq \mathbf{W}_{La} \leq \mathbf{S}_{L,\max} \\ \Delta \mathbf{V}_{\min} \leq \mathbf{A}_u \times \mathbf{W}_a \leq \Delta \mathbf{V}_{\max} \\ \Delta \mathbf{V}_{\min} \leq \mathbf{A}_u(k) \times \mathbf{W}_a \leq \Delta \mathbf{V}_{\max}, \forall \psi_k \in \Psi \\ a \in \{1, 2, 3, \dots\} \end{array} \right. \quad (8)$$

where α is the serial number of a boundary point. $|\text{Val}(\mathbf{W}_{DG\alpha})|$ represents the total DG output of a boundary point \mathbf{W}_a . \mathbf{A}_{Le}^{N-1} , \mathbf{A}_{DGe}^{N-1} and \mathbf{c}_e^{N-1} are equal load coefficient matrix, DG coefficient matrix, and constant vector considering N-0 and N-1 criterion, respectively. \mathbf{A}_{Lne}^{N-1} , \mathbf{A}_{DGne}^{N-1} and \mathbf{c}_{ne}^{N-1} are respectively without equal consideration of load coefficient matrix, DG coefficient matrix, and constant vector considering N-0 and N-1 criterion. $\|\mathbf{A}_{DGe}^{N-1}\|$ means that the largest value among the sums of each column of the matrix is selected. Since the elements in the matrix are all greater than or equal to 0, $\|\mathbf{A}_{DGe}^{N-1}\| < 0$ is used to indicate that every column in \mathbf{A}_{DGe}^{N-1} has an element that is not 0, that is, the equality constraints cover all DG variables. $\mathbf{S}_{L,\min}$ is the vector of node valley load, and $\mathbf{S}_{L,\max}$ is the vector of node peak load. $\mathbf{S}_{L,\min} \leq \mathbf{W}_{La} \leq \mathbf{S}_{L,\max}$ means that the node load should be between the peak and valley of the node load curve. \mathbf{A}_u is the voltage offset coefficient matrix in normal operation, $\mathbf{A}_u(k)$ is the voltage offset coefficient matrix after the component ψ_k quitting operation, and the determination method of the elements in the two matrices is shown in Appendix C. $\Delta \mathbf{V}_{\min}$ and $\Delta \mathbf{V}_{\max}$ refer to the lower voltage limit and the upper voltage limit, respectively.

The upper TAC curve describes the DG accommodation capability of the distribution network that meets the N-1 criterion with various distributions of load and DG, that is, the maximum allowable DG output. The fluctuating characteristics of uncontrolled DG do not affect the TAC curve because the model considers the range of DG output variation.

3.2. The lower TAC curve model

Similar to the upper TAC curve model, the lower TAC curve model is established. See Appendix B for the specific process.

The existing distribution network planning principle requires that feeder load does not exceed the feeder capacity, and all loads are completely powered by substations. Thus, the lower TAC curve in the existing distribution network is generally constant at 0, this paper focuses on the upper TAC curve.

4. The TAC curve model solution and plotting method

The TAC curve model is solved and plotted based on a sampling algorithm [8]. The DG boundary is continuous, so the number of boundary points is infinite. A finite number of points representing the whole curve is generated and sorted through sampling, avoiding randomness and disorder of the acquisition of boundary points. The TAC curve solution and plotting process are shown in Fig. 1. The detailed calculation process is described in Appendix C.

5. Case study

5.1. Simple test systems

Single tie-line is the most basic structure of distribution network, and it is the basis to explore the distribution network with more complicated contact relations and more diversified load transfer paths. To study the

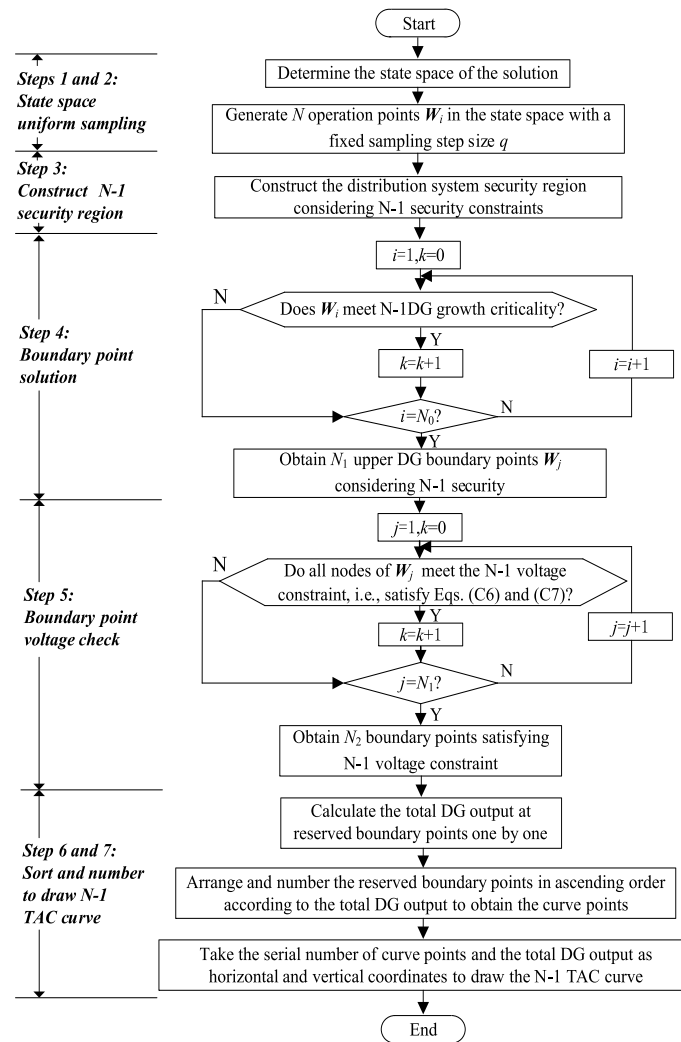


Fig. 1. The TAC curve solution and plotting process.

TAC curve under N-1 criterion, distribution network structures with power transfer capability through closing the normally open tie switches are designed as in Fig. 2. To study TAC curves with different DG and load distribution, three cases with different access locations of DG and load are designed. The detailed information is as follows:

Case1: DG units are connected to one feeder, and the end of the feeder is the load node.

Case2: DG units are connected to one feeder, and the end of the feeder is the DG node.

Case3: DG units are connected to both feeders.

The system parameters of the above three cases are shown in Table 1. The system parameters of these three examples are the same in the study.

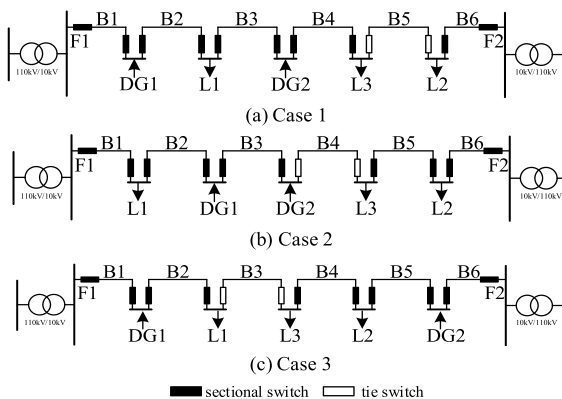


Fig. 2. The structures of three case networks.

Table 1
Parameters of the case networks.

System parameters		Parameter values
Voltage level		10kV
Feeder	Type	LGJ-185
	Capacity of each section	7.0MVA
	Length of each section	1.0 km
Substation transformer	Capacity	40MVA
Power load	Power range	[0,7.00] MVA
	Power factor	0.90
Power generation from each DG unit	Power range	[-7.00,0] MVA
	Power factor	0.95
Slack bus	Location	The root of the feeder
	Assumed voltage	1.0 p.u.
Voltage fluctuation range		[0.93, 1.07] p.u.
The network loss coefficient r		2 %

5.2. The TAC curve calculation and plotting

According to the method in Section IV, the calculation process of Case1 is as follows:

Step 1: According to the DG and load power ranges, the state space is:

$$\Theta = \left\{ \mathbf{W} \left| \begin{array}{l} 0 \leq S_{Lx} \leq 7\text{MVA}, \forall 1 \leq x \leq 3 \\ -7\text{MVA} \leq S_{Dgy} \leq 0, \forall 1 \leq y \leq 2 \end{array} \right. \right\} \quad (9)$$

Step 2: Take a sampling step of 0.2MVA, sample at equal intervals in the state space, and generate a uniformly distributed set of operating states \mathbf{W} ($N = 36^5 = 60,466,176$) to be determined.

Step 3: Use the DC power flow model Eq. (5) to construct the N-1 security region expression of Case1 (See Appendix D).

According to Eq. (6), the expression of security region considering N-1 criterion for Case1 is converted into the coefficient matrix of DG output \mathbf{A}_{DGne}^{N-1} and load output \mathbf{A}_{Line}^{N-1} . Constant vectors are generated using the maximum power of DG or load nodes (See Appendix D).

Step 4: The coefficient matrix obtained in step 3 and the operating state \mathbf{W} determined in step 2 are substituted into Eq. (7). The 443,556 operating states that meet Eq. (7) and are located at the upper DG boundary are screened out from the set of operating states and denoted as \mathbf{W}' .

Step5: Substitute \mathbf{W}' into equations $\Delta \mathbf{V}_{\min} \leq \Delta \mathbf{V} = \mathbf{A}_u \times \mathbf{W} \leq \Delta \mathbf{V}_{\max}$ and $\Delta \mathbf{V}_{\min} \leq \Delta \mathbf{V}(k) = \mathbf{A}_u(k) \times \mathbf{W} \leq \Delta \mathbf{V}_{\max}, \forall \psi_k \in \Psi$ from Eq. (8), and for 10 kV distribution network, ΔV_{\min} and ΔV_{\max} are -7% and 7% , respectively [6]. As a result, the 61,435 boundary points are deleted because they don't meet the voltage constraints; and the 382,121 boundary points meet the voltage constraints. Their detailed data, maximum voltage offset, and corresponding nodes are shown in Table D1 and Table D2 in Appendix D.

Step 6: Calculate the total DG output at the reserved operating states, sort and number them according to their size. Plot the N-1 TAC curve with the sorted serial number as the horizontal coordinate and the total DG output as the vertical coordinate, as shown in Fig. 3.

To distinguish from the TAC curves in this paper, the TAC curve considering N-0 criterion are termed as TAC0 curve [11]. In order to compare the TAC curve and the TAC0 curve, the TAC0 curve is calculated using the method provided in [11], and the data are summarized in Table D3 in Appendix D. The results of the TAC curves and the TAC0 curves for the three case distribution networks are compared in Fig. 3.

As shown in Fig. 3:

i) The TAC curve varies from 7MVA to 14MVA. The reason is as follows:

When DG1 and DG2 are connected to the same feeder segment and there is no load downstream of the feeder segment to help accommodate DG, the sum of outputs of DG1 and DG2 must meet the capacity constraint of the feeder segment by 7MVA. In this situation, the

minimum value of the TAC curve is 7MVA.

When there is a 7MVA load downstream of the feeder segment which is connected to DG1 and DG2, the load can help accommodate DG to meet the capacity constraint of the feeder segment. In this situation, the maximum TAC curve is 14MVA. In particular, 14MVA is the theoretical value, and the access load will be $<7\text{MVA}$ in practice, for example, if the access load is 5MVA, the maximum TAC curve is 12MVA.

ii) The TAC curve is lower than the TAC0 curve. The reason is as follows:

Firstly, the fault of the feeder segment causes the load to transfer, thus increasing the reverse power flow and limiting accommodation capability. Fig. 3(a) corresponds to Case1. When Case1 operates normally, branch B1 has the largest reverse power flow. After the failure of B4, the load L3 transfers to feeder F2, the downstream load of branch B1 is reduced, and the reverse power flow will further increase, thus limiting the accommodation capability and making the TAC curve decrease compared with the TAC0 curve.

Secondly, the fault of the feeder segment causes DG to transfer, thus increasing the reverse power flow and limiting accommodation capability. Fig. 3(c) corresponds to Case3. When Case3 operates normally, branch B1 and B6 have the largest reverse power flow. When B1 or B6 fails, the reverse power flow of the feeder transfers to the other feeder through the tie line, and only one side of the feeder is borne by the reverse power flow. As a result, the reverse power flow constraint of the feeder will be more stringent, so that the TAC curve will decrease compared with the TAC0 curve.

iii) In some situations, the TAC curve and the TAC0 curve are a horizontal line. The reason is as follows:

As shown in Fig. 3(b), since DG1 and DG2 are connected to the feeder segment B2, and there is no load downstream of B2 to help accommodate DG, the sum of outputs of DG1 and DG2 must meet the capacity constraint of the feeder segment by 7MVA, so the maximum TAC curve value can only be 7MVA. In addition, the distribution network with the structure of Case2 will not have the situation that the reverse power flow of the feeder segment will further increase due to the load transfer after failure, thus limiting the accommodation capability, that is, the N-1 criterion constraint is not more stringent than the N-0 criterion constraint, so the TAC curve is almost unchanged compared with the TAC0 curve.

It should be noted that the volatility of DG output does not affect the TAC curve, because the TAC curve only reflects the maximum output range of DG under the criterion constraint, and the volatility of DG output is reflected in the change of the operating state of the system.

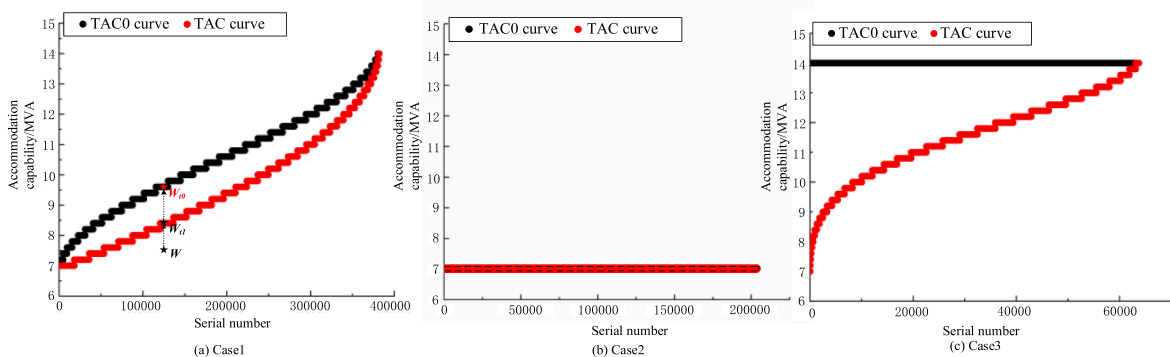


Fig. 3. The TAC0 and TAC curves of the three cases.

5.3. Rules about the TAC curve

1) Rules between TAC curve and TAC0 curve

Fig. 3 shows that the TAC curves of all cases are lower than the TAC0 curves or remain unchanged compared with TAC0 curves. The reason is that TAC curve must meet both N-0 and N-1 criterion constraints. When the N-1 criterion constraint is more stringent than the N-0 criterion constraint, the TAC curve is lower than the TAC0 curve. When the N-1 criterion constraint is no more stringent than the N-0 criterion constraint, the TAC curve remains the same as the TAC0 curve.

5.3.1. Influencing factors and rules on TAC curve

Network structure, voltage regulation measures, feeder load, etc., may affect the TAC curve of the distribution network [20]. However, network structure and voltage regulation measures of the established distribution network are usually unchanged. Therefore, this paper studies the influence of the power load on the TAC curve.

Literature [20] proposed that under normal operation, TAC curve is only related to the downstream load of the bottleneck feeder segment. Considering N-0 criterion, the bottleneck feeder segment is the one that carries the maximum reverse power flow of DG. When the reverse power flow through it reaches the upper limit of the capacity, the output of any DG cannot be increased, and the distribution network reaches the limit state of the accommodation of DG [20].

Firstly, it is necessary to define the bottleneck feeder segment considering N-1 criterion. In this paper, the feeder segment, which has the greatest limitation on the accommodation capability in all operating states, is defined as the bottleneck feeder segment. Case1 is used for an example to explain the definition of bottleneck feeder segment under N-1 criterion, see Appendix E.

The TAC curves of Case1, Case2, Case3 are shown in Fig. 4.

Table 2 shows the bottleneck feeder segment and downstream load number of each case, considering N-1 criterion. Since each load is the same, the number of loads reflects the size of the load.

As can be seen from Fig. 4 and Table 2, the accommodation capability of Case2, Case1 and Case3 increases successively, TAC curve increases successively, and the downstream load of the bottleneck feeder segment increases successively. Therefore, the general rule of influence of feeder load on TAC curve can be obtained: The accommodation capability of TAC curve is positively correlated with the downstream load of the bottleneck feeder segment. The reason for the rule is that: Increasing the load downstream of the bottleneck feeder segment will reduce the reverse power flow through the bottleneck feeder segment,

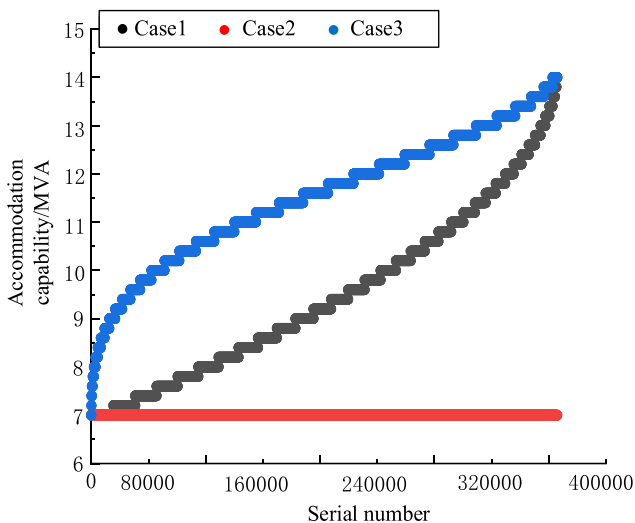


Fig. 4. The change of TAC curves of 3 cases.

Table 2

Bottleneck feeder segments and number of downstream load nodes of each case considering N-1 criterion.

Case	Bottleneck feeder segments	Number of downstream load nodes
Case1	B1, B4	1
Case2	B2, B4	0
Case3	B1, B6	3

thus increasing the DG output limit, which is the operating state of the TAC curve, and therefore improves the accommodation capability of the TAC curve.

Similarly, TAC0 curve also satisfies this rule, and the specific analysis process is shown in Appendix F.

5.4. Applications of the TAC curves

5.4.1. Planning of DG integration

TAC curves can determine the DG accommodation margin at the current operating state. The DG accommodation margin is how much DG output can be further integrated in the existing distribution networks [20].

For example, in Case1, an operating state $\mathbf{W} = [1.4, 0.2, 1.2, -2.6, -4.8]$. In order to calculate the accommodation margin considering N-0 criterion [11], the TAC upper limit curve data considering N-0 criterion in Table D3 (Appendix D) is consulted from the data of the operating state \mathbf{W} , and the curve point is $\mathbf{W}_{t0} = [1.4, 0.2, 1.2, -3.8, -5.8]$. Consequently, the accommodation margin considering N-0 criterion is: $\mathbf{M}_0 = \mathbf{W}_{t0} - \mathbf{W} = |-3.8 - (-2.6)| + |-5.8 - (-4.8)| = 1.2 + 1.0 = 2.2\text{MVA}$.

By referring to Table D1 in Appendix D using the data of operating state \mathbf{W} , it can be seen that the N-1 TAC curve point corresponding to operating state \mathbf{W} is $\mathbf{W}_{t1} = [1.4, 0.2, 1.2, -3.2, -5.2]$. As a result, the accommodation margin considering N-1 criterion is: $\mathbf{M}_1 = \mathbf{W}_{t1} - \mathbf{W} = |-3.2 - (-2.6)| + |-5.2 - (-4.8)| = 0.6 + 0.4 = 1.0\text{MVA}$.

In practical application, the accommodation margin can provide a reference for the planning of DG integration in the distribution network. The accommodation margin considering N-1 criterion is 1.0MVA, which means that considering the N-1 criterion, the two feeders in the Case 1 distribution network can also accept a maximum of 1.0MVA of new DG power based on the operating state. If the security constrain is relaxed and the accommodation margin increases without considering N-1 criterion, the distribution network can allow new DG power up to 2.2MVA.

5.4.2. Planning of load integration

Based on the positive correlation between the TAC curve and the downstream load size of the bottleneck feeder segment, it can be used to guide the new load integration in practice. The TAC curve is improved by increasing the downstream load of the bottleneck feeder segment with a new integrated load.

For example, in Case1, the new integrated load increases the downstream load L1 of the bottleneck feeder segment considering N-0 and N-1 criterion by 2.0MVA. Following the steps presented in Section V., Part B, the TAC/TAC0 curves for Case 1 after increasing L1 can be calculated and plotted, as shown in Fig. 5. The detailed data is shown in Tables G1 and G2 in Appendix G.

Fig. 5 shows that increasing the downstream load L1 of the bottleneck feeder segment improves both TAC0 and TAC curves. The average accommodation capability of all operating states on the TAC curve after increasing L1 is 10.47 MVA (detailed data is in Table G1 in Appendix G), and the average accommodation capability of all operating states on the TAC curve before increasing L1 is 9.33 MVA (detailed data is in Table D1 in Appendix D). The difference between 10.47 MVA and 9.33 MVA is the average value of the TAC curve increased by 1.14 MVA after increasing L1. Similarly, the average value of TAC0 curve increased by 1.0 MVA after increasing L1. The reason for the increase in the average values of the TAC/TAC0 curves after increasing the downstream load of the

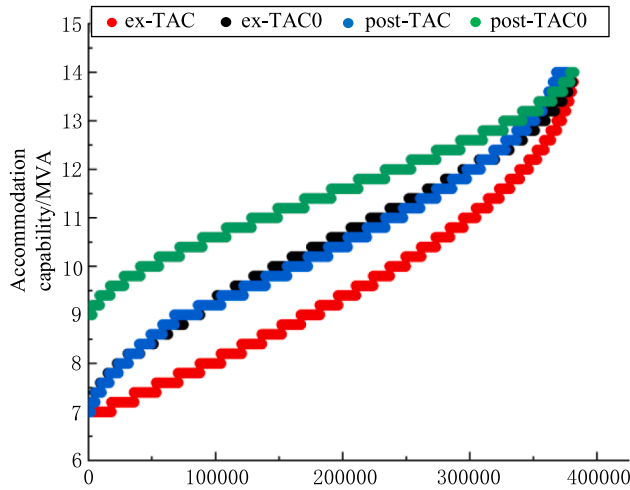


Fig. 5. Effect of the downstream load of the bottleneck feeder segment increases on the TAC curve of Case1.

bottleneck feeder segment is as follows:

Increasing the downstream load of the bottleneck feeder segment reduces the reverse power flow through the bottleneck feeder segment, thereby increasing the DG accommodation limit (for a detailed analysis, see Section 2 of Part C). In Case 1, the downstream load of the bottleneck feeder segment is L1, so increasing L1 by integrating new loads can improve TAC/TAC0 curves.

Increasing the TAC curve by increasing the downstream load of the bottleneck feeder segment can improve the accommodation capability and load rate of the system simultaneously. In the actual project, this measure can be implemented in power supply service expansion involving new load access.

5.4.3. Guidance of the replacement of distribution network wires

The bottleneck feeder segment is an important factor limiting the TAC curve. In practical application, the TAC curve can be improved by upgrading the bottleneck feeder segment by replacing the conductor.

For example, in Case1, bottleneck feeder segments B1 and B4 are upgraded from 7MVA to 8MVA according to Table 2 and Table F1. Following the steps presented in Section V., Part B, the TAC/TAC0 curves for Case 1 after the expansion of bottleneck feeder segments B1 and B4 can be calculated and plotted, as shown in Fig. 6. The detailed

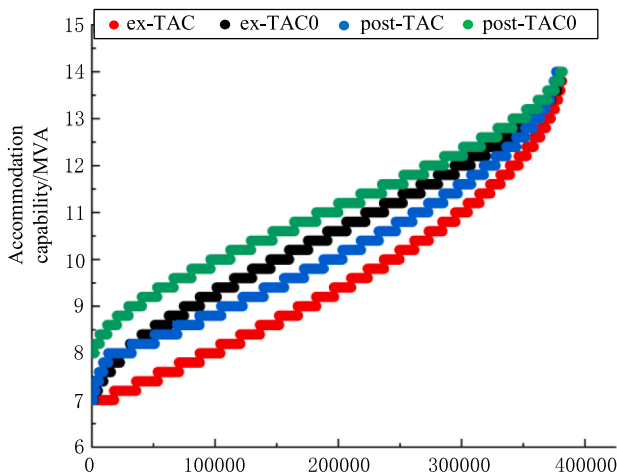


Fig. 6. Effect of bottleneck feeder segment upgrading on the TAC curve of Case1.

data is shown in Tables G3 and G4 in Appendix G.

Fig. 6 shows that expanding the capacity of the bottleneck feeder segment improves TAC/TAC0 curves. The average accommodation capability of all operating states on the TAC curve after the expansion of bottleneck feeder segments B1 and B4 is 9.83 MVA (detailed data is in Table G3 in Appendix G), and the average accommodation capability of all operating states on the TAC curve before the expansion is 9.33 MVA (detailed data is in Table D1 in Appendix D). The difference between 9.83MVA and 9.33MVA is the average value of the TAC curve increased by 0.5 MVA after the expansion. Similarly, the average value of the TAC0 curve increases by 0.81 MVA after the expansion. The reason for the increase in the average values of the TAC/TAC0 curves after expanding the capacity of the bottleneck feeder segments is as follows:

The capacity of the bottleneck feeder segment is a main factor limiting the TAC/TAC0 curve. The TAC/TAC0 curve can be improved by replacing the conductor to expand the bottleneck feeder segment. The bottleneck feeder segments of Case1 are B1 and B4, so the TAC/TAC0 curve can be increased by replacing the conductor to expand the capacity of B1 and B4.

In engineering practice, this method can be implemented in the replacement of wires in the reconstruction project and the accommodation capability of the system can be increased by replacing the bottleneck feeder segment with a larger capacity.

5.5. IEEE-RBTS-BUS4 test system

The modified IEEE-RBTS-BUS4 distribution network is used to verify the proposed model and method. Parameters, network structure and TAC/TAC0 curves of the case are shown in Appendix H. The calculation performance of the simple cases and IEEE-RBTS-BUS4 case is shown in Table 3.

It can be seen from Table 3:

With the increase of network size, although the time to solve TAC curve increases significantly, it remains acceptable. The reasons are as follows: (i) There are infinitely many TAC curve operating states, and the proposed algorithm can obtain a finite number of sampling points that can represent the complete TAC curve operating states through sampling. With the increase of network size, the proposed algorithm increases the number of sampling points, so the calculation time has increased. (ii) At present, TAC curves are mostly used in distribution network planning, and the priority is to ensure curve accuracy rather than calculation time.

For larger cases, decouple dimensionality reduction technique can be used [21], in which identifies and decouples feeders that are not electrically connected after an N-1 fault. As a result, the high-dimensional security region of the original network is divided into multiple lower-dimensional security regions, which significantly reduces the computational complexity.

6. Conclusions

This paper proposes a TAC curve to describe the capability of a distribution network to accommodate DG. The proposed TAC curves can completely and accurately evaluate the maximum allowable DG output within the distribution network. Due to the high security requirements

Table 3
Computational performance of different scale cases.

Case	Number of nodes	TAC Curve	
		Calculation time/s	Number of operating states
Simple test system	5	900.36	382,121
IEEE-RBTS-BUS4	42	2460.85	642,567

for urban distribution networks, this paper proposes the TAC curve considering N-1 criterion, and fill the gap that the existing research on the TAC curves only considers N-0 criterion. The main contributions are as follows:

- 1) The TAC curve model considering N-1 criterion and its solution method are proposed. The TAC curves are generated based on the analysis of the operating states on the DG security boundary. To obtain the DG security boundary, the DC power flow model is used first. The AC power flow simulation is then used to verify and correct the boundary points due to the prominent problem of voltage violation after DG integration.
- 2) Two rules of TAC curve are found. Firstly, the N-1 TAC curve is lower than the N-0 TAC curve or remains unchanged compared with the N-0 TAC curve. Secondly, the power load is a main factor that affects the TAC curve. It is found that the accommodation capability reflected in the TAC curve is positively correlated with the downstream load of the bottleneck feeder segment.
- 3) In engineering applications, the proposed TAC curves can be used for the planning of DG and load integration. Additionally, they can provide guidance on upgrading the bottleneck feeder segments, rather than the segments across the whole network, to improve the accommodation capability with lower cost.

This paper is an important supplement to the TAC curve model proposed in previous studies. In the future, we will further investigate other forms of the TAC curve, design of DG access planning scheme and other aspects.

CRediT authorship contribution statement

Jun Xiao: Conceptualization. **Luyan Xue:** Writing – original draft. **Xun Jiang:** Validation, Supervision. **Chuanqi Wang:** Project administration. **Haishen Liang:** Funding acquisition. **Kangli Wang:** Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary materials

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Data availability

Data will be made available on request.

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