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Energy storage configuration model for reliability services of active distribution networks

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ABSTRACT

The volatility introduced by the integration of renewable energy poses challenges to the reliability of power supply, increasing the demand for energy storage in distribution networks. Shared energy storage in distribution networks can participate in energy storage allocation as a provider of reliability ancillary services. This paper proposes a novel Nash bargaining based energy storage coordinated allocation method to fully incentivize shared energy storage to participate in reliability services within the distribution network. First, an analytical reliability assessment model is constructed and embedded into the energy storage allocation model, where the impact of renewable energy uncertainty is described using chance constraints. Considering the interests of both the distribution network and shared energy storage operators, a Nash bargaining based energy storage coordinated allocation and benefit sharing mechanism is established, which is then transformed into a mixed-integer linear programming (MILP) model for efficient solution. Case studies show that the proposed method, through cooperation between the distribution system operator and shared energy storage operators, significantly reduces investment cost of energy storage and ensures a rational distribution of the benefits obtained.

KEYWORDS

Distribution network, energy storage, game theory, reliability.

With the continuous development of the economy and society, customers' demand for reliable power supply is steadily increasing. Simultaneously, the integration of renewable energy resources introduces uncertainties that pose new challenges to reliable power supply. Energy storage systems are considered to be an effective means of improving the reliability of power supply[1]. In addition to energy storage, which is built by the distribution system operator, there are energy storage resources, which are owned by other entities, such as shared energy storage, that can be shared with the distribution system operator. By reasonably guiding shared energy storage to participate in reliability ancillary services, a win-win situation can be achieved, maximizing social benefits. Therefore, it is necessary to establish a coordinated allocation and benefit sharing model between the distribution system operator and shared energy storage, which is based on the quantification of the contribution of energy storage to reliability improvement in the context of renewable energy integration.

A major drawback of renewable energy is its intermittent nature, which means that it cannot provide the reliable power supply as traditional energy sources[2]. Energy storage can mitigate this impact on reliable power supply in the distribution network by providing charging and discharging capabilities[3]. Therefore, distribution system operators often combine renewable energy with energy storage devices. Paul et al.[4] proposed a multi-objective planning framework to determine the optimal capacity for the coordinated operation of wind farms and battery energy storage systems. Based on the Monte Carlo method, Liu et al.[5] established a techno-economic evaluation method for solar power plants that integrates the uncertainty of input variables. Zhu et al.[6] provided guidelines for energy storage planning under various scenarios to accommodate the penetration of renewable energy, taking into account the energy storage policies of different countries. Based on the stochastic unit commitment model with energy storage, the impact of increasing renewable penetration on the attractiveness of energy storage was analyzed in Ref. [7]. Although distribution system operators have the need to install energy storage, the costs associated with installation can be prohibitively high. One way to address this drawback is to encourage cooperation between distribution system operators and energy storage owners. Through such cooperation, the reliability level of distribution networks integrating renewable energy can be effectively improved, reducing the need for additionally constructing large-scale energy storage and providing cost reductions for participants[8].

Inspired by the aforementioned needs, the concept of shared energy storage has been proposed[9]. Shared energy storage refers to the energy storage unit that is jointly used and maintained by multiple entities, thereby achieving optimal utilization and fair cost sharing[10]. Unlike the direct construction and management of energy storage by distribution system operators, shared energy storage involves multiple stakeholders. Therefore, its allocation and management must consider the interests of all parties, making it more suitable for a market-based business model[11].

The literature already includes relevant research on planning methods for shared energy storage. In Ref. [12], a bi-level model was proposed to optimize the size and operation of shared energy storage in renewable generation systems. Considering the renewable correlation on the supply side, a planning method for shared energy storage was presented by Wang et al.[13]. Rodrigues et al.[14] analyzed the impact of different ownership structures of battery energy storage systems on their optimal size and show that energy sharing can reduce energy costs for customers. Zhao et al.[15] constructed a two-stage optimization model for energy storage investment that determines the pricing strategy of the storage aggregator and the capacity purchased by customers.

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Nomenclature

Indices	
$(\cdot)^{\mathrm{CB}}$	Index for the stage from circuit breaker tripping to automatic operation of the device.
$(\cdot)^{\mathrm{AS}}$	Index for the stage from automatic operation of the device to manual operation of the device.
$(\cdot)^{\mathrm{MS}}$	Index for the stage from manual operation of the device to fault repair.
$(\cdot)^{\mathrm{NO}}$	Index for normal operating state.
Set	
Ψ_{B}	Set of branches.
Ψ_{N}	Set of nodes.
Ψ_{SS}	Set of substation nodes.
Ψ_{T}	Set of transformers.
Ψ_{LN}	Rental fees for shared energy storage during coordinated allocation.
Ψ_{ES}	Set of energy storages.
$\Psi_{\mathrm{ES}}^{\mathrm{Can}}$	Set of candidate new energy storage installations.
$\Psi_{\mathrm{ES}}^{\mathrm{Exi}}$	Set of existing shared energy storage.
Ψ_i	Set of nodes connected to node *i*
Λ^{C}	Set of energy storage construction capacities.
Λ^{E}	Set of energy storage rental capacities.
Parameters	
$C_{\mathrm{Con}}^{a,I}$	Cost of constructing type *a* energy storage.
$C_{i,\mathrm{Ren}}$	Unit price of renting energy storage at node *i*.
M	A sufficiently large positive constant.
N	Number of bits for binary encoding of rental prices.
NC_i	Number of customers of node *i*.
P_i	Active load demand of node *i*.
$P_{i,\mathrm{disch}}^{\max}$	Maximum discharge power of the energy storage *i*.
r_{ij}	Resistance of branch *ij*.
$S_{i,\mathrm{Con}}^{a}$	Capacity of energy storage construction type *a* at node *i*.
$S_{i,\mathrm{Ren}}^{c}$	Capacity of energy storage rental option *c* at node *i*.
S_{ij}^{C}	Capacity of branch *ij*.
S_{Tr}^{C}	Capacity of transformer *Tr*.
x_{ij}	Inductance of branch *ij*.
$x_{i,ij}^{\mathrm{CB}}$	Equal to 1 when circuit breaker is placed at end *i* of branch *ij*.
$x_{i,ij}^{\mathrm{AS}}$	Equal to 1 when automatic switch is placed at end *i* of branch *ij*.
$x_{i,ij}^{\mathrm{SW}}$	Equal to 1 when switch is placed at end *i* of branch *ij*.
w_i	Load annual energy demand of node *i*.

α	Confidence level of the result.
λ_{xy}	Failure rate of branch *xy*.
η_i	Power factor of node *i*.
$\overline{(\cdot)}/\underline{(\cdot)}$	Upper and lower bounds of the variable.
$Pr()$	Probability.
Continuous variables	
C_{con}	Energy storage construction cost for the distribution system operator during coordinated allocation.
C_{ren}	Set of load nodes.
C_{con}^{ori}	Energy storage construction cost for the distribution system operator without considering the rental of shared energy storage.
CID_i	Customer interruption duration of node *i*.
CIF_i	Customer interruption frequency of node *i*.
F_i^{xy}	$F_i^{xy} \in [0,1]$ and equals to 0 when node *i* loses power supply due to a fault at branch *xy*.
F_{ij}^{xy}	$F_{ij}^{xy} \in [0,1]$ and equals to 0 when branch *ij* loses power supply due to a fault at branch *xy*.
$P_{\mathrm{de},i,xy}$	Shed active demand of node *i* due to a fault at branch *xy*.
$P_{i,g}^{t}$	Active power generation of renewable energy at node *i* in stage *t*.
$P_{i,e}^{xy,t}$	Active charging power of energy storage at node *i* in stage *t* due to a fault at branch *xy*.
$P_{de,i,g}^{xy,t}$	Renewable energy active power curtailment at node *i* in stage *t* due to a fault at branch *xy*
P_{ij}^{xy}	Active power flow through branch *ij* (from node *i* to node *j*) due to a fault at branch *xy*.
$P_{tr^{\mathrm{out}}}^{xy}$	Active power flow through the outlet branch of transformer *Tr* due to a fault at branch *xy*.
$P_{Tr}^{xy,t}$	Active power of transformer *Tr* in stage *t* due to a fault at branch *xy*.
$Q_{Tr}^{xy,t}$	Reactive power of transformer *Tr* in stage *t* due to a fault at branch *xy*.
Q_{ij}^{xy}	Reactive power flow through branch *ij* (from node *i* to node *j*) due to a fault at branch *xy*.
$Q_{tr^{\mathrm{out}}}^{xy}$	Reactive power flow through the outlet branch of transformer *Tr* due to a fault at branch *xy*.
$SOC_{i,xy}$	State of Charge of energy storage *i* due to a fault at branch *xy*.
U_i^{xy}	Square voltage at node *i* due to a fault at branch *xy*.
U^{ss}	Square voltage of substation node.
Binary variables	
$b_{i,ij}^{xy}$	Equal to 1 when circuit breaker placed at end *i* of branch *ij* is closed due to a fault at branch *xy*.
e_i^{c}	Indicates whether to rent option *c* energy storage at node *i* (1 for renting, otherwise 0)
l_i^{a}	Indicates whether to construct type *a* energy storage at node *i* (1 for constructing, otherwise 0)
p_i^{k}	Indicates whether the *k*th bit of the rental price encoding for energy storage at node *i* is 1 (1 if yes, otherwise 0)
p_i^{xy}	Equal to 1 when node *i* is affected by the outage due to a fault at branch *xy*.
q_i^{xy}	Equal to 1 when the node *i* remains in the power outage state due to a fault at branch *xy*.
$s_{i,ij}^{xy}$	Equal to 1 when switch placed at end *i* of branch *ij* is closed due to a fault in branch *xy*.

Considering the role of energy storage in improving power quality and peak shaving, storage owners can also create value by providing ancillary services to the power grid. The coordinated scheduling strategy of energy storage and wind turbines was studied by Simla and Stanek[16] to mitigate the impact of wind power uncertainty on conventional generation and reduce energy losses. To address the risks posed by distributed generation to the power distribution network, Sedghi et al.[17] proposed an energy storage planning model that considers reliability costs to determine the optimal location and capacity of energy storage. The model is solved via the hybrid tabu search/particle swarm optimization (TS/PSO) algorithm. However, these studies model the distribution system operator and the energy storage owner as a whole and lack a detailed discussion of the contribution levels and benefit sharing among different stakeholders.

Game theory, as a method for handling the allocation of interests among multiple stakeholders, has been widely applied in power systems[18]. It can provide a structured framework to analyze and optimize decision-making processes, helping ensure fair and efficient distribution of benefits, managing conflicts of interest and improving overall system performance. Andoni et al.[19] adopted the Stackelberg-Cournot game model to simulate the interactions among participants investing in the network, renewable generation and storage capacity. Valinejad et al.[20] proposed a bi-level framework that considers demand response to plan distributed energy resources and energy storage and examined the impact of coalition formation on generation resource planning. To address the proliferation of renewable energy and energy storage systems in community households, Chiş and Koivunen[8] proposed a joint cost optimization method. Owners of renewable energy and energy storage systems can form alliances and share the cost savings among the members based on the Shapley value. In Ref. [21], Han et al. employed cooperative game theory to form energy alliances and optimize the operation of energy storage systems, addressing the load balancing issues of the power grid caused by distributed renewable generation. Xu et al.[22] proposed a coordinated optimization model for shared energy storage systems and microgrids. In the model, Nash bargaining is used to allocate benefits among different stakeholders to maintain long-term cooperation. Nash bargaining was also used in Ref. [23] for the power management of multiple virtual energy storage systems to increase the penetration of renewable energy by leveraging competitive objective functions.

Shared energy storage can profit in two ways: by charging service fees or by arbitraging from charging and discharging. The first method involves receiving compensation for providing ancillary services to the power grid. The second method entails the system engaging in energy trading with users or the grid based on electricity prices to generate profits[22]. In this paper, our focus is on the first method, where shared energy storage generates profit by providing reliability services to the distribution network. As shared energy storage is often not owned by the distribution system operator, there are different interests between the distribution system operator and the storage operators[24]. Therefore, game theory is used to allocate benefits among different participants, guiding the distribution system operator in the rational construction and utilization of energy storage to enhance the reliability of the distribution system.

This paper proposes a novel energy storage allocation model in the context of shared energy storage participating in the distribution network reliability service. The allocation model proposed in this paper can reduce energy storage construction costs through cooperation between the distribution network operator and the shared energy storage, achieving mutual benefits for both parties and thereby contributing to overall social welfare. The main contributions of this paper are twofold:

(1) The integration of renewable energy poses challenges to the reliable operation of distribution networks, and existing energy storage allocation models for power supply reliability services lack risk quantification associated with renewable energy uncertainties. This paper embeds the analytical reliability assessment model into the energy storage allocation model, taking into account the post-fault charging and discharging of energy storage, as well as switch-dependent network reconfiguration. The proposed model can provide energy storage allocation solutions with the specified reliability confidence level.

(2) To fully mobilize energy storage resources in the power distribution network, a coordinated allocation and benefit sharing model between the distribution system operator and shared energy storage is established based on Nash bargaining. By reasonably guiding shared energy storage to participate in reliability services, storage investment costs can be saved, and maximum social benefits can be achieved. The model is linearized via the Big-M method and binary encoding, and finally a mixed-integer linear programming (MILP) model is established that can be solved efficiently.

The remainder of this paper is organized as follows. Section 1 presents the mathematical model of the Nash bargaining-based coordinated allocation method and how the model is linearized. In Section 2, numerical tests are reported and analyzed. Finally, conclusions are drawn in Section 3.

1 Mathematical model

A distribution network must provide the safe and reliable power supply. When faults occur in the distribution branches, the network can isolate the fault and transfer the load through various switching devices, such as circuit breakers, automatic switches, and manual switches. The integration of renewable energy presents new challenges for reliable power supply. The distribution system operator can mitigate the impact of renewable energy uncertainties on reliable operation by installing energy storage systems. By cooperating with shared energy storage owners, the distribution system operator can reduce investment costs, whereas shared energy storage can earn compensation for providing ancillary services. The distribution network operator and the shared energy storage operator can negotiate and establish an agreement in advance to specify the amount of storage capacity to be rented. In this way, a portion of the available capacity of the shared energy storage system is reserved specifically for providing reliability services. This section constructs the game model between the distribution system operator and shared energy storage, including objective functions, game strategies, and the analytical allocation model with adjustable risk levels.

1.1 Objective function

The proposed coordinated optimization model for energy storage allocation aims to maximize the benefits for both the distribution network and the shared energy storage operators, striving for the greatest social welfare. To maintain cooperation between the parties, the profits are distributed via Nash bargaining, ensuring fairness and rationality in the allocation of benefits. The objective function form for Nash bargaining is as follows:

$$\max_{(s,d)\in U} f=(s-s^{*})(d-d^{*}) \tag{1}$$

Here, s and d are the possible strategies of the game participants, U is the set of strategies, and s^{*} and d^{*} are the breakdown points of the bargaining.

For the distribution system operator, the strategy includes the locations and capacities for energy storage that need to be constructed by the network operator to address the integration of renewable energy, as well as the capacity to be rented from existing shared energy storage owned by private investors. For the shared energy storage owner, the strategy involves setting the rental price for the storage capacity. The decision variables include the capacity of energy storage constructed by the distribution network operator, the capacity rented from the shared energy storage, and the unit rental price offered by the shared energy storage. The breakdown point of bargaining is defined as the scenario where the distribution network operator independently constructs energy storage, whereas the shared energy storage operator does not earn profit through capacity renting. The objective function of the coordinated energy storage allocation model is expanded as follows:

$$\max f = [C_{\text{con}}^{ori} - (C_{\text{con}} + C_{\text{ren}})] \cdot C_{\text{ren}} \tag{2}$$

Here, C_{con}^{ori} is the energy storage construction cost for the distribution system operator without considering the rental of shared energy storage. C_{con} and C_{ren} are the energy storage construction cost for the distribution system operator and the rental fees for shared energy storage with coordinated allocation, respectively.

$$C_{\text{con}} = \sum_{i \in \Psi_{\text{ES}}^{\text{Can}}} \sum_{a \in \Lambda^{\text{C}}} C_{\text{Con}}^{a,I} l_i^a \tag{3}$$

$$C_{\text{ren}} = \sum_{i \in \Psi_{\text{ES}}^{\text{Exi}}} \sum_{c \in \Lambda^{\text{E}}} \sum_{k=1}^{N} 2^k p_i^k C_{i,\text{Ren}} S_{i,\text{Ren}}^c e_i^c \tag{4}$$

Constraint (3) indicates that the total construction cost of energy storage is the product of the construction costs for different storage capacities and the binary variables indicating the types of storage capacities actually constructed. Constraint (4) indicates that the rental cost of energy storage is the product of the unit capacity rental price and the actual rented capacity. To linearize the model, we use binary encoding to discretize the rental price offers for shared energy storage.

1.2 Constraints

The constraints include fault flow constraints, operational constraints, energy storage constraints, and equipment availability constraints.

The first part consists of fault flow constraints during the operation of circuit breakers and sectionalizing switches to analytically describe the outage scope at different stages after faults occur. The fault flow constraints during the operation of circuit breakers are as follows:

$$F_{xy}^{xy,\text{CB}} = 0 \tag{5}$$

$$\begin{cases} -(2 - b_{i,ij}^{xy} - s_{i,ij}^{\text{NO}})M + F_i^{xy,CB} \leqslant F_{ij}^{xy,\text{CB}} \\ F_{ij}^{xy,\text{CB}} \leqslant (2 - b_{i,ij}^{xy} - s_{i,ij}^{\text{NO}})M + F_i^{xy,\text{CB}} \end{cases}, \forall ij \in \Psi_{\text{B}} \tag{6}$$

$$\begin{cases} -(2 - b_{j,ij}^{xy} - s_{j,ij}^{\text{NO}})M + F_j^{xy,\text{CB}} \leqslant F_{ij}^{xy,\text{CB}} \\ F_{ij}^{xy,\text{CB}} \leqslant (2 - b_{j,ij}^{xy} - s_{j,ij}^{\text{NO}})M + F_j^{xy,\text{CB}} \end{cases}, \forall ij \in \Psi_{\text{B}} \tag{7}$$

$$b_{i,ij}^{xy} \geqslant F_{ij}^{xy,\text{CB}}, ij \in \Psi_{\text{B}} \tag{8}$$

$$b_{j,ij}^{xy} \geqslant F_{ij}^{xy,\text{CB}}, ij \in \Psi_{\text{B}} \tag{9}$$

$$F_i^{xy,\text{CB}} = 1, \forall i \in \Psi_{\text{SS}} \tag{10}$$

$$p_i^{xy} = 1 - F_i^{xy,\text{CB}}, \forall i \in \Psi_{\text{N}} \tag{11}$$

$$P_{\text{de},i,xy}^{\text{CB}} \leqslant P_i, \forall i \in \Psi_{\text{N}} \tag{12}$$

As shown in constraint (5), the source of the fault flow is the faulted branch. Constraints (6)–(9) are fault current propagation constraints during the circuit breaker operation stage and represent the extent of fault impact. Constraints (10) and (11) restrict the relationship between the extent of the fault impact and the outage status of nodes during the circuit breaker operation stage. Constraint (12) limits the maximum value of load shedding.

The fault current constraints during the operation of sectionalizing switches, including automatic switches and manually operated switches, are as follows:

$$F_{xy}^{xy,t} = 0 \tag{13}$$

$$\begin{cases} -(1 - s_{i,ij}^{xy,t})M + F_i^{xy,t} \leqslant F_{ij}^{xy,t} \\ F_{ij}^{xy,t} \leqslant (1 - s_{i,ij}^{xy,t})M + F_i^{xy,t} \end{cases}, \forall ij \in \boldsymbol{\Psi_B} \tag{14}$$

$$\begin{cases} -(1 - s_{j,ij}^{xy,t})M + F_j^{xy,t} \leqslant F_{ij}^{xy,t} \\ F_{ij}^{xy,t} \leqslant (1 - s_{j,ij}^{xy,t})M + F_j^{xy,t} \end{cases}, \forall ij \in \Psi_{\text{B}} \tag{15}$$

$$F_i^{xy,t} = 1, \forall i \in \Psi_{\text{SS}} \tag{16}$$

$$q_i^{xy,t} = 1 - F_i^{xy,t}, \forall i \in \Psi_{\text{N}} \tag{17}$$

$$P_{\text{de},i,xy}^{t} \leqslant P_i, \forall i \in \Psi_{\text{N}} \tag{18}$$

$$q_i^{xy,\text{AS}} \frac{P_i - P_{\text{de},i,xy}^{\text{AS}}}{P_i} + \frac{P_{\text{de},i,xy}^{\text{AS}}}{P_i} \leqslant p_i^{xy} \frac{P_i - P_{\text{de},i,xy}^{\text{CB}}}{P_i} + \frac{P_{\text{de},i,xy}^{\text{CB}}}{P_i}, \forall i \in \Psi_{\text{N}} \tag{19}$$

$$q_i^{xy,\text{MS}} \frac{P_i - P_{\text{de},i,xy}^{\text{MS}}}{P_i} + \frac{P_{\text{de},i,xy}^{\text{MS}}}{P_i} \leqslant q_i^{xy,\text{AS}} \frac{P_i - P_{\text{de},i,xy}^{\text{AS}}}{P_i} + \frac{P_{\text{de},i,xy}^{\text{AS}}}{P_i}, \forall i \in \Psi_{\text{N}} \tag{20}$$

Here, $t = AS, MS$. Constraints (13)–(15) represent the source and extent of the fault flow during the sectionalizing switch operation stage. Constraints (16) and (17) represent the node outage conditions during this stage, whereas constraint (18) is the load shedding constraint. Constraints (19) and (20) restrict the relationship between node outage statuses across different stages, ensuring that nodes restored to service in the previous stage cannot experience outages again in the subsequent stage.

The second part consists of operational constraints, including power flow constraints, power balance constraints, radial operation constraints, and voltage and capacity safety constraints.

$$-M(1 - F_{ij}^{xy,t}) \leqslant U_j^{xy,t} - U_i^{xy,t} + 2(r_{ij} P_{ij}^{xy,t} + x_{ij} Q_{ij}^{xy,t}), \forall ij \in \Psi_{\text{B}} \tag{21}$$

$$U_j^{xy,t} - U_i^{xy,t} + 2(r_{ij} P_{ij}^{xy,t} + x_{ij} Q_{ij}^{xy,t}) \leqslant M(1 - F_{ij}^{xy,t}), \forall ij \in \Psi_{\text{B}} \tag{22}$$

$$\underline{U} \leqslant U_i^{xy,t} \leqslant \overline{U}, \forall i \in \Psi_{\text{N}} \tag{23}$$

$$U_i^{xy,t} = U^{ss}, \forall i \in \Psi_{\text{SS}} \tag{24}$$

$$\sum\nolimits_{i \in \Psi_{\text{LN}}} (1 - q_i^{xy,t}) = \sum\nolimits_{ij \in \Psi_{\text{B}}} F_{ij}^{xy,t} \tag{25}$$

$$P_i^{xy,t} = \left(P_i - P_{\text{de},i,xy}^{t}\right)\left(1 - q_i^{xy,t}\right), \forall i \in \Psi_{\text{N}} \tag{26}$$

$$Q_i^{xy,t} = \left(Q_i - Q_{\text{de},i,xy}^{t}\right)\left(1 - q_i^{xy,t}\right), \forall i \in \Psi_{\text{N}} \tag{27}$$

$$Q_{\text{de},i,xy}^{t} = \eta_i P_{\text{de},i,xy}^{t}, \forall i \in \Psi_{\text{N}} \tag{28}$$

$$\Pr(P_i^{xy,t} - P_{i,g}^t - P_{i,e}^{xy,t} + P_{\mathrm{de},i,g}^{xy,t} \leqslant \textstyle\sum_{j\in\Psi_i} P_{ij}^{xy,t}) \geqslant \alpha,\ \forall i \in \Psi_{\mathrm{N}} \tag{29}$$

$$\Pr(Q_i^{xy,t} - Q_{i,g}^t - Q_{i,e}^{xy,t} + Q_{\mathrm{de},i,g}^{xy,t} \leqslant \textstyle\sum_{j\in\Psi_i} Q_{ij}^{xy,t}) \geqslant \alpha,\ \forall i \in \Psi_{\mathrm{N}} \tag{30}$$

$$\Pr(P_{i,g}^t - P_i^{xy,t} + P_{i,e}^{xy,t} - P_{\mathrm{de},i,g}^{xy,t} \leqslant -\textstyle\sum_{j\in\Psi_i} P_{ij}^{xy,t}) \geqslant \alpha,\ \forall i \in \Psi_{\mathrm{N}} \tag{31}$$

$$\Pr(Q_{i,g}^t - Q_i^{xy,t} + Q_{i,e}^{xy,t} - Q_{\mathrm{de},i,g}^{xy,t} \leqslant -\textstyle\sum_{j\in\Psi_i} Q_{ij}^{xy,t}) \geqslant \alpha,\ \forall i \in \Psi_{\mathrm{N}} \tag{32}$$

$$\begin{cases} -Ms_{k,ij}^{xy,t} \leqslant P_{ij}^{xy,t} \leqslant Ms_{k,ij}^{xy,t} \\ -Ms_{k,ij}^{xy,t} \leqslant Q_{ij}^{xy,t} \leqslant Ms_{k,ij}^{xy,t} \end{cases}, \forall ij \in \Psi_{\mathrm{B}} \tag{33}$$

$$P_{Tr}^{xy,t} = P_{tr^{out}}^{xy,t}, Q_{Tr}^{xy,t} = Q_{tr^{out}}^{xy,t}, \forall Tr \in \Psi_{\mathrm{T}} \tag{34}$$

$$\begin{cases} -S_{DE}^C \leqslant P_{DE}^{xy,t} \leqslant S_{DE}^C \\ -S_{DE}^C \leqslant Q_{DE}^{xy,t} \leqslant S_{DE}^C \end{cases} \tag{35}$$

$$-\sqrt{2}S_{DE}^C \leqslant P_{DE}^{xy,t} \pm Q_{DE}^{xy,t} \leqslant \sqrt{2}S_{DE}^C \tag{36}$$

Here, $DE = \{ij, Tr\}, \forall ij \in \Psi_{\mathrm{B}}, \forall Tr \in \Psi_{\mathrm{T}}$, which represents branches and transformers. $k = i, j$ represent the two ends of branch ij. Constraints (21)–(24) are linearized power flow constraints and node voltage constraints. Constraint (25) is the radial operation constraint. Constraints (26)–(28) describe the relationship between the remaining load at a node and its outage status. Constraints (29)–(32) represent power balance constraints in the form of chance constraints to address the uncertainties introduced by the integration of renewable energy. Constraints (33)–(36) represent branch and transformer capacity constraints. The chance constraints are transformed into deterministic constraints by solving the corresponding quantile conditions[25].

The third part includes constraints on energy storage construction and operation.

$$SOC_i^{\mathrm{NO}} = \sum_{a\in\Lambda^{\mathrm{E}}} S_{i,\mathrm{Con}}^a \cdot l_i^a, i \in \Psi_{\mathrm{ES}}^{\mathrm{Can}} \tag{37}$$

$$SOC_i^{\mathrm{NO}} = \sum_{c\in\Lambda^{\mathrm{C}}} S_{i,\mathrm{Ren}}^c \cdot e_i^c, i \in \Psi_{\mathrm{ES}}^{\mathrm{Exi}} \tag{38}$$

$$SOC_{i,xy}^{\mathrm{AS}} = SOC_i^{\mathrm{NO}} - P_{i,e}^{xy,\mathrm{CB}} \cdot \tau_{xy}^{\mathrm{AST}}, i \in \Psi_{\mathrm{ES}} \tag{39}$$

$$SOC_{i,xy}^{\mathrm{MS}} = SOC_i^{\mathrm{AS}} - P_{i,e}^{xy,\mathrm{AS}} \cdot (\tau_{xy}^{\mathrm{MST}} - \tau_{xy}^{\mathrm{AST}}), i \in \Psi_{\mathrm{ES}} \tag{40}$$

$$SOC_{i,xy}^{\mathrm{RP}} = SOC_i^{\mathrm{MS}} - P_{i,e}^{xy,\mathrm{MS}} \cdot (\tau_{xy}^{\mathrm{RPT}} - \tau_{xy}^{\mathrm{MST}}), i \in \Psi_{\mathrm{ES}} \tag{41}$$

$$0 \leqslant SOC_{i,xy}^t \leqslant SOC_i^{\mathrm{NO}}, i \in \Psi_{\mathrm{ES}} \tag{42}$$

$$-P_{i,\mathrm{disch}}^{\max} \leqslant P_{i,e}^{xy,t} \leqslant P_{i,\mathrm{disch}}^{\max}, i \in \Psi_{\mathrm{ES}} \tag{43}$$

Here, τ_{xy}^{AST}, τ_{xy}^{MST} and τ_{xy}^{RPT} represent the automatic equipment action time, manual operation time, and fault repair time, respectively. t represents different stages. Constraint (37) represents the energy storage construction at candidate nodes in the distribution network. Constraint (38) represents the capacity rental status of shared energy storage. Constraints (39)–(43) represent the state-of-charge (SOC) constraints of energy storage across different stages, describing the relationship between the energy stored and the charging and discharging power.

The fourth part comprises logical constraints on the status and installation of switching devices.

$$b_{i,ij}^{xy} \geqslant 1 - x_{i,ij}^{\mathrm{CB}}, \forall ij \in \Psi_{\mathrm{B}} \tag{44}$$

$$b_{j,ij}^{xy} \geqslant 1 - x_{j,ij}^{\mathrm{CB}}, \forall ij \in \Psi_{\mathrm{B}} \tag{45}$$

$$s_{i,ij}^{xy,\mathrm{AS}} \geqslant b_{i,ij}^{xy} + s_{i,ij}^{\mathrm{NO}} - 1 - x_{i,ij}^{\mathrm{AS}}, \forall ij \in \Psi_{\mathrm{B}} \tag{46}$$

$$s_{j,ij}^{xy,\mathrm{AS}} \geqslant b_{j,ij}^{xy} + s_{j,ij}^{\mathrm{NO}} - 1 - x_{j,ij}^{\mathrm{AS}}, \forall ij \in \Psi_{\mathrm{B}} \tag{47}$$

$$s_{i,ij}^{xy,\mathrm{AS}} \leqslant b_{i,ij}^{xy} + s_{i,ij}^{\mathrm{NO}} - 1 + x_{i,ij}^{\mathrm{AS}}, \forall ij \in \Psi_{\mathrm{B}} \tag{48}$$

$$s_{j,ij}^{xy,\mathrm{AS}} \leqslant b_{j,ij}^{xy} + s_{j,ij}^{\mathrm{NO}} - 1 + x_{j,ij}^{\mathrm{AS}}, \forall ij \in \Psi_{\mathrm{B}} \tag{49}$$

$$s_{i,ij}^{xy,\mathrm{MS}} \geqslant 1 - x_{i,ij}^{\mathrm{SW}}, \forall ij \in \Psi_{\mathrm{B}} \tag{50}$$

$$s_{j,ij}^{xy,\mathrm{MS}} \geqslant 1 - x_{j,ij}^{\mathrm{SW}}, \forall ij \in \Psi_{\mathrm{B}} \tag{51}$$

Constraints (44) and (51) indicate that the switching states of circuit breakers, automatic switches, and manual switches are restricted by their installation status.

Finally, reliability index constraints exist. First, the reliability indices of the nodes are calculated, and the system indices are computed on this basis.

$$CIF_i = \textstyle\sum_{xy\in\Psi_{\mathrm{B}}} \lambda_{xy} p_i^{xy}, \forall i \in \Psi_{\mathrm{N}} \tag{52}$$

$$CID_i = \sum_{xy\in\Psi_{\mathrm{B}}} \lambda_{xy} \begin{bmatrix} \tau_{xy}^{\mathrm{AST}} \left(p_i^{xy} \dfrac{P_i - P_{\mathrm{de},i,xy}^{\mathrm{CB}}}{P_i} + \dfrac{P_{\mathrm{de},i,xy}^{\mathrm{CB}}}{P_i} \right) \\ +(\tau_{xy}^{\mathrm{MST}} - \tau_{xy}^{\mathrm{AST}}) \left(q_i^{xy,\mathrm{AS}} \dfrac{P_i - P_{\mathrm{de},i,xy}^{\mathrm{AS}}}{P_i} + \dfrac{P_{\mathrm{de},i,xy}^{\mathrm{AS}}}{P_i} \right) \\ +(\tau_{xy}^{\mathrm{RPT}} - \tau_{xy}^{\mathrm{MST}}) \left(q_i^{xy,\mathrm{MS}} \dfrac{P_i - P_{\mathrm{de},i,xy}^{\mathrm{MS}}}{P_i} + \dfrac{P_{\mathrm{de},i,xy}^{\mathrm{MS}}}{P_i} \right) \end{bmatrix}, \forall i \in \Psi_{\mathrm{N}} \tag{53}$$

$$SAIDI = \frac{\sum_{i\in\Psi_{\mathrm{N}}} NC_i \cdot CID_i}{\sum_{i\in\Psi_{\mathrm{N}}} NC_i} \tag{54}$$

$$SAIFI = \frac{\sum_{i\in\Psi_{\mathrm{N}}} NC_i \cdot CIF_i}{\sum_{i\in\Psi_{\mathrm{N}}} NC_i} \tag{55}$$

$$EENS = \sum_{i\in\Psi_{\mathrm{N}}} \frac{CID_i}{8760} w_i \tag{56}$$

$$SAIDI \leqslant \varepsilon_{\mathrm{SAIDI}} \tag{57}$$

$$SAIFI \leqslant \varepsilon_{\mathrm{SAIFI}} \tag{58}$$

$$EENS \leqslant \varepsilon_{\mathrm{EENS}} \tag{59}$$

Constraints (52) and (53) calculate the customer interruption frequency (CIF) and customer interruption duration (CID) on the basis of the node outage status and load shedding at different stages. Constraints (54)–(56) calculate the system reliability indices, including the system average interruption duration index (SAIDI), system average interruption frequency index (SAIFI) and expected energy not supplied (EENS), on the basis of the node outage indices. Constraints (57)–(59) represent the restrictions on the system reliability indices. Here, $\varepsilon_{\mathrm{SAIDI}}$ is the SAIDI index, $\varepsilon_{\mathrm{SAIFI}}$

is the SAIFI index, and $\varepsilon_{\mathrm{EENS}}$ is the EENS index.

The objective function can be expanded as follows:

$$\begin{aligned}\max f = {} & C_{\mathrm{con}}^{ori} \cdot \sum_{i\in\Psi_{\mathrm{ES}}^{\mathrm{Exi}}} \sum_{c\in\Lambda^{\mathrm{E}}} \sum_{k=1}^{N} 2^k p_i^k C_{i,\mathrm{Ren}} S_{i,\mathrm{Ren}}^c e_i^c \\ & - \sum_{i\in\Psi_{\mathrm{ES}}^{\mathrm{Can}}} \sum_{a\in\Lambda^{\mathrm{C}}} \sum_{j\in\Psi_{\mathrm{ES}}^{\mathrm{Exi}}} \sum_{c\in\Lambda^{\mathrm{C}}} \sum_{k=1}^{N} C_{\mathrm{Con}}^{a,I} C_{j,\mathrm{Ren}} S_{j,\mathrm{Ren}}^c 2^k l_i^a p_j^k e_j^c \\ & - \sum_{i\in\Psi_{\mathrm{ES}}^{\mathrm{Exi}}} \sum_{c\in\Lambda^{\mathrm{E}}} \sum_{k=1}^{N} \sum_{j\in\Psi_{\mathrm{ES}}^{\mathrm{Exi}}} \sum_{d\in\Lambda^{\mathrm{E}}} \sum_{m=1}^{N} 2^k p_i^k C_{i,\mathrm{Ren}} S_{i,\mathrm{Ren}}^c e_i^c 2^m p_j^m C_{j,\mathrm{Ren}} S_{j,\mathrm{Ren}}^d e_j^d\end{aligned} \tag{60}$$

The current objective function contains product terms of multiple binary variables, making it unsolvable directly. We use the Big-M method to transform the objective function into a linear form for more efficient solving.

Taking the product term $p_i^k e_i^c$ as an example, we introduce an intermediate variable. The product term can be determined by the following constraints:

$$\begin{aligned} & -M\cdot(1-p_i^k) - M\cdot(1-e_i^c) \leqslant x_i^{a,c} \\ & x_i^{a,c} \leqslant M\cdot(1-p_i^k) + M\cdot(1-e_i^c) \\ & -M\cdot p_i^k \leqslant x_i^{a,c} \leqslant M\cdot p_i^k \\ & -M\cdot e_i^c \leqslant x_i^{a,c} \leqslant M\cdot e_i^c \end{aligned} \tag{61}$$

Here, $x_i^{a,c}$, $y_{i,j}^{a,c,k}$, $z^{c,k,d,m}$, p_j^m, and e_j^d are auxiliary variables. The other product terms are handled similarly by introducing intermediate variables and adding the following constraints:

$$\begin{aligned} & -M\cdot(1-l_i^a) - M\cdot(1-p_j^k) - M\cdot(1-e_j^c) \leqslant y_{i,j}^{a,c,k} \\ & y_{i,j}^{a,c,k} \leqslant M\cdot(1-l_i^a) + M\cdot(1-p_j^k) + M\cdot(1-e_j^c) \\ & -M\cdot l_i^a \leqslant y_{i,j}^{a,c,k} \leqslant M\cdot l_i^a \\ & -M\cdot p_j^k \leqslant y_{i,j}^{a,c,k} \leqslant M\cdot p_j^k \\ & -M\cdot e_j^c \leqslant y_{i,j}^{a,c,k} \leqslant M\cdot e_j^c \end{aligned} \tag{62}$$

$$\begin{aligned} & -M\cdot(1-p_i^k) - M\cdot(1-e_i^c) - M\cdot(1-p_j^m) - M\cdot(1-e_j^d) \leqslant z_{i,j}^{c,k,d,m} \\ & z_{i,j}^{c,k,d,m} \leqslant M\cdot(1-p_i^k) + M\cdot(1-e_i^c) + M\cdot(1-p_j^m) + M\cdot(1-e_j^d) \\ & -M\cdot p_i^k \leqslant z_{i,j}^{c,k,d,m} \leqslant M\cdot p_i^k \\ & -M\cdot e_i^c \leqslant z_{i,j}^{c,k,d,m} \leqslant M\cdot e_i^c \\ & -M\cdot p_j^m \leqslant z_{i,j}^{c,k,d,m} \leqslant M\cdot p_j^m \\ & -M\cdot e_j^d \leqslant z_{i,j}^{c,k,d,m} \leqslant M\cdot e_j^d \end{aligned} \tag{63}$$

After processing, the objective function can be organized into the following form:

$$\begin{aligned}\max f = {} & C_{con}^{ori} \cdot \sum_{i\in\Psi_{ES}^{Exi}} \sum_{c\in\Lambda^{E}} \sum_{k=1}^{N} 2^k C_{i,Ren} S_{i,Ren}^c x_i^{a,c} \\ & - \sum_{i\in\Psi_{ES}^{Can}} \sum_{a\in\Lambda^{C}} \sum_{j\in\Psi_{ES}^{Exi}} \sum_{c\in\Lambda^{E}} \sum_{k=1}^{N} 2^k C_{Con}^{a,I} C_{j,Ren} S_{j,Ren}^c y_{i,j}^{a,c,k} \\ & - \sum_{i\in\Psi_{ES}^{Exi}} \sum_{c\in\Lambda^{E}} \sum_{k=1}^{N} 2^k C_{i,Ren} S_{i,Ren}^c 2^m C_{j,Ren} S_{j,Ren}^d z_{i,j}^{c,k,d,m}\end{aligned} \tag{64}$$

The entire model is a mixed-integer linear programming (MILP) model, which can be efficiently solved.

2 Numerical tests

The energy storage allocation model is demonstrated in this section. The formulated MILP problems are solved via GUROBI 9.0.

2.1 54-Node test system

This tested system includes 50 load nodes, 4 substation nodes, and 4 tie branches, as depicted in Figure 1. Circuit breakers and sectionalizing switches are installed on the feeder, some of which have been upgraded to be automatically operable. The interruption durations for manual switching and total repair are set as 0.5 h and 3 h, respectively. The confidence level for the reliability indicator is set to 0.95. There are 5 candidate locations for energy storage construction in the distribution network, and renewable energy is integrated at nodes 7, 9, 15, 22, 28, 36, and 42. Existing energy storage devices are located at nodes 16, 35, 37, and 47.

In the case study, the effectiveness of cooperation between distribution system operators and shared energy storage is evaluated. Under the same reliability requirements, the results of energy storage configuration by distribution system operators alone and in cooperation with shared energy storage are listed in Table 1. The results indicate that by collaborating with shared energy storage operators, distribution system operators can reduce part of the

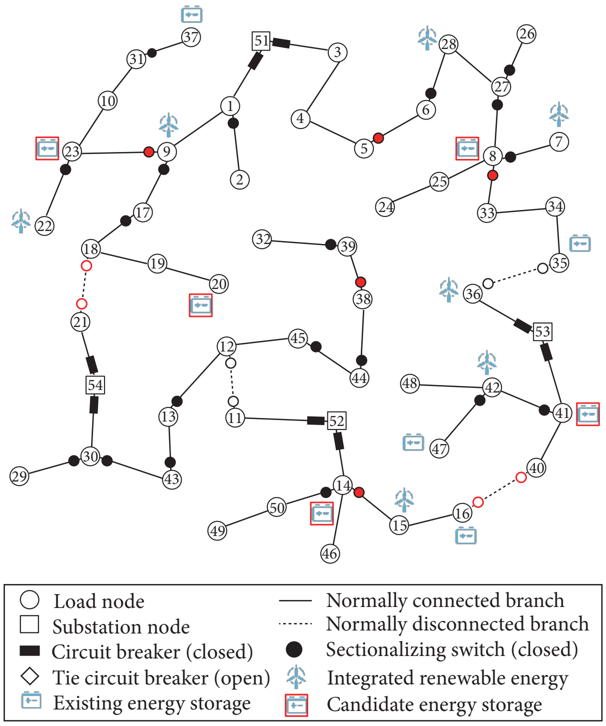

Figure 1 One-line diagram of the 54-node system.

Table 1 Comparison of energy storage configurations considering coordinated optimization

	Node	Uncoordinated	Coordinated
Rented shared energy storage capacity (MWh)	16	—	0.75
	35	—	0.6
	37	—	1.2
	47	—	0.5
Constructed energy storage capacity (MWh)	8	0.6	0.6
	14	0.6	0.6
	20	1.6	0.8
	23	1.8	1
	41	0.6	0.6
Investment cost of distribution system operators (k$)		260	220

energy storage construction costs by renting the capacity of existing shared energy storage. Compared with the construction of new energy storage facilities, renting existing storage is more cost effective. This cooperation reduces the investment costs for distribution system operators while providing financial benefits to shared energy storage operators offering auxiliary services.

2.2 Rental price of energy storage under different reliability requirements

The shared energy storage operator can determine the rental price per unit of storage capacity based on actual demand. The rental prices for energy storage at different nodes vary with the reliability requirements. To investigate the impact of system reliability requirements on the rental pricing strategy of shared energy storage operators in the game, Figure 2 presents the rental prices at different nodes corresponding to various system reliability requirements.

As shown in Figure, shared energy storage operators can flexibly adjust their pricing strategies in the game based on system reliability requirements. As the system reliability requirements increase, although fluctuations in rental pricing exist across different nodes, there is an overall upward trend. This trend aligns with the interests of shared energy storage operators. Therefore, the coordinated configuration method proposed in this paper can guide shared energy storage operators in offering reasonable rental prices.

2.3 Results of energy storage coordinated allocation under different reliability requirements

To investigate the impact of varying system reliability requirements on the energy storage configuration, we establish different reliability requirement scenarios (Cases A–D). The construction and rental status of energy storage are listed in Table 2.

The results in Table 2 indicate that as reliability requirements increase, the distribution system operator can initially meet the demand by increasing the renting of shared energy storage. Subsequently, the coordinated approach involving the combination of newly built storage and rented storage can be adopted. The investment cost comparison of whether the distribution system operator engages in coordinated configuration under different reliability requirements is shown in Figure 3.

Figure 3 shows that, compared with constructing energy storage independently, collaborating with shared energy storage for a coordinated configuration can reduce the investment costs for the distribution system operator. The extent of cost reduction increases with higher system reliability requirements. Additionally, the importance of shared energy storage becomes more pronounced as reliability requirements increase. Under higher reliability demands, the proportion of total investment accounted for by renting shared energy storage rises, highlighting the significance of collaboration with shared energy storage for distribution system operators.

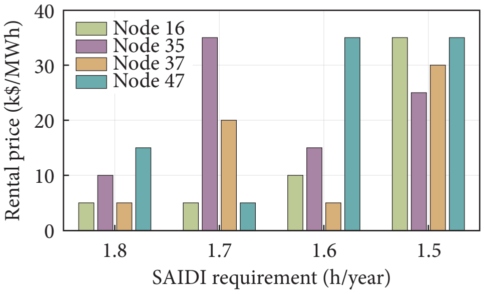

Figure 2 **Rental prices for energy storage at different nodes.**

Table 2 **Energy storage configuration under different reliability requirements**

Case	SAIDI requirement (h/year)	Construction capacity (MWh)	Rental capacity (MWh)
Case A	1.8	3	2.6
Case B	1.7	3	2.8
Case C	1.6	3.6	3.05
Case D	1.5	4.4	3.2

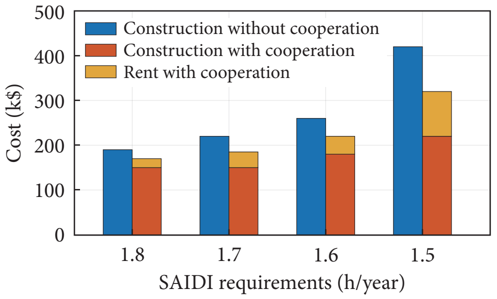

Figure 3 **Comparison of costs under coordinated allocation with different reliability requirements.**

3 Conclusions

This paper presents an energy storage coordinated configuration method for distribution networks and shared energy storage operators, focused on providing reliability ancillary services. Considering the involvement of different stakeholders, a Nash bargaining-based model for energy storage coordination and benefit allocation is developed, incorporating chance constraints to characterize the impact of renewable energy uncertainty. The model is ultimately transformed into a mixed-integer linear programming (MILP) problem for solution. Case studies under different reliability requirements demonstrate the rationality of the benefit allocation and the effectiveness of the cost savings achieved by the proposed method.

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Additional information

Declaration of competing interest

The authors have no competing interests to declare that are relevant to the content of this article.

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