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Inference of Abstraction for Grounded Predicate Logic

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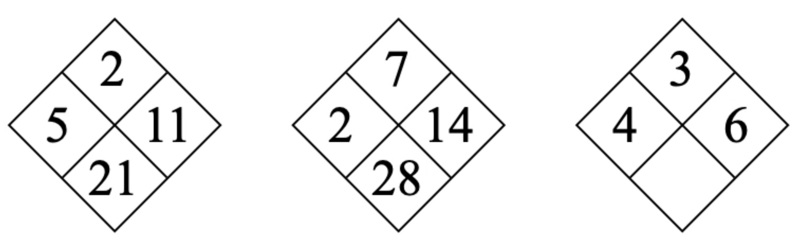
ABSTRACT An important open question in AI is what simple and natural principle enables a machine to ground logical reasoning in data for meaningful abstraction. This paper explores a conceptually new approach to combining probabilistic reasoning and predicative symbolic reasoning over data. We return to the era of reasoning with a full joint distribution before the advent of Bayesian networks. We then discuss that a full joint distribution over models of exponential size in propositional logic and of infinite size in predicate logic should be simply derived from a full joint distribution over data of linear size. We show that the same process is not only enough to generalise the logical consequence relation of predicate logic but also to provide a new perspective to rethink well-known limitations such as the undecidability of predicate logic, the symbol grounding problem and the principle of explosion. The reproducibility of this theoretical work is fully demonstrated by the included proofs.

MOTIVATING EXAMPLE

Humans can think abstractly and logically from experiences.

Example 1

What number fits in the blank?



18 because of the following knowledge that can be expressed in predicate logic.

$$top \times left + right = bottom$$

Example 2

- Alice and Bob did not blame each other.
- One day Alice blamed Bob, and she blamed herself afterwards.
- Alice and Bob blamed each other on another day.

Carol remembers these senes. One day, she wanted to blame Bob, but she didn't. Why?

She knows that someone will blame her because of the knowledge, expressed in predicate logic.

$$\forall x(\exists y(Blames(x,y)) \to \exists z(Blames(z,x)))$$

Fundamental question

For agents to reason abstractly and logically from experiences, what mathematical principle can ground predicate logic in data?

RELATED WORK

Probabilistic logic learning: inductive logic programming (ILP, Muggleton, 91, Nienhuys-Cheng & Wolf, 97), probabilistic logic programming, (PLP, Sato, 95),

Statistical relational learning: Bayesian networks (BN, Pearl, 88), probabilistic relational model (PRM, Friedman et al., 96), Markov logic networks (MLN, Richardson & Domingos, 06), etc.

Fundamental problems

- **Symbol grounding / inference grounding**

No simple theory explains how pieces of abstract knowledge (e.g., symbols and networks) and even inferences emerge from concrete data.

- **Separation of learning and reasoning**

Learning is the process of extracting knowledge from data, while reasoning is the process of drawing new conclusions from that knowledge. Learning methods cannot be used as reasoning methods, and vice versa.

- **Reasoning under anomalies**

Agents cannot reason from inconsistent or impossible information, due to the principle of explosion in formal logic and division by zero in probability theory.

- **Huge hypothesis spaces**

The size of a probability distribution over models is infinite in predicate logic and exponential in propositional logic. The existing solutions demand independence and conditional independence assumptions.

OUR SOLUTION

✅ Modelling how knowledge emerges from data (see →)

❎ Modelling logical/probabilistic knowledge and inference over such knowledge (see ↓)

BN: $rain \rightarrow wet$

Rain	p(Wet=1|Rain)
0	0.1
1	0.95

ILP:

$Fish(x) \leftarrow Swims(x)$

$Swims(sharky)$

$Fish(sharky)$

MLN:

$1.5 : smokes(x) \Rightarrow cancer(x)$

$2.0 : friends(x,y) \Rightarrow (smokes(x) \Leftrightarrow smokes(y))$

PLP:

alarm :- earthquake.

0.2 : earthquake.

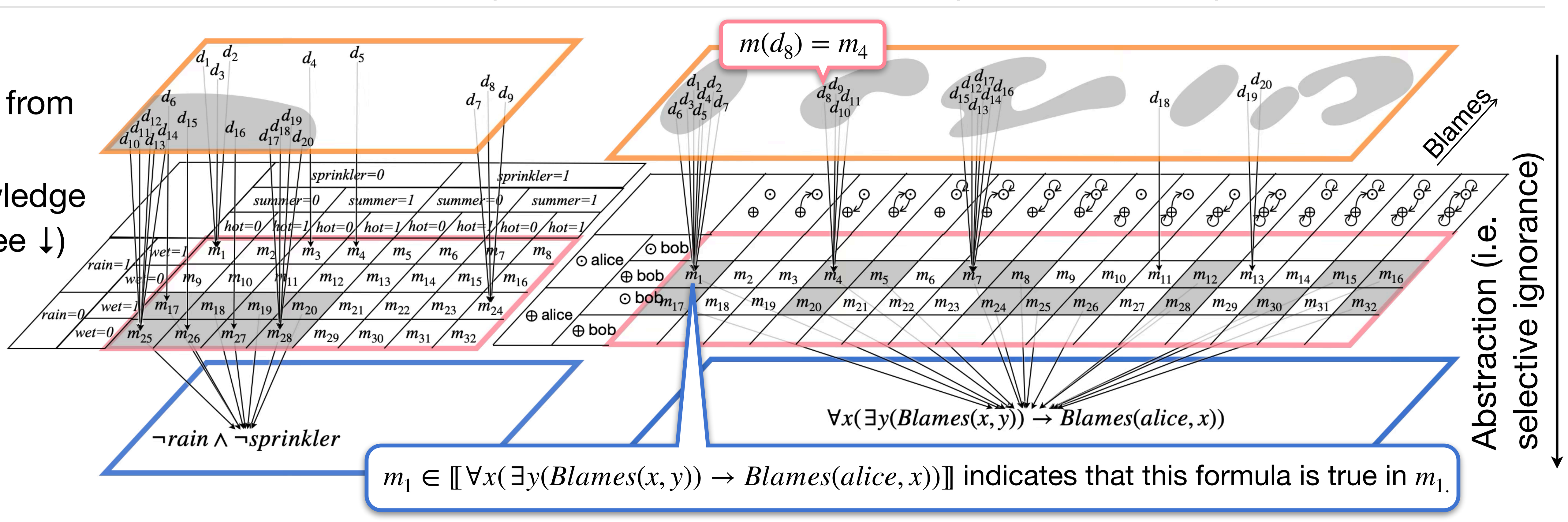


Fig: Inference of abstraction for propositional logic (left: Kido, 25) and predicate logic (right). Top: data d_k. Middle: models m_n (i.e. pairs of domains and valuation functions). Bottom: formula x_i.

Definition (Data distribution)

$$p(d_k) \overset{\text{Def.1}}{=} \frac{1}{\#\text{ Data}}$$

i.e., data is distributed uniformly, e.g., $p(d_8) = 1/20$

Definition (Model distribution)

$$p(m_n|d_k) \overset{\text{Def.2}}{=} \begin{cases} 1 & \text{if } m_n = m(d_k) \\ 0 & \text{otherwise} \end{cases}$$

i.e., each data point supports a single model, e.g., $p(m_4|d_8) = 1$

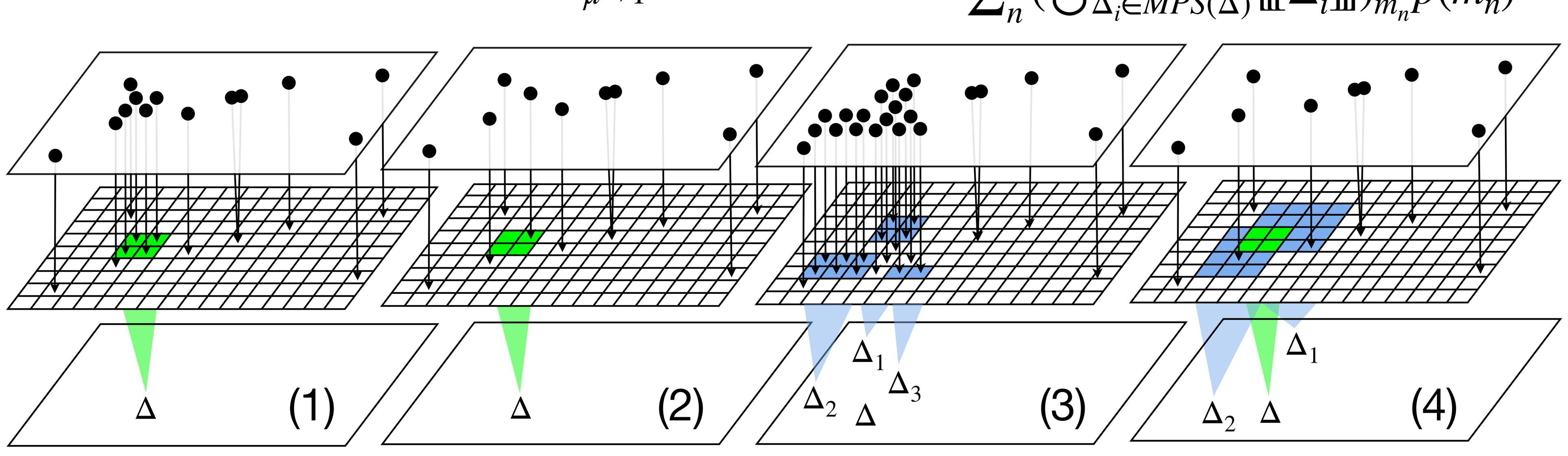
Definition (Knowledge distribution) Let $\mu \in [0.5, 1]$.

$$p(x_1|x_2, \cdots, x_I, m_n, d_k) \overset{\text{Def.3}}{=} \begin{cases} \mu & \text{if } m_n \in [\![x_1]\!] \\ 1-\mu & \text{otherwise} \end{cases} \overset{\text{Prop.1}}{=} p(x_1|m_n)$$

i.e., the truth value obeys the semantics of predicate logic, e.g., $p(Blames(bob,alice)|m_2) = p(Blames(bob,alice)|m(d_8)) = \mu$

EVALUATIONS

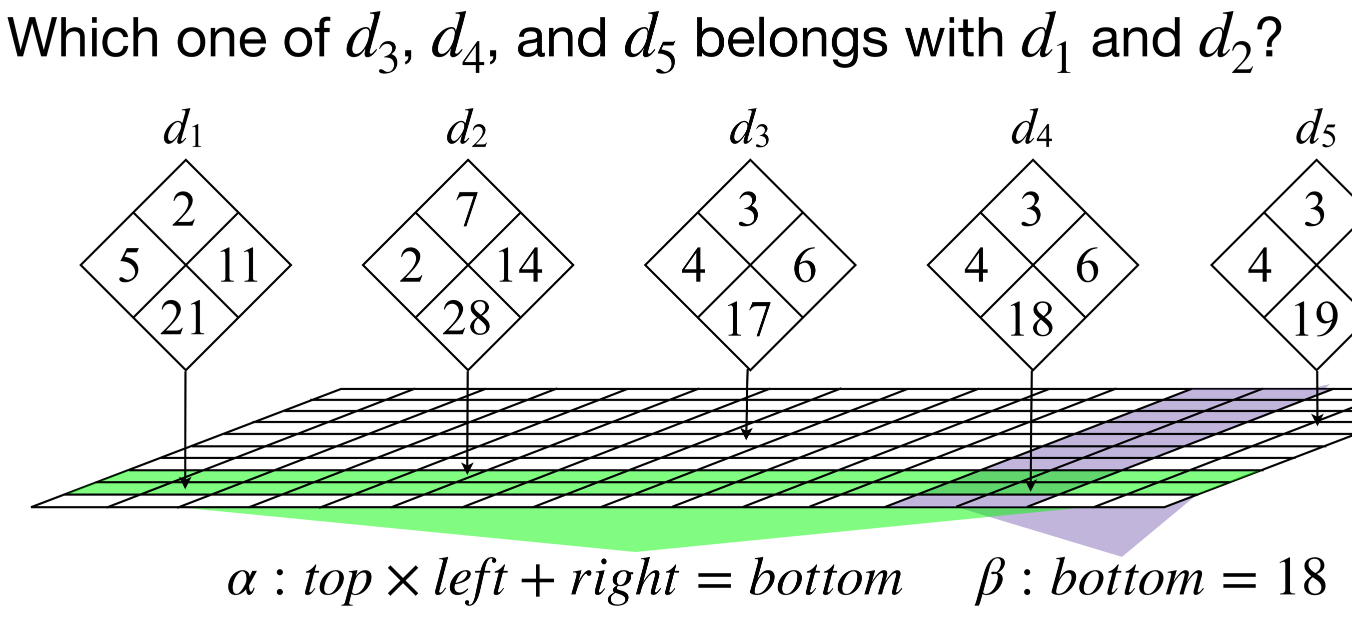
Fig: $\lim_{\mu \to 1} p(\alpha|\Delta) \overset{\text{Thm.2}}{=} \frac{\sum_n [\![\alpha]\!]_{m_n} (\bigcup_{\Delta_i \in MPS(\Delta)} [\![\Delta_i]\!])_{m_n} p(m_n)}{\sum_n (\bigcup_{\Delta_i \in MPS(\Delta)} [\![\Delta_i]\!])_{m_n} p(m_n)}$



Reasoning under	Results: $\lim_{\mu \to 1} p(\alpha\|\Delta) = 1$ iff	Assumptions
(1) Consistency	$\Delta \models \alpha$ (Cor.5)	$[\![\Delta]\!] = [\![\Delta]\!] \neq \emptyset$ from the two below
(2) Possibility	$\Delta \models \alpha$ (Cor.4)	$[\![\Delta]\!] \neq \emptyset$
(3) Inconsistency	$\forall \Delta_i \in MCS(\Delta) . \Delta_i \models \alpha$ (Cor.3)	$\bigcup_{\Delta_i \in MPS(\Delta)} [\![\Delta_i]\!] = \bigcup_{\Delta_i \in MCS(\Delta)} [\![\Delta_i]\!]$
(4) Impossibility	$\forall \Delta_i \in MPS(\Delta) . \Delta_i \models \alpha$ (Cor.2)	No assumption
Uncertainty	$\lim_{\mu \to 1} p(\alpha\|\Delta) \in [0,1]$	Same as above

Example

Which one of d_3, d_4, and d_5 belongs with d_1 and d_2?



Top-down reasoning with $d_{1\text{-}2}$

$$p(\alpha) = \sum_{n=1}^{\infty} \sum_{k=1}^{2} p(\alpha, m_n, d_k) = \sum_{k=1}^{2} p(\alpha|m(d_k))p(d_k) = \frac{1}{2} \sum_{k=1}^{2} [\![\alpha]\!]_{m(d_k)} = 1$$

Bottom-up and top-down reasoning with $d_{3\text{-}5}$

$$p(\beta|\alpha) = \frac{\sum_{n=1}^{\infty} \sum_{k=3}^{5} p(\beta, \alpha, m_n, d_k)}{\sum_{n=1}^{\infty} \sum_{k=3}^{5} p(\alpha, m_n, d_k)} = \frac{\sum_{k=3}^{5} p(\beta|m(d_k))p(\alpha|m(d_k))}{\sum_{k=3}^{5} p(\alpha|m(d_k))} = \frac{1}{1}$$