IMA Journal of Management Mathematics (2025) **36**, 649–667
https://doi.org/10.1093/imaman/dpaf035
Advance Access publication on 11 September 2025

Simple Economic Order Quantity heuristics for stochastic inventory control

RUUD H. TEUNTER[†]
Department of Operations, University of Groningen, Zernike Campus, Duisenberg Building, Nettelbosje 2, 9747 AE Groningen, Netherlands
[†]Corresponding author. Email: r.h.teunter@rug.nl

M. ZIED BABAI
Kedge Business School, Campus Bordeaux, 680 Cours de la Libération, 33405 Talence, France

AND

ARIS A. SYNTETOS
PARC Institute of Manufacturing, Logistics and Inventory, Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, UK

[Received on 09 January 2025; accepted on 02 September 2025]

Accepted by: Phil Scarf

Derived for deterministic demand, the Economic Order Quantity (EOQ) formula remains a popular method for stochastic demand, typically in combination with an order level. Textbooks split the inventory cost into the 'cycle inventory' cost (the cost of holding half the order quantity) and the cost of safety stock (order level minus expected lead time demand), showing that the EOQ minimizes the total cost of ordering and the cycle-inventory cost. However, under stochastic demand, the EOQ is smaller than the optimal order quantity and often much smaller. A number of authors have suggested exact procedures for determining the optimal order quantity (and order level), but the derivations (and resulting procedures) are complicated, in contrast with the intuitively appealing nature of EOQ and its simplicity. This paper presents an alternative approximation, leading to closed-form order quantity formulas under both a cost and service objective for normally distributed lead-time demand. It splits inventory costs, but (i) uses that the average cycle stock is less than half of the order quantity due to backorders, and (ii) considers inventory left-over at the end of a cycle instead of safety stock. A numerical investigation shows that the approximation is very accurate, with a cost error of <0.02% on average. For the traditional EOQ formula, the cost error is considerable, going up to 6% in some cases, and so it is worthwhile in many real-life situations to use our newly proposed formulas. Moreover, for teaching and training purposes, the adaptations help understand why the EOQ is suboptimal.

Keywords: inventory; EOQ; cost model; service model; software; teaching OM and OR.

1. Introduction

The aim of our research is to offer alternatives to the Economic Order Quantity (EOQ) formula, one of the major approaches for production and inventory management (Moily, 2015; Nakhaeinejad, 2024) used in industry to balance stock investment and fixed replenishment cost efficiently (Silver et al., 2009). This is upon recognition that EOQ calculations fall short of demand realities, i.e. that demand is (almost) always stochastic. There are, of course, other strong assumptions underlying the EOQ that researchers have challenged and for which adaptations have been proposed. For instance, Hill (1999) analyses how the optimal order quantity deviates from the EOQ towards the end of a product life cycle, and Chang et al. (2011) include imperfect quality and inspection. However, we argue that certainty of demand is the strongest assumption, since (in our experience) it never applies in real life settings. Previous research (reviewed in the next section) has shown for multiple settings that, as a result, the EOQ is always smaller than the optimal order quantity. The intuitive explanation for this is as follows. At the end of each cycle, backorder costs may be incurred. Therefore, increasing the order quantity—and thereby decreasing the number of cycles per time unit—has two cost benefits: this reduces the backordering costs as well as the fixed replenishment costs per time unit. The EOQ ignores the first benefit and only balances the fixed replenishment cost reduction with the holding cost increase. By underestimating the total benefit of an order quantity increase, the EOQ methodology leads to an order quantity that is less than the optimum order quantity.

Some procedures have been proposed in the literature to address this issue (e.g. Hadley & Whitin, 1963; Axsäter, 2015) all of which however are complicated and as such in direct contrast with the intuitively appealing nature of EOQ and its simplicity. It is indeed this intuitive appeal that renders the EOQ one of the most frequently used methods in (i) practice but also in (ii) classrooms and training rooms. With regard to practice, the EOQ is used both in re-order level systems, but very much so also in re-order level order-up-to (OUT) level systems, where the difference between the two parameters is approximated by the EOQ (Porteus, 1985). For education, EOQ is the preferred means to teach inventory related matters in generalist, say Operations or MBA, courses, and to organize lab experiments on behavioral issues in inventory (Stangl & Thonemann, 2017; Perera et al., 2020).

The classic EOQ formula developed by Harris (1913) is given by EOQ $= \sqrt{2K\mu/h}$ where μ denotes the demand rate, h the holding cost per item per time unit and K the fixed cost per replenishment order (so, independent of how many units are ordered). It is easy to show that this is the optimal order quantity if the demand rate is constant and continuous, the lead-time is constant and backorders are not allowed. These assumptions never apply in practice, though. In particular, the demand rate is always uncertain. Nevertheless, in both theory (inventory courses) and practice, the EOQ formula is often used to determine the order quantity under stochastic demand (Rumyantsev & Netessine, 2007). It is then typically combined with a safety stock and a corresponding order level, R, resulting in an (R, Q) policy that places a new order for Q items, when the inventory position (on hand + on order − backorders) drops to R (Berling & Marklund, 2006). It is also used in min max inventory systems, where the re-order and OUT level are seldom both/jointly optimized, but rather replenishments rely upon an optimized re-order level to which the EOQ is added to calculate the OUT level.

As will be discussed in detail in the next section, previous research has shown that the relative cost error of using the EOQ under stochastic demand is often small, but also that it can be more than 10%. So, especially in settings with low profit margins (e.g. retail), carefully considering whether or not to adopt the EOQ is worthwhile.

1.1. *Contribution and organization of the paper*

With this in mind, in this paper, we revisit the EOQ in the presence of demand uncertainty, considering the case that demands during stock-out periods are backordered. The objective is to find the values for R and Q that (approximately) minimize the total (holding and fixed replenishment) cost per time unit, where we concentrate on approximating the optimal order quantity. We consider both a cost model and a service model. The cost model includes a backordering cost b per unit and time unit, whilst the service model instead specifies a target fill rate, denoted by β. The latter model is more practical, given the difficulty of estimating the backordering cost in real life. Most of the inventory control literature considers cost models, though, to avoid having to optimize under a constraint (Hadley & Whitin, 1963; Axsäter, 2015). The cost and service model are very closely related. Indeed, it is well known that for most demand processes (including Normal lead-time demand, which is considered most), the optimal fill rate is given by $\frac{b}{b+h}$ (see, e.g. Axsäter, 2015). Interestingly, though, it will appear from our results that setting the target fill rate to that fraction in the service model does not give the same optimal order quantity as in the cost model.

As our literature review in the next section shows, a number of authors have presented methods for calculating the optimal order quantity (and the corresponding optimal order level) exactly. However, these are typically non-intuitive, complex and often iterative. We, instead, take a more intuitive approach that splits the inventory holding costs into cycle holding costs and 'left-over' holding costs (at the end of a cycle). This mimics the traditional reasoning of deriving the EOQ based on cycle holding costs only and considering 'safety stocks' to set the order level. Considering left-over stocks instead of safety stocks leads to better holding cost approximations, as we will explain in our analysis. For both the cost and service model, we derive closed-form order quantity formulas. A numerical analysis shows these to be very accurate. Furthermore, comparing these formulas to each other and to the traditional EOQ formula provides many insights.

Our main contributions are as follows. First, we derive simple and accurate closed-form expressions for near-optimal order quantities for both the cost and service model. Second, comparing them to the traditional EOQ formula we provide many insights. In particular, the cost and service formulas show: (i) how the suboptimality of the EOQ is related to both overestimating the cycle stock and ignoring the reduction in safety stocks when placing larger orders, and (ii) that even if the target service level for the service model is set to the optimal level for the cost model, the optimal order quantities differ. Third, the intuitive approach for splitting the stock into cycle stock and left-over stock is easy to include in inventory management courses, providing valuable insights into how demand uncertainty affects the optimal order quantity. Fourth, although we consider Normal lead-time demand, most of our analysis holds for any lead-time demand distribution, and the part that does depend on a specific distribution can be adapted to other ones—although the difficulty of doing so depends on the type of distribution.

The remainder of the paper is organized as follows. In the next section we review the literature, distinguishing between the cost and service approach. In Section 3 we present the new order quantity formulae. Sections 4 and 5 discuss the EOQ under a fill rate constraint and with backorder costs, respectively. We conduct a numerical investigation in Section 6 and offer important insights for inventory theory and practice. We conclude in Section 7, where we also discuss the next steps of research.

2. Literature review

If the lead-time is negligible and backorders are not allowed, then the traditional EOQ formula is still optimal under stochastic demand if that follows a renewal process (Chao, 1992; Maddah & Noueihed, 2017). However, this is no longer true if the lead-time is positive. Backorders can then occur and, related, an important distinction in the stochastic inventory control literature is between a cost approach, where backorders are penalized, and a service approach, specifying a service level constraint. The service approach is arguably more practical, as backordering costs are typically very hard to quantify (Jiang et al., 2019). However, as optimization under a constraint is typically harder, the cost approach is dominant in the inventory control literature. We next discuss each approach. Our attention is mainly on studies considering the optimization of the (R,Q) policy. Moreover, we focus on the classic setting with a fixed cost per replenishment order, a holding cost per item per time unit and either a backordering cost per item per time unit or a fill rate constraint.

2.1. *Cost approach*

Authors have proven for various settings with stochastic demand that the optimal order quantity is always greater than that EOQ. Zheng (1992, Theorem 2) does so for the standard (R, Q) policy, and Christou et al. (2020, Proposition 5.3.) extend this result to the (r, nQ, T) policy (also known as (R, s, Q) policy). Rao (2003, Theorem 12) proves that for the base stock (R, S) policy with varying order quantities, the average order quantity is always greater than EOQ. This result is extended by Lagodimos et al. (2018, Proposition 5.5) for the base stock policy under discrete time, using the discrete time version of the EOQ.

For the standard (R, Q) policy that we also consider, various techniques have been suggested for determining the optimal values for policy parameters R and Q (see e.g. Hadley & Whitin, 1963; Federgruen & Zheng, 1992; Axsäter, 2015). However, although not computationally intensive, these approaches are fairly complex and often iterative, and in our experience seldom implemented—even in dedicated inventory software. Instead, a two-step approach is often used, where the order quantity is determined first, after which the order level is optimized given that order quantity.

Most commonly, the order quantity (in the first step) is determined using the classic EOQ formula. Several authors have analyzed the performance of this approach. Zheng (1992) derives an upper bound of $1/8 = 0.125$ on the relative cost error resulting from the use of the deterministic EOQ formula as a heuristic solution for a stochastic demand. Axsäter (1996) derives a slightly stronger upper bound of $\left(\sqrt{5}-2\right)/2 \approx 0.118$. He also shows that this bound is tight by presenting examples where the cost error is arbitrarily close to the lower bound.

Gallego (1998) shows that the optimal order quantity is always between one and two times the standard EOQ and uses that to show that an alternative order quantity (in the first step) of $\sqrt{2}$ times the EOQ leads a better lower bound of 1.061 on the relative cost error.

2.2. *Service approach*

Silver and Wilson (1972) derive values for R and Q based on approximating the on-hand inventory as $\frac{Q}{2} + \left(R - \mu_L\right)$, where μ_L denotes mean demand during the lead-time. However, as will also become apparent from our analysis, the first term overestimates the average cycle stock while the second term underestimates the on-hand stock at the end of a cycle (i.e. the safety stock). Moreover, Silver and Wilson do not determine closed-form expressions for R and Q, but a system of two nonlinear equations that need to be solved.

Yano (1985) presents both an exact algorithm and a heuristic for optimizing R and Q. She also shows global convergence of the exact algorithm. The heuristic is simpler in that it avoids the numerical enumeration of integrals. However, both the exact algorithm and the heuristic are iterative procedures. Alstrøm (2001) also develops an approximate analysis under a fill rate constraint, but it is again rather complex and does not lead to a closed-form expression.

Axsäter (2006) argues, as we do, that simplicity is important for practical applications, and present an alternative two-step procedure for normally distributed lead-time demand that has a negligible cost error. However, the order quantity approximation is not closed-form and determining it involves looking up an intermediate result from a table for a grid of parameter values (and interpolating in between those values).

Platt et al. (1997) derive several heuristics for determining R and Q. They do so by applying Lagrangian relaxation on the exact cost function under the fill rate constraint, and then using limiting behavior (with EOQ converging to zero or infinity) of the loss function. One of these has a closed form expression for the order quantity, namely $\frac{1}{\beta}\sqrt{\frac{2K\mu}{h} + \sigma_L^2}$, where σ_L denotes the standard deviation of

TABLE 1 *Notations*

Model parameters	
K	Fixed cost per replenishment order
h	Holding cost per unit per time unit
μ	Demand rate (per time unit)
σ	Standard deviation of the demand (per time unit)
L	Replenishment lead-time
μ_L	Expected lead time demand
σ_L	Standard deviation of lead time demand
$f(u)$	Density function of lead-time demand
β	Fill rate, i.e. fraction of demands satisfied immediately
b	Backordering cost per unit per time unit (for the cost model)
Other notations	
R	Order level
Q	Order quantity
$E[B]$	Expected number of backorders that occur in a cycle
α	Probability of no backorders when a replenishment order arrives, aka Cycle Service Level
E_O	Expected number of orders placed per time unit
E_B	Expected number of backorders (at any time)
E_{CS}	Expected cycle stock per time unit (at any time)
E_{LS}	Expected left-over stock per time unit (at any time)
C_O	Expected fixed replenishment cost per time unit
C_B	Expected backordering cost per time unit
C_{CS}	Expected cycle stock holding cost per time unit
C_{LS}	Expected left-over stock holding cost per time unit
C_T	Expected total cost per time unit

lead-time demand and β is the required fill rate. We will compare this formula to the new ones resulting from our analysis in Section 4.

3. Approximations for cost components

The notations used in this and remaining sections are listed in Table 1. For completeness, we include here notations that have also already been defined, Moreover, and for ease of presentation, the derivative of some function $f(.)$ with respect to the order quantity Q is denoted by $f'(.)$, i.e. $f'(.) \equiv \frac{d}{dQ}f(.)$.

This section presents the building blocks for deriving EOQ formulae in later sections. These are (approximate) expressions for (derivatives of): the expected number of orders per time unit (Section 3.1), the expected backorder level (Section 3.2), the expected cycle inventories (Section 3.3) and the expected left-over inventories (Section 3.4). The latter are closely linked to safety stock inventories but are not the same. Safety stock inventories are usually defined as the difference between the re-order level and the expected lead-time demand. Left-over inventories are, as the name indicates, the (expected) left-over stock just before a new batch arrives. Therefore, left-over stocks are directly linked to inventory costs, which does not apply to safety stocks. Perhaps the easiest way to see this is that for a zero safety stock, the expected left-over stock is still positive; in fact, half of the cycles then ends with positive left-over stock.

3.1. *Expected orders per time unit*

The average duration of an order cycle is $\frac{Q}{\mu}$, and so the expected number of orders per time unit is easily obtained as:

$$E_O = \frac{\mu}{Q},$$

and so

$$E_O' = -\frac{\mu}{Q^2}. \tag{2}$$

3.2. *Expected backorder level*

From the definition of fill rate β, we have that from every batch of Q ordered items, on average $(1-\beta)\,Q$ arrive late. So, when an order arrives, the expected number of backordered units is $E\,[B] = (1-\beta)\,Q$. Hence, during a cycle, the number of backordered units is zero until the stock runs out (if that happens), and then increases from zero to (on average) $(1-\beta)\,Q$. Therefore, during the latter part of a cycle with positive backorders, the average number of backordered units is $\frac{1}{2}\,(1-\beta)\,Q$. Moreover, as the ready rate equals the fill rate (for normal lead-time demand), the expected fraction of time with positive backorders is $(1-\beta)$. Hence, the expected backorder level is approximately

$$E_B \approx \frac{1}{2}(1-\beta)^2 Q,$$

and so

$$E_B' \approx \frac{1}{2}(1-\beta)^2. \tag{3}$$

3.3. *Expected cycle inventory level*

When an order arrives, the expected number of backordered units is $E\,[B] = (1-\beta)\,Q$ and so the expected cycle stock just after an order arrival is $Q - (1-\beta)\,Q = \beta Q$. Hence, for an 'average cycle', the cycle stock varies between βQ and 0 and is $\frac{1}{2}\beta Q$ on average when positive. Moreover, a fill rate of β implies that stock on hand is positive for fraction β of the time, so that the cycle stock can also only be positive for fraction β of the time. Therefore, the expected cycle stock level is approximately

$$E_{CH} \approx \frac{1}{2}\beta Q \times \beta = \frac{1}{2}\beta^2 Q,$$

and so

$$E_{CH}' \approx \frac{1}{2}\beta^2. \tag{4}$$

3.4. *Expected left-over inventory level*

The expected left-over stock equals

$$E_{LH} = \int_{-\infty}^{R} f(u)\,(R-u)\,du.$$

As indicated by this expression, the expected left-over stock is dependent on the lead-time demand distribution. In what follows, we first show how its derivative is related to service levels for any demand distribution, and then use that to find a closed-form approximation for normal lead-time demand.

The effect of the order quantity on the expected left-over stock can be cut up into three steps: (i) given the required/optimal fill rate, a larger order quantity implies a longer cycle and so more expected backorders per cycle, (ii) more expected backorders per cycle implies a lower order level, and (iii) a lower order level reduces the expected left-over stock. Correspondingly, we can write the derivative of E_{LH} as

$$E'_{LH} \equiv \frac{dE_{LH}}{dQ} = \frac{dE_{LH}}{dR} \times \frac{dR}{dE\,[B]} \times \frac{dE\,[B]}{dQ}.$$

Rewriting the three terms (inverted for the second) on the RHS as

$$\frac{dLE_{LH}}{dR} = \frac{d}{dR}\int_{-\infty}^{R} f(u)\,(R-u)\,du = \int_{-\infty}^{R} f(u)du = \alpha,$$

and

$$\frac{dE\,[B]}{dR} = \frac{d}{dR}\int_{R}^{\infty} f(u)\,(u-R)\,du = -\int_{R}^{\infty} f(u)du = -\,(1-\alpha)\,,$$
$$\frac{dE\,[B]}{dQ} = \frac{d}{dQ}\,[(1-\beta)\,Q\,] = (1-\beta)\,,$$

we get

$$E'_{LH} = -\,(1-\beta)\,\frac{\alpha}{(1-\alpha)}. \tag{5}$$

As discussed before, the difficulty in analyzing the left-over stock is that the lead-time distribution plays a crucial role, which can also be seen from Equation (5) as the relation between the cycle service level α and the fill rate β depends on that distribution. Fortunately, for the most commonly assumed Normal lead time demand, we can use the following approximate result from statistics, formulated in terms of demand.

> (Shore, 1982): Consider some 'safety factor' x. For a standard Normal demand distribution, if the probability of zero loss (i.e. that demand is at most x) equals α, then the expected loss (i.e. the expected demand above x) is approximately $0.4115\frac{(1-\alpha)}{\alpha}$.

We remark that Shore (1982) shows that this is especially accurate for $\alpha \geq \frac{1}{2}$, i.e. for non-negative safety stocks, which applies to almost all real-life situations.

Since the expected loss of any Normal distribution is proportional to the standard deviation, Shore's approximation translates to

$$\frac{E\,[B]}{\sigma_L} \approx 0.4115\frac{(1-\alpha)}{\alpha}.$$

Using $E\,[B] = (1-\beta)\,Q$, this gives

$$(1-\beta)\,\frac{\alpha}{(1-\alpha)} \approx \frac{0.4115\sigma_L}{Q}.$$

Inserting this into Equation (5), we get

$$E'_{LH} \approx -\frac{0.4115\sigma_L}{Q}. \tag{6}$$

4. EOQ under a fill rate constraint

Under a fill rate constraint, backorder costs are not considered. So

$$C_T = C_O + C_{CH} + C_{LH} = KE_O + hE_{CH} + hE_{LH},$$

which gives

$$C'_T = KE'_O + hE'_{CH} + hE'_{LH}.$$

Using Equations (2), (4), and (6), we then get:

$$C'_T \approx -\frac{K\mu}{Q^2} + \frac{1}{2}h\beta^2 - h\frac{0.4115\sigma_L}{Q} \tag{7}$$

,which is zero for

$$\tilde{Q}_{\text{service}} = \frac{0.4115\sigma_L}{\beta^2} + \sqrt{\left(\frac{1}{\beta}\sqrt{\frac{2K\mu}{h}}\right)^2 + \left(\frac{0.4115\sigma_L}{\beta^2}\right)^2} = \frac{0.4115\sigma_L}{\beta^2} + \sqrt{\left(\frac{1}{\beta}\text{EOQ}\right)^2 + \left(\frac{0.4115\sigma_L}{\beta^2}\right)^2}. \tag{8}$$

Note that there are two factors that cause $\tilde{Q}_{\text{service}}$ to be larger than the traditional EOQ. Firstly, different from the EOQ analysis, the term β^2 in Equation (7) considers that (i) the expected maximum cycle stock (after satisfying backorders upon arrival) is fraction β of the ordered amount, and (ii) cycle stock can only be positive for fraction β of the time. As a result, $\tilde{Q}_{\text{service}}$ equals at least $\frac{1}{\beta}$EOQ. Secondly, if a larger order quantity implies that more backorders are allowed per cycle, the order level (and with it the expected left-over stock) can be reduced. This benefit of applying a larger order quantity increases with demand uncertainty, explaining why $\tilde{Q}_{\text{service}}$ increases with the lead-time demand variance.

The numerical results of Section 6 will show that $\tilde{Q}_{\text{service}}$ is very close to the optimal order quantity and can be much higher than the EOQ.

5. EOQ with backorder costs

Including backorder costs, the total cost becomes

$$C_T = C_O + C_{CH} + C_{LH} + C_B = KE_O + hE_{CH} + bE_B,$$

which gives

$$C_T' = KE_O' + hE_{CH}' + hE_{LH}' + bE_B'.$$

Using Equations (2)–(6), we then get:

$$C_T' \approx -\frac{K\mu}{Q^2} + \frac{1}{2}h\beta^2 - h\frac{0.4115\sigma_L}{Q} + \frac{1}{2}b(1-\beta)^2. \tag{9}$$

As mentioned in the introduction, it is well-known that the optimal fill rate for the cost model by $\frac{b}{b+h}$. Setting β in Equation (9) to that level, we get

$$C_T' \approx -\frac{K\mu}{Q^2} + \frac{1}{2}h\left(\frac{b}{b+h}\right)^2 - h\frac{0.4115\sigma_L}{Q} + \frac{1}{2}b\left(\frac{h}{b+h}\right)^2,$$

which is zero for

$$\begin{aligned}\widetilde{Q}_{\text{cost}} &= 0.4115\sigma_L\frac{b+h}{b} + \sqrt{2K\mu\frac{b+h}{bh} + \left(0.4115\sigma_L\frac{b+h}{b}\right)^2} \\ &= 0.4115\sigma_L\frac{b+h}{b} + \sqrt{\left(\sqrt{\frac{b+h}{b}}\text{EOQ}\right)^2 + \left(0.4115\sigma_L\frac{b+h}{b}\right)^2}. \end{aligned} \tag{10}$$

Note that for $\sigma_L = 0$, this gives $\widetilde{Q}_{\text{cost}} = \sqrt{\frac{b+h}{b}}\text{EOQ}$, which is indeed the optimal order quantity without demand uncertainty (see, e.g. Axsäter, 2015). Moreover, compared to that deterministic solution, $\widetilde{Q}_{\text{cost}}$ increases via the term $0.4115\sigma_L\frac{b+h}{b}$. The logic is that increases in either σ_L or $\frac{b+h}{b}$ imply a higher optimal safety stock (via a wider demand distribution and a larger optimal fill rate, respectively), and a larger order quantity allows for a reduction of the safety stock.

An interesting result from comparing $\widetilde{Q}_{\text{service}}$ in Equation (8) and $\widetilde{Q}_{\text{cost}}$ in Equation (10) is that when we set the target fill rate β for the service model to the optimal fill rate $\beta^* = \frac{b}{b+h}$ for the cost model, the service model has a larger approximately optimal order quantity. In fact, if all β^2 terms in Equation (8) are replaced by β^*, then Equation (10) is obtained. This difference exists, because the cost model considers backorder duration (implying that backorder costs are only incurred during a typically small part of an order cycle), but the service model does not. It raises the more general question of how to model backorder costs for inventory management, which we will discuss in Section 7.

TABLE 2 *Instances considered in the numerical investigation*

Parameter	Value range
μ	100
σ	10, 30, 50
L	$1, 3, 5$
K	$25, 100, 400$
β	90%, 95%, 98%
h	1
$b/(b+h)$	90%, 95%, 98%

6. Numerical investigation

In this section, a numerical investigation quantifies the benefits of using the proposed EOQ approximations~~,~~, referred hereafter to as *New Approx.*, compared to using the traditional EOQ approximation and using the Platt et al. approximation. Note that for the cost model, the formula of Platt et al. (1997) is adapted by setting the fill rate equal to $b/(b+h)$.

Using exact expressions from the literature for the cost and fill rate of the (R,Q) policy, given for completeness in Appendix A, we determine (by complete search) the optimal unrestricted order quantity, order level, and corresponding minimal cost; as well as the optimal order level and corresponding cost for the different EOQ approximations. The percentage cost gap (i.e. increase vs. optimal) for an approximation is denoted hereafter by ΔCost.

We vary the values of the parameters in order to analyze their impact on the performance of all the considered approximations. We consider a demand that has a mean $\mu = 100$ and a standard deviation $\sigma = 10,\ 30,\ 50$. The lead-time is varied as $L = 1, 3, 5$. The fixed replenishment cost per order $K = 25, 100, 400$. Under the fill rate constraint, we consider realistic targets of $\beta = 90\%, 95\%, 98\%$. With a backorder cost, we assume that the unit holding cost is normalized to $h = 1$ and the backorder unit cost is $\frac{b}{(b+h)} = 90\%, 95\%, 98\%$. A summary of the values considered in the different instances is provided in Table 2.

Hence, both under a fill rate constraint and with a backorder cost, we consider $3 \times 3 \times 3 \times 3 = 81$ instances. The detailed results (values) for all control parameter combinations are provided in Tables B1–B3 and C1–C3 of Appendices B and C for the service and cost model, respectively. In Table 3, for both the service model (LHS) and the cost model (RHS), we show the average (per fill rate or backorder cost setting) absolute error in the order quantity for the three considered approximations.

The results in Table 3 show that the order quantities resulting from the New approximation are much closer to optimal than those resulting from the EOQ and from Platt et al.'s approximation. The EOQ performs particularly poor, with quantity errors between 14.5% and 23.1% for all considered fill rate / cost settings. Platt et al.'s approximation performs much better, but the quantity errors are still at least 3.8% and go up to 7.0%. The new approximation has quantity errors below 2% for all considered settings.

As shown in Table 4, the cost errors show the same pattern, but are much smaller than the quantity errors, indicating that the cost function is quite flat around the optimum (as also discussed in textbooks such as that by Axsäter, 2015.

Also in terms of cost, the new approximation outperforms both the EOQ method and the Platt et al. approximation. It leads to an average cost gap (over the optimal value) that does not exceed 0.02%, which shows that the new approximation leads to a near-optimal solution. The Platt et al. approximation

TABLE 3 *Quantity error: absolute percentage deviation of an approximation compared to the optimal order quantity, averaged for considered fill rate and backorder cost settings*

β (%)	% Quantity error (absolute)			$\frac{b}{b+h}$ (%)	% Quantity error (absolute)		
	EOQ (%)	New approx. (%)	Platt et al. approx. (%)		EOQ (%)	New approx. (%)	Platt et al. approx. (%)
90	23.1	1.7	6.5	90	19.0	1.2	3.8
95	19.3	1.3	7.0	95	16.6	1.2	4.7
98	16.3	1.2	6.8	98	14.5	1.6	5.3

TABLE 4 *Cost error: percentage cost increase of using an approximation compared to the optimal order quantity, averaged for considered fill rate and backorder cost settings*

β (%)	% Cost error			$\frac{b}{b+h}$ (%)	% Cost error		
	EOQ (%)	New approx. (%)	Platt et al. approx. (%)		EOQ (%)	New approx. (%)	Platt et al. approx. (%)
90	2.86	0.02	0.23	90	1.65	0.01	0.06
95	1.76	0.01	0.23	95	1.13	0.01	0.09
98	1.10	0.01	0.18	98	0.78	0.01	0.10

performs less well but still good, with an average cost gap varying from 0.06% to 0.23%. The EOQ method has an average cost gap that goes up to 2.86% under the fill rate constraint and up to 1.65% with a backorder cost.

Box plots of the cost gaps in Figs 1 and 2 show the cost gap variation over the considered instances under a service constraint and with a backorder cost, respectively.

Under a fill rate constraint, the maximum cost gap of the new approximation is 0.2%, which is obtained for the lowest service $\beta = 90\%$ under a standard deviation $\sigma = 50$, a lead-time $L = 5$ and a fixed replenishment cost $K = 25$. The maximum cost gap of the Platt et al. approximation is 0.5%, whereas that of EOQ goes up to 6.1%. With a backorder cost, the maximum cost gap reaches 0.2% under our new approximation, 0.3% under the adapted Platt et al. approx. and 3.8% under EOQ. The largest cost gaps are observed for instances with a low target service level or a low backorder cost value, a high standard deviation of the demand (i.e. $\sigma = 50$) and a relatively long lead-time (i.e. $L = 3, 5$).

7. Discussion and conclusion

The success of the EOQ formula can largely be attributed, in our opinion, to both its intuitive derivation and its simplicity for implementation. The latter may seem less relevant now than it was in especially the pre-computer era, but most inventory control software packages that we know of still use simple (EOQ-based) instead of exact, iterative procedures that have been presented in the literature (Scarf et al., 2024). As to the intuitive derivation, the EOQ can easily be shown to minimize the cyclic ordering and inventory holding costs under deterministic demand, and this can still be applied under stochastic demand if safety stock costs are ignored.

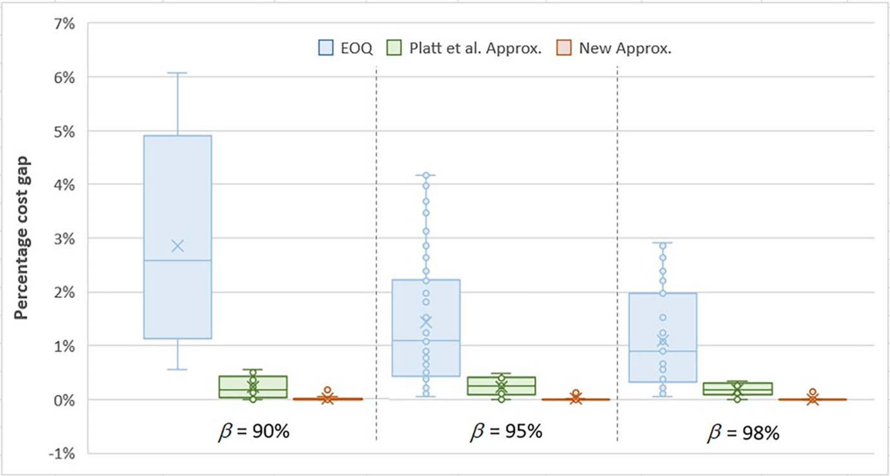

FIG. 1 Box plots of the percentage cost gap under a service constraint.

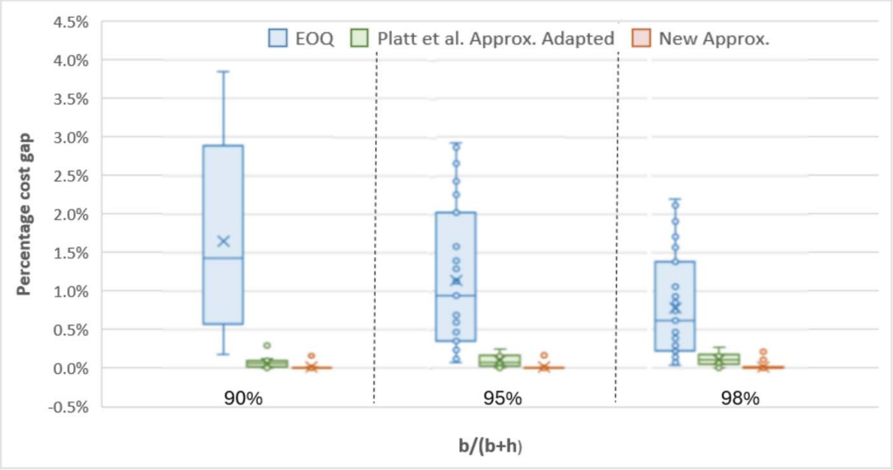

FIG. 2 Box plots of the percentage cost gap with a backorder cost.

We retained the logic of splitting the inventory costs into cycle costs and demand uncertainty related costs but adapted the traditional analysis in two ways. First, we 'corrected' the inventory cycle costs, considering that the average cycle inventory level is less than half of the order quantity, since part of an arriving batch is sometimes used to satisfy backorder and there are also periods with no inventory at all. Second, we considered left-over inventories instead of safety stocks and included them in approximating the optimal order quantity.

The first adaptation clearly leads to an increase in the order quantity compared to the EOQ. The same holds for the second adaptation, since a larger order quantity implies fewer order arrivals with associated safety stock (and backorder) costs. Aside from the exact analysis, this explains why the EOQ formula results in order quantities that are always too low under stochastic demand, which we feel is important to explain in inventory control courses and textbooks.

The most difficult part of approximating the optimal order quantity concerns the left-over inventory costs, since these depend on the specific lead-time demand distribution. We considered Normal lead-time demand, as is most often done in the literature, and used well-known statistical results on approximating the Normal loss function. This led to new, closed-form formulas that approximate the optimal order quantity for both a cost model (with a backorder cost per item per time unit) and a service model (with a fill rate constraint). A numerical investigation showed these to be very accurate, with an average cost error of 0.01% compared to 2% for the EOQ formula. Although the EOQ formula is reasonably robust, the 2% cost reduction may still be worthwhile in many practical situations and come at no added (computational) difficult as the newly proposed formulas are closed-form and simple.

An interesting observation is that setting the required fill rate for the service model equal to the optimal fill rate level for the cost model does not result in the same (approximately) optimal order quantities as for the cost model. The explanation is that the fill rate depends only on the fraction of backorders and not their duration, but the cost model does charge backorder costs per time unit. So, in a way, using the fill rate to measure service is not in line with charging backorder costs per time unit. Instead, the fill rate service measure is more in line with a fixed cost per backorder. Interestingly, though, for that cost structure there is no easy relation between cost parameters and the optimal fill rate. Future research could also consider deriving (nearly) optimal order quantity formulas when there is a fixed cost per backorder, or more generally, a fixed and time-dependent backorder cost. Furthermore, analyzing the relations between inventory cost models and service definitions is interesting for inventory control in general.

Future research can also address the sub-optimality of the EOQ formula for non-continuous demand. Our analysis, as most of the inventory control literature, considered continuous review and continuous (Normal lead-time) demand, implying that an order is always placed when the inventory position drops to the order level. Under periodic review and/or lumpy demand, the inventory position may drop below the order level and the so-called undershoot plays a role. Such situations are harder to analyze in general but deserve attention. Logical starting points would be to either consider periodic review where demand during the review-period plus lead-time is still Normal or consider continuous review for lumpy demand modelled by a compound Poisson process (Babai et al., 2025).

Data availability

No new data have been generated for the purposes of this article. For any information regarding the numerical investigation, interested readers are invited to contact the corresponding author.

References

Alstrøm, P. (2001) Numerical computation of inventory policies, based on the EOQ/σx value for order-point systems. *Int. J. Prod. Econ.*, **71**, 235–245.

Axsäter, S. (1996) Using the deterministic EOQ formula in stochastic inventory control. *Manag. Sci.*, **42**, 830–834.

Axsäter, S. (2006) A simple procedure for determining order quantities under a fill rate constraint and normally distributed lead-time demand. *Eur. J. Oper. Res.*, **174**, 480–491.

Axsäter, S. (2015) *Inventory Control (Vol. 225). International Series in Operations Research & Management Science*, 3rd edn. New York, USA: Springer.

Babai, M. Z., Syntetos, A. A. & Teunter, R. (2025) Fifty years of inventory research from a forecasting perspective. *Eur. J. Oper. Res*. https://doi.org/10.1016/j.ejor.2025.07.003.

Berling, P. & Marklund, J. (2006) Heuristic coordination of decentralized inventory systems using induced backorder costs. *Prod. Oper. Manag.*, **15**, 294–310.

Chang, H. C. & Ho, C. H. (2011) A note on solving the EOQ model with imperfect quality subject to in-house inspection. *IMA J. Manag. Math.*, **22**, 301–306.

Chao, H. P. (1992) The EQQ model with stochastic demand and discounting. *Eur. J. Oper. Res.*, **59**, 434–443.

Christou, I. T., Skouri, K. & Lagodimos, A. G. (2020) Fast evaluation of a periodic review inventory policy. *Comput. Ind. Eng.*, **144**, 106369.

Federgruen, A. & Zheng, Y. S. (1992) An efficient algorithm for computing an optimal (r, Q) policy in continuous review stochastic inventory systems. *Oper. Res.*, **40**, 808–813.

Gallego, G. (1998) New bounds and heuristics for (Q, r) policies. *Manag. Sci.*, **44**, 219–233.

Hadley, G. & Whitin, T. M. (1963) *Analysis of Inventory Systems*. Englewood Cliffs, NY: Prentice-Hall.

Harris, F. W. (1913) *How Many Parts to Make at Once*. Factory, The Magazine of Management, **10**, 135–136, 152.

Hill, R. M. (1999) Production planning towards the end of a product life cycle. *IMA J. Manag. Math.*, **10**, 165–176.

Jiang, Y., Shi, C. & Shen, S. (2019) Service level constrained inventory systems. *Prod. Oper. Manag.*, **28**, 2365–2389.

Lagodimos, A. G., Skouri, K., Christou, I. T. & Chountalas, P. T. (2018) The discrete-time EOQ model: solution and implications. *Eur. J. Oper. Res.*, **266**, 112–121.

Maddah, B. & Noueihed, N. (2017) EOQ holds under stochastic demand, a technical note. *Appl. Math. Model.*, **45**, 205–208.

Moily, J. P. (2015) Economic manufacturing quantity and its integrating implications. *Prod. Oper. Manag.*, **24**, 1696–1705.

Nakhaeinejad, M. (2024) The economic production quantity model with optimal single sampling inspection. *IMA J. Manag. Math.*, **35**, 463–485.

Perera, H. N., Fahimnia, B. & Tokar, T. (2020) Inventory and ordering decisions: a systematic review on research driven through behavioral experiments. *Int. J. Oper. Prod. Manag.*, **40**, 997–1039.

Platt, D. E., Robinson, L. W. & Freund, R. B. (1997) Tractable (Q, R) heuristic models for constrained service levels. *Manag. Sci.*, **43**, 951–965.

Porteus, E. L. (1985) Numerical comparisons of inventory policies for periodic review systems. *Oper. Res.*, **33**, 132–152.

Rao, U. S. (2003) Properties of the periodic review (R, T) inventory control policy for stationary, stochastic demand. *Manuf. Serv. Oper. Manag.*, **5**, 37–53.

Rumyantsev, S. & Netessine, S. (2007) What can be learned from classical inventory models? A cross-industry exploratory investigation. *Manuf. Serv. Oper. Manag.*, **9**, 409–429.

Scarf, P. A., Syntetos, A. A. & Teunter, R. (2024) Joint maintenance and spare-parts inventory models: a review and discussion of practical stock-keeping rules. *IMA J. Manag. Math.*, **35**, 83–109.

Silver, E. A. & Wilson, T. G. (1972) Cost penalties of simplified procedures for selecting reorder points and order quantities. *Int Tech Conf Proc*. Washington, DC: American Production and Inventory Control Society, pp. 219–234.

Silver, E. A., Bischak, D. P. & Da Silveira, G. J. (2009) An efficient method for calculating the minimum distance from an operating point to a specific (hyperbolic) efficient frontier. *IMA J. Manag. Math.*, **20**, 251–261.

Stangl, T. & Thonemann, U. W. (2017) Equivalent inventory metrics: a behavioral perspective. *Manuf. Serv. Oper. Manag.*, **19**, 472–488.

Yano, C. A. (1985) New algorithms for (Q, r) systems with complete backordering using a fill-rate criterion. *Naval Res Logistics Quart*, **32**, 675–688.

Zheng, Y. S. (1992) On properties of stochastic inventory systems. *Manag. Sci.*, **38**, 87–103.

Appendix A. Exact analysis

Let us denote the standard Normal density and distribution function by $\Phi(x)$ and $\varphi(x)$, respectively, and define the 'loss function':

$$G(x) = \int_x^{\infty} (v - x)\ \varphi(v)dv = \varphi(x) - x\,(1 - \Phi(x)) \tag{A.1}$$

as well as

$$H(x) = \int_x^\infty G(v)dv = \frac{1}{2}\left[\left(x^2+1\right)\left(1-\Phi(x)-x\varphi(x)\right]\right. \tag{A.2}$$

As shown in e.g. Section 5.9.2 of Axsäter (2015), the cost per time unit for the cost approach (so including backorder costs) is

$$C_T = \frac{K\mu}{Q} + h\left(R+\frac{Q}{2}-\mu_L\right) + (h+b)\frac{{\sigma_L}^2}{Q}\left[H\left(\frac{R-\mu_L}{\sigma_L}\right) - H\left(\frac{R+Q-\mu_L}{\sigma_L}\right)\right], \tag{A.3}$$

from which the cost per time unit for the service approach is easily obtained by setting b to zero, and the fill rate is

$$\beta = 1 - \frac{\sigma_L}{Q}\left[G\left(\frac{R-\mu_L}{\sigma_L}\right) - G\left(\frac{R+Q-\mu_L}{\sigma_L}\right)\right]. \tag{A.4}$$

Appendix B. Detailed numerical results, service system

TABLE B1 *Comparative results under the fill rate $\beta = 90\%$*

				Optimal solution			EOQ			New approx.			Platt et al. approx.		
μ	σ	L	K	Q	R	Cost	Q	R	ΔCost (%)	Q	R	ΔCost (%)	Q	R	ΔCost (%)
100	10	1	25	82.2	93.3	65.6	70.7	94.9	1.1	83.8	93.1	0.0	79.3	93.7	0.1
100	30	1	25	95.6	105.2	80.9	70.7	111.5	3.6	95.3	105.2	0.0	85.3	107.6	0.5
100	50	1	25	108.4	121.8	101.8	70.7	134.0	5.3	108.0	121.9	0.0	96.2	125.5	0.4
100	10	1	100	158.6	84.4	127.8	141.4	86.2	0.7	162.3	84.0	0.0	157.5	84.5	0.0
100	30	1	100	170.9	90.9	136.9	141.4	95.9	1.7	173.1	90.5	0.0	160.6	92.5	0.2
100	50	1	100	184.4	103.1	152.6	141.4	112.9	3.0	184.6	103.1	0.0	166.7	106.9	0.4
100	10	1	400	314.5	68.6	254.7	282.8	71.7	0.6	319.4	68.1	0.0	314.5	68.6	0.0
100	30	1	400	322.5	70.3	258.2	282.8	75.1	0.9	329.9	69.4	0.0	316.0	71.1	0.0
100	50	1	400	335.1	77.0	267.6	282.8	85.1	1.4	340.7	76.2	0.0	319.1	79.4	0.1
100	10	3	25	87.0	296.7	70.1	70.7	299.7	2.0	87.9	296.5	0.0	80.9	297.8	0.2
100	30	3	25	109.5	323.6	104.0	70.7	336.3	5.4	109.3	323.7	0.0	97.5	327.3	0.4
100	50	3	25	127.3	357.1	144.8	70.7	378.7	6.1	134.0	354.8	0.1	124.2	358.2	0.0
100	10	3	100	162.7	285.7	130.0	141.4	288.5	1.0	166.2	285.3	0.0	158.3	286.3	0.0
100	30	3	100	185.7	304.5	154.3	141.4	314.8	3.1	185.7	304.5	0.0	167.4	308.5	0.5
100	50	3	100	208.6	331.9	188.9	141.4	352.3	4.9	207.2	332.3	0.0	184.3	338.8	0.5
100	10	3	400	316.1	268.6	255.2	282.8	272.1	0.6	323.2	267.9	0.0	314.9	268.8	0.0
100	30	3	400	336.4	277.9	268.8	282.8	286.3	1.5	341.8	277.1	0.0	319.5	280.5	0.1
100	50	3	400	359.6	297.2	293.7	282.8	313.3	2.6	361.3	296.9	0.0	328.7	303.4	0.4
100	10	5	25	90.4	499.8	74.0	70.7	504.0	2.7	90.7	499.7	0.0	82.4	501.4	0.4
100	30	5	25	117.9	537.7	121.4	70.7	554.6	5.9	119.7	537.1	0.0	108.3	540.9	0.2
100	50	5	25	137.9	583.3	176.0	70.7	610.3	5.9	153.7	577.5	0.2	147.0	579.9	0.1
100	10	5	100	165.9	487.4	132.3	141.4	491.0	1.3	168.9	487.0	0.0	159.1	488.4	0.1
100	30	5	100	195.9	515.8	168.7	141.4	530.4	4.0	194.9	516.0	0.0	173.9	521.4	0.5
100	50	5	100	223.6	554.4	216.9	141.4	582.0	5.6	223.9	554.3	0.0	200.3	561.7	0.3
100	10	5	400	318.3	469.0	256.1	282.8	472.9	0.7	325.8	468.2	0.0	315.3	469.3	0.0
100	30	5	400	346.5	485.5	278.7	282.8	497.0	1.9	350.2	484.8	0.0	323.0	489.6	0.2
100	50	5	400	376.8	514.6	315.9	282.8	537.5	3.4	376.2	514.7	0.0	337.9	523.6	0.5

TABLE B2 *Comparative results under the fill rate $\beta = 95\%$*

μ	σ	L	K	**Optimal solution**			**EOQ**			**New approx.**			**Platt et al. approx.**		
				Q	R	**Cost**	Q	R	**ΔCost** (%)	Q	R	**ΔCost** (%)	Q	R	**ΔCost** (%)
100	10	1	25	79.0	100.1	71.5	70.7	100.9	0.6	79.1	100.1	0.0	75.2	100.5	0.1
100	30	1	25	90.7	120.0	93.7	70.7	124.3	2.2	89.4	120.3	0.0	80.9	122.0	0.5
100	50	1	25	101.9	144.5	121.1	70.7	153.5	3.5	100.6	144.8	0.0	91.2	147.4	0.3
100	10	1	100	152.2	94.1	136.3	141.4	94.9	0.3	153.5	94.0	0.0	149.2	94.3	0.0
100	30	1	100	163.8	108.5	152.3	141.4	111.5	1.0	163.2	108.6	0.0	152.2	110.0	0.2
100	50	1	100	175.5	128.8	174.8	141.4	135.3	1.8	173.4	129.1	0.0	157.9	132.0	0.4
100	10	1	400	299.1	85.4	269.2	282.8	86.2	0.2	302.3	85.2	0.0	297.9	85.4	0.0
100	30	1	400	310.2	93.5	278.5	282.8	95.9	0.4	311.7	93.4	0.0	299.4	94.4	0.1
100	50	1	400	321.9	108.2	294.9	282.8	112.9	0.8	321.4	108.3	0.0	302.3	110.5	0.2
100	10	3	25	83.3	306.4	78.6	70.7	308.3	1.2	82.7	306.5	0.0	76.6	307.4	0.3
100	30	3	25	102.9	347.0	124.0	70.7	356.5	3.5	101.8	347.3	0.0	92.4	349.9	0.3
100	50	3	25	118.9	393.8	176.2	70.7	411.0	4.2	123.7	392.2	0.0	117.7	394.2	0.0
100	10	3	100	156.5	298.2	141.0	141.4	299.7	0.5	157.0	298.2	0.0	150.0	298.9	0.1
100	30	3	100	176.7	330.9	177.2	141.4	337.8	1.9	174.4	331.3	0.0	158.6	334.3	0.4
100	50	3	100	196.5	372.0	223.2	141.4	386.8	3.1	193.5	372.7	0.0	174.6	377.5	0.4
100	10	3	400	302.9	287.2	271.4	282.8	288.5	0.2	305.7	287.0	0.0	298.3	287.5	0.0
100	30	3	400	323.0	309.9	296.8	282.8	314.9	0.8	322.4	310.0	0.0	302.7	312.4	0.2
100	50	3	400	343.2	343.2	333.6	282.8	353.7	1.5	339.8	343.8	0.0	311.4	348.5	0.4
100	10	5	25	86.2	511.5	84.2	70.7	514.3	1.6	85.3	511.7	0.0	78.1	513.0	0.4
100	30	5	25	110.3	567.0	146.4	70.7	580.0	4.0	111.1	566.8	0.0	102.6	569.3	0.1
100	50	5	25	128.5	629.5	215.7	70.7	651.6	4.2	141.2	625.2	0.1	139.3	625.8	0.1
100	10	5	100	159.4	502.0	145.1	141.4	504.0	0.7	159.4	502.0	0.0	150.7	502.9	0.1
100	30	5	100	185.5	548.2	196.6	141.4	558.4	2.5	182.6	548.9	0.0	164.8	552.8	0.5
100	50	5	100	209.8	604.3	259.4	141.4	625.1	3.7	208.3	604.7	0.0	189.8	609.9	0.2
100	10	5	400	305.8	489.3	273.8	282.8	491.0	0.3	308.1	489.1	0.0	298.7	489.8	0.0
100	30	5	400	331.8	523.6	312.0	282.8	530.8	1.1	329.9	523.9	0.0	306.0	527.3	0.3
100	50	5	400	358.0	570.6	364.2	282.8	586.1	2.1	353.0	571.6	0.0	320.1	578.1	0.5

TABLE B3 *Comparative results under the fill rate $\beta = 98\%$*

μ	σ	L	K	**Optimal solution**			**EOQ**			**New approx.**			**Platt et al. approx.**		
				Q	R	**Cost**	Q	R	**ΔCost** (%)	Q	R	**ΔCost** (%)	Q	R	**ΔCost** (%)
100	10	1	25	77.0	106.6	77.6	70.7	107.1	0.3	76.6	106.6	0.0	72.9	106.9	0.1
100	30	1	25	86.9	135.5	108.0	70.7	138.5	1.3	86.1	135.7	0.0	78.4	137.0	0.3
100	50	1	25	96.5	168.7	143.3	70.7	175.4	2.2	96.7	168.7	0.0	88.4	170.7	0.2
100	10	1	100	148.8	102.2	143.9	141.4	102.6	0.1	148.7	102.2	0.0	144.7	102.4	0.0
100	30	1	100	158.9	126.1	168.8	141.4	128.0	0.6	157.7	126.2	0.0	147.5	127.3	0.2
100	50	1	100	168.8	155.4	199.5	141.4	159.8	1.1	167.3	155.6	0.0	153.1	157.9	0.3
100	10	1	400	292.2	96.7	279.8	282.8	97.0	0.1	292.9	96.7	0.0	288.8	96.8	0.0
100	30	1	400	302.8	114.6	298.4	282.8	115.9	0.2	301.8	114.7	0.0	290.2	115.4	0.1
100	50	1	400	312.8	138.9	323.6	282.8	141.7	0.4	310.8	139.0	0.0	293.1	140.7	0.2
100	10	3	25	80.6	316.3	87.8	70.7	317.5	0.7	80.0	316.4	0.0	74.3	317.1	0.3

(continued)

TABLE B3 *Continued.*

μ	σ	L	K	Optimal solution Q	R	Cost	EOQ Q	R	ΔCost (%)	New approx. Q	R	ΔCost (%)	Platt et al. approx. Q	R	ΔCost (%)
100	30	3	25	97.4	372.1	146.9	70.7	379.1	2.3	97.8	372.0	0.0	89.5	374.0	0.2
100	50	3	25	111.7	433.7	212.6	70.7	447.3	2.8	118.2	431.8	0.0	114.1	433.0	0.0
100	10	3	100	152.6	309.9	151.9	141.4	310.7	0.3	151.9	310.0	0.0	145.4	310.4	0.1
100	30	3	100	169.7	358.4	202.7	141.4	363.1	1.1	168.3	358.7	0.0	153.7	361.0	0.3
100	50	3	100	186.8	414.7	262.4	141.4	425.4	2.0	186.1	414.9	0.0	169.2	418.6	0.3
100	10	3	400	296.2	302.1	285.4	282.8	302.7	0.1	296.1	302.1	0.0	289.2	302.4	0.0
100	30	3	400	313.8	341.4	326.3	282.8	344.5	0.5	311.7	341.6	0.0	293.4	343.4	0.2
100	50	3	400	330.9	390.5	377.6	282.8	397.4	0.9	328.1	390.9	0.0	301.8	394.6	0.3
100	10	5	25	83.8	523.6	95.5	70.7	525.5	0.9	82.4	523.8	0.0	75.7	524.8	0.3
100	30	5	25	104.1	598.6	175.2	70.7	608.6	2.6	106.4	598.0	0.0	99.5	599.9	0.0
100	50	5	25	120.5	680.1	261.9	70.7	698.0	2.9	134.5	675.6	0.1	135.0	675.5	0.1
100	10	5	100	154.4	516.1	158.3	141.4	517.2	0.4	154.2	516.1	0.0	146.1	516.8	0.2
100	30	5	100	177.2	582.4	228.0	141.4	589.6	1.5	175.9	582.7	0.0	159.7	585.8	0.3
100	50	5	100	198.4	657.9	308.4	141.4	673.5	2.4	200.0	657.5	0.0	184.0	661.6	0.1
100	10	5	400	298.4	506.7	290.2	282.8	507.5	0.1	298.4	506.7	0.0	289.5	507.2	0.0
100	30	5	400	321.3	562.1	347.9	282.8	566.7	0.6	318.8	562.4	0.0	296.6	565.0	0.3
100	50	5	400	343.4	629.2	418.3	282.8	639.7	1.2	340.5	629.7	0.0	310.3	634.7	0.3

Appendix C. Detailed numerical results, cost system

TABLE C1 *Comparative results for cost ratio b/b + h = 90%*

μ	σ	L	K	Optimal solution Q	R	Cost	EOQ Q	R	ΔCost (%)	New approx. Q	R	ΔCost (%)	Platt et al. approx. adapted Q	R	ΔCost (%)
100	10	1	25	78.9	93.7	72.7	70.7	94.9	0.6	79.2	93.7	0.0	79.3	93.7	0.0
100	30	1	25	90.2	106.4	96.7	70.7	111.5	2.1	89.5	106.6	0.0	85.3	107.6	0.1
100	50	1	25	101.0	124.0	126.0	70.7	134.0	3.2	100.8	124.1	0.0	96.2	125.5	0.1
100	10	1	100	152.3	85.1	137.4	141.4	86.2	0.3	153.7	84.9	0.0	157.5	84.5	0.1
100	30	1	100	163.5	92.0	155.6	141.4	95.9	0.9	163.4	92.1	0.0	160.6	92.5	0.0
100	50	1	100	174.8	105.2	180.0	141.4	112.9	1.7	173.7	105.4	0.0	166.7	106.9	0.1
100	10	1	400	300.0	70.0	270.0	282.8	71.7	0.2	302.7	69.7	0.0	314.5	68.6	0.1
100	30	1	400	310.2	71.7	281.9	282.8	75.1	0.4	312.2	71.5	0.0	316.0	71.1	0.0
100	50	1	400	321.4	79.1	300.4	282.8	85.1	0.7	321.9	79.0	0.0	319.1	79.4	0.0
100	10	3	25	83.1	297.4	80.4	70.7	299.7	1.1	82.9	297.4	0.0	80.9	297.8	0.0
100	30	3	25	102.0	325.9	129.1	70.7	336.3	3.3	102.0	325.8	0.0	97.5	327.2	0.1
100	50	3	25	117.4	360.7	184.4	70.7	378.7	3.8	124.0	358.3	0.0	124.2	358.2	0.1
100	10	3	100	156.4	286.5	142.9	141.4	288.5	0.5	157.2	286.4	0.0	158.3	286.3	0.0
100	30	3	100	176.0	306.6	182.6	141.4	314.8	1.8	174.7	306.9	0.0	167.4	308.5	0.1
100	50	3	100	195.1	335.7	231.8	141.4	352.3	2.9	193.8	336.0	0.0	184.3	338.8	0.1
100	10	3	400	303.3	269.9	273.3	282.8	272.1	0.2	306.2	269.7	0.0	314.9	268.8	0.1
100	30	3	400	322.5	280.0	302.5	282.8	286.3	0.8	322.8	279.9	0.0	319.5	280.5	0.0
100	50	3	400	342.1	300.6	342.8	282.8	313.3	1.4	340.4	301.0	0.0	328.7	303.4	0.1

(continued)

TABLE C1 *Continued.*

				Optimal solution			EOQ			New approx.			Platt et al. approx. adapted		
μ	σ	L	K	Q	R	Cost	Q	R	ΔCost (%)	Q	R	ΔCost (%)	Q	R	ΔCost (%)
100	10	5	25	85.9	500.7	86.6	70.7	504.0	1.5	85.5	500.7	0.0	82.4	501.4	0.1
100	30	5	25	109.2	540.6	152.8	70.7	554.5	3.7	111.3	539.9	0.0	108.3	540.9	0.0
100	50	5	25	126.7	587.5	226.1	70.7	610.3	3.8	141.5	581.9	0.2	147.0	579.9	0.3
100	10	5	100	159.2	488.3	147.6	141.4	491.0	0.6	159.6	488.3	0.0	159.1	488.4	0.0
100	30	5	100	184.5	518.7	203.4	141.4	530.4	2.3	182.9	519.0	0.0	173.9	521.4	0.1
100	50	5	100	207.9	559.3	270.3	141.4	582.0	3.4	208.7	559.0	0.0	200.3	561.7	0.0
100	10	5	400	306.0	470.4	276.3	282.8	472.9	0.3	308.5	470.1	0.0	315.3	469.3	0.0
100	30	5	400	331.0	488.1	319.2	282.8	497.0	1.1	330.4	488.2	0.0	323.0	489.6	0.0
100	50	5	400	356.4	519.3	375.7	282.8	537.4	1.9	353.6	519.9	0.0	337.9	523.6	0.1

TABLE C2 *Comparative results for cost ratio b/b + h = 95%*

				Optimal solution			EOQ			New approx.			Platt et al. approx. adapted		
μ	σ	L	K	Q	R	Cost (%)	Q	R	ΔCost (%)	Q	R	ΔCost	Q	R	ΔCost (%)
100	10	1	25	77.2	100.3	77.5	70.7	100.9	0.3	77.0	100.3	0.0	75.2	100.5	0.0
100	30	1	25	87.2	120.7	107.9	70.7	124.3	1.4	86.7	120.8	0.0	80.9	122.0	0.2
100	50	1	25	96.9	145.8	143.3	70.7	153.5	2.3	97.4	145.7	0.0	91.2	147.4	0.1
100	10	1	100	149.3	94.3	143.6	141.4	94.9	0.1	149.5	94.3	0.0	149.2	94.3	0.0
100	30	1	100	159.4	109.1	168.5	141.4	111.5	0.6	158.7	109.2	0.0	152.2	110.0	0.1
100	50	1	100	169.4	129.9	199.3	141.4	135.3	1.1	168.4	130.1	0.0	157.9	132.0	0.2
100	10	1	400	293.3	85.7	279.0	282.8	86.2	0.1	294.6	85.6	0.0	297.9	85.4	0.0
100	30	1	400	303.7	94.0	297.7	282.8	95.9	0.2	303.5	94.1	0.0	299.4	94.4	0.0
100	50	1	400	313.8	109.2	323.0	282.8	112.9	0.5	312.7	109.3	0.0	302.3	110.5	0.1
100	10	3	25	80.9	306.8	87.6	70.7	308.3	0.7	80.4	306.8	0.0	76.6	307.4	0.1
100	30	3	25	97.7	348.4	146.9	70.7	356.5	2.3	98.5	348.2	0.0	92.4	349.9	0.1
100	50	3	25	112.0	396.1	212.7	70.7	411.0	2.9	119.2	393.7	0.1	117.7	394.2	0.0
100	10	3	100	153.0	298.6	151.6	141.4	299.7	0.3	152.8	298.6	0.0	150.0	298.9	0.0
100	30	3	100	170.4	332.1	202.5	141.4	337.8	1.2	169.3	332.3	0.0	158.6	334.3	0.2
100	50	3	100	187.4	374.3	262.3	141.4	386.8	2.0	187.4	374.2	0.0	174.6	377.5	0.1
100	10	3	400	297.1	287.5	284.7	282.8	288.5	0.1	297.8	287.5	0.0	298.3	287.5	0.0
100	30	3	400	314.8	310.9	325.7	282.8	314.8	0.5	313.6	311.0	0.0	302.7	312.4	0.1
100	50	3	400	332.1	345.0	377.1	282.8	353.7	0.9	330.1	345.3	0.0	311.4	348.5	0.2
100	10	5	25	83.4	512.0	95.4	70.7	514.3	1.0	82.9	512.1	0.0	78.1	512.9	0.2
100	30	5	25	104.3	568.8	175.2	70.7	580.0	2.7	107.2	567.9	0.0	102.6	569.3	0.0
100	50	5	25	120.6	632.4	262.1	70.7	651.6	2.9	135.7	627.1	0.2	139.3	625.8	0.2
100	10	5	100	155.6	502.4	158.0	141.4	504.0	0.4	155.1	502.4	0.0	150.7	502.9	0.0
100	30	5	100	177.9	549.9	227.9	141.4	558.4	1.6	177.0	550.0	0.0	164.8	552.8	0.2
100	50	5	100	199.0	607.3	308.4	141.4	625.1	2.4	201.4	606.6	0.0	189.8	609.9	0.0
100	10	5	400	299.8	489.7	289.5	282.8	491.0	0.2	300.0	489.7	0.0	298.7	489.8	0.0
100	30	5	400	322.4	524.9	347.3	282.8	530.8	0.7	320.7	525.2	0.0	306.0	527.3	0.1
100	50	5	400	344.6	573.3	417.9	282.8	586.1	1.3	342.6	573.6	0.0	320.1	578.1	0.2

TABLE C3 *Comparative results for cost ratio b/b + h = 98%*

				Optimal solution			EOQ			New approx.			Platt et al. approx. adapted		
μ	σ	L	K	Q	R	Cost	Q	R	ΔCost (%)	Q	R	ΔCost (%)	Q	R	ΔCost (%)
100	10	1	25	75.9	106.6	82.6	70.7	107.1	0.2	75.8	106.7	0.0	72.9	106.9	0.1
100	30	1	25	84.6	135.9	120.5	70.7	138.5	0.9	85.1	135.8	0.0	78.4	137.0	0.2
100	50	1	25	93.2	169.6	163.1	70.7	175.4	1.6	95.4	169.0	0.0	88.4	170.7	0.1
100	10	1	100	147.3	102.3	149.6	141.4	102.6	0.1	147.1	102.3	0.0	144.7	102.4	0.0
100	30	1	100	156.2	126.4	182.6	141.4	128.0	0.4	156.0	126.4	0.0	147.5	127.3	0.1
100	50	1	100	164.9	156.0	220.9	141.4	159.8	0.7	165.4	155.9	0.0	153.1	157.9	0.2
100	10	1	400	289.9	96.8	286.7	282.8	97.0	0.0	289.9	96.8	0.0	288.8	96.8	0.0
100	30	1	400	299.2	114.8	314.1	282.8	115.9	0.1	298.6	114.9	0.0	290.2	115.4	0.0
100	50	1	400	308.0	139.3	347.3	282.8	141.7	0.3	307.5	139.3	0.0	293.1	140.7	0.1
100	10	3	25	79.1	316.5	95.6	70.7	317.5	0.5	79.1	316.5	0.0	74.3	317.1	0.1
100	30	3	25	94.0	373.0	167.4	70.7	379.1	1.6	96.5	372.3	0.0	89.5	374.1	0.0
100	50	3	25	106.9	435.2	245.5	70.7	447.3	2.1	116.5	432.3	0.1	114.1	433.0	0.1
100	10	3	100	150.6	310.1	160.7	141.4	310.7	0.2	150.3	310.1	0.0	145.4	310.4	0.1
100	30	3	100	165.7	359.1	224.8	141.4	363.1	0.8	166.3	359.0	0.0	153.7	361.0	0.2
100	50	3	100	180.7	416.0	297.1	141.4	425.4	1.4	183.8	415.3	0.0	169.2	418.6	0.1
100	10	3	400	293.4	302.2	295.6	282.8	302.7	0.1	293.1	302.2	0.0	289.2	302.4	0.0
100	30	3	400	308.9	341.9	350.8	282.8	344.4	0.3	308.4	341.9	0.0	293.4	343.4	0.1
100	50	3	400	323.9	391.4	415.4	282.8	397.4	0.6	324.4	391.4	0.0	301.8	394.6	0.2
100	10	5	25	81.3	523.9	105.2	70.7	525.5	0.6	81.4	523.9	0.0	75.7	524.7	0.2
100	30	5	25	99.9	599.7	201.1	70.7	608.6	1.9	104.9	598.4	0.0	99.5	599.9	0.0
100	50	5	25	115.0	682.0	303.8	70.7	698.1	2.2	132.4	676.3	0.2	135.0	675.5	0.3
100	10	5	100	152.9	516.2	169.1	141.4	517.2	0.2	152.6	516.2	0.0	146.1	516.8	0.1
100	30	5	100	172.3	583.4	255.7	141.4	589.6	1.1	173.8	583.1	0.0	159.7	585.8	0.2
100	50	5	100	191.2	659.7	352.2	141.4	673.5	1.7	197.3	658.2	0.0	184.0	661.6	0.0
100	10	5	400	295.8	506.8	302.6	282.8	507.5	0.1	295.3	506.8	0.0	289.5	507.2	0.0
100	30	5	400	315.5	562.7	378.2	282.8	566.6	0.4	315.3	562.8	0.0	296.6	565.0	0.1
100	50	5	400	334.9	630.5	465.5	282.8	639.7	0.9	336.5	630.3	0.0	310.3	634.7	0.2