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Fair assignment of urban–rural electric vehicle shared charging using queuing theory

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ABSTRACT

The widespread adoption of electric vehicles (EVs) is crucial for decarbonising transportation and achieving global net-zero goals. However, a significant challenge in this transition is ensuring equitable access to charging infrastructure, particularly when addressing the simultaneous charging needs of urban residents and rural visitors in urban areas. This is a critical aspect often overlooked in existing literature. This study formulates the problem of implementing a charging system for urban areas that can support both urban and rural users as a multi-objective integer linear programming (ILP) model. This approach uniquely achieves fairness by reducing congestion in urban charging systems to ensure sufficient charging capacity for rural residents visiting the area. Specifically, the expected mean waiting time for all users is minimised. Concurrently, the travel distance for urban residents to their assigned charging stations is also minimised, thereby ensuring sufficient charging capacity for rural residents visiting the area. An ILP solver was employed to evaluate the proposed model across various problem instances, including a detailed case study of Cardiff city, UK. Results demonstrate the significant advantages of this assignment model: for a simulated scenario with 36 charging stations in Cardiff's urban centre, the model reduced the mean waiting time by approximately 7 min per user (from 16.6 to 9.6 min) and decreased the average travel distance for urban users by 2.25 km (from 3.6 to 1.35 km) compared to a baseline approach. Further experiments across different charging station densities consistently showed that the optimisation model reduced mean waiting times by up to 12.8 min and average travel distances by up to 3.3 km. This research provides a robust, data-driven framework that enables more equitable and efficient EV charging infrastructure planning, facilitating a truly inclusive transition to electric vehicles for both urban and rural communities.

1. Introduction

Decarbonisation is becoming increasingly important worldwide. The UK government, for instance, emphasises decarbonisation and has introduced policies such as the Net Zero Strategy, aiming to decarbonise all UK economic sectors by 2050 (Government, 2021a). Specific policies, such as the Road to Zero, outline a strategy for cleaner road transport, focusing on the transportation sector (Government, 2021b).

The UK government has introduced a ban on the sale of new petrol and diesel cars by 2035 (Government, 2021c). As such, the UK's transportation sector has begun the process of transitioning away from conventional petrol and diesel vehicles, internal combustion engine vehicles (ICEVs), towards electric vehicles (EVs). Given recent advancements in environmentally friendly battery manufacturing (Krajinska, 2021), this transition to EVs plays a vital role in the UK's overall net-zero goal, which aims to balance greenhouse gas emissions with their removal from the atmosphere.

Significant distinctions exist in the transition to EVs between rural and urban areas, creating an urban/rural divide. For example, the rural population in the UK is more reliant on private transport than those living in urban areas. Urban residents typically have greater access to public transport. The rural population also tend to travel further distances to access services. For example, in Wales, the number of charging stations in rural areas is small compared to urban areas (Welsh Government, 2020). This means a person's location affects the feasibility of transitioning to an EV. Rural residents may struggle with convenient vehicle charging, while urban residents typically have easier access to public charging stations.

Beyond infrastructure availability, the urban/rural divide is also defined by potential pricing differences. Rural EV users, who are more likely to have off-street parking, can often take advantage of discounted

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overnight home charging rates. This raises questions about how urban EV users, who may depend more on public charging, can be ensured equitable access to charging options close to home and work without incurring penalties due to higher public charging tariffs or peak-time pricing. Therefore, location affects not only the convenience of transitioning to an EV but also the financial implications of doing so.

While a significant body of work has focused on the optimal placement of EV charging stations (He et al., 2018; Giménez-Gaydou et al., 2016; Bitencourt et al., 2021), there is limited research on the fair and efficient assignment of users to existing infrastructure, particularly with a focus on urban–rural disparities. Existing fairness-focused studies either model general transportation equity (He et al., 2019) or prioritise agent-level preferences in assignment without accounting for congestion effects (Lesca et al., 2019; Aziz et al., 2015). This urban–rural disparity in infrastructure and adoption has been increasingly recognised as a key challenge in EV deployment (Maybury et al., 2022; Kukutschka and Schmitt, 2021). These socioeconomic and pricing disparities complicate equitable access, underscoring the importance of fairness in the planning and allocation of charging infrastructure (Hopkins et al., 2023; Charlier and Hogan, 2023). This paper responds to these gaps by proposing a novel assignment model that explicitly incorporates fairness metrics tied to user queuing time and charging station accessibility. In contrast to traditional location optimisation, this model focuses on how users should be allocated to existing charging infrastructure in a way that reduces congestion while addressing regional inequalities. Unlike existing studies that mainly optimise for proximity or minimise travel cost (Giménez-Gaydou et al., 2016; He et al., 2018), our method introduces user prioritisation and demand balancing under shared infrastructure constraints, enabling a more equitable user experience.

To achieve a transition to EVs that is fair for all vehicle users, it is essential to account for the differences between rural and urban areas in any mathematical model that optimises the transition. However, a research gap exists in this area. While some articles mention the disparity in EV adoption between rural and urban areas, few models explicitly account for it, and it is rarely their primary focus (Maybury et al., 2022). Moreover, prior models rarely incorporate queuing theory to account for wait times at charging stations — a critical factor in shared urban environments (Cao et al., 2022; Said and Mouftah, 2019). Current models that incorporate fairness often do so in the context of allocation or assignment problems, but few explicitly integrate queuing dynamics and the urban–rural divide in EV charging (Aziz et al., 2015; He et al., 2019). Thus, the unique contribution of this study is to propose a mathematical model that optimises the UK's transition to EVs with a focus on the urban/rural divide. It is worth noting that this urban/rural divide is not unique to EVs and has existed in other technological transitions, such as the transition to high-speed internet access. Furthermore, more research is needed on how urban areas can facilitate the charging needs of both residents and visitors (Charly et al., 2023). The equitable rollout of charging stations is a well-established research topic, playing an important part in the transition to EVs (Hopkins et al., 2023). However, existing works typically do not consider this urban/rural divide. Our model fills this research gap by combining fairness-aware user assignment with congestion-sensitive queuing theory, offering a replicable and adaptable approach for infrastructure providers and policymakers. This focus aims to ensure fairness in the UK's transition to EVs. The measure of fairness used in this study is access to charging stations plus the waiting times for users to charge their EVs at these charging stations. Waiting times and queuing effects have been shown to significantly impact user satisfaction and system efficiency in EV charging contexts (Cao et al., 2022; Vandet and Rich, 2022).

We formulate the problem of creating a fair integrated urban/rural EV charging system as a mathematical optimisation problem. The optimisation problem in question is an assignment problem focusing on fairness and integrates queuing theory. The main objective of the optimisation problem is to minimise the average queue time in the system with an optimal assignment of urban users to shared charging stations, where the queue time is the time spent waiting in a queue to charge an EV. It is hypothesised that optimising this assignment and implementing the optimal solution will reduce system congestion. This would then ensure rural residents visiting urban areas can charge their EVs in a desirable manner. Hence this is the aspect of 'fairness' that is considered in this model. More specifically, the measure of fairness is the time a user must wait in a queue to charge their EV. This is achieved by assigning each urban resident to a priority charging station which acts as their home charging station and does not incur any additional fees. It is assumed that the charging station locations are already optimised or predetermined. Therefore, this problem is distinct from the well-researched facility location problem (Farahani and Hekmatfar, 2009; Cornuéjols et al., 1983; Corcoran and Gagarin, 2021). This distinction holds even in the context of charging station location optimisation (He et al., 2018; Giménez-Gaydou et al., 2016; Bitencourt et al., 2021). While location optimisation is a critical field, this study uniquely focuses on optimally assigning users to existing stations with fairness and queuing time as primary criteria (Cao et al., 2022; Said and Mouftah, 2019).

The novelty of our contribution lies in this shift from optimising infrastructure locations to optimising equitable infrastructure access. We demonstrate that this approach can better serve both system efficiency and fairness goals, especially under scenarios of limited infrastructure expansion. Our framework is designed to support evidence-based policymaking by evaluating how different assignment rules, congestion pricing, or charging demand forecasts impact urban–rural equity in real-world settings.

The key contributions of this work are as follows:

- Introduces a novel multi-objective ILP model that explicitly accounts for urban–rural fairness by minimising queuing times and travel distances.
- Generates optimal assignments of users to charging stations in urban areas, enabling fair access through implementable strategies for shared home charging.
- Integrates queuing models to manage system congestion and evaluate the impact of policies like congestion charges. This integration addresses a critical gap in modelling EV infrastructure fairness (Cao et al., 2022; Vandet and Rich, 2022).
- Enables scenario analysis using charging station density variations to guide future expansion and investment decisions.
- Offers practical utility for city planners, policymakers, and infrastructure providers through clear outputs and policy-relevant levers.
- Ensures equitable charging access for rural EV users by balancing urban congestion and infrastructure usage.
- Adapts to varying data spatial resolutions (e.g., Census OAs), station densities, and can be extended with more realistic arrival patterns or charger specifications.
- Opens pathways for exploring dynamic demand, individual-level user modelling, richer charger features, and joint optimisation of location and assignment.

The layout of the remainder of the paper is as follows. Section 2 provides an overview of related literature in the field. Section 3 describes the methodology, including the model formulation and a description of the proposed charging system. Section 4 presents an evaluation of the proposed model with respect to both random and real world problem instances by performing simulations with respect to both. Finally, Section 5 will cover the model's advantages, limitations, and practical implications, while Section 6 will summarise the main contributions of this research.

2. Related work

Previous research has explored different aspects of the problem considered in this study. As such, in this section, the main topics reviewed will be fairness in optimisation problems, the urban/rural divide in a transition to EVs, queuing theory related to EV charging, and shared EV charging models.

Fairness has been considered in the context of many different optimisation problems. Although this study focuses on fairness in transportation, it is a universal property that applies to many systems. The *fair assignment problem* has been considered in previous studies, with the definition of 'fairness' varying from field to field. Alfonsetti et al. (2015) focused on optimising fairness, defined as 'social benefit', to users whilst assigning them to car parking spaces. Lesca et al. (2019) proposed a new method to solve a fair one-to-one assignment problem, which is the problem of assigning n objects to n agents, with each agent only having one object assigned to it. Aziz et al. (2015) considered the fair assignment problem where fairness is incorporated by considering agent preferences. Xu et al. (2016) consider fairness in cyberphysical systems, defined as the average social benefit to agents.

The urban/rural divide discussed in Section 1 has not been the main focus of any models concerning EV adoption in the past, but it has been considered in a number of models. Jia et al. (2020) used machine learning methods to build a prediction model focusing on predicting EV demand across the United States. The urban/rural divide was incorporated with a household related variable in the model, "urban–rural", which denoted whether the area in question is urban or rural. Mulholland et al. (2018) developed a consumer choice model to analyse the impacts of retracting low carbon transport subsidies. In this study, the consumers were split into urban and rural categories using a EuroStat 2014 regional dataset.

As mentioned in the introduction to this article, the model proposed in this work uses a queuing theory model. Cao et al. (2022) developed a Cyber–Physical–Social System to introduce the consideration of queuing theory in the users' process of choosing an EV charging station. Duan et al. (2022) proposed a user equilibrium traffic assignment model, incorporating queuing theory to optimise the planning scheme for charging service providers (CSPs). Kumar et al. (2022) used a queuing model to predict charging demand. Vandet and Rich (2022) derived an approximation of expected queuing time from queuing theory in order to plan the optimal placement and sizing of EV charging infrastructure. Said and Mouftah (2019) used a queuing model to develop an EV charging management protocol. Lu and Hua (2015) incorporated queuing theory in a location-sizing model to tackle the constraint of number of EV charging spaces.

The concept of shared community charging models for EVs has been widely considered in past research regarding EV adoption. Wang et al. (2019) analysed the practicality of shared charging for EVs using a dataset based on the Chinese city Shenzhen. Koç et al. (2019) incorporated shared charging into the electric vehicle routing problem (EVRP). The EVRP is the problem of optimising the routes of a fleet of EVs based on EV-specific constraints, such as the driving range of the EVs in question and available charging infrastructure of the focus area. Gong et al. (2019) focused on identifying optimal locations for shared charging stations. Zhang et al. (2023) focused on employing shared charging stations to satisfy the demands of those living in multi-unit dwellings (MUDs), such as apartment buildings, and produced a model to minimise the total waiting time of the charging stations.

3. Methodology

The model developed in this work considers the different characteristics between EV charging in rural and urban areas. There are two different types of users considered: rural users residing in rural areas and urban users residing in urban areas. It is assumed that rural users have access to slow charging at home and limited fast charging close to their residence. Also, urban destinations may be a long distance away depending on how far their homes are from the urban area in question is. Therefore rural users will require access to charging stations at destinations in urban areas. In contrast, it is assumed that urban users have limited access to home charging due to a large number of people living in apartment blocks and more populated residential areas, with less space for parking and therefore EV charging. Therefore urban users will have access to *shared home charging*, which is a public charging station near the residence, prioritising the urban users that live nearby. Any user may use any public charging station, but an additional 'congestion charge' will be applicable to those urban users who are using a charging station that is not their priority charging station. The purpose of this fee is to incentivise charging behaviour, with respect to shared charging in urban areas, that improves the experience of charging EVs for all users. The charging system aims to minimise congestion at individual urban charging stations. This provides rural users with the opportunity to charge their EVs when visiting urban areas. Concurrently, it ensures urban users are not disadvantaged, allowing them to charge their EVs in a desirable manner, measured by minimal waiting times and distance travelled to charge points.

The goal of this optimisation model is to contribute to achieving fairness in the charging system. In this case, fairness is defined as all users – urban and rural residents – being able to charge their EV in a desirable manner. Therefore it is argued that minimising the average queue time in the system, i.e reducing the congestion in the system, contributes to fairness. This is because reduced congestion in the overall system allows rural visitors from any direction to charge their EV without having to wait a long time for a charger to be free.

The remainder of this section is structured as follows: an overview of the model formulation is given in Section 3.1, where Table 1 contains definitions of all variables and parameters of the model. In Section 3.2, the objective function of the model is derived, and in Section 3.3, the final ILP is formulated.

3.1. Model formulation overview

The main purpose of the model presented in this study is to reduce congestion in an urban charging system. This ensures that sufficient capacity remains for visiting rural residents to charge their EVs. Therefore the problem of minimising congestion in an urban charging system in a manner that is fair to all users is proposed, where congestion is measured as the average queuing time. This problem is formulated as an ILP assignment problem as follows. Groups of urban users living in the same geographical region are assigned to charging station queues corresponding to charging stations of pre-set locations. The assignment of rural users to charging stations is not directly modelled or optimised. Instead, the optimisation model produces a result that ensures fairness for rural users visiting urban areas by guaranteeing available charging capacity. This is achieved by minimising congestion in the system with respect to shared charging in urban areas. This assignment is obtained using an ILP optimisation model, which is now described.

The ILP model has assignment variables and is parameterised by a number of constants, which are a function of the environment to which the model is applied. The variables and parameters in question are summarised in Table 1 and described as follows. The urban area is partitioned into regions $U = \{u_1, \dots, u_m\}$, that have respective populations $S = \{s_1, \dots, s_m\}$, corresponding to urban users residing at m regions. These urban users are assigned to charging station queues $Q = \{q_1, \dots, q_n\}$, that have respective sizes $P = \{p_1, \dots, p_n\}$, corresponding to the set of n charging stations. The size of a charging station is defined as the number of charge points at that station. Let D denote a distance matrix containing values d_{ij} corresponding to the shortest distance from region u_i to queue q_j; the maximum distance between a user and its assigned charging station, $\max_d$; the frequency with which an urban resident charges their EV on average, f; the average time taken by an urban user to charge their EV, t; and the number of hours a day

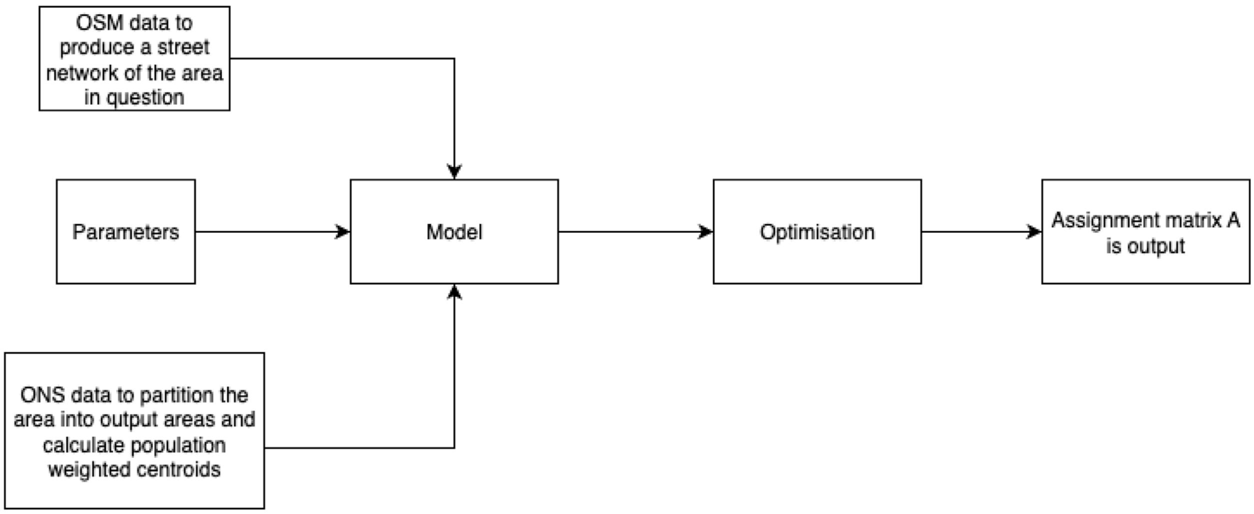

Fig. 1. Flowchart illustrating the process of applying the assignment model to a given scenario with parameters U, S, Q, P, W_j, D, max_d, f, t, h, λ_1 and λ_2, defined in Table 1.

that the assignment is active for, h, which allows the congestion control measures to be in effect during times which the urban area may have a significant number of visitors from rural areas needing to charge their EVs. There is a set of decision variables, that is the assignment matrix $A = \{a_{ij} \in \mathbb{N} \mid i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}\}$, with elements representing the number of users from region u_i assigned to queue q_j. Then the charging station corresponding to queue q_j becomes their *shared home charging station*. To address road network congestion in residential areas, there is an auxiliary variable representing the maximum number of residents from area u_i assigned to a single charging station, $\max_i$. The ILP model formulation and optimisation process are illustrated in Fig. 1.

The practical applicability of the model is built upon several key assumptions, designed to reflect realistic scenarios while ensuring computational tractability. Firstly, this approach assumes that the set of potential charging station locations aligns with existing car park infrastructure (demonstrated in Section 4), representing a pragmatic choice for urban planning given available land and electrical grid connections. Secondly, for the purpose of simulating varied infrastructure densities and future growth, the variable τ is introduced; a fraction of these car parks designated as active charging stations. This allows for flexible scenario analysis, from current limited infrastructure to future widespread deployment. Thirdly, concerning user behaviour at charging stations, we assume a First-Come-First-Served (FCFS) queuing discipline and that EV arrivals approximate a Poisson process. While real-world arrival patterns can exhibit more complexity (e.g., peak-hour surges), the Poisson assumption is widely adopted in queuing theory for modelling random arrival events and provides a robust basis for estimating average waiting times, especially for aggregated demand analysis (Gross et al., 2011; Allen, 2014). Lastly, to incentivise optimal charging behaviour and manage demand, the system proposes a 'congestion charge' for urban users opting for a non-assigned station. This mechanism reflects common urban demand management strategies aimed at influencing public resource utilisation.

This formulation is demonstrated on the following simple example, which is a network of urban regions and charging stations, and can be seen in Fig. 2. The network has nodes representing charging stations in red and nodes representing regions of urban users in black. The set of charging station queues is $Q = \{q_1, q_2\}$ and the set of regions of users is $U = \{u_1, u_2, u_3, u_4, u_5\}$. It also has weighted edges where edge weights represent d_{ij}, the shortest path lengths between the regions of urban users and charging stations. Suppose there are 8 charging points at charging station 1, and 4 charging points at charging station 2, so $P = \{p_1, p_2\} = \{8, 4\}$. Suppose the regions of urban users have populations 3, 4, 5, 3, 4 respectively, so $S = \{3, 4, 5, 3, 4\}$.

Table 1
Definitions of key parameters and decision variables used in the multi-objective ILP model for a fair shared urban–rural charging system. Parameters define the system's static properties and user characteristics, while variables represent the decisions optimised by the model.

Parameter	Description
$U = \{u_i \mid i \in \{1, \ldots, m\}\}$	Groups of urban users corresponding to m regions
$S = \{s_i \mid i \in \{1, \ldots, m\}\}$	s_i = population of urban group u_i
$Q = \{q_j \mid j \in \{1, \ldots, n\}\}$	Charging station queues corresponding to n charging stations
$P = \{p_j \mid j \in \{1, \ldots, n\}\}$	p_j = number of charge points at charging station queue q_j
W_j	Average waiting time in queue q_j
$D = \{d_{ij} \mid i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}\}$	d_{ij} = shortest distance from location i to charging station j
$\max_d$	The maximum acceptable value of the shortest distance between the location of an urban user and their assigned charging station
f	The frequency with which an urban resident charges their EV
t	The average time taken by an urban user to charge their EV
h	The number of hours a day that the assignment is active for
λ_1, λ_2	Constants to determine the penalty associated with the corresponding slack variables
Variable	**Description**
$A = \{a_{ij} \mid i \in \{1, \ldots, m\}, j \in \{1, \ldots, n\}\}$	a_{ij} = number of users from group u_i assigned to charging station queue q_j
$\max_i$	The maximum number of residents from area i assigned to a single charging station
B_1, B_2	Slack variables

From the network, we can obtain each shortest distance d_{ij} from urban region u_i to queue q_j. Then we form the matrix D of distances d_{ij} to input into the model:

$$D = \begin{pmatrix} 5 & 8 \\ 8 & 7 \\ 9 & 3 \\ 1 & 5 \\ 6 & 10 \end{pmatrix} \tag{1}$$

The objective of the model is to minimise the average queuing time over the set of queues Q, the average distance between the regions

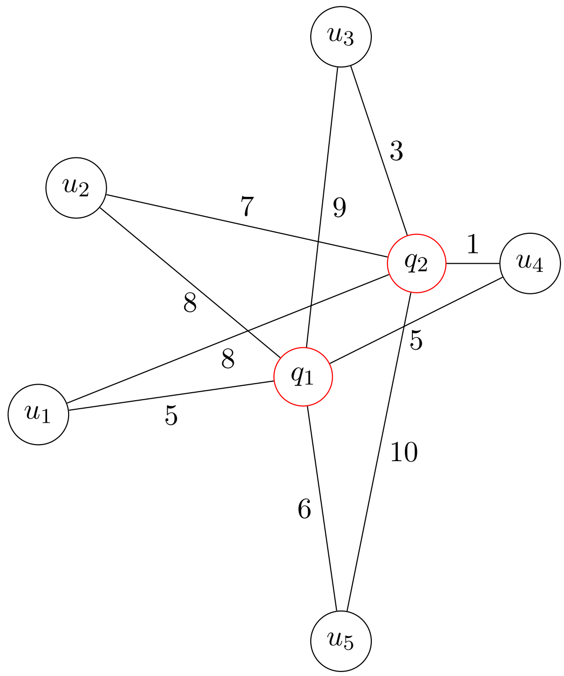

Fig. 2. Simple network example demonstrating the ILP model's inputs: black nodes $\{u_1 - u_5\}$ represent urban regions with populations $S = \{3,4,5,3,4\}$, red nodes $\{q_1, q_2\}$ denote charging stations with capacities $P = \{8,4\}$ charge points, and weighted edges show the shortest travel distances d_{ij}.

of users and their assigned charging stations d_{ij}, and the sum of the maximum number of urban residents assigned to each charging station from any given area. The remainder of the formulation, including the objective function derivation and the final ILP, is now described and demonstrated on the simple example given above.

3.2. The objective function

The problem of performing a fair assignment of users to charging station queues has three dimensions. The three dimensions in question are queuing time, distance travelled, and road traffic congestion. Each of these dimensions is modelled as an individual term in the optimisation objective function. The first term of the objective function minimises the average queue time in the urban charging system, and aims to fulfil the main purpose of the model by reducing congestion in the system to ensure there is capacity left for visiting rural residents to charge their EVs. The second term minimises the sum of the distance between urban residents and their assigned charging station, as it is preferable to charge a vehicle close to the home. The third term minimises the sum of the maximum numbers of urban residents assigned to each charging station from any given area. The purpose of this term is to reduce road traffic congestion by ensuring a more uniform distribution of charging station assignments from each area. To achieve this, the maximum element of each row of the assignment matrix is minimised, where each row corresponds to an assignment of users from a particular area to charging stations.

The derivation of the average queue time in the charging system is based on queuing theory concepts. It is assumed that the charging stations follow a first come first served (FCFS) queuing system, where there is one queue per charging station, and there are multiple 'servers', where servers correspond to charging points at the charging station. Each charging station queue is assumed to be a Poisson queue, where arrivals are random and independent of each other. This is a common and tractable assumption for modelling such systems in operations research (Gross et al., 2011). Fig. 3 gives an example of the general process of a charging station queue. Let W_j be the mean waiting time in queue q_j. Then the mean waiting time for a user is:

$$\frac{1}{n}\sum_{j=1}^{n} W_j \tag{2}$$

Recall that $P = \{p_1, \dots, p_n\}$ is a set of integer values where p_j is the number of charging points at the charging station associated with queue q_j. Let $h \in [0, 24]$ be a real value. This value represents the number of hours per day a charging station requires measures to control congestion. The measure in question encourages urban residents to use their assigned charging station. This aims to minimise overall congestion in the system. Let f be the average number of times per day an electric vehicle owned by an urban user needs to be charged, and $t \in \mathbb{R}$ be the average time taken to charge an electric vehicle. Then each queue $q_j \in Q$ is served by a number of charge points p_j. Additionally, a number of urban residents, $\sum_{i=1}^{m} a_{ij} \in \mathbb{Z}$, are assigned to it. If a user is assigned to a queue, the corresponding charging station acts as their 'shared home charging station'. This station can then be used without accruing any additional fees. Then the expected mean waiting time of the system is as follows:

$$\frac{1}{n}\sum_{j=1}^{n} W_j \approx \frac{1}{n}\frac{ft}{h^2}\sum_{j=1}^{n}\frac{\sum_{i=1}^{m} a_{ij}}{p_j} \approx \sum_{j=1}^{n}\frac{\sum_{i=1}^{m} a_{ij}}{p_j} \tag{3}$$

The h^2 term in Eq. (3) reflects the non-linear impact of increasing a charging station's active congestion control hours on queuing time. In multi-server queuing systems, such as the charging stations, waiting times are highly sensitive to system utilisation. As utilisation approaches 100%, waiting times increase dramatically. The h variable represents the hours per day during which congestion control measures are active. It essentially acts as a proxy for the effective throughput available to manage demand.

The second term of the objective function has the goal of minimising the distance travelled by urban residents to their assigned charging stations. Therefore the second term is the average distance between each assignment of urban residents to charging stations as follows:

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij} d_{ij}}{mn} \tag{4}$$

The third term of the objective function has the purpose of ensuring a uniform assignment of urban users from a given area to the set of charging stations. This is achieved by minimising the maximum value of each set of assignment variables from a given area as follows:

$$\sum_{i=1}^{m} \max_i \tag{5}$$

The objective function is therefore composed of the terms in Eqs. (3), (4) and (5).

3.3. Final ILP

The assignment matrix A models the assignment of groups of users U to charging station queues Q as a multi-objective optimisation problem, where w_1 and w_2 provide the weighting factors of the 3 dimensions of the objective function, as follows:

$$\text{Min}\ \ w_1\Big(\sum_{j=1}^{n}\frac{\sum_{i=1}^{m} a_{ij}}{p_j}\Big) + w_2\frac{(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}d_{ij})}{mn} + (1 - w_1 - w_2)\Big(\sum_{i=1}^{m}\max_i\Big) + \lambda_1 B_1 + \lambda_2 B_2 \tag{6a}$$

$$\sum_j a_{ij} p_j = B_1 + fts_i, \qquad i = 1, \dots, m, \tag{6b}$$

$$\sum_i a_{ij} + B_2 = \frac{1}{ft} p_j, \qquad j = 1, \dots, n, \tag{6c}$$

$$\sum_j a_{ij} \geq s_i, \qquad i = 1, \dots, m, \tag{6d}$$

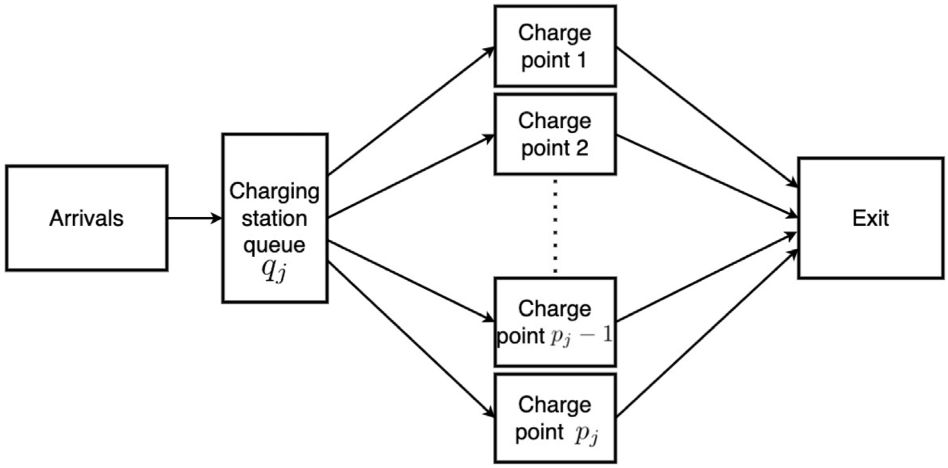

Fig. 3. Illustrative representation of the queuing process at a charging station with multiple charge points. This diagram conceptualises how vehicles (customers) arrive, wait in a single queue, and are served by available charge points (servers), which forms the basis for calculating average waiting times in the optimisation model's objective function.

$$a_{ij} = 0 \quad \text{if} \quad d_{ij} \geq max_d \tag{6e}$$

$$a_{ij} \geq 0, \quad \forall i, \forall j \tag{6f}$$

$$B_1, B_2 \geq 0 \tag{6g}$$

where the variables and parameters are defined in Table 1.

Constraints (6b) and (6c) have the purpose of ensuring that there is sufficient capacity to satisfy demand at the charging stations. Constraint (6d) ensures that every urban resident is assigned to a charging station. Constraint (6e) ensures that urban residents are not assigned to a charging station further than $\max_d$ from their residence.

When this ILP is applied to the simple example in Fig. 2, the resulting optimal assignment of groups of urban users to charging stations is shown in Fig. 4. The number of urban users from each group assigned to each charging station can be seen on the edges. For example, from urban region 1, 2 users are assigned to charging station 1, and 1 user is assigned to charging station 2.

4. Results

In Section 1, it was hypothesised that the proposed model would provide a fair result, where fairness is measured by ensuring low waiting times for both rural and urban residents. In this section, the hypothesis is evaluated empirically. Firstly, the data used for the remainder of the article is introduced. Next, the method formulated in Section 3 is applied to a case study of Cardiff City. The optimal assignment of urban users to queues is obtained. Then the method is applied to a number of random instances to test its scalability and the results are recorded.

4.1. Data

The data sources used in the remainder of the article are OpenStreetMap (OSM) (Haklay and Weber, 2008) and the Office for National Statistics (ONS) (U.K. Government, 2021). The motivation for the use of these data sources is that they are free and open for the entire UK. Each of the data sources is now described in turn.

OpenStreetMap

OSM is a free, open source map of the world, created and contributed to by volunteers, made possible by a collaborative editing software. For the purpose of this study, a map of the street network of Cardiff City Centre was downloaded from OSM, and used to construct a graphical representation of street networks, where nodes are street intersections or dead-ends, and edges represent the street segments between these nodes. OSM data was retrieved using the Python package OSMnx (Boeing, 2022).

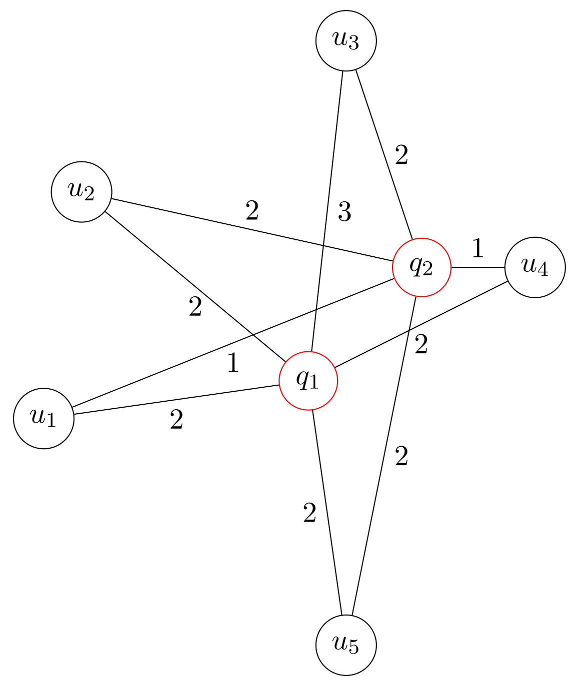

Fig. 4. Optimal assignment of urban user groups to charging stations resulting from the Integer Linear Programming (ILP) model applied to the simple network example. Black nodes $\{u_1 - u_5\}$ represent urban user regions, red nodes $\{q_1, q_2\}$ are charging stations, and the numerical edge weights indicate the optimised number of users assigned from each urban region to a specific charging station.

Office for National Statistics

The ONS is the UK's national statistical institution which gathers, analyses and publishes statistical data covering a wide range of economical, social and demographic issues. In this study, ONS data was utilised to partition Cardiff City Centre into output areas (OAs), which is a small geographical unit created by the ONS to provide a standardised geographic framework for the collection and analysis of small area statistics in the UK. OAs are the lowest area of geographical region for census statistics, and therefore allowed for the finest level of analysis in the optimisation model. Each OA consists of approximately

Fig. 5. Geographical street network of Cardiff city centre, extracted from OpenStreetMap (OSM) for the case study analysis. This network, covering an area within 1100 m of the city's centre, provides the underlying infrastructure for modelling EV charging station placement and user assignment within the optimisation framework.

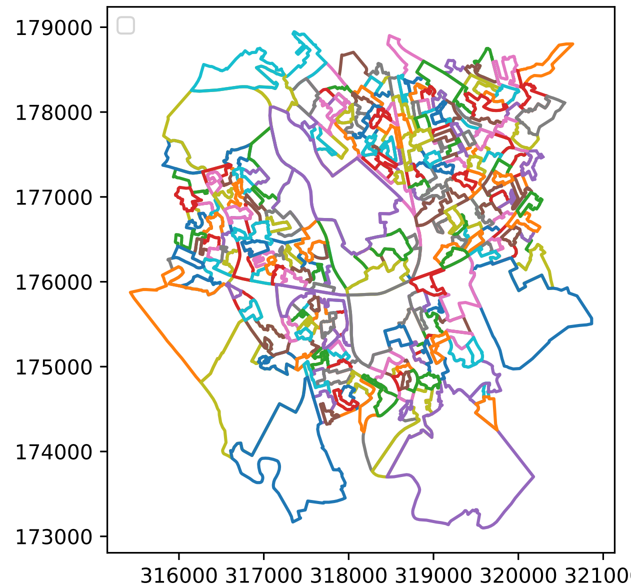

Fig. 6. Geographical partitioning of Cardiff city centre into Census Output Areas (OAs), utilised as distinct urban user regions for the EV charging model. These OAs, derived from ONS Census data, represent the smallest geographical units for statistical analysis in the UK, allowing for a fine-grained spatial representation of urban user groups within the optimisation framework.

125 households, and contains a population of approximately 300 (Government, 2001). The average area of the OAs in the data used in this study is approximately $0.065\,\mathrm{km}^2$.

To incorporate population parameters, car ownership data was also obtained from ONS (Office for National Statistics, 2022), as those who do not own a car will not need to be assigned to a charging station, and therefore can be omitted from the model. Each OA was then assigned a population parameter, detailed in Section 4.2.

4.2. Case study: Cardiff City Centre

To apply the method to a real-world scenario, Cardiff's urban centre was selected as the study area. Cardiff, the capital city of Wales, is chosen as a study area following the Welsh Government's recent statement emphasising the importance of the uptake of EVs in carrying out the Net Zero Plan (Senedd Research, 2023). For the purpose of the study, a street network was modelled using a graph, where nodes represent locations and edges represent paths between locations. As such, a street network of Cardiff was created from OSM using the Python package OSMnx (Boeing, 2022), which can be seen in Fig. 5. This street network is a bounding box of size 1100 m from centre point (51.479363, −3.176703).

Using ONS Census data for Wales, the area considered was partitioned into OAs, shown in Fig. 6. Potential charging station locations were assumed to be car park locations obtained from Ordnance Survey Points of Interest (Ordnance Survey (GB), 2022). To simplify computations, residents of each OA were assumed to reside at the centre point of the OA. The centre points considered are population weighted centroids (PWCs) obtained from ONS. PWCs were calculated using a polygon median centroid algorithm in ArcGIS 10.0, and give a centroid value less influenced by outliers than usual centroid algorithms which give the mean spatial centroid. Fig. 7 shows the street network with car park locations corresponding to potential charging station locations in red and PWC points in blue.

Each OA, corresponding to a region of urban users, was assigned a population parameter based on the ONS car ownership data (Office for National Statistics, 2022). Recall each group of users in region u_i has population s_i. Let H_n denote the number of households in region

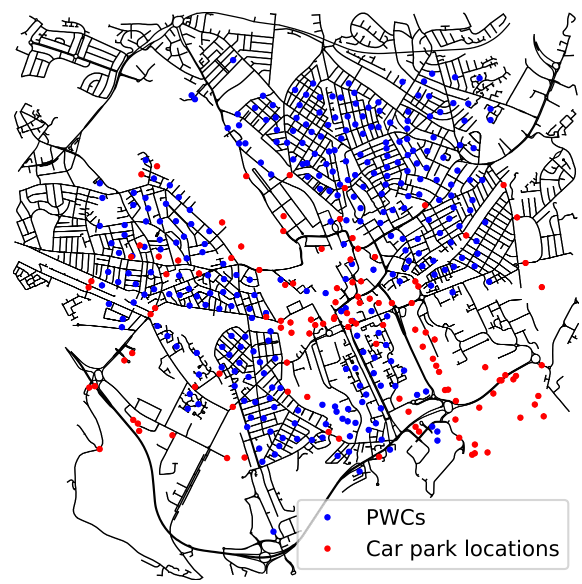

Fig. 7. Street network of Cardiff city centre overlaid with key locations for the EV charging optimisation study. The red points indicate potential charging station locations, derived from Ordnance Survey Points of Interest data (specifically car parks). The blue points represent population-weighted centroids (PWCs) of the Output Areas, serving as the assumed residence points for urban users in the model.

u_i with n cars. While the car ownership data accounts for households with up to three cars, our model extends this to a maximum of ten cars per household to ensure completeness. Then:

$$s_i = \sum_{n=0}^{2} n \cdot H_n + \sum_{n=3}^{10} 3 \cdot H_n \tag{7}$$

It is assumed that people travel along the shortest path between their home and charging stations. The shortest distance between each PWC and each charging station is calculated and stored as a distance matrix d_{ij}. The graphical network indicating the distances in d_{ij} is added onto the street map as an overlay network.

To approximate the potential number of chargers n_i at each queue q_j, a random array containing values between 5 and 150 is used. Based on survey data, it is assumed that urban residents charge their vehicle 0.5 times a day (Zuo, 2019), therefore $f = 0.5$. It is also assumed that there are significantly less rural visitors to the urban area in the night time, and so there is a need to minimise congestion in the system for 12 h a day, therefore $h = 12$. It is assumed that the average time spent charging an EV for an urban resident is four hours, therefore $t = 4$.

Urban residents are assigned to charging stations which then become their 'shared home charging station'. Rural residents are not assigned to a charging station and are therefore able to choose the charging station most convenient for them during visits to the urban area.

4.2.1. Optimisation

The assignment of urban residents to charging stations is obtained by running the ILP using Gurobi optimisation software (Gurobi Optimization, 2022), which employs a branch and cut algorithm to solve individual problem instances (Mitchell, 2002).

A solver time limit of 600 s is implemented. If an optimal solution is not found within this timeframe, the best feasible solution obtained so far (the incumbent solution) is returned. The quality of this solution is assessed using the *optimality gap*, which quantifies how close the incumbent solution is to the theoretical optimum. This is defined as follows:

$$\text{Optimality Gap (\%)} = \frac{Z_{\text{incumbent}} - Z_{\text{bound}}}{Z_{\text{incumbent}}} \times 100\% \tag{8}$$

where $Z_{\text{incumbent}}$ is the objective value of the best feasible solution found, and Z_{bound} is the best known bound on the optimal value (a lower bound in this case as it is a minimisation problem). The optimality gap is automatically reported by the Gurobi solver, providing a measure of how close the returned solution is to optimality at the point of solver termination.

The third term in Eq. (5) was implemented using Gurobi function max_(), which calculates the maximum value of a set of decision variables.

It is proposed that this optimal assignment could be used to inform a charging policy based on financial incentives. For example, in order to minimise the overall congestion in the system by encouraging urban residents to use their assigned home charging station, additional fees are introduced for urban residents choosing to charge at a charging station which is not their assigned home charging station. Alternatively, urban residents can use their assigned 'shared home charging station' at a discounted rate. Rural residents incur no extra fee and are able to charge at any charging station which is convenient upon their visit to the urban area.

4.2.2. Results

As the set of potential charging station locations is equal to the set of existing car park locations, a random partition of car park locations is used to create a set of charging station locations to ensure a realistic ratio of charging stations to users. This is achieved by considering a random fraction $\tau \in [0, 1]$ of car parks from the car park data and setting these car parks as charging station locations.

The outputs of the model are the objective function value, which includes the expected mean waiting time in the charging system in minutes, the average distance between an urban user and their assigned charging station, and an assignment matrix demonstrating how many urban residents from each urban area are assigned to each charging station. As this assignment matrix is too large to display, the results of the model are demonstrated by running it once as shown in Section 3.3, and once with a baseline model where a number of constraints are removed. The baseline model runs without the considerations of the expected mean waiting time and the distance between urban users and their assigned charging stations in the objective function. The mean waiting time and average distance between urban users and their assigned charging station are then compared for both models.

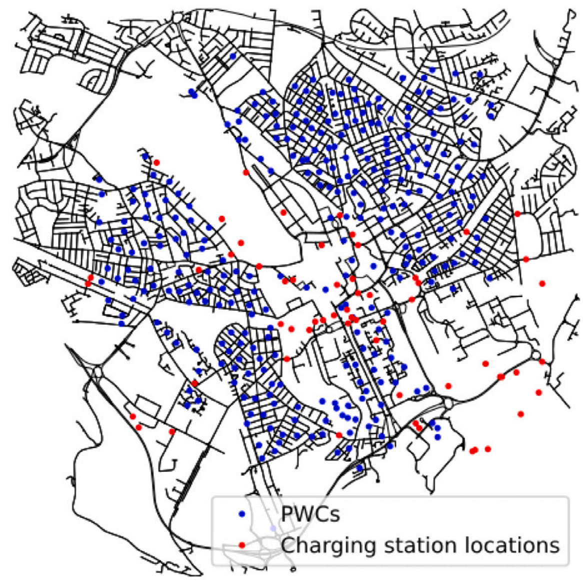

Fig. 8. Spatial distribution of potential EV charging station locations (red) and urban population weighted centroid (PWC) points (blue) within the Cardiff city centre street network, representing a scenario where 50% ($\tau = 0.5$) of existing car park locations are designated as charging stations for the optimisation model.

Table 2
Comparative performance of the proposed optimisation model against a baseline model for a scenario with $\tau = 0.5$ (50% car parks as charging stations). The table presents key outcome metrics: the mean expected waiting time (in minutes) for users in the charging system and the average travel distance (in kilometres) between urban users and their assigned charging stations.

Scenario	Mean waiting time	Mean distance
Optimisation model	9.6 mins	1.35 km
Baseline model	16.6 mins	3.6 km

For the instance shown in Fig. 8, where $\tau = 0.5$, there are 315 urban regions and 36 charging stations considered. The populations of the urban areas, corresponding to car ownership data, range between 0 and 255. The number of charging points at the charging stations range between 13 and 148.

For $\tau = 0.5$, Table 2 shows the results of the model implemented as shown in Section 3.3 and the results of the baseline model. It can be seen that with the baseline model, the mean waiting time is 7 min longer per user, and the distance between an urban resident and their assigned charging station is 2.25 km further away on average. This is a more desirable result for urban users, as they have to queue for less time on average to charge their vehicle, and also have to travel less distance to their assigned charging station. It is also a more desirable result for rural users visiting the urban area as the charging system is less congested and they have to queue for less time on average to charge their vehicle.

Further, this comparison of results from the optimisation model vs the baseline model can be repeated in experiments where a random fraction $\tau \in [0, 1]$ of car park locations are assumed to be charging stations. The results of these experiments are recorded in Table 3. The results of this experiment support the hypothesis that the optimisation model achieves reduced congestion in the charging system by minimising the mean waiting time, and also ensures that urban users do not have to travel as far to their assigned charging station. Furthermore,

Table 3
Detailed comparison of the proposed optimisation model's performance versus the baseline model across varying charging station densities, represented by the fraction τ of car parks designated as charging stations (ranging from 0.1 to 1.0). The table presents the mean expected waiting time (in minutes) for users and the average travel distance (in kilometres) for urban users to their assigned charging stations for each τ value.

τ/scenario	Optimisation model	Baseline model
1.0	4.8 mins 1.3 km	8.2 mins 4.6 km
0.9	5.6 mins 1.3 km	8.3 mins 4.4 km
0.8	6.1 mins 1.3 km	10 mins 4.2 km
0.7	7.2 mins 1.3 km	10.5 mins 4.0 km
0.6	7.1 mins 1.3 km	12.3 mins 3.9 km
0.5	9.6 mins 1.35 km	16.6 mins 3.6 km
0.4	10.2 mins 1.3 km	17.2 mins 3.5 km
0.3	17.7 mins 1.7 km	22.3 mins 3.4 km
0.2	19.5 mins 1.5 km	29.6 mins 3.3 km
0.1	38.2 mins 1.6 km	51 mins 3.4 km

this experiment could be useful for a predictive analysis. Whilst it may be unrealistic to assume that all car parks, or a large number of car parks, in an urban area contain charge points at present, the number of charge points is expected to increase in the coming years as the roll out of EVs and subsequent expansion of charging infrastructure continues.

4.3. Analysis of random instances

To evaluate the scalability of the proposed method, in this section the model is applied to a number of random problem instances of increasing size. For a given number of urban areas m and charging stations n we define a corresponding random problem instance using the following approach. Values m and n are varied, whilst the other parameters are set as follows: $f = 0.5$, $h = 12$, $t = 1$. max_d varies where $max_d \in [1.5, 2.5]$. P varies with $p_j \in \mathbb{Z}, p_j \in [1, 50]$. D varies with $d_{i,j} \in \mathbb{R}, d_{i,j} \in [0, 3]$. S varies with $s_i \in \mathbb{Z}, s_i \in [1, 200]$.

For each pair of varied parameters (m, n), a corresponding random model is created and executed 10 times. In Table 4, the number of optimal solutions found is recorded, along with the average expected waiting time over the 10 executions in minutes. If there are no optimal solutions found, a dash ('–') is recorded in place of the average expected waiting time.

5. Discussion

5.1. Advantages and limitations of the proposed model

This proposed multi-objective ILP model offers several significant advantages for advancing equitable and efficient EV charging infrastructure planning. The primary advantage is its unique focus on achieving fairness through congestion reduction, explicitly accounting for the distinct charging needs of both urban residents and rural visitors. By minimising average queuing times across the entire system, the model directly addresses a critical problem for EV users and ensures that urban infrastructure can adequately serve transient rural demand without imposing undue waiting burdens. This explicit modelling of the urban–rural dynamic for equitable access represents a novel contribution, as existing literature often overlooks this specific aspect.

Furthermore, the model concurrently minimises travel distances for urban residents to their assigned shared home charging stations. This enhances convenience for daily use and reduces unnecessary vehicle miles travelled, aligning with broader sustainable mobility objectives. The model's ability to generate optimal assignment plans for urban users provides a clear, actionable road-map for city planners and system operators, translating directly into implementable strategies. Its inherent flexibility to incorporate varying charging station densities (via the τ parameter) makes it a powerful predictive analysis tool for future infrastructure planning, enabling proactive decision-making as EV adoption continues to grow. The robust integration of queuing theory provides a data-driven framework for quantitatively assessing and managing congestion.

Despite these strengths, the model has certain limitations that delineate avenues for future research. A key aspect is the current reliance on aggregated urban resident data based on Census OAs, which is a pragmatic choice given individual-level privacy concerns. While effective for macroscopic planning, this aggregation could potentially obscure finer-grained individual user behaviours. Similarly, assuming Poisson arrivals for charging stations is a standard and computationally tractable approach in queuing theory, but may not fully capture the complex temporal variations of real-world peak-hour demand. For example, there may be busier time periods during lunch hours or after the workday. Future enhancements could explore more sophisticated arrival distributions or incorporate time-varying parameters to reflect fluctuating demand patterns. Another challenge encountered was the inherent difficulty in obtaining granular data on the specific charging patterns of rural EV users visiting urban areas. This lack of detailed data limits a precise quantification of how much additional rural demand the optimised system can accommodate, although the model's congestion reduction still ensures capacity for them. Lastly, the model's reliance on a 'congestion charge' as a financial incentive to guide user behaviour suggests that future work could explore the effectiveness and impact of different financial or non-financial incentive mechanisms.

5.2. Practical implications and stakeholder utility

This optimisation scheme is primarily designed with the potential to provide tangible benefits for city planners, local authorities, urban mobility departments, and charging infrastructure providers. Its practical outcomes offer direct utility in both the strategic planning and operational management of public EV charging networks, focusing on an inclusive transition to electric vehicles.

- **Optimal User Assignment for Fair Access:** The model provides a clear, actionable output: the assignment matrix (a_{ij}). This indicates precisely how many urban residents from each geographical area should be 'assigned' to specific public charging stations as their 'shared home charging stations'. City planners can use this to structure and implement local charging schemes, potentially through digital platforms or community incentive programs that encourage equitable access for all users.
- **Informed Infrastructure Sizing and Investment:** By treating the number of charging points (p_j) as a variable parameter, the model allows stakeholders to perform 'what-if' analyses. City planners can use this framework to determine the optimal number of charge points required at different charging locations to meet projected demand while meeting target waiting times. This directly guides investment decisions and infrastructure roll-out strategies.

Table 4
Scalability analysis of the proposed optimisation model across varied problem instance sizes (urban areas, *m*, and charging stations, *n*). For each configuration, the table reports the number of optimal solutions found, the number of infeasible solutions, and the average expected waiting time (in minutes) over 10 independent executions, demonstrating the model's performance and solvability.

m \ n	5	10	20	50	100	200
100	6, 4, 28	10, 0, 18.1	10, 0, 9.3	10, 0, 3.9	10, 0, 1.9	10, 0, 1.1
200	3, 7, 102.2	10, 0, 42.9	10, 0, 23.3	10, 0, 7.1	10, 0, 3.4	10, 1, 1.8
300	4, 6, 117.7	10, 0, 59	10, 0, 27.9	10, 0, 10.7	10, 0, 5.3	10, 0, 2.8
400	2, 8, 56.3	10, 0, 73.1	10, 0, 35.8	10, 0, 16	10, 0, 6.6	10, 0, 3.3
500	1, 9, 131	10, 0, 105.2	10, 0, 41.4	10, 0, 16.7	10, 0, 9.4	10, 0, 4
600	1, 9, 240.5	10, 0, 100.5	10, 0, 59.9	10, 0, 24.3	10, 0, 10	10, 0, 4.8
700	1, 9, 230.1	10, 0, 94.2	10, 0, 69.2	10, 0, 25.1	10, 0, 11	10, 0, 5.9
800	0, 10, -	10, 0, 131.7	10, 0, 64.1	10, 0, 26.9	10, 0, 11.4	10, 0, 6
900	0, 10, -	10, 0, 130	10, 0, 81.4	10, 0, 29.7	10, 0, 13.8	10, 0, 6.2
1000	0, 10, -	10, 0, 194	10, 0, 86.6	10, 0, 29.1	10, 0, 17.4	10, 0, 7.1

- **Effective Congestion Management Strategy:** The model quantifies the impact of policies, such as the proposed 'congestion charge', on reducing overall system congestion. Stakeholders can leverage these insights to design dynamic pricing models or priority access policies that encourage efficient use of charging resources, especially during peak hours. This ensures critical availability for rural visitors who may not have alternative charging options within the urban area.
- **Predictive Analysis for Future Growth:** The comprehensive experiments across varying τ values (Table 3) offer a powerful predictive analysis tool. System operators and policymakers can input projected future charging station densities or planned car park conversions to forecast expected mean waiting times and travel distances. This enables proactive infrastructure expansion and resource allocation, preventing future bottlenecks and ensuring preparedness for increasing EV adoption. For example, if a policy goal is to maintain a mean waiting time below 10 min, the model can indicate the minimum charging station density required.
- **Quantifying and Ensuring Equity:** Fundamentally, the model's most significant practical outcome is its ability to plan and manage a charging network that promotes equitable access. By strategically reducing congestion and optimising urban user assignments, the system ensures that rural residents visiting urban areas are not unfairly disadvantaged by long queues or unavailable chargers. This direct contribution to fairness facilitates a smoother and more inclusive transition to EVs across all communities.

5.3. Future research directions

Building upon this work, several promising avenues for future research emerge. One key area involves exploring the effects of different financial incentives on the success of congestion minimisation. For instance, varying the proposed 'congestion charge' or investigating non-financial incentives could provide valuable insights into user behavioural responses.

Further enhancing the model's realism could involve incorporating temporal constraints to account for dynamic demand patterns throughout the day, beyond the static average arrival rate. Additionally, expanding the model's functionality to include different charger attributes, such as varying voltage ratings and charging speeds, would enable a more nuanced optimisation of charger allocation.

While this study focuses on urban users at an aggregated level (OA populations), future work could aim for a finer analysis using individual-level data. However, this would need to carefully address potential privacy concerns and computational challenges, possibly requiring the adoption of heuristic-based methods to solve larger assignment matrices.

Finally, while this study assumes a pre-set positioning of charging stations (making it distinct from the facility location problem), future research could consider a joint optimisation approach that simultaneously tackles both the optimal placement of new charging stations and the user assignment problem presented here. This would offer a more comprehensive framework for EV charging system design and planning.

6. Conclusion

This article developed a multi-objective ILP optimisation model, integrating queuing theory, to address the fair assignment of users to EV charging stations. The model's primary aim is to minimise average queuing time within urban charging systems, ensuring sufficient capacity for both urban residents and visiting rural EV users. Our methodology effectively reduces congestion and optimises urban user assignments to shared charging stations.

Applied to a real-world scenario like Cardiff, the model demonstrated significant practical advantages. For instance, in a specific case with 36 charging stations in Cardiff's urban centre, the model notably reduced mean waiting times for users and decreased average travel distances for urban users compared to a baseline. Further experiments consistently showed the model's capability to reduce both waiting times and travel distances across various charging station densities.

In essence, this research provides a robust, data-driven framework for more equitable and efficient EV charging infrastructure planning. It facilitates a genuinely inclusive transition to electric vehicles for both urban and rural communities by optimising access and minimising wait times, thereby promoting fairness in EV charging.

CRediT authorship contribution statement

Lucy Maybury: Writing – review & editing, Writing – original draft, Resources, Investigation, Formal analysis, Data curation, Conceptualization. **Padraig Corcoran:** Writing – review & editing, Visualization, Validation, Supervision, Software, Resources, Project administration, Investigation, Funding acquisition, Conceptualization. **Liana Cipcigan:** Supervision, Resources, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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